TRAUTWINE

# MANUAL DEL INGENIERO

TRADUCIDO AL ESPAÑOL

Y CONVERTIDO AL SISTEMA MÉTRICO

POR

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GOVERNMENT OF INDIA
DEPARTMENT OF ARCHAEOLOGY
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# PRÓLOGO

*a la 1ª EDICIÓN (1914).*

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Con el objeto de aportar algún esfuerzo de consideración tendente al adelanto de la hermosa ciencia del Ingeniero Constructor, emprendimos hace algunos años y hemos llevado á cabo, la traducción del inmejorable *Manual del Ingeniero* escrito en inglés por John C. Trautwine, de los E. E. U. U. de América, y convertido además todas sus fórmulas, tablas y ejemplos, que naturalmente trae dicho autor en sistema inglés, al sistema métrico.

Con este trabajo creemos haber llenado el gran vacío que dificulta, de modo notable, la práctica expedita y certera de los constructores Españoles é Hispano Americanos, por la falta de Manuales y Formularios verdaderamente prácticos, tanto por el criterio que los inspira como por el método y exposición de las reglas y fórmulas que han de usarse en los diversos y complejos casos que á diario se presentan al Ingeniero Constructor. Y son aquellas precisamente las cualidades que han dado al Trautwine el éxito y celebridad insólitos; éxito comprobado irrefutablemente por los (140 000) *ciento cuarenta mil ejemplares* que hasta ahora se han vendido de dicha obra.

Pero la culminante característica de este formulario está en el inmenso y laborioso acopio de resultados escogidos entre todos los innumerables trabajos de investigación experimental llevados á cabo en estos últimos

años por los más célebres Constructores y las más reputadas Asociaciones para extender sobre base firmísima la Construcción Moderna.

Además el Trautwine no solo supera á las obras análogas en español por la cantidad de materia que contiene, por lo adecuadas de sus fórmulas, por la sencillez y claridad, al alcance de técnicos y profanos, de su exposición, de sus diagramas y tablas, sino que es muy superior á todos los formularios franceses que son los más consultados por los Constructores Hispano Americanos. Otra de las ventajas que se derivan de la publicación de esta obra en español es sin duda que nos pone en relación, nos hace conocer los importantes y eficaces progresos de la ingeniería Norte-Americana é Inglesa y los admirables trabajos de sus corporaciones científicas, generalmente tan desconocidos para los Ingenieros y Constructores que hablan español, pues es muy sabido que una gran parte aprenden el francés pero muy pocos el inglés.

Como cada día se estrechan más las relaciones entre los Constructores é Ingenieros Ingleses é Hispano Americanos, ya que ambos idiomas van extendiéndose constantemente, y son muchas las obras y empresas en que trabajan juntos Ingenieros ingleses y españoles y más frecuentes aun los casos en que tenemos que hacer pedidos ó estudiar proyectos que nos obligan á conocer el sistema inglés en toda su amplitud, hemos dejado casi odas las fórmulas inglesas al lado de sus equivalentes métricas, lo que en nada estorba la lectura en el sistema métrico. Es éste quizás el unico texto de esta índole que contiene ambos sistemas.

Otra insuperable ventaja, en este Manual, es sin duda la de hacer que sus prácticos consejos, dictámenes y fórmulas vayan siempre precedidos de una exposición completa y sintética de toda la teoría que sirve á aquellos de fundamento, desarrollada en capítulos de una síntesis y claridad admirables que traen á la memoria los elementos indispensables para hacer conscientes y fecundas las aplicaciones de las fórmulas y los principios, dando al teórico el empirismo que le da iniciativa y éxito y al

empírico la teoría que le da la fé y la deliberación tan necesarias en el espíritu del Constructor.

Por no introducir alteraciones que podrían quizás desvirtuar una obra de tan notoria reputación y fama, no hemos ni siquiera modificado su estilo, aunque siempre claro y preciso, á veces quizás demasiado simple y llano.

El tipo de letra, la división y distribución de las materias, la numeración de los capítulos, párrafos, etc., todo es idéntico al texto inglés; y lo es de tal manera, que los Ingenieros que hablan inglés y poco el español y no conocen su terminología y viceversa, pueden estudiarla y aprenderla cotejando los capítulos de número á número y de renglón á renglón.

Hemos adoptado el práctico sistema de poner en letra gruesa al principio ó dentro del párrafo ó capítulo, la palabra ó frase más importante facilitando así la rápida consulta de una cuestión cualquiera.

También en el índice, como en todo lo demás, hemos seguido al autor que no copia siempre fielmente el título de los capítulos ó párrafos sino que solo menciona á veces la más importante de las materias tratadas en la página citada.

Los autores de esta obra se han consagrado con el mayor ahinco y laboriosidad á mejorar cada una de las ediciones sucesivas, á tal punto, que las primeras ediciones solo tenían como la mitad del contenido de esta última (19ª). Como seiscientas páginas de materia nueva y como doscientas revisadas y modificadas se encuentran en la última edición, resultado de unos 19 ó 20 capítulos nuevos agregados y otros tantos casi en u totalidad escritos de nuevo.

Nosotros en las 1272 páginas que forman la obra, hemos reemplazado solo unas diez ó doce páginas de materias por las razones que en debido lugar exponemos; hemos agregado algunas páginas con los más importantes problemas del trazado de curvas, del mismo autor, pues de otro modo quedaba incompleto el capítulo de « Ferrocarriles ». Sólo hemos suprimido los capítulos que tratan de Biblio-

grafía y en parte la lista de precios pues sólo tienen importancia para los Ingenieros Norteamericanos.

No se escapa á nuestros colegas las dificultades de diversa índole que hemos tenido que vencer, sobre todo en la adecuada elección de multitud de términos técnicos, llegando en muchos casos á tener que crearlos; puesto que el gran progreso alcanzado sobre todo en los Estados Unidos por las ciencias é industrias que se relacionan con el extensísimo Arte de Construir, ha hecho que multitud de sus términos y expresiones, no tengan aún su equivalente fijo en español.

ALBERTO SMITH.

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NOTA IMPORTANTE :

Siguiendo el sistema del Autor, hemos adoptado *el punto*, para separar los números enteros de los decimales y la coma para separar los enteros entre sí.

# PRÓLOGO

DE LA

*SEGUNDA EDICIÓN* (3ª emisión) *traducida de la* (20ª) *última edición del texto Inglés.*

Despues de la magnífica acogida con que el pùblico ha recibido las dos emisiones de la primera edición de esta traducción, no hemos omitido esfuerzo ni gasto para traer a esta edición todas las mejoras de la última del texto inglés o sea la 20ª del año de 1919. Ademas nos hemos esmerado en la corrección de todas las erratas, inevitables en las primeras ediciones de obras tan complejas como esta y, como lo observará el lector, hemos mejorado, no solo la calidad del papel sino también la de la encuadernación, pasta, etc.

En efecto : la actual edición trae como 340 páginas nuevas, sobre todo en el ramo de ferrocariles, y 16 páginas sobre los principios y fórmulas del Derecho en relación con las prácticas de la Construccion, materia esta que no existe en el texto inglés ni que sepamos en ninguna otra, de la índole de esta obra, en los otros idiomas. Mas adelante exponemos los motivos que nos han inducido a esta innovación que juzgamos mui útil.

Volviendo al capítulo de ferrocarriles, haremos una sucinta comparación que dé al lector una idea de la mayor, de la mui grande, extensión dada a algunas materias y ademas de los capítulos enteramente nuevos que aparecen en esta edición :

| | PÁGINAS En la edición anterior. | Enesta edición. | |
|---|---|---|---|
| Via | 15 | 44 | |
| Desvios y Cruceros | 25 | 52 | |
| Curvas circulares | 20 | 93 | |
| Curvas en espiral | | 18 | Nuevo. |
| Señales | | 12 | Nuevo. |
| Estaciones y patios | 10 | 16 | |
| Materiel rodante | 10 | 17 | |
| Resistencia en los trenes | | 14 | Nuevo. |
| Dinámica de los trenes | | 11 | Nuevo. |
| Costo de explotación | | 14 | Nuevo. |
| Costo de construcción | | 14 | Nuevo. |
| El Derecho en relación con la Construcción | | 24 | Nuevo. |
| Páginas | 80 | 329 | |

Los mas nuevos y recientes procedimientos prácticos de la Ingenieria están en esta edición y sobre todo los preceptos y prácticas adoptados por la Asociación Americana de Ingenieros de Ferrocarriles (American Railway Engineering Association).

Aparecen nuevas tablas de : Radios de curvas con sus logaritmos; Cuerdas Largas; Ordenadas Medias y funciones de las curvas de (1°) un grado.

La discusión y práctica del replanteo de curvas es enteramente nuevo asi como el estudio y trazado de curvas Espirales, que es el recomendado por la ya citada A.A. de I. de F. Fíjese el lector en la importancia práctica de los también nuevos capítulos sobre Señales; Resistencia y Dinámica de los Trenes; Costo de Construcción y Explotación y el de los preceptos del Derecho en relación con la Construcción en general, tan util a los que dirigen obras.

En toda la parte nueva de esta obra, como en las anteriores ediciones han seguido sus autores el sistema de mantenerla por su claridad y fácil lenguaje al alcance, no solo de Técnicos é Ingenieros sino también de los jefes de trabajo, maestros de obras, etc., que posean simples elementos de Aritmética, Geometria, etc. Este ha sido el secreto del éxito incomparable de esta obra.

Como consecuencia de la introducción de mucha materia nueva y de las modificaciones y mejoras en muchos capítulos hemos tenido que cambiar el Indice aprovechando esta ocasión para hacerlo mucho mas minucioso.

Volviendo al capítulo sobre el Derecho y las construcciones observamos que los ingenieros y jefes de obras en el estudio y construcción de éstas, son quizás entre todos los profesionales los que mas necesitan conocer los preceptos del Derecho que se relacionan con el ejercicio de su profesión. Desde el emplazamiento de sus construcciones, durante la ejecución y luego en la explotación, están expuestos a vulnerar el derecho ajeno por la falta de conocimientos, siquiera generales y someros de aquellas prescripciones. Cuantos litigios han resultado de tales deficiencias!

Es por esto que hemos creido conducente y eficaz agregar en esta nueva edición las muy breves apuntaciones, excesivamente sintéticas, que ha tenido la bondad de suministrarnos el notable Jurisconsulto venezolano, Doctor José Loreto Arismendi, cuya ilustrada colaboración jurídica hemos solicitado y a quien reiteramos nuestra gratitud por su trabajo.

Los principios expuestos por el Doctor Arismendi están basados sobre el Derecho Francés, en atención a que éste es el que ha servido generalmente de modelo para calcar sus leyes las naciones de habla española.

Asi como, por la índole de este Manual, no se entra en razonamientos y conexiones extensos y constitutivos de un estudio completo de la técnica de la ingenieria, dando solo las fórmulas y la exposición sintética de las teorias y principios, asi también, en el apéndice juridico de que tratamos, sólo se formulan mui brevemente algunos dictámenes que, íntimamente ligados con las materias de esta obra, sirvan como de advertencia útiles para orientar y provocar consultas y más amplios estudios, cuando fuere necesario.

Suplicamos á todos nuestros bondadosos lectores nos excusen las faltas y errores que hayamos podido cometer y nos envíen por correo á la siguinte dirección: Alberto SMITH c/o TRAUTWINE C°, 257. S. 4th St — Philadelphia (Penn'a), todas las observaciones que crean pertinentes, contribuyendo así, con nosotros, al mejoramiento de la obra.

Paris, octubre 1921.

ALBERTO SMITH.

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*Nota.* — En este Indice una T entre paréntesis (T) indica una tabla.


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# MATEMATICAS

## SÍMBOLOS MATEMÁTICOS

$+$ Más, positivo, adición.

$-$ Menos, negativo, sustracción.

$\pm$ Más ó menos, positivos ó negativos. Ej., $\sqrt{a^2} = \pm a$.

$\times$ Multiplicado por. Ej., $x \times y = x.y = xy$; $3 \times 4 = 12$.

$\div$ $:$ $/$ Dividido por. Ej., $a \div b = a : b = a / b = \frac{a}{b}$.

$::$ Proporción. Ej., $a : b :: c : d$, $a$ está con $b$ como $c$ está con d.

$=$ Igualdad, es igual; $a = b$; $a$ es igual $b$.

$>$ Mayor que. Ej., $6 > 5$.

$<$ Menor que. Ej., $5 < 6$.

$\neq$ No es igual.

$\lessgtr$ Mayor ó menor que.

$\ngtr$ No es mayor que.

$\nless$ No es menor que.

$\geqslant$ Es igual ó mayor que.

$\leqslant$ Es igual ó menor que.

$\propto$ Es proporcional, varía con.

$\infty$ Infinito.

$\perp$ Es perpendicular á.

$\angle$ Angulo.

$\backsim$ Es semejante á.

$\parallel$ Es paralelo á.

$\surd$ $\sqrt{\ }$ Raíz de. Ej., $\sqrt{a}$ ó $\sqrt{a}$ = raíz cuadrada de $a$; $\sqrt[3]{a}$ = raíz cúbica de $a$; $\sqrt[n]{a}$ = raíz $n$ de $a$.

$( )$ $[ ]$ $\{ \}$ Paréntesis. Las cantidades incluídas dentro de estos paréntesis, deben tomarse en conjunto.

$\because$ De aquí que. Por esta razón.

$\therefore$ Puesto que. Ya que.

$^\circ$ Grados.

$'$ Minutos de arco, pie.

$''$ Segundos de arco, pulgadas.

Los $'$ $''$ $'''$ minutos y segundos de tiempo se expresan : *min, seg.* etc. Prima, segunda, tercera, etc. Ej., $a'$, $a$ prima; $b''$, $b$ segunda; $b''$ $b$ tercera.

$\pi$ $\frac{\text{Circunferencia}}{\text{Diámetro}}$ = Relación de la circunferencia al diámetro 3.14159265.

E Módulo de elasticidad.

$e$ (epsilon). Base de los logaritmos neperianos ó hiperbólicos.

g Aceleración debida á la gravedad; aproximadamente igual á 32.2 pies por segundo = 9.81 metros por segundo.

## ALFABETO GRIEGO

Insertamos este alfabeto en obsequio de las personas que consultan libros científicos donde se emplean letras griegas.

| Letras griegas. | | Nombre. | Equivalente aproximado. | Generalmente usados para designar. |
|---|---|---|---|---|
| Mayúsculas. | Minúsculas. | | | |
| A | α | Alpha | a | Angulos, coeficientes. |
| B | β | Beta | b | — — |
| Γ | γ | Gamma | g | — gravedad. |
| Δ | δ | Delta | d | — densidad, variación. |
| E | ε | Epsilon | e (corta) | Base de los logaritmos hiperbólicos 2.7182818. Excentricidad en las secciones cónicas. |
| Z | ζ | Zeta | z | Coordenadas, coeficientes. |
| H | η | Eta | e (larga) | — — |
| Θ | θ ϑ | Theta | th | Angulos. |
| I | ι | Iota | i | |
| K | κ | Kappa | k | |
| Λ | λ | Lambda | l | Angulos, coeficientes, latitud. |
| M | μ | Mu | m | — — |
| N | ν | Nu | n | — |
| Ξ | ξ | Xi | x | Coordenadas. |
| O | ο | Omicron | o (corta) | |
| Π | π | Pi | p | Circunferencia ÷ diámetro.* |
| P | ρ | Rho | r | Radio, razón. |
| Σ | σ ς | Sigma | s | Distancia, espacio.† |
| T | τ | Tau | t | Temperatura, tiempo. |
| Υ | υ | Upsilon | u ó y | |
| Φ | φ | Phi | ph | Angulos, coeficientes. |
| X | χ | Chi | ch | |
| Ψ | ψ | Psi | ps | Angulos. |
| Ω | ω | Omega | o (larga) | Velocidades angulares. |

★ La letra minúscula π se emplea universalmente para designar el número de veces = 3.14159265...) que el diámetro de un círculo está contenido en su circunferencia, ó el radio en la semicircunferencia. En la medida circular de los ángulos, un ángulo se mide por el número de veces que el radio del círculo á que pertenece el ángulo, está contenido en el arco que subtende el ángulo. Como π representa el número de veces que el radio está contenido en la semicircunferencia, π es la medida en radios del arco de 180° ó una semicircunferencia.

Algunos matemáticos emplean la mayúscula Π (pi) para expresar el producto obtenido multiplicando entre sí los números 1, 2, 3, 4, etc.; por ej. $\Pi 4 = 1 \times 2 \times 3 \times 4 = 24$.

† La mayúscula Σ (sigma) se usa para expresar una suma. Por ejemplo, si en un sistema de fuerzas paralelas representamos cada fuerza por F, su resultante, como es igual á la suma algebráica de las fuerzas, se expresará así: $R = \Sigma F$.

# ARITMÉTICA

## FACTORES Y MÚLTIPLOS

(1) **Los factores** de cualquier número $n$ son los números cuyo producto $= n$. Ej. 17 y 4 son factores de 68; también 34 y 2; también 17, 2 y 2.

(2) **Números primos** son los que no tienen factores, excepto ellos mismos y la unidad; ej. 2, 3, 5, 19, 233.

(3) **El factor común, el común divisor**, ó **la medida común** de dos ó más números es un número divisor exacto de cada uno de ellos. Ej. 3 es divisor común de 6, 12 y 18.

(4) **El máximo factor común**, ó **máximo divisor común** de dos ó más números, se expresa así: **M. F. C.** ó **M. D. C.** Ej. 6 es **M. F. C.** de 6, 12 y 18.

(5) **Para encontrar el M. F. C.** de dos ó más números, búsquense los números ó factores primos de cada uno y multiplíquense juntos los que sean comunes á todos, tomando cada factor una sola vez. Ej. búsquese el M. F. C. de los números 78, 126, 234.

$$78 = 2\times3\times13$$
$$126 = 2\times3\times3\times7$$
$$234 = 2\times3\times3\times13$$

El M. F. C. $= 2\times3 = 6$.

(6) **Para encontrar el M. F. C.** de dos números muy grandes divídase el mayor por el menor, el menor por el primer residuo A; A por el segundo residuo B; B por el tercer residuo C y así sucesivamente hasta que no haya residuo. El último divisor es el M. F. C. Ej. búsquese el M. F. C. de 575 y 782.

575)782(1
575
A 207)575(2
414
B 161)207(1
161
C 46)161(3
138
D 23)46(2
46
0

M. F. C. = D = 23.

(7) **El múltiplo común** de dos ó más números, es un número exactamente divisible por cada uno de ellos.

(8) **El mínimo múltiplo común** de dos ó más números se expresa así: M. M. C.

(9) **Para encontrar el M. M. C.** de dos ó más números, se buscan los números primos de cada uno. Multiplíquense juntos los factores, tomando cada uno tantas veces cuantas esté repetido en el número que más lo contiene. Ej. para los números 7, 30 y 48.

$$7 = 7$$
$$30 = 2\times3\times5$$
$$48 = 2\times2\times2\times2\times3$$
$$\text{M. M. C.} = 7\times2\times2\times2\times2\times3\times5 = 1680.$$

(10) **Para encontrar el M.M. C.** de dos números grandes; búsquese el M. F. C., como se dijo arriba, y dividiendo á cada número por él, búsquese el otro factor y entonces hágase el producto de los factores, como antes. Ej. búsquese el M. M. C. de 575 y 782. Como arriba el

$$\text{M. F. C.} = 23;\ \frac{575}{23} = 25;\ \text{y}\ \frac{782}{23} = 34;\ \text{Luego}$$

$$575 = 23\times25$$
$$782 = 23\times34$$

y M. M. C. $= 23\times25\times34 = 19{,}550$.

## FRACCIONES

(1) **El común denominador** de dos ó más fracciones es el múltiple común de sus denominadores.

(2) **El mínimo común denominador** ó M. C. D. de dos ó más fracciones es el M. M. C. de sus denominadores.

(3) **Para reducir á un común denominador,** sea

N = el nuevo numerador de cualquier fracción.
n = el antiguo numerador.
d = el antiguo denominador.
C = el común denominador.

Entonces $$N = n\frac{C}{d}.$$

Así, $\frac{3}{4}, \frac{5}{6}, \frac{7}{8}$. C = M. M. C. de los denominadores = 24.

$$\frac{3}{4} = \frac{3 \times \frac{24}{4}}{4 \times \frac{24}{4}} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}; \quad \frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}; \quad \frac{7}{8} = \frac{7 \times 3}{8 \times 3} = \frac{21}{24}.$$

Si los denominadores no tienen factor común, entonces C = al producto de todos los denominadores; $\frac{C}{d}$ = al producto, P, de los *otros* denominadores y N = Pn.

Ej., $\frac{2}{3}, \frac{1}{4}, \frac{5}{7}$. C = 84.

$$\frac{2}{3} = \frac{2 \times 4 \times 7}{84} = \frac{56}{84}; \quad \frac{1}{4} = \frac{1 \times 3 \times 7}{84} = \frac{21}{84}; \quad \frac{5}{7} = \frac{5 \times 3 \times 4}{84} = \frac{60}{84}.$$

(4) **Suma y resta.** Si es necesario redúzcanse las fracciones á un común denominador, el menor será mejor. Súmense ó réstense los numeradores. Ej.

$$\frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1; \quad \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1; \quad \frac{3}{4} + \frac{5}{9} = \frac{27}{36} + \frac{20}{36} = \frac{47}{36} = 1\frac{11}{36};$$

$$\frac{3}{4} + \frac{7}{8} = \frac{6}{8} + \frac{7}{8} = \frac{13}{8} = 1\frac{5}{8}.$$

$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}; \quad \frac{3}{4} - \frac{5}{9} = \frac{27}{36} - \frac{20}{36} = \frac{7}{36}; \quad \frac{7}{8} - \frac{3}{4} = \frac{7}{8} - \frac{6}{8} = \frac{1}{8}.$$

(5) **Multiplicación.** Multipliquense todos les numeradores y todos los denominadores. El producto es una fracción que tiene por numerador el producto de los numeradores y por denominador el producto de los denominadores. Si se quiere simplificar, divídanse el numerador y denominador del quebrado producto por su M. D. C. Ej.

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}; \quad \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}; \quad \frac{3}{4} \times \frac{5}{9} \times \frac{2}{3} = \frac{5}{18};$$

$$\frac{4}{7} \times 1\frac{2}{3} = \frac{25}{7} \times \frac{5}{3} = \frac{125}{21} = 5\frac{20}{21}; \quad \frac{2}{3} \times \frac{3}{5} = \frac{2}{5};$$

$$\frac{3}{4} \text{ de } \frac{1}{2} \text{ de } \frac{5}{3} \text{ de } \frac{7}{5} = \frac{3}{4} \times \frac{1}{2} \times \frac{5}{3} \times \frac{7}{5} = \frac{7}{8}.$$

(6) **División.** Multipliquense por el quebrado divisor invertido.

$$\frac{1}{2} \div \frac{1}{2} = \frac{1}{2} \times \frac{2}{1} = \frac{2}{2} = 1; \quad \frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \times \frac{4}{1} = \frac{3}{1} = 3$$

$$\frac{3}{4} \div \frac{5}{9} = \frac{3}{4} \times \frac{9}{5} = \frac{27}{20} = 1\frac{7}{20};$$

$$3\frac{4}{7} \div 1\frac{2}{3} = \frac{25}{7} \div \frac{5}{3} = \frac{25}{7} \times \frac{3}{5} = \frac{5}{7} \times \frac{3}{1} = \frac{15}{7} = 2\frac{1}{7};$$

$$5 \div \frac{7}{8}\frac{8}{7} = 5 \times = \frac{40}{7} = 5\frac{5}{7}.$$

(7) Se dice que una fracción está **simplificada** cuando su numerador y denominador no tienen factor común. Ej.

$$\frac{34}{85} \text{ simplificada} = \frac{2}{5}.$$

(8) Para simplificar una fracción ó reducirla á **su más sencilla expresión.** Divídase el numerador y denominador por su M. F. C. Ej. simplifíquese á $\frac{34}{85}$.

$$\text{M. F .C. de } 34 \text{ y } 85 = 17; \quad \frac{34}{85} = \frac{34 \div 17}{85 \div 17} = \frac{2}{5}.$$

## DECIMALES

(9) **Multiplicación.** Las cifras decimales del producto son iguales á la suma de las que contienen los factores. Ej.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Factores: | 100 | × 3 | × 3.5 | × 0.004 | × 465.21 | = | 1953.882000 | | |
| Número de decimales: | 0 | + 0 | + 1 | + 3 | + 2 | = | | 6 | |

(10) **División.** El número de cifras decimales del cociente = á las que contier el dividendo menos las que contiene el divisor. Ej.

$$\frac{5.125}{4.1} = 1.25; \quad \frac{5}{4} = \frac{5.00}{4} = 1.25; \quad \frac{3}{4} = \frac{3.00}{4} = 0.75; \quad \frac{0.42}{0.0021} = \frac{0.4200}{0.0021} = 200$$

Cuando el divisor es una fracción ó un número mixto, debemos multiplicar á ambos, divisor y dividendo, por la menor potencia de 10 que convierta al divisor en número entero. Ej.

$$\frac{2.679454}{0.0062} = \frac{26,794.54}{62} = 432.17.$$

(11) **Para reducir una fracción ordinaria á fracción decimal,** divídase el numerador por el denominador. Ej. $\frac{32}{40} = 0.8$; $1\frac{3}{5} = \frac{8}{5} = 1.6$.

**Tabla 1. Fracciones decimales equivalentes á las fracciones ordinarias.**

| Octavos. | Dieciseisavos. | Treintaidosavos. | Sesentaicuatroavos. | | Octavos. | Dieciseisavos. | Treintaidosavos. | Sesentaicuatroavos. | |
|---|---|---|---|---|---|---|---|---|---|
| | | | 1 | .015625 | | | | 33 | .515625 |
| | | 1 | 2 | .03125 | | | 17 | 34 | .53125 |
| | | | 3 | .046875 | | | | 35 | .546875 |
| | 1 | 2 | 4 | .0625 | | 9 | 18 | 36 | .5625 |
| | | | 5 | .078125 | | | | 37 | .578125 |
| | | 3 | 6 | .09375 | | | 19 | 38 | .59375 |
| | | | 7 | .109375 | | | | 39 | .609375 |
| 1 | 2 | 4 | 8 | .125 | 5 | 10 | 20 | 40 | .625 |
| | | | 9 | .140625 | | | | 41 | .640625 |
| | | 5 | 10 | .15625 | | | 21 | 42 | .65625 |
| | | | 11 | .171875 | | | | 43 | .671875 |
| | 3 | 6 | 12 | .1875 | | 11 | 22 | 44 | .6875 |
| | | | 13 | .203125 | | | | 45 | .703125 |
| | | 7 | 14 | .21875 | | | 23 | 46 | .71875 |
| | | | 15 | .234375 | | | | 47 | .734375 |
| 2 | 4 | 8 | 16 | .25 | 6 | 12 | 24 | 48 | .75 |
| | | | 17 | .265625 | | | | 49 | .765625 |
| | | 9 | 18 | .28125 | | | 25 | 50 | .78125 |
| | | | 19 | .296875 | | | | 51 | .796875 |
| | 5 | 10 | 20 | .3125 | | 13 | 26 | 52 | .8125 |
| | | | 21 | .328125 | | | | 53 | .828125 |
| | | 11 | 22 | .34375 | | | 27 | 54 | .84375 |
| | | | 23 | .359375 | | | | 55 | .859375 |
| 3 | 6 | 12 | 24 | .375 | 7 | 14 | 28 | 56 | .875 |
| | | | 25 | .390625 | | | | 57 | .890625 |
| | | 13 | 26 | .40625 | | | 29 | 58 | .90625 |
| | | | 27 | .421875 | | | | 59 | .921875 |
| | 7 | 14 | 28 | .4375 | | 15 | 30 | 60 | .9375 |
| | | | 29 | .453125 | | | | 61 | .953125 |
| | | 15 | 30 | .46875 | | | 31 | 62 | .96875 |
| | | | 31 | .484375 | | | | 63 | .984375 |
| 4 | 8 | 16 | 32 | .5 | 8 | 16 | 32 | 64 | 1. |

**(12) Para reducir una fracción decimal á fracción ordinaria.** Póngasele por denominador 1 y redúzcase á su más sencilla expresión la fracción que resulte. Ejemplo,

$$0.25 = \frac{0.25}{1.00} = \frac{25}{100} = \frac{1}{4};\quad 0.75 = \frac{75}{100} = \frac{3}{4};\quad 0.890625 = \frac{890625}{1000000} = \frac{57}{64}.$$

**(13) Se llaman fracciones periódicas,** aquellas en que hay una ó más decimales que se repiten indefinidamente. Ej. $\frac{1}{3} = 0.3333\ldots\ldots$, $\frac{10}{7} = 1.428571428571\ldots\ldots$ La parte periódica se señala así : $0.\overline{3}$, $1.\overline{428571}$; ó así : 0.*3, 1.*428571.....

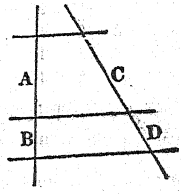
## RAZÓN Y PROPORCIÓN

**(1) Razón.** La razón entre dos cantidades como A y B, se expresa por su cociente, $\frac{A}{B}$ ó $\frac{B}{A}$. La razón entre 10 y 5 es $= \frac{10}{5} = 2$; entre 5 y 10 $= \frac{5}{10} = 0.5$.

(2) **La doble razón** es la razón entre los cuadrados de los números. Ej. $\frac{A^2}{B^2}$ es la doble razón entre A y B.

(3) **Proporción** es la igualdad de razones. Ej. $\frac{10}{5} = \frac{12}{6} = \frac{1266}{633} = 2$. En la figura que representa segmentos A, B, C, D, entre líneas paralelas; A : B :: C : D, ó $\frac{A}{B} = \frac{C}{D}$.

(4) El primero y cuarto términos A y D se llaman **extremos**; y el segundo y tercero B y C se llaman **medios**. El primer término A ó C de cada razón, se llama **antecedente** y el segundo B ó D **consecuente**. El término D se llama la **cuarta proporcional** de A, B, y C.




(5) En una proporción, A : B = C : D, se tiene :

Producto de los extremos = producto de los medios. AD = BC.

Alternando. $\frac{A}{B} = \frac{C}{D}$; $\frac{A}{C} = \frac{B}{D}$.

Invirtiendo. $\frac{B}{A} = \frac{D}{C}$; $\frac{B}{D} = \frac{A}{C}$; $\frac{D}{B} = \frac{C}{A}$.

Componiendo sumando. $\frac{A+B}{A} = \frac{C+D}{C}$; $\frac{A+B}{B} = \frac{C+D}{D}$.

Componiendo restando. $\frac{A-B}{A} = \frac{C-D}{C}$; $\frac{A-B}{B} = \frac{C-D}{D}$.

Componiendo dividiendo. $\frac{A+B}{A-B} = \frac{C+D}{C-D}$.

También se tiene :

$$\frac{mA}{mB} = \frac{A}{B} = \frac{C}{D} = \frac{nA}{nB} = \frac{nC}{nD}; \quad \frac{mA}{nB} = \frac{mC}{nD}; \quad \frac{A^n}{B^n} = \frac{C^n}{D^n}; \quad \frac{\sqrt[n]{A}}{\sqrt[n]{B}} = \frac{\sqrt[n]{C}}{\sqrt[n]{D}}.$$

(6) Si en la proporción A : B = C : D, se tiene que B = C = $m$, entonces, $\frac{A}{m} = \frac{m}{D}$; ó $m^2 = AD$, ó $m = \sqrt{AD}$.

(7) En este caso se dice que $m$ es **media proporcional** entre A y D; y D se llama la **tercera proporcional** entre A y $m$.

Una **proporción continua** es una serie de razones iguales como :

$$A : B = C : D = E . F, \text{ etc.} = R; \quad \frac{A}{B} = \frac{C}{D} = \frac{E}{F}, \text{ etc.} = R.$$

En la proporción continua,

$$\frac{A + C + E + \text{etc.}}{B + D + F + \text{etc.}} = \frac{A}{B} = \frac{C}{D} = \frac{E}{F}, \text{ etc.} = R.$$

Si $\frac{A}{B} = \frac{C}{D}$; $\frac{A'}{B'} = \frac{C'}{D'}$; $\frac{A''}{B''} = \frac{C''}{D''}$; entonces $\frac{A\,A'\,A''}{B\,B'\,B''} = \frac{C\,C'\,C''}{D\,D'\,D''}$, etc.

**8)** Sean A, B y C tres números cualesquiera, entonces

$$\frac{A}{C} = \frac{A}{B} \cdot \frac{B}{C}, \text{ y } \frac{A}{B} = \frac{A}{C} \cdot \frac{C}{B}.$$

**(9) Proporción recíproca ó inversa.** — Dos cantidades, se dice que están en razón inversa, cuando la razón $\frac{A}{B}$ de dos valores A y B de la una es igual á la recíproca $\frac{B}{A'}$ de la razón de otros dos valores A', B' correspondientes, de la otra. Ej.: Sea A = a una velocidad de dos millas por hora, y B = 3 millas por hora. Entonces el tiempo requerido por milla en horas, será respectivamente $A' = \frac{1}{A} = \frac{1}{2}$, $B' = \frac{1}{B} = \frac{1}{3}$. En este caso A : B = B' : A', ó bien

$$\frac{A}{B} = \frac{B'}{A'}, \text{ ó } \frac{2}{3} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3} = 1 \div \frac{A'}{B}.$$

**(10)** Si dos números variables A y B son recíprocamente proporcionales, de modo que A' : B' = B'' : A'' el producto A' A'' de cualquiera de los dos valores de uno de los números, es igual al producto B' B'' de los dos valores correspondientes del otro.

**(11)** La aplicación de la proporción á los problemas prácticos, se llama á veces **Regla de tres.** Ejemplo de una **regla de tres simple** : si tres hombres cargan 10,000 ladrillos en cierto tiempo, ¿cuántos pueden cargar 6 hombres en el mismo tiempo? Como están 3 con 6 hombres, así están 10,000 con 20,000 ladrillos; ó bien $10{,}000 \times \frac{6}{3} = 20{,}000$ ladrillos.

Si 3 hombres invierten 10 horas para cargar cierto número de ladrillos, ¿cuántas horas invertirán 6 hombres para cargar el mismo número? Como están 6 con 3 hombres, así están 10 horas con 5 horas, ó bien 10 horas $\times \frac{3}{6} = 5$ horas.

**(12) Regla de tres compuesta.** — Si 3 hombres cargan 4,000 ladrillos en 2 días, ¿cuántos hombres pueden cargar 12,000 en 3 días? En este caso 4,000 ladrillos requieren 3 hombres en 2 días ó 6 hombres en un día, 12,000 necesitarán tres veces más, ó sean $6 \times \frac{12{,}000}{4{,}000} = 6 \times 3 = 18$ hombres en un día ó 18 *hombres-días;* y como el trabajo se va á hacer en 3 días, serán $\frac{18}{3} = 6$ hombres que se necesitan.

## PROGRESIÓN

**(1)** Se dice que una serie de números están en *progresión aritmética* cuando cada número difiere del que le precede, en una misma cantidad. Ej. : — 2, — 1, 0, 1, 2, 3, etc., aquí la diferencia es 1; ó bien 4, 3, 2, 1, 0, — 1, — 2, etc., en que la diferencia es = — 1; ó bien — 4, — 2, 0, 2, 4, 6, etc., en que la diferencia es 2; ó bien 1½, 1¼, 1, ¾, ½, ¼, 0, — ¼, — ½, etc., en que la diferencia = — ¼.

**(2)** En cualesquiera de estas series los números se llaman términos. Sean, $a$ el primer término; $l$ el último; $d$ la diferencia constante; $n$ el número de términos, y $s$ la suma de estos términos. Entonces

| Se desea conocer. | Conociendo. | |
|---|---|---|
| | $a\ d\ n$ | $l = a + (n-1)\,d$ |
| | $a\ d\ s$ | $l = -\frac{1}{2}d \pm \sqrt{2ds + \left(a - \frac{1}{2}d\right)^2}$ |
| | $a\ d\ n$ | $s = \frac{1}{2}n\,[2a + (n-1)\,d]$ |

| | | |
|---|---|---|
| $a$ | $d\ l\ s$ | $a = \frac{1}{2}d \pm \sqrt{\left(l + \frac{1}{2}d\right)^2 - 2ds}$ |
| $n$ | $a\ d\ s$ | $n = \frac{d - 2a \pm \sqrt{(2a - d)^2 + 8ds}}{2d}$ |
| $n$ | $d\ l\ s$ | $n = \frac{2l + d \pm \sqrt{(2l + d)^2 - 8ds}}{2d}$ |

§ (3) **Progresión geométrica.** Se dice que una serie de números están en progresión geométrica, cuando uno cualquiera de ellos dividido por el anterior da el mismo cociente. Ej. : $\frac{1}{9}$, $\frac{1}{3}$, 1, 3, 9, 27, etc., aquí el cociente es 3; ó bien 48, 24, 12, 6, 3, $1\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, etc., aquí el cociente es $= \frac{1}{2}$.

(4) Sea $a$ el primer término; $l$ el último; $r$ la razón ó cociente constante; $n$ el número de términos y $s$ la suma de los términos. Entonces

| Se desea conocer. | Conociendo. | |
|---|---|---|
| $l$ | $a\ r\ n$ | $l = ar^{n-1}$ * |
| $l$ | $a\ r\ s$ | $l = \frac{a + (r-1)s}{r}$ |
| $l$ | $r\ n\ s$ | $l = \frac{(r-1)sr^{n-1}}{r^n - 1}$ |
| $s$ | $a\ n\ l$ | $s = \frac{\sqrt[n-1]{l^n} - \sqrt[n-1]{a^n}}{\sqrt[n-1]{l} - \sqrt[n-1]{a}}$ |
| $s$ | $r\ n\ l$ | $s = \frac{lr^n - l}{r^n - r^{n-1}}$ |
| $r$ | $a\ n\ l$ | $r = \sqrt[n-1]{l \div a}$ |

## PERMUTACIÓN, Etc.

(1) **Permutación.** Las permutaciones sirven para encontrar en cuántas posiciones puede ser dispuesto en hilera un número cualquiera de cosas. Para hacer esto, multipliquense juntos todos los números sucesivos desde 1 hasta el número de objetos, así : ¿En cuántas posiciones en una misma línea pueden colocarse 9 objetos?

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362{,}880 \text{ posiciones.}$$

(2) **Combinaciones.** ¿Cuántas combinaciones diferentes pueden formarse con un número de objetos, entrando en cada combinación cierto número? Para saber esto, fíjese primero la cifra que indica el número de objetos y después la serie de números que siguen cuya diferencia sea 1 hasta que haya *por todo* tantos números cuantos objetos van á entrar en cada combinación. Luego, á partir del número de objetos que van á entrar en cada combinación, fórmese otra serie descendente, con la misma diferencia de 1. Multipliquense juntos todos los números de la primera operación, para formar un producto y todos los de la segunda para formar otro y divídase el primer producto por el segundo.

Ejemplo : ¿Cuántas combinaciones diferentes de 4 números cada una pueden hacerse con los nueve números 1, 2, 3, 4, 5, 6, 7, 8 y 9 ó 9 objetos cualesquiera?

$$\frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4} = \frac{3024}{24} = 126 \text{ combinaciones.}$$

* Las ecuaciones que contienen potencias ó radicales son á veces resueltas con más facilidad por los logaritmos.

**(3) Aligación.** Sirve para encontrar el valor de una mezcla de diferentes ingredientes cuando la cantidad y valor de cada uno de estos últimos son conocidos.

Ejemplo : ¿Cuál es el valor de una libra de la mezcla de 20 libras de á 15 centavos la libra con 30 libras de á 25 centavos la libra?

| lbs. | cts. | cts. |
|---|---|---|
| $20 \times 15 =$ | | 300 |
| $30 \times 25 =$ | | 750 |
| 50 lbs. | | 1,050 cts. |

por tanto $\frac{1,050}{50} = 21$ cts. Resultado.

## TANTO POR CIENTO (Porcentaje), INTERÉS, ANUALIDADES

**(1)** La razón se expresa á menudo con la palabra *por*. Así, hablando de una pendiente, se dice: 105.6 pies (=30 m. 48) por milla ó por 5,280 pies (=1,680 m. 38). Cuando los dos términos de la razón se refieren á cantidades de la misma especie y denominacion, la razón se expresa á menudo en un *porcentage* ó *tanto por ciento*. Así, una pendiente de 105.6 pies por milla ó por 5,280 pies, equivale á una pendiente de 0.02 de pie por pie* ó de metro por metro, ó de legua por legua, etc.; es decir, de 2 *por* 100.

**(2)** Un *cincuentavo* ó 1 por 50, equivale á 2 por 100, ó dos centésimos. Del mismo modo $\frac{1}{4} = 25$ por ciento, $\frac{3}{4} = 3 \times 25$ por ciento $= 75$ por ciento, etc. Por tanto, para encontrar el tanto por ciento equivalente á una razón, dividase 100 veces el primer término por el segundo. Ej. : En un concreto (argamasa de cemento) de 1 parte de cemento, 2 de arena y 5 de piedra picada, por todo 8 partes (por peso)†.

| | | | |
|---|---|---|---|
| Cemento | $= \frac{1}{8} = 0.125 =$ | 12.5 | por ciento del total. |
| Arena | $= \frac{2}{8} = 0.250 =$ | 25.0 | » » |
| Piedra | $= \frac{5}{8} = 0.625 =$ | 62.5 | » » |
| Concreto | $= \frac{8}{8} = 1.000 =$ | 100.0 | por ciento del total. |

**(3)** El tanto por ciento tiene una amplia aplicación en asuntos de dinero: y el pago de servicios en las operaciones se basa á menudo en el monto de las sumas que se negocian. Por ejemplo : A un agente comprador ó vendedor de valores hay que pagarle una comisión que forma un tanto por ciento del valor de la cosa negociada; el premio pagado por aseguro, es un tanto por ciento del valor de los bienes asegurados.

### Interés.

**(4)** Interés es la renta que se paga por el dinero prestado. La suma prestada es el **capital** y el número de centavos pagados mensual ó anualmente por cada peso ó unidad de moneda, ó el número de pesos por cada cien pesos, es el **interés.** Éste siempre se fija en un tanto por ciento y se expresa con este signo %;

---

* Las fracciones como $\frac{1}{8}, \frac{5}{16}$, etc., ó las decimales equivalentes .125, .3125, etc., están comparadas con la *unidad* ó 1 ; pero al tratarse de un *porcentaje* el primer término de la razón ó cociente se compara con *100 unidades* del segundo término ó dividendo. Se cometen muchos errores por olvidar esta distinción. Ej. : .06 (seis por ciento) se lee equivocadamente á veces : seis centésimos de uno por ciento ó *seis centesimos* por ciento.

† Para las proporciones por volúmenes véanse en el capítulo *Argamasa-Cemento* los números 40 y siguientes y los números 5 y siguientes del capítulo *Pruebas* de la misma sección.

así una suma al 3% anual, expresa que cada cien unidades devengan de interés 3 unidades por año.

(5) Si el interés se paga al prestamista á proporción que se produce, se dice que el dinero está á **interés simple;** pero, si el interés se va periódicamente agregando al capital, para que, aumentándolo, devengue á su vez interés, se dice que el dinero está á **interés compuesto.**

## Interés simple.

(6) Al fin del año el interés del capital P á la rata ó interés de $r$ por unidad será $Pr$ y la suma A de capital é intereses será

$$A=P+Pr=P\ (1+r)$$

(7) Al término de un número de años $n$, el interés será $Prn$ (véase el lado derecho de la figura 1) y,

$$A=P+Prn=P\ (1+rn)$$

Ejemplo : Sea P=$865.32; $r$=3 por ciento ó 0.03 y $n$=1 año, 3 meses y 10 días=1 año y 100 días=$1\frac{100}{365}$ años=1.274 años. Entonces $A=P\ (1+rn)=$ $=865.32\times(1+0.03\times1.274)=865.32\times1.03822=\$898.39$.

(8) Para conocer el valor actual P de una suma capitalizada, que se entregará dentro de $n$ años, se tendrá despejando á P en la ecuación dicha

$$P=\frac{A}{1+rn}$$

Ejemplo : ¿Cuál es el valor actual P de una suma que en un año, 3 meses y 10 días al 3% anual se convierte en $898.39. Se tiene $P=\frac{898.39}{1+0.03\times1.274}=865.32$.

(9) En el comercio se calculan los intereses aproximadamente, suponiendo el año de 12 meses de á 30 días. Así el 6% de interés anual en 2 meses=60 días=1 por ciento; 1 mes=30 días=½%; 6 días=0.1 por ciento. Ej. : ¿Cuál es el interés de $1,264.35 en 5 meses, 28 días al 5% anual?

| | |
|---|---|
| Capital | $1,264.35 |
| Intereses, 2 meses, 1% | 12.64 |
| — 2 — 1% | 12.64 |
| — 1 — ½% | 6.32 |
| — 20 días, ⅓% | 4.21 |
| — 6 — 0.1% | 1.26 |
| — 2 — 1/30% | 0.42 |
| Intereses al 6% | $37.49 |
| Se deduce un sexto | 6.25 |
| Interés al 5% | $31.24 |

## Ecuación de pagos.

(10) A le debe á B $1,200; de los cuales 400 deben pagarse en 3 meses; 500 en 4 meses y 300 en 6 meses. Todas devengan interés hasta ser pagados, pero se ha convenido en que la suma se pague junta. Se pregunta : ¿En qué tiempo debe pagársela para que ninguno se perjudique?

| | meses. |
|---|---|
| 400 × 3 | = 1,200 |
| 500 × 4 | = 2,000 |
| 300 × 6 | = 1,800 |
| 1,200 | 5,000 |

Tiempo medio $\frac{5,000}{1,200}$ = 4 1/6 meses. Respuesta.

## Interés compuesto.

**(11)** Se acostumbra acumular los intereses por año, por semestre ó por trimestre Si por años (véase el lado izquierdo de la fig. 1).

al final de 1 año $A=P\,(1+r)$

— 2 años $A=P\,(1+r)\,(1+r)=P\,(1+r)^2$

— $n$ — $A=P\,(1+r)^n$ y

$$P=\frac{A}{(1+r)^n}=A\,(1+r)^{-n}$$

$$\frac{A}{P}=(1+r)^n$$

**(12)** Si los intereses se acumulan $q$ veces por año, se tendrá

$$\frac{A}{P}=\left(1+\frac{r}{q}\right)^{qn}.$$

**(13)** E capital P, se llama á veces **valor actual** de la suma A. Así, en la tabla que sigue $1.00 es el **valor actual** de $2.191 dentro de 20 años al 4% anual al interés compuesto.

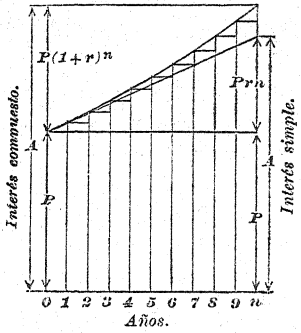



**Fig. 2.**

## Tabla 2. Interés compuesto.

Valor de una unidad monetaria cualquiera á interés compuesto.

| Años. | 3 % | 3½ % | 4 % | 4½ % | 5 % | 5½ % | 6 % | 6½ % |
|---|---|---|---|---|---|---|---|---|
| 1 | 1.030 | 1.035 | 1.040 | 1.045 | 1.050 | 1.055 | 1.060 | 1.065 |
| 2 | 1.061 | 1.071 | 1.082 | 1.092 | 1.103 | 1.113 | 1.124 | 1.134 |
| 3 | 1.093 | 1.109 | 1.125 | 1.141 | 1.158 | 1.174 | 1.191 | 1.208 |
| 4 | 1.126 | 1.148 | 1.170 | 1.193 | 1.216 | 1.239 | 1.262 | 1.286 |
| 5 | 1.159 | 1.188 | 1.217 | 1.246 | 1.276 | 1.307 | 1.338 | 1.370 |
| 6 | 1.194 | 1.229 | 1.265 | 1.302 | 1.340 | 1.379 | 1.419 | 1.459 |
| 7 | 1.230 | 1.272 | 1.316 | 1.361 | 1.407 | 1.455 | 1.504 | 1.554 |
| 8 | 1.267 | 1.317 | 1.369 | 1.422 | 1.477 | 1.535 | 1.594 | 1.655 |
| 9 | 1.305 | 1.363 | 1.423 | 1.486 | 1.551 | 1.619 | 1.689 | 1.763 |
| 10 | 1.344 | 1.411 | 1.480 | 1.553 | 1.629 | 1.708 | 1.791 | 1.877 |
| 11 | 1.384 | 1.460 | 1.539 | 1.623 | 1.710 | 1.802 | 1.898 | 1.999 |
| 12 | 1.426 | 1.511 | 1.601 | 1.696 | 1.796 | 1.901 | 2.012 | 2.129 |
| 13 | 1.469 | 1.564 | 1.665 | 1.772 | 1.886 | 2.006 | 2.133 | 2.267 |
| 14 | 1.513 | 1.619 | 1.732 | 1.852 | 1.980 | 2.116 | 2.261 | 2.415 |
| 15 | 1.558 | 1.675 | 1.801 | 1.935 | 2.079 | 2.232 | 2.397 | 2.572 |
| 16 | 1.605 | 1.734 | 1.873 | 2.022 | 2.183 | 2.355 | 2.540 | 2.739 |
| 17 | 1.653 | 1.795 | 1.948 | 2.113 | 2.292 | 2.485 | 2.693 | 2.917 |
| 18 | 1.702 | 1.858 | 2.026 | 2.208 | 2.407 | 2.621 | 2.854 | 3.107 |
| 19 | 1.754 | 1.923 | 2.107 | 2.308 | 2.527 | 2.766 | 3.026 | 3.309 |
| 20 | 1.806 | 1.990 | 2.191 | 2.412 | 2.653 | 2.918 | 3.207 | 3.524 |
| 21 | 1.860 | 2.059 | 2.279 | 2.520 | 2.786 | 3.078 | 3.400 | 3.753 |
| 22 | 1.916 | 2.132 | 2.370 | 2.634 | 2.925 | 3.248 | 3.604 | 3.997 |
| 23 | 1.974 | 2.206 | 2.465 | 2.752 | 3.072 | 3.426 | 3.820 | 4.256 |
| 24 | 2.033 | 2.283 | 2.563 | 2.876 | 3.225 | 3.615 | 4.049 | 4.533 |
| 25 | 2.094 | 2.363 | 2.666 | 3.005 | 3.386 | 3.813 | 4.292 | 4.823 |
| 26 | 2.157 | 2.446 | 2.772 | 3.141 | 3.556 | 4.023 | 4.549 | 5.141 |
| 27 | 2.221 | 2.532 | 2.883 | 3.282 | 3.733 | 4.244 | 4.822 | 5.476 |
| 28 | 2.288 | 2.620 | 2.999 | 3.430 | 3.920 | 4.478 | 5.112 | 5.832 |
| 29 | 2.357 | 2.712 | 3.119 | 3.584 | 4.116 | 4.724 | 5.418 | 6.211 |
| 30 | 2.427 | 2.807 | 3.243 | 3.745 | 4.322 | 4.984 | 5.743 | 6.614 |
| 31 | 2.500 | 2.905 | 3.373 | 3.914 | 4.538 | 5.258 | 6.088 | 7.044 |
| 32 | 2.575 | 3.007 | 3.508 | 4.090 | 4.765 | 5.547 | 6.453 | 7.502 |
| 33 | 2.652 | 3.112 | 3.648 | 4.274 | 5.003 | 5.852 | 6.841 | 7.990 |
| 34 | 2.732 | 3.221 | 3.794 | 4.466 | 5.253 | 6.174 | 7.251 | 8.509 |
| 35 | 2.814 | 3.334 | 3.946 | 4.667 | 5.516 | 6.514 | 7.686 | 9.062 |
| 36 | 2.898 | 3.450 | 4.104 | 4.877 | 5.792 | 6.872 | 8.147 | 9.651 |
| 37 | 2.985 | 3.571 | 4.268 | 5.097 | 6.081 | 7.250 | 8.636 | 10.279 |
| 38 | 3.075 | 3.696 | 4.439 | 5.326 | 6.385 | 7.649 | 9.154 | 10.947 |
| 39 | 3.167 | 3.825 | 4.616 | 5.566 | 6.705 | 8.069 | 9.704 | 11.658 |
| 40 | 3.262 | 3.959 | 4.801 | 5.816 | 7.040 | 8.513 | 10.286 | 12.416 |

El interés compuesto de M unidades cualesquiera á cualquier rata $r$ en años = M × el interés compuesto de la unidad á la misma rata $r$ y por en años

### Anualidad, fondo de reserva, amortización, depreciación.

(**14**) Hay casos en que cierta suma o capital pagado una sola vez, se le destina á cumular sus propios intereses, simples o compuestos; en otros se hacen pagos períodicos, comunmente anuales, llama dos **anualidades**, para acumular también sus intereses, generalmente compuestos.

(**15**) Por ejemplo, se pone aparte ó se deposita una suma anual que con sus intereses compuestos forma un **fondo de reserva** destinado á cubrir una deuda.

Este sistema se emplea á veces para pagar á plazos las construcciones ú obras de ingeniería. Este proceso se llama de **amortización**.

**(16)** Al valorar la obra se hace un cálculo para su **depreciación**, con el tiempo, uso, etc. Hecha esta apreciación se calcula la vida ó duración de la obra, en años $n$ y se busca la anualidad $p$, que á la rata $r$ de interés compuesto, sea suficiente para reponer la obra al cabo del tiempo $n$, cuando el uso, etc., la hayan destruído.

**(17)** El **valor actual** ó **capitalización** $W$, fig. 2, de una anualidad $p$ para un número dado de años $n$, es una suma que, si se la coloca ahora, á interés compuesto, á la rata $r$, llega, al cabo de dicho tiempo, á valer la misma suma A, que se alcanzó con dicha anualidad.

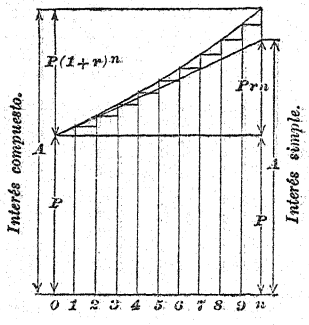



Fig. 1.

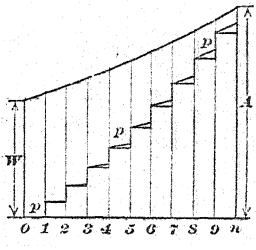



Fig. 2.

**(18) Ecuaciones para interés compuesto y anualidades** (véanse figs. 1 y 2).

| | | |
|---|---|---|
| P = capital; | $r$ = interés; | $n$ = número de años; |
| A = suma total; | $p$ = anualidad; | $W$ = valor actual. |

Se supone el interés compuesto y las anualidades depositadas al final de cada año.

### Interés compuesto.

(1) El monto A de una unidad monetaria cualquiera al cabo de $n$ años (véase **11**) es $A=(1+r)^n$.

(2) Por tanto, como el valor actual de la suma $(1+r)^n$ pagadera á los $n$ años es 1 (dólar, franco, etc.) (véase **(1)**); siguiendo el mismo razonamiento la unidad $1, por ejemplo, pagadero á los $n$ años, vale hoy $W = \dfrac{1}{(1+r)^n} = (1+r)^{-n}$.

### Anualidades.

3) Considerada $r$ como una anualidad que se acumula, al cabo de $n$ años, valdrá $(1+r)^n-1$, pues hay que rebajar el capital primitivo 1, para que quede sólo la acumulación de la anualidad $r$. Si la anualidad $r$ vale á los $n$ años $(1+r)^n-1$ **el monto A de la anualidad** de $1, valdrá á los $n$ años

$$A = \frac{(1+r)^n - 1}{r}.$$

(4) Para **el valor actual** $W$ **de una anualidad** de $1 durante $n$ años, se tiene, de las ecuaciones (1) y (3) :

$$(1+r)^n : 1 = \frac{(1+r)^n - 1}{r} : W. \quad \text{Así } W = \frac{(1+r)^n - 1}{(1+r)^n r} = \frac{1 - \frac{1}{(r+1)^n}}{r}.$$

(Véase tabla 3.)

(5) La anualidad durante $n$ años que corresponde á lo que puede ganar $1, es

$$p = \frac{1}{W} = \frac{r}{1 - \frac{1}{(1+r)^n}}.$$

(6) La anualidad que en $n$ años llega á valor $1, es :

$$p' = p - r = \frac{1}{W} - r = \frac{r}{1 - \frac{1}{(1+r)^n}} - r = \frac{r}{(1+r)^n - 1}.$$

**Tabla 3. — Valor actual de una anualidad de 1000** (dólares francos, etc.). Véase ecuación 4.

| | A interés compuesto. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Años. | 2½ % | 3 % | 3½ % | 4 % | 4½ % | 5 % | 5½ % | 6 % |
| 5 | 4,646 | 4,580 | 4,515 | 4,452 | 4,390 | 4,329 | 4,268 | 4,212 |
| 10 | 8,752 | 8,530 | 8,316 | 8,111 | 7,913 | 7,722 | 7,538 | 7,360 |
| 15 | 12,381 | 11,938 | 11,517 | 11,118 | 10,740 | 10,380 | 10,037 | 9,712 |
| 20 | 15,589 | 14,877 | 14,212 | 13,590 | 13,008 | 12,462 | 11,950 | 11,470 |
| 25 | 18,424 | 17,413 | 16,482 | 15,622 | 14,828 | 14,094 | 13,414 | 12,783 |
| 30 | 20,930 | 19,600 | 18,392 | 17,292 | 16,289 | 15,372 | 14,534 | 13,765 |
| 35 | 23,145 | 21,487 | 20,000 | 18,664 | 17,461 | 16,374 | 15,391 | 14,498 |
| 40 | 25,103 | 23,115 | 21,355 | 19,793 | 18,401 | 17,159 | 16,045 | 15,046 |
| 45 | 26,833 | 24,519 | 22,495 | 20,720 | 19,156 | 17,774 | 16,548 | 15,456 |
| 50 | 28,362 | 25,730 | 23,456 | 21,482 | 19,762 | 18,256 | 16,932 | 15,762 |
| 100 | 36,614 | 31,599 | 27,655 | 24,505 | 21,950 | 19,848 | 18,096 | 15,618 |

(**19**) En la construcción de obras y sus mejoras ó refacciones se acostumbra agregar al costo y gastos ordinarios de conservación y reparaciones : 1.° El interés del costo de la obra; 2.° Un márgen por depreciación, y algunas veces : 3.° Una anualidad que forme un fondo de reserva ó previsión para extinguir la deuda contraída por la construcción. La capitalización, del total anual de gastos así obtenido, se considera como el verdadero costo de la construcción. Todos los elementos del costo se reducen así á una base común y se hacen verdaderamente comparables.

(**20**) Así se estimó en 1889 el costo de las mejoras hechas al acueducto de Filadelfia, la rata $r$ del interés, se hizo al 3%, la depreciación se calculó como se expresa más abajo. Bajo el nombre de « Duración » se aprecia el tiempo que puede durar en buen servicio, cada elemento de la construcción, aparatos, etc., y como « Anualidad » la suma que se aparta anualmente para reemplazar, al destruirse cada $1,000 en objetos deteriorados.

| Valor actual | Anualidad | | Valor actual | | Anualidad | |
|---|---|---|---|---|---|---|
| * Porque, W | : $1.00 | :: | $1.00 | : | $p$. | Así $p = \frac{1}{W}$. |
| Ecuación (4) | | | Ecuación (5) | | | |
| Anualidad | Monto de la suma | | Anualidad | Monto de la suma | | |
| † Porque, $r$ : | $(1+r)^n - 1$ :: | | $p'$ . | $1.00. | Así, $p' = \frac{r}{(1+r)^n - 1}$ | |

‡ *Informe de* Rudolph Hering, S.M. Gray y J. M. Wilson.

| Construcción, aparatos, etc. | Duración en años. | Anualidad $ |
|---|---|---|
| Conductos de mampostería, lechos de filtros, receptáculos. | Indefinida | 0.00 |
| Edificio | 100 | 1.65 |
| Tuberías de hierro, rieles, etc. | 80 | 3.11 |
| Tubos de acero, válvulas, compuertas, etc. | 35 | 16.54 |
| Máquinas y bombas | 30 | 21.02 |
| Calderas, planta eléctrica, tranvías, equipo | 20 | 37.22 |
| Líneas telefónicas, aparatos, etc. | 10 | 87.24 |

**(21)** Sobre estas bases, se calcularon dos proyectos, capaces cada uno de suministrar 450 millones de galones (1,705,500 metros cúbicos) por día. Se los comparó así:

**Agua sin filtrar, por acueductos.**

| | | |
|---|---|---|
| *Primer costo.* | | |
| Receptáculos de depósito | | $30,900,000 |
| Acueductos | | 47,730,000 |
| Distribución | | 3,555,000 |
| Estanques distribuidores | | 1,000,000 |
| Total | | $83,185,000 |
| *Por año.* | | |
| Interés sobre $83,185,000 | | $2,495,550 |
| Depreciación | | 198,640 |
| *Manipulaciones y sostenimiento.* | | |
| Análisis é inspección | $41,61 | |
| Reparaciones ordinarias | 49,150 | |
| Bombas, salarios, etc. | 140,770 | |
| | | 231,540 |
| | | $2,925,730 |

**Agua de río tomada dentro de los límites de la ciudad y filtrada.**

| | | |
|---|---|---|
| *Primer costo.* | | |
| Filtros | | $23,174,680 |
| Canales | | 10,980,000 |
| Total | | $34,154,680 |
| *Por año.* | | |
| Interés sobre $34,154,680 | | $1,024,640 |
| Depreciación | | 205,540 |
| *Manipulaciones y sostenimiento.* | | |
| Bombas | $1,216,021 | |
| Filtración | 525,600 | |
| | | 1,741,621 |
| | | $2,971,801 |

Se observa que, no obstante que el primer costo del proyecto por filtro es mucho menor de la mitad del proyecto por acueducto, la proporción que hay que cargar por sus elementos más fácilmente destructibles, es algo mayor; al mismo tiempo, la manipulación y sostenimiento es más de 7 veces mayor, y el total de su gasto anual un poco mayor.

**Tabla 4. — Anualidad para amortizar** $1,000. (Véase ecuación 6.)

| | A interés compuesto. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Años. | 1 % | 2 % | 2½ % | 3 % | 3½ % | 4 % | 5 % | 6 % |
| 5 | 196.04 | 192.16 | 190.24 | 188 36 | 186.49 | 184.63 | 180.98 | 177.39 |
| 10 | 95.58 | 91.33 | 89.25 | 87.23 | 85.24 | 83.29 | 79.50 | 75.87 |
| 15 | 62.12 | 57.83 | 55.77 | 53.77 | 51.82 | 49.94 | 46.34 | 42.96 |
| 20 | 45.42 | 41.15 | 39.14 | 37.22 | 35.36 | 33.58 | 30.24 | 27.18 |
| 25 | 35.41 | 31.22 | 29.27 | 27.43 | 25.67 | 24.01 | 20.95 | 18.23 |
| 30 | 28.75 | 24.65 | 22.78 | 21.02 | 19.37 | 17.83 | 15.05 | 12.65 |
| 35 | 24.00 | 20.00 | 18.20 | 16.54 | 15.00 | 13.58 | 11.07 | 8.97 |
| 40 | 20.46 | 16.55 | 14.84 | 13.26 | 11.83 | 10.52 | 8.28 | 6.46 |
| 45 | 17.71 | 13.91 | 12.27 | 10.79 | 9.45 | 8.26 | 6.26 | 4.70 |
| 50 | 15.51 | 11.82 | 10.26 | 8.87 | 7.63 | 6.55 | 4.78 | 3.44 |
| 60 | 12.24 | 8.77 | 7.35 | 6.13 | 5.09 | 4.20 | 2.83 | 1.88 |
| 70 | 9.93 | 6.67 | 5.40 | 4.34 | 3.46 | 2.74 | 1.70 | 1.03 |
| 80 | 8.22 | 5.16 | 4.03 | 3.11 | 2.38 | 1.81 | 1.03 | 0.573 |
| 90 | 6.91 | 4.05 | 3.04 | 2.26 | 1.66 | 1.21 | 0.627 | 0.318 |
| 100 | 5.87 | 3.20 | 2.31 | 1.65 | 1.16 | 0.808 | 0.383 | 0.177 |

## SISTEMA DUODECIMAL

(1) En el sistema árabe, se toma 10 como base, pero en el duodecimal, la base es 12 ó una « docena ». 10 es divisible sólo por 5 y por 2; y 12 es divisible dos veces por 2; y una por 3, por 4 y por 6. Esta es la razón de la popularidad de la docena como base de numeración; en los pesos como en la libra de Troy de 12 onzas; en las longitudes como en el pie en 12 pulgadas; en el año en 12 meses; en la mitad del día en 12 horas y en la moneda, en el chelín inglés en 12 peniques.

(2) **Duodecimales.** En este sistema se mide el lado de los rectángulos en pies y pulgadas; y las áreas, al multiplicar una dimensión por otra vienen en pies cuadrados y doceavos de pie cuadrado (que equivale á 12 pulgadas cuadradas, puesto que el pie tiene 144 pulgadas cuadradas) y en pulgadas cuadradas; pero los lados deben tomarse en pies y décimos de pie y en la multiplicación, así más práctica, se puede reducir la fracción decimal de los pies á pulgadas cuadradas multiplicándola por 144.

**Tabla de números inversos.** (Véanse las explicaciones después de las tablas.)

| No. | Inverso. | No. | Inverso. | No. | Inverso. | No. | Inverso. |
|---|---|---|---|---|---|---|---|
| 1 | 1.000000000 | 56 | .017857143 | 111 | .009009009 | 166 | .006024096 |
| 2 | 0.500000000 | 57 | .017543860 | 112 | .008928571 | 167 | .005988024 |
| 3 | .333333333 | 58 | .017241379 | 113 | .008849558 | 168 | .005952381 |
| 4 | .250000000 | 59 | .016949153 | 114 | .008771930 | 169 | .005917160 |
| 5 | .200000000 | 60 | .016666667 | 115 | .008695652 | 170 | .005882353 |
| 6 | .166666667 | 61 | .016393443 | 116 | .008620690 | 171 | .005847953 |
| 7 | .142857143 | 62 | .016129032 | 117 | .008547009 | 172 | .005813953 |
| 8 | .125000000 | 63 | .015873016 | 118 | .008474576 | 173 | .005780347 |
| 9 | .111111111 | 64 | .015625000 | 119 | .008403361 | 174 | .005747126 |
| 10 | .100000000 | 65 | .015384615 | 120 | .008333333 | 175 | .005714286 |
| 11 | .090909091 | 66 | .015151515 | 121 | .008264463 | 176 | .005681818 |
| 12 | .083333333 | 67 | .014925373 | 122 | .008196721 | 177 | .005649718 |
| 13 | .076923077 | 68 | .014705882 | 123 | .008130081 | 178 | .005617978 |
| 14 | .071428571 | 69 | .014492754 | 124 | .008064516 | 179 | .005586592 |
| 15 | .066666667 | 70 | .014285714 | 125 | .008000000 | 180 | .005555556 |
| 16 | .062500000 | 71 | .014084507 | 126 | .007936508 | 181 | .005524862 |
| 17 | .058823529 | 72 | .013888889 | 127 | .007874016 | 182 | .005494505 |
| 18 | .055555556 | 73 | .013698630 | 128 | .007812500 | 183 | .005464481 |
| 19 | .052631579 | 74 | .013513514 | 129 | .007751938 | 184 | .005434783 |
| 20 | .050000000 | 75 | .013333333 | 130 | .007692308 | 185 | .005405405 |
| 21 | .047619048 | 76 | .013157895 | 131 | .007633588 | 186 | .005376344 |
| 22 | .045454545 | 77 | .012987013 | 132 | .007575758 | 187 | .005347594 |
| 23 | .043478261 | 78 | .012820513 | 133 | .007518797 | 188 | .005319149 |
| 24 | .041666667 | 79 | .012658228 | 134 | .007462687 | 189 | .005291005 |
| 25 | .040000000 | 80 | .012500000 | 135 | .007407407 | 190 | .005263158 |
| 26 | .038461538 | 81 | .012345679 | 136 | .007352941 | 191 | .005235602 |
| 27 | .037037037 | 82 | .012195122 | 137 | .007299270 | 192 | .005208333 |
| 28 | .035714286 | 83 | .012048193 | 138 | .007246377 | 193 | .005181347 |
| 29 | .034482759 | 84 | .011904762 | 139 | .007194245 | 194 | .005154639 |
| 30 | .033333333 | 85 | .011764706 | 140 | .007142857 | 195 | .005128205 |
| 31 | .032258065 | 86 | .011627907 | 141 | .007092199 | 196 | .005102041 |
| 32 | .031250000 | 87 | .011494253 | 142 | .007042254 | 197 | .005076142 |
| 33 | .030303030 | 88 | .011363636 | 143 | .006993007 | 198 | .005050505 |
| 34 | .029411765 | 89 | .011235955 | 144 | .006944444 | 199 | .005025126 |
| 35 | .028571429 | 90 | .011111111 | 145 | .006896552 | 200 | .005000000 |
| 36 | .027777778 | 91 | .010989011 | 146 | .006849315 | 201 | .004975124 |
| 37 | .027027027 | 92 | .010869565 | 147 | .006802721 | 202 | .004950495 |
| 38 | .026315789 | 93 | .010752688 | 148 | .006756757 | 203 | .004926108 |
| 39 | .025641026 | 94 | .010638298 | 149 | .006711409 | 204 | .004901961 |
| 40 | .025000000 | 95 | .010526316 | 150 | .006666667 | 205 | .004878049 |
| 41 | .024390244 | 96 | .010416667 | 151 | .006622517 | 206 | .004854369 |
| 42 | .023809524 | 97 | .010309278 | 152 | .006578947 | 207 | .004830918 |
| 43 | .023255814 | 98 | .010204082 | 153 | .006535948 | 208 | .004807692 |
| 44 | .022727273 | 99 | .010101010 | 154 | .006493506 | 209 | .004784689 |
| 45 | .022222222 | 100 | .010000000 | 155 | .006451613 | 210 | .004761905 |
| 46 | .021739130 | 101 | .009900990 | 156 | .006410256 | 211 | .004739336 |
| 47 | .021276600 | 102 | .009803922 | 157 | .006369427 | 212 | .004716981 |
| 48 | .020833333 | 103 | .009708738 | 158 | .006329114 | 213 | .004694836 |
| 49 | .020408163 | 104 | .009615385 | 159 | .006289308 | 214 | .004672897 |
| 50 | .020000000 | 105 | .009523810 | 160 | .006250000 | 215 | .004651163 |
| 51 | .019607843 | 106 | .009433962 | 161 | .006211180 | 216 | .004629630 |
| 52 | .019230769 | 107 | .009345794 | 162 | .006172840 | 217 | .004608295 |
| 53 | .018867925 | 108 | .009259259 | 163 | .006134969 | 218 | .004587156 |
| 54 | .018518519 | 109 | .009174312 | 164 | .006097561 | 219 | .004566210 |
| 55 | .018181818 | 110 | .009090909 | 165 | .006060606 | 220 | .004545455 |

**Tabla de números inversos.** (Véanse las explicaciones después de las tablas.

| No. | Inverso. | No. | Inverso. | No. | Inverso. | No. | Inverso. |
|---|---|---|---|---|---|---|---|
| 221 | .004524887 | 276 | .003623188 | 331 | .003021148 | 386 | .002590674 |
| 222 | .004504505 | 277 | .003610108 | 332 | .003012048 | 387 | .002583979 |
| 223 | .004484305 | 278 | .003597122 | 333 | .003003003 | 388 | .002577320 |
| 224 | .004464286 | 279 | .003584229 | 334 | .002994012 | 389 | .002570694 |
| 225 | .004444444 | 280 | .003571429 | 335 | .002985075 | 390 | .002564103 |
| 226 | .004424779 | 281 | .003558719 | 336 | .002976190 | 391 | .002557545 |
| 227 | .004405286 | 282 | .003546099 | 337 | .002967359 | 392 | .002551020 |
| 228 | .004385965 | 283 | .003533569 | 338 | .002958580 | 393 | .002544529 |
| 229 | .004366812 | 284 | .003521127 | 339 | .002949853 | 394 | .002538071 |
| 230 | .004347826 | 285 | .003508772 | 340 | .002941176 | 395 | .002531646 |
| 231 | .004329004 | 286 | .003496503 | 341 | .002932551 | 396 | .002525253 |
| 232 | .004310345 | 287 | .003484321 | 342 | .002923977 | 397 | .002518892 |
| 233 | .004291845 | 288 | .003472222 | 343 | .002915452 | 398 | .002512563 |
| 234 | .004273504 | 289 | .003460208 | 344 | .002906977 | 399 | .002506266 |
| 235 | .004255319 | 290 | .003448276 | 345 | .002898551 | 400 | .002500000 |
| 236 | .004237288 | 291 | .003436426 | 346 | .002890173 | 401 | .002493766 |
| 237 | .004219409 | 292 | .003424658 | 347 | .002881844 | 402 | .002487562 |
| 238 | .004201681 | 293 | .003412969 | 348 | .002873563 | 403 | .002481390 |
| 239 | .004184100 | 294 | .003401361 | 349 | .002865330 | 404 | .002475248 |
| 240 | .004166667 | 295 | .003389831 | 350 | .002857143 | 405 | .002469136 |
| 241 | .004149378 | 296 | .003378378 | 351 | .002849003 | 406 | .002463054 |
| 242 | .004132231 | 297 | .003367003 | 352 | .002840909 | 407 | .002457002 |
| 243 | .004115226 | 298 | .003355705 | 353 | .002832861 | 408 | .002450980 |
| 244 | .004098361 | 299 | .003344482 | 354 | .002824859 | 409 | .002444988 |
| 245 | .004081633 | 300 | .003333333 | 355 | .002816901 | 410 | .002439024 |
| 246 | .004065041 | 301 | .003322259 | 356 | .002808989 | 411 | .002433090 |
| 247 | .004048583 | 302 | .003311258 | 357 | .002801120 | 412 | .002427184 |
| 248 | .004032258 | 303 | .003300330 | 358 | .002793296 | 413 | .002421308 |
| 249 | .004016064 | 304 | .003289474 | 359 | .002785515 | 414 | .002415459 |
| 250 | .004000000 | 305 | .003278689 | 360 | .002777778 | 415 | .002409639 |
| 251 | .003984064 | 306 | .003267974 | 361 | .002770083 | 416 | .002403846 |
| 252 | .003968254 | 307 | .003257329 | 362 | .002762431 | 417 | .002398082 |
| 253 | .003952569 | 308 | .003246753 | 363 | .002754821 | 418 | .002392344 |
| 254 | .003937008 | 309 | .003236246 | 364 | .002747253 | 419 | .002386635 |
| 255 | .003921569 | 310 | .003225806 | 365 | .002739726 | 420 | .002380952 |
| 256 | .003906250 | 311 | .003215434 | 366 | .002732240 | 421 | .002375297 |
| 257 | .003891051 | 312 | .003205128 | 367 | .002724796 | 422 | .002369668 |
| 258 | .003875969 | 313 | .003194888 | 368 | .002717391 | 423 | .002364066 |
| 259 | .003861004 | 314 | .003184713 | 369 | .002710027 | 424 | .002358491 |
| 260 | .003846154 | 315 | .003174603 | 370 | .002702703 | 425 | .002352941 |
| 261 | .003831418 | 316 | .003164557 | 371 | .002695418 | 426 | .002347418 |
| 262 | .003816794 | 317 | .003154574 | 372 | .002688172 | 427 | .002341920 |
| 263 | .003802281 | 318 | .003144654 | 373 | .002680965 | 428 | .002336449 |
| 264 | .003787879 | 319 | .003134796 | 374 | .002673797 | 429 | .002331002 |
| 265 | .003773585 | 320 | .003125000 | 375 | .002666667 | 430 | .002325581 |
| 266 | .003759398 | 321 | .003115265 | 376 | .002659574 | 431 | .002320186 |
| 267 | .003745318 | 322 | .003105590 | 377 | .002652520 | 432 | .002314815 |
| 268 | .003731343 | 323 | .003095975 | 378 | .002645503 | 433 | .002309469 |
| 269 | .003717472 | 324 | .003086420 | 379 | .002638522 | 434 | .002304147 |
| 270 | .003703704 | 325 | .003076923 | 380 | .002631579 | 435 | .002298851 |
| 271 | .003690037 | 326 | .003067485 | 381 | .002624672 | 436 | .002293578 |
| 272 | .003676471 | 327 | .003058104 | 382 | .002617801 | 437 | .002288330 |
| 273 | .003663004 | 328 | .003048780 | 383 | .002610966 | 438 | .002283105 |
| 274 | .003649635 | 329 | .003039514 | 384 | .002604167 | 439 | .002277904 |
| 275 | .003636364 | 330 | .003030303 | 385 | .002597403 | 440 | .002272727 |

**Tabla de números inversos.** (Véanse las explicaciones después de las tablas.

| No. | Inverso. | No. | Inverso. | No. | Inverso. | No. | Inverso. |
|---|---|---|---|---|---|---|---|
| 441 | .002267574 | 496 | .002016129 | 551 | .001814882 | 606 | .001650165 |
| 442 | .002262443 | 497 | .002012072 | 552 | .001811594 | 607 | .001647446 |
| 443 | .002257336 | 498 | .002008032 | 553 | .001808318 | 608 | .001644737 |
| 444 | .002252252 | 499 | .002004008 | 554 | .001805054 | 609 | .001642036 |
| 445 | .002247191 | 500 | .002000000 | 555 | .001801802 | 610 | .001639344 |
| 446 | .002242152 | 501 | .001996008 | 556 | .001798561 | 611 | .001636661 |
| 447 | .002237136 | 502 | .001992032 | 557 | .001795332 | 612 | .001633987 |
| 448 | .002232143 | 503 | .001988072 | 558 | .001792115 | 613 | .001631321 |
| 449 | .002227171 | 504 | .001984127 | 559 | .001788909 | 614 | .001628664 |
| 450 | .002222222 | 505 | .001980198 | 560 | .001785714 | 615 | .001626016 |
| 451 | .002217295 | 506 | .001976285 | 561 | .001782531 | 616 | .001623377 |
| 452 | .002212389 | 507 | .001972387 | 562 | .001779359 | 617 | .001620746 |
| 453 | .002207506 | 508 | .001968504 | 563 | .001776199 | 618 | .001618123 |
| 454 | .002202643 | 509 | .001964637 | 564 | .001773050 | 619 | .001615509 |
| 455 | .002197802 | 510 | .001960784 | 565 | .001769912 | 620 | .001612903 |
| 456 | .002192982 | 511 | .001956947 | 566 | .001766784 | 621 | .001610306 |
| 457 | .002188184 | 512 | .001953125 | 567 | .001763668 | 622 | .001607717 |
| 458 | .002183406 | 513 | .001949318 | 568 | .001760563 | 623 | .001605136 |
| 459 | .002178649 | 514 | .001945525 | 569 | .001757469 | 624 | .001602564 |
| 460 | .002173913 | 515 | .001941748 | 570 | .001754386 | 625 | .001600000 |
| 461 | .002169197 | 516 | .001937984 | 571 | .001751313 | 626 | .001597444 |
| 462 | .002164502 | 517 | .001934236 | 572 | .001748252 | 627 | .001594896 |
| 463 | .002159827 | 518 | .001930502 | 573 | .001745201 | 628 | .001592357 |
| 464 | .002155172 | 519 | .001926782 | 574 | .001742160 | 629 | .001589825 |
| 465 | .002150538 | 520 | .001923077 | 575 | .001739130 | 630 | .001587302 |
| 466 | .002145923 | 521 | .001919386 | 576 | .001736111 | 631 | .001584786 |
| 467 | .002141328 | 522 | .001915709 | 577 | .001733102 | 632 | .001582278 |
| 468 | .002136752 | 523 | .001912046 | 578 | .001730104 | 633 | .001579779 |
| 469 | .002132196 | 524 | .001908397 | 579 | .001727116 | 634 | .001577287 |
| 470 | .002127660 | 525 | .001904762 | 580 | .001724138 | 635 | .001574803 |
| 471 | .002123142 | 526 | .001901141 | 581 | .001721170 | 636 | .001572327 |
| 472 | .002118644 | 527 | .001897533 | 582 | .001718213 | 637 | .001569859 |
| 473 | .002114165 | 528 | .001893939 | 583 | .001715266 | 638 | .001567398 |
| 474 | .002109705 | 529 | .001890359 | 584 | .001712329 | 639 | .001564945 |
| 475 | .002105263 | 530 | .001886792 | 585 | .001709402 | 640 | .001562500 |
| 476 | .002100840 | 531 | .001883239 | 586 | .001706485 | 641 | .001560062 |
| 477 | .002096436 | 532 | .001879699 | 587 | .001703578 | 642 | .001557632 |
| 478 | .002092050 | 533 | .001876173 | 588 | .001700680 | 643 | .001555210 |
| 479 | .002087683 | 534 | .001872659 | 589 | .001697793 | 644 | .001552795 |
| 480 | .002083333 | 535 | .001869159 | 590 | .001694915 | 645 | .001550388 |
| 481 | .002079002 | 536 | .001865672 | 591 | .001692047 | 646 | .001547988 |
| 482 | .002074689 | 537 | .001862197 | 592 | .001689189 | 647 | .001545595 |
| 483 | .002070393 | 538 | .001858736 | 593 | .001686341 | 648 | .001543210 |
| 484 | .002066116 | 539 | .001855288 | 594 | .001683502 | 649 | .001540832 |
| 485 | .002061856 | 540 | .001851852 | 595 | .001680672 | 650 | .001538462 |
| 486 | .002057613 | 541 | .001848429 | 596 | .001677852 | 651 | .001536098 |
| 487 | .002053388 | 542 | .001845018 | 597 | .001675042 | 652 | .001533742 |
| 488 | .002049180 | 543 | .001841621 | 598 | .001672241 | 653 | .001531394 |
| 489 | .002044990 | 544 | .001838235 | 599 | .001669449 | 654 | .001529052 |
| 490 | .002040816 | 545 | .001834862 | 600 | .001666667 | 655 | .001526718 |
| 491 | .002036660 | 546 | .001831502 | 601 | .001663894 | 656 | .001524390 |
| 492 | .002032520 | 547 | .001828154 | 602 | .001661130 | 657 | .001522070 |
| 493 | .002028398 | 548 | .001824818 | 603 | .001658375 | 658 | .001519757 |
| 494 | .002024291 | 549 | .001821494 | 604 | .001655629 | 659 | .001517451 |
| 495 | .002020202 | 550 | .001818182 | 605 | .001652893 | 660 | .001515152 |

**Tabla de números inversos.** (Véanse las explicaciones después de las tablas.)

| No. | Inverso. | No. | Inverso. | No. | Inverso. | No. | Inverso. |
|---|---|---|---|---|---|---|---|
| 661 | .001512859 | 716 | .001396648 | 771 | .001297017 | 826 | .001210654 |
| 662 | .001510574 | 717 | .001394700 | 772 | .001295337 | 827 | .001209190 |
| 663 | .001508296 | 718 | .001392758 | 773 | .001293661 | 828 | .001207729 |
| 664 | .001506024 | 719 | .001390821 | 774 | .001291990 | 829 | .001206273 |
| 665 | .001503759 | 720 | .001388889 | 775 | .001290323 | 830 | .001204819 |
| 666 | .001501502 | 721 | .001386963 | 776 | .001288660 | 831 | .001203369 |
| 667 | .001499250 | 722 | .001385042 | 777 | .001287001 | 832 | .001201923 |
| 668 | .001497006 | 723 | .001383126 | 778 | .001285347 | 833 | .001200480 |
| 669 | .001494768 | 724 | .001381215 | 779 | .001283697 | 834 | .001199041 |
| 670 | .001492537 | 725 | .001379310 | 780 | .001282051 | 835 | .001197605 |
| 671 | .001490313 | 726 | .001377410 | 781 | .001280410 | 836 | .001196172 |
| 672 | .001488095 | 727 | .001375516 | 782 | .001278772 | 837 | .001194743 |
| 673 | .001485884 | 728 | .001373626 | 783 | .001277139 | 838 | .001193317 |
| 674 | .001483680 | 729 | .001371742 | 784 | .001275510 | 839 | .001191895 |
| 675 | .001481481 | 730 | .001369863 | 785 | .001273885 | 840 | .001190476 |
| 676 | .001479290 | 731 | .001367989 | 786 | .001272265 | 841 | .001189061 |
| 677 | .001477105 | 732 | .001366120 | 787 | .001270648 | 842 | .001187648 |
| 678 | .001474926 | 733 | .001364256 | 788 | .001269036 | 843 | .001186240 |
| 679 | .001472754 | 734 | .001362398 | 789 | .001267427 | 844 | .001184834 |
| 680 | .001470588 | 735 | .001360544 | 790 | .001265823 | 845 | .001183432 |
| 681 | .001468429 | 736 | .001358696 | 791 | .001264223 | 846 | .001182033 |
| 682 | .001466276 | 737 | .001356852 | 792 | .001262626 | 847 | .001180688 |
| 683 | .001464129 | 738 | .001355014 | 793 | .001261034 | 848 | .001179245 |
| 684 | .001461988 | 739 | .001353180 | 794 | .001259446 | 849 | .001177856 |
| 685 | .001459854 | 740 | .001351351 | 795 | .001257862 | 850 | .001176471 |
| 686 | .001457726 | 741 | .001349528 | 796 | .001256281 | 851 | .001175088 |
| 687 | .001455604 | 742 | .001347709 | 797 | .001254705 | 852 | .001173709 |
| 688 | .001453488 | 743 | .001345895 | 798 | .001253133 | 853 | .001172333 |
| 689 | .001451379 | 744 | .001344086 | 799 | .001251564 | 854 | .001170960 |
| 690 | .001449275 | 745 | .001342282 | 800 | .001250000 | 855 | .001169591 |
| 691 | .001447178 | 746 | .001340483 | 801 | .001248439 | 856 | .001168224 |
| 692 | .001445087 | 747 | .001338688 | 802 | .001246883 | 857 | .001166861 |
| 693 | .001443001 | 748 | .001336898 | 803 | .001245330 | 858 | .001165501 |
| 694 | .001440922 | 749 | .001335113 | 804 | .001243781 | 859 | .001164144 |
| 695 | .001438849 | 750 | .001333333 | 805 | .001242236 | 860 | .001162791 |
| 696 | .001436782 | 751 | .001331558 | 806 | .001240695 | 861 | .001161440 |
| 697 | .001434720 | 752 | .001329787 | 807 | .001239157 | 862 | .001160093 |
| 698 | .001432665 | 753 | .001328021 | 808 | .001237624 | 863 | .001158749 |
| 699 | .001430615 | 754 | .001326260 | 809 | .001236094 | 864 | .001157407 |
| 700 | .001428571 | 755 | .001324503 | 810 | .001234568 | 865 | .001156069 |
| 701 | .001426534 | 756 | .001322751 | 811 | .001233046 | 866 | .001154734 |
| 702 | .001424501 | 757 | .001321004 | 812 | .001231527 | 867 | .001153403 |
| 703 | .001422475 | 758 | .001319261 | 813 | .001230012 | 868 | .001152074 |
| 704 | .001420455 | 759 | .001317523 | 814 | .001228501 | 869 | .001150748 |
| 705 | .001418440 | 760 | .001315789 | 815 | .001226994 | 870 | .001149425 |
| 706 | .001416431 | 761 | .001314060 | 816 | .001225490 | 871 | .001148106 |
| 707 | .001414427 | 762 | .001312336 | 817 | .001223990 | 872 | .001146789 |
| 708 | .001412429 | 763 | .001310616 | 818 | .001222494 | 873 | .001145475 |
| 709 | .001410437 | 764 | .001308901 | 819 | .001221001 | 874 | .001144165 |
| 710 | .001408451 | 765 | .001307190 | 820 | .001219512 | 875 | .001142857 |
| 711 | .001406470 | 766 | .001305483 | 821 | .001218027 | 876 | .001141553 |
| 712 | .001404494 | 767 | .001303781 | 822 | .001216545 | 877 | .001140251 |
| 713 | .001402525 | 768 | .001302083 | 823 | .001215067 | 878 | .001138952 |
| 714 | .001400560 | 769 | .001300390 | 824 | .001213592 | 879 | .001137656 |
| 715 | .001398601 | 770 | .001298701 | 825 | .001212121 | 880 | .001136364 |

**Tabla de números inversos.** (Véanse las explicaciones después de las tablas.)

| No. | Inverso. | No. | Inverso. | No. | Inverso. | No. | Inverso. |
|---|---|---|---|---|---|---|---|
| 881 | .001135074 | 911 | .001097695 | 941 | .001062699 | 971 | .001029866 |
| 882 | .001133787 | 912 | .001096491 | 942 | .001061571 | 972 | .001028807 |
| 883 | .001132503 | 913 | .001095290 | 943 | .001060445 | 973 | .001027749 |
| 884 | .001131222 | 914 | .001094092 | 944 | .001059322 | 974 | .001026694 |
| 885 | .001129944 | 915 | .001092896 | 945 | .001058201 | 975 | .001025641 |
| 886 | .001128668 | 916 | .001091703 | 946 | .001057082 | 976 | .001024590 |
| 887 | .001127396 | 917 | .001090513 | 947 | .001055966 | 977 | .001023541 |
| 888 | .001126126 | 918 | .001089325 | 948 | .001054852 | 978 | .001022495 |
| 889 | .001124859 | 919 | .001088139 | 949 | .001053741 | 979 | .001021450 |
| 890 | .001123596 | 920 | .001086957 | 950 | .001052632 | 980 | .001020408 |
| 891 | .001122334 | 921 | .001085776 | 951 | .001051525 | 981 | .001019368 |
| 892 | .001121076 | 922 | .001084599 | 952 | .001050420 | 982 | .001018330 |
| 893 | .001119821 | 923 | .001083424 | 953 | .001049318 | 983 | .001017294 |
| 894 | .001118568 | 924 | .001082251 | 954 | .001048218 | 984 | .001016260 |
| 895 | .001117318 | 925 | .001081081 | 955 | .001047120 | 985 | .001015228 |
| 896 | .001116071 | 926 | .001079914 | 956 | .001046025 | 986 | .001014199 |
| 897 | .001114827 | 927 | .001078749 | 957 | .001044932 | 987 | .001013171 |
| 898 | .001113586 | 928 | .001077586 | 958 | .001043841 | 988 | .001012146 |
| 899 | .001112347 | 929 | .001076426 | 959 | .001042753 | 989 | .001011122 |
| 900 | .001111111 | 930 | .001075269 | 960 | .001041667 | 990 | .001010101 |
| 901 | .001109878 | 931 | .001074114 | 961 | .001040583 | 991 | .001009082 |
| 902 | .001108647 | 932 | .001072961 | 962 | .001039501 | 992 | .001008065 |
| 903 | .001107420 | 933 | .001071811 | 963 | .001038422 | 993 | .001007049 |
| 904 | .001106195 | 934 | .001070664 | 964 | .001037344 | 994 | .001006036 |
| 905 | .001104972 | 935 | .001069519 | 965 | .001036269 | 995 | .001005025 |
| 906 | .001103753 | 936 | .001068376 | 966 | .001035197 | 996 | .001004016 |
| 907 | .001102536 | 937 | .001067236 | 967 | .001034126 | 997 | .001003009 |
| 908 | .001101322 | 938 | .001066098 | 968 | .001033058 | 998 | .001002004 |
| 909 | .001100110 | 939 | .001064963 | 969 | .001031992 | 999 | .001001001 |
| 910 | .001098901 | 940 | .001063830 | 970 | .001030928 | 1000 | .001000000 |

## NÚMEROS INVERSOS

(a) **El inverso de un número es** el número que resulta de dividir la unidad por el número de que se trata. Si $n$ es un número cualquiera, el inverso de $n = \frac{1}{n}$. Así, el inverso de $40 = \frac{1}{40} = 0.025$; inverso de $0.4 = \frac{1}{0.4} = 2.5$, etc.

Por consiguiente inver de $\frac{a}{b} = \frac{b}{a}$ porque el inver de $\frac{a}{b} = 1 \div \frac{a}{b} = 1 \times \frac{b}{a} = \frac{b}{a}$.

Como una yarda es=36 pulgadas, 1 pulgada es $= \frac{1}{36}$ de yarda=0.027777778 de yarda, por tanto el inver de 36 es=0.027777778. Asimismo una columna de agua de un pie da una presión de 0.4335 libras por pulgada cuadrada. Por consiguiente una presión de una libra por pulgada cuadrada corresponde á una columna de $\frac{1}{0.4335}$ pies=2.306805. El inver de 0.4335 es 2.306805. (Véase **h** más abajo.)

(b) Se deduce que si cualquier número en la columna encabezada « No. » es el denominador de una fracción cuyo numerador es 1, el inver correspondiente es el valor de esa fracción expresado en decimales *. Así $\frac{1}{32} = 0.03125$. Por consi-

* Los números 2 y 5 y sus potencias y productos son los únicos cuyos inver pueden expresarse en decimales exactas.

guiente **para reducir una fracción ordinaria á decimal** : multiplíquese el inver del denominador por el numerador. Así $\frac{17}{32} = 0.53125$ porque el recíproco de 32 es=0.03125 y 0.03125×17 es=0.53125.

(c) Viceversa, si el inver de un número $n$ se toma como número, el número $n$ mismo viene á ser el inver. De otro modo : el inver de $\frac{1}{n} = n$. Así, el inver de $0.205 = \frac{1}{40}$ es 40; el inver de $2.5 = \frac{1}{0.4}$ es 0.4, etc., etc.

(d) El producto de cualquier número por su inver es igual á la unidad, es decir $n \times \frac{1}{n} = \frac{n}{n} = 1$.

(e) Por consiguiente **para evitarnos el trabajo de dividir** podemos multiplicar el dividendo por el inver del divisor, puesto que $\frac{a}{n} = a \times \frac{1}{n}$.

Ej. : 200÷48750=200×inver de 48750=200×0.00002051282 (véase **h** más abajo)=0.004102564.....

(f) Cualquier número $a \div$ por el inver de un número, $n$ es $= a \div \frac{1}{n} = an$. Por consiguiente $a \div$ inver de $a$ es $= a \div \frac{1}{a} = a \times \frac{a}{1} = a^2$. Así el inver de 2 = 0.5 y $\frac{2}{\text{inver } 2} = \frac{2}{0.5} = 4 = 2^2$.

(g) Los números en la tabla anterior se extienden desde 1 hasta 1,000, pero los inver **de los múltiples de estos números por 10** pueden deducirse de la tabla agregando un cero entre la coma y la primera cifra decimal por cada cero agregado al número. Así :

Inver 390=0.002564103
— 3,900=0.0002564103.
— 39,000=0.00002564103.

Y los inver de los **números que contengan decimales** pueden deducirse de la tabla corriendo la coma en el inver de la tabla un lugar hacia la derecha por cada cifra decimal que contenga el número. Así :

Inver 227 = 0.004405286
— 22.7 = 0.04405286
— 2.27 = 0.4405286
— 0.227 = 4.405286
— 0.0227=44.05286

(h) El inver **de un número de más de tres cifras** puede tomarse de la tabla aproximadamente por interpolación. Así para encontrar el inver de 236.4

Inver de 236=0.004237288
— de 237=0.004219409
Diferencias : 1, 0.000017879; 236.4 — 236=0.4.
Luego 0.4×0.000017879=0.000007152
el inver de 236=0.004237288
menos 0.000007152
=Inver de 236.4=0.004230136 por interpolación.

El inver exacto es 0.004230118.

(i) Los inversos de los números que no están en la tabla pueden encontrarse **por medio de logaritmos**. Así, para encontrar el inver $= 236.4 = \frac{1}{236.4}$.

Log 1 =0.000000
Réstese el Log 236.4=2.373647
$\bar{3}.626353$=Log 0.00423012
Inver 236.4=$0.0042301^2$

Para encontrar el inver de $\frac{8424}{236.4} = \frac{236.4}{8424}$.

Log 236.4 = 2.373647
Réstese el Log 8424 = 3.925518

$\bar{2}.448129$ = Log de 0.0280627

Inver $\frac{8424}{236.4} = 0.0280627$.

(j) Para los números 10, 100, 1,000, etc., el puesto que ocupa á la derecha de la coma la primera cifra significativa en el número inver correspondiente es igual al número de ceros que contiene el número; pero para todos los otros números es gual al numero de cifras de la parte entera de la fracción. Así inver de 143.7 = = 0.0069....., etc. En este caso el número de cifras en la parte entera (143) es 3 y la primera cifra significativa (6) del inver 0.0069 ocupa el tercer puesto

## Raíces cuadradas y cúbicas de los números de 0.1 á 28.

Sin errores.

| No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. | No. | Raíz cuad. | Raíz cúbica. | No. | Raíz cuad. | Raíz cúbica. |
|---|---|---|---|---|---|---|---|---|---|---|
| .1 | .01 | .001 | .316 | .464 | .7 | 2.387 | 1.786 | .4 | 3.661 | 2.375 |
| .15 | .0225 | .0034 | .387 | .531 | .8 | 2.408 | 1.797 | .6 | 3.688 | 2.387 |
| .2 | .04 | .008 | .447 | .585 | .9 | 2.429 | 1.807 | .8 | 3.715 | 2.399 |
| .25 | .0625 | .0156 | .500 | .630 | 6. | 2.449 | 1.817 | 14. | 3.742 | 2.410 |
| .3 | .09 | .027 | .548 | .669 | .1 | 2.470 | 1.827 | .2 | 3.768 | 2.422 |
| .35 | .1225 | .0429 | .592 | .705 | .2 | 2.490 | 1.837 | .4 | 3.795 | 2.433 |
| .4 | .16 | .064 | .633 | .737 | .3 | 2.510 | 1.847 | .6 | 3.821 | 2.444 |
| .45 | .2025 | .0911 | .671 | .766 | .4 | 2.530 | 1.857 | .8 | 3.847 | 2.455 |
| .5 | .25 | .125 | .707 | .794 | .5 | 2.550 | 1.866 | 15. | 3.873 | 2.466 |
| .55 | .3025 | .1664 | .742 | .819 | .6 | 2.569 | 1.876 | .2 | 3.899 | 2.477 |
| .6 | .36 | .216 | .775 | .843 | .7 | 2.588 | 1.885 | .4 | 3.924 | 2.488 |
| .65 | .4225 | .2746 | .806 | .866 | .8 | 2.608 | 1.895 | .6 | 3.950 | 2.499 |
| .7 | .49 | .343 | .837 | .888 | .9 | 2.627 | 1.904 | .8 | 3.975 | 2.509 |
| .75 | .5625 | .4219 | .866 | .909 | 7. | 2.646 | 1.913 | 16. | 4. | 2.520 |
| .8 | .64 | .512 | .894 | .928 | .1 | 2.665 | 1.922 | .2 | 4.025 | 2.530 |
| .85 | .7225 | .6141 | .922 | .947 | .2 | 2.683 | 1.931 | .4 | 4.050 | 2.541 |
| .9 | .81 | .729 | .949 | .965 | .3 | 2.702 | 1.940 | .6 | 4.074 | 2.551 |
| .95 | .9025 | .8574 | .975 | .983 | .4 | 2.720 | 1.949 | .8 | 4.099 | 2.561 |
| 1. | 1.000 | 1.000 | 1.000 | 1.000 | .5 | 2.739 | 1.957 | 17. | 4.123 | 2.571 |
| .05 | 1.103 | 1.158 | 1.025 | 1.016 | .6 | 2.757 | 1.966 | .2 | 4.147 | 2.581 |
| 1.1 | 1.210 | 1.331 | 1.049 | 1.032 | .7 | 2.775 | 1.975 | .4 | 4.171 | 2.591 |
| .15 | 1.323 | 1.521 | 1.072 | 1.048 | .8 | 2.793 | 1.983 | .6 | 4.195 | 2.601 |
| 1.2 | 1.440 | 1.728 | 1.095 | 1.063 | .9 | 2.811 | 1.992 | .8 | 4.219 | 2.611 |
| .25 | 1.563 | 1.963 | 1.118 | 1.077 | 8. | 2.828 | 2.000 | 18. | 4.243 | 2.621 |
| 1.3 | 1.690 | 2.197 | 1.140 | 1.091 | .1 | 2.846 | 2.008 | .2 | 4.266 | 2.630 |
| .35 | 1.823 | 2.460 | 1.162 | 1.105 | .2 | 2.864 | 2.017 | .4 | 4.290 | 2.640 |
| 1.4 | 1.960 | 2.744 | 1.183 | 1.119 | .3 | 2.881 | 2.025 | .6 | 4.313 | 2.650 |
| .45 | 2.103 | 3.049 | 1.204 | 1.132 | .4 | 2.898 | 2.033 | .8 | 4.336 | 2.659 |
| 1.5 | 2.250 | 3.375 | 1.225 | 1.145 | .5 | 2.915 | 2.041 | 19. | 4.359 | 2.668 |
| .55 | 2.403 | 3.724 | 1.245 | 1.157 | .6 | 2.933 | 2.049 | .2 | 4.382 | 2.678 |
| 1.6 | 2.560 | 4.096 | 1.265 | 1.170 | .7 | 2.950 | 2.057 | .4 | 4.405 | 2.687 |
| .65 | 2.723 | 4.492 | 1.285 | 1.182 | .8 | 2.966 | 2.065 | .6 | 4.427 | 2.696 |
| 1.7 | 2.890 | 4.913 | 1.304 | 1.193 | .9 | 2.983 | 2.072 | .8 | 4.450 | 2.705 |
| .75 | 3.063 | 5.359 | 1.323 | 1.205 | 9. | 3. | 2.080 | 20. | 4.472 | 2.714 |
| 1.8 | 3.240 | 5.832 | 1.342 | 1.216 | .1 | 3.017 | 2.088 | .2 | 4.494 | 2.723 |
| .85 | 3.423 | 6.332 | 1.360 | 1.228 | .2 | 3.033 | 2.095 | .4 | 4.517 | 2.732 |
| 1.9 | 3.610 | 6.859 | 1.378 | 1.239 | .3 | 3.050 | 2.103 | .6 | 4.539 | 2.741 |
| .95 | 3.803 | 7.415 | 1.396 | 1.249 | .4 | 3.066 | 2.110 | .8 | 4.561 | 2.750 |
| 2. | 4.000 | 8.000 | 1.414 | 1.260 | .5 | 3.082 | 2.118 | 21. | 4.583 | 2.759 |
| .1 | 4.410 | 9.261 | 1.449 | 1.281 | .6 | 3.098 | 2.125 | .2 | 4.604 | 2.768 |
| .2 | 4.840 | 10.65 | 1.483 | 1.301 | .7 | 3.114 | 2.133 | .4 | 4.626 | 2.776 |
| .3 | 5.290 | 12.17 | 1.517 | 1.320 | .8 | 3.130 | 2.140 | .6 | 4.648 | 2.785 |
| .4 | 5.760 | 13.82 | 1.549 | 1.339 | .9 | 3.146 | 2.147 | .8 | 4.669 | 2.794 |
| .5 | 6.250 | 15.63 | 1.581 | 1.357 | 10. | 3.162 | 2.154 | 22. | 4.690 | 2.802 |
| .6 | 6.760 | 17.58 | 1.612 | 1.375 | .1 | 3.178 | 2.162 | .2 | 4.712 | 2.810 |
| .7 | 7.290 | 19.68 | 1.643 | 1.392 | .2 | 3.194 | 2.169 | .4 | 4.733 | 2.819 |
| .8 | 7.840 | 21.95 | 1.673 | 1.409 | .3 | 3.209 | 2.176 | .6 | 4.754 | 2.827 |
| .9 | 8.410 | 24.39 | 1.703 | 1.426 | .4 | 3.225 | 2.183 | .8 | 4.775 | 2.836 |
| 3. | 9. | 27. | 1.732 | 1.442 | .5 | 3.240 | 2.190 | 23. | 4.796 | 2.844 |
| .1 | 9.61 | 29.79 | 1.761 | 1.458 | .6 | 3.256 | 2.197 | .2 | 4.817 | 2.852 |
| .2 | 10.24 | 32.77 | 1.789 | 1.474 | .7 | 3.271 | 2.204 | .4 | 4.837 | 2.860 |
| .3 | 10.89 | 35.94 | 1.817 | 1.489 | .8 | 3.286 | 2.210 | .6 | 4.858 | 2.868 |
| .4 | 11.56 | 39.30 | 1.844 | 1.504 | .9 | 3.302 | 2.217 | .8 | 4.879 | 2.876 |
| .5 | 12.25 | 42.88 | 1.871 | 1.518 | 11. | 3.317 | 2.224 | 24. | 4.899 | 2.884 |
| .6 | 12.96 | 46.66 | 1.897 | 1.533 | .1 | 3.332 | 2.231 | .2 | 4.919 | 2.892 |
| .7 | 13.69 | 50.65 | 1.924 | 1.547 | .2 | 3.347 | 2.237 | .4 | 4.940 | 2.900 |
| .8 | 14.44 | 54.87 | 1.949 | 1.560 | .3 | 3.362 | 2.244 | .6 | 4.960 | 2.908 |
| .9 | 15.21 | 59.32 | 1.975 | 1.574 | .4 | 3.376 | 2.251 | .8 | 4.980 | 2.916 |
| 4. | 16. | 64. | 2. | 1.587 | .5 | 3.391 | 2.257 | 25. | 5. | 2.924 |
| .1 | 16.81 | 68.92 | 2.025 | 1.601 | .6 | 3.406 | 2.264 | .2 | 5.020 | 2.932 |
| .2 | 17.64 | 74.09 | 2.049 | 1.618 | .7 | 3.421 | 2.270 | .4 | 5.040 | 2.940 |
| .3 | 18.49 | 79.51 | 2.074 | 1.626 | .8 | 3.435 | 2.277 | .6 | 5.060 | 2.947 |
| .4 | 19.36 | 85.18 | 2.098 | 1.639 | .9 | 3.450 | 2.283 | .8 | 5.079 | 2.955 |
| .5 | 20.25 | 91.13 | 2.121 | 1.651 | 12. | 3.464 | 2.289 | 26. | 5.099 | 2.962 |
| .6 | 21.16 | 97.34 | 2.145 | 1.663 | .1 | 3.479 | 2.296 | .2 | 5.119 | 2.970 |
| .7 | 22.09 | 103.8 | 2.168 | 1.675 | .2 | 3.493 | 2.302 | .4 | 5.138 | 2.978 |
| .8 | 23.04 | 110.6 | 2.191 | 1.687 | .3 | 3.507 | 2.308 | .6 | 5.158 | 2.985 |
| .9 | 24.01 | 117.6 | 2.214 | 1.698 | .4 | 3.521 | 2.315 | .8 | 5.177 | 2.993 |
| 5. | 25. | 125. | 2.236 | 1.710 | .5 | 3.536 | 2.321 | 27. | 5.196 | 3.000 |
| .1 | 26.01 | 132.7 | 2.258 | 1.721 | .6 | 3.550 | 2.327 | .2 | 5.215 | 3.007 |
| .2 | 27.04 | 140.6 | 2.280 | 1.732 | .7 | 3.564 | 2.333 | .4 | 5.235 | 3.015 |
| .3 | 28.09 | 148.9 | 2.302 | 1.744 | .8 | 3.578 | 2.339 | .6 | 5.254 | 3.022 |
| .4 | 29.16 | 157.5 | 2.324 | 1.754 | .9 | 3.592 | 2.345 | .8 | 5.273 | 3.029 |
| .5 | 30.25 | 166.4 | 2.345 | 1.765 | 13. | 3.606 | 2.351 | 28. | 5.292 | 3.037 |
| .6 | 31.36 | 175.6 | 2.366 | 1.776 | .2 | 3.633 | 2.363 | .2 | 5.310 | 3.044 |

## Tabla de los cuadrados, cubos, raíces cuadradas y cúbicas de los números de 1 á 1,000.

Observación á la tabla siguiente: cada vez que, por el valor de la quinta cifra decimal, ha sido necesario aumentarle una unidad á la cuarta, se ha hecho así.

Sin errores.

| No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. | No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1.0000 | 1.0000 | 61 | 3721 | 226981 | 7.8102 | 3.9365 |
| 2 | 4 | 8 | 1.4142 | 1.2599 | 62 | 3844 | 238328 | 7.8740 | 3.9579 |
| 3 | 9 | 27 | 1.7321 | 1.4422 | 63 | 3969 | 250047 | 7.9373 | 3.9791 |
| 4 | 16 | 64 | 2.0000 | 1.5874 | 64 | 4096 | 262144 | 8.0000 | 4. |
| 5 | 25 | 125 | 2.2361 | 1.7100 | 65 | 4225 | 274625 | 8.0623 | 4.0207 |
| 6 | 36 | 216 | 2.4495 | 1.8171 | 66 | 4356 | 287496 | 8.1240 | 4.0412 |
| 7 | 49 | 343 | 2.6458 | 1.9129 | 67 | 4489 | 300763 | 8.1854 | 4.0615 |
| 8 | 64 | 512 | 2.8284 | 2.0000 | 68 | 4624 | 314432 | 8.2462 | 4.0817 |
| 9 | 81 | 729 | 3.0000 | 2.0801 | 69 | 4761 | 328509 | 8.3066 | 4.1016 |
| 10 | 100 | 1000 | 3.1623 | 2.1544 | 70 | 4900 | 343000 | 8.3666 | 4.1213 |
| 11 | 121 | 1331 | 3.3166 | 2.2240 | 71 | 5041 | 357911 | 8.4261 | 4.1408 |
| 12 | 144 | 1728 | 3.4641 | 2.2894 | 72 | 5184 | 373248 | 8.4853 | 4.1602 |
| 13 | 169 | 2197 | 3.6056 | 2.3513 | 73 | 5329 | 389017 | 8.5440 | 4.1798 |
| 14 | 196 | 2744 | 3.7417 | 2.4101 | 74 | 5476 | 405224 | 8.6023 | 4.1983 |
| 15 | 225 | 3375 | 3.8730 | 2.4662 | 75 | 5625 | 421875 | 8.6603 | 4.2172 |
| 16 | 256 | 4096 | 4.0000 | 2.5198 | 76 | 5776 | 438976 | 8.7178 | 4.2358 |
| 17 | 289 | 4913 | 4.1231 | 2.5713 | 77 | 5929 | 456533 | 8.7750 | 4.2543 |
| 18 | 324 | 5832 | 4.2426 | 2.6207 | 78 | 6084 | 474552 | 8.8318 | 4.2727 |
| 19 | 361 | 6859 | 4.3589 | 2.6684 | 79 | 6241 | 493039 | 8.8882 | 4.2908 |
| 20 | 400 | 8000 | 4.4721 | 2.7144 | 80 | 6400 | 512000 | 8.9443 | 4.3089 |
| 21 | 441 | 9261 | 4.5826 | 2.7589 | 81 | 6561 | 531441 | 9. | 4.3267 |
| 22 | 484 | 10648 | 4.6904 | 2.8020 | 82 | 6724 | 551368 | 9.0554 | 4.3445 |
| 23 | 529 | 12167 | 4.7958 | 2.8439 | 83 | 6889 | 571787 | 9.1104 | 4.3621 |
| 24 | 576 | 13824 | 4.8990 | 2.8845 | 84 | 7056 | 592704 | 9.1652 | 4.3795 |
| 25 | 625 | 15625 | 5.0000 | 2.9240 | 85 | 7225 | 614125 | 9.2195 | 4.3968 |
| 26 | 676 | 17576 | 5.0990 | 2.9625 | 86 | 7396 | 636056 | 9.2736 | 4.4140 |
| 27 | 729 | 19683 | 5.1962 | 3.0000 | 87 | 7569 | 658503 | 9.3274 | 4.4310 |
| 28 | 784 | 21952 | 5.2915 | 3.0366 | 88 | 7744 | 681472 | 9.3808 | 4.4480 |
| 29 | 841 | 24389 | 5.3852 | 3.0723 | 89 | 7921 | 704969 | 9.4340 | 4.4647 |
| 30 | 900 | 27000 | 5.4772 | 3.1072 | 90 | 8100 | 729000 | 9.4868 | 4.4814 |
| 31 | 961 | 29791 | 5.5678 | 3.1414 | 91 | 8281 | 753571 | 9.5394 | 4.4979 |
| 32 | 1024 | 32768 | 5.6569 | 3.1748 | 92 | 8464 | 778688 | 9.5917 | 4.5144 |
| 33 | 1089 | 35937 | 5.7446 | 3.2075 | 93 | 8649 | 804357 | 9.6437 | 4.5307 |
| 34 | 1156 | 39304 | 5.8310 | 3.2396 | 94 | 8836 | 830584 | 9.6954 | 4.5468 |
| 35 | 1225 | 42875 | 5.9161 | 3.2711 | 95 | 9025 | 857375 | 9.7468 | 4.5629 |
| 36 | 1296 | 46656 | 6.0000 | 3.3019 | 96 | 9216 | 884736 | 9.7980 | 4.5789 |
| 37 | 1369 | 50653 | 6.0828 | 3.3322 | 97 | 9409 | 912673 | 9.8489 | 4.5947 |
| 38 | 1444 | 54872 | 6.1644 | 3.3620 | 98 | 9604 | 941192 | 9.8995 | 4.6104 |
| 39 | 1521 | 59319 | 6.2450 | 3.3912 | 99 | 9801 | 970299 | 9.9499 | 4.6261 |
| 40 | 1600 | 64000 | 6.3246 | 3.4200 | 100 | 10000 | 1000000 | 10. | 4.6416 |
| 41 | 1681 | 68921 | 6.4031 | 3.4482 | 101 | 10201 | 1030301 | 10.0499 | 4.6570 |
| 42 | 1764 | 74088 | 6.4807 | 3.4760 | 102 | 10404 | 1061208 | 10.0995 | 4.6723 |
| 43 | 1849 | 79507 | 6.5574 | 3.5034 | 103 | 10609 | 1092727 | 10.1489 | 4.6875 |
| 44 | 1936 | 85184 | 6.6332 | 3.5303 | 104 | 10816 | 1124864 | 10.1980 | 4.7027 |
| 45 | 2025 | 91125 | 6.7082 | 3.5569 | 105 | 11025 | 1157625 | 10.2470 | 4.7177 |
| 46 | 2116 | 97336 | 6.7823 | 3.5830 | 106 | 11236 | 1191016 | 10.2956 | 4.7326 |
| 47 | 2209 | 103823 | 6.8557 | 3.6088 | 107 | 11449 | 1225043 | 10.3441 | 4.7475 |
| 48 | 2304 | 110592 | 6.9282 | 3.6342 | 108 | 11664 | 1259712 | 10.3923 | 4.7622 |
| 49 | 2401 | 117649 | 7.0000 | 3.6593 | 109 | 11881 | 1295029 | 10.4403 | 4.7769 |
| 50 | 2500 | 125000 | 7.0711 | 3.6840 | 110 | 12100 | 1331000 | 10.4881 | 4.7914 |
| 51 | 2601 | 132651 | 7.1414 | 3.7084 | 111 | 12321 | 1367631 | 10.5357 | 4.8059 |
| 52 | 2704 | 140608 | 7.2111 | 3.7325 | 112 | 12544 | 1404928 | 10.5830 | 4.8203 |
| 53 | 2809 | 148877 | 7.2801 | 3.7563 | 113 | 12769 | 1442897 | 10.6301 | 4.8346 |
| 54 | 2916 | 157464 | 7.3485 | 3.7798 | 114 | 12996 | 1481544 | 10.6771 | 4.8488 |
| 55 | 3025 | 166375 | 7.4162 | 3.8030 | 115 | 13225 | 1520875 | 10.7238 | 4.8629 |
| 56 | 3136 | 175616 | 7.4833 | 3.8259 | 116 | 13456 | 1560896 | 10.7703 | 4.8770 |
| 57 | 3249 | 185193 | 7.5498 | 3.8485 | 117 | 13689 | 1601613 | 10.8167 | 4.8910 |
| 58 | 3364 | 195112 | 7.6158 | 3.8709 | 118 | 13924 | 1643032 | 10.8628 | 4.9049 |
| 59 | 3481 | 205379 | 7.6811 | 3.8930 | 119 | 14161 | 1685159 | 10.9087 | 4.9187 |
| 60 | 3600 | 216000 | 7.7460 | 3.9149 | 120 | 14400 | 1728000 | 10.9545 | 4.9324 |

## Tabla de los cuadrados, cubos, raíces cuadradas y cúbicas de los números de 1 á 1,000.

| No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. | No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. |
|---|---|---|---|---|---|---|---|---|---|
| 121 | 14641 | 1771561 | 11. | 4.9461 | 186 | 34596 | 6434856 | 13.6382 | 5.7083 |
| 122 | 14884 | 1815848 | 11.0454 | 4.9597 | 187 | 34969 | 6539203 | 13.6748 | 5.7185 |
| 123 | 15129 | 1860867 | 11.0905 | 4.9732 | 188 | 35344 | 6644672 | 13.7113 | 5.7287 |
| 124 | 15376 | 1906624 | 11.1355 | 4.9866 | 189 | 35721 | 6751269 | 13.7477 | 5.7388 |
| 125 | 15625 | 1953125 | 11.1803 | 5. | 190 | 36100 | 6859000 | 13.7840 | 5.7489 |
| 126 | 15876 | 2000376 | 11.2250 | 5.0133 | 191 | 36481 | 6967871 | 13.8203 | 5.7590 |
| 127 | 16129 | 2048383 | 11.2694 | 5.0265 | 192 | 36864 | 7077888 | 13.8564 | 5.7690 |
| 128 | 16384 | 2097152 | 11.3137 | 5.0397 | 193 | 37249 | 7189057 | 13.8924 | 5.7790 |
| 129 | 16641 | 2146689 | 11.3578 | 5.0528 | 194 | 37636 | 7301384 | 13.9284 | 5.7890 |
| 130 | 16900 | 2197000 | 11.4018 | 5.0658 | 195 | 38025 | 7414875 | 13.9642 | 5.7989 |
| 131 | 17161 | 2248091 | 11.4455 | 5.0788 | 196 | 38416 | 7529536 | 14. | 5.8088 |
| 132 | 17424 | 2299968 | 11.4891 | 5.0916 | 197 | 38809 | 7645373 | 14.0357 | 5.8186 |
| 133 | 17689 | 2352637 | 11.5326 | 5.1045 | 198 | 39204 | 7762392 | 14.0712 | 5.8285 |
| 134 | 17956 | 2406104 | 11.5758 | 5.1172 | 199 | 39601 | 7880599 | 14.1067 | 5.8383 |
| 135 | 18225 | 2460375 | 11.6190 | 5.1299 | 200 | 40000 | 8000000 | 14.1421 | 5.8480 |
| 136 | 18496 | 2515456 | 11.6619 | 5.1426 | 201 | 40401 | 8120601 | 14.1774 | 5.8578 |
| 137 | 18769 | 2571353 | 11.7047 | 5.1551 | 202 | 40804 | 8242408 | 14.2127 | 5.8675 |
| 138 | 19044 | 2628072 | 11.7473 | 5.1676 | 203 | 41209 | 8365427 | 14.2478 | 5.8771 |
| 139 | 19321 | 2685619 | 11.7898 | 5.1801 | 204 | 41616 | 8489664 | 14.2829 | 5.8868 |
| 140 | 19600 | 2744000 | 11.8322 | 5.1925 | 205 | 42025 | 8615125 | 14.3178 | 5.8964 |
| 141 | 19881 | 2803221 | 11.8743 | 5.2048 | 206 | 42436 | 8741816 | 14.3527 | 5.9059 |
| 142 | 20164 | 2863288 | 11.9164 | 5.2171 | 207 | 42849 | 8869743 | 14.3875 | 5.9155 |
| 143 | 20449 | 2924207 | 11.9583 | 5.2293 | 208 | 43264 | 8998912 | 14.4222 | 5.9250 |
| 144 | 20736 | 2985984 | 12. | 5.2415 | 209 | 43681 | 9129329 | 14.4568 | 5.9345 |
| 145 | 21025 | 3048625 | 12.0416 | 5.2536 | 210 | 44100 | 9261000 | 14.4914 | 5.9439 |
| 146 | 21316 | 3112136 | 12.0830 | 5.2656 | 211 | 44521 | 9393931 | 14.5258 | 5.9533 |
| 147 | 21609 | 3176523 | 12.1244 | 5.2776 | 212 | 44944 | 9528128 | 14.5602 | 5.9627 |
| 148 | 21904 | 3241792 | 12.1655 | 5.2896 | 213 | 45369 | 9663597 | 14.5945 | 5.9721 |
| 149 | 22201 | 3307949 | 12.2066 | 5.3015 | 214 | 45796 | 9800344 | 14.6287 | 5.9814 |
| 150 | 22500 | 3375000 | 12.2474 | 5.3133 | 215 | 46225 | 9938375 | 14.6629 | 5.9907 |
| 151 | 22801 | 3442951 | 12.2882 | 5.3251 | 216 | 46656 | 10077696 | 14.6969 | 6. |
| 152 | 23104 | 3511808 | 12.3288 | 5.3368 | 217 | 47089 | 10218313 | 14.7309 | 6.0092 |
| 153 | 23409 | 3581577 | 12.3693 | 5.3485 | 218 | 47524 | 10360232 | 14.7648 | 6.0185 |
| 154 | 23716 | 3652264 | 12.4097 | 5.3601 | 219 | 47961 | 10503459 | 14.7986 | 6.0277 |
| 155 | 24025 | 3723875 | 12.4499 | 5.3717 | 220 | 48400 | 10648000 | 14.8324 | 6.0368 |
| 156 | 24336 | 3796416 | 12.4900 | 5.3832 | 221 | 48841 | 10793861 | 14.8661 | 6.0459 |
| 157 | 24649 | 3869893 | 12.5300 | 5.3947 | 222 | 49284 | 10941048 | 14.8997 | 6.0550 |
| 158 | 24964 | 3944312 | 12.5698 | 5.4061 | 223 | 49729 | 11089567 | 14.9332 | 6.0641 |
| 159 | 25281 | 4019679 | 12.6095 | 5.4175 | 224 | 50176 | 11239424 | 14.9666 | 6.0732 |
| 160 | 25600 | 4096000 | 12.6491 | 5.4288 | 225 | 50625 | 11390625 | 15. | 6.0822 |
| 161 | 25921 | 4173281 | 12.6886 | 5.4401 | 226 | 51076 | 11543176 | 15.0333 | 6.0912 |
| 162 | 26244 | 4251528 | 12.7279 | 5.4514 | 227 | 51529 | 11697083 | 15.0665 | 6.1002 |
| 163 | 26569 | 4330747 | 12.7671 | 5.4626 | 228 | 51984 | 11852352 | 15.0997 | 6.1091 |
| 164 | 26896 | 4410944 | 12.8062 | 5.4737 | 229 | 52441 | 12008989 | 15.1327 | 6.1180 |
| 165 | 27225 | 4492125 | 12.8452 | 5.4848 | 230 | 52900 | 12167000 | 15.1658 | 6.1269 |
| 166 | 27556 | 4574296 | 12.8841 | 5.4959 | 231 | 53361 | 12326391 | 15.1987 | 6.1358 |
| 167 | 27889 | 4657463 | 12.9228 | 5.5069 | 232 | 53824 | 12487168 | 15.2315 | 6.1446 |
| 168 | 28224 | 4741632 | 12.9615 | 5.5178 | 233 | 54289 | 12649337 | 15.2643 | 6.1534 |
| 169 | 28561 | 4826809 | 13. | 5.5288 | 234 | 54756 | 12812904 | 15.2971 | 6.1622 |
| 170 | 28900 | 4913000 | 13.0384 | 5.5397 | 235 | 55225 | 12977875 | 15.3297 | 6.1710 |
| 171 | 29241 | 5000211 | 13.0767 | 5.5505 | 236 | 55696 | 13144256 | 15.3623 | 6.1797 |
| 172 | 29584 | 5088448 | 13.1149 | 5.5613 | 237 | 56169 | 13312053 | 15.3948 | 6.1885 |
| 173 | 29929 | 5177717 | 13.1529 | 5.5721 | 238 | 56644 | 13481272 | 15.4272 | 6.1972 |
| 174 | 30276 | 5268024 | 13.1909 | 5.5828 | 239 | 57121 | 13651919 | 15.4596 | 6.2058 |
| 175 | 30625 | 5359375 | 13.2288 | 5.5934 | 240 | 57600 | 13824000 | 15.4919 | 6.2145 |
| 176 | 30976 | 5451776 | 13.2665 | 5.6041 | 241 | 58081 | 13997521 | 15.5242 | 6.2231 |
| 177 | 31329 | 5545233 | 13.3041 | 5.6147 | 242 | 58564 | 14172488 | 15.5563 | 6.2317 |
| 178 | 31684 | 5639752 | 13.3417 | 5.6252 | 243 | 59049 | 14348907 | 15.5885 | 6.2403 |
| 179 | 32041 | 5735339 | 13.3791 | 5.6357 | 244 | 59536 | 14526784 | 15.6205 | 6.2488 |
| 180 | 32400 | 5832000 | 13.4164 | 5.6462 | 245 | 60025 | 14706125 | 15.6525 | 6.2573 |
| 181 | 32761 | 5929741 | 13.4536 | 5.6567 | 246 | 60516 | 14886936 | 15.6844 | 6.2658 |
| 182 | 33124 | 6028568 | 13.4907 | 5.6671 | 247 | 61009 | 15069223 | 15.7162 | 6.2743 |
| 183 | 33489 | 6128487 | 13.5277 | 5.6774 | 248 | 61504 | 15252992 | 15.7480 | 6.2828 |
| 184 | 33856 | 6229504 | 13.5647 | 5.6877 | 249 | 62001 | 15438249 | 15.7797 | 6.2912 |
| 185 | 34225 | 6331625 | 13.6015 | 5.6980 | 250 | 62500 | 15625000 | 15.8114 | 6.2996 |

## Tabla de los cuadrados, cubos, raíces cuadradas y cúbicas de los números de 1 á 1,000.

| No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. | No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. |
|---|---|---|---|---|---|---|---|---|---|
| 251 | 63001 | 15813251 | 15.8430 | 6.3080 | 316 | 99856 | 31554496 | 17.7764 | 6.8113 |
| 252 | 63504 | 16003008 | 15.8745 | 6.3164 | 317 | 100489 | 31855013 | 17.8045 | 6.8185 |
| 253 | 64009 | 16194277 | 15.9060 | 6.3247 | 318 | 101124 | 32157432 | 17.8326 | 6.8256 |
| 254 | 64516 | 16387064 | 15.9374 | 6.3330 | 319 | 101761 | 32461759 | 17.8606 | 6.8328 |
| 255 | 65025 | 16581375 | 15.9687 | 6.3413 | 320 | 102400 | 32768000 | 17.8885 | 6.8399 |
| 256 | 65536 | 16777216 | 16. | 6.3496 | 321 | 103041 | 33076161 | 17.9165 | 6.8470 |
| 257 | 66049 | 16974593 | 16.0312 | 6.3579 | 322 | 103684 | 33386248 | 17.9444 | 6.8541 |
| 258 | 66564 | 17173512 | 16.0624 | 6.3661 | 323 | 104329 | 33698267 | 17.9722 | 6.8612 |
| 259 | 67081 | 17373979 | 16.0935 | 6.3743 | 324 | 104976 | 34012224 | 18. | 6.8683 |
| 260 | 67600 | 17576000 | 16.1245 | 6.3825 | 325 | 105625 | 34328125 | 18.0278 | 6.8753 |
| 261 | 68121 | 17779581 | 16.1555 | 6.3907 | 326 | 106276 | 34645976 | 18.0555 | 6.8824 |
| 262 | 68644 | 17984728 | 16.1864 | 6.3988 | 327 | 106929 | 34965783 | 18.0831 | 6.8894 |
| 263 | 69169 | 18191447 | 16.2173 | 6.4070 | 328 | 107584 | 35287552 | 18.1108 | 6.8964 |
| 264 | 69696 | 18399744 | 16.2481 | 6.4151 | 329 | 108241 | 35611289 | 18.1384 | 6.9034 |
| 265 | 70225 | 18609625 | 16.2788 | 6.4232 | 330 | 108900 | 35937000 | 18.1659 | 6.9104 |
| 266 | 70756 | 18821096 | 16.3095 | 6.4312 | 331 | 109561 | 36264691 | 18.1934 | 6.9174 |
| 267 | 71289 | 19034163 | 16.3401 | 6.4393 | 332 | 110224 | 36594368 | 18.2209 | 6.9244 |
| 268 | 71824 | 19248832 | 16.3707 | 6.4473 | 333 | 110889 | 36926037 | 18.2483 | 6.9313 |
| 269 | 72361 | 19465109 | 16.4012 | 6.4553 | 334 | 111556 | 37259704 | 18.2757 | 6.9382 |
| 270 | 72900 | 19683000 | 16.4317 | 6.4633 | 335 | 112225 | 37595375 | 18.3030 | 6.9451 |
| 271 | 73441 | 19902511 | 16.4621 | 6.4713 | 336 | 112896 | 37933056 | 18.3303 | 6.9521 |
| 272 | 73984 | 20123648 | 16.4924 | 6.4792 | 337 | 113569 | 38272753 | 18.3576 | 6.9589 |
| 273 | 74529 | 20346417 | 16.5227 | 6.4872 | 338 | 114244 | 38614472 | 18.3848 | 6.9658 |
| 274 | 75076 | 20570824 | 16.5529 | 6.4951 | 339 | 114921 | 38958219 | 18.4120 | 6.9727 |
| 275 | 75625 | 20796875 | 16.5831 | 6.5030 | 340 | 115600 | 39304000 | 18.4391 | 6.9795 |
| 276 | 76176 | 21024576 | 16.6132 | 6.5108 | 341 | 116281 | 39651821 | 18.4662 | 6.9864 |
| 277 | 76729 | 21253933 | 16.6433 | 6.5187 | 342 | 116964 | 40001688 | 18.4932 | 6.9932 |
| 278 | 77284 | 21484952 | 16.6733 | 6.5265 | 343 | 117649 | 40353607 | 18.5203 | 7. |
| 279 | 77841 | 21717639 | 16.7033 | 6.5343 | 344 | 118336 | 40707584 | 18.5472 | 7.0068 |
| 280 | 78400 | 21952000 | 16.7332 | 6.5421 | 345 | 119025 | 41063625 | 18.5742 | 7.0136 |
| 281 | 78961 | 22188041 | 16.7631 | 6.5499 | 346 | 119716 | 41421736 | 18.6011 | 7.0203 |
| 282 | 79524 | 22425768 | 16.7929 | 6.5577 | 347 | 120409 | 41781923 | 18.6279 | 7.0271 |
| 283 | 80089 | 22665187 | 16.8226 | 6.5654 | 348 | 121104 | 42144192 | 18.6548 | 7.0338 |
| 284 | 80656 | 22906304 | 16.8523 | 6.5731 | 349 | 121801 | 42508549 | 18.6815 | 7.0406 |
| 285 | 81225 | 23149125 | 16.8819 | 6.5808 | 350 | 122500 | 42875000 | 18.7083 | 7.0473 |
| 286 | 81796 | 23393656 | 16.9115 | 6.5885 | 351 | 123201 | 43243551 | 18.7350 | 7.0540 |
| 287 | 82369 | 23639903 | 16.9411 | 6.5962 | 352 | 123904 | 43614208 | 18.7617 | 7.0607 |
| 288 | 82944 | 23887872 | 16.9706 | 6.6039 | 353 | 124609 | 43986977 | 18.7883 | 7.0674 |
| 289 | 83521 | 24137569 | 17. | 6.6115 | 354 | 125316 | 44361864 | 18.8149 | 7.0740 |
| 290 | 84100 | 24389000 | 17.0294 | 6.6191 | 355 | 126025 | 44738875 | 18.8414 | 7.0807 |
| 291 | 84681 | 24642171 | 17.0587 | 6.6267 | 356 | 126736 | 45118016 | 18.8680 | 7.0873 |
| 292 | 85264 | 24897088 | 17.0880 | 6.6343 | 357 | 127449 | 45499293 | 18.8944 | 7.0940 |
| 293 | 85849 | 25153757 | 17.1172 | 6.6419 | 358 | 128164 | 45882712 | 18.9209 | 7.1006 |
| 294 | 86436 | 25412184 | 17.1464 | 6.6494 | 359 | 128881 | 46268279 | 18.9473 | 7.1072 |
| 295 | 87025 | 25672375 | 17.1756 | 6.6569 | 360 | 129600 | 46656000 | 18.9737 | 7.1138 |
| 296 | 87616 | 25934336 | 17.2047 | 6.6644 | 361 | 130321 | 47045881 | 19. | 7.1204 |
| 297 | 88209 | 26198073 | 17.2337 | 6.6719 | 362 | 131044 | 47437928 | 19.0263 | 7.1269 |
| 298 | 88804 | 26463592 | 17.2627 | 6.6794 | 363 | 131769 | 47832147 | 19.0526 | 7.1335 |
| 299 | 89401 | 26730899 | 17.2916 | 6.6869 | 364 | 132496 | 48228544 | 19.0788 | 7.1400 |
| 300 | 90000 | 27000000 | 17.3205 | 6.6943 | 365 | 133225 | 48627125 | 19.1050 | 7.1466 |
| 301 | 90601 | 27270901 | 17.3494 | 6.7018 | 366 | 133956 | 49027896 | 19.1311 | 7.1531 |
| 302 | 91204 | 27543608 | 17.3781 | 6.7092 | 367 | 134689 | 49430863 | 19.1572 | 7.1596 |
| 303 | 91809 | 27818127 | 17.4069 | 6.7166 | 368 | 135424 | 49836032 | 19.1833 | 7.1661 |
| 304 | 92416 | 28094464 | 17.4356 | 6.7240 | 369 | 136161 | 50243409 | 19.2094 | 7.1726 |
| 305 | 93025 | 28372625 | 17.4642 | 6.7313 | 370 | 136900 | 50653000 | 19.2354 | 7.1791 |
| 306 | 93636 | 28652616 | 17.4929 | 6.7387 | 371 | 137641 | 51064811 | 19.2614 | 7.1855 |
| 307 | 94249 | 28934443 | 17.5214 | 6.7460 | 372 | 138384 | 51478848 | 19.2873 | 7.1920 |
| 308 | 94864 | 29218112 | 17.5499 | 6.7533 | 373 | 139129 | 51895117 | 19.3132 | 7.1984 |
| 309 | 95481 | 29503629 | 17.5784 | 6.7606 | 374 | 139876 | 52313624 | 19.3391 | 7.2048 |
| 310 | 96100 | 29791000 | 17.6068 | 6.7679 | 375 | 140625 | 52734375 | 19.3649 | 7.2112 |
| 311 | 96721 | 30080231 | 17.6352 | 6.7752 | 376 | 141376 | 53157376 | 19.3907 | 7.2177 |
| 312 | 97344 | 30371328 | 17.6635 | 6.7824 | 377 | 142129 | 53582633 | 19.4165 | 7.2240 |
| 313 | 97969 | 30664297 | 17.6918 | 6.7897 | 378 | 142884 | 54010152 | 19.4422 | 7.2304 |
| 314 | 98596 | 30959144 | 17.7200 | 6.7969 | 379 | 143641 | 54439939 | 19.4679 | 7.2368 |
| 315 | 99225 | 31255875 | 17.7482 | 6.8041 | 380 | 144400 | 54872000 | 19.4936 | 7.2432 |

## Tabla de los cuadrados, cubos, raíces cuadradas y cúbicas de los números de 1 á 1,000.

| No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. | No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. |
|---|---|---|---|---|---|---|---|---|---|
| 381 | 145161 | 55306341 | 19.5192 | 7.2495 | 446 | 198916 | 88716536 | 21.1187 | 7.6403 |
| 382 | 145924 | 55742968 | 19.5448 | 7.2558 | 447 | 199809 | 89314623 | 21.1424 | 7.6460 |
| 383 | 146689 | 56181887 | 19.5704 | 7.2622 | 448 | 200704 | 89915392 | 21.1660 | 7.6517 |
| 384 | 147456 | 56623104 | 19.5959 | 7.2685 | 449 | 201601 | 90518849 | 21.1896 | 7.6574 |
| 385 | 148225 | 57066625 | 19.6214 | 7.2748 | 450 | 202500 | 91125000 | 21.2132 | 7.6631 |
| 386 | 148996 | 57512456 | 19.6469 | 7.2811 | 451 | 203401 | 91733851 | 21.2368 | 7.6688 |
| 387 | 149769 | 57960603 | 19.6723 | 7.2874 | 452 | 204304 | 92345408 | 21.2603 | 7.6744 |
| 388 | 150544 | 58411072 | 19.6977 | 7.2936 | 453 | 205209 | 92959677 | 21.2838 | 7.6801 |
| 389 | 151321 | 58863869 | 19.7231 | 7.2999 | 454 | 206116 | 93576664 | 21.3073 | 7.6857 |
| 390 | 152100 | 59319000 | 19.7484 | 7.3061 | 455 | 207025 | 94196375 | 21.3307 | 7.6914 |
| 391 | 152881 | 59776471 | 19.7737 | 7.3124 | 456 | 207936 | 94818816 | 21.3542 | 7.6970 |
| 392 | 153664 | 60236288 | 19.7990 | 7.3186 | 457 | 208849 | 95443993 | 21.3776 | 7.7026 |
| 393 | 154449 | 60698457 | 19.8242 | 7.3248 | 458 | 209764 | 96071912 | 21.4009 | 7.7082 |
| 394 | 155236 | 61162984 | 19.8494 | 7.3310 | 459 | 210681 | 96702579 | 21.4243 | 7.7138 |
| 395 | 156025 | 61629875 | 19.8746 | 7.3372 | 460 | 211600 | 97336000 | 21.4476 | 7.7194 |
| 396 | 156816 | 62099136 | 19.8997 | 7.3434 | 461 | 212521 | 97972181 | 21.4709 | 7.7250 |
| 397 | 157609 | 62570773 | 19.9249 | 7.3496 | 462 | 213444 | 98611128 | 21.4942 | 7.7306 |
| 398 | 158404 | 63044792 | 19.9499 | 7.3558 | 463 | 214369 | 99252847 | 21.5174 | 7.7362 |
| 399 | 159201 | 63521199 | 19.9750 | 7.3619 | 464 | 215296 | 99897344 | 21.5407 | 7.7418 |
| 400 | 160000 | 64000000 | 20. | 7.3681 | 465 | 216225 | 100544625 | 21.5639 | 7.7473 |
| 401 | 160801 | 64481201 | 20.0250 | 7.3742 | 466 | 217156 | 101194696 | 21.5870 | 7.7529 |
| 402 | 161604 | 64964808 | 20.0499 | 7.3803 | 467 | 218089 | 101847563 | 21.6102 | 7.7584 |
| 403 | 162409 | 65450827 | 20.0749 | 7.3864 | 468 | 219024 | 102503232 | 21.6333 | 7.7639 |
| 404 | 163216 | 65939264 | 20.0998 | 7.3925 | 469 | 219961 | 103161709 | 21.6564 | 7.7695 |
| 405 | 164025 | 66430125 | 20.1246 | 7.3986 | 470 | 220900 | 103823000 | 21.6795 | 7.7750 |
| 406 | 164836 | 66923416 | 20.1494 | 7.4047 | 471 | 221841 | 104487111 | 21.7025 | 7.7805 |
| 407 | 165649 | 67419143 | 20.1742 | 7.4108 | 472 | 222784 | 105154048 | 21.7256 | 7.7860 |
| 408 | 166464 | 67917312 | 20.1990 | 7.4169 | 473 | 223729 | 105823817 | 21.7486 | 7.7915 |
| 409 | 167281 | 68417929 | 20.2237 | 7.4229 | 474 | 224676 | 106496424 | 21.7715 | 7.7970 |
| 410 | 168100 | 68921000 | 20.2485 | 7.4290 | 475 | 225625 | 107171875 | 21.7945 | 7.8025 |
| 411 | 168921 | 69426531 | 20.2731 | 7.4350 | 476 | 226576 | 107850176 | 21.8174 | 7.8079 |
| 412 | 169744 | 69934528 | 20.2978 | 7.4410 | 477 | 227529 | 108531333 | 21.8403 | 7.8134 |
| 413 | 170569 | 70444997 | 20.3224 | 7.4470 | 478 | 228484 | 109215352 | 21.8632 | 7.8188 |
| 414 | 171396 | 70957944 | 20.3470 | 7.4530 | 479 | 229441 | 109902239 | 21.8861 | 7.8243 |
| 415 | 172225 | 71473375 | 20.3715 | 7.4590 | 480 | 230400 | 110592000 | 21.9089 | 7.8297 |
| 416 | 173056 | 71991296 | 20.3961 | 7.4650 | 481 | 231361 | 111284641 | 21.9317 | 7.8352 |
| 417 | 173889 | 72511713 | 20.4206 | 7.4710 | 482 | 232324 | 111980168 | 21.9545 | 7.8406 |
| 418 | 174724 | 73034632 | 20.4450 | 7.4770 | 483 | 233289 | 112678587 | 21.9773 | 7.8460 |
| 419 | 175561 | 73560059 | 20.4695 | 7.4829 | 484 | 234256 | 113379904 | 22. | 7.8514 |
| 420 | 176400 | 74088000 | 20.4939 | 7.4889 | 485 | 235225 | 114084125 | 22.0227 | 7.8568 |
| 421 | 177241 | 74618461 | 20.5183 | 7.4948 | 486 | 236196 | 114791256 | 22.0454 | 7.8622 |
| 422 | 178084 | 75151448 | 20.5426 | 7.5007 | 487 | 237169 | 115501303 | 22.0681 | 7.8676 |
| 423 | 178929 | 75686967 | 20.5670 | 7.5067 | 488 | 238144 | 116214272 | 22.0907 | 7.8730 |
| 424 | 179776 | 76225024 | 20.5913 | 7.5126 | 489 | 239121 | 116930169 | 22.1133 | 7.8784 |
| 425 | 180625 | 76765625 | 20.6155 | 7.5185 | 490 | 240100 | 117649000 | 22.1359 | 7.8837 |
| 426 | 181476 | 77308776 | 20.6398 | 7.5244 | 491 | 241081 | 118370771 | 22.1585 | 7.8891 |
| 427 | 182329 | 77854483 | 20.6640 | 7.5302 | 492 | 242064 | 119095488 | 22.1811 | 7.8944 |
| 428 | 183184 | 78402752 | 20.6882 | 7.5361 | 493 | 243049 | 119823157 | 22.2036 | 7.8998 |
| 429 | 184041 | 78953589 | 20.7123 | 7.5420 | 494 | 244036 | 120553784 | 22.2261 | 7.9051 |
| 430 | 184900 | 79507000 | 20.7364 | 7.5478 | 495 | 245025 | 121287375 | 22.2486 | 7.9105 |
| 431 | 185761 | 80062991 | 20.7605 | 7.5537 | 496 | 246016 | 122023936 | 22.2711 | 7.9158 |
| 432 | 186624 | 80621568 | 20.7846 | 7.5595 | 497 | 247009 | 122763473 | 22.2935 | 7.9211 |
| 433 | 187489 | 81182737 | 20.8087 | 7.5654 | 498 | 248004 | 123505992 | 22.3159 | 7.9264 |
| 434 | 188356 | 81746504 | 20.8327 | 7.5712 | 499 | 249001 | 124251499 | 22.3383 | 7.9317 |
| 435 | 189225 | 82312875 | 20.8567 | 7.5770 | 500 | 250000 | 125000000 | 22.3607 | 7.9370 |
| 436 | 190096 | 82881856 | 20.8806 | 7.5828 | 501 | 251001 | 125751501 | 22.3830 | 7.9423 |
| 437 | 190969 | 83453453 | 20.9045 | 7.5886 | 502 | 252004 | 126506008 | 22.4054 | 7.9476 |
| 438 | 191844 | 84027672 | 20.9284 | 7.5944 | 503 | 253009 | 127263527 | 22.4277 | 7.9528 |
| 439 | 192721 | 84604519 | 20.9523 | 7.6001 | 504 | 254016 | 128024064 | 22.4499 | 7.9581 |
| 440 | 193600 | 85184000 | 20.9762 | 7.6059 | 505 | 255025 | 128787625 | 22.4722 | 7.9634 |
| 441 | 194481 | 85766121 | 21. | 7.6117 | 506 | 256036 | 129554216 | 22.4944 | 7.9686 |
| 442 | 195364 | 86350888 | 21.0238 | 7.6174 | 507 | 257049 | 130323843 | 22.5167 | 7.9739 |
| 443 | 196249 | 86938307 | 21.0476 | 7.6232 | 508 | 258064 | 131096512 | 22.5389 | 7.9791 |
| 444 | 197136 | 87528384 | 21.0713 | 7.6289 | 509 | 259081 | 131872229 | 22.5610 | 7.9843 |
| 445 | 198025 | 88121125 | 21.0950 | 7.6346 | 510 | 260100 | 132651000 | 22.5832 | 7.9895 |

## Tabla de los cuadrados, cubos, raíces cuadradas y cúbicas de los números de 1 á 1,000.

| No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. | No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. |
|---|---|---|---|---|---|---|---|---|---|
| 511 | 261121 | 133432831 | 22.6053 | 7.9948 | 576 | 331776 | 191102976 | 24. | 8.3203 |
| 512 | 262144 | 134217728 | 22.6274 | 8. | 577 | 332929 | 192100033 | 24.0208 | 8.3251 |
| 513 | 263169 | 135005697 | 22.6495 | 8.0052 | 578 | 334084 | 193100552 | 24.0416 | 8.3300 |
| 514 | 264196 | 135796744 | 22.6716 | 8.0104 | 579 | 335241 | 194104539 | 24.0624 | 8.3348 |
| 515 | 265225 | 136590875 | 22.6936 | 8.0156 | 580 | 336400 | 195112000 | 24.0832 | 8.3396 |
| 516 | 266256 | 137388096 | 22.7156 | 8.0208 | 581 | 337561 | 196122941 | 24.1039 | 8.3443 |
| 517 | 267289 | 138188413 | 22.7376 | 8.0260 | 582 | 338724 | 197137368 | 24.1247 | 8.3491 |
| 518 | 268324 | 138991832 | 22.7596 | 8.0311 | 583 | 339889 | 198155287 | 24.1454 | 8.3539 |
| 519 | 269361 | 139798359 | 22.7816 | 8.0363 | 584 | 341056 | 199176704 | 24.1661 | 8.3587 |
| 520 | 270400 | 140608000 | 22.8035 | 8.0415 | 585 | 342225 | 200201625 | 24.1868 | 8.3634 |
| 521 | 271441 | 141420761 | 22.8254 | 8.0466 | 586 | 343396 | 201230056 | 24.2074 | 8.3682 |
| 522 | 272484 | 142236648 | 22.8473 | 8.0517 | 587 | 344569 | 202262003 | 24.2281 | 8.3730 |
| 523 | 273529 | 143055667 | 22.8692 | 8.0569 | 588 | 345744 | 203297472 | 24.2487 | 8.3777 |
| 524 | 274576 | 143877824 | 22.8910 | 8.0620 | 589 | 346921 | 204336469 | 24.2693 | 8.3825 |
| 525 | 275625 | 144703125 | 22.9129 | 8.0671 | 590 | 348100 | 205379000 | 24.2899 | 8.3872 |
| 526 | 276676 | 145531576 | 22.9347 | 8.0723 | 591 | 349281 | 206425071 | 24.3105 | 8.3919 |
| 527 | 277729 | 146363183 | 22.9565 | 8.0774 | 592 | 350464 | 207474688 | 24.3311 | 8.3967 |
| 528 | 278784 | 147197952 | 22.9783 | 8.0825 | 593 | 351649 | 208527857 | 24.3516 | 8.4014 |
| 529 | 279841 | 148035889 | 23. | 8.0876 | 594 | 352836 | 209584584 | 24.3721 | 8.4061 |
| 530 | 280900 | 148877000 | 23.0217 | 8.0927 | 595 | 354025 | 210644875 | 24.3926 | 8.4108 |
| 531 | 281961 | 149721291 | 23.0434 | 8.0978 | 596 | 355216 | 211708736 | 24.4131 | 8.4155 |
| 532 | 283024 | 150568768 | 23.0651 | 8.1028 | 597 | 356409 | 212776173 | 24.4336 | 8.4202 |
| 533 | 284089 | 151419437 | 23.0868 | 8.1079 | 598 | 357604 | 213847192 | 24.4540 | 8.4249 |
| 534 | 285156 | 152273304 | 23.1084 | 8.1130 | 599 | 358801 | 214921799 | 24.4745 | 8.4296 |
| 535 | 286225 | 153130375 | 23.1301 | 8.1180 | 600 | 360000 | 216000000 | 24.4949 | 8.4343 |
| 536 | 287296 | 153990656 | 23.1517 | 8.1231 | 601 | 361201 | 217081801 | 24.5153 | 8.4390 |
| 537 | 288369 | 154854153 | 23.1733 | 8.1281 | 602 | 362404 | 218167208 | 24.5357 | 8.4437 |
| 538 | 289444 | 155720872 | 23.1948 | 8.1332 | 603 | 363609 | 219256227 | 24.5561 | 8.4484 |
| 539 | 290521 | 156589819 | 23.2164 | 8.1382 | 604 | 364816 | 220348864 | 24.5764 | 8.4530 |
| 540 | 291600 | 157464000 | 23.2379 | 8.1433 | 605 | 366025 | 221445125 | 24.5967 | 8.4577 |
| 541 | 292681 | 158340421 | 23.2594 | 8.1483 | 606 | 367236 | 222545016 | 24.6171 | 8.4623 |
| 542 | 293764 | 159220088 | 23.2809 | 8.1533 | 607 | 368449 | 223648543 | 24.6374 | 8.4670 |
| 543 | 294849 | 160103007 | 23.3024 | 8.1583 | 608 | 369664 | 224755712 | 24.6577 | 8.4716 |
| 544 | 295936 | 160989184 | 23.3238 | 8.1633 | 609 | 370881 | 225866529 | 24.6779 | 8.4763 |
| 545 | 297025 | 161878625 | 23.3452 | 8.1683 | 610 | 372100 | 226981000 | 24.6982 | 8.4809 |
| 546 | 298116 | 162771336 | 23.3666 | 8.1733 | 611 | 373321 | 228099131 | 24.7184 | 8.4856 |
| 547 | 299209 | 163667323 | 23.3880 | 8.1783 | 612 | 374544 | 229220928 | 24.7386 | 8.4902 |
| 548 | 300304 | 164566592 | 23.4094 | 8.1833 | 613 | 375769 | 230346397 | 24.7588 | 8.4948 |
| 549 | 301401 | 165469149 | 23.4307 | 8.1882 | 614 | 376996 | 231475544 | 24.7790 | 8.4994 |
| 550 | 302500 | 166375000 | 23.4521 | 8.1932 | 615 | 378225 | 232608375 | 24.7992 | 8.5040 |
| 551 | 303601 | 167284151 | 23.4734 | 8.1982 | 616 | 379456 | 233744896 | 24.8193 | 8.5086 |
| 552 | 304704 | 168196608 | 23.4947 | 8.2031 | 617 | 380689 | 234885113 | 24.8395 | 8.5132 |
| 553 | 305809 | 169112377 | 23.5160 | 8.2081 | 618 | 381924 | 236029032 | 24.8596 | 8.5178 |
| 554 | 306916 | 170031464 | 23.5372 | 8.2130 | 619 | 383161 | 237176659 | 24.8797 | 8.5224 |
| 555 | 308025 | 170953875 | 23.5584 | 8.2180 | 620 | 384400 | 238328000 | 24.8998 | 8.5270 |
| 556 | 309136 | 171879616 | 23.5797 | 8.2229 | 621 | 385641 | 239483061 | 24.9199 | 8.5316 |
| 557 | 310249 | 172808693 | 23.6008 | 8.2278 | 622 | 386884 | 240641848 | 24.9399 | 8.5362 |
| 558 | 311364 | 173741112 | 23.6220 | 8.2327 | 623 | 388129 | 241804367 | 24.9600 | 8.5408 |
| 559 | 312481 | 174676879 | 23.6432 | 8.2377 | 624 | 389376 | 242970624 | 24.9800 | 8.5453 |
| 560 | 313600 | 175616000 | 23.6643 | 8.2426 | 625 | 390625 | 244140625 | 25. | 8.5499 |
| 561 | 314721 | 176558481 | 23.6854 | 8.2475 | 626 | 391876 | 245314376 | 25.0200 | 8.5544 |
| 562 | 315844 | 177504328 | 23.7065 | 8.2524 | 627 | 393129 | 246491883 | 25.0400 | 8.5590 |
| 563 | 316969 | 178453547 | 23.7276 | 8.2573 | 628 | 394384 | 247673152 | 25.0599 | 8.5635 |
| 564 | 318096 | 179406144 | 23.7487 | 8.2621 | 629 | 395641 | 248858189 | 25.0799 | 8.5681 |
| 565 | 319225 | 180362125 | 23.7697 | 8.2670 | 630 | 396900 | 250047000 | 25.0998 | 8.5726 |
| 566 | 320356 | 181321496 | 23.7908 | 8.2719 | 631 | 398161 | 251239591 | 25.1197 | 8.5772 |
| 567 | 321489 | 182284263 | 23.8118 | 8.2768 | 632 | 399424 | 252435968 | 25.1396 | 8.5817 |
| 568 | 322624 | 183250432 | 23.8328 | 8.2816 | 633 | 400689 | 253636137 | 25.1595 | 8.5862 |
| 569 | 323761 | 184220009 | 23.8537 | 8.2865 | 634 | 401956 | 254840104 | 25.1794 | 8.5907 |
| 570 | 324900 | 185193000 | 23.8747 | 8.2913 | 635 | 403225 | 256047875 | 25.1992 | 8.5952 |
| 571 | 326041 | 186169411 | 23.8956 | 8.2962 | 636 | 404496 | 257259456 | 25.2190 | 8.5997 |
| 572 | 327184 | 187149248 | 23.9165 | 8.3010 | 637 | 405769 | 258474853 | 25.2389 | 8.6043 |
| 573 | 328329 | 188132517 | 23.9374 | 8.3059 | 638 | 407044 | 259694072 | 25.2587 | 8.6088 |
| 574 | 329476 | 189119224 | 23.9583 | 8.3107 | 639 | 408321 | 260917119 | 25.2784 | 8.6133 |
| 575 | 330625 | 190109375 | 23.9792 | 8.3155 | 640 | 409600 | 262144000 | 25.2982 | 8.6177 |

## Tabla de los cuadrados, cubos, raíces cuadradas y cúbicas de los números de 1 á 1,000.

| No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. | No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. |
|---|---|---|---|---|---|---|---|---|---|
| 641 | 410881 | 263374721 | 25.3180 | 8.6222 | 706 | 498436 | 351895816 | 26.5707 | 8.9043 |
| 642 | 412164 | 264609288 | 25.3377 | 8.6287 | 707 | 499849 | 353393243 | 26.5895 | 8.9085 |
| 643 | 413449 | 265847707 | 25.3574 | 8.6352 | 708 | 501264 | 354894912 | 26.6083 | 8.9127 |
| 644 | 414736 | 267089984 | 25.3772 | 8.6357 | 709 | 502681 | 356400829 | 26.6271 | 8.9169 |
| 645 | 416025 | 268336125 | 25.3969 | 8.6401 | 710 | 504100 | 357911000 | 26.6458 | 8.9211 |
| 646 | 417316 | 269586136 | 25.4165 | 8.6446 | 711 | 505521 | 359425431 | 26.6646 | 8.9253 |
| 647 | 418609 | 270840023 | 25.4362 | 8.6490 | 712 | 506944 | 360944128 | 26.6833 | 8.9295 |
| 648 | 419904 | 272097792 | 25.4558 | 8.6535 | 713 | 508369 | 362467097 | 26.7021 | 8.9337 |
| 649 | 421201 | 273359449 | 25.4755 | 8.6579 | 714 | 509796 | 363994344 | 26.7208 | 8.9378 |
| 650 | 422500 | 274625000 | 25.4951 | 8.6624 | 715 | 511225 | 365525875 | 26.7395 | 8.9420 |
| 651 | 423801 | 275894451 | 25.5147 | 8.6668 | 716 | 512656 | 367061696 | 26.7582 | 8.9462 |
| 652 | 425104 | 277167808 | 25.5343 | 8.6713 | 717 | 514089 | 368601813 | 26.7769 | 8.9503 |
| 653 | 426409 | 278445077 | 25.5539 | 8.6757 | 718 | 515524 | 370146232 | 26.7955 | 8.9545 |
| 654 | 427716 | 279726264 | 25.5734 | 8.6801 | 719 | 516961 | 371694959 | 26.8142 | 8.9587 |
| 655 | 429025 | 281011375 | 25.5930 | 8.6845 | 720 | 518400 | 373248000 | 26.8328 | 8.9628 |
| 656 | 430336 | 282300416 | 25.6125 | 8.6890 | 721 | 519841 | 374805361 | 26.8514 | 8.9670 |
| 657 | 431649 | 283593393 | 25.6320 | 8.6934 | 722 | 521284 | 376367048 | 26.8701 | 8.9711 |
| 658 | 432964 | 284890312 | 25.6515 | 8.6978 | 723 | 522729 | 377933067 | 26.8887 | 8.9752 |
| 659 | 434281 | 286191179 | 25.6710 | 8.7022 | 724 | 524176 | 379503424 | 26.9072 | 8.9794 |
| 660 | 435600 | 287496000 | 25.6905 | 8.7066 | 725 | 525625 | 381078125 | 26.9258 | 8.9835 |
| 661 | 436921 | 288804781 | 25.7099 | 8.7110 | 726 | 527076 | 382657176 | 26.9444 | 8.9876 |
| 662 | 438244 | 290117528 | 25.7294 | 8.7154 | 727 | 528529 | 384240583 | 26.9629 | 8.9918 |
| 663 | 439569 | 291434247 | 25.7488 | 8.7198 | 728 | 529984 | 385828352 | 26.9815 | 8.9959 |
| 664 | 440896 | 292754944 | 25.7682 | 8.7241 | 729 | 531441 | 387420489 | 27. | 9. |
| 665 | 442225 | 294079625 | 25.7876 | 8.7285 | 730 | 532900 | 389017000 | 27.0185 | 9.0041 |
| 666 | 443556 | 295408296 | 25.8070 | 8.7329 | 731 | 534361 | 390617891 | 27.0370 | 9.0082 |
| 667 | 444889 | 296740963 | 25.8263 | 8.7373 | 732 | 535824 | 392223168 | 27.0555 | 9.0123 |
| 668 | 446224 | 298077632 | 25.8457 | 8.7416 | 733 | 537289 | 393832837 | 27.0740 | 9.0164 |
| 669 | 447561 | 299418309 | 25.8650 | 8.7460 | 734 | 538756 | 395446904 | 27.0924 | 9.0205 |
| 670 | 448900 | 300763000 | 25.8844 | 8.7503 | 735 | 540225 | 397065375 | 27.1109 | 9.0246 |
| 671 | 450241 | 302111711 | 25.9037 | 8.7547 | 736 | 541696 | 398688256 | 27.1293 | 9.0287 |
| 672 | 451584 | 303464448 | 25.9230 | 8.7590 | 737 | 543169 | 400315553 | 27.1477 | 9.0328 |
| 673 | 452929 | 304821217 | 25.9422 | 8.7634 | 738 | 544644 | 401947272 | 27.1662 | 9.0369 |
| 674 | 454276 | 306182024 | 25.9615 | 8.7677 | 739 | 546121 | 403583419 | 27.1846 | 9.0410 |
| 675 | 455625 | 307546875 | 25.9808 | 8.7721 | 740 | 547600 | 405224000 | 27.2029 | 9.0450 |
| 676 | 456976 | 308915776 | 26. | 8.7764 | 741 | 549081 | 406869021 | 27.2213 | 9.0491 |
| 677 | 458329 | 310288733 | 26.0192 | 8.7807 | 742 | 550564 | 408518488 | 27.2397 | 9.0532 |
| 678 | 459684 | 311665752 | 26.0384 | 8.7850 | 743 | 552049 | 410172407 | 27.2580 | 9.0572 |
| 679 | 461041 | 313046839 | 26.0576 | 8.7893 | 744 | 553536 | 411830784 | 27.2764 | 9.0613 |
| 680 | 462400 | 314432000 | 26.0768 | 8.7937 | 745 | 555025 | 413493625 | 27.2947 | 9.0654 |
| 681 | 463761 | 315821241 | 26.0960 | 8.7980 | 746 | 556516 | 415160936 | 27.3130 | 9.0694 |
| 682 | 465124 | 317214568 | 26.1151 | 8.8023 | 747 | 558009 | 416832723 | 27.3313 | 9.0735 |
| 683 | 466489 | 318611987 | 26.1343 | 8.8066 | 748 | 559504 | 418508992 | 27.3496 | 9.0775 |
| 684 | 467856 | 320013504 | 26.1534 | 8.8109 | 749 | 561001 | 420189749 | 27.3679 | 9.0816 |
| 685 | 469225 | 321419125 | 26.1725 | 8.8152 | 750 | 562500 | 421875000 | 27.3861 | 9.0856 |
| 686 | 470596 | 322828856 | 26.1916 | 8.8194 | 751 | 564001 | 423564751 | 27.4044 | 9.0896 |
| 687 | 471969 | 324242703 | 26.2107 | 8.8237 | 752 | 565504 | 425259008 | 27.4226 | 9.0937 |
| 688 | 473344 | 325660672 | 26.2298 | 8.8280 | 753 | 567009 | 426957777 | 27.4408 | 9.0977 |
| 689 | 474721 | 327082769 | 26.2488 | 8.8323 | 754 | 568516 | 428661064 | 27.4591 | 9.1017 |
| 690 | 476100 | 328509000 | 26.2679 | 8.8366 | 755 | 570025 | 430368875 | 27.4773 | 9.1057 |
| 691 | 477481 | 329939371 | 26.2869 | 8.8408 | 756 | 571536 | 432081216 | 27.4955 | 9.1098 |
| 692 | 478864 | 331373888 | 26.3059 | 8.8451 | 757 | 573049 | 433798093 | 27.5136 | 9.1138 |
| 693 | 480249 | 332812557 | 26.3249 | 8.8493 | 758 | 574564 | 435519512 | 27.5318 | 9.1178 |
| 694 | 481636 | 334255384 | 26.3439 | 8.8536 | 759 | 576081 | 437245479 | 27.5500 | 9.1218 |
| 695 | 483025 | 335702375 | 26.3629 | 8.8578 | 760 | 577600 | 438976000 | 27.5681 | 9.1258 |
| 696 | 484416 | 337153536 | 26.3818 | 8.8621 | 761 | 579121 | 440711081 | 27.5862 | 9.1298 |
| 697 | 485809 | 338609873 | 26.4008 | 8.8663 | 762 | 580644 | 442450728 | 27.6043 | 9.1338 |
| 698 | 487204 | 340068392 | 26.4197 | 8.8706 | 763 | 582169 | 444194947 | 27.6225 | 9.1378 |
| 699 | 488601 | 341532099 | 26.4386 | 8.8748 | 764 | 583696 | 445943744 | 27.6405 | 9.1418 |
| 700 | 490000 | 343000000 | 26.4575 | 8.8790 | 765 | 585225 | 447697125 | 27.6586 | 9.1458 |
| 701 | 491401 | 344472101 | 26.4764 | 8.8833 | 766 | 586756 | 449455096 | 27.6767 | 9.1498 |
| 702 | 492804 | 345948408 | 26.4953 | 8.8875 | 767 | 588289 | 451217663 | 27.6948 | 9.1537 |
| 703 | 494209 | 347428927 | 26.5141 | 8.8917 | 768 | 589824 | 452984832 | 27.7128 | 9.1577 |
| 704 | 495616 | 348913664 | 26.5330 | 8.8959 | 769 | 591361 | 454756609 | 27.7308 | 9.1617 |
| 705 | 497025 | 350402625 | 26.5518 | 8.9001 | 770 | 592900 | 456533000 | 27.7489 | 9.1657 |

## Tabla de los cuadrados, cubos, raíces cuadradas y cúbicas de los números de 1 á 1,000.

| No | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. | No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. |
|---|---|---|---|---|---|---|---|---|---|
| 771 | 594441 | 458314011 | 27.7669 | 9.1696 | 836 | 698896 | 584277056 | 28.9137 | 9.4204 |
| 772 | 595984 | 460099648 | 27.7849 | 9.1736 | 837 | 700569 | 586376253 | 28.9310 | 9.4241 |
| 773 | 597529 | 461889917 | 27.8029 | 9.1775 | 838 | 702244 | 588480172 | 28.9482 | 9.4279 |
| 774 | 599076 | 463684824 | 27.8209 | 9.1815 | 839 | 703921 | 590589719 | 28.9655 | 9.4316 |
| 775 | 600625 | 465484375 | 27.8388 | 9.1855 | 840 | 705600 | 592704000 | 28.9828 | 9.4354 |
| 776 | 602176 | 467288576 | 27.8568 | 9.1894 | 841 | 707281 | 594823321 | 29. | 9.4391 |
| 777 | 603729 | 469097433 | 27.8747 | 9.1933 | 842 | 708964 | 596947688 | 29.0172 | 9.4429 |
| 778 | 605284 | 470910952 | 27.8927 | 9.1973 | 843 | 710649 | 599077107 | 29.0345 | 9.4466 |
| 779 | 606841 | 472729139 | 27.9106 | 9.2012 | 844 | 712336 | 601211584 | 29.0517 | 9.4503 |
| 780 | 608400 | 474552000 | 27.9285 | 9.2052 | 845 | 714025 | 603351125 | 29.0689 | 9.4541 |
| 781 | 609961 | 476379541 | 27.9464 | 9.2091 | 846 | 715716 | 605495736 | 29.0861 | 9.4578 |
| 782 | 611524 | 478211768 | 27.9643 | 9.2130 | 847 | 717409 | 607645423 | 29.1033 | 9.4615 |
| 783 | 613089 | 480048687 | 27.9821 | 9.2170 | 848 | 719104 | 609800192 | 29.1204 | 9.4652 |
| 784 | 614656 | 481890304 | 28. | 9.2209 | 849 | 720801 | 611960049 | 29.1376 | 9.4690 |
| 785 | 616225 | 483736625 | 28.0179 | 9.2248 | 850 | 722500 | 614125000 | 29.1548 | 9.4727 |
| 786 | 617796 | 485587656 | 28.0357 | 9.2287 | 851 | 724201 | 616295051 | 29.1719 | 9.4764 |
| 787 | 619369 | 487443403 | 28.0535. | 9.2326 | 852 | 725904 | 618470208 | 29.1890 | 9.4801 |
| 788 | 620944 | 489303872 | 28.0713 | 9.2365 | 853 | 727609 | 620650477 | 29.2062 | 9.4838 |
| 789 | 622521 | 491169069 | 28.0891 | 9.2404 | 854 | 729316 | 622835864 | 29.2233 | 9.4875 |
| 790 | 624100 | 493039000 | 28.1069 | 9.2443 | 855 | 731025 | 625026375 | 29.2404 | 9.4912 |
| 791 | 625681 | 494913671 | 28.1247 | 9.2482 | 856 | 732736 | 627222016 | 29.2575 | 9.4949 |
| 792 | 627264 | 496793088 | 28.1425 | 9.2521 | 857 | 734449 | 629422793 | 29.2746 | 9.4986 |
| 793 | 628849 | 498677257 | 28.1603 | 9.2560 | 858 | 736164 | 631628712 | 29.2916 | 9.5023 |
| 794 | 630436 | 500566184 | 28.1780 | 9.2598 | 859 | 737881 | 633839779 | 29.3087 | 9.5060 |
| 795 | 632025 | 502459875 | 28.1957 | 9.2638 | 860 | 739600 | 636056000 | 29.3258 | 9.5097 |
| 796 | 633616 | 504358336 | 28.2135 | 9.2677 | 861 | 741321 | 638277381 | 29.3428 | 9.5134 |
| 797 | 635209 | 506261573 | 28.2312 | 9.2716 | 862 | 743044 | 640503928 | 29.3598 | 9.5171 |
| 798 | 636804 | 508169592 | 28.2489 | 9.2754 | 863 | 744769 | 642735647 | 29.3769 | 9.5207 |
| 799 | 638401 | 510082399 | 28.2666 | 9.2793 | 864 | 746496 | 644972544 | 29.3939 | 9.5244 |
| 800 | 640000 | 512000000 | 28.2843 | 9.2832 | 865 | 748225 | 647214625 | 29.4109 | 9.5281 |
| 801 | 641601 | 513922401 | 28.3019 | 9.2870 | 866 | 749956 | 649461896 | 29.4279 | 9.5317 |
| 802 | 643204 | 515849608 | 28.3196 | 9.2909 | 867 | 751689 | 651714363 | 29.4449 | 9.5354 |
| 803 | 644809 | 517781627 | 28.3373 | 9.2948 | 868 | 753424 | 653972032 | 29.4618 | 9.5391 |
| 804 | 646416 | 519718464 | 28.3549 | 9.2986 | 869 | 755161 | 656234909 | 29.4788 | 9.5427 |
| 805 | 648025 | 521660125 | 28.3725 | 9.3025 | 870 | 756900 | 658503000 | 29.4958 | 9.5464 |
| 806 | 649636 | 523606616 | 28.3901 | 9.3063 | 871 | 758641 | 660776311 | 29.5127 | 9.5501 |
| 807 | 651249 | 525557943 | 28.4077 | 9.3102 | 872 | 760384 | 663054848 | 29.5296 | 9.5537 |
| 808 | 652864 | 527514112 | 28.4253 | 9.3140 | 873 | 762129 | 665338617 | 29.5466 | 9.5574 |
| 809 | 654481 | 529475129 | 28.4429 | 9.3179 | 874 | 763876 | 667627624 | 29.5635 | 9.5610 |
| 810 | 656100 | 531441000 | 28.4605 | 9.3217 | 875 | 765625 | 669921875 | 29.5804 | 9.5647 |
| 811 | 657721 | 533411731 | 28.4781 | 9.3255 | 876 | 767376 | 672221376 | 29.5973 | 9.5683 |
| 812 | 659344 | 535387328 | 28.4956 | 9.3294 | 877 | 769129 | 674526133 | 29.6142 | 9.5719 |
| 813 | 660969 | 537367797 | 28.5132 | 9.3332 | 878 | 770884 | 676836152 | 29.6311 | 9.5756 |
| 814 | 662596 | 539353144 | 28.5307 | 9.3370 | 879 | 772641 | 679151439 | 29.6479 | 9.5792 |
| 815 | 664225 | 541343375 | 28.5482 | 9.3408 | 880 | 774400 | 681472000 | 29.6648 | 9.5828 |
| 816 | 665856 | 543338496 | 28.5657 | 9.3447 | 881 | 776161 | 683797841 | 29.6816 | 9.5865 |
| 817 | 667489 | 545338513 | 28.5832 | 9.3485 | 882 | 777924 | 686128968 | 29.6985 | 9.5901 |
| 818 | 669124 | 547343432 | 28.6007 | 9.3523 | 883 | 779689 | 688465387 | 29.7153 | 9.5937 |
| 819 | 670761 | 549353259 | 28.6182 | 9.3561 | 884 | 781456 | 690807104 | 29.7321 | 9.5973 |
| 820 | 672400 | 551368000 | 28.6356 | 9.3599 | 885 | 783225 | 693154125 | 29.7489 | 9.6010 |
| 821 | 674041 | 553387661 | 28.6531 | 9.3637 | 886 | 784996 | 695506456 | 29.7658 | 9.6046 |
| 822 | 675684 | 555412248 | 28.6705 | 9.3675 | 887 | 786769 | 697864103 | 29.7825 | 9.6082 |
| 823 | 677329 | 557441767 | 28.6880 | 9.3713 | 888 | 788544 | 700227072 | 29.7993 | 9.6118 |
| 824 | 678976 | 559476224 | 28.7054 | 9.3751 | 889 | 790321 | 702595369 | 29.8161 | 9.6154 |
| 825 | 680625 | 561515625 | 28.7228 | 9.3789 | 890 | 792100 | 704969000 | 29.8329 | 9.6190 |
| 826 | 682276 | 563559976 | 28.7402 | 9.3827 | 891 | 793881 | 707347971 | 29.8496 | 9.6226 |
| 827 | 683929 | 565609283 | 28.7576 | 9.3865 | 892 | 795664 | 709732288 | 29.8664 | 9.6262 |
| 828 | 685584 | 567663552 | 28.7750 | 9.3902 | 893 | 797449 | 712121957 | 29.8831 | 9.6298 |
| 829 | 687241 | 569722789 | 28.7924 | 9.3940 | 894 | 799236 | 714516984 | 29.8998 | 9.6334 |
| 830 | 688900 | 571787000 | 28.8097 | 9.3978 | 895 | 801025 | 716917375 | 29.9166 | 9.6370 |
| 831 | 690561 | 573856191 | 28.8271 | 9.4016 | 896 | 802816 | 719323136 | 29.9333 | 9.6406 |
| 832 | 692224 | 575930368 | 28.8444 | 9.4053 | 897 | 804609 | 721734273 | 29.9500 | 9.6442 |
| 833 | 693889 | 578009537 | 28.8617 | 9.4091 | 898 | 806404 | 724150792 | 29.9666 | 9.6477 |
| 834 | 695556 | 580093704 | 28.8791 | 9.4129 | 899 | 808201 | 726572699 | 29.9833 | 9.6513 |
| 835 | 697225 | 582182875 | 28.8964 | 9.4166 | 900 | 810000 | 729000000 | 30. | 9.6549 |

## Tabla de los cuadrados, cubos, raíces cuadradas y cúbicas de los números de 1 á 1,000.

| No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz cúbica. | No. | Cuadrado. | Cubo. | Raíz cuad. | Raíz. cúbica. |
|---|---|---|---|---|---|---|---|---|---|
| 901 | 811801 | 731432701 | 30.0167 | 9.6585 | 951 | 904401 | 860085351 | 30.8383 | 9.8336 |
| 902 | 813604 | 733870808 | 30.0333 | 9.6620 | 952 | 906304 | 862801408 | 30.8545 | 9.8374 |
| 903 | 815409 | 736314327 | 30.0500 | 9.6656 | 953 | 908209 | 865523177 | 30.8707 | 9.8408 |
| 904 | 817216 | 738763264 | 30.0666 | 9.6692 | 954 | 910116 | 868250664 | 30.8869 | 9.8445 |
| 905 | 819025 | 741217625 | 30.0832 | 9.6727 | 955 | 912025 | 870983875 | 30.9031 | 9.8477 |
| 906 | 820836 | 743677416 | 30.0998 | 9.6763 | 956 | 913936 | 873722816 | 30.9192 | 9.8511 |
| 907 | 822649 | 746142643 | 30.1164 | 9.6799 | 957 | 915849 | 876467493 | 30.9354 | 9.8546 |
| 908 | 824464 | 748613312 | 30.1330 | 9.6834 | 958 | 917764 | 879217912 | 30.9516 | 9.8580 |
| 909 | 826281 | 751089429 | 30.1496 | 9.6870 | 959 | 919681 | 881974079 | 30.9677 | 9.8614 |
| 910 | 828100 | 753571000 | 30.1662 | 9.6905 | 960 | 921600 | 884736000 | 30.9839 | 9.8648 |
| 911 | 829921 | 756058031 | 30.1828 | 9.6941 | 961 | 923521 | 887503681 | 31. | 9.8683 |
| 912 | 831744 | 758550528 | 30.1993 | 9.6976 | 962 | 925444 | 890277128 | 31.0161 | 9.8717 |
| 913 | 833569 | 761048497 | 30.2159 | 9.7012 | 963 | 927369 | 893056347 | 31.0322 | 9.8751 |
| 914 | 835396 | 763551944 | 30.2324 | 9.7047 | 964 | 929296 | 895841344 | 31.0483 | 9.8785 |
| 915 | 837225 | 766060875 | 30.2490 | 9.7082 | 965 | 931225 | 898632125 | 31.0644 | 9.8819 |
| 916 | 839056 | 768575296 | 30.2655 | 9.7118 | 966 | 933156 | 901428696 | 31.0805 | 9.8854 |
| 917 | 840889 | 771095213 | 30.2820 | 9.7153 | 967 | 935089 | 904231063 | 31.0966 | 9.8888 |
| 918 | 842724 | 773620632 | 30.2985 | 9.7188 | 968 | 937024 | 907039232 | 31.1127 | 9.8922 |
| 919 | 844561 | 776151559 | 30.3150 | 9.7224 | 969 | 938961 | 909853209 | 31.1288 | 9.8956 |
| 920 | 846400 | 778688000 | 30.3315 | 9.7259 | 970 | 940900 | 912673000 | 31.1448 | 9.8990 |
| 921 | 848241 | 781229961 | 30.3480 | 9.7294 | 971 | 942841 | 915498611 | 31.1609 | 9.9024 |
| 922 | 850084 | 783777448 | 30.3645 | 9.7329 | 972 | 944784 | 918330048 | 31.1769 | 9.9058 |
| 923 | 851929 | 786330467 | 30.3809 | 9.7364 | 973 | 946729 | 921167317 | 31.1929 | 9.9092 |
| 924 | 853776 | 788889024 | 30.3974 | 9.7400 | 974 | 948676 | 924010424 | 31.2090 | 9.9126 |
| 925 | 855625 | 791453125 | 30.4138 | 9.7435 | 975 | 950625 | 926859375 | 31.2250 | 9.9160 |
| 926 | 857476 | 794022776 | 30.4302 | 9.7470 | 976 | 952576 | 929714176 | 31.2410 | 9.9194 |
| 927 | 859329 | 796597983 | 30.4467 | 9.7505 | 977 | 954529 | 932574833 | 31.2570 | 9.9227 |
| 928 | 861184 | 799178752 | 30.4631 | 9.7540 | 978 | 956484 | 935441352 | 31.2730 | 9.9261 |
| 929 | 863041 | 801765089 | 30.4795 | 9.7575 | 979 | 958441 | 938313739 | 31.2890 | 9.9295 |
| 930 | 864900 | 804357000 | 30.4959 | 9.7610 | 980 | 960400 | 941192000 | 31.3050 | 9.9329 |
| 931 | 866761 | 806954491 | 30.5123 | 9.7645 | 981 | 962361 | 944076141 | 31.3209 | 9.9363 |
| 932 | 868624 | 809557568 | 30.5287 | 9.7680 | 982 | 964324 | 946966168 | 31.3369 | 9.9396 |
| 933 | 870489 | 812166237 | 30.5450 | 9.7715 | 983 | 966289 | 949862087 | 31.3528 | 9.9430 |
| 934 | 872356 | 814780504 | 30.5614 | 9.7750 | 984 | 968256 | 952763904 | 31.3688 | 9.9464 |
| 935 | 874225 | 817400375 | 30.5778 | 9.7785 | 985 | 970225 | 955671625 | 31.3847 | 9.9497 |
| 936 | 876096 | 820025856 | 30.5941 | 9.7819 | 986 | 972196 | 958585256 | 31.4006 | 9.9531 |
| 937 | 877969 | 822656953 | 30.6105 | 9.7854 | 987 | 974169 | 961504803 | 31.4166 | 9.9565 |
| 938 | 879844 | 825293672 | 30.6268 | 9.7889 | 988 | 976144 | 964430272 | 31.4325 | 9.9598 |
| 939 | 881721 | 827936019 | 30.6431 | 9.7924 | 989 | 978121 | 967361669 | 31.4484 | 9.9632 |
| 940 | 883600 | 830584000 | 30.6594 | 9.7959 | 990 | 980100 | 970299000 | 31.4643 | 9.9666 |
| 941 | 885481 | 833237621 | 30.6757 | 9.7993 | 991 | 982081 | 973242271 | 31.4802 | 9.9699 |
| 942 | 887364 | 835896888 | 30.6920 | 9.8028 | 992 | 984064 | 976191488 | 31.4960 | 9.9733 |
| 943 | 889249 | 838561807 | 30.7083 | 9.8063 | 993 | 986049 | 979146657 | 31.5119 | 9.9766 |
| 944 | 891136 | 841232384 | 30.7246 | 9.8097 | 994 | 988036 | 982107784 | 31.5278 | 9.9800 |
| 945 | 893025 | 843908625 | 30.7409 | 9.8132 | 995 | 990025 | 985074875 | 31.5436 | 9.9833 |
| 946 | 894916 | 846590536 | 30.7571 | 9.8167 | 996 | 992016 | 988047936 | 31.5595 | 9.9866 |
| 947 | 896809 | 849278123 | 30.7734 | 9.8201 | 997 | 994009 | 991026973 | 31.5753 | 9.9900 |
| 948 | 898704 | 851971392 | 30.7896 | 9.8236 | 998 | 996004 | 994011992 | 31.5911 | 9.9933 |
| 949 | 900601 | 854670349 | 30.8058 | 9.8270 | 999 | 998001 | 997002999 | 31.6070 | 9.9967 |
| 950 | 902500 | 857375000 | 30.8221 | 9.8305 | 1000 | 1000000 | 1000000000 | 31.6228 | 10. |

**Para encontrar el cuadrado ó cubo de cualquier número entero que termina en ceros.** Primero, suprímanse los ceros. Búsquese, en la tabla, el cuadrado ó cubo, según sea el caso, del número sin los ceros. Si es cuadrado agréguense después dos ceros, por cada uno de los que tenía el número y tres si es cubo. Ej., 90500 ; $905^2$=819025. Agréguense 2 veces 2 ceros; se obtiene : 81902500000. Para 90500; $905^4$=741217625; agréguense 3 veces dos ceros; se obtiene : 741217625000000.

## Raíces cuadradas y cúbicas de los números 1,000 á 10,000.

Sin errores.

| Núm. | R. cuad. | R. cúb. | Núm. | R. cuad. | R. cúb. | Núm. | R. cuad. | R. cúb. | Núm. | R. cuad. | R. cúb. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1005 | 31.70 | 10.02 | 1405 | 37.48 | 11.20 | 1805 | 42.49 | 12.18 | 2205 | 46.96 | 13.02 |
| 1010 | 31.78 | 10.03 | 1410 | 37.55 | 11.21 | 1810 | 42.54 | 12.19 | 2210 | 47.01 | 13.03 |
| 1015 | 31.86 | 10.05 | 1415 | 37.62 | 11.23 | 1815 | 42.60 | 12.20 | 2215 | 47.06 | 13.04 |
| 1020 | 31.94 | 10.07 | 1420 | 37.68 | 11.24 | 1820 | 42.66 | 12.21 | 2220 | 47.12 | 13.05 |
| 1025 | 32.02 | 10.08 | 1425 | 37.75 | 11.25 | 1825 | 42.72 | 12.22 | 2225 | 47.17 | 13.05 |
| 1030 | 32.09 | 10.10 | 1430 | 37.82 | 11.27 | 1830 | 42.78 | 12.23 | 2230 | 47.22 | 13.06 |
| 1035 | 32.17 | 10.12 | 1435 | 37.88 | 11.28 | 1835 | 42.84 | 12.24 | 2235 | 47.28 | 13.07 |
| 1040 | 32.25 | 10.13 | 1440 | 37.95 | 11.29 | 1840 | 42.90 | 12.25 | 2240 | 47.33 | 13.08 |
| 1045 | 32.33 | 10.15 | 1445 | 38.01 | 11.31 | 1845 | 42.95 | 12.26 | 2245 | 47.38 | 13.09 |
| 1050 | 32.40 | 10.16 | 1450 | 38.08 | 11.32 | 1850 | 43.01 | 12.28 | 2250 | 47.43 | 13.10 |
| 1055 | 32.48 | 10.18 | 1455 | 38.14 | 11.33 | 1855 | 43.07 | 12.29 | 2255 | 47.49 | 13.11 |
| 1060 | 32.56 | 10.20 | 1460 | 38.21 | 11.34 | 1860 | 43.13 | 12.30 | 2260 | 47.54 | 13.12 |
| 1065 | 32.63 | 10.21 | 1465 | 38.28 | 11.36 | 1865 | 43.19 | 12.31 | 2265 | 47.59 | 13.13 |
| 1070 | 32.71 | 10.23 | 1470 | 38.34 | 11.37 | 1870 | 43.24 | 12.32 | 2270 | 47.64 | 13.14 |
| 1075 | 32.79 | 10.24 | 1475 | 38.41 | 11.38 | 1875 | 43.30 | 12.33 | 2275 | 47.70 | 13.15 |
| 1080 | 32.86 | 10.26 | 1480 | 38.47 | 11.40 | 1880 | 43.36 | 12.34 | 2280 | 47.75 | 13.16 |
| 1085 | 32.94 | 10.28 | 1485 | 38.54 | 11.41 | 1885 | 43.42 | 12.35 | 2285 | 47.80 | 13.17 |
| 1090 | 33.02 | 10.29 | 1490 | 38.60 | 11.42 | 1890 | 43.47 | 12.36 | 2290 | 47.85 | 13.18 |
| 1095 | 33.09 | 10.31 | 1495 | 38.67 | 11.43 | 1895 | 43.53 | 12.37 | 2295 | 47.91 | 13.19 |
| 1100 | 33.17 | 10.32 | 1500 | 38.73 | 11.45 | 1900 | 43.59 | 12.39 | 2300 | 47.96 | 13.20 |
| 1105 | 33.24 | 10.34 | 1505 | 38.79 | 11.46 | 1905 | 43.65 | 12.40 | 2305 | 48.01 | 13.21 |
| 1110 | 33.32 | 10.35 | 1510 | 38.86 | 11.47 | 1910 | 43.70 | 12.41 | 2310 | 48.06 | 13.22 |
| 1115 | 33.39 | 10.37 | 1515 | 38.92 | 11.49 | 1915 | 43.76 | 12.42 | 2315 | 48.11 | 13.23 |
| 1120 | 33.47 | 10.38 | 1520 | 38.99 | 11.50 | 1920 | 43.82 | 12.43 | 2320 | 48.17 | 13.24 |
| 1125 | 33.54 | 10.40 | 1525 | 39.05 | 11.51 | 1925 | 43.87 | 12.44 | 2325 | 48.22 | 13.25 |
| 1130 | 33.62 | 10.42 | 1530 | 39.12 | 11.52 | 1930 | 43.93 | 12.45 | 2330 | 48.27 | 13.26 |
| 1135 | 33.69 | 10.43 | 1535 | 39.18 | 11.54 | 1935 | 43.99 | 12.46 | 2335 | 48.32 | 13.27 |
| 1140 | 33.76 | 10.45 | 1540 | 39.24 | 11.55 | 1940 | 44.05 | 12.47 | 2340 | 48.37 | 13.28 |
| 1145 | 33.84 | 10.46 | 1545 | 39.31 | 11.56 | 1945 | 44.10 | 12.48 | 2345 | 48.43 | 13.29 |
| 1150 | 33.91 | 10.48 | 1550 | 39.37 | 11.57 | 1950 | 44.16 | 12.49 | 2350 | 48.48 | 13.30 |
| 1155 | 33.99 | 10.49 | 1555 | 39.43 | 11.59 | 1955 | 44.22 | 12.50 | 2355 | 48.53 | 13.30 |
| 1160 | 34.06 | 10.51 | 1560 | 39.50 | 11.60 | 1960 | 44.27 | 12.51 | 2360 | 48.58 | 13.31 |
| 1165 | 34.13 | 10.52 | 1565 | 39.56 | 11.61 | 1965 | 44.33 | 12.53 | 2365 | 48.63 | 13.32 |
| 1170 | 34.21 | 10.54 | 1570 | 39.62 | 11.62 | 1970 | 44.38 | 12.54 | 2370 | 48.68 | 13.33 |
| 1175 | 34.28 | 10.55 | 1575 | 39.69 | 11.63 | 1975 | 44.44 | 12.55 | 2375 | 48.73 | 13.34 |
| 1180 | 34.35 | 10.57 | 1580 | 39.75 | 11.65 | 1980 | 44.50 | 12.56 | 2380 | 48.79 | 13.35 |
| 1185 | 34.42 | 10.58 | 1585 | 39.81 | 11.66 | 1985 | 44.55 | 12.57 | 2385 | 48.84 | 13.36 |
| 1190 | 34.50 | 10.60 | 1590 | 39.87 | 11.67 | 1990 | 44.61 | 12.58 | 2390 | 48.89 | 13.37 |
| 1195 | 34.57 | 10.61 | 1595 | 39.94 | 11.68 | 1995 | 44.67 | 12.59 | 2395 | 48.94 | 13.38 |
| 1200 | 34.64 | 10.63 | 1600 | 40.00 | 11.70 | 2000 | 44.72 | 12.60 | 2400 | 48.99 | 13.39 |
| 1205 | 34.71 | 10.64 | 1605 | 40.06 | 11.71 | 2005 | 44.78 | 12.61 | 2405 | 49.04 | 13.40 |
| 1210 | 34.79 | 10.66 | 1610 | 40.12 | 11.72 | 2010 | 44.83 | 12.62 | 2410 | 49.09 | 13.41 |
| 1215 | 34.86 | 10.67 | 1615 | 40.19 | 11.73 | 2015 | 44.89 | 12.63 | 2415 | 49.14 | 13.42 |
| 1220 | 34.93 | 10.69 | 1620 | 40.25 | 11.74 | 2020 | 44.94 | 12.64 | 2420 | 49.19 | 13.43 |
| 1225 | 35.00 | 10.70 | 1625 | 40.31 | 11.76 | 2025 | 45.00 | 12.65 | 2425 | 49.24 | 13.43 |
| 1230 | 35.07 | 10.71 | 1630 | 40.37 | 11.77 | 2030 | 45.06 | 12.66 | 2430 | 49.30 | 13.44 |
| 1235 | 35.14 | 10.73 | 1635 | 40.44 | 11.78 | 2035 | 45.11 | 12.67 | 2435 | 49.35 | 13.45 |
| 1240 | 35.21 | 10.74 | 1640 | 40.50 | 11.79 | 2040 | 45.17 | 12.68 | 2440 | 49.40 | 13.46 |
| 1245 | 35.28 | 10.76 | 1645 | 40.56 | 11.80 | 2045 | 45.22 | 12.69 | 2445 | 49.45 | 13.47 |
| 1250 | 35.36 | 10.77 | 1650 | 40.62 | 11.82 | 2050 | 45.28 | 12.70 | 2450 | 49.50 | 13.48 |
| 1255 | 35.43 | 10.79 | 1655 | 40.68 | 11.83 | 2055 | 45.33 | 12.71 | 2460 | 49.60 | 13.50 |
| 1260 | 35.50 | 10.80 | 1660 | 40.74 | 11.84 | 2060 | 45.39 | 12.72 | 2470 | 49.70 | 13.52 |
| 1265 | 35.57 | 10.82 | 1665 | 40.80 | 11.85 | 2065 | 45.44 | 12.73 | 2480 | 49.80 | 13.54 |
| 1270 | 35.64 | 10.83 | 1670 | 40.87 | 11.86 | 2070 | 45.50 | 12.74 | 2490 | 49.90 | 13.55 |
| 1275 | 35.71 | 10.84 | 1675 | 40.93 | 11.88 | 2075 | 45.55 | 12.75 | 2500 | 50.00 | 13.57 |
| 1280 | 35.78 | 10.86 | 1680 | 40.99 | 11.89 | 2080 | 45.61 | 12.77 | 2510 | 50.10 | 13.59 |
| 1285 | 35.85 | 10.87 | 1685 | 41.05 | 11.90 | 2085 | 45.66 | 12.78 | 2520 | 50.20 | 13.61 |
| 1290 | 35.92 | 10.89 | 1690 | 41.11 | 11.91 | 2090 | 45.72 | 12.79 | 2530 | 50.30 | 13.63 |
| 1295 | 35.99 | 10.90 | 1695 | 41.17 | 11.92 | 2095 | 45.77 | 12.80 | 2540 | 50.40 | 13.64 |
| 1300 | 36.06 | 10.91 | 1700 | 41.23 | 11.93 | 2100 | 45.83 | 12.81 | 2550 | 50.50 | 13.66 |
| 1305 | 36.12 | 10.93 | 1705 | 41.29 | 11.95 | 2105 | 45.88 | 12.82 | 2560 | 50.60 | 13.68 |
| 1310 | 36.19 | 10.94 | 1710 | 41.35 | 11.96 | 2110 | 45.93 | 12.83 | 2570 | 50.70 | 13.70 |
| 1315 | 36.26 | 10.96 | 1715 | 41.41 | 11.97 | 2115 | 45.99 | 12.84 | 2580 | 50.79 | 13.72 |
| 1320 | 36.33 | 10.97 | 1720 | 41.47 | 11.98 | 2120 | 46.04 | 12.85 | 2590 | 50.89 | 13.73 |
| 1325 | 36.40 | 10.98 | 1725 | 41.53 | 11.99 | 2125 | 46.10 | 12.86 | 2600 | 50.99 | 13.75 |
| 1330 | 36.47 | 11.00 | 1730 | 41.59 | 12.00 | 2130 | 46.15 | 12.87 | 2610 | 51.09 | 13.77 |
| 1335 | 36.54 | 11.01 | 1735 | 41.65 | 12.02 | 2135 | 46.21 | 12.88 | 2620 | 51.19 | 13.79 |
| 1340 | 36.61 | 11.02 | 1740 | 41.71 | 12.03 | 2140 | 46.26 | 12.89 | 2630 | 51.28 | 13.80 |
| 1345 | 36.67 | 11.04 | 1745 | 41.77 | 12.04 | 2145 | 46.31 | 12.90 | 2640 | 51.38 | 13.82 |
| 1350 | 36.74 | 11.05 | 1750 | 41.83 | 12.05 | 2150 | 46.37 | 12.91 | 2650 | 51.48 | 13.84 |
| 1355 | 36.81 | 11.07 | 1755 | 41.89 | 12.06 | 2155 | 46.42 | 12.92 | 2660 | 51.58 | 13.86 |
| 1360 | 36.88 | 11.08 | 1760 | 41.95 | 12.07 | 2160 | 46.48 | 12.93 | 2670 | 51.67 | 13.87 |
| 1365 | 36.95 | 11.09 | 1765 | 42.01 | 12.09 | 2165 | 46.53 | 12.94 | 2680 | 51.77 | 13.89 |
| 1370 | 37.01 | 11.11 | 1770 | 42.07 | 12.10 | 2170 | 46.58 | 12.95 | 2690 | 51.87 | 13.91 |
| 1375 | 37.08 | 11.12 | 1775 | 42.13 | 12.11 | 2175 | 46.64 | 12.96 | 2700 | 51.96 | 13.92 |
| 1380 | 37.15 | 11.13 | 1780 | 42.19 | 12.12 | 2180 | 46.69 | 12.97 | 2710 | 52.06 | 13.94 |
| 1385 | 37.22 | 11.15 | 1785 | 42.25 | 12.13 | 2185 | 46.74 | 12.98 | 2720 | 52.15 | 13.96 |
| 1390 | 37.28 | 11.16 | 1790 | 42.31 | 12.14 | 2190 | 46.80 | 12.99 | 2730 | 52.25 | 13.98 |
| 1395 | 37.35 | 11.17 | 1795 | 42.37 | 12.15 | 2195 | 46.85 | 13.00 | 2740 | 52.35 | 13.99 |
| 1400 | 37.42 | 11.19 | 1800 | 42.43 | 12.16 | 2200 | 46.90 | 13.01 | 2750 | 52.44 | 14.01 |

## Raíces cuadradas y cúbicas de los números 1,000 á 10,000.

| Núm. | R. cuad. | R. cúb. | Núm. | R. cuad. | R. cúb. | Núm. | R. cuad. | R. cúb. | Núm. | R. cuad. | R. cúb. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2760 | 52.54 | 14.03 | 3550 | 59.58 | 15.25 | 4340 | 65.88 | 16.31 | 5130 | 71.62 | 17.25 |
| 2770 | 52.63 | 14.04 | 3560 | 59.67 | 15.27 | 4350 | 65.95 | 16.32 | 5140 | 71.69 | 17.26 |
| 2780 | 52.73 | 14.06 | 3570 | 59.75 | 15.28 | 4360 | 66.03 | 16.34 | 5150 | 71.76 | 17.27 |
| 2790 | 52.82 | 14.08 | 3580 | 59.83 | 15.30 | 4370 | 66.11 | 16.35 | 5160 | 71.83 | 17.28 |
| 2800 | 52.92 | 14.09 | 3590 | 59.92 | 15.31 | 4380 | 66.18 | 16.36 | 5170 | 71.90 | 17.29 |
| 2810 | 53.01 | 14.11 | 3600 | 60.00 | 15.33 | 4390 | 66.26 | 16.37 | 5180 | 71.97 | 17.30 |
| 2820 | 53.10 | 14.13 | 3610 | 60.08 | 15.34 | 4400 | 66.33 | 16.39 | 5190 | 72.04 | 17.31 |
| 2830 | 53.20 | 14.14 | 3620 | 60.17 | 15.35 | 4410 | 66.41 | 16.40 | 5200 | 72.11 | 17.32 |
| 2840 | 53.29 | 14.16 | 3630 | 60.25 | 15.37 | 4420 | 66.48 | 16.41 | 5210 | 72.18 | 17.34 |
| 2850 | 53.39 | 14.18 | 3640 | 60.33 | 15.38 | 4430 | 66.56 | 16.42 | 5220 | 72.25 | 17.35 |
| 2860 | 53.48 | 14.19 | 3650 | 60.42 | 15.40 | 4440 | 66.63 | 16.44 | 5230 | 72.32 | 17.36 |
| 2870 | 53.57 | 14.21 | 3660 | 60.50 | 15.41 | 4450 | 66.71 | 16.45 | 5240 | 72.39 | 17.37 |
| 2880 | 53.67 | 14.23 | 3670 | 60.58 | 15.42 | 4460 | 66.78 | 16.46 | 5250 | 72.46 | 17.38 |
| 2890 | 53.76 | 14.24 | 3680 | 60.66 | 15.44 | 4470 | 66.86 | 16.47 | 5260 | 72.53 | 17.39 |
| 2900 | 53.85 | 14.26 | 3690 | 60.75 | 15.45 | 4480 | 66.93 | 16.49 | 5270 | 72.59 | 17.40 |
| 2910 | 53.94 | 14.28 | 3700 | 60.83 | 15.47 | 4490 | 67.01 | 16.50 | 5280 | 72.66 | 17.41 |
| 2920 | 54.04 | 14.29 | 3710 | 60.91 | 15.48 | 4500 | 67.08 | 16.51 | 5290 | 72.73 | 17.42 |
| 2930 | 54.13 | 14.31 | 3720 | 60.99 | 15.49 | 4510 | 67.16 | 16.52 | 5300 | 72.80 | 17.44 |
| 2940 | 54.22 | 14.33 | 3730 | 61.07 | 15.51 | 4520 | 67.23 | 16.53 | 5310 | 72.87 | 17.45 |
| 2950 | 54.31 | 14.34 | 3740 | 61.16 | 15.52 | 4530 | 67.31 | 16.55 | 5320 | 72.94 | 17.46 |
| 2960 | 54.41 | 14.36 | 3750 | 61.24 | 15.54 | 4540 | 67.38 | 16.56 | 5330 | 73.01 | 17.47 |
| 2970 | 54.50 | 14.37 | 3760 | 61.32 | 15.55 | 4550 | 67.45 | 16.57 | 5340 | 73.08 | 17.48 |
| 2980 | 54.59 | 14.39 | 3770 | 61.40 | 15.56 | 4560 | 67.53 | 16.58 | 5350 | 73.14 | 17.49 |
| 2990 | 54.68 | 14.41 | 3780 | 61.48 | 15.58 | 4570 | 67.60 | 16.59 | 5360 | 73.21 | 17.50 |
| 3000 | 54.77 | 14.42 | 3790 | 61.56 | 15.59 | 4580 | 67.68 | 16.61 | 5370 | 73.28 | 17.51 |
| 3010 | 54.86 | 14.44 | 3800 | 61.64 | 15.60 | 4590 | 67.75 | 16.62 | 5380 | 73.35 | 17.52 |
| 3020 | 54.95 | 14.45 | 3810 | 61.73 | 15.62 | 4600 | 67.82 | 16.63 | 5390 | 73.42 | 17.53 |
| 3030 | 55.05 | 14.47 | 3820 | 61.81 | 15.63 | 4610 | 67.90 | 16.64 | 5400 | 73.48 | 17.54 |
| 3040 | 55.14 | 14.49 | 3830 | 61.89 | 15.65 | 4620 | 67.97 | 16.66 | 5410 | 73.55 | 17.55 |
| 3050 | 55.23 | 14.50 | 3840 | 61.97 | 15.66 | 4630 | 68.04 | 16.67 | 5420 | 73.62 | 17.57 |
| 3060 | 55.32 | 14.52 | 3850 | 62.05 | 15.67 | 4640 | 68.12 | 16.68 | 5430 | 73.69 | 17.58 |
| 3070 | 55.41 | 14.53 | 3860 | 62.13 | 15.69 | 4650 | 68.19 | 16.69 | 5440 | 73.76 | 17.59 |
| 3080 | 55.50 | 14.55 | 3870 | 62.21 | 15.70 | 4660 | 68.26 | 16.70 | 5450 | 73.82 | 17.60 |
| 3090 | 55.59 | 14.57 | 3880 | 62.29 | 15.71 | 4670 | 68.34 | 16.71 | 5460 | 73.89 | 17.61 |
| 3100 | 55.68 | 14.58 | 3890 | 62.37 | 15.73 | 4680 | 68.41 | 16.73 | 5470 | 73.96 | 17.62 |
| 3110 | 55.77 | 14.60 | 3900 | 62.45 | 15.74 | 4690 | 68.48 | 16.74 | 5480 | 74.03 | 17.63 |
| 3120 | 55.86 | 14.61 | 3910 | 62.53 | 15.75 | 4700 | 68.56 | 16.75 | 5490 | 74.09 | 17.64 |
| 3130 | 55.95 | 14.63 | 3920 | 62.61 | 15.77 | 4710 | 68.63 | 16.76 | 5500 | 74.16 | 17.65 |
| 3140 | 56.04 | 14.64 | 3930 | 62.69 | 15.78 | 4720 | 68.70 | 16.77 | 5510 | 74.23 | 17.66 |
| 3150 | 56.12 | 14.66 | 3940 | 62.77 | 15.79 | 4730 | 68.77 | 16.79 | 5520 | 74.30 | 17.67 |
| 3160 | 56.21 | 14.67 | 3950 | 62.85 | 15.81 | 4740 | 68.85 | 16.80 | 5530 | 74.36 | 17.68 |
| 3170 | 56.30 | 14.69 | 3960 | 62.93 | 15.82 | 4750 | 68.92 | 16.81 | 5540 | 74.43 | 17.69 |
| 3180 | 56.39 | 14.71 | 3970 | 63.01 | 15.83 | 4760 | 68.99 | 16.82 | 5550 | 74.50 | 17.71 |
| 3190 | 56.48 | 14.72 | 3980 | 63.09 | 15.85 | 4770 | 69.07 | 16.83 | 5560 | 74.57 | 17.72 |
| 3200 | 56.57 | 14.74 | 3990 | 63.17 | 15.86 | 4780 | 69.14 | 16.85 | 5570 | 74.63 | 17.73 |
| 3210 | 56.66 | 14.75 | 4000 | 63.25 | 15.87 | 4790 | 69.21 | 16.86 | 5580 | 74.70 | 17.74 |
| 3220 | 56.75 | 14.77 | 4010 | 63.32 | 15.89 | 4800 | 69.28 | 16.87 | 5590 | 74.77 | 17.75 |
| 3230 | 56.83 | 14.78 | 4020 | 63.40 | 15.90 | 4810 | 69.35 | 16.88 | 5600 | 74.83 | 17.76 |
| 3240 | 56.92 | 14.80 | 4030 | 63.48 | 15.91 | 4820 | 69.43 | 16.89 | 5610 | 74.90 | 17.77 |
| 3250 | 57.01 | 14.81 | 4040 | 63.56 | 15.93 | 4830 | 69.50 | 16.90 | 5620 | 74.97 | 17.78 |
| 3260 | 57.10 | 14.83 | 4050 | 63.64 | 15.94 | 4840 | 69.57 | 16.92 | 5630 | 75.03 | 17.79 |
| 3270 | 57.18 | 14.84 | 4060 | 63.72 | 15.95 | 4850 | 69.64 | 16.93 | 5640 | 75.10 | 17.80 |
| 3280 | 57.27 | 14.86 | 4070 | 63.80 | 15.97 | 4860 | 69.71 | 16.94 | 5650 | 75.17 | 17.81 |
| 3290 | 57.36 | 14.87 | 4080 | 63.87 | 15.98 | 4870 | 69.79 | 16.95 | 5660 | 75.23 | 17.82 |
| 3300 | 57.45 | 14.89 | 4090 | 63.95 | 15.99 | 4880 | 69.86 | 16.96 | 5670 | 75.30 | 17.83 |
| 3310 | 57.53 | 14.90 | 4100 | 64.03 | 16.01 | 4890 | 69.93 | 16.97 | 5680 | 75.37 | 17.84 |
| 3320 | 57.62 | 14.92 | 4110 | 64.11 | 16.02 | 4900 | 70.00 | 16.98 | 5690 | 75.43 | 17.85 |
| 3330 | 57.71 | 14.93 | 4120 | 64.19 | 16.03 | 4910 | 70.07 | 17.00 | 5700 | 75.50 | 17.86 |
| 3340 | 57.79 | 14.95 | 4130 | 64.27 | 16.04 | 4920 | 70.14 | 17.01 | 5710 | 75.56 | 17.87 |
| 3350 | 57.88 | 14.96 | 4140 | 64.34 | 16.06 | 4930 | 70.21 | 17.02 | 5720 | 75.63 | 17.88 |
| 3360 | 57.97 | 14.98 | 4150 | 64.42 | 16.07 | 4940 | 70.29 | 17.03 | 5730 | 75.70 | 17.89 |
| 3370 | 58.05 | 14.99 | 4160 | 64.50 | 16.08 | 4950 | 70.36 | 17.04 | 5740 | 75.76 | 17.90 |
| 3380 | 58.14 | 15.01 | 4170 | 64.58 | 16.10 | 4960 | 70.43 | 17.05 | 5750 | 75.83 | 17.92 |
| 3390 | 58.22 | 15.02 | 4180 | 64.65 | 16.11 | 4970 | 70.50 | 17.07 | 5760 | 75.89 | 17.93 |
| 3400 | 58.31 | 15.04 | 4190 | 64.73 | 16.12 | 4980 | 70.57 | 17.08 | 5770 | 75.96 | 17.94 |
| 3410 | 58.40 | 15.05 | 4200 | 64.81 | 16.13 | 4990 | 70.64 | 17.09 | 5780 | 76.03 | 17.95 |
| 3420 | 58.48 | 15.07 | 4210 | 64.88 | 16.15 | 5000 | 70.71 | 17.10 | 5790 | 76.09 | 17.96 |
| 3430 | 58.57 | 15.08 | 4220 | 64.96 | 16.16 | 5010 | 70.78 | 17.11 | 5800 | 76.16 | 17.97 |
| 3440 | 58.65 | 15.10 | 4230 | 65.04 | 16.17 | 5020 | 70.85 | 17.12 | 5810 | 76.22 | 17.98 |
| 3450 | 58.74 | 15.11 | 4240 | 65.12 | 16.19 | 5030 | 70.92 | 17.13 | 5820 | 76.29 | 17.99 |
| 3460 | 58.82 | 15.12 | 4250 | 65.19 | 16.20 | 5040 | 70.99 | 17.15 | 5830 | 76.35 | 18.00 |
| 3470 | 58.91 | 15.14 | 4260 | 65.27 | 16.21 | 5050 | 71.06 | 17.16 | 5840 | 76.42 | 18.01 |
| 3480 | 58.99 | 15.15 | 4270 | 65.35 | 16.22 | 5060 | 71.13 | 17.17 | 5850 | 76.49 | 18.02 |
| 3490 | 59.08 | 15.17 | 4280 | 65.42 | 16.24 | 5070 | 71.20 | 17.18 | 5860 | 76.55 | 18.03 |
| 3500 | 59.16 | 15.18 | 4290 | 65.50 | 16.25 | 5080 | 71.27 | 17.19 | 5870 | 76.62 | 18.04 |
| 3510 | 59.25 | 15.20 | 4300 | 65.57 | 16.26 | 5090 | 71.34 | 17.20 | 5880 | 76.68 | 18.05 |
| 3520 | 59.33 | 15.21 | 4310 | 65.65 | 16.27 | 5100 | 71.41 | 17.21 | 5890 | 76.75 | 18.06 |
| 3530 | 59.41 | 15.23 | 4320 | 65.73 | 16.29 | 5110 | 71.48 | 22 | 5900 | 76.81 | 18.07 |
| 3540 | 59.50 | 15.24 | 4330 | 65.80 | 16.30 | 5120 | 71.55 | 24 | 5910 | 76.88 | 18.08 |

## Raíces cuadradas y cúbicas de los números 1,000 á 10,000.

| Núm. | R. cuad. | R. cúb. | Núm. | R. cuad. | R. cúb. | Núm. | R. cuad. | R. cúb. | Núm. | R. cuad. | R. cúb. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 5920 | 76.94 | 18.09 | 6710 | 81.91 | 18.86 | 7500 | 86.60 | 19.57 | 8290 | 91.05 | 20.24 |
| 5930 | 77.01 | 18.10 | 6720 | 81.98 | 18.87 | 7510 | 86.66 | 19.58 | 8300 | 91.10 | 20.25 |
| 5940 | 77.07 | 18.11 | 6730 | 82.04 | 18.88 | 7520 | 86.72 | 19.59 | 8310 | 91.16 | 20.26 |
| 5950 | 77.14 | 18.12 | 6740 | 82.10 | 18.89 | 7530 | 86.78 | 19.60 | 8320 | 91.21 | 20.26 |
| 5960 | 77.20 | 18.13 | 6750 | 82.16 | 18.90 | 7540 | 86.83 | 19.61 | 8330 | 91.27 | 20.27 |
| 5970 | 77.27 | 18.14 | 6760 | 82.22 | 18.91 | 7550 | 86.89 | 19.62 | 8340 | 91.32 | 20.28 |
| 5980 | 77.33 | 18.15 | 6770 | 82.28 | 18.92 | 7560 | 86.95 | 19.63 | 8350 | 91.38 | 20.29 |
| 5990 | 77.40 | 18.16 | 6780 | 82.34 | 18.93 | 7570 | 87.01 | 19.64 | 8360 | 91.43 | 20.30 |
| 6000 | 77.46 | 18.17 | 6790 | 82.40 | 18.94 | 7580 | 87.06 | 19.64 | 8370 | 91.49 | 20.30 |
| 6010 | 77.52 | 18.18 | 6800 | 82.46 | 18.95 | 7590 | 87.12 | 19.65 | 8380 | 91.54 | 20.31 |
| 6020 | 77.59 | 18.19 | 6810 | 82.52 | 18.95 | 7600 | 87.18 | 19.66 | 8390 | 91.60 | 20.32 |
| 6030 | 77.65 | 18.20 | 6820 | 82.58 | 18.96 | 7610 | 87.24 | 19.67 | 8400 | 91.65 | 20.33 |
| 6040 | 77.72 | 18.21 | 6830 | 82.64 | 18.97 | 7620 | 87.29 | 19.68 | 8410 | 91.71 | 20.34 |
| 6050 | 77.78 | 18.22 | 6840 | 82.70 | 18.98 | 7630 | 87.35 | 19.69 | 8420 | 91.76 | 20.34 |
| 6060 | 77.85 | 18.23 | 6850 | 82.76 | 18.99 | 7640 | 87.41 | 19.70 | 8430 | 91.82 | 20.35 |
| 6070 | 77.91 | 18.24 | 6860 | 82.83 | 19.00 | 7650 | 87.46 | 19.70 | 8440 | 91.87 | 20.36 |
| 6080 | 77.97 | 18.25 | 6870 | 82.89 | 19.01 | 7660 | 87.52 | 19.71 | 8450 | 91.92 | 20.37 |
| 6090 | 78.04 | 18.26 | 6880 | 82.95 | 19.02 | 7670 | 87.58 | 19.72 | 8460 | 91.98 | 20.38 |
| 6100 | 78.10 | 18.27 | 6890 | 83.01 | 19.03 | 7680 | 87.64 | 19.73 | 8470 | 92.03 | 20.38 |
| 6110 | 78.17 | 18.28 | 6900 | 83.07 | 19.04 | 7690 | 87.69 | 19.74 | 8480 | 92.09 | 20.39 |
| 6120 | 78.23 | 18.29 | 6910 | 83.13 | 19.05 | 7700 | 87.75 | 19.75 | 8490 | 92.14 | 20.40 |
| 6130 | 78.29 | 18.30 | 6920 | 83.19 | 19.05 | 7710 | 87.81 | 19.76 | 8500 | 92.20 | 20.41 |
| 6140 | 78.36 | 18.31 | 6930 | 83.25 | 19.07 | 7720 | 87.86 | 19.76 | 8510 | 92.25 | 20.42 |
| 6150 | 78.42 | 18.32 | 6940 | 83.31 | 19.07 | 7730 | 87.92 | 19.77 | 8520 | 92.30 | 20.42 |
| 6160 | 78.49 | 18.33 | 6950 | 83.37 | 19.08 | 7740 | 87.98 | 19.78 | 8530 | 92.36 | 20.43 |
| 6170 | 78.55 | 18.34 | 6960 | 83.43 | 19.09 | 7750 | 88.03 | 19.79 | 8540 | 92.41 | 20.44 |
| 6180 | 78.61 | 18.35 | 6970 | 83.49 | 19.10 | 7760 | 88.09 | 19.80 | 8550 | 92.47 | 20.45 |
| 6190 | 78.68 | 18.36 | 6980 | 83.55 | 19.11 | 7770 | 88.15 | 19.81 | 8560 | 92.52 | 20.46 |
| 6200 | 78.74 | 18.37 | 6990 | 83.61 | 19.12 | 7780 | 88.20 | 19.81 | 8570 | 92.57 | 20.46 |
| 6210 | 78.80 | 18.38 | 7000 | 83.67 | 19.13 | 7790 | 88.26 | 19.82 | 8580 | 92.63 | 20.47 |
| 6220 | 78.87 | 18.39 | 7010 | 83.73 | 19.14 | 7800 | 88.32 | 19.83 | 8590 | 92.68 | 20.48 |
| 6230 | 78.93 | 18.40 | 7020 | 83.79 | 19.15 | 7810 | 88.37 | 19.84 | 8600 | 92.74 | 20.49 |
| 6240 | 78.99 | 18.41 | 7030 | 83.85 | 19.16 | 7820 | 88.43 | 19.85 | 8610 | 92.79 | 20.50 |
| 6250 | 79.06 | 18.42 | 7040 | 83.90 | 19.17 | 7830 | 88.49 | 19.86 | 8620 | 92.84 | 20.50 |
| 6260 | 79.12 | 18.43 | 7050 | 83.96 | 19.17 | 7840 | 88.54 | 19.87 | 8630 | 92.90 | 20.51 |
| 6270 | 79.18 | 18.44 | 7060 | 84.02 | 19.18 | 7850 | 88.60 | 19.87 | 8640 | 92.95 | 20.52 |
| 6280 | 79.25 | 18.45 | 7070 | 84.08 | 19.19 | 7860 | 88.66 | 19.88 | 8650 | 93.01 | 20.53 |
| 6290 | 79.31 | 18.46 | 7080 | 84.14 | 19.20 | 7870 | 88.71 | 19.89 | 8660 | 93.06 | 20.54 |
| 6300 | 79.37 | 18.47 | 7090 | 84.20 | 19.21 | 7880 | 88.77 | 19.90 | 8670 | 93.11 | 20.54 |
| 6310 | 79.44 | 18.48 | 7100 | 84.26 | 19.22 | 7890 | 88.83 | 19.91 | 8680 | 93.17 | 20.55 |
| 6320 | 79.50 | 18.49 | 7110 | 84.32 | 19.23 | 7900 | 88.88 | 19.92 | 8690 | 93.22 | 20.56 |
| 6330 | 79.56 | 18.50 | 7120 | 84.38 | 19.24 | 7910 | 88.94 | 19.92 | 8700 | 93.27 | 20.57 |
| 6340 | 79.62 | 18.51 | 7130 | 84.44 | 19.25 | 7920 | 88.99 | 19.93 | 8710 | 93.33 | 20.57 |
| 6350 | 79.69 | 18.52 | 7140 | 84.50 | 19.26 | 7930 | 89.05 | 19.94 | 8720 | 93.38 | 20.58 |
| 6360 | 79.75 | 18.53 | 7150 | 84.56 | 19.26 | 7940 | 89.11 | 19.95 | 8730 | 93.43 | 20.59 |
| 6370 | 79.81 | 18.54 | 7160 | 84.62 | 19.27 | 7950 | 89.16 | 19.96 | 8740 | 93.49 | 20.60 |
| 6380 | 79.87 | 18.55 | 7170 | 84.68 | 19.28 | 7960 | 89.22 | 19.97 | 8750 | 93.54 | 20.61 |
| 6390 | 79.94 | 18.56 | 7180 | 84.73 | 19.29 | 7970 | 89.27 | 19.97 | 8760 | 93.59 | 20.61 |
| 6400 | 80.00 | 18.57 | 7190 | 84.79 | 19.30 | 7980 | 89.33 | 19.98 | 8770 | 93.65 | 20.62 |
| 6410 | 80.06 | 18.58 | 7200 | 84.85 | 19.31 | 7990 | 89.39 | 19.99 | 8780 | 93.70 | 20.63 |
| 6420 | 80.12 | 18.59 | 7210 | 84.91 | 19.32 | 8000 | 89.44 | 20.00 | 8790 | 93.75 | 20.64 |
| 6430 | 80.19 | 18.60 | 7220 | 84.97 | 19.33 | 8010 | 89.50 | 20.01 | 8800 | 93.81 | 20.65 |
| 6440 | 80.25 | 18.60 | 7229 | 85.03 | 19.34 | 8020 | 89.55 | 20.02 | 8810 | 93.86 | 20.65 |
| 6450 | 80.31 | 18.61 | 7240 | 85.09 | 19.35 | 8030 | 89.61 | 20.02 | 8820 | 93.91 | 20.66 |
| 6460 | 80.37 | 18.62 | 7250 | 85.15 | 19.35 | 8040 | 89.67 | 20.03 | 8830 | 93.97 | 20.67 |
| 6470 | 80.44 | 18.63 | 7260 | 85.21 | 19.36 | 8050 | 89.72 | 20.04 | 8840 | 94.02 | 20.68 |
| 6480 | 80.50 | 18.64 | 7270 | 85.26 | 19.37 | 8060 | 89.78 | 20.05 | 8850 | 94.07 | 20.68 |
| 6490 | 80.56 | 18.65 | 7280 | 85.32 | 19.38 | 8070 | 89.83 | 20.06 | 8860 | 94.13 | 20.69 |
| 6500 | 80.62 | 18.66 | 7290 | 85.38 | 19.39 | 8080 | 89.89 | 20.07 | 8870 | 94.18 | 20.70 |
| 6510 | 80.68 | 18.67 | 7300 | 85.44 | 19.40 | 8090 | 89.94 | 20.07 | 8880 | 94.23 | 20.71 |
| 6520 | 80.75 | 18.68 | 7310 | 85.50 | 19.41 | 8100 | 90.00 | 20.08 | 8890 | 94.29 | 20.72 |
| 6530 | 80.81 | 18.69 | 7320 | 85.56 | 19.42 | 8110 | 90.06 | 20.09 | 8900 | 94.34 | 20.72 |
| 6540 | 80.87 | 18.70 | 7330 | 85.62 | 19.43 | 8120 | 90.11 | 20.10 | 8910 | 94.39 | 20.73 |
| 6550 | 80.93 | 18.71 | 7340 | 85.67 | 19.43 | 8130 | 90.17 | 20.11 | 8920 | 94.45 | 20.74 |
| 6560 | 80.99 | 18.72 | 7350 | 85.73 | 19.44 | 8140 | 90.22 | 20.12 | 8930 | 94.50 | 20.75 |
| 6570 | 81.06 | 18.73 | 7360 | 85.79 | 19.45 | 8150 | 90.28 | 20.12 | 8940 | 94.55 | 20.75 |
| 6580 | 81.12 | 18.74 | 7370 | 85.85 | 19.46 | 8160 | 90.33 | 20.13 | 8950 | 94.60 | 20.76 |
| 6590 | 81.18 | 18.75 | 7380 | 85.91 | 19.47 | 8170 | 90.39 | 20.14 | 8960 | 94.66 | 20.77 |
| 6600 | 81.24 | 18.76 | 7390 | 85.97 | 19.48 | 8180 | 90.44 | 20.15 | 8970 | 94.71 | 20.78 |
| 6610 | 81.30 | 18.77 | 7400 | 86.02 | 19.49 | 8190 | 90.50 | 20.16 | 8980 | 94.76 | 20.79 |
| 6620 | 81.36 | 18.78 | 7410 | 86.08 | 19.50 | 8200 | 90.55 | 20.17 | 8990 | 94.82 | 20.79 |
| 6630 | 81.42 | 18.79 | 7420 | 86.14 | 19.50 | 8210 | 90.61 | 20.17 | 9000 | 94.87 | 20.80 |
| 6640 | 81.49 | 18.80 | 7430 | 86.20 | 19.51 | 8220 | 90.66 | 20.18 | 9010 | 94.92 | 20.81 |
| 6650 | 81.55 | 18.81 | 7440 | 86.26 | 19.52 | 8230 | 90.72 | 20.19 | 9020 | 94.97 | 20.82 |
| 6660 | 81.61 | 18.81 | 7450 | 86.31 | 19.53 | 8240 | 90.77 | 20.20 | 9030 | 95.03 | 20.82 |
| 6670 | 81.67 | 18.82 | 7460 | 86.37 | 19.54 | 8250 | 90.83 | 20 21 | 9040 | 95.08 | 20.83 |
| 6680 | 81.73 | 18.83 | 7470 | 86.43 | 19.55 | 8260 | 90.88 | 20.21 | 9050 | 95.13 | 20.84 |
| 6690 | 81.79 | 18.84 | 7480 | 86.49 | 19.56 | 8270 | 90.94 | 20.22 | 9060 | 95.18 | 20.85 |
| 6700 | 81.85 | 18.85 | 7490 | 86.54 | 19.57 | 8280 | 90.99 | 20.23 | 9070 | 95.24 | 20.85 |

## Raíces cuadradas y cúbicas de los números 1,000 á 10,000.

| Núm. | R. cuad. | R. cúb. | Núm. | R. cuad. | R. cúb. | Núm. | R. cuad. | R. cúb. | Núm. | R. cuad. | R. cúb. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 9080 | 95.29 | 20.86 | 9320 | 96.54 | 21.04 | 9550 | 97.72 | 21.22 | 9780 | 98.89 | 21.39 |
| 9090 | 95.34 | 20.87 | 9330 | 96.59 | 21.05 | 9560 | 97.78 | 21.22 | 9790 | 98.94 | 21.39 |
| 9100 | 95.39 | 20.88 | 9340 | 96.64 | 21.06 | 9570 | 97.83 | 21.23 | 9800 | 98.99 | 21.40 |
| 9110 | 95.45 | 20.89 | 9350 | 96.70 | 21.07 | 9580 | 97.88 | 21.24 | 9810 | 99.05 | 21.41 |
| 9120 | 95.50 | 20.89 | 9360 | 96.75 | 21.07 | 9590 | 97.93 | 21.25 | 9820 | 99.10 | 21.41 |
| 9130 | 95.55 | 20.90 | 9370 | 96.80 | 21.08 | 9600 | 97.98 | 21.25 | 9830 | 99.15 | 21.42 |
| 9140 | 95.60 | 20.91 | 9380 | 96.85 | 21.09 | 9610 | 98.03 | 21.26 | 9840 | 99.20 | 21.43 |
| 9150 | 95.66 | 20.92 | 9390 | 96.90 | 21.10 | 9620 | 98.08 | 21.27 | 9850 | 99.25 | 21.44 |
| 9160 | 95.71 | 20.92 | 9400 | 96.95 | 21.10 | 9630 | 98.13 | 21.28 | 9860 | 99.30 | 21.44 |
| 9170 | 95.76 | 20.93 | 9410 | 97.01 | 21.11 | 9640 | 98.18 | 21.28 | 9870 | 99.35 | 21.45 |
| 9180 | 95.81 | 20.94 | 9420 | 97.06 | 21.12 | 9650 | 98.23 | 21.29 | 9880 | 99.40 | 21.46 |
| 9190 | 95.86 | 20.95 | 9430 | 97.11 | 21.13 | 9660 | 98.29 | 21.30 | 9890 | 99.45 | 21.47 |
| 9200 | 95.92 | 20.95 | 9440 | 97.16 | 21.13 | 9670 | 98.34 | 21.30 | 9900 | 99.50 | 21.47 |
| 9210 | 95.97 | 20.96 | 9450 | 97.21 | 21.14 | 9680 | 98.39 | 21.31 | 9910 | 99.55 | 21.48 |
| 9220 | 96.02 | 20.97 | 9460 | 97.26 | 21.15 | 9690 | 98.44 | 21.32 | 9920 | 99.60 | 21.49 |
| 9230 | 96.07 | 20.98 | 9470 | 97.31 | 21.16 | 9700 | 98.49 | 21.33 | 9930 | 99.65 | 21.49 |
| 9240 | 96.12 | 20.98 | 9480 | 97.37 | 21.16 | 9710 | 98.54 | 21.33 | 9940 | 99.70 | 21.50 |
| 9250 | 96.18 | 20.99 | 9490 | 97.42 | 21.17 | 9720 | 98.59 | 21.34 | 9950 | 99.75 | 21.51 |
| 9260 | 96.23 | 21.00 | 9500 | 97.47 | 21.18 | 9730 | 98.64 | 21.35 | 9960 | 99.80 | 21.52 |
| 9270 | 96.28 | 21.01 | 9510 | 97.52 | 21.19 | 9740 | 98.69 | 21.36 | 9970 | 99.85 | 21.52 |
| 9280 | 96.33 | 21.01 | 9520 | 97.57 | 21.19 | 9750 | 98.74 | 21.36 | 9980 | 99.90 | 21.53 |
| 9290 | 96.38 | 21.02 | 9530 | 97.62 | 21.20 | 9760 | 98.79 | 21.37 | 9990 | 99.95 | 21.54 |
| 9300 | 96.44 | 21.03 | 9540 | 97.67 | 21.21 | 9770 | 98.84 | 21.38 | 10000 | 100.00 | 21.54 |
| 9310 | 96.49 | 21.04 | | | | | | | | | |

**Para encontrar las raíces cuadradas ó cúbicas de números grandes, que no están contenidos en la columna de números de la tabla, se procede así :**

Dichas raíces pueden ser algunas veces tomadas de la tabla directamente, considerando como columnas de números las columnas de las potencias, y las de los números como las de raíces. Así, si se desea la raíz cuad de 25281, búsquese primero ese número en la columna de los cuadrados y frente á él en la columna de los números está 159 que es su raíz cuadrada. Para la raíz cúbica de 857375, búsquese este número en la columna de los cubos y opuesto á él, en la columna de los números, está 95, que es su raíz cúbica. Cuando no está contenido, en la columna de los cuads ó cúbs, el número exacto, podemos usar en su lugar el número más cercano á él, si no se requiere grande exactitud; pero cuando ésta es requerida se emplean los métodos siguientes :

### Para la raíz cuadrada.

Esta regla es aplicable tanto á los números enteros como á aquellos que son decimales en parte. Primero tómese de la tabla y de la manera explicada el número más cercano al número dado, también su raíz cuadrada; multiplíquese este número de la tabla por 3 y agréguese al producto el número dado. Llámese la suma A. Multiplíquese entonces el número dado por 3, agréguese al producto el número de la tabla y llámese la suma B. Entonces

A : B : : la raíz de la tabla : la raíz requerida.

Ej. : Supongamos que el número dado sea 946.53. Encontramos, en este caso, que el número más cercano de la tabla es 947 y su raíz cuad en la tabla 30.7734. Por consiguiente

$$\left.\begin{array}{l} 947 \\ 3 \\ \hline 2841 \\ 946.53 = \text{núm dado.} \\ \hline 3787.53 = \text{A.} \end{array}\right\} \text{ y } \left\{\begin{array}{l} 946.53 = \text{núm dado.} \\ 3 \\ \hline 2839.59 \\ 947 = \text{núm de la tabla.} \\ \hline 3786.59 = \text{B.} \end{array}\right.$$

Entonces A B raíz de la tabla. raíz que se busca.

3787.53 : 3786.59 : : 30.7734 : 30.7657+

Si se extrae la raíz exactamente es también 30.7657 +.

### Para la raíz cúbica.

Esta regla es aplicable tanto á números enteros como á los que tienen una parte decimal. Tómense primero en la tabla el número más cercano al número dado y su raíz cúbica; multiplíquese este número de la tabla por 2, agréguese al producto el número dado y llámese la suma A. Multiplíquese entonces el número dado por 2 y agréguese al producto el número de la tabla y llámese la suma B. Entonces

A : B : : la raíz de la tabla : la raíz que se busca.

Ej. : Supongamos que el número dado sea 7,368. Encontramos, en este caso, que el número de la tabla más cercano (en la columna de cubos) es 6,859 y su raíz cúbica, 19. Por consiguiente

$$\left.\begin{array}{l} 6859 = \text{núm de la tabla.} \\ \quad 2 \\ \hline 13718 \\ \quad 7368 = \text{núm dado.} \\ \hline 21086 = \text{A.} \end{array}\right\} \text{y} \left\{\begin{array}{l} 7368 = \text{núm dado.} \\ \quad 2 \\ \hline 14736 \\ \quad 6859 = \text{núm de la tabla.} \\ \hline 21595 = \text{B.} \end{array}\right.$$

| A | | B | | raíz de la tabla. | | raíz que se busca. |
|---|---|---|---|---|---|---|
| 21,086 | : | 21,595 | : : | 19 | : | 19.4585 |

La raíz extraída algebraicamente es 19.4588. El ingeniero rara vez necesita mayor grado de exactitud, por tanto es preferible este procedimiento al procedimiento laborioso ordinario.

### Para encontrar la raíz cuadrada de una fracción decimal.

Muy simple y exacto hasta la tercera cifra inclusive. Si el número no contiene, por lo menos cinco cifras, *contando desde la primera cifra significativa inclusive*, agréguense uno ó más ceros á fin de hacer cinco; si después de esto el número total no se puede separar de dos en dos cifras, agréguese otro cero á fin de poderlo hacer. Comenzando entonces por la primera cifra significativa é incluyéndola, se supone que el número es entero. Se busca en la tabla el número más cercano á éste; se toma su raíz cuadrada en la tabla y se corre el punto en esta raíz de la tabla, hacia la izquierda, tantos lugares cuantas cifras haya en la mitad de las cifras decimales del número modificado como se dijo.

Ej. : ¿Cuál es la raíz cuadrada de la fracción decimal 0.002? En este caso, con el objeto de tener, por lo menos, cinco cifras decimales, contando desde la primera cifra significativa (2) é incluyéndola, agréguense ceros así : 0.00.20.000 ; pero como no se pueden separar de dos en dos, agréguese otro cero así: 0.00.20.00.00. Comenzando entonces par la cifra significativa (2), supóngase que el numero es : 200000. El más cercano á éste en la tabla es 199809 y la raíz cuadrada de éste es 447. El número decimal, modificado, fué : 0.00.20.00.00; tiene ocho cifras; la mitad, cuatro; córrase por tanto el punto en la raíz 447 cuatro lugares hacia la izquierda : haciéndolo 0.0447. Esta es la raíz cuadrada de 0.002, exacta hasta la tercera cifra inclusive.

**Encontrar la raíz cúbica de una fracción decimal.** Muy simple y exacta hasta la tercera cifra. Si el número no contiene por lo menos cinco cifras á contar de la primera significativa inclusive, agréguense uno ó más ceros para formar cinco; si después de esto el número no se puede separar de tres en tres, agréguense uno ó más ceros para poderlo hacer, comenzando entonces por la primera cifra significativa inclusive y se considera el número desde ella á la derecha. Se buséa en la tabla el número más cercano á éste; se toma su raíz cúbica en la tabla y se corre el punto hacia la izquierda tantos lugares cuantas cifras haya en la tercera parte de las que contiene el número modificado.

Ej. : ¿Cuál es la raíz cúbica de la fracción decimal 0.002? En este caso, con el objeto de tener, por lo menos, cinco cifras decimales, contando desde la primera cifra significativa (2), agréguense ceros así : 0.002.000.0, pero como no se pueden separar de tres en tres, agréguense dos ceros más así 0.002.000.000. Considerando entonces desde la primera cifra significativa (2) supóngase que el número es : 2.000.000. El cubo más cercano de este número en la tabla en la columna de los cubos es 2000376 y su raíz cúbica es 126. El número decima

tal como se modificó finalmente es : 0.002000000; tiene nueve cifras, la tercera parte es 3; córrase por tanto el punto en la raíz 126 tres lugares hacia la izquierda, lo que da : 0.126. Esta es la raíz cúbica requerida de la decimal 0.002, exacta hasta la tercera cifra inclusive.

## Raíces quintas y quintas potencias.

| Poten-cia. | No. ó raíz. | Poten-cia. | No. ó raíz. | Poten-cia. | No. ó raíz. | Poten-cia. | No. ó raíz. | Poten-cia. | No. ó raíz. | Poten-cia. | No. ó raíz. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| .0000100 | .1 | .000142 | .170 | .004219 | .335 | .077760 | .60 | .695688 | .93 | 8.11868 | 1.52 |
| | | .000164 | .175 | .004544 | .340 | .084480 | .61 | .733904 | .94 | 8.66171 | 1.54 |
| .0000110 | .102 | .000189 | .18 | .004888 | .345 | .091613 | .62 | .773781 | .95 | 9.23896 | 1.56 |
| | | .000217 | .185 | .005252 | .350 | .099244 | .63 | .815373 | .96 | 9.84658 | 1.58 |
| .0000122 | .104 | .000248 | .190 | .005638 | .355 | .107374 | .64 | .858734 | .97 | 10.4858 | 1.60 |
| | | .000282 | .195 | .006047 | .360 | .116029 | .65 | .903921 | .98 | 11.1577 | 1.62 |
| .0000134 | .106 | .000320 | .200 | .006478 | .365 | .125233 | .66 | .950960 | .99 | 11.8637 | 1.64 |
| | | .000362 | .205 | .006934 | .370 | .135012 | .67 | 1. | 1. | 12.6049 | 1.66 |
| .0000147 | .108 | .000408 | .210 | .007416 | .375 | .145393 | .68 | 1.10408 | 1.02 | 13.3828 | 1.68 |
| .0000161 | .110 | .000459 | .215 | .007924 | .380 | .156403 | .69 | 1.21665 | 1.04 | 14.1986 | 1.70 |
| .0000176 | .112 | .000515 | .220 | .008459 | .385 | .168070 | .70 | 1.33823 | 1.06 | 15.0537 | 1.72 |
| .0000193 | .114 | .000577 | .225 | .009022 | .390 | .180423 | .71 | 1.46933 | 1.08 | 15.9495 | 1.74 |
| .0000210 | .116 | .000644 | .230 | .009616 | .395 | .193492 | .72 | 1.61051 | 1.10 | 16.8874 | 1.76 |
| .0000229 | .118 | .000717 | .235 | .010240 | .400 | .207307 | .73 | 1.76234 | 1.12 | 17.8690 | 1.78 |
| .0000249 | .120 | .000796 | .240 | .011586 | .41 | .221901 | .74 | 1.92541 | 1.14 | 18.8957 | 1.80 |
| .0000270 | .122 | .000883 | .245 | .013089 | .42 | .237305 | .75 | 2.10034 | 1.16 | 19.9690 | 1.82 |
| .0000293 | .124 | .000977 | .250 | .014701 | .43 | .253553 | .76 | 2.28775 | 1.18 | 21.0906 | 1.84 |
| .0000318 | .126 | .001078 | .255 | .016492 | .44 | .270678 | .77 | 2.48832 | 1.20 | 22.2620 | 1.86 |
| .0000344 | .128 | .001188 | .260 | .018453 | .45 | .288717 | .78 | 2.70271 | 1.22 | 23.4849 | 1.88 |
| .0000371 | .130 | .001307 | .265 | .020596 | .46 | .307706 | .79 | 2.93163 | 1.24 | 24.7610 | 1.90 |
| .0000401 | .132 | .001435 | .270 | .022935 | .47 | .327680 | .80 | 3.17580 | 1.26 | 26.0919 | 1.92 |
| .0000432 | .134 | .001573 | .275 | .025480 | .48 | .348678 | .81 | 3.43597 | 1.28 | 27.4795 | 1.94 |
| .0000465 | .136 | .001721 | .280 | .028248 | .49 | .370740 | .82 | 3.71293 | 1.30 | 28.9255 | 1.96 |
| .0000500 | .138 | .001880 | .285 | .031250 | .50 | .393904 | .83 | 4.00746 | 1.32 | 30.4317 | 1.98 |
| .0000538 | .140 | .002051 | .290 | .034503 | .51 | .418212 | .84 | 4.32040 | 1.34 | 32.0000 | 2.00 |
| .0000578 | .142 | .002234 | .295 | .038020 | .52 | .443705 | .85 | 4.65259 | 1.36 | 33.2051 | 2.05 |
| .0000619 | .144 | .002430 | .300 | .041820 | .53 | .470427 | .86 | 5.00490 | 1.38 | 40.8410 | 2.10 |
| .0000663 | .146 | .002639 | .305 | .045917 | .54 | .498421 | .87 | 5.37824 | 1.40 | 45.9401 | 2.15 |
| .0000710 | .148 | .002863 | .310 | .050328 | .55 | .527732 | .88 | 5.77353 | 1.42 | 51.5363 | 2.20 |
| .0000754 | .150 | .003101 | .315 | .055073 | .56 | .558406 | .89 | 6.19174 | 1.44 | 57.6650 | 2.25 |
| .0000895 | .155 | .003355 | .320 | .060169 | .57 | .590490 | .90 | 6.63383 | 1.46 | 64.3634 | 2.30 |
| .000105 | .160 | .003626 | .325 | .065636 | .58 | .624032 | .91 | 7.10082 | 1.48 | 71.6703 | 2.35 |
| .000122 | .165 | .003914 | .330 | .071493 | .59 | .659082 | .92 | 7.59375 | 1.50 | 79.6252 | 2.40 |

## Raíces quintas y quintas potencias.

| No. | Potencia. | No. ó raíz. | Potencia. | No. ó raíz. | Potencia. | No. ó raíz. | Potencia. | No. ó raíz. | Potencia. | No. ó raíz. | Potencia. | No. ó raíz. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 88.2735 | 2.45 | 2824.75 | 4.90 | 85873 | 9.80 | 2609193 | 19.4 | 20511149 | 29.0 | 69165024 | 48. |
| 97.6562 | 2.50 | 2971.84 | 4.95 | 90392 | 9.90 | 2747949 | 19.5 | 21228256 | 29.2 | 70328437 | 49. |
| 107.320 | 2.55 | 3125.00 | 5.00 | 95099 | 10.00 | 2892767 | 19.6 | 21965275 | 29.4 | 71537176 | 50. |
| 118.014 | 2.60 | 3450.25 | 5.10 | 100000 | 10.0 | 3043168 | 19.8 | 22722628 | 29.6 | 72792057 | 51. |
| 130.096 | 2.65 | 3802.04 | 5.20 | 110408 | 10.2 | 3200000 | 20.0 | 23500128 | 29.8 | 74093878 | 52. |
| 143.489 | 2.70 | 4181.35 | 5.30 | 121665 | 10.4 | 3362324 | 20.2 | 24300000 | 30.0 | 75442299 | 53. |
| 157.276 | 2.75 | 4591.65 | 5.40 | 133823 | 10.6 | 3530309 | 20.4 | 25123631 | 30.5 | 77760000 | 55. |
| 172.104 | 2.80 | 5032.84 | 5.50 | 146933 | 10.8 | 3703977 | 20.6 | 25971951 | 31.0 | 80156961 | 57. |
| 188.029 | 2.85 | 5507.32 | 5.60 | 161051 | 11.0 | 3883299 | 20.8 | 26845964 | 31.5 | 82631281 | 59. |
| 205.111 | 2.90 | 6016.92 | 5.70 | 176234 | 11.2 | 4068101 | 21.0 | 27744132 | 32.0 | 85184364 | 61. |
| 223.414 | 2.95 | 6563.57 | 5.80 | 192541 | 11.4 | 4258322 | 21.2 | 28669082 | 32.5 | 87818424 | 63. |
| 243.000 | 3.00 | 7149.24 | 5.90 | 210034 | 11.6 | 4454166 | 21.4 | 29618393 | 33.0 | 90535005 | 65. |
| 263.936 | 3.05 | 7776.00 | 6.00 | 228776 | 11.8 | 4701850 | 21.6 | 30591410 | 33.5 | 93335576 | 66. |
| 286.292 | 3.10 | 8445.96 | 6.10 | 248832 | 12.0 | 4923597 | 21.8 | 31589424 | 34.0 | 96221107 | 68. |
| 310.136 | 3.15 | 9161.33 | 6.20 | 270271 | 12.2 | 5153632 | 22.0 | 32612875 | 34.5 | 99193356 | 69. |
| 335.544 | 3.20 | 9924.37 | 6.30 | 293163 | 12.4 | 5392186 | 22.2 | 33662175 | 35.0 | 102253349 | 71. |
| 362.591 | 3.25 | 10737 | 6.40 | 317580 | 12.6 | 5639493 | 22.4 | 34737167 | 35.5 | 105407000 | 73. |
| 391.354 | 3.30 | 11603 | 6.50 | 343597 | 12.8 | 5895793 | 22.6 | 35836716 | 36.0 | 108629351 | 74. |
| 421.419 | 3.35 | 12523 | 6.60 | 371293 | 13.0 | 6161327 | 22.8 | 36961857 | 36.5 | 111948176 | 76. |
| 454.354 | 3.40 | 13501 | 6.70 | 400746 | 13.2 | 6436343 | 23.0 | 38113947 | 37.0 | 115363799 | 78. |
| 488.760 | 3.45 | 14539 | 6.80 | 432040 | 13.4 | 6721093 | 23.2 | 39292716 | 37.5 | 118900654 | 79. |
| 525.219 | 3.50 | 15640 | 6.90 | 465259 | 13.6 | 7015834 | 23.4 | 40498168 | 38.0 | 122546875 | 81. |
| 563.822 | 3.55 | 16807 | 7.00 | 500490 | 13.8 | 7320825 | 23.6 | 41731005 | 38.5 | 126305253 | 83. |
| 604.662 | 3.60 | 18042 | 7.10 | 537824 | 14.0 | 7636332 | 23.8 | 42991499 | 39.0 | 130178415 | 85. |
| 647.835 | 3.65 | 19349 | 7.20 | 577353 | 14.2 | 7962624 | 24.0 | 44279852 | 39.5 | 134171436 | 87. |
| 693.440 | 3.70 | 20731 | 7.30 | 619174 | 14.4 | 8299976 | 24.2 | 45596000 | 40.0 | 138285399 | 89. |
| 741.577 | 3.75 | 22190 | 7.40 | 663383 | 14.6 | 8648666 | 24.4 | 46940203 | 40.5 | 142526000 | 90. |
| 792.352 | 3.80 | 23730 | 7.50 | 710082 | 14.8 | 9008978 | 24.6 | 48313856 | 41.0 | 146878401 | 92. |
| 845.870 | 3.85 | 25355 | 7.60 | 759375 | 15.0 | 9381200 | 24.8 | 49716200 | 41.5 | 151359520 | 94. |
| 902.242 | 3.90 | 27068 | 7.70 | 811368 | 15.2 | 9765625 | 25.0 | 51148232 | 42.0 | 155968232 | 96. |
| 961.580 | 3.95 | 28872 | 7.80 | 866171 | 15.4 | 10162550 | 25.2 | 52610250 | 42.5 | 160677910 | 98. |
| 1024.00 | 4.00 | 30771 | 7.90 | 923896 | 15.6 | 10571478 | 25.4 | 54102443 | 43.0 | 165505425 | 99. |
| 1089.62 | 4.05 | 32768 | 8.00 | 984658 | 15.8 | 10995116 | 25.6 | 55626033 | 43.5 | 170427018 | 101. |
| 1158.56 | 4.10 | 34868 | 8.10 | 1048576 | 16.0 | 11431377 | 25.8 | 57180172 | 44.0 | 175464209 | 103. |
| 1230.95 | 4.15 | 37074 | 8.20 | 1115771 | 16.2 | 11881376 | 26.0 | 58765858 | 44.5 | 180627018 | 105. |
| 1306.91 | 4.20 | 39390 | 8.30 | 1186367 | 16.4 | 12345437 | 26.2 | 60384145 | 45.0 | 185884945 | 107. |
| 1386.58 | 4.25 | 41821 | 8.40 | 1260493 | 16.6 | 12823886 | 26.4 | 62035045 | 45.5 | 191249000 | 109. |
| 1470.08 | 4.30 | 44371 | 8.50 | 1338278 | 16.8 | 13317055 | 26.6 | 63720162 | 46.0 | 196721451 | 111. |
| 1557.57 | 4.35 | 47043 | 8.60 | 1419857 | 17.0 | 13825281 | 26.8 | 65440615 | 46.5 | 202301852 | 113. |
| 1649.16 | 4.40 | 49842 | 8.70 | 1505366 | 17.2 | 14348907 | 27.0 | 67196036 | 47.0 | 207990232 | 115. |
| 1745.02 | 4.45 | 52773 | 8.80 | 1594947 | 17.4 | 14888280 | 27.2 | 68987161 | 47.5 | 213786343 | 117. |
| 1845.28 | 4.50 | 55841 | 8.90 | 1688742 | 17.6 | 15443752 | 27.4 | 70815836 | 48.0 | 219693978 | 119. |
| 1950.10 | 4.55 | 59049 | 9.00 | 1786899 | 17.8 | 16015681 | 27.6 | 72681843 | 48.5 | 225713269 | 121. |
| 2059.63 | 4.60 | 62403 | 9.10 | 1889568 | 18.0 | 16604430 | 27.8 | 74585999 | 49.0 | 231847102 | 123. |
| 2174.03 | 4.65 | 65908 | 9.20 | 1996903 | 18.2 | 17210368 | 28.0 | 76530121 | 49.5 | 238095798 | 125. |
| 2293.45 | 4.70 | 69569 | 9.30 | 2109061 | 18.4 | 17833868 | 28.2 | 78514000 | 50.0 | 244460000 | 127. |
| 2418.07 | 4.75 | 73390 | 9.40 | 2226203 | 18.6 | 18475309 | 28.4 | 80538251 | 51. | | |
| 2548.04 | 4.80 | 77378 | 9.50 | 2348493 | 18.8 | 19135075 | 28.6 | 82604032 | 52. | | |
| 2683.54 | 4.85 | 81537 | 9.60 | 2476099 | 19.0 | 19813557 | 28.8 | 84712493 | 53. | | |

## Raíz cuadrada de la quinta potencia de los números.

$\sqrt{n^5}$, ó potencia 5/2 de los números, $n$.

(Véase la tabla siguiente.)

La columna encabezada 12 $n$, facilita el uso de la tabla en los casos, por ej., en que la cantidad está dada en *pulgadas* y cuando se desea obtener la potencia $^5/_2$ de la misma cantidad en *pies*. Supongamos, por ej., que se tengan tubos de ½ pulg. y que se desee la potencia $^5/_2$ del diámetro en pies. Encuéntrese $^5/_2$ (diám. en pulg) en la columna 12 $n$, opuesto á éste en la colum. $n$ está 0.041666 (diám en pies) y en la columna $n$ $^5/_2$, 0.00035 (la potencia $^5/_2$ del diám. 0.041666 en pies).

Los valores de $n$ que terminan en 0 ó en 5 son exactos. Todos los otros terminan en decimales que se repiten, por ej., $n$=0.052083 significa que $n$= 0.05208333.....

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*N. del T.* — Cuando el diámetro del tubo se tenga en *centímetros* ó *decímetros*, ó *metros*, ó cualquiera de sus fracciones, sirve esta misma tabla para encontrar las respectivas « raíces cuad de las quintas potencias » prescindiendo de la columna 12 $n$ y cuando sólo las columnas de los números $n$ y la de los $n^{\frac{5}{2}}$.

## Raíces cuadradas de la quinta potencia de los números.

(Véase la explicación anterior.)

| 12 n | n | $n^{\frac{5}{2}}$ | 12 n | n | $n^{\frac{5}{2}}$ | 12 n | n | $n^{\frac{5}{2}}$ | 12 n | n | $n^{\frac{5}{2}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ¼ | 0.020833 | 0.000063 | 22 | 1.8333 | 4.5510 | 84 | 7.000 | 129.64 | 468 | 39 | 9499 |
| ⅜ | 0.031250 | 0.000173 | 23 | 1.9166 | 5.0859 | 85 | 7.083 | 133.53 | 480 | 40 | 10119 |
| ½ | 0.041666 | 0.000354 | 24 | 2.0000 | 5.6569 | 86 | 7.166 | 137.50 | 492 | 41 | 10764 |
| ⅝ | 0.052083 | 0.000619 | 25 | 2.0833 | 6.2647 | 87 | 7.250 | 141.53 | 504 | 42 | 11432 |
| ¾ | 0.062500 | 0.000977 | 26 | 2.1666 | 6.9100 | 88 | 7.333 | 145.63 | 516 | 43 | 12125 |
| ⅞ | 0.072916 | 0.001436 | 27 | 2.2500 | 7.5938 | 89 | 7.416 | 149.80 | 528 | 44 | 12842 |
| 1 | 0.083333 | 0.002005 | 28 | 2.3333 | 8.3165 | 90 | 7.500 | 154.05 | 540 | 45 | 13584 |
| ⅛ | 0.093750 | 0.002691 | 29 | 2.4166 | 9.0791 | 91 | 7.583 | 158.36 | 552 | 46 | 14351 |
| ¼ | 0.104166 | 0.003502 | 30 | 2.5000 | 9.8821 | 92 | 7.666 | 162.75 | 564 | 47 | 15144 |
| ⅜ | 0.114583 | 0.004444 | 31 | 2.5833 | 10.726 | 93 | 7.750 | 167.21 | 576 | 48 | 15964 |
| ½ | 0.125000 | 0.005524 | 32 | 2.6666 | 11.612 | 94 | 7.833 | 171.74 | 588 | 49 | 16807 |
| ⅝ | 0.135416 | 0.006743 | 33 | 2.7500 | 12.541 | 95 | 7.916 | 176.34 | 600 | 50 | 17678 |
| ¾ | 0.145833 | 0.008122 | 34 | 2.8333 | 13.513 | 96 | 8.000 | 181.02 | 612 | 51 | 18575 |
| ⅞ | 0.156250 | 0.009651 | 35 | 2.9166 | 14.528 | 97 | 8.083 | 185.77 | 624 | 52 | 19499 |
| 2 | 0.166666 | 0.011340 | 36 | 3.0000 | 15.588 | 98 | 8.166 | 190.60 | 636 | 53 | 20450 |
| ¼ | 0.187500 | 0.015223 | 37 | 3.0833 | 16.694 | 99 | 8.250 | 195.49 | 648 | 54 | 21428 |
| ½ | 0.208333 | 0.019811 | 38 | 3.1666 | 17.845 | 100 | 8.333 | 200.47 | 660 | 55 | 22434 |
| ¾ | 0.229166 | 0.025141 | 39 | 3.2500 | 19.042 | 102 | 8.50 | 210.64 | 672 | 56 | 23468 |
| 3 | 0.250000 | 0.031250 | 40 | 3.3333 | 20.286 | 105 | 8.75 | 226.47 | 684 | 57 | 24529 |
| ¼ | 0.270833 | 0.038173 | 41 | 3.4166 | 21.578 | 108 | 9.00 | 243.00 | 696 | 58 | 25619 |
| ½ | 0.291666 | 0.045943 | 42 | 3.5000 | 22.918 | 111 | 9.25 | 260.23 | 708 | 59 | 26738 |
| ¾ | 0.312500 | 0.054592 | 43 | 3.5833 | 24.306 | 114 | 9.50 | 278.17 | 720 | 60 | 27885 |
| 4 | 0.333333 | 0.064150 | 44 | 3.6666 | 25.744 | 117 | 9.75 | 296.83 | 732 | 61 | 29062 |
| ¼ | 0.354166 | 0.074648 | 45 | 3.7500 | 27.232 | 120 | 10.0 | 316.23 | 744 | 62 | 30268 |
| ½ | 0.375000 | 0.086115 | 46 | 3.8333 | 28.770 | 126 | 10.5 | 357.25 | 756 | 63 | 31503 |
| ¾ | 0.395833 | 0.098578 | 47 | 3.9166 | 30.359 | 132 | 11.0 | 401.31 | 768 | 64 | 32768 |
| 5 | 0.416666 | 0.11207 | 48 | 4.0000 | 32.000 | 138 | 11.5 | 448.48 | 780 | 65 | 34063 |
| ¼ | 0.437500 | 0.12660 | 49 | 4.0833 | 33.693 | 144 | 12.0 | 498.83 | 792 | 66 | 35388 |
| ½ | 0.458333 | 0.14222 | 50 | 4.1666 | 35.438 | 150 | 12.5 | 552.43 | 804 | 67 | 36744 |
| ¾ | 0.479166 | 0.15893 | 51 | 4.2500 | 37.237 | 156 | 13.0 | 609.34 | 816 | 68 | 38130 |
| 6 | 0.500000 | 0.17673 | 52 | 4.3333 | 39.089 | 162 | 13.5 | 669.63 | 828 | 69 | 39548 |
| ½ | 0.541666 | 0.21594 | 53 | 4.4166 | 40.996 | 168 | 14.0 | 733.37 | 840 | 70 | 40996 |
| 7 | 0.583333 | 0.25989 | 54 | 4.5000 | 42.957 | 174 | 14.5 | 800.61 | 852 | 71 | 42476 |
| ½ | 0.625000 | 0.30882 | 55 | 4.5833 | 44.973 | 180 | 15.0 | 871.42 | 864 | 72 | 43988 |
| 8 | 0.666666 | 0.36289 | 56 | 4.6666 | 47.045 | 186 | 15.5 | 945.87 | 876 | 73 | 45531 |
| ½ | 0.708333 | 0.42227 | 57 | 4.7500 | 49.174 | 192 | 16.0 | 1024.0 | 888 | 74 | 47106 |
| 9 | 0.750000 | 0.48714 | 58 | 4.8333 | 51.359 | 198 | 16.5 | 1105.9 | 900 | 75 | 48714 |
| ¼ | 0.791666 | 0.55764 | 59 | 4.9166 | 53.602 | 204 | 17.0 | 1191.6 | 912 | 76 | 50354 |
| 10 | 0.833333 | 0.63394 | 60 | 5.0000 | 55.902 | 210 | 17.5 | 1281.1 | 924 | 77 | 52027 |
| ½ | 0.875000 | 0.71618 | 61 | 5.0833 | 58.260 | 216 | 18.0 | 1374.6 | 936 | 78 | 53732 |
| 11 | 0.916666 | 0.80451 | 62 | 5.1666 | 60.677 | 222 | 18.5 | 1472.1 | 948 | 79 | 55471 |
| ½ | 0.958333 | 0.89907 | 63 | 5.2500 | 63.154 | 228 | 19.0 | 1573.6 | 960 | 80 | 57243 |
| 12 | 1.000000 | 1.00000 | 64 | 5.3333 | 65.690 | 234 | 19.5 | 1679.1 | 972 | 81 | 59049 |
| ½ | 1.041666 | 1.1074 | 65 | 5.4166 | 68.286 | 240 | 20 | 1788.9 | 984 | 82 | 60888 |
| 13 | 1.083333 | 1.2215 | 66 | 5.5000 | 70.943 | 252 | 21 | 2020.9 | 996 | 83 | 62762 |
| ½ | 1.125000 | 1.3424 | 67 | 5.5833 | 73.660 | 264 | 22 | 2270.2 | 1008 | 84 | 64669 |
| 14 | 1.166666 | 1.4702 | 68 | 5.6666 | 76.440 | 276 | 23 | 2537.0 | 1020 | 85 | 66611 |
| ½ | 1.208333 | 1.6050 | 69 | 5.7500 | 79.281 | 288 | 24 | 2821.8 | 1032 | 86 | 68588 |
| 15 | 1.250000 | 1.7469 | 70 | 5.8333 | 82.185 | 300 | 25 | 3125.0 | 1044 | 87 | 70599 |
| ½ | 1.291666 | 1.8962 | 71 | 5.9166 | 85.152 | 312 | 26 | 3446.9 | 1056 | 88 | 72645 |
| 16 | 1.333333 | 2.0528 | 72 | 6.0000 | 88.182 | 324 | 27 | 3788.0 | 1068 | 89 | 74727 |
| ½ | 1.375000 | 2.2170 | 73 | 6.0833 | 91.276 | 336 | 28 | 4148.5 | 1080 | 90 | 76843 |
| 17 | 1.416666 | 2.3887 | 74 | 6.1666 | 94.434 | 348 | 29 | 4528.9 | 1092 | 91 | 78996 |
| ½ | 1.458333 | 2.5683 | 75 | 6.2500 | 97.656 | 360 | 30 | 4929.5 | 1104 | 92 | 81184 |
| 18 | 1.500000 | 2.7557 | 76 | 6.3333 | 100.94 | 372 | 31 | 5350.6 | 1116 | 93 | 83408 |
| ½ | 1.541666 | 2.9510 | 77 | 6.4166 | 104.30 | 384 | 32 | 5792.6 | 1128 | 94 | 85668 |
| 19 | 1.583333 | 3.1545 | 78 | 6.5000 | 107.72 | 396 | 33 | 6255.8 | 1140 | 95 | 87965 |
| ½ | 1.625000 | 3.3662 | 79 | 6.5833 | 111.20 | 408 | 34 | 6740.6 | 1152 | 96 | 90298 |
| 20 | 1.666666 | 3.5861 | 80 | 6.6666 | 114.76 | 420 | 35 | 7247.2 | 1164 | 97 | 92668 |
| ½ | 1.708333 | 3.8144 | 81 | 6.7500 | 118.37 | 432 | 36 | 7776.0 | 1176 | 98 | 95075 |
| 21 | 1.750000 | 4.0513 | 82 | 6.8333 | 122.06 | 444 | 37 | 8327.3 | 1188 | 99 | 97519 |
| ½ | 1.791666 | 4.2968 | 83 | 6.9166 | 125.82 | 456 | 38 | 8901.4 | 1200 | 100 | 100000 |

## LOGARITMOS

(**1**) Las tablas de logaritmos facilitan mucho las operaciones de la multiplicación, división, elevación á potencias y extracción de raíces de los números *.

(**2**) Las tablas, pgs 82 á 95, contienen los logaritmos **vulgares, decimales** ó **de Briggs** de los números. El logaritmo *vulgar* de un número es el exponente de la potencia de 10 que produce el número. Véase (18). Ej., $1{,}000 = 10^{?}$ y log 1,000 (logaritmo de 1,000) = 3.00 000. Del mismo modo $28.7 = 10^{1.45\ 788}$, y log 28.7 = 1.45 788.

(**3**) En general sean A y B dos números cualesquiera y $k$ un exponente cualquiera. Se tendrá :

$$(1)\ \text{Log } AB = \log A + \log B; \qquad (2)\ \text{Log } \frac{A}{B} = \log A - \log B;$$

$$(3)\ \text{Log } A^k = k\ (\log A); \qquad (4)\ \text{Log } \sqrt[k]{A} = \frac{\log A}{k}$$

ó bien expresado en lenguaje vulgar : (1) El log de un producto = á la suma de los logs de sus factores; (2) el log de un cociente = al log del dividendo — log del divisor; (3) el log de una potencia = al log del número multiplicado por el exponente; (4) el log de una raíz = al log del número dividido por el exponente.

(**4**) De lo dicho se deduce que :

| | |
|---|---|
| Log 100 = log $10^2$ = 2.00 000 | Log 0.1 = log $10^{-1}$ = $\bar{1}$.00 000 + |
| Log 10 = log $10^1$ = 1.00 000 | Log 0.01 = log $10^{-2}$ = $\bar{2}$.00 000 |
| Log 1 = log $10^0$ = 0.00 000 + | Log 0.001 = log $10^{-3}$ = $\bar{3}$.00 000 |

(**5**) Cada log vulgar es un número mixto, compuesto de una parte *entera* que precede al punto decimal y que se llama la **característica** ó **índice** y de una fracción que sigue al punto decimal, llamada **mantisa**. Las tablas dan solamente la mantisa de cada log; la característica se encuentra como lo indicaremos después. La mantisa es siempre positiva. La característica es igual al número que representa el puesto que ocupa la primera cifra significativa, contando desde el puesto de las *unidades* como *cero;* y es positivo para todos los números mixtos y negativo para las fracciones. Ej. :

$$\underset{4\ 3\ 2\ 1\ 0}{\text{Log } 2\,4\,6\,0\,0} = \bar{4}.39\,094; \qquad \underset{0\ 1\ 2\ 3\ 4}{\text{Log } 0.0\,0\,0\,0\,4} = \bar{5}.60\,206.$$

Por ejemplo :

| | |
|---|---|
| Log 2,870 = 3.45 788 | Log 0.287 = $\bar{1}$.45 788 + |
| Log 287 = 2.45 788 | Log 0.0287 = $\bar{2}$.45 788 |
| Log 28.7 = 1.45 788 | Log 0.00287 = $\bar{3}$.45 788 |
| Log 2.87 = 0.45 788 | Log 0.000287 = $\bar{4}$.45 788 |

Debemos advertir (y compruébese en los ejemplos últimos) que la mantisa permanece la misma para cualquier número expresado por las mismas cifras significativas, cualquiera que sea la situación del punto decimal, la posición de este punto sólo afecta al valor de la característica.

(**6**) Divídase el número en 2 factores de manera que uno sea una potencia entera de 10 y el otro resulte mayor que uno pero menor que 10. Entonces el exponente ó índice de la potencia de 10, es la característica del log y el log del otro factor es la mantisa. Ej. : $2{,}870 = 1{,}000 \times 2.87 = 10^3 \times 2.87$ y el logaritmo de 2,870 (3.45 788) es la suma del exponente 3 y del logaritmo de 2.87 que es 0.45 788.

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* Por no ser los logaritmos cantidades exactas, las operaciones que se ejecutan con ellos, están sometidas á cierta inexactitud, sobre todo cuando se mult, á un log por un número grande, puesto que entonces el error del log se mult tambien. Los log de sólo cinco decimales en la mantisa son suficientes para cálculos con números enteros de 4 ó 5 cifras. Es claro que hay más exactitud con tablas que den los logs hasta la séptima decimal.

† $\text{Log } 1 = \log \frac{10}{10} = \log 10 - \log 10 = 1 - 1 = 0$; ó bien $1 = 10^0$.

$\text{Log } 0.1 = \log \frac{1}{10} = \log 1 - \log 10 = 0 - 1 = \bar{1}.0$; ó bien $0.1 = 10^{-1}$.

‡ 0.287 = 2.87 ÷ 10. Por tanto log 0.287 = log 2.87 — log 10 = 0.45788 — 1, lo que se ha convenido en escribir así : $\bar{1}$.45 788. Véase (16). Del mismo modo log 0.0287 = log 2.87 — log 100 = 0.45 788 — 2 = $\bar{2}$.45 788.

(7) **Para encontrar el log de un número.** La tabla corta en las pags 80, 81, dan los logs de los números hasta 1,000. Todas las tablas pags 80 á 93 dan

(1) La mantisa para cada número de..... 1,000 á 1,750
(2) La mantisa para los números pares..... 1,750 á 3,750
(3) La mantisa para los quintos números de.... 3,750 á 10,000

(8) Los logs de **los números intermedios** á los que da la tabla se encuentran por simple proporción. El procedimiento está explicado en los ejemplos que se dan en conexión con las tablas, pero generalmente basta, en cuanto á exactitud, con tomar el del número más próximo en la tabla sin hacer interpolaciones.

El **antilogaritmo** es el **número** correspondiente á un logaritmo dado. Ej., log 2=0.30 103; y el antilogaritmo de 0.30 103=2. Habitualmente se escribe « $\log^{-1}$ 0.30 103=2 ».

(9) **Multiplicación.** Para multiplicar dos ó más números entre sí, súmense sus logs y búsquese el antilog de la suma. — Véase proporción (11) más abajo.

(10) **División.** Réstese del log del dividendo, el del divisor y búsquese el antilog del residuo. — Véase (11).

El inverso de cualquier número, $n$, $= \frac{1}{n}$. Véase pág. 54. Ej., inverso de $2 = \frac{1}{2} = 0.5$. Por tanto log del inverso de $n = \log \frac{1}{n} = \log 1 - \log n = 0 - \log n$.

Del mismo modo : log del inverso de $\frac{m}{n} = \log \frac{1}{m \div n} = 0 - \log \frac{m}{n}$. Ej., $\log \frac{6.3023}{23057} = 0 - \log \frac{23057}{6.3023} = 0 - 3.56\ 331 = \bar{4}.43\ 669$. Por tanto $n^{2-1} = n^1 = \frac{n^2}{n}$ $n^{1-1} = n^0 = \frac{n}{n} = 1$, $n^{0-1} = n^{-1} = \frac{1}{n}$, y $n^{0-2} = n^{-2} = \frac{1}{n^2}$ de donde se sigue que $\log n^{-1} = \log \frac{1}{n}$ = al log del inverso de $n$; $\log n^{-2} = \log \frac{1}{n^2} = \log$ del inverso de $n^2$, etc.

(11) **Proporción.** Ej., 6.3023 : 290.19=1260.7 : ?

| | | |
|---|---|---|
| Múltipl números. Súmense logs | Log 290.19 | =2.46 269 |
| | Log 1260.7 | =3.10 062 |
| | Log 290.19×1260.7 | =5.56 331 |
| Divídanse núm. Réstense logs. | Log 6.3023 | =0.79 950 |
| | Log 58,051 | =4.76 381 |

El verdadero valor es : 58049.05+.

(12) En vez de restar el log del divisor agréguese su **cologaritmo** ó **complemento aritmético** que es el log del inverso del divisor = 0 — log divisor = 10 — log divisor — 10. Ej. :

$$\frac{1523}{3.332 \times 8.655} = ?$$

| | |
|---|---|
| Log 1523 | = 3.18 270 |
| Colog 3.332=10 — log 3.332 — 10=10 — 0.52 270 — 10= | 9.47 730 — 10 |
| Colog 8.655=10 — log 8.655 — 10=10 — 0.93 727 — 10= | 9.06 273 — 10 |
| Suma de logs y cologs | =21.72 273 — 20 |
| | =Log 52.813= 1.72 273 |

El verdadero valor es : 52.8114+.

(13) Para encontrar **las potencias de los números.** Multiplíquese el log del número por el exponente de la potencia y búsquese el antilog del producto. Ej. : $36^3$=?

Log 36=1.55 630; 1.55 630×3=4.66 890. Antilog. 4.66 890=46656.

(14) Para encontrar las raíces de los números, se *divide* el log del número por el exponente de la raíz y se busca el antilog del cociente. Ej. :

$\sqrt[3]{46656}$ =? Log 46656 = 4.66 890. 4.66 890 ÷ 3= 1.55 630. Antilog 1.55 630=36

**(15) Potencias y raíces de fracciones. Características negativas.** Téngase presente que la *mantisa* es siempre positiva. Ej. (Log 0.048 = 2.68 124 = 0.68 124 — 2).

(1) $0.048^{1.5} = ?$

$1.5 \times (0.68\ 124 - 2) = 1.02\ 186 - 3 = \bar{2}.02\ 186 = \log 0.01052 = \log 0.048^{1.5}$.

(2) $\sqrt[3]{0.0048} = ?$ $\bar{3}.68\ 124 \div 3 = \bar{1}.22\ 708 = \log 0.1687 = \log \sqrt[3]{0.0048}$.

(3) Pero si la característica negativa no es exactamente divisible por el exponente, la división dará resultados erróneos; en estos casos, agréguese y réstese a lo cualquier múltiplo del exponente mayor que la característica. Ej. $\sqrt[3]{.00048} = ?$

Log 0.00048 = $\bar{4}.68\ 124 = 6 + \bar{4}.68\ 124 - 6$

$= 2.68\ 124 - 6$

Dividiendo por 3 $\quad 0.89\ 375 - 2$

$= \bar{2}.89\ 375 \quad = \log 0.0783 = \log \sqrt[3]{0.00048}$.

**(16)** Para evitar inconvenientes con el uso de las **características negativas** se acostumbra **modificarlas**, agregándoles 10 unidades y después se deduce de la suma de los logs tantas veces 10 cuantas veces se agregó. Ej. : Multiplicando ó dividiendo 7425 por 0.25, se tiene

| | Multipl. | Dividiendo. |
|---|---|---|
| Log 7425 | = 3.87 070 | =3.87 070 |
| Log 0.25 | = $\bar{1}$.39 794 | =$\bar{1}$.39 794 |
| | 3.26 864 | 4.47 276 |
| ó bien Log 7425 | = 3.87 070 | =3.87 070 |
| Log modificado de 0.25 | = 9.39 794 — 10 | =9.39 794 — 10 |
| | 13.26 864 — 10 | =$\bar{6}$.47 276 + 10 |
| | = 3.26 864 | =4.47 276 |

En muchos casos puede olvidarse la sustracción de las decenas agregadas, pero por la naturaleza de estos cálculos, el aumento producido por el olvido es tan grande que difícilmente puede pasar inadvertido.

**(17) Para dividir un log modificado** agréguesele tal múltiplo de 10 que resulte la suma excediendo al verdadero log en 10 veces el divisor. Ej. : para dividir el log 0.00048 por 3. Log 0.00048 = 4.68 124, que dividido por 3, = 2.89 375. Véase (15).

| | |
|---|---|
| Log 0.00048 | = $\bar{4}.68\ 124$ |
| Log modificado 0.00048 | = $\bar{6}.68\ 124 - 10$ |
| Agregando | $2 \times 10 = 20 \quad - 20$ |
| Dividiendo por | 3) $\bar{26}.68\ \bar{124} - 30$ |
| Se obtiene | $8.89\ 375 - 10 = \bar{2}.89\ 375$. |

**(18)** Excepto 1, cualquier número puede ser la base de un sistema de logaritmos. La base de los logs llamados **hiperbólicos** ó **Neperianos** ó **naturales**, muy usados en ingeniería naval, es

$$1 + \frac{1}{1} + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3} + \frac{1}{1 \times 2 \times 3 \times 4} + \ldots = 2.71\ 828 +$$

y se los llama $e$ (epsilon) ó *e*.

$M = \log_{10} e$ (log vulgar de *e*) = 0.43 429; $\frac{1}{M} = \log_e 10$ (log hiper de 10) = 2.30 259

Para cualquier número *n*,

$$\log_e n = \frac{\log_{10} n}{M} = 2.30259 \times \log_{10} n; \quad \log_{10} n = M \log_e n = 0.43429 \log_e n.$$

**(19)** Cualquiera que sea la base elegida para un sistema de logs, la mantisa del log de cualquier número guarda una **relación constante** con la de cualquier

otro. Ej.: En cualquier sistema de logs el log 4 es siempre $2 \times \log 2$, igual $^2/_3 \times \log 8$, etc., etc.

(20) **El logaritmo del seno, tangente**, etc., de los ángulos es el log del número que representa el seno, tangente, etc., de esos ángulos. Ej.: seno de 30° = 0.50000 y el log sen 30° = log 0.50000 = 1.69897, comúnmente escrito 9.69897 — 10 ó simplemente 9.69897.

Véanse las **tablas logarítmicas** de los senos, tang., etc., pags 151, etc.

(21) Como ninguna potencia de un número positivo puede ser negativa, los números negativos propiamente no tienen log, pero **las operaciones con números negativos** pueden no obstante practicarse con logs, tratando todos los números como si fueran positivos, teniendo cuidado de usar el signo adecuado + ó — en el resultado. Por ejemplo, si se busca la 3.ª potencia de — 2). Log. 2 = 0.30 103; y $3 \times 0.30\ 103 = 0.90\ 309 = \log 8$; pero $(-2)^3$ debe ser negativo, por tanto $(-2)^3 = -8$.

## Carta y regla logarítmica.

(1) Por medio de una carta ó diagrama se hacen gráficamente las operaciones logarítmicas y mecánicamente, por medio de una regla corrediza *. No obstante, con este sistema y aun con cartas amplias y buenas reglas no se logra exactitud más allá de la 3.ª ó 4.ª cifra. Su uso facilita mucho los cálculos en Hidráulica y otros ramos de la Ingeniería.

(2) La **carta logarítmica** consiste primeramente en un cuadrado * sobre cuyos lados las distancias 1 — 2, 1 — 3, etc. están marcadas en relación con los logs (0.30 103, 0.47 712, etc.) de los números 2, 3, etc. Generalmente se usa el papel cuadriculado poniendo en cada línea de la cuadrícula el log correspondiente en lugar de los números. Las líneas que representan los números se trazan en sus respectivos lugares. — (Véanse las líneas de los logs y la de los números correspondientes debajo y á la izquierda del diagrama. *N. del T.)*

(3) La **regla corrediza logarítmica** consiste generalmente en 4 escalas A, B, C y D, véase (17); las escalas A y D se construyen en la regla fija, y la B y C en la lengueta ó regla movediza. Como en el diagrama las escalas son *logarítmicas*, véase (2), pero marcadas con los *números* que corresponden á los logaritmos. Las escalas A y B son iguales como lo son las C y D, pero una distancia dada en A ó B, representa un log doble del de C ó D. Véase (4). Por tanto, cada número marcado en A es el *cuadrado* del que le corresponde ó coincide en D.

(4) Una escala logarítmica simple está generalmente numerada de 1 á 10, ó de 10 á 100; pero debe tomársela como representando cualquier serie comprendida entre números representados por $10^n$ á $10^{n+1}$; como por ej. de $0.1=10^{-1}$ á $1.0=10^0$ (en que $n=-1$); ó de $1.0=10^0$ á $10.0=10^1$ (en que $n=0$) ó de 10.0 á 100 (en que $n=1$), etc. Aquí $n$ y $n+1$ son las características de los correspondientes logaritmos.

Una escala simple puede servir sin embargo para todos los valores, desde $o$ hasta el infinito; pero por conveniencia se agregan á veces varias escalas contiguas.

Cuando una línea llega al límite del cuadrado, ella continúa en el punto directamente opuesto. Por ej.: en el caso de la línea $x^{2}/_{3}(=\sqrt[3]{x^2}.)$ (Véase el diagrama.)

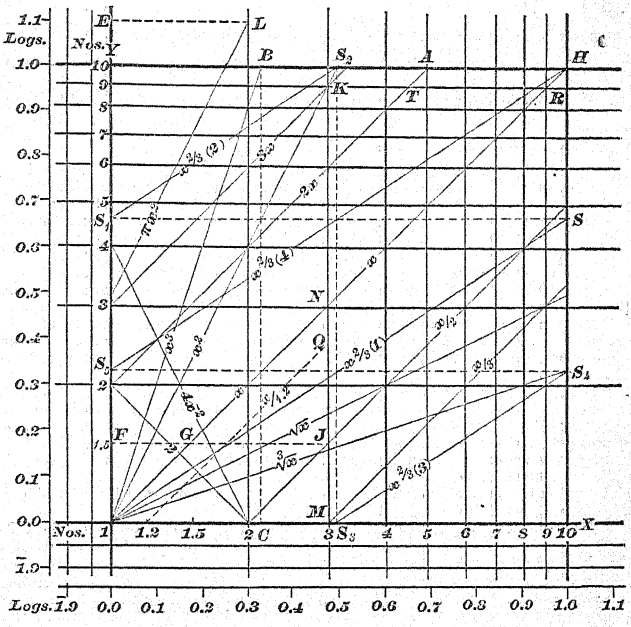
| Línea marcada $x^{2/3}$ | Entre. | Corresponde á los valores | |
|---|---|---|---|
| | | De $x$ desde. | De $x^{2/3}$ desde. |
| (1) | 1 y S | 1 á 10 | 1 á 4.64 |
| (2) | $S_1$ y $S_2$ | 10 á 31.62 | 4.64 á 10 |
| (3) | $S_2$ y $S_3$ | 31.62 á 100 | 10 á 21.54 |
| (4) | $S_3$ y H | 100 á 1000 | 21.54 á 100 |

Obsérvese que los números marcados en cualquier escala deben ser tomados como diez veces mayores que los números correspondientes de la escala inmediata anterior y la característica, en consecuencia, aumentada en 1, y viceversa. Así en el diagrama log. 1.5+log 2=distancias (1–1.5+1–2)=log 3=distancia 1–M. Pero log 15+log 20=(log 1.5+log 10)+(log 2+log 10)=(1–1.5+1–10)+(1–2+1–10), de tal manera que la característica del log que resulta está aumentada en 2, y el 3 que representa el producto de 15 y 20 está realmente en el segundo cuadrado á la derecha del de la figura. Al tratarse de las raíces de los números, recuérdese que, multiplicando ó dividiendo el número por 0.1, 10, 100, etc. (y cambiando por tanto la característica de su log) cambia también la mantisa del log de su *raíz*. Ej.: $\sqrt[3]{2.7}=1.39\ldots$ (log=0.14 379), pero $\sqrt[3]{27}=3$ (log=0.47 712) y $\sqrt[3]{270}=6.46\ldots$ (log= 0.81 023). La carta ó la regla dan *todas* estas raíces *posibles* y se debe tener buen cuidado en escoger la que

* La relación entre las mantisas de los logs de cualesquiera números, siendo constante, para cualquier sistema de logs, las relaciones entre las distancias respectivas en la carta ó en la regla corrediza, es la misma para todos los sistemas, y el uso de aquéllas es independiente del sistema de logs empleado.

conviene al caso. La mayor parte de las operaciones exceden del límite de una escala y las facilidades que puede dar el uso de estos instrumentos dependen de la habilidad en pasar fácil y prontamente de una escala á otra. Esta habilidad se alcanza por la práctica ayudada por un conocimiento completo de los principios que envuelve. Cuando se tiene que practicar una serie de operaciones distintas, se usa una marca ó indicador corredizo, que lo trae la misma regla para evitar que un movimiento de la corredera cambie lo marcado. Las reglas se venden con sus instrucciones.

(5) **Multiplicación y división.** Ej. : $2 \times 1.5 = ?$ Sobre la línea 1–X en la carta ó sobre C ó D en la regla la distancia 1–1.5 representa en escala el log (0.17 609) de 1.5 y 1–2 representa el log (0.30 103) de 2. Si se suman estas dos distancias agregándole la distancia 1–2 á la 1–1.5 sobre las 1–X de la carta ó colocando la regla C sobre la D como está en la figura, se obtiene la distancia $1\text{–}3 = 0.47\,712$ = á la mantisa del log 3 ó del log $(2 \times 1.5)$ *. Al contrario, para dividir á 3 por 2, gráfica ó mecánicamente se resta de la distancia 1–3 la 1–2.




(6) **En la carta logarítmica** por ser iguales las escalas de ambos ejes 1–X, 1–Y, la línea 1–H bisectriz y diagonal del cuadrado forma un ángulo de 45° con cada eje (tan 45° = 1), † y es también bisectriz de todos los cuadrados formados sobre coordenadas iguales. Todos los puntos de la diagonal $x$ que están sobre los 2, 3, etc. del eje 1–X corresponden á los opuestos 2, 3, etc., en el eje 1–Y. Véase (4).

* En la regla corrediza, tal como aparece, *cada* número en D = 1.5 × el número con que coincide en C.

(†) Al tratarse de las tangentes de los ángulos, en la carta logarítmica, deben tomarse las distancias sobre la escala proporcional de los logaritmos y no sobre la de los números que son de mera conveniencia; por ej.: trazando la línea 1-B se tiene que tang $\text{C1B} = \frac{\text{BC}}{\text{C1}} = \frac{1.0}{0.33}$ y no $\frac{10}{2.15}$.

**(7)** Si se trazan las líneas 2–A, 3–K, etc. (marcadas 2 $x$, 3 $x$, etc.) paralelas á 1–H y, *sobre* ella, los puntos de esas líneas situados sobre cualquier punto $x$ del eje 1–X estarán respectivamente opuestos á los números que dan el producto 2 $x$, 3 $x$, etc., sobre el eje 1–Y; así también las líneas trazadas del mismo modo, *debajo* de 1–H por los puntos 2, 3, etc. de 1–X, darán respectivamente los valores $\frac{x}{2}$, $\frac{x}{3}$, etc., en 1–Y; las líneas semejantes trazadas debajo de 1–H, por los puntos 2, 3, etc. de 1–X, dan los valores de $\frac{x}{2}$, $\frac{x}{3}$, etc. Si estas líneas $\frac{x}{2}$, $\frac{x}{3}$, etc. se prolongan hacia abajo, cortarán respectivamente á 1–Y (prolongada) en $0.5\left(=\frac{1}{2}\right)$, $0.33...\left(=\frac{1}{3}\right)$*. Véase (*4*).

**(8) Potencias y raíces.** Si se traza la línea $x^2$ por el punto 1, formando un ángulo $S_2$ 1–X cuya $\text{tang}=\frac{S_1-S_2}{S_2-1}=\frac{1}{0.5}=2$, dará valores de $x^2$. Así la vertical por 3 en 1–X corta la línea $x^2$ en un punto cuyo opuesto en 1–Y es $9=3^2$. Del mismo modo la línea $x^3$ (tang=3) da valores de $x^3$; y la línea $\left(\text{tang}=\frac{1}{3}\right)$ da valores de $x^{\frac{1}{3}}$ ó $\sqrt[3]{x}$. Véase (*4*).

**(9)** Cualquier ecuación de la forma $y=Cx^n$ en que $\log y=\log C+n \log x$ (como: área del círculo$=\pi\times$radio$^2$), está representada en una carta logarítmica por una línea recta trazada de modo que la tangente del ángulo que forme con 1–X sea$=n$ y que corte en un punto a la línea 1–Y cuyo valor represente á C. Ej.: la línea marcada $\pi x^2$ (tang$=2$) es línea de cuadrados y, habiendo sido trazada desde $\pi$ (=3.14) en 1–Y da los valores de $\pi x^2$. Así: para un círculo de radio 2, se encuentra en la línea $\pi x^2$ sobre el punto 2, un punto L cuyo opuesto E ó 12.57 es el área de dicho círculo †. Al revés, teniendo el área 12.57 se obtiene por el diagrama el radio$=2$.

**(10)** Cuando, con un diagrama se van á resolver muchas ecuaciones de una sola especie como $y=Cx^n$, en que C es un coeficiente variable y $n$ un exponente constante, trácense sobre la hoja paralelas á cortos intervalos que formen todas el mismo ángulo con 1–X.

**(11)** A cualquier log como 1–3 (=log 3), se puede substituir su igual, M–N ó 3–N, que concurren á la diagonal central 1–H marcada $x$; y, entonces, puesto que, por ej., 1–1.2=N–Q, 1–3=N–K, etc., se puede agregar cualquier log (como 1–3), moviendo *hacia arriba*, desde la línea $x$ (como de N hacia K) ó hacia *la derecha*, y *restar* cualquier log (como 1–1.2) moviendo *hacia abajo* (como de N hacia Q) ó hacia *la izquierda*.

Esto facilita la realización de una serie de operaciones. Ej.: Multipliquese $1.5\times2$ (=3) por 3 (=9) y divídase por 2 (=4.5).

F–G=1–F=log 1.5. Agréguese G–J=1–2=log 2; súmese F–J=log 3=1–3=M–N. Agréguese N–K=1–3=log 3; súma=M–K= log 9=1–9 =9– R. Réstese R–T=1–2=log 2; residuo =9–T=log. 4.5.

**(12) Exponentes negativos.** — Si la $x$ está en el divisor la línea se inclinará en opuesta dirección ó hacia abajo de izquierda á derecha. Así la línea 4–2 que parte de 4 sobre 1–Y y forma con 1–X el ángulo X, 2, 4, cuya $\text{tang}=\frac{602...}{-301...}=-2$, representa la ecuación $y=\frac{4}{x^2}=4x^{-2}$.

**(13)** Si las líneas de productos, potencias y raíces $Cx$, $x^n$, y $\sqrt[n]{x}$, etc., están trazadas con ángulos, cuyas tangentes son menores en 1 que los ángulos formados por las correspondientes líneas en nuestra figura, los resultados deben leerse directamente en las líneas oblicuas trazadas paralelas á 2–2. Las líneas ($Cx$) que

---

* En cada una de estas líneas, el producto de los dos números en sus extremos es=10. Ej.: en la línea 2–A, $2\times5=10$; en la 3–K, $3\times3.33...=10$, etc. Esto constituye una tabla de *inversos*.

(†) Con los buenos diagramas y reglas no se debe contar con exactitud más allá de tercera ó cuarta cifra.

dan múltiples y submúltiples de la primera potencia de $x$ se hacen entonces horizontales (tang=0).

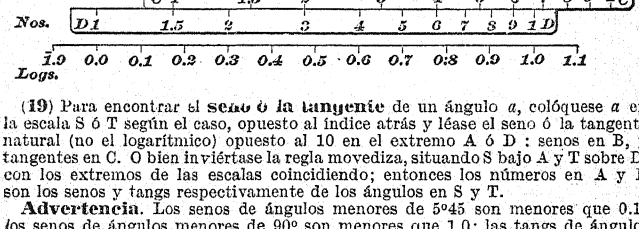
(14) **Potencias y raíces con la regla corrediza.** — Las escalas C y D por ser dobles de las escalas A y B, cuando coinciden sus extremos forman una tabla de cuadrados y raíces. Véase (3). Moviendo la parte corrediza se resuelven ecuaciones de la forma $y=(Cx)^2$, y también $y=Cx^4$. Entonces, con la regla como está cada número en A es = al cuadrado de (1.5×el número con que coincide en C); con 1 en B opuesto a 1.5 en A, cada número en A es=1.5×el cuadrado del número con que coincide en C.

(15) Como $x^3=x^2\times x$, encontramos las terceras potencias ó cubos, colocando la regla con 1 en B opuesto á $x^2$ en A (por tanto opuesto á $x$ en D), véase (3), y leyendo $x^3$ en A opuesto á $x$ en B. Ej.: $1.5^3=?$ Sitúese 1 en B opuesto 1.5 en D: por tanto opuesto $1.5^2$ (=2.25) en A. Entonces en A opuesto 1.5 en B se encuentra $3.375=1.5^3$. O bien inviértanse los extremos de la regla corrediza. Sitúese 1.5 en B opuesto 1.5 en D, y por tanto opuesto $1.5^2=2.25$ en A. Entonces agregando log 1.5 (en B) al log 2.25 en A, se encuentra 3.375 ($=1.5^3$) en A opuesto 1 en B

(16) Al contrario: para encontrar á $\sqrt[3]{x}$, se invierte la regla corrediza (á su posición normal) hasta que se encuentre en B opuesto á $x$ en A, el mismo número que se tiene en D opuesto á 1 en C y este número será $=\sqrt[3]{x}$. O bien inviértanse los extremos de la regla corrediza,* sitúese 1 en C opuesto á $x$ en A y encuéntrese, en B, un número que coincida con su igual en D; este número es $=\sqrt[3]{x}$. Véanse también (17) (18).

(17) En la parte atrás de la regla se traza generalmente una escala de logaritmos (véase la escala debajo de la regla en la figura) y dos escalas de ángulos marcadas « S » y « T » respectivamente, para encontrar los senos de los ángulos mayores que 0°34′ y tangentes de ángulos entre 5°42′ y 45°.

(18) Colocando 1 en C, opuesto á cualquier número $x$ en D (con la regla móvil en su posición normal) el log $x$ se lee en la escala de los logaritmos, por medio de un índice en la parte atrás de la regla. Deben usarse los logaritmos para encontrar potencias y raíces.




(19) Para encontrar el **seno ó la tangente** de un ángulo $a$, colóquese $a$ en la escala S ó T según el caso, opuesto al índice atrás y léase el seno ó la tangente natural (no el logarítmico) opuesto al 10 en el extremo A ó D: senos en B, y tangentes en C. O bien inviértase la regla movediza, situando S bajo A y T sobre D, con los extremos de las escalas coincidiendo; entonces los números en A y D son los senos y tangs respectivamente de los ángulos en S y T.

**Advertencia.** Los senos de ángulos menores de 5°45 son menores que 0.1; los senos de ángulos menores de 90° son menores que 1.0; las tangs de ángulos entre 5°42 y 45° están entre 0.1 y 1.0.

(20) Engrasando los filos de la regla corrediza y la cavidad donde corre, se hace más fácil su manejo, á veces incómodo. Si la regla móvil está muy floja, debe ahondarse la cavidad y colocarse, á guisa de resorte, una laminilla de acero delgada en la canal, y apoyada en la regla.

* Con la regla móvil así invertida y con los extremos de las escalas coincidiendo, los números en A y B son *inversos* (pág. 51), co.. o también los de C y D.

## LOGARITMOS DE LOS NÚMEROS HASTA 1,000.

Para el uso de los logaritmos, véanse págs 72 á 75.
Para los logs con cinco cifras de los números hasta 10,000, véanse las págs 82 á 93.

| No. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | No. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | | 0000 | 3010 | 4771 | 6021 | 6990 | 7782 | 8451 | 9031 | 9542 | 0 |
| 1 | 0000 | 0414 | 0792 | 1139 | 1461 | 1761 | 2041 | 2304 | 2553 | 2788 | 1 |
| 2 | 3010 | 3222 | 3424 | 3617 | 3802 | 3979 | 4150 | 4314 | 4472 | 4624 | 2 |
| 3 | 4771 | 4914 | 5052 | 5185 | 5315 | 5441 | 5563 | 5682 | 5798 | 5911 | 3 |
| 4 | 6021 | 6128 | 6232 | 6335 | 6435 | 6532 | 6628 | 6721 | 6812 | 6902 | 4 |
| 5 | 6990 | 7076 | 7160 | 7243 | 7324 | 7404 | 7482 | 7559 | 7634 | 7709 | 5 |
| 6 | 7782 | 7853 | 7924 | 7993 | 8062 | 8129 | 8195 | 8261 | 8325 | 8388 | 6 |
| 7 | 8451 | 8513 | 8573 | 8633 | 8692 | 8751 | 8808 | 8865 | 8921 | 8976 | 7 |
| 8 | 9031 | 9085 | 9138 | 9191 | 9243 | 9294 | 9345 | 9395 | 9445 | 9494 | 8 |
| 9 | 9542 | 9590 | 9638 | 9685 | 9731 | 9777 | 9823 | 9868 | 9912 | 9956 | 9 |
| 10 | 0000 | 0043 | 0086 | 0128 | 0170 | 0212 | 0253 | 0294 | 0334 | 0374 | 10 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0755 | 11 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1038 | 1072 | 1106 | 12 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 | 13 |
| 14 | 1461 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 | 14 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 | 15 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 | 2175 | 2201 | 2227 | 2253 | 2279 | 16 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 | 17 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 2718 | 2742 | 2765 | 18 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 | 19 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 | 20 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 3404 | 21 |
| 22 | 3424 | 3444 | 3464 | 3483 | 3502 | 3522 | 3541 | 3560 | 3579 | 3598 | 22 |
| 23 | 3617 | 3636 | 3655 | 3674 | 3692 | 3711 | 3729 | 3747 | 3766 | 3784 | 23 |
| 24 | 3802 | 3820 | 3838 | 3856 | 3874 | 3892 | 3909 | 3927 | 3945 | 3962 | 24 |
| 25 | 3979 | 3997 | 4014 | 4031 | 4048 | 4065 | 4082 | 4099 | 4116 | 4133 | 25 |
| 26 | 4150 | 4166 | 4183 | 4200 | 4216 | 4232 | 4249 | 4265 | 4281 | 4298 | 26 |
| 27 | 4314 | 4330 | 4346 | 4362 | 4378 | 4393 | 4409 | 4425 | 4440 | 4456 | 27 |
| 28 | 4472 | 4487 | 4502 | 4518 | 4533 | 4548 | 4564 | 4579 | 4594 | 4609 | 28 |
| 29 | 4624 | 4639 | 4654 | 4669 | 4683 | 4698 | 4713 | 4728 | 4742 | 4757 | 29 |
| 30 | 4771 | 4786 | 4800 | 4814 | 4829 | 4843 | 4857 | 4871 | 4886 | 4900 | 30 |
| 31 | 4914 | 4928 | 4942 | 4955 | 4969 | 4983 | 4997 | 5011 | 5024 | 5038 | 31 |
| 32 | 5052 | 5065 | 5079 | 5092 | 5105 | 5119 | 5132 | 5145 | 5159 | 5172 | 32 |
| 33 | 5185 | 5198 | 5211 | 5224 | 5237 | 5250 | 5263 | 5276 | 5289 | 5302 | 33 |
| 34 | 5315 | 5328 | 5340 | 5353 | 5366 | 5378 | 5391 | 5403 | 5416 | 5428 | 34 |
| 35 | 5441 | 5453 | 5465 | 5478 | 5490 | 5502 | 5515 | 5527 | 5539 | 5551 | 35 |
| 36 | 5563 | 5575 | 5587 | 5599 | 5611 | 5623 | 5635 | 5647 | 5658 | 5670 | 36 |
| 37 | 5682 | 5694 | 5705 | 5717 | 5729 | 5740 | 5752 | 5763 | 5775 | 5786 | 37 |
| 38 | 5798 | 5809 | 5821 | 5832 | 5843 | 5855 | 5866 | 5877 | 5888 | 5899 | 38 |
| 39 | 5911 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 5988 | 5999 | 6010 | 39 |
| 40 | 6021 | 6031 | 6042 | 6053 | 6064 | 6075 | 6085 | 6096 | 6107 | 6117 | 40 |
| 41 | 6128 | 6138 | 6149 | 6160 | 6170 | 6180 | 6191 | 6201 | 6212 | 6222 | 41 |
| 42 | 6232 | 6243 | 6253 | 6263 | 6274 | 6284 | 6294 | 6304 | 6314 | 6325 | 42 |
| 43 | 6335 | 6345 | 6355 | 6365 | 6375 | 6385 | 6395 | 6405 | 6415 | 6425 | 43 |
| 44 | 6435 | 6444 | 6454 | 6464 | 6474 | 6484 | 6493 | 6503 | 6513 | 6522 | 44 |
| 45 | 6532 | 6542 | 6551 | 6561 | 6571 | 6580 | 6590 | 6599 | 6609 | 6618 | 45 |
| 46 | 6628 | 6637 | 6646 | 6656 | 6665 | 6675 | 6684 | 6693 | 6702 | 6712 | 46 |
| 47 | 6721 | 6730 | 6739 | 6749 | 6758 | 6767 | 6776 | 6785 | 6794 | 6803 | 47 |
| 48 | 6812 | 6821 | 6830 | 6839 | 6848 | 6857 | 6866 | 6875 | 6884 | 6893 | 48 |
| 49 | 6902 | 6911 | 6920 | 6928 | 6937 | 6946 | 6955 | 6964 | 6972 | 6981 | 49 |
| 50 | 6990 | 6998 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 | 50 |

## LOGARITMOS DE LOS NÚMEROS HASTA 1,000. (Continuación.)

Para el uso de los logaritmos, véanse págs 72 á 75.
Para los logs con cinco cifras de los números hasta 10,000, véanse las págs 82 á 93.

| No. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | No. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 6990 | 6998 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 | 50 |
| 51 | 7076 | 7084 | 7093 | 7101 | 7110 | 7118 | 7126 | 7135 | 7143 | 7152 | 51 |
| 52 | 7160 | 7168 | 7177 | 7185 | 7193 | 7202 | 7210 | 7218 | 7226 | 7235 | 52 |
| 53 | 7243 | 7251 | 7259 | 7267 | 7275 | 7284 | 7292 | 7300 | 7308 | 7316 | 53 |
| 54 | 7324 | 7332 | 7340 | 7348 | 7356 | 7364 | 7372 | 7380 | 7388 | 7396 | 54 |
| 55 | 7404 | 7412 | 7419 | 7427 | 7435 | 7443 | 7451 | 7459 | 7466 | 7474 | 55 |
| 56 | 7482 | 7490 | 7497 | 7505 | 7513 | 7520 | 7528 | 7536 | 7543 | 7551 | 56 |
| 57 | 7559 | 7566 | 7574 | 7582 | 7589 | 7597 | 7604 | 7612 | 7619 | 7627 | 57 |
| 58 | 7634 | 7642 | 7649 | 7657 | 7664 | 7672 | 7679 | 7686 | 7694 | 7701 | 58 |
| 59 | 7709 | 7716 | 7723 | 7731 | 7738 | 7745 | 7752 | 7760 | 7767 | 7774 | 59 |
| 60 | 7782 | 7789 | 7796 | 7803 | 7810 | 7818 | 7825 | 7832 | 7839 | 7846 | 60 |
| 61 | 7853 | 7860 | 7868 | 7875 | 7882 | 7889 | 7896 | 7903 | 7910 | 7917 | 61 |
| 62 | 7924 | 7931 | 7938 | 7945 | 7952 | 7959 | 7966 | 7973 | 7980 | 7987 | 62 |
| 63 | 7993 | 8000 | 8007 | 8014 | 8021 | 8028 | 8035 | 8041 | 8048 | 8055 | 63 |
| 64 | 8062 | 8069 | 8075 | 8082 | 8089 | 8096 | 8102 | 8109 | 8116 | 8122 | 64 |
| 65 | 8129 | 8136 | 8142 | 8149 | 8156 | 8162 | 8169 | 8176 | 8182 | 8189 | 65 |
| 66 | 8195 | 8202 | 8209 | 8215 | 8222 | 8228 | 8235 | 8241 | 8248 | 8254 | 66 |
| 67 | 8261 | 8267 | 8274 | 8280 | 8287 | 8293 | 8299 | 8306 | 8312 | 8319 | 67 |
| 68 | 8325 | 8331 | 8338 | 8344 | 8351 | 8357 | 8363 | 8370 | 8376 | 8382 | 68 |
| 69 | 8388 | 8395 | 8401 | 8407 | 8414 | 8420 | 8426 | 8432 | 8439 | 8445 | 69 |
| 70 | 8451 | 8457 | 8463 | 8470 | 8476 | 8482 | 8488 | 8494 | 8500 | 8506 | 70 |
| 71 | 8513 | 8519 | 8525 | 8531 | 8537 | 8543 | 8549 | 8555 | 8561 | 8567 | 71 |
| 72 | 8573 | 8579 | 8585 | 8591 | 8597 | 8603 | 8609 | 8615 | 8621 | 8627 | 72 |
| 73 | 8633 | 8639 | 8645 | 8651 | 8657 | 8663 | 8669 | 8675 | 8681 | 8686 | 73 |
| 74 | 8692 | 8698 | 8704 | 8710 | 8716 | 8722 | 8727 | 8733 | 8739 | 8745 | 74 |
| 75 | 8751 | 8756 | 8762 | 8768 | 8774 | 8779 | 8785 | 8791 | 8797 | 8802 | 75 |
| 76 | 8808 | 8814 | 8820 | 8825 | 8831 | 8837 | 8842 | 8848 | 8854 | 8859 | 76 |
| 77 | 8865 | 8871 | 8876 | 8882 | 8887 | 8893 | 8899 | 8904 | 8910 | 8915 | 77 |
| 78 | 8921 | 8927 | 8932 | 8938 | 8943 | 8949 | 8954 | 8960 | 8965 | 8971 | 78 |
| 79 | 8976 | 8982 | 8987 | 8993 | 8998 | 9004 | 9009 | 9015 | 9020 | 9025 | 79 |
| 80 | 9031 | 9036 | 9042 | 9047 | 9053 | 9058 | 9063 | 9069 | 9074 | 9079 | 80 |
| 81 | 9085 | 9090 | 9096 | 9101 | 9106 | 9112 | 9117 | 9122 | 9128 | 9133 | 81 |
| 82 | 9138 | 9143 | 9149 | 9154 | 9159 | 9165 | 9170 | 9175 | 9180 | 9186 | 82 |
| 83 | 9191 | 9196 | 9201 | 9206 | 9212 | 9217 | 9222 | 9227 | 9232 | 9238 | 83 |
| 84 | 9243 | 9248 | 9253 | 9258 | 9263 | 9269 | 9274 | 9279 | 9284 | 9289 | 84 |
| 85 | 9294 | 9299 | 9304 | 9309 | 9315 | 9320 | 9325 | 9330 | 9335 | 9340 | 85 |
| 86 | 9345 | 9350 | 9355 | 9360 | 9365 | 9370 | 9375 | 9380 | 9385 | 9390 | 86 |
| 87 | 9395 | 9400 | 9405 | 9410 | 9415 | 9420 | 9425 | 9430 | 9435 | 9440 | 87 |
| 88 | 9445 | 9450 | 9455 | 9460 | 9465 | 9469 | 9474 | 9479 | 9484 | 9489 | 88 |
| 89 | 9494 | 9499 | 9504 | 9509 | 9513 | 9518 | 9523 | 9528 | 9533 | 9538 | 89 |
| 90 | 9542 | 9547 | 9552 | 9557 | 9562 | 9566 | 9571 | 9576 | 9581 | 9586 | 90 |
| 91 | 9590 | 9595 | 9600 | 9605 | 9609 | 9614 | 9619 | 9624 | 9628 | 9633 | 91 |
| 92 | 9638 | 9643 | 9647 | 9652 | 9657 | 9661 | 9666 | 9671 | 9675 | 9680 | 92 |
| 93 | 9685 | 9689 | 9694 | 9699 | 9703 | 9708 | 9713 | 9717 | 9722 | 9727 | 93 |
| 94 | 9731 | 9736 | 9741 | 9745 | 9750 | 9754 | 9759 | 9764 | 9768 | 9773 | 94 |
| 95 | 9777 | 9782 | 9786 | 9791 | 9795 | 9800 | 9805 | 9809 | 9814 | 9818 | 95 |
| 96 | 9823 | 9827 | 9832 | 9836 | 9841 | 9845 | 9850 | 9854 | 9859 | 9863 | 96 |
| 97 | 9868 | 9872 | 9877 | 9881 | 9886 | 9890 | 9894 | 9899 | 9903 | 9908 | 97 |
| 98 | 9912 | 9917 | 9921 | 9926 | 9930 | 9934 | 9939 | 9943 | 9948 | 9952 | 98 |
| 99 | 9956 | 9961 | 9965 | 9969 | 9974 | 9978 | 9983 | 9987 | 9991 | 9996 | 99 |
| 100 | 0000 | 0004 | 0009 | 0013 | 0017 | 0022 | 0026 | 0030 | 0035 | 0039 | 100 |

## Logaritmos vulgares ó de Briggs. Base=10.

| No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1000** | 00000 | 43 | **1050** | 02119 | 41 | **1100** | 04139 | 40 | **1150** | 06070 | 38 | **1200** | 07918 | 36 |
| 01 | 043 | 44 | 51 | 160 | 42 | 01 | —179 | 39 | 51 | —108 | 38 | 01 | 954 | 36 |
| 02 | —087 | 43 | 52 | —202 | 41 | 02 | 218 | 39 | 52 | 145 | 37 | 02 | 990 | 37 |
| 03 | 130 | 43 | 53 | —243 | 41 | 03 | —258 | 40 | 53 | —183 | 38 | 03 | 08027 | 36 |
| 04 | 173 | 43 | 54 | 284 | 41 | 04 | —297 | 39 | 54 | —221 | 37 | 04 | —063 | 36 |
| 05 | —217 | 44 | 55 | 325 | 41 | 05 | 336 | 39 | 55 | 258 | 38 | 05 | —099 | 36 |
| 06 | —260 | 43 | 56 | 366 | 41 | 06 | —376 | 40 | 56 | —295 | 37 | 06 | —135 | 36 |
| 07 | —303 | 43 | 57 | 407 | 42 | 07 | —415 | 39 | 57 | 333 | 38 | 67 | —171 | 36 |
| 08 | 346 | 43 | 58 | —449 | 41 | 08 | —454 | 39 | 58 | —371 | 37 | 68 | —207 | 36 |
| 09 | 389 | 43 | 59 | —490 | 41 | 09 | 493 | 39 | 59 | 408 | 38 | 09 | —243 | 36 |
| **1010** | 432 | 43 | **1060** | —531 | 41 | **1110** | 532 | 39 | **1160** | —446 | 37 | **1210** | —279 | 35 |
| 11 | 475 | 43 | 61 | —572 | 41 | 11 | 571 | 39 | 61 | 483 | 38 | 11 | 314 | 36 |
| 12 | 518 | 43 | 62 | 612 | 40 | 12 | 610 | 39 | 62 | —521 | 38 | 12 | 350 | 36 |
| 13 | —561 | 43 | 63 | 653 | 41 | 13 | —650 | 40 | 63 | —558 | 37 | 13 | 386 | 36 |
| 14 | —604 | 43 | 64 | 694 | 41 | 14 | —689 | 39 | 64 | 595 | 37 | 14 | —422 | 36 |
| 15 | —647 | 43 | 65 | —735 | 41 | 15 | 727 | 38 | 65 | —632 | 38 | 15 | —458 | 36 |
| 16 | 689 | 42 | 66 | —776 | 41 | 16 | 766 | 39 | 66 | —670 | 37 | 16 | 493 | 35 |
| 17 | 732 | 43 | 67 | 816 | 40 | 17 | 805 | 39 | 67 | 707 | 37 | 17 | 529 | 36 |
| 18 | —775 | 43 | 68 | 857 | 41 | 18 | 844 | 39 | 68 | 744 | 37 | 18 | —565 | 36 |
| 19 | 817 | 42 | 69 | —898 | 41 | 19 | 883 | 39 | 69 | 781 | 37 | 19 | 600 | 35 |
| | | 43 | | | 41 | | | | | | 38 | | | 36 |
| **1020** | 860 | 43 | **1070** | 938 | 41 | **1120** | —922 | 39 | **1170** | —819 | 37 | **1220** | —636 | 36 |
| 21 | —903 | 42 | 71 | —979 | 40 | 21 | —961 | 38 | 71 | —856 | 37 | 21 | —672 | 36 |
| 22 | 945 | 43 | 72 | 03019 | 41 | 22 | 999 | 38 | 72 | —893 | 37 | 22 | 707 | 35 |
| 23 | —988 | 42 | 73 | —060 | 40 | 23 | 05038 | 39 | 73 | —930 | 37 | 23 | —743 | 36 |
| 24 | 01030 | 42 | 74 | 100 | 41 | 24 | —077 | 39 | 74 | —967 | 37 | 24 | 778 | 35 |
| 25 | 072 | 43 | 75 | —141 | 40 | 25 | 115 | 38 | 75 | 07004 | 37 | 25 | —814 | 36 |
| 26 | —115 | 42 | 76 | 181 | 41 | 26 | —154 | 39 | 76 | —041 | 37 | 26 | 849 | 35 |
| 27 | 157 | 42 | 77 | —222 | 40 | 27 | 192 | 38 | 77 | —078 | 37 | 27 | 884 | 36 |
| 28 | 199 | 42 | 78 | —262 | 40 | 28 | —231 | 39 | 78 | —115 | 37 | 28 | —920 | 35 |
| 29 | —242 | 43 | 79 | 302 | 40 | 29 | 269 | 38 | 79 | 151 | 36 | 29 | 955 | 35 |
| | | 42 | | | 40 | | | 39 | | | 37 | | | 36 |
| **1030** | —284 | 42 | **1080** | 342 | 41 | **1130** | —308 | 38 | **1180** | 188 | 37 | **1230** | —991 | 35 |
| 31 | —326 | 42 | 81 | —383 | 40 | 31 | 346 | 38 | 81 | —225 | 37 | 31 | 09026 | 35 |
| 32 | —368 | 42 | 82 | —423 | 40 | 32 | —385 | 38 | 82 | —262 | 37 | 32 | 061 | 35 |
| 33 | 410 | 42 | 83 | —463 | 40 | 33 | —423 | 38 | 83 | 298 | 36 | 33 | 096 | 35 |
| 34 | 452 | 42 | 84 | —503 | 40 | 34 | 461 | 39 | 84 | 335 | 37 | 34 | —132 | 36 |
| 35 | 494 | 42 | 85 | —543 | 40 | 35 | —500 | 38 | 85 | —372 | 37 | 35 | —167 | 35 |
| 36 | —536 | 42 | 86 | —583 | 40 | 36 | —538 | 38 | 86 | 408 | 36 | 36 | —202 | 35 |
| 37 | —578 | 42 | 87 | —623 | 40 | 37 | 576 | 38 | 87 | 445 | 37 | 37 | —237 | 35 |
| 38 | —620 | 42 | 88 | —663 | 40 | 38 | 614 | 38 | 88 | —482 | 37 | 38 | 272 | 35 |
| 39 | —662 | 41 | 89 | —703 | 40 | 39 | 652 | 38 | 89 | 518 | 36 | 39 | 307 | 35 |
| | | | | | | | | | | | 37 | | | 35 |
| **1040** | 703 | 42 | **1090** | —743 | 39 | **1140** | 690 | 39 | **1190** | —555 | 36 | **1240** | 342 | 35 |
| 41 | 745 | 42 | 91 | 782 | 40 | 41 | —729 | 38 | 91 | 591 | 37 | 41 | 377 | 35 |
| 42 | —787 | 42 | 92 | 822 | 40 | 42 | —767 | 38 | 92 | —628 | 36 | 42 | 412 | 35 |
| 43 | 828 | 41 | 93 | 862 | 40 | 43 | —805 | 38 | 93 | 664 | 36 | 43 | 447 | 35 |
| 44 | 870 | 42 | 94 | —902 | 39 | 44 | —843 | 38 | 94 | 700 | 36 | 44 | 482 | 35 |
| 45 | —912 | 42 | 95 | 941 | 40 | 45 | —881 | 38 | 95 | —737 | 37 | 45 | —517 | 35 |
| 46 | 953 | 41 | 96 | 981 | 40 | 46 | 918 | 37 | 96 | 773 | 36 | 46 | —552 | 35 |
| 47 | —995 | 42 | 97 | 04021 | 39 | 47 | 956 | 38 | 97 | 809 | 36 | 47 | —587 | 34 |
| 48 | 02036 | 41 | 98 | 060 | 40 | 48 | 994 | 38 | 98 | —846 | 37 | 48 | 621 | 35 |
| 49 | —078 | 42 | 99 | —100 | 39 | 49 | 06032 | 38 | 99 | —882 | 36 | 49 | 656 | 35 |
| | | 41 | | | 39 | | | 38 | | | 36 | | | 35 |

**Ejemplo**: Encontrar el log 11826:
log 11830=07298
dif. = 10 **36**
log 11820=07262
11826 — 11820=**6**
dif. para **6** debajo de **36**=22
log 11826=
07262+22 = 07284.

| | **44** | **43** | **42** | **41** | **40** | **39** | **38** | **37** | **36** | **35** | **34** | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 3 | **1** |
| **2** | 9 | 9 | 8 | 8 | 8 | 8 | 8 | 7 | 7 | 7 | 7 | **2** |
| **3** | 13 | 13 | 13 | 12 | 12 | 12 | 11 | 11 | 11 | 11 | 10 | **3** |
| **4** | 18 | 17 | 17 | 16 | 16 | 16 | 15 | 15 | 14 | 14 | 14 | **4** |
| **5** | 22 | 22 | 21 | 21 | 20 | 20 | 19 | 19 | 18 | 18 | 17 | **5** |
| **6** | 26 | 26 | 25 | 25 | 24 | 23 | 23 | 22 | 22 | 21 | 20 | **6** |
| **7** | 31 | 30 | 29 | 29 | 28 | 27 | 27 | 26 | 25 | 25 | 24 | **7** |
| **8** | 35 | 34 | 34 | 33 | 32 | 31 | 30 | 30 | 29 | 28 | 27 | **8** |
| **9** | 40 | 39 | 38 | 37 | 36 | 35 | 34 | 33 | 32 | 32 | 31 | **9** |

## Logaritmos vulgares ó de Briggs. Base=10.

| No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1250** | 09691 | 35 | **1300** | 11394 | 34 | **1350** | 13033 | 33 | **1400** | 14613 | 31 | **1450** | 16137 | 30 |
| 51 | —726 | 34 | 01 | —428 | 33 | 51 | —066 | 32 | 01 | —644 | 31 | 51 | —167 | 30 |
| 52 | 760 | 35 | 02 | 461 | 33 | 52 | —098 | 32 | 02 | —675 | 31 | 52 | —197 | 30 |
| 53 | 795 | 35 | 03 | 494 | 34 | 53 | —130 | 32 | 03 | —706 | 31 | 53 | —227 | 30 |
| 54 | —830 | 34 | 04 | —528 | 33 | 54 | —162 | 32 | 04 | —737 | 31 | 54 | 256 | 29 |
| 55 | 864 | 35 | 05 | 561 | 33 | 55 | —194 | 32 | 05 | —768 | 31 | 55 | 286 | 30 |
| 56 | —899 | 35 | 06 | 594 | 34 | 56 | —226 | 32 | 06 | —799 | 30 | 56 | 316 | 30 |
| 57 | —934 | 34 | 07 | —628 | 33 | 57 | —258 | 32 | 07 | 829 | 31 | 57 | —346 | 30 |
| 58 | 968 | 35 | 08 | —661 | 33 | 58 | —290 | 32 | 08 | 860 | 31 | 58 | —376 | 30 |
| 59 | 10003 | 34 | 09 | —694 | 33 | 59 | —322 | 32 | 09 | 891 | 31 | 59 | —406 | 29 |
| **1260** | 037 | 35 | **1310** | 727 | 33 | **1360** | —354 | 32 | **1410** | —922 | 31 | **1460** | 435 | 30 |
| 61 | —072 | 34 | 11 | 760 | 33 | 61 | —386 | 32 | 11 | —953 | 31 | 61 | 465 | 30 |
| 62 | —106 | 34 | 12 | 793 | 33 | 62 | —418 | 32 | 12 | 983 | 30 | 62 | —495 | 29 |
| 63 | 140 | 34 | 13 | 826 | 34 | 63 | —450 | 32 | 13 | 15014 | 31 | 63 | 524 | 30 |
| 64 | —175 | 35 | 14 | —860 | 33 | 64 | 481 | 31 | 14 | —045 | 31 | 64 | 554 | 30 |
| 65 | 209 | 34 | 15 | —893 | 33 | 65 | 513 | 32 | 15 | —076 | 31 | 65 | —584 | 30 |
| 66 | 243 | 34 | 16 | —926 | 33 | 66 | 545 | 32 | 16 | 106 | 30 | 66 | 613 | 29 |
| 67 | —278 | 35 | 17 | —959 | 33 | 67 | —577 | 32 | 17 | —137 | 31 | 67 | 643 | 30 |
| 68 | —312 | 34 | 18 | —992 | 33 | 68 | —609 | 32 | 18 | —168 | 31 | 68 | —673 | 30 |
| 69 | 346 | 34 | 19 | 12024 | 32 | 69 | 640 | 31 | 19 | 198 | 30 | 69 | 702 | 29 |
| | | | | | 33 | | | 32 | | | 31 | | | |
| **1270** | 380 | 35 | **1320** | 057 | 33 | **1370** | 672 | 32 | **1420** | —229 | 30 | **1470** | —732 | 29 |
| 71 | —415 | 34 | 21 | 090 | 33 | 71 | —704 | 31 | 21 | 259 | 31 | 71 | 761 | 30 |
| 72 | —449 | 34 | 22 | 123 | 33 | 72 | 735 | 32 | 22 | —290 | 31 | 72 | —791 | 29 |
| 73 | —483 | 34 | 23 | —156 | 33 | 73 | 767 | 32 | 23 | 320 | 30 | 73 | 820 | 30 |
| 74 | —517 | 34 | 24 | —189 | 33 | 74 | —799 | 32 | 24 | —351 | 31 | 74 | —850 | 29 |
| 75 | 551 | 34 | 25 | —222 | 32 | 75 | 830 | 31 | 25 | 381 | 30 | 75 | 879 | 30 |
| 76 | 585 | 34 | 26 | 254 | 33 | 76 | —862 | 32 | 26 | —412 | 31 | 76 | —909 | 29 |
| 77 | 619 | 34 | 27 | 287 | 33 | 77 | 893 | 31 | 27 | 442 | 30 | 77 | 938 | 29 |
| 78 | 653 | 34 | 28 | —320 | 32 | 78 | —925 | 32 | 28 | —473 | 31 | 78 | 967 | 30 |
| 79 | 687 | 34 | 29 | 352 | 33 | 79 | 956 | 31 | 29 | 503 | 30 | 79 | —997 | 29 |
| | | | | | | | | 32 | | | 31 | | | |
| **1280** | —721 | 34 | **1330** | 385 | 33 | **1380** | —988 | 31 | **1430** | —534 | 30 | **1480** | 17026 | 30 |
| 81 | —755 | 34 | 31 | —418 | 32 | 81 | 14019 | 32 | 31 | —564 | 30 | 81 | —056 | 29 |
| 82 | —789 | 34 | 32 | 450 | 33 | 82 | —051 | 31 | 32 | 594 | 31 | 82 | —085 | 29 |
| 83 | —823 | 34 | 33 | 483 | 33 | 83 | 082 | 32 | 33 | —625 | 30 | 83 | 114 | 29 |
| 84 | —857 | 33 | 34 | —516 | 32 | 84 | —114 | 31 | 34 | —655 | 30 | 84 | 143 | 30 |
| 85 | 890 | 34 | 35 | 548 | 33 | 85 | —145 | 31 | 35 | 685 | 30 | 85 | —173 | 29 |
| 86 | 924 | 34 | 36 | —581 | 32 | 86 | 176 | 32 | 36 | 715 | 30 | 86 | —202 | 29 |
| 87 | —958 | 34 | 37 | 613 | 33 | 87 | —208 | 31 | 37 | —746 | 31 | 87 | 231 | 29 |
| 88 | —992 | 33 | 38 | —646 | 32 | 88 | —239 | 31 | 38 | —776 | 30 | 88 | 260 | 29 |
| 89 | 11025 | 34 | 39 | 678 | 32 | 89 | 270 | 31 | 39 | 806 | 30 | 89 | 289 | 30 |
| **1290** | —059 | 34 | **1340** | 710 | 33 | **1390** | 301 | 32 | **1440** | 836 | 30 | **1490** | —319 | 29 |
| 91 | —093 | 34 | 41 | —743 | 32 | 91 | —333 | 31 | 41 | 866 | 31 | 91 | —348 | 29 |
| 92 | 126 | 34 | 42 | 775 | 33 | 92 | —364 | 31 | 42 | —897 | 30 | 92 | —377 | 29 |
| 93 | —160 | 33 | 43 | —808 | 32 | 93 | 395 | 31 | 43 | —927 | 30 | 93 | —406 | 29 |
| 94 | 193 | 34 | 44 | —840 | 32 | 94 | 426 | 31 | 44 | —957 | 30 | 94 | 435 | 29 |
| 95 | —227 | 34 | 45 | 872 | 33 | 95 | 457 | 32 | 45 | —987 | 30 | 95 | 464 | 29 |
| 96 | —261 | 33 | 46 | —905 | 32 | 96 | —489 | 31 | 46 | 16017 | 30 | 96 | 493 | 29 |
| 97 | —294 | 33 | 47 | —937 | 32 | 97 | —520 | 31 | 47 | —047 | 30 | 97 | 522 | 29 |
| 98 | 327 | 34 | 48 | —969 | 32 | 98 | —551 | 31 | 48 | —077 | 30 | 98 | 551 | 29 |
| 99 | —361 | 33 | 49 | 13001 | 32 | 99 | —582 | 31 | 49 | —107 | 30 | 99 | 580 | 29 |

**Ejemplo**: Encontrar log 12605:
log 12610=10072
dif. = 10 **35**
log 12600=10037
12605 — 12600 = **5**
dif. para **5** debajo **35**=18
log 12605=
10037+18=10055.

| | 35 | 34 | 33 | 32 | 31 | 30 | 29 | |
|---|---|---|---|---|---|---|---|---|
| **1** | 4 | 3 | 3 | 3 | 3 | 3 | 3 | **1** |
| **2** | 7 | 7 | 7 | 6 | 6 | 6 | 6 | **2** |
| **3** | 11 | 10 | 10 | 10 | 9 | 9 | 9 | **3** |
| **4** | 14 | 14 | 13 | 13 | 12 | 12 | 12 | **4** |
| **5** | 18 | 17 | 17 | 16 | 16 | 15 | 15 | **5** |
| **6** | 21 | 20 | 20 | 19 | 19 | 18 | 17 | **6** |
| **7** | 25 | 24 | 23 | 22 | 22 | 21 | 20 | **7** |
| **8** | 28 | 27 | 26 | 26 | 25 | 24 | 23 | **8** |
| **9** | 32 | 31 | 30 | 29 | 28 | 27 | 26 | **9** |

**Un guión** antes ó después de un log indica que su verdadero valor es *menor* que el tabular en *menos* de la mitad de la unidad correspondiente al último puesto. Ej.:
log 1366=1354507
log 1367=1357685

## Logaritmos vulgares ó de Briggs. Base=10.

| No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1500** | 17609 | 29 | **1550** | 19033 | 28 | **1600** | 20412 | 27 | **1650** | 21748 | 27 | **1700** | 23045 | 25 |
| 01 | 638 | 29 | 51 | 061 | 28 | 01 | 439 | 27 | 51 | —775 | 26 | 01 | 070 | 26 |
| 02 | —667 | 29 | 52 | 089 | 28 | 02 | 466 | 27 | 52 | 801 | 26 | 02 | —096 | 25 |
| 03 | —696 | 29 | 53 | 117 | 28 | 03 | 493 | 27 | 53 | 827 | 27 | 03 | 121 | 26 |
| 04 | —725 | 29 | 54 | 145 | 28 | 04 | 520 | 27 | 54 | —854 | 26 | 04 | —147 | 25 |
| 05 | —754 | 28 | 55 | 173 | 28 | 05 | —548 | 28 | 55 | —880 | 26 | 05 | 172 | 26 |
| 06 | 782 | 29 | 56 | —201 | 28 | 06 | —575 | 27 | 56 | 906 | 26 | 06 | —198 | 25 |
| 07 | 811 | 29 | 57 | —229 | 28 | 07 | —602 | 27 | 57 | 932 | 26 | 07 | 223 | 26 |
| 08 | 840 | 29 | 58 | —257 | 28 | 08 | —629 | 27 | 58 | 958 | 27 | 08 | —249 | 25 |
| 09 | —869 | 29 | 59 | —285 | 28 | 09 | —656 | 27 | 59 | —985 | 26 | 09 | 274 | 26 |
| **1510** | —898 | 28 | **1560** | 312 | 28 | **1610** | —683 | 27 | **1660** | 22011 | 26 | **1710** | —300 | 25 |
| 11 | 926 | 29 | 61 | 340 | 28 | 11 | —710 | 27 | 61 | —037 | 26 | 11 | 325 | 25 |
| 12 | 955 | 29 | 62 | 368 | 28 | 12 | —737 | 26 | 62 | 063 | 26 | 12 | 350 | 26 |
| 13 | —984 | 29 | 63 | —396 | 28 | 13 | 763 | 27 | 63 | 089 | 26 | 13 | —376 | 25 |
| 14 | 18013 | 28 | 64 | —424 | 27 | 14 | 790 | 27 | 64 | 115 | 26 | 14 | 401 | 25 |
| 15 | 041 | 28 | 65 | 451 | 28 | 15 | 817 | 27 | 65 | 141 | 26 | 15 | 426 | 26 |
| 16 | —070 | 29 | 66 | 479 | 28 | 16 | 844 | 27 | 66 | 167 | 26 | 16 | —452 | 25 |
| 17 | —099 | 29 | 67 | —507 | 28 | 17 | 871 | 27 | 67 | —194 | 27 | 17 | 477 | 25 |
| 18 | 127 | 28 | 68 | —535 | 27 | 18 | —898 | 27 | 68 | —220 | 26 | 18 | 502 | 26 |
| 19 | —156 | 29 | 69 | 562 | 28 | 19 | —925 | 27 | 69 | —246 | 26 | 19 | —528 | 25 |
| **1520** | 184 | 29 | **1570** | —590 | 28 | **1620** | —952 | 26 | **1670** | —272 | 26 | **1720** | —553 | 25 |
| 21 | —213 | 28 | 71 | —618 | 27 | 21 | 978 | 27 | 71 | —298 | 26 | 21 | 578 | 25 |
| 22 | 241 | 29 | 72 | 645 | 28 | 22 | 21005 | 27 | 72 | —324 | 26 | 22 | 603 | 26 |
| 23 | —270 | 28 | 73 | —673 | 27 | 23 | —032 | 27 | 73 | —350 | 26 | 23 | —629 | 25 |
| 24 | 298 | 29 | 74 | 700 | 28 | 24 | —059 | 26 | 74 | —376 | 25 | 24 | —654 | 25 |
| 25 | —327 | 28 | 75 | 728 | 28 | 25 | 085 | 27 | 75 | 401 | 26 | 25 | —679 | 25 |
| 26 | 355 | 29 | 76 | —756 | 27 | 26 | 112 | 27 | 76 | 427 | 26 | 26 | 704 | 25 |
| 27 | —384 | 28 | 77 | 783 | 28 | 27 | —139 | 26 | 77 | 453 | 26 | 27 | 729 | 25 |
| 28 | 412 | 29 | 78 | —811 | 27 | 28 | 165 | 27 | 78 | 479 | 26 | 28 | 754 | 25 |
| 29 | —441 | 28 | 79 | 838 | 28 | 29 | 192 | 27 | 79 | 505 | 26 | 29 | 779 | 26 |
| **1530** | 469 | 29 | **1580** | —866 | 27 | **1630** | —219 | 26 | **1680** | 531 | 26 | **1730** | —805 | 25 |
| 31 | —498 | 28 | 81 | 893 | 28 | 31 | 245 | 27 | 81 | —557 | 26 | 31 | —830 | 25 |
| 32 | —526 | 28 | 82 | —921 | 27 | 32 | 272 | 27 | 82 | —583 | 26 | 32 | —855 | 25 |
| 33 | 554 | 29 | 83 | 948 | 28 | 33 | —299 | 26 | 83 | 608 | 25 | 33 | —880 | 25 |
| 34 | —583 | 28 | 84 | —976 | 27 | 34 | 325 | 27 | 84 | 634 | 26 | 34 | —905 | 25 |
| 35 | —611 | 28 | 85 | 20003 | 27 | 35 | —352 | 26 | 85 | —660 | 26 | 35 | —930 | 25 |
| 36 | 639 | 28 | 86 | 030 | 27 | 36 | 378 | 27 | 86 | —686 | 26 | 36 | —955 | 25 |
| 37 | 667 | 29 | 87 | —058 | 28 | 37 | —405 | 26 | 87 | —712 | 25 | 37 | —980 | 25 |
| 38 | —696 | 28 | 88 | 085 | 27 | 38 | 431 | 27 | 88 | 737 | 26 | 38 | 24005 | 25 |
| 39 | —724 | 28 | 89 | 112 | 27 | 39 | —458 | 26 | 89 | —763 | 26 | 39 | —030 | 25 |
| **1540** | 752 | 28 | **1590** | —140 | 27 | **1640** | 484 | 27 | **1690** | —789 | 25 | **1740** | —055 | 25 |
| 41 | 780 | 28 | 91 | 167 | 27 | 41 | —511 | 26 | 91 | 814 | 26 | 41 | —080 | 25 |
| 42 | 808 | 29 | 92 | 194 | 27 | 42 | 537 | 27 | 92 | 840 | 26 | 42 | —105 | 25 |
| 43 | —837 | 28 | 93 | —222 | 28 | 43 | —564 | 27 | 93 | —866 | 26 | 43 | —130 | 25 |
| 44 | —865 | 28 | 94 | —249 | 27 | 44 | 590 | 26 | 94 | 891 | 27 | 44 | —155 | 25 |
| 45 | —893 | 28 | 95 | 276 | 27 | 45 | —617 | 27 | 95 | —917 | 26 | 45 | —180 | 25 |
| 46 | —921 | 28 | 96 | 303 | 27 | 46 | —643 | 26 | 96 | —943 | 26 | 46 | 204 | 24 |
| 47 | 949 | 28 | 97 | 330 | 27 | 47 | 669 | 26 | 97 | 968 | 25 | 47 | 229 | 25 |
| 48 | 977 | 28 | 98 | —358 | 27 | 48 | —696 | 27 | 98 | —994 | 26 | 48 | 254 | 25 |
| 49 | 19005 | 28 | 99 | —385 | 27 | 49 | 722 | 26 | 99 | 23019 | 25 | 49 | —279 | 25 |

**Ejemplo**: Para encontrar log 15414:
log 15420=18808
dif. = 10 **28**
log 15410=18780
15414 — 15410=4
dif. para 4 debajo **28**
=11
log 15414=
18780+11=18791.

| | **29** | **28** | **27** | **26** | **25** | **24** | |
|---|---|---|---|---|---|---|---|
| **1** | 3 | 3 | 3 | 3 | 3 | 2 | **1** |
| **2** | 6 | 6 | 5 | 5 | 5 | 5 | **2** |
| **3** | 9 | 8 | 8 | 8 | 8 | 7 | **3** |
| **4** | 12 | 11 | 11 | 10 | 10 | 10 | **4** |
| **5** | 15 | 14 | 14 | 13 | 13 | 12 | **5** |
| **6** | 17 | 17 | 16 | 16 | 15 | 14 | **6** |
| **7** | 20 | 20 | 19 | 18 | 18 | 17 | **7** |
| **8** | 23 | 22 | 22 | 21 | 20 | 19 | **8** |
| **9** | 26 | 25 | 24 | 23 | 23 | 22 | **9** |

**Un guión** antes ó despues de un log indica que su verdadero valor es *menor* que el tabular en *menos* de la mitad de la unidad correspondiente al último puesto. Ej.:
log 1562=1936810
log 1563=1939590

## Logaritmos vulgares ó de Briggs. Base=10.

| No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1750** | 24304 | 49 | **1850** | 26717 | 47 | **1950** | 29003 | 45 | **2050** | 31175 | 43 | **2150** | 33244 | 40 |
| 52 | 353 | 50 | 52 | 764 | 47 | 52 | —048 | 44 | 52 | —218 | 42 | 52 | 284 | 41 |
| 54 | —403 | 49 | 54 | —811 | 47 | 54 | 092 | 45 | 54 | 260 | 42 | 54 | —325 | 40 |
| 56 | 452 | 50 | 56 | —858 | 47 | 56 | —137 | 44 | 56 | 302 | 43 | 56 | —365 | 40 |
| 58 | —502 | 49 | 58 | —905 | 46 | 58 | 181 | 45 | 58 | —345 | 42 | 58 | 405 | 40 |
| **1760** | 551 | 50 | **1860** | 951 | 47 | **1960** | —226 | 44 | **2060** | —387 | 42 | **2160** | 445 | 41 |
| 62 | —601 | 49 | 62 | —998 | 47 | 62 | —270 | 44 | 62 | —429 | 42 | 62 | —486 | 40 |
| 64 | —650 | 49 | 64 | 27045 | 46 | 64 | 314 | 44 | 64 | —471 | 42 | 64 | —526 | 40 |
| 66 | 699 | 49 | 66 | 091 | 47 | 66 | 358 | 45 | 66 | 513 | 42 | 66 | —566 | 40 |
| 68 | 748 | 49 | 68 | —138 | 46 | 68 | —403 | 44 | 68 | 555 | 42 | 68 | —606 | 40 |
| **1770** | 797 | 49 | **1870** | 184 | 47 | **1970** | —447 | 44 | **2070** | 597 | 42 | **2170** | —646 | 40 |
| 72 | 846 | 49 | 72 | —231 | 46 | 72 | —491 | 44 | 72 | —639 | 42 | 72 | —686 | 40 |
| 74 | 895 | 49 | 74 | —277 | 46 | 74 | —535 | 44 | 74 | —681 | 42 | 74 | —726 | 40 |
| 76 | 944 | 49 | 76 | 323 | 46 | 76 | —579 | 44 | 76 | —723 | 42 | 76 | —766 | 40 |
| 78 | 993 | 49 | 78 | —370 | 47 | 78 | —623 | 44 | 78 | —765 | 41 | 78 | —806 | 40 |
| **1780** | 25042 | 49 | **1880** | —416 | 46 | **1980** | —667 | 43 | **2080** | 806 | 42 | **2180** | —846 | 39 |
| 82 | —091 | 48 | 82 | —462 | 46 | 82 | 710 | 44 | 82 | 848 | 42 | 82 | 885 | 40 |
| 84 | 139 | 49 | 84 | 508 | 46 | 84 | 754 | 44 | 84 | —890 | 41 | 84 | 925 | 40 |
| 86 | 188 | 49 | 86 | 554 | 46 | 86 | —798 | 44 | 86 | 931 | 42 | 86 | 965 | 40 |
| 88 | —237 | 48 | 88 | 600 | 46 | 88 | —842 | 43 | 88 | 973 | 42 | 88 | 34005 | 39 |
| **1790** | 285 | 49 | **1890** | 646 | 46 | **1990** | 885 | 44 | **2090** | 32015 | 41 | **2190** | 044 | 40 |
| 92 | —334 | 48 | 92 | 692 | 46 | 92 | —929 | 44 | 92 | 056 | 42 | 92 | 084 | 40 |
| 94 | 382 | 49 | 94 | —738 | 46 | 94 | —973 | 43 | 94 | —098 | 41 | 94 | —124 | 39 |
| 96 | —431 | 48 | 96 | —784 | 46 | 96 | 30016 | 44 | 96 | 139 | 42 | 96 | 163 | 40 |
| 98 | —479 | 48 | 98 | —830 | 46 | 98 | —060 | 43 | 98 | —181 | 41 | 98 | —203 | 39 |
| **1800** | 527 | 48 | **1900** | 875 | 46 | **2000** | —103 | 43 | **2100** | —222 | 41 | **2200** | 242 | 40 |
| 02 | 575 | 49 | 02 | 921 | 46 | 02 | 146 | 44 | 02 | 263 | 42 | 02 | —282 | 39 |
| 04 | —624 | 48 | 04 | —967 | 45 | 04 | —190 | 43 | 04 | —305 | 41 | 04 | 321 | 40 |
| 06 | —672 | 48 | 06 | 28012 | 46 | 06 | 233 | 43 | 06 | —346 | 41 | 06 | —361 | 39 |
| 08 | —720 | 48 | 08 | —058 | 45 | 08 | 276 | 44 | 08 | 387 | 41 | 08 | —400 | 39 |
| **1810** | —768 | 48 | **1910** | 103 | 46 | **2010** | —320 | 43 | **2110** | 428 | 41 | **2210** | 439 | 40 |
| 12 | —816 | 48 | 12 | —149 | 45 | 12 | —363 | 43 | 12 | 469 | 41 | 12 | —479 | 39 |
| 14 | —864 | 48 | 14 | 194 | 46 | 14 | —406 | 43 | 14 | 510 | 42 | 14 | —518 | 39 |
| 16 | —912 | 47 | 16 | —240 | 45 | 16 | 449 | 43 | 16 | —552 | 41 | 16 | —557 | 39 |
| 18 | 959 | 48 | 18 | —285 | 45 | 18 | 492 | 43 | 18 | —593 | 41 | 18 | 596 | 39 |
| **1820** | 26007 | 48 | **1920** | 330 | 45 | **2020** | 535 | 43 | **2120** | —634 | 41 | **2220** | 635 | 39 |
| 22 | —055 | 47 | 22 | 375 | 46 | 22 | 578 | 43 | 22 | —675 | 40 | 22 | 674 | 39 |
| 24 | 102 | 48 | 24 | —421 | 45 | 24 | 621 | 43 | 24 | 715 | 41 | 24 | 713 | 40 |
| 26 | 150 | 48 | 26 | —466 | 45 | 26 | —664 | 43 | 26 | 756 | 41 | 26 | —753 | 39 |
| 28 | —198 | 47 | 28 | —511 | 45 | 28 | —707 | 43 | 28 | 797 | 41 | 28 | —792 | 38 |
| **1830** | 245 | 48 | **1930** | —556 | 45 | **2030** | —750 | 42 | **2130** | —838 | 41 | **2230** | 830 | 39 |
| 32 | —293 | 47 | 32 | —601 | 45 | 32 | 792 | 43 | 32 | —879 | 40 | 32 | 869 | 39 |
| 34 | —340 | 47 | 34 | —646 | 45 | 34 | 835 | 43 | 34 | 919 | 41 | 34 | 908 | 39 |
| 36 | 387 | 48 | 36 | —691 | 44 | 36 | —878 | 42 | 36 | 960 | 41 | 36 | 947 | 39 |
| 38 | —435 | 47 | 38 | 735 | 45 | 38 | 920 | 43 | 38 | 33001 | 40 | 38 | 986 | 39 |
| **1840** | —482 | 47 | **1940** | 780 | 45 | **2040** | 963 | 43 | **2140** | 041 | 41 | **2240** | 35025 | 39 |
| 42 | —529 | 47 | 42 | —825 | 45 | 42 | 31006 | 42 | 42 | —082 | 40 | 42 | —064 | 38 |
| 44 | 576 | 47 | 44 | —870 | 44 | 44 | 048 | 43 | 44 | 122 | 41 | 44 | 102 | 39 |
| 46 | 623 | 47 | 46 | 914 | 45 | 46 | —091 | 42 | 46 | —163 | 40 | 46 | —141 | 39 |
| 48 | 670 | 47 | 48 | —959 | 44 | 48 | —133 | 42 | 48 | 203 | 41 | 48 | —180 | 38 |

Encontr. log 18117:
log 18120=25816
dif. 20 48
log 18100=25768
18117—18100=17
Debajo 48
dif. para 10=24
— — 7=17
— — 17=41
log 18117=
25768+41=25809.

| | 50 | 49 | 48 | 47 | 46 | 45 | 44 | 43 | 42 | 41 | 40 | 39 | 38 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | **1** |
| **2** | 5 | 5 | 5 | 5 | 5 | 5 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | **2** |
| **3** | 8 | 7 | 7 | 7 | 7 | 7 | 7 | 6 | 6 | 6 | 6 | 6 | 6 | **3** |
| **4** | 10 | 10 | 10 | 9 | 9 | 9 | 9 | 9 | 8 | 8 | 8 | 8 | 8 | **4** |
| **5** | 13 | 12 | 12 | 12 | 12 | 11 | 11 | 11 | 11 | 10 | 10 | 10 | 10 | **5** |
| **6** | 15 | 15 | 14 | 14 | 14 | 14 | 13 | 13 | 13 | 12 | 12 | 12 | 11 | **6** |
| **7** | 18 | 17 | 17 | 16 | 16 | 16 | 15 | 15 | 15 | 14 | 14 | 14 | 13 | **7** |
| **8** | 20 | 20 | 19 | 19 | 18 | 18 | 18 | 17 | 17 | 16 | 16 | 16 | 15 | **8** |
| **9** | 23 | 22 | 22 | 21 | 21 | 20 | 20 | 19 | 19 | 18 | 18 | 18 | 17 | **9** |
| **10** | 25 | 25 | 24 | 24 | 23 | 23 | 22 | 22 | 21 | 21 | 20 | 20 | 19 | **10** |

## Logaritmos vulgares ó de Briggs. Base=10.

| No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2250** | 35218 | 39 | **2350** | 37107 | 37 | **2450** | 38917 | 35 | **2550** | 40654 | 34 | **2650** | 42325 | 32 |
| 52 | —257 | 38 | 52 | —144 | 37 | 52 | 952 | 35 | 52 | 688 | 34 | 52 | 357 | 33 |
| 54 | 295 | 39 | 54 | —181 | 37 | 54 | 987 | 35 | 54 | 722 | 34 | 54 | 390 | 33 |
| 56 | —334 | 39 | 56 | —218 | 37 | 56 | 39023 | 36 | 56 | 756 | 34 | 56 | —423 | 33 |
| 58 | 372 | 38 | 58 | 254 | 36 | 58 | 058 | 35 | 58 | 790 | 34 | 58 | 455 | 32 |
| | | 39 | | | 37 | | | 36 | | | 34 | | | 33 |
| **2260** | —411 | 38 | **2360** | 291 | 37 | **2460** | —094 | 35 | **2560** | —824 | 34 | **2660** | 488 | 33 |
| 62 | 449 | 39 | 62 | —328 | 37 | 62 | —129 | 35 | 62 | —858 | 34 | 62 | —521 | 32 |
| 64 | —488 | 38 | 64 | —365 | 37 | 64 | 164 | 35 | 64 | —892 | 34 | 64 | 553 | 32 |
| 66 | —526 | 38 | 66 | 401 | 36 | 66 | 199 | 35 | 66 | —926 | 34 | 66 | 586 | 33 |
| 68 | 564 | 38 | 68 | 438 | 37 | 68 | —235 | 36 | 68 | —960 | 33 | 68 | —619 | 32 |
| | | 39 | | | 37 | | | 35 | | | 34 | | | 32 |
| **2270** | —603 | 38 | **2370** | —475 | 36 | **2470** | —270 | 35 | **2570** | 993 | 34 | **2670** | 651 | 33 |
| 72 | —641 | 38 | 72 | 511 | 37 | 72 | —305 | 35 | 72 | 41027 | 34 | 72 | —684 | 32 |
| 74 | 679 | 38 | 74 | 548 | 37 | 74 | —340 | 35 | 74 | —061 | 34 | 74 | 716 | 33 |
| 76 | 717 | 38 | 76 | —585 | 37 | 76 | 375 | 35 | 76 | —095 | 33 | 76 | —749 | 32 |
| 78 | 755 | 38 | 78 | 621 | 36 | 78 | 410 | 35 | 78 | 128 | 34 | 78 | 781 | 32 |
| | | 38 | | | 37 | | | 35 | | | 34 | | | 32 |
| **2280** | 793 | 39 | **2380** | —658 | 36 | **2480** | 445 | 35 | **2580** | —162 | 34 | **2680** | 813 | 33 |
| 82 | —832 | 38 | 82 | 694 | 37 | 82 | 480 | 35 | 82 | —196 | 33 | 82 | —846 | 33 |
| 84 | —870 | 38 | 84 | —731 | 36 | 84 | 515 | 35 | 84 | 229 | 34 | 84 | 878 | 33 |
| 86 | —908 | 38 | 86 | 767 | 36 | 86 | 550 | 35 | 86 | —263 | 33 | 86 | —911 | 32 |
| 88 | —946 | 38 | 88 | 803 | 36 | 88 | 585 | 35 | 88 | 296 | 34 | 88 | —943 | 32 |
| | | 37 | | | 37 | | | 35 | | | 34 | | | 32 |
| **2290** | —984 | 37 | **2390** | —840 | 36 | **2490** | —620 | 35 | **2590** | —330 | 33 | **2690** | 975 | 33 |
| 92 | 36021 | 38 | 92 | 876 | 36 | 92 | —655 | 35 | 92 | 363 | 34 | 92 | 48008 | 33 |
| 94 | 059 | 38 | 94 | 912 | 37 | 94 | —690 | 34 | 94 | —397 | 33 | 94 | —040 | 32 |
| 96 | 097 | 38 | 96 | —949 | 36 | 96 | 724 | 35 | 96 | 430 | 34 | 96 | —072 | 32 |
| 98 | 135 | 38 | 98 | —985 | 36 | 98 | 759 | 35 | 98 | —464 | 33 | 98 | 104 | 32 |
| | | 38 | | | 36 | | | 35 | | | 33 | | | 32 |
| **2300** | —173 | 38 | **2400** | 38021 | 36 | **2500** | 794 | 35 | **2600** | 497 | 34 | **2700** | 136 | 33 |
| 02 | —211 | 37 | 02 | 057 | 36 | 02 | —829 | 34 | 02 | —531 | 33 | 02 | —169 | 32 |
| 04 | 248 | 38 | 04 | 093 | 37 | 04 | 863 | 35 | 04 | 564 | 33 | 04 | —201 | 32 |
| 06 | —286 | 38 | 06 | —130 | 36 | 06 | 898 | 35 | 06 | 597 | 34 | 06 | —233 | 32 |
| 08 | —324 | 37 | 08 | —166 | 36 | 08 | —933 | 34 | 08 | —631 | 33 | 08 | —265 | 32 |
| | | 38 | | | 36 | | | 34 | | | 33 | | | 32 |
| **2310** | 361 | 38 | **2410** | —202 | 36 | **2510** | 967 | 35 | **2610** | 664 | 33 | **2710** | —297 | 32 |
| 12 | —399 | 37 | 12 | —238 | 36 | 12 | 40002 | 35 | 12 | 697 | 34 | 12 | —329 | 32 |
| 14 | 436 | 38 | 14 | —274 | 36 | 14 | —037 | 35 | 14 | —731 | 33 | 14 | —361 | 32 |
| 16 | —474 | 37 | 16 | —310 | 36 | 16 | 071 | 34 | 16 | —764 | 33 | 16 | —393 | 32 |
| 18 | 511 | 38 | 18 | —346 | 36 | 18 | —106 | 35 | 18 | —797 | 33 | 18 | —425 | 32 |
| **2320** | —549 | 37 | **2420** | —382 | 35 | **2520** | 140 | 34 | **2620** | 830 | 33 | **2720** | —457 | 32 |
| 22 | 586 | 38 | 22 | 417 | 36 | 22 | —175 | 35 | 22 | 863 | 33 | 22 | —489 | 32 |
| 24 | —624 | 37 | 24 | 453 | 36 | 24 | —209 | 34 | 24 | 896 | 33 | 24 | —521 | 32 |
| 26 | —661 | 37 | 26 | 489 | 36 | 26 | 243 | 34 | 26 | 929 | 34 | 26 | —553 | 31 |
| 28 | 698 | 38 | 28 | —525 | 36 | 28 | —278 | 35 | 28 | —963 | 33 | 28 | 584 | 32 |
| | | 38 | | | 35 | | | 34 | | | 33 | | | 32 |
| **2330** | —736 | 37 | **2430** | —561 | 35 | **2530** | 312 | 34 | **2630** | —996 | 33 | **2730** | 616 | 32 |
| 32 | —773 | 37 | 32 | 596 | 36 | 32 | 346 | 35 | 32 | 42029 | 33 | 32 | 648 | 32 |
| 34 | 810 | 37 | 34 | 632 | 36 | 34 | —381 | 34 | 34 | —062 | 33 | 34 | —680 | 32 |
| 36 | 847 | 37 | 36 | —668 | 35 | 36 | —415 | 34 | 36 | —095 | 33 | 36 | —712 | 31 |
| 38 | 884 | 38 | 38 | 703 | 36 | 38 | 449 | 34 | 38 | 127 | 32 | 38 | 743 | 32 |
| | | 38 | | | 36 | | | 34 | | | 33 | | | 32 |
| **2340** | —922 | 37 | **2440** | —739 | 36 | **2540** | 483 | 35 | **2640** | 160 | 33 | **2740** | 775 | 32 |
| 42 | —959 | 37 | 42 | —775 | 35 | 42 | —518 | 34 | 42 | 193 | 33 | 42 | —807 | 31 |
| 44 | —996 | 37 | 44 | 810 | 36 | 44 | —552 | 34 | 44 | 226 | 33 | 44 | 838 | 32 |
| 46 | 37033 | 37 | 46 | —846 | 35 | 46 | —586 | 34 | 46 | —259 | 33 | 46 | 870 | 32 |
| 48 | —070 | 37 | 48 | 881 | 35 | 48 | —620 | 34 | 48 | —292 | 33 | 48 | —902 | 31 |
| | | 37 | | | 35 | | | 34 | | | 33 | | | 31 |

Encontrar log 23335 :
log 23340=36810
dif. 20 **37**
log 23320 = 36773
23335 — 233320=**15**
Debajo **37**
dif. para 10=19
— — 5= 9
— — **15**=28
log 23335=
36773+28=36801.

| | **39** | **38** | **37** | **36** | **35** | **34** | **33** | **32** | **31** | |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | **1** |
| **2** | 4 | 4 | 4 | 4 | 4 | 3 | 3 | 3 | 3 | **2** |
| **3** | 6 | 6 | 6 | 5 | 5 | 5 | 5 | 5 | 5 | **3** |
| **4** | 8 | 8 | 7 | 7 | 7 | 7 | 7 | 6 | 6 | **4** |
| **5** | 10 | 10 | 9 | 9 | 9 | 8 | 8 | 8 | 8 | **5** |
| **6** | 12 | 11 | 11 | 11 | 11 | 10 | 10 | 10 | 9 | **6** |
| **7** | 14 | 13 | 13 | 13 | 12 | 12 | 12 | 11 | 11 | **7** |
| **8** | 16 | 15 | 15 | 14 | 14 | 14 | 13 | 13 | 12 | **8** |
| **9** | 18 | 17 | 17 | 16 | 16 | 15 | 15 | 14 | 14 | **9** |
| **10** | 20 | 19 | 19 | 18 | 18 | 17 | 17 | 16 | 16 | **10** |

## Logaritmos vulgares ó de Briggs. Base=10.

| No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2750** | 43933 | 32 | **2850** | 45484 | 31 | **2950** | 46982 | 30 | **3050** | 48430 | 28 | **3150** | 49831 | 28 |
| 52 | —965 | 31 | 52 | —515 | 30 | 52 | 47012 | 29 | 52 | 458 | 29 | 52 | —859 | 27 |
| 54 | 996 | 32 | 54 | 545 | 31 | 54 | 041 | 29 | 54 | —487 | 29 | 54 | 886 | 28 |
| 56 | 44028 | 31 | 56 | —576 | 30 | 56 | 070 | 30 | 56 | 515 | 29 | 56 | —914 | 27 |
| 58 | 059 | 32 | 58 | 606 | 31 | 58 | —100 | 29 | 58 | —544 | 28 | 58 | 941 | 28 |
| **2760** | —091 | 31 | **2860** | —637 | 30 | **2960** | 129 | 30 | **3060** | 572 | 29 | **3160** | —969 | 27 |
| 62 | 122 | 32 | 62 | —667 | 30 | 62 | —159 | 29 | 62 | —601 | 28 | 62 | 996 | 28 |
| 64 | —154 | 31 | 64 | 697 | 31 | 64 | —188 | 29 | 64 | —629 | 28 | 64 | 50024 | 27 |
| 66 | 185 | 32 | 66 | —728 | 30 | 66 | 217 | 29 | 66 | 657 | 29 | 66 | 051 | 28 |
| 68 | —217 | 31 | 68 | —758 | 30 | 68 | 246 | 30 | 68 | —686 | 28 | 68 | —079 | 27 |
| **2770** | —248 | 31 | **2870** | 788 | 30 | **2970** | —276 | 29 | **3070** | —714 | 28 | **3170** | —106 | 27 |
| 72 | 279 | 32 | 72 | 818 | 31 | 72 | —305 | 29 | 72 | 742 | 28 | 72 | 133 | 28 |
| 74 | —311 | 31 | 74 | —849 | 30 | 74 | 334 | 29 | 74 | 770 | 29 | 74 | —161 | 27 |
| 76 | —342 | 31 | 76 | —879 | 30 | 76 | 363 | 29 | 76 | —799 | 28 | 76 | 188 | 27 |
| 78 | 373 | 31 | 78 | 909 | 30 | 78 | 392 | 30 | 78 | —827 | 28 | 78 | 215 | 26 |
| **2780** | 404 | 32 | **2880** | 939 | 30 | **2980** | —422 | 29 | **3080** | 855 | 28 | **3180** | —243 | 27 |
| 82 | —436 | 31 | 82 | 969 | 31 | 82 | —451 | 29 | 82 | 883 | 28 | 82 | 270 | 27 |
| 84 | —467 | 31 | 84 | 46000 | 30 | 84 | —480 | 29 | 84 | 911 | 29 | 84 | 297 | 28 |
| 86 | 498 | 31 | 86 | —030 | 30 | 86 | —509 | 29 | 86 | —940 | 28 | 86 | —325 | 27 |
| 88 | 529 | 31 | 88 | —060 | 30 | 88 | 538 | 29 | 88 | —968 | 28 | 88 | —352 | 27 |
| **2790** | 560 | 32 | **2890** | —090 | 30 | **2990** | 567 | 29 | **3090** | —996 | 28 | **3190** | 379 | 27 |
| 92 | —592 | 31 | 92 | —120 | 30 | 92 | 596 | 29 | 92 | 49024 | 28 | 92 | 406 | 27 |
| 94 | —623 | 31 | 94 | —150 | 30 | 94 | 625 | 29 | 94 | 052 | 28 | 94 | 433 | 28 |
| 96 | —654 | 31 | 96 | —180 | 30 | 96 | 654 | 29 | 96 | 080 | 28 | 96 | —461 | 27 |
| 98 | —685 | 31 | 98 | —210 | 30 | 98 | 683 | 29 | 98 | 108 | 28 | 98 | —488 | 27 |
| **2800** | —716 | 31 | **2900** | —240 | 30 | **3000** | 712 | 29 | **3100** | 136 | 28 | **3200** | —515 | 27 |
| 02 | —747 | 31 | 02 | —270 | 30 | 02 | 741 | 29 | 02 | 164 | 28 | 02 | 542 | 27 |
| 04 | —778 | 31 | 04 | —300 | 30 | 04 | —770 | 29 | 04 | 192 | 28 | 04 | 569 | 27 |
| 06 | —809 | 31 | 06 | —330 | 29 | 06 | —799 | 29 | 06 | 220 | 28 | 06 | 596 | 27 |
| 08 | —840 | 31 | 08 | 359 | 30 | 08 | —828 | 29 | 08 | 248 | 28 | 08 | 623 | 28 |
| **2810** | —871 | 31 | **2910** | 389 | 30 | **3010** | —857 | 28 | **3110** | 276 | 28 | **3210** | —651 | 27 |
| 12 | —902 | 31 | 12 | 419 | 30 | 12 | 885 | 29 | 12 | —304 | 28 | 12 | —678 | 27 |
| 14 | 932 | 30 | 14 | —449 | 30 | 14 | 914 | 29 | 14 | —332 | 28 | 14 | —705 | 27 |
| 16 | 963 | 31 | 16 | —479 | 30 | 16 | 943 | 29 | 16 | —360 | 28 | 16 | —732 | 27 |
| 18 | 994 | 31 | 18 | —509 | 29 | 18 | —972 | 29 | 18 | —388 | 27 | 18 | —759 | 27 |
| **2820** | 45025 | 31 | **2920** | 538 | 30 | **3020** | 48001 | 28 | **3120** | 415 | 28 | **3220** | —786 | 27 |
| 22 | —056 | 30 | 22 | 568 | 30 | 22 | 029 | 29 | 22 | 443 | 28 | 22 | —813 | 27 |
| 24 | 086 | 31 | 24 | —598 | 29 | 24 | 058 | 29 | 24 | 471 | 28 | 24 | —840 | 26 |
| 26 | 117 | 31 | 26 | 627 | 30 | 26 | —087 | 29 | 26 | —499 | 28 | 26 | 866 | 27 |
| 28 | —148 | 31 | 28 | 657 | 30 | 28 | —116 | 28 | 28 | —527 | 27 | 28 | 893 | 27 |
| **2830** | —179 | 30 | **2930** | —687 | 29 | **3030** | 144 | 29 | **3130** | 554 | 28 | **3230** | 920 | 27 |
| 32 | 209 | 31 | 32 | 716 | 30 | 32 | —173 | 29 | 32 | 582 | 28 | 32 | 947 | 27 |
| 34 | —240 | 31 | 34 | 746 | 30 | 34 | —202 | 28 | 34 | —610 | 28 | 34 | 974 | 27 |
| 36 | —271 | 30 | 36 | —776 | 29 | 36 | 230 | 29 | 36 | —638 | 27 | 36 | 51001 | 27 |
| 38 | 301 | 31 | 38 | 805 | 30 | 38 | —259 | 28 | 38 | 665 | 28 | 38 | —028 | 27 |
| **2840** | —332 | 30 | **2940** | —835 | 29 | **3040** | 287 | 29 | **3140** | —693 | 28 | **3240** | —055 | 27 |
| 42 | 362 | 31 | 42 | 864 | 30 | 42 | —316 | 28 | 42 | —721 | 27 | 42 | 081 | 26 |
| 44 | —393 | 30 | 44 | —894 | 29 | 44 | 344 | 29 | 44 | 748 | 28 | 44 | 108 | 27 |
| 46 | 423 | 31 | 46 | 923 | 30 | 46 | —373 | 28 | 46 | —776 | 27 | 46 | —135 | 27 |
| 48 | —454 | 30 | 48 | —953 | 29 | 48 | 401 | 29 | 48 | 803 | 27 | 48 | —162 | 26 |

Encontr. log 29019:
log 29020 = 46270
dif. 20 **30**
log 29000 = 46240
29019 — 29000 = **19**
Debajo **30**
dif. para 10 = 15
— — 9 = 14
— — **19** = 29
Log 29019 =
46240 + 29 = 46269.

| | **32** | **31** | **30** | **29** | **28** | **27** | **26** | |
|---|---|---|---|---|---|---|---|---|
| **1** | 2 | 2 | 2 | 1 | 1 | 1 | 1 | **1** |
| **2** | 3 | 3 | 3 | 3 | 3 | 3 | 3 | **2** |
| **3** | 5 | 5 | 5 | 4 | 4 | 4 | 4 | **3** |
| **4** | 6 | 6 | 6 | 6 | 6 | 5 | 5 | **4** |
| **5** | 8 | 8 | 8 | 7 | 7 | 7 | 7 | **5** |
| **6** | 10 | 9 | 9 | 9 | 8 | 8 | 8 | **6** |
| **7** | 11 | 11 | 11 | 10 | 10 | 9 | 9 | **7** |
| **8** | 13 | 12 | 12 | 12 | 11 | 11 | 10 | **8** |
| **9** | 14 | 14 | 14 | 13 | 13 | 12 | 12 | **9** |
| **10** | 16 | 16 | 15 | 15 | 14 | 14 | 13 | **10** |

**Un guión** antes ó después de un log indica que su verdadero valor es *menor* que el tabular en *menos* de la mitad de la unidad correspondiente al último puesto. Ej.:

log 3128 = 4952667
log 3130 = 4955443

## Logaritmos vulgares ó de Briggs. Base=10.

| No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **3250** | 51188 | 27 | **3350** | 52504 | 26 | **3450** | 53782 | 25 | **3550** | 55023 | 24 | **3650** | 56229 | 24 |
| 52 | 215 | 27 | 52 | 530 | 26 | 52 | 807 | 25 | 52 | 047 | 24 | 52 | 253 | 24 |
| 54 | —242 | 26 | 54 | 556 | 26 | 54 | 832 | 25 | 54 | —072 | 25 | 54 | —277 | 24 |
| 56 | 268 | 27 | 56 | 582 | 26 | 56 | 857 | 25 | 56 | 096 | 24 | 56 | —301 | 24 |
| 58 | 295 | 27 | 58 | 608 | 26 | 58 | 882 | 25 | 58 | —121 | 25 | 58 | 324 | 23 |
| **3260** | —322 | 26 | **3360** | —634 | 26 | **3460** | —908 | 25 | **3560** | —145 | 24 | **3660** | 348 | 24 |
| 62 | 348 | 27 | 62 | —660 | 26 | 62 | —933 | 25 | 62 | 169 | 24 | 62 | —372 | 24 |
| 64 | 375 | 27 | 64 | —686 | 26 | 64 | —958 | 25 | 64 | —194 | 25 | 64 | —396 | 24 |
| 66 | —402 | 26 | 66 | 711 | 25 | 66 | —983 | 25 | 66 | 218 | 24 | 66 | 419 | 23 |
| 68 | 428 | 27 | 68 | 737 | 26 | 68 | 54008 | 25 | 68 | 242 | 24 | 68 | —443 | 24 |
| **3270** | —455 | 26 | **3370** | —763 | 26 | **3470** | —033 | 25 | **3570** | —267 | 25 | **3670** | —467 | 24 |
| 72 | 481 | 27 | 72 | —789 | 26 | 72 | —058 | 25 | 72 | 291 | 24 | 72 | 490 | 23 |
| 74 | —508 | 26 | 74 | —815 | 26 | 74 | —083 | 25 | 74 | 315 | 24 | 74 | —514 | 24 |
| 76 | 534 | 27 | 76 | 840 | 25 | 76 | —108 | 25 | 76 | —340 | 25 | 76 | —538 | 24 |
| 78 | —561 | 26 | 78 | —866 | 26 | 78 | —133 | 25 | 78 | 364 | 24 | 78 | 561 | 23 |
| **3280** | 587 | 27 | **3380** | —892 | 26 | **3480** | —158 | 25 | **3580** | 388 | 24 | **3680** | —585 | 24 |
| 82 | —614 | 27 | 82 | 917 | 25 | 82 | —183 | 25 | 82 | —413 | 24 | 82 | 608 | 24 |
| 84 | 640 | 26 | 84 | 943 | 26 | 84 | —208 | 25 | 84 | —437 | 24 | 84 | —632 | 24 |
| 86 | —667 | 27 | 86 | —969 | 26 | 86 | —233 | 25 | 86 | 461 | 24 | 86 | —656 | 23 |
| 88 | 693 | 26 | 88 | 994 | 25 | 88 | —258 | 25 | 88 | 485 | 24 | 88 | 679 | 24 |
| **3290** | —720 | 26 | **3390** | 53020 | 26 | **3490** | —283 | 24 | **3590** | 509 | 25 | **3690** | —703 | 23 |
| 92 | —746 | 26 | 92 | —046 | 25 | 92 | 307 | 25 | 92 | —534 | 24 | 92 | 726 | 24 |
| 94 | 772 | 27 | 94 | 071 | 26 | 94 | 332 | 25 | 94 | —558 | 24 | 94 | —750 | 23 |
| 96 | —799 | 26 | 96 | —097 | 25 | 96 | 357 | 25 | 96 | —582 | 24 | 96 | 773 | 24 |
| 98 | 825 | 26 | 98 | 122 | 26 | 98 | —382 | 25 | 98 | 606 | 24 | 98 | —797 | 23 |
| **3300** | 851 | 27 | **3400** | —148 | 25 | **3500** | —407 | 25 | **3600** | 630 | 24 | **3700** | 820 | 24 |
| 02 | —878 | 26 | 02 | 173 | 26 | 02 | —432 | 24 | 02 | 654 | 24 | 02 | —844 | 23 |
| 04 | 904 | 26 | 04 | —199 | 25 | 04 | 456 | 25 | 04 | 678 | 25 | 04 | 867 | 24 |
| 06 | 930 | 27 | 06 | 224 | 26 | 06 | 481 | 25 | 06 | —703 | 24 | 06 | —891 | 23 |
| 08 | —957 | 26 | 08 | —250 | 25 | 08 | —506 | 25 | 08 | —727 | 24 | 08 | —914 | 23 |
| **3310** | —983 | 26 | **3410** | 275 | 26 | **3510** | —531 | 24 | **3610** | —751 | 24 | **3710** | 937 | 24 |
| 12 | 52009 | 26 | 12 | —301 | 25 | 12 | 555 | 25 | 12 | —775 | 24 | 12 | —961 | 23 |
| 14 | 035 | 26 | 14 | 326 | 26 | 14 | 580 | 25 | 14 | —799 | 24 | 14 | 984 | 24 |
| 16 | 061 | 27 | 16 | —352 | 25 | 16 | —605 | 25 | 16 | —823 | 24 | 16 | 57008 | 23 |
| 18 | —088 | 26 | 18 | 377 | 26 | 18 | —630 | 25 | 18 | —847 | 24 | 18 | —031 | 23 |
| **3320** | —114 | 26 | **3420** | —403 | 25 | **3520** | 654 | 24 | **3620** | —871 | 24 | **3720** | 054 | 24 |
| 22 | —140 | 26 | 22 | 428 | 25 | 22 | —679 | 25 | 22 | —895 | 24 | 22 | —078 | 23 |
| 24 | 166 | 26 | 24 | 453 | 26 | 24 | —704 | 25 | 24 | —919 | 24 | 24 | —101 | 23 |
| 26 | 192 | 26 | 26 | —479 | 25 | 26 | 728 | 24 | 26 | —943 | 24 | 26 | 124 | 24 |
| 28 | 218 | 26 | 28 | 504 | 25 | 28 | —753 | 25 | 28 | —967 | 24 | 28 | —148 | 23 |
| **3330** | 244 | 26 | **3430** | 529 | 26 | **3530** | 777 | 24 | **3630** | —991 | 24 | **3730** | —171 | 23 |
| 32 | 270 | 27 | 32 | —555 | 25 | 32 | 802 | 25 | 32 | 56015 | 23 | 32 | 194 | 23 |
| 34 | —297 | 26 | 34 | 580 | 25 | 34 | —827 | 25 | 34 | 038 | 24 | 34 | 217 | 24 |
| 36 | —323 | 26 | 36 | 605 | 26 | 36 | 851 | 24 | 36 | 062 | 24 | 36 | —241 | 23 |
| 38 | —349 | 26 | 38 | —631 | 25 | 38 | —876 | 25 | 38 | 086 | 24 | 38 | —264 | 23 |
| **3340** | —375 | 26 | **3440** | —656 | 25 | **3540** | 900 | 24 | **3640** | 110 | 24 | **3740** | 287 | 23 |
| 42 | —401 | 26 | 42 | 681 | 25 | 42 | —925 | 25 | 42 | —134 | 24 | 42 | 310 | 24 |
| 44 | —427 | 26 | 44 | 706 | 26 | 44 | 949 | 24 | 44 | —158 | 24 | 44 | —334 | 23 |
| 46 | —453 | 26 | 46 | —732 | 25 | 46 | —974 | 25 | 46 | —182 | 24 | 46 | —357 | 23 |
| 48 | —479 | 25 | 48 | —757 | 25 | 48 | 998 | 24 | 48 | 205 | 23 | 48 | —380 | 23 |

Encontrar log 36114 :
log 36120=55775
log 36100=55751
dif. 20 24
36114—36100=14
Debajo **24**
dif. para 10=12
— — 4= 5
— — 14=17
Log 36114=
55751+17=55768.

| | 27 | 26 | 25 | 24 | 23 | |
|---|---|---|---|---|---|---|
| **1** | 1 | 1 | 1 | 1 | 1 | **1** |
| **2** | 3 | 3 | 3 | 2 | 2 | **2** |
| **3** | 4 | 4 | 4 | 4 | 3 | **3** |
| **4** | 5 | 5 | 5 | 5 | 5 | **4** |
| **5** | 7 | 7 | 6 | 6 | 6 | **5** |
| **6** | 8 | 8 | 8 | 7 | 7 | **6** |
| **7** | 10 | 9 | 9 | 8 | 8 | **7** |
| **8** | 11 | 10 | 10 | 10 | 9 | **8** |
| **9** | 12 | 12 | 11 | 11 | 10 | **9** |
| **10** | 14 | 13 | 13 | 12 | 12 | **10** |

**Un guión** antes ó después de un log indica que su verdadero valor es *menor* que el tabular en *menos* de la mitad de la unidad correspondiente al último puesto. Ej. :
Log 3490=5428254
Log 3492=5430742.

## Logaritmos vulgares ó de Briggs. Base=10.

| No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **3750** | 57403 | | **4000** | 60206 | | **4250** | 62839 | | **4500** | 65321 | | **4750** | 67669 | |
| 55 | —461 | 58 | 05 | 260 | 54 | 55 | —890 | 51 | 05 | 369 | 48 | 55 | 715 | 46 |
| 60 | —519 | 58 | 10 | 314 | 54 | 60 | —941 | 51 | 10 | —418 | 49 | 60 | —761 | 46 |
| 65 | 576 | 57 | 15 | —369 | 55 | 65 | —992 | 51 | 15 | —466 | 48 | 65 | 806 | 45 |
| 70 | 634 | 58 | 20 | —423 | 54 | 70 | 63043 | 51 | 20 | —514 | 48 | 70 | —852 | 46 |
| 75 | —692 | 58 | 25 | —477 | 54 | 75 | —094 | 51 | 25 | —562 | 48 | 75 | 897 | 45 |
| 80 | 749 | 57 | 30 | —531 | 54 | 80 | 144 | 50 | 30 | —610 | 48 | 80 | —943 | 46 |
| 85 | —807 | 58 | 35 | 584 | 53 | 85 | 195 | 51 | 35 | —658 | 48 | 85 | 988 | 45 |
| 90 | —864 | 57 | 40 | 638 | 54 | 90 | —246 | 51 | 40 | —706 | 48 | 90 | 68034 | 46 |
| 95 | 921 | 57 | 45 | —692 | 54 | 95 | 296 | 50 | 45 | 753 | 47 | 95 | —079 | 45 |
| **3800** | 978 | 57 | **4050** | —746 | 54 | **4300** | —347 | 51 | **4550** | 801 | 48 | **4800** | 124 | 45 |
| 05 | 58035 | 57 | 55 | 799 | 53 | 05 | 397 | 50 | 55 | —849 | 48 | 05 | 169 | 45 |
| 10 | 092 | 57 | 60 | —853 | 54 | 10 | —448 | 51 | 60 | 896 | 47 | 10 | —215 | 46 |
| 15 | 149 | 57 | 65 | 906 | 53 | 15 | 498 | 50 | 65 | 944 | 48 | 15 | —260 | 45 |
| 20 | 206 | 57 | 70 | 959 | 53 | 20 | 548 | 50 | 70 | —992 | 48 | 20 | —305 | 45 |
| 25 | 263 | 57 | 75 | 61013 | 54 | 25 | —599 | 51 | 75 | 66039 | 47 | 25 | —350 | 45 |
| 30 | —320 | 57 | 80 | 066 | 53 | 30 | —649 | 50 | 80 | —087 | 48 | 30 | —395 | 45 |
| 35 | —377 | 57 | 85 | 119 | 53 | 35 | —699 | 50 | 85 | —134 | 47 | 35 | —440 | 45 |
| 40 | 433 | 56 | 90 | 172 | 53 | 40 | —749 | 50 | 90 | 181 | 47 | 40 | —485 | 45 |
| 45 | —490 | 57 | 95 | 225 | 53 | 45 | —799 | 50 | 95 | —229 | 48 | 45 | 529 | 44 |
| **3850** | 546 | 56 | **4100** | 278 | 53 | **4350** | —849 | 50 | **4600** | —276 | 47 | **4850** | 574 | 45 |
| 55 | 602 | 56 | 05 | 331 | 53 | 55 | —899 | 50 | 05 | —323 | 47 | 55 | —619 | 45 |
| 60 | —659 | 57 | 10 | 384 | 53 | 60 | —949 | 50 | 10 | 370 | 47 | 60 | —664 | 44 |
| 65 | —715 | 56 | 15 | —437 | 53 | 65 | 998 | 49 | 15 | 417 | 47 | 65 | 708 | 45 |
| 70 | 771 | 56 | 20 | —490 | 53 | 70 | 64048 | 50 | 20 | 464 | 47 | 70 | —753 | 44 |
| 75 | 827 | 56 | 25 | 542 | 52 | 75 | —098 | 50 | 25 | 511 | 47 | 75 | 797 | 45 |
| 80 | 883 | 56 | 30 | 595 | 53 | 80 | 147 | 49 | 30 | 558 | 47 | 80 | —842 | 44 |
| 85 | 939 | 56 | 35 | —648 | 53 | 85 | —197 | 50 | 35 | —605 | 47 | 85 | 886 | 45 |
| 90 | —995 | 56 | 40 | 700 | 52 | 90 | 246 | 49 | 40 | —652 | 47 | 90 | —931 | 44 |
| 95 | 59051 | 55 | 45 | 752 | 52 | 95 | —296 | 50 | 45 | —699 | 46 | 95 | 975 | 45 |
| **3900** | 106 | 56 | **4150** | —805 | 53 | **4400** | 345 | 49 | **4650** | 745 | 47 | **4900** | 69020 | 44 |
| 05 | 162 | 56 | 55 | 857 | 52 | 05 | —395 | 50 | 55 | —792 | 47 | 05 | —064 | 44 |
| 10 | —218 | 56 | 60 | 909 | 52 | 10 | —444 | 49 | 60 | —839 | 46 | 10 | 108 | 44 |
| 15 | 273 | 55 | 65 | —962 | 53 | 15 | 493 | 49 | 65 | 885 | 47 | 15 | 152 | 44 |
| 20 | —329 | 56 | 70 | 62014 | 52 | 20 | 542 | 49 | 70 | —932 | 46 | 20 | —197 | 45 |
| 25 | —384 | 55 | 75 | —066 | 52 | 25 | 591 | 49 | 75 | 978 | 47 | 25 | —241 | 44 |
| 30 | 439 | 55 | 80 | —118 | 52 | 30 | 640 | 49 | 80 | 67025 | 46 | 30 | —285 | 44 |
| 35 | 494 | 55 | 85 | —170 | 52 | 35 | 689 | 49 | 85 | —071 | 46 | 35 | —329 | 44 |
| 40 | —550 | 56 | 90 | 221 | 51 | 40 | 738 | 49 | 90 | 117 | 46 | 40 | —373 | 44 |
| 45 | —605 | 55 | 95 | 273 | 52 | 45 | 787 | 49 | 95 | —164 | 47 | 45 | —417 | 44 |
| **3950** | —660 | 55 | **4200** | —325 | 52 | **4450** | 836 | 49 | **4700** | —210 | 46 | **4950** | —461 | 43 |
| 55 | —715 | 55 | 05 | —377 | 51 | 55 | —885 | 48 | 05 | —256 | 46 | 55 | 504 | 44 |
| 60 | —770 | 54 | 10 | 428 | 52 | 60 | 933 | 49 | 10 | 302 | 46 | 60 | 548 | 44 |
| 65 | 824 | 55 | 15 | —480 | 51 | 65 | 982 | 49 | 15 | 348 | 46 | 65 | —592 | 44 |
| 70 | 879 | 55 | 20 | 531 | 52 | 70 | 65031 | 48 | 20 | 394 | 46 | 70 | —636 | 43 |
| 75 | —934 | 54 | 25 | —583 | 51 | 75 | 079 | 49 | 25 | 440 | 46 | 75 | 679 | 44 |
| 80 | 988 | 54 | 30 | 634 | 51 | 80 | —128 | 48 | 30 | 486 | 46 | 80 | —723 | 44 |
| 85 | 60043 | 55 | 35 | 685 | 52 | 85 | 176 | 49 | 35 | —532 | 46 | 85 | —767 | 43 |
| 90 | 097 | 54 | 40 | —737 | 51 | 90 | —225 | 48 | 40 | —578 | 46 | 90 | 810 | 44 |
| 95 | —152 | 55 | 45 | —788 | 51 | 95 | —273 | 48 | 45 | —624 | 45 | 95 | —854 | 43 |
| | | 54 | | | | | | | | | | | | |

| | 58 | 57 | 56 | 55 | 54 | 53 | 52 | 51 | 50 | 49 | 48 | 47 | 46 | 45 | 44 | 43 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 1.2 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | **1** |
| **2** | 2.3 | 2.3 | 2.2 | 2.2 | 2.2 | 2.1 | 2.1 | 2.0 | 2.0 | 2.0 | 1.9 | 1.9 | 1.8 | 1.8 | 1.8 | 1.7 | **2** |
| **3** | 3.5 | 3.4 | 3.4 | 3.3 | 3.2 | 3.2 | 3.1 | 3.1 | 3.0 | 2.9 | 2.9 | 2.8 | 2.8 | 2.7 | 2.6 | 2.6 | **3** |
| **4** | 4.6 | 4.6 | 4.5 | 4.4 | 4.3 | 4.2 | 4.2 | 4.1 | 4.0 | 3.9 | 3.8 | 3.8 | 3.7 | 3.6 | 3.5 | 3.4 | **4** |
| **5** | 5.8 | 5.7 | 5.6 | 5.5 | 5.4 | 5.3 | 5.2 | 5.1 | 5.0 | 4.9 | 4.8 | 4.7 | 4.6 | 4.5 | 4.4 | 4.3 | **5** |
| **6** | 7.0 | 6.8 | 6.7 | 6.6 | 6.5 | 6.4 | 6.2 | 6.1 | 6.0 | 5.9 | 5.8 | 5.6 | 5.5 | 5.4 | 5.3 | 5.2 | **6** |
| **7** | 8.1 | 8.0 | 7.8 | 7.7 | 7.6 | 7.4 | 7.3 | 7.1 | 7.0 | 6.9 | 6.7 | 6.6 | 6.4 | 6.3 | 6.2 | 6.0 | **7** |
| **8** | 9.3 | 9.1 | 9.0 | 8.8 | 8.6 | 8.5 | 8.3 | 8.2 | 8.0 | 7.8 | 7.7 | 7.5 | 7.4 | 7.2 | 7.0 | 6.9 | **8** |
| **9** | 10.4 | 10.3 | 10.1 | 9.9 | 9.7 | 9.5 | 9.4 | 9.2 | 9.0 | 8.8 | 8.6 | 8.5 | 8.3 | 8.1 | 7.9 | 7.7 | **9** |
| **10** | 11.6 | 11.4 | 11.2 | 11.0 | 10.8 | 10.6 | 10.4 | 10.2 | 10.0 | 9.8 | 9.6 | 9.4 | 9.2 | 9.0 | 8.8 | 8.6 | **10** |

## Logaritmos vulgares ó de Briggs. Base=10.

| No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **5000** | 69897 | 43 | **5250** | 72016 | 41 | **5500** | 74036 | 40 | **5750** | 75967 | 38 | **6000** | 77815 | 36 |
| 05 | 940 | 43 | 55 | 057 | 41 | 05 | —076 | 40 | 55 | 76005 | 37 | 05 | 851 | 36 |
| 10 | —984 | 44 | 60 | —099 | 42 | 10 | 115 | 39 | 60 | 042 | 37 | 10 | 887 | 37 |
| 15 | 70027 | 43 | 65 | —140 | 41 | 15 | —155 | 40 | 65 | —080 | 38 | 15 | —924 | 36 |
| 20 | 070 | 43 | 70 | 181 | 41 | 20 | —194 | 39 | 70 | —118 | 38 | 20 | —960 | 36 |
| 25 | —114 | 44 | 75 | 222 | 41 | 25 | 233 | 39 | 75 | 155 | 37 | 25 | —996 | 36 |
| 30 | —157 | 43 | 80 | 263 | 41 | 30 | —273 | 40 | 80 | —193 | 38 | 30 | 78032 | 36 |
| 35 | —200 | 43 | 85 | 304 | 41 | 35 | —312 | 39 | 85 | 230 | 37 | 35 | —068 | 36 |
| 40 | 243 | 43 | 90 | —346 | 42 | 40 | —351 | 39 | 90 | —268 | 38 | 40 | —104 | 36 |
| 45 | 286 | 43 | 95 | —387 | 41 | 45 | 390 | 39 | 95 | 305 | 37 | 45 | —140 | 36 |
| **5050** | 329 | 43 | **5300** | —428 | 41 | **5550** | 429 | 39 | **5800** | —343 | 38 | **6050** | —176 | 36 |
| 55 | 372 | 43 | 05 | —469 | 40 | 55 | 468 | 39 | 05 | 380 | 37 | 55 | 211 | 35 |
| 60 | 415 | 43 | 10 | 509 | 41 | 60 | 507 | 40 | 10 | —418 | 38 | 60 | 247 | 36 |
| 65 | —458 | 43 | 15 | 550 | 41 | 65 | —547 | 39 | 15 | —455 | 37 | 65 | 283 | 36 |
| 70 | —501 | 43 | 20 | 591 | 41 | 70 | —586 | 38 | 20 | 492 | 37 | 70 | —319 | 36 |
| 75 | —544 | 43 | 25 | —632 | 41 | 75 | 624 | 39 | 25 | —530 | 38 | 75 | —355 | 36 |
| 80 | 586 | 42 | 30 | —673 | 41 | 80 | 663 | 39 | 30 | —567 | 37 | 80 | 390 | 35 |
| 85 | 629 | 43 | 35 | 713 | 40 | 85 | 702 | 39 | 35 | 604 | 37 | 85 | 426 | 36 |
| 90 | —672 | 43 | 40 | 754 | 41 | 90 | 741 | 39 | 40 | 641 | 37 | 90 | —462 | 36 |
| 95 | 714 | 42 | 45 | —795 | 41 | 95 | 780 | 39 | 45 | 678 | 37 | 95 | 497 | 35 |
| | | 43 | | | 40 | | | 39 | | | 38 | | | 36 |
| **5100** | 757 | 43 | **5350** | 835 | 41 | **5600** | —819 | 39 | **5850** | —716 | 37 | **6100** | —533 | 36 |
| 05 | —800 | 42 | 55 | —876 | 40 | 05 | —858 | 38 | 55 | —753 | 37 | 05 | —569 | 35 |
| 10 | 842 | 43 | 60 | 916 | 41 | 10 | 896 | 38 | 60 | —790 | 37 | 10 | 604 | 36 |
| 15 | —885 | 42 | 65 | —957 | 40 | 15 | —935 | 39 | 65 | —827 | 37 | 15 | —640 | 35 |
| 20 | —927 | 42 | 70 | 997 | 41 | 20 | —974 | 39 | 70 | —864 | 37 | 20 | 675 | 36 |
| 25 | 969 | 43 | 75 | 73038 | 41 | 25 | 75012 | 38 | 75 | —901 | 37 | 25 | —711 | 35 |
| 30 | 71012 | 42 | 80 | 078 | 40 | 30 | —051 | 39 | 80 | —938 | 37 | 30 | 746 | 35 |
| 35 | 054 | 42 | 85 | —119 | 41 | 35 | 089 | 38 | 85 | —975 | 37 | 35 | 781 | 36 |
| 40 | 096 | 42 | 90 | —159 | 40 | 40 | —128 | 39 | 90 | 77012 | 37 | 40 | —817 | 35 |
| 45 | —139 | 43 | 95 | 199 | 40 | 45 | 166 | 38 | 95 | 048 | 36 | 45 | 852 | 36 |
| | | 42 | | | 40 | | | 39 | | | 37 | | | |
| **5150** | —181 | 42 | **5400** | 239 | 41 | **5650** | —205 | 38 | **5900** | 085 | 37 | **6150** | —888 | 35 |
| 55 | —223 | 42 | 05 | —280 | 40 | 55 | 243 | 39 | 05 | —122 | 37 | 55 | —923 | 35 |
| 60 | —265 | 42 | 10 | —320 | 40 | 60 | —282 | 38 | 10 | —159 | 36 | 60 | 958 | 35 |
| 65 | 307 | 42 | 15 | —360 | 40 | 65 | —320 | 38 | 15 | 195 | 37 | 65 | 993 | 36 |
| 70 | 349 | 42 | 20 | —400 | 40 | 70 | 358 | 39 | 20 | 232 | 37 | 70 | 79029 | 35 |
| 75 | 391 | 42 | 25 | —440 | 40 | 75 | —397 | 38 | 25 | —269 | 37 | 75 | —064 | 35 |
| 80 | —433 | 42 | 30 | —480 | 40 | 80 | —435 | 38 | 30 | 305 | 36 | 80 | —099 | 35 |
| 85 | —475 | 42 | 35 | —520 | 40 | 85 | 473 | 38 | 35 | 342 | 37 | 85 | —134 | 35 |
| 90 | —517 | 42 | 40 | —560 | 40 | 90 | 511 | 38 | 40 | —379 | 36 | 90 | 169 | 35 |
| 95 | —559 | 42 | 45 | —600 | 40 | 95 | 549 | 38 | 45 | 415 | 37 | 95 | 204 | 35 |
| | | 41 | | | 40 | | | 38 | | | 37 | | | 35 |
| **5200** | 600 | 42 | **5450** | —640 | 39 | **5700** | 587 | 39 | **5950** | —452 | 36 | **6200** | 239 | 35 |
| 05 | 642 | 42 | 55 | 679 | 40 | 05 | —626 | 38 | 55 | 488 | 37 | 05 | 274 | 35 |
| 10 | —684 | 41 | 60 | 719 | 40 | 10 | —664 | 38 | 60 | —525 | 36 | 10 | 309 | 35 |
| 15 | 725 | 42 | 65 | 759 | 40 | 15 | —702 | 38 | 65 | 561 | 36 | 15 | 344 | 35 |
| 20 | 767 | 42 | 70 | —799 | 39 | 20 | —740 | 38 | 70 | 597 | 37 | 20 | 379 | 35 |
| 25 | —809 | 41 | 75 | 838 | 40 | 25 | —778 | 37 | 75 | —634 | 36 | 25 | —414 | 35 |
| 30 | 850 | 42 | 80 | 878 | 40 | 30 | 815 | 38 | 80 | 670 | 36 | 30 | —449 | 35 |
| 35 | —892 | 41 | 85 | —918 | 39 | 35 | 853 | 38 | 85 | 706 | 36 | 35 | —484 | 34 |
| 40 | 933 | 42 | 90 | 957 | 40 | 40 | 891 | 38 | 90 | —743 | 37 | 40 | 518 | 35 |
| 45 | —975 | 42 | 95 | —997 | 39 | 45 | 929 | 38 | 95 | —779 | 36 | 45 | 553 | 35 |
| | | 41 | | | | | | | | | | | | |

| | 44 | 43 | 42 | 41 | 40 | 39 | 38 | 37 | 36 | 35 | 34 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.9 | 0.9 | 0.8 | 0.8 | 0.8 | 0.8 | 0.8 | 0.7 | 0.7 | 0.7 | 0.7 |
| 2 | 1.8 | 1.7 | 1.7 | 1.6 | 1.6 | 1.6 | 1.5 | 1.5 | 1.4 | 1.4 | 1.4 |
| 3 | 2.6 | 2.6 | 2.5 | 2.5 | 2.4 | 2.3 | 2.3 | 2.2 | 2.2 | 2.1 | 2.0 |
| 4 | 3.5 | 3.4 | 3.4 | 3.3 | 3.2 | 3.1 | 3.0 | 3.0 | 2.9 | 2.8 | 2.7 |
| 5 | 4.4 | 4.3 | 4.2 | 4.1 | 4.0 | 3.9 | 3.8 | 3.7 | 3.6 | 3.5 | 3.4 |
| 6 | 5.3 | 5.2 | 5.0 | 4.9 | 4.8 | 4.7 | 4.6 | 4.4 | 4.3 | 4.2 | 4.1 |
| 7 | 6.2 | 6.0 | 5.9 | 5.7 | 5.6 | 5.5 | 5.3 | 5.2 | 5.0 | 4.9 | 4.8 |
| 8 | 7.0 | 6.9 | 6.7 | 6.6 | 6.4 | 6.2 | 6.1 | 5.9 | 5.8 | 5.6 | 5.4 |
| 9 | 7.9 | 7.7 | 7.6 | 7.4 | 7.2 | 7.0 | 6.8 | 6.7 | 6.5 | 6.3 | 6.1 |
| 10 | 8.8 | 8.6 | 8.4 | 8.2 | 8.0 | 7.8 | 7.6 | 7.4 | 7.2 | 7.0 | 6.8 |

Encontrar log 58636

log 58650=76827

log 58600=76790

dif. 50 37

58636 — 58600=36

Debajo **37**

Dif. para 10×3=22.0

— — 6= 4.4

— — **36**=26.4

log 58636=

767916+026= 78

## Logaritmos vulgares ó de Briggs. Base=10.

| No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **6250** | 79588 | 35 | **6500** | 81291 | 34 | **6750** | 82930 | 33 | **7000** | 84510 | 31 | **7250** | 86034 | 30 |
| 55 | —623 | 34 | 05 | —325 | 33 | 55 | —963 | 32 | 05 | —541 | 31 | 55 | —064 | 30 |
| 60 | 657 | 35 | 10 | 358 | 33 | 60 | —995 | 32 | 10 | —572 | 31 | 60 | —094 | 30 |
| 65 | 692 | 35 | 15 | 391 | 34 | 65 | 83027 | 32 | 15 | —603 | 31 | 65 | —124 | 29 |
| 70 | —727 | 34 | 20 | —425 | 33 | 70 | —059 | 32 | 20 | —634 | 31 | 70 | 153 | 30 |
| 75 | 761 | 35 | 25 | 458 | 33 | 75 | —091 | 32 | 25 | —665 | 31 | 75 | 183 | 30 |
| 80 | —796 | 35 | 30 | 491 | 34 | 80 | —123 | 32 | 30 | —696 | 30 | 80 | 213 | 30 |
| 85 | —831 | 34 | 35 | —525 | 33 | 85 | —155 | 32 | 35 | 726 | 31 | 85 | —243 | 30 |
| 90 | 865 | 35 | 40 | —558 | 33 | 90 | —187 | 32 | 40 | 757 | 31 | 90 | —273 | 30 |
| 95 | —900 | 34 | 45 | —591 | 33 | 95 | —219 | 32 | 45 | 788 | 31 | 95 | —303 | 29 |
| **6300** | 934 | 35 | **6550** | 624 | 33 | **6800** | —251 | 32 | **7050** | —819 | 31 | **7300** | 332 | 30 |
| 05 | —969 | 34 | 55 | 657 | 33 | 05 | —283 | 32 | 55 | —850 | 30 | 05 | 362 | 30 |
| 10 | 80003 | 34 | 60 | 690 | 33 | 10 | —315 | 32 | 60 | 880 | 31 | 10 | —392 | 29 |
| 15 | 037 | 34 | 65 | 723 | 34 | 15 | —347 | 31 | 65 | 911 | 31 | 15 | 421 | 30 |
| 20 | —072 | 35 | 70 | —757 | 33 | 20 | 378 | 32 | 70 | —942 | 31 | 20 | 451 | 30 |
| 25 | 106 | 34 | 75 | —790 | 33 | 25 | 410 | 32 | 75 | —973 | 30 | 25 | —481 | 29 |
| 30 | 140 | 34 | 80 | —823 | 33 | 30 | 442 | 32 | 80 | 85003 | 31 | 30 | 510 | 30 |
| 35 | —175 | 35 | 85 | —856 | 33 | 35 | —474 | 32 | 85 | —034 | 31 | 35 | 540 | 30 |
| 40 | —209 | 34 | 90 | —889 | 33 | 40 | —506 | 32 | 90 | —065 | 30 | 40 | —570 | 29 |
| 45 | 243 | 34 | 95 | 921 | 33 | 45 | 537 | 31 | 95 | 095 | 31 | 45 | 599 | 30 |
| **6350** | 277 | 34 | **6600** | 954 | 33 | **6850** | 569 | 32 | **7100** | —126 | 30 | **7350** | —629 | 29 |
| 55 | —312 | 35 | 05 | 987 | 33 | 55 | —601 | 31 | 05 | 156 | 31 | 55 | 658 | 30 |
| 60 | —346 | 34 | 10 | 82020 | 33 | 60 | 632 | 32 | 10 | —187 | 30 | 60 | —688 | 29 |
| 65 | —380 | 34 | 15 | —053 | 33 | 65 | 664 | 32 | 15 | 217 | 31 | 65 | 717 | 30 |
| 70 | —414 | 34 | 20 | —086 | 33 | 70 | —696 | 31 | 20 | —248 | 30 | 70 | —747 | 29 |
| 75 | 448 | 34 | 25 | —119 | 32 | 75 | 727 | 32 | 25 | 278 | 31 | 75 | 776 | 30 |
| 80 | 482 | 34 | 30 | 151 | 33 | 80 | —759 | 31 | 30 | —309 | 30 | 80 | —806 | 29 |
| 85 | 516 | 34 | 35 | 184 | 33 | 85 | 790 | 32 | 35 | 339 | 31 | 85 | 835 | 29 |
| 90 | 550 | 34 | 40 | —217 | 32 | 90 | —822 | 31 | 40 | —370 | 30 | 90 | 864 | 30 |
| 95 | 584 | 34 | 45 | 249 | 33 | 95 | 853 | 32 | 45 | 400 | 31 | 95 | —894 | 29 |
| **6400** | —618 | 34 | **6650** | 282 | 33 | **6900** | —885 | 31 | **7150** | —431 | 30 | **7400** | 923 | 30 |
| 05 | —652 | 34 | 55 | —315 | 32 | 05 | 916 | 32 | 55 | —461 | 30 | 05 | —953 | 29 |
| 10 | —686 | 34 | 60 | 347 | 33 | 10 | —948 | 31 | 60 | 491 | 31 | 10 | —982 | 29 |
| 15 | —720 | 34 | 65 | 380 | 33 | 15 | 979 | 32 | 65 | —522 | 30 | 15 | 87011 | 29 |
| 20 | —754 | 34 | 70 | —413 | 32 | 20 | 84011 | 31 | 70 | —552 | 30 | 20 | 040 | 30 |
| 25 | 787 | 33 | 75 | 445 | 33 | 25 | —042 | 31 | 75 | 582 | 30 | 25 | —070 | 29 |
| 30 | 821 | 34 | 80 | —478 | 32 | 30 | 073 | 32 | 80 | 612 | 31 | 30 | —099 | 29 |
| 35 | —855 | 34 | 85 | 510 | 33 | 35 | —105 | 31 | 85 | —643 | 30 | 35 | 128 | 29 |
| 40 | —889 | 33 | 90 | —543 | 32 | 40 | —136 | 31 | 90 | —673 | 30 | 40 | 157 | 29 |
| 45 | 922 | 34 | 95 | 575 | 32 | 45 | 167 | 31 | 95 | 703 | 30 | 45 | 186 | 30 |
| **6450** | —956 | 34 | **6700** | 607 | 33 | **6950** | 198 | 32 | **7200** | 733 | 30 | **7450** | —216 | 29 |
| 55 | —990 | 33 | 05 | —640 | 32 | 55 | —230 | 31 | 05 | 763 | 31 | 55 | —245 | 29 |
| 60 | 81023 | 34 | 10 | 672 | 33 | 60 | —261 | 31 | 10 | —794 | 30 | 60 | —274 | 29 |
| 65 | —057 | 34 | 15 | —705 | 32 | 65 | 292 | 31 | 15 | —824 | 30 | 65 | —303 | 29 |
| 70 | 090 | 33 | 20 | —737 | 32 | 70 | 323 | 31 | 20 | —854 | 30 | 70 | 332 | 29 |
| 75 | —124 | 34 | 25 | 769 | 33 | 75 | 354 | 32 | 25 | —884 | 30 | 75 | 361 | 29 |
| 80 | —158 | 33 | 30 | —802 | 32 | 80 | —386 | 31 | 30 | —914 | 30 | 80 | 390 | 29 |
| 85 | —191 | 33 | 35 | —834 | 32 | 85 | —417 | 31 | 35 | —944 | 30 | 85 | 419 | 29 |
| 90 | 224 | 33 | 40 | —866 | 32 | 90 | —448 | 31 | 40 | —974 | 30 | 90 | 448 | 29 |
| 95 | —258 | 33 | 45 | 898 | 32 | 95 | —479 | 31 | 45 | 86004 | 30 | 95 | 477 | 29 |

Encontrar log 63023:
log 63050=79969
log 63000=79934
dif. 50 35
63023 — 63000=23
Debajo **35**
dif. para 10×2=14.0
— — 3= 2.1
— — 23=16.1
Log 63023=
9934+16=79950.

| | 35 | 34 | 33 | 32 | 31 | 30 | 29 | |
|---|---|---|---|---|---|---|---|---|
| **1** | 0.7 | 0.7 | 0.7 | 0.6 | 0.6 | 0.6 | 0.6 | **1** |
| **2** | 1.4 | 1.4 | 1.3 | 1.3 | 1.2 | 1.2 | 1.2 | **2** |
| **3** | 2.1 | 2.0 | 2.0 | 1.9 | 1.9 | 1.8 | 1.7 | **3** |
| **4** | 2.8 | 2.7 | 2.6 | 2.6 | 2.5 | 2.4 | 2.3 | **4** |
| **5** | 3.5 | 3.4 | 3.3 | 3.2 | 3.1 | 3.0 | 2.9 | **5** |
| **6** | 4.2 | 4.1 | 4.0 | 3.8 | 3.7 | 3.6 | 3.5 | **6** |
| **7** | 4.9 | 4.8 | 4.6 | 4.5 | 4.3 | 4.2 | 4.1 | **7** |
| **8** | 5.6 | 5.4 | 5.3 | 5.1 | 5.0 | 4.8 | 4.6 | **8** |
| **9** | 6.3 | 6.1 | 5.9 | 5.8 | 5.6 | 5.4 | 5.2 | **9** |
| **10** | 7.0 | 6.8 | 6.6 | 6.4 | 6.2 | 6.0 | 5.8 | **10** |

**Un guión** antes ó después de un log indica que su verdadero valor es *menor* que el tabular en *menos* de la mitad de la unidad correspondiente al último puesto. Ej.:
Log 7400=8692317
Log 7405=8695251.

## Logaritmos vulgares ó de Briggs. Base=10.

| No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **7500** | 87506 | 29 | **7750** | 88930 | 28 | **8000** | 90309 | 27 | **8250** | 91645 | 27 | **8500** | 92942 | 25 |
| 05 | 535 | 29 | 55 | 958 | 28 | 05 | 336 | 27 | 55 | —672 | 26 | 05 | 967 | 26 |
| 10 | —564 | 29 | 60 | 986 | 28 | 10 | 363 | 27 | 60 | 698 | 26 | 10 | —993 | 25 |
| 15 | —593 | 29 | 65 | 89014 | 28 | 15 | 390 | 27 | 65 | 724 | 27 | 15 | 93018 | 26 |
| 20 | —622 | 29 | 70 | 042 | 28 | 20 | 417 | 28 | 70 | —751 | 26 | 20 | —044 | 25 |
| 25 | —651 | 28 | 75 | 070 | 28 | 25 | —445 | 27 | 75 | —777 | 26 | 25 | 069 | 26 |
| 30 | 679 | 29 | 80 | —098 | 28 | 30 | —472 | 27 | 80 | 803 | 26 | 30 | —095 | 25 |
| 35 | 708 | 29 | 85 | —126 | 28 | 35 | —499 | 27 | 85 | 829 | 26 | 35 | 120 | 26 |
| 40 | 737 | 29 | 90 | —154 | 28 | 40 | —526 | 27 | 90 | 855 | 27 | 40 | —146 | 25 |
| 45 | —766 | 29 | 95 | —182 | 27 | 45 | —553 | 27 | 95 | —882 | 26 | 45 | 171 | 26 |
| **7550** | —795 | 28 | **7800** | 209 | 28 | **8050** | —580 | 27 | **8300** | —908 | 26 | **8550** | —197 | 25 |
| 55 | 823 | 29 | 05 | 237 | 28 | 55 | —607 | 27 | 05 | —934 | 26 | 55 | 222 | 25 |
| 60 | 852 | 29 | 10 | 265 | 28 | 60 | —634 | 26 | 10 | 960 | 26 | 60 | 247 | 26 |
| 65 | —881 | 29 | 15 | —293 | 28 | 65 | 660 | 27 | 15 | 986 | 26 | 65 | —273 | 25 |
| 70 | —910 | 28 | 20 | —321 | 27 | 70 | 687 | 27 | 20 | 92012 | 26 | 70 | 298 | 25 |
| 75 | 938 | 29 | 25 | 348 | 28 | 75 | 714 | 27 | 25 | 038 | 27 | 75 | 323 | 26 |
| 80 | —967 | 29 | 30 | 376 | 28 | 80 | 741 | 27 | 30 | —065 | 26 | 80 | —349 | 25 |
| 85 | —996 | 28 | 35 | —404 | 28 | 85 | 768 | 27 | 35 | —091 | 26 | 85 | 374 | 25 |
| 90 | 88024 | 29 | 40 | —432 | 27 | 90 | —795 | 27 | 40 | —117 | 26 | 90 | 399 | 26 |
| 95 | —053 | 28 | 45 | 459 | 28 | 95 | —822 | 27 | 45 | —143 | 26 | 95 | —425 | 25 |
| **7600** | 081 | 29 | **7850** | —487 | 28 | **8100** | —849 | 26 | **8350** | —169 | 26 | **8600** | —450 | 25 |
| 05 | —110 | 28 | 55 | —515 | 27 | 05 | 875 | 27 | 55 | —195 | 26 | 05 | 475 | 25 |
| 10 | 138 | 29 | 60 | 542 | 28 | 10 | 902 | 27 | 60 | —221 | 26 | 10 | 500 | 26 |
| 15 | —167 | 28 | 65 | —570 | 27 | 15 | —929 | 27 | 65 | —247 | 26 | 15 | —526 | 25 |
| 20 | 195 | 29 | 70 | 597 | 28 | 20 | —956 | 26 | 70 | —273 | 25 | 20 | —551 | 25 |
| 25 | —224 | 28 | 75 | 625 | 28 | 25 | 982 | 27 | 75 | 298 | 26 | 25 | —576 | 25 |
| 30 | 252 | 29 | 80 | —653 | 27 | 30 | 91009 | 27 | 80 | 324 | 26 | 30 | 601 | 25 |
| 35 | —281 | 28 | 85 | 680 | 28 | 35 | —036 | 26 | 85 | 350 | 26 | 35 | 626 | 25 |
| 40 | 309 | 29 | 90 | —708 | 27 | 40 | 062 | 27 | 90 | 376 | 26 | 40 | 651 | 25 |
| 45 | —338 | 28 | 95 | 735 | 28 | 45 | 089 | 27 | 95 | 402 | 26 | 45 | 676 | 26 |
| **7650** | 366 | 29 | **7900** | —763 | 27 | **8150** | —116 | 26 | **8400** | —428 | 26 | **8650** | —702 | 25 |
| 55 | —395 | 28 | 05 | 790 | 28 | 55 | 142 | 27 | 05 | —454 | 26 | 55 | —727 | 25 |
| 60 | —423 | 28 | 10 | —818 | 27 | 60 | 169 | 27 | 10 | —480 | 25 | 60 | —752 | 25 |
| 65 | 451 | 29 | 15 | 845 | 28 | 65 | —196 | 26 | 15 | 505 | 26 | 65 | —777 | 25 |
| 70 | —480 | 28 | 20 | —873 | 27 | 70 | 222 | 27 | 20 | 531 | 26 | 70 | —802 | 25 |
| 75 | —508 | 28 | 25 | —900 | 27 | 75 | —249 | 26 | 25 | —557 | 26 | 75 | —827 | 25 |
| 80 | 536 | 28 | 30 | 927 | 28 | 80 | 275 | 27 | 30 | —583 | 26 | 80 | —852 | 25 |
| 85 | 564 | 29 | 35 | —955 | 27 | 85 | —302 | 26 | 35 | —609 | 25 | 85 | —877 | 25 |
| 90 | —593 | 28 | 40 | 982 | 27 | 90 | 328 | 27 | 40 | 634 | 26 | 90 | —902 | 25 |
| 95 | —621 | 28 | 45 | 90009 | 28 | 95 | —355 | 26 | 45 | —660 | 26 | 95 | —927 | 25 |
| **7700** | 649 | 28 | **7950** | —037 | 27 | **8200** | 381 | 27 | **8450** | —686 | 25 | **8700** | —952 | 25 |
| 05 | 677 | 28 | 55 | 064 | 27 | 05 | —408 | 26 | 55 | 711 | 26 | 05 | —977 | 25 |
| 10 | 705 | 29 | 60 | 091 | 28 | 10 | 434 | 27 | 60 | 737 | 26 | 10 | 94002 | 25 |
| 15 | —734 | 28 | 65 | —119 | 27 | 15 | —461 | 26 | 65 | —763 | 25 | 15 | —027 | 25 |
| 20 | —762 | 28 | 70 | —146 | 27 | 20 | 487 | 27 | 70 | 788 | 26 | 20 | —052 | 25 |
| 25 | —790 | 28 | 75 | 173 | 27 | 25 | —514 | 26 | 75 | —814 | 26 | 25 | —077 | 24 |
| 30 | —818 | 28 | 80 | 200 | 27 | 30 | —540 | 26 | 80 | —840 | 25 | 30 | 101 | 25 |
| 35 | 846 | 28 | 85 | 227 | 27 | 35 | 566 | 27 | 85 | 865 | 26 | 35 | 126 | 25 |
| 40 | 874 | 28 | 90 | —255 | 27 | 40 | —593 | 26 | 90 | —891 | 25 | 40 | 151 | 25 |
| 45 | 902 | 28 | 95 | —282 | 27 | 45 | 619 | 26 | 95 | 916 | 26 | 45 | —176 | 25 |

Encontrar log 83678:
log 83700=92273
log 83650=92247
dif. 50 26
83678 — 83650=**28**
Debajo de **26**
dif. para 10×2=10.0
— — 8= 4.2
— — **28**=14.2
Log 83678=
92247+14=92261.

| | 29 | 28 | 27 | 26 | 25 | 24 | |
|---|---|---|---|---|---|---|---|
| **1** | 0.6 | 0.6 | 0.5 | 0.5 | 0.5 | 0.5 | **1** |
| **2** | 1.2 | 1.1 | 1.1 | 1.0 | 1.0 | 1.0 | **2** |
| **3** | 1.7 | 1.7 | 1.6 | 1.6 | 1.5 | 1.4 | **3** |
| **4** | 2.3 | 2.2 | 2.2 | 2.1 | 2.0 | 1.9 | **4** |
| **5** | 2.9 | 2.8 | 2.7 | 2.6 | 2.5 | 2.4 | **5** |
| **6** | 3.5 | 3.4 | 3.2 | 3.1 | 3.0 | 2.9 | **6** |
| **7** | 4.1 | 3.9 | 3.8 | 3.6 | 3.5 | 3.4 | **7** |
| **8** | 4.6 | 4.5 | 4.3 | 4.2 | 4.0 | 3.8 | **8** |
| **9** | 5.2 | 5.0 | 4.9 | 4.7 | 4.5 | 4.3 | **9** |
| **10** | 5.8 | 5.6 | 5.4 | 5.2 | 5.0 | 4.8 | **10** |

**Un guión** antes ó después de un log indica que su verdadero valor es *menor* que el tabular en *menos* de la mitad de la unidad correspondiente al último puesto. Ej.:
Log 8325=9203842
Log 8330=9206450.

## Logaritmos vulgares ó de Briggs. Base=10.

| No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. | No. | Log. | Dif. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **8750** | 94201 | 25 | **9000** | 95424 | 24 | **9250** | 96614 | 24 | **9500** | 97772 | 23 | **9750** | 98900 | 23 |
| 55 | —226 | 24 | 05 | 448 | 24 | 55 | —638 | 23 | 05 | 795 | 23 | 55 | —923 | 23 |
| 60 | 250 | 25 | 10 | 472 | 24 | 60 | 661 | 24 | 10 | 818 | 23 | 60 | —945 | 22 |
| 65 | 275 | 25 | 15 | —497 | 25 | 65 | —685 | 23 | 15 | —841 | 23 | 65 | 967 | 22 |
| 70 | —300 | 25 | 20 | —521 | 24 | 70 | —708 | 23 | 20 | —864 | 22 | 70 | 989 | 23 |
| 75 | —325 | 24 | 25 | —545 | 24 | 75 | 731 | 24 | 25 | 886 | 23 | 75 | 99012 | 22 |
| 80 | 349 | 25 | 30 | —569 | 24 | 80 | —755 | 23 | 30 | 909 | 23 | 80 | —034 | 22 |
| 85 | 374 | 25 | 35 | —593 | 24 | 85 | 778 | 24 | 35 | 932 | 23 | 85 | 056 | 22 |
| 90 | —399 | 25 | 40 | —617 | 24 | 90 | —802 | 23 | 40 | —955 | 23 | 90 | 078 | 22 |
| 95 | —424 | 24 | 45 | —641 | 24 | 95 | —825 | 23 | 45 | —978 | 22 | 95 | 100 | 22 |
| **8800** | 448 | 25 | **9050** | —665 | 24 | **9300** | 848 | 24 | **9550** | 98000 | 23 | **9800** | —123 | 23 |
| 05 | —473 | 25 | 55 | —689 | 24 | 05 | —872 | 23 | 55 | 023 | 23 | 05 | —145 | 22 |
| 10 | —498 | 24 | 60 | —713 | 24 | 10 | —895 | 23 | 60 | —046 | 22 | 10 | —167 | 22 |
| 15 | 522 | 25 | 65 | —737 | 24 | 15 | 918 | 24 | 65 | 068 | 23 | 15 | 189 | 22 |
| 20 | —547 | 24 | 70 | —761 | 24 | 20 | —942 | 23 | 70 | 091 | 23 | 20 | 211 | 22 |
| 25 | 571 | 25 | 75 | —785 | 24 | 25 | —965 | 23 | 75 | —114 | 23 | 25 | 233 | 22 |
| 30 | 596 | 25 | 80 | —809 | 23 | 30 | 988 | 23 | 80 | —137 | 22 | 30 | 255 | 22 |
| 35 | —621 | 24 | 85 | 832 | 24 | 35 | 97011 | 24 | 85 | 159 | 23 | 35 | 277 | 22 |
| 40 | 645 | 25 | 90 | 856 | 24 | 40 | —035 | 23 | 90 | —182 | 22 | 40 | —300 | 23 |
| 45 | —670 | 24 | 95 | 880 | 24 | 45 | —058 | 23 | 95 | 204 | 23 | 45 | —322 | 22 |
| **8850** | 694 | 25 | **9100** | 904 | 24 | **9350** | 081 | 23 | **9600** | 227 | 23 | **9850** | —344 | 22 |
| 55 | —719 | 24 | 05 | —928 | 24 | 55 | 104 | 24 | 05 | —250 | 22 | 55 | —366 | 22 |
| 60 | 743 | 25 | 10 | —952 | 24 | 60 | —128 | 23 | 10 | 272 | 23 | 60 | —388 | 22 |
| 65 | —768 | 24 | 15 | —976 | 23 | 65 | —151 | 23 | 15 | —295 | 23 | 65 | —410 | 22 |
| 70 | 792 | 25 | 20 | 999 | 24 | 70 | —174 | 23 | 20 | —318 | 22 | 70 | —432 | 22 |
| 75 | —817 | 24 | 25 | 96023 | 24 | 75 | 197 | 23 | 25 | 340 | 23 | 75 | —454 | 22 |
| 80 | 841 | 25 | 30 | 047 | 24 | 80 | 220 | 23 | 30 | —363 | 22 | 80 | —476 | 22 |
| 85 | —866 | 24 | 35 | —071 | 24 | 85 | 243 | 24 | 35 | 385 | 23 | 85 | —498 | 22 |
| 90 | 890 | 25 | 40 | —095 | 23 | 90 | —267 | 23 | 40 | —408 | 22 | 90 | —520 | 22 |
| 95 | —915 | 24 | 45 | 118 | 24 | 95 | —290 | 23 | 45 | 430 | 23 | 95 | —542 | 22 |
| **8900** | 939 | 24 | **9150** | 142 | 24 | **9400** | —313 | 23 | **9650** | —453 | 22 | **9900** | —564 | 21 |
| 05 | 963 | 25 | 55 | —166 | 24 | 05 | —336 | 23 | 55 | 475 | 23 | 05 | 585 | 22 |
| 10 | —988 | 24 | 60 | —190 | 23 | 10 | —359 | 23 | 60 | —498 | 22 | 10 | 607 | 22 |
| 15 | 95012 | 24 | 65 | 213 | 24 | 15 | 382 | 23 | 65 | 520 | 23 | 15 | 629 | 22 |
| 20 | 036 | 25 | 70 | —237 | 24 | 20 | 405 | 23 | 70 | —543 | 22 | 20 | 651 | 22 |
| 25 | —061 | 24 | 75 | —261 | 23 | 25 | 428 | 23 | 75 | 565 | 23 | 25 | 673 | 22 |
| 30 | 085 | 24 | 80 | 284 | 24 | 30 | 451 | 23 | 80 | —588 | 22 | 30 | —695 | 22 |
| 35 | 109 | 25 | 85 | —308 | 24 | 35 | 474 | 23 | 85 | —610 | 22 | 35 | — 717 | 22 |
| 40 | —134 | 24 | 90 | —332 | 23 | 40 | 497 | 23 | 90 | 632 | 23 | 40 | — 739 | 22 |
| 45 | 158 | 24 | 95 | 355 | 24 | 45 | 520 | 23 | 95 | —655 | 22 | 45 | 760 | 21 |
| **8950** | 182 | 25 | **9200** | —379 | 23 | **9450** | 543 | 23 | **9700** | 677 | 23 | **9950** | 782 | 22 |
| 55 | —207 | 24 | 05 | 402 | 24 | 55 | 566 | 23 | 05 | —700 | 22 | 55 | 804 | 22 |
| 60 | —231 | 24 | 10 | —426 | 24 | 60 | 589 | 23 | 10 | —722 | 22 | 60 | —826 | 22 |
| 65 | 255 | 24 | 15 | —450 | 23 | 65 | 612 | 23 | 15 | 744 | 23 | 65 | —848 | 22 |
| 70 | 279 | 24 | 20 | 473 | 24 | 70 | —635 | 23 | 20 | —767 | 22 | 70 | —870 | 21 |
| 75 | 303 | 25 | 25 | —497 | 23 | 75 | —658 | 23 | 25 | —789 | 22 | 75 | 891 | 22 |
| 80 | —328 | 24 | 30 | 520 | 24 | 80 | —681 | 23 | 30 | 811 | 23 | 80 | 913 | 22 |
| 85 | —352 | 24 | 35 | —544 | 23 | 85 | —704 | 23 | 35 | —834 | 22 | 85 | —935 | 22 |
| 90 | —376 | 24 | 40 | 567 | 24 | 90 | —727 | 23 | 40 | —856 | 22 | 90 | —957 | 22 |
| 95 | 400 | 24 | 45 | —591 | 23 | 95 | 749 | 23 | 45 | 878 | 22 | 95 | 978 | 22 |

Encontrar log 95544:
log 95550=98023
log 95500=98000
dif. 50 23
95544 — 95500=44
Debajo de 23
dif. para 10×4=18.0
— — 4= 1.8
— — 44=19.8
Log 95554=
98000+20=98020.

| | 25 | 24 | 23 | 22 | 21 |
|---|---|---|---|---|---|
| 1 | 0.5 | 0.5 | 0.5 | 0.4 | 0.4 |
| 2 | 1.0 | 1.0 | 0.9 | 0.9 | 0.8 |
| 3 | 1.5 | 1.4 | 1.4 | 1.3 | 1.3 |
| 4 | 2.0 | 1.9 | 1.8 | 1.8 | 1.7 |
| 5 | 2.5 | 2.4 | 2.3 | 2.2 | 2.1 |
| 6 | 3.0 | 2.9 | 2.8 | 2.6 | 2.5 |
| 7 | 3.5 | 3.4 | 3.2 | 3.1 | 2.9 |
| 8 | 4.0 | 3.8 | 3.7 | 3.5 | 3.4 |
| 9 | 4.5 | 4.3 | 4.1 | 4.0 | 3.8 |
| 10 | 5.0 | 4.8 | 4.6 | 4.4 | 4.2 |

**Un guión** antes ó después de un log indica que su verdadero valor es *menor* que el tabular en *menos* de la mitad de la unidad correspondiente al último puesto. Ej.:
Log 9600=9822712
Log 9605=9824974.

# GEOMETRIA

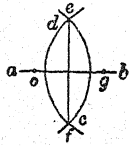
Definición de **líneas, figuras planas y sólidos.** — Hablando estrictamente, una **línea** es simplemente una longitud ó distancia. Las líneas que trazamos en el papel tienen no sólo longitud sino también anchura y espesor, sin embargo son el símbolo más conveniente que podemos emplear para designar una línea geométrica. Línea vertical es la que se dirige hacia el centro de gravedad de la tierra, y horizontal, la que corta en ángulo recto la vertical. Una figura plana es un plano encerrado por líneas rectas ó curvas que se denominan perímetro, circunferencia ó periferia. Se confunde á menudo el perímetro con la figura misma como cuando se habla de trazar círculos, cuadrados, etc., y lo que trazamos son sus perímetros. Geométricamente hablando una figura tiene longitud y latitud, pero no espesor. **Sólido** es todo lo que tiene longitud, latitud ó anchura y espesor.

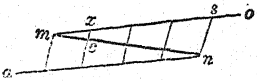
Las figuras ó sólidos semejantes no han de ser necesariamente del mismo tamaño, pero sí precisamente de la misma **forma**. Así, dos cuadrados cualesquiera son semejantes entre sí, como también dos círculos ó cubos cualquiera que sea su diferencia de tamaño. Cuando no sólo son de la misma forma sino del mismo tamaño, se dice que son **iguales**.

**Las líneas** están entre sí como sus **longitudes**, pero las áreas ó superficies de **figuras** semejantes están entre sí como los **cuadrados** de sus líneas homólogas. Los **sólidos** semejantes están entre sí como los **cubos** de sus líneas homólogas.

**Advertencia.** — Por simples que aparezcan las operaciones siguientes, se requiere gran cuidado y buenos instrumentos á fin de obtener en la práctica resultados exactos. Algunas de ellas pueden ejecutarse mucho mejor por medio de un triángulo metálico que tenga un ángulo recto. En el terreno, la cinta de agrimensor, cadena ó una percha, sustituirá á los compases de división y regla que se usan en el dibujo.

**Para dividir una línea dada** *ab* **en dos partes iguales.** — De sus extremos *a*, *b*, como centros y con cualquier radio mayor que la mitad de *ab* descríbanse los arcos *c*, *d*, y únase *e* con *f*. Si la línea *ab* es muy larga, trácense primero distancias iguales *ao*, *bg* en cada lado desde los extremos, y procédase entonces como si *og* fuera la línea que tiene que dividirse. O bien mídase *ab* para determinar su centro.



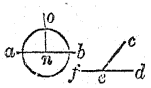

**Para dividir una línea** *mn* **en cualquier número de partes iguales.** De *mn* trácense dos líneas paralelas cualesquiera *mo*, *na*, hasta una distancia indefinida y sobre ellas desde *m*, *n* márquese el número requerido de partes iguales de una longitud cualquiera y conveniente; después únanse los puntos correspondientes así marcados. O bien trácese sólo una línea como *mo* y únase entonces *s* con *n*, trazando las otras líneas paralelas á ésta.

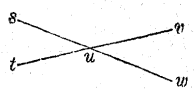
**Para dividir una línea dada** *mn* **en dos partes que deben estar entre sí en una proporción dada** se aplica el mismo principio anterior. Por ejemplo: en la proporción de 1 á 3. Trácese primero una línea cualquiera *mo* y con cualquiera apertura del compás hágase *mx* igual á una división y *xs* igual á tres de estas divisiones. Unanse *sn* y paralela á ella trácese *xc*. Entonces *mc* es á *cn* como 1 es á 3.

## ÁNGULOS

Cuando dos líneas rectas se encuentran con cualquiera inclinación, esa inclinación se llama **ángulo** y se mide por los grados contenidos en un arco de círculo que tenga por centro el punto donde se cortan las dos líneas. Como todo círculo, grande ó pequeño, se supone dividido en 360 grados, se deduce que á cualquier número de grados de un círculo pequeño corresponderán igual número en el mayor.

Cuando dos líneas rectas como *on*, *ab* se encuentran de tal manera que la inclinación *ona* es igual á la inclinación *onb*, se dice que las dos líneas son **perpendiculares** entre sí y los ángulos *ona*, *onb* se llaman **ángulos rectos** y son

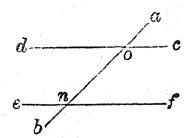


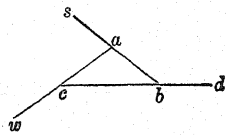




iguales á 90 grados ó á una cuarta parte de la circunferencia del círculo. Cualquier ángulo, como *ced*, más pequeño que un ángulo recto se llama **agudo** y uno *cef* mayor que un ángulo recto se llama **obtuso**. Cuando una línea encuentra otra como en la figura siguiente los dos ángulos en el mismo lado de cualquiera de las dos líneas se llaman **adyacentes**. Así : *vus* y *vuw* son adyacentes; también *tus* y *tuw*; *sut* y *suv*, *wut* y *wuv*. La suma de dos ángulos adyacentes es siempre igual á dos ángulos rectos ó á 180°. Por tanto, si conocemos el número de grados que contiene uno de ellos y lo restamos de 180° obtenemos el otro.

Cuando dos líneas rectas se cruzan formando cuatro ángulos, cualquier par de estos ángulos, situados en direcciones opuestas, se llaman **opuestos por el vértice** tales como *suv*, *tuw*. Los ángulos opuestos por el vértice son iguales.

Cuando una línea recta *ab* corta dos líneas paralelas *dc*, *ef*, los ángulos **alternos** que forman una especie de Z son iguales entre sí. Así, los ángulos *don*, *onf*, son iguales como también lo son *con*, *one*. También la suma de los dos ángulos inte-

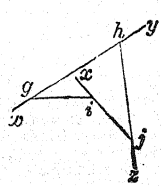



riores en el mismo lado de *ab* es igual á dos ángulos rectos ó 180°; así : *con*+*onf*=180°; también *don*+*one*=180°.

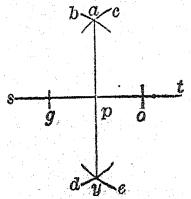
Se llaman **ángulos internos** los formados dentro de cualquier figura por la intersección de dos de sus lados como los ángulos *cab*, *abc*, *bca*, de este triángulo. La suma de los ángulos internos de un polígono de cualquier número de lados, es igual á tantas veces dos ángulos rectos menos cuatro, como lados tiene el polígono. Así, un triángulo tiene 3 lados; el doble de este número es 6, y seis ángulos rectos, ó 6×90°=540°, de los cuales se restan cuatro ángulos rectos ó 360°; y quedan 180°, que es el número de grados que suman los ángulos de un triángulo. Este principio sirve para probar las medidas hechas en los ángulos de cualquiera figura; porque si la suma de todos los ángulos medidos, no está de acuerdo con la regla anterior, es prueba de que hemos cometido error.

**Angulo externo** en un polígono, es cualquier ángulo como *abd* formado *fuera* de la figura por el encuentro de cualquier lado, *ab*, con la prolongación



del lado adyacente *bd;* lo mismo son los ángulos *cas*, *bcw*. La suma de todos los ángulos exteriores en cualquier polígono, sea cual fuere su número de lados, es igual á 360°; pero si alguno de los ángulos es *reentrante* como *gij*, el ángulo **interior** *gij* excede 180°, y el ángulo *gix* siendo = 180° — el ángulo interior es negativo. Así *abd* + *bcw* + *cas* = 360°; y, *yhj* + *zji* — *gix* + *igw* = 360°; los ángulos como *a*, *b*, *c*, *g*, *h*, *j*, cuyos vértices apuntan hacia afuera, se llaman **salientes.**

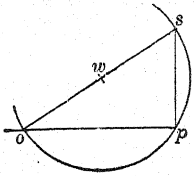
**En cualquier punto dado**, *p*, **de una línea** *st*, **levantar una perpendicular** *pa*. Tomando *p* por centro, fíjense con el compás dos distancias iguales, *po* y *pg*. Luego, tomando *o* y *g* como centros, trácense con una abertura mayor




que la mitad de *og* los dos arcos *b* y *c* y únase *a* con *p*. O todavía mejor: descríbanse cuatro arcos y únase *a* con *y*.

O de otro modo, desde *p* con cualquier escala conveniente descríbanse dos arcos pequeños *g*, *a*, cualquiera de los dos con un radio 3 y el otro con un radio 4. Entonces de *g* con un radio 5, descríbase el arco *b*. Unase *p* con *a*.

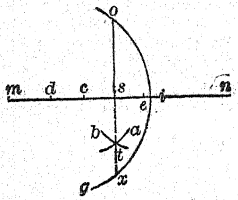
**Si el punto** *p* **está en un extremo de la línea ó muy cerca de él,**

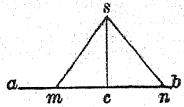



**extiéndase la línea si es posible** y procédase como arriba. Pero si esto no puede hacerse, de un punto conveniente *w*, ábrase el compás hasta *p*, y descríbase el semicírculo *spo;* por *ow* trácese *ows;* y únase *p* con *s*.

O úsese el procedimiento anterior con los radios 3, 4, 5.

**De un punto dado** *o* **bajar una perpendicular** *os*, **á una línea dada** *mn*. De *o* mídanse hasta la línea *mn* dos distancias iguales, *oc*, *oe*, y desde *c* y *e*,

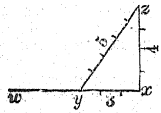


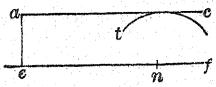




como centros, con cualquiera abertura mayor que la mitad de *ce*, descríbanse los dos arcos *a* y *b*, y únanse *ot*. O desde cualquier punto *d* sobre la línea ábrase el compás hasta *o* y descríbase el arco *og;* hágase *ix* igual *io;* y únanse *ox*.

**Si la línea**, *ab*, **está en el terreno** y se quiere levantar una perpendicular desde *c*, mídanse primero dos distancias iguales cualesquiera, *cm*, *cn*.

En *mn*, sujétense los extremos de un pedazo de cuerda, cinta de agrimensor, ó cadena, *msn*, luego átese la cuerda, etc., como se demuestra en *msn*, siendo su centro *s* y entonces *sc* será la perpendicular que se busca. Si la perpendicular *xz* debe levantarse en el extremo de la línea *wx*, mídase primero *xy* sobre la línea igual á tres unidades cualesquiera y sujetando el extremo de una cinta de agrimensor en *x*, se marcan nueve unidades en *y*; sujétese ahora la marca de cuatro unidades

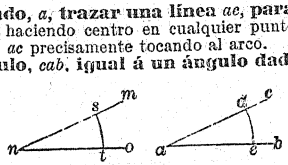



en *z* manteniendo á *zx*, *xy* igualmente tensos. Entonces *zx* será la perpendicular que se requiere porque 3, 4 y 5 forman los lados de un triángulo rectángulo. En lugar de 3, 4 y 5 se pueden usar cualesquiera múltiples de estos números, tales como 6 8 y 10; 9, 12 y 15, etc.

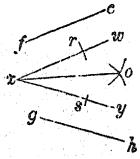
**Por un punto dado, *a*, trazar una línea *ac*, paralela á otra línea *ef*.** Descríbase un arco *t* haciendo centro en cualquier punto de la línea *ef* con la distancia *ae* y trácese *ac* precisamente tocando al arco.

**Formar un ángulo, *cab*, igual á un ángulo dado *mno*, en cualquier**




**punto, *a*, de una línea, *ab*.** Desde *n* y *a* con cualquier radio, descríbanse los arcos *st*, *de*, mídase *st*, hágase *ed*, igual á él; y por *ad*, trácese *ac*.

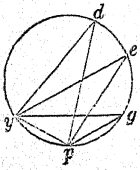
**Dividir cualquier ángulo, *wxy*, en dos partes iguales.** Trácense desde *x* dos distancias iguales cualesquiera, *xr*, *xs*, de *r*, *s*, descríbanse, con cualquier radio, dos arcos que se intercepten en *o* y únase *o* con *x*. Si los dos lados del ángulo no se cortan como sucede en *cf*, *gh*, extiéndanse hasta que se encuentren ó

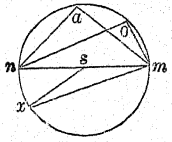



trácense líneas *xw*, *xy* paralelas y á distancias iguales de ellas de modo que se encuentren, y entonces se procederá como antes.

Todos los ángulos como *nam*, *nom*, cuyo vértice se encuentra en la circunferencia de un semicírculo y cuyos lados terminan en los extremos de un diámetro *nm*, son ángulos rectos ó como se dice generalmente: **todos los ángulos inscritos en un semicírculo son ángulos rectos.** Un ángulo *nsx* cuyo vértice está en el centro de un círculo, es dos veces mayor que un ángulo *nmx* apoyado en la circunferencia; cuando los lados de ambos comprenden el mismo arco *nx*.

Todos los ángulos, como *ydp*, *yep*, *ygp*, que están en la circunferencia de un

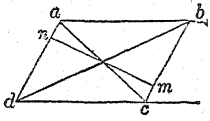


círculo y cuyos lados corresponden al mismo arco $yp$, son iguales entre sí; ó como se dice usualmente : todos los ángulos inscritos en el mismo segmento de un círculo, son iguales.

El **complemento** de un ángulo es lo que le falta para llegar á 90°. Así, el complemento de 80° es 90° — 80°=10° y el de 210° es 90° — 210° = — 120°. El **suplemento** de un ángulo es lo que le falta para llegar á 180°. El suplemento de 80° es 180° — 80°=100°; y el de 210° es 180° — 210° = — 30°. Pero generalmente podemos prescindir de anteponer los signos + ó — á los complementos y suplementos y llamar el complemento de un ángulo, su diferencia con 90° y el suplemento su diferencia con 180°.

**Ángulos de un paralelogramo.** Paralelogramo es cualquiera figura rectilínea de cuatro lados cuyos lados opuestos son iguales, como *abcd*. Cualquiera línea trazada á través de un paralelogramo, entre dos ángulos opuestos se llama *diagonal*, como *ac*, *bd*. Una diagonal divide el paralelogramo en dos partes iguales y también cualquiera línea *mn* trazada por el centro de una de las dos diagonales; además la línea *mn* misma queda dividida en dos partes iguales por la diagonal. Dos diagonales se cortan entre sí, y dividen también al paralelogramo en cuatro triángulos de iguales áreas. La suma de los dos ángulos á los extremos de uno cualquiera de los lados es=180°. Así $dab+abc=abc+bcd=180°$; y la suma de los cuatro ángulos *dab*, *abc*, *bcd*, *cda*=360°.




La suma de los cuadrados de los cuatro lados es igual á la suma de los cuadrados de las dos diagonales.

### Para reducir minutos y segundos á grados y fracciones decimales de un grado, etc.

En cualquier ángulo dado :

| | | | |
|---|---|---|---|
| **Número de grados** | = Número de minutos | ÷ | 60. |
| | = — de segundos | ÷ | 3,600. |
| **Número de minutos** | = — de grados | × | 60. |
| | = — de segundos | ÷ | 60. |
| **Número de segundos** | = — de grados | × | 3,600. |
| | = — de minutos | × | 60. |

## Tabla de minutos y segundos en decimales de un grado y de segundos en decimales de un minuto.

(Las columnas de min y gr se emplean y responden también para segundos y minutos.)

En cada equivalente **el último dígito se repite indefinidamente.**
Véase * debajo :

| Mins. | Grad. | Mins. | Grad. | Mins. | Grad. | Segs. | Grad. | Segs. | Grad. | Segs. | Grad. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.016 | 21 | 0.350 | 41 | 0.683 | 1 | 0.00027 | 21 | 0.00583 | 41 | 0.01138 |
| 2 | 0.033 | 22 | 0.366 | 42 | 0.700 | 2 | 0.00055 | 22 | 0.00611 | 42 | 0.01166 |
| 3 | 0.050 | 23 | 0.383 | 43 | 0.716 | 3 | 0.00083 | 23 | 0.00638 | 43 | 0.01194 |
| 4 | 0.066 | 24 | 0.400 | 44 | 0.733 | 4 | 0.00111 | 24 | 0.00666 | 44 | 0.01222 |
| 5 | 0.083 | 25 | 0.416 | 45 | 0.750 | 5 | 0.00138 | 25 | 0.00694 | 45 | 0.01250 |
| 6 | 0.100 | 26 | 0.433 | 46 | 0.766 | 6 | 0.00166 | 26 | 0.00722 | 46 | 0.01277 |
| 7 | 0.116 | 27 | 0.450 | 47 | 0.783 | 7 | 0.00194 | 27 | 0.00750 | 47 | 0.01305 |
| 8 | 0.133 | 28 | 0.466 | 48 | 0.800 | 8 | 0.00222 | 28 | 0.00777 | 48 | 0.01333 |
| 9 | 0.150 | 29 | 0.483 | 49 | 0.816 | 9 | 0.00250 | 29 | 0.00805 | 49 | 0.01361 |
| 10 | 0.166 | 30 | 0.500 | 50 | 0.833 | 10 | 0.00277 | 30 | 0.00833 | 50 | 0.01388 |
| 11 | 0.183 | 31 | 0.516 | 51 | 0.850 | 11 | 0.00305 | 31 | 0.00861 | 51 | 0.01416 |
| 12 | 0.200 | 32 | 0.533 | 52 | 0.866 | 12 | 0.00333 | 32 | 0.00888 | 52 | 0.01444 |
| 13 | 0.216 | 33 | 0.550 | 53 | 0.883 | 13 | 0.00361 | 33 | 0.00916 | 53 | 0.01472 |
| 14 | 0.233 | 34 | 0.566 | 54 | 0.900 | 14 | 0.00388 | 34 | 0.00944 | 54 | 0.01500 |
| 15 | 0.250 | 35 | 0.583 | 55 | 0.916 | 15 | 0.00416 | 35 | 0.00972 | 55 | 0.01527 |
| 16 | 0.266 | 36 | 0.600 | 56 | 0.933 | 16 | 0.00444 | 36 | 0.01000 | 56 | 0.01555 |
| 17 | 0.283 | 37 | 0.616 | 57 | 0.950 | 17 | 0.00472 | 37 | 0.01027 | 57 | 0.01583 |
| 18 | 0.300 | 38 | 0.633 | 58 | 0.966 | 18 | 0.00500 | 38 | 0.01055 | 58 | 0.01611 |
| 19 | 0.316 | 39 | 0.650 | 59 | 0.983 | 19 | 0.00527 | 39 | 0.01083 | 59 | 0.01638 |
| 20 | 0.333 | 40 | 0.666 | 60 | 1.000 | 20 | 0.00555 | 40 | 0.01111 | 60 | 0.01666 |

| Segs. | Mins. | Segs. | Mins. | Segs. | Mins. | Segs. | Grad. | Segs. | Grad. | Segs. | Grad. |
|---|---|---|---|---|---|---|---|---|---|---|---|

* Cada equivalente tiene una **decimal periódica.** Ejemplo :

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 2 | minutos | = 0.033333..... | grados | 12 | segundos | = 0.200000..... | minuto |
| 7 | — | = 0.116666..... | — | 1 | — | = 0.000277..... | grado |
| 12 | — | = 0.200000..... | — | 50 | — | = 0.013888 ..... | — |

## MEDIDA APROXIMADA DE ÁNGULOS

**(1) Los cuatro dedos de la mano** formando ángulos rectos con el brazo y á distancia del largo del brazo de los ojos, cubren 7 grados y un ángulo de 7° corresponde como á 12.2 pies en 100 pies ó á 12.2 metros en 100 metros.

**(2) Se miden aproximadamente ángulos por medio de una regla de dos pies**, sea en un trazado ó entre objetos distantes en el terreno, así: si las puntas interiores de una regla ordinaria de dos pies de largo (articulada en su mitad, como se usa, *N. del T.*) se abren con las separaciones que indica la columna de pulgadas en la próxima tabla, las dos reglas quedarán inclinadas entre sí los ángulos que determina la columna de ángulos de la misma tabla. No se debe contar con gran exactitud en este sistema y mucho menos después que el ángulo pasa de 105°, porque el error se aumenta á proporción que se hace mayor la abertura.

Los ángulos para aberturas intermedias de aquellos que se dan, pueden calcularse por medio de una simple proporción.

**Tabla de ángulos correspondientes á aberturas de una regla de 2 pies.** (Original.)

| Pulg. | Gr | Min. | Pulg. | Gr | Min. | Pulg. | Gr | Min. | Pulg. | Gr | Min. | Pulg. | Gr | Min. | Pulg. | Gr | Min. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ¼ | 1 | 12 | 4¼ | 20 | 24 | 8¼ | 40 | 18 | 12¼ | 61 | 23 | 16¼ | 85 | 14 | 20¼ | 115 | 5 |
| | 1 | 48 | | 21 | | | 40 | 51 | | 62 | 5 | | 86 | 3 | | 116 | 12 |
| ⅓ | 2 | 24 | ⅓ | 21 | 37 | ⅓ | 41 | 29 | ⅓ | 62 | 47 | ⅓ | 86 | 52 | ⅓ | 117 | 20 |
| | 3 | 00 | | 22 | 13 | | 42 | 7 | | 63 | 28 | | 87 | 41 | | 118 | 59 |
| ¾ | 3 | 36 | ¾ | 22 | 50 | ¾ | 42 | 46 | ¾ | 64 | 11 | ¾ | 88 | 31 | ¾ | 119 | 40 |
| | 4 | 11 | | 23 | 27 | | 43 | 24 | | 64 | 53 | | 89 | 21 | | 120 | 52 |
| 1 | 4 | 47 | 5 | 24 | 3 | 9 | 44 | 3 | 13 | 65 | 35 | 17 | 90 | 12 | 21 | 122 | 6 |
| | 5 | 23 | | 24 | 39 | | 44 | 42 | | 66 | 18 | | 91 | 3 | | 123 | 20 |
| ¼ | 5 | 58 | ¼ | 25 | 16 | ¼ | 45 | 21 | ¼ | 67 | 1 | ¼ | 91 | 54 | ¼ | 124 | 56 |
| | 6 | 34 | | 25 | 53 | | 45 | 59 | | 67 | 44 | | 92 | 46 | | 125 | 54 |
| ½ | 7 | 10 | ½ | 26 | 30 | ½ | 46 | 38 | ½ | 68 | 28 | ½ | 93 | 38 | ½ | 127 | 14 |
| | 7 | 46 | | 27 | 7 | | 47 | 17 | | 69 | 12 | | 94 | 31 | | 128 | 35 |
| ¾ | 8 | 22 | ¾ | 27 | 44 | ¾ | 47 | 56 | ¾ | 69 | 55 | ¾ | 95 | 24 | ¾ | 129 | 59 |
| | 8 | 58 | | 28 | 21 | | 48 | 35 | | 70 | 38 | | 96 | 17 | | 131 | 25 |
| 2 | 9 | 34 | 6 | 28 | 58 | 10 | 49 | 15 | 14 | 71 | 22 | 18 | 97 | 11 | 22 | 132 | 53 |
| | 10 | 10 | | 29 | 35 | | 49 | 54 | | 72 | 6 | | 98 | 5 | | 134 | 24 |
| ¼ | 10 | 46 | ¼ | 30 | 11 | ¼ | 50 | 34 | ¼ | 72 | 51 | ¼ | 99 | 00 | ¼ | 135 | 58 |
| | 11 | 22 | | 30 | 49 | | 51 | 13 | | 73 | 36 | | 99 | 55 | | 137 | 35 |
| ½ | 11 | 58 | ½ | 31 | 26 | ½ | 51 | 53 | ½ | 74 | 21 | ½ | 100 | 51 | ½ | 139 | 16 |
| | 12 | 34 | | 32 | 3 | | 52 | 33 | | 75 | 6 | | 101 | 48 | | 141 | 1 |
| ¾ | 13 | 10 | ¾ | 32 | 40 | ¾ | 53 | 13 | ¾ | 75 | 51 | ¾ | 102 | 45 | ¾ | 142 | 51 |
| | 13 | 46 | | 33 | 17 | | 53 | 52 | | 76 | 36 | | 103 | 43 | | 144 | 46 |
| 3 | 14 | 22 | 7 | 33 | 54 | 11 | 54 | 34 | 15 | 77 | 22 | 19 | 104 | 41 | 23 | 146 | 48 |
| | 14 | 58 | | 34 | 33 | | 55 | 14 | | 78 | 8 | | 105 | 40 | | 148 | 58 |
| ¼ | 15 | 34 | ¼ | 35 | 10 | ¼ | 55 | 55 | ¼ | 78 | 54 | ¼ | 106 | 39 | ¼ | 151 | 11 |
| | 16 | 10 | | 35 | 47 | | 56 | 35 | | 79 | 40 | | 107 | | | 153 | 45 |
| ½ | 16 | 46 | ½ | 36 | 25 | ½ | 57 | 16 | ½ | 80 | 27 | ½ | 108 | 41 | ½ | 156 | 34 |
| | 17 | 22 | | 37 | 3 | | 57 | 57 | | 81 | 14 | | 109 | 42 | | 159 | 45 |
| ¾ | 17 | 59 | ¾ | 37 | 41 | ¾ | 58 | 38 | ¾ | 82 | 2 | ¾ | 110 | 46 | ¾ | 163 | 27 |
| | 18 | 35 | | 38 | 19 | | 59 | 19 | | 82 | 49 | | 111 | 49 | | 168 | 18 |
| 4 | 19 | 12 | 8 | 38 | 57 | 12 | 60 | 00 | 16 | 83 | 37 | 20 | 112 | 53 | 24 | 180 | 00 |
| | 19 | 48 | | 39 | 35 | | 60 | 41 | | 84 | 26 | | 113 | 58 | | | |

**(3) Con la misma tabla usando pies en lugar de pulgadas.** De cualquier punto mídanse 12 pies hacia cada objeto y colóquense marcas, mídanse las distancias en pies entre estas marcas. Supóngase que la primera columna de la tabla esté en pies en lugar de pulgadas. Entonces frente á la distancia en pies estará el ángulo.

⅛ de pie = 1.5 pulgadas.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 pulg. = .083 pies. | 4 pulg. = .333 pies | 7 pulg = .583 pies. | 10 pulg = .833 pies. |
| 2 — = .167 — | 5 — = .416 — | 8 — = .667 — | 11 — = .917 — |
| 3 — = .25 — | 6 — = .5 — | 9 — = .75 — | 12 — = 1.0 — |

**(4) De otro modo, mídase hacia * cada objeto** 100 ó cualquier número

* Si hay inconveniente en medir hacia los objetos, mídase directamente desde ellos.

de pies y colóquense marcas y mídanse las distancias en pies entre las marcas. Entonces

$$\text{Seno de la mitad del ángulo} = \frac{\text{la mitad de la distancia entre las marcas}}{\text{la distancia medida hacia uno de los objetos.}}$$

Búsquese este seno en la tabla de senos, tangs, etc., págs 106, etc., tómese el ángulo correspondiente y multipliquese por 2.

*N. del T.* — Esta misma tabla es útil empleando el sistema métrico. En efecto: dos reglas de un pie de longitud cada una, dobladas en ángulo, se reemplazan por sus equivalentes de 305 milímetros = 1 pie cada una y para medir las distancias entre sus puntas interiores se sabe que 1 pulg = 25.4 milímetros. Multiplicando, pues, el número de pulgs de la tabla necesarias para obtener un ángulo × 25.4 se tiene el número de milímetros de la abertura. — Ej.: se quiere un ángulo de 75° — 51', la distancia, según la tabla, es 14 pulg — 3/4 = 14.75, lo que × 25.4 será = 374.65 milímetros. Separando las puntas 375 mm se tendrá el ang buscado. Al contrario, si se tiene un ángulo dado y se quiere saber su valor: mídase en mm la distancia entre las puntas, divídase por 25.4 y el cociente será las pulgs que en la tabla darán (enfrente) el ang. — Ej.: dist entre las puntas 375 mm divid por 25.4 = 14.76 ó sean 14 3/4 pulgs. El ángulo que le corresponde en la tabla es = 75° — 51'.

# TRIGONOMETRÍA PLANA

Para las **figuras**, véase la pág. 104*a*.

**1. Medida común de los ángulos.** Si se divide una circunferencia en 360 arcos iguales, cada uno de esos arcos ó el ángulo del centro subtendido por el arco es un **grado** (°). Cada grado está subdividido en 60 **minutos** (′), y cada minuto en 60 **segundos** (″). Un ángulo **recto** es un ángulo de 90°.

**2. Medida circular de los ángulos.** El arco cuya longitud es igual al radio, ó el ángulo del centro subtendido por dicho arco se llama un **radial.** (*N. del T.* abajo.) Como la semicircunferencia (véase adelante capítulo *Círculos*) es = π × radio = π radiales = 180° se tiene :

$$\textbf{1 radial} = \frac{180°}{\pi} = \frac{180°}{3.14159} = 57.2957795...°. \quad \text{Log} = 1.758\ 1226.$$

1 grado = .017 453 292 520 radial.
1 minuto = .000 290 888 209 —
1 segundo = .000 004 848 137 —

**3. Relación entre el arco y el radio.** En cualquier ángulo la longitud del arco con relación al radio se llama *el arco* del ángulo. Ej.: fig. 1 y 3, arco* A = ZB / OZ. Para otras relaciones angulares, véanse §§ 6 y 10.

**4. Angulos positivos y negativos.** — En la fig. 1* supóngase el radio OZ, describiendo, parte ó todo el círculo, en dirección de la flecha, ó sea hacia la izquierda, en sentido **contrario** al movimiento de las agujas de un reloj; los arcos ó ángulos ZB, BC, ZBC, así descriptos, se consideran **positivos.** Los trazados en sentido contrario, de Z hacia E, como ZED, etc., son **negativos.** Ej. : arco ZBCDE = + 280°; arco ZE = — 80°.

**5. Complemento, suplemento, etc.** Para cualquier ángulo **A** se tiene **complemento** de A = 90° — A; **suplemento** de A = 180° — A.

**6. Funciones angulares ó razones angulares.** Líneas trigonométricas son las razones entre los lados de un triángulo rectángulo. Fig. 2. Las razones principales de un ángulo A son : el **seno** (sen A), **coseno** (cos A), **tangente** (tang A), **secante** (sec A), **cosecante** (cosec A) y **cotangente A** (cot A). Para otras razones véanse §§ 3 y 10.

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* Para las figuras, véase la pág. 104*a*.
(*N. del T.* — Creemos útil introducir esta expresión en español.

En la fig. 2 :*

$$\textbf{Sen}\quad A = \frac{a}{c} = \frac{\text{lado opuesto}}{\text{hipotenusa}}; \qquad \textbf{cosec}\, A = \frac{1}{\text{sen } A} = \frac{c}{a} = \frac{\text{hipotenusa}}{\text{lado opuesto}};$$

$$\textbf{Cos}\quad A = \frac{b}{c} = \frac{\text{lado adyacente}}{\text{hipotenusa}}; \qquad \textbf{sec}\, A = \frac{1}{\cos A} = \frac{c}{b} = \frac{\text{hipotenusa}}{\text{lado adyacente}};$$

$$\textbf{Tang}\ A = \frac{a}{b} = \frac{\text{lado opuesto}}{\text{lado adyacente}}; \qquad \textbf{cot}\, A = \frac{1}{\text{tang } A} = \frac{b}{a} = \frac{\text{lado adyacente}}{\text{lado opuesto}}.$$

**7.** Si se representa el denominador en cada razón (**6**) por una línea de longitud 1, entonces la longitud de la línea que representa el numerador, da el valor de la razón. Ej. : sea en la fig. 3* el radio OB=1; el ángulo ZOU=90°; ZOB un ángulo cualquiera A y sean MB y ZB perpend á OZ. Entonces

$$\textbf{sen}\ \ A = MB; \quad \textbf{cos}\ A = OM = UB; \quad \textbf{tang}\ A = ZB'$$
$$\textbf{cosec}\ A = OB''; \quad \textbf{sec}\ A = OB'; \quad \textbf{cot}\ \ A = UB''$$

El seno, coseno, etc., expresados así, se llaman seno, coseno, etc., **naturales** para el **radio 1.**

$$\cos A = \text{sen } (90° - A) = \text{seno (del complemento de A)}$$
$$\text{cot } A = \text{tang } (90° - A) = \text{tang (del complemento de A)}$$
$$\text{cosec } A = \text{sec } (90° - A) = \text{sec (del complemento de A)}.$$

Véase § 13.

**8. Signos positivos y negativos.** Fig. 4. Supóngase el círculo dividido en 4 cuadrantes, I, II, III y IV empezando de la derecha del diámetro horizontal hacia arriba y á la izquierda en sentido contrario á las agujas de un reloj. Entonces las líneas verticales y horizontales, medidas del diám horizontal hacia arriba y del vertical hacia la derecha, respectivamente son positivas; por el contrario, las líneas medidas de dichos diámetros hacia abajo ó hacia la izquierda son negativas; pero el radio en cualquier cuadrante que esté, siempre que se mida del centro hacia fuera, es positivo.

Así :

Seno (y cosecante) *positivos* en los cuadrantes superiores;
Coseno (y secante) — en el cuadrante de la derecha; ó bien:
Tang (y cotang) — en el I y III cuadrante;

| **En los cuadrantes** | **I** | **II** | **III** | **IV** |
|---|---|---|---|---|
| Incluyendo ángulos de | 0° á 90° | 90° á 180° | 180° á 270° | 270° á 360° |
| Seno y cosecante | + | + | — | — |
| Tang y cotang | + | — | + | — |
| Secante y coseno | + | — | — | + |

* Para las figuras, véase pag. 104*a*.

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**9. Valor numérico de las funciones de ciertos ángulos.**

| | 0° | 30° | 45° | 60° | 90° | 120° | 180° | 270° | 360° |
|---|---|---|---|---|---|---|---|---|---|
| Sen | 0 | $\frac{1}{2}$ | $\frac{1}{2}\sqrt{2}$ | $\frac{1}{2}\sqrt{3}$ | 1 | $\frac{1}{2}\sqrt{3}$ | 0 | $-1$ | 0 |
| Tang | 0 | $\frac{1}{3}\sqrt{3}$ | 1 | $\sqrt{3}$ | $\infty$ | $-\sqrt{3}$ | 0 | $\infty$ | 0 |
| Sec | 1 | $\frac{2}{3}\sqrt{3}$ | $\sqrt{2}$ | 2 | $\infty$ | $-2$ | $-1$ | $\infty$ | 1 |
| Cos | 1 | $\frac{1}{2}\sqrt{3}$ | $\frac{1}{2}\sqrt{2}$ | $\frac{1}{2}$ | 0 | $-\frac{1}{2}$ | $-1$ | 0 | 1 |
| $Sen^2$ | 0 | $\frac{1}{4}$ | $\frac{2}{4}$ | $\frac{3}{4}$ | 1 | $\frac{3}{4}$ | 0 | 1 | 0 |
| $Tang^2$ | 0 | $\frac{1}{3}$ | 1 | 3 | $\infty$ | 3 | 0 | $\infty$ | 0 |
| $Sec^2$ | 1 | $\frac{4}{3}$ | 2 | 4 | $\infty$ | 4 | 1 | $\infty$ | 1 |
| $Cos^2$ | 1 | $\frac{3}{4}$ | $\frac{2}{4}$ | $\frac{1}{4}$ | 0 | $\frac{1}{4}$ | 1 | 0 | 1 |

Para ecuaciones entre funciones angulares, véase 14 á 19.

**10. Otras funciones.** Véase también 3 y 6.

En fig. 3 BN=cuerda 2A= 2 sen A; y **cuerda A**=2 sen (A/2).*

**El seno-verso** MZ, fig. 3, de un áng A=1 — cos A.

Se usa mucho en los trazados de curvas de ferrocarriles.

**El coseno-verso** UU′ = 1 — sen A, fig. 3, de un ángulo A es el seno-verso del complemento BOU de A. Por sus definiciones, seno-verso A (=1 — cos A) y coseno-verso A (=1 — sen A) son siempre positivos. En los 4 cuadrantes, fig. 4, sus valores cambian así:

| Cuadrantes | I | II | III | IV |
|---|---|---|---|---|
| Incluyendo ángulos de | 0° á 90° | 90° á 180° | 180° á 270° | 270° á 360° |
| Seno-verso | 0 á 1 | 1 á 2 | 2 á 1 | 1 á 0 |
| Coseno-verso | 1 á 0 | 0 á 1 | 1 á 2 | 2 á 1 |

**La parte externa de la secante** (exsecante) BB′, fig. 3, de un ángulo A=sec A — 1. Ésta se usa también, como dijimos del seno-verso, en los trazados de curvas para ferrocarriles.

**Tablas de senos, tangs, cotangs, cosenos.**

Para los senos, etc., logarítmicos, véanse las tablas de las funciones logarítmicas trigonométricas ya cerca del final de esta obra.

**11-12. Tabla de líneas naturales.** Las tablas (pág 108-152), contienen los senos, tangentes, cotangs y cosenos naturales de los ángulos de 0° á 90°, de minuto en minuto. Para los ángulos intermedios puede encontrarse el valor de las citadas líneas, con bastante exactitud para casi todos los casos, por una simple proporción. **Para las líneas que no da la tabla,** se tiene:

Secante A=1/cos A; Seno-verso A=1 — cos A;
Cosecante A=1/sen A; Coseno-verso A=1 — sen A;
Cuerda A=2 sen (A/2); Secante externa A=sec A — 1=(1/cos A — 1).

* Para las figs., véase pág. 104*a*.

### Líneas suplementarias y complementarias.

**13.** Siendo A un ángulo cualquiera. Se tiene fig. 5.

$$\begin{aligned}
\text{Sen}\quad A &= \text{sen}\ (180^\circ + A) = \text{sen}\ (180^\circ - A) = \text{sen}\ (A - 180^\circ) \\
&= \cos\ (90^\circ + A) = \cos\ (90^\circ - A) = \cos\ (A - 90^\circ); \\
\text{Cos}\quad A &= \cos\ (180^\circ + A) = \cos\ (180^\circ - A) = \cos\ (A - 180^\circ) \\
&= \text{sen}\ (90^\circ + A) = \text{sen}\ (90^\circ - A) = \text{sen}\ (A - 90^\circ); \\
\text{Tang}\ A &= \text{tang}\ (180^\circ + A) = \text{tang}\ (180^\circ - A) = \text{tang}\ (A - 180^\circ) \\
&= \cot\ (90^\circ + A) = \cot\ (90^\circ - A) = \cot\ (A - 90^\circ).
\end{aligned}$$

### Propiedades de las líneas trigonométricas, figs. 6, 7, 8.

**14. Para cualquier ángulo A** se tiene :

$$\text{Tang}\ A = \frac{\text{sen}\ A}{\cos A}; \qquad \text{sen}^2 A + \cos^2 A = 1;$$

$$1 + \text{tang}^2 A = \sec^2 A; \qquad 1 + \cot^2 A = \text{cosec}^2 A;$$

$$\text{cot}\ A = \frac{1}{\text{tang}\ A} = \frac{\cos A}{\text{sen}\ A}; \quad \text{sec}\ A = \frac{1}{\cos A}; \quad \text{cosec}\ A = \frac{1}{\text{sen}\ A} = \frac{\sec A}{\text{tang}\ A}.$$

De la fig. 4 se deduce sen $(-A) = -\text{sen}\ A$; tang $(-A) = -\text{tang}\ A$; y cos $(-A) = \cos A$.

**15. Para dos ángulos cualesquiera A y B** podría demostrarse que :

$$\text{sen}\ (A+B) = \text{sen}\ A.\cos B + \cos A.\text{sen}\ B;$$
$$\text{y... cos}\ (A+B) = \cos A.\cos B - \text{sen}\ A.\text{sen}\ B;$$

de donde dividiendo miembro á miembro las dos ecuaciones y después dividiendo el numerador y denominador del segundo miembro por cos A cos B, se obtendrá, haciendo las debidas sustituciones :

$$\text{tang}\ (A + B) = \frac{\text{sen}\ (A + B)}{\cos\ (A + B)} = \frac{\text{tang}\ A + \text{tang}\ B}{1 - \text{tang}\ A\ \text{tang}\ B}.$$

**16.** Haciendo A=B en las fórmulas últimas para (A+B) se tiene :

$$\text{sen}\ 2A = 2\ \text{sen}\ A.\cos A$$
$$\cos 2A = \cos^2 A - \text{sen}^2 A = 1 - 2\ \text{sen}^2 A = 2\cos^2 A - 1;$$

$$\text{tang}\ 2A = \frac{2\ \text{tang}\ A}{1 - \text{tang}^2 A}.$$

y poniendo en lugar de B, —B, se tiene; véase final 14.

$$\text{sen}\ (A - B) = \text{sen}\ A\cos B - \cos A\ \text{sen}\ B;$$
$$\cos (A - B) = \cos A\cos B + \text{sen}\ A\ \text{sen}\ B;$$

$$\text{tang}\ (A - B) = \frac{\text{tang}\ A - \text{tang}\ B}{1 + \text{tang}\ A.\ \text{tang}\ B}.$$

**17.** Como $\cos 2A = 1 - 2\,\text{sen}^2 A$; se tiene poniendo $A/2$ por A; $\cos A = 1 - 2\,\text{sen}^2 A/2$; de donde $\text{sen}\ \frac{A}{2} = \sqrt{\frac{1 - \cos A}{2}}$ y como $\cos 2A = 2\cos^2 A - 1$; se tiene $2\cos^2 \frac{A}{2} = \cos A + 1$; de donde $\cos \frac{A}{2} = \sqrt{\frac{1 + \cos A}{2}}$ y como $\text{tang}\ \frac{A}{2} = \frac{\text{sen}\ A/2}{\cos A/2} = \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{1 - \cos A}{\text{sen}\ A} = \frac{\text{sen}\ A}{1 + \cos A}$.

**18. Fórmulas que transforman una suma ó diferencia en producto:**

$$\text{sen}\ A + \text{sen}\ B = 2\ \text{sen}\ \frac{A + B}{2}.\ \cos \frac{A - B}{2}.$$

$$\text{sen}\ A - \text{sen}\ B = 2\ \text{sen}\ \frac{A - B}{2}.\ \cos \frac{A + B}{2}.$$

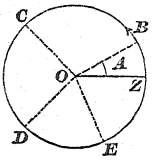


Fig. 1.

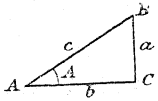


Fig. 2.

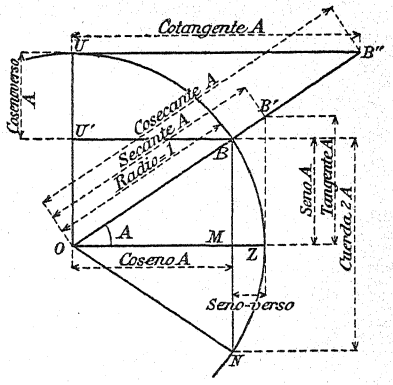


Fig. 3.

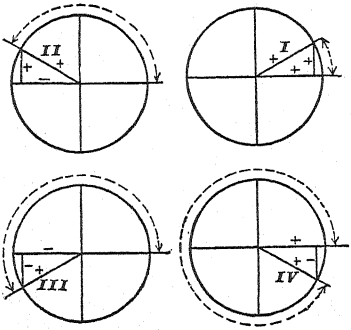


Fig. 4.

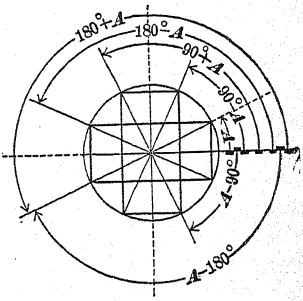


Fig. 5.

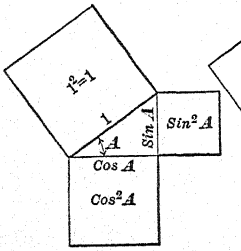


Fig. 6

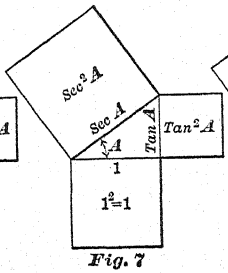


Fig. 7

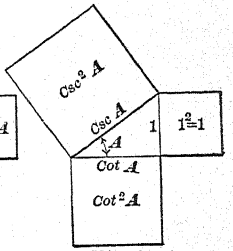


Fig. 8

(*Obs. del T.* — *Sin* en inglés equivale á *seno* en español.)

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} . \cos \frac{A-B}{2} .$$

$$\cos A - \cos B = -2 \operatorname{sen} \frac{A+B}{2} . \operatorname{sen} \frac{A+B}{2} .$$

Véase también (20) para las fórmulas usadas en la resolución de los triángulos.

**19.** Cualquier línea trigonométrica en función de otra, figs. 6, 7 y 8.

En función del seno :

$$\cos^2 A = 1 - \operatorname{sen}^2 A ; \cos A = \sqrt{1 - \operatorname{sen}^2 A} :$$

$$\operatorname{tang} A = \frac{\operatorname{sen} A}{\cos A} = \frac{\operatorname{sen} A}{\sqrt{1 - \operatorname{sen}^2 A}}$$

$$\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1 - \operatorname{sen}^2 A}}$$

En función de la tangente :

$$\sec^2 A = 1 + \operatorname{tang}^2 A ; \sec = \sqrt{1 + \operatorname{tang}^2 A};$$

$$\cos A = \frac{1}{\sec A} = \frac{1}{\sqrt{1 + \operatorname{tang}^2 A}}$$

$$\operatorname{sen} A = \frac{\operatorname{tang} A}{\sec A} = \frac{\operatorname{tang} A}{\sqrt{1 + \operatorname{tang}^2 A}}$$

**20. Fórmulas usadas en la resolución de los triángulos.** Fig. 9.

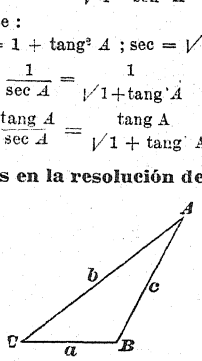


**Fig. 9.**

$$\frac{\operatorname{sen} A}{\operatorname{sen} B} = \frac{a}{b}; \; a^2 = b^2 + c^2 - 2bc \cos A;$$

$$\operatorname{tang} \frac{A-B}{2} = \frac{a-b}{a+b} . \operatorname{tang} \frac{A+B}{2} ;$$

$$\operatorname{tang} \frac{A}{2} = \frac{r}{s-a} ; \text{ aquí } s = \frac{a+b+c}{2} \text{ y } r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

$= $ al radio del círculo inscripto.

$$\text{Área del triángulo} = \frac{ab \operatorname{sen} C}{2} = rs = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \frac{\operatorname{sen} A \operatorname{sen} C b^2}{2 \operatorname{sen} B} .$$

(Véanse también los capítulos Triángulos y Trigonometría Plana.)

## TABLAS DE SENOS, TANGENTES, COTANGENTES Y COSENOS (Para los logaritmos de estas líneas véanse las tablas respectivas pag. 151 etc.)

| 0° | | | | | | 0° | | | | | | 0° | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·0000000 | ·000000 | Infinite. | 1·000000 | 60 | 21 | ·0061086 | ·006108 | 163·7001 | ·9999813 | 39 | 41 | ·0119261 | ·011927 | 83·84350 | ·9999289 | 19 |
| 1 | ·0002909 | ·000291 | 3437·746 | 1·000000 | 59 | 22 | ·0063995 | ·006399 | 156·2590 | ·9999795 | 38 | 42 | ·0122170 | ·012217 | 81·84704 | ·9999254 | 18 |
| 2 | ·0005818 | ·000582 | 1718·873 | ·9999998 | 58 | 23 | ·0066904 | ·006690 | 149·4650 | ·9999776 | 37 | 43 | ·0125079 | ·012508 | 79·94343 | ·9999218 | 17 |
| 3 | ·0008727 | ·000872 | 1145·915 | ·9999996 | 57 | 24 | ·0069813 | ·006981 | 143·2371 | ·9999756 | 36 | 44 | ·0127987 | ·012799 | 78·12634 | ·9999181 | 16 |
| 4 | ·0011636 | ·001163 | 859·4363 | ·9999993 | 56 | 25 | ·0072721 | ·007272 | 137·5075 | ·9999736 | 35 | 45 | ·0130896 | ·013090 | 76·39000 | ·9999143 | 15 |
| 5 | ·0014544 | ·001454 | 687·5488 | ·9999989 | 55 | 26 | ·0075630 | ·007563 | 132·2185 | ·9999714 | 34 | 46 | ·0133805 | ·013381 | 74·72916 | ·9999105 | 14 |
| 6 | ·0017453 | ·001745 | 572·9572 | ·9999985 | 54 | 27 | ·0078539 | ·007854 | 127·3213 | ·9999692 | 33 | 47 | ·0136713 | ·013672 | 73·13899 | ·9999065 | 13 |
| 7 | ·0020362 | ·002036 | 491·1060 | ·9999979 | 53 | 28 | ·0081448 | ·008145 | 122·7739 | ·9999668 | 32 | 48 | ·0139622 | ·013963 | 71·61507 | ·9999025 | 12 |
| 8 | ·0023271 | ·002327 | 429·7175 | ·9999973 | 52 | 29 | ·0084357 | ·008436 | 118·5401 | ·9999644 | 31 | 49 | ·0142530 | ·014254 | 70·15334 | ·9998984 | 11 |
| 9 | ·0026180 | ·002618 | 381·9709 | ·9999966 | 51 | 30 | ·0087265 | ·008726 | 114·5886 | ·9999619 | 30 | 50 | ·0145439 | ·014545 | 68·75008 | ·9998942 | 10 |
| 10 | ·0029089 | ·002908 | 343·7737 | ·9999958 | 50 | 31 | ·0090174 | ·009017 | 110·8920 | ·9999593 | 29 | 51 | ·0148348 | ·014836 | 67·40185 | ·9998900 | 9 |
| 11 | ·0031998 | ·003199 | 312·5213 | ·9999949 | 49 | 32 | ·0093083 | ·009308 | 107·4264 | ·9999567 | 28 | 52 | ·0151256 | ·015127 | 66·10547 | ·9998856 | 8 |
| 12 | ·0034907 | ·003490 | 286·4777 | ·9999939 | 48 | 33 | ·0095992 | ·009599 | 104·1709 | ·9999539 | 27 | 53 | ·0154165 | ·015418 | 64·85800 | ·9998812 | 7 |
| 13 | ·0037815 | ·003781 | 264·4408 | ·9999928 | 47 | 34 | ·0098900 | ·009890 | 101·1069 | ·9999511 | 26 | 54 | ·0157073 | ·015709 | 63·65674 | ·9998766 | 6 |
| 14 | ·0040724 | ·004072 | 245.5519 | ·9999917 | 46 | 35 | ·0101809 | ·010181 | 98·21794 | ·9999482 | 25 | 55 | ·0159982 | ·016000 | 62·49915 | ·9998720 | 5 |
| 15 | ·0043633 | ·004363 | 229·1816 | ·9999905 | 45 | 36 | ·0104718 | ·010472 | 95·48947 | ·9999452 | 24 | 56 | ·0162890 | ·016291 | 61·38290 | ·9998673 | 4 |
| 16 | ·0046542 | ·004654 | 214·8576 | ·9999892 | 44 | 37 | ·0107627 | ·010763 | 92·90848 | ·9999421 | 23 | 57 | ·0165799 | ·016582 | 60·30582 | ·9998625 | 3 |
| 17 | ·0049451 | ·004945 | 202·2187 | ·9999878 | 43 | 38 | ·0110535 | ·011054 | 90·46333 | ·9999389 | 22 | 58 | ·0168707 | ·016873 | 59·26587 | ·9998577 | 2 |
| 18 | ·0052360 | ·005236 | 190·9841 | ·9999863 | 42 | 39 | ·0113444 | ·011345 | 88·14357 | ·9999357 | 21 | 59 | ·0171616 | ·017164 | 58·26117 | ·9998527 | 1 |
| 19 | ·0055268 | ·005526 | 180·9322 | ·9999847 | 41 | 40 | ·0116353 | ·011636 | 85·93979 | ·9999323 | 20 | 60 | ·0174524 | ·017455 | 57·28996 | ·9998477 | 0 |
| 20 | ·0058177 | ·005817 | 171·8854 | ·9999831 | 40 | | | | | | | | | | | | |
| | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
| | | | | | 89° | | | | | | 89° | | | | | | 89° |

| 1° | | | | | | | 1° | | | | | | 1° | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | |
| 0 | ·0174524 | ·017455 | 57·28996 | ·9998477 | 60 | 21 | ·0235598 | ·023566 | 42·43346 | ·9997224 | 39 | 41 | ·0293756 | ·029388 | 34·02730 | ·9995684 | 19 | |
| 1 | ·0177432 | ·017746 | 56·35059 | ·9998426 | 59 | 22 | ·0238506 | ·023857 | 41·91579 | ·9997156 | 38 | 42 | ·0296662 | ·029679 | 33·69350 | ·9995599 | 18 | |
| 2 | ·0180341 | ·018037 | 55·44151 | ·9998374 | 58 | 23 | ·0241414 | ·024148 | 41·41058 | ·9997086 | 37 | 43 | ·0299570 | ·029970 | 33·36619 | ·9995512 | 17 | |
| 3 | ·0183249 | ·018328 | 54·56130 | ·9998321 | 57 | 24 | ·0244322 | ·024439 | 40·91741 | ·9997015 | 36 | 44 | ·0302478 | ·030261 | 33·04517 | ·9995424 | 16 | |
| 4 | ·0186158 | ·018619 | 53·70858 | ·9998267 | 56 | 25 | ·0247230 | ·024730 | 40·43583 | ·9996943 | 35 | 45 | ·0305385 | ·030552 | 32·73026 | ·9995336 | 15 | |
| 5 | ·0189066 | ·018910 | 52·88211 | ·9998213 | 55 | 26 | ·0250138 | ·025021 | 39·96546 | ·9996871 | 34 | 46 | ·0308293 | ·030843 | 32·42129 | ·9995247 | 14 | |
| 6 | ·0191974 | ·019201 | 52·08067 | ·9998157 | 54 | 27 | ·0253046 | ·025312 | 39·50589 | ·9996798 | 33 | 47 | ·0311200 | ·031135 | 32·11809 | ·9995157 | 13 | |
| 7 | ·0194883 | ·019492 | 51·30315 | ·9998101 | 53 | 28 | ·0255954 | ·025603 | 39·05677 | ·9996724 | 32 | 48 | ·0314108 | ·031426 | 31·82051 | ·9995066 | 12 | |
| 8 | ·0197791 | ·019783 | 50·54850 | ·9998044 | 52 | 29 | ·0258862 | ·025894 | 38·61773 | ·9996649 | 31 | 49 | ·0317015 | ·031717 | 31·52839 | ·9994974 | 11 | |
| 9 | ·0200699 | ·020074 | 49·81572 | ·9997986 | 51 | 30 | ·0261769 | ·026185 | 38·18845 | ·9996573 | 30 | 50 | ·0319922 | ·032008 | 31·24157 | ·9994881 | 10 | |
| 10 | ·0203608 | ·020365 | 49·10388 | ·9997927 | 50 | 31 | ·0264677 | ·026477 | 37·76861 | ·9996497 | 29 | 51 | ·0322830 | ·032299 | 30·95992 | ·9994788 | 9 | |
| 11 | ·0206516 | ·020656 | 48·41208 | ·9997867 | 49 | 32 | ·0267585 | ·026768 | 37·35789 | ·9996419 | 28 | 52 | ·0325737 | ·032591 | 30·68330 | ·9994693 | 8 | |
| 12 | ·0209424 | ·020947 | 47·73950 | ·9997807 | 48 | 33 | ·0270493 | ·027059 | 36·95600 | ·9996341 | 27 | 53 | ·0328644 | ·032882 | 30·41158 | ·9994598 | 7 | |
| 13 | ·0212332 | ·021238 | 47·08534 | ·9997745 | 47 | 34 | ·0273401 | ·027350 | 36·56265 | ·9996262 | 26 | 54 | ·0331552 | ·033173 | 30·14461 | ·9994502 | 6 | |
| 14 | ·0215241 | ·021529 | 46·44886 | ·9997683 | 46 | 35 | ·0276309 | ·027641 | 36·17759 | ·9996182 | 25 | 55 | ·0334459 | ·033464 | 29·88229 | ·9994405 | 5 | |
| 15 | ·0218149 | ·021820 | 45·82935 | ·9997620 | 45 | 36 | ·0279216 | ·027932 | 35·80055 | ·9996101 | 24 | 56 | ·0337366 | ·033755 | 29·62449 | ·9994308 | 4 | |
| 16 | ·0221057 | ·022111 | 45·22614 | ·9997556 | 44 | 37 | ·0282124 | ·028223 | 35·43128 | ·9996020 | 23 | 57 | ·0340274 | ·034047 | 29·37110 | ·9994209 | 3 | |
| 17 | ·0223965 | ·022402 | 44·63859 | ·9997492 | 43 | 38 | ·0285032 | ·028514 | 35·06954 | ·9995937 | 22 | 58 | ·0343181 | ·034338 | 29·12200 | ·9994110 | 2 | |
| 18 | ·0226873 | ·022693 | 44·06611 | ·9997426 | 42 | 39 | ·0287940 | ·028805 | 34·71511 | ·9995854 | 21 | 59 | ·0346088 | ·034629 | 28·87708 | ·9994009 | 1 | |
| 19 | ·0229781 | ·022984 | 43·50812 | ·9997360 | 41 | 40 | ·0290847 | ·029097 | 34·36777 | ·9995770 | 20 | 60 | ·0348995 | ·034920 | 28·63625 | ·9993908 | 0 | |
| 20 | ·0232690 | ·023275 | 42·96407 | ·9997292 | 40 | | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·0348995 | ·034920 | 28·63625 | ·9993908 | 60 | 21 | ·0410037 | ·041038 | 24·36750 | ·9991590 | 39 | 41 | ·0468159 | ·046867 | 21.33685 | ·9989035 | 19 |
| 1 | ·0351902 | ·035212 | 28·39939 | ·9993806 | 59 | 22 | ·0412944 | ·041329 | 24·19571 | ·9991470 | 38 | 42 | ·0471065 | ·047158 | 21·20494 | ·9988899 | 18 |
| 2 | ·0354809 | ·035503 | 28·16642 | ·9993704 | 58 | 23 | ·0415850 | ·041621 | 24·02632 | ·9991350 | 37 | 43 | ·0473970 | ·047450 | 21·07466 | ·9988761 | 17 |
| 3 | ·0357716 | ·035794 | 27·93723 | ·9993600 | 57 | 24 | ·0418757 | ·041912 | 23·85927 | ·9991228 | 36 | 44 | ·0476876 | ·047741 | 20·94596 | ·9988623 | 16 |
| 4 | ·0360623 | ·036085 | 27·71174 | ·9993495 | 56 | 25 | ·0421663 | ·042203 | 23·69453 | ·9991106 | 35 | 45 | ·0479781 | ·048033 | 20·81882 | ·9988484 | 15 |
| 5 | ·0363530 | ·036377 | 27·48985 | ·9993390 | 55 | 26 | ·0424569 | ·042495 | 23·53205 | ·9990983 | 34 | 46 | ·0482687 | ·048325 | 20·69322 | ·9988344 | 14 |
| 6 | ·0366437 | ·036668 | 27·27148 | ·9993284 | 54 | 27 | ·0427475 | ·042786 | 23·37177 | ·9990859 | 33 | 47 | ·0485592 | ·048616 | 20·56911 | ·9988203 | 13 |
| 7 | ·0369344 | ·036959 | 27·05655 | ·9993177 | 53 | 28 | ·0430382 | ·043078 | 23·21366 | ·9990734 | 32 | 48 | ·0488498 | ·048908 | 20·44648 | ·9988061 | 12 |
| 8 | ·0372251 | ·037250 | 26·84498 | ·9993069 | 52 | 29 | ·0433288 | ·043369 | 23·05767 | ·9990609 | 31 | 49 | ·0491403 | ·049199 | 20·32530 | ·9987919 | 11 |
| 9 | ·0375158 | ·037542 | 26·63669 | ·9992960 | 51 | 30 | ·0436194 | ·043660 | 22·90376 | ·9990482 | 30 | 50 | ·0494308 | ·049491 | 20·20555 | ·9987775 | 10 |
| 10 | ·0378065 | ·037833 | 26·43160 | ·9992851 | 50 | 31 | ·0439100 | ·043952 | 22·75189 | ·9990355 | 29 | 51 | ·0497214 | ·049782 | 20·08719 | ·9987631 | 9 |
| 11 | ·0380971 | ·038124 | 26·22963 | ·9992740 | 49 | 32 | ·0442006 | ·044243 | 22·60201 | ·9990227 | 28 | 52 | ·0500119 | ·050074 | 19·97021 | ·9987486 | 8 |
| 12 | ·0383878 | ·038416 | 26·03073 | ·9992629 | 48 | 33 | ·0444912 | ·044535 | 22·45409 | ·9990098 | 27 | 53 | ·0503024 | ·050366 | 19·85459 | ·9987340 | 7 |
| 13 | ·0386785 | ·038707 | 25·83482 | ·9992517 | 47 | 34 | ·0447818 | ·044826 | 22·30809 | ·9989968 | 26 | 54 | ·0505929 | ·050657 | 19·74029 | ·9987194 | 6 |
| 14 | ·0389692 | ·038998 | 25·64183 | ·9992404 | 46 | 35 | ·0450724 | ·045118 | 22·16398 | ·9989837 | 25 | 55 | ·0508835 | ·050949 | 19·62729 | ·9987046 | 5 |
| 15 | ·0392598 | ·039290 | 25·45170 | ·9992290 | 45 | 36 | ·0453630 | ·045409 | 22·02171 | ·9989706 | 24 | 56 | ·0511740 | ·051241 | 19·51558 | ·9986898 | 4 |
| 16 | ·0395505 | ·039581 | 25·26436 | ·9992176 | 44 | 37 | ·0456536 | ·045701 | 21·88125 | ·9989573 | 23 | 57 | ·0514645 | ·051532 | 19·40513 | ·9986748 | 3 |
| 17 | ·0398411 | ·039872 | 25·07975 | ·9992060 | 43 | 38 | ·0459442 | ·045992 | 21·74256 | ·9989440 | 22 | 58 | ·0517550 | ·051824 | 19·29592 | ·9986598 | 2 |
| 18 | ·0401318 | ·040164 | 24·89782 | ·9991944 | 42 | 39 | ·0462347 | ·046284 | 21·60563 | ·9989306 | 21 | 59 | ·0520455 | ·052116 | 19·18793 | ·9986447 | 1 |
| 19 | ·0404224 | ·040455 | 24·71851 | ·9991827 | 41 | 40 | ·0465253 | ·046575 | 21·47040 | ·9989171 | 20 | 60 | ·0523360 | ·052407 | 19·08113 | ·9986295 | 0 |
| 20 | ·0407131 | ·040746 | 24·54175 | ·9991709 | 40 | | | | | | | | | | | | |
| | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | |
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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | | | Sen. | Tang. | Cotang. | Cos. | |
| 0 | ·0523360 | ·052407 | 19·08113 | ·9986295 | 60 | 21 | ·0584352 | ·058535 | 17·08372 | ·9982912 | 39 | 41 | ·0642420 | ·064375 | 15·53398 | ·9979343 | 19 |
| 1 | ·0526264 | ·052699 | 18·97552 | ·9986143 | 59 | 22 | ·0587256 | ·058827 | 16·99895 | ·9982742 | 38 | 42 | ·0645323 | ·064667 | 15·46381 | ·9979156 | 18 |
| 2 | ·0529169 | ·052991 | 18·87106 | ·9985989 | 58 | 23 | ·0590160 | ·059119 | 16·91502 | ·9982570 | 37 | 43 | ·0648226 | ·064959 | 15·39427 | ·9978968 | 17 |
| 3 | ·0532074 | ·053282 | 18·76775 | ·9985835 | 57 | 24 | ·0593064 | ·059410 | 16·83191 | ·9982398 | 36 | 44 | ·0651129 | ·065251 | 15·32535 | ·9978779 | 16 |
| 4 | ·0534979 | ·053574 | 18·66556 | ·9985680 | 56 | 25 | ·0595967 | ·059702 | 16·74961 | ·9982225 | 35 | 45 | ·0654031 | ·065543 | 15·25705 | ·9978589 | 15 |
| 5 | ·0537883 | ·053866 | 18·56447 | ·9985524 | 55 | 26 | ·0598871 | ·059994 | 16·66811 | ·9982052 | 34 | 46 | ·0656934 | ·065835 | 15·18934 | ·9978399 | 14 |
| 6 | ·0540788 | ·054158 | 18·46447 | ·9985367 | 54 | 27 | ·0601775 | ·060286 | 16·58739 | ·9981877 | 33 | 47 | ·0659836 | ·066127 | 15·12224 | ·9978207 | 13 |
| 7 | ·0543693 | ·054449 | 18·36553 | ·9985209 | 53 | 28 | ·0604678 | ·060578 | 16·50745 | ·9981701 | 32 | 48 | ·0662739 | ·066419 | 15·05572 | ·9978015 | 12 |
| 8 | ·0546597 | ·054741 | 18·26765 | ·9985050 | 52 | 29 | ·0607582 | ·060870 | 16·42827 | ·9981525 | 31 | 49 | ·0665641 | ·066712 | 14·98978 | ·9977821 | 11 |
| 9 | ·0549502 | ·055033 | 18·17080 | ·9984891 | 51 | 30 | ·0610485 | ·061162 | 16·34985 | ·9981348 | 30 | 50 | ·0668544 | ·067004 | 14·92441 | ·9977627 | 10 |
| 10 | ·0552406 | ·055325 | 18·07497 | ·9984731 | 50 | 31 | ·0613389 | ·061454 | 16·27217 | ·9981170 | 29 | 51 | ·0671446 | ·067296 | 14·85961 | ·9977433 | 9 |
| 11 | ·0555311 | ·055616 | 17·98015 | ·9984570 | 49 | 32 | ·0616292 | ·061746 | 16·19522 | ·9980991 | 28 | 52 | ·0674349 | ·067588 | 14·79537 | ·9977237 | 8 |
| 12 | ·0558215 | ·055908 | 17·88631 | ·9984408 | 48 | 33 | ·0619196 | ·062038 | 16·11899 | ·9980811 | 27 | 53 | ·0677251 | ·067880 | 14·73167 | ·9977040 | 7 |
| 13 | ·0561119 | ·056200 | 17·79344 | ·9984245 | 47 | 34 | ·0622099 | ·062330 | 16·04348 | ·9980631 | 26 | 54 | ·0680153 | ·068173 | 14·66852 | ·9976843 | 6 |
| 14 | ·0564024 | ·056492 | 17·70152 | ·9984081 | 46 | 35 | ·0625002 | ·062622 | 15·96866 | ·9980450 | 25 | 55 | ·0683055 | ·068465 | 14·60591 | ·9976645 | 5 |
| 15 | ·0566928 | ·056784 | 17·61055 | ·9983917 | 45 | 36 | ·0627905 | ·062914 | 15·89454 | ·9980267 | 24 | 56 | ·0685957 | ·068757 | 14·54383 | ·9976445 | 4 |
| 16 | ·0569832 | ·057075 | 17·52051 | ·9983751 | 44 | 37 | ·0630808 | ·063206 | 15·82110 | ·9980084 | 23 | 57 | ·0688859 | ·069049 | 14·48227 | ·9976245 | 3 |
| 17 | ·0572736 | ·057367 | 17·43138 | ·9983585 | 43 | 38 | ·0633711 | ·063498 | 15·74833 | ·9979900 | 22 | 58 | ·0691761 | ·069342 | 14·42123 | ·9976045 | 2 |
| 18 | ·0575640 | ·057659 | 17·34315 | ·9983418 | 42 | 39 | ·0636614 | ·063790 | 15·67623 | ·9979716 | 21 | 59 | ·0694663 | ·069634 | 14·36069 | ·9975843 | 1 |
| 19 | ·0578544 | ·057951 | 17·25580 | ·9983250 | 41 | 40 | ·0639517 | ·064082 | 15·60478 | ·9979530 | 20 | 60 | ·0697565 | ·069926 | 14·30066 | ·9975641 | 0 |
| 20 | ·0581448 | ·058243 | 17·16933 | ·9983082 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | | Cos. | Cotang. | Tang. | Sen. | ′ |
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| ′ | Sen. | Tang. | Coang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·0697565 | ·069926 | 14·30066 | ·9975641 | 60 | 21 | ·0758489 | ·076068 | 13·14612 | ·9971193 | 39 | 41 | ·0816486 | ·081922 | 12·20671 | ·9966612 | 19 |
| 1 | ·0700467 | ·070219 | 14·24113 | ·9975437 | 59 | 22 | ·0761390 | ·076360 | 13·09575 | ·9970972 | 38 | 42 | ·0819385 | ·082215 | 12·16323 | ·9966374 | 18 |
| 2 | ·0703368 | ·070511 | 14·18209 | ·9975233 | 58 | 23 | ·0764290 | ·076653 | 13.04576 | ·9970750 | 37 | 43 | ·0822284 | ·082507 | 12·12006 | ·9966135 | 17 |
| 3 | ·0706270 | ·070803 | 14·12353 | ·9975028 | 57 | 24 | ·0767190 | ·076945 | 12·99616 | ·9970528 | 36 | 44 | ·0825183 | ·082800 | 12·07719 | ·9965895 | 16 |
| 4 | ·0709171 | ·071096 | 14·06545 | ·9974822 | 56 | 25 | ·0770091 | ·077238 | 12·94692 | ·9970304 | 35 | 45 | ·0828082 | ·083093 | 12·03462 | ·9965655 | 15 |
| 5 | ·0712073 | ·071388 | 14·00785 | ·9974615 | 55 | 26 | ·0772991 | ·077531 | 12·89805 | ·9970080 | 34 | 46 | ·0830981 | ·083386 | 11·99234 | ·9965414 | 14 |
| 6 | ·0714974 | ·071680 | 13·95071 | ·9974408 | 54 | 27 | ·0775891 | ·077823 | 12·84955 | ·9969854 | 33 | 47 | ·0833880 | ·083679 | 11·95037 | ·9965172 | 13 |
| 7 | ·0717876 | ·071973 | 13·89404 | ·9974199 | 53 | 28 | ·0778791 | ·078116 | 12·80141 | ·9969628 | 32 | 48 | ·0836778 | ·083972 | 11·90868 | ·9964929 | 12 |
| 8 | ·0720777 | ·072265 | 13·83782 | ·9973990 | 52 | 29 | ·0781691 | ·078409 | 12·75363 | ·9969401 | 31 | 49 | ·0839677 | ·084265 | 11·86728 | ·9964685 | 11 |
| 9 | ·0723678 | ·072558 | 13·78205 | ·9973780 | 51 | 30 | ·0784591 | ·078701 | 12·70620 | ·9969173 | 30 | 50 | ·0842576 | ·084558 | 11·82616 | ·9964440 | 10 |
| 10 | ·0726580 | ·072850 | 13·72673 | ·9973569 | 50 | 31 | ·0787491 | ·078994 | 12·65912 | ·9968945 | 29 | 51 | ·0845474 | ·084851 | 11·78533 | ·9964195 | 9 |
| 11 | ·0729481 | ·073143 | 13·67185 | ·9973357 | 49 | 32 | ·0790391 | ·079287 | 12·61239 | ·9968715 | 28 | 52 | ·0848373 | ·085144 | 11·74477 | ·9963948 | 8 |
| 12 | ·0732382 | ·073435 | 13·61740 | ·9973145 | 48 | 33 | ·0793290 | ·079579 | 12·56599 | ·9968485 | 27 | 53 | ·0851271 | ·085437 | 11·70450 | ·9963701 | 7 |
| 13 | ·0735283 | ·073727 | 13·56339 | ·9972931 | 47 | 34 | ·0796190 | ·079872 | 12·51994 | ·9968254 | 26 | 54 | ·0854169 | ·085730 | 11·66449 | ·9963453 | 6 |
| 14 | ·0738184 | ·074020 | 13·50979 | ·9972717 | 46 | 35 | ·0799090 | ·080165 | 12·47422 | ·9968022 | 25 | 55 | ·0857067 | ·086023 | 11·62476 | ·9963204 | 5 |
| 15 | ·0741085 | ·074312 | 13·45662 | ·9972502 | 45 | 36 | ·0801989 | ·080458 | 12·42883 | ·9967789 | 24 | 56 | ·0859966 | ·086316 | 11·58529 | ·9962954 | 4 |
| 16 | ·0743986 | ·074605 | 13·40386 | ·9972286 | 44 | 37 | ·0804889 | ·080750 | 12·38376 | ·9967555 | 23 | 57 | ·0862864 | ·086609 | 11·54609 | ·9962704 | 3 |
| 17 | ·0746887 | ·074897 | 13·35151 | ·9972069 | 43 | 38 | ·0807788 | ·081043 | 12·33902 | ·9967321 | 22 | 58 | ·0865762 | ·086902 | 11·50715 | ·9962452 | 2 |
| 18 | ·0749787 | ·075190 | 13·29957 | ·9971851 | 42 | 39 | ·0810687 | ·081336 | 12·29460 | ·9967085 | 21 | 59 | ·0868660 | ·087195 | 11·46847 | ·9962200 | 1 |
| 19 | ·0752688 | ·075482 | 13·24803 | ·9971633 | 41 | 40 | ·0813587 | ·081629 | 12·25050 | ·9966849 | 20 | 60 | ·0871557 | ·087488 | 11·43005 | ·9961947 | 0 |
| 20 | ·0755589 | ·075775 | 13·19688 | ·9971413 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·0871557 | ·087488 | 11·43005 | ·9961947 | 60 | 21 | ·0932395 | ·093647 | 10·67834 | ·9956437 | 39 | 41 | ·0990303 | ·099519 | 10·04828 | ·9950844 | 19 |
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| 2 | ·0877353 | ·088074 | 11·35397 | ·9961438 | 58 | 23 | ·0938187 | ·094234 | 10·61184 | ·9955892 | 37 | 43 | ·0996092 | ·100107 | 9·989305 | ·9950266 | 17 |
| 3 | ·0880251 | ·088368 | 11·31630 | ·9961183 | 57 | 24 | ·0941083 | ·094527 | 10·57889 | ·9955620 | 36 | 44 | ·0998986 | ·100400 | 9·960072 | ·9949976 | 16 |
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| 5 | ·0886046 | ·088954 | 11·24171 | ·9960669 | 55 | 26 | ·0946875 | ·095114 | 10·51360 | ·9955070 | 34 | 46 | ·1004775 | ·100988 | 9·902112 | ·9949393 | 14 |
| 6 | ·0888943 | ·089247 | 11·20478 | ·9960411 | 54 | 27 | ·0949771 | ·095408 | 10·48126 | ·9954794 | 33 | 47 | ·1007669 | ·101282 | 9·873382 | ·9949101 | 13 |
| 7 | ·0891840 | ·089540 | 11·16808 | ·9960152 | 53 | 28 | ·0952666 | ·095701 | 10·44911 | ·9954517 | 32 | 48 | ·1010563 | ·101576 | 9·844816 | ·9948807 | 12 |
| 8 | ·0894738 | ·089834 | 11·13163 | ·9959892 | 52 | 29 | ·0955562 | ·095995 | 10·41715 | ·9954240 | 31 | 49 | ·1013457 | ·101870 | 9·816414 | ·9948513 | 11 |
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| 12 | ·0906326 | ·091007 | 10·98815 | ·9958844 | 48 | 33 | ·0967144 | ·097169 | 10·29125 | ·9953122 | 27 | 53 | ·1025032 | ·103046 | 9·704407 | ·9947327 | 7 |
| 13 | ·0909223 | ·091300 | 10·95285 | ·9958580 | 47 | 34 | ·0970039 | ·097463 | 10·26024 | ·9952840 | 26 | 54 | ·1027925 | ·103339 | 9·676800 | ·9947028 | 6 |
| 14 | ·0912119 | ·091593 | 10·91777 | ·9958315 | 46 | 35 | ·0972934 | ·097757 | 10·22942 | ·9952557 | 25 | 55 | ·1030819 | ·103634 | 9·649347 | ·9946729 | 5 |
| 15 | ·0915016 | ·091887 | 10·88292 | ·9958049 | 45 | 36 | ·0975829 | ·098050 | 10·19878 | ·9952274 | 24 | 56 | ·1033712 | ·103928 | 9·622048 | ·9946428 | 4 |
| 16 | ·0917913 | ·092180 | 10·84828 | ·9957783 | 44 | 37 | ·0978724 | ·098344 | 10·16833 | ·9951990 | 23 | 57 | ·1036605 | ·104222 | 9·594902 | ·9946127 | 3 |
| 17 | ·0920809 | ·092473 | 10·81387 | ·9957515 | 43 | 38 | ·0981619 | ·098638 | 10·13805 | ·9951705 | 22 | 58 | ·1039499 | ·104516 | 9·567906 | ·9945825 | 2 |
| 18 | ·0923706 | ·092767 | 10·77967 | ·9957247 | 42 | 39 | ·0984514 | ·098932 | 10·10795 | ·9951419 | 21 | 59 | ·1042392 | ·104810 | 9·541061 | ·9945523 | 1 |
| 19 | ·0926602 | ·093060 | 10·74568 | ·9956978 | 41 | 40 | ·0987408 | ·099225 | 10·07803 | ·9951132 | 20 | 60 | ·1045285 | ·105104 | 9·514364 | ·9945219 | 0 |
| 20 | ·0929499 | ·093354 | 10·71191 | ·9956708 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | Cos. | Cotang. | Tang. | Sen. | |
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III

| 6° | | | | | | 6° | | | | | | 6° | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·1045285 | ·105104 | 9·514364 | ·9945219 | 60 | 21 | ·1106017 | ·111284 | 8·985984 | ·9938648 | 39 | 41 | ·1163818 | ·117178 | 8·534017 | ·9932045 | 19 |
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| 3 | ·1053963 | ·105986 | 9·435153 | ·9944303 | 57 | 24 | ·1114689 | ·112168 | 8·915200 | ·9937679 | 36 | 44 | ·1172485 | ·118062 | 8·470065 | ·9931026 | 16 |
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| 5 | ·1059748 | ·106575 | 9·383066 | ·9943688 | 55 | 26 | ·1120471 | ·112757 | 8·868620 | ·9937029 | 34 | 46 | ·1178263 | ·118652 | 8·427953 | ·9930342 | 14 |
| 6 | ·1062641 | ·106869 | 9·357235 | ·9943379 | 54 | 27 | ·1123361 | ·113051 | 8·845510 | ·9936703 | 33 | 47 | ·1181151 | ·118947 | 8·407051 | ·9929999 | 13 |
| 7 | ·1065533 | ·107163 | 9·331545 | ·9943070 | 53 | 28 | ·1126252 | ·113346 | 8·822518 | ·9936375 | 32 | 48 | ·1184040 | ·119242 | 8·386251 | ·9929655 | 12 |
| 8 | ·1068425 | ·107457 | 9·305993 | ·9942760 | 52 | 29 | ·1129142 | ·113641 | 8·799644 | ·9936047 | 31 | 49 | ·1186928 | ·119537 | 8·365553 | ·9929310 | 11 |
| 9 | ·1071316 | ·107751 | 9·280580 | ·9942448 | 51 | 30 | ·1132032 | ·113935 | 8·776887 | ·9935719 | 30 | 50 | ·1189816 | ·119832 | 8·344955 | ·9928965 | 10 |
| 10 | ·1074210 | ·108046 | 9·255303 | ·9942136 | 50 | 31 | ·1134922 | ·114230 | 8·754246 | ·9935389 | 29 | 51 | ·1192704 | ·120127 | 8·324457 | ·9928618 | 9 |
| 11 | ·1077102 | ·108340 | 9·230162 | ·9941823 | 49 | 32 | ·1137812 | ·114525 | 8·731719 | ·9935058 | 28 | 52 | ·1195593 | ·120423 | 8·304058 | ·9928271 | 8 |
| 12 | ·1079994 | ·108634 | 9·205156 | ·9941510 | 48 | 33 | ·1140702 | ·114819 | 8·709307 | ·9934727 | 27 | 53 | ·1198481 | ·120718 | 8·283757 | ·9927922 | 7 |
| 13 | ·1082885 | ·108929 | 9·180283 | ·9941195 | 47 | 34 | ·1143592 | ·115114 | 8·687008 | ·9934395 | 26 | 54 | ·1201368 | ·121013 | 8·263554 | ·9927573 | 6 |
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| 15 | ·1088669 | ·109517 | 9·130934 | ·9940563 | 45 | 36 | ·1149372 | ·115703 | 8·642747 | ·9933728 | 24 | 56 | ·1207144 | ·121603 | 8·223438 | ·9926873 | 4 |
| 16 | ·1091560 | ·109812 | 9·106456 | ·9940246 | 44 | 37 | ·1152261 | ·115998 | 8·620783 | ·9933393 | 23 | 57 | ·1210031 | ·121898 | 8·203523 | ·9926521 | 3 |
| 17 | ·1094452 | ·110106 | 9·082107 | ·9939928 | 43 | 38 | ·1155151 | ·116293 | 8·598929 | ·9933057 | 22 | 58 | ·1212919 | ·122194 | 8·183704 | ·9926169 | 2 |
| 18 | ·1097343 | ·110401 | 9·057886 | ·9939610 | 42 | 39 | ·1158040 | ·116588 | 8·577183 | ·9932721 | 21 | 59 | ·1215806 | ·122489 | 8·163978 | ·9925816 | 1 |
| 19 | ·1100234 | ·110695 | 9·033793 | ·9939290 | 41 | 40 | ·1160929 | ·116883 | 8·555546 | ·9932384 | 20 | 60 | ·1218693 | ·122784 | 8·144346 | ·9925462 | 0 |
| 20 | ·1103126 | ·110989 | 9·009826 | ·9938969 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | | ′ | Sen. | Tang. | Cotang. | Cos. | | ′ | Sen. | Tang. | Cotang. | Cos. | | ′ |
| 0 | ·1218693 | ·122784 | 8·144346 | ·9925462 | 60 | 21 | ·1279302 | ·128990 | 7·752536 | ·9917832 | 39 | 41 | ·1336979 | ·134909 | 7·412397 | ·9910221 | 19 | 0 |
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| 3 | ·1227355 | ·123670 | 8·086004 | ·9924394 | 57 | 24 | ·1287956 | ·129877 | 7·699573 | ·9916712 | 36 | 44 | ·1345627 | ·135797 | 7·363891 | ·9909051 | 16 | 3 |
| 4 | ·1230241 | ·123965 | 8·066739 | ·9924037 | 56 | 25 | ·1290841 | ·130173 | 7·682076 | ·9916337 | 35 | 45 | ·1348509 | ·136094 | 7·347861 | ·9908659 | 15 | 4 |
| 5 | ·1233128 | ·124261 | 8·047564 | ·9923679 | 55 | 26 | ·1293725 | ·130469 | 7·664658 | ·9915961 | 34 | 46 | ·1351392 | ·136390 | 7·331898 | ·9908266 | 14 | 5 |
| 6 | ·1236015 | ·124556 | 8·028479 | ·9923319 | 54 | 27 | ·1296609 | ·130764 | 7·647317 | ·9915584 | 33 | 47 | ·1354274 | ·136686 | 7·316004 | ·9907873 | 13 | 6 |
| 7 | ·1238901 | ·124852 | 8·009483 | ·9922959 | 53 | 28 | ·1299494 | ·131060 | 7·630053 | ·9915206 | 32 | 48 | ·1357156 | ·136983 | 7·300178 | ·9907478 | 12 | 7 |
| 8 | ·1241788 | ·125147 | 7·990575 | ·9922599 | 52 | 29 | ·1302378 | ·131356 | 7·612865 | ·9914828 | 31 | 49 | ·1360038 | ·137279 | 7·284418 | ·9907083 | 11 | 8 |
| 9 | ·1244674 | ·125442 | 7·971755 | ·9922237 | 51 | 30 | ·1305262 | ·131652 | 7·595754 | ·9914449 | 30 | 50 | ·1362919 | ·137575 | 7·268725 | ·9906687 | 10 | 9 |
| 10 | ·1247560 | ·125738 | 7·953022 | ·9921874 | 50 | 31 | ·1308146 | ·131948 | 7·578717 | ·9914069 | 29 | 51 | ·1365801 | ·137872 | 7·253098 | ·9906290 | 9 | 10 |
| 11 | ·1 50446 | ·126033 | 7·934375 | ·9921511 | 49 | 32 | ·1311030 | ·132244 | 7·561756 | ·9913689 | 28 | 52 | ·1368683 | ·138168 | 7·237537 | ·9905893 | 8 | 11 |
| 12 | · 3332 | ·126329 | 7·915815 | ·9921147 | 48 | 33 | ·1313913 | ·132540 | 7·544869 | ·9913306 | 27 | 53 | ·1371564 | ·138465 | 7·222042 | ·9905494 | 7 | 12 |
| 13 | · 218 | ·126624 | 7·897339 | ·9920782 | 47 | 34 | ·1316797 | ·132836 | 7·528057 | ·9912923 | 26 | 54 | ·1374445 | ·138761 | 7·206611 | ·9905095 | 6 | 13 |
| 14 | · | ·126920 | 7·878948 | ·9920416 | 46 | 35 | ·1319681 | ·133132 | 7·511317 | ·9912540 | 25 | 55 | ·1377327 | ·139058 | 7·191245 | ·9904694 | 5 | 14 |
| 15 | · | ·127216 | 7·860642 | ·9920049 | 45 | 36 | ·1322564 | ·133128 | 7·494651 | ·9912155 | 24 | 56 | ·1380208 | ·139354 | 7·175943 | ·9904293 | 4 | 15 |
| 16 | ·04875 | ·127511 | 7·842419 | ·9919682 | 44 | 37 | ·1325447 | ·133724 | 7·478057 | ·9911770 | 23 | 57 | ·1383089 | ·139651 | 7·160705 | ·9903891 | 3 | 16 |
| 17 | ·1267761 | ·127807 | 7·824279 | ·9919314 | 43 | 38 | ·1328330 | ·134020 | 7·461535 | ·9911384 | 22 | 58 | ·1385970 | ·139947 | 7·145530 | ·9903489 | 2 | 17 |
| 18 | ·1270646 | ·128103 | 7·806221 | ·9918944 | 42 | 39 | ·1331213 | ·134316 | 7·445085 | ·9910997 | 21 | 59 | ·1388850 | ·140244 | 7·130419 | ·9903085 | 1 | 18 |
| 19 | ·1273531 | ·128398 | 7·788245 | ·9918574 | 41 | 40 | ·1334096 | ·134612 | 7·428706 | ·9910610 | 20 | 60 | ·1391731 | ·140540 | 7·115369 | ·9902681 | 0 | 19 |
| 20 | ·1276416 | ·128694 | 7·770350 | ·9918204 | 40 | | | | | | | | | | | | | 20 |
| ′ | Cos. | Cotang. | Tang. | Sen. | | ′ | Cos. | Cotang. | Tang. | Sen. | | ′ | Cos. | Cotang. | Tang. | Sen. | | ′ |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·1391731 | ·140540 | 7·115369 | ·9902684 | 60 | 21 | ·1452197 | ·1467[illegible]5 | 6·813122 | ·9893994 | 39 | 41 | ·1509733 | ·152723 | 6·547767 | ·9885378 | 19 |
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| 3 | ·1400372 | ·141430 | 7·070593 | ·9901462 | 57 | 24 | ·1460830 | ·147667 | 6·771986 | ·9892723 | 36 | 44 | ·1518359 | ·153617 | 6·509698 | ·9884057 | 16 |
| 4 | ·1403252 | ·141727 | 7·055790 | ·9901055 | 56 | 25 | ·1463708 | ·147964 | 6·758382 | ·9892298 | 35 | 45 | ·1521234 | ·153914 | 6·497104 | ·9883615 | 15 |
| 5 | ·1406132 | ·142024 | 7·041048 | ·9900646 | 55 | 26 | ·1466585 | ·148261 | 6·744831 | ·9891872 | 34 | 46 | ·1524109 | ·154212 | 6·484558 | ·9883172 | 14 |
| 6 | ·1409012 | ·142321 | 7·026366 | ·9900237 | 54 | 27 | ·1469463 | ·148559 | 6·731334 | ·9891445 | 33 | 47 | ·1526984 | ·154510 | 6·472059 | ·9882728 | 13 |
| 7 | ·1411892 | ·142617 | 7·011744 | ·9899826 | 53 | 28 | ·1472340 | ·148856 | 6·717889 | ·9891017 | 32 | 48 | ·1529858 | ·154808 | 6·459607 | ·9882284 | 12 |
| 8 | ·1414772 | ·142914 | 6·997180 | ·9899415 | 52 | 29 | ·1475217 | ·149153 | 6·704496 | ·9890588 | 31 | 49 | ·1532733 | ·155106 | 6·447201 | ·9881838 | 11 |
| 9 | ·1417651 | ·143211 | 6·982678 | ·9899003 | 51 | 30 | ·1478094 | ·149451 | 6·691156 | ·9890159 | 30 | 50 | ·1535607 | ·155404 | 6·434842 | ·9881392 | 10 |
| 10 | ·1420531 | ·143508 | 6·968233 | ·9898590 | 50 | 31 | ·1480971 | ·149748 | 6·677867 | ·9889728 | 29 | 51 | ·1538482 | ·155701 | 6·422530 | ·9880945 | 9 |
| 11 | ·1423410 | ·143805 | 6·953847 | ·9898177 | 49 | 32 | ·1483848 | ·150045 | 6·664630 | ·9889297 | 28 | 52 | ·1541356 | ·155999 | 6·410263 | ·9880497 | 8 |
| 12 | ·1426289 | ·144102 | 6·939519 | ·9897762 | 48 | 33 | ·1486724 | ·150343 | 6·651444 | ·9888865 | 27 | 53 | ·1544230 | ·156297 | 6·398042 | ·9880048 | 7 |
| 13 | ·1429168 | ·144399 | 6·925248 | ·9897347 | 47 | 34 | ·1489601 | ·150640 | 6·638310 | ·9888432 | 26 | 54 | ·1547104 | ·156595 | 6·385866 | ·9879599 | 6 |
| 14 | ·1432047 | ·144696 | 6·911035 | ·9896931 | 46 | 35 | ·1492477 | ·150938 | 6·625225 | ·9887998 | 25 | 55 | ·1549978 | ·156893 | 6·373735 | ·9879148 | 5 |
| 15 | ·1434926 | ·144993 | 6·896879 | ·9896514 | 45 | 36 | ·1495353 | ·151235 | 6·612191 | ·9887564 | 24 | 56 | ·1552851 | ·157191 | 6·361650 | ·9878697 | 4 |
| 16 | ·1437805 | ·145290 | 6·882780 | ·9896096 | 44 | 37 | ·1498230 | ·151533 | 6·599208 | ·9887128 | 23 | 57 | ·1555725 | ·157490 | 6·349609 | ·9878245 | 3 |
| 17 | ·1440684 | ·145587 | 6·868737 | ·9895677 | 43 | 38 | ·1501106 | ·151830 | 6·586273 | ·9886692 | 22 | 58 | ·1558598 | ·157788 | 6·337612 | ·9877792 | 2 |
| 18 | ·1443562 | ·145884 | 6·854750 | ·9895258 | 42 | 39 | ·1503981 | ·152128 | 6·573389 | ·9886255 | 21 | 59 | ·1561472 | ·158086 | 6·325660 | ·9877338 | 1 |
| 19 | ·1446440 | ·146181 | 6·840819 | ·9894838 | 41 | 40 | ·1506857 | ·152426 | 6·560553 | ·9885817 | 20 | 60 | ·1564345 | ·158384 | 6·313751 | ·9876883 | 0 |
| 20 | ·1449319 | ·146478 | 6·826943 | ·9894416 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. |
| 0 | ·1564345 | ·158384 | 6·313751 | ·9876883 | 60 | ·1624650 | ·164652 | 6·073397 | ·9867143 | 60 | ·1682026 | ·170633 | 5·860505 | ·9857524 |
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| 3 | ·1572963 | ·159279 | 6·278286 | ·9875514 | 57 | ·1633260 | ·165548 | 6·040510 | ·9865722 | 57 | ·1690628 | ·171532 | 5·829817 | ·9856053 |
| 4 | ·1575836 | ·159577 | 6·266551 | ·9875057 | 56 | ·1636129 | ·165847 | 6·029624 | ·9865246 | 56 | ·1693495 | ·171831 | 5·819657 | ·9855561 |
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| 6 | ·1581581 | ·160174 | 6·243208 | ·9874138 | 54 | ·1641868 | ·166445 | 6·007967 | ·9864293 | 54 | ·1699228 | ·172430 | 5·799440 | ·9854574 |
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| 8 | ·1587325 | ·160770 | 6·220034 | ·9873216 | 52 | ·1647607 | ·167043 | 5·986461 | ·9863336 | 52 | ·1704961 | ·173029 | 5·779359 | ·9853583 |
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| 11 | ·1595940 | ·161666 | 6·185586 | ·9871827 | 49 | ·1656214 | ·167940 | 5·954481 | ·9861894 | 49 | ·1713560 | ·173928 | 5·749488 | ·9852092 |
| 12 | ·1598812 | ·161964 | 6·174186 | ·9871363 | 48 | ·1659082 | ·168239 | 5·943895 | ·9861412 | 48 | ·1716425 | ·174228 | 5·739598 | ·9851593 |
| 13 | ·1601683 | ·162263 | 6·162827 | ·9870897 | 47 | ·1661951 | ·168539 | 5·933345 | ·9860929 | 47 | ·1719291 | ·174527 | 5·729741 | ·9851093 |
| 14 | ·1604555 | ·162561 | 6·151508 | ·9870431 | 46 | ·1664819 | ·168838 | 5·922832 | ·9860445 | 46 | ·1722156 | ·174827 | 5·719917 | ·9850593 |
| 15 | ·1607426 | ·162860 | 6·140230 | ·9869964 | 45 | ·1667687 | ·169137 | 5·912355 | ·9859960 | 45 | ·1725022 | ·175127 | 5·710125 | ·9850091 |
| 16 | ·1610297 | ·163159 | 6·128992 | ·9869496 | 44 | ·1670556 | ·169436 | 5·901913 | ·9859475 | 44 | ·1727887 | ·175427 | 5·700366 | ·9849589 |
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| 18 | ·1616038 | ·163756 | 6·106636 | ·9868557 | 42 | ·1676291 | ·170035 | 5·881138 | ·9858501 | 42 | ·1733617 | ·176027 | 5·680944 | ·9848582 |
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| 20 | ·1621779 | ·164353 | 6·084438 | ·9867615 | 40 | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | Cos. | Cotang. | Tang. | Sen. |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·1736482 | ·176327 | 5·671281 | ·9848078 | 60 | ·1796607 | ·182632 | 5·475478 | ·9837286 | 39 | ·1853808 | ·188650 | 5·300801 | ·9826668 | 19 |
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| 4 | ·1747939 | ·177527 | 5·632947 | ·9846050 | 56 | ·1808052 | ·183835 | 5·439659 | ·9835189 | 35 | ·1865240 | ·189855 | 5·267151 | ·9824504 | 15 |
| 5 | ·1750803 | ·177827 | 5·623442 | ·9845542 | 55 | ·1810913 | ·184135 | 5·430775 | ·9834663 | 34 | ·1868098 | ·190157 | 5·258803 | ·9823961 | 14 |
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| 9 | ·1762258 | ·179027 | 5·585730 | ·9843498 | 51 | ·1822355 | ·185339 | 5·395517 | ·9832549 | 30 | ·1879529 | ·191363 | 5·225664 | ·9821781 | 10 |
| 10 | ·1765121 | ·179327 | 5·576378 | ·9842985 | 50 | ·1825215 | ·185639 | 5·386771 | ·9832019 | 29 | ·1882385 | ·191664 | 5·217442 | ·9821234 | 9 |
| 11 | ·1767984 | ·179628 | 5·567057 | ·9842471 | 49 | ·1828075 | ·185940 | 5·378053 | ·9831487 | 28 | ·1885241 | ·191966 | 5·209245 | ·9820686 | 8 |
| 12 | ·1770847 | ·179928 | 5·557766 | ·9841956 | 48 | ·1830935 | ·186241 | 5·369363 | ·9830955 | 27 | ·1888098 | ·192268 | 5·201073 | ·9820137 | 7 |
| 13 | ·1773710 | ·180228 | 5·548505 | ·9841441 | 47 | ·1833795 | ·186542 | 5·360699 | ·9830422 | 26 | ·1890954 | ·192569 | 5·192926 | ·9819587 | 6 |
| 14 | ·1776573 | ·180529 | 5·539274 | ·9840924 | 46 | ·1836654 | ·186843 | 5·352062 | ·9829888 | 25 | ·1893811 | ·192871 | 5·184803 | ·9819037 | 5 |
| 15 | ·1779435 | ·180829 | 5·530072 | ·9840407 | 45 | ·1839514 | ·187144 | 5·343452 | ·9829353 | 24 | ·1896667 | ·193173 | 5·176705 | ·9818485 | 4 |
| 16 | ·1782298 | ·181129 | 5·520900 | ·9839889 | 44 | ·1842373 | ·187446 | 5·334869 | ·9828818 | 23 | ·1899523 | ·193474 | 5·168631 | ·9817933 | 3 |
| 17 | ·1785160 | ·181430 | 5·511757 | ·9839370 | 43 | ·1845232 | ·187747 | 5·326313 | ·9828282 | 22 | ·1902379 | ·193776 | 5·160581 | ·9817380 | 2 |
| 18 | ·1788022 | ·181730 | 5·502644 | ·9838850 | 42 | ·1848091 | ·188048 | 5·317783 | ·9827744 | 21 | ·1905234 | ·194078 | 5·152555 | ·9816826 | 1 |
| 19 | ·1790884 | ·182031 | 5·493560 | ·9838330 | 41 | ·1850949 | ·188349 | 5·309279 | ·9827206 | 20 | ·1908090 | ·194380 | 5·144554 | ·9816272 | 0 |
| 20 | ·1793746 | ·182331 | 5·484505 | ·9837808 | 40 | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | | Cos. | Cotang. | Tang. | Sen. | | Cos. | Cotang. | Tang. | Sen. | ′ |
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| 11° | | | | | | 11° | | | | | | 11° | | | | | |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·1908090 | ·194380 | 5·144554 | ·9816272 | 60 | 21 | ·1968018 | ·200727 | 4·981881 | ·9804433 | 39 | 41 | ·2025024 | ·206786 | 4·835901 | ·9792818 | 19 |
| 1 | ·1910945 | ·194682 | 5·136576 | ·9815716 | 59 | 22 | ·1970870 | ·201030 | 4·974321 | ·9803860 | 38 | 42 | ·2027873 | ·207090 | 4·828817 | ·9792228 | 18 |
| 2 | ·1913801 | ·194984 | 5·128622 | ·9815160 | 58 | 23 | ·1973722 | ·201332 | 4·966903 | ·9803286 | 37 | 43 | ·2030721 | ·207393 | 4·821753 | ·9791638 | 17 |
| 3 | ·1916656 | ·195286 | 5·120692 | ·9814603 | 57 | 24 | ·1976573 | ·201635 | 4·959447 | ·9802712 | 36 | 44 | ·2033569 | ·207696 | 4·814709 | ·9791047 | 16 |
| 4 | ·1919510 | ·195588 | 5·112785 | ·9814045 | 56 | 25 | ·1979425 | ·201938 | 4·952012 | ·9802136 | 35 | 45 | ·2036418 | ·208000 | 4·807685 | ·9790455 | 15 |
| 5 | ·1922365 | ·195890 | 5·104902 | ·9813486 | 55 | 26 | ·1982276 | ·202240 | 4·944599 | ·9801560 | 34 | 46 | ·2039265 | ·208303 | 4·800680 | ·9789862 | 14 |
| 6 | ·1925220 | ·196192 | 5·097042 | ·9812927 | 54 | 27 | ·1985127 | ·202543 | 4·937206 | ·9800983 | 33 | 47 | ·2042113 | ·208607 | 4·793695 | ·9789268 | 13 |
| 7 | ·1928074 | ·196494 | 5·089206 | ·9812366 | 53 | 28 | ·1987978 | ·202846 | 4·929835 | ·9800405 | 32 | 48 | ·2044961 | ·208910 | 4·786730 | ·9788674 | 12 |
| 8 | ·1930928 | ·196796 | 5·081392 | ·9811805 | 52 | 29 | ·1990829 | ·203149 | 4·922485 | ·9799827 | 31 | 49 | ·2047808 | ·209214 | 4·779783 | ·9788079 | 11 |
| 9 | ·1933782 | ·197098 | 5·073602 | ·9811243 | 51 | 30 | ·1993679 | ·203452 | 4·915157 | ·9799247 | 30 | 50 | ·2050655 | ·209518 | 4·772856 | ·9787483 | 10 |
| 10 | ·1936636 | ·197400 | 5·065835 | ·9810680 | 50 | 31 | ·1996530 | ·203755 | 4·907849 | ·9798667 | 29 | 51 | ·2053502 | ·209821 | 4·765949 | ·9786886 | 9 |
| 11 | ·1939490 | ·197703 | 5·058090 | ·9810116 | 49 | 32 | ·1999380 | ·204058 | 4·900562 | ·9798086 | 28 | 52 | ·2056349 | ·210125 | 4·759060 | ·9786288 | 8 |
| 12 | ·1942344 | ·198005 | 5·050369 | ·9809552 | 48 | 33 | ·2002230 | ·204361 | 4·893295 | ·9797504 | 27 | 53 | ·2059195 | ·210429 | 4·752190 | ·9785689 | 7 |
| 13 | ·1945197 | ·198307 | 5·042670 | ·9808986 | 47 | 34 | ·2005080 | ·204664 | 4·886049 | ·9796921 | 26 | 54 | ·2062042 | ·210733 | 4·745340 | ·9785090 | 6 |
| 14 | ·1948050 | ·198610 | 5·034993 | ·9808420 | 46 | 35 | ·2007930 | ·204967 | 4·878824 | ·9796337 | 25 | 55 | ·2064888 | ·211036 | 4·738508 | ·9784490 | 5 |
| 15 | ·1950903 | ·198912 | 5·027339 | ·9807853 | 45 | 36 | ·2010779 | ·205270 | 4·871620 | ·9795752 | 24 | 56 | ·2067734 | ·211340 | 4·731695 | ·9783889 | 4 |
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| 17 | ·1956609 | ·199517 | 5·012098 | ·9806716 | 43 | 38 | ·2016478 | ·205876 | 4·857271 | ·9794581 | 22 | 58 | ·2073426 | ·211948 | 4·718125 | ·9782684 | 2 |
| 18 | ·1959461 | ·199819 | 5·004511 | ·9806147 | 42 | 39 | ·2019327 | ·206180 | 4·850128 | ·9793994 | 21 | 59 | ·2076272 | ·212252 | 4·711368 | ·9782080 | 1 |
| 19 | ·1962314 | ·200122 | 4·996945 | ·9805576 | 41 | 40 | ·2022176 | ·206483 | 4·843004 | ·9793406 | 20 | 60 | ·2079117 | ·212556 | 4·704630 | ·9781476 | 0 |
| 20 | ·1965166 | ·200424 | 4·989402 | ·9805005 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ ′ | | Cos. | Cotang. | Tang. | Sen. | ′ ′ | | Cos. | Cotang. | Tang. | Sen. | ′ |
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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·2079117 | ·212556 | 4·704630 | ·9781476 | 60 | 21 | ·2138829 | ·218949 | 4·567261 | ·9768593 | 39 | 41 | ·2195624 | ·225054 | 4·443376 | ·9755985 | 19 |
| 1 | ·2081962 | ·212860 | 4·697910 | ·9780871 | 59 | 22 | ·2141671 | ·219254 | 4·560911 | ·9767970 | 38 | 42 | ·2198462 | ·225359 | 4·437350 | ·9755345 | 18 |
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| 3 | ·2087652 | ·213468 | 4·684524 | ·9779658 | 57 | 24 | ·2147353 | ·219864 | 4·548260 | ·9766723 | 36 | 44 | ·2204137 | ·225971 | 4·425343 | ·9754065 | 16 |
| 4 | ·2090497 | ·213773 | 4·677859 | ·9779050 | 56 | 25 | ·2150194 | ·220169 | 4·541960 | ·9766098 | 35 | 45 | ·2206974 | ·226276 | 4·419364 | ·9753423 | 15 |
| 5 | ·2093341 | ·214077 | 4·671212 | ·9778441 | 55 | 26 | ·2153035 | ·220474 | 4·535677 | ·9765472 | 34 | 46 | ·2209811 | ·226582 | 4·413399 | ·9752781 | 14 |
| 6 | ·2096186 | ·214381 | 4·664583 | ·9777832 | 54 | 27 | ·2155876 | ·220779 | 4·529410 | ·9764845 | 33 | 47 | ·2212648 | ·226888 | 4·407450 | ·9752138 | 13 |
| 7 | ·2099030 | ·214685 | 4·657972 | ·9777222 | 53 | 28 | ·2158716 | ·221084 | 4·523160 | ·9764217 | 32 | 48 | ·2215485 | ·227194 | 4·401516 | ·9751494 | 12 |
| 8 | ·2101874 | ·214990 | 4·651378 | ·9776611 | 52 | 29 | ·2161556 | ·221389 | 4·516926 | ·9763589 | 31 | 49 | ·2218321 | ·227500 | 4·395597 | ·9750849 | 11 |
| 9 | ·2104718 | ·215294 | 4·644803 | ·9775999 | 51 | 30 | ·2164396 | ·221694 | 4·510708 | ·9762960 | 30 | 50 | ·2221158 | ·227806 | 4·389694 | ·9750203 | 10 |
| 10 | ·2107561 | ·215598 | 4·638245 | ·9775386 | 50 | 31 | ·2167236 | ·221999 | 4·504507 | ·9762330 | 29 | 51 | ·2223994 | ·228112 | 4·383805 | ·9749556 | 9 |
| 11 | ·2110405 | ·215903 | 4·631705 | ·9774773 | 49 | 32 | ·2170076 | ·222305 | 4·498322 | ·9761699 | 28 | 52 | ·2226830 | ·228418 | 4·377931 | ·9748909 | 8 |
| 12 | ·2113248 | ·216207 | 4·625183 | ·9774159 | 48 | 33 | ·2172915 | ·222610 | 4·492153 | ·9761067 | 27 | 53 | ·2229666 | ·228724 | 4·372073 | ·9748261 | 7 |
| 13 | ·2116091 | ·216512 | 4·618678 | ·9773544 | 47 | 34 | ·2175754 | ·222915 | 4·486000 | ·9760435 | 26 | 54 | ·2232501 | ·229030 | 4·366229 | ·9747612 | 6 |
| 14 | ·2118934 | ·216816 | 4·612190 | ·9772928 | 46 | 35 | ·2178593 | ·223221 | 4·479863 | ·9759802 | 25 | 55 | ·2235337 | ·229336 | 4·360400 | ·9746962 | 5 |
| 15 | ·2121777 | ·217121 | 4·605720 | ·9772311 | 45 | 36 | ·2181432 | ·223526 | 4·473742 | ·9759168 | 24 | 56 | ·2238172 | ·229642 | 4·354586 | ·9746313 | 4 |
| 16 | ·2124619 | ·217425 | 4·599268 | ·9771693 | 44 | 37 | ·2184271 | ·223831 | 4·467637 | ·9758533 | 23 | 57 | ·2241007 | ·229949 | 4·348786 | ·9745660 | 3 |
| 17 | ·2127462 | ·217730 | 4·592832 | ·9771075 | 43 | 38 | ·2187110 | ·224137 | 4·461548 | ·9757897 | 22 | 58 | ·2243842 | ·230255 | 4·343001 | ·9745008 | 2 |
| 18 | ·2130304 | ·218035 | 4·586414 | ·9770456 | 42 | 39 | ·2189948 | ·224442 | 4·455475 | ·9757260 | 21 | 59 | ·2246676 | ·230561 | 4·337231 | ·9744355 | 1 |
| 19 | ·2133146 | ·218340 | 4·580012 | ·9769836 | 41 | 40 | ·2192786 | ·224748 | 4·449418 | ·9756623 | 20 | 60 | ·2249511 | ·230868 | 4·331475 | ·9743701 | 0 |
| 20 | ·2135988 | ·218644 | 4·573628 | ·9769215 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | | | Cos. | Cotang. | Tang. | Sen. | | | Cos. | Cotang. | Tang. | Sen. | ′ |
| | | | | 77° | | | | | | 77° | | | | | | 77° | |

| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | ·2249511 | ·230868 | 4·331475 | ·9743701 | 60 | 21 | ·2308989 | ·237311 | 4·213869 | ·9729777 | 39 | 41 | ·2365555 | ·243465 | 4·107356 | ·9716180 | 19 |
| 1 | ·2252345 | ·231174 | 4·325734 | ·9743046 | 59 | 22 | ·2311819 | ·237618 | 4·208419 | ·9729105 | 38 | 42 | ·2368381 | ·243773 | 4·102164 | ·9715491 | 18 |
| 2 | ·2255179 | ·231481 | 4·320007 | ·9742390 | 58 | 23 | ·2314649 | ·237926 | 4·202983 | ·9728432 | 37 | 43 | ·2371207 | ·244081 | 4·096985 | ·9714802 | 17 |
| 3 | ·2258013 | ·231787 | 4·314295 | ·9741734 | 57 | 24 | ·2317479 | ·238233 | 4·197560 | ·9727759 | 36 | 44 | ·2374033 | ·244390 | 4·091817 | ·9714112 | 16 |
| 4 | ·2260846 | ·232094 | 4·308597 | ·9741077 | 56 | 25 | ·2320309 | ·238541 | 4·192151 | ·9727084 | 35 | 45 | ·2376859 | ·244698 | 4·086662 | ·9713421 | 15 |
| 5 | ·2263680 | ·232400 | 4·302913 | ·9740419 | 55 | 26 | ·2323138 | ·238848 | 4·186754 | ·9726409 | 34 | 46 | ·2379684 | ·245006 | 4·081519 | ·9712729 | 14 |
| 6 | ·2266513 | ·232707 | 4·297244 | ·9739760 | 54 | 27 | ·2325967 | ·239156 | 4·181371 | ·9725733 | 33 | 47 | ·2382510 | ·245315 | 4·076389 | ·9712036 | 13 |
| 7 | ·2269346 | ·233014 | 4·291588 | ·9739100 | 53 | 28 | ·2328796 | ·239463 | 4·176001 | ·9725056 | 32 | 48 | ·2385335 | ·245623 | 4·071270 | ·9711343 | 12 |
| 8 | ·2272179 | ·233320 | 4·285947 | ·9738439 | 52 | 29 | ·2331625 | ·239771 | 4·170644 | ·9724378 | 31 | 49 | ·2388159 | ·245932 | 4·066164 | ·9710649 | 11 |
| 9 | ·2275012 | ·233627 | 4·280319 | ·9737778 | 51 | 30 | ·2334454 | ·240078 | 4·165299 | ·9723699 | 30 | 50 | ·2390984 | ·246240 | 4·061070 | ·9709953 | 10 |
| 10 | ·2277844 | ·233934 | 4·274706 | ·9737116 | 50 | 31 | ·2337282 | ·240386 | 4·159968 | ·9723020 | 29 | 51 | ·2393808 | ·246549 | 4·055987 | ·9709258 | 9 |
| 11 | ·2280677 | ·234241 | 4·269107 | ·9736453 | 49 | 32 | ·2340110 | ·240694 | 4·154650 | ·9722339 | 28 | 52 | ·2396633 | ·246857 | 4·050917 | ·9708561 | 8 |
| 12 | ·2283509 | ·234547 | 4·263521 | ·9735789 | 48 | 33 | ·2342938 | ·241001 | 4·149344 | ·9721658 | 27 | 53 | ·2399457 | ·247166 | 4·045859 | ·9707863 | 7 |
| 13 | ·2286341 | ·234854 | 4·257950 | ·9735124 | 47 | 34 | ·2345766 | ·241309 | 4·144051 | ·9720976 | 26 | 54 | ·2402280 | ·247475 | 4·040812 | ·9707165 | 6 |
| 14 | ·2289172 | ·235161 | 4·252392 | ·9734458 | 46 | 35 | ·2348594 | ·241617 | 4·138771 | ·9720294 | 25 | 55 | ·2405104 | ·247783 | 4·035777 | ·9706466 | 5 |
| 15 | ·2292004 | ·235468 | 4·246848 | ·9733792 | 45 | 36 | ·2351421 | ·241925 | 4·133504 | ·9719610 | 24 | 56 | ·2407927 | ·248092 | 4·030755 | ·9705766 | 4 |
| 16 | ·2294835 | ·235775 | 4·241317 | ·9733125 | 44 | 37 | ·2354248 | ·242233 | 4·128249 | ·9718926 | 23 | 57 | ·2410751 | ·248401 | 4·025744 | ·9705065 | 3 |
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| 18 | ·2300497 | ·236390 | 4·230297 | ·9731789 | 42 | 39 | ·2359902 | ·242849 | 4·117778 | ·9717554 | 21 | 59 | ·2416396 | ·249019 | 4·015757 | ·9703660 | 1 |
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| 20 | ·2306159 | ·237004 | 4·219331 | ·9730449 | 40 | | | | | | | | | | | | |
| | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |

| 14° | | | | | 14° | | | | | 14° | | | | |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·2419219 | ·249328 | 4.010780 | ·9702957 | 60 | 21 | ·2478445 | ·255826 | 3·908901 | ·9687998 | 39 | 41 | ·2534766 | ·262034 | 3·816295 | ·9673415 | 19 |
| 1 | ·2422041 | ·249637 | 4·005816 | ·9702253 | 59 | 22 | ·2481263 | ·256136 | 3·904171 | ·9687277 | 38 | 42 | ·2537579 | ·262345 | 3·811773 | ·9672678 | 18 |
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| 5 | ·2433329 | ·250873 | 3·986073 | ·9699428 | 55 | 26 | ·2492533 | ·257376 | 3·885357 | ·9684383 | 34 | 46 | ·2548832 | ·263589 | 3·793783 | ·9669718 | 14 |
| 6 | ·2436150 | ·251182 | 3·981166 | ·9698720 | 54 | 27 | ·2495350 | ·257686 | 3·880680 | ·9683658 | 33 | 47 | ·2551645 | ·263900 | 3·789310 | ·9668977 | 13 |
| 7 | ·2438971 | ·251491 | 3·976271 | ·9698011 | 53 | 28 | ·2498167 | ·257997 | 3·876014 | ·9682931 | 32 | 48 | ·2554458 | ·264211 | 3·784848 | ·9668234 | 12 |
| 8 | ·2441792 | ·251801 | 3·971386 | ·9697301 | 52 | 29 | ·2500984 | ·258307 | 3·871358 | ·9682204 | 31 | 49 | ·2557270 | ·264522 | 3·780395 | ·9667490 | 11 |
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| 12 | ·2453074 | ·253038 | 3·951961 | ·9694453 | 48 | 33 | ·2512248 | ·259548 | 3·852839 | ·9679288 | 27 | 53 | ·2568517 | ·265768 | 3·762680 | ·9664508 | 7 |
| 13 | ·2455894 | ·253348 | 3·947133 | ·9693740 | 47 | 34 | ·2515063 | ·259859 | 3·848235 | ·9678557 | 26 | 54 | ·2571328 | ·266079 | 3·758276 | ·9663761 | 6 |
| 14 | ·2458713 | ·253658 | 3·942315 | ·9693025 | 46 | 35 | ·2517879 | ·260169 | 3·843642 | ·9677825 | 25 | 55 | ·2574139 | ·266390 | 3·753881 | ·9663012 | 5 |
| 15 | ·2461533 | ·253967 | 3·937509 | ·9692309 | 45 | 36 | ·2520694 | ·260480 | 3·839059 | ·9677092 | 24 | 56 | ·2576950 | ·266702 | 3·749496 | ·9662263 | 4 |
| 16 | ·2464352 | ·254277 | 3·932714 | ·9691593 | 44 | 37 | ·2523508 | ·260791 | 3·834486 | ·9676358 | 23 | 57 | ·2579760 | ·267014 | 3·745120 | ·9661513 | 3 |
| 17 | ·2467171 | ·254587 | 3·927929 | ·9690875 | 43 | 38 | ·2526323 | ·261101 | 3·829923 | ·9675624 | 22 | 58 | ·2582570 | ·267325 | 3·740754 | ·9660762 | 2 |
| 18 | ·2469990 | ·254896 | 3·923156 | ·9690157 | 42 | 39 | ·2529137 | ·261412 | 3·825370 | ·9674888 | 21 | 59 | ·2585381 | ·267637 | 3·736398 | ·9660011 | 1 |
| 19 | ·2472809 | ·255206 | 3·918393 | ·9689438 | 41 | 40 | ·2531952 | ·261723 | 3·820828 | ·9674152 | 20 | 60 | ·2588190 | ·267949 | 3·732050 | ·9659258 | 0 |
| 20 | ·2475627 | ·255516 | 3·913642 | ·9688719 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·2588190 | ·267949 | 3·732050 | ·9659258 | 60 | 21 | ·2647147 | ·274507 | 3·642891 | ·9643268 | 39 | 41 | ·2703204 | ·280773 | 3·561590 | ·9627704 | 19 |
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| 2 | ·2593810 | ·268572 | 3·723384 | ·9657751 | 58 | 23 | ·2652757 | ·275133 | 3·634606 | ·9641726 | 37 | 43 | ·2708805 | ·281401 | 3·553644 | ·9626130 | 17 |
| 3 | ·2596619 | ·268884 | 3·719065 | ·9656996 | 57 | 24 | ·2655561 | ·275445 | 3·630477 | ·9640954 | 36 | 44 | ·2711605 | ·281715 | 3·549684 | ·9625342 | 16 |
| 4 | ·2599428 | ·269196 | 3·714756 | ·9656240 | 56 | 25 | ·2658366 | ·275758 | 3·626356 | ·9640181 | 35 | 45 | ·2714404 | ·282029 | 3·545732 | ·9624552 | 15 |
| 5 | 2602237 | ·269508 | 3·710455 | ·9655484 | 55 | 26 | ·2661170 | ·276071 | 3·622244 | ·9639407 | 34 | 46 | ·2717204 | ·282343 | 3·541788 | ·9623762 | 14 |
| 6 | ·2605045 | ·269820 | 3·706164 | ·9654726 | 54 | 27 | ·2663973 | ·276385 | 3·618141 | ·9638633 | 33 | 47 | ·2720003 | ·282657 | 3·537852 | ·9622972 | 13 |
| 7 | ·2607853 | ·270132 | 3·701883 | ·9653968 | 53 | 28 | ·2666777 | ·276698 | 3·614046 | ·9637858 | 32 | 48 | ·2722802 | ·282971 | 3·533925 | ·9622180 | 12 |
| 8 | ·2610662 | ·270444 | 3·697610 | ·9653209 | 52 | 29 | ·2669581 | ·277011 | 3·609960 | ·9637081 | 31 | 49 | ·2725601 | ·283285 | 3·530005 | ·9621387 | 11 |
| 9 | ·2613469 | ·270757 | 3·693346 | ·9652449 | 51 | 30 | ·2672384 | ·277324 | 3·605883 | ·9636305 | 30 | 50 | ·2728400 | ·283599 | 3·526093 | ·9620594 | 10 |
| 10 | ·2616277 | ·271069 | 3·689092 | ·9651689 | 50 | 31 | ·2675187 | ·277637 | 3·601814 | ·9635527 | 29 | 51 | ·2731198 | ·283914 | 3·522190 | ·9619800 | 9 |
| 11 | ·2619085 | ·271381 | 3·684847 | ·9650927 | 49 | 32 | ·2677989 | ·277951 | 3·597754 | ·9634748 | 28 | 52 | ·2733997 | ·284228 | 3·518294 | ·9619005 | 8 |
| 12 | ·2621892 | ·271694 | 3·680611 | ·9650165 | 48 | 33 | ·2680792 | ·278264 | 3·593702 | ·9633969 | 27 | 53 | ·2736794 | ·284543 | 3·514407 | ·9618210 | 7 |
| 13 | ·2624699 | ·272006 | 3·676384 | ·9649402 | 47 | 34 | ·2683594 | ·278578 | 3·589659 | ·9633189 | 26 | 54 | ·2739592 | ·284857 | 3·510527 | ·9617413 | 6 |
| 14 | ·2627506 | ·272318 | 3·672166 | ·9648638 | 46 | 35 | ·2686396 | ·278891 | 3·585624 | ·9632408 | 25 | 55 | ·2742390 | ·285172 | 3·506655 | ·9616616 | 5 |
| 15 | ·2630312 | ·272631 | 3·667957 | ·9647873 | 45 | 36 | ·2689198 | ·279205 | 3·581597 | ·9631626 | 24 | 56 | ·2745187 | ·285486 | 3·502791 | ·9615818 | 4 |
| 16 | ·2633118 | ·272943 | 3·663757 | ·9647108 | 44 | 37 | ·2692000 | ·279518 | 3·577579 | ·9630843 | 23 | 57 | ·2747984 | ·285801 | 3·498935 | ·9615019 | 3 |
| 17 | ·2635925 | ·273256 | 3·659566 | ·9646341 | 43 | 38 | ·2694801 | ·279832 | 3·573569 | ·9630060 | 22 | 58 | ·2750781 | ·286115 | 3·495087 | ·9614219 | 2 |
| 18 | ·2638730 | ·273569 | 3·655384 | ·9645574 | 42 | 39 | ·2697602 | ·280145 | 3·569568 | ·9629275 | 21 | 59 | ·2753577 | ·286430 | 3·491247 | ·9613418 | 1 |
| 19 | ·2641536 | ·275881 | 3·651211 | ·9644806 | 41 | 40 | ·2700403 | ·280459 | 3·565574 | ·9628490 | 20 | 60 | ·2756374 | ·286745 | 3·487414 | ·9612617 | 0 |
| 20 | ·2644342 | ·274194 | 3·647046 | ·9644037 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
| | 74° | | | | | | 74° | | | | | | 74° | | | | |

| 16° | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | ·2756374 | ·286745 | 3·487414 | ·9612617 | 60 | 21 | ·2815042 | ·293368 | 3·408688 | ·9595600 | 39 | 41 | ·2870819 | ·299697 | 3·336699 | ·9579060 | 19 |
| 1 | ·2759170 | ·287060 | 3·483589 | ·9611815 | 59 | 22 | ·2817833 | ·293683 | 3·405021 | ·9594781 | 38 | 42 | ·2873605 | ·300014 | 3·333173 | ·9578225 | 18 |
| 2 | ·2761965 | ·287375 | 3·479772 | ·9611012 | 58 | 23 | ·2820624 | ·293999 | 3·401361 | ·9593961 | 37 | 43 | ·2876391 | ·300331 | 3·329654 | ·9577389 | 17 |
| 3 | ·2764761 | ·287690 | 3·475963 | ·9610208 | 57 | 24 | ·2823415 | ·294316 | 3·397708 | ·9593140 | 36 | 44 | ·2879177 | ·300648 | 3·326141 | ·9576552 | 16 |
| 4 | ·2767556 | ·288005 | 3·472161 | ·9609403 | 56 | 25 | ·2826205 | ·294632 | 3·394063 | ·9592318 | 35 | 45 | ·2881963 | ·300965 | 3·322636 | ·9575714 | 15 |
| 5 | ·2770352 | ·288320 | 3·468367 | ·9608598 | 55 | 26 | ·2828995 | ·294948 | 3·390424 | ·9591496 | 34 | 46 | ·2884748 | ·301283 | 3·319137 | ·9574875 | 14 |
| 6 | ·2773147 | ·288635 | 3·464581 | ·9607792 | 54 | 27 | ·2831785 | ·295264 | 3·386793 | ·9590672 | 33 | 47 | ·2887533 | ·301600 | 3·315645 | ·9574035 | 13 |
| 7 | ·2775941 | ·288950 | 3·460802 | ·9606984 | 53 | 28 | ·2834575 | ·295580 | 3·383169 | ·9589848 | 32 | 48 | ·2890318 | ·301917 | 3·312159 | ·9573195 | 12 |
| 8 | ·2778736 | ·289265 | 3·457031 | ·9606177 | 52 | 29 | ·2837364 | ·295897 | 3·379553 | ·9589023 | 31 | 49 | ·2893103 | ·302235 | 3·308681 | ·9572354 | 11 |
| 9 | ·2781530 | ·289580 | 3·453267 | ·9605368 | 51 | 30 | ·2840153 | ·296213 | 3·375943 | ·9588197 | 30 | 50 | ·2895887 | ·302552 | 3·305209 | ·9571512 | 10 |
| 10 | ·2784324 | ·289896 | 3·449512 | ·9604558 | 50 | 31 | ·2842942 | ·296529 | 3·372340 | ·9587371 | 29 | 51 | ·2898671 | ·302870 | 3·301743 | ·9570669 | 9 |
| 11 | ·2787118 | ·290211 | 3·445763 | ·9603748 | 49 | 32 | ·2845731 | ·296846 | 3·368745 | ·9586543 | 28 | 52 | ·2901455 | ·303187 | 3·298285 | ·9569825 | 8 |
| 12 | ·2789911 | ·290526 | 3·442022 | ·9602937 | 48 | 33 | ·2848520 | ·297163 | 3·365156 | ·9585715 | 27 | 53 | ·2904239 | ·303505 | 3·294833 | ·9568981 | 7 |
| 13 | ·2792704 | ·290842 | 3·438289 | ·9602125 | 47 | 34 | ·2851308 | ·297479 | 3·361575 | ·9584886 | 26 | 54 | ·2907022 | ·303823 | 3·291387 | ·9568136 | 6 |
| 14 | ·2795497 | ·291157 | 3·434563 | ·9601312 | 46 | 35 | ·2854096 | ·297796 | 3·358000 | ·9584056 | 25 | 55 | ·2909805 | ·304141 | 3·287948 | ·9567290 | 5 |
| 15 | ·2798290 | ·291473 | 3·430844 | ·9600499 | 45 | 36 | ·2856884 | ·298112 | 3·354433 | ·9583226 | 24 | 56 | ·2912588 | ·304458 | 3·284516 | ·9566443 | 4 |
| 16 | ·2801083 | ·291789 | 3·427133 | ·9599684 | 44 | 37 | ·2859671 | ·298429 | 3·350872 | ·9582394 | 23 | 57 | ·2915371 | ·304776 | 3·281090 | ·9565595 | 3 |
| 17 | ·2803875 | ·292104 | 3·423429 | ·9598869 | 43 | 38 | ·2862458 | ·298746 | 3·347319 | ·9581562 | 22 | 58 | ·2918153 | ·305094 | 3·277671 | ·9564747 | 2 |
| 18 | ·2806667 | ·292420 | 3·419733 | ·9598053 | 42 | 39 | ·2865246 | ·299063 | 3·343772 | ·9580729 | 21 | 59 | ·2920935 | ·305412 | 3·274258 | ·9563898 | 1 |
| 19 | ·2809459 | ·292736 | 3·416044 | ·9597236 | 41 | 40 | ·2868032 | ·299380 | 3·340232 | ·9579895 | 20 | 60 | ·2923717 | ·305730 | 3·270852 | ·9563048 | 0 |
| 20 | ·2812251 | ·293052 | 3·412362 | ·9596418 | 40 | | | | | | | | | | | | |

| 73° | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | 73° |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | ·2923717 | ·305730 | 3·270852 | ·9563048 | 60 | 21 | ·2982079 | ·312422 | 3·200789 | ·9545009 | 39 | 41 | ·3037559 | ·318820 | 3·136563 | ·9527499 | 19 |
| 1 | ·2926499 | ·306048 | 3·267452 | 9562197 | 59 | 22 | ·2984856 | ·312742 | 3·197521 | ·9544141 | 38 | 42 | ·3040331 | ·319140 | 3·133414 | ·9526615 | 18 |
| 2 | ·2929280 | ·306367 | 3·264059 | ·9561345 | 58 | 23 | ·2987632 | ·313061 | 3·194259 | ·9543273 | 37 | 43 | ·3043102 | ·319461 | 3·130270 | ·9525730 | 17 |
| 3 | ·2932061 | ·306685 | 3·260672 | ·9560492 | 57 | 24 | ·2990408 | ·313381 | 3·191003 | ·9542403 | 36 | 44 | ·3045872 | ·319781 | 3·127131 | ·9524844 | 16 |
| 4 | ·2934842 | ·307003 | 3·257292 | ·9559639 | 56 | 25 | ·2993184 | ·313700 | 3·187754 | ·9541533 | 35 | 45 | ·3048643 | ·320102 | 3·123999 | ·9523958 | 15 |
| 5 | ·2937623 | ·307321 | 3·253918 | ·9558785 | 55 | 26 | ·2995959 | ·314020 | 3·184510 | ·9540662 | 34 | 46 | ·3051413 | ·320423 | 3·120872 | ·9523071 | 14 |
| 6 | ·2940403 | ·307640 | 3·250550 | ·9557930 | 54 | 27 | ·2998734 | ·314339 | 3·181272 | ·9539790 | 33 | 47 | ·3054183 | ·320744 | 3·117750 | ·9522183 | 13 |
| 7 | ·2943183 | ·307958 | 3·247189 | ·9557074 | 53 | 28 | ·3001509 | ·314659 | 3·178040 | ·9538917 | 32 | 48 | ·3056953 | ·321064 | 3·114635 | ·9521294 | 12 |
| 8 | ·2945963 | ·308277 | 3·243834 | ·9556218 | 52 | 29 | ·3004284 | ·314979 | 3·174814 | ·9538044 | 31 | 49 | ·3059723 | ·321385 | 3·111525 | ·9520404 | 11 |
| 9 | ·2948743 | ·308595 | 3·240486 | ·9555361 | 51 | 30 | ·3007058 | ·315298 | 3·171594 | ·9537170 | 30 | 50 | ·3062492 | ·321706 | 3·108421 | ·9519514 | 10 |
| 10 | ·2951522 | ·308914 | 3·237143 | ·9554502 | 50 | 31 | ·3009832 | ·315618 | 3 168380 | ·9536294 | 29 | 51 | ·3065261 | ·322027 | 3·105322 | ·9518623 | 9 |
| 11 | ·2954302 | ·309233 | 3·233807 | ·9553643 | 49 | 32 | ·3012606 | ·315938 | 3·165172 | ·9535418 | 28 | 52 | ·3068030 | ·322348 | 3·102229 | ·9517731 | 8 |
| 12 | ·2957081 | ·309551 | 3·230478 | ·9552784 | 48 | 33 | ·3015380 | ·316258 | 3·161970 | ·9534542 | 27 | 53 | ·3070798 | ·322670 | 3·099141 | ·9516838 | 7 |
| 13 | ·2959859 | ·309870 | 3·227154 | ·9551923 | 47 | 34 | ·3018153 | ·316578 | 3·158774 | ·9533664 | 26 | 54 | ·3073566 | ·322991 | 3·096059 | ·9515944 | 6 |
| 14 | ·2962638 | ·310189 | 3·223837 | ·9551062 | 46 | 35 | ·3020926 | ·316898 | 3·155584 | ·9532786 | 25 | 55 | ·3076334 | ·323312 | 3·092983 | ·9515050 | 5 |
| 15 | ·2965416 | ·310508 | 3·220526 | ·9550199 | 45 | 36 | ·3023699 | ·317218 | 3·152399 | ·9531907 | 24 | 56 | ·3079102 | ·323633 | 3·089912 | ·9514154 | 4 |
| 16 | ·2968194 | ·310827 | 3·217221 | ·9549336 | 44 | 37 | ·3026471 | ·317538 | 3·149220 | ·9531027 | 23 | 57 | ·3081869 | ·323955 | 3·086846 | ·9513258 | 3 |
| 17 | ·2970971 | ·311146 | 3·213922 | ·9548473 | 43 | 38 | ·3029244 | ·317859 | 3·146047 | ·9530146 | 22 | 58 | ·3084636 | ·324276 | 3·083786 | ·9512361 | 2 |
| 18 | ·2973749 | ·311465 | 3·210630 | ·9547608 | 42 | 39 | ·3032016 | ·318179 | 3·142880 | ·9529264 | 21 | 59 | ·3087403 | ·324598 | 3·080732 | ·9511464 | 1 |
| 19 | ·2976526 | ·311784 | 3·207344 | ·9546743 | 41 | 40 | ·3034788 | ·318499 | 3·139719 | ·9528382 | 20 | 60 | ·3090170 | ·324919 | 3·077683 | ·9510565 | 0 |
| 20 | ·2979303 | ·312103 | 3·204063 | ·9545876 | 40 | | | | | | | | | | | | |

| ′ | Cos. | Cotang. | Tang. | Sen. | | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
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| 18° | | | | | | 18° | | | | | | 18° | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·3090170 | ·324919 | 3·077683 | ·9510565 | 60 | 21 | ·3148209 | ·331686 | 3·014892 | ·9491511 | 39 | 41 | ·3203374 | ·338157 | 2·957205 | ·9473035 | 19 |
| 1 | ·3092936 | ·325241 | 3·074640 | ·9509666 | 59 | 22 | ·3150969 | ·332009 | 3·011960 | ·9490595 | 38 | 42 | ·3206130 | ·338481 | 2·954372 | ·9472103 | 18 |
| 2 | ·3095702 | ·325563 | 3·071602 | ·9508766 | 58 | 23 | ·3153730 | ·332332 | 3·009033 | ·9489678 | 37 | 43 | ·3208885 | ·338805 | 2·951545 | ·9471170 | 17 |
| 3 | ·3098468 | ·325884 | 3·068569 | ·9507865 | 57 | 24 | ·3156490 | ·332655 | 3·006110 | ·9488760 | 36 | 44 | ·3211640 | ·339129 | 2·948722 | ·9470236 | 16 |
| 4 | ·3101234 | ·326206 | 3·065542 | ·9506963 | 56 | 25 | ·3159250 | ·332978 | 3·003193 | ·9487842 | 35 | 45 | ·3214395 | ·339454 | 2·945905 | ·9469301 | 15 |
| 5 | ·3103999 | ·326528 | 3·062520 | ·9506061 | 55 | 26 | ·3162010 | ·333302 | 3·000282 | ·9486922 | 34 | 46 | ·3217149 | ·339778 | 2·943092 | ·9468366 | 14 |
| 6 | ·3106764 | ·326850 | 3·059503 | ·9505157 | 54 | 27 | ·3164770 | ·333625 | 2·997375 | ·9486002 | 33 | 47 | ·3219903 | ·340103 | 2·940284 | ·9467430 | 13 |
| 7 | ·3109529 | ·327172 | 3·056492 | ·9504253 | 53 | 28 | ·3167529 | ·333948 | 2·994473 | ·9485081 | 32 | 48 | ·3222657 | ·340427 | 2·937480 | ·9466493 | 12 |
| 8 | ·3112294 | ·327494 | 3·053487 | ·9503348 | 52 | 29 | ·3170288 | ·334271 | 2·991576 | ·9484159 | 31 | 49 | ·3225411 | ·340752 | 2·934682 | ·9465555 | 11 |
| 9 | ·3115058 | ·327816 | 3·050486 | ·9502443 | 51 | 30 | ·3173047 | ·334595 | 2·988685 | ·9483237 | 30 | 50 | ·3228164 | ·341077 | 2·931888 | ·9464616 | 10 |
| 10 | ·3117822 | ·328138 | 3·047491 | ·9501536 | 50 | 31 | ·3175805 | ·334918 | 2·985798 | ·9482313 | 29 | 51 | ·3230917 | ·341401 | 2·929099 | ·9463677 | 9 |
| 11 | ·3120586 | ·328461 | 3·044501 | ·9500629 | 49 | 32 | ·3178563 | ·335242 | 2·982916 | ·9481389 | 28 | 52 | ·3233670 | ·341726 | 2·926315 | ·9462736 | 8 |
| 12 | ·3123349 | ·328783 | 3·041517 | ·9499721 | 48 | 33 | ·3181321 | ·335566 | 2·980040 | ·9480464 | 27 | 53 | ·3236422 | ·342051 | 2·923535 | ·9461795 | 7 |
| 13 | ·3126112 | ·329105 | 3·038538 | ·9498812 | 47 | 34 | ·3184079 | ·335889 | 2·977168 | ·9479538 | 26 | 54 | ·3239174 | ·342376 | 2·920761 | ·9460854 | 6 |
| 14 | ·3128875 | ·329428 | 3·035564 | ·9497902 | 46 | 35 | ·3186836 | ·336213 | 2·974301 | ·9478612 | 25 | 55 | ·3241926 | ·342701 | 2·917990 | ·9459911 | 5 |
| 15 | ·3131638 | ·329750 | 3·032595 | ·9496991 | 45 | 36 | ·3189593 | ·336537 | 2·971439 | ·9477684 | 24 | 56 | ·3244678 | ·343026 | 2·915225 | ·9458968 | 4 |
| 16 | ·3134400 | ·330073 | 3·029632 | ·9496080 | 44 | 37 | ·3192350 | ·336861 | 2·968583 | ·9476756 | 23 | 57 | ·3247429 | ·343351 | 2·912464 | ·9458023 | 3 |
| 17 | 3137163 | ·330395 | 3·026673 | ·9495168 | 43 | 38 | ·3195106 | ·337185 | 2·965731 | ·9475827 | 22 | 58 | ·3250180 | ·343677 | 2·909708 | ·9457078 | 2 |
| 18 | ·3139925 | ·330718 | 3·023720 | ·9494255 | 42 | 39 | ·3197863 | ·337509 | 2·962884 | ·9474897 | 21 | 59 | ·3252931 | ·344002 | 2·906957 | ·9456132 | 1 |
| 19 | ·3142686 | ·331041 | 3·020772 | ·9493341 | 41 | 40 | ·3200619 | ·337833 | 2·960042 | ·9473966 | 20 | 60 | ·3255682 | ·344327 | 2·904210 | ·9455186 | 0 |
| 20 | ·3145448 | ·331363 | 3·017830 | ·9492426 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
| | 71° | | | | | | 71° | | | | | | 71° | | | | |

| 19° | | | | | | | 19° | | | | | | 19° | | | | | |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·3255682 | ·344327 | 2·904210 | ·9455186 | 60 | 21 | ·3313379 | ·351175 | 2·847583 | ·9435122 | 39 | 41 | ·3368214 | ·357723 | 2·795453 | ·9415686 | 19 |
| 1 | ·3258432 | ·344653 | 2·901468 | ·9454238 | 59 | 22 | ·3316123 | ·351501 | 2·844935 | ·9434157 | 38 | 42 | ·3370953 | ·358051 | 2·792891 | ·9414705 | 18 |
| 2 | ·3261182 | ·344978 | 2·898731 | ·9453290 | 58 | 23 | ·3318867 | ·351828 | 2·842292 | ·9433192 | 37 | 43 | ·3373691 | ·358380 | 2·790333 | ·9413724 | 17 |
| 3 | ·3263932 | ·345304 | 2·895998 | ·9452341 | 57 | 24 | ·3321611 | ·352155 | 2·839653 | ·9432227 | 36 | 44 | ·3376429 | ·358708 | 2·787780 | ·9412743 | 16 |
| 4 | ·3266681 | ·345629 | 2·893270 | ·9451391 | 56 | 25 | ·3324355 | ·352482 | 2·837019 | ·9431260 | 35 | 45 | ·3379167 | ·359036 | 2·785230 | ·9411760 | 15 |
| 5 | ·3269430 | ·345955 | 2·890546 | ·9450441 | 55 | 26 | ·3327098 | ·352809 | 2·834389 | ·9430293 | 34 | 46 | ·3381905 | ·359365 | 2·782685 | ·9410777 | 14 |
| 6 | ·3272179 | ·346281 | 2·887827 | ·9449489 | 54 | 27 | ·3329841 | ·353136 | 2·831763 | ·9429324 | 33 | 47 | ·3384642 | ·359693 | 2·780144 | ·9409793 | 13 |
| 7 | ·3274928 | ·346606 | 2·885113 | ·9448537 | 53 | 28 | ·3332584 | ·353464 | 2·829142 | ·9428355 | 32 | 48 | ·3387379 | ·360022 | 2·777606 | ·9408808 | 12 |
| 8 | ·3277676 | ·346932 | 2·882403 | ·9447584 | 52 | 29 | ·3335326 | ·353791 | 2·826525 | ·9427386 | 31 | 49 | ·3390116 | ·360350 | 2·775073 | ·9407822 | 11 |
| 9 | ·3280424 | ·347258 | 2·879697 | ·9446630 | 51 | 30 | ·3338069 | ·354118 | 2·823912 | ·9426415 | 30 | 50 | ·3392852 | ·360679 | 2·772544 | ·9406835 | 10 |
| 10 | ·3283172 | ·347584 | 2·876997 | ·9445675 | 50 | 31 | ·3340810 | ·354446 | 2·821304 | ·9425444 | 29 | 51 | ·3395589 | ·361008 | 2·770019 | ·9405848 | 9 |
| 11 | ·3285919 | ·347910 | 2·874300 | ·9444720 | 49 | 32 | ·3343552 | ·354773 | 2·818700 | ·9424471 | 28 | 52 | ·3398325 | ·361337 | 2·767499 | ·9404860 | 8 |
| 12 | ·3288666 | ·348236 | 2·871608 | ·9443764 | 48 | 33 | ·3346293 | ·355101 | 2·816100 | ·9423498 | 27 | 53 | ·3401060 | ·361666 | 2·764982 | ·9403871 | 7 |
| 13 | ·3291413 | ·348563 | 2·868921 | ·9442807 | 47 | 34 | ·3349034 | ·355428 | 2·813504 | ·9422525 | 26 | 54 | ·3403796 | ·361994 | 2·762469 | ·9402881 | 6 |
| 14 | ·3294160 | ·348889 | 2·866238 | ·9441849 | 46 | 35 | ·3351775 | ·355756 | 2·810913 | ·9421550 | 25 | 55 | ·3406531 | ·362324 | 2·759960 | ·9401891 | 5 |
| 15 | ·3296906 | ·349215 | 2·863560 | ·9440890 | 45 | 36 | ·3354516 | ·356084 | 2·808326 | ·9420575 | 24 | 56 | ·3409265 | ·362653 | 2·757456 | ·9400899 | 4 |
| 16 | ·3299653 | ·349542 | 2·860886 | ·9439931 | 44 | 37 | ·3357256 | ·356411 | 2·805743 | ·9419598 | 23 | 57 | ·3412000 | ·362982 | 2·754955 | ·9399907 | 3 |
| 17 | ·3302398 | ·349868 | 2·858216 | ·9438971 | 43 | 38 | ·3359996 | ·356739 | 2·803164 | ·9418621 | 22 | 58 | ·3414734 | ·363311 | 2·752458 | ·9398914 | 2 |
| 18 | ·3305144 | ·350195 | 2·855551 | ·9438010 | 42 | 39 | ·3362735 | ·357067 | 2·800590 | ·9417644 | 21 | 59 | ·3417468 | ·363640 | 2·749966 | ·9397921 | 1 |
| 19 | ·3307889 | ·350521 | 2·852891 | ·9437048 | 41 | 40 | ·3365475 | ·357395 | 2·798019 | ·9416665 | 20 | 60 | ·3420201 | ·363970 | 2·747477 | ·9396926 | 0 |
| 20 | ·3310634 | ·350848 | 2·850234 | ·9436085 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
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| 20° | | | | | 20° | | | | | 20° | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·3420201 | ·363970 | 2·747477 | ·9396926 | 60 | 21 | ·3477540 | ·370693 | 2·696118 | ·9375858 | 39 | 41 | ·3532027 | ·377536 | 2·648753 | ·9355468 | 19 |
| 1 | ·3422935 | ·364299 | 2·744992 | ·9395931 | 59 | 22 | ·3480267 | ·371234 | 2·693714 | ·9374846 | 38 | 42 | ·3534748 | ·377868 | 2·646423 | ·9354440 | 18 |
| 2 | ·3425668 | ·364629 | 2·742512 | ·9394935 | 58 | 23 | ·3482994 | ·371565 | 2·691314 | ·9373833 | 37 | 43 | ·3537469 | ·378201 | 2·644096 | ·9353412 | 17 |
| 3 | ·3428400 | ·364958 | 2·740035 | ·9393938 | 57 | 24 | ·3485720 | ·371896 | 2·688919 | ·9372820 | 36 | 44 | ·3540190 | ·378533 | 2·641774 | ·9352382 | 16 |
| 4 | ·3431133 | ·365288 | 2·737562 | ·9392940 | 56 | 25 | ·3488447 | ·372227 | 2·686526 | ·9371806 | 35 | 45 | ·3542910 | ·378866 | 2·639454 | ·9351352 | 15 |
| 5 | ·3433865 | ·365618 | 2·735093 | ·9391942 | 55 | 26 | ·3491173 | ·372559 | 2·684138 | ·9370790 | 34 | 46 | ·3545630 | ·379198 | 2·637139 | ·9350321 | 14 |
| 6 | ·3436597 | ·365948 | 2·732628 | ·9390943 | 54 | 27 | ·3493898 | ·372890 | 2·681753 | ·9369774 | 33 | 47 | ·3548350 | ·379531 | 2·634827 | ·9349289 | 13 |
| 7 | ·3439329 | ·366277 | 2·730167 | ·9389943 | 53 | 28 | ·3496624 | ·373221 | 2·679372 | ·9368758 | 32 | 48 | ·3551070 | ·379864 | 2·632519 | ·9348257 | 12 |
| 8 | ·3442060 | ·366607 | 2·727710 | ·9388942 | 52 | 29 | ·3499349 | ·373553 | 2·676995 | ·9367740 | 31 | 49 | ·3553789 | ·380197 | 2·630213 | ·9347223 | 11 |
| 9 | ·3444791 | ·366937 | 2·725256 | ·9387940 | 51 | 30 | ·3502074 | ·373884 | 2·674621 | ·9366722 | 30 | 50 | ·3556508 | ·380530 | 2·627912 | ·9346189 | 10 |
| 10 | ·3447521 | ·367268 | 2·722807 | ·9386938 | 50 | 31 | ·3504798 | ·374216 | 2·672251 | ·9365703 | 29 | 51 | ·3559226 | ·380 8 | 2·625614 | ·9345154 | 9 |
| 11 | ·3450252 | ·367598 | 2·720362 | ·9385934 | 49 | 32 | ·3507523 | ·374547 | 2·669885 | ·9364683 | 28 | 52 | ·3561944 | ·381196 | 2·623319 | ·9344119 | 8 |
| 12 | ·3452982 | ·367928 | 2·717920 | ·9384930 | 48 | 33 | ·3510246 | ·374879 | 2·667522 | ·9363662 | 27 | 53 | ·3564662 | ·381529 | 2·621028 | ·9343082 | 7 |
| 13 | ·3455712 | ·368258 | 2·715482 | ·9383925 | 47 | 34 | ·3512970 | ·375211 | 2·665163 | ·9362641 | 26 | 54 | ·3567380 | ·381862 | 2·618741 | ·9342045 | 6 |
| 14 | ·3458441 | ·368589 | 2·713048 | ·9382920 | 46 | 35 | ·3515693 | ·375543 | 2·662808 | ·9361618 | 25 | 55 | ·3570097 | ·382196 | 2·616457 | ·9341007 | 5 |
| 15 | ·3461171 | ·368919 | 2·710618 | ·9381913 | 45 | 36 | ·3518416 | ·375875 | 2·660456 | ·9360595 | 24 | 56 | ·3572814 | ·382529 | 2·614176 | ·9339968 | 4 |
| 16 | ·3463900 | ·369250 | 2·708192 | ·9380906 | 44 | 37 | ·3521139 | ·376207 | 2·658108 | ·9359571 | 23 | 57 | ·3575531 | ·382863 | 2·611899 | ·9338928 | 3 |
| 17 | ·3466628 | ·369580 | 2·705769 | ·9379898 | 43 | 38 | ·3523862 | ·376539 | 2·655764 | ·9358547 | 22 | 58 | ·3578248 | ·383196 | 2·609625 | ·9337888 | 2 |
| 18 | ·3469357 | ·369911 | 2·703351 | ·9378889 | 42 | 39 | ·3526584 | ·376871 | 2·653423 | ·9357521 | 21 | 59 | ·3580964 | ·383530 | 2·607355 | ·9336846 | 1 |
| 19 | ·3472085 | ·370242 | 2·700936 | ·9377880 | 41 | 40 | ·3529306 | ·377203 | 2·651086 | ·9356495 | 20 | 60 | ·3583679 | ·383864 | 2·605089 | ·9335804 | 0 |
| 20 | ·3474812 | ·370572 | 2·698525 | ·9376869 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·3583679 | ·383864 | 2·605089 | ·9335804 | 60 | 21 | ·3640641 | ·390889 | 2·558268 | ·9313739 | 39 | 41 | ·3694765 | ·397611 | 2·516018 | ·9292401 | 19 |
| 1 | ·3586395 | ·384197 | 2·602825 | ·9334761 | 59 | 22 | ·3643351 | ·391224 | 2·556075 | ·9312679 | 38 | 42 | ·3697468 | ·397946 | 2·512889 | ·9291326 | 18 |
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| 3 | ·3591825 | ·384865 | 2·598309 | ·9332673 | 57 | 24 | ·3648768 | ·391895 | 2·551699 | ·9310558 | 36 | 44 | ·3702872 | ·398622 | 2·508639 | ·9289173 | 16 |
| 4 | ·3594540 | ·385199 | 2·596056 | ·9331628 | 56 | 25 | ·3651476 | ·392231 | 2·549516 | ·9309496 | 35 | 45 | ·3705574 | ·398959 | 2·506519 | ·9288096 | 15 |
| 5 | ·3597254 | ·385533 | 2·593806 | ·9330582 | 55 | 26 | ·3654184 | ·392567 | 2·547335 | ·9308434 | 34 | 46 | ·3708276 | ·399296 | 2·504403 | ·9287017 | 14 |
| 6 | ·3599968 | ·385867 | 2·591560 | ·9329535 | 54 | 27 | ·3656891 | ·392902 | 2·545159 | ·9307370 | 33 | 47 | ·3710977 | ·399634 | 2·502289 | ·9285938 | 13 |
| 7 | ·3602682 | ·386202 | 2·589317 | ·9328488 | 53 | 28 | ·3659599 | ·393238 | 2·542985 | ·9306306 | 32 | 48 | ·3713678 | ·399971 | 2·500178 | ·9284858 | 12 |
| 8 | ·3605395 | ·386536 | 2·587078 | ·9327439 | 52 | 29 | ·3662306 | ·393574 | 2·540815 | ·9305241 | 31 | 49 | ·3716379 | ·400308 | 2·498070 | ·9283778 | 11 |
| 9 | ·3608108 | ·386870 | 2·584842 | ·9326390 | 51 | 30 | ·3665012 | ·393910 | 2·538647 | ·9304176 | 30 | 50 | ·3719079 | ·400646 | 2·495966 | ·9282696 | 10 |
| 10 | ·3610821 | ·387205 | 2·582609 | ·9325340 | 50 | 31 | ·3667719 | ·394246 | 2·536483 | ·9303109 | 29 | 51 | ·3721780 | ·400984 | 2·493864 | ·9281614 | 9 |
| 11 | ·3613534 | ·387539 | 2·580380 | ·9324290 | 49 | 32 | ·3670425 | ·394582 | 2·534323 | ·9302042 | 28 | 52 | ·3724479 | ·401321 | 2·491766 | ·9280531 | 8 |
| 12 | ·3616246 | ·387874 | 2·578153 | ·9323238 | 48 | 33 | ·3673130 | ·394918 | 2·532165 | ·9300974 | 27 | 53 | ·3727179 | ·401659 | 2·489670 | ·9279447 | 7 |
| 13 | ·3618958 | ·388209 | 2·575931 | ·9322186 | 47 | 34 | ·3675836 | ·395255 | 2·530011 | ·9299905 | 26 | 54 | ·3729878 | ·401997 | 2·487578 | ·9278363 | 6 |
| 14 | ·3621669 | ·388543 | 2·573711 | ·9321133 | 46 | 35 | ·3678541 | ·395591 | 2·527859 | ·9298835 | 25 | 55 | ·3732577 | ·402335 | 2·485488 | ·9277277 | 5 |
| 15 | ·3624380 | ·388878 | 2·571495 | ·9320079 | 45 | 36 | ·3681246 | ·395928 | 2·525711 | ·9297765 | 24 | 56 | ·3735275 | ·402673 | 2·483402 | ·9276191 | 4 |
| 16 | ·3627091 | ·389213 | 2·569283 | ·9319024 | 44 | 37 | ·3683950 | ·396264 | 2·523566 | ·9296694 | 23 | 57 | ·3737973 | ·403011 | 2·481319 | ·9275104 | 3 |
| 17 | ·3629802 | ·389548 | 2·567073 | ·9317969 | 43 | 38 | ·3686654 | ·396601 | 2·521424 | ·9295622 | 22 | 58 | ·3740671 | ·403349 | 2·479238 | ·9274016 | 2 |
| 18 | ·3632512 | ·389883 | 2·564867 | ·9316912 | 42 | 39 | ·3689358 | ·396937 | 2·519286 | ·9294549 | 21 | 59 | ·3743369 | ·403687 | 2·477161 | ·9272928 | 1 |
| 19 | ·3635222 | ·390218 | 2·562664 | ·9315855 | 41 | 40 | ·3692061 | ·397274 | 2·517150 | ·9293475 | 20 | 60 | ·3746066 | ·404026 | 2·475086 | ·9271839 | 0 |
| 20 | ·3637932 | ·390554 | 2·560464 | ·9314797 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·3746066 | ·404026 | 2·475086 | ·9271839 | 60 | 21 | ·3802634 | ·411149 | 2·432204 | ·9248782 | 39 | 41 | ·3856377 | ·417967 | 2·392531 | ·9226503 | 19 |
| 1 | ·3748763 | ·404364 | 2·473015 | ·9270748 | 59 | 22 | ·3805324 | ·411489 | 2·430193 | ·9247676 | 38 | 42 | ·3859060 | ·418309 | 2·390576 | ·9225381 | 18 |
| 2 | ·3751459 | ·404703 | 2·470947 | ·9269658 | 58 | 23 | ·3808014 | ·411830 | 2·428186 | ·9246568 | 37 | 43 | ·3861744 | ·418650 | 2·388625 | ·9224258 | 17 |
| 3 | ·3754156 | ·405041 | 2·468881 | ·9268566 | 57 | 24 | ·3810704 | ·412170 | 2·426181 | ·9245460 | 36 | 44 | ·3864427 | ·418992 | 2·386675 | ·9223134 | 16 |
| 4 | ·3756852 | ·405380 | 2·466819 | ·9267474 | 56 | 25 | ·3813393 | ·412510 | 2·424180 | ·9244351 | 35 | 45 | ·3867110 | ·419334 | 2·384729 | ·9222010 | 15 |
| 5 | ·3759547 | ·405719 | 2·464759 | ·9266380 | 55 | 26 | ·3816082 | ·412851 | 2·422181 | ·9243242 | 34 | 46 | ·3869792 | ·419676 | 2·382785 | ·9220884 | 14 |
| 6 | ·3762243 | ·406057 | 2·462703 | ·9265286 | 54 | 27 | ·3818770 | ·413191 | 2·420185 | ·9242131 | 33 | 47 | ·3872474 | ·420019 | 2·380844 | ·9219758 | 13 |
| 7 | ·3764938 | ·406396 | 2·460649 | ·9264192 | 53 | 28 | ·3821459 | ·413532 | 2·418191 | ·9241020 | 32 | 48 | ·3875156 | ·420361 | 2·378906 | ·9218632 | 12 |
| 8 | ·3767632 | ·406735 | 2·458598 | ·9263096 | 52 | 29 | ·3824147 | ·413872 | 2·416201 | ·9239908 | 31 | 49 | ·3877837 | ·420703 | 2·376970 | ·9217504 | 11 |
| 9 | ·3770327 | ·407074 | 2·456551 | ·9262000 | 51 | 30 | ·3826834 | ·414213 | 2·414213 | ·9238795 | 30 | 50 | ·3880518 | ·421046 | 2·375037 | ·9216375 | 10 |
| 10 | ·3773021 | ·407413 | 2·454506 | ·9260902 | 50 | 31 | ·3829522 | ·414554 | 2·412228 | ·9237682 | 29 | 51 | ·3883199 | ·421388 | 2·373106 | ·9215246 | 9 |
| 11 | ·3775714 | ·407753 | 2·452464 | ·9259805 | 49 | 32 | ·3832209 | ·414895 | 2·410246 | ·9236567 | 28 | 52 | ·3885880 | ·421731 | 2·371179 | ·9214116 | 8 |
| 12 | ·3778408 | ·408092 | 2·450425 | ·9258706 | 48 | 33 | ·3834895 | ·415236 | 2·408267 | ·9235452 | 27 | 53 | ·3888560 | ·422073 | 2·369254 | ·9212986 | 7 |
| 13 | ·3781101 | ·408431 | 2·448389 | ·9257606 | 47 | 34 | ·3837582 | ·415577 | 2·406290 | ·9234336 | 26 | 54 | ·3891240 | ·422416 | 2·367331 | ·9211854 | 6 |
| 14 | ·3783794 | ·408771 | 2·446355 | ·9256506 | 46 | 35 | ·3840268 | ·415918 | 2·404316 | ·9233220 | 25 | 55 | ·3893919 | ·422759 | 2·365411 | ·9210722 | 5 |
| 15 | ·3786486 | ·409110 | 2·444325 | ·9255405 | 45 | 36 | ·3842953 | ·416259 | 2·402345 | ·9232102 | 24 | 56 | ·3896598 | ·423102 | 2·363494 | ·9209589 | 4 |
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| 18 | ·3794562 | ·410129 | 2·438251 | ·9252097 | 42 | 39 | ·3851008 | ·417284 | 2·396449 | ·9228745 | 21 | 59 | ·3904633 | ·424131 | 2·357759 | ·9206185 | 1 |
| 19 | ·3797253 | ·410469 | 2·436233 | ·9250993 | 41 | 40 | ·3853693 | ·417625 | 2·394488 | ·9227624 | 20 | 60 | ·3907311 | ·424474 | 2·355852 | ·9205049 | 0 |
| 20 | ·3799944 | ·410809 | 2·434217 | ·9249888 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
| | | | | 67° | | | | | | 67° | | | | | | 67° | |

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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | | ′ | Sen. | Tang. | Cotang. | Cos. | | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·3907311 | ·424474 | 2·355852 | ·9205049 | 60 | 21 | ·3963468 | ·431703 | 2·316407 | ·9181009 | 39 | 41 | ·4016814 | ·438622 | 2·279865 | ·9157795 | 19 |
| 1 | ·3909989 | ·424818 | 2·353948 | ·9203912 | 59 | 22 | ·3966139 | ·432048 | 2·314557 | ·9179855 | 38 | 42 | ·4019478 | ·438969 | 2·278063 | ·9156626 | 18 |
| 2 | ·3912666 | ·425161 | 2·352046 | ·9202774 | 58 | 23 | ·3968809 | ·432393 | 2·312709 | ·9178701 | 37 | 43 | ·4022141 | ·439316 | 2·276264 | ·9155456 | 17 |
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| 4 | ·3918019 | ·425848 | 2·348251 | ·9200496 | 56 | 25 | ·3974148 | ·433084 | 2·309020 | ·9176391 | 35 | 45 | ·4027467 | ·440010 | 2·272672 | ·9153115 | 15 |
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| 7 | ·3926047 | ·426880 | 2·342578 | ·9197073 | 53 | 28 | ·3982155 | ·434120 | 2·303506 | ·9172919 | 32 | 48 | ·4035453 | ·441052 | 2·267303 | ·9149597 | 12 |
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| 9 | ·3931397 | ·427568 | 2·338809 | ·9194788 | 51 | 30 | ·3987491 | ·434812 | 2·299842 | ·9170601 | 30 | 50 | ·4040775 | ·441747 | 2·263735 | ·9147247 | 10 |
| 10 | ·3934071 | ·427912 | 2·336928 | ·9193644 | 50 | 31 | ·3990158 | ·435158 | 2·298014 | ·9169440 | 29 | 51 | ·4043436 | ·442095 | 2·261955 | ·9146072 | 9 |
| 11 | ·3936745 | ·428256 | 2·335050 | ·9192499 | 49 | 32 | ·3992825 | ·435504 | 2·296188 | ·9168279 | 28 | 52 | ·4046096 | ·442443 | 2·260177 | ·9144895 | 8 |
| 12 | ·3939419 | ·428600 | 2·333174 | ·9191353 | 48 | 33 | ·3995492 | ·435850 | 2·294365 | ·9167118 | 27 | 53 | ·4048756 | ·442791 | 2·258401 | ·9143718 | 7 |
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| 14 | ·3944766 | ·429289 | 2·329431 | ·9189060 | 46 | 35 | ·4000825 | ·436542 | 2·290725 | ·9164791 | 25 | 55 | ·4054075 | ·443487 | 2·254857 | ·9141361 | 5 |
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| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ ′ |
| 0 | ·4067366 | ·445228 | 2·246036 | ·9135455 | 60 | 21 | ·4123096 | ·452568 | 2·209611 | ·9110438 | 39 | 41 | ·4176028 | ·459596 | 2·175822 | ·9086297 | 19 |
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| 3 | ·4075337 | ·446274 | 2·240772 | ·9131902 | 57 | 24 | ·4131044 | ·453620 | 2·204487 | ·9106837 | 36 | 44 | ·4183956 | ·460653 | 2·170828 | ·9082649 | 16 |
| 4 | ·4077993 | ·446623 | 2·239021 | ·9130716 | 56 | 25 | ·4133693 | ·453970 | 2·202784 | ·9105635 | 35 | 45 | ·4186597 | ·461006 | 2·169167 | ·9081432 | 15 |
| 5 | ·4080649 | ·446972 | 2·237273 | ·9129529 | 55 | 26 | ·4136342 | ·454321 | 2·201083 | ·9104432 | 34 | 46 | ·4189239 | ·461359 | 2·167509 | ·9080214 | 14 |
| 6 | ·4083305 | ·447321 | 2·235528 | ·9128342 | 54 | 27 | ·4138990 | ·454672 | 2·199384 | ·9103228 | 33 | 47 | ·4191880 | ·461711 | 2·165852 | ·9078995 | 13 |
| 7 | ·4085960 | ·447670 | 2·233784 | ·9127154 | 53 | 28 | ·4141638 | ·455023 | 2·197687 | ·9102024 | 32 | 48 | ·4194521 | ·462064 | 2·164198 | ·9077775 | 12 |
| 8 | ·4088615 | ·448020 | 2·232043 | ·9125965 | 52 | 29 | ·4144285 | ·455375 | 2·195992 | ·9100819 | 31 | 49 | ·4197161 | ·462417 | 2·162546 | ·9076554 | 11 |
| 9 | ·4091269 | ·448369 | 2·230301 | ·9124775 | 51 | 30 | ·4146932 | ·455726 | 2·194299 | ·9099613 | 30 | 50 | ·4199801 | ·462771 | 2·160895 | ·9075333 | 10 |
| 10 | ·4093923 | ·448718 | 2·228567 | ·9123584 | 50 | 31 | ·4149579 | ·456077 | 2·192609 | ·9098406 | 29 | 51 | ·4202441 | ·463124 | 2·159247 | ·9074111 | 9 |
| 11 | ·4096577 | ·449068 | 2·226833 | ·9122393 | 49 | 32 | ·4152226 | ·456429 | 2·190921 | ·9097199 | 28 | 52 | ·4205080 | ·463477 | 2·157601 | ·9072888 | 8 |
| 12 | ·4099230 | ·449417 | 2·225100 | ·9121201 | 48 | 33 | ·4154872 | ·456780 | 2·189234 | ·9095990 | 27 | 53 | ·4207719 | ·463831 | 2·155957 | ·9071665 | 7 |
| 13 | ·4101883 | ·449767 | 2·223370 | ·9120008 | 47 | 34 | ·4157517 | ·457132 | 2·187551 | ·9094781 | 26 | 54 | ·4210358 | ·464184 | 2·154315 | ·9070440 | 6 |
| 14 | ·4104536 | ·450117 | 2·221643 | ·9118815 | 46 | 35 | ·4160163 | ·457483 | 2·185869 | ·9093572 | 25 | 55 | ·4212996 | ·464538 | 2·152675 | ·9069215 | 5 |
| 15 | ·4107189 | ·450467 | 2·219917 | ·9117620 | 45 | 36 | ·4162808 | ·457835 | 2·184189 | ·9092361 | 24 | 56 | ·4215634 | ·464891 | 2·151037 | ·9067989 | 4 |
| 16 | ·4109841 | ·450817 | 2·218194 | ·9116425 | 44 | 37 | ·4165453 | ·458187 | 2·182511 | ·9091150 | 23 | 57 | ·4218272 | ·465245 | 2·149402 | ·9066762 | 3 |
| 17 | ·4112492 | ·451167 | 2·216473 | ·9115229 | 43 | 38 | ·4168097 | ·458539 | 2·180836 | ·9089938 | 22 | 58 | ·4220909 | ·465599 | 2·147768 | ·9065535 | 2 |
| 18 | ·4115144 | ·451517 | 2·214754 | ·9114033 | 42 | 39 | ·4170741 | ·458891 | 2·179163 | ·9088725 | 21 | 59 | ·4223546 | ·465953 | 2·146136 | ·9064307 | 1 |
| 19 | ·4117795 | ·451867 | 2·213037 | ·9112835 | 41 | 40 | ·4173385 | ·459243 | 2·177492 | ·9087511 | 20 | 60 | ·4226183 | ·466307 | 2·144506 | ·9063078 | 0 |
| 20 | ·4120445 | ·452217 | 2·211323 | ·9111637 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | | | Cos. | Cotang. | Tang. | Sen. | ′ ′ | | Cos. | Cotang. | Tang. | Sen. | |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | ·4226183 | ·466307 | 2·144506 | ·9063078 | 60 | 21 | ·4281467 | ·473765 | 2·110747 | ·9037093 | 39 | 41 | ·4333976 | ·480909 | 2·079394 | ·9012031 | 19 |
| 1 | ·4228819 | ·466661 | 2·142879 | ·9061848 | 59 | 22 | ·4284095 | ·474122 | 2·109161 | ·9035847 | 38 | 42 | ·4336591 | ·481267 | 2·077846 | ·9010770 | 18 |
| 2 | ·4231455 | ·467016 | 2·141253 | ·9060618 | 58 | 23 | ·4286723 | ·474478 | 2·107577 | ·9034600 | 37 | 43 | ·4339212 | ·481625 | 2·076300 | ·9009508 | 17 |
| 3 | ·4234090 | ·467370 | 2·139630 | ·9059386 | 57 | 24 | ·4289351 | ·474834 | 2·105995 | ·9033353 | 36 | 44 | ·4341832 | ·481984 | 2·074756 | ·9008246 | 16 |
| 4 | ·4236725 | ·467725 | 2·138008 | ·9058154 | 56 | 25 | ·4291979 | ·475191 | 2·104415 | ·9032105 | 35 | 45 | ·4344453 | ·482342 | 2·073214 | ·9006982 | 15 |
| 5 | ·4239360 | ·468079 | 2·136389 | ·9056922 | 55 | 26 | ·4294606 | ·475548 | 2·102836 | ·9030856 | 34 | 46 | ·4347072 | ·482701 | 2·071674 | ·9005718 | 14 |
| 6 | ·4241994 | ·468434 | 2·134771 | ·9055688 | 54 | 27 | ·4297233 | ·475904 | 2·101260 | ·9029606 | 33 | 47 | ·4349692 | ·483060 | 2·070135 | ·9004453 | 13 |
| 7 | ·4244628 | ·468789 | 2·133155 | ·9054454 | 53 | 28 | ·4299859 | ·476261 | 2·099686 | ·9028356 | 32 | 48 | ·4352311 | ·483418 | 2·068599 | ·9003188 | 12 |
| 8 | ·4247262 | ·469143 | 2·131542 | ·9053219 | 52 | 29 | ·4302485 | ·476618 | 2·098114 | ·9027105 | 31 | 49 | ·4354930 | ·483777 | 2·067064 | ·9001921 | 11 |
| 9 | ·4249895 | ·469498 | 2·129930 | ·9051983 | 51 | 30 | ·4305111 | ·476975 | 2·096543 | ·9025853 | 30 | 50 | ·4357548 | ·484136 | 2·065531 | ·9000654 | 10 |
| 10 | ·4252528 | ·469853 | 2·128321 | ·9050746 | 50 | 31 | ·4307736 | ·477332 | 2·094975 | ·9024600 | 29 | 51 | ·4360166 | ·484495 | 2·064000 | ·8999386 | 9 |
| 11 | ·4255161 | ·470209 | 2·126713 | ·9049509 | 49 | 32 | ·4310361 | ·477689 | 2·093408 | ·9023347 | 28 | 52 | ·4362784 | ·484855 | 2·062471 | ·8998117 | 8 |
| 12 | ·4257793 | ·470564 | 2·125108 | ·9048271 | 48 | 33 | ·4312986 | ·478047 | 2·091843 | ·9022092 | 27 | 53 | ·4365401 | ·485214 | 2·060944 | ·8996848 | 7 |
| 13 | ·4260425 | ·470919 | 2·123504 | ·9047032 | 47 | 34 | ·4315610 | ·478404 | 2·090280 | ·9020838 | 26 | 54 | ·4368018 | ·485573 | 2·059418 | ·8995578 | 6 |
| 14 | ·4263056 | ·471275 | 2·121903 | ·9045792 | 46 | 35 | ·4318234 | ·478762 | 2·088720 | ·9019582 | 25 | 55 | ·4370634 | ·485933 | 2·057895 | ·8994307 | 5 |
| 15 | ·4265687 | ·471630 | 2·120303 | ·9044551 | 45 | 36 | ·4320857 | ·479119 | 2·087161 | ·9018325 | 24 | 56 | ·4373251 | ·486293 | 2·056373 | ·8993035 | 4 |
| 16 | ·4268318 | ·471986 | 2·118705 | ·9043310 | 44 | 37 | ·4323481 | ·479477 | 2·085603 | ·9017068 | 23 | 57 | ·4375866 | ·486652 | 2·054853 | ·8991763 | 3 |
| 17 | ·4270949 | ·472342 | 2·117110 | ·9042068 | 43 | 38 | ·4326103 | ·479835 | 2·084048 | ·9015810 | 22 | 58 | ·4378482 | ·487012 | 2·053334 | ·8990489 | 2 |
| 18 | ·4273579 | ·472697 | 2·115516 | ·9040825 | 42 | 39 | ·4328726 | ·480193 | 2·082495 | ·9014551 | 21 | 59 | ·4381097 | ·487372 | 2·051818 | ·8989215 | 1 |
| 19 | ·4276208 | ·473053 | 2·113924 | ·9039582 | 41 | 40 | ·4331348 | ·480551 | 2·080943 | ·9013292 | 20 | 60 | ·4383711 | ·487732 | 2·050303 | ·8987940 | 0 |
| 20 | ·4278838 | ·473409 | 2·112334 | ·9038338 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |

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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·4383711 | ·487732 | 2·050303 | ·8987940 | 60 | 21 | ·4438534 | ·495317 | 2·018908 | ·8960994 | 39 | 41 | ·4490591 | ·502583 | 1·989720 | ·8935021 | 19 |
| 1 | ·4386326 | ·488092 | 2·048791 | ·8986665 | 59 | 22 | ·4441140 | ·495679 | 2·017433 | ·8959703 | 38 | 42 | ·4493190 | ·502947 | 1·988278 | ·8933714 | 18 |
| 2 | ·4388940 | ·488453 | 2·047280 | ·8985389 | 58 | 23 | ·4443746 | ·496041 | 2·015959 | ·8958411 | 37 | 43 | ·4495789 | ·503312 | 1·986838 | ·8932406 | 17 |
| 3 | ·4391553 | ·488813 | 2·045770 | ·8984112 | 57 | 24 | ·4446352 | ·496404 | 2·014486 | ·8957118 | 36 | 44 | ·4498387 | ·503676 | 1·985400 | ·8931098 | 16 |
| 4 | ·4394166 | ·489173 | 2·044263 | ·8982834 | 56 | 25 | ·4448957 | ·496766 | 2·013016 | ·8955824 | 35 | 45 | ·4500984 | ·504041 | 1·983963 | ·8929789 | 15 |
| 5 | ·4396779 | ·489534 | 2·042757 | ·8981555 | 55 | 26 | ·4451562 | ·497129 | 2·011547 | ·8954529 | 34 | 46 | ·4503582 | ·504406 | 1·982528 | ·8928480 | 14 |
| 6 | ·4399392 | ·489894 | 2·041254 | ·8980276 | 54 | 27 | ·4454167 | ·497492 | 2·010080 | ·8953234 | 33 | 47 | ·4506179 | ·504771 | 1·981095 | ·8927169 | 13 |
| 7 | ·4402004 | ·490255 | 2·039751 | ·8978996 | 53 | 28 | ·4456771 | ·497855 | 2·008615 | ·8951938 | 32 | 48 | ·4508775 | ·505136 | 1·979663 | ·8925858 | 12 |
| 8 | ·4404615 | ·490616 | 2·038251 | ·8977715 | 52 | 29 | ·4459375 | ·498218 | 2·007151 | ·8950641 | 31 | 49 | ·4511372 | ·505501 | 1·978233 | ·8924546 | 11 |
| 9 | ·4407227 | ·490977 | 2·036753 | ·8976433 | 51 | 30 | ·4461978 | ·498581 | 2·005689 | ·8949344 | 30 | 50 | ·4513967 | ·505866 | 1·976805 | ·8923234 | 10 |
| 10 | ·4409838 | ·491338 | 2·035256 | ·8975151 | 50 | 31 | ·4464581 | ·498944 | 2·004229 | ·8948045 | 29 | 51 | ·4516563 | ·506232 | 1·975378 | ·8921920 | 9 |
| 11 | ·4412448 | ·491699 | 2·033761 | ·8973868 | 49 | 32 | ·4467184 | ·499308 | 2·002771 | ·8946746 | 28 | 52 | ·4519158 | ·506597 | 1·973953 | ·8920606 | 8 |
| 12 | ·4415059 | ·492061 | 2·032268 | ·8972584 | 48 | 33 | ·4469786 | ·499671 | 2·001314 | ·8945446 | 27 | 53 | ·4521753 | ·506963 | 1·972529 | ·8919291 | 7 |
| 13 | ·4417668 | ·492422 | 2·030776 | ·8971299 | 47 | 34 | ·4472388 | ·500035 | 1·999859 | ·8944146 | 26 | 54 | ·4524347 | ·507329 | 1·971107 | ·8917975 | 6 |
| 14 | ·4420278 | ·492783 | 2·029287 | ·8970014 | 46 | 35 | ·4474990 | ·500398 | 1·998405 | ·8942844 | 25 | 55 | ·4526941 | ·507694 | 1·969687 | ·8916659 | 5 |
| 15 | ·4422887 | ·493145 | 2·027799 | ·8968727 | 45 | 36 | ·4477591 | ·500762 | 1·996953 | ·8941542 | 24 | 56 | ·4529535 | ·508060 | 1·968268 | ·8915342 | 4 |
| 16 | ·4425496 | ·493507 | 2·026313 | ·8967440 | 44 | 37 | ·4480192 | ·501126 | 1·995503 | ·8940240 | 23 | 57 | ·4532128 | ·508426 | 1·966851 | ·8914024 | 3 |
| 17 | ·4428104 | ·493868 | 2·024828 | ·8966153 | 43 | 38 | ·4482792 | ·501490 | 1·994055 | ·8938936 | 22 | 58 | ·4534721 | ·508792 | 1·965436 | ·8912705 | 2 |
| 18 | ·4430712 | ·494230 | 2·023346 | ·8964864 | 42 | 39 | ·4485392 | ·501854 | 1·992608 | ·8937632 | 21 | 59 | ·4537313 | ·509159 | 1·964022 | ·8911385 | 1 |
| 19 | ·4433319 | ·494592 | 2·021865 | ·8963575 | 41 | 40 | ·4487992 | ·502218 | 1·991163 | ·8936326 | 20 | 60 | ·4539905 | ·509525 | 1·962610 | ·8910065 | 0 |
| 20 | ·4435927 | ·494954 | 2·020386 | ·8962285 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·4539905 | ·509525 | 1·962610 | ·8910065 | 60 | ·4594248 | 517244 | 1·933323 | ·8882166 | 39 | ·4645845 | ·524640 | 1·906066 | ·8855288 | 19 |
| 1 | ·4542497 | ·509891 | 1·961200 | ·8908744 | 59 | ·4596832 | ·517612 | 1·931945 | ·8880830 | 38 | ·4648420 | ·525011 | 1.904719 | ·8853936 | 18 |
| 2 | ·4545088 | ·510258 | 1·959791 | ·8907423 | 58 | ·4599415 | ·517981 | 1·930569 | ·8879492 | 37 | ·4650996 | ·525382 | 1.903373 | ·8852584 | 17 |
| 3 | ·4547679 | ·510625 | 1·958382 | ·8906100 | 57 | ·4601998 | ·518350 | 1·929195 | ·8878154 | 36 | ·4653571 | ·525754 | 1·902029 | ·8851230 | 16 |
| 4 | ·4550269 | ·510991 | 1·956978 | ·8904777 | 56 | ·4604580 | ·518719 | 1·927822 | ·8876815 | 35 | ·4656145 | ·526125 | 1 900687 | ·8849876 | 15 |
| 5 | ·4552859 | ·511358 | 1·955573 | ·8903453 | 55 | ·4607162 | ·519089 | 1·926451 | ·8875475 | 34 | ·4658719 | ·526496 | 1·899346 | ·8848522 | 14 |
| 6 | ·4555449 | ·511725 | 1·954171 | ·8902128 | 54 | ·4609744 | ·519458 | 1·925081 | ·8874134 | 33 | ·4661293 | ·526868 | 1·898006 | ·8847166 | 13 |
| 7 | ·4558038 | ·512093 | 1·952770 | ·8900803 | 53 | ·4612325 | ·519827 | 1·923713 | ·8872793 | 32 | ·4663866 | ·527240 | 1·896668 | ·8845810 | 12 |
| 8 | ·4560627 | ·512460 | 1·951371 | ·8899476 | 52 | ·4614906 | ·520197 | 1·922347 | ·8871451 | 31 | ·4666439 | ·527612 | 1·895332 | ·8844453 | 11 |
| 9 | ·4563216 | ·512827 | 1·949973 | ·8898149 | 51 | ·4617486 | ·520567 | 1·920982 | ·8870108 | 30 | ·4669012 | ·527983 | 1·893997 | ·8843095 | 10 |
| 10 | ·4565804 | ·513195 | 1·948577 | ·8896822 | 50 | ·4620066 | ·520936 | 1·919618 | ·8868765 | 29 | ·4671584 | ·528356 | 1·892663 | ·8841736 | 9 |
| 11 | ·4568392 | ·513562 | 1·947182 | ·8895493 | 49 | ·4622646 | ·521306 | 1·918256 | ·8867420 | 28 | ·4674156 | ·528728 | 1·891331 | ·8840377 | 8 |
| 12 | ·4570979 | ·513930 | 1·945789 | ·8894164 | 48 | ·4625225 | ·521676 | 1·916896 | ·8866075 | 27 | ·4676727 | ·529100 | 1·890000 | ·8839017 | 7 |
| 13 | ·4573566 | ·514298 | 1·944398 | ·8892834 | 47 | ·4627804 | ·522046 | 1·915537 | ·8864730 | 26 | ·4679298 | ·529472 | 1·888671 | ·8837656 | 6 |
| 14 | ·4576153 | ·514665 | 1·943008 | ·8891503 | 46 | ·4630382 | ·522417 | 1·914179 | ·8863383 | 25 | ·4681869 | ·529845 | 1·887343 | ·8836295 | 5 |
| 15 | ·4578739 | ·515033 | 1·941620 | ·8890171 | 45 | ·4632960 | ·522787 | 1·912823 | ·8862036 | 24 | ·4684439 | ·530217 | 1·886017 | ·8834933 | 4 |
| 16 | ·4581325 | ·515401 | 1·940233 | ·8888839 | 44 | ·4635538 | ·523157 | 1·911469 | ·8860688 | 23 | ·4687009 | ·530590 | 1·884692 | ·8833569 | 3 |
| 17 | ·4583910 | ·515770 | 1·938848 | ·8887506 | 43 | ·4638115 | ·523528 | 1·910116 | ·8859339 | 22 | ·4689578 | ·530963 | 1·883369 | ·8832206 | 2 |
| 18 | ·4586496 | ·516138 | 1·937464 | ·8886172 | 42 | ·4640692 | ·523899 | 1·908764 | ·8857989 | 21 | ·4692147 | ·531336 | 1·882047 | ·8830841 | 1 |
| 19 | ·4589080 | ·516506 | 1·936082 | ·8884838 | 41 | ·4643269 | ·524269 | 1·907414 | ·8856639 | 20 | ·4694716 | ·531709 | 1·880726 | ·8829476 | 0 |
| 20 | ·4591665 | ·516875 | 1·934702 | ·8883503 | 40 | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | |
| 0 | ·4694716 | ·531709 | 1·880726 | ·8829476 | 60 | 21 | ·4748564 | ·539570 | 1·853325 | ·8800633 | 39 | 41 | ·4799683 | ·547106 | 1·827799 | ·8772859 | 19 | |
| 1 | ·4697284 | ·532082 | 1·879407 | ·8828110 | 59 | 22 | ·4751124 | ·539946 | 1·852035 | ·8799251 | 38 | 42 | ·4802235 | ·547484 | 1·826537 | ·8771462 | 18 | |
| 2 | ·4699852 | ·532455 | 1·878089 | ·8826743 | 58 | 23 | ·4753683 | ·540322 | 1·850747 | ·8797869 | 37 | 43 | ·4804786 | ·547862 | 1·825276 | ·8770064 | 17 | |
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| 8 | ·4715250 | ·534698 | 1·870214 | ·8818527 | 52 | 29 | ·4769031 | ·542579 | 1·843049 | ·8789559 | 31 | 49 | ·4820086 | ·550133 | 1·817740 | ·8761665 | 11 | |
| 9 | ·4717815 | ·535072 | 1·868906 | ·8817155 | 51 | 30 | ·4771588 | ·542955 | 1·841770 | ·8788171 | 30 | 50 | ·4822634 | ·550512 | 1·816489 | ·8760263 | 10 | |
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| 12 | ·4725508 | ·536195 | 1·864992 | ·8813035 | 48 | 33 | ·4779255 | ·544086 | 1·837944 | ·8784004 | 27 | 53 | ·4830277 | ·551650 | 1·812743 | ·8756051 | 7 | |
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| 14 | ·4730634 | ·536944 | 1·862389 | ·8810284 | 46 | 35 | ·4784364 | ·544840 | 1·835399 | ·8781222 | 25 | 55 | ·4835370 | ·552409 | 1·810252 | ·8753239 | 5 | |
| 15 | ·4733197 | ·537319 | 1·861090 | ·8808907 | 45 | 36 | ·4786919 | ·545217 | 1·834129 | ·8779830 | 24 | 56 | ·4837916 | ·552789 | 1·809008 | ·8751832 | 4 | |
| 16 | ·4735759 | ·537694 | 1·859792 | ·8807530 | 44 | 37 | ·4789472 | ·545595 | 1·832861 | ·8778437 | 23 | 57 | ·4840462 | ·553168 | 1·807766 | ·8750425 | 3 | |
| 17 | ·4738321 | ·538069 | 1·858496 | ·8806152 | 43 | 38 | ·4792026 | ·545972 | 1·831593 | ·8777043 | 22 | 58 | ·4843007 | ·553548 | 1·806525 | ·8749016 | 2 | |
| 18 | ·4740882 | ·538444 | 1·857201 | ·8804774 | 42 | 39 | ·4794579 | ·546350 | 1·830327 | ·8775649 | 21 | 59 | ·4845552 | ·553928 | 1·805286 | ·8747607 | 1 | |
| 19 | ·4743443 | ·538819 | 1·855908 | ·8803394 | 41 | 40 | ·4797131 | ·546728 | 1·829062 | ·8774254 | 20 | 60 | ·4848096 | ·554309 | 1·804047 | ·8746197 | 0 | |
| 20 | ·4746004 | ·539195 | 1·854615 | ·8802014 | 40 | | | | | | | | | | | | | |
| | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | |
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| 29° | | | | | | 29° | | | | | | 29° | | | | | |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·4848096 | ·554309 | 1·804047 | ·8746197 | 60 | 21 | ·4901433 | ·562321 | 1·778340 | ·8716419 | 39 | 41 | ·4952060 | ·570004 | 1·754372 | ·8687756 | 19 |
| 1 | ·4850640 | ·554689 | 1·802810 | ·8744786 | 59 | 22 | ·4903968 | ·562704 | 1·777130 | ·8714993 | 38 | 42 | ·4954587 | ·570389 | 1·753186 | ·8686315 | 18 |
| 2 | ·4853184 | ·555069 | 1·801575 | ·8743375 | 58 | 23 | ·4906503 | ·563087 | 1·775921 | ·8713566 | 37 | 43 | ·4957113 | ·570775 | 1·752002 | ·8684874 | 17 |
| 3 | ·4855727 | ·555450 | 1·800340 | ·8741963 | 57 | 24 | ·4909038 | ·563471 | 1·774714 | ·8712138 | 36 | 44 | ·4959639 | ·571161 | 1·750819 | ·8683431 | 16 |
| 4 | ·4858270 | ·555831 | 1·799107 | ·8740550 | 56 | 25 | ·4911572 | ·563854 | 1·773507 | ·8710710 | 35 | 45 | ·4962165 | ·571547 | 1·749637 | ·8681988 | 15 |
| 5 | ·4860812 | ·556211 | 1·797875 | ·8739137 | 55 | 26 | ·4914105 | ·564237 | 1·772302 | ·8709281 | 34 | 46 | ·4964690 | ·571933 | 1·748456 | ·8680544 | 14 |
| 6 | ·4863354 | ·556592 | 1·796645 | ·8737722 | 54 | 27 | ·4916638 | ·564621 | 1·771098 | ·8707851 | 33 | 47 | ·4967215 | ·572319 | 1·747276 | ·8679100 | 13 |
| 7 | ·4865895 | ·556973 | 1·795416 | ·8736307 | 53 | 28 | .4919171 | ·565005 | 1·769895 | ·8706420 | 32 | 48 | ·4969740 | ·572705 | 1·746098 | ·8677655 | 12 |
| 8 | ·4868436 | ·557355 | 1·794188 | ·8734891 | 52 | 29 | ·4921704 | ·565388 | 1·768694 | ·8704989 | 31 | 49 | ·4972264 | ·573091 | 1·744921 | ·8676209 | 11 |
| 9 | ·4870977 | ·557736 | 1·792961 | ·8733475 | 51 | 30 | ·4924236 | ·565772 | 1·767494 | ·8703557 | 30 | 50 | ·4974787 | ·573478 | 1·743745 | ·8674762 | 10 |
| 10 | ·4873517 | ·558117 | 1·791736 | ·8732058 | 50 | 31 | ·4926767 | ·566156 | 1·766295 | ·8702124 | 29 | 51 | ·4977310 | ·573864 | 1·742570 | ·8673314 | 9 |
| 11 | ·4876057 | ·558499 | 1·790512 | ·8730640 | 49 | 32 | ·4929298 | ·566541 | 1·765097 | ·8700691 | 28 | 52 | ·4979833 | .574251 | 1·741396 | ·8671866 | 8 |
| 12 | ·4878597 | ·558881 | 1·789289 | ·8729221 | 48 | 33 | ·4931829 | ·566925 | 1·763900 | ·8699256 | 27 | 53 | ·4982355 | ·574638 | 1·740224 | ·8670417 | 7 |
| 13 | ·4881136 | ·559262 | 1·788067 | ·8727801 | 47 | 34 | ·4934359 | ·567309 | 1·762705 | ·8697821 | 26 | 54 | ·4984877 | ·575025 | 1·739053 | ·8668967 | 6 |
| 14 | ·4883674 | ·559644 | 1·786847 | ·8726381 | 46 | 35 | ·4936889 | ·567694 | 1·761511 | ·8696386 | 25 | 55 | ·4987399 | ·575412 | 1·737883 | ·8667517 | 5 |
| 15 | ·4886212 | ·560026 | 1·785628 | ·8724960 | 45 | 36 | ·4939419 | ·568079 | 1·760318 | ·8694949 | 24 | 56 | ·4989920 | ·575799 | 1·736714 | ·8666066 | 4 |
| 16 | ·4888750 | ·560409 | 1·784410 | ·8723538 | 44 | 37 | ·4941948 | ·568463 | 1·759126 | ·8693512 | 23 | 57 | ·4992441 | ·576187 | 1·735546 | ·8664614 | 3 |
| 17 | ·4891288 | ·560791 | 1·783194 | ·8722116 | 43 | 38 | ·4944476 | ·568848 | 1·757936 | ·8692074 | 22 | 58 | ·4994961 | ·576574 | 1·734380 | ·8663161 | 2 |
| 18 | ·4893825 | ·561173 | 1·781979 | ·8720693 | 42 | 39 | ·4947005 | ·569233 | 1·756747 | ·8690636 | 21 | 59 | ·4997481 | ·576962 | 1·733214 | ·8661708 | 1 |
| 19 | ·4896361 | ·561556 | 1·780765 | ·8719269 | 41 | 40 | ·4949532 | ·569619 | 1·755559 | ·8689196 | 20 | 60 | ·5000000 | ·577350 | 1·732050 | ·8660254 | 0 |
| 20 | ·4898897 | ·561939 | 1·779552 | ·8717844 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | | | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | |
| | 60° | | | | | | 60° | | | | | | 60° | | | | |

| 30° | | | | | 30° | | | | | 30° | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·5000000 | ·577350 | 1·732050 | ·8660254 | 60 | 21 | ·5052809 | ·585524 | 1·707871 | ·8629549 | 39 | 41 | ·5102928 | ·593363 | 1·685308 | ·8600007 | 19 |
| 1 | ·5002519 | ·577738 | 1·730887 | ·8658799 | 59 | 22 | ·5055319 | ·585914 | 1·706732 | ·8628079 | 38 | 42 | ·5105429 | ·593756 | 1·684191 | ·8598523 | 18 |
| 2 | ·5005037 | ·578126 | 1·729726 | ·8657344 | 58 | 23 | ·5057828 | ·586305 | 1·705595 | ·8626608 | 37 | 43 | ·5107930 | ·594150 | 1·683076 | ·8597037 | 17 |
| 3 | ·5007556 | ·578514 | 1·728565 | ·8655887 | 57 | 24 | ·5060338 | ·586696 | 1·704458 | ·8625137 | 36 | 44 | ·5110431 | ·594543 | 1·681962 | ·8595551 | 16 |
| 4 | ·5010073 | ·578902 | 1·727406 | ·8654430 | 56 | 25 | ·5062846 | ·587087 | 1·703323 | ·8623664 | 35 | 45 | ·5112931 | ·594937 | 1·680848 | ·8594064 | 15 |
| 5 | ·5012591 | ·579291 | 1·726247 | ·8652973 | 55 | 26 | ·5065355 | ·587478 | 1·702189 | ·8622191 | 34 | 46 | ·5115431 | ·595331 | 1·679736 | ·8592576 | 14 |
| 6 | ·5015107 | ·579679 | 1·725090 | ·8651514 | 54 | 27 | ·5067863 | ·587870 | 1·701055 | ·8620717 | 33 | 47 | ·5117930 | ·595725 | 1·678625 | ·8591088 | 13 |
| 7 | ·5017624 | ·580068 | 1·723934 | ·8650055 | 53 | 28 | ·5070370 | ·588261 | 1·699923 | ·8619243 | 32 | 48 | ·5120429 | ·596119 | 1·677515 | ·8589599 | 12 |
| 8 | ·5020140 | ·580457 | 1·722779 | ·8648595 | 52 | 29 | ·5072877 | ·588653 | 1·698792 | ·8617768 | 31 | 49 | ·5122927 | ·596514 | 1·676406 | ·8588109 | 11 |
| 9 | ·5022655 | ·580846 | 1·721626 | ·8647134 | 51 | 30 | ·5075384 | ·589045 | 1·697663 | ·8616292 | 30 | 50 | ·5125425 | ·596908 | 1·675298 | ·8586619 | 10 |
| 10 | ·5025170 | ·581235 | 1·720473 | ·8645673 | 50 | 31 | ·5077890 | ·589436 | 1·696534 | ·8614815 | 29 | 51 | ·5127923 | ·597303 | 1·674192 | ·8585127 | 9 |
| 11 | ·5027685 | ·581624 | 1·719322 | ·8644211 | 49 | 32 | ·5080396 | ·589828 | 1·695406 | ·8613337 | 28 | 52 | ·5130420 | ·597697 | 1·673086 | ·8583635 | 8 |
| 12 | ·5030199 | ·582013 | 1·718172 | ·8642748 | 48 | 33 | ·5082901 | ·590221 | 1·694280 | ·8611859 | 27 | 53 | ·5132916 | ·598092 | 1·671981 | ·8582143 | 7 |
| 13 | ·5032713 | ·582403 | 1·717023 | ·8641284 | 47 | 34 | ·5085406 | ·590613 | 1·693155 | ·8610380 | 26 | 54 | ·5135413 | ·598487 | 1·670878 | ·8580649 | 6 |
| 14 | ·5035227 | ·582793 | 1·715875 | ·8639820 | 46 | 35 | ·5087910 | ·591005 | 1·692030 | ·8608901 | 25 | 55 | ·5137908 | ·598882 | 1·669775 | ·8579155 | 5 |
| 15 | ·5037740 | ·583182 | 1·714728 | ·8638355 | 45 | 36 | ·5090414 | ·591398 | 1·690907 | ·8607420 | 24 | 56 | ·5140404 | ·599278 | 1·668674 | ·8577660 | 4 |
| 16 | ·5040252 | ·583572 | 1·713582 | ·8636889 | 44 | 37 | ·5092918 | ·591791 | 1·689785 | ·8605939 | 23 | 57 | ·5142899 | ·599673 | 1·667574 | ·8576164 | 3 |
| 17 | ·5042765 | ·583962 | 1·712438 | ·8635423 | 43 | 38 | ·5095421 | ·592183 | 1·688664 | ·8604457 | 22 | 58 | ·5145393 | ·600069 | 1·666474 | ·8574668 | 2 |
| 18 | ·5045276 | ·584352 | 1·711294 | ·8633956 | 42 | 39 | ·5097924 | ·592576 | 1·687544 | ·8602975 | 21 | 59 | ·5147887 | ·600464 | 1·665376 | ·8573171 | 1 |
| 19 | ·5047788 | ·584743 | 1·710152 | ·8632488 | 41 | 40 | ·5100426 | ·592969 | 1·686426 | ·8601491 | 20 | 60 | ·5150381 | ·600860 | 1·664279 | ·8571673 | 0 |
| 20 | ·5050298 | ·585133 | 1·709011 | ·8631019 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
| | 59° | | | | | | 59° | | | | | | 59° | | | | |

| 31° | | | | | 31° | | | | | 31° | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. |
| 0 | ·5150381 | ·600860 | 1·664279 | ·8571673 | 60 | 21 | ·5202646 | ·609205 | 1·641482 | ·8540051 | 39 | 41 | ·5252241 | ·617210 | 1·620192 | ·8509639 |
| 1 | ·5152874 | ·601256 | 1·663183 | ·8570174 | 59 | 22 | ·5205130 | ·609604 | 1·640408 | ·8538538 | 38 | 42 | ·5254717 | ·617612 | 1·619138 | ·8508111 |
| 2 | ·5155367 | ·601652 | 1·662088 | ·8568675 | 58 | 23 | ·5207613 | ·610003 | 1·639335 | ·8537023 | 37 | 43 | ·5257191 | ·618014 | 1·618085 | ·8506582 |
| 3 | ·5157859 | ·602049 | 1·660994 | ·8567175 | 57 | 24 | ·5210096 | ·610402 | 1·638263 | ·8535508 | 36 | 44 | ·5259665 | ·618416 | 1·617033 | ·8505053 |
| 4 | ·5160351 | ·602445 | 1·659901 | ·8565674 | 56 | 25 | ·5212579 | ·610801 | 1·637191 | ·8533992 | 35 | 45 | ·5262139 | ·618818 | 1·615982 | ·8503522 |
| 5 | ·5162842 | ·602841 | 1·658809 | ·8564173 | 55 | 26 | ·5215061 | ·611201 | 1·636121 | ·8532475 | 34 | 46 | ·5264613 | ·619221 | 1·614932 | ·8501991 |
| 6 | ·5165333 | ·603238 | 1·657718 | ·8562671 | 54 | 27 | ·5217543 | ·611601 | 1·635052 | ·8530958 | 33 | 47 | ·5267085 | ·619623 | 1·613882 | ·8500459 |
| 7 | ·5167824 | ·603635 | 1·656629 | ·8561168 | 53 | 28 | ·5220024 | ·612000 | 1·633984 | ·8529440 | 32 | 48 | ·5269558 | ·620026 | 1·612834 | ·8498927 |
| 8 | ·5170314 | ·604032 | 1·655540 | ·8559664 | 52 | 29 | ·5222505 | ·612400 | 1·632917 | ·8527921 | 31 | 49 | ·5272030 | ·620429 | 1·611787 | ·8497394 |
| 9 | ·5172804 | ·604429 | 1·654452 | ·8558160 | 51 | 30 | ·5224986 | ·612800 | 1·631851 | ·8526402 | 30 | 50 | ·5274502 | ·620832 | 1·610741 | ·8495860 |
| 10 | ·5175293 | ·604826 | 1·653366 | ·8556655 | 50 | 31 | ·5227466 | ·613201 | 1·630786 | ·8524881 | 29 | 51 | ·5276973 | ·621235 | 1·609696 | ·8494325 |
| 11 | ·5177782 | ·605224 | 1·652280 | ·8555149 | 49 | 32 | ·5229945 | ·613601 | 1·629722 | ·8523360 | 28 | 52 | ·5279443 | ·621638 | 1·608652 | ·8492790 |
| 12 | ·5180270 | ·605621 | 1·651196 | ·8553643 | 48 | 33 | ·5232424 | ·614001 | 1·628659 | ·8521839 | 27 | 53 | ·5281914 | ·622041 | 1·607609 | ·8491254 |
| 13 | ·5182758 | ·606019 | 1·650112 | ·8552135 | 47 | 34 | ·5234903 | ·614402 | 1·627597 | ·8520316 | 26 | 54 | ·5284383 | ·622445 | 1·606567 | ·8489717 |
| 14 | ·5185246 | ·606417 | 1·649030 | ·8550627 | 46 | 35 | ·5237381 | ·614803 | 1·626536 | ·8518793 | 25 | 55 | ·5286853 | ·622848 | 1·605526 | ·8488179 |
| 15 | ·5187733 | ·606814 | 1·647949 | ·8549119 | 45 | 36 | ·5239859 | ·615204 | 1·625476 | ·8517269 | 24 | 56 | ·5289322 | ·623252 | 1·604485 | ·8486641 |
| 16 | ·5190219 | ·607213 | 1·646868 | ·8547609 | 44 | 37 | ·5242336 | ·615605 | 1·624417 | ·8515745 | 23 | 57 | ·5291790 | ·623656 | 1·603446 | ·8485102 |
| 17 | ·5192705 | ·607611 | 1·645789 | ·8546099 | 43 | 38 | ·5244813 | ·616006 | 1·623359 | ·8514219 | 22 | 58 | ·5294258 | ·624060 | 1·602408 | ·8483562 |
| 18 | ·5195191 | ·608009 | 1·644711 | ·8544588 | 42 | 39 | ·5247290 | ·616407 | 1·622302 | ·8512693 | 21 | 59 | ·5296726 | ·624465 | 1·601370 | ·8482022 |
| 19 | ·5197676 | ·608408 | 1·643633 | ·8543077 | 41 | 40 | ·5249766 | ·616809 | 1·621246 | ·8511167 | 20 | 60 | ·5299193 | ·624869 | 1·600334 | ·8480481 |
| 20 | ·5200161 | ·608806 | 1·642557 | ·8541564 | 40 | | | | | | | | | | | |
| | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. |
| | 58° | | | | | | 58° | | | | | | 58° | | | |

| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | ·5299193 | ·624869 | 1·600334 | ·8480481 | 60 | 21 | ·5350898 | ·633395 | 1·578791 | ·8447952 | 39 | 41 | ·5399955 | ·641577 | 1·558657 | ·8416679 | 19 |
| 1 | ·5301659 | ·625273 | 1·599299 | ·8478939 | 59 | 22 | ·5353355 | ·633803 | 1·577776 | ·8446395 | 38 | 42 | ·5402403 | ·641988 | 1·557660 | ·8415108 | 18 |
| 2 | ·5304125 | ·625678 | 1·598264 | ·8477397 | 58 | 23 | ·5355812 | ·634211 | 1·576761 | ·8444838 | 37 | 43 | ·5404851 | ·642399 | 1·556663 | ·8413536 | 17 |
| 3 | ·5306591 | ·626083 | 1·597231 | ·8475853 | 57 | 24 | ·5358268 | ·634619 | 1·575747 | ·8443279 | 36 | 44 | ·5407298 | ·642810 | 1·555668 | ·8411963 | 16 |
| 4 | ·5309057 | ·626488 | 1·596198 | ·8474309 | 56 | 25 | ·5360724 | ·635027 | 1·574735 | ·8441720 | 35 | 45 | ·5409745 | ·643221 | 1·554674 | ·8410390 | 15 |
| 5 | ·5311521 | ·626893 | 1·595167 | ·8472765 | 55 | 26 | ·5363179 | ·635435 | 1·573723 | ·8440161 | 34 | 46 | ·5412191 | ·643632 | 1·553680 | ·8408816 | 14 |
| 6 | ·5313986 | ·627298 | 1·594136 | ·8471219 | 54 | 27 | ·5365634 | ·635844 | 1·572712 | ·8438600 | 33 | 47 | ·5414637 | ·644044 | 1·552688 | ·8407241 | 13 |
| 7 | ·5316450 | ·627704 | 1·593107 | ·8469673 | 53 | 28 | ·5368089 | ·636252 | 1·571702 | ·8437039 | 32 | 48 | ·5417082 | ·644456 | 1·551696 | ·8405666 | 12 |
| 8 | ·5318913 | ·628109 | 1·592078 | ·8468126 | 52 | 29 | ·5370543 | ·636661 | 1·570693 | ·8435477 | 31 | 49 | ·5419527 | ·644867 | 1·550705 | ·8404090 | 11 |
| 9 | ·5321376 | ·628515 | 1·591050 | ·8466579 | 51 | 30 | ·5372996 | ·637070 | 1·569685 | ·8433914 | 30 | 50 | ·5421971 | ·645279 | 1·549715 | ·8402513 | 10 |
| 10 | ·5323839 | ·628921 | 1·590023 | ·8465030 | 50 | 31 | ·5375449 | ·637479 | 1·568678 | ·8432351 | 29 | 51 | ·5424415 | ·645691 | 1·548726 | ·8400936 | 9 |
| 11 | ·5326301 | ·629327 | 1·588997 | ·8463481 | 49 | 32 | ·5377902 | ·637888 | 1·567672 | ·8430787 | 28 | 52 | ·5426859 | ·646104 | 1·547738 | ·8399357 | 8 |
| 12 | ·5328763 | ·629733 | 1·587973 | ·8461932 | 48 | 33 | ·5380354 | ·638297 | 1·566666 | ·8429222 | 27 | 53 | ·5429302 | ·646516 | 1·546751 | ·8397778 | 7 |
| 13 | ·5331224 | ·630139 | 1·586949 | ·8460381 | 47 | 34 | ·5382806 | ·638707 | 1·565662 | ·8427657 | 26 | 54 | ·5431744 | ·646929 | 1·545764 | ·8396199 | 6 |
| 14 | ·5333685 | ·630546 | 1·585926 | ·8458830 | 46 | 35 | ·5385257 | ·639116 | 1·564659 | ·8426091 | 25 | 55 | ·5434187 | ·647341 | 1·544779 | ·8394618 | 5 |
| 15 | ·5336145 | ·630953 | 1·584904 | ·8457278 | 45 | 36 | ·5387708 | ·639526 | 1·563656 | ·8424524 | 24 | 56 | ·5436628 | ·647754 | 1·543794 | ·8393037 | 4 |
| 16 | ·5338605 | ·631359 | 1·583883 | ·8455726 | 44 | 37 | ·5390158 | ·639936 | 1·562654 | ·8422956 | 23 | 57 | ·5439069 | ·648167 | 1·542810 | ·8391455 | 3 |
| 17 | ·5341065 | ·631766 | 1·582862 | ·8454172 | 43 | 38 | ·5392608 | ·640346 | 1·561654 | ·8421388 | 22 | 58 | ·5441510 | ·648580 | 1·541828 | ·8389873 | 2 |
| 18 | ·5343523 | ·632173 | 1·581843 | ·8452618 | 42 | 39 | ·5395058 | ·640756 | 1·560654 | ·8419819 | 21 | 59 | ·5443951 | ·648994 | 1·540846 | ·8388290 | 1 |
| 19 | ·5345982 | ·632581 | 1·580825 | ·8451064 | 41 | 40 | ·5397507 | ·641167 | 1·559655 | ·8418249 | 20 | 60 | ·5446390 | ·649407 | 1·539865 | ·8386706 | 0 |
| 20 | ·5348440 | ·632988 | 1·579807 | ·8449508 | 40 | | | | | | | | | | | | |
| | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |

| 33° | | | | | 33° | | | | | 33. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·5446390 | ·649407 | 1·539865 | ·8386706 | 60 | 21 | ·5497520 | ·658127 | 1·519463 | ·8353279 | 39 | 41 | ·5546024 | ·666496 | 1·500382 | ·8321155 | 19 |
| 1 | ·5448830 | ·649821 | 1·538884 | ·8385121 | 59 | 22 | ·5499950 | ·658544 | 1·518501 | ·8351680 | 38 | 42 | ·5548444 | ·666917 | 1·499436 | ·8319541 | 18 |
| 2 | ·5451269 | ·650235 | 1·537905 | ·8383536 | 58 | 23 | ·5502379 | ·658961 | 1·517540 | ·8350080 | 37 | 43 | ·5550864 | ·667337 | 1·498492 | ·8317927 | 17 |
| 3 | ·5453707 | ·650649 | 1·536927 | ·8381950 | 57 | 24 | ·5504807 | ·659378 | 1·516579 | ·8348479 | 36 | 44 | ·5553283 | ·667758 | 1·497548 | ·8316312 | 16 |
| 4 | ·5456145 | ·651063 | 1·535949 | ·8380363 | 56 | 25 | ·5507236 | ·659796 | 1·515620 | ·8346877 | 35 | 45 | ·5555702 | ·668178 | 1·496605 | ·8314696 | 15 |
| 5 | ·5458583 | ·651477 | 1·534972 | ·8378775 | 55 | 26 | ·5509663 | ·660213 | 1·514661 | ·8345275 | 34 | 46 | ·5558121 | ·668599 | 1·495663 | ·8313080 | 14 |
| 6 | ·5461020 | ·651891 | 1·533996 | ·8377187 | 54 | 27 | ·5512091 | ·660631 | 1·513703 | ·8343672 | 33 | 47 | ·5560539 | ·669020 | 1·494722 | ·8311463 | 13 |
| 7 | ·5463456 | ·652306 | 1·533021 | ·8375598 | 53 | 28 | ·5514518 | ·661049 | 1·512746 | ·8342068 | 32 | 48 | ·5562956 | ·669441 | 1·493782 | ·8309845 | 12 |
| 8 | ·5465892 | ·652721 | 1·532047 | ·8374009 | 52 | 29 | ·5516944 | ·661467 | 1·511790 | ·8340463 | 31 | 49 | ·5565373 | ·669863 | 1·492842 | ·8308226 | 11 |
| 9 | ·5468328 | ·653136 | 1·531074 | ·8372418 | 51 | 30 | ·5519370 | ·661885 | 1·510835 | ·8338858 | 30 | 50 | ·5567790 | ·670284 | 1·491903 | ·8306607 | 10 |
| 10 | ·5470763 | ·653551 | 1·530102 | ·8370827 | 50 | 31 | ·5521795 | ·662304 | 1·509880 | ·8337252 | 29 | 51 | ·5570206 | ·670706 | 1·490965 | ·8304987 | 9 |
| 11 | ·5473198 | ·653966 | 1·529130 | ·8369236 | 49 | 32 | ·5524220 | ·662722 | 1·508927 | ·8335646 | 28 | 52 | ·5572621 | ·671128 | 1·490028 | ·8303366 | 8 |
| 12 | ·5475632 | ·654381 | 1·528160 | ·8367643 | 48 | 33 | ·5526645 | ·663141 | 1·507974 | ·8334038 | 27 | 53 | ·5575036 | ·671550 | 1·489092 | ·8301745 | 7 |
| 13 | ·5478066 | ·654797 | 1·527190 | ·8366050 | 47 | 34 | ·5529069 | ·663560 | 1·507022 | ·8332430 | 26 | 54 | ·5577451 | ·671972 | 1·488157 | ·8300123 | 6 |
| 14 | ·5480499 | ·655212 | 1·526221 | ·8364456 | 46 | 35 | ·5531492 | ·663979 | 1·506071 | ·8330822 | 25 | 55 | ·5579865 | ·672394 | 1·487222 | ·8298500 | 5 |
| 15 | ·5482932 | ·655628 | 1·525253 | ·8362862 | 45 | 36 | ·5533915 | ·664398 | 1·505121 | ·8329212 | 24 | 56 | ·5582279 | ·672816 | 1·486288 | ·8296877 | 4 |
| 16 | ·5485365 | ·656044 | 1·524286 | ·8361266 | 44 | 37 | ·5536338 | ·664817 | 1·504171 | ·8327602 | 23 | 57 | ·5584692 | ·673239 | 1·485355 | ·8295252 | 3 |
| 17 | ·5487797 | ·656460 | 1·523320 | ·8359670 | 43 | 38 | ·5538760 | ·665237 | 1·503222 | ·8325991 | 22 | 58 | ·5587105 | ·673662 | 1·484423 | ·8293628 | 2 |
| 18 | ·5490228 | ·656877 | 1·522354 | ·8358074 | 42 | 39 | ·5541182 | ·665657 | 1·502275 | ·8324380 | 21 | 59 | ·5589517 | ·674085 | 1·483491 | ·8292002 | 1 |
| 19 | ·5492659 | ·657293 | 1·521389 | ·8356476 | 41 | 40 | ·5543603 | ·666076 | 1·501328 | ·8322768 | 20 | 60 | ·5591929 | ·674508 | 1·482561 | ·8290376 | 0 |
| 20 | ·5495090 | ·657710 | 1·520426 | ·8354878 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
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| 34° | | | | | | 34° | | | | | | 34° | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·5591929 | ·674508 | 1·482561 | ·8290376 | 60 | 21 | ·5642467 | ·683433 | 1·463200 | ·8256062 | 39 | 41 | ·5690403 | ·692002 | 1·445081 | ·8223094 | 19 |
| 1 | ·5594340 | ·674931 | 1·481631 | ·8288749 | 59 | 22 | ·5644869 | ·683860 | 1·462287 | ·8254420 | 38 | 42 | ·5692795 | ·692432 | 1·444183 | ·8221440 | 18 |
| 2 | ·5596751 | ·675355 | 1·480702 | ·8287121 | 58 | 23 | ·5647270 | ·684287 | 1·461374 | ·8252778 | 37 | 43 | ·5695187 | ·692863 | 1·443286 | ·8219784 | 17 |
| 3 | ·5599162 | ·675779 | 1·479773 | ·8285493 | 57 | 24 | ·5649670 | ·684714 | 1·460463 | ·8251135 | 36 | 44 | ·5697577 | ·693293 | 1·442389 | ·8218127 | 16 |
| 4 | ·5601572 | ·676202 | 1·478846 | ·8283864 | 56 | 25 | ·5652070 | ·685141 | 1·459552 | ·8249491 | 35 | 45 | ·5699968 | ·693724 | 1·441494 | ·8216469 | 15 |
| 5 | ·5603981 | ·676626 | 1·477919 | ·8282234 | 55 | 26 | ·5654469 | ·685569 | 1·458642 | ·8247847 | 34 | 46 | ·5702357 | ·694155 | 1·440599 | ·8214811 | 14 |
| 6 | ·5606390 | ·677050 | 1·476993 | ·8280603 | 54 | 27 | ·5656868 | ·685996 | 1·457732 | ·8246202 | 33 | 47 | ·5704747 | ·694586 | 1·439704 | ·8213152 | 13 |
| 7 | ·5608798 | ·677475 | 1·476068 | ·8278972 | 53 | 28 | ·5659267 | ·686424 | 1·456824 | ·8244556 | 32 | 48 | ·5707136 | ·695018 | 1·438811 | ·8211492 | 12 |
| 8 | ·5611206 | ·677899 | 1·475144 | ·8277340 | 52 | 29 | ·5661665 | ·686852 | 1·455916 | ·8242909 | 31 | 49 | ·5709524 | ·695449 | 1·437918 | ·8209832 | 11 |
| 9 | ·5613614 | ·678324 | 1·474221 | ·8275708 | 51 | 30 | ·5664062 | ·687281 | 1·455009 | ·8241262 | 30 | 50 | ·5711912 | ·695881 | 1·437026 | ·8208170 | 10 |
| 10 | ·5616021 | ·678749 | 1·473298 | ·8274074 | 50 | 31 | ·5666459 | ·687709 | 1·454102 | ·8239614 | 29 | 51 | ·5714299 | ·696313 | 1·436135 | ·8206509 | 9 |
| 11 | ·5618428 | ·679174 | 1·472376 | ·8272440 | 49 | 32 | ·5668856 | ·688137 | 1·453197 | ·8237965 | 28 | 52 | ·5716686 | ·696745 | 1·435245 | ·8204846 | 8 |
| 12 | ·5620834 | ·679599 | 1·471455 | ·8270806 | 48 | 33 | ·5671252 | ·688566 | 1·452292 | ·8236316 | 27 | 53 | ·5719073 | ·697177 | 1·434355 | ·8203183 | 7 |
| 13 | ·5623239 | ·680024 | 1·470535 | ·8269170 | 47 | 34 | ·5673648 | ·688995 | 1·451388 | ·8234666 | 26 | 54 | ·5721459 | ·697609 | 1·433466 | ·8201519 | 6 |
| 14 | ·5625645 | ·680450 | 1·469615 | ·8267534 | 46 | 35 | ·5676043 | ·689424 | 1·450485 | ·8233015 | 25 | 55 | ·5723844 | ·698042 | 1·432578 | ·8199854 | 5 |
| 15 | ·5628049 | ·680875 | 1·468696 | ·8265897 | 45 | 36 | ·5678437 | ·689853 | 1·449582 | ·8231364 | 24 | 56 | ·5726229 | ·698474 | 1·431690 | ·8198189 | 4 |
| 16 | ·5630453 | ·681301 | 1·467778 | ·8264260 | 44 | 37 | ·5680832 | ·690283 | 1·448680 | ·8229712 | 23 | 57 | ·5728614 | ·698907 | 1.430803 | ·8196523 | 3 |
| 17 | ·5632857 | ·681727 | 1·466861 | ·8262622 | 43 | 38 | ·5683225 | ·690712 | 1·447779 | ·8228059 | 22 | 58 | ·5730998 | ·699340 | 1.429917 | ·8194856 | 2 |
| 18 | ·5635260 | ·682153 | 1·465945 | ·8260983 | 42 | 39 | ·5685619 | ·691142 | 1·446879 | ·8226405 | 21 | 59 | ·5733381 | ·699774 | 1.429032 | ·8193189 | 1 |
| 19 | ·5637663 | ·682580 | 1·465029 | ·8259343 | 41 | 40 | ·5688011 | ·691572 | 1·445980 | ·8224751 | 20 | 60 | ·5735764 | ·700207 | 1.428148 | ·8191520 | 0 |
| 20 | ·5640066 | ·683006 | 1·464114 | ·8257703 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
|---|---|---|---|---|---|
| 0 | ·5735764 | ·700207 | 1·428148 | ·8191520 | 60 |
| 1 | ·5738147 | ·700641 | 1·427264 | ·8189852 | 59 |
| 2 | ·5740529 | ·701074 | 1·426381 | ·8188182 | 58 |
| 3 | ·5742911 | ·701508 | 1·425498 | ·8186512 | 57 |
| 4 | ·5745292 | ·701943 | 1·424617 | ·8184841 | 56 |
| 5 | ·5747672 | ·702377 | 1·423736 | ·8183169 | 55 |
| 6 | ·5750053 | ·702811 | 1·422856 | ·8181497 | 54 |
| 7 | ·5752432 | ·703246 | 1·421976 | ·8179824 | 53 |
| 8 | ·5754811 | ·703681 | 1·421097 | ·8178151 | 52 |
| 9 | ·5757190 | ·704116 | 1·420220 | ·8176476 | 51 |
| 10 | ·5759568 | ·704551 | 1·419342 | ·8174801 | 50 |
| 11 | ·5761946 | ·704986 | 1·418465 | ·8173125 | 49 |
| 12 | ·5764323 | ·705422 | 1·417590 | ·8171449 | 48 |
| 13 | ·5766700 | ·705858 | 1·416715 | ·8169772 | 47 |
| 14 | ·5769076 | ·706294 | 1·415840 | ·8168094 | 46 |
| 15 | ·5771452 | ·706730 | 1·414967 | ·8166416 | 45 |
| 16 | ·5773827 | ·707166 | 1·414094 | ·8164736 | 44 |
| 17 | ·5776202 | ·707602 | 1·413222 | ·8163056 | 43 |
| 18 | ·5778576 | ·708039 | 1·412350 | ·8161376 | 42 |
| 19 | ·5780950 | ·708476 | 1·411479 | ·8159695 | 41 |
| 20 | ·5783323 | ·708913 | 1·410609 | 8158013 | 40 |

| ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
|---|---|---|---|---|---|
| 21 | ·5785696 | ·709350 | 1·409740 | ·8156330 | 39 |
| 22 | ·5788069 | ·709787 | 1·408871 | ·8154647 | 38 |
| 23 | ·5790440 | ·710225 | 1·408003 | ·8152963 | 37 |
| 24 | ·5792812 | ·710663 | 1·407136 | ·8151278 | 36 |
| 25 | ·5795183 | ·711100 | 1·406270 | ·8149593 | 35 |
| 26 | ·5797553 | ·711539 | 1·405404 | ·8147906 | 34 |
| 27 | ·5799923 | ·711977 | 1·404539 | ·8146220 | 33 |
| 28 | ·5802292 | ·712415 | 1·403674 | ·8144532 | 32 |
| 29 | ·5804661 | ·712854 | 1·402811 | ·8142844 | 31 |
| 30 | ·5807030 | ·713293 | 1·401948 | ·8141155 | 30 |
| 31 | ·5809397 | ·713732 | 1·401086 | ·8139466 | 29 |
| 32 | ·5811765 | ·714171 | 1·400224 | ·8137775 | 28 |
| 33 | ·5814132 | ·714610 | 1·399363 | ·8136084 | 27 |
| 34 | ·5816498 | ·715050 | 1·398503 | ·8134393 | 26 |
| 35 | ·5818864 | ·715490 | 1·397644 | ·8132701 | 25 |
| 36 | ·5821230 | ·715929 | 1·396785 | ·8131008 | 24 |
| 37 | ·5823595 | ·716369 | 1·395927 | ·8129314 | 23 |
| 38 | ·5825959 | ·716810 | 1·395069 | ·8127620 | 22 |
| 39 | ·5828323 | ·717250 | 1·394213 | ·8125925 | 21 |
| 40 | ·5830687 | ·717691 | 1·393357 | ·8124229 | 20 |

| ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
|---|---|---|---|---|---|
| 41 | ·5833050 | ·718131 | 1·392501 | ·8122532 | 19 |
| 42 | ·5835412 | ·718572 | 1·391647 | ·8120835 | 18 |
| 43 | ·5837774 | ·719014 | 1·390793 | ·8119137 | 17 |
| 44 | ·5840136 | ·719455 | 1·389940 | ·8117439 | 16 |
| 45 | ·5842497 | ·719897 | 1·389087 | ·8115740 | 15 |
| 46 | ·5844857 | ·720338 | 1·388235 | ·8114040 | 14 |
| 47 | ·5847217 | ·720780 | 1·387384 | ·8112339 | 13 |
| 48 | ·5849577 | ·721222 | 1·386534 | ·8110638 | 12 |
| 49 | ·5851936 | ·721665 | 1·385684 | ·8108936 | 11 |
| 50 | ·5854294 | ·722107 | 1·384835 | ·8107234 | 10 |
| 51 | ·5856652 | ·722550 | 1·383986 | ·8105530 | 9 |
| 52 | ·5859010 | ·722993 | 1·383139 | ·8103826 | 8 |
| 53 | ·5861367 | ·723436 | 1·382292 | ·8102122 | 7 |
| 54 | ·5863724 | ·723879 | 1·381445 | ·8100416 | 6 |
| 55 | ·5866080 | ·724322 | 1·380600 | ·8098710 | 5 |
| 56 | ·5868435 | ·724766 | 1·379755 | ·8097004 | 4 |
| 57 | ·5870790 | ·725210 | 1·378910 | ·8095296 | 3 |
| 58 | ·5873145 | ·725654 | 1·378067 | ·8093588 | 2 |
| 59 | ·5875499 | ·726098 | 1·377224 | ·8091879 | 1 |
| 60 | ·5877853 | ·726542 | 1·376381 | ·8090170 | 0 |

| 36° | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | ·5877853 | ·726542 | 1·376381 | ·8090170 | 60 | 21 | ·5927163 | ·735917 | 1·358848 | ·8054113 | 39 | 41 | ·5973919 | ·744924 | 1·342417 | ·8019495 | 19 |
| 1 | ·5880206 | ·726987 | 1·375540 | ·8088460 | 59 | 22 | ·5929505 | ·736366 | 1·358020 | ·8052389 | 38 | 42 | ·5976251 | ·745377 | 1·341602 | ·8017756 | 18 |
| 2 | ·5882558 | ·727431 | 1·374699 | ·8086749 | 58 | 23 | ·5931847 | ·736814 | 1·357193 | ·8050664 | 37 | 43 | ·5978583 | ·745829 | 1·340788 | ·8016018 | 17 |
| 3 | ·5884910 | ·727876 | 1·373859 | ·8085037 | 57 | 24 | ·5934189 | ·737263 | 1·356367 | ·8048938 | 36 | 44 | ·5980915 | ·746282 | 1·339975 | ·8014278 | 16 |
| 4 | ·5887262 | ·728321 | 1·373019 | ·8083325 | 56 | 25 | ·5936530 | ·737712 | 1·355541 | ·8047211 | 35 | 45 | ·5983246 | ·746735 | 1·339162 | ·8012538 | 15 |
| 5 | 5889613 | ·728767 | 1·372180 | ·8081612 | 55 | 26 | ·5938871 | ·738162 | 1·354716 | ·8045484 | 34 | 46 | ·5985577 | ·747188 | 1·338350 | ·8010797 | 14 |
| 6 | ·5891964 | ·729212 | 1·371342 | ·8079899 | 54 | 27 | ·5941211 | ·738611 | 1·353891 | ·8043756 | 33 | 47 | ·5987906 | ·747642 | 1·337538 | ·8009056 | 13 |
| 7 | 5894314 | ·729658 | 1·370504 | ·8078185 | 53 | 28 | ·5943550 | ·739061 | 1·353068 | ·8042028 | 32 | 48 | ·5990236 | ·748095 | 1·336727 | ·8007314 | 12 |
| 8 | ·5896663 | ·730104 | 1·369667 | ·8076470 | 52 | 29 | ·5945889 | ·739511 | 1·352244 | ·8040299 | 31 | 49 | ·5992565 | ·748549 | 1·335917 | ·8005571 | 11 |
| 9 | ·5899012 | ·730550 | 1·368831 | ·8074754 | 51 | 30 | ·5948228 | ·739961 | 1·351422 | ·8038569 | 30 | 50 | ·5994893 | ·749003 | 1·335107 | ·8003827 | 10 |
| 10 | ·5901361 | ·730996 | 1·367995 | ·8073038 | 50 | 31 | ·5950566 | ·740411 | 1·350600 | ·8036838 | 29 | 51 | ·5997221 | ·749457 | 1·334298 | ·8002083 | 9 |
| 11 | ·5903709 | ·731442 | 1·367161 | ·8071321 | 49 | 32 | ·5952904 | ·740861 | 1·349779 | ·8035107 | 28 | 52 | ·5999549 | ·749911 | 1·333490 | ·8000338 | 8 |
| 12 | ·5906057 | ·731889 | 1·366326 | ·8069603 | 48 | 33 | ·5955241 | ·741312 | 1·348958 | ·8033375 | 27 | 53 | ·6001876 | ·750366 | 1·332682 | ·7998593 | 7 |
| 13 | ·5908404 | ·732336 | 1·365493 | ·8067885 | 47 | 34 | ·5957577 | ·741763 | 1·348139 | ·8031642 | 26 | 54 | ·6004202 | ·750821 | 1·331875 | ·7996847 | 6 |
| 14 | ·5910750 | ·732783 | 1·364660 | ·8066166 | 46 | 35 | ·5959913 | ·742214 | 1·347319 | ·8029909 | 25 | 55 | ·6006528 | ·751276 | 1·331068 | ·7995100 | 5 |
| 15 | ·5913096 | ·733230 | 1·363827 | ·8064446 | 45 | 36 | ·5962249 | ·742665 | 1·346501 | ·8028175 | 24 | 56 | ·6008854 | ·751731 | 1·330262 | ·7993352 | 4 |
| 16 | ·5915442 | ·733677 | 1·362996 | ·8062726 | 44 | 37 | ·5964584 | ·743117 | 1.345683 | ·8026440 | 23 | 57 | ·6011179 | ·752186 | 1·329457 | ·7991604 | 3 |
| 17 | ·5917787 | ·734125 | 1·362165 | ·8061005 | 43 | 38 | ·5966918 | ·743568 | 1.344865 | ·8024705 | 22 | 58 | ·6013503 | ·752642 | 1·328652 | ·7989855 | 2 |
| 18 | ·5920132 | ·734573 | 1·361335 | ·8059283 | 42 | 39 | ·5969252 | ·744020 | 1.344049 | ·8022969 | 21 | 59 | ·6015827 | ·753098 | 1·327848 | ·7988105 | 1 |
| 19 | ·5922476 | ·735021 | 1·360505 | ·8057560 | 41 | 40 | ·5971586 | ·744472 | 1.343233 | ·8021232 | 20 | 60 | ·6018150 | ·753554 | 1·327044 | ·7986355 | 0 |
| 20 | ·5924819 | ·735469 | 1·359676 | ·8055837 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
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|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·6018150 | ·753554 | 1·327044 | ·7986355 | 60 | 21 | ·6066824 | ·763175 | 1·310314 | ·7949444 | 39 | 41 | ·6112969 | ·772423 | 1·294627 | ·7914014 | 19 |
| 1 | ·6020473 | ·754010 | 1·326242 | ·7984604 | 59 | 22 | ·6069136 | ·763636 | 1·309523 | ·7947678 | 38 | 42 | ·6115270 | ·772887 | 1·293848 | ·7912235 | 18 |
| 2 | ·6022795 | ·754466 | 1·325439 | ·7982853 | 58 | 23 | ·6071447 | ·764096 | 1·308734 | ·7945913 | 37 | 43 | ·6117572 | ·773352 | 1·293071 | ·7910456 | 17 |
| 3 | ·6025117 | ·754923 | 1·324638 | ·7981100 | 57 | 24 | ·6073758 | ·764557 | 1·307945 | ·7944146 | 36 | 44 | ·6119873 | ·773817 | 1·292294 | ·7908676 | 16 |
| 4 | ·6027439 | ·755379 | 1·323837 | ·7979347 | 56 | 25 | ·6076069 | ·765018 | 1·307157 | ·7942379 | 35 | 45 | ·6122173 | ·774282 | 1·291517 | ·7906896 | 15 |
| 5 | ·6029760 | ·755836 | 1·323036 | ·7977594 | 55 | 26 | ·6078379 | ·765480 | 1·306369 | ·7940611 | 34 | 46 | ·6124473 | ·774748 | 1·290742 | ·7905115 | 14 |
| 6 | ·6032080 | ·756294 | 1·322237 | ·7975839 | 54 | 27 | ·6080689 | ·765941 | 1·305582 | ·7938843 | 33 | 47 | ·6126772 | ·775213 | 1·289966 | ·7903333 | 13 |
| 7 | ·6034400 | ·756751 | 1·321437 | ·7974084 | 53 | 28 | ·6082998 | ·766403 | 1·304796 | ·7937074 | 32 | 48 | ·6129071 | ·775679 | 1·289192 | ·7901550 | 12 |
| 8 | ·6036719 | ·757209 | 1·320639 | ·7972329 | 52 | 29 | ·6085306 | ·766864 | 1·304010 | ·7935304 | 31 | 49 | ·6131369 | ·776145 | 1·288418 | ·7899767 | 11 |
| 9 | ·6039038 | ·757666 | 1·319841 | ·7970572 | 51 | 30 | ·6087614 | ·767327 | 1·303225 | ·7933533 | 30 | 50 | ·6133666 | ·776611 | 1·287644 | ·7897983 | 10 |
| 10 | ·6041356 | ·758124 | 1·319044 | ·7968815 | 50 | 31 | ·6089922 | ·767789 | 1·302440 | ·7931762 | 29 | 51 | ·6135964 | ·777078 | 1·286871 | ·7896198 | 9 |
| 11 | ·6043674 | ·758582 | 1·318247 | ·7967058 | 49 | 32 | ·6092229 | ·768251 | 1·301656 | ·7929990 | 28 | 52 | ·6138260 | ·777544 | 1·286099 | ·7894413 | 8 |
| 12 | ·6045991 | ·759041 | 1·317451 | ·7965299 | 48 | 33 | ·6094535 | ·768714 | 1·300873 | ·7928218 | 27 | 53 | ·6140556 | ·778011 | 1·285327 | ·7892627 | 7 |
| 13 | ·6048308 | ·759499 | 1·316655 | ·7963540 | 47 | 34 | ·6096841 | ·769177 | 1·300090 | ·7926445 | 26 | 54 | ·6142852 | ·778478 | 1·284556 | ·7890841 | 6 |
| 14 | ·6050624 | ·759958 | 1·315861 | ·7961780 | 46 | 35 | ·6099147 | ·769640 | 1·299308 | ·7924671 | 25 | 55 | ·6145147 | ·778946 | 1·283786 | ·7889054 | 5 |
| 15 | ·6052940 | ·760417 | 1.315066 | ·7960020 | 45 | 36 | ·6101452 | ·770103 | 1·298526 | ·7922896 | 24 | 56 | ·6147442 | ·779413 | 1·283016 | ·7887266 | 4 |
| 16 | ·6055255 | ·760876 | 1·314273 | ·7958259 | 44 | 37 | ·6103756 | ·770567 | 1·297745 | ·7921121 | 23 | 57 | ·6149736 | ·779881 | 1·282246 | ·7885477 | 3 |
| 17 | ·6057570 | ·761336 | 1.313480 | ·7956497 | 43 | 38 | ·6106060 | ·771030 | 1·296964 | ·7919345 | 22 | 58 | ·6152029 | ·780349 | 1·281477 | ·7883688 | 2 |
| 18 | ·6059884 | ·761795 | 1.312687 | ·7954735 | 42 | 39 | ·6108363 | ·771494 | 1·296185 | ·7917569 | 21 | 59 | ·6154322 | ·780817 | 1·280709 | ·7881898 | 1 |
| 19 | ·6062198 | ·762255 | 1.311895 | .7952972 | 41 | 40 | ·6110666 | ·771958 | 1·295405 | ·7915792 | 20 | 60 | ·6156615 | ·781285 | 1·279941 | ·7880108 | 0 |
| 20 | ·6064511 | ·762715 | 1.311104 | ·7951208 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ |
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| 0 | ·6156615 | ·781285 | 1·279941 | ·7880108 | 60 | 21 |
| 1 | ·6158907 | ·781754 | 1·279174 | ·7878316 | 59 | 22 |
| 2 | ·6161198 | ·782222 | 1·278407 | ·7876524 | 58 | 23 |
| 3 | ·6163489 | ·782691 | 1·277641 | ·7874732 | 57 | 24 |
| 4 | ·6165780 | ·783161 | 1·276876 | ·7872939 | 56 | 25 |
| 5 | ·6168069 | ·783630 | 1·276111 | ·7871145 | 55 | 26 |
| 6 | ·6170359 | ·784100 | 1·275347 | ·7869350 | 54 | 27 |
| 7 | ·6172648 | ·784570 | 1·274583 | ·7867555 | 53 | 28 |
| 8 | ·6174936 | ·785040 | 1·273820 | ·7865759 | 52 | 29 |
| 9 | ·6177224 | ·785510 | 1·273057 | ·7863963 | 51 | 30 |
| 10 | ·6179511 | ·785980 | 1·272295 | ·7862165 | 50 | 31 |
| 11 | ·6181798 | ·786451 | 1·271534 | ·7860367 | 49 | 32 |
| 12 | ·6184084 | ·786922 | 1·270773 | ·7858569 | 48 | 33 |
| 13 | ·6186370 | ·787393 | 1·270013 | ·7856770 | 47 | 34 |
| 14 | ·6188655 | ·787864 | 1·269253 | ·7854970 | 46 | 35 |
| 15 | ·6190939 | ·788336 | 1·268494 | ·7853169 | 45 | 36 |
| 16 | ·6193224 | ·788808 | 1·267735 | ·7851368 | 44 | 37 |
| 17 | ·6195507 | ·789280 | 1·266977 | ·7849566 | 43 | 38 |
| 18 | ·6197790 | ·789752 | 1·266219 | ·7847764 | 42 | 39 |
| 19 | ·6200073 | ·790224 | 1·265462 | ·7845961 | 41 | 40 |
| 20 | ·6202355 | ·790697 | 1·264706 | ·7844157 | 40 | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ |

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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ |
|---|---|---|---|---|---|---|
| 21 | ·6204636 | ·791170 | 1·263950 | ·7842352 | 39 | 41 |
| 22 | ·6206917 | ·791643 | 1·263195 | ·7840547 | 38 | 42 |
| 23 | ·6209198 | ·792116 | 1·262440 | ·7838741 | 37 | 43 |
| 24 | ·6211478 | ·792590 | 1·261686 | ·7836935 | 36 | 44 |
| 25 | ·6213757 | ·793064 | 1·260932 | ·7835127 | 35 | 45 |
| 26 | ·6216036 | ·793537 | 1·260179 | ·7833320 | 34 | 46 |
| 27 | ·6218314 | ·794012 | 1·259426 | ·7831511 | 33 | 47 |
| 28 | ·6220592 | ·794486 | 1·258674 | ·7829702 | 32 | 48 |
| 29 | ·6222870 | ·794961 | 1·257923 | ·7827892 | 31 | 49 |
| 30 | ·6225146 | ·795435 | 1·257172 | ·7826082 | 30 | 50 |
| 31 | ·6227423 | ·795911 | 1·256421 | ·7824270 | 29 | 51 |
| 32 | ·6229698 | ·796386 | 1·255672 | ·7822459 | 28 | 52 |
| 33 | ·6231974 | ·796861 | 1·254922 | ·7820646 | 27 | 53 |
| 34 | ·6234248 | ·797337 | 1·254174 | ·7818833 | 26 | 54 |
| 35 | ·6236522 | ·797813 | 1·253426 | ·7817019 | 25 | 55 |
| 36 | ·6238796 | ·798289 | 1·252679 | ·7815205 | 24 | 56 |
| 37 | ·6241069 | ·798765 | 1·251931 | ·7813390 | 23 | 57 |
| 38 | ·6243342 | ·799242 | 1·251184 | ·7811574 | 22 | 58 |
| 39 | ·6245614 | ·799719 | 1·250438 | ·7809757 | 21 | 59 |
| 40 | ·6247885 | ·800196 | 1·249693 | ·7807940 | 20 | 60 |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ |

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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
|---|---|---|---|---|---|
| 41 | ·6250156 | ·800673 | 1·248948 | ·7806123 | 19 |
| 42 | ·6252427 | ·801151 | 1·248204 | ·7804304 | 18 |
| 43 | ·6254696 | ·801628 | 1·247460 | ·7802485 | 17 |
| 44 | ·6256966 | ·802106 | 1·246716 | ·7800665 | 16 |
| 45 | ·6259235 | ·802584 | 1·245974 | ·7798845 | 15 |
| 46 | ·6261503 | ·803063 | 1·245232 | ·7797024 | 14 |
| 47 | ·6263771 | ·803541 | 1·244490 | ·7795202 | 13 |
| 48 | ·6266038 | ·804020 | 1·243749 | ·7793380 | 12 |
| 49 | ·6268305 | ·804499 | 1·243008 | ·7791557 | 11 |
| 50 | ·6270571 | ·804979 | 1·242268 | ·7789733 | 10 |
| 51 | ·6272837 | ·805458 | 1·241529 | ·7787909 | 9 |
| 52 | ·6275102 | ·805938 | 1·240790 | ·7786084 | 8 |
| 53 | ·6277366 | ·806418 | 1·240051 | ·7784258 | 7 |
| 54 | ·6279631 | ·806898 | 1·239313 | ·7782431 | 6 |
| 55 | ·6281894 | ·807378 | 1·238576 | ·7780604 | 5 |
| 56 | ·6284157 | ·807859 | 1·237839 | ·7778777 | 4 |
| 57 | ·6286420 | ·808340 | 1·237103 | ·7776949 | 3 |
| 58 | ·6288682 | ·808821 | 1·236367 | ·7775120 | 2 |
| 59 | ·6290943 | ·809302 | 1·235631 | ·7773290 | 1 |
| 60 | ·6293204 | ·809784 | 1·234897 | ·7771460 | 0 |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ |

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| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | ·6293204 | ·809784 | 1·234897 | ·7771460 | 60 | 21 | ·6340559 | ·819948 | 1·219588 | ·7732872 | 39 | 41 | ·6385440 | ·829724 | 1·205219 | ·7695353 | 19 |
| 1 | ·6295464 | ·810265 | 1·234162 | ·7769629 | 59 | 22 | ·6342808 | ·820435 | 1·218865 | ·7731027 | 38 | 42 | ·6387678 | ·830216 | 1·204505 | ·7693996 | 18 |
| 2 | ·6297724 | ·810747 | 1·233429 | ·7767797 | 58 | 23 | ·6345057 | ·820922 | 1·218142 | ·7729182 | 37 | 43 | ·6389916 | ·830707 | 1·203793 | ·7692137 | 17 |
| 3 | ·6299983 | ·811230 | 1·232696 | ·7765965 | 57 | 24 | ·6347305 | ·821409 | 1·217419 | ·7727336 | 36 | 44 | ·6392153 | ·831199 | 1·203081 | ·7690278 | 16 |
| 4 | ·6302242 | ·811712 | 1·231963 | ·7764132 | 56 | 25 | ·6349553 | ·821896 | 1·216698 | ·7725489 | 35 | 45 | ·6394390 | ·831691 | 1·202369 | ·7688418 | 15 |
| 5 | ·6304500 | ·812195 | 1·231231 | ·7762298 | 55 | 26 | ·6351800 | ·822384 | 1·215976 | ·7723642 | 34 | 46 | ·6396626 | ·832183 | 1·201658 | ·7686558 | 14 |
| 6 | ·6306758 | ·812678 | 1·230499 | ·7760464 | 54 | 27 | ·6354046 | ·822871 | 1·215256 | ·7721794 | 33 | 47 | ·6398862 | ·832675 | 1·200947 | ·7684697 | 13 |
| 7 | ·6309015 | ·813161 | 1·229768 | ·7758629 | 53 | 28 | ·6356292 | ·823359 | 1·214535 | ·7719945 | 32 | 48 | ·6401097 | ·833168 | 1·200237 | ·7682835 | 12 |
| 8 | ·6311272 | ·813644 | 1·229038 | ·7756794 | 52 | 29 | ·6358537 | ·823847 | 1·213816 | ·7718096 | 31 | 49 | ·6403332 | ·833661 | 1·199527 | ·7680973 | 11 |
| 9 | ·6313528 | ·814128 | 1·228308 | ·7754957 | 51 | 30 | ·6360782 | ·824336 | 1·213097 | ·7716246 | 30 | 50 | ·6405566 | ·834154 | 1·198818 | ·7679110 | 10 |
| 10 | ·6315784 | ·814611 | 1·227578 | ·7753121 | 50 | 31 | ·6363026 | ·824825 | 1·212378 | ·7714395 | 29 | 51 | ·6407799 | ·834648 | 1·198109 | ·7677246 | 9 |
| 11 | ·6318039 | ·815095 | 1·226849 | ·7751283 | 49 | 32 | ·6365270 | ·825314 | 1·211660 | ·7712544 | 28 | 52 | ·6410032 | ·835141 | 1·197401 | ·7675382 | 8 |
| 12 | ·6320293 | ·815580 | 1·226121 | ·7749445 | 48 | 33 | ·6367513 | ·825803 | 1·210942 | ·7710692 | 27 | 53 | ·6412264 | ·835635 | 1·196693 | ·7673517 | 7 |
| 13 | ·6322547 | ·816064 | 1·225393 | ·7747606 | 47 | 34 | ·6369756 | ·826292 | 1·210225 | ·7708840 | 26 | 54 | ·6414496 | ·836129 | 1·195986 | ·7671652 | 6 |
| 14 | ·6324800 | ·816549 | 1·224665 | ·7745767 | 46 | 35 | ·6371998 | ·826782 | 1·209508 | ·7706986 | 25 | 55 | ·6416728 | ·836624 | 1·195279 | ·7669785 | 5 |
| 15 | ·6327053 | ·817034 | 1·223938 | ·7743926 | 45 | 36 | ·6374240 | ·827271 | 1·208792 | ·7705132 | 24 | 56 | ·6418958 | ·837118 | 1·194573 | ·7667918 | 4 |
| 16 | ·6329306 | ·817519 | 1·223212 | ·7742086 | 44 | 37 | ·6376481 | ·827762 | 1·208076 | ·7703278 | 23 | 57 | ·6421189 | ·837613 | 1·193867 | ·7666051 | 3 |
| 17 | ·6331557 | ·818004 | 1·222486 | ·7740244 | 43 | 38 | ·6378721 | ·828252 | 1·207361 | ·7701423 | 22 | 58 | ·6423418 | ·838108 | 1·193162 | ·7664183 | 2 |
| 18 | ·6333809 | ·818490 | 1·221761 | ·7738402 | 42 | 39 | ·6380961 | ·828742 | 1·206646 | ·7699567 | 21 | 59 | ·6425647 | ·838604 | 1·192457 | ·7662314 | 1 |
| 19 | ·6336059 | ·818976 | 1·221036 | ·7736559 | 41 | 40 | ·6383201 | ·829233 | 1·205932 | ·7697710 | 20 | 60 | ·6427876 | ·839099 | 1·191753 | ·7660444 | 0 |
| 20 | ·6338310 | ·819462 | 1·220312 | ·7734716 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |

| 40° | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | | 40° | Sen. | Tang. | Cotang. | Cos. | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | ·6427876 | ·839099 | 1·191753 | ·7660444 | 60 | 21 | ·6474551 | ·849563 | 1·177075 | ·7621036 | 39 | 41 | ·6518778 | ·859629 | 1·163291 | ·7583240 | 19 |
| 1 | ·6430104 | ·839595 | 1·191049 | ·7658574 | 59 | 22 | ·6476767 | ·850064 | 1·176382 | ·7619152 | 38 | 42 | ·6520984 | ·860135 | 1·162607 | ·7581343 | 18 |
| 2 | ·6432332 | ·840091 | 1.190346 | ·7656704 | 58 | 23 | ·6478984 | ·850565 | 1·175688 | ·7617268 | 37 | 43 | 6523189 | ·860641 | 1·161923 | ·7579446 | 17 |
| 3 | ·6434559 | ·840587 | 1·189643 | ·7654832 | 57 | 24 | ·6481199 | ·851066 | 1·174996 | ·7615383 | 36 | 44 | ·6525394 | ·861148 | 1·161240 | ·7577548 | 16 |
| 4 | ·6436785 | ·841084 | 1·188941 | ·7652960 | 56 | 25 | ·6483414 | ·851568 | 1·174303 | ·7613497 | 35 | 45 | ·6527598 | ·861655 | 1·160557 | ·7575650 | 15 |
| 5 | ·6439011 | ·841581 | 1·188239 | ·7651087 | 55 | 26 | ·6485628 | ·852070 | 1·173612 | ·7611611 | 34 | 46 | ·6529801 | ·862162 | 1·159874 | ·7573751 | 14 |
| 6 | ·6441236 | ·842078 | 1·187538 | ·7649214 | 54 | 27 | ·6487842 | ·852572 | 1·172920 | ·7609724 | 33 | 47 | ·6532004 | ·862669 | 1·159192 | ·7571851 | 13 |
| 7 | ·6443461 | ·842575 | 1·186837 | ·7647340 | 53 | 28 | ·6490056 | ·853075 | 1·172229 | ·7607837 | 32 | 48 | ·6534206 | ·863176 | 1·158511 | ·7569951 | 12 |
| 8 | ·6445685 | ·843073 | 1·186136 | ·7645465 | 52 | 29 | ·6492268 | ·853577 | 1·171539 | ·7605949 | 31 | 49 | ·6536408 | ·863684 | 1·157830 | ·7568050 | 11 |
| 9 | ·6447909 | ·843570 | 1·185437 | ·7643590 | 51 | 30 | ·6494480 | ·854080 | 1·170849 | ·7604060 | 30 | 50 | ·6538609 | ·864192 | 1·157149 | ·7566148 | 10 |
| 10 | ·6450132 | ·844068 | 1·184737 | ·7641714 | 50 | 31 | ·6496692 | ·854583 | 1·170160 | ·7602170 | 29 | 51 | ·6540810 | ·864700 | 1·156469 | ·7564246 | 9 |
| 11 | ·6452355 | ·844567 | 1·184038 | ·7639838 | 49 | 32 | ·6498903 | ·855087 | 1·169471 | ·7600280 | 28 | 52 | ·6543010 | ·865209 | 1·155789 | ·7562343 | 8 |
| 12 | ·6454577 | ·845065 | 1·183340 | ·7637960 | 48 | 33 | ·6501114 | ·855591 | 1·168782 | ·7598389 | 27 | 53 | ·6545209 | ·865718 | 1·155110 | ·7560439 | 7 |
| 13 | ·6456798 | ·845564 | 1·182642 | ·7636082 | 47 | 34 | ·6503324 | ·856095 | 1·168094 | ·7596498 | 26 | 54 | ·6547408 | ·866227 | 1·154431 | ·7558535 | 6 |
| 14 | ·6459019 | ·846063 | 1·181944 | ·7634204 | 46 | 35 | ·6505533 | ·856599 | 1·167407 | ·7594606 | 25 | 55 | ·6549607 | ·866736 | 1·153753 | ·7556630 | 5 |
| 15 | ·6461240 | ·846562 | 1·181247 | ·7632325 | 45 | 36 | ·6507742 | ·857103 | 1·166720 | ·7592713 | 24 | 56 | ·6551804 | ·867246 | 1·153075 | ·7554724 | 4 |
| 16 | ·6463460 | ·847062 | 1·180551 | ·7630445 | 44 | 37 | ·6509951 | ·857608 | 1·166033 | ·7590820 | 23 | 57 | ·6554002 | ·867755 | 1·152397 | ·7552818 | 3 |
| 17 | ·6465679 | ·847561 | 1·179855 | ·7628564 | 43 | 38 | ·6512158 | ·858113 | 1·165347 | ·7588926 | 22 | 58 | ·6556198 | ·868265 | 1·151721 | ·7550911 | 2 |
| 18 | ·6467898 | ·848061 | 1·179159 | ·7626683 | 42 | 39 | ·6514366 | ·858618 | 1·164661 | ·7587031 | 21 | 59 | ·6558395 | ·868776 | 1·151044 | ·7549004 | 1 |
| 19 | ·6470116 | ·848561 | 1·178464 | ·7624802 | 41 | 40 | ·6516572 | ·859124 | 1·163976 | ·7585136 | 20 | 60 | ·6560590 | ·869286 | 1·150368 | ·7547096 | 0 |
| 20 | ·6472334 | ·849062 | 1·177769 | ·7622919 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | | | Cos. | Cotang. | Tang. | Sen. | | | Cos. | Cotang. | Tang. | Sen. | ′ |
| | 49° | | | | | | | 49° | | | | | | 49° | | | |

| 41° | | | | | | 41° | | | | | | 41° | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·6560590 | ·869286 | 1·150368 | ·7547096 | 60 | 21 | ·6606570 | ·880068 | 1·136274 | ·7506879 | 39 | 41 | ·6650131 | ·890445 | 1·123032 | ·7468317 | 19 |
| 1 | ·6562785 | ·869797 | 1·149692 | ·7545187 | 59 | 22 | ·6608754 | ·880585 | 1·135608 | ·7504957 | 38 | 42 | ·6652304 | ·890967 | 1·122375 | ·7466382 | 18 |
| 2 | ·6564980 | ·870308 | 1·149017 | ·7543278 | 58 | 23 | ·6610936 | ·881101 | 1·134942 | ·7503034 | 37 | 43 | ·6654475 | ·891489 | 1·121718 | ·7464446 | 17 |
| 3 | 6567174 | ·870820 | 1·148342 | ·7541368 | 57 | 24 | ·6613119 | ·881618 | 1·134277 | ·7501111 | 36 | 44 | ·6656646 | ·892011 | 1·121061 | ·7462510 | 16 |
| 4 | ·6569367 | ·871331 | 1·147668 | ·7539457 | 56 | 25 | ·6615300 | ·882135 | 1·133612 | ·7499187 | 35 | 45 | ·6658817 | ·892534 | 1·120405 | ·7460574 | 15 |
| 5 | ·6571560 | ·871843 | 1·146994 | ·7537546 | 55 | 26 | ·6617482 | ·882653 | 1·132947 | ·7497262 | 34 | 46 | ·6660987 | ·893056 | 1·119749 | ·7458636 | 14 |
| 6 | ·6573752 | ·872355 | 1·146321 | ·7535634 | 54 | 27 | ·6619662 | ·883170 | 1·132283 | ·7495337 | 33 | 47 | ·6663156 | ·893579 | 1·119094 | ·7456699 | 13 |
| 7 | ·6575944 | ·872868 | 1·145648 | ·7533721 | 53 | 28 | ·6621842 | ·883688 | 1·131620 | ·7493411 | 32 | 48 | ·6665325 | ·894103 | 1·118439 | ·7454760 | 12 |
| 8 | ·6578135 | ·873380 | 1·144976 | ·7531808 | 52 | 29 | ·6624022 | ·884206 | 1·130957 | ·7491484 | 31 | 49 | ·6667493 | ·894626 | 1·117784 | ·7452821 | 11 |
| 9 | ·6580326 | ·873893 | 1·144304 | ·7529894 | 51 | 30 | ·6626200 | ·884725 | 1·130294 | ·7489557 | 30 | 50 | ·6669661 | ·895150 | 1·117130 | ·7450881 | 10 |
| 10 | ·6582516 | ·874406 | 1·143632 | ·7527980 | 50 | 31 | ·6628379 | ·885244 | 1·129632 | ·7487629 | 29 | 51 | ·6671828 | ·895674 | 1·116476 | ·7448941 | 9 |
| 11 | ·6584706 | ·874920 | 1·142961 | ·7526065 | 49 | 32 | ·6630557 | ·885763 | 1·128970 | ·7485701 | 28 | 52 | ·6673994 | ·896199 | 1·115823 | ·7446999 | 8 |
| 12 | ·6586895 | ·875433 | 1·142290 | ·7524149 | 48 | 33 | ·6632734 | ·886282 | 1·128308 | ·7483772 | 27 | 53 | ·6676160 | ·896723 | 1·115170 | ·7445058 | 7 |
| 13 | ·6589083 | ·875947 | 1·141620 | ·7522233 | 47 | 34 | ·6634910 | ·886801 | 1·127647 | ·7481842 | 26 | 54 | ·6678326 | ·897248 | 1·114518 | ·7443115 | 6 |
| 14 | ·6591271 | ·876462 | 1·140950 | ·7520316 | 46 | 35 | ·6637087 | ·887321 | 1·126987 | ·7479912 | 25 | 55 | ·6680490 | ·897773 | 1·113866 | ·7441173 | 5 |
| 15 | ·6593458 | ·876976 | 1·140281 | ·7518398 | 45 | 36 | ·6639262 | ·887841 | 1·126327 | ·7477981 | 24 | 56 | ·6682655 | ·898299 | 1·113214 | ·7439229 | 4 |
| 16 | ·6595645 | ·877491 | 1·139612 | ·7516480 | 44 | 37 | ·6641437 | ·888361 | 1·125667 | ·7476049 | 23 | 57 | ·6684818 | ·898825 | 1·112563 | ·7437285 | 3 |
| 17 | ·6597831 | ·878006 | 1·138944 | ·7514561 | 43 | 38 | ·6643612 | ·888882 | 1·125008 | ·7474117 | 22 | 58 | ·6686981 | ·899351 | 1·111912 | ·7435340 | 2 |
| 18 | ·6600017 | ·878521 | 1·138276 | ·7512641 | 42 | 39 | ·6645785 | ·889403 | 1·124349 | ·7472184 | 21 | 59 | ·6689144 | ·899877 | 1·111262 | ·7433394 | 1 |
| 19 | ·6602202 | ·879037 | 1·137608 | ·7510721 | 41 | 40 | ·6647959 | ·889924 | 1·123690 | ·7470251 | 20 | 60 | ·6691306 | ·900404 | 1·110612 | ·7431448 | 0 |
| 20 | ·6604386 | ·879552 | 1·136941 | ·7508800 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
| | 48° | | | | | | 48° | | | | | | 48° | | | | |

| 42° | | | | | | 42° | | | | | | 42° | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·6691306 | ·900404 | 1·110612 | ·7431448 | 60 | 21 | ·6736577 | ·911526 | 1·097060 | ·7390435 | 39 | 41 | ·6779459 | ·922235 | 1.084322 | ·7351118 | 19 |
| 1 | ·6693468 | ·900930 | 1·109963 | ·7429502 | 59 | 22 | ·6738727 | ·912059 | 1·096420 | ·7388475 | 38 | 42 | ·6781597 | ·922773 | 1·083689 | ·7349146 | 18 |
| 2 | ·6695628 | ·901458 | 1·109314 | ·7427554 | 58 | 23 | ·6740876 | ·912592 | 1·095779 | ·7386515 | 37 | 43 | ·6783734 | ·923312 | 1·083057 | ·7347173 | 17 |
| 3 | ·6697789 | ·901985 | 1·108665 | ·7425606 | 57 | 24 | ·6743024 | ·913125 | 1·095139 | ·7384553 | 36 | 44 | ·6785871 | ·923851 | 1·082425 | ·7345199 | 16 |
| 4 | ·6699948 | ·902513 | 1·108017 | ·7423658 | 56 | 25 | ·6745172 | ·913659 | 1·094500 | ·7382592 | 35 | 45 | ·6788007 | ·924391 | 1·081793 | ·7343225 | 15 |
| 5 | ·6702108 | ·903041 | 1·107369 | ·7421708 | 55 | 26 | ·6747319 | ·914192 | 1·093861 | ·7380629 | 34 | 46 | ·6790143 | ·924930 | 1·081162 | ·7341250 | 14 |
| 6 | ·6704266 | ·903569 | 1·106721 | ·7419758 | 54 | 27 | ·6749466 | ·914727 | 1·093222 | ·7378666 | 33 | 47 | ·6792278 | ·925470 | 1·080532 | ·7339275 | 13 |
| 7 | ·6706424 | ·904097 | 1·106075 | ·7417808 | 53 | 28 | ·6751612 | ·915261 | 1·092584 | ·7376703 | 32 | 48 | ·6794413 | ·926010 | 1·079901 | ·7337299 | 12 |
| 8 | ·6708582 | ·904626 | 1·105428 | ·7415857 | 52 | 29 | ·6753757 | ·915796 | 1·091946 | ·7374738 | 31 | 49 | ·6796547 | ·926550 | 1·079271 | ·7335322 | 11 |
| 9 | ·6710739 | ·905155 | 1·104782 | ·7413905 | 51 | 30 | ·6755902 | ·916331 | 1·091308 | ·7372773 | 30 | 50 | ·6798681 | ·927091 | 1·078642 | ·7333345 | 10 |
| 10 | ·6712895 | ·905685 | 1·104136 | ·7411953 | 50 | 31 | ·6758046 | ·916866 | 1·090671 | ·7370808 | 29 | 51 | ·6800813 | ·927632 | 1·078013 | ·7331367 | 9 |
| 11 | ·6715051 | ·906214 | 1·103491 | ·7410000 | 49 | 32 | ·6760190 | ·917402 | 1·090034 | ·7368842 | 28 | 52 | ·6802946 | ·928173 | 1·077384 | ·7329388 | 8 |
| 12 | ·6717206 | ·906744 | 1·102846 | ·7408046 | 48 | 33 | ·6762333 | ·917937 | 1·089398 | ·7366875 | 27 | 53 | ·6805078 | ·928715 | 1·076756 | ·7327409 | 7 |
| 13 | ·6719361 | ·907274 | 1·102201 | ·7406092 | 47 | 34 | ·6764476 | ·918474 | 1·088762 | ·7364908 | 26 | 54 | ·6807209 | ·929257 | 1·076128 | ·7325429 | 6 |
| 14 | ·6721515 | ·907805 | 1·101557 | ·7404137 | 46 | 35 | ·6766618 | ·919010 | 1·088126 | ·7362940 | 25 | 55 | ·6809339 | ·929799 | 1·075500 | ·7323449 | 5 |
| 15 | ·6725668 | ·908336 | 1·100914 | ·7402181 | 45 | 36 | ·6768760 | ·919547 | 1·087491 | ·7360971 | 24 | 56 | ·6811469 | ·930342 | 1·074873 | ·7321467 | 4 |
| 16 | ·6725821 | ·908867 | 1·100270 | ·7400225 | 44 | 37 | ·6770901 | ·920084 | 1·086857 | ·7359002 | 23 | 57 | ·6813599 | ·930884 | 1·074246 | ·7319486 | 3 |
| 17 | ·6727973 | ·909398 | 1·099628 | ·7398268 | 43 | 38 | ·6773041 | ·920621 | 1·086222 | ·7357032 | 22 | 58 | ·6815728 | ·931428 | 1·073620 | ·7317503 | 2 |
| 18 | ·6730125 | ·909920 | 1·098985 | ·7396311 | 42 | 39 | ·6775181 | ·921159 | 1·085588 | ·7355061 | 21 | 59 | ·6817856 | ·931971 | 1·072994 | ·7315521 | 1 |
| 19 | ·6732276 | ·910461 | 1·098343 | ·7394353 | 41 | 40 | ·6777320 | ·921696 | 1·084955 | ·7353090 | 20 | 60 | ·6819984 | ·932515 | 1·072368 | ·7313537 | 0 |
| 20 | ·6734427 | ·910994 | 1·097702 | ·7392394 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
| | | | | 47° | | | | | 47° | | | | | | 47° | | |

| ′ | Sen. | Tang. | Cotang. | Cos. | | ′ | Sen. | Tang. | Cotang. | Cos. | | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | ·6819984 | ·932515 | 1·072368 | ·7313537 | 60 | 21 | ·6864532 | ·944001 | 1·059320 | ·7271740 | 39 | 41 | ·6906721 | ·955064 | 1·047049 | ·7231681 | 19 |
| 1 | ·6822111 | ·933059 | 1·071743 | ·7311553 | 59 | 22 | ·6866647 | ·944551 | 1·058703 | ·7269743 | 38 | 42 | ·6908824 | ·955620 | 1·046440 | ·7229671 | 18 |
| 2 | ·6824237 | ·933603 | 1·071118 | ·7309568 | 58 | 23 | ·6868761 | ·945102 | 1·058086 | ·7267745 | 37 | 43 | ·6910927 | ·956177 | 1·045831 | ·7227661 | 17 |
| 3 | ·6826363 | ·934147 | 1·070494 | ·7307583 | 57 | 24 | ·6870875 | ·945653 | 1·057470 | ·7265747 | 36 | 44 | ·6913029 | ·956734 | 1·045222 | ·7225651 | 16 |
| 4 | ·6828489 | ·934692 | 1·069870 | ·7305597 | 56 | 25 | ·6872988 | ·946204 | 1·056854 | ·7263748 | 35 | 45 | ·6915131 | ·957291 | 1·044613 | ·7223640 | 15 |
| 5 | ·6830613 | ·935238 | 1·069246 | ·7303610 | 55 | 26 | ·6875101 | ·946755 | 1·056238 | ·7261748 | 34 | 46 | ·6917232 | ·957849 | 1·044005 | ·7221628 | 14 |
| 6 | ·6832738 | ·935783 | 1·068623 | ·7301623 | 54 | 27 | ·6877213 | ·947307 | 1·055623 | ·7259748 | 33 | 47 | ·6919332 | ·958407 | 1·043397 | ·7219615 | 13 |
| 7 | ·6834861 | ·936329 | 1·068000 | ·7299635 | 53 | 28 | ·6879325 | ·947859 | 1·055008 | ·7257747 | 32 | 48 | ·6921432 | ·958965 | 1·042790 | ·7217602 | 12 |
| 8 | ·6836984 | ·936875 | 1·067377 | ·7297646 | 52 | 29 | ·6881435 | ·948411 | 1·054394 | ·7255746 | 31 | 49 | ·6923531 | ·959524 | 1·042183 | ·7215589 | 11 |
| 9 | ·6839107 | ·937421 | 1·066755 | ·7295657 | 51 | 30 | ·6883546 | ·948964 | 1·053780 | ·7253744 | 30 | 50 | ·6925630 | ·960082 | 1·041576 | ·7213574 | 10 |
| 10 | ·6841229 | ·937968 | 1·066134 | ·7293668 | 50 | 31 | ·6885655 | ·949517 | 1·053166 | ·7251741 | 29 | 51 | ·6927728 | ·960642 | 1·040970 | ·7211559 | 9 |
| 11 | ·6843350 | ·938515 | 1·065512 | ·7291677 | 49 | 32 | ·6887765 | ·950070 | 1·052553 | ·7249738 | 28 | 52 | ·6929825 | ·961201 | 1·040364 | ·7209544 | 8 |
| 12 | ·6845471 | ·939062 | 1·064891 | ·7289686 | 48 | 33 | ·6889873 | ·950624 | 1·051940 | ·7247734 | 27 | 53 | ·6931922 | ·961761 | 1·039758 | ·7207528 | 7 |
| 13 | ·6847591 | ·939610 | 1·064271 | ·7287695 | 47 | 34 | ·6891981 | ·951178 | 1·051327 | ·7245729 | 26 | 54 | ·6934018 | ·962321 | 1·039153 | ·7205511 | 6 |
| 14 | ·6849711 | ·940157 | 1·063651 | ·7285703 | 46 | 35 | ·6894089 | ·951732 | 1·050715 | ·7243724 | 25 | 55 | ·6936114 | ·962881 | 1·038548 | ·7203494 | 5 |
| 15 | ·6851830 | ·940706 | 1·063031 | ·7283710 | 45 | 36 | ·6896195 | ·952287 | 1·050103 | ·7241719 | 24 | 56 | ·6938209 | ·963442 | 1·037944 | ·7201476 | 4 |
| 16 | ·6853948 | ·941254 | 1·062411 | ·7281716 | 44 | 37 | ·6898302 | ·952842 | 1·049492 | ·7239712 | 23 | 57 | ·6940304 | ·964003 | 1·037340 | ·7199457 | 3 |
| 17 | ·6856066 | ·941803 | 1·061792 | ·7279722 | 43 | 38 | ·6900407 | ·953397 | 1·048880 | ·7237705 | 22 | 58 | ·6942398 | ·964565 | 1·036736 | ·7197438 | 2 |
| 18 | ·6858184 | ·942352 | 1·061174 | ·7277728 | 42 | 39 | ·6902512 | ·953952 | 1·048270 | ·7235698 | 21 | 59 | ·6944491 | ·965126 | 1·036133 | ·7195419 | 1 |
| 19 | ·6860300 | ·942901 | 1·060556 | ·7275732 | 41 | 40 | ·6904617 | ·954508 | 1·047659 | ·7233690 | 20 | 60 | ·6946584 | ·965688 | 1·035530 | ·7193398 | 0 |
| 20 | ·6862416 | ·943451 | 1·059938 | ·7273736 | 40 | | | | | | | | | | | | |
| ′ | Cos. | Cotang. | Tang. | Sen. | | ′ | Cos. | Cotang. | Tang. | Sen. | | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |

| 44° | | | | | 44° | | | | | 44° | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ | Sen. | Tang. | Cotang. | Cos. | ′ |
| 0 | ·6946584 | ·965688 | 1·035530 | ·7193398 | 60 | 21 | ·6990396 | ·977564 | 1·022950 | ·7150830 | 39 | 41 | ·7031879 | ·989006 | 1·011115 | ·7110041 | 19 |
| 1 | ·6948676 | ·966251 | 1·034927 | ·7191377 | 59 | 22 | ·6992476 | ·978133 | 1·022355 | ·7148796 | 38 | 42 | ·7033947 | ·989582 | 1·010527 | ·7107995 | 18 |
| 2 | ·6950767 | ·966813 | 1·034325 | ·7189355 | 58 | 23 | ·6994555 | ·978702 | 1·021760 | ·7146762 | 37 | 43 | ·7036014 | ·990158 | 1·009939 | ·7105948 | 17 |
| 3 | 6952858 | ·967376 | 1·033723 | ·7187333 | 57 | 24 | ·6996633 | ·979272 | 1·021166 | ·7144727 | 36 | 44 | ·7038081 | ·990734 | 1·009352 | ·7103901 | 16 |
| 4 | ·6954949 | ·967939 | 1·033122 | ·7185310 | 56 | 25 | ·6998711 | ·979842 | 1·020572 | ·7142691 | 35 | 45 | ·7040147 | ·991311 | 1·008764 | ·7101854 | 15 |
| 5 | ·6957039 | ·968503 | 1·032520 | ·7183287 | 55 | 26 | ·7000789 | ·980412 | 1·019978 | ·7140655 | 34 | 46 | ·7042213 | ·991888 | 1·008178 | ·7099806 | 14 |
| 6 | ·6959128 | ·969067 | 1·031919 | ·7181263 | 54 | 27 | ·7002866 | ·980983 | 1·019385 | ·7138618 | 33 | 47 | ·7044278 | ·992465 | 1·007591 | ·7097757 | 13 |
| 7 | ·6961217 | ·969631 | 1·031319 | ·7179238 | 53 | 28 | ·7004942 | ·981554 | 1·018792 | ·7136581 | 32 | 48 | ·7046342 | ·993042 | 1·007005 | ·7095707 | 12 |
| 8 | ·6963305 | ·970196 | 1·030719 | ·7177213 | 52 | 29 | ·7007018 | ·982125 | 1·018199 | ·7134543 | 31 | 49 | ·7048406 | ·993620 | 1·006420 | ·7093657 | 11 |
| 9 | ·6965392 | ·970761 | 1·030119 | ·7175187 | 51 | 30 | ·7009093 | ·982697 | 1·017607 | ·7132504 | 30 | 50 | ·7050469 | ·994199 | 1·005834 | ·7091607 | 10 |
| 10 | ·6967479 | ·971326 | 1·029520 | ·7173161 | 50 | 31 | ·7011167 | ·983269 | 1·017015 | ·7130465 | 29 | 51 | ·7052532 | ·994777 | 1·005249 | ·7089556 | 9 |
| 11 | ·6969565 | ·971891 | 1·028921 | ·7171134 | 49 | 32 | ·7013241 | ·983841 | 1·016423 | ·7128426 | 28 | 52 | ·7054594 | ·995356 | 1·004665 | ·7087504 | 8 |
| 12 | ·6971651 | ·972457 | 1·028322 | ·7169106 | 48 | 33 | ·7015314 | ·984414 | 1·015832 | ·7126385 | 27 | 53 | ·7056655 | ·995935 | 1·004080 | ·7085451 | 7 |
| 13 | ·6973736 | ·973023 | 1·027724 | ·7167078 | 47 | 34 | ·7017387 | ·984987 | 1·015241 | ·7124344 | 26 | 54 | ·7058716 | ·996515 | 1·003496 | ·7083398 | 6 |
| 14 | ·6975821 | ·973590 | 1·027126 | ·7165049 | 46 | 35 | ·7019459 | ·985560 | 1·014651 | ·7122303 | 25 | 55 | ·7060776 | ·997095 | 1·002913 | ·7081345 | 5 |
| 15 | ·6977905 | ·974156 | 1·026528 | ·7163019 | 45 | 36 | ·7021531 | ·986133 | 1·014061 | ·7120260 | 24 | 56 | ·7062835 | ·997675 | 1·002329 | ·7079291 | 4 |
| 16 | ·6979988 | ·974724 | 1·025931 | ·7160989 | 44 | 37 | ·7023601 | ·986707 | 1·013471 | ·7118218 | 23 | 57 | ·7064894 | ·998256 | 1·001746 | ·7077236 | 3 |
| 17 | ·6982071 | ·975291 | 1·025334 | ·7158959 | 43 | 38 | ·7025672 | ·987282 | 1·012881 | ·7116174 | 22 | 58 | ·7066953 | ·998837 | 1·001164 | ·7075180 | 2 |
| 18 | ·6984153 | ·975859 | 1·024738 | ·7156927 | 42 | 39 | ·7027741 | ·987856 | 1·012292 | ·7114130 | 21 | 59 | ·7069011 | ·999418 | 1·000581 | ·7073124 | 1 |
| 19 | ·6986234 | ·976427 | 1·024141 | ·7154895 | 41 | 40 | ·7029811 | ·988431 | 1·011703 | ·7112086 | 20 | 60 | ·7071068 | 1·00000 | 1·000000 | ·7071068 | 0 |
| 20 | ·6988315 | ·976995 | 1·023546 | ·7152863 | 40 | | | | | | | | | | | | |
| | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ | ′ | Cos. | Cotang. | Tang. | Sen. | ′ |
| | 45° | | | | | | 45° | | | | | | 45° | | | | |

# LOGARITMOS DE LAS FUNCIONES TRIGONOMÉTRICAS

(1) **El logaritmo del seno, tang, etc.** (log sen, log tan, etc.), de un arco es el *logaritmo* del seno, tang, etc., *natural* de dicho arco. Así:

sen nat 37°= .60182; y
log sen 37°=log .60182 =1.779 463
= .779 463 — 1
=9.779 463 — 10.

En las tablas, con el objeto de evitar las cantidades negativas, se emplea la *tercera* forma (log sen 37°=9.779 463 — 10); pero el « — 10 » se sobrentiende que está omitido.

Damos **dos tablas** de los logs de las funciones trigonométricas, es decir: una

« **tabla principal** », págs. 151*j* á 152*x*
y una
« **tabla especial** », págs. 151*c* á 151*i*.

La **principal** da dichos valores para cada **minuto** del cuadrante de 0° á 90°.

La **especial**, los da á **pequeños intervalos**, para ángulos pequeños y grandes (de 0° á 2° y de 88° á 90°), en los cuales ciertas funciones cambian tan rápidamente, que á los intervalos de un minuto entre los arcos corresponden valores de las funciones demasiado amplios para interpolaciones satisfactorias. Véanse §§ 7 á 9.

## Tabla principal.

(2) **Esta tabla**, págs. 151*j* á 152*x* da con 6 decimales el

**log sen, log cos, log tan y log cot** *

del arco de minuto en minuto ó sea de 0°0′ á 89°60′ (90°). Véase también (6) más abajo.

(3) Para arcos de 0° á 45°; **léase hacia abajo**, como en la columna izquierda de los minutos, usando los títulos que están á la cabeza de las columnas. Para arcos de 45° á 90°, **léase hacia arriba**, como en la columna de la derecha, usando los títulos que están al pie de las columnas.

(4) **Ejemplo.** Encontrar el log del sen 7°34′. En la cabeza de la pág. 151*o*, á la izquierda, en tipo grueso, se encuentra 7°; y debajo de él, en la primera columna ó columna de los minutos (leyendo *hacia abajo*), encontramos 34; opuesto á éste en la próxima ó segunda col, *bajo* « Seno », encontramos 9.119 519, el log buscado de 7°34′.

(5) **Ejemplo.** Encontrar el log cos 82°26′. Al pie de la pág. 151*o*, á la derecha, en tipo grueso, encontramos 82°; y sobre éstos, en la col de la derecha ó col de los minutos (leyendo hacia arriba), encontramos 26; opuesto al cual, en la segunda columna de izquierda á derecha **sobre** « Coseno », encontramos 9.119 519, que es el log cos 82°26′ buscado.

(6) **Para arcos entre 90° y 180°**, tenemos las relaciones siguientes:

sen $a$ = sen (180 — $a$); cos $a$ = 0 — cos (180° — $a$);
tan $a$ = 0 — tan (180° — $a$); cot $a$ = 0 — cot (180° — $a$).

---

* **Secante** $a = \frac{1}{\cos a}$; log sec $a$ = 0 — log cos $a$. **Cosecante** $a = \frac{1}{\text{sen } a}$; log cosec = 0 — log sen $a$.

Por tanto, log sen, log cos, log tan y log cot de un ángulo $a$, entre 90° y 180°, son numéricamente iguales respectivamente á los log sen, log cos, log tan, log cot, **del suplemento** $180° - a$. En la tabla principal, estas funciones se encuentran directamente usando los encabezamientos en tipo grueso, en los ángulos de la parte baja de la izquierda y alta de la derecha; notando que, la columna de los minutos, á la *izquierda*, pertenece á los grados de la izquierda (ya estén á la cabeza ó pie de la columna, así como la columna de los minutos á la derecha pertenece á los grados de la derecha.

Así: log sen 172°26′ = 9.119 519;
log (0 — cos 172°26′) = 9.996 202.

(7) **Para arcos intermedios de los dados en la tabla principal** (págs. 151*j* á 152*x*), si el arco no está entre 0° y 2° ó entre 88° y 90°, ó entre 178° y 180° (véanse §§ 8 y 9), **interpólese** por medio de la columna encabezada D. 1″ (que significa, diferencia de las funciones para 1 segundo en los ángulos) como en el ejemplo siguiente:

| Dado. | Se busca. | La tabla da D. 1″ = 15.87 y 9.118 567 = | Resultado. |
|---|---|---|---|
| $a$ = 7°33′12″ | log sen $a$ | log sen 7°33′0″ | 9.118 567 + 12 D. 1″ = 9.118 757; |
| $a$ = 82°26′48″ | log cos $a$ | log cos 82°27′0″ | 9.118 567 + 12 D. 1″ = 9.118 757; |
| log sen $a$ = 9.118 757 | $a$ | log sen 7°33′0″ | 7°33′0″ + 190 * D.1″ = 7°33′12″; |
| log cos $a$ = 9.118 757 | $a$ | log cos 82°27′0″ | 82°27′0″ — 190 * D.1″ = 82°26′48″. |

**Tabla especial, págs. 151*c* á 151*i*.**

(8) Para el log sen, tan, cot, de los arcos de 0°0′ á 1°60′ (= 2°0′); y para los log cos, cot, tan, de arcos de 88°0′ á 89°60′ (= 90°0′); la diferencia entre los sucesivos logaritmos varía tan rápidamente, que el uso de la columna D. 1″, con intervalos de 1 minuto entre arcos tabulares, daría resultados de insuficiente exactitud.

(9) Para estos casos damos la **tabla especial de 5 decimales**, págs. 1083 á 1090, con más pequeños intervalos entre los arcos, así:

| para arcos de | intervalo | páginas. |
|---|---|---|
| 0°00′ á 0°18′ | 1 segundo | 1083 — 5 |
| 0°18′ á 0°30′ | 3 segundos | 1086 — 7 |
| 0°30′ á 0°40′ | 5 — | 1088 |
| 0°40′ á 0°60′ | 10 — | 1089 |
| 1° 0′ á 1°60′ | 30 — | 1090 |

(10) **Tangentes de ángulos próximos á 90°, y cotangentes de ángulos próximos á 0°**, no se dan en la tabla especial, pero tenemos:

$$\log \tan a = 0 - \log \cot a; \qquad \log \cot a = 0 - \log \tan a.$$

(11) Con funciones de sólo cinco decimales, tenemos:

en arcos de 0° á 0°18′; log sen $a$ = log tan $a$ = 0 — log cot $a$;
— 89°42′ á 90°; log cos $a$ = log cot $a$ = 0 — log tan $a$.

(12) Para arcos de 0°0′ á 0°18′ (á un segundo de intervalo), por supuesto que no se necesita de interpolación, para segundos completos. Para los otros ángulos arriba mencionados, el mayor **error debido á la interpolación** es menor que 1 en la quinta decimal. En la tabla principal, log sen 2° 0′ 30″, el **error debido á la interpolación**, es como .4 de la quinta cifra.

**Ejemplos** de interpolación usando la tabla especial.

La tabla da

Dado, $a$ = 1°48′10″ } log sen 1°48′30″ = 8.49 908
Se pide, log sen $a$ } log sen 1°48′ 0″ = 8.49 708
dif 0° 0′30″ 0.00 200

$$\text{Resultado} = \log \text{ sen } a = 8.49\ 708 + \frac{10}{30} \times 200 = 8.49\ 775$$

* 9.118 757 — 9.118 567 = .000 190.

| | | La tabla de |
|---|---|---|
| Dado, log sen $a$ = 8.49775 | 8.49 908 | = log sen 1°48'30" |
| Se pide, $a$ | 8.49 708 | = log sen 1°48' 0" |
| | dif 0.00 200 | 0° 0'30" |

$$\text{Resultado} = a = 1°48'0'' + \frac{67^*}{200} \times 30'' = 1°48'10''$$

---

* 67 = 8.49 775 — 8.49 708.

(*N. del T.* — En las tablas que siguen en encuentra *sine* en inglés por *seno* y *cosine* por *coseno*, etc.; pero como se comprende tan facilmente las lineas á que se refieren, hemos pasado sobre eso para aprovechar en esta parte los mismos clisés del autor inglés.)

| Log sine = Log tangent = 0 − Log cotangent. | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0° | 0′ | 1′ | 2′ | 3′ | 4′ | 5′ | |
| 0″ | −∞ | 6.46373 | 6.76476 | 6.94085 | 7.06579 | 7.16270 | 60″ |
| 1″ | 4.68557 | 6.47090 | 6.76836 | 6.94325 | 7.06759 | 7.16414 | 59″ |
| 2″ | 4.98660 | 6.47797 | 6.77193 | 6.94565 | 7.06939 | 7.16558 | 58″ |
| 3″ | 5.16270 | 6.48492 | 6.77548 | 6.94803 | 7.07118 | 7.16702 | 57″ |
| 4″ | 5.28763 | 6.49175 | 6.77900 | 6.95039 | 7.07296 | 7.16845 | 56″ |
| 5″ | 5.38454 | 6.49849 | 6.78248 | 6.95275 | 7.07474 | 7.16987 | 55″ |
| 6″ | 5.46373 | 6.50512 | 6.78595 | 6.95509 | 7.07651 | 7.17130 | 54″ |
| 7″ | 5.53067 | 6.51165 | 6.78938 | 6.95742 | 7.07827 | 7.17271 | 53″ |
| 8″ | 5.58866 | 6.51808 | 6.79278 | 6.95973 | 7.08003 | 7.17413 | 52″ |
| 9″ | 5.63982 | 6.52442 | 6.79616 | 6.96204 | 7.08177 | 7.17553 | 51″ |
| 10″ | 5.68557 | 6.53067 | 6.79952 | 6.96433 | 7.08351 | 7.17694 | 50″ |
| 11″ | 5.72697 | 6.53683 | 6.80285 | 6.96661 | 7.08525 | 7.17834 | 49″ |
| 12″ | 5.76476 | 6.54291 | 6.80615 | 6.96888 | 7.08698 | 7.17973 | 48″ |
| 13″ | 5.79952 | 6.54890 | 6.80943 | 6.97113 | 7.08870 | 7.18112 | 47″ |
| 14″ | 5.83170 | 6.55481 | 6.81268 | 6.97338 | 7.09041 | 7.18250 | 46″ |
| 15″ | 5.86167 | 6.56064 | 6.81591 | 6.97561 | 7.09211 | 7.18389 | 45″ |
| 16″ | 5.88969 | 6.56639 | 6.81911 | 6.97783 | 7.09381 | 7.18526 | 44″ |
| 17″ | 5.91602 | 6.57207 | 6.82230 | 6.98004 | 7.09551 | 7.18663 | 43″ |
| 18″ | 5.94085 | 6.57767 | 6.82545 | 6.98224 | 7.09719 | 7.18800 | 42″ |
| 19″ | 5.96433 | 6.58320 | 6.82859 | 6.98443 | 7.09887 | 7.18937 | 41″ |
| 20″ | 5.98660 | 6.58866 | 6.83170 | 6.98660 | 7.10055 | 7.19072 | 40″ |
| 21″ | 6.00779 | 6.59406 | 6.83479 | 6.98877 | 7.10222 | 7.19208 | 39″ |
| 22″ | 6.02800 | 6.59939 | 6.83786 | 6.99093 | 7.10388 | 7.19343 | 38″ |
| 23″ | 6.04730 | 6.60465 | 6.84091 | 6.99307 | 7.10553 | 7.19478 | 37″ |
| 24″ | 6.06579 | 6.60985 | 6.84394 | 6.99521 | 7.10718 | 7.19612 | 36″ |
| 25″ | 6.08351 | 6.61499 | 6.84694 | 6.99733 | 7.10882 | 7.19746 | 35″ |
| 26″ | 6.10055 | 6.62007 | 6.84993 | 6.99944 | 7.11046 | 7.19879 | 34″ |
| 27″ | 6.11694 | 6.62509 | 6.85289 | 7.00155 | 7.11209 | 7.20012 | 33″ |
| 28″ | 6.13273 | 6.63006 | 6.85584 | 7.00364 | 7.11371 | 7.20145 | 32″ |
| 29″ | 6.14797 | 6.63496 | 6.85876 | 7.00572 | 7.11533 | 7.20277 | 31″ |
| 30″ | 6.16270 | 6.63982 | 6.86167 | 7.00779 | 7.11694 | 7.20409 | 30″ |
| 31″ | 6.17694 | 6.64462 | 6.86455 | 7.00986 | 7.11854 | 7.20540 | 29″ |
| 32″ | 6.19072 | 6.64936 | 6.86742 | 7.01191 | 7.12014 | 7.20671 | 28″ |
| 33″ | 6.20409 | 6.65406 | 6.87027 | 7.01395 | 7.12174 | 7.20802 | 27″ |
| 34″ | 6.21705 | 6.65870 | 6.87310 | 7.01599 | 7.12333 | 7.20932 | 26″ |
| 35″ | 6.22964 | 6.66330 | 6.87591 | 7.01801 | 7.12491 | 7.21062 | 25″ |
| 36″ | 6.24188 | 6.66785 | 6.87870 | 7.02003 | 7.12648 | 7.21191 | 24″ |
| 37″ | 6.25378 | 6.67235 | 6.88147 | 7.02203 | 7.12805 | 7.21320 | 23″ |
| 38″ | 6.26536 | 6.67680 | 6.88423 | 7.02403 | 7.12962 | 7.21449 | 22″ |
| 39″ | 6.27664 | 6.68121 | 6.88697 | 7.02602 | 7.13118 | 7.21577 | 21″ |
| 40″ | 6.28763 | 6.68557 | 6.88969 | 7.02800 | 7.13273 | 7.21705 | 20″ |
| 41″ | 6.29836 | 6.68990 | 6.89240 | 7.02997 | 7.13428 | 7.21833 | 19″ |
| 42″ | 6.30882 | 6.69418 | 6.89509 | 7.03193 | 7.13582 | 7.21960 | 18″ |
| 43″ | 6.31904 | 6.69841 | 6.89776 | 7.03388 | 7.13736 | 7.22087 | 17″ |
| 44″ | 6.32903 | 6.70261 | 6.90042 | 7.03582 | 7.13889 | 7.22213 | 16″ |
| 45″ | 6.33879 | 6.70676 | 6.90306 | 7.03776 | 7.14042 | 7.22339 | 15″ |
| 46″ | 6.34833 | 6.71088 | 6.90568 | 7.03968 | 7.14194 | 7.22465 | 14″ |
| 47″ | 6.35767 | 6.71496 | 6.90829 | 7.04160 | 7.14346 | 7.22590 | 13″ |
| 48″ | 6.36682 | 6.71900 | 6.91088 | 7.04351 | 7.14497 | 7.22715 | 12″ |
| 49″ | 6.37577 | 6.72300 | 6.91346 | 7.04541 | 7.14647 | 7.22840 | 11″ |
| 50″ | 6.38454 | 6.72697 | 6.91602 | 7.04730 | 7.14797 | 7.22964 | 10″ |
| 51″ | 6.39315 | 6.73090 | 6.91857 | 7.04919 | 7.14947 | 7.23088 | 9″ |
| 52″ | 6.40158 | 6.73479 | 6.92110 | 7.05106 | 7.15096 | 7.23212 | 8″ |
| 53″ | 6.40985 | 6.73865 | 6.92362 | 7.05293 | 7.15244 | 7.23335 | 7″ |
| 54″ | 6.41797 | 6.74248 | 6.92612 | 7.05479 | 7.15392 | 7.23458 | 6″ |
| 55″ | 6.42594 | 6.74627 | 6.92861 | 7.05664 | 7.15540 | 7.23580 | 5″ |
| 56″ | 6.43376 | 6.75003 | 6.93109 | 7.05849 | 7.15687 | 7.23702 | 4″ |
| 57″ | 6.44145 | 6.75376 | 6.93355 | 7.06032 | 7.15833 | 7.23824 | 3″ |
| 58″ | 6.44900 | 6.75746 | 6.93599 | 7.06215 | 7.15979 | 7.23946 | 2″ |
| 59″ | 6.45643 | 6.76112 | 6.93843 | 7.06397 | 7.16125 | 7.24067 | 1″ |
| 60″ | 6.46373 | 6.76476 | 6.94085 | 7.06579 | 7.16270 | 7.24188 | 0″ |
| | 59′ | 58′ | 57′ | 56′ | 55′ | 54′ | 89° |
| Log cosine = Log cotangent = 0 − Log tangent. | | | | | | | |

Log sine = Log tangent = 0 − Log cotangent.

| 0° | 6′ | 7′ | 8′ | 9′ | 10′ | 11′ | |
|---|---|---|---|---|---|---|---|
| 0″ | 7.24188 | 7.30882 | 7.36682 | 7.41797 | 7.46373 | 7.50512 | 60″ |
| 1″ | 7.24308 | 7.30986 | 7.36772 | 7.41877 | 7.46445 | 7.50578 | 59″ |
| 2″ | 7.24428 | 7.31089 | 7.36862 | 7.41957 | 7.46517 | 7.50643 | 58″ |
| 3″ | 7.24548 | 7.31191 | 7.36952 | 7.42037 | 7.46589 | 7.50709 | 57″ |
| 4″ | 7.24668 | 7.31294 | 7.37042 | 7.42117 | 7.46661 | 7.50774 | 56″ |
| 5″ | 7.24787 | 7.31396 | 7.37132 | 7.42197 | 7.46733 | 7.50840 | 55″ |
| 6″ | 7.24906 | 7.31498 | 7.37221 | 7.42277 | 7.46805 | 7.50905 | 54″ |
| 7″ | 7.25024 | 7.31600 | 7.37310 | 7.42356 | 7.46876 | 7.50970 | 53″ |
| 8″ | 7.25142 | 7.31702 | 7.37399 | 7.42435 | 7.46948 | 7.51035 | 52″ |
| 9″ | 7.25260 | 7.31803 | 7.37488 | 7.42515 | 7.47019 | 7.51100 | 51″ |
| 10″ | 7.25378 | 7.31904 | 7.37577 | 7.42594 | 7.47090 | 7.51165 | 50″ |
| 11″ | 7.25495 | 7.32005 | 7.37666 | 7.42673 | 7.47162 | 7.51230 | 49″ |
| 12″ | 7.25612 | 7.32106 | 7.37754 | 7.42751 | 7.47233 | 7.51294 | 48″ |
| 13″ | 7.25728 | 7.32206 | 7.37842 | 7.42830 | 7.47303 | 7.51359 | 47″ |
| 14″ | 7.25845 | 7.32306 | 7.37930 | 7.42908 | 7.47374 | 7.51423 | 46″ |
| 15″ | 7.25961 | 7.32406 | 7.38018 | 7.42987 | 7.47445 | 7.51488 | 45″ |
| 16″ | 7.26076 | 7.32506 | 7.38106 | 7.43065 | 7.47515 | 7.51552 | 44″ |
| 17″ | 7.26192 | 7.32606 | 7.38193 | 7.43143 | 7.47586 | 7.51616 | 43″ |
| 18″ | 7.26307 | 7.32705 | 7.38280 | 7.43221 | 7.47656 | 7.51680 | 42″ |
| 19″ | 7.26421 | 7.32804 | 7.38368 | 7.43299 | 7.47726 | 7.51744 | 41″ |
| 20″ | 7.26536 | 7.32903 | 7.38454 | 7.43376 | 7.47797 | 7.51808 | 40″ |
| 21″ | 7.26650 | 7.33001 | 7.38541 | 7.43454 | 7.47867 | 7.51872 | 39″ |
| 22″ | 7.26764 | 7.33100 | 7.38628 | 7.43531 | 7.47936 | 7.51936 | 38″ |
| 23″ | 7.26877 | 7.33198 | 7.38714 | 7.43608 | 7.48006 | 7.51999 | 37″ |
| 24″ | 7.26991 | 7.33296 | 7.38801 | 7.43685 | 7.48076 | 7.52063 | 36″ |
| 25″ | 7.27104 | 7.33393 | 7.38887 | 7.43762 | 7.48145 | 7.52126 | 35″ |
| 26″ | 7.27216 | 7.33491 | 7.38972 | 7.43839 | 7.48215 | 7.52190 | 34″ |
| 27″ | 7.27329 | 7.33588 | 7.39058 | 7.43916 | 7.48284 | 7.52253 | 33″ |
| 28″ | 7.27441 | 7.33685 | 7.39144 | 7.43992 | 7.48353 | 7.52316 | 32″ |
| 29″ | 7.27552 | 7.33782 | 7.39229 | 7.44069 | 7.48422 | 7.52379 | 31″ |
| 30″ | 7.27664 | 7.33879 | 7.39314 | 7.44145 | 7.48491 | 7.52442 | 30″ |
| 31″ | 7.27775 | 7.33975 | 7.39400 | 7.44221 | 7.48560 | 7.52505 | 29″ |
| 32″ | 7.27886 | 7.34071 | 7.39484 | 7.44297 | 7.48629 | 7.52568 | 28″ |
| 33″ | 7.27997 | 7.34167 | 7.39569 | 7.44373 | 7.48698 | 7.52631 | 27″ |
| 34″ | 7.28107 | 7.34263 | 7.39654 | 7.44449 | 7.48766 | 7.52693 | 26″ |
| 35″ | 7.28217 | 7.34359 | 7.39738 | 7.44524 | 7.48835 | 7.52756 | 25″ |
| 36″ | 7.28327 | 7.34454 | 7.39822 | 7.44600 | 7.48903 | 7.52818 | 24″ |
| 37″ | 7.28437 | 7.34549 | 7.39907 | 7.44675 | 7.48971 | 7.52881 | 23″ |
| 38″ | 7.28546 | 7.34644 | 7.39990 | 7.44750 | 7.49039 | 7.52943 | 22″ |
| 39″ | 7.28655 | 7.34739 | 7.40074 | 7.44825 | 7.49108 | 7.53005 | 21″ |
| 40″ | 7.28763 | 7.34833 | 7.40158 | 7.44900 | 7.49175 | 7.53067 | 20″ |
| 41″ | 7.28872 | 7.34928 | 7.40241 | 7.44975 | 7.49243 | 7.53129 | 19″ |
| 42″ | 7.28980 | 7.35022 | 7.40324 | 7.45050 | 7.49311 | 7.53191 | 18″ |
| 43″ | 7.29088 | 7.35116 | 7.40408 | 7.45124 | 7.49379 | 7.53253 | 17″ |
| 44″ | 7.29196 | 7.35209 | 7.40491 | 7.45199 | 7.49446 | 7.53315 | 16″ |
| 45″ | 7.29303 | 7.35303 | 7.40573 | 7.45273 | 7.49513 | 7.53376 | 15″ |
| 46″ | 7.29410 | 7.35396 | 7.40656 | 7.45347 | 7.49581 | 7.53438 | 14″ |
| 47″ | 7.29517 | 7.35489 | 7.40739 | 7.45421 | 7.49648 | 7.53499 | 13″ |
| 48″ | 7.29623 | 7.35582 | 7.40821 | 7.45495 | 7.49715 | 7.53561 | 12″ |
| 49″ | 7.29730 | 7.35675 | 7.40903 | 7.45569 | 7.49782 | 7.53622 | 11″ |
| 50″ | 7.29836 | 7.35767 | 7.40985 | 7.45643 | 7.49849 | 7.53683 | 10″ |
| 51″ | 7.29942 | 7.35860 | 7.41067 | 7.45716 | 7.49916 | 7.53744 | 9″ |
| 52″ | 7.30047 | 7.35952 | 7.41149 | 7.45790 | 7.49982 | 7.53805 | 8″ |
| 53″ | 7.30152 | 7.36044 | 7.41230 | 7.45863 | 7.50049 | 7.53866 | 7″ |
| 54″ | 7.30257 | 7.36135 | 7.41312 | 7.45936 | 7.50115 | 7.53927 | 6″ |
| 55″ | 7.30362 | 7.36227 | 7.41393 | 7.46009 | 7.50182 | 7.53988 | 5″ |
| 56″ | 7.30467 | 7.36318 | 7.41474 | 7.46082 | 7.50248 | 7.54049 | 4″ |
| 57″ | 7.30571 | 7.36409 | 7.41555 | 7.46155 | 7.50314 | 7.54109 | 3″ |
| 58″ | 7.30675 | 7.36500 | 7.41636 | 7.46228 | 7.50380 | 7.54170 | 2″ |
| 59″ | 7.30779 | 7.36591 | 7.41716 | 7.46300 | 7.50446 | 7.54230 | 1″ |
| 60″ | 7.30882 | 7.36682 | 7.41797 | 7.46373 | 7.50512 | 7.54291 | 0″ |
| | 53′ | 52′ | 51′ | 50′ | 49′ | 48′ | 89° |

Log cosine = Log cotangent = 0 − Log tangent.

| Log sine = Log tangent = 0 − Log cotangent. | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0° | 12′ | 13′ | 14′ | 15′ | 16′ | 17′ | |
| 0″ | 7.54291 | 7.57767 | 7.60985 | 7.63982 | 7.66784 | 7.69417 | 60″ |
| 1″ | 7.54351 | 7.57822 | 7.61037 | 7.64030 | 7.66830 | 7.69460 | 59″ |
| 2″ | 7.54411 | 7.57878 | 7.61089 | 7.64078 | 7.66875 | 7.69502 | 58″ |
| 3″ | 7.54471 | 7.57934 | 7.61140 | 7.64126 | 7.66920 | 7.69545 | 57″ |
| 4″ | 7.54531 | 7.57989 | 7.61192 | 7.64174 | 7.66965 | 7.69587 | 56″ |
| 5″ | 7.54591 | 7.58044 | 7.61243 | 7.64222 | 7.67010 | 7.69630 | 55″ |
| 6″ | 7.54651 | 7.58100 | 7.61294 | 7.64270 | 7.67055 | 7.69672 | 54″ |
| 7″ | 7.54711 | 7.58155 | 7.61346 | 7.64318 | 7.67100 | 7.69714 | 53″ |
| 8″ | 7.54771 | 7.58210 | 7.61397 | 7.64366 | 7.67145 | 7.69757 | 52″ |
| 9″ | 7.54830 | 7.58265 | 7.61448 | 7.64414 | 7.67190 | 7.69799 | 51″ |
| 10″ | 7.54890 | 7.58320 | 7.61499 | 7.64461 | 7.67235 | 7.69841 | 50″ |
| 11″ | 7.54949 | 7.58375 | 7.61550 | 7.64509 | 7.67279 | 7.69883 | 49″ |
| 12″ | 7.55009 | 7.58430 | 7.61601 | 7.64557 | 7.67324 | 7.69925 | 48″ |
| 13″ | 7.55068 | 7.58485 | 7.61652 | 7.64604 | 7.67369 | 7.69967 | 47″ |
| 14″ | 7.55127 | 7.58539 | 7.61703 | 7.64652 | 7.67413 | 7.70009 | 46″ |
| 15″ | 7.55186 | 7.58594 | 7.61754 | 7.64699 | 7.67458 | 7.70051 | 45″ |
| 16″ | 7.55245 | 7.58649 | 7.61805 | 7.64747 | 7.67502 | 7.70093 | 44″ |
| 17″ | 7.55304 | 7.58703 | 7.61855 | 7.64794 | 7.67547 | 7.70135 | 43″ |
| 18″ | 7.55363 | 7.58758 | 7.61906 | 7.64842 | 7.67591 | 7.70177 | 42″ |
| 19″ | 7.55422 | 7.58812 | 7.61957 | 7.64889 | 7.67636 | 7.70219 | 41″ |
| 20″ | 7.55481 | 7.58866 | 7.62007 | 7.64936 | 7.67680 | 7.70261 | 40″ |
| 21″ | 7.55539 | 7.58921 | 7.62058 | 7.64983 | 7.67724 | 7.70302 | 39″ |
| 22″ | 7.55598 | 7.58975 | 7.62108 | 7.65030 | 7.67768 | 7.70344 | 38″ |
| 23″ | 7.55656 | 7.59029 | 7.62158 | 7.65078 | 7.67813 | 7.70386 | 37″ |
| 24″ | 7.55715 | 7.59083 | 7.62209 | 7.65125 | 7.67857 | 7.70427 | 36″ |
| 25″ | 7.55773 | 7.59137 | 7.62259 | 7.65172 | 7.67901 | 7.70469 | 35″ |
| 26″ | 7.55831 | 7.59191 | 7.62309 | 7.65218 | 7.67945 | 7.70510 | 34″ |
| 27″ | 7.55889 | 7.59245 | 7.62359 | 7.65265 | 7.67989 | 7.70552 | 33″ |
| 28″ | 7.55948 | 7.59299 | 7.62409 | 7.65312 | 7.68033 | 7.70593 | 32″ |
| 29″ | 7.56006 | 7.59352 | 7.62459 | 7.65359 | 7.68077 | 7.70635 | 31″ |
| 30″ | 7.56064 | 7.59406 | 7.62509 | 7.65406 | 7.68121 | 7.70676 | 30″ |
| 31″ | 7.56121 | 7.59459 | 7.62559 | 7.65452 | 7.68165 | 7.70718 | 29″ |
| 32″ | 7.56179 | 7.59513 | 7.62609 | 7.65499 | 7.68208 | 7.70759 | 28″ |
| 33″ | 7.56237 | 7.59566 | 7.62659 | 7.65546 | 7.68252 | 7.70800 | 27″ |
| 34″ | 7.56295 | 7.59620 | 7.62709 | 7.65592 | 7.68296 | 7.70841 | 26″ |
| 35″ | 7.56352 | 7.59673 | 7.62758 | 7.65639 | 7.68340 | 7.70883 | 25″ |
| 36″ | 7.56410 | 7.59726 | 7.62808 | 7.65685 | 7.68383 | 7.70924 | 24″ |
| 37″ | 7.56467 | 7.59780 | 7.62857 | 7.65731 | 7.68427 | 7.70965 | 23″ |
| 38″ | 7.56524 | 7.59833 | 7.62907 | 7.65778 | 7.68470 | 7.71006 | 22″ |
| 39″ | 7.56582 | 7.59886 | 7.62956 | 7.65824 | 7.68514 | 7.71047 | 21″ |
| 40″ | 7.56639 | 7.59939 | 7.63006 | 7.65870 | 7.68557 | 7.71088 | 20″ |
| 41″ | 7.56696 | 7.59992 | 7.63055 | 7.65916 | 7.68601 | 7.71129 | 19″ |
| 42″ | 7.56753 | 7.60045 | 7.63104 | 7.65962 | 7.68644 | 7.71170 | 18″ |
| 43″ | 7.56810 | 7.60097 | 7.63153 | 7.66009 | 7.68687 | 7.71211 | 17″ |
| 44″ | 7.56867 | 7.60150 | 7.63203 | 7.66055 | 7.68731 | 7.71251 | 16″ |
| 45″ | 7.56924 | 7.60203 | 7.63252 | 7.66101 | 7.68774 | 7.71292 | 15″ |
| 46″ | 7.56980 | 7.60255 | 7.63301 | 7.66146 | 7.68817 | 7.71333 | 14″ |
| 47″ | 7.57037 | 7.60308 | 7.63350 | 7.66192 | 7.68860 | 7.71374 | 13″ |
| 48″ | 7.57094 | 7.60360 | 7.63399 | 7.66238 | 7.68903 | 7.71414 | 12″ |
| 49″ | 7.57150 | 7.60413 | 7.63448 | 7.66284 | 7.68946 | 7.71455 | 11″ |
| 50″ | 7.57206 | 7.60465 | 7.63496 | 7.66330 | 7.68989 | 7.71496 | 10″ |
| 51″ | 7.57263 | 7.60517 | 7.63545 | 7.66375 | 7.69032 | 7.71536 | 9″ |
| 52″ | 7.57319 | 7.60570 | 7.63594 | 7.66421 | 7.69075 | 7.71577 | 8″ |
| 53″ | 7.57375 | 7.60622 | 7.63643 | 7.66467 | 7.69118 | 7.71617 | 7″ |
| 54″ | 7.57431 | 7.60674 | 7.63691 | 7.66512 | 7.69161 | 7.71658 | 6″ |
| 55″ | 7.57488 | 7.60726 | 7.63740 | 7.66558 | 7.69204 | 7.71698 | 5″ |
| 56″ | 7.57544 | 7.60778 | 7.63788 | 7.66603 | 7.69247 | 7.71739 | 4″ |
| 57″ | 7.57599 | 7.60830 | 7.63837 | 7.66649 | 7.69289 | 7.71779 | 3″ |
| 58″ | 7.57655 | 7.60882 | 7.63885 | 7.66694 | 7.69332 | 7.71819 | 2″ |
| 59″ | 7.57711 | 7.60934 | 7.63933 | 7.66739 | 7.69375 | 7.71859 | 1″ |
| 60″ | 7.57767 | 7.60985 | 7.63982 | 7.66784 | 7.69417 | 7.71900 | 0″ |
| | 47′ | 46′ | 45′ | 44′ | 43′ | 42′ | 89° |
| Log cosine = Log cotangent = 0 − Log tangent. | | | | | | | |

| 0° | Log sin | D 1″ | Log tan* | |
|---|---|---|---|---|
| 18′ | 7.71900 | 40 | 7.71900 | 42′ |
| 3″ | 7.72020 | 40 | 7.72021 | 57″ |
| 6″ | 7.72140 | 40 | 7.72141 | 54″ |
| 9″ | 7.72260 | 40 | 7.72261 | 51″ |
| 12″ | 7.72380 | 40 | 7.72380 | 48″ |
| 15″ | 7.72499 | 40 | 7.72499 | 45″ |
| 18″ | 7.72618 | 40 | 7.72618 | 42″ |
| 21″ | 7.72736 | 40 | 7.72737 | 39″ |
| 24″ | 7.72854 | 39 | 7.72855 | 36″ |
| 27″ | 7.72972 | 39 | 7.72973 | 33″ |
| 30″ | 7.73090 | 39 | 7.73090 | 30″ |
| 33″ | 7.73207 | 39 | 7.73207 | 27″ |
| 36″ | 7.73324 | 39 | 7.73324 | 24″ |
| 39″ | 7.73440 | 39 | 7.73441 | 21″ |
| 42″ | 7.73557 | 39 | 7.73557 | 18″ |
| 45″ | 7.73673 | 39 | 7.73673 | 15″ |
| 48″ | 7.73788 | 38 | 7.73789 | 12″ |
| 51″ | 7.73904 | 38 | 7.73904 | 9″ |
| 54″ | 7.74019 | 38 | 7.74019 | 6″ |
| 57″ | 7.74133 | 38 | 7.74134 | 3″ |
| 19′ | 7.74248 | 38 | 7.74248 | 41′ |
| 3″ | 7.74362 | 38 | 7.74363 | 57″ |
| 6″ | 7.74476 | 38 | 7.74476 | 54″ |
| 9″ | 7.74589 | 38 | 7.74590 | 51″ |
| 12″ | 7.74703 | 38 | 7.74703 | 48″ |
| 15″ | 7.74815 | 38 | 7.74816 | 45″ |
| 18″ | 7.74928 | 38 | 7.74929 | 42″ |
| 21″ | 7.75040 | 37 | 7.75041 | 39″ |
| 24″ | 7.75153 | 37 | 7.75153 | 36″ |
| 27″ | 7.75264 | 37 | 7.75265 | 33″ |
| 30″ | 7.75376 | 37 | 7.75377 | 30″ |
| 33″ | 7.75487 | 37 | 7.75488 | 27″ |
| 36″ | 7.75598 | 37 | 7.75599 | 24″ |
| 39″ | 7.75709 | 37 | 7.75709 | 21″ |
| 42″ | 7.75819 | 37 | 7.75820 | 18″ |
| 45″ | 7.75929 | 37 | 7.75930 | 15″ |
| 48″ | 7.76039 | 37 | 7.76040 | 12″ |
| 51″ | 7.76148 | 36 | 7.76149 | 9″ |
| 54″ | 7.76258 | 36 | 7.76258 | 6″ |
| 57″ | 7.76367 | 36 | 7.76367 | 3″ |
| 20′ | 7.76475 | 36 | 7.76476 | 40′ |
| 3″ | 7.76584 | 36 | 7.76585 | 57″ |
| 6″ | 7.76692 | 36 | 7.76693 | 54″ |
| 9″ | 7.76800 | 36 | 7.76801 | 51″ |
| 12″ | 7.76908 | 36 | 7.76908 | 48″ |
| 15″ | 7.77015 | 36 | 7.77016 | 45″ |
| 18″ | 7.77122 | 36 | 7.77123 | 42″ |
| 21″ | 7.77229 | 36 | 7.77230 | 39″ |
| 24″ | 7.77335 | 36 | 7.77336 | 36″ |
| 27″ | 7.77442 | 35 | 7.77442 | 33″ |
| 30″ | 7.77548 | 35 | 7.77549 | 30″ |
| 33″ | 7.77654 | 35 | 7.77654 | 27″ |
| 36″ | 7.77759 | 35 | 7.77760 | 24″ |
| 39″ | 7.77864 | 35 | 7.77865 | 21″ |
| 42″ | 7.77969 | 35 | 7.77970 | 18″ |
| 45″ | 7.78074 | 35 | 7.78075 | 15″ |
| 48″ | 7.78179 | 35 | 7.78179 | 12″ |
| 51″ | 7.78283 | 35 | 7.78284 | 9″ |
| 54″ | 7.78387 | 35 | 7.78388 | 6″ |
| 57″ | 7.78491 | 35 | 7.78492 | 3″ |
| 21′ | 7.78594 | | 7.78595 | 39′ |
| | Log cos | D 1″ | Log cot* | 89° |

| 0° | Log sin | D 1″ | Log tan* | |
|---|---|---|---|---|
| 21′ | 7.78594 | 34 | 7.78595 | 39′ |
| 3″ | 7.78698 | 34 | 7.78698 | 57″ |
| 6″ | 7.78801 | 34 | 7.78801 | 54″ |
| 9″ | 7.78903 | 34 | 7.78904 | 51″ |
| 12″ | 7.79006 | 34 | 7.79007 | 48″ |
| 15″ | 7.79108 | 34 | 7.79109 | 45″ |
| 18″ | 7.79210 | 34 | 7.79211 | 42″ |
| 21″ | 7.79312 | 34 | 7.79313 | 39″ |
| 24″ | 7.79414 | 34 | 7.79415 | 36″ |
| 27″ | 7.79515 | 34 | 7.79516 | 33″ |
| 30″ | 7.79616 | 34 | 7.79617 | 30″ |
| 33″ | 7.79717 | 34 | 7.79718 | 27″ |
| 36″ | 7.79818 | 34 | 7.79819 | 24″ |
| 39″ | 7.79918 | 33 | 7.79919 | 21″ |
| 42″ | 7.80018 | 33 | 7.80019 | 18″ |
| 45″ | 7.80118 | 33 | 7.80119 | 15″ |
| 48″ | 7.80218 | 33 | 7.80219 | 12″ |
| 51″ | 7.80317 | 33 | 7.80318 | 9″ |
| 54″ | 7.80417 | 33 | 7.80418 | 6″ |
| 57″ | 7.80516 | 33 | 7.80517 | 3″ |
| 22′ | 7.80615 | 33 | 7.80615 | 38′ |
| 3″ | 7.80713 | 33 | 7.80714 | 57″ |
| 6″ | 7.80812 | 33 | 7.80812 | 54″ |
| 9″ | 7.80910 | 33 | 7.80911 | 51″ |
| 12″ | 7.81008 | 33 | 7.81009 | 48″ |
| 15″ | 7.81105 | 33 | 7.81106 | 45″ |
| 18″ | 7.81203 | 32 | 7.81204 | 42″ |
| 21″ | 7.81300 | 32 | 7.81301 | 39″ |
| 24″ | 7.81397 | 32 | 7.81398 | 36″ |
| 27″ | 7.81494 | 32 | 7.81495 | 33″ |
| 30″ | 7.81591 | 32 | 7.81591 | 30″ |
| 33″ | 7.81687 | 32 | 7.81688 | 27″ |
| 36″ | 7.81783 | 32 | 7.81784 | 24″ |
| 39″ | 7.81879 | 32 | 7.81880 | 21″ |
| 42″ | 7.81975 | 32 | 7.81976 | 18″ |
| 45″ | 7.82070 | 32 | 7.82071 | 15″ |
| 48″ | 7.82166 | 32 | 7.82167 | 12″ |
| 51″ | 7.82261 | 32 | 7.82262 | 9″ |
| 54″ | 7.82356 | 32 | 7.82357 | 6″ |
| 57″ | 7.82451 | 32 | 7.82452 | 3″ |
| 23′ | 7.82545 | 31 | 7.82546 | 37′ |
| 3″ | 7.82639 | 31 | 7.82640 | 57″ |
| 6″ | 7.82733 | 31 | 7.82734 | 54″ |
| 9″ | 7.82827 | 31 | 7.82828 | 51″ |
| 12″ | 7.82921 | 31 | 7.82922 | 48″ |
| 15″ | 7.83015 | 31 | 7.83016 | 45″ |
| 18″ | 7.83108 | 31 | 7.83109 | 42″ |
| 21″ | 7.83201 | 31 | 7.83202 | 39″ |
| 24″ | 7.83294 | 31 | 7.83295 | 36″ |
| 27″ | 7.83387 | 31 | 7.83388 | 33″ |
| 30″ | 7.83479 | 31 | 7.83480 | 30″ |
| 33″ | 7.83571 | 31 | 7.83572 | 27″ |
| 36″ | 7.83663 | 31 | 7.83664 | 24″ |
| 39″ | 7.83755 | 31 | 7.83756 | 21″ |
| 42″ | 7.83847 | 31 | 7.83848 | 18″ |
| 45″ | 7.83939 | 30 | 7.83940 | 15″ |
| 48″ | 7.84030 | 30 | 7.84031 | 12″ |
| 51″ | 7.84121 | 30 | 7.84122 | 9″ |
| 54″ | 7.84212 | 30 | 7.84213 | 6″ |
| 57″ | 7.84303 | 30 | 7.84304 | 3″ |
| 24′ | 7.84393 | | 7.84394 | 36′ |
| | Log cos | D 1″ | Log cot* | 89° |

*Log cot A = 0 − log tan A; Log tan A = 0 − log cot A.

| 0° | Log sin | D 1″ | Log tan* | |
|---|---|---|---|---|
| 24′ | 7.84393 | | 7.84394 | 36′ |
| 3″ | 7.84484 | 30 | 7.84485 | 57″ |
| 6″ | 7.84574 | 30 | 7.84575 | 54″ |
| 9″ | 7.84664 | 30 | 7.84665 | 51″ |
| 12″ | 7.84754 | 30 | 7.84755 | 48″ |
| 15″ | 7.84843 | 30 | 7.84845 | 45″ |
| 18″ | 7.84933 | 30 | 7.84934 | 42″ |
| 21″ | 7.85022 | 30 | 7.85023 | 39″ |
| 24″ | 7.85111 | 30 | 7.85112 | 36″ |
| 27″ | 7.85200 | 30 | 7.85201 | 33″ |
| 30″ | 7.85289 | 30 | 7.85290 | 30″ |
| 33″ | 7.85377 | 30 | 7.85379 | 27″ |
| 36″ | 7.85466 | 29 | 7.85467 | 24″ |
| 39″ | 7.85554 | 29 | 7.85555 | 21″ |
| 42″ | 7.85642 | 29 | 7.85643 | 18″ |
| 45″ | 7.85730 | 29 | 7.85731 | 15″ |
| 48″ | 7.85817 | 29 | 7.85819 | 12″ |
| 51″ | 7.85905 | 29 | 7.85906 | 9″ |
| 54″ | 7.85992 | 29 | 7.85993 | 6″ |
| 57″ | 7.86079 | 29 | 7.86080 | 3″ |
| 25′ | 7.86166 | 29 | 7.86167 | 35′ |
| 3″ | 7.86253 | 29 | 7.86254 | 57″ |
| 6″ | 7.86340 | 29 | 7.86341 | 54″ |
| 9″ | 7.86426 | 29 | 7.86427 | 51″ |
| 12″ | 7.86512 | 29 | 7.86513 | 48″ |
| 15″ | 7.86598 | 29 | 7.86600 | 45″ |
| 18″ | 7.86684 | 29 | 7.86685 | 42″ |
| 21″ | 7.86770 | 29 | 7.86771 | 39″ |
| 24″ | 7.86856 | 29 | 7.86857 | 36″ |
| 27″ | 7.86941 | 28 | 7.86942 | 33″ |
| 30″ | 7.87026 | 28 | 7.87027 | 30″ |
| 33″ | 7.87111 | 28 | 7.87113 | 27″ |
| 36″ | 7.87196 | 28 | 7.87197 | 24″ |
| 39″ | 7.87281 | 28 | 7.87282 | 21″ |
| 42″ | 7.87366 | 28 | 7.87367 | 18″ |
| 45″ | 7.87450 | 28 | 7.87451 | 15″ |
| 48″ | 7.87534 | 28 | 7.87535 | 12″ |
| 51″ | 7.87618 | 28 | 7.87619 | 9″ |
| 54″ | 7.87702 | 28 | 7.87703 | 6″ |
| 57″ | 7.87786 | 28 | 7.87787 | 3″ |
| 26′ | 7.87870 | 28 | 7.87871 | 34′ |
| 3″ | 7.87953 | 28 | 7.87954 | 57″ |
| 6″ | 7.88036 | 28 | 7.88038 | 54″ |
| 9″ | 7.88119 | 28 | 7.88121 | 51″ |
| 12″ | 7.88202 | 28 | 7.88204 | 48″ |
| 15″ | 7.88285 | 28 | 7.88286 | 45″ |
| 18″ | 7.88368 | 27 | 7.88369 | 42″ |
| 21″ | 7.88450 | 27 | 7.88452 | 39″ |
| 24″ | 7.88533 | 27 | 7.88534 | 36″ |
| 27″ | 7.88615 | 27 | 7.88616 | 33″ |
| 30″ | 7.88697 | 27 | 7.88698 | 30″ |
| 33″ | 7.88779 | 27 | 7.88780 | 27″ |
| 36″ | 7.88860 | 27 | 7.88862 | 24″ |
| 39″ | 7.88942 | 27 | 7.88943 | 21″ |
| 42″ | 7.89023 | 27 | 7.89025 | 18″ |
| 45″ | 7.89105 | 27 | 7.89106 | 15″ |
| 48″ | 7.89186 | 27 | 7.89187 | 12″ |
| 51″ | 7.89267 | 27 | 7.89268 | 9″ |
| 54″ | 7.89347 | 27 | 7.89349 | 6″ |
| 57″ | 7.89428 | 27 | 7.89429 | 3″ |
| 27′ | 7.89509 | | 7.89510 | 33′ |
| | Log cos | D 1″ | Log cot* | 89° |

| 0° | Log sin | D 1″ | Log tan* | |
|---|---|---|---|---|
| 27′ | 7.89509 | | 7.89510 | 33′ |
| 3″ | 7.89589 | 27 | 7.89590 | 57″ |
| 6″ | 7.89669 | 27 | 7.89670 | 54″ |
| 9″ | 7.89749 | 27 | 7.89751 | 51″ |
| 12″ | 7.89829 | 27 | 7.89830 | 48″ |
| 15″ | 7.89909 | 27 | 7.89910 | 45″ |
| 18″ | 7.89988 | 27 | 7.89990 | 42″ |
| 21″ | 7.90068 | 26 | 7.90069 | 39″ |
| 24″ | 7.90147 | 26 | 7.90149 | 36″ |
| 27″ | 7.90226 | 26 | 7.90228 | 33″ |
| 30″ | 7.90305 | 26 | 7.90307 | 30″ |
| 33″ | 7.90384 | 26 | 7.90386 | 27″ |
| 36″ | 7.90463 | 26 | 7.90464 | 24″ |
| 39″ | 7.90542 | 26 | 7.90543 | 21″ |
| 42″ | 7.90620 | 26 | 7.90622 | 18″ |
| 45″ | 7.90698 | 26 | 7.90700 | 15″ |
| 48″ | 7.90777 | 26 | 7.90778 | 12″ |
| 51″ | 7.90855 | 26 | 7.90856 | 9″ |
| 54″ | 7.90933 | 26 | 7.90934 | 6″ |
| 57″ | 7.91010 | 26 | 7.91012 | 3″ |
| 28′ | 7.91088 | 26 | 7.91089 | 32′ |
| 3″ | 7.91165 | 26 | 7.91167 | 57″ |
| 6″ | 7.91243 | 26 | 7.91244 | 54″ |
| 9″ | 7.91320 | 26 | 7.91321 | 51″ |
| 12″ | 7.91397 | 26 | 7.91399 | 48″ |
| 15″ | 7.91474 | 26 | 7.91475 | 45″ |
| 18″ | 7.91551 | 26 | 7.91552 | 42″ |
| 21″ | 7.91627 | 26 | 7.91629 | 39″ |
| 24″ | 7.91704 | 26 | 7.91705 | 36″ |
| 27″ | 7.91780 | 25 | 7.91782 | 33″ |
| 30″ | 7.91857 | 25 | 7.91858 | 30″ |
| 33″ | 7.91933 | 25 | 7.91934 | 27″ |
| 36″ | 7.92009 | 25 | 7.92010 | 24″ |
| 39″ | 7.92085 | 25 | 7.92086 | 21″ |
| 42″ | 7.92160 | 25 | 7.92162 | 18″ |
| 45″ | 7.92236 | 25 | 7.92237 | 15″ |
| 48″ | 7.92311 | 25 | 7.92313 | 12″ |
| 51″ | 7.92387 | 25 | 7.92388 | 9″ |
| 54″ | 7.92462 | 25 | 7.92463 | 6″ |
| 57″ | 7.92537 | 25 | 7.92539 | 3″ |
| 29′ | 7.92612 | 25 | 7.92613 | 31′ |
| 3″ | 7.92687 | 25 | 7.92688 | 57″ |
| 6″ | 7.92761 | 25 | 7.92763 | 54″ |
| 9″ | 7.92836 | 25 | 7.92838 | 51″ |
| 12″ | 7.92910 | 25 | 7.92912 | 48″ |
| 15″ | 7.92985 | 25 | 7.92986 | 45″ |
| 18″ | 7.93059 | 25 | 7.93060 | 42″ |
| 21″ | 7.93133 | 25 | 7.93134 | 39″ |
| 24″ | 7.93207 | 25 | 7.93208 | 36″ |
| 27″ | 7.93281 | 25 | 7.93282 | 33″ |
| 30″ | 7.93354 | 25 | 7.93356 | 30″ |
| 33″ | 7.93428 | 25 | 7.93429 | 27″ |
| 36″ | 7.93501 | 24 | 7.93503 | 24″ |
| 39″ | 7.93575 | 24 | 7.93576 | 21″ |
| 42″ | 7.93648 | 24 | 7.93649 | 18″ |
| 45″ | 7.93721 | 24 | 7.93722 | 15″ |
| 48″ | 7.93794 | 24 | 7.93795 | 12″ |
| 51″ | 7.93867 | 24 | 7.93868 | 9″ |
| 54″ | 7.93939 | 24 | 7.93941 | 6″ |
| 57″ | 7.94012 | 24 | 7.94013 | 3″ |
| 30′ | 7.94084 | | 7.94086 | 30′ |
| | Log cos | D 1″ | Log cot* | 89° |

*Log cot A = 0 − log tan A; Log tan A = 0 − log cot A.

| 0° | Log sin | D 1″ | Log tan* | |
|---|---|---|---|---|
| 30′ | 7.94084 | 24.1 | 7.94086 | 30′ |
| 5″ | 7.94205 | 24.0 | 7.94206 | 55″ |
| 10″ | 7.94325 | 24.0 | 7.94326 | 50″ |
| 15″ | 7.94445 | 23.9 | 7.94446 | 45″ |
| 20″ | 7.94564 | 23.8 | 7.94566 | 40″ |
| 25″ | 7.94683 | 23.8 | 7.94685 | 35″ |
| 30″ | 7.94802 | 23.7 | 7.94804 | 30″ |
| 35″ | 7.94921 | 23.6 | 7.94922 | 25″ |
| 40″ | 7.95039 | 23.6 | 7.95040 | 20″ |
| 45″ | 7.95157 | 23.5 | 7.95158 | 15″ |
| 50″ | 7.95274 | 23.4 | 7.95276 | 10″ |
| 55″ | 7.95391 | 23.4 | 7.95393 | 5″ |
| 31′ | 7.95508 | 23.3 | 7.95510 | 29′ |
| 5″ | 7.95625 | 23.3 | 7.95627 | 55″ |
| 10″ | 7.95741 | 23.2 | 7.95743 | 50″ |
| 15″ | 7.95857 | 23.1 | 7.95859 | 45″ |
| 20″ | 7.95973 | 23.1 | 7.95974 | 40″ |
| 25″ | 7.96088 | 23.0 | 7.96090 | 35″ |
| 30″ | 7.96203 | 23.0 | 7.96205 | 30″ |
| 35″ | 7.96318 | 22.9 | 7.96320 | 25″ |
| 40″ | 7.96432 | 22.8 | 7.96434 | 20″ |
| 45″ | 7.96546 | 22.8 | 7.96548 | 15″ |
| 50″ | 7.96660 | 22.7 | 7.96662 | 10″ |
| 55″ | 7.96774 | 22.7 | 7.96776 | 5″ |
| 32′ | 7.96887 | 22.6 | 7.96889 | 28′ |
| 5″ | 7.97000 | 22.5 | 7.97002 | 55″ |
| 10″ | 7.97113 | 22.5 | 7.97114 | 50″ |
| 15″ | 7.97225 | 22.4 | 7.97227 | 45″ |
| 20″ | 7.97337 | 22.4 | 7.97339 | 40″ |
| 25″ | 7.97449 | 22.3 | 7.97451 | 35″ |
| 30″ | 7.97560 | 22.2 | 7.97562 | 30″ |
| 35″ | 7.97672 | 22.2 | 7.97673 | 25″ |
| 40″ | 7.97782 | 22.1 | 7.97784 | 20″ |
| 45″ | 7.97893 | 22.1 | 7.97895 | 15″ |
| 50″ | 7.98003 | 22.0 | 7.98005 | 10″ |
| 55″ | 7.98114 | 22.0 | 7.98116 | 5″ |
| 33′ | 7.98223 | 21.9 | 7.98225 | 27′ |
| 5″ | 7.98333 | 21.8 | 7.98335 | 55″ |
| 10″ | 7.98442 | 21.8 | 7.98444 | 50″ |
| 15″ | 7.98551 | 21.7 | 7.98553 | 45″ |
| 20″ | 7.98660 | 21.7 | 7.98662 | 40″ |
| 25″ | 7.98768 | 21.6 | 7.98770 | 35″ |
| 30″ | 7.98876 | 21.6 | 7.98878 | 30″ |
| 35″ | 7.98984 | 21.5 | 7.98986 | 25″ |
| 40″ | 7.99092 | 21.5 | 7.99094 | 20″ |
| 45″ | 7.99199 | 21.4 | 7.99201 | 15″ |
| 50″ | 7.99306 | 21.4 | 7.99308 | 10″ |
| 55″ | 7.99413 | 21.3 | 7.99415 | 5″ |
| 34′ | 7.99520 | 21.3 | 7.99522 | 26′ |
| 5″ | 7.99626 | 21.2 | 7.99628 | 55″ |
| 10″ | 7.99732 | 21.2 | 7.99734 | 50″ |
| 15″ | 7.99838 | 21.1 | 7.99840 | 45″ |
| 20″ | 7.99943 | 21.1 | 7.99946 | 40″ |
| 25″ | 8.00049 | 21.0 | 8.00051 | 35″ |
| 30″ | 8.00154 | 21.0 | 8.00156 | 30″ |
| 35″ | 8.00259 | 20.9 | 8.00261 | 25″ |
| 40″ | 8.00363 | 20.9 | 8.00365 | 20″ |
| 45″ | 8.00467 | 20.8 | 8.00470 | 15″ |
| 50″ | 8.00571 | 20.8 | 8.00574 | 10″ |
| 55″ | 8.00675 | 20.7 | 8.00677 | 5″ |
| 35′ | 8.00779 | 20.7 | 3.00781 | 25′ |
| | Log cos | D 1″ | Log cot* | 89° |

| 0° | Log sin | D 1″ | Log tan* | |
|---|---|---|---|---|
| 35′ | 8.00779 | 20.7 | 8.00781 | 25′ |
| 5″ | 8.00882 | 20.6 | 8.00884 | 55″ |
| 10″ | 8.00985 | 20.5 | 8.00987 | 50″ |
| 15″ | 8.01088 | 20.5 | 8.01090 | 45″ |
| 20″ | 8.01190 | 20.5 | 8.01193 | 40″ |
| 25″ | 8.01293 | 20.4 | 8.01295 | 35″ |
| 30″ | 8.01395 | 20.4 | 8.01397 | 30″ |
| 35″ | 8.01497 | 20.3 | 8.01499 | 25″ |
| 40″ | 8.01598 | 20.3 | 8.01600 | 20″ |
| 45″ | 8.01699 | 20.2 | 8.01702 | 15″ |
| 50″ | 8.01801 | 20.2 | 8.01803 | 10″ |
| 55″ | 8.01901 | 20.1 | 8.01904 | 5″ |
| 36′ | 8.02002 | 20.1 | 8.02004 | 24′ |
| 5″ | 8.02102 | 20.0 | 8.02105 | 55″ |
| 10″ | 8.02203 | 20.0 | 8.02205 | 50″ |
| 15″ | 8.02303 | 20.0 | 8.02305 | 45″ |
| 20″ | 8.02402 | 20.0 | 8.02405 | 40″ |
| 25″ | 8.02502 | 19.9 | 8.02504 | 35″ |
| 30″ | 8.02601 | 19.9 | 8.02604 | 30″ |
| 35″ | 8.02700 | 19.8 | 8.02703 | 25″ |
| 40″ | 8.02799 | 19.8 | 8.02801 | 20″ |
| 45″ | 8.02898 | 19.7 | 8.02900 | 15″ |
| 50″ | 8.02996 | 19.7 | 8.02998 | 10″ |
| 55″ | 8.03094 | 19.6 | 8.03097 | 5″ |
| 37′ | 8.03192 | 19.6 | 8.03194 | 23′ |
| 5″ | 8.03290 | 19.5 | 8.03292 | 55″ |
| 10″ | 8.03387 | 19.5 | 8.03390 | 50″ |
| 15″ | 8.03484 | 19.4 | 8.03487 | 45″ |
| 20″ | 8.03581 | 19.4 | 8.03584 | 40″ |
| 25″ | 8.03678 | 19.4 | 8.03681 | 35″ |
| 30″ | 8.03775 | 19.3 | 8.03777 | 30″ |
| 35″ | 8.03871 | 19.3 | 8.03874 | 25″ |
| 40″ | 8.03967 | 19.2 | 8.03970 | 20″ |
| 45″ | 8.04063 | 19.2 | 8.04066 | 15″ |
| 50″ | 8.04159 | 19.1 | 8.04162 | 10″ |
| 55″ | 8.04255 | 19.1 | 8.04257 | 5″ |
| 38′ | 8.04350 | 19.1 | 8.04353 | 22′ |
| 5″ | 8.04445 | 19.0 | 8.04448 | 55″ |
| 10″ | 8.04540 | 19.0 | 8.04543 | 50″ |
| 15″ | 8.04635 | 18.9 | 8.04638 | 45″ |
| 20″ | 8.04729 | 18.9 | 8.04732 | 40″ |
| 25″ | 8.04824 | 18.9 | 8.04826 | 35″ |
| 30″ | 8.04918 | 18.8 | 8.04921 | 30″ |
| 35″ | 8.05012 | 18.8 | 8.05014 | 25″ |
| 40″ | 8.05105 | 18.7 | 8.05108 | 20″ |
| 45″ | 8.05199 | 18.7 | 8.05202 | 15″ |
| 50″ | 8.05292 | 18.7 | 8.05295 | 10″ |
| 55″ | 8.05385 | 18.6 | 8.05388 | 5″ |
| 39′ | 8.05478 | 18.6 | 8.05481 | 21′ |
| 5″ | 8.05571 | 18.5 | 8.05574 | 55″ |
| 10″ | 8.05663 | 18.5 | 8.05666 | 50″ |
| 15″ | 8.05756 | 18.5 | 8.05758 | 45″ |
| 20″ | 8.05848 | 18.4 | 8.05851 | 40″ |
| 25″ | 8.05940 | 18.4 | 8.05943 | 35″ |
| 30″ | 8.06031 | 18.3 | 8.06034 | 30″ |
| 35″ | 8.06123 | 18.3 | 8.06126 | 25″ |
| 40″ | 8.06214 | 18.3 | 8.06217 | 20″ |
| 45″ | 8.06305 | 18.2 | 8.06308 | 15″ |
| 50″ | 8.06396 | 18.2 | 8.06399 | 10″ |
| 55″ | 8.06487 | 18.2 | 8.06490 | 5″ |
| 40′ | 8.06578 | 18.1 | 8.06581 | 20′ |
| | Log cos | D 1″ | Log cot* | 89° |

*Log cot A = 0 − log tan A; Log tan A = 0 − log cot A.

| 0° | Log sin | D 1″ | Log tan* | |
|---|---|---|---|---|
| 40′ | 8.06578 | | 8.06581 | 20′ |
| 10″ | 8.06758 | 18.1 | 8.06761 | 50″ |
| 20″ | 8.06938 | 18.0 | 8.06941 | 40″ |
| 30″ | 8.07117 | 17.9 | 8.07120 | 30″ |
| 40″ | 8.07295 | 17.8 | 8.07299 | 20″ |
| 50″ | 8.07473 | 17.7 | 8.07476 | 10″ |
| 41′ | 8.07650 | 17.7 | 8.07653 | 19′ |
| 10″ | 8.07826 | 17.6 | 8.07829 | 50″ |
| 20″ | 8.08002 | 17.6 | 8.08005 | 40″ |
| 30″ | 8.08176 | 17.5 | 8.08180 | 30″ |
| 40″ | 8.08350 | 17.4 | 8.08354 | 20″ |
| 50″ | 8.08524 | 17.3 | 8.08527 | 10″ |
| 42′ | 8.08696 | 17.3 | 8.08700 | 18′ |
| 10″ | 8.08868 | 17.2 | 8.08872 | 50″ |
| 20″ | 8.09040 | 17.1 | 8.09043 | 40″ |
| 30″ | 8.09210 | 17.1 | 8.09214 | 30″ |
| 40″ | 8.09380 | 17.0 | 8.09384 | 20″ |
| 50″ | 8.09550 | 16.9 | 8.09553 | 10″ |
| 43′ | 8.09718 | 16.9 | 8.09722 | 17′ |
| 10″ | 8.09886 | 16.8 | 8.09890 | 50″ |
| 20″ | 8.10054 | 16.7 | 8.10057 | 40″ |
| 30″ | 8.10220 | 16.7 | 8.10224 | 30″ |
| 40″ | 8.10386 | 16.6 | 8.10390 | 20″ |
| 50″ | 8.10552 | 16.5 | 8.10555 | 10″ |
| 44′ | 8.10717 | 16.5 | 8.10720 | 16′ |
| 10″ | 8.10881 | 16.4 | 8.10884 | 50″ |
| 20″ | 8.11044 | 16.4 | 8.11048 | 40″ |
| 30″ | 8.11207 | 16.3 | 8.11211 | 30″ |
| 40″ | 8.11370 | 16.2 | 8.11373 | 20″ |
| 50″ | 8.11531 | 16.2 | 8.11535 | 10″ |
| 45′ | 8.11693 | 16.1 | 8.11696 | 15′ |
| 10″ | 8.11853 | 16.1 | 8.11857 | 50″ |
| 20″ | 8.12013 | 16.0 | 8.12017 | 40″ |
| 30″ | 8.12172 | 15.9 | 8.12176 | 30″ |
| 40″ | 8.12331 | 15.9 | 8.12335 | 20″ |
| 50″ | 8.12489 | 15.8 | 8.12493 | 10″ |
| 46′ | 8.12647 | 15.8 | 8.12651 | 14′ |
| 10″ | 8.12804 | 15.7 | 8.12808 | 50″ |
| 20″ | 8.12961 | 15.6 | 8.12965 | 40″ |
| 30″ | 8.13117 | 15.6 | 8.13121 | 30″ |
| 40″ | 8.13272 | 15.5 | 8.13276 | 20″ |
| 50″ | 8.13427 | 15.5 | 8.13431 | 10″ |
| 47′ | 8.13581 | 15.4 | 8.13585 | 13′ |
| 10″ | 8.13735 | 15.4 | 8.13739 | 50″ |
| 20″ | 8.13888 | 15.3 | 8.13892 | 40″ |
| 30″ | 8.14041 | 15.3 | 8.14045 | 30″ |
| 40″ | 8.14193 | 15.2 | 8.14197 | 20″ |
| 50″ | 8.14344 | 15.2 | 8.14348 | 10″ |
| 48′ | 8.14495 | 15.1 | 8.14500 | 12′ |
| 10″ | 8.14646 | 15.1 | 8.14650 | 50″ |
| 20″ | 8.14796 | 15.0 | 8.14800 | 40″ |
| 30″ | 8.14945 | 15.0 | 8.14950 | 30″ |
| 40″ | 8.15094 | 14.9 | 8.15099 | 20″ |
| 50″ | 8.15243 | 14.9 | 8.15247 | 10″ |
| 49′ | 8.15391 | 14.8 | 8.15395 | 11′ |
| 10″ | 8.15538 | 14.8 | 8.15543 | 50″ |
| 20″ | 8.15685 | 14.7 | 8.15690 | 40″ |
| 30″ | 8.15832 | 14.6 | 8.15836 | 30″ |
| 40″ | 8.15978 | 14.6 | 8.15982 | 20″ |
| 50″ | 8.16123 | 14.6 | 8.16128 | 10″ |
| 50′ | 8.16268 | 14.5 | 8.16273 | 10′ |

| 0° | Log sin | D 1″ | Log tan* | |
|---|---|---|---|---|
| 50′ | 8.16268 | | 8.16273 | 60″ |
| 10″ | 8.16413 | 14.5 | 8.16417 | 50″ |
| 20″ | 8.16557 | 14.4 | 8.16561 | 40″ |
| 30″ | 8.16700 | 14.4 | 8.16705 | 30″ |
| 40″ | 8.16843 | 14.3 | 8.16848 | 20″ |
| 50″ | 8.16986 | 14.3 | 8.16991 | 10″ |
| 51′ | 8.17128 | 14.2 | 8.17133 | 9′ |
| 10″ | 8.17270 | 14.2 | 8.17275 | 50″ |
| 20″ | 8.17411 | 14.1 | 8.17416 | 40″ |
| 30″ | 8.17552 | 14.1 | 8.17557 | 30″ |
| 40″ | 8.17692 | 14.0 | 8.17697 | 20″ |
| 50″ | 8.17832 | 14.0 | 8.17837 | 10″ |
| 52′ | 8.17971 | 13.9 | 8.17976 | 8′ |
| 10″ | 8.18110 | 13.9 | 8.18115 | 50″ |
| 20″ | 8.18249 | 13.9 | 8.18254 | 40″ |
| 30″ | 8.18387 | 13.8 | 8.18392 | 30″ |
| 40″ | 8.18525 | 13.8 | 8.18530 | 20″ |
| 50″ | 8.18662 | 13.7 | 8.18667 | 10″ |
| 53′ | 8.18798 | 13.7 | 8.18804 | 7′ |
| 10″ | 8.18935 | 13.6 | 8.18940 | 50″ |
| 20″ | 8.19071 | 13.6 | 8.19076 | 40″ |
| 30″ | 8.19206 | 13.6 | 8.19212 | 30″ |
| 40″ | 8.19341 | 13.5 | 8.19347 | 20″ |
| 50″ | 8.19476 | 13.5 | 8.19481 | 10″ |
| 54′ | 8.19610 | 13.4 | 8.19616 | 6′ |
| 10″ | 8.19744 | 13.4 | 8.19749 | 50″ |
| 20″ | 8.19877 | 13.3 | 8.19883 | 40″ |
| 30″ | 8.20010 | 13.3 | 8.20016 | 30″ |
| 40″ | 8.20143 | 13.3 | 8.20149 | 20″ |
| 50″ | 8.20275 | 13.2 | 8.20281 | 10″ |
| 55′ | 8.20407 | 13.2 | 8.20413 | 5′ |
| 10″ | 8.20538 | 13.1 | 8.20544 | 50″ |
| 20″ | 8.20669 | 13.1 | 8.20675 | 40″ |
| 30″ | 8.20800 | 13.1 | 8.20806 | 30″ |
| 40″ | 8.20930 | 13.0 | 8.20936 | 20″ |
| 50″ | 8.21060 | 13.0 | 8.21066 | 10″ |
| 56′ | 8.21189 | 12.9 | 8.21195 | 4′ |
| 10″ | 8.21319 | 12.9 | 8.21324 | 50″ |
| 20″ | 8.21447 | 12.9 | 8.21453 | 40″ |
| 30″ | 8.21576 | 12.8 | 8.21581 | 30″ |
| 40″ | 8.21703 | 12.8 | 8.21709 | 20″ |
| 50″ | 8.21831 | 12.8 | 8.21837 | 10″ |
| 57′ | 8.21958 | 12.7 | 8.21964 | 3′ |
| 10″ | 8.22085 | 12.7 | 8.22091 | 50″ |
| 20″ | 8.22211 | 12.6 | 8.22217 | 40″ |
| 30″ | 8.22337 | 12.6 | 8.22343 | 30″ |
| 40″ | 8.22463 | 12.6 | 8.22469 | 20″ |
| 50″ | 8.22588 | 12.5 | 8.22595 | 10″ |
| 58′ | 8.22713 | 12.5 | 8.22720 | 2′ |
| 10″ | 8.22838 | 12.5 | 8.22844 | 50″ |
| 20″ | 8.22962 | 12.4 | 8.22968 | 40″ |
| 30″ | 8.23086 | 12.4 | 8.23092 | 30″ |
| 40″ | 8.23210 | 12.4 | 8.23216 | 20″ |
| 50″ | 8.23333 | 12.3 | 8.23339 | 10″ |
| 59′ | 8.23456 | 12.3 | 8.23462 | 1′ |
| 10″ | 8.23578 | 12.3 | 8.23585 | 50″ |
| 20″ | 8.23700 | 12.2 | 8.23707 | 40″ |
| 30″ | 8.23822 | 12.2 | 8.23829 | 30″ |
| 40″ | 8.23944 | 12.1 | 8.23950 | 20″ |
| 50″ | 8.24065 | 12.1 | 8.24071 | 10″ |
| 60′ | 8.24186 | 12.1 | 8.24192 | 0′ |

| | Log cos | D 1″ | Log cot* | 89° |
|---|---|---|---|---|

| | Log cos | D 1″ | Log cot* | 89° |
|---|---|---|---|---|

*Log cot A = 0 − log tan A; Log tan A = 0 − log cot A.

| 1° | Log sin | D 1″ | Log tan* | | 1° | Log sin | D 1″ | Log tan* | |
|---|---|---|---|---|---|---|---|---|---|
| 0′ | 8.24186 | 12.01 | 8.24192 | 60′ | 30′ | 8.41792 | 8.02 | 8.41807 | 30′ |
| 0′30″ | 8.24546 | 11.92 | 8.24553 | 59′30″ | 30′30″ | 8.42032 | 7.97 | 8.42048 | 29′30″ |
| 1′ | 8.24903 | 11.81 | 8.24910 | 59′ | 31′ | 8.42272 | 7.93 | 8.42287 | 29′ |
| 1′30″ | 8.25258 | 11.72 | 8.25265 | 58′30″ | 31′30″ | 8.42510 | 7.89 | 8.42525 | 28′30″ |
| 2′ | 8.25609 | 11.62 | 8.25616 | 58′ | 32′ | 8.42746 | 7.84 | 8.42762 | 28′ |
| 2′30″ | 8.25958 | 11.53 | 8.25965 | 57′30″ | 32′30″ | 8.42982 | 7.81 | 8.42997 | 27′30″ |
| 3′ | 8.26304 | 11.44 | 8.26312 | 57′ | 33′ | 8.43216 | 7.76 | 8.43232 | 27′ |
| 3′30″ | 8.26648 | 11.35 | 8.26655 | 56′30″ | 33′30″ | 8.43448 | 7.71 | 8.43464 | 26′30″ |
| 4′ | 8.26988 | 11.27 | 8.26996 | 56′ | 34′ | 8.43680 | 7.67 | 8.43696 | 26′ |
| 4′30″ | 8.27326 | 11.18 | 8.27334 | 55′30″ | 34′30″ | 8.43910 | 7.64 | 8.43927 | 25′30″ |
| 5′ | 8.27661 | 11.10 | 8.27669 | 55′ | 35′ | 8.44139 | 7.60 | 8.44156 | 25′ |
| 5′30″ | 8.27994 | 11.01 | 8.28002 | 54′30″ | 35′30″ | 8.44367 | 7.56 | 8.44384 | 24′30″ |
| 6′ | 8.28324 | 10.92 | 8.28332 | 54′ | 36′ | 8.44594 | 7.52 | 8.44611 | 24′ |
| 6′30″ | 8.28652 | 10.84 | 8.28660 | 53′30″ | 36′30″ | 8.44820 | 7.47 | 8.44837 | 23′30″ |
| 7′ | 8.28977 | 10.76 | 8.28986 | 53′ | 37′ | 8.45044 | 7.44 | 8.45061 | 23′ |
| 7′30″ | 8.29300 | 10.68 | 8.29309 | 52′30″ | 37′30″ | 8.45267 | 7.40 | 8.45285 | 22′30″ |
| 8′ | 8.29621 | 10.60 | 8.29629 | 52′ | 38′ | 8.45489 | 7.36 | 8.45507 | 22′ |
| 8′30″ | 8.29939 | 10.53 | 8.29947 | 51′30″ | 38′30″ | 8.45710 | 7.33 | 8.45728 | 21′30″ |
| 9′ | 8.30255 | 10.45 | 8.30263 | 51′ | 39′ | 8.45930 | 7.29 | 8.45948 | 21′ |
| 9′30″ | 8.30568 | 10.37 | 8.30577 | 50′30″ | 39′30″ | 8.46149 | 7.25 | 8.46167 | 20′30″ |
| 10′ | 8.30879 | 10.30 | 8.30888 | 50′ | 40′ | 8.46366 | 7.22 | 8.46385 | 20′ |
| 10′30″ | 8.31188 | 10.23 | 8.31198 | 49′30″ | 40′30″ | 8.46583 | 7.18 | 8.46602 | 19′30″ |
| 11′ | 8.31495 | 10.16 | 8.31505 | 49′ | 41′ | 8.46799 | 7.14 | 8.46817 | 19′ |
| 11′30″ | 8.31800 | 10.09 | 8.31809 | 48′30″ | 41′30″ | 8.47013 | 7.11 | 8.47032 | 18′30″ |
| 12′ | 8.32103 | 10.02 | 8.32112 | 48′ | 42′ | 8.47226 | 7.08 | 8.47245 | 18′ |
| 12′30″ | 8.32403 | 9.95 | 8.32413 | 47′30″ | 42′30″ | 8.47439 | 7.04 | 8.47458 | 17′30″ |
| 13′ | 8.32702 | 9.88 | 8.32711 | 47′ | 43′ | 8.47650 | 7.01 | 8.47669 | 17′ |
| 13′30″ | 8.32998 | 9.81 | 8.33008 | 46′30″ | 43′30″ | 8.47860 | 6.97 | 8.47880 | 16′30″ |
| 14′ | 8.33292 | 9.75 | 8.33302 | 46′ | 44′ | 8.48069 | 6.94 | 8.48089 | 16′ |
| 14′30″ | 8.33585 | 9.68 | 8.33595 | 45′30″ | 44′30″ | 8.48278 | 6.91 | 8.48298 | 15′30″ |
| 15′ | 8.33875 | 9.61 | 8.33886 | 45′ | 45′ | 8.48485 | 6.88 | 8.48505 | 15′ |
| 15′30″ | 8.34164 | 9.55 | 8.34174 | 44′30″ | 45′30″ | 8.48691 | 6.84 | 8.48711 | 14′30″ |
| 16′ | 8.34450 | 9.50 | 8.34461 | 44′ | 46′ | 8.48896 | 6.81 | 8.48917 | 14′ |
| 16′30″ | 8.34735 | 9.43 | 8.34746 | 43′30″ | 46′30″ | 8.49101 | 6.78 | 8.49121 | 13′30″ |
| 17′ | 8.35018 | 9.37 | 8.35029 | 43′ | 47′ | 8.49304 | 6.74 | 8.49325 | 13′ |
| 17′30″ | 8.35299 | 9.31 | 8.35310 | 42′30″ | 47′30″ | 8.49506 | 6.71 | 8.49528 | 12′30″ |
| 18′ | 8.35578 | 9.25 | 8.35590 | 42′ | 48′ | 8.49708 | 6.68 | 8.49729 | 12′ |
| 18′30″ | 8.35856 | 9.19 | 8.35867 | 41′30″ | 48′30″ | 8.49908 | 6.66 | 8.49930 | 11′30″ |
| 19′ | 8.36132 | 9.13 | 8.36143 | 41′ | 49′ | 8.50108 | 6.62 | 8.50130 | 11′ |
| 19′30″ | 8.36405 | 9.08 | 8.36417 | 40′30″ | 49′30″ | 8.50307 | 6.59 | 8.50329 | 10′30″ |
| 20′ | 8.36678 | 9.02 | 8.36689 | 40′ | 50′ | 8.50504 | 6.56 | 8.50527 | 10′ |
| 20′30″ | 8.36948 | 8.96 | 8.36960 | 39′30″ | 50′30″ | 8.50701 | 6.54 | 8.50724 | 9′30″ |
| 21′ | 8.37217 | 8.91 | 8.37229 | 39′ | 51′ | 8.50897 | 6.50 | 8.50920 | 9′ |
| 21′30″ | 8.37484 | 8.86 | 8.37497 | 38′30″ | 51′30″ | 8.51092 | 6.47 | 8.51115 | 8′30″ |
| 22′ | 8.37750 | 8.80 | 8.37762 | 38′ | 52′ | 8.51287 | 6.44 | 8.51310 | 8′ |
| 22′30″ | 8.38014 | 8.74 | 8.38026 | 37′30″ | 52′30″ | 8.51480 | 6.42 | 8.51503 | 7′30″ |
| 23′ | 8.38276 | 8.69 | 8.38289 | 37′ | 53′ | 8.51673 | 6.39 | 8.51696 | 7′ |
| 23′30″ | 8.38537 | 8.64 | 8.38550 | 36′30″ | 53′30″ | 8.51864 | 6.36 | 8.51888 | 6′30″ |
| 24′ | 8.38796 | 8.58 | 8.38809 | 36′ | 54′ | 8.52055 | 6.33 | 8.52079 | 6′ |
| 24′30″ | 8.39054 | 8.54 | 8.39067 | 35′30″ | 54′30″ | 8.52245 | 6.30 | 8.52269 | 5′30″ |
| 25′ | 8.39310 | 8.49 | 8.39323 | 35′ | 55′ | 8.52434 | 6.28 | 8.52459 | 5′ |
| 25′30″ | 8.39565 | 8.44 | 8.39578 | 34′30″ | 55′30″ | 8.52623 | 6.25 | 8.52647 | 4′30″ |
| 26′ | 8.39818 | 8.39 | 8.39832 | 34′ | 96′ | 8.52810 | 6.22 | 8.52835 | 4′ |
| 26′30″ | 8.40070 | 8.34 | 8.40083 | 33′30″ | 56′30″ | 8.52997 | 6.20 | 8.53022 | 3′30″ |
| 27′ | 8.40320 | 8.30 | 8.40334 | 33′ | 57′ | 8.53183 | 6.17 | 8.53208 | 3′ |
| 27′30″ | 8.40569 | 8.24 | 8.40583 | 32′30″ | 57′30″ | 8.53368 | 6.15 | 8.53393 | 2′30″ |
| 28′ | 8.40816 | 8.20 | 8.40830 | 32′ | 58′ | 8.53552 | 6.12 | 8.53578 | 2′ |
| 28′30″ | 8.41062 | 8.16 | 8.41077 | 31′30″ | 58′30″ | 8.53736 | 6.09 | 8.53762 | 1′30″ |
| 29′ | 8.41307 | 8.10 | 8.41321 | 31′ | 59′ | 8.53919 | 6.07 | 8.53945 | 1′ |
| 29′30″ | 8.41550 | 8.07 | 8.41565 | 30′30″ | 59′30″ | 8.54101 | 6.04 | 8.54127 | 0′30″ |
| 30′ | 8.41792 | | 8.41807 | 30′ | 60′ | 8.54282 | | 8.54308 | 0′ |
| | Log cos | D 1″ | Log cot* | 88° | | Log cos | D 1″ | Log cot* | 88° |

*Log cot A = 0 − log tan A; Log tan A = 0 − log cot A.

| " | ' | Seno. | D. 1". | Coseno. | Tang. | Cotang. | ' |
|---|---|---|---|---|---|---|---|
| 0 | 0 | Inf. neg. | | ten | Inf. neg. | Inf. pos. | 60 |
| 60 | 1 | 6.463726 | | ten | 6.463726 | 13.536274 | 59 |
| 120 | 2 | .764756 | | ten | .764756 | .235244 | 58 |
| 180 | 3 | 6.940847 | | ten | 6.940847 | 13.059153 | 57 |
| 240 | 4 | 7.065786 | | ten | 7.065786 | 12.934214 | 56 |
| 300 | 5 | .162696 | | ten | .162696 | .837304 | 55 |
| 360 | 6 | .241877 | .02 | 9.999999 | .241878 | .758122 | 54 |
| 420 | 7 | .308824 | .00 | .999999 | .308825 | .691175 | 53 |
| 480 | 8 | .366816 | .00 | .999999 | .366817 | .633183 | 52 |
| 540 | 9 | .417968 | .00 | .999999 | .417970 | .582030 | 51 |
| 600 | 10 | .463726 | .02 | .999998 | .463727 | .536273 | 50 |
| 660 | 11 | 7.505118 | .00 | 9.999998 | 7.505120 | 12.494880 | 49 |
| 720 | 12 | .542906 | .02 | .999997 | .542909 | .457091 | 48 |
| 780 | 13 | .577668 | .00 | .999997 | .577672 | .422328 | 47 |
| 840 | 14 | .609853 | .02 | .999996 | .609857 | .390143 | 46 |
| 900 | 15 | .639816 | .00 | .999996 | .639820 | .360180 | 45 |
| 960 | 16 | .667845 | .02 | .999995 | .667849 | .332151 | 44 |
| 1020 | 17 | .694173 | .00 | .999995 | .694179 | .305821 | 43 |
| 1080 | 18 | .718997 | .02 | .999994 | .719003 | .280997 | 42 |
| 1140 | 19 | .742478 | .02 | .999993 | .742484 | .257516 | 41 |
| 1200 | 20 | .764754 | .00 | .999993 | .764761 | .235239 | 40 |
| 1260 | 21 | 7.785943 | .02 | 9.999992 | 7.785951 | 12.214049 | 39 |
| 1320 | 22 | .806146 | .03 | .999991 | .806155 | .193845 | 38 |
| 1380 | 23 | .825451 | .02 | .999990 | .825460 | .174540 | 37 |
| 1440 | 24 | .843934 | .02 | .999989 | .843944 | .156056 | 36 |
| 1500 | 25 | .861662 | .00 | .999989 | .861674 | .138326 | 35 |
| 1560 | 26 | .878695 | .02 | .999988 | .878708 | .121292 | 34 |
| 1620 | 27 | .895085 | .02 | .999987 | .895099 | .104901 | 33 |
| 1680 | 28 | .910879 | .02 | .999986 | .910894 | .089106 | 32 |
| 1740 | 29 | .926119 | .02 | .999985 | .926134 | .073866 | 31 |
| 1800 | 30 | .940842 | .03 | .999983 | .940858 | .059143 | 30 |
| 1860 | 31 | 7.955082 | .02 | 9.999982 | 7.955100 | 12.044900 | 29 |
| 1920 | 32 | .968870 | .02 | .999981 | .968889 | .031111 | 28 |
| 1980 | 33 | .982233 | .02 | .999980 | .982253 | .017747 | 27 |
| 2040 | 34 | 7.995198 | .02 | .999979 | 7.995219 | 12.004781 | 26 |
| 2100 | 35 | 8.007787 | .03 | .999977 | 8.007809 | 11.992191 | 25 |
| 2160 | 36 | .020021 | .02 | .999976 | .020044 | .979956 | 24 |
| 2220 | 37 | .031919 | .02 | .999975 | .031945 | .968055 | 23 |
| 2280 | 38 | .043501 | .03 | .999973 | .043527 | .956473 | 22 |
| 2340 | 39 | .054781 | .02 | .999972 | .054809 | .945191 | 21 |
| 2400 | 40 | .065776 | .02 | .999971 | .065806 | .934194 | 20 |
| 2460 | 41 | 8.076500 | .03 | 9.999969 | 8.076531 | 11.923469 | 19 |
| 2520 | 42 | .086965 | .02 | .999968 | .086997 | .913003 | 18 |
| 2580 | 43 | .097183 | .03 | .999966 | .097217 | .902783 | 17 |
| 2640 | 44 | .107167 | .03 | .999964 | .107203 | .892797 | 16 |
| 2700 | 45 | .116926 | .02 | .999963 | .116963 | .883037 | 15 |
| 2760 | 46 | .126471 | .03 | .999961 | .126510 | .873490 | 14 |
| 2820 | 47 | .135810 | .03 | .999959 | .135851 | .864149 | 13 |
| 2880 | 48 | .144953 | .02 | .999958 | .144996 | .855004 | 12 |
| 2940 | 49 | .153907 | .03 | .999956 | .153952 | .846048 | 11 |
| 3000 | 50 | .162681 | .03 | .999954 | .162727 | .837273 | 10 |
| 3060 | 51 | 8.171280 | .03 | 9.999952 | 8.171328 | 11.828672 | 9 |
| 3120 | 52 | .179713 | .03 | .999950 | .179763 | .820237 | 8 |
| 3180 | 53 | .187985 | .03 | .999948 | .188036 | .811964 | 7 |
| 3240 | 54 | .196102 | .03 | .999946 | .196156 | .803844 | 6 |
| 3300 | 55 | .204070 | .03 | .999944 | .204126 | .795874 | 5 |
| 3360 | 56 | .211895 | .03 | .999942 | .211953 | .788047 | 4 |
| 3420 | 57 | .219581 | .03 | .999940 | .219641 | .780359 | 3 |
| 3480 | 58 | .227134 | .03 | .999938 | .227195 | .772805 | 2 |
| 3540 | 59 | .234557 | .03 | .999936 | .234621 | .765379 | 1 |
| 3600 | 60 | 8.241855 | .03 | 9.999934 | 8.241921 | 11.758079 | 0 |
| " | ' | Coseno. | D. 1". | Seno. | Cotang. | Tang. | ' |

Para **valores intermedios**, véanse págs. **151***e* á **151***h* inclusive.

(*N. del T.* — *ten*, significa *diez*.)

| ″ | ′ | Seno. | D. 1″. | Coseno. | Tang. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|
| 3600 | 0 | 8.241855 | .03 | 9.999934 | 8.241921 | 11.758079 | 60 |
| 3660 | 1 | .249033 | .05 | .999932 | .249102 | .750898 | 59 |
| 3720 | 2 | .256094 | .03 | .999929 | .256165 | .743835 | 58 |
| 3780 | 3 | .263042 | .03 | .999927 | .263115 | .736885 | 57 |
| 3840 | 4 | .269881 | .05 | .999925 | .269956 | .730044 | 56 |
| 3900 | 5 | .276614 | .03 | .999922 | .276691 | .723309 | 55 |
| 3960 | 6 | .283243 | .03 | .999920 | .283323 | .716677 | 54 |
| 4020 | 7 | .289773 | .05 | .999918 | .289856 | .710144 | 53 |
| 4080 | 8 | .296207 | .03 | .999915 | .296292 | .703708 | 52 |
| 4140 | 9 | .302546 | .05 | .999913 | .302634 | .697366 | 51 |
| 4200 | 10 | .308794 | | .999910 | .308884 | .691116 | 50 |
| 4260 | 11 | 8.314954 | .05 | 9.999907 | 8.315046 | 11.684954 | 49 |
| 4320 | 12 | .321027 | .03 | .999905 | .321122 | .678878 | 48 |
| 4380 | 13 | .327016 | .05 | .999902 | .327114 | .672886 | 47 |
| 4440 | 14 | .332924 | .05 | .999899 | .333025 | .666975 | 46 |
| 4500 | 15 | .338753 | .03 | .999897 | .338856 | .661144 | 45 |
| 4560 | 16 | .344504 | .05 | .999894 | .344610 | .655390 | 44 |
| 4620 | 17 | .350181 | .05 | .999891 | .350289 | .649711 | 43 |
| 4680 | 18 | .355783 | .05 | .999888 | .355895 | .644105 | 42 |
| 4740 | 19 | .361315 | .05 | .999885 | .361430 | .638570 | 41 |
| 4800 | 20 | .366777 | | .999882 | .366895 | .633105 | 40 |
| 4860 | 21 | 8.372171 | .05 | 9.999879 | 8.372292 | 11.627708 | 39 |
| 4920 | 22 | .377499 | .05 | .999876 | .377622 | .622378 | 38 |
| 4980 | 23 | .382762 | .05 | .999873 | .382889 | .617111 | 37 |
| 5040 | 24 | .387963 | .05 | .999870 | .388092 | .611908 | 36 |
| 5100 | 25 | .393101 | .05 | .999867 | .393234 | .606766 | 35 |
| 5160 | 26 | .398179 | .05 | .999864 | .398315 | .601685 | 34 |
| 5220 | 27 | .403199 | .05 | .999861 | .403338 | .596662 | 33 |
| 5280 | 28 | .408161 | .07 | .999858 | .408304 | .591696 | 32 |
| 5340 | 29 | .413068 | .05 | .999854 | .413213 | .586787 | 31 |
| 5400 | 30 | .417919 | | .999851 | .418068 | .581932 | 30 |
| 5460 | 31 | 8.422717 | .05 | 9.999848 | 8.422869 | 11.577131 | 29 |
| 5520 | 32 | .427462 | .07 | .999844 | .427618 | .572382 | 28 |
| 5580 | 33 | .432156 | .05 | .999841 | .432315 | .567685 | 27 |
| 5640 | 34 | 436800 | .05 | .999838 | .436962 | .563038 | 26 |
| 5700 | 35 | .441394 | .07 | .999834 | .441560 | .558440 | 25 |
| 5760 | 36 | .445941 | .05 | .999831 | .446110 | .553890 | 24 |
| 5820 | 37 | .450440 | .07 | .999827 | .450613 | .549387 | 23 |
| 5880 | 38 | .454893 | .05 | .999824 | .455070 | .544930 | 22 |
| 5940 | 39 | .459301 | .07 | .999820 | .459481 | .540519 | 21 |
| 6000 | 40 | .463665 | | .999816 | .463849 | .536151 | 20 |
| 6060 | 41 | 8.467985 | .05 | 9.999813 | 8.468172 | 11.531828 | 19 |
| 6120 | 42 | .472263 | .07 | .999809 | .472454 | .527546 | 18 |
| 6180 | 43 | .476498 | .07 | .999805 | .476693 | .523307 | 17 |
| 6240 | 44 | .480693 | .07 | .999801 | .480892 | .519108 | 16 |
| 6300 | 45 | .484848 | .05 | .999797 | .485050 | .514950 | 15 |
| 6360 | 46 | .488963 | .07 | .999794 | .489170 | .510830 | 14 |
| 6420 | 47 | .493040 | .07 | .999790 | .493250 | .506750 | 13 |
| 6480 | 48 | .497078 | .07 | .999786 | .497293 | .502707 | 12 |
| 6540 | 49 | .501080 | .07 | .999782 | .501298 | .498702 | 11 |
| 6600 | 50 | .505045 | | .999778 | .505267 | .494733 | 10 |
| 6660 | 51 | 8.508974 | .07 | 9.999774 | 8.509200 | 11.490800 | 9 |
| 6720 | 52 | .512867 | .08 | .999769 | .513098 | .486902 | 8 |
| 6780 | 53 | .516726 | .07 | .999765 | .516961 | .483039 | 7 |
| 6840 | 54 | .520551 | .07 | .999761 | .520790 | .479210 | 6 |
| 6900 | 55 | .524343 | .07 | .999757 | .524586 | .475414 | 5 |
| 6960 | 56 | .528102 | .08 | .999753 | .528349 | .471651 | 4 |
| 7020 | 57 | .531828 | .07 | .999748 | .532080 | .467920 | 3 |
| 7080 | 58 | .535523 | .07 | .999744 | .535779 | .464221 | 2 |
| 7140 | 59 | .539186 | .08 | .999740 | .539447 | .460553 | 1 |
| 7200 | 60 | 8.542819 | | 9.999735 | 8.543084 | 11.456916 | 0 |
| ″ | ′ | Coseno. | D. 1″. | Seno. | Cotang. | Tang. | ′ |

Para valores intermedios, véase pág. 1090.

Para valores intermedios, pág. 151i

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 8 542819 | 60.05 | 9.999735 | .07 | 8.543084 | 60.12 | 11.456916 | 60 |
| 1 | .546222 | 59.55 | .999731 | .08 | .546691 | 59.62 | .453309 | 59 |
| 2 | .549995 | 59.07 | .999726 | .07 | .550268 | 59.15 | .449732 | 58 |
| 3 | .553539 | 58.58 | .999722 | .08 | .553817 | 58.65 | .446183 | 57 |
| 4 | .557054 | 58.10 | .999717 | .07 | .557336 | 58.20 | .442664 | 56 |
| 5 | .560540 | 57.65 | .999713 | .08 | .560828 | 57.72 | .439172 | 55 |
| 6 | .563999 | 57.20 | .999708 | .07 | .564291 | 57.27 | .435709 | 54 |
| 7 | .567431 | 56.75 | .999704 | .08 | .567727 | 56.83 | .432273 | 53 |
| 8 | .570836 | 56.30 | .999699 | .08 | .571137 | 56.38 | .428863 | 52 |
| 9 | .574214 | 55.87 | .999694 | .08 | .574520 | 55.95 | .425480 | 51 |
| 10 | .577566 | 55.43 | .999689 | .07 | .577877 | 55.52 | .422123 | 50 |
| 11 | 8.580892 | 55.02 | 9.999685 | .08 | 8.581208 | 55.10 | 11.418792 | 49 |
| 12 | .584193 | 54.60 | .999680 | .08 | .584514 | 54.68 | .415486 | 48 |
| 13 | .587469 | 54.20 | .999675 | .08 | .587795 | 54.27 | .412205 | 47 |
| 14 | .590721 | 53.78 | .999670 | .08 | .591051 | 53.87 | .408949 | 46 |
| 15 | .593948 | 53.40 | .999665 | .08 | .594283 | 53.48 | .405717 | 45 |
| 16 | .597152 | 53.00 | .999660 | .08 | .597492 | 53.08 | .402508 | 44 |
| 17 | .600332 | 52.62 | .999655 | .08 | .600677 | 52.70 | .399323 | 43 |
| 18 | .603489 | 52.23 | .999650 | .08 | .603839 | 52.32 | .396161 | 42 |
| 19 | .606623 | 51.85 | .999645 | .08 | .606978 | 51.93 | .393022 | 41 |
| 20 | .609734 | 51.48 | .999640 | .08 | .610094 | 51.58 | .389906 | 40 |
| 21 | 8.612823 | 51.13 | 9.999635 | .10 | 8.613189 | 51.22 | 11.386811 | 39 |
| 22 | .615891 | 50.77 | .999629 | .08 | .616262 | 50.85 | .383738 | 38 |
| 23 | .618937 | 50.42 | .999624 | .08 | .619313 | 50.50 | .380687 | 37 |
| 24 | .621962 | 50.05 | .999619 | .08 | .622343 | 50.15 | .377657 | 36 |
| 25 | .624965 | 49.72 | .999614 | .10 | .625352 | 49.80 | .374648 | 35 |
| 26 | .627948 | 49.38 | .999608 | .08 | .628340 | 49.47 | .371660 | 34 |
| 27 | .630911 | 49.05 | .999603 | .10 | .631308 | 49.13 | .368692 | 33 |
| 28 | .633854 | 48.70 | .999597 | .08 | .634256 | 48.80 | .365744 | 32 |
| 29 | .636776 | 48.40 | .999592 | .10 | .637184 | 48.48 | .362816 | 31 |
| 30 | .639680 | 48.05 | .999586 | .08 | .640093 | 48.15 | .359907 | 30 |
| 31 | 8.642563 | 47.75 | 9.999581 | .10 | 8.642982 | 47.85 | 11.357018 | 29 |
| 32 | .645428 | 47.43 | .999575 | .08 | .645853 | 47.52 | .354147 | 28 |
| 33 | .648274 | 47.13 | .999570 | .10 | .648704 | 47.22 | .351296 | 27 |
| 34 | .651102 | 46.82 | .999564 | .10 | .651537 | 46.92 | .348463 | 26 |
| 35 | .653911 | 46.52 | .999558 | .08 | .654352 | 46.62 | .345648 | 25 |
| 36 | .656702 | 46.22 | .999553 | .10 | .657149 | 46.32 | .342851 | 24 |
| 37 | .659475 | 45.92 | .999547 | .10 | .659928 | 46.02 | .340072 | 23 |
| 38 | .662230 | 45.63 | .999541 | .10 | .662689 | 45.73 | .337311 | 22 |
| 39 | .664968 | 45.35 | .999535 | .10 | .665433 | 45.45 | .334567 | 21 |
| 40 | .667689 | 45.07 | .999529 | .08 | .668160 | 45.17 | .331840 | 20 |
| 41 | 8.670393 | 44.78 | 9.999524 | .10 | 8.670870 | 44.88 | 11.329130 | 19 |
| 42 | .673080 | 44.52 | .999518 | .10 | .673563 | 44.60 | .326437 | 18 |
| 43 | .675751 | 44.23 | .999512 | .10 | .676239 | 44.35 | .323761 | 17 |
| 44 | .678405 | 43.97 | .999506 | .10 | .678900 | 44.07 | .321100 | 16 |
| 45 | .681043 | 43.70 | .999500 | .12 | .681544 | 43.80 | .318456 | 15 |
| 46 | .683665 | 43.45 | .999493 | .10 | .684173 | 43.53 | .315828 | 14 |
| 47 | .686272 | 43.18 | .999487 | .10 | .686784 | 43.28 | .313216 | 13 |
| 48 | .688863 | 42.92 | .999481 | .10 | .689381 | 43.03 | .310619 | 12 |
| 49 | .691438 | 42.67 | .999475 | .10 | .691963 | 42.77 | .308037 | 11 |
| 50 | .693998 | 42.42 | .999469 | .10 | .694529 | 42.53 | .305471 | 10 |
| 51 | 8.696543 | 42.17 | 9.999463 | .12 | 8.697081 | 42.27 | 11.302919 | 9 |
| 52 | .699073 | 41.93 | .999456 | .10 | .699617 | 42.03 | .300383 | 8 |
| 53 | .701589 | 41.68 | .999450 | .12 | .702139 | 41.78 | .297861 | 7 |
| 54 | .704090 | 41.45 | .999443 | .10 | .704646 | 41.57 | .295354 | 6 |
| 55 | .706577 | 41.20 | .999437 | .10 | .707140 | 41.30 | .292860 | 5 |
| 56 | .709049 | 40.97 | .999431 | .12 | .709618 | 41.08 | .290382 | 4 |
| 57 | .711507 | 40.75 | .999424 | .10 | .712083 | 40.85 | .287917 | 3 |
| 58 | .713952 | 40.52 | .999418 | .12 | .714534 | 40.63 | .285466 | 2 |
| 59 | .716383 | 40.28 | .999411 | .12 | .716972 | 40.40 | .283028 | 1 |
| 60 | 8.718800 | | 9.999404 | | 8.719396 | | 11.280604 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 8.718800 | 40.07 | 9.999404 | .10 | 8.719396 | 40.17 | 11.280604 | 60 |
| 1 | .721204 | 39.85 | .999398 | .12 | .721806 | 39.97 | .278194 | 59 |
| 2 | .723595 | 39.62 | .999391 | .12 | .724204 | 39.73 | .275796 | 58 |
| 3 | .725972 | 39.42 | .999384 | .10 | .726588 | 39.52 | .273412 | 57 |
| 4 | .728337 | 39.18 | .999378 | .12 | .728959 | 39.30 | .271041 | 56 |
| 5 | .730688 | 38.98 | .999371 | .12 | .731317 | 39.10 | .268683 | 55 |
| 6 | .733027 | 38.78 | .999364 | .12 | .733663 | 38.88 | .266337 | 54 |
| 7 | .735354 | 38.55 | .999357 | .12 | .735996 | 38.68 | .264004 | 53 |
| 8 | .737667 | 38.37 | .999350 | .12 | .738317 | 38.48 | .261683 | 52 |
| 9 | .739969 | 38.17 | .999343 | .12 | .740626 | 38.27 | .259374 | 51 |
| 10 | .742259 | 37.95 | .999336 | .12 | .742922 | 38.08 | .257078 | 50 |
| 11 | 8.744536 | 37.77 | 9.999329 | .12 | 8.745207 | 37.87 | 11.254793 | 49 |
| 12 | .746802 | 37.55 | .999322 | .12 | .747479 | 37.68 | .252521 | 48 |
| 13 | .749055 | 37.37 | .999315 | .12 | .749740 | 37.48 | .250260 | 47 |
| 14 | .751297 | 37.18 | .999308 | .12 | .751989 | 37.30 | .248011 | 46 |
| 15 | .753528 | 36.98 | .999301 | .12 | .754227 | 37.10 | .245773 | 45 |
| 16 | .755747 | 36.80 | .999294 | .12 | .756453 | 36.92 | .243547 | 44 |
| 17 | .757955 | 36.60 | .999287 | .13 | .758668 | 36.73 | .241332 | 43 |
| 18 | .760151 | 36.43 | .999279 | .12 | .760872 | 36.55 | .239128 | 42 |
| 19 | .762337 | 36.23 | .999272 | .12 | .763065 | 36.35 | .236935 | 41 |
| 20 | .764511 | 36.07 | .999265 | .13 | .765246 | 36.18 | .234754 | 40 |
| 21 | 8.766675 | 35.88 | 9.999257 | .12 | 8.767417 | 36.02 | 11.232583 | 39 |
| 22 | .768828 | 35.70 | .999250 | .13 | .769578 | 35.82 | .230422 | 38 |
| 23 | .770970 | 35.52 | .999242 | .12 | .771727 | 35.65 | .228273 | 37 |
| 24 | .773101 | 35.37 | .999235 | .13 | .773866 | 35.48 | .226134 | 36 |
| 25 | .775223 | 35.17 | .999227 | .12 | .775995 | 35.32 | .224005 | 35 |
| 26 | .777333 | 35.02 | .999220 | .13 | .778114 | 35.13 | .221886 | 34 |
| 27 | .779434 | 34.83 | .999212 | .12 | .780222 | 34.97 | .219778 | 33 |
| 28 | .781524 | 34.68 | .999205 | .13 | .782320 | 34.80 | .217680 | 32 |
| 29 | .783605 | 34.50 | .999197 | .13 | .784408 | 34.63 | .215592 | 31 |
| 30 | .785675 | 34.35 | .999189 | .13 | .786486 | 34.47 | .213514 | 30 |
| 31 | 8.787736 | 34.18 | 9.999181 | .12 | 8.788554 | 34.32 | 11.211446 | 29 |
| 32 | .789787 | 34.02 | .999174 | .13 | .790613 | 34.15 | .209387 | 28 |
| 33 | .791828 | 33.85 | .999166 | .13 | .792662 | 33.98 | .207338 | 27 |
| 34 | .793859 | 33.70 | .999158 | .13 | .794701 | 33.83 | .205299 | 26 |
| 35 | .795881 | 33.55 | .999150 | .13 | .796731 | 33.68 | .203269 | 25 |
| 36 | .797894 | 33.38 | .999142 | .13 | .798752 | 33.52 | .201248 | 24 |
| 37 | .799897 | 33.25 | .999134 | .13 | .800763 | 33.37 | .199237 | 23 |
| 38 | .801892 | 33.07 | .999126 | .13 | .802765 | 33.22 | .197235 | 22 |
| 39 | .803876 | 32.93 | .999118 | .13 | .804758 | 33.07 | .195242 | 21 |
| 40 | .805852 | 32.78 | .999110 | .13 | .806742 | 32.92 | .193258 | 20 |
| 41 | 8.807819 | 32.63 | 9.999102 | .13 | 8.808717 | 32.77 | 11.191283 | 19 |
| 42 | .809777 | 32.48 | .999094 | .13 | .810683 | 32.63 | .189317 | 18 |
| 43 | .811726 | 32.35 | .999086 | .13 | .812641 | 32.47 | .187359 | 17 |
| 44 | .813667 | 32.20 | .999077 | .15 | .814589 | 32.33 | .185411 | 16 |
| 45 | .815599 | 32.05 | .999069 | .13 | .816529 | 32.20 | .183471 | 15 |
| 46 | .817522 | 31.90 | .999061 | .13 | .818461 | 32.05 | .181539 | 14 |
| 47 | .819436 | 31.78 | .999053 | .15 | .820384 | 31.90 | .179616 | 13 |
| 48 | .821343 | 31.62 | .999044 | .13 | .822298 | 31.78 | .177702 | 12 |
| 49 | .823240 | 31.50 | .999036 | .15 | .824205 | 31.63 | .175795 | 11 |
| 50 | .825130 | 31.35 | .999027 | .13 | .826103 | 31.48 | .173897 | 10 |
| 51 | 8.827011 | 31.22 | 9.999019 | .15 | 8.827992 | 31.37 | 11.172008 | 9 |
| 52 | .828884 | 31.08 | .999010 | .13 | .829874 | 31.23 | .170126 | 8 |
| 53 | .830749 | 30.97 | .999002 | .15 | .831748 | 31.08 | .168252 | 7 |
| 54 | .832607 | 30.82 | .998993 | .15 | .833613 | 30.97 | .166387 | 6 |
| 55 | .834456 | 30.68 | .998984 | .13 | .835471 | 30.83 | .164529 | 5 |
| 56 | .836297 | 30.55 | .998976 | .15 | .837321 | 30.70 | .162679 | 4 |
| 57 | .838130 | 30.43 | .998967 | .15 | .839163 | 30.58 | .160837 | 3 |
| 58 | .839956 | 30.30 | .998958 | .13 | .840998 | 30.45 | .159002 | 2 |
| 59 | .841774 | 30.18 | .998950 | .15 | .842825 | 30.32 | .157175 | 1 |
| 60 | 8.843585 | | 9.998941 | | 8.844644 | | 11.155356 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 8.843585 | 30.03 | 9.998941 | .15 | 8.844644 | 30.18 | 11.155356 | 60 |
| 1 | .845887 | 29.93 | .998932 | .15 | .846955 | 30.08 | .153345 | 59 |
| 2 | .847188 | 29.80 | .998923 | .15 | .848260 | 29.95 | .151740 | 58 |
| 3 | .848971 | 29.67 | .998914 | .15 | .850057 | 29.82 | .149943 | 57 |
| 4 | .850751 | 29.57 | .998905 | .15 | .851846 | 29.70 | .148154 | 56 |
| 5 | .852525 | 29.43 | .998896 | .15 | .853628 | 29.58 | .146372 | 55 |
| 6 | .854291 | 29.30 | .998887 | .15 | .855403 | 29.47 | .144597 | 54 |
| 7 | .856049 | 29.20 | .998878 | .15 | .857171 | 29.35 | .142829 | 53 |
| 8 | .857801 | 29.08 | .998869 | .15 | .858932 | 29.23 | .141068 | 52 |
| 9 | .859546 | 28.95 | .998860 | .15 | .860686 | 29.12 | .139314 | 51 |
| 10 | .861283 | 28.85 | .998851 | .17 | .862433 | 29.00 | .137567 | 50 |
| 11 | 8.863014 | 28.73 | 9.998841 | .15 | 8.864173 | 28.88 | 11.135827 | 49 |
| 12 | .864738 | 28.62 | .998832 | .15 | .865906 | 28.77 | .134094 | 48 |
| 13 | .866455 | 28.50 | .998823 | .17 | .867632 | 28.65 | .132368 | 47 |
| 14 | .868165 | 28.38 | .998813 | .15 | .869351 | 28.55 | .130649 | 46 |
| 15 | .869868 | 28.28 | .998804 | .15 | .871064 | 28.43 | .128936 | 45 |
| 16 | .871565 | 28.17 | .998795 | .17 | .872770 | 28.32 | .127230 | 44 |
| 17 | .873255 | 28.05 | .998785 | .15 | .874469 | 28.22 | .125531 | 43 |
| 18 | .874938 | 27.95 | .998776 | .17 | .876162 | 28.12 | .123838 | 42 |
| 19 | .876615 | 27.83 | .998766 | .15 | .877849 | 28.00 | .122151 | 41 |
| 20 | .878285 | 27.73 | .998757 | .17 | .879529 | 27.88 | .120471 | 40 |
| 21 | 8.879949 | 27.63 | 9.998747 | .15 | 8.881202 | 27.78 | 11.118798 | 39 |
| 22 | .881607 | 27.52 | .998738 | .17 | .882869 | 27.68 | .117131 | 38 |
| 23 | .883258 | 27.42 | .998728 | .17 | .884530 | 27.58 | .115470 | 37 |
| 24 | .884903 | 27.32 | .998718 | .17 | .886185 | 27.47 | .113815 | 36 |
| 25 | .886542 | 27.20 | .998708 | .15 | .887833 | 27.38 | .112167 | 35 |
| 26 | .888174 | 27.12 | .998699 | .17 | .889476 | 27.27 | .110524 | 34 |
| 27 | .889801 | 27.00 | .998689 | .17 | .891112 | 27.17 | .108888 | 33 |
| 28 | .891421 | 26.90 | .998679 | .17 | .892742 | 27.07 | .107258 | 32 |
| 29 | .893035 | 26.80 | .998669 | .17 | .894366 | 26.97 | .105634 | 31 |
| 30 | .894643 | 26.72 | .998659 | .17 | .895984 | 26.87 | .104016 | 30 |
| 31 | 8.896246 | 26.60 | 9.998649 | .17 | 8.897596 | 26.78 | 11.102404 | 29 |
| 32 | .897842 | 26.50 | .998639 | .17 | .899203 | 26.67 | .100797 | 28 |
| 33 | .899432 | 26.42 | .998629 | .17 | .900803 | 26.58 | .099197 | 27 |
| 34 | .901017 | 26.32 | .998619 | .17 | .902398 | 26.48 | .097602 | 26 |
| 35 | .902596 | 26.22 | .998609 | .17 | .903987 | 26.38 | .096013 | 25 |
| 36 | .904169 | 26.12 | .998599 | .17 | .905570 | 26.28 | .094430 | 24 |
| 37 | .905736 | 26.02 | .998589 | .18 | .907147 | 26.20 | .092853 | 23 |
| 38 | .907297 | 25.93 | .998578 | .17 | .908719 | 26.10 | .091281 | 22 |
| 39 | .908853 | 25.85 | .998568 | .17 | .910285 | 26.02 | .089715 | 21 |
| 40 | .910404 | 25.75 | .998558 | .17 | .911846 | 25.93 | .088154 | 20 |
| 41 | 8.911949 | 25.65 | 9.998548 | .18 | 8.913401 | 25.83 | 11.086599 | 19 |
| 42 | .913488 | 25.57 | .998537 | .17 | .914951 | 25.73 | .085049 | 18 |
| 43 | .915022 | 25.47 | .998527 | .18 | .916495 | 25.63 | .083505 | 17 |
| 44 | .916550 | 25.38 | .998516 | .17 | .918034 | 25.57 | .081966 | 16 |
| 45 | .918073 | 25.30 | .998506 | .18 | .919568 | 25.47 | .080432 | 15 |
| 46 | .919591 | 25.20 | .998495 | .17 | .921096 | 25.38 | .078904 | 14 |
| 47 | .921103 | 25.12 | .998485 | .18 | .922619 | 25.28 | .077381 | 13 |
| 48 | .922610 | 25.03 | .998474 | .17 | .924136 | 25.22 | .075864 | 12 |
| 49 | .924112 | 24.95 | .998464 | .18 | .925649 | 25.12 | .074351 | 11 |
| 50 | .925609 | 24.85 | .998453 | .18 | .927156 | 25.03 | .072844 | 10 |
| 51 | 8.927100 | 24.78 | 9.998442 | .18 | 8.928658 | 24.95 | 11.071342 | 9 |
| 52 | .928587 | 24.68 | .998431 | .17 | .930155 | 24.87 | .069845 | 8 |
| 53 | .930068 | 24.60 | .998421 | .18 | .931647 | 24.78 | .068353 | 7 |
| 54 | .931544 | 24.52 | .998410 | .18 | .933134 | 24.70 | .066866 | 6 |
| 55 | .933015 | 24.43 | .998399 | .18 | .934616 | 24.62 | .065384 | 5 |
| 56 | .934481 | 24.35 | .998388 | .18 | .936093 | 24.53 | .063907 | 4 |
| 57 | .935942 | 24.27 | .998377 | .18 | .937565 | 24.45 | .062435 | 3 |
| 58 | .937398 | 24.20 | .998366 | .18 | .939032 | 24.37 | .060968 | 2 |
| 59 | .938850 | 24.10 | .998355 | .18 | .940494 | 24.30 | .059506 | 1 |
| 60 | 8.940296 | | 9.998344 | | 8.941952 | | 11.058048 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 8.940296 | | 9.998344 | .18 | 8.941952 | | 11.058048 | 60 |
| 1 | .941738 | 24.03 | .998333 | .18 | .943404 | 24.20 | .056596 | 59 |
| 2 | .943174 | 23.93 | .998322 | .18 | .944852 | 24.13 | .055148 | 58 |
| 3 | .944606 | 23.87 | .998311 | .18 | .946295 | 24.05 | .053705 | 57 |
| 4 | .946034 | 23.80 | .998300 | .18 | .947734 | 23.98 | .052266 | 56 |
| 5 | .947456 | 23.70 | .998289 | .20 | .949168 | 23.90 | .050832 | 55 |
| 6 | .948874 | 23.63 | .998277 | .18 | .950597 | 23.82 | .049403 | 54 |
| 7 | .950287 | 23.55 | .998266 | .18 | .952021 | 23.73 | .047979 | 53 |
| 8 | .951696 | 23.48 | .998255 | .20 | .953441 | 23.67 | .046559 | 52 |
| 9 | .953100 | 23.40 | .998243 | .18 | .954856 | 23.58 | .045144 | 51 |
| 10 | .954499 | 23.32 | .998232 | .20 | .956267 | 23.52 | .043733 | 50 |
| 11 | 8.955894 | 23.25 | 9.998220 | .18 | 8.957674 | 23.45 | 11.042326 | 49 |
| 12 | .957284 | 23.17 | .998209 | .20 | .959075 | 23.35 | .040925 | 48 |
| 13 | .958670 | 23.10 | .998197 | .18 | .960473 | 23.30 | .039527 | 47 |
| 14 | .960052 | 23.03 | .998186 | .20 | .961866 | 23.22 | .038134 | 46 |
| 15 | .961429 | 22.95 | .998174 | .18 | .963255 | 23.15 | .036745 | 45 |
| 16 | .962801 | 22.87 | .998163 | .20 | .964639 | 23.07 | .035361 | 44 |
| 17 | .964170 | 22.82 | .998151 | .20 | .966019 | 23.00 | .033981 | 43 |
| 18 | .965534 | 22.73 | .998139 | .20 | .967394 | 22.92 | .032606 | 42 |
| 19 | .966893 | 22.65 | .998128 | .18 | .968766 | 22.87 | .031234 | 41 |
| 20 | .968249 | 22.60 | .998116 | .20 | .970133 | 22.78 | .029867 | 40 |
| 21 | 8.969600 | 22.52 | 9.998104 | .20 | 8.971496 | 22.72 | 11.028504 | 39 |
| 22 | .970947 | 22.45 | .998092 | .20 | .972855 | 22.65 | .027145 | 38 |
| 23 | .972289 | 22.37 | .998080 | .20 | .974209 | 22.57 | .025791 | 37 |
| 24 | .973628 | 22.32 | .998068 | .20 | .975560 | 22.52 | .024440 | 36 |
| 25 | .974962 | 22.23 | .998056 | .20 | .976906 | 22.43 | .023094 | 35 |
| 26 | .976293 | 22.18 | .998044 | .20 | .978248 | 22.37 | .021752 | 34 |
| 27 | .977619 | 22.10 | .998032 | .20 | .979586 | 22.30 | .020414 | 33 |
| 28 | .978941 | 22.03 | .998020 | .20 | .980921 | 22.25 | .019079 | 32 |
| 29 | .980259 | 21.97 | .998008 | .20 | .982251 | 22.17 | .017749 | 31 |
| 30 | .981573 | 21.90 | .997996 | .20 | .983577 | 22.10 | .016423 | 30 |
| 31 | 8.982883 | 21.83 | 9.997984 | .20 | 8.984899 | 22.03 | 11.015101 | 29 |
| 32 | .984189 | 21.77 | .997972 | .22 | .986217 | 21.97 | .013783 | 28 |
| 33 | .985491 | 21.72 | .997959 | .20 | .987532 | 21.92 | .012468 | 27 |
| 34 | .986789 | 21.65 | .997947 | .20 | .988842 | 21.83 | .011158 | 26 |
| 35 | .988083 | 21.57 | .997935 | .20 | .990149 | 21.78 | .009851 | 25 |
| 36 | .989374 | 21.52 | .997922 | .22 | .991451 | 21.70 | .008549 | 24 |
| 37 | .990660 | 21.43 | .997910 | .20 | .992750 | 21.65 | .007250 | 23 |
| 38 | .991943 | 21.38 | .997897 | .22 | .994045 | 21.58 | .005955 | 22 |
| 39 | .993222 | 21.32 | .997885 | .20 | .995337 | 21.53 | .004663 | 21 |
| 40 | .994497 | 21.25 | .997872 | .22 | .996624 | 21.45 | .003376 | 20 |
| 41 | 8.995768 | 21.18 | 9.997860 | .22 | 8.997908 | 21.40 | 11.002092 | 19 |
| 42 | .997036 | 21.12 | .997847 | .20 | 8.999188 | 21.33 | 11.000812 | 18 |
| 43 | .998299 | 21.05 | .997835 | .22 | 9.000465 | 21.28 | 10.999535 | 17 |
| 44 | 8.999560 | 21.02 | .997822 | .22 | .001738 | 21.22 | .998262 | 16 |
| 45 | 9.000816 | 20.93 | .997809 | .22 | .003007 | 21.15 | .996993 | 15 |
| 46 | .002069 | 20.88 | .997797 | .20 | .004272 | 21.08 | .995728 | 14 |
| 47 | .003318 | 20.82 | .997784 | .22 | .005534 | 21.03 | .994466 | 13 |
| 48 | .004563 | 20.75 | .997771 | .22 | .006792 | 20.97 | .993208 | 12 |
| 49 | .005805 | 20.70 | .997758 | .22 | .008047 | 20.92 | .991953 | 11 |
| 50 | .007044 | 20.65 | .997745 | .22 | .009298 | 20.85 | .990702 | 10 |
| 51 | 9.008278 | 20.57 | 9.997732 | .22 | 9.010546 | 20.80 | 10.989454 | 9 |
| 52 | .009510 | 20.53 | .997719 | .22 | .011790 | 20.73 | .988210 | 8 |
| 53 | .010737 | 20.45 | .997706 | .23 | .013031 | 20.68 | .986969 | 7 |
| 54 | .011962 | 20.42 | .997693 | .23 | .014268 | 20.62 | .985732 | 6 |
| 55 | .013182 | 20.33 | .997680 | .22 | .015502 | 20.57 | .984498 | 5 |
| 56 | .014400 | 20.30 | .997667 | .23 | .016732 | 20.50 | .983268 | 4 |
| 57 | .015613 | 20.22 | .997654 | .22 | .017959 | 20.45 | .982041 | 3 |
| 58 | .016824 | 20.18 | .997641 | .23 | .019183 | 20.40 | .980817 | 2 |
| 59 | .018031 | 20.12 | .997628 | .23 | .020403 | 20.33 | .979597 | 1 |
| 60 | 9.019235 | 20.07 | 9.997614 | .23 | 9.021620 | 20.28 | 10.978380 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D.1″. | Coseno. | D.1″. | Tang. | D.1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.019235 | 20.00 | 9.997614 | .22 | 9.021620 | 20.23 | 10.978380 | 60 |
| 1 | .020435 | 19.95 | .997601 | .22 | .022834 | 20.17 | .977166 | 59 |
| 2 | .021632 | 19.88 | .997588 | .22 | .024044 | 20.12 | .975956 | 58 |
| 3 | .022825 | 19.85 | .997574 | .23 | .025251 | 20.07 | .974749 | 57 |
| 4 | .024016 | 19.78 | .997561 | .22 | .026455 | 20.00 | .973545 | 56 |
| 5 | .025203 | 19.72 | .997547 | .23 | .027655 | 19.95 | .972345 | 55 |
| 6 | .026386 | 19.68 | .997534 | .23 | .028852 | 19.90 | .971148 | 54 |
| 7 | .027567 | 19.62 | .997520 | .23 | .030046 | 19.85 | .969954 | 53 |
| 8 | .028744 | 19.57 | .997507 | .23 | .031237 | 19.80 | .968763 | 52 |
| 9 | .029918 | 19.52 | .997493 | .22 | .032425 | 19.73 | .967575 | 51 |
| 10 | .031089 | 19.47 | .997480 | .23 | .033609 | 19.70 | .966391 | 50 |
| 11 | 9.032257 | 19.40 | 9.997466 | .23 | 9.034791 | 19.63 | 10.965209 | 49 |
| 12 | .033421 | 19.35 | .997452 | .22 | .035969 | 19.58 | .964031 | 48 |
| 13 | .034582 | 19.32 | .997439 | .23 | .037144 | 19.53 | .962856 | 47 |
| 14 | .035741 | 19.25 | .997425 | .23 | .038316 | 19.48 | .961684 | 46 |
| 15 | .036896 | 19.20 | .997411 | .23 | .039485 | 19.43 | .960515 | 45 |
| 16 | .038048 | 19.15 | .997397 | .23 | .040651 | 19.37 | .959349 | 44 |
| 17 | .039197 | 19.08 | .997383 | .23 | .041813 | 19.33 | .958187 | 43 |
| 18 | .040342 | 19.05 | .997369 | .23 | .042973 | 19.28 | .957027 | 42 |
| 19 | .041485 | 19.00 | .997355 | .23 | .044130 | 19.22 | .955870 | 41 |
| 20 | .042625 | 18.95 | .997341 | .23 | .045284 | 19.17 | .954716 | 40 |
| 21 | 9.043762 | 18.88 | 9.997327 | .23 | 9.046434 | 19.13 | 10.953566 | 39 |
| 22 | .044895 | 18.85 | .997313 | .23 | .047582 | 19.08 | .952418 | 38 |
| 23 | .046026 | 18.80 | .997299 | .23 | .048727 | 19.03 | .951273 | 37 |
| 24 | .047154 | 18.75 | .997285 | .23 | .049869 | 18.98 | .950131 | 36 |
| 25 | .048279 | 18.68 | .997271 | .23 | .051008 | 18.93 | .948992 | 35 |
| 26 | .049400 | 18.65 | .997257 | .23 | .052144 | 18.88 | .947856 | 34 |
| 27 | .050519 | 18.60 | .997242 | .23 | .053277 | 18.83 | .946723 | 33 |
| 28 | .051635 | 18.57 | .997228 | .23 | .054407 | 18.80 | .945593 | 32 |
| 29 | .052749 | 18.50 | .997214 | .23 | .055535 | 18.73 | .944465 | 31 |
| 30 | .053859 | 18.45 | .997199 | .23 | .056659 | 18.70 | .943341 | 30 |
| 31 | 9.054966 | 18.42 | 9.997185 | .23 | 9.057781 | 18.65 | 10.942219 | 29 |
| 32 | .056071 | 18.35 | .997170 | .23 | .058900 | 18.60 | .941100 | 28 |
| 33 | .057172 | 18.32 | .997156 | .23 | .060016 | 18.57 | .939984 | 27 |
| 34 | .058271 | 18.27 | .997141 | .23 | .061130 | 18.50 | .938870 | 26 |
| 35 | .059367 | 18.22 | .997127 | .23 | .062240 | 18.47 | .937760 | 25 |
| 36 | .060460 | 18.18 | .997112 | .23 | .063348 | 18.43 | .936652 | 24 |
| 37 | .061551 | 18.13 | .997098 | .23 | .064453 | 18.38 | .935547 | 23 |
| 38 | .062639 | 18.08 | .997083 | .23 | .065556 | 18.32 | .934444 | 22 |
| 39 | .063724 | 18.03 | .997068 | .23 | .066655 | 18.28 | .933345 | 21 |
| 40 | .064806 | 17.98 | .997053 | .23 | .067752 | 18.25 | .932248 | 20 |
| 41 | 9.065885 | 17.95 | 9.997039 | .23 | 9.068846 | 18.20 | 10.931154 | 19 |
| 42 | .066962 | 17.90 | .997024 | .23 | .069938 | 18.15 | .930062 | 18 |
| 43 | .068036 | 17.85 | .997009 | .23 | .071027 | 18.10 | .928973 | 17 |
| 44 | .069107 | 17.82 | .996994 | .23 | .072113 | 18.07 | .927887 | 16 |
| 45 | .070176 | 17.77 | .996979 | .23 | .073197 | 18.02 | .926803 | 15 |
| 46 | .071242 | 17.73 | .996964 | .23 | .074278 | 17.97 | .925722 | 14 |
| 47 | .072306 | 17.67 | .996949 | .23 | .075356 | 17.93 | .924644 | 13 |
| 48 | .073366 | 17.63 | .996934 | .23 | .076432 | 17.88 | .923568 | 12 |
| 49 | .074424 | 17.60 | .996919 | .23 | .077505 | 17.85 | .922495 | 11 |
| 50 | .075480 | 17.55 | .996904 | .23 | .078576 | 17.80 | .921424 | 10 |
| 51 | 9.076533 | 17.50 | 9.996889 | .23 | 9.079644 | 17.77 | 10.920356 | 9 |
| 52 | .077583 | 17.47 | .996874 | .23 | .080710 | 17.72 | .919290 | 8 |
| 53 | .078631 | 17.42 | .996858 | .23 | .081773 | 17.67 | .918227 | 7 |
| 54 | .079676 | 17.38 | .996843 | .23 | .082833 | 17.63 | .917167 | 6 |
| 55 | .080719 | 17.33 | .996828 | .23 | .083891 | 17.60 | .916109 | 5 |
| 56 | .081759 | 17.30 | .996812 | .23 | .084947 | 17.55 | .915053 | 4 |
| 57 | .082797 | 17.25 | .996797 | .23 | .086000 | 17.50 | .914000 | 3 |
| 58 | .083832 | 17.20 | .996782 | .23 | .087050 | 17.47 | .912950 | 2 |
| 59 | .084864 | 17.17 | .996766 | .23 | .088098 | 17.43 | .911902 | 1 |
| 60 | 9.085894 | | 9.996751 | .23 | 9.089144 | | 10.910856 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ' | Seno. | D. 1". | Coseno. | D. 1". | Tang. | D. 1". | Cotang. | ' |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.085894 | | 9.996751 | | 9.089144 | | | |
| 1 | .086922 | 17.13 | .996735 | .27 | .090187 | 17.33 | 10.910856 | 60 |
| 2 | .087947 | 17.08 | .996720 | .25 | .091228 | 17.35 | .909813 | 59 |
| 3 | .088970 | 17.05 | .996704 | .27 | .092266 | 17.30 | .908772 | 58 |
| 4 | .089990 | 17.00 | .996688 | .27 | .093302 | 17.27 | .907734 | 57 |
| 5 | .091008 | 16.97 | .996673 | .25 | .094336 | 17.23 | .906698 | 56 |
| 6 | .092024 | 16.93 | .996657 | .27 | .095367 | 17.18 | .905664 | 55 |
| 7 | .093037 | 16.88 | .996641 | .27 | .096395 | 17.13 | .904633 | 54 |
| 8 | .094047 | 16.83 | .996625 | .27 | .097422 | 17.12 | .903605 | 53 |
| 9 | .095056 | 16.82 | .996610 | .25 | .098446 | 17.07 | .902578 | 52 |
| 10 | .096062 | 16.77 | .996594 | .27 | .099468 | 17.03 | .901554 | 51 |
| 11 | 9.097065 | 16.72 | 9.996578 | .27 | 9.100487 | 16.98 | 10.900532 | 50 |
| 12 | .098066 | 16.68 | .996562 | .27 | .101504 | 16.95 | .899513 | 49 |
| 13 | .099065 | 16.65 | .996546 | .27 | .102519 | 16.92 | .898496 | 48 |
| 14 | .100062 | 16.62 | .996530 | .27 | .103532 | 16.88 | .897481 | 47 |
| 15 | .101056 | 16.57 | .996514 | .27 | .104542 | 16.83 | .896468 | 46 |
| 16 | .102048 | 16.53 | .996498 | .27 | .105550 | 16.80 | .895458 | 45 |
| 17 | .103037 | 16.48 | .996482 | .27 | .106556 | 16.77 | .894450 | 44 |
| 18 | .104025 | 16.47 | .996465 | .28 | .107559 | 16.72 | .893444 | 43 |
| 19 | .105010 | 16.42 | .996449 | .27 | .108560 | 16.68 | .892441 | 42 |
| 20 | .105992 | 16.37 | .996433 | .27 | .109559 | 16.65 | .891440 | 41 |
| 21 | 9.106973 | 16.35 | 9.996417 | .27 | 9.110556 | 16.62 | .890441 | 40 |
| 22 | .107951 | 16.30 | .996400 | .28 | .111551 | 16.58 | 10.889444 | 39 |
| 23 | .108927 | 16.27 | .996384 | .27 | .112543 | 16.53 | .888449 | 38 |
| 24 | .109901 | 16.23 | .996368 | .27 | .113533 | 16.50 | .887457 | 37 |
| 25 | .110873 | 16.20 | .996351 | .28 | .114521 | 16.47 | .886467 | 36 |
| 26 | .111842 | 16.15 | .996335 | .27 | .115507 | 16.43 | .885479 | 35 |
| 27 | .112809 | 16.12 | .996318 | .28 | .116491 | 16.40 | .884493 | 34 |
| 28 | .113774 | 16.08 | .996302 | .27 | .117472 | 16.35 | .883509 | 33 |
| 29 | .114737 | 16.05 | .996285 | .28 | .118452 | 16.33 | .882528 | 32 |
| 30 | .115698 | 16.02 | .996269 | .27 | .119429 | 16.28 | .881548 | 31 |
| 31 | 9.116656 | 15.97 | 9.996252 | .28 | 9.120404 | 16.25 | .880571 | 30 |
| 32 | .117613 | 15.95 | .996235 | .28 | .121377 | 16.22 | 10.879596 | 29 |
| 33 | .118567 | 15.90 | .996219 | .27 | .122348 | 16.18 | .878623 | 28 |
| 34 | .119519 | 15.87 | .996202 | .28 | .123317 | 16.15 | .877652 | 27 |
| 35 | .120469 | 15.83 | .996185 | .28 | .124284 | 16.12 | .876683 | 26 |
| 36 | .121417 | 15.80 | .996168 | .28 | .125249 | 16.08 | .875716 | 25 |
| 37 | .122362 | 15.75 | .996151 | .28 | .126211 | 16.03 | .874751 | 24 |
| 38 | .123306 | 15.73 | .996134 | .28 | .127172 | 16.02 | .873789 | 23 |
| 39 | .124248 | 15.70 | .996117 | .28 | .128130 | 15.97 | .872828 | 22 |
| 40 | .125187 | 15.65 | .996100 | .28 | .129087 | 15.95 | .871870 | 21 |
| 41 | 9.126125 | 15.63 | 9.996083 | .28 | 9.130041 | 15.90 | .870913 | 20 |
| 42 | .127060 | 15.58 | .996066 | .28 | .130994 | 15.88 | 10.869959 | 19 |
| 43 | .127993 | 15.55 | .996049 | .28 | .131944 | 15.83 | .869006 | 18 |
| 44 | .128925 | 15.53 | .996032 | .28 | .132893 | 15.82 | .868056 | 17 |
| 45 | .129854 | 15.48 | .996015 | .28 | .133839 | 15.77 | .867107 | 16 |
| 46 | .130781 | 15.45 | .995998 | .28 | .134784 | 15.75 | .866161 | 15 |
| 47 | .131706 | 15.42 | .995980 | .30 | .135726 | 15.70 | .865216 | 14 |
| 48 | .132630 | 15.40 | .995963 | .28 | .136667 | 15.68 | .864274 | 13 |
| 49 | .133551 | 15.35 | .995946 | .28 | .137605 | 15.63 | .863333 | 12 |
| 50 | .134470 | 15.32 | .995928 | .30 | .138542 | 15.62 | .862395 | 11 |
| 51 | 9.135387 | 15.28 | 9.995911 | .28 | 9.139476 | 15.57 | .861458 | 10 |
| 52 | .136303 | 15.27 | .995894 | .30 | .140409 | 15.55 | 10.860524 | 9 |
| 53 | .137216 | 15.22 | .995876 | .28 | .141340 | 15.52 | .859591 | 8 |
| 54 | .138128 | 15.20 | .995859 | .30 | .142269 | 15.48 | .858660 | 7 |
| 55 | .139037 | 15.15 | .995841 | .30 | .143196 | 15.45 | .857731 | 6 |
| 56 | .139944 | 15.12 | .995823 | .30 | .144121 | 15.42 | .856804 | 5 |
| 57 | .140850 | 15.10 | .995806 | .28 | .145044 | 15.38 | .855879 | 4 |
| 58 | .141754 | 15.07 | .995788 | .30 | .145966 | 15.37 | .854956 | 3 |
| 59 | .142655 | 15.03 | .995771 | .28 | .146885 | 15.32 | .854034 | 2 |
| 60 | 9.143555 | 15.00 | 9.995753 | .30 | 9.147803 | 15.30 | 10.852197 | 0 |
| | Coseno. | D. 1". | Seno. | D. 1". | Cotang. | D. 1". | Tang. | ' |

| ′ | Seno. | D. 1″. | seno. | D. 1″. | Cog. | D. 1″ | Tang. | Cotang | . | ′ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 9.143555 | 14.97 | 9.995758 | .30 | 9.147803 | 15.25 | 10.852197 | | | 60 |
| 1 | .144453 | 14.93 | .995735 | .30 | .148718 | 15.23 | .851282 | | | 59 |
| 2 | .145349 | 14.90 | .995717 | .30 | .149632 | 15.20 | .850368 | | | 58 |
| 3 | .146243 | 14.88 | .995699 | .30 | .150544 | 15.17 | .849456 | | | 57 |
| 4 | .147136 | 14.83 | .995681 | .28 | .151454 | 15.15 | .848546 | | | 56 |
| 5 | .148026 | 14.82 | .995664 | .30 | .152363 | 15.10 | .847637 | | | 55 |
| 6 | .148915 | 14.78 | .995646 | .30 | .153269 | 15.08 | .846731 | | | 54 |
| 7 | .149802 | 14.73 | .995628 | .30 | .154174 | 15.05 | .845826 | | | 53 |
| 8 | .150686 | 14.72 | .995610 | .32 | .155077 | 15.02 | .844923 | | | 52 |
| 9 | .151569 | 14.70 | .995591 | .30 | .155978 | 14.98 | .844022 | | | 51 |
| 10 | .152451 | 14.65 | .995573 | .30 | .156877 | 14.97 | .843123 | | | 50 |
| 11 | 9.153330 | 14.63 | 9.995555 | .30 | 9.157775 | 14.93 | 10.842225 | | | 49 |
| 12 | .154208 | 14.58 | .995537 | .30 | .158671 | 14.90 | .841329 | | | 48 |
| 13 | .155083 | 14.57 | .995519 | .30 | .159565 | 14.87 | .840435 | | | 47 |
| 14 | .155957 | 14.55 | .995501 | .32 | .160457 | 14.83 | .839543 | | | 46 |
| 15 | .156830 | 14.50 | .995482 | .30 | .161347 | 14.82 | .838653 | | | 45 |
| 16 | .157700 | 14.48 | .995464 | .30 | .162236 | 14.78 | .837764 | | | 44 |
| 17 | .158569 | 14.43 | .995446 | .32 | .163123 | 14.75 | .836877 | | | 43 |
| 18 | .159435 | 14.43 | .995427 | .30 | .164008 | 14.73 | .835992 | | | 42 |
| 19 | .160301 | 14.38 | .995409 | .32 | .164892 | 14.70 | .835108 | | | 41 |
| 20 | .161164 | 14.35 | .995390 | .30 | .165774 | 14.67 | .834226 | | | 40 |
| 21 | 9.162025 | 14.33 | 9.995372 | .32 | 9.166654 | 14.63 | 10.833346 | | | 39 |
| 22 | .162885 | 14.30 | .995353 | .32 | .167532 | 14.62 | .832468 | | | 38 |
| 23 | .163743 | 14.28 | .995334 | .30 | .168409 | 14.58 | .831591 | | | 37 |
| 24 | .164600 | 14.23 | .995316 | .32 | .169284 | 14.55 | .830716 | | | 36 |
| 25 | .165454 | 14.22 | .995297 | .32 | .170157 | 14.53 | .829843 | | | 35 |
| 26 | .166307 | 14.20 | .995278 | .30 | .171029 | 14.50 | .828971 | | | 34 |
| 27 | .167159 | 14.15 | .995260 | .32 | .171899 | 14.47 | .828101 | | | 33 |
| 28 | .168008 | 14.13 | .995241 | .32 | .172767 | 14.45 | .827233 | | | 32 |
| 29 | .168856 | 14.10 | .995222 | .32 | .173634 | 14.42 | .826366 | | | 31 |
| 30 | .169702 | 14.08 | .995203 | .32 | .174499 | 14.38 | .825501 | | | 30 |
| 31 | 9.170547 | 14.03 | 9.995184 | .32 | 9.175362 | 14.37 | 10.824638 | | | 29 |
| 32 | .171389 | 14.02 | .995165 | .32 | .176224 | 14.33 | .823776 | | | 28 |
| 33 | .172230 | 14.00 | .995146 | .32 | .177084 | 14.30 | .822916 | | | 27 |
| 34 | .173070 | 13.97 | .995127 | .32 | .177942 | 14.28 | .822058 | | | 26 |
| 35 | .173908 | 13.93 | .995108 | .32 | .178799 | 14.27 | .821201 | | | 25 |
| 36 | .174744 | 13.90 | .995089 | .32 | .179655 | 14.22 | .820345 | | | 24 |
| 37 | .175578 | 13.88 | .995070 | .32 | .180508 | 14.20 | .819492 | | | 23 |
| 38 | .176411 | 13.85 | .995051 | .32 | .181360 | 14.18 | .818640 | | | 22 |
| 39 | .177242 | 13.83 | .995032 | .32 | .182211 | 14.13 | .817789 | | | 21 |
| 40 | .178072 | 13.80 | .995013 | .33 | .183059 | 14.13 | .816941 | | | 20 |
| 41 | 9.178900 | 13.77 | 9.994993 | .32 | 9.183907 | 14.08 | 10.816093 | | | 19 |
| 42 | .179726 | 13.75 | .994974 | .32 | .184752 | 14.08 | .815248 | | | 18 |
| 43 | .180551 | 13.72 | .994955 | .33 | .185597 | 14.03 | .814403 | | | 17 |
| 44 | .181374 | 13.70 | .994935 | .32 | .186439 | 14.03 | .813561 | | | 16 |
| 45 | .182196 | 13.67 | .994916 | .33 | .187280 | 14.00 | .812720 | | | 15 |
| 46 | .183016 | 13.63 | .994896 | .32 | .188120 | 13.97 | .811880 | | | 14 |
| 47 | .183834 | 13.62 | .994877 | .33 | .188958 | 13.93 | .811042 | | | 13 |
| 48 | .184651 | 13.62 | .994857 | .32 | .189794 | 13.92 | .810206 | | | 12 |
| 49 | .185466 | 13.58 | .994838 | .33 | .190629 | 13.88 | .809371 | | | 11 |
| 50 | .186280 | 13.57 | .994818 | .33 | .191462 | 13.87 | .808538 | | | 10 |
| 51 | 9.187092 | 13.52 | 9.994798 | .32 | 9.192294 | 13.83 | 10.807706 | | | 9 |
| 52 | .187903 | 13.48 | .994779 | .33 | .193124 | 13.82 | .806876 | | | 8 |
| 53 | .188712 | 13.45 | .994759 | .33 | .193953 | 13.78 | .806047 | | | 7 |
| 54 | .189519 | 13.43 | .994739 | .32 | .194780 | 13.77 | .805220 | | | 6 |
| 55 | .190325 | 13.42 | .994720 | .33 | .195606 | 13.73 | .804394 | | | 5 |
| 56 | .191130 | 13.38 | .994700 | .33 | .196430 | 13.72 | .803570 | | | 4 |
| 57 | .191933 | 13.35 | .994680 | .33 | .197253 | 13.68 | .802747 | | | 3 |
| 58 | .192734 | 13.33 | .994660 | .33 | .198074 | 13.67 | .801926 | | | 2 |
| 59 | .193534 | 13.30 | .994640 | .33 | .198894 | 13.65 | .801106 | | | 1 |
| 60 | 9.194332 | | 9.994620 | .33 | 9.199713 | | 10.800287 | | | 0 |
| | Coseno. | D. | . | Seno. | D. | . | Cotang. | D. | ″. | Tang. |

| ′ | Seno. | D. 1″. | Coseno. | D. 1′. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.194332 | | 9.994620 | | 9.199713 | | 10.800287 | 60 |
| 1 | .195129 | 13.28 | .994600 | .33 | .200529 | 13.60 | .799471 | 59 |
| 2 | .195925 | 13.27 | .994580 | .33 | .201345 | 13.60 | .798655 | 58 |
| 3 | .196719 | 13.23 | .994560 | .33 | .202159 | 13.57 | .797841 | 57 |
| 4 | .197511 | 13.20 | .994540 | .33 | .202971 | 13.53 | .797029 | 56 |
| 5 | .198302 | 13.18 | .994519 | .35 | .203782 | 13.52 | .796218 | 55 |
| 6 | .199091 | 13.15 | .994499 | .33 | .204592 | 13.50 | .795408 | 54 |
| 7 | .199879 | 13.13 | .994479 | .33 | .205400 | 13.47 | .794600 | 53 |
| 8 | .200666 | 13.12 | .994459 | .33 | .206207 | 13.45 | .793793 | 52 |
| 9 | .201451 | 13.08 | .994438 | .35 | .207013 | 13.43 | .792987 | 51 |
| 10 | .202234 | 13.05 | .994418 | .33 | .207817 | 13.40 | .792183 | 50 |
| 11 | 9.203017 | 13.05 | 9.994398 | .33 | 9.208619 | 13.37 | 10.791381 | 49 |
| 12 | .203797 | 13.00 | .994377 | .35 | .209420 | 13.35 | .790580 | 48 |
| 13 | .204577 | 13.00 | .994357 | .33 | .210220 | 13.33 | .789780 | 47 |
| 14 | .205354 | 12.95 | .994336 | .35 | .211018 | 13.30 | .788982 | 46 |
| 15 | .206131 | 12.95 | .994316 | .33 | .211815 | 13.28 | .788185 | 45 |
| 16 | .206906 | 12.93 | .994295 | .35 | .212611 | 13.27 | .787389 | 44 |
| 17 | .207679 | 12.88 | .994274 | .35 | .213405 | 13.23 | .786595 | 43 |
| 18 | .208452 | 12.88 | .994254 | .33 | .214198 | 13.22 | .785802 | 42 |
| 19 | .209222 | 12.83 | .994233 | .35 | .214989 | 13.18 | .785011 | 41 |
| 20 | .209992 | 12.83 | .994212 | .35 | .215780 | 13.18 | .784220 | 40 |
| 21 | 9.210760 | 12.80 | 9.994191 | .35 | 9.216568 | 13.13 | 10.783432 | 39 |
| 22 | .211526 | 12.77 | .994171 | .33 | .217356 | 13.13 | .782644 | 38 |
| 23 | .212291 | 12.75 | .994150 | .35 | .218142 | 13.10 | .781858 | 37 |
| 24 | .213055 | 12.73 | .994129 | .35 | .218926 | 13.07 | .781074 | 36 |
| 25 | .213818 | 12.72 | .994108 | .35 | .219710 | 13.07 | .780290 | 35 |
| 26 | .214579 | 12.68 | .994087 | .35 | .220492 | 13.03 | .779508 | 34 |
| 27 | .215338 | 12.65 | .994066 | .35 | .221272 | 13.00 | .778728 | 33 |
| 28 | .216097 | 12.65 | .994045 | .35 | .222052 | 13.00 | .777948 | 32 |
| 29 | .216854 | 12.62 | .994024 | .35 | .222830 | 12.97 | .777170 | 31 |
| 30 | .217609 | 12.58 | .994003 | .35 | .223607 | 12.95 | .776393 | 30 |
| 31 | 9.218363 | 12.57 | 9.993982 | .37 | 9.224382 | 12.92 | 10.775618 | 29 |
| 32 | .219116 | 12.55 | .993960 | .35 | .225156 | 12.90 | .774844 | 28 |
| 33 | .219868 | 12.53 | .993939 | .35 | .225929 | 12.88 | .774071 | 27 |
| 34 | .220618 | 12.50 | .993918 | .35 | .226700 | 12.85 | .773300 | 26 |
| 35 | .221367 | 12.48 | .993897 | .35 | .227471 | 12.85 | .772529 | 25 |
| 36 | .222115 | 12.47 | .993875 | .37 | .228239 | 12.80 | .771761 | 24 |
| 37 | .222861 | 12.43 | .993854 | .35 | .229007 | 12.80 | .770993 | 23 |
| 38 | .223606 | 12.42 | .993832 | .37 | .229773 | 12.77 | .770227 | 22 |
| 39 | .224349 | 12.38 | .993811 | .35 | .230539 | 12.77 | .769461 | 21 |
| 40 | .225092 | 12.35 | .993789 | .37 | .231302 | 12.73 | .768698 | 20 |
| 41 | 9.225833 | 12.33 | 9.993768 | .37 | 9.232065 | 12.72 | 10.767935 | 19 |
| 42 | .226573 | 12.30 | .993746 | .35 | .232826 | 12.68 | .767174 | 18 |
| 43 | .227311 | 12.28 | .993725 | .37 | .233586 | 12.67 | .766414 | 17 |
| 44 | .228048 | 12.27 | .993703 | .37 | .234345 | 12.65 | .765655 | 16 |
| 45 | .228784 | 12.23 | .993681 | .37 | .235103 | 12.63 | .764897 | 15 |
| 46 | .229518 | 12.23 | .993660 | .35 | .235859 | 12.60 | .764141 | 14 |
| 47 | .230252 | 12.20 | .993638 | .37 | .236614 | 12.58 | .763386 | 13 |
| 48 | .230984 | 12.18 | .993616 | .37 | .237368 | 12.57 | .762632 | 12 |
| 49 | .231715 | 12.15 | .993594 | .37 | .238120 | 12.53 | .761880 | 11 |
| 50 | .232444 | 12.13 | .993572 | .37 | .238872 | 12.53 | .761128 | 10 |
| 51 | 9.233172 | 12.12 | 9.993550 | .37 | 9.239622 | 12.50 | 10.760378 | 9 |
| 52 | .233899 | 12.10 | .993528 | .37 | .240371 | 12.48 | .759629 | 8 |
| 53 | .234625 | 12.07 | .993506 | .37 | .241118 | 12.45 | .758882 | 7 |
| 54 | .235349 | 12.07 | .993484 | .37 | .241865 | 12.45 | .758135 | 6 |
| 55 | .236073 | 12.03 | .993462 | .37 | .242610 | 12.42 | .757390 | 5 |
| 56 | .236795 | 12.00 | .993440 | .37 | .243354 | 12.40 | .756646 | 4 |
| 57 | .237515 | 12.00 | .993418 | .37 | .244097 | 12.38 | .755903 | 3 |
| 58 | .238235 | 11.97 | .993396 | .37 | .244839 | 12.37 | .755161 | 2 |
| 59 | .238953 | 11.95 | .993374 | .38 | .245579 | 12.33 | .754421 | 1 |
| 60 | 9.239670 | | 9.993351 | | 9.246319 | 12.33 | 10.753681 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″ | Tang. | ′ |

| ' | Seno. | D. 1". | Coseno. | D. 1". | Tang. | D. 1". | Cotang. | ' |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | 10.753681 | 60 |
| 0 | 9.239670 | 11.93 | 9.993351 | .37 | 9.246319 | 12.30 | .752943 | 59 |
| 1 | .240386 | 11.92 | .993329 | .37 | .247057 | 12.28 | .752206 | 58 |
| 2 | .241101 | 11.88 | .993307 | .38 | .247794 | 12.27 | .751470 | 57 |
| 3 | .241814 | 11.87 | .993284 | .37 | .248530 | 12.23 | .750736 | 56 |
| 4 | .242526 | 11.85 | .993262 | .37 | .249264 | 12.23 | .750002 | 55 |
| 5 | .243237 | 11.83 | .993240 | .38 | .249998 | 12.20 | .749270 | 54 |
| 6 | .243947 | 11.82 | .993217 | .37 | .250730 | 12.18 | .748539 | 53 |
| 7 | .244656 | 11.78 | .993195 | .38 | .251461 | 12.17 | .747809 | 52 |
| 8 | .245363 | 11.77 | .993172 | .38 | .252191 | 12.15 | .747080 | 51 |
| 9 | .246069 | 11.77 | .993149 | .37 | .252920 | 12.13 | .746352 | 50 |
| 10 | .246775 | 11.72 | .993127 | .38 | .253648 | 12.10 | | |
| | | | | | | | 10.745626 | 49 |
| 11 | 9.247478 | 11.72 | 9.993104 | .38 | 9.254374 | 12.10 | .744900 | 48 |
| 12 | .248181 | 11.70 | .993081 | .37 | .255100 | 12.07 | .744176 | 47 |
| 13 | .248883 | 11.67 | .993059 | .38 | .255824 | 12.05 | .743453 | 46 |
| 14 | .249583 | 11.65 | .993036 | .38 | .256547 | 12.03 | .742731 | 45 |
| 15 | .250282 | 11.63 | .993013 | .38 | .257269 | 12.02 | .742010 | 44 |
| 16 | .250980 | 11.62 | .992990 | .38 | .257990 | 12.00 | .741290 | 43 |
| 17 | .251677 | 11.60 | .992967 | .38 | .258710 | 11.98 | .740571 | 42 |
| 18 | .252373 | 11.57 | .992944 | .38 | .259429 | 11.95 | .739854 | 41 |
| 19 | .253067 | 11.57 | .992921 | .38 | .260146 | 11.95 | .739137 | 40 |
| 20 | .253761 | 11.53 | .992898 | .38 | .260863 | 11.92 | | |
| | | | | | | | 10.738422 | 39 |
| 21 | 9.254453 | 11.53 | 9.992875 | .38 | 9.261578 | 11.90 | .737708 | 38 |
| 22 | .255144 | 11.50 | .992852 | .38 | .262292 | 11.88 | .736995 | 37 |
| 23 | .255834 | 11.48 | .992829 | .38 | .263005 | 11.87 | .736283 | 36 |
| 24 | .256523 | 11.47 | .992806 | .38 | .263717 | 11.85 | .735572 | 35 |
| 25 | .257211 | 11.45 | .992783 | .40 | .264428 | 11.83 | .734862 | 34 |
| 26 | .257898 | 11.42 | .992759 | .38 | .265138 | 11.82 | .734153 | 33 |
| 27 | .258583 | 11.42 | .992736 | .38 | .265847 | 11.80 | .733445 | 32 |
| 28 | .259268 | 11.38 | .992713 | .38 | .266555 | 11.77 | .732739 | 31 |
| 29 | .259951 | 11.37 | .992690 | .40 | .267261 | 11.77 | .732033 | 30 |
| 30 | .260633 | 11.35 | .992666 | .38 | .267967 | 11.73 | | |
| | | | | | | | 10.731329 | 29 |
| 31 | 9.261314 | 11.33 | 9.992643 | .40 | 9.268671 | 11.73 | .730625 | 28 |
| 32 | .261994 | 11.32 | .992619 | .38 | .269375 | 11.70 | .729923 | 27 |
| 33 | .262673 | 11.30 | .992596 | .40 | .270077 | 11.70 | .729221 | 26 |
| 34 | .263351 | 11.27 | .992572 | .38 | .270779 | 11.67 | .728521 | 25 |
| 35 | .264027 | 11.27 | .992549 | .40 | .271479 | 11.65 | .727822 | 24 |
| 36 | .264703 | 11.23 | .992525 | .40 | .272178 | 11.63 | .727124 | 23 |
| 37 | .265377 | 11.23 | .992501 | .38 | .272876 | 11.62 | .726427 | 22 |
| 38 | .266051 | 11.20 | .992478 | .40 | .273573 | 11.60 | .725731 | 21 |
| 39 | .266723 | 11.20 | .992454 | .40 | .274269 | 11.58 | .725036 | 20 |
| 40 | .267395 | 11.17 | .992430 | .40 | .274964 | 11.57 | | |
| | | | | | | | 10.724342 | 19 |
| 41 | 9.268065 | 11.15 | 9.992406 | .40 | 9.275658 | 11.55 | .723649 | 18 |
| 42 | .268734 | 11.13 | .992382 | .38 | .276351 | 11.53 | .722957 | 17 |
| 43 | .269402 | 11.12 | .992359 | .40 | .277043 | 11.52 | .722266 | 16 |
| 44 | .270069 | 11.10 | .992335 | .40 | .277734 | 11.50 | .721576 | 15 |
| 45 | .270735 | 11.08 | .992311 | .40 | .278424 | 11.48 | .720887 | 14 |
| 46 | .271400 | 11.07 | .992287 | .40 | .279113 | 11.47 | .720199 | 13 |
| 47 | .272064 | 11.03 | .992263 | .40 | .279801 | 11.45 | .719512 | 12 |
| 48 | .272726 | 11.03 | .992239 | .42 | .280488 | 11.43 | .718826 | 11 |
| 49 | .273388 | 11.02 | .992214 | .40 | .281174 | 11.40 | .718142 | 10 |
| 50 | .274049 | 10.98 | .992190 | .40 | .281858 | 11.40 | | |
| | | | | | | | 10.717458 | 9 |
| 51 | 9.274708 | 10.98 | 9.992166 | .40 | 9.282542 | 11.38 | .716775 | 8 |
| 52 | .275367 | 10.97 | .992142 | .40 | .283225 | 11.37 | .716093 | 7 |
| 53 | .276025 | 10.93 | .992118 | .42 | .283907 | 11.35 | .715412 | 6 |
| 54 | .276681 | 10.93 | .992093 | .40 | .284588 | 11.33 | .714732 | 5 |
| 55 | .277337 | 10.90 | .992069 | .42 | .285268 | 11.32 | .714053 | 4 |
| 56 | .277991 | 10.90 | .992044 | .40 | .285947 | 11.28 | .713376 | 3 |
| 57 | .278645 | 10.90 | .992020 | .40 | .286624 | 11.28 | .712699 | 2 |
| 58 | .279297 | 10.87 | .991996 | .42 | .287301 | 11.27 | .712023 | 1 |
| 59 | .279948 | 10.85 | .991971 | .40 | .287977 | 11.25 | .711348 | 0 |
| 60 | 9.280599 | 10.85 | 9.991947 | | 9.288652 | | | |
| ' | Coseno. | D. 1". | Seno. | D. 1". | Cotang. | D.1". | Tang. | ' |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.280599 | | 9.991947 | | 9.288652 | | 10.711348 | 60 |
| 1 | .281248 | 10.82 | .991922 | .42 | .289326 | 11.23 | .710674 | 59 |
| 2 | .281897 | 10.82 | .991897 | .42 | .289999 | 11.22 | .710001 | 58 |
| 3 | .282544 | 10.78 | .991873 | .40 | .290671 | 11.20 | .709329 | 57 |
| 4 | .283190 | 10.77 | .991848 | .42 | .291342 | 11.18 | .708658 | 56 |
| 5 | .283836 | 10.77 | .991823 | .42 | .292013 | 11.18 | .707987 | 55 |
| 6 | .284480 | 10.73 | .991799 | .40 | .292682 | 11.15 | .707318 | 54 |
| 7 | .285124 | 10.73 | .991774 | .42 | .293350 | 11.13 | .706650 | 53 |
| 8 | .285766 | 10.70 | .991749 | .42 | .294017 | 11.12 | .705983 | 52 |
| 9 | .286408 | 10.70 | .991724 | .42 | .294684 | 11.12 | .705316 | 51 |
| 10 | .287048 | 10.67 | .991699 | .42 | .295349 | 11.08 | .704651 | 50 |
| | | 10.67 | | .42 | | 11.07 | | |
| 11 | 9.287688 | 10.63 | 9.991674 | .42 | 9.296013 | 11.07 | 10.703987 | 49 |
| 12 | .288326 | 10.63 | .991649 | .42 | .296677 | 11.03 | .703323 | 48 |
| 13 | .288964 | 10.60 | .991624 | .42 | .297339 | 11.03 | .702661 | 47 |
| 14 | .289600 | 10.60 | .991599 | .42 | .298001 | 11.02 | .701999 | 46 |
| 15 | .290236 | 10.57 | .991574 | .42 | .298662 | 11.00 | .701338 | 45 |
| 16 | .290870 | 10.57 | .991549 | .42 | .299322 | 10.97 | .700678 | 44 |
| 17 | .291504 | 10.55 | .991524 | .43 | .299980 | 10.97 | .700020 | 43 |
| 18 | .292137 | 10.52 | .991498 | .42 | .300638 | 10.95 | .699362 | 42 |
| 19 | .292768 | 10.52 | .991473 | .42 | .301295 | 10.93 | .698705 | 41 |
| 20 | .293399 | 10.50 | .991448 | .43 | .301951 | 10.93 | .698049 | 40 |
| 21 | 9.294029 | 10.48 | 9.991422 | .42 | 9.302607 | 10.90 | 10.697393 | 39 |
| 22 | .294658 | 10.47 | .991397 | .42 | .303261 | 10.88 | .696739 | 38 |
| 23 | .295286 | 10.45 | .991372 | .42 | .303914 | 10.88 | .696086 | 37 |
| 24 | .295913 | 10.43 | .991346 | .43 | .304567 | 10.85 | .695433 | 36 |
| 25 | .296539 | 10.42 | .991321 | .42 | .305218 | 10.85 | .694782 | 35 |
| 26 | .297164 | 10.40 | .991295 | .43 | .305869 | 10.83 | .694131 | 34 |
| 27 | .297788 | 10.40 | .991270 | .42 | .306519 | 10.82 | .693481 | 33 |
| 28 | .298412 | 10.37 | .991244 | .43 | .307168 | 10.80 | .692832 | 32 |
| 29 | .299034 | 10.35 | .991218 | .43 | .307816 | 10.78 | .692184 | 31 |
| 30 | .299655 | 10.35 | .991193 | .42 | .308463 | 10.77 | .691537 | 30 |
| | | 10.32 | | .43 | | 10.75 | | |
| 31 | 9.300276 | 10.32 | 9.991167 | .43 | 9.309109 | 10.75 | 10.690891 | 29 |
| 32 | .300895 | 10.30 | .991141 | .43 | .309754 | 10.72 | .690246 | 28 |
| 33 | .301514 | 10.27 | .991115 | .42 | .310399 | 10.72 | .689601 | 27 |
| 34 | .302132 | 10.27 | .991090 | .43 | .311042 | 10.70 | .688958 | 26 |
| 35 | .302748 | 10.27 | .991064 | .43 | .311685 | 10.68 | .688315 | 25 |
| 36 | .303364 | 10.25 | .991038 | .43 | .312327 | 10.67 | .687673 | 24 |
| 37 | .303979 | 10.23 | .991012 | .43 | .312968 | 10.67 | .687032 | 23 |
| 38 | .304593 | 10.23 | .990986 | .43 | .313608 | 10.65 | .686392 | 22 |
| 39 | .305207 | 10.20 | .990960 | .43 | .314247 | 10.63 | .685753 | 21 |
| 40 | .305819 | 10.18 | .990934 | .43 | .314885 | 10.63 | .685115 | 20 |
| | | 10.18 | | .43 | | 10.60 | | |
| 41 | 9.306430 | 10.15 | 9.990908 | .43 | 9.315523 | 10.60 | 10.684477 | 19 |
| 42 | .307041 | 10.15 | .990882 | .45 | .316159 | 10.58 | .683841 | 18 |
| 43 | .307650 | 10.13 | .990855 | .43 | .316795 | 10.57 | .683205 | 17 |
| 44 | .308259 | 10.12 | .990829 | .43 | .317430 | 10.55 | .682570 | 16 |
| 45 | .308867 | 10.12 | .990803 | .43 | .318064 | 10.55 | .681936 | 15 |
| 46 | .309474 | 10.10 | .990777 | .45 | .318697 | 10.53 | .681303 | 14 |
| 47 | .310080 | 10.08 | .990750 | .43 | .319330 | 10.52 | .680670 | 13 |
| 48 | .310685 | 10.07 | .990724 | .45 | .319961 | 10.50 | .680039 | 12 |
| 49 | .311289 | 10.07 | .990697 | .43 | .320592 | 10.50 | .679408 | 11 |
| 50 | .311893 | 10.03 | .990671 | .43 | .321222 | 10.48 | .678778 | 10 |
| | | 10.03 | | .45 | | 10.47 | | |
| 51 | 9.312495 | 10.02 | 9.990645 | .45 | 9.321851 | 10.45 | 10.678149 | 9 |
| 52 | .313097 | 9.98 | .990618 | .43 | .322479 | 10.45 | .677521 | 8 |
| 53 | .313698 | 10.00 | .990591 | .45 | .323106 | 10.42 | .676894 | 7 |
| 54 | .314297 | 9.97 | .990565 | .45 | .323732 | 10.42 | .676267 | 6 |
| 55 | .314897 | 9.95 | .990538 | .45 | .324358 | 10.40 | .675642 | 5 |
| 56 | .315495 | 9.95 | .990511 | .43 | .324983 | 10.40 | .675017 | 4 |
| 57 | .316092 | 9.95 | .990485 | .45 | .325607 | 10.37 | .674393 | 3 |
| 58 | .316689 | 9.92 | .990458 | .45 | .326231 | 10.37 | .673769 | 2 |
| 59 | .317284 | 9.92 | .990431 | .45 | .326853 | 10.37 | .673147 | 1 |
| 60 | 9.317879 | | 9.990404 | | 9.327475 | | 10.672525 | 0 |

| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |
|---|---|---|---|---|---|---|---|---|

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.317879 | 9.90 | 9.990404 | .43 | 9.327475 | 10.33 | 10.672525 | 60 |
| 1 | .318473 | 9.88 | .990378 | .45 | .328095 | 10.33 | .671905 | 59 |
| 2 | .319066 | 9.87 | .990351 | .45 | .328715 | 10.32 | .671285 | 58 |
| 3 | .319658 | 9.85 | .990324 | .45 | .329334 | 10.32 | .670666 | 57 |
| 4 | .320249 | 9.85 | .990297 | .45 | .329953 | 10.28 | .670047 | 56 |
| 5 | .320840 | 9.83 | .990270 | .45 | .330570 | 10.28 | .669430 | 55 |
| 6 | .321430 | 9.82 | .990243 | .47 | .331187 | 10.27 | .668813 | 54 |
| 7 | .322019 | 9.80 | .990215 | .45 | .331803 | 10.25 | .668197 | 53 |
| 8 | .322607 | 9.78 | .990188 | .45 | .332418 | 10.25 | .667582 | 52 |
| 9 | .323194 | 9.77 | .990161 | .45 | .333033 | 10.22 | .666967 | 51 |
| 10 | .323780 | 9.77 | .990134 | .45 | .333646 | 10.22 | .666354 | 50 |
| 11 | 9.324366 | 9.73 | 9.990107 | .47 | 9.334259 | 10.20 | 10.665741 | 49 |
| 12 | .324950 | 9.73 | .990079 | .45 | .334871 | 10.18 | .665129 | 48 |
| 13 | .325534 | 9.72 | .990052 | .45 | .335482 | 10.18 | .664518 | 47 |
| 14 | .326117 | 9.72 | .990025 | .47 | .336093 | 10.15 | .663907 | 46 |
| 15 | .326700 | 9.68 | .989997 | .45 | .336702 | 10.15 | .663298 | 45 |
| 16 | .327281 | 9.68 | .989970 | .47 | .337311 | 10.13 | .662689 | 44 |
| 17 | .327862 | 9.67 | .989942 | .45 | .337919 | 10.13 | .662081 | 43 |
| 18 | .328442 | 9.65 | .989915 | .47 | .338527 | 10.10 | .661473 | 42 |
| 19 | .329021 | 9.63 | .989887 | .45 | .339133 | 10.10 | .660867 | 41 |
| 20 | .329599 | 9.62 | .989860 | .47 | .339739 | 10.08 | .660261 | 40 |
| 21 | 9.330176 | 9.62 | 9.989832 | .47 | 9.340344 | 10.07 | 10.659656 | 39 |
| 22 | .330753 | 9.60 | .989804 | .45 | .340948 | 10.07 | .659052 | 38 |
| 23 | .331329 | 9.57 | .989777 | .47 | .341552 | 10.05 | .658448 | 37 |
| 24 | .331903 | 9.58 | .989749 | .47 | .342155 | 10.03 | .657845 | 36 |
| 25 | .332478 | 9.55 | .989721 | .47 | .342757 | 10.02 | .657243 | 35 |
| 26 | .333051 | 9.55 | .989693 | .47 | .343358 | 10.00 | .656642 | 34 |
| 27 | .333624 | 9.52 | .989665 | .47 | .343958 | 10.00 | .656042 | 33 |
| 28 | .334195 | 9.53 | .989637 | .45 | .344558 | 9.98 | .655442 | 32 |
| 29 | .334767 | 9.50 | .989610 | .47 | .345157 | 9.97 | .654843 | 31 |
| 30 | .335337 | 9.48 | .989582 | .48 | .345755 | 9.97 | .654245 | 30 |
| 31 | 9.335906 | 9.48 | 9.989553 | .47 | 9.346353 | 9.93 | 10.653647 | 29 |
| 32 | .336475 | 9.47 | .989525 | .47 | .346949 | 9.93 | .653051 | 28 |
| 33 | .337043 | 9.45 | .989497 | .47 | .347545 | 9.93 | .652455 | 27 |
| 34 | .337610 | 9.43 | .989469 | .47 | .348141 | 9.90 | .651859 | 26 |
| 35 | .338176 | 9.43 | .989441 | .47 | .348735 | 9.90 | .651265 | 25 |
| 36 | .338742 | 9.42 | .989413 | .47 | .349329 | 9.88 | .650671 | 24 |
| 37 | .339307 | 9.40 | .989385 | .48 | .349922 | 9.87 | .650078 | 23 |
| 38 | .339871 | 9.38 | .989356 | .47 | .350514 | 9.87 | .649486 | 22 |
| 39 | .340434 | 9.37 | .989328 | .47 | .351106 | 9.85 | .648894 | 21 |
| 40 | .340996 | 9.37 | .989300 | .48 | .351697 | 9.83 | .648303 | 20 |
| 41 | 9.341558 | 9.35 | 9.989271 | .47 | 9.352287 | 9.82 | 10.647713 | 19 |
| 42 | .342119 | 9.33 | .989243 | .48 | .352876 | 9.82 | .647124 | 18 |
| 43 | .342679 | 9.33 | .989214 | .47 | .353465 | 9.80 | .646535 | 17 |
| 44 | .343239 | 9.30 | .989186 | .48 | .354053 | 9.78 | .645947 | 16 |
| 45 | .343797 | 9.30 | .989157 | .48 | .354640 | 9.78 | .645360 | 15 |
| 46 | .344355 | 9.28 | .989128 | .47 | .355227 | 9.77 | .644773 | 14 |
| 47 | .344912 | 9.28 | .989100 | .48 | .355813 | 9.75 | .644187 | 13 |
| 48 | .345469 | 9.25 | .989071 | .48 | .356398 | 9.73 | .643602 | 12 |
| 49 | .346024 | 9.25 | .989042 | .47 | .356982 | 9.73 | .643018 | 11 |
| 50 | .346579 | 9.25 | .989014 | .48 | .357566 | 9.72 | .642434 | 10 |
| 51 | 9.347134 | 9.22 | 9.988985 | .48 | 9.358149 | 9.70 | 10.641851 | 9 |
| 52 | .347687 | 9.22 | .988956 | .48 | .358731 | 9.70 | .641269 | 8 |
| 53 | .348240 | 9.20 | .988927 | .48 | .359313 | 9.67 | .640687 | 7 |
| 54 | .348792 | 9.18 | .988898 | .48 | .359893 | 9.68 | .640107 | 6 |
| 55 | .349343 | 9.17 | .988869 | .48 | .360474 | 9.65 | .639526 | 5 |
| 56 | .349893 | 9.17 | .988840 | .48 | .361053 | 9.65 | .638947 | 4 |
| 57 | .350443 | 9.15 | .988811 | .48 | .361632 | 9.63 | .638368 | 3 |
| 58 | .350992 | 9.13 | .988782 | .48 | .362210 | 9.62 | .637790 | 2 |
| 59 | .351540 | 9.13 | .988753 | .48 | .362787 | 9.62 | .637213 | 1 |
| 60 | 9.352088 | | 9.988724 | | 9.363364 | | 10.636636 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.352088 | | 9.988724 | | 9.363364 | | 10.636636 | 60 |
| 1 | .352635 | 9.12 | .988695 | .48 | .363940 | 9.60 | .636060 | 59 |
| 2 | .353181 | 9.10 | .988666 | .48 | .364515 | 9.58 | .635485 | 58 |
| 3 | .353726 | 9.08 | .988636 | .50 | .365090 | 9.58 | .634910 | 57 |
| 4 | .354271 | 9.08 | .988607 | .48 | .365664 | 9.57 | .634336 | 56 |
| 5 | .354815 | 9.07 | .988578 | .48 | .366237 | 9.55 | .633763 | 55 |
| 6 | .355358 | 9.05 | .988548 | .50 | .366810 | 9.55 | .633190 | 54 |
| 7 | .355901 | 9.05 | .988519 | .48 | .367382 | 9.53 | .632618 | 53 |
| 8 | .356443 | 9.03 | .988489 | .50 | .367953 | 9.52 | .632047 | 52 |
| 9 | .356984 | 9.02 | .988460 | .48 | .368524 | 9.52 | .631476 | 51 |
| 10 | .357524 | 9.00 | .988430 | .50 | .369094 | 9.50 | .630906 | 50 |
| 11 | 9.358064 | 9.00 | 9.988401 | .48 | 9.369663 | 9.48 | 10.630337 | 49 |
| 12 | .358603 | 8.98 | .988371 | .50 | .370232 | 9.48 | .629768 | 48 |
| 13 | .359141 | 8.97 | .988342 | .48 | .370799 | 9.45 | .629201 | 47 |
| 14 | .359678 | 8.95 | .988312 | .50 | .371367 | 9.47 | .628633 | 46 |
| 15 | .360215 | 8.95 | .988282 | .50 | .371933 | 9.43 | .628067 | 45 |
| 16 | .360752 | 8.92 | .988253 | .48 | .372499 | 9.43 | .627501 | 44 |
| 17 | .361287 | 8.92 | .988223 | .50 | .373064 | 9.42 | .626936 | 43 |
| 18 | .361822 | 8.90 | .988193 | .50 | .373629 | 9.42 | .626371 | 42 |
| 19 | .362356 | 8.88 | .988163 | .50 | .374193 | 9.40 | .625807 | 41 |
| 20 | .362889 | 8.88 | .988133 | .50 | .374756 | 9.38 | .625244 | 40 |
| 21 | 9.363422 | 8.87 | 9.988103 | .50 | 9.375319 | 9.37 | 10.624681 | 39 |
| 22 | .363954 | 8.85 | .988073 | .50 | .375881 | 9.35 | .624119 | 38 |
| 23 | .364485 | 8.85 | .988043 | .50 | .376442 | 9.35 | .623558 | 37 |
| 24 | .365016 | 8.83 | .988013 | .50 | .377003 | 9.33 | .622997 | 36 |
| 25 | .365546 | 8.82 | .987983 | .50 | .377563 | 9.32 | .622437 | 35 |
| 26 | .366075 | 8.82 | .987953 | .52 | .378122 | 9.32 | .621878 | 34 |
| 27 | .366604 | 8.78 | .987922 | .50 | .378681 | 9.30 | .621319 | 33 |
| 28 | .367131 | 8.80 | .987892 | .50 | .379239 | 9.30 | .620761 | 32 |
| 29 | .367659 | 8.77 | .987862 | .50 | .379797 | 9.28 | .620203 | 31 |
| 30 | .368185 | 8.77 | .987832 | .52 | .380354 | 9.27 | .619646 | 30 |
| 31 | 9.368711 | 8.75 | 9.987801 | .50 | 9.380910 | 9.27 | 10.619090 | 29 |
| 32 | .369236 | 8.75 | .987771 | .52 | .381466 | 9.23 | .618534 | 28 |
| 33 | .369761 | 8.72 | .987740 | .50 | .382020 | 9.25 | .617980 | 27 |
| 34 | .370285 | 8.72 | .987710 | .52 | .382575 | 9.23 | .617425 | 26 |
| 35 | .370808 | 8.70 | .987679 | .50 | .383129 | 9.22 | .616871 | 25 |
| 36 | .371330 | 8.70 | .987649 | .52 | .383682 | 9.20 | .616318 | 24 |
| 37 | .371852 | 8.68 | .987618 | .50 | .384234 | 9.20 | .615766 | 23 |
| 38 | .372373 | 8.68 | .987588 | .52 | .384786 | 9.18 | .615214 | 22 |
| 39 | .372894 | 8.67 | .987557 | .52 | .385337 | 9.18 | .614663 | 21 |
| 40 | .373414 | 8.65 | .987526 | .50 | .385888 | 9.17 | .614112 | 20 |
| 41 | 9.373933 | 8.65 | 9.987496 | .52 | 9.386438 | 9.15 | 10.613562 | 19 |
| 42 | .374452 | 8.63 | .987465 | .52 | .386987 | 9.15 | .613013 | 18 |
| 43 | .374970 | 8.62 | .987434 | .52 | .387536 | 9.13 | .612464 | 17 |
| 44 | .375487 | 8.60 | .987403 | .52 | .388084 | 9.12 | .611916 | 16 |
| 45 | .376003 | 8.60 | .987372 | .52 | .388631 | 9.12 | .611369 | 15 |
| 46 | .376519 | 8.60 | .987341 | .52 | .389178 | 9.10 | .610822 | 14 |
| 47 | .377035 | 8.57 | .987310 | .52 | .389724 | 9.10 | .610276 | 13 |
| 48 | .377549 | 8.57 | .987279 | .52 | .390270 | 9.08 | .609730 | 12 |
| 49 | .378063 | 8.57 | .987248 | .52 | .390815 | 9.08 | .609185 | 11 |
| 50 | .378577 | 8.53 | .987217 | .52 | .391360 | 9.05 | .608640 | 10 |
| 51 | 9.379089 | 8.53 | 9.987186 | .52 | 9.391903 | 9.07 | 10.608097 | 9 |
| 52 | .379601 | 8.53 | .987155 | .52 | .392447 | 9.03 | .607553 | 8 |
| 53 | .380113 | 8.52 | .987124 | .53 | .392989 | 9.03 | .607011 | 7 |
| 54 | .380624 | 8.50 | .987092 | .52 | .393531 | 9.03 | .606469 | 6 |
| 55 | .381134 | 8.48 | .987061 | .52 | .394073 | 9.02 | .605927 | 5 |
| 56 | .381643 | 8.48 | .987030 | .53 | .394614 | 9.00 | .605386 | 4 |
| 57 | .382152 | 8.48 | .986998 | .52 | .395154 | 9.00 | .604846 | 3 |
| 58 | .382661 | 8.45 | .986967 | .52 | .395694 | 8.98 | .604306 | 2 |
| 59 | .383168 | 8.45 | .986936 | .53 | .396233 | 8.97 | .603767 | 1 |
| 60 | 9.383675 | | 9.986904 | | 9.396771 | | 10.603229 | 0 |
| | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.383675 | 8.45 | 9.986904 | .53 | 9.396771 | 8.97 | 10.603229 | 60 |
| 1 | .384182 | 8.42 | .986873 | .53 | .397309 | 8.95 | .602691 | 59 |
| 2 | .384687 | 8.42 | .986841 | .53 | .397846 | 8.95 | .602154 | 58 |
| 3 | .385192 | 8.42 | .986809 | .52 | .398383 | 8.93 | .601617 | 57 |
| 4 | .385697 | 8.40 | .986778 | .53 | .398919 | 8.93 | .601081 | 56 |
| 5 | .386201 | 8.38 | .986746 | .53 | .399455 | 8.92 | .600545 | 55 |
| 6 | .386704 | 8.38 | .986714 | .52 | .399990 | 8.90 | .600010 | 54 |
| 7 | .387207 | 8.37 | .986682 | .53 | .400524 | 8.90 | .599476 | 53 |
| 8 | .387709 | 8.35 | .986651 | .53 | .401058 | 8.88 | .598942 | 52 |
| 9 | .388210 | 8.35 | .986619 | .53 | .401591 | 8.88 | .598409 | 51 |
| 10 | .388711 | 8.33 | .986587 | .53 | .402124 | 8.87 | .597876 | 50 |
| 11 | 9.389211 | 8.33 | 9.986555 | .53 | 9.402656 | 8.85 | 10.597344 | 49 |
| 12 | .389711 | 8.32 | .986523 | .53 | .403187 | 8.85 | .596813 | 48 |
| 13 | .390210 | 8.30 | .986491 | .53 | .403718 | 8.85 | .596282 | 47 |
| 14 | .390708 | 8.30 | .986459 | .53 | .404249 | 8.82 | .595751 | 46 |
| 15 | .391206 | 8.28 | .986427 | .53 | .404778 | 8.82 | .595222 | 45 |
| 16 | .391703 | 8.27 | .986395 | .53 | .405308 | 8.80 | .594692 | 44 |
| 17 | .392199 | 8.27 | .986363 | .53 | .405836 | 8.80 | .594164 | 43 |
| 18 | .392695 | 8.27 | .986331 | .53 | .406364 | 8.80 | .593636 | 42 |
| 19 | .393191 | 8.27 | .986299 | .55 | .406892 | 8.78 | .593108 | 41 |
| 20 | .393685 | 8.23 | .986266 | .53 | .407419 | 8.77 | .592581 | 40 |
| 21 | 9.394179 | 8.23 | 9.986234 | .53 | 9.407945 | 8.77 | 10.592055 | 39 |
| 22 | .394673 | 8.22 | .986202 | .55 | .408471 | 8.75 | .591529 | 38 |
| 23 | .395166 | 8.20 | .986169 | .53 | .408996 | 8.75 | .591004 | 37 |
| 24 | .395658 | 8.20 | .986137 | .53 | .409521 | 8.73 | .590479 | 36 |
| 25 | .396150 | 8.18 | .986104 | .53 | .410045 | 8.73 | .589955 | 35 |
| 26 | .396641 | 8.18 | .986072 | .55 | .410569 | 8.72 | .589431 | 34 |
| 27 | .397132 | 8.15 | .986039 | .53 | .411092 | 8.72 | .588908 | 33 |
| 28 | .397621 | 8.17 | .986007 | .55 | .411615 | 8.70 | .588385 | 32 |
| 29 | .398111 | 8.15 | .985974 | .53 | .412137 | 8.68 | .587863 | 31 |
| 30 | .398600 | 8.13 | .985942 | .55 | .412658 | 8.68 | .587342 | 30 |
| 31 | 9.399088 | 8.12 | 9.985909 | .55 | 9.413179 | 8.67 | 10.586821 | 29 |
| 32 | .399575 | 8.12 | .985876 | .55 | .413699 | 8.67 | .586301 | 28 |
| 33 | .400062 | 8.12 | .985843 | .53 | .414219 | 8.65 | .585781 | 27 |
| 34 | .400549 | 8.10 | .985811 | .55 | .414738 | 8.65 | .585262 | 26 |
| 35 | .401035 | 8.08 | .985778 | .55 | .415257 | 8.63 | .584743 | 25 |
| 36 | .401520 | 8.08 | .985745 | .53 | .415775 | 8.62 | .584225 | 24 |
| 37 | .402005 | 8.07 | .985712 | .55 | .416293 | 8.62 | .583707 | 23 |
| 38 | .402489 | 8.07 | .985679 | .55 | .416810 | 8.60 | .583190 | 22 |
| 39 | .402972 | 8.05 | .985646 | .55 | .417326 | 8.60 | .582674 | 21 |
| 40 | .403455 | 8.05 | .985613 | .55 | .417842 | 8.60 | .582158 | 20 |
| 41 | 9.403938 | 8.03 | 9.985580 | .55 | 9.418358 | 8.58 | 10 581642 | 19 |
| 42 | .404420 | 8.02 | .985547 | .55 | .418873 | 8.57 | .581127 | 18 |
| 43 | .404901 | 8.02 | .985514 | .57 | .419387 | 8.57 | .580613 | 17 |
| 44 | .405382 | 8.00 | .985480 | .55 | .419901 | 8.57 | .580099 | 16 |
| 45 | .405862 | 7.98 | .985447 | .55 | .420415 | 8.55 | .579585 | 15 |
| 46 | .406341 | 7.98 | .985414 | .55 | .420927 | 8.55 | .579073 | 14 |
| 47 | .406820 | 7.98 | .985381 | .57 | .421440 | 8.53 | .578560 | 13 |
| 48 | .407299 | 7.97 | .985347 | .55 | .421952 | 8.52 | .578048 | 12 |
| 49 | .407777 | 7.95 | .985314 | .57 | .422463 | 8.52 | .577537 | 11 |
| 50 | .408254 | 7.95 | .985280 | .57 | .422974 | 8.50 | .577026 | 10 |
| 51 | 9.408731 | 7.93 | 9.985247 | .57 | 9.423484 | 8.48 | 10.576516 | 9 |
| 52 | .409207 | 7.93 | .985213 | .55 | .423993 | 8.50 | .576007 | 8 |
| 53 | .409682 | 7.92 | .985180 | .57 | .424503 | 8.47 | .575497 | 7 |
| 54 | .410157 | 7.92 | .985146 | .55 | .425011 | 8.47 | .574989 | 6 |
| 55 | .410632 | 7.90 | .985113 | .57 | .425519 | 8.47 | .574481 | 5 |
| 56 | .411106 | 7.88 | .985079 | .57 | .426027 | 8.45 | .573973 | 4 |
| 57 | .411579 | 7.88 | .985045 | .57 | .426534 | 8.45 | .573466 | 3 |
| 58 | .412052 | 7.87 | .985011 | .55 | .427041 | 8.43 | .572959 | 2 |
| 59 | .412524 | 7.87 | .984978 | .57 | .427547 | 8.42 | .572453 | 1 |
| 60 | 9.412996 | | 9.984944 | | 9.428052 | | 10.571948 | 0 |
| ′ | Coseno. | D. 1″ | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.412996 | | 9.984944 | | 9.428052 | | 10.571948 | 60 |
| 1 | .413467 | 7.85 | .984910 | .57 | .428558 | 8.43 | .571442 | 59 |
| 2 | .413938 | 7.85 | .984876 | .57 | .429062 | 8.40 | .570938 | 58 |
| 3 | .414408 | 7.83 | .984842 | .57 | .429566 | 8.40 | .570434 | 57 |
| 4 | .414878 | 7.83 | .984808 | .57 | .430070 | 8.38 | .569930 | 56 |
| 5 | .415347 | 7.82 | .984774 | .57 | .430573 | 8.38 | .569427 | 55 |
| 6 | .415815 | 7.80 | .984740 | .57 | .431075 | 8.37 | .568925 | 54 |
| 7 | .416283 | 7.80 | .984706 | .57 | .431577 | 8.37 | .568423 | 53 |
| 8 | .416751 | 7.80 | .984672 | .57 | .432079 | 8.37 | .567921 | 52 |
| 9 | .417217 | 7.77 | .984638 | .57 | .432580 | 8.35 | .567420 | 51 |
| 10 | .417684 | 7.77 | .984603 | .57 | .433080 | 8.33 | .566920 | 50 |
| 11 | 9.418150 | 7.75 | 9.984569 | .57 | 9.433580 | 8.33 | 10.566420 | 49 |
| 12 | .418615 | 7.73 | .984535 | .58 | .434080 | 8.32 | .565920 | 48 |
| 13 | .419079 | 7.75 | .984500 | .57 | .434579 | 8.32 | .565421 | 47 |
| 14 | .419544 | 7.72 | .984466 | .57 | .435078 | 8.30 | .564923 | 46 |
| 15 | .420007 | 7.72 | .984432 | .58 | .435576 | 8.28 | .564424 | 45 |
| 16 | .420470 | 7.72 | .984397 | .57 | .436073 | 8.28 | .563927 | 44 |
| 17 | .420933 | 7.72 | .984363 | .58 | .436570 | 8.28 | .563430 | 43 |
| 18 | .421395 | 7.70 | .984328 | .57 | .437067 | 8.27 | .562933 | 42 |
| 19 | .421857 | 7.70 | .984294 | .58 | .437563 | 8.27 | .562437 | 41 |
| 20 | .422318 | 7.67 | .984259 | .58 | .438059 | 8.25 | .561941 | 40 |
| 21 | 9.422778 | 7.67 | 9.984224 | .57 | 9.438554 | 8.23 | 10.561446 | 39 |
| 22 | .423238 | 7.65 | .984190 | .58 | .439048 | 8.25 | .560952 | 38 |
| 23 | .423697 | 7.65 | .984155 | .58 | .439543 | 8.23 | .560457 | 37 |
| 24 | .424156 | 7.65 | .984120 | .58 | .440036 | 8.22 | .559964 | 36 |
| 25 | .424615 | 7.65 | .984085 | .58 | .440529 | 8.22 | .559471 | 35 |
| 26 | .425073 | 7.63 | .984050 | .58 | .441022 | 8.20 | .558978 | 34 |
| 27 | .425530 | 7.62 | .984015 | .57 | .441514 | 8.20 | .558486 | 33 |
| 28 | .425987 | 7.62 | .983981 | .58 | .442006 | 8.18 | .557994 | 32 |
| 29 | .426443 | 7.60 | .983946 | .58 | .442497 | 8.18 | .557503 | 31 |
| 30 | .426899 | 7.58 | .983911 | .60 | .442988 | 8.18 | .557012 | 30 |
| 31 | 9.427354 | 7.58 | 9.983875 | .58 | 9.443479 | 8.15 | 10.556521 | 29 |
| 32 | .427809 | 7.57 | .983840 | .58 | .443968 | 8.17 | .556032 | 28 |
| 33 | .428263 | 7.57 | .983805 | .58 | .444458 | 8.15 | .555542 | 27 |
| 34 | .428717 | 7.55 | .983770 | .58 | .444947 | 8.13 | .555053 | 26 |
| 35 | .429170 | 7.55 | .983735 | .58 | .445435 | 8.13 | .554565 | 25 |
| 36 | .429623 | 7.53 | .983700 | .60 | .445923 | 8.13 | .554077 | 24 |
| 37 | .430075 | 7.53 | .983664 | .58 | .446411 | 8.12 | .553589 | 23 |
| 38 | .430527 | 7.53 | .983629 | .58 | .446898 | 8.10 | .553102 | 22 |
| 39 | .430978 | 7.52 | .983594 | .60 | .447384 | 8.10 | .552616 | 21 |
| 40 | .431429 | 7.50 | .983558 | .58 | .447870 | 8.10 | .552130 | 20 |
| 41 | 9.431879 | 7.50 | 9.983523 | .60 | 9.448356 | 8.08 | 10.551644 | 19 |
| 42 | .432329 | 7.48 | .983487 | .58 | .448841 | 8.08 | .551159 | 18 |
| 43 | .432778 | 7.47 | .983452 | .60 | .449326 | 8.07 | .550674 | 17 |
| 44 | .433226 | 7.48 | .983416 | .58 | .449810 | 8.07 | .550190 | 16 |
| 45 | .433675 | 7.45 | .983381 | .60 | .450294 | 8.05 | .549706 | 15 |
| 46 | .434122 | 7.45 | .983345 | .60 | .450777 | 8.05 | .549223 | 14 |
| 47 | .434569 | 7.45 | .983309 | .60 | .451260 | 8.05 | .548740 | 13 |
| 48 | .435016 | 7.45 | .983273 | .58 | .451743 | 8.03 | .548257 | 12 |
| 49 | .435462 | 7.43 | .983238 | .60 | .452225 | 8.03 | .547775 | 11 |
| 50 | .435908 | 7.42 | .983202 | .60 | .452706 | 8.02 | .547294 | 10 |
| 51 | 9.436353 | 7.42 | 9.983166 | .60 | 9.453187 | 8.02 | 10.546813 | 9 |
| 52 | .436798 | 7.40 | .983130 | .60 | .453668 | 8.00 | .546332 | 8 |
| 53 | .437242 | 7.40 | .983094 | .60 | .454148 | 8.00 | .545852 | 7 |
| 54 | .437686 | 7.38 | .983058 | .60 | .454628 | 7.98 | .545372 | 6 |
| 55 | .438129 | 7.38 | .983022 | .60 | .455107 | 7.98 | .544893 | 5 |
| 56 | .438572 | 7.37 | .982986 | .60 | .455586 | 7.97 | .544414 | 4 |
| 57 | .439014 | 7.37 | .982950 | .60 | .456064 | 7.97 | .543936 | 3 |
| 58 | .439456 | 7.35 | .982914 | .60 | .456542 | 7.95 | .543458 | 2 |
| 59 | .439897 | 7.35 | .982878 | .60 | .457019 | 7.95 | .542981 | 1 |
| 60 | 9.440338 | | 9.982842 | | 9.457496 | | 10.542504 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.440338 | | 9.982842 | | 9.457496 | | 10.542504 | 60 |
| 1 | .440778 | 7.33 | .982805 | .62 | .457973 | 7.95 | .542027 | 59 |
| 2 | .441218 | 7.33 | .982769 | .60 | .458449 | 7.93 | .541551 | 58 |
| 3 | .441658 | 7.33 | .982733 | .62 | .458925 | 7.93 | .541075 | 57 |
| 4 | .442096 | 7.30 | .982696 | .60 | .459400 | 7.92 | .540600 | 56 |
| 5 | .442535 | 7.32 | .982660 | .60 | .459875 | 7.92 | .540125 | 55 |
| 6 | .442973 | 7.30 | .982624 | .62 | .460349 | 7.90 | .539651 | 54 |
| 7 | .443410 | 7.28 | .982587 | .60 | .460823 | 7.90 | .539177 | 53 |
| 8 | .443847 | 7.28 | .982551 | .62 | .461297 | 7.90 | .538703 | 52 |
| 9 | .444284 | 7.28 | .982514 | .62 | .461770 | 7.88 | .538230 | 51 |
| 10 | .444720 | 7.27 | .982477 | .60 | .462242 | 7.87 | .537758 | 50 |
| 11 | 9.445155 | 7.25 | 9.982441 | .62 | 9.462715 | 7.88 | 10.537285 | 49 |
| 12 | .445590 | 7.25 | .982404 | .62 | .463186 | 7.85 | .536814 | 48 |
| 13 | .446025 | 7.23 | .982367 | .60 | .463658 | 7.87 | .536342 | 47 |
| 14 | .446459 | 7.23 | .982331 | .62 | .464128 | 7.83 | .535872 | 46 |
| 15 | .446893 | 7.22 | .982294 | .62 | .464599 | 7.85 | .535401 | 45 |
| 16 | .447326 | 7.22 | .982257 | .62 | .465069 | 7.83 | .534931 | 44 |
| 17 | .447759 | 7.20 | .982220 | .62 | .465539 | 7.83 | .534461 | 43 |
| 18 | .448191 | 7.20 | .982183 | .62 | .466008 | 7.82 | .533992 | 42 |
| 19 | .448623 | 7.18 | .982146 | .62 | .466477 | 7.82 | .533523 | 41 |
| 20 | .449054 | 7.18 | .982109 | .62 | .466945 | 7.80 | .533055 | 40 |
| 21 | 9.449485 | 7.17 | 9.982072 | .62 | 9.467413 | 7.80 | 10.532587 | 39 |
| 22 | .449915 | 7.17 | .982035 | .62 | .467880 | 7.78 | .532120 | 38 |
| 23 | .450345 | 7.17 | .981998 | .62 | .468347 | 7.78 | .531653 | 37 |
| 24 | .450775 | 7.15 | .981961 | .62 | .468814 | 7.78 | .531186 | 36 |
| 25 | .451204 | 7.13 | .981924 | .63 | .469280 | 7.77 | .530720 | 35 |
| 26 | .451632 | 7.13 | .981886 | .62 | .469746 | 7.75 | .530254 | 34 |
| 27 | .452060 | 7.13 | .981849 | .62 | .470211 | 7.75 | .529789 | 33 |
| 28 | .452488 | 7.12 | .981812 | .63 | .470676 | 7.75 | .529324 | 32 |
| 29 | .452915 | 7.12 | .981774 | .62 | .471141 | 7.73 | .528859 | 31 |
| 30 | .453342 | 7.10 | .981737 | .62 | .471605 | 7.73 | .528395 | 30 |
| 31 | 9.453768 | 7.10 | 9.981700 | .63 | 9.472069 | 7.72 | 10.527931 | 29 |
| 32 | .454194 | 7.08 | .981662 | .62 | .472532 | 7.72 | .527468 | 28 |
| 33 | .454619 | 7.08 | .981625 | .63 | .472995 | 7.70 | .527005 | 27 |
| 34 | .455044 | 7.08 | .981587 | .63 | .473457 | 7.70 | .526543 | 26 |
| 35 | .455469 | 7.07 | .981549 | .62 | .473919 | 7.70 | .526081 | 25 |
| 36 | .455893 | 7.05 | .981512 | .63 | .474381 | 7.68 | .525619 | 24 |
| 37 | .456316 | 7.05 | .981474 | .63 | .474842 | 7.68 | .525158 | 23 |
| 38 | .456739 | 7.05 | .981436 | .62 | .475303 | 7.67 | .524697 | 22 |
| 39 | .457162 | 7.03 | .981399 | .63 | .475763 | 7.67 | .524237 | 21 |
| 40 | .457584 | 7.03 | .981361 | .63 | .476223 | 7.67 | .523777 | 20 |
| 41 | 9.458006 | 7.02 | 9.981323 | .63 | 9.476683 | 7.65 | 10.523317 | 19 |
| 42 | .458427 | 7.02 | .981285 | .63 | .477142 | 7.65 | .522858 | 18 |
| 43 | .458848 | 7.00 | .981247 | .63 | .477601 | 7.63 | .522399 | 17 |
| 44 | .459268 | 7.00 | .981209 | .63 | .478059 | 7.63 | .521941 | 16 |
| 45 | .459688 | 7.00 | .981171 | .63 | .478517 | 7.63 | .521483 | 15 |
| 46 | .460108 | 6.98 | .981133 | .63 | .478975 | 7.63 | .521025 | 14 |
| 47 | .460527 | 6.98 | .981095 | .63 | .479432 | 7.62 | .520568 | 13 |
| 48 | .460946 | 6.97 | .981057 | .63 | .479889 | 7.62 | .520111 | 12 |
| 49 | .461364 | 6.97 | .981019 | .63 | .480345 | 7.60 | .519655 | 11 |
| 50 | .461782 | 6.95 | .980981 | .65 | .480801 | 7.60 | .519199 | 10 |
| 51 | 9.462199 | 6.95 | 9.980942 | .63 | 9.481257 | 7.58 | 10.518743 | 9 |
| 52 | .462616 | 6.93 | .980904 | .63 | .481712 | 7.58 | .518288 | 8 |
| 53 | .463032 | 6.93 | .980866 | .65 | .482167 | 7.57 | .517833 | 7 |
| 54 | .463448 | 6.93 | .980827 | .63 | .482621 | 7.57 | .517379 | 6 |
| 55 | .463864 | 6.92 | .980789 | .65 | .483075 | 7.57 | .516925 | 5 |
| 56 | .464279 | 6.92 | .980750 | .63 | .483529 | 7.55 | .516471 | 4 |
| 57 | .464694 | 6.90 | .980712 | .65 | .483982 | 7.55 | .516018 | 3 |
| 58 | .465108 | 6.90 | .980673 | .63 | .484435 | 7.53 | .515565 | 2 |
| 59 | .465522 | 6.88 | .980635 | .65 | .484887 | 7.53 | .515113 | 1 |
| 60 | 9.465935 | 6.88 | 9.980596 | .65 | 9.485339 | | 10.514661 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.465935 | | 9.980596 | | 9.485339 | | 10.514661 | 60 |
| 1 | .466348 | 6.88 | .980558 | .63 | .485791 | 7.53 | .514209 | 59 |
| 2 | .466761 | 6.88 | .980519 | .65 | .486242 | 7.52 | .513758 | 58 |
| 3 | .467173 | 6.87 | .980480 | .65 | .486693 | 7.52 | .513307 | 57 |
| 4 | .467585 | 6.87 | .980442 | .63 | .487143 | 7.50 | .512857 | 56 |
| 5 | .467996 | 6.85 | .980403 | .65 | .487593 | 7.50 | .512407 | 55 |
| 6 | .468407 | 6.85 | .980364 | .65 | .488043 | 7.50 | .511957 | 54 |
| 7 | .468817 | 6.83 | .980325 | .65 | .488492 | 7.48 | .511508 | 53 |
| 8 | .469227 | 6.83 | .980286 | .65 | .488941 | 7.48 | .511059 | 52 |
| 9 | .469637 | 6.83 | .980247 | .65 | .489390 | 7.48 | .510610 | 51 |
| 10 | .470046 | 6.82 | .980208 | .65 | .489838 | 7.47 | .510162 | 50 |
| 11 | 9.470455 | | 9.980169 | .65 | 9.490286 | 7.45 | 10.509714 | 49 |
| 12 | .470863 | 6.80 | .980130 | .65 | .490733 | 7.45 | .509267 | 48 |
| 13 | .471271 | 6.80 | .980091 | .65 | .491180 | 7.45 | .508820 | 47 |
| 14 | .471679 | 6.80 | .980052 | .67 | .491627 | 7.43 | .508373 | 46 |
| 15 | .472086 | 6.78 | .980012 | .65 | .492073 | 7.43 | .507927 | 45 |
| 16 | .472492 | 6.77 | .979973 | .65 | .492519 | 7.43 | .507481 | 44 |
| 17 | .472898 | 6.77 | .979934 | .65 | .492965 | 7.42 | .507035 | 43 |
| 18 | .473304 | 6.77 | .979895 | .65 | .493410 | 7.42 | .506590 | 42 |
| 19 | .473710 | 6.77 | .979855 | .67 | .493854 | 7.40 | .506146 | 41 |
| 20 | .474115 | 6.75 | .979816 | .65 | .494299 | 7.42 | .505701 | 40 |
| 21 | 9.474519 | 6.73 | 9.979776 | .67 | 9.494743 | 7.38 | 10.505257 | 39 |
| 22 | .474923 | 6.73 | .979737 | .65 | .495186 | 7.40 | .504814 | 38 |
| 23 | .475327 | 6.72 | .979697 | .67 | .495630 | 7.38 | .504370 | 37 |
| 24 | .475730 | 6.72 | .979658 | .65 | .496073 | 7.37 | .503927 | 36 |
| 25 | .476133 | 6.72 | .979618 | .67 | .496515 | 7.37 | .503485 | 35 |
| 26 | .476536 | 6.70 | .979579 | .65 | .496957 | 7.37 | .503043 | 34 |
| 27 | .476938 | 6.70 | .979539 | .67 | .497399 | 7.37 | .502601 | 33 |
| 28 | .477340 | 6.70 | .979499 | .67 | .497841 | 7.37 | .502159 | 32 |
| 29 | .477741 | 6.68 | .979459 | .67 | .498282 | 7.35 | .501718 | 31 |
| 30 | .478142 | 6.68 | .979420 | .65 | .498722 | 7.33 | .501278 | 30 |
| 31 | 9.478542 | 6.67 | 9.979380 | .67 | 9.499163 | 7.33 | 10.500837 | 29 |
| 32 | .478942 | 6.67 | .979340 | .67 | .499603 | 7.32 | .500397 | 28 |
| 33 | .479342 | 6.67 | .979300 | .67 | .500042 | 7.32 | .499958 | 27 |
| 34 | .479741 | 6.65 | .979260 | .67 | .500481 | 7.32 | .499519 | 26 |
| 35 | .480140 | 6.65 | .979220 | .67 | .500920 | 7.32 | .499080 | 25 |
| 36 | .480539 | 6.65 | .979180 | .67 | .501359 | 7.30 | .498641 | 24 |
| 37 | .480937 | 6.63 | .979140 | .67 | .501797 | 7.30 | .498203 | 23 |
| 38 | .481334 | 6.62 | .979100 | .67 | .502235 | 7.28 | .497765 | 22 |
| 39 | .481731 | 6.62 | .979059 | .68 | .502672 | 7.28 | .497328 | 21 |
| 40 | .482128 | 6.62 | .979019 | .67 | .503109 | 7.28 | .496891 | 20 |
| 41 | 9.482525 | 6.60 | 9.978979 | .67 | 9.503546 | 7.27 | 10.496454 | 19 |
| 42 | .482921 | 6.58 | .978939 | .68 | .503982 | 7.27 | .496018 | 18 |
| 43 | .483316 | 6.60 | .978898 | .67 | .504418 | 7.27 | .495582 | 17 |
| 44 | .483712 | 6.58 | .978858 | .68 | .504854 | 7.25 | .495146 | 16 |
| 45 | .484107 | 6.57 | .978817 | .67 | .505289 | 7.25 | .494711 | 15 |
| 46 | .484501 | 6.57 | .978777 | .67 | .505724 | 7.25 | .494276 | 14 |
| 47 | .484895 | 6.57 | .978737 | .68 | .506159 | 7.23 | .493841 | 13 |
| 48 | .485289 | 6.55 | .978696 | .68 | .506593 | 7.23 | .493407 | 12 |
| 49 | .485682 | 6.55 | .978655 | .67 | .507027 | 7.23 | .492973 | 11 |
| 50 | .486075 | 6.53 | .978615 | .68 | .507460 | 7.22 | .492540 | 10 |
| 51 | 9.486467 | 6.55 | 9.978574 | .68 | 9.507893 | 7.22 | 10.492107 | 9 |
| 52 | .486860 | 6.52 | .978533 | .67 | .508326 | 7.22 | .491674 | 8 |
| 53 | .487251 | 6.53 | .978493 | .68 | .508759 | 7.20 | .491241 | 7 |
| 54 | .487643 | 6.52 | .978452 | .68 | .509191 | 7.18 | .490809 | 6 |
| 55 | .488034 | 6.50 | .978411 | .68 | .509622 | 7.20 | .490378 | 5 |
| 56 | .488424 | 6.50 | .978370 | .68 | .510054 | 7.18 | .489946 | 4 |
| 57 | .488814 | 6.50 | 978329 | .68 | .510485 | 7.18 | .489515 | 3 |
| 58 | .489204 | 6.48 | .978288 | .68 | .510916 | 7.17 | .489084 | 2 |
| 59 | .489593 | 6.48 | .978247 | .68 | .511346 | 7.17 | .488654 | 1 |
| 60 | 9.489982 | | 9.978206 | | 9.511776 | | 10.488224 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.489982 | 6.48 | 9.978206 | .68 | 9.511776 | 7.17 | 10.488224 | 60 |
| 1 | .490371 | 6.47 | .978165 | .68 | .512206 | 7.15 | .487794 | 59 |
| 2 | .490759 | 6.47 | .978124 | .68 | .512635 | 7.15 | .487365 | 58 |
| 3 | .491147 | 6.47 | .978083 | .68 | .513064 | 7.15 | .486936 | 57 |
| 4 | .491535 | 6.45 | .978042 | .68 | .513493 | 7.13 | .486507 | 56 |
| 5 | .491922 | 6.43 | .978001 | .70 | .513921 | 7.13 | .486079 | 55 |
| 6 | .492308 | 6.45 | .977959 | .68 | .514349 | 7.13 | .485651 | 54 |
| 7 | .492695 | 6.43 | .977918 | .68 | .514777 | 7.12 | .485223 | 53 |
| 8 | .493081 | 6.42 | .977877 | .70 | .515204 | 7.12 | .484796 | 52 |
| 9 | .493466 | 6.42 | .977835 | .68 | .515631 | 7.10 | .484369 | 51 |
| 10 | .493851 | 6.42 | .977794 | .70 | .516057 | 7.12 | .483943 | 50 |
| 11 | 9.494236 | 6.42 | 9.977752 | .68 | 9.516484 | 7.10 | 10.483516 | 49 |
| 12 | .494621 | 6.40 | .977711 | .70 | .516910 | 7.08 | .483090 | 48 |
| 13 | .495005 | 6.38 | .977669 | .68 | .517335 | 7.10 | .482665 | 47 |
| 14 | .495388 | 6.40 | .977628 | .70 | .517761 | 7.08 | .482239 | 46 |
| 15 | .495772 | 6.37 | .977586 | .70 | .518186 | 7.07 | .481814 | 45 |
| 16 | .496154 | 6.38 | .977544 | .68 | .518610 | 7.07 | .481390 | 44 |
| 17 | .496537 | 6.37 | .977503 | .70 | .519034 | 7.07 | .480966 | 43 |
| 18 | .496919 | 6.37 | .977461 | .70 | .519458 | 7.07 | .480542 | 42 |
| 19 | .497301 | 6.35 | .977419 | .70 | .519882 | 7.05 | .480118 | 41 |
| 20 | .497682 | 6.35 | .977377 | .70 | .520305 | 7.05 | .479695 | 40 |
| 21 | 9.498064 | 6.33 | 9.977335 | .70 | 9.520728 | 7.05 | 10.479272 | 39 |
| 22 | .498444 | 6.35 | .977293 | .70 | .521151 | 7.03 | .478849 | 38 |
| 23 | .498825 | 6.32 | .977251 | .70 | .521573 | 7.03 | .478427 | 37 |
| 24 | .499204 | 6.33 | .977209 | .70 | .521995 | 7.03 | .478005 | 36 |
| 25 | .499584 | 6.32 | .977167 | .70 | .522417 | 7.02 | .477583 | 35 |
| 26 | .499963 | 6.32 | .977125 | .70 | .522838 | 7.02 | .477162 | 34 |
| 27 | .500342 | 6.32 | .977083 | .70 | .523259 | 7.02 | .476741 | 33 |
| 28 | .500721 | 6.30 | .977041 | .70 | .523680 | 7.02 | .476320 | 32 |
| 29 | .501099 | 6.28 | .976999 | .70 | .524100 | 7.00 | .475900 | 31 |
| 30 | .501476 | 6.30 | .976957 | .72 | .524520 | 7.00 | .475480 | 30 |
| 31 | 9.501854 | 6.28 | 9.976914 | .70 | 9.524940 | 6.98 | 10.475060 | 29 |
| 32 | .502231 | 6.27 | .976872 | .70 | .525359 | 6.98 | .474641 | 28 |
| 33 | .502607 | 6.28 | .976830 | .72 | .525778 | 6.98 | .474222 | 27 |
| 34 | .502984 | 6.27 | .976787 | .70 | .526197 | 6.97 | .473803 | 26 |
| 35 | .503360 | 6.25 | .976745 | .72 | .526615 | 6.97 | .473385 | 25 |
| 36 | .503735 | 6.25 | .976702 | .70 | .527033 | 6.97 | .472967 | 24 |
| 37 | .504110 | 6.25 | .976660 | .72 | .527451 | 6.95 | .472549 | 23 |
| 38 | .504485 | 6.25 | .976617 | .72 | .527868 | 6.95 | .472132 | 22 |
| 39 | .504860 | 6.23 | .976574 | .70 | .528285 | 6.95 | .471715 | 21 |
| 40 | .505234 | 6.23 | .976532 | .72 | .528702 | 6.95 | .471298 | 20 |
| 41 | 9.505608 | 6.22 | 9.976489 | .72 | 9.529119 | 6.93 | 10.470881 | 19 |
| 42 | .505981 | 6.22 | .976446 | .70 | .529535 | 6.93 | .470465 | 18 |
| 43 | .506354 | 6.22 | .976404 | .72 | .529951 | 6.92 | .470049 | 17 |
| 44 | .506727 | 6.20 | .976361 | .72 | .530366 | 6.92 | .469634 | 16 |
| 45 | .507099 | 6.20 | .976318 | .72 | .530781 | 6.92 | .469219 | 15 |
| 46 | .507471 | 6.20 | .976275 | .72 | .531196 | 6.92 | .468804 | 14 |
| 47 | .507843 | 6.18 | .976232 | .72 | .531611 | 6.90 | .468389 | 13 |
| 48 | .508214 | 6.18 | .976189 | .72 | .532025 | 6.90 | .467975 | 12 |
| 49 | .508585 | 6.18 | .976146 | .72 | .532439 | 6.90 | .467561 | 11 |
| 50 | .508956 | 6.17 | .976103 | .72 | .532853 | 6.88 | .467147 | 10 |
| 51 | 9.509326 | 6.17 | 9.976060 | .72 | 9.533266 | 6.88 | 10.466734 | 9 |
| 52 | .509696 | 6.15 | .976017 | .72 | .533679 | 6.88 | .466321 | 8 |
| 53 | .510065 | 6.15 | .975974 | .73 | .534092 | 6.88 | .465908 | 7 |
| 54 | .510434 | 6.15 | .975930 | .72 | .534504 | 6.87 | .465496 | 6 |
| 55 | .510803 | 6.15 | .975887 | .73 | .534916 | 6.87 | .465084 | 5 |
| 56 | .511172 | 6.13 | .975844 | .73 | .535328 | 6.85 | .464672 | 4 |
| 57 | .511540 | 6.12 | .975800 | .72 | .535739 | 6.85 | .464261 | 3 |
| 58 | .511907 | 6.13 | .975757 | .73 | .536150 | 6.85 | .463850 | 2 |
| 59 | .512275 | 6.12 | .975714 | .73 | .536561 | 6.85 | .463439 | 1 |
| 60 | 9.512642 | | 9.975670 | | 9.536972 | | 10.463028 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.512642 | 6.12 | 9.975670 | .72 | 9.536972 | 6.83 | 10.463028 | 60 |
| 1 | .513009 | 6.10 | .975627 | .73 | .537382 | 6.83 | .462618 | 59 |
| 2 | .513375 | 6.10 | .975583 | .73 | .537792 | 6.83 | .462208 | 58 |
| 3 | .513741 | 6.10 | .975539 | .73 | .538202 | 6.83 | .461798 | 57 |
| 4 | .514107 | 6.08 | .975496 | .73 | .538611 | 6.82 | .461389 | 56 |
| 5 | .514472 | 6.08 | .975452 | .73 | .539020 | 6.82 | .460980 | 55 |
| 6 | .514837 | 6.08 | .975408 | .72 | .539429 | 6.80 | .460571 | 54 |
| 7 | .515202 | 6.07 | .975365 | .73 | .539837 | 6.80 | .460163 | 53 |
| 8 | .515566 | 6.07 | .975321 | .73 | .540245 | 6.80 | .459755 | 52 |
| 9 | .515930 | 6.07 | .975277 | .73 | .540653 | 6.80 | .459347 | 51 |
| 10 | .516294 | 6.05 | .975233 | .73 | .541061 | 6.78 | .458939 | 50 |
| 11 | 9.516657 | 6.05 | 9.975189 | .73 | 9.541468 | 6.78 | 10.458532 | 49 |
| 12 | .517020 | 6.03 | .975145 | .73 | .541875 | 6.77 | .458125 | 48 |
| 13 | .517382 | 6.05 | .975101 | .73 | .542281 | 6.78 | .457719 | 47 |
| 14 | .517745 | 6.03 | .975057 | .73 | .542688 | 6.77 | .457312 | 46 |
| 15 | .518107 | 6.02 | .975013 | .73 | .543094 | 6.75 | .456906 | 45 |
| 16 | .518468 | 6.02 | .974969 | .73 | .543499 | 6.77 | .456501 | 44 |
| 17 | .518829 | 6.02 | .974925 | .75 | .543905 | 6.75 | .456095 | 43 |
| 18 | .519190 | 6.03 | .974880 | .73 | .544310 | 6.75 | .455690 | 42 |
| 19 | .519551 | 6.00 | .974836 | .73 | .544715 | 6.73 | .455285 | 41 |
| 20 | .519911 | 6.00 | .974792 | .73 | .545119 | 6.75 | .454881 | 40 |
| 21 | 9.520271 | 6.00 | 9.974748 | .75 | 9.545524 | 6.73 | 10.454476 | 39 |
| 22 | .520631 | 5.98 | .974703 | .73 | .545928 | 6.72 | .454072 | 38 |
| 23 | .520990 | 5.98 | .974659 | .73 | .546331 | 6.73 | .453669 | 37 |
| 24 | .521349 | 5.97 | .974614 | .75 | .546735 | 6.72 | .453265 | 36 |
| 25 | .521707 | 5.98 | .974570 | .73 | .547138 | 6.72 | .452862 | 35 |
| 26 | .522066 | 5.97 | .974525 | .73 | .547540 | 6.70 | .452460 | 34 |
| 27 | .522424 | 5.95 | .974481 | .75 | .547943 | 6.72 | .452057 | 33 |
| 28 | .522781 | 5.95 | .974436 | .75 | .548345 | 6.70 | .451655 | 32 |
| 29 | .523138 | 5.95 | .974391 | .73 | .548747 | 6.70 | .451253 | 31 |
| 30 | .523495 | 5.95 | .974347 | .75 | .549149 | 6.70 | .450851 | 30 |
| 31 | 9.523852 | 5.93 | 9.974302 | .75 | 9.549550 | 6.68 | 10.450450 | 29 |
| 32 | .524208 | 5.93 | .974257 | .75 | .549951 | 6.68 | .450049 | 28 |
| 33 | .524564 | 5.93 | .974212 | .75 | .550352 | 6.67 | .449648 | 27 |
| 34 | .524920 | 5.93 | .974167 | .75 | .550752 | 6.68 | .449248 | 26 |
| 35 | .525275 | 5.92 | .974122 | .75 | .551153 | 6.65 | .448847 | 25 |
| 36 | .525630 | 5.90 | .974077 | .75 | .551552 | 6.67 | .448448 | 24 |
| 37 | .525984 | 5.92 | .974032 | .75 | .551953 | 6.65 | .448048 | 23 |
| 38 | .526339 | 5.90 | .973987 | .75 | .552351 | 6.65 | .447649 | 22 |
| 39 | .526693 | 5.88 | .973942 | .75 | .552750 | 6.65 | .447250 | 21 |
| 40 | .527046 | 5.90 | .973897 | .75 | .553149 | 6.65 | .446851 | 20 |
| 41 | 9.527400 | 5.88 | 9.973852 | .75 | 9.553548 | 6.63 | 10.446452 | 19 |
| 42 | .527753 | 5.87 | .973807 | .77 | .553946 | 6.63 | .446054 | 18 |
| 43 | .528105 | 5.88 | .973761 | .75 | .554344 | 6.62 | .445656 | 17 |
| 44 | .528458 | 5.87 | .973716 | .75 | .554741 | 6.63 | .445259 | 16 |
| 45 | .528810 | 5.87 | .973671 | .75 | .555139 | 6.62 | .444861 | 15 |
| 46 | .529161 | 5.85 | .973625 | .77 | .555536 | 6.62 | .444464 | 14 |
| 47 | .529513 | 5.87 | .973580 | .75 | .555933 | 6.62 | .444067 | 13 |
| 48 | .529864 | 5.85 | .973535 | .77 | .556329 | 6.60 | .443671 | 12 |
| 49 | .530215 | 5.85 | .973489 | .75 | .556725 | 6.60 | .443275 | 11 |
| 50 | .530565 | 5.83 | .973444 | .77 | .557121 | 6.60 | .442879 | 10 |
| 51 | 9.530915 | 5.83 | 9.973398 | .77 | 9.557517 | 6.60 | 10.442483 | 9 |
| 52 | .531265 | 5.82 | .973352 | .75 | .557913 | 6.58 | .442087 | 8 |
| 53 | .531614 | 5.82 | .973307 | .77 | .558308 | 6.58 | .441692 | 7 |
| 54 | .531963 | 5.82 | .973261 | .77 | .558703 | 6.57 | .441297 | 6 |
| 55 | .532312 | 5.82 | .973215 | .77 | .559097 | 6.57 | .440903 | 5 |
| 56 | .532661 | 5.80 | .973169 | .77 | .559491 | 6.57 | .440509 | 4 |
| 57 | .533009 | 5.80 | .973124 | .75 | .559885 | 6.57 | .440115 | 3 |
| 58 | .533357 | 5.78 | .973078 | .77 | .560279 | 6.57 | .439721 | 2 |
| 59 | .533704 | 5.80 | .973032 | .77 | .560673 | 6.55 | .439327 | 1 |
| 60 | 9.534052 | | 9.972986 | | 9.561066 | | 10.438934 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.534052 | 5.78 | 9.972986 | .77 | 9.561066 | 6.55 | 10.438934 | 60 |
| 1 | .534399 | 5.77 | .972940 | .77 | .561459 | 6.53 | .438541 | 59 |
| 2 | .534745 | 5.78 | .972894 | .77 | .561851 | 6.55 | .438149 | 58 |
| 3 | .535092 | 5.77 | .972848 | .77 | .562244 | 6.53 | .437756 | 57 |
| 4 | .535438 | 5.75 | .972802 | .78 | .562636 | 6.53 | .437364 | 56 |
| 5 | .535783 | 5.77 | .972755 | .77 | .563028 | 6.52 | .436972 | 55 |
| 6 | .536129 | 5.75 | .972709 | .77 | .563419 | 6.53 | .436581 | 54 |
| 7 | .536474 | 5.73 | .972663 | .77 | .563811 | 6.52 | .436189 | 53 |
| 8 | .536818 | 5.75 | .972617 | .78 | .564202 | 6.52 | .435798 | 52 |
| 9 | .537163 | 5.73 | .972570 | .77 | .564593 | 6.50 | .435407 | 51 |
| 10 | .537507 | 5.73 | .972524 | .77 | .564983 | 6.50 | .435017 | 50 |
| 11 | 9.537851 | 5.72 | 9.972478 | .78 | 9.565373 | 6.50 | 10.434627 | 49 |
| 12 | .538194 | 5.73 | .972431 | .77 | .565763 | 6.50 | .434237 | 48 |
| 13 | .538538 | 5.70 | .972385 | .78 | .566153 | 6.48 | .433847 | 47 |
| 14 | .538880 | 5.72 | .972338 | .78 | .566542 | 6.50 | .433458 | 46 |
| 15 | .539223 | 5.70 | .972291 | .77 | .566932 | 6.47 | .433068 | 45 |
| 16 | .539565 | 5.70 | .972245 | .78 | .567320 | 6.48 | .432680 | 44 |
| 17 | .539907 | 5.68 | .972198 | .78 | .567709 | 6.48 | .432291 | 43 |
| 18 | .540249 | 5.68 | .972151 | .77 | .568098 | 6.47 | .431902 | 42 |
| 19 | .540590 | 5.68 | .972105 | .78 | .568486 | 6.45 | .431514 | 41 |
| 20 | .540931 | 5.68 | .972058 | .78 | .568873 | 6.47 | .431127 | 40 |
| 21 | 9.541272 | 5.68 | 9.972011 | .78 | 9.569261 | 6.45 | 10.430739 | 39 |
| 22 | .541613 | 5.67 | .971964 | .78 | .569648 | 6.45 | .430352 | 38 |
| 23 | .541953 | 5.67 | .971917 | .78 | .570035 | 6.45 | .429965 | 37 |
| 24 | .542293 | 5.65 | .971870 | .78 | .570422 | 6.45 | .429578 | 36 |
| 25 | .542632 | 5.65 | .971823 | .78 | .570809 | 6.43 | .429191 | 35 |
| 26 | .542971 | 5.65 | .971776 | .78 | .571195 | 6.43 | .428805 | 34 |
| 27 | .543310 | 5.65 | .971729 | .78 | .571581 | 6.43 | .428419 | 33 |
| 28 | .543649 | 5.63 | .971682 | .78 | .571967 | 6.42 | .428033 | 32 |
| 29 | .543987 | 5.63 | .971635 | .78 | .572352 | 6.43 | .427648 | 31 |
| 30 | .544325 | 5.63 | .971588 | .80 | .572738 | 6.42 | .427262 | 30 |
| 31 | 9.544663 | 5.62 | 9.971540 | .78 | 9.573123 | 6.40 | 10.426877 | 29 |
| 32 | .545000 | 5.63 | .971493 | .78 | .573507 | 6.42 | .426493 | 28 |
| 33 | .545338 | 5.60 | .971446 | .80 | .573892 | 6.40 | .426108 | 27 |
| 34 | .545674 | 5.62 | .971398 | .78 | .574276 | 6.40 | .425724 | 26 |
| 35 | .546011 | 5.60 | .971351 | .80 | .574660 | 6.40 | .425340 | 25 |
| 36 | .546347 | 5.60 | .971303 | .78 | .575044 | 6.38 | .424956 | 24 |
| 37 | .546683 | 5.60 | .971256 | .80 | .575427 | 6.38 | .424573 | 23 |
| 38 | .547019 | 5.60 | .971208 | .78 | .575810 | 6.38 | .424190 | 22 |
| 39 | .547354 | 5.58 | .971161 | .80 | .576193 | 6.38 | .423807 | 21 |
| 40 | .547689 | 5.58 | .971113 | .78 | .576576 | 6.38 | .423424 | 20 |
| 41 | 9.548024 | 5.58 | 9.971066 | .80 | 9.576959 | 6.37 | 10.423041 | 19 |
| 42 | .548359 | 5.57 | .971018 | .80 | .577341 | 6.37 | .422659 | 18 |
| 43 | .548693 | 5.57 | .970970 | .80 | .577723 | 6.35 | .422277 | 17 |
| 44 | .549027 | 5.55 | .970922 | .80 | .578104 | 6.37 | .421896 | 16 |
| 45 | .549360 | 5.55 | .970874 | .78 | .578486 | 6.35 | .421514 | 15 |
| 46 | .549693 | 5.55 | .970827 | .80 | .578867 | 6.35 | .421133 | 14 |
| 47 | .550026 | 5.55 | .970779 | .80 | .579248 | 6.35 | .420752 | 13 |
| 48 | .550359 | 5.55 | .970731 | .80 | .579629 | 6.33 | .420371 | 12 |
| 49 | .550692 | 5.53 | .970683 | .80 | .580009 | 6.33 | .419991 | 11 |
| 50 | .551024 | 5.53 | .970635 | .82 | .580389 | 6.33 | .419611 | 10 |
| 51 | 9.551356 | 5.52 | 9.970586 | .80 | 9.580769 | 6.33 | 10.419231 | 9 |
| 52 | .551687 | 5.52 | .970538 | .80 | .581149 | 6.32 | .418851 | 8 |
| 53 | .552018 | 5.52 | .970490 | .80 | .581528 | 6.32 | .418472 | 7 |
| 54 | .552349 | 5.52 | .970442 | .80 | .581907 | 6.32 | .418093 | 6 |
| 55 | .552680 | 5.50 | .970394 | .82 | .582286 | 6.32 | .417714 | 5 |
| 56 | .553010 | 5.52 | .970345 | .80 | .582665 | 6.32 | .417335 | 4 |
| 57 | .553341 | 5.48 | .970297 | .80 | .583044 | 6.30 | .416956 | 3 |
| 58 | .553670 | 5.50 | .970249 | .82 | .583422 | 6.30 | .416578 | 2 |
| 59 | .554000 | 5.48 | .970200 | .80 | .583800 | 6.28 | .416200 | 1 |
| 60 | 9.554329 | | 9.970152 | | 9.584177 | | 10.415823 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.554329 | | 9.970152 | | 9.584177 | | 10.415823 | 60 |
| 1 | .554658 | 5.48 | .970103 | .82 | .584555 | 6.30 | .415445 | 59 |
| 2 | .554987 | 5.48 | .970055 | .80 | .584932 | 6.28 | .415068 | 58 |
| 3 | .555315 | 5.47 | .970006 | .82 | .585309 | 6.28 | .414691 | 57 |
| 4 | .555643 | 5.47 | .969957 | .82 | .585686 | 6.28 | .414314 | 56 |
| 5 | .555971 | 5.47 | .969909 | .80 | .586062 | 6.27 | .413938 | 55 |
| 6 | .556299 | 5.47 | .969860 | .82 | .586439 | 6.28 | .413561 | 54 |
| 7 | .556626 | 5.45 | .969811 | .82 | .586815 | 6.27 | .413185 | 53 |
| 8 | .556953 | 5.45 | .969762 | .82 | .587190 | 6.25 | .412810 | 52 |
| 9 | .557280 | 5.45 | .969714 | .80 | .587566 | 6.27 | .412434 | 51 |
| 10 | .557606 | 5.43 | .969665 | .82 | .587941 | 6.25 | .412059 | 50 |
| 11 | 9.557932 | 5.43 | 9.969616 | .82 | 9.588316 | 6.25 | 10.411684 | 49 |
| 12 | .558258 | 5.42 | .969567 | .82 | .588691 | 6.25 | .411309 | 48 |
| 13 | .558583 | 5.43 | .969518 | .82 | .589066 | 6.23 | .410934 | 47 |
| 14 | .558909 | 5.42 | .969469 | .82 | .589440 | 6.23 | .410560 | 46 |
| 15 | .559234 | 5.40 | .969420 | .82 | .589814 | 6.23 | .410186 | 45 |
| 16 | .559558 | 5.42 | .969370 | .83 | .590188 | 6.23 | .409812 | 44 |
| 17 | .559883 | 5.40 | .969321 | .82 | .590562 | 6.22 | .409438 | 43 |
| 18 | .560207 | 5.40 | .969272 | .82 | .590935 | 6.22 | .409065 | 42 |
| 19 | .560531 | 5.40 | .969223 | .83 | .591308 | 6.22 | .408692 | 41 |
| 20 | .560855 | 5.38 | .969173 | .82 | .591681 | 6.22 | .408319 | 40 |
| 21 | 9.561178 | 5.38 | 9.969124 | .82 | 9.592054 | 6.20 | 10.407946 | 39 |
| 22 | .561501 | 5.38 | .969075 | .83 | .592426 | 6.22 | .407574 | 38 |
| 23 | .561824 | 5.37 | .969025 | .82 | .592799 | 6.20 | .407201 | 37 |
| 24 | .562146 | 5.37 | .968976 | .83 | .593171 | 6.18 | .406829 | 36 |
| 25 | .562468 | 5.37 | .968926 | .83 | .593542 | 6.20 | .406458 | 35 |
| 26 | .562790 | 5.37 | .968877 | .82 | .593914 | 6.18 | .406086 | 34 |
| 27 | .563112 | 5.37 | .968827 | .83 | .594285 | 6.18 | .405715 | 33 |
| 28 | .563433 | 5.35 | .968777 | .83 | .594656 | 6.18 | .405344 | 32 |
| 29 | .563755 | 5.37 | .968728 | .83 | .595027 | 6.18 | .404973 | 31 |
| 30 | .564075 | 5.33 | .968678 | .83 | .595398 | 6.17 | .404602 | 30 |
| 31 | 9.564396 | 5.33 | 9.968628 | .83 | 9.595768 | 6.17 | 10.404232 | 29 |
| 32 | .564716 | 5.33 | .968578 | .83 | .596138 | 6.17 | .403862 | 28 |
| 33 | .565036 | 5.33 | .968528 | .82 | .596508 | 6.17 | .403492 | 27 |
| 34 | .565356 | 5.33 | .968479 | .83 | .596878 | 6.15 | .403122 | 26 |
| 35 | .565676 | 5.33 | .968429 | .83 | .597247 | 6.15 | .402753 | 25 |
| 36 | .565995 | 5.32 | .968379 | .83 | .597616 | 6.15 | .402384 | 24 |
| 37 | .566314 | 5.32 | .968329 | .85 | .597985 | 6.15 | .402015 | 23 |
| 38 | .566632 | 5.30 | .968278 | .83 | .598354 | 6.13 | .401646 | 22 |
| 39 | .566951 | 5.32 | .968228 | .83 | .598722 | 6.15 | .401278 | 21 |
| 40 | .567269 | 5.30 | .968178 | .83 | .599091 | 6.13 | .400909 | 20 |
| 41 | 9.567587 | 5.28 | 9.968128 | .83 | 9.599459 | 6.13 | 10.400541 | 19 |
| 42 | .567904 | 5.30 | .968078 | .85 | .599827 | 6.12 | .400173 | 18 |
| 43 | .568222 | 5.28 | .968027 | .83 | .600194 | 6.13 | .399806 | 17 |
| 44 | .568539 | 5.28 | .967977 | .83 | .600562 | 6.12 | .399438 | 16 |
| 45 | .568856 | 5.28 | .967927 | .83 | .600929 | 6.12 | .399071 | 15 |
| 46 | .569172 | 5.27 | .967876 | .85 | .601296 | 6.12 | .398704 | 14 |
| 47 | .569488 | 5.27 | .967826 | .83 | .601663 | 6.12 | .398337 | 13 |
| 48 | .569804 | 5.27 | .967775 | .85 | .602029 | 6.10 | .397971 | 12 |
| 49 | .570120 | 5.27 | .967725 | .83 | .602395 | 6.10 | .397605 | 11 |
| 50 | .570435 | 5.25 | .967674 | .85 | .602761 | 6.10 | .397239 | 10 |
| 51 | 9.570751 | 5.27 | 9.967624 | .83 | 9.603127 | 6.10 | 10.396873 | 9 |
| 52 | .571066 | 5.25 | .967573 | .85 | .603493 | 6.08 | .396507 | 8 |
| 53 | .571380 | 5.23 | .967522 | .85 | .603858 | 6.08 | .396142 | 7 |
| 54 | .571695 | 5.25 | .967471 | .83 | .604223 | 6.08 | .395777 | 6 |
| 55 | .572009 | 5.23 | .967421 | .85 | .604588 | 6.08 | .395412 | 5 |
| 56 | .572323 | 5.23 | .967370 | .85 | .604953 | 6.07 | .395047 | 4 |
| 57 | .572636 | 5.22 | .967319 | .85 | .605317 | 6.08 | .394683 | 3 |
| 58 | .572950 | 5.23 | .967268 | .85 | .605682 | 6.07 | .394318 | 2 |
| 59 | .573263 | 5.22 | .967217 | .85 | .606046 | 6.07 | .393954 | 1 |
| 60 | 9.573575 | 5.20 | 9.967166 | .85 | 9.606410 | | 10.393590 | 0 |
| | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.573575 | 5.22 | 9.967166 | .85 | 9.606410 | 6.05 | 10.393590 | 60 |
| 1 | .573888 | 5.20 | .967115 | .85 | .606773 | 6.07 | .393227 | 59 |
| 2 | .574200 | 5.20 | .967064 | .85 | .607137 | 6.05 | .392863 | 58 |
| 3 | .574512 | 5.20 | .967013 | .87 | .607500 | 6.05 | .392500 | 57 |
| 4 | .574824 | 5.20 | .966961 | .85 | .607863 | 6.03 | .392137 | 56 |
| 5 | .575136 | 5.18 | .966910 | .85 | .608225 | 6.05 | .391775 | 55 |
| 6 | .575447 | 5.18 | .966859 | .85 | .608588 | 6.03 | .391412 | 54 |
| 7 | .575758 | 5.18 | .966808 | .85 | .608950 | 6.03 | .391050 | 53 |
| 8 | .576069 | 5.17 | .966756 | .85 | .609312 | 6.03 | .390688 | 52 |
| 9 | .576379 | 5.17 | .966705 | .87 | .609674 | 6.03 | .390326 | 51 |
| 10 | .576689 | 5.17 | .966653 | .85 | .610036 | 6.02 | .389964 | 50 |
| 11 | 9.576999 | 5.17 | 9.966602 | .87 | 9.610397 | 6.03 | 10.389603 | 49 |
| 12 | .577309 | 5.15 | .966550 | .85 | .610759 | 6.02 | .389241 | 48 |
| 13 | .577618 | 5.15 | .966499 | .87 | .611120 | 6.00 | .388880 | 47 |
| 14 | .577927 | 5.15 | .966447 | .87 | .611480 | 6.02 | .388520 | 46 |
| 15 | .578236 | 5.15 | .966395 | .85 | .611841 | 6.00 | .388159 | 45 |
| 16 | .578545 | 5.13 | .966344 | .87 | .612201 | 6.00 | .387799 | 44 |
| 17 | .578853 | 5.15 | .966292 | .87 | .612561 | 6.00 | .387439 | 43 |
| 18 | .579162 | 5.13 | .966240 | .87 | .612921 | 6.00 | .387079 | 42 |
| 19 | .579470 | 5.12 | .966188 | .87 | .613281 | 6.00 | .386719 | 41 |
| 20 | .579777 | 5.13 | .966136 | .85 | .613641 | 5.98 | .386359 | 40 |
| 21 | 9.580085 | 5.12 | 9.966085 | .87 | 9.614000 | 5.98 | 10.386000 | 39 |
| 22 | .580392 | 5.12 | .966033 | .87 | .614359 | 5.98 | .385641 | 38 |
| 23 | .580699 | 5.10 | .965981 | .87 | .614718 | 5.98 | .385282 | 37 |
| 24 | .581005 | 5.12 | .965929 | .88 | .615077 | 5.97 | .384923 | 36 |
| 25 | .581312 | 5.10 | .965876 | .87 | .615435 | 5.97 | .384565 | 35 |
| 26 | .581618 | 5.10 | .965824 | .87 | .615793 | 5.97 | .384207 | 34 |
| 27 | .581924 | 5.10 | .965772 | .87 | .616151 | 5.97 | .383849 | 33 |
| 28 | .582229 | 5.08 | .965720 | .87 | .616509 | 5.97 | .383491 | 32 |
| 29 | .582535 | 5.10 | .965668 | .88 | .616867 | 5.97 | .383133 | 31 |
| 30 | .582840 | 5.08 | .965615 | .87 | .617224 | 5.95 | .382776 | 30 |
| 31 | 9.583145 | 5.07 | 9.965563 | .87 | 9.617582 | 5.95 | 10.382418 | 29 |
| 32 | .583449 | 5.08 | .965511 | .88 | .617939 | 5.93 | .382061 | 28 |
| 33 | .583754 | 5.07 | .965458 | .87 | .618295 | 5.95 | .381705 | 27 |
| 34 | .584058 | 5.05 | .965406 | .88 | .618652 | 5.93 | .381348 | 26 |
| 35 | .584361 | 5.07 | .965353 | .87 | .619008 | 5.93 | .380992 | 25 |
| 36 | .584665 | 5.05 | .965301 | .88 | .619364 | 5.93 | .380636 | 24 |
| 37 | .584968 | 5.07 | .965248 | .88 | .619720 | 5.93 | .380280 | 23 |
| 38 | .585272 | 5.03 | .965195 | .87 | .620076 | 5.93 | .379924 | 22 |
| 39 | .585574 | 5.05 | .965143 | .88 | .620432 | 5.92 | .379568 | 21 |
| 40 | .585877 | 5.03 | .965090 | .88 | .620787 | 5.92 | .379213 | 20 |
| 41 | 9.586179 | 5.05 | 9.965037 | .88 | 9.621142 | 5.92 | 10.378858 | 19 |
| 42 | .586482 | 5.02 | .964984 | .88 | .621497 | 5.92 | .378503 | 18 |
| 43 | .586783 | 5.03 | .964931 | .87 | .621852 | 5.92 | .378148 | 17 |
| 44 | .587085 | 5.02 | .964879 | .88 | .622207 | 5.90 | .377793 | 16 |
| 45 | .587386 | 5.02 | .964826 | .88 | .622561 | 5.90 | .377439 | 15 |
| 46 | .587688 | 5.03 | .964773 | .88 | .622915 | 5.90 | .377085 | 14 |
| 47 | .587989 | 5.02 | .964720 | .90 | .623269 | 5.90 | .376731 | 13 |
| 48 | .588289 | 5.00 | .964666 | .88 | .623623 | 5.88 | .376377 | 12 |
| 49 | .588590 | 5.02 | .964613 | .88 | .623976 | 5.90 | .376024 | 11 |
| 50 | .588890 | 5.00 | .964560 | .88 | .624330 | 5.88 | .375670 | 10 |
| 51 | 9.589190 | 4.98 | 9.964507 | .88 | 9.624683 | 5.88 | 10.375317 | 9 |
| 52 | .589489 | 5.00 | .964454 | .90 | .625036 | 5.87 | .374964 | 8 |
| 53 | .589789 | 4.98 | .964400 | .88 | .625388 | 5.88 | .374612 | 7 |
| 54 | .590088 | 4.98 | .964347 | .88 | .625741 | 5.87 | .374259 | 6 |
| 55 | .590387 | 4.98 | .964294 | .90 | .626093 | 5.87 | .373907 | 5 |
| 56 | .590686 | 4.98 | .964240 | .88 | .626445 | 5.87 | .373555 | 4 |
| 57 | .590984 | 4.97 | .964187 | .90 | .626797 | 5.87 | .373203 | 3 |
| 58 | .591282 | 4.97 | .964133 | .88 | .627149 | 5.87 | .372851 | 2 |
| 59 | .591580 | 4.97 | .964080 | .90 | .627501 | 5.85 | .372499 | 1 |
| 60 | 9.591878 | | 9.964026 | | 9.627852 | | 10.372148 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.591878 | 4.97 | 9.964026 | .90 | 9.627852 | 5.85 | 10.372148 | 60 |
| 1 | .592176 | 4.95 | .963972 | .88 | .628203 | 5.85 | .371797 | 59 |
| 2 | .592473 | 4.95 | .963919 | .90 | .628554 | 5.85 | .371446 | 58 |
| 3 | .592770 | 4.95 | .963865 | .90 | .628905 | 5.85 | .371095 | 57 |
| 4 | .593067 | 4.95 | .963811 | .90 | .629255 | 5.83 | .370745 | 56 |
| 5 | .593363 | 4.93 | .963757 | .90 | .629606 | 5.85 | .370394 | 55 |
| 6 | .593659 | 4.93 | .963704 | .88 | .629956 | 5.83 | .370044 | 54 |
| 7 | .593955 | 4.93 | .963650 | .90 | .630306 | 5.83 | .369694 | 53 |
| 8 | .594251 | 4.93 | .963596 | .90 | .630656 | 5.82 | .369344 | 52 |
| 9 | .594547 | 4.92 | .963542 | .90 | .631005 | 5.83 | .368995 | 51 |
| 10 | .594842 | 4.92 | .963488 | .90 | .631355 | 5.82 | .368645 | 50 |
| 11 | 9.595137 | 4.92 | 9.963434 | .92 | 9.631704 | 5.82 | 10.368296 | 49 |
| 12 | .595432 | 4.92 | .963379 | .90 | .632053 | 5.82 | .367947 | 48 |
| 13 | .595727 | 4.92 | .963325 | .90 | .632402 | 5.80 | .367598 | 47 |
| 14 | .596021 | 4.90 | .963271 | .90 | .632750 | 5.82 | .367250 | 46 |
| 15 | .596315 | 4.90 | .963217 | .90 | .633099 | 5.80 | .366901 | 45 |
| 16 | .596609 | 4.90 | .963163 | .92 | .633447 | 5.80 | .366553 | 44 |
| 17 | .596903 | 4.88 | .963108 | .90 | .633795 | 5.80 | .366205 | 43 |
| 18 | .597196 | 4.90 | .963054 | .92 | .634143 | 5.78 | .365857 | 42 |
| 19 | .597490 | 4.88 | .962999 | .90 | .634490 | 5.80 | .365510 | 41 |
| 20 | .597783 | 4.87 | .962945 | .92 | .634838 | 5.78 | .365162 | 40 |
| 21 | 9.598075 | 4.88 | 9.962890 | .90 | 9.635185 | 5.78 | 10.364815 | 39 |
| 22 | .598368 | 4.87 | .962836 | .92 | .635532 | 5.78 | .364468 | 38 |
| 23 | .598660 | 4.87 | .962781 | .90 | .635879 | 5.78 | .364121 | 37 |
| 24 | .598952 | 4.87 | .962727 | .92 | .636226 | 5.77 | .363774 | 36 |
| 25 | .599244 | 4.87 | .962672 | .92 | .636572 | 5.78 | .363428 | 35 |
| 26 | .599535 | 4.85 | .962617 | .92 | .636919 | 5.77 | .363081 | 34 |
| 27 | .599827 | 4.85 | .962563 | .90 | .637265 | 5.77 | .362735 | 33 |
| 28 | .600118 | 4.85 | .962508 | .92 | .637611 | 5.75 | .362389 | 32 |
| 29 | .600409 | 4.85 | .962453 | .92 | .637956 | 5.77 | .362044 | 31 |
| 30 | .600700 | 4.83 | .962398 | .92 | .638302 | 5.75 | .361698 | 30 |
| 31 | 9.600990 | 4.83 | 9.962343 | .92 | 9.638647 | 5.75 | 10.361353 | 29 |
| 32 | .601280 | 4.83 | .962288 | .92 | .638992 | 5.75 | .361008 | 28 |
| 33 | .601570 | 4.83 | .962233 | .92 | .639337 | 5.75 | .360663 | 27 |
| 34 | .601860 | 4.83 | .962178 | .92 | .639682 | 5.75 | .360318 | 26 |
| 35 | .602150 | 4.82 | .962123 | .93 | .640027 | 5.73 | .359973 | 25 |
| 36 | .602439 | 4.82 | .962067 | .92 | .640371 | 5.75 | .359629 | 24 |
| 37 | .602728 | 4.82 | .962012 | .92 | .640716 | 5.73 | .359284 | 23 |
| 38 | .603017 | 4.80 | .961957 | .92 | .641060 | 5.73 | .358940 | 22 |
| 39 | .603305 | 4.82 | .961902 | .92 | .641404 | 5.72 | .358596 | 21 |
| 40 | .603594 | 4.80 | .961846 | .93 | .641747 | 5.73 | .358253 | 20 |
| 41 | 9.603882 | 4.80 | 9.961791 | .93 | 9.642091 | 5.72 | 10.357909 | 19 |
| 42 | .604170 | 4.78 | .961735 | .92 | .642434 | 5.72 | .357566 | 18 |
| 43 | .604457 | 4.80 | .961680 | .93 | .642777 | 5.72 | .357223 | 17 |
| 44 | .604745 | 4.78 | .961624 | .92 | .643120 | 5.72 | .356880 | 16 |
| 45 | .605032 | 4.78 | .961569 | .93 | .643463 | 5.72 | .356537 | 15 |
| 46 | .605319 | 4.78 | .961513 | .92 | .643806 | 5.70 | .356194 | 14 |
| 47 | .605606 | 4.77 | .961458 | .93 | .644148 | 5.70 | .355852 | 13 |
| 48 | .605892 | 4.78 | .961402 | .93 | .644490 | 5.70 | .355510 | 12 |
| 49 | .606179 | 4.77 | .961346 | .93 | .644832 | 5.70 | .355168 | 11 |
| 50 | .606465 | 4.77 | .961290 | .92 | .645174 | 5.70 | .354826 | 10 |
| 51 | 9.606751 | 4.75 | 9.961235 | .93 | 9.645516 | 5.68 | 10.354484 | 9 |
| 52 | .607036 | 4.77 | .961179 | .93 | .645857 | 5.70 | .354143 | 8 |
| 53 | .607322 | 4.75 | .961123 | .93 | .646199 | 5.68 | .353801 | 7 |
| 54 | .607607 | 4.75 | .961067 | .93 | .646540 | 5.68 | .353460 | 6 |
| 55 | .607892 | 4.75 | .961011 | .93 | .646881 | 5.68 | .353119 | 5 |
| 56 | .608177 | 4.73 | .960955 | .93 | .647222 | 5.67 | .352778 | 4 |
| 57 | .608461 | 4.73 | .960899 | .93 | .647562 | 5.68 | .352438 | 3 |
| 58 | .608745 | 4.73 | .960843 | .95 | .647903 | 5.67 | .352097 | 2 |
| 59 | .609029 | 4.73 | .960786 | .93 | .648243 | 5.67 | .351757 | 1 |
| 60 | 9.609313 | | 9.960730 | | 9.648583 | | 10.351417 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.609312 | 4.73 | 9.960730 | .93 | 9.648583 | 5.67 | 10.351417 | 60 |
| 1 | .609597 | 4.72 | .960674 | .93 | .648923 | 5.67 | .351077 | 59 |
| 2 | .609880 | 4.73 | .960618 | .93 | .649263 | 5.67 | .350737 | 58 |
| 3 | .610164 | 4.72 | .960561 | .93 | .649602 | 5.65 | .350398 | 57 |
| 4 | .610447 | 4.70 | .960505 | .93 | .649942 | 5.67 | .350058 | 56 |
| 5 | .610729 | 4.72 | .960448 | .93 | .650281 | 5.65 | .349719 | 55 |
| 6 | .611012 | 4.70 | .960392 | .93 | .650620 | 5.65 | .349380 | 54 |
| 7 | .611294 | 4.70 | .960335 | .93 | .650959 | 5.65 | .349041 | 53 |
| 8 | .611576 | 4.70 | .960279 | .93 | .651297 | 5.63 | .348703 | 52 |
| 9 | .611858 | 4.70 | .960222 | .93 | .651636 | 5.65 | .348364 | 51 |
| 10 | .612140 | 4.68 | .960165 | .93 | .651974 | 5.63 | .348026 | 50 |
| 11 | 9.612421 | 4.68 | 9.960109 | .95 | 9.652312 | 5.63 | 10.347688 | 49 |
| 12 | .612702 | 4.68 | .960052 | .95 | .652650 | 5.63 | .347350 | 48 |
| 13 | .612983 | 4.68 | .959995 | .95 | .652988 | 5.63 | .347012 | 47 |
| 14 | .613264 | 4.68 | .959938 | .93 | .653326 | 5.62 | .346674 | 46 |
| 15 | .613545 | 4.67 | .959882 | .95 | .653663 | 5.62 | .346337 | 45 |
| 16 | .613825 | 4.67 | .959825 | .95 | .654000 | 5.62 | .346000 | 44 |
| 17 | .614105 | 4.67 | .959768 | .95 | .654337 | 5.62 | .345663 | 43 |
| 18 | .614385 | 4.67 | .959711 | .95 | .654674 | 5.62 | .345326 | 42 |
| 19 | .614665 | 4.67 | .959654 | .95 | .655011 | 5.62 | .344989 | 41 |
| 20 | .614944 | 4.65 | .959596 | .97 | .655348 | 5.60 | .344652 | 40 |
| 21 | 9.615223 | 4.65 | 9.959539 | .95 | 9.655684 | 5.60 | 10.344316 | 39 |
| 22 | .615503 | 4.65 | .959482 | .95 | .656020 | 5.60 | .343980 | 38 |
| 23 | .615781 | 4.65 | .959425 | .95 | .656356 | 5.60 | .343644 | 37 |
| 24 | .616060 | 4.63 | .959368 | .97 | .656692 | 5.60 | .343308 | 36 |
| 25 | .616338 | 4.63 | .959310 | .95 | .657028 | 5.60 | .342972 | 35 |
| 26 | .616616 | 4.63 | .959253 | .97 | .657364 | 5.58 | .342636 | 34 |
| 27 | .616894 | 4.63 | .959195 | .95 | .657699 | 5.58 | .342301 | 33 |
| 28 | .617172 | 4.63 | .959138 | .95 | .658034 | 5.58 | .341966 | 32 |
| 29 | .617450 | 4.62 | .959080 | .97 | .658369 | 5.58 | .341631 | 31 |
| 30 | .617727 | 4.63 | .959023 | .97 | .658704 | 5.58 | .341296 | 30 |
| 31 | 9.618004 | 4.62 | 9.958965 | .95 | 9.659039 | 5.57 | 10.340961 | 29 |
| 32 | .618281 | 4.62 | .958908 | .97 | .659373 | 5.58 | .340627 | 28 |
| 33 | .618558 | 4.60 | .958850 | .97 | .659708 | 5.57 | .340292 | 27 |
| 34 | .618834 | 4.60 | .958792 | .97 | .660042 | 5.57 | .339958 | 26 |
| 35 | .619110 | 4.60 | .958734 | .95 | .660376 | 5.57 | .339624 | 25 |
| 36 | .619386 | 4.60 | .958677 | .97 | .660710 | 5.55 | .339290 | 24 |
| 37 | .619662 | 4.60 | .958619 | .97 | .661043 | 5.57 | .338957 | 23 |
| 38 | .619938 | 4.58 | .958561 | .97 | .661377 | 5.55 | .338623 | 22 |
| 39 | .620213 | 4.58 | .958503 | .97 | .661710 | 5.55 | .338290 | 21 |
| 40 | .620488 | 4.58 | .958445 | .97 | .662043 | 5.55 | .337957 | 20 |
| 41 | 9.620763 | 4.58 | 9.958387 | .97 | 9.662376 | 5.55 | 10.337624 | 19 |
| 42 | .621038 | 4.58 | .958329 | .97 | .662709 | 5.55 | .337291 | 18 |
| 43 | .621313 | 4.57 | .958271 | .97 | .663042 | 5.55 | .336958 | 17 |
| 44 | .621587 | 4.57 | .958213 | .98 | .663375 | 5.53 | .336625 | 16 |
| 45 | .621861 | 4.57 | .958154 | .97 | .663707 | 5.53 | .336293 | 15 |
| 46 | .622135 | 4.57 | .958096 | .97 | .664039 | 5.53 | .335961 | 14 |
| 47 | .622409 | 4.55 | .958038 | .98 | .664371 | 5.53 | .335629 | 13 |
| 48 | .622682 | 4.57 | .957979 | .97 | .664703 | 5.53 | .335297 | 12 |
| 49 | .622956 | 4.55 | .957921 | .97 | .665035 | 5.53 | .334965 | 11 |
| 50 | .623229 | 4.55 | .957863 | .98 | .665366 | 5.53 | .334634 | 10 |
| 51 | 9.623502 | 4.53 | 9.957804 | .97 | 9.665698 | 5.52 | 10.334302 | 9 |
| 52 | .623774 | 4.55 | .957746 | .98 | .666029 | 5.52 | .333971 | 8 |
| 53 | .624047 | 4.53 | .957687 | .98 | .666360 | 5.52 | .333640 | 7 |
| 54 | .624319 | 4.53 | .957628 | .97 | .666691 | 5.50 | .333309 | 6 |
| 55 | .624591 | 4.53 | .957570 | .98 | .667021 | 5.52 | .332979 | 5 |
| 56 | .624863 | 4.53 | .957511 | .98 | .667352 | 5.50 | .332648 | 4 |
| 57 | .625135 | 4.53 | .957452 | .98 | .667682 | 5.50 | .332318 | 3 |
| 58 | .625406 | 4.53 | .957393 | .98 | .668013 | 5.50 | .331987 | 2 |
| 59 | .625677 | 4.53 | .957335 | .97 | .668343 | 5.50 | .331657 | 1 |
| 60 | 9.625948 | 4.52 | 9.957276 | .98 | 9.668673 | 5.50 | 10.331327 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.625948 | | 9.957276 | | 9.668673 | | 10.331327 | 60 |
| 1 | .626219 | 4.52 | .957217 | .98 | .669002 | 5.48 | .330998 | 59 |
| 2 | .626490 | 4.52 | .957158 | .98 | .669332 | 5.50 | .330668 | 58 |
| 3 | .626760 | 4.50 | .957099 | .98 | .669661 | 5.48 | .330339 | 57 |
| 4 | .627030 | 4.50 | .957040 | .98 | .669991 | 5.50 | .330009 | 56 |
| 5 | .627300 | 4.50 | .956981 | .98 | .670320 | 5.48 | .329680 | 55 |
| 6 | .627570 | 4.50 | .956921 | 1.00 | .670649 | 5.48 | .329351 | 54 |
| 7 | .627840 | 4.50 | .956862 | .98 | .670977 | 5.47 | .329023 | 53 |
| 8 | .628109 | 4.48 | .956803 | .98 | .671306 | 5.48 | .328694 | 52 |
| 9 | .628378 | 4.48 | .956744 | .98 | .671635 | 5.48 | .328365 | 51 |
| 10 | .628647 | 4.48 | .956684 | 1.00 | .671963 | 5.47 | .328037 | 50 |
| 11 | 9.628916 | 4.48 | 9.956625 | .98 | 9.672291 | 5.47 | 10.327709 | 49 |
| 12 | .629185 | 4.47 | .956566 | 1.00 | .672619 | 5.47 | .327381 | 48 |
| 13 | .629453 | 4.47 | .956506 | .98 | .672947 | 5.45 | .327053 | 47 |
| 14 | .629721 | 4.47 | .956447 | 1.00 | .673274 | 5.47 | .326726 | 46 |
| 15 | .629989 | 4.47 | .956387 | 1.00 | .673602 | 5.45 | .326398 | 45 |
| 16 | .630257 | 4.47 | .956327 | 1.00 | .673929 | 5.47 | .326071 | 44 |
| 17 | .630524 | 4.45 | .956268 | .98 | .674257 | 5.45 | .325743 | 43 |
| 18 | .630792 | 4.47 | .956208 | 1.00 | .674584 | 5.45 | .325416 | 42 |
| 19 | .631059 | 4.45 | .956148 | 1.00 | .674911 | 5.43 | .325089 | 41 |
| 20 | .631326 | 4.45 | .956089 | .98 | .675237 | 5.45 | .324763 | 40 |
| 21 | 9.631593 | 4.43 | 9.956029 | 1.00 | 9.675564 | 5.43 | 10.324436 | 39 |
| 22 | .631859 | 4.43 | .955969 | 1.00 | .675890 | 5.45 | .324110 | 38 |
| 23 | .632125 | 4.45 | .955909 | 1.00 | .676217 | 5.43 | .323783 | 37 |
| 24 | .632392 | 4.43 | .955849 | 1.00 | .676543 | 5.43 | .323457 | 36 |
| 25 | .632658 | 4.43 | .955789 | 1.00 | .676869 | 5.43 | .323131 | 35 |
| 26 | .632923 | 4.42 | .955729 | 1.00 | .677194 | 5.42 | .322806 | 34 |
| 27 | .633189 | 4.43 | .955669 | 1.00 | .677520 | 5.43 | .322480 | 33 |
| 28 | .633454 | 4.42 | .955609 | 1.00 | .677846 | 5.43 | .322154 | 32 |
| 29 | .633719 | 4.42 | .955548 | .98 | .678171 | 5.42 | .321829 | 31 |
| 30 | .633984 | 4.42 | .955488 | 1.00 | .678496 | 5.42 | .321504 | 30 |
| 31 | 9.634249 | 4.42 | 9.955428 | 1.00 | 9.678821 | 5.42 | 10.321179 | 29 |
| 32 | .634514 | 4.40 | .955368 | 1.02 | .679146 | 5.42 | .320854 | 28 |
| 33 | .634778 | 4.40 | .955307 | 1.00 | .679471 | 5.40 | .320529 | 27 |
| 34 | .635042 | 4.40 | .955247 | 1.02 | .679795 | 5.42 | .320205 | 26 |
| 35 | .635306 | 4.40 | .955186 | 1.00 | .680120 | 5.40 | .319880 | 25 |
| 36 | .635570 | 4.40 | .955126 | 1.02 | .680444 | 5.40 | .319556 | 24 |
| 37 | .635834 | 4.40 | .955065 | 1.02 | .680768 | 5.40 | .319232 | 23 |
| 38 | .636097 | 4.38 | .955005 | 1.00 | .681092 | 5.40 | .318908 | 22 |
| 39 | .636360 | 4.38 | .954944 | 1.02 | .681416 | 5.40 | .318584 | 21 |
| 40 | .636623 | 4.38 | .954883 | 1.00 | .681740 | 5.38 | .318260 | 20 |
| 41 | 9.636886 | 4.37 | 9.954823 | 1.02 | 9.682063 | 5.40 | 10.317937 | 19 |
| 42 | .637148 | 4.38 | .954762 | 1.02 | .682387 | 5.38 | .317613 | 18 |
| 43 | .637411 | 4.37 | .954701 | 1.02 | .682710 | 5.38 | .317290 | 17 |
| 44 | .637673 | 4.37 | .954640 | 1.02 | .683033 | 5.38 | .316967 | 16 |
| 45 | .637935 | 4.37 | .954579 | 1.02 | .683356 | 5.38 | .316644 | 15 |
| 46 | .638197 | 4.37 | .954518 | 1.02 | .683679 | 5.37 | .316321 | 14 |
| 47 | .638458 | 4.35 | .954457 | 1.02 | .684001 | 5.38 | .315999 | 13 |
| 48 | .638720 | 4.37 | .954396 | 1.02 | .684324 | 5.37 | .315676 | 12 |
| 49 | .638981 | 4.35 | .954335 | 1.02 | .684646 | 5.37 | .315354 | 11 |
| 50 | .639242 | 4.35 | .954274 | 1.02 | .684968 | 5.37 | .315032 | 10 |
| 51 | 9.639503 | 4.35 | 9.954213 | 1.02 | 9.685290 | 5.37 | 10.314710 | 9 |
| 52 | .639764 | 4.33 | .954152 | 1.03 | .685612 | 5.37 | .314388 | 8 |
| 53 | .640024 | 4.33 | .954090 | 1.02 | .685934 | 5.35 | .314066 | 7 |
| 54 | .640284 | 4.33 | .954029 | 1.02 | .686255 | 5.37 | .313745 | 6 |
| 55 | .640544 | 4.33 | .953968 | 1.03 | .686577 | 5.35 | .313423 | 5 |
| 56 | .640804 | 4.33 | .953906 | 1.02 | .686898 | 5.35 | .313102 | 4 |
| 57 | .641064 | 4.33 | .953845 | 1.03 | .687219 | 5.35 | .312781 | 3 |
| 58 | .641324 | 4.33 | .953783 | 1.03 | .687540 | 5.35 | .312460 | 2 |
| 59 | .641583 | 4.32 | .953722 | 1.02 | .687861 | 5.35 | .312139 | 1 |
| 60 | 9.641842 | 4.32 | 9.953660 | 1.03 | 9.688182 | 5.35 | 10.311818 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D 1″. | Tang. | D. 1″. | Cotang. | |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.641842 | 4.32 | 9.953660 | 1.02 | 9.688182 | 5.33 | 10.311818 | 60 |
| 1 | .642101 | 4.32 | .953599 | 1.03 | .688502 | 5.32 | .311498 | 59 |
| 2 | .642360 | 4.30 | .953537 | 1.03 | .688823 | 5.33 | .311177 | 58 |
| 3 | .642618 | 4.32 | .953475 | 1.03 | .689143 | 5.33 | .310857 | 57 |
| 4 | .642877 | 4.30 | .953413 | 1.02 | .689463 | 5.33 | .310537 | 56 |
| 5 | .643135 | 4.30 | .953352 | 1.03 | .689783 | 5.33 | .310217 | 55 |
| 6 | .643393 | 4.30 | .953290 | 1.03 | .690103 | 5.33 | .309897 | 54 |
| 7 | .643650 | 4.28 | .953228 | 1.03 | .690423 | 5.32 | .309577 | 53 |
| 8 | .643908 | 4.30 | .953166 | 1.03 | .690742 | 5.33 | .309258 | 52 |
| 9 | .644165 | 4.28 | .953104 | 1.03 | .691062 | 5.32 | .308938 | 51 |
| 10 | .644423 | 4.30 | .953042 | 1.03 | .691381 | 5.32 | .308619 | 50 |
| 11 | 9.644680 | 4.27 | 9.952980 | 1.03 | 9.691700 | 5.32 | 10.308300 | 49 |
| 12 | .644936 | 4.28 | .952918 | 1.05 | .692019 | 5.32 | .307981 | 48 |
| 13 | .645193 | 4.28 | .952855 | 1.03 | .692338 | 5.30 | .307662 | 47 |
| 14 | .645450 | 4.27 | .952793 | 1.03 | .692656 | 5.32 | .307344 | 46 |
| 15 | .645706 | 4.27 | .952731 | 1.03 | .692975 | 5.30 | .307025 | 45 |
| 16 | .645962 | 4.27 | .952669 | 1.03 | .693293 | 5.32 | .306707 | 44 |
| 17 | .646218 | 4.27 | .952606 | 1.05 | .693612 | 5.30 | .306388 | 43 |
| 18 | .646474 | 4.27 | .952544 | 1.03 | .693930 | 5.30 | .306070 | 42 |
| 19 | .646729 | 4.25 | .952481 | 1.05 | .694248 | 5.30 | .305752 | 41 |
| 20 | .646984 | 4.27 | .952419 | 1.05 | .694566 | 5.28 | .305434 | 40 |
| 21 | 9.647240 | 4.23 | 9.952356 | 1.03 | 9.694883 | 5.30 | 10.305117 | 39 |
| 22 | .647494 | 4.25 | .952294 | 1.05 | .695201 | 5.28 | .304799 | 38 |
| 23 | .647749 | 4.25 | .952231 | 1.05 | .695518 | 5.30 | .304482 | 37 |
| 24 | .648004 | 4.23 | .952168 | 1.03 | .695836 | 5.28 | .304164 | 36 |
| 25 | .648258 | 4.23 | .952106 | 1.05 | .696153 | 5.28 | .303847 | 35 |
| 26 | .648512 | 4.23 | .952043 | 1.05 | .696470 | 5.28 | .303530 | 34 |
| 27 | .648766 | 4.23 | .951980 | 1.05 | .696787 | 5.27 | .303213 | 33 |
| 28 | .649020 | 4.23 | .951917 | 1.05 | .697103 | 5.28 | .302897 | 32 |
| 29 | .649274 | 4.23 | .951854 | 1.05 | .697420 | 5.27 | .302580 | 31 |
| 30 | .649527 | 4.23 | .951791 | 1.05 | .697736 | 5.28 | .302264 | 30 |
| 31 | 9.649781 | 4.22 | 9.951728 | 1.05 | 9.698053 | 5.27 | 10.301947 | 29 |
| 32 | .650034 | 4.22 | .951665 | 1.05 | .698369 | 5.27 | .301631 | 28 |
| 33 | .650287 | 4.20 | .951602 | 1.05 | .698685 | 5.27 | .301315 | 27 |
| 34 | .650539 | 4.22 | .951539 | 1.05 | .699001 | 5.25 | .300999 | 26 |
| 35 | .650792 | 4.20 | .951476 | 1.07 | .699316 | 5.27 | .300684 | 25 |
| 36 | .651044 | 4.22 | .951412 | 1.05 | .699632 | 5.25 | .300368 | 24 |
| 37 | .651297 | 4.20 | .951349 | 1.05 | .699947 | 5.27 | .300053 | 23 |
| 38 | .651549 | 4.18 | .951286 | 1.07 | .700263 | 5.25 | .299737 | 22 |
| 39 | .651800 | 4.20 | .951222 | 1.05 | .700578 | 5.25 | .299422 | 21 |
| 40 | .652052 | 4.20 | .951159 | 1.05 | .700893 | 5.25 | .299107 | 20 |
| 41 | 9.652304 | 4.18 | 9.951096 | 1.07 | 9.701208 | 5.25 | 10.298792 | 19 |
| 42 | .652555 | 4.18 | .951032 | 1.07 | .701523 | 5.23 | .298477 | 18 |
| 43 | .652806 | 4.18 | .950968 | 1.05 | .701837 | 5.25 | .298163 | 17 |
| 44 | .653057 | 4.18 | .950905 | 1.07 | .702152 | 5.23 | .297848 | 16 |
| 45 | .653308 | 4.17 | .950841 | 1.05 | .702466 | 5.25 | .297534 | 15 |
| 46 | .653558 | 4.17 | .950778 | 1.07 | .702781 | 5.23 | .297219 | 14 |
| 47 | .653808 | 4.18 | .950714 | 1.07 | .703095 | 5.23 | .296905 | 13 |
| 48 | .654059 | 4.17 | .950650 | 1.07 | .703409 | 5.23 | .296591 | 12 |
| 49 | .654309 | 4.15 | .950586 | 1.07 | .703723 | 5.23 | .296278 | 11 |
| 50 | .654558 | 4.17 | .950522 | 1.07 | .704036 | 5.23 | .295964 | 10 |
| 51 | 9.654808 | 4.17 | 9.950458 | 1.07 | 9.704350 | 5.22 | 10.295650 | 9 |
| 52 | .655058 | 4.15 | .950394 | 1.07 | .704663 | 5.23 | .295337 | 8 |
| 53 | .655307 | 4.15 | .950330 | 1.07 | .704976 | 5.23 | .295024 | 7 |
| 54 | .655556 | 4.15 | .950266 | 1.07 | .705290 | 5.22 | .294710 | 6 |
| 55 | .655805 | 4.15 | .950202 | 1.07 | .705603 | 5.22 | .294397 | 5 |
| 56 | .656054 | 4.13 | .950138 | 1.07 | .705916 | 5.20 | .294084 | 4 |
| 57 | .656302 | 4.15 | .950074 | 1.07 | .706228 | 5.22 | .293772 | 3 |
| 58 | .656551 | 4.13 | .950010 | 1.08 | .706541 | 5.22 | .293459 | 2 |
| 59 | .656799 | 4.13 | .949945 | 1.07 | .706854 | 5.20 | .293146 | 1 |
| 60 | 9.657047 | | 9.949881 | | 9.707166 | | 10.292834 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.657047 | | 9.949881 | | 9.707166 | | 10.292834 | 60 |
| 1 | .657295 | 4.13 | .949816 | 1.08 | .707478 | 5.20 | .292522 | 59 |
| 2 | .657542 | 4.12 | .949752 | 1.07 | .707790 | 5.20 | .292210 | 58 |
| 3 | .657790 | 4.13 | .949688 | 1.07 | .708102 | 5.20 | .291898 | 57 |
| 4 | .658037 | 4.12 | .949623 | 1.08 | .708414 | 5.20 | .291586 | 56 |
| 5 | .658284 | 4.12 | .949558 | 1.08 | .708726 | 5.20 | .291274 | 55 |
| 6 | .658531 | 4.12 | .949494 | 1.07 | .709037 | 5.18 | .290963 | 54 |
| 7 | .658778 | 4.12 | .949429 | 1.08 | .709349 | 5.20 | .290651 | 53 |
| 8 | .659025 | 4.12 | .949364 | 1.08 | .709660 | 5.18 | .290340 | 52 |
| 9 | .659271 | 4.10 | .949300 | 1.07 | .709971 | 5.18 | .290029 | 51 |
| 10 | .659517 | 4.10 | .949235 | 1.08 | .710282 | 5.18 | .289718 | 50 |
| 11 | 9.659763 | 4.10 | 9.949170 | 1.08 | 9.710593 | 5.18 | 10.289407 | 49 |
| 12 | .660009 | 4.10 | .949105 | 1.08 | .710904 | 5.18 | .289096 | 48 |
| 13 | .660255 | 4.10 | .949040 | 1.08 | .711215 | 5.17 | .288785 | 47 |
| 14 | .660501 | 4.08 | .948975 | 1.08 | .711525 | 5.18 | .288475 | 46 |
| 15 | .660746 | 4.08 | .948910 | 1.08 | .711836 | 5.17 | .288164 | 45 |
| 16 | .660991 | 4.08 | .948845 | 1.08 | .712146 | 5.17 | .287854 | 44 |
| 17 | .661236 | 4.08 | .948780 | 1.08 | .712456 | 5.17 | .287544 | 43 |
| 18 | .661481 | 4.08 | .948715 | 1.08 | .712766 | 5.17 | .287234 | 42 |
| 19 | .661726 | 4.08 | .948650 | 1.08 | .713076 | 5.17 | .286924 | 41 |
| 20 | .661970 | 4.07 | .948584 | 1.10 | .713386 | 5.17 | .286614 | 40 |
| 21 | 9.662214 | 4.08 | 9.948519 | 1.08 | 9.713696 | 5.15 | 10.286304 | 39 |
| 22 | .662459 | 4.07 | .948454 | 1.10 | .714005 | 5.15 | .285995 | 38 |
| 23 | .662703 | 4.05 | .948388 | 1.08 | .714314 | 5.17 | .285686 | 37 |
| 24 | .662946 | 4.07 | .948323 | 1.10 | .714624 | 5.15 | .285376 | 36 |
| 25 | .663190 | 4.05 | .948257 | 1.08 | .714933 | 5.15 | .285067 | 35 |
| 26 | .663433 | 4.07 | .948192 | 1.10 | .715242 | 5.15 | .284758 | 34 |
| 27 | .663677 | 4.05 | .948126 | 1.10 | .715551 | 5.15 | .284449 | 33 |
| 28 | .663920 | 4.05 | .948060 | 1.10 | .715860 | 5.15 | .284140 | 32 |
| 29 | .664163 | 4.05 | .947995 | 1.08 | .716168 | 5.13 | .283832 | 31 |
| 30 | .664406 | 4.05 | .947929 | 1.10 | .716477 | 5.15 | .283523 | 30 |
| 31 | 9.664648 | 4.03 | 9.947863 | 1.10 | 9.716785 | 5.13 | 10.283215 | 29 |
| 32 | .664891 | 4.05 | .947797 | 1.10 | .717093 | 5.13 | .282907 | 28 |
| 33 | .665133 | 4.03 | .947731 | 1.10 | .717401 | 5.13 | .282599 | 27 |
| 34 | .665375 | 4.03 | .947665 | 1.08 | .717709 | 5.13 | .282291 | 26 |
| 35 | .665617 | 4.03 | .947600 | 1.12 | .718017 | 5.13 | .281983 | 25 |
| 36 | .665859 | 4.02 | .947533 | 1.10 | .718325 | 5.13 | .281675 | 24 |
| 37 | .666100 | 4.03 | .947467 | 1.10 | .718633 | 5.13 | .281367 | 23 |
| 38 | .666342 | 4.02 | .947401 | 1.10 | .718940 | 5.12 | .281060 | 22 |
| 39 | .666583 | 4.02 | .947335 | 1.10 | .719248 | 5.13 | .280752 | 21 |
| 40 | .666824 | 4.02 | .947269 | 1.10 | .719555 | 5.12 | .280445 | 20 |
| 41 | 9.667065 | 4.00 | 9.947203 | 1.12 | 9.719862 | 5.12 | 10.280138 | 19 |
| 42 | .667305 | 4.02 | .947136 | 1.10 | .720169 | 5.12 | .279831 | 18 |
| 43 | .667546 | 4.00 | .947070 | 1.10 | .720476 | 5.12 | .279524 | 17 |
| 44 | .667786 | 4.02 | .947004 | 1.10 | .720783 | 5.12 | .279217 | 16 |
| 45 | .668027 | 4.00 | .946937 | 1.12 | .721089 | 5.10 | .278911 | 15 |
| 46 | .668267 | 3.98 | .946871 | 1.12 | .721396 | 5.12 | .278604 | 14 |
| 47 | .668506 | 4.00 | .946804 | 1.10 | .721702 | 5.10 | .278298 | 13 |
| 48 | .668746 | 4.00 | .946738 | 1.12 | .722009 | 5.12 | .277991 | 12 |
| 49 | .668986 | 4.00 | .946671 | 1.12 | .722315 | 5.10 | .277685 | 11 |
| 50 | .669225 | 3.98 | .946604 | 1.12 | .722621 | 5.10 | .277379 | 10 |
| 51 | 9.669464 | 3.98 | 9.946538 | 1.12 | 9.722927 | 5.08 | 10.277073 | 9 |
| 52 | .669703 | 3.98 | .946471 | 1.12 | .723232 | 5.10 | .276768 | 8 |
| 53 | .669942 | 3.98 | .946404 | 1.12 | .723538 | 5.10 | .276462 | 7 |
| 54 | .670181 | 3.98 | .946337 | 1.12 | .723844 | 5.10 | .276156 | 6 |
| 55 | .670419 | 3.97 | .946270 | 1.12 | .724149 | 5.08 | .275851 | 5 |
| 56 | .670658 | 3.98 | .946203 | 1.12 | .724454 | 5.10 | .275546 | 4 |
| 57 | .670896 | 3.97 | .946136 | 1.12 | .724760 | 5.08 | .275240 | 3 |
| 58 | .671134 | 3.97 | .946069 | 1.12 | .725065 | 5.08 | .274935 | 2 |
| 59 | .671372 | 3.97 | .946002 | 1.12 | .725370 | 5.08 | .274630 | 1 |
| 60 | 9.671609 | 3.95 | 9.945935 | 1.12 | 9.725674 | 5.07 | 10.274326 | 0 |
| | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.671609 | 3.97 | 9.945935 | 1.12 | 9.725674 | 5.08 | 10.274326 | 60 |
| 1 | .671847 | 3.95 | .945868 | 1.13 | .725979 | 5.08 | .274021 | 59 |
| 2 | .672084 | 3.95 | .945800 | 1.12 | .726284 | 5.07 | .273716 | 58 |
| 3 | .672321 | 3.95 | .945733 | 1.12 | .726588 | 5.07 | .273412 | 57 |
| 4 | .672558 | 3.95 | .945666 | 1.13 | .726892 | 5.05 | .273108 | 56 |
| 5 | .672795 | 3.95 | .945598 | 1.12 | .727197 | 5.07 | .272803 | 55 |
| 6 | .673032 | 3.93 | .945531 | 1.12 | .727501 | 5.07 | .272499 | 54 |
| 7 | .673268 | 3.95 | .945464 | 1.13 | .727805 | 5.07 | .272195 | 53 |
| 8 | .673505 | 3.93 | .945396 | 1.13 | .728109 | 5.05 | .271891 | 52 |
| 9 | .673741 | 3.93 | .945328 | 1.12 | .728412 | 5.07 | .271588 | 51 |
| 10 | .673977 | 3.93 | .945261 | 1.13 | .728716 | 5.07 | .271284 | 50 |
| 11 | 9.674213 | 3.92 | 9.945193 | 1.13 | 9.729020 | 5.05 | 10.270980 | 49 |
| 12 | .674448 | 3.93 | .945125 | 1.12 | .729323 | 5.05 | .270677 | 48 |
| 13 | .674684 | 3.92 | .945058 | 1.13 | .729626 | 5.05 | .270374 | 47 |
| 14 | .674919 | 3.93 | .944990 | 1.13 | .729929 | 5.07 | .270071 | 46 |
| 15 | .675155 | 3.92 | .944922 | 1.13 | .730233 | 5.03 | .269767 | 45 |
| 16 | .675390 | 3.90 | .944854 | 1.13 | .730535 | 5.05 | .269465 | 44 |
| 17 | .675624 | 3.92 | .944786 | 1.13 | .730838 | 5.05 | .269162 | 43 |
| 18 | .675859 | 3.92 | .944718 | 1.13 | .731141 | 5.05 | .268859 | 42 |
| 19 | .676094 | 3.90 | .944650 | 1.13 | .731444 | 5.03 | .268556 | 41 |
| 20 | .676328 | 3.90 | .944582 | 1.13 | .731746 | 5.03 | .268254 | 40 |
| 21 | 9.676562 | 3.90 | 9.944514 | 1.13 | 9.732048 | 5.05 | 10.267952 | 39 |
| 22 | .676796 | 3.90 | .944446 | 1.15 | .732351 | 5.03 | .267649 | 38 |
| 23 | .677030 | 3.90 | .944377 | 1.13 | .732653 | 5.03 | .267347 | 37 |
| 24 | .677264 | 3.90 | .944309 | 1.13 | .732955 | 5.03 | .267045 | 36 |
| 25 | .677498 | 3.88 | .944241 | 1.15 | .733257 | 5.02 | .266743 | 35 |
| 26 | .677731 | 3.88 | .944172 | 1.13 | .733558 | 5.03 | .266442 | 34 |
| 27 | .677964 | 3.88 | .944104 | 1.13 | .733860 | 5.03 | .266140 | 33 |
| 28 | .678197 | 3.88 | .944036 | 1.15 | .734162 | 5.02 | .265838 | 32 |
| 29 | .678430 | 3.88 | .943967 | 1.13 | .734463 | 5.02 | .265537 | 31 |
| 30 | .678663 | 3.87 | .943899 | 1.15 | .734764 | 5.03 | .265236 | 30 |
| 31 | 9.678895 | 3.88 | 9.943830 | 1.15 | 9.735066 | 5.02 | 10.264934 | 29 |
| 32 | .679128 | 3.87 | .943761 | 1.13 | .735367 | 5.02 | .264633 | 28 |
| 33 | .679360 | 3.87 | .943693 | 1.15 | .735668 | 5.02 | .264332 | 27 |
| 34 | .679592 | 3.87 | .943624 | 1.15 | .735969 | 5.00 | .264031 | 26 |
| 35 | .679824 | 3.87 | .943555 | 1.15 | .736269 | 5.02 | .263731 | 25 |
| 36 | .680056 | 3.87 | .943486 | 1.15 | .736570 | 5.00 | .263430 | 24 |
| 37 | .680288 | 3.85 | .943417 | 1.15 | .736870 | 5.02 | .263130 | 23 |
| 38 | .680519 | 3.85 | .943348 | 1.15 | .737171 | 5.00 | .262829 | 22 |
| 39 | .680750 | 3.87 | .943279 | 1.15 | .737471 | 5.00 | .262529 | 21 |
| 40 | .680982 | 3.85 | .943210 | 1.15 | .737771 | 5.00 | .262229 | 20 |
| 41 | 9.681213 | 3.83 | 9.943141 | 1.15 | 9.738071 | 5.00 | 10.261929 | 19 |
| 42 | .681443 | 3.85 | .943072 | 1.15 | .738371 | 5.00 | .261629 | 18 |
| 43 | .681674 | 3.85 | .943003 | 1.15 | .738671 | 5.00 | .261329 | 17 |
| 44 | .681905 | 3.85 | .942934 | 1.17 | .738971 | 5.00 | .261029 | 16 |
| 45 | .682135 | 3.83 | .942864 | 1.15 | .739271 | 4.98 | .260729 | 15 |
| 46 | .682365 | 3.83 | .942795 | 1.15 | .739570 | 5.00 | .260430 | 14 |
| 47 | .682595 | 3.83 | .942726 | 1.17 | .739870 | 4.98 | .260130 | 13 |
| 48 | .682825 | 3.83 | .942656 | 1.15 | .740169 | 4.98 | .259831 | 12 |
| 49 | .683055 | 3.82 | .942587 | 1.17 | .740468 | 4.98 | .259532 | 11 |
| 50 | .683284 | 3.83 | .942517 | 1.15 | .740767 | 4.98 | .259233 | 10 |
| 51 | 9.683514 | 3.82 | 9.942448 | 1.17 | 9.741066 | 4.98 | 10.258934 | 9 |
| 52 | .683743 | 3.82 | .942378 | 1.17 | .741365 | 4.98 | .258635 | 8 |
| 53 | .683972 | 3.82 | .942308 | 1.15 | .741664 | 4.97 | .258336 | 7 |
| 54 | .684201 | 3.82 | .942239 | 1.17 | .741963 | 4.98 | .258038 | 6 |
| 55 | .684430 | 3.80 | .942169 | 1.17 | .742261 | 4.97 | .257739 | 5 |
| 56 | .684658 | 3.82 | .942099 | 1.17 | .742559 | 4.98 | .257441 | 4 |
| 57 | .684887 | 3.80 | .942029 | 1.17 | .742858 | 4.97 | .257142 | 3 |
| 58 | .685115 | 3.80 | .941959 | 1.17 | .743156 | 4.97 | .256844 | 2 |
| 59 | .685343 | 3.80 | .941889 | 1.17 | .743454 | 4.97 | .256546 | 1 |
| 60 | 9.685571 | | 9.941819 | | 9.743752 | | 10.256248 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.685571 | 3.80 | 9.941819 | 1.17 | 9.743752 | 4.97 | 10.256248 | 60 |
| 1 | .685799 | 3.80 | .941749 | 1.17 | .744050 | 4.97 | .255950 | 59 |
| 2 | .686027 | 3.78 | .941679 | 1.17 | .744348 | 4.95 | .255652 | 58 |
| 3 | .686254 | 3.80 | .941609 | 1.17 | .744645 | 4.97 | .255355 | 57 |
| 4 | .686482 | 3.78 | .941539 | 1.17 | .744943 | 4.95 | .255057 | 56 |
| 5 | .686709 | 3.78 | .941469 | 1.18 | .745240 | 4.97 | .254760 | 55 |
| 6 | .686936 | 3.78 | .941398 | 1.17 | .745538 | 4.95 | .254462 | 54 |
| 7 | .687163 | 3.78 | .941328 | 1.17 | .745835 | 4.95 | .254165 | 53 |
| 8 | .687389 | 3.77 | .941258 | 1.18 | .746132 | 4.95 | .253868 | 52 |
| 9 | .687616 | 3.78 | .941187 | 1.17 | .746429 | 4.95 | .253571 | 51 |
| 10 | .687843 | 3.77 | .941117 | 1.18 | .746726 | 4.95 | .253274 | 50 |
| 11 | 9.688069 | 3.77 | 9.941046 | 1.18 | 9.747023 | 4.93 | 10.252977 | 49 |
| 12 | .688295 | 3.77 | .940975 | 1.17 | .747319 | 4.95 | .252681 | 48 |
| 13 | .688521 | 3.77 | .940905 | 1.18 | .747616 | 4.95 | .252384 | 47 |
| 14 | .688747 | 3.75 | .940834 | 1.18 | .747913 | 4.93 | .252087 | 46 |
| 15 | .688972 | 3.77 | .940763 | 1.17 | .748209 | 4.93 | .251791 | 45 |
| 16 | .689198 | 3.75 | .940693 | 1.18 | .748505 | 4.93 | .251495 | 44 |
| 17 | .689423 | 3.75 | .940622 | 1.18 | .748801 | 4.93 | .251199 | 43 |
| 18 | .689648 | 3.75 | .940551 | 1.18 | .749097 | 4.93 | .250903 | 42 |
| 19 | .689873 | 3.75 | .940480 | 1.18 | .749393 | 4.93 | .250607 | 41 |
| 20 | .690098 | 3.75 | .940409 | 1.18 | .749689 | 4.93 | .250311 | 40 |
| 21 | 9.690323 | 3.75 | 9.940338 | 1.18 | 9.749985 | 4.93 | 10.250015 | 39 |
| 22 | .690548 | 3.73 | .940267 | 1.18 | .750281 | 4.93 | .249719 | 38 |
| 23 | .690772 | 3.73 | .940196 | 1.18 | .750576 | 4.92 | .249424 | 37 |
| 24 | .690996 | 3.73 | .940125 | 1.18 | .750872 | 4.93 | .249128 | 36 |
| 25 | .691220 | 3.73 | .940054 | 1.20 | .751167 | 4.92 | .248833 | 35 |
| 26 | .691444 | 3.73 | .939982 | 1.18 | .751462 | 4.92 | .248538 | 34 |
| 27 | .691668 | 3.73 | .939911 | 1.18 | .751757 | 4.92 | .248243 | 33 |
| 28 | .691892 | 3.73 | .939840 | 1.20 | .752052 | 4.92 | .247948 | 32 |
| 29 | .692115 | 3.73 | .939768 | 1.18 | .752347 | 4.92 | .247653 | 31 |
| 30 | .692339 | 3.72 | .939697 | 1.20 | .752642 | 4.92 | .247358 | 30 |
| 31 | 9.692562 | 3.72 | 9.939625 | 1.18 | 9.752937 | 4.90 | 10.247063 | 29 |
| 32 | .692785 | 3.72 | .939554 | 1.20 | .753231 | 4.92 | .246769 | 28 |
| 33 | .693008 | 3.72 | .939482 | 1.20 | .753526 | 4.90 | .246474 | 27 |
| 34 | .693231 | 3.70 | .939410 | 1.18 | .753820 | 4.92 | .246180 | 26 |
| 35 | .693453 | 3.72 | .939339 | 1.20 | .754115 | 4.90 | .245885 | 25 |
| 36 | .693676 | 3.70 | .939267 | 1.20 | .754409 | 4.90 | .245591 | 24 |
| 37 | .693898 | 3.70 | .939195 | 1.20 | .754703 | 4.90 | .245297 | 23 |
| 38 | .694120 | 3.70 | .939123 | 1.18 | .754997 | 4.90 | .245003 | 22 |
| 39 | .694342 | 3.70 | .939052 | 1.20 | .755291 | 4.90 | .244709 | 21 |
| 40 | .694564 | 3.70 | .938980 | 1.20 | .755585 | 4.88 | .244415 | 20 |
| 41 | 9.694786 | 3.68 | 9.938908 | 1.20 | 9.755878 | 4.90 | 10.244122 | 19 |
| 42 | .695007 | 3.70 | .938836 | 1.22 | .756172 | 4.88 | .243828 | 18 |
| 43 | .695229 | 3.68 | .938763 | 1.20 | .756465 | 4.90 | .243535 | 17 |
| 44 | .695450 | 3.68 | .938691 | 1.20 | .756759 | 4.88 | .243241 | 16 |
| 45 | .695671 | 3.68 | .938619 | 1.20 | .757052 | 4.88 | .242948 | 15 |
| 46 | .695892 | 3.68 | .938547 | 1.20 | .757345 | 4.88 | .242655 | 14 |
| 47 | .696113 | 3.68 | .938475 | 1.20 | .757638 | 4.88 | .242362 | 13 |
| 48 | .696334 | 3.68 | .938402 | 1.22 | .757931 | 4.88 | .242069 | 12 |
| 49 | .696554 | 3.67 | .938330 | 1.20 | .758224 | 4.88 | .241776 | 11 |
| 50 | .696775 | 3.68 | .938258 | 1.20 | .758517 | 4.88 | .241483 | 10 |
| 51 | 9.696995 | 3.67 | 9.938185 | 1.22 | 9.758810 | 4.87 | 10.241190 | 9 |
| 52 | .697215 | 3.67 | .938113 | 1.20 | .759102 | 4.88 | .240898 | 8 |
| 53 | .697435 | 3.67 | .938040 | 1.22 | .759395 | 4.87 | .240605 | 7 |
| 54 | .697654 | 3.65 | .937967 | 1.22 | .759687 | 4.87 | .240313 | 6 |
| 55 | .697874 | 3.67 | .937895 | 1.20 | .759979 | 4.88 | .240021 | 5 |
| 56 | .698094 | 3.67 | .937822 | 1.22 | .760272 | 4.87 | .239728 | 4 |
| 57 | .698313 | 3.65 | .937749 | 1.22 | .760564 | 4.87 | .239436 | 3 |
| 58 | .698532 | 3.65 | .937676 | 1.20 | .760856 | 4.87 | .239144 | 2 |
| 59 | .698751 | 3.65 | .937604 | 1.22 | .761148 | 4.85 | .238852 | 1 |
| 60 | 9.698970 | 3.65 | 9.937531 | 1.22 | 9.761439 | 4.87 | 10.238561 | 0 |
| ′ | Coseno | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.698970 | 3.65 | 9.937531 | 1.22 | 9.761439 | 4.87 | 10.238561 | 60 |
| 1 | .699189 | 3.63 | .937458 | 1.22 | .761731 | 4.87 | .238269 | 59 |
| 2 | .699407 | 3.63 | .937385 | 1.22 | .762023 | 4.85 | .237977 | 58 |
| 3 | .699626 | 3.63 | .937312 | 1.23 | .762314 | 4.87 | .237686 | 57 |
| 4 | .699844 | 3.63 | .937238 | 1.22 | .762606 | 4.85 | .237394 | 56 |
| 5 | .700062 | 3.63 | .937165 | 1.22 | .762897 | 4.85 | .237103 | 55 |
| 6 | .700280 | 3.63 | .937092 | 1.22 | .763188 | 4.85 | .236813 | 54 |
| 7 | .700498 | 3.63 | .937019 | 1.22 | .763479 | 4.85 | .236521 | 53 |
| 8 | .700716 | 3.62 | .936946 | 1.23 | .763770 | 4.85 | .236230 | 52 |
| 9 | .700933 | 3.63 | .936872 | 1.22 | .764061 | 4.85 | .235939 | 51 |
| 10 | .701151 | 3.62 | .936799 | 1.23 | .764352 | 4.85 | .235648 | 50 |
| 11 | 9.701368 | 3.62 | 9.936725 | 1.22 | 9.764643 | 4.83 | 10.235357 | 49 |
| 12 | .701585 | 3.62 | .936652 | 1.23 | .764933 | 4.85 | .235067 | 48 |
| 13 | .701802 | 3.62 | .936578 | 1.22 | .765224 | 4.83 | .234776 | 47 |
| 14 | .702019 | 3.62 | .936505 | 1.23 | .765514 | 4.85 | .234486 | 46 |
| 15 | .702236 | 3.60 | .936431 | 1.23 | .765805 | 4.83 | .234195 | 45 |
| 16 | .702452 | 3.62 | .936357 | 1.22 | .766095 | 4.83 | .233905 | 44 |
| 17 | .702669 | 3.62 | .936284 | 1.23 | .766385 | 4.83 | .233615 | 43 |
| 18 | .702885 | 3.60 | .936210 | 1.23 | .766675 | 4.83 | .233325 | 42 |
| 19 | .703101 | 3.60 | .936136 | 1.23 | .766965 | 4.83 | .233035 | 41 |
| 20 | .703317 | 3.60 | .936062 | 1.23 | .767255 | 4.83 | .232745 | 40 |
| 21 | 9.703533 | 3.60 | 9.935988 | 1.23 | 9.767545 | 4.82 | 10.232455 | 39 |
| 22 | .703749 | 3.58 | .935914 | 1.23 | .767834 | 4.83 | .232166 | 38 |
| 23 | .703964 | 3.58 | .935840 | 1.23 | .768124 | 4.83 | .231876 | 37 |
| 24 | .704179 | 3.60 | .935766 | 1.23 | .768414 | 4.82 | .231586 | 36 |
| 25 | .704395 | 3.58 | .935692 | 1.23 | .768703 | 4.82 | .231297 | 35 |
| 26 | .704610 | 3.58 | .935618 | 1.25 | .768992 | 4.82 | .231008 | 34 |
| 27 | .704825 | 3.58 | .935543 | 1.23 | .769281 | 4.82 | .230719 | 33 |
| 28 | .705040 | 3.57 | .935469 | 1.23 | .769571 | 4.82 | .230429 | 32 |
| 29 | .705254 | 3.58 | .935395 | 1.25 | .769860 | 4.80 | .230140 | 31 |
| 30 | .705469 | 3.57 | .935320 | 1.23 | .770148 | 4.82 | .229852 | 30 |
| 31 | 9.705683 | 3.58 | 9.935246 | 1.25 | 9.770437 | 4.82 | 10.229563 | 29 |
| 32 | .705898 | 3.57 | .935171 | 1.23 | .770726 | 4.82 | .229274 | 28 |
| 33 | .706112 | 3.57 | .935097 | 1.25 | .771015 | 4.80 | .228985 | 27 |
| 34 | .706326 | 3.55 | .935022 | 1.23 | .771303 | 4.82 | .228697 | 26 |
| 35 | .706539 | 3.57 | .934948 | 1.25 | .771592 | 4.82 | .228408 | 25 |
| 36 | .706753 | 3.57 | .934873 | 1.25 | .771880 | 4.80 | .228120 | 24 |
| 37 | .706967 | 3.57 | .934798 | 1.25 | .772168 | 4.80 | .227832 | 23 |
| 38 | .707180 | 3.55 | .934723 | 1.23 | .772457 | 4.82 | .227543 | 22 |
| 39 | .707393 | 3.55 | .934649 | 1.25 | .772745 | 4.80 | .227255 | 21 |
| 40 | .707606 | 3.55 | .934574 | 1.25 | .773033 | 4.80 | .226967 | 20 |
| 41 | 9.707819 | 3.55 | 9.934499 | 1.25 | 9.773321 | 4.78 | 10.226679 | 19 |
| 42 | .708032 | 3.55 | .934424 | 1.25 | .773608 | 4.80 | .226392 | 18 |
| 43 | .708245 | 3.55 | .934349 | 1.25 | .773896 | 4.80 | .226104 | 17 |
| 44 | .708458 | 3.53 | .934274 | 1.25 | .774184 | 4.78 | .225816 | 16 |
| 45 | .708670 | 3.53 | .934199 | 1.27 | .774471 | 4.80 | .225529 | 15 |
| 46 | .708882 | 3.53 | .934123 | 1.25 | .774759 | 4.78 | .225241 | 14 |
| 47 | .709094 | 3.53 | .934048 | 1.25 | .775046 | 4.78 | .224954 | 13 |
| 48 | .709306 | 3.53 | .933973 | 1.25 | .775333 | 4.80 | .224667 | 12 |
| 49 | .709518 | 3.53 | .933898 | 1.27 | .775621 | 4.78 | .224379 | 11 |
| 50 | .709730 | 3.52 | .933822 | 1.25 | .775908 | 4.78 | .224092 | 10 |
| 51 | 9.709941 | 3.53 | 9.933747 | 1.27 | 9.776195 | 4.78 | 10.223805 | 9 |
| 52 | .710153 | 3.52 | .933671 | 1.25 | .776482 | 4.77 | .223518 | 8 |
| 53 | .710364 | 3.52 | .933596 | 1.27 | .776768 | 4.78 | .223232 | 7 |
| 54 | .710575 | 3.52 | .933520 | 1.25 | .777055 | 4.78 | .222945 | 6 |
| 55 | .710786 | 3.52 | .933445 | 1.27 | .777342 | 4.77 | .222658 | 5 |
| 56 | .710997 | 3.52 | .933369 | 1.27 | .777628 | 4.78 | .222372 | 4 |
| 57 | .711208 | 3.52 | .933293 | 1.27 | .777915 | 4.77 | .222085 | 3 |
| 58 | .711419 | 3.50 | .933217 | 1.27 | .778201 | 4.78 | .221799 | 2 |
| 59 | .711629 | 3.50 | .933141 | 1.25 | .778488 | 4.77 | .221512 | 1 |
| 60 | 9.711839 | | 9.933066 | | 9.778774 | | 10.221226 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.711 839 | | 9.933066 | | 9.778774 | | 10.221226 | 60 |
| 1 | .711950 | 3.52 | .932990 | 1.27 | .779060 | 4.77 | .220940 | 59 |
| 2 | .712060 | 3.50 | .932914 | 1.27 | .779346 | 4.77 | .220654 | 58 |
| 3 | .712169 | 3.48 | .932838 | 1.27 | .779632 | 4.77 | .220368 | 57 |
| 4 | .712279 | 3.50 | .932762 | 1.28 | .779918 | 4.77 | .220082 | 56 |
| 5 | .712389 | 3.48 | .932685 | 1.27 | .780203 | 4.75 | .219797 | 55 |
| 6 | .712498 | 3.50 | .932609 | 1.27 | .780489 | 4.77 | .219511 | 54 |
| 7 | .712608 | 3.48 | .932533 | 1.27 | .780775 | 4.77 | .219225 | 53 |
| 8 | .712717 | 3.48 | .932457 | 1.28 | .781060 | 4.75 | .218940 | 52 |
| 9 | .712726 | 3.48 | .932380 | 1.27 | .781346 | 4.77 | .218654 | 51 |
| 10 | .712935 | 3.48 | .932304 | 1.27 | .781631 | 4.75 | .218369 | 50 |
| 11 | 9.714144 | 3.47 | 9.932228 | 1.28 | 9.781916 | 4.75 | 10.218084 | 49 |
| 12 | .714352 | 3.48 | .932151 | 1.27 | .782201 | 4.75 | .217799 | 48 |
| 13 | .714561 | 3.47 | .932075 | 1.28 | .782486 | 4.75 | .217514 | 47 |
| 14 | .714769 | 3.48 | .931998 | 1.28 | .782771 | 4.75 | .217229 | 46 |
| 15 | .714978 | 3.47 | .931921 | 1.27 | .783056 | 4.75 | .216944 | 45 |
| 16 | .715186 | 3.47 | .931845 | 1.28 | .783341 | 4.75 | .216659 | 44 |
| 17 | .715394 | 3.47 | .931768 | 1.28 | .783626 | 4.73 | .216374 | 43 |
| 18 | .715602 | 3.47 | .931691 | 1.28 | .783910 | 4.75 | .216090 | 42 |
| 19 | .715809 | 3.45 | .931614 | 1.28 | .784195 | 4.73 | .215805 | 41 |
| 20 | .716017 | 3.47 | .931537 | 1.28 | .784479 | 4.75 | .215521 | 40 |
| 21 | 9.716224 | 3.47 | 9.931460 | 1.28 | 9.784764 | 4.73 | 10.215236 | 39 |
| 22 | .716432 | 3.45 | .931383 | 1.28 | .785048 | 4.73 | .214952 | 38 |
| 23 | .716639 | 3.45 | .931306 | 1.28 | .785332 | 4.73 | .214668 | 37 |
| 24 | .716846 | 3.45 | .931229 | 1.28 | .785616 | 4.73 | .214384 | 36 |
| 25 | .717053 | 3.43 | .931152 | 1.28 | .785900 | 4.73 | .214100 | 35 |
| 26 | .717259 | 3.45 | .931075 | 1.28 | .786184 | 4.73 | .213816 | 34 |
| 27 | .717466 | 3.45 | .930998 | 1.28 | .786468 | 4.73 | .213532 | 33 |
| 28 | .717673 | 3.45 | .930921 | 1.30 | .786752 | 4.73 | .213248 | 32 |
| 29 | .717879 | 3.43 | .930843 | 1.28 | .787036 | 4.73 | .212964 | 31 |
| 30 | .718085 | 3.43 | .930766 | 1.30 | .787319 | 4.73 | .212681 | 30 |
| 31 | 9.718291 | 3.43 | 9.930688 | 1.28 | 9.787603 | 4.72 | 10.212397 | 29 |
| 32 | .718497 | 3.43 | .930611 | 1.30 | .787886 | 4.73 | .212114 | 28 |
| 33 | .718703 | 3.43 | .930533 | 1.28 | .788170 | 4.72 | .211830 | 27 |
| 34 | .718909 | 3.42 | .930456 | 1.30 | .788453 | 4.72 | .211547 | 26 |
| 35 | .719114 | 3.43 | .930378 | 1.30 | .788736 | 4.72 | .211264 | 25 |
| 36 | .719320 | 3.42 | .930300 | 1.28 | .789019 | 4.72 | .210981 | 24 |
| 37 | .719525 | 3.42 | .930223 | 1.30 | .789302 | 4.72 | .210698 | 23 |
| 38 | .719730 | 3.42 | .930145 | 1.30 | .789585 | 4.72 | .210415 | 22 |
| 39 | .719935 | 3.42 | .930067 | 1.30 | .789868 | 4.72 | .210132 | 21 |
| 40 | .720140 | 3.42 | .929989 | 1.30 | .790151 | 4.72 | .209849 | 20 |
| 41 | 9.720345 | 3.40 | 9.929911 | 1.30 | 9.790434 | 4.70 | 10.209566 | 19 |
| 42 | .720549 | 3.42 | .929833 | 1.30 | .790716 | 4.72 | .209284 | 18 |
| 43 | .720754 | 3.40 | .929755 | 1.30 | .790999 | 4.70 | .209001 | 17 |
| 44 | .720958 | 3.40 | .929677 | 1.30 | .791281 | 4.70 | .208719 | 16 |
| 45 | .721162 | 3.40 | .929599 | 1.30 | .791563 | 4.72 | .208437 | 15 |
| 46 | .721366 | 3.40 | .929521 | 1.32 | .791846 | 4.70 | .208154 | 14 |
| 47 | .721570 | 3.40 | .929442 | 1.30 | .792128 | 4.70 | .207872 | 13 |
| 48 | .721774 | 3.40 | .929364 | 1.30 | .792410 | 4.70 | .207590 | 12 |
| 49 | .721978 | 3.38 | .929286 | 1.32 | .792692 | 4.70 | .207308 | 11 |
| 50 | .722181 | 3.40 | .929207 | 1.30 | .792974 | 4.70 | .207026 | 10 |
| 51 | 9.722385 | 3.38 | 9.929129 | 1.32 | 9.793256 | 4.70 | 10.206744 | 9 |
| 52 | .722588 | 3.38 | .929050 | 1.30 | .793538 | 4.68 | .206462 | 8 |
| 53 | .722791 | 3.38 | .928972 | 1.32 | .793819 | 4.70 | .206181 | 7 |
| 54 | .722994 | 3.38 | .928893 | 1.30 | .794101 | 4.70 | .205899 | 6 |
| 55 | .723197 | 3.38 | .928815 | 1.32 | .794383 | 4.68 | .205617 | 5 |
| 56 | .723400 | 3.38 | .928736 | 1.32 | .794664 | 4.70 | .205336 | 4 |
| 57 | .723603 | 3.37 | .928657 | 1.32 | .794946 | 4.68 | .205054 | 3 |
| 58 | .723805 | 3.37 | .928578 | 1.32 | .795227 | 4.68 | .204773 | 2 |
| 59 | .724007 | 3.38 | .928499 | 1.32 | .795508 | 4.68 | .204492 | 1 |
| 60 | 9.724210 | | 9.928420 | | 9.795789 | | 10.204211 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.724210 | 3.37 | 9.928420 | 1.30 | 9.795789 | 4.68 | 10.204211 | 60 |
| 1 | .724412 | 3.37 | .928342 | 1.32 | .796070 | 4.68 | .203930 | 59 |
| 2 | .724614 | 3.37 | .928263 | 1.33 | .796351 | 4.68 | .203649 | 58 |
| 3 | .724816 | 3.35 | .928183 | 1.32 | .796632 | 4.68 | .203368 | 57 |
| 4 | .725017 | 3.37 | .928104 | 1.32 | .796913 | 4.68 | .203087 | 56 |
| 5 | .725219 | 3.37 | .928025 | 1.32 | .797194 | 4.67 | .202806 | 55 |
| 6 | .725420 | 3.35 | .927946 | 1.32 | .797474 | 4.68 | .202526 | 54 |
| 7 | .725622 | 3.37 | .927867 | 1.32 | .797755 | 4.68 | .202245 | 53 |
| 8 | .725823 | 3.35 | .927787 | 1.33 | .798036 | 4.67 | .201964 | 52 |
| 9 | .726024 | 3.35 | .927708 | 1.32 | .798316 | 4.67 | .201684 | 51 |
| 10 | .726225 | 3.35 | .927629 | 1.33 | .798596 | 4.68 | .201404 | 50 |
| 11 | 9.726426 | 3.33 | 9.927549 | 1.32 | 9.798877 | 4.67 | 10.201123 | 49 |
| 12 | .726626 | 3.35 | .927470 | 1.33 | .799157 | 4.67 | .200843 | 48 |
| 13 | .726827 | 3.33 | .927390 | 1.33 | .799437 | 4.67 | .200563 | 47 |
| 14 | .727027 | 3.35 | .927310 | 1.33 | .799717 | 4.67 | .200283 | 46 |
| 15 | .727228 | 3.33 | .927231 | 1.33 | .799997 | 4.67 | .200003 | 45 |
| 16 | .727428 | 3.33 | .927151 | 1.33 | .800277 | 4.67 | .199723 | 44 |
| 17 | .727628 | 3.33 | .927071 | 1.33 | .800557 | 4.65 | .199443 | 43 |
| 18 | .727828 | 3.32 | .926991 | 1.33 | .800836 | 4.67 | .199164 | 42 |
| 19 | .728027 | 3.33 | .926911 | 1.33 | .801116 | 4.67 | .198884 | 41 |
| 20 | .728227 | 3.33 | .926831 | 1.33 | .801396 | 4.65 | .198604 | 40 |
| 21 | 9.728427 | 3.32 | 9.926751 | 1.33 | 9.801675 | 4.67 | 10.198325 | 39 |
| 22 | .728626 | 3.32 | .926671 | 1.33 | .801955 | 4.65 | .198045 | 38 |
| 23 | .728825 | 3.32 | .926591 | 1.33 | .802234 | 4.65 | .197766 | 37 |
| 24 | .729024 | 3.32 | .926511 | 1.33 | .802513 | 4.65 | .197487 | 36 |
| 25 | .729223 | 3.32 | .926431 | 1.33 | .802792 | 4.67 | .197208 | 35 |
| 26 | .729422 | 3.32 | .926351 | 1.35 | .803072 | 4.65 | .196928 | 34 |
| 27 | .729621 | 3.32 | .926270 | 1.33 | .803351 | 4.65 | .196649 | 33 |
| 28 | .729820 | 3.32 | .926190 | 1.33 | .803630 | 4.65 | .196370 | 32 |
| 29 | .730018 | 3.30 | .926110 | 1.33 | .803909 | 4.63 | .196091 | 31 |
| 30 | .730217 | 3.30 | .926029 | 1.33 | .804187 | 4.65 | .195813 | 30 |
| 31 | 9.730415 | 3.30 | 9.925949 | 1.35 | 9.804466 | 4.65 | 10.195534 | 29 |
| 32 | .730613 | 3.30 | .925868 | 1.33 | .804745 | 4.63 | .195255 | 28 |
| 33 | .730811 | 3.30 | .925788 | 1.35 | .805023 | 4.65 | .194977 | 27 |
| 34 | .731009 | 3.28 | .925707 | 1.35 | .805302 | 4.63 | .194698 | 26 |
| 35 | .731206 | 3.30 | .925626 | 1.35 | .805580 | 4.65 | .194420 | 25 |
| 36 | .731404 | 3.30 | .925545 | 1.35 | .805859 | 4.63 | .194141 | 24 |
| 37 | .731602 | 3.30 | .925465 | 1.33 | .806137 | 4.63 | .193863 | 23 |
| 38 | .731799 | 3.28 | .925384 | 1.35 | .806415 | 4.63 | .193585 | 22 |
| 39 | .731996 | 3.28 | .925303 | 1.35 | .806693 | 4.63 | .193307 | 21 |
| 40 | .732193 | 3.28 | .925222 | 1.35 | .806971 | 4.63 | .193029 | 20 |
| 41 | 9.732390 | 3.28 | 9.925141 | 1.35 | 9.807249 | 4.63 | 10.192751 | 19 |
| 42 | .732587 | 3.28 | .925060 | 1.35 | .807527 | 4.63 | .192473 | 18 |
| 43 | .732784 | 3.27 | .924979 | 1.37 | .807805 | 4.63 | .192195 | 17 |
| 44 | .732980 | 3.28 | .924897 | 1.35 | .808083 | 4.63 | .191917 | 16 |
| 45 | .733177 | 3.27 | .924816 | 1.35 | .808361 | 4.62 | .191639 | 15 |
| 46 | .733373 | 3.27 | .924735 | 1.35 | .808638 | 4.63 | .191362 | 14 |
| 47 | .733569 | 3.27 | .924654 | 1.35 | .808916 | 4.63 | .191084 | 13 |
| 48 | .733765 | 3.27 | .924572 | 1.37 | .809193 | 4.62 | .190807 | 12 |
| 49 | .733961 | 3.27 | .924491 | 1.35 | .809471 | 4.63 | .190529 | 11 |
| 50 | .734157 | 3.27 | .924409 | 1.37 | .809748 | 4.62 | .190252 | 10 |
| 51 | 9.734353 | 3.27 | 9.924328 | 1.35 | 9.810025 | 4.62 | 10.189975 | 9 |
| 52 | .734549 | 3.25 | .924246 | 1.37 | .810302 | 4.63 | .189698 | 8 |
| 53 | .734744 | 3.25 | .924164 | 1.35 | .810580 | 4.62 | .189420 | 7 |
| 54 | .734939 | 3.27 | .924083 | 1.37 | .810857 | 4.62 | .189143 | 6 |
| 55 | .735135 | 3.25 | .924001 | 1.37 | .811134 | 4.60 | .188866 | 5 |
| 56 | .735330 | 3.25 | .923919 | 1.37 | .811410 | 4.62 | .188590 | 4 |
| 57 | .735525 | 3.23 | .923837 | 1.37 | .811687 | 4.62 | .188313 | 3 |
| 58 | .735719 | 3.25 | .923755 | 1.37 | .811964 | 4.62 | .188036 | 2 |
| 59 | .735914 | 3.25 | .923673 | 1.37 | .812241 | 4.60 | .187759 | 1 |
| 60 | 9.736109 | | 9.923591 | | 9.812517 | | 10.187483 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.736109 | | 9.923591 | | 9.812517 | | 10.187483 | 60 |
| 1 | .736203 | 3.23 | .923509 | 1.37 | .812794 | 4.62 | .187206 | 59 |
| 2 | .736493 | 3.25 | .923427 | 1.37 | .813070 | 4.60 | .186930 | 58 |
| 3 | .736692 | 3.23 | .923345 | 1.37 | .813347 | 4.62 | .186653 | 57 |
| 4 | .736886 | 3.23 | .923263 | 1.37 | .813623 | 4.60 | .186377 | 56 |
| 5 | .737080 | 3.23 | .923181 | 1.38 | .813899 | 4.60 | .186101 | 55 |
| 6 | .737274 | 3.23 | .923098 | 1.37 | .814176 | 4.62 | .185824 | 54 |
| 7 | .737467 | 3.22 | .923016 | 1.37 | .814452 | 4.60 | .185548 | 53 |
| 8 | .737661 | 3.23 | .922933 | 1.38 | .814728 | 4.60 | .185272 | 52 |
| 9 | .737855 | 3.23 | .922851 | 1.37 | .815004 | 4.60 | .184996 | 51 |
| 10 | .738048 | 3.22 | .922768 | 1.37 | .815280 | 4.58 | .184720 | 50 |
| 11 | 9.738241 | 3.22 | 9.922686 | 1.38 | 9.815555 | 4.60 | 10.184445 | 49 |
| 12 | .738434 | 3.22 | .922603 | 1.38 | .815831 | 4.60 | .184169 | 48 |
| 13 | .738627 | 3.22 | .922520 | 1.37 | .816107 | 4.58 | .183893 | 47 |
| 14 | .738820 | 3.22 | .922438 | 1.38 | .816382 | 4.60 | .183618 | 46 |
| 15 | .739013 | 3.22 | .922355 | 1.38 | .816658 | 4.58 | .183342 | 45 |
| 16 | .739206 | 3.22 | .922272 | 1.38 | .816933 | 4.60 | .183067 | 44 |
| 17 | .739398 | 3.20 | .922189 | 1.38 | .817209 | 4.58 | .182791 | 43 |
| 18 | .739590 | 3.22 | .922106 | 1.38 | .817484 | 4.58 | .182516 | 42 |
| 19 | .739783 | 3.20 | .922023 | 1.38 | .817759 | 4.58 | .182241 | 41 |
| 20 | .739975 | 3.20 | .921940 | 1.38 | .818035 | 4.60 | .181965 | 40 |
| 21 | 9.740167 | 3.20 | 9.921857 | 1.38 | 9.818310 | 4.58 | 10.181690 | 39 |
| 22 | .740359 | 3.18 | .921774 | 1.38 | .818585 | 4.58 | .181415 | 38 |
| 23 | .740550 | 3.20 | .921691 | 1.40 | .818860 | 4.58 | .181140 | 37 |
| 24 | .740742 | 3.20 | .921607 | 1.38 | .819135 | 4.58 | .180865 | 36 |
| 25 | .740934 | 3.18 | .921524 | 1.38 | .819410 | 4.57 | .180590 | 35 |
| 26 | .741125 | 3.18 | .921441 | 1.40 | .819684 | 4.58 | .180316 | 34 |
| 27 | .741316 | 3.20 | .921357 | 1.40 | .819959 | 4.58 | .180041 | 33 |
| 28 | .741508 | 3.18 | .921274 | 1.38 | .820234 | 4.58 | .179766 | 32 |
| 29 | .741699 | 3.17 | .921190 | 1.40 | .820508 | 4.57 | .179492 | 31 |
| 30 | .741889 | 3.18 | .921107 | 1.38 | .820783 | 4.58 | .179217 | 30 |
| 31 | 9.742080 | 3.18 | 9.921023 | 1.40 | 9.821057 | 4.57 | 10.178943 | 29 |
| 32 | .742271 | 3.18 | .920939 | 1.38 | .821332 | 4.58 | .178668 | 28 |
| 33 | .742462 | 3.17 | .920856 | 1.40 | .821606 | 4.57 | .178394 | 27 |
| 34 | .742652 | 3.17 | .920772 | 1.40 | .821880 | 4.57 | .178120 | 26 |
| 35 | .742842 | 3.18 | .920688 | 1.40 | .822154 | 4.58 | .177846 | 25 |
| 36 | .743033 | 3.17 | .920604 | 1.40 | .822429 | 4.57 | .177571 | 24 |
| 37 | .743223 | 3.17 | .920520 | 1.40 | .822703 | 4.57 | .177297 | 23 |
| 38 | .743413 | 3.15 | .920436 | 1.40 | .822977 | 4.57 | .177023 | 22 |
| 39 | .743602 | 3.17 | .920352 | 1.40 | .823251 | 4.55 | .176749 | 21 |
| 40 | .743792 | 3.17 | .920268 | 1.40 | .823524 | 4.57 | .176476 | 20 |
| 41 | 9.743982 | 3.15 | 9.920184 | 1.42 | 9.823798 | 4.57 | 10.176202 | 19 |
| 42 | .744171 | 3.17 | .920099 | 1.40 | .824072 | 4.55 | .175928 | 18 |
| 43 | .744361 | 3.17 | .920015 | 1.40 | .824345 | 4.57 | .175655 | 17 |
| 44 | .744550 | 3.15 | .919931 | 1.40 | .824619 | 4.57 | .175381 | 16 |
| 45 | .744739 | 3.15 | .919846 | 1.42 | .824893 | 4.55 | .175107 | 15 |
| 46 | .744928 | 3.15 | .919762 | 1.42 | .825166 | 4.55 | .174834 | 14 |
| 47 | .745117 | 3.15 | .919677 | 1.40 | .825439 | 4.57 | .174561 | 13 |
| 48 | .745306 | 3.13 | .919593 | 1.42 | .825713 | 4.55 | .174287 | 12 |
| 49 | .745494 | 3.15 | .919508 | 1.40 | .825986 | 4.55 | .174014 | 11 |
| 50 | .745683 | 3.13 | .919424 | 1.42 | .826259 | 4.55 | .173741 | 10 |
| 51 | 9.745871 | 3.15 | 9.919339 | 1.42 | 9.826532 | 4.55 | 10.173468 | 9 |
| 52 | .746060 | 3.13 | .919254 | 1.42 | .826805 | 4.55 | .173195 | 8 |
| 53 | .746248 | 3.13 | .919169 | 1.40 | .827078 | 4.55 | .172922 | 7 |
| 54 | .746436 | 3.13 | .919085 | 1.42 | .827351 | 4.55 | .172649 | 6 |
| 55 | .746624 | 3.13 | .919000 | 1.42 | .827624 | 4.55 | .172376 | 5 |
| 56 | .746812 | 3.12 | .918915 | 1.42 | .827897 | 4.55 | .172103 | 4 |
| 57 | .746999 | 3.13 | .918830 | 1.42 | .828170 | 4.53 | .171830 | 3 |
| 58 | .747187 | 3.12 | .918745 | 1.43 | .828442 | 4.55 | .171558 | 2 |
| 59 | .747374 | 3.13 | .918659 | 1.42 | .828715 | 4.53 | .171285 | 1 |
| 60 | 9.747562 | | 9.918574 | | 9.828987 | | 10.171013 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ' | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ' |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.747562 | 3.12 | 9.918574 | 1.42 | 9.828987 | 4.55 | 10.171013 | 60 |
| 1 | .747749 | 3.13 | .918489 | 1.42 | .829260 | 4.53 | .170740 | 59 |
| 2 | .747936 | 3.12 | .918404 | 1.43 | .829532 | 4.55 | .170468 | 58 |
| 3 | .748123 | 3.12 | .918318 | 1.42 | .829805 | 4.53 | .170195 | 57 |
| 4 | .748310 | 3.12 | .918233 | 1.43 | .830077 | 4.53 | .169923 | 56 |
| 5 | .748497 | 3.12 | .918147 | 1.42 | .830349 | 4.53 | .169651 | 55 |
| 6 | .748683 | 3.10 | .918062 | 1.43 | .830621 | 4.53 | .169379 | 54 |
| 7 | .748870 | 3.12 | .917976 | 1.42 | .830893 | 4.53 | .169107 | 53 |
| 8 | .749056 | 3.10 | .917891 | 1.43 | .831165 | 4.53 | .168835 | 52 |
| 9 | .749243 | 3.12 | .917805 | 1.43 | .831437 | 4.53 | .168563 | 51 |
| 10 | .749429 | 3.10 | .917719 | 1.42 | .831709 | 4.53 | .168291 | 50 |
| 11 | 9.749615 | 3.10 | 9.917634 | 1.43 | 9.831981 | 4.53 | 10.168019 | 49 |
| 12 | .749801 | 3.10 | .917548 | 1.43 | .832253 | 4.53 | .167747 | 48 |
| 13 | .749987 | 3.08 | .917462 | 1.43 | .832525 | 4.52 | .167475 | 47 |
| 14 | .750172 | 3.10 | .917376 | 1.43 | .832796 | 4.53 | .167204 | 46 |
| 15 | .750358 | 3.08 | .917290 | 1.43 | .833068 | 4.52 | .166932 | 45 |
| 16 | .750543 | 3.08 | .917204 | 1.43 | .833339 | 4.53 | .166661 | 44 |
| 17 | .750729 | 3.10 | .917118 | 1.43 | .833611 | 4.52 | .166389 | 43 |
| 18 | .750914 | 3.08 | .917032 | 1.43 | .833882 | 4.53 | .166118 | 42 |
| 19 | .751099 | 3.08 | .916946 | 1.45 | .834154 | 4.52 | .165846 | 41 |
| 20 | .751284 | 3.08 | .916859 | 1.43 | .834425 | 4.52 | .165575 | 40 |
| 21 | 9.751469 | 3.08 | 9.916773 | 1.43 | 9.834696 | 4.52 | 10.165304 | 39 |
| 22 | .751654 | 3.08 | .916687 | 1.45 | .834967 | 4.52 | .165033 | 38 |
| 23 | .751839 | 3.08 | .916600 | 1.43 | .835238 | 4.52 | .164762 | 37 |
| 24 | .752023 | 3.07 | .916514 | 1.43 | .835509 | 4.52 | .164491 | 36 |
| 25 | .752208 | 3.08 | .916427 | 1.45 | .835780 | 4.52 | .164220 | 35 |
| 26 | .752392 | 3.07 | .916341 | 1.43 | .836051 | 4.52 | .163949 | 34 |
| 27 | .752576 | 3.07 | .916254 | 1.45 | .836322 | 4.52 | .163678 | 33 |
| 28 | .752760 | 3.07 | .916167 | 1.43 | .836593 | 4.52 | .163407 | 32 |
| 29 | .752944 | 3.07 | .916081 | 1.45 | .836864 | 4.50 | .163136 | 31 |
| 30 | .753128 | 3.07 | .915994 | 1.45 | .837134 | 4.52 | .162866 | 30 |
| 31 | 9.753312 | 3.05 | 9.915907 | 1.45 | 9.837405 | 4.50 | 10.162595 | 29 |
| 32 | .753495 | 3.07 | .915820 | 1.45 | .837675 | 4.52 | .162325 | 28 |
| 33 | .753679 | 3.07 | .915733 | 1.45 | .837946 | 4.50 | .162054 | 27 |
| 34 | .753862 | 3.07 | .915646 | 1.45 | .838216 | 4.52 | .161784 | 26 |
| 35 | .754046 | 3.07 | .915559 | 1.45 | .838487 | 4.50 | .161513 | 25 |
| 36 | .754229 | 3.05 | .915472 | 1.45 | .838757 | 4.50 | .161243 | 24 |
| 37 | .754412 | 3.05 | .915385 | 1.47 | .839027 | 4.50 | .160973 | 23 |
| 38 | .754595 | 3.05 | .915297 | 1.45 | .839297 | 4.52 | .160703 | 22 |
| 39 | .754778 | 3.03 | .915210 | 1.45 | .839568 | 4.50 | .160432 | 21 |
| 40 | .754960 | 3.05 | .915123 | 1.47 | .839838 | 4.50 | .160162 | 20 |
| 41 | 9.755143 | 3.05 | 9.915035 | 1.45 | 9.840108 | 4.50 | 10.159892 | 19 |
| 42 | .755326 | 3.03 | .914948 | 1.47 | .840378 | 4.50 | .159622 | 18 |
| 43 | .755508 | 3.03 | .914860 | 1.45 | .840648 | 4.48 | .159352 | 17 |
| 44 | .755690 | 3.03 | .914773 | 1.47 | .840917 | 4.50 | .159083 | 16 |
| 45 | .755872 | 3.03 | .914685 | 1.47 | .841187 | 4.50 | .158813 | 15 |
| 46 | .756054 | 3.03 | .914598 | 1.45 | .841457 | 4.50 | .158543 | 14 |
| 47 | .756236 | 3.03 | .914510 | 1.47 | .841727 | 4.50 | .158273 | 13 |
| 48 | .756418 | 3.03 | .914422 | 1.47 | .841996 | 4.48 | .158004 | 12 |
| 49 | .756600 | 3.03 | .914334 | 1.47 | .842266 | 4.50 | .157734 | 11 |
| 50 | .756782 | 3.02 | .914246 | 1.47 | .842535 | 4.50 | .157465 | 10 |
| 51 | 9.756963 | 3.02 | 9.914158 | 1.47 | 9.842805 | 4.48 | 10.157195 | 9 |
| 52 | .757144 | 3.03 | .914070 | 1.47 | .843074 | 4.48 | .156926 | 8 |
| 53 | .757326 | 3.02 | .913982 | 1.47 | .843343 | 4.48 | .156657 | 7 |
| 54 | .757507 | 3.02 | .913894 | 1.47 | .843612 | 4.50 | .156388 | 6 |
| 55 | .757688 | 3.02 | .913806 | 1.47 | .843882 | 4.48 | .156118 | 5 |
| 56 | .757869 | 3.02 | .913718 | 1.47 | .844151 | 4.48 | .155849 | 4 |
| 57 | .758050 | 3.02 | .913630 | 1.47 | .844420 | 4.48 | .155580 | 3 |
| 58 | .758230 | 3.00 | .913541 | 1.48 | .844689 | 4.48 | .155311 | 2 |
| 59 | .758411 | 3.02 | .913453 | 1.47 | .844958 | 4.48 | .155042 | 1 |
| 60 | 9.758591 | 3.00 | 9.913365 | 1.47 | 9.845227 | 4.48 | 10.154773 | 0 |
| ' | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ' |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.758591 | 3.02 | 9.913365 | 1.48 | 9.845227 | 4.48 | 10.154773 | 60 |
| 1 | .758772 | 3.00 | .913276 | 1.48 | .845496 | 4.47 | .154504 | 59 |
| 2 | .758952 | 3.00 | .913187 | 1.48 | .845764 | 4.48 | .154236 | 58 |
| 3 | .759132 | 3.00 | .913099 | 1.47 | .846033 | 4.48 | .153967 | 57 |
| 4 | .759312 | 3.00 | .913010 | 1.48 | .846302 | 4.47 | .153698 | 56 |
| 5 | .759492 | 3.00 | .912922 | 1.47 | .846570 | 4.48 | .153430 | 55 |
| 6 | .759672 | 3.00 | .912833 | 1.48 | .846839 | 4.48 | .153161 | 54 |
| 7 | .759852 | 2.98 | .912744 | 1.48 | .847108 | 4.47 | .152892 | 53 |
| 8 | .760031 | 3.00 | .912655 | 1.48 | .847376 | 4.47 | .152624 | 52 |
| 9 | .760211 | 2.98 | .912566 | 1.48 | .847644 | 4.48 | .152356 | 51 |
| 10 | .760390 | 2.98 | .912477 | 1.48 | .847913 | 4.47 | .152087 | 50 |
| 11 | 9.760569 | 2.98 | 9.912388 | 1.48 | 9.848181 | 4.47 | 10.151819 | 49 |
| 12 | .760748 | 2.98 | .912299 | 1.48 | .848449 | 4.47 | .151551 | 48 |
| 13 | .760927 | 2.98 | .912210 | 1.48 | .848717 | 4.48 | .151283 | 47 |
| 14 | .761106 | 2.98 | .912121 | 1.48 | .848986 | 4.47 | .151014 | 46 |
| 15 | .761285 | 2.98 | .912031 | 1.50 | .849254 | 4.47 | .150746 | 45 |
| 16 | .761464 | 2.97 | .911942 | 1.48 | .849522 | 4.47 | .150478 | 44 |
| 17 | .761642 | 2.98 | .911853 | 1.50 | .849790 | 4.45 | .150210 | 43 |
| 18 | .761821 | 2.97 | .911763 | 1.48 | .850057 | 4.47 | .149943 | 42 |
| 19 | .761999 | 2.97 | .911674 | 1.50 | .850325 | 4.47 | .149675 | 41 |
| 20 | .762177 | 2.98 | .911584 | 1.48 | .850593 | 4.47 | .149407 | 40 |
| 21 | 9.762356 | 2.97 | 9.911495 | 1.50 | 9.850861 | 4.47 | 10.149139 | 39 |
| 22 | .762534 | 2.97 | .911405 | 1.50 | .851129 | 4.45 | .148871 | 38 |
| 23 | .762712 | 2.95 | .911315 | 1.48 | .851396 | 4.47 | .148604 | 37 |
| 24 | .762889 | 2.97 | .911226 | 1.50 | .851664 | 4.45 | .148336 | 36 |
| 25 | .763067 | 2.97 | .911136 | 1.50 | .851931 | 4.47 | .148069 | 35 |
| 26 | .763245 | 2.97 | .911046 | 1.50 | .852199 | 4.47 | .147801 | 34 |
| 27 | .763422 | 2.95 | .910956 | 1.50 | .852466 | 4.45 | .147534 | 33 |
| 28 | .763600 | 2.97 | .910866 | 1.50 | .852733 | 4.47 | .147267 | 32 |
| 29 | .763777 | 2.95 | .910776 | 1.50 | .853001 | 4.45 | .146999 | 31 |
| 30 | .763954 | 2.95 | .910686 | 1.50 | .853268 | 4.45 | .146732 | 30 |
| 31 | 9.764131 | 2.95 | 9.910596 | 1.50 | 9.853535 | 4.45 | 10.146465 | 29 |
| 32 | .764308 | 2.95 | .910506 | 1.52 | .853802 | 4.45 | .146198 | 28 |
| 33 | .764485 | 2.95 | .910415 | 1.50 | .854069 | 4.45 | .145931 | 27 |
| 34 | .764662 | 2.93 | .910325 | 1.50 | .854336 | 4.45 | .145664 | 26 |
| 35 | .764838 | 2.95 | .910235 | 1.52 | .854603 | 4.45 | .145397 | 25 |
| 36 | .765015 | 2.93 | .910144 | 1.50 | .854870 | 4.45 | .145130 | 24 |
| 37 | .765191 | 2.93 | .910054 | 1.52 | .855137 | 4.45 | .144863 | 23 |
| 38 | .765367 | 2.93 | .909963 | 1.50 | .855404 | 4.45 | .144596 | 22 |
| 39 | .765544 | 2.95 | .909873 | 1.52 | .855671 | 4.45 | .144329 | 21 |
| 40 | .765720 | 2.93 | .909782 | 1.52 | .855938 | 4.43 | .144062 | 20 |
| 41 | 9.765896 | 2.93 | 9.909691 | 1.50 | 9.856204 | 4.45 | 10.143796 | 19 |
| 42 | .766072 | 2.92 | .909601 | 1.52 | .856471 | 4.43 | .143529 | 18 |
| 43 | .766247 | 2.93 | .909510 | 1.52 | .856737 | 4.45 | .143263 | 17 |
| 44 | .766423 | 2.92 | .909419 | 1.52 | .857004 | 4.43 | .142996 | 16 |
| 45 | .766598 | 2.93 | .909328 | 1.52 | .857270 | 4.45 | .142730 | 15 |
| 46 | .766774 | 2.92 | .909237 | 1.52 | .857537 | 4.43 | .142463 | 14 |
| 47 | .766949 | 2.92 | .909146 | 1.52 | .857803 | 4.43 | .142197 | 13 |
| 48 | .767124 | 2.93 | .909055 | 1.52 | .858069 | 4.43 | .141931 | 12 |
| 49 | .767300 | 2.92 | .908964 | 1.52 | .858336 | 4.45 | .141664 | 11 |
| 50 | .767475 | 2.90 | .908873 | 1.53 | .858602 | 4.43 | .141398 | 10 |
| 51 | 9.767649 | 2.92 | 9.908781 | 1.52 | 9.858868 | 4.43 | 10.141132 | 9 |
| 52 | .767824 | 2.92 | .908690 | 1.52 | .859134 | 4.43 | .140866 | 8 |
| 53 | .767999 | 2.90 | .908599 | 1.53 | .859400 | 4.43 | .140600 | 7 |
| 54 | .768173 | 2.92 | .908507 | 1.52 | .859666 | 4.43 | .140334 | 6 |
| 55 | .768348 | 2.90 | .908416 | 1.53 | .859932 | 4.43 | .140068 | 5 |
| 56 | .768522 | 2.92 | .908324 | 1.52 | .860198 | 4.43 | .139802 | 4 |
| 57 | .768697 | 2.90 | .908233 | 1.53 | .860464 | 4.43 | .139536 | 3 |
| 58 | .768871 | 2.90 | .908141 | 1.53 | .860730 | 4.42 | .139270 | 2 |
| 59 | .769045 | 2.90 | .908049 | 1.52 | .860995 | 4.43 | .139005 | 1 |
| 60 | 9.769219 | | 9.907958 | | 9.861261 | | 10.138739 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.769219 | | 9.907958 | | 9.861261 | | 10.138739 | 60 |
| 1 | .769393 | 2.90 | .907866 | 1.53 | .861527 | 4.43 | .138473 | 59 |
| 2 | .769566 | 2.88 | .907774 | 1.53 | .861792 | 4.42 | .138208 | 58 |
| 3 | .769740 | 2.90 | .907682 | 1.53 | .862058 | 4.43 | .137942 | 57 |
| 4 | .769913 | 2.90 | .907590 | 1.53 | .862323 | 4.43 | .137677 | 56 |
| 5 | .770087 | 2.88 | .907498 | 1.53 | .862589 | 4.42 | .137411 | 55 |
| 6 | .770260 | 2.88 | .907406 | 1.53 | .862854 | 4.42 | .137146 | 54 |
| 7 | .770433 | 2.88 | .907314 | 1.53 | .863119 | 4.42 | .136881 | 53 |
| 8 | .770606 | 2.88 | .907222 | 1.53 | .863385 | 4.43 | .136615 | 52 |
| 9 | .770779 | 2.88 | .907129 | 1.55 | .863650 | 4.42 | .136350 | 51 |
| 10 | .770952 | 2.88 | .907037 | 1.53 | .863915 | 4.42 | .136085 | 50 |
| 11 | 9.771125 | 2.88 | 9.906945 | 1.55 | 9.864180 | 4.42 | 10.135820 | 49 |
| 12 | .771298 | 2.87 | .906852 | 1.53 | .864445 | 4.42 | .135555 | 48 |
| 13 | .771470 | 2.88 | .906760 | 1.55 | .864710 | 4.42 | .135290 | 47 |
| 14 | .771643 | 2.87 | .906667 | 1.53 | .864975 | 4.42 | .135025 | 46 |
| 15 | .771815 | 2.87 | .906575 | 1.55 | .865240 | 4.42 | .134760 | 45 |
| 16 | .771987 | 2.87 | .906482 | 1.55 | .865505 | 4.42 | .134495 | 44 |
| 17 | .772159 | 2.87 | .906389 | 1.55 | .865770 | 4.42 | .134230 | 43 |
| 18 | .772331 | 2.87 | .906296 | 1.53 | .866035 | 4.42 | .133965 | 42 |
| 19 | .772503 | 2.87 | .906204 | 1.53 | .866300 | 4.42 | .133700 | 41 |
| 20 | .772675 | 2.87 | .906111 | 1.55 | .866564 | 4.42 | .133436 | 40 |
| 21 | 9.772847 | 2.85 | 9.906018 | 1.55 | 9.866829 | 4.42 | 10.133171 | 39 |
| 22 | .773018 | 2.87 | .905925 | 1.55 | .867094 | 4.40 | .132906 | 38 |
| 23 | .773190 | 2.85 | .905832 | 1.55 | .867358 | 4.42 | .132642 | 37 |
| 24 | .773361 | 2.87 | .905739 | 1.57 | .867623 | 4.40 | .132377 | 36 |
| 25 | .773533 | 2.85 | .905645 | 1.55 | .867887 | 4.42 | .132113 | 35 |
| 26 | .773704 | 2.85 | .905552 | 1.55 | .868152 | 4.40 | .131848 | 34 |
| 27 | .773875 | 2.85 | .905459 | 1.55 | .868416 | 4.40 | .131584 | 33 |
| 28 | .774046 | 2.85 | .905366 | 1.55 | .868680 | 4.40 | .131320 | 32 |
| 29 | .774217 | 2.85 | .905272 | 1.57 | .868945 | 4.42 | .131055 | 31 |
| 30 | .774388 | 2.83 | .905179 | 1.57 | .869209 | 4.40 | .130791 | 30 |
| 31 | 9.774558 | 2.85 | 9.905085 | 1.55 | 9.869473 | 4.40 | 10.130527 | 29 |
| 32 | .774729 | 2.83 | .904992 | 1.57 | .869737 | 4.40 | .130263 | 28 |
| 33 | .774899 | 2.85 | .904898 | 1.57 | .870001 | 4.40 | .129999 | 27 |
| 34 | .775070 | 2.83 | .904804 | 1.55 | .870265 | 4.40 | .129735 | 26 |
| 35 | .775240 | 2.83 | .904711 | 1.57 | .870529 | 4.40 | .129471 | 25 |
| 36 | .775410 | 2.83 | .904617 | 1.57 | .870793 | 4.40 | .129207 | 24 |
| 37 | .775580 | 2.83 | .904523 | 1.57 | .871057 | 4.40 | .128943 | 23 |
| 38 | .775750 | 2.83 | .904429 | 1.57 | .871321 | 4.40 | .128679 | 22 |
| 39 | .775920 | 2.83 | .904335 | 1.57 | .871585 | 4.40 | .128415 | 21 |
| 40 | .776090 | 2.83 | .904241 | 1.57 | .871849 | 4.38 | .128151 | 20 |
| 41 | 9.776259 | 2.83 | 9.904147 | 1.57 | 9.872112 | 4.40 | 10.127888 | 19 |
| 42 | .776429 | 2.82 | .904053 | 1.57 | .872376 | 4.40 | .127624 | 18 |
| 43 | .776598 | 2.83 | .903959 | 1.57 | .872640 | 4.40 | .127360 | 17 |
| 44 | .776768 | 2.82 | .903864 | 1.57 | .872903 | 4.40 | .127097 | 16 |
| 45 | .776937 | 2.82 | .903770 | 1.57 | .873167 | 4.38 | .126833 | 15 |
| 46 | .777106 | 2.82 | .903676 | 1.58 | .873430 | 4.40 | .126570 | 14 |
| 47 | .777275 | 2.82 | .903581 | 1.57 | .873694 | 4.38 | .126306 | 13 |
| 48 | .777444 | 2.82 | .903487 | 1.57 | .873957 | 4.38 | .126043 | 12 |
| 49 | .777613 | 2.80 | .903392 | 1.58 | .874220 | 4.40 | .125780 | 11 |
| 50 | .777781 | 2.82 | .903298 | 1.57 | .874484 | 4.38 | .125516 | 10 |
| 51 | 9.777950 | 2.82 | 9.903203 | 1.58 | 9.874747 | 4.38 | 10.125253 | 9 |
| 52 | .778119 | 2.80 | .903108 | 1.57 | .875010 | 4.38 | .124990 | 8 |
| 53 | .778287 | 2.80 | .903014 | 1.58 | .875273 | 4.40 | .124727 | 7 |
| 54 | .778455 | 2.82 | .902919 | 1.58 | .875537 | 4.38 | .124463 | 6 |
| 55 | .778624 | 2.80 | .902824 | 1.58 | .875800 | 4.38 | .124200 | 5 |
| 56 | .778792 | 2.80 | .902729 | 1.58 | .876063 | 4.38 | .123937 | 4 |
| 57 | .778960 | 2.80 | .902634 | 1.58 | .876326 | 4.38 | .123674 | 3 |
| 58 | .779128 | 2.80 | .902539 | 1.58 | .876589 | 4.38 | .123411 | 2 |
| 59 | .779295 | 2.78 | .902444 | 1.58 | .876852 | 4.38 | .123148 | 1 |
| 60 | 9.779463 | 2.80 | 9.902349 | 1.58 | 9.877114 | 4.37 | 10.122886 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.779463 | 2.80 | 9.902349 | 1.60 | 9.877114 | 4.38 | 10.122886 | 60 |
| 1 | .779631 | 2.78 | .902253 | 1.58 | .877377 | 4.38 | .122623 | 59 |
| 2 | .779798 | 2.80 | .902158 | 1.58 | .877640 | 4.38 | .122360 | 58 |
| 3 | .779966 | 2.78 | .902063 | 1.60 | .877903 | 4.37 | .122097 | 57 |
| 4 | .780133 | 2.78 | .901967 | 1.58 | .878165 | 4.38 | .121835 | 56 |
| 5 | .780300 | 2.78 | .901872 | 1.60 | .878428 | 4.38 | .121572 | 55 |
| 6 | .780467 | 2.78 | .901776 | 1.58 | .878691 | 4.37 | .121309 | 54 |
| 7 | .780634 | 2.78 | .901681 | 1.60 | .878953 | 4.38 | .121047 | 53 |
| 8 | .780801 | 2.78 | .901585 | 1.58 | .879216 | 4.37 | .120784 | 52 |
| 9 | .780968 | 2.77 | .901490 | 1.60 | .879478 | 4.38 | .120522 | 51 |
| 10 | .781134 | 2.78 | .901394 | 1.60 | .879741 | 4.37 | .120259 | 50 |
| 11 | 9.781301 | 2.78 | 9.901298 | 1.60 | 9.880003 | 4.37 | 10.119997 | 49 |
| 12 | .781468 | 2.77 | .901202 | 1.60 | .880265 | 4.38 | .119735 | 48 |
| 13 | .781634 | 2.77 | .901106 | 1.60 | .880528 | 4.37 | .119472 | 47 |
| 14 | .781800 | 2.77 | .901010 | 1.60 | .880790 | 4.37 | .119210 | 46 |
| 15 | .781966 | 2.77 | .900914 | 1.60 | .881052 | 4.37 | .118948 | 45 |
| 16 | .782132 | 2.77 | .900818 | 1.60 | .881314 | 4.38 | .118686 | 44 |
| 17 | .782298 | 2.77 | .900722 | 1.60 | .881577 | 4.37 | .118423 | 43 |
| 18 | .782464 | 2.77 | .900626 | 1.62 | .881839 | 4.37 | .118161 | 42 |
| 19 | .782630 | 2.77 | .900529 | 1.60 | .882101 | 4.37 | .117899 | 41 |
| 20 | .782796 | 2.75 | .900433 | 1.60 | .882363 | 4.37 | .117637 | 40 |
| 21 | 9.782961 | 2.77 | 9.900337 | 1.62 | 9.882625 | 4.37 | 10.117375 | 39 |
| 22 | .783127 | 2.75 | .900240 | 1.60 | .882887 | 4.35 | .117113 | 38 |
| 23 | .783292 | 2.77 | .900144 | 1.62 | .883148 | 4.37 | .116852 | 37 |
| 24 | .783458 | 2.75 | .900047 | 1.60 | .883410 | 4.37 | .116590 | 36 |
| 25 | .783623 | 2.75 | .899951 | 1.62 | .883672 | 4.37 | .116328 | 35 |
| 26 | .783788 | 2.75 | .899854 | 1.62 | .883934 | 4.37 | .116066 | 34 |
| 27 | .783953 | 2.75 | .899757 | 1.62 | .884196 | 4.35 | .115804 | 33 |
| 28 | .784118 | 2.73 | .899660 | 1.60 | .884457 | 4.37 | .115543 | 32 |
| 29 | .784282 | 2.75 | .899564 | 1.62 | .884719 | 4.35 | .115281 | 31 |
| 30 | .784447 | 2.75 | .899467 | 1.62 | .884980 | 4.37 | .115020 | 30 |
| 31 | 9.784612 | 2.73 | 9.899370 | 1.62 | 9.885242 | 4.37 | 10.114758 | 29 |
| 32 | .784776 | 2.75 | .899273 | 1.62 | .885504 | 4.35 | .114496 | 28 |
| 33 | .784941 | 2.73 | .899176 | 1.63 | .885765 | 4.35 | .114235 | 27 |
| 34 | .785105 | 2.73 | .899078 | 1.62 | .886026 | 4.37 | .113974 | 26 |
| 35 | .785269 | 2.73 | .898981 | 1.62 | .886288 | 4.35 | .113712 | 25 |
| 36 | .785433 | 2.73 | .898884 | 1.62 | .886549 | 4.37 | .113451 | 24 |
| 37 | .785597 | 2.73 | .898787 | 1.63 | .886811 | 4.35 | .113189 | 23 |
| 38 | .785761 | 2.73 | .898689 | 1.62 | .887072 | 4.35 | .112928 | 22 |
| 39 | .785925 | 2.73 | .898592 | 1.63 | .887333 | 4.35 | .112667 | 21 |
| 40 | .786089 | 2.72 | .898494 | 1.62 | .887594 | 4.35 | .112406 | 20 |
| 41 | 9.786252 | 2.73 | 9.898397 | 1.63 | 9.887855 | 4.35 | 10.112145 | 19 |
| 42 | .786416 | 2.72 | .898299 | 1.62 | .888116 | 4.37 | .111884 | 18 |
| 43 | .786579 | 2.72 | .898202 | 1.63 | .888378 | 4.35 | .111622 | 17 |
| 44 | .786742 | 2.73 | .898104 | 1.63 | .888639 | 4.35 | .111361 | 16 |
| 45 | .786906 | 2.73 | .898006 | 1.63 | .888900 | 4.35 | .111100 | 15 |
| 46 | .787069 | 2.72 | .897908 | 1.63 | .889161 | 4.33 | .110839 | 14 |
| 47 | .787232 | 2.72 | .897810 | 1.63 | .889421 | 4.35 | .110579 | 13 |
| 48 | .787395 | 2.72 | .897712 | 1.63 | .889682 | 4.35 | .110318 | 12 |
| 49 | .787557 | 2.70 | .897614 | 1.63 | .889943 | 4.35 | .110057 | 11 |
| 50 | .787720 | 2.72 | .897516 | 1.63 | .890204 | 4.35 | .109796 | 10 |
| 51 | 9.787883 | 2.70 | 9.897418 | 1.63 | 9.890465 | 4.33 | 10.109535 | 9 |
| 52 | .788045 | 2.72 | .897320 | 1.63 | .890725 | 4.35 | .109275 | 8 |
| 53 | .788208 | 2.70 | .897222 | 1.65 | .890986 | 4.35 | .109014 | 7 |
| 54 | .788370 | 2.70 | .897123 | 1.63 | .891247 | 4.33 | .108753 | 6 |
| 55 | .788532 | 2.70 | .897025 | 1.65 | .891507 | 4.35 | .108493 | 5 |
| 56 | .788694 | 2.70 | .896926 | 1.63 | .891768 | 4.33 | .108232 | 4 |
| 57 | .788856 | 2.70 | .896828 | 1.65 | .892028 | 4.35 | .107972 | 3 |
| 58 | .789018 | 2.70 | .896729 | 1.63 | .892289 | 4.33 | .107711 | 2 |
| 59 | .789180 | 2.70 | .896631 | 1.65 | .892549 | 4.35 | .107451 | 1 |
| 60 | 9.789342 | | 9.896532 | | 9.892810 | | 10.107190 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.789342 | 2.70 | 9.896332 | 1.65 | 9.892810 | 4.33 | 10.107190 | 60 |
| 1 | .789504 | 2.68 | .896433 | 1.68 | .893070 | 4.35 | .106930 | 59 |
| 2 | .789665 | 2.70 | .896335 | 1.65 | .893331 | 4.33 | .106669 | 58 |
| 3 | .789827 | 2.68 | .896236 | 1.65 | .893591 | 4.33 | .106409 | 57 |
| 4 | .789988 | 2.68 | .896137 | 1.65 | .893851 | 4.33 | .106149 | 56 |
| 5 | .790149 | 2.68 | .896038 | 1.65 | .894111 | 4.35 | .105889 | 55 |
| 6 | .790310 | 2.68 | .895939 | 1.65 | .894372 | 4.33 | .105628 | 54 |
| 7 | .790471 | 2.68 | .895840 | 1.65 | .894632 | 4.33 | .105368 | 53 |
| 8 | .790632 | 2.68 | .895741 | 1.67 | .894892 | 4.33 | .105108 | 52 |
| 9 | .790793 | 2.68 | .895641 | 1.65 | .895152 | 4.33 | .104848 | 51 |
| 10 | .790954 | 2.68 | .895542 | 1.65 | .895412 | 4.33 | .104588 | 50 |
| 11 | 9.791115 | 2.67 | 9.895443 | 1.67 | 9.895672 | 4.33 | 10.104328 | 49 |
| 12 | .791275 | 2.68 | .895343 | 1.65 | .895932 | 4.33 | .104068 | 48 |
| 13 | .791436 | 2.67 | .895244 | 1.65 | .896192 | 4.33 | .103808 | 47 |
| 14 | .791596 | 2.68 | .895145 | 1.67 | .896452 | 4.33 | .103548 | 46 |
| 15 | .791757 | 2.67 | .895045 | 1.67 | .896712 | 4.32 | .103288 | 45 |
| 16 | .791917 | 2.67 | .894945 | 1.65 | .896971 | 4.33 | .103029 | 44 |
| 17 | .792077 | 2.67 | .894846 | 1.67 | .897231 | 4.33 | .102769 | 43 |
| 18 | .792237 | 2.67 | .894746 | 1.67 | .897491 | 4.33 | .102509 | 42 |
| 19 | .792397 | 2.67 | .894646 | 1.67 | .897751 | 4.32 | .102249 | 41 |
| 20 | .792557 | 2.65 | .894546 | 1.67 | .898010 | 4.33 | .101990 | 40 |
| 21 | 9.792716 | 2.67 | 9.894446 | 1.67 | 9.898270 | 4.33 | 10.101730 | 39 |
| 22 | .792876 | 2.65 | .894346 | 1.67 | .898530 | 4.32 | .101470 | 38 |
| 23 | .793035 | 2.67 | .894246 | 1.67 | .898789 | 4.33 | .101211 | 37 |
| 24 | .793195 | 2.65 | .894146 | 1.67 | .899049 | 4.32 | .100951 | 36 |
| 25 | .793354 | 2.67 | .894046 | 1.67 | .899308 | 4.33 | .100692 | 35 |
| 26 | .793514 | 2.65 | .893946 | 1.67 | .899568 | 4.32 | .100432 | 34 |
| 27 | .793673 | 2.65 | .893846 | 1.68 | .899827 | 4.33 | .100173 | 33 |
| 28 | .793832 | 2.65 | .893745 | 1.67 | .900087 | 4.32 | .099913 | 32 |
| 29 | .793991 | 2.65 | .893645 | 1.68 | .900346 | 4.32 | .099654 | 31 |
| 30 | .794150 | 2.63 | .893544 | 1.67 | .900605 | 4.32 | .099395 | 30 |
| 31 | 9.794308 | 2.65 | 9.893444 | 1.68 | 9.900864 | 4.33 | 10.099136 | 29 |
| 32 | .794467 | 2.65 | .893343 | 1.67 | .901124 | 4.32 | .098876 | 28 |
| 33 | .794626 | 2.63 | .893243 | 1.68 | .901383 | 4.32 | .098617 | 27 |
| 34 | .794784 | 2.63 | .893142 | 1.68 | .901642 | 4.32 | .098358 | 26 |
| 35 | .794942 | 2.63 | .893041 | 1.68 | .901901 | 4.32 | .098099 | 25 |
| 36 | .795101 | 2.65 | .892940 | 1.68 | .902160 | 4.32 | .097840 | 24 |
| 37 | .795259 | 2.63 | .892839 | 1.67 | .902420 | 4.32 | .097580 | 23 |
| 38 | .795417 | 2.63 | .892739 | 1.68 | .902679 | 4.32 | .097321 | 22 |
| 39 | .795575 | 2.63 | .892638 | 1.70 | .902938 | 4.32 | .097062 | 21 |
| 40 | .795733 | 2.63 | .892536 | 1.68 | .903197 | 4.32 | .096803 | 20 |
| 41 | 9.795891 | 2.63 | 9.892435 | 1.68 | 9.903456 | 4.30 | 10.096544 | 19 |
| 42 | .796049 | 2.62 | .892334 | 1.68 | .903714 | 4.32 | .096286 | 18 |
| 43 | .796206 | 2.63 | .892233 | 1.68 | .903973 | 4.32 | .096027 | 17 |
| 44 | .796364 | 2.62 | .892132 | 1.70 | .904232 | 4.32 | .095768 | 16 |
| 45 | .796521 | 2.63 | .892030 | 1.68 | .904491 | 4.32 | .095509 | 15 |
| 46 | .796679 | 2.62 | .891929 | 1.70 | .904750 | 4.30 | .095250 | 14 |
| 47 | .796836 | 2.62 | .891827 | 1.68 | .905008 | 4.32 | .094992 | 13 |
| 48 | .796993 | 2.62 | .891726 | 1.70 | .905267 | 4.32 | .094733 | 12 |
| 49 | .797150 | 2.62 | .891624 | 1.70 | .905526 | 4.32 | .094474 | 11 |
| 50 | .797307 | 2.62 | .891523 | 1.70 | .905785 | 4.30 | .094215 | 10 |
| 51 | 9.797464 | 2.62 | 9.891421 | 1.70 | 9.906043 | 4.32 | 10.093957 | 9 |
| 52 | .797621 | 2.60 | .891319 | 1.70 | .906302 | 4.30 | .093698 | 8 |
| 53 | .797777 | 2.62 | .891217 | 1.70 | .906560 | 4.32 | .093440 | 7 |
| 54 | .797934 | 2.62 | .891115 | 1.70 | .906819 | 4.30 | .093181 | 6 |
| 55 | .798091 | 2.60 | .891013 | 1.70 | .907077 | 4.32 | .092923 | 5 |
| 56 | .798247 | 2.60 | .890911 | 1.70 | .907336 | 4.30 | .092664 | 4 |
| 57 | .798403 | 2.62 | .890809 | 1.70 | .907594 | 4.32 | .092406 | 3 |
| 58 | .798560 | 2.60 | .890707 | 1.70 | .907852 | 4.30 | .092147 | 2 |
| 59 | .798716 | 2.60 | .890605 | 1.70 | .908111 | 4.30 | .091889 | 1 |
| 60 | 9.798872 | | 9.890503 | | 9.908369 | | 10.091631 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ' | Seno. | D. 1". | Coseno. | D. 1". | Tang. | D. 1". | Cotang. | ' |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.798872 | | 9.890503 | | 9.908369 | | 10.091631 | 60 |
| 1 | .799028 | 2.60 | .890400 | 1.72 | .908628 | 4.32 | .091372 | 59 |
| 2 | .799184 | 2.60 | .890298 | 1.70 | .908886 | 4.30 | .091114 | 58 |
| 3 | .799339 | 2.58 | .890195 | 1.72 | .909144 | 4.30 | .090856 | 57 |
| 4 | .799495 | 2.60 | .890093 | 1.70 | .909402 | 4.30 | .090598 | 56 |
| 5 | .799651 | 2.60 | .889990 | 1.72 | .909660 | 4.30 | .090340 | 55 |
| 6 | .799806 | 2.58 | .889888 | 1.70 | .909918 | 4.30 | .090082 | 54 |
| 7 | .799962 | 2.60 | .889785 | 1.72 | .910177 | 4.32 | .089823 | 53 |
| 8 | .800117 | 2.58 | .889682 | 1.72 | .910435 | 4.30 | .089565 | 52 |
| 9 | .800272 | 2.58 | .889579 | 1.72 | .910693 | 4.30 | .089307 | 51 |
| 10 | .800427 | 2.58 | .889477 | 1.70 | .910951 | 4.30 | .089049 | 50 |
| 11 | 9.800582 | | 9.889374 | | 9.911209 | | 10.088791 | 49 |
| 12 | .800737 | 2.58 | .889271 | 1.72 | .911467 | 4.30 | .088533 | 48 |
| 13 | .800892 | 2.58 | .889168 | 1.72 | .911725 | 4.30 | .088275 | 47 |
| 14 | .801047 | 2.58 | .889064 | 1.73 | .911982 | 4.28 | .088018 | 46 |
| 15 | .801201 | 2.57 | .888961 | 1.72 | .912240 | 4.30 | .087760 | 45 |
| 16 | .801356 | 2.58 | .888858 | 1.72 | .912498 | 4.30 | .087502 | 44 |
| 17 | .801511 | 2.58 | .888755 | 1.72 | .912756 | 4.30 | .087244 | 43 |
| 18 | .801665 | 2.57 | .888651 | 1.73 | .913014 | 4.30 | .086986 | 42 |
| 19 | .801819 | 2.57 | .888548 | 1.72 | .913271 | 4.28 | .086729 | 41 |
| 20 | .801973 | 2.57 | .888444 | 1.73 | .913529 | 4.30 | .086471 | 40 |
| 21 | 9.802128 | 2.58 | 9.888341 | 1.72 | 9.913787 | 4.30 | 10.086213 | 39 |
| 22 | .802282 | 2.57 | .888237 | 1.73 | .914044 | 4.28 | .085956 | 38 |
| 23 | .802436 | 2.57 | .888134 | 1.72 | .914302 | 4.30 | .085698 | 37 |
| 24 | .802589 | 2.55 | .888030 | 1.73 | .914560 | 4.30 | .085440 | 36 |
| 25 | .802743 | 2.57 | .887926 | 1.73 | .914817 | 4.28 | .085183 | 35 |
| 26 | .802897 | 2.57 | .887822 | 1.73 | .915075 | 4.30 | .084925 | 34 |
| 27 | .803050 | 2.55 | .887718 | 1.73 | .915332 | 4.28 | .084668 | 33 |
| 28 | .803204 | 2.57 | .887614 | 1.73 | .915590 | 4.30 | .084410 | 32 |
| 29 | .803357 | 2.55 | .887510 | 1.73 | .915847 | 4.28 | .084153 | 31 |
| 30 | .803511 | 2.57 | .887406 | 1.73 | .916104 | 4.28 | .083896 | 30 |
| 31 | 9.803664 | 2.55 | 9.887302 | 1.73 | 9.916362 | 4.30 | 10.083638 | 29 |
| 32 | .803817 | 2.55 | .887198 | 1.75 | .916619 | 4.28 | .083381 | 28 |
| 33 | .803970 | 2.55 | .887093 | 1.73 | .916877 | 4.30 | .083123 | 27 |
| 34 | .804123 | 2.55 | .886989 | 1.73 | .917134 | 4.28 | .082866 | 26 |
| 35 | .804276 | 2.55 | .886885 | 1.73 | .917391 | 4.28 | .082609 | 25 |
| 36 | .804428 | 2.55 | .886780 | 1.75 | .917648 | 4.28 | .082352 | 24 |
| 37 | .804581 | 2.55 | .886676 | 1.73 | .917906 | 4.30 | .082094 | 23 |
| 38 | .804734 | 2.55 | .886571 | 1.75 | .918163 | 4.28 | .081837 | 22 |
| 39 | .804886 | 2.53 | .886466 | 1.75 | .918420 | 4.28 | .081580 | 21 |
| 40 | .805039 | 2.55 | .886362 | 1.73 | .918677 | 4.28 | .081323 | 20 |
| 41 | 9.805191 | 2.53 | 9.886257 | 1.75 | 9.918934 | 4.28 | 10.081066 | 19 |
| 42 | .805343 | 2.53 | .886152 | 1.75 | .919191 | 4.28 | .080809 | 18 |
| 43 | .805495 | 2.53 | .886047 | 1.75 | .919448 | 4.28 | .080552 | 17 |
| 44 | .805647 | 2.53 | .885942 | 1.75 | .919705 | 4.28 | .080295 | 16 |
| 45 | .805799 | 2.53 | .885837 | 1.75 | .919962 | 4.28 | .080038 | 15 |
| 46 | .805951 | 2.53 | .885732 | 1.75 | .920219 | 4.28 | .079781 | 14 |
| 47 | .806103 | 2.52 | .885627 | 1.75 | .920476 | 4.28 | .079524 | 13 |
| 48 | .806254 | 2.53 | .885522 | 1.77 | .920733 | 4.28 | .079267 | 12 |
| 49 | .806406 | 2.52 | .885416 | 1.75 | .920990 | 4.28 | .079010 | 11 |
| 50 | .806557 | 2.53 | .885311 | 1.77 | .921247 | 4.27 | .078753 | 10 |
| 51 | 9.806709 | 2.52 | 9.885205 | 1.75 | 9.921503 | 4.28 | 10.078497 | 9 |
| 52 | .806860 | 2.52 | .885100 | 1.77 | .921760 | 4.28 | .078240 | 8 |
| 53 | .807011 | 2.53 | .884994 | 1.75 | .922017 | 4.28 | .077983 | 7 |
| 54 | .807163 | 2.52 | .884889 | 1.77 | .922274 | 4.28 | .077726 | 6 |
| 55 | .807314 | 2.52 | .884783 | 1.77 | .922530 | 4.27 | .077470 | 5 |
| 56 | .807465 | 2.52 | .884677 | 1.77 | .922787 | 4.28 | .077213 | 4 |
| 57 | .807615 | 2.50 | .884572 | 1.75 | .923044 | 4.28 | .076956 | 3 |
| 58 | .807766 | 2.52 | .884466 | 1.77 | .923300 | 4.27 | .076700 | 2 |
| 59 | .807917 | 2.52 | .884360 | 1.77 | .923557 | 4.28 | .076443 | 1 |
| 60 | 9.808067 | 2.50 | 9.884254 | 1.77 | 9.923814 | 4.28 | 10.076186 | 0 |
| ' | Coseno. | D. 1". | Seno. | D. 1". | Cotang. | D. 1". | Tang. | ' |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.808067 | 2.52 | 9.884254 | 1.77 | 9.923814 | 4.27 | 10.076186 | 60 |
| 1 | .808218 | 2.50 | .884148 | 1.77 | .924070 | 4.28 | .075930 | 59 |
| 2 | .808368 | 2.52 | .884042 | 1.77 | .924327 | 4.27 | .075673 | 58 |
| 3 | .808519 | 2.50 | .883936 | 1.78 | .924583 | 4.28 | .075417 | 57 |
| 4 | .808669 | 2.50 | .883829 | 1.77 | .924840 | 4.27 | .075160 | 56 |
| 5 | .808819 | 2.50 | .883723 | 1.77 | .925096 | 4.27 | .074904 | 55 |
| 6 | .808969 | 2.50 | .883617 | 1.78 | .925352 | 4.28 | .074648 | 54 |
| 7 | .809119 | 2.50 | .883510 | 1.77 | .925609 | 4.27 | .074391 | 53 |
| 8 | .809269 | 2.50 | .883404 | 1.78 | .925865 | 4.28 | .074135 | 52 |
| 9 | .809419 | 2.50 | .883297 | 1.77 | .926122 | 4.27 | .073878 | 51 |
| 10 | .809569 | 2.48 | .883191 | 1.78 | .926378 | 4.27 | .073622 | 50 |
| 11 | 9.809718 | 2.50 | 9.883084 | 1.78 | 9.926634 | 4.27 | 10.073366 | 49 |
| 12 | .809868 | 2.48 | .882977 | 1.77 | .926890 | 4.28 | .073110 | 48 |
| 13 | .810017 | 2.50 | .882871 | 1.78 | .927147 | 4.27 | .072853 | 47 |
| 14 | .810167 | 2.48 | .882764 | 1.78 | .927403 | 4.27 | .072597 | 46 |
| 15 | .810316 | 2.48 | .882657 | 1.78 | .927659 | 4.27 | .072341 | 45 |
| 16 | .810465 | 2.48 | .882550 | 1.78 | .927915 | 4.27 | .072085 | 44 |
| 17 | .810614 | 2.48 | .882443 | 1.78 | .928171 | 4.27 | .071829 | 43 |
| 18 | .810763 | 2.48 | .882336 | 1.78 | .928427 | 4.28 | .071573 | 42 |
| 19 | .810912 | 2.48 | .882229 | 1.80 | .928684 | 4.27 | .071316 | 41 |
| 20 | .811061 | 2.48 | .882121 | 1.78 | .928940 | 4.27 | .071060 | 40 |
| 21 | 9.811210 | 2.47 | 9.882014 | 1.78 | 9.929196 | 4.27 | 10.070804 | 39 |
| 22 | .811358 | 2.48 | .881907 | 1.80 | .929452 | 4.27 | .070548 | 38 |
| 23 | .811507 | 2.47 | .881799 | 1.78 | .929708 | 4.27 | .070292 | 37 |
| 24 | .811655 | 2.48 | .881692 | 1.80 | .929964 | 4.27 | .070036 | 36 |
| 25 | .811804 | 2.47 | .881584 | 1.78 | .930220 | 4.25 | .069780 | 35 |
| 26 | .811952 | 2.47 | .881477 | 1.80 | .930475 | 4.27 | .069525 | 34 |
| 27 | .812100 | 2.47 | .881369 | 1.80 | .930731 | 4.27 | .069269 | 33 |
| 28 | .812248 | 2.47 | .881261 | 1.80 | .930987 | 4.27 | .069013 | 32 |
| 29 | .812396 | 2.47 | .881153 | 1.78 | .931243 | 4.27 | .068757 | 31 |
| 30 | .812544 | 2.47 | .881046 | 1.80 | .931499 | 4.27 | .068501 | 30 |
| 31 | 9.812692 | 2.47 | 9.880938 | 1.80 | 9.931755 | 4.25 | 10.068245 | 29 |
| 32 | .812840 | 2.47 | .880830 | 1.80 | .932010 | 4.27 | .067990 | 28 |
| 33 | .812988 | 2.45 | .880722 | 1.82 | .932266 | 4.27 | .067734 | 27 |
| 34 | .813135 | 2.47 | .880613 | 1.80 | .932522 | 4.27 | .067478 | 26 |
| 35 | .813283 | 2.45 | .880505 | 1.80 | .932778 | 4.25 | .067222 | 25 |
| 36 | .813430 | 2.47 | .880397 | 1.80 | .933033 | 4.27 | .066967 | 24 |
| 37 | .813578 | 2.45 | .880289 | 1.82 | .933289 | 4.27 | .066711 | 23 |
| 38 | .813725 | 2.45 | .880180 | 1.80 | .933545 | 4.25 | .066455 | 22 |
| 39 | .813872 | 2.45 | .880072 | 1.82 | .933800 | 4.27 | .066200 | 21 |
| 40 | .814019 | 2.45 | .879963 | 1.80 | .934056 | 4.25 | .065944 | 20 |
| 41 | 9.814166 | 2.45 | 9.879855 | 1.82 | 9.934311 | 4.27 | 10.065689 | 19 |
| 42 | .814313 | 2.45 | .879746 | 1.82 | .934567 | 4.25 | .065433 | 18 |
| 43 | .814460 | 2.45 | .879637 | 1.80 | .934822 | 4.27 | .065178 | 17 |
| 44 | .814607 | 2.43 | .879529 | 1.82 | .935078 | 4.25 | .064922 | 16 |
| 45 | .814753 | 2.45 | .879420 | 1.82 | .935333 | 4.27 | .064667 | 15 |
| 46 | .814900 | 2.43 | .879311 | 1.82 | .935589 | 4.25 | .064411 | 14 |
| 47 | .815046 | 2.45 | .879202 | 1.82 | .935844 | 4.27 | .064156 | 13 |
| 48 | .815193 | 2.43 | .879093 | 1.82 | .936100 | 4.25 | .063900 | 12 |
| 49 | .815339 | 2.43 | .878984 | 1.82 | .936355 | 4.27 | .063645 | 11 |
| 50 | .815485 | 2.45 | .878875 | 1.82 | .936611 | 4.25 | .063389 | 10 |
| 51 | 9.815632 | 2.43 | 9.878766 | 1.83 | 9.936866 | 4.25 | 10.063134 | 9 |
| 52 | .815778 | 2.43 | .878656 | 1.82 | .937121 | 4.27 | .062879 | 8 |
| 53 | .815924 | 2.42 | .878547 | 1.82 | .937377 | 4.25 | .062623 | 7 |
| 54 | .816069 | 2.43 | .878438 | 1.83 | .937632 | 4.25 | .062368 | 6 |
| 55 | .816215 | 2.43 | .878328 | 1.82 | .937887 | 4.25 | .062113 | 5 |
| 56 | .816361 | 2.43 | .878219 | 1.83 | .938142 | 4.27 | .061858 | 4 |
| 57 | .816507 | 2.43 | .878109 | 1.83 | .938398 | 4.25 | .061602 | 3 |
| 58 | .816652 | 2.42 | .877999 | 1.82 | .938653 | 4.25 | .061347 | 2 |
| 59 | .816798 | 2.43 | .877890 | 1.83 | .938908 | 4.25 | .061092 | 1 |
| 60 | 9.816943 | 2.42 | 9.877780 | 1.83 | 9.939163 | | 10.060837 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

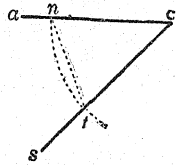
| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.816943 | | 9.877780 | | 9.939163 | | 10.060837 | 60 |
| 1 | .817088 | 2.42 | .877670 | 1.83 | .939418 | 4.25 | .060582 | 59 |
| 2 | .817233 | 2.42 | .877560 | 1.83 | .939673 | 4.25 | .060327 | 58 |
| 3 | .817379 | 2.43 | .877450 | 1.83 | .939928 | 4.25 | .060072 | 57 |
| 4 | .817524 | 2.42 | .877340 | 1.83 | .940183 | 4.25 | .059817 | 56 |
| 5 | .817668 | 2.40 | .877230 | 1.83 | .940439 | 4.27 | .059561 | 55 |
| 6 | .817813 | 2.42 | .877120 | 1.83 | .940694 | 4.25 | .059306 | 54 |
| 7 | .817958 | 2.42 | .877010 | 1.83 | .940949 | 4.25 | .059051 | 53 |
| 8 | .818103 | 2.42 | .876899 | 1.85 | .941204 | 4.25 | .058796 | 52 |
| 9 | .818247 | 2.40 | .876789 | 1.83 | .941459 | 4.25 | .058541 | 51 |
| 10 | .818392 | 2.42 | .876678 | 1.85 | .941713 | 4.23 | .058287 | 50 |
| | | 2.40 | | 1.83 | | 4.25 | | |
| 11 | 9.818536 | 2.42 | 9.876568 | 1.85 | 9.941968 | 4.25 | 10.058032 | 49 |
| 12 | .818681 | 2.40 | .876457 | 1.83 | .942223 | 4.25 | .057777 | 48 |
| 13 | .818825 | 2.40 | .876347 | 1.85 | .942478 | 4.25 | .057522 | 47 |
| 14 | .818969 | 2.40 | .876236 | 1.85 | .942733 | 4.25 | .057267 | 46 |
| 15 | .819113 | 2.40 | .876125 | 1.85 | .942988 | 4.25 | .057012 | 45 |
| 16 | .819257 | 2.40 | .876014 | 1.83 | .943243 | 4.25 | .056757 | 44 |
| 17 | .819401 | 2.40 | .875904 | 1.85 | .943498 | 4.23 | .056502 | 43 |
| 18 | .819545 | 2.40 | .875793 | 1.85 | .943753 | 4.25 | .056248 | 42 |
| 19 | .819689 | 2.38 | .875682 | 1.85 | .944007 | 4.25 | .055993 | 41 |
| 20 | .819832 | 2.40 | .875571 | 1.87 | .944262 | 4.25 | .055738 | 40 |
| 21 | 9.819976 | 2.40 | 9.875459 | 1.85 | 9.944517 | 4.23 | 10.055483 | 39 |
| 22 | .820120 | 2.38 | .875348 | 1.85 | .944771 | 4.25 | .055229 | 38 |
| 23 | .820263 | 2.38 | .875237 | 1.85 | .945026 | 4.25 | .054974 | 37 |
| 24 | .820406 | 2.40 | .875126 | 1.87 | .945281 | 4.23 | .054719 | 36 |
| 25 | .820550 | 2.38 | .875014 | 1.85 | .945535 | 4.25 | .054465 | 35 |
| 26 | .820693 | 2.38 | .874903 | 1.87 | .945790 | 4.25 | .054210 | 34 |
| 27 | .820836 | 2.38 | .874791 | 1.85 | .946045 | 4.23 | .053955 | 33 |
| 28 | .820979 | 2.38 | .874680 | 1.87 | .946299 | 4.25 | .053701 | 32 |
| 29 | .821122 | 2.38 | .874568 | 1.87 | .946554 | 4.23 | .053446 | 31 |
| 30 | .821265 | 2.37 | .874456 | 1.87 | .946808 | 4.25 | .053192 | 30 |
| 31 | 9.821407 | 2.38 | 9.874344 | 1.87 | 9.947063 | 4.25 | 10.052937 | 29 |
| 32 | .821550 | 2.38 | .874232 | 1.85 | .947318 | 4.23 | .052682 | 28 |
| 33 | .821693 | 2.37 | .874121 | 1.87 | .947572 | 4.25 | .052428 | 27 |
| 34 | .821835 | 2.37 | .874009 | 1.87 | .947827 | 4.23 | .052173 | 26 |
| 35 | .821977 | 2.38 | .873896 | 1.88 | .948081 | 4.23 | .051919 | 25 |
| 36 | .822120 | 2.37 | .873784 | 1.87 | .948335 | 4.25 | .051665 | 24 |
| 37 | .822262 | 2.37 | .873672 | 1.87 | .948590 | 4.23 | .051410 | 23 |
| 38 | .822404 | 2.37 | .873560 | 1.87 | .948844 | 4.25 | .051156 | 22 |
| 39 | .822546 | 2.37 | .873448 | 1.88 | .949099 | 4.23 | .050901 | 21 |
| 40 | .822688 | 2.37 | .873335 | 1.87 | .949353 | 4.25 | .050647 | 20 |
| 41 | 9.822830 | 2.37 | 9.873223 | 1.88 | 9.949608 | 4.23 | 10.050392 | 19 |
| 42 | .822972 | 2.37 | .873110 | 1.87 | .949862 | 4.23 | .050138 | 18 |
| 43 | .823114 | 2.35 | .872998 | 1.88 | .950116 | 4.25 | .049884 | 17 |
| 44 | .823255 | 2.37 | .872885 | 1.88 | .950371 | 4.23 | .049629 | 16 |
| 45 | .823397 | 2.37 | .872772 | 1.88 | .950625 | 4.23 | .049375 | 15 |
| 46 | .823539 | 2.35 | .872659 | 1.87 | .950879 | 4.23 | .049121 | 14 |
| 47 | .823680 | 2.35 | .872547 | 1.88 | .951133 | 4.25 | .048867 | 13 |
| 48 | .823821 | 2.37 | .872434 | 1.88 | .951388 | 4.23 | .048612 | 12 |
| 49 | .823963 | 2.35 | .872321 | 1.88 | .951642 | 4.23 | .048358 | 11 |
| 50 | .824104 | 2.35 | .872208 | 1.88 | .951896 | 4.23 | .048104 | 10 |
| 51 | 9.824245 | 2.35 | 9.872095 | 1.90 | 9.952150 | 4.25 | 10.047850 | 9 |
| 52 | .824386 | 2.35 | .871981 | 1.88 | .952405 | 4.23 | .047595 | 8 |
| 53 | .824527 | 2.35 | .871868 | 1.88 | .952659 | 4.23 | .047341 | 7 |
| 54 | .824668 | 2.33 | .871755 | 1.90 | .952913 | 4.23 | .047087 | 6 |
| 55 | .824808 | 2.35 | .871641 | 1.88 | .953167 | 4.23 | .046833 | 5 |
| 56 | .824949 | 2.35 | .871528 | 1.90 | .953421 | 4.23 | .046579 | 4 |
| 57 | .825090 | 2.33 | .871414 | 1.88 | .953675 | 4.23 | .046325 | 3 |
| 58 | .825230 | 2.35 | .871301 | 1.90 | .953929 | 4.23 | .046071 | 2 |
| 59 | .825371 | 2.33 | .871187 | 1.90 | .954183 | 4.23 | .045817 | 1 |
| 60 | 9.825511 | | 9.871073 | | 9.954437 | | 10.045563 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.825511 | 2.33 | 9.871073 | 1.88 | 9.954437 | 4.23 | 10.045563 | 60 |
| 1 | .825651 | 2.33 | .870960 | 1.90 | .954691 | 4.23 | .045309 | 59 |
| 2 | .825791 | 2.33 | .870846 | 1.90 | .954946 | 4.23 | .045054 | 58 |
| 3 | .825931 | 2.33 | .870732 | 1.90 | .955200 | 4.23 | .044800 | 57 |
| 4 | .826071 | 2.33 | .870618 | 1.90 | .955454 | 4.23 | .044546 | 56 |
| 5 | .826211 | 2.33 | .870504 | 1.90 | .955708 | 4.23 | .044292 | 55 |
| 6 | .826351 | 2.33 | .870390 | 1.90 | .955961 | 4.23 | .044039 | 54 |
| 7 | .826491 | 2.33 | .870276 | 1.92 | .956215 | 4.23 | .043785 | 53 |
| 8 | .826631 | 2.32 | .870161 | 1.90 | .956469 | 4.23 | .043531 | 52 |
| 9 | .826770 | 2.33 | .870047 | 1.90 | .956723 | 4.23 | .043277 | 51 |
| 10 | .826910 | 2.32 | .869933 | 1.92 | .956977 | 4.23 | .043023 | 50 |
| 11 | 9.827049 | 2.33 | 9.869818 | 1.90 | 9.957231 | 4.23 | 10.042769 | 49 |
| 12 | .827189 | 2.32 | .869704 | 1.92 | .957485 | 4.23 | .042515 | 48 |
| 13 | .827328 | 2.32 | .869589 | 1.92 | .957739 | 4.23 | .042261 | 47 |
| 14 | .827467 | 2.32 | .869474 | 1.90 | .957993 | 4.23 | .042007 | 46 |
| 15 | .827606 | 2.32 | .869360 | 1.92 | .958247 | 4.22 | .041753 | 45 |
| 16 | .827745 | 2.32 | .869245 | 1.92 | .958500 | 4.23 | .041500 | 44 |
| 17 | .827884 | 2.32 | .869130 | 1.92 | .958754 | 4.23 | .041246 | 43 |
| 18 | .828023 | 2.32 | .869015 | 1.92 | .959008 | 4.23 | .040992 | 42 |
| 19 | .828162 | 2.32 | .868900 | 1.92 | .959262 | 4.23 | .040738 | 41 |
| 20 | .828301 | 2.30 | .868785 | 1.92 | .959516 | 4.22 | .040484 | 40 |
| 21 | 9.828439 | 2.32 | 9.868670 | 1.92 | 9.959769 | 4.23 | 10.040231 | 39 |
| 22 | .828578 | 2.30 | .868555 | 1.92 | .960023 | 4.23 | .039977 | 38 |
| 23 | .828716 | 2.32 | .868440 | 1.93 | .960277 | 4.22 | .039723 | 37 |
| 24 | .828855 | 2.30 | .868324 | 1.93 | .960530 | 4.23 | .039470 | 36 |
| 25 | .828993 | 2.30 | .868209 | 1.93 | .960784 | 4.23 | .039216 | 35 |
| 26 | .829131 | 2.30 | .868093 | 1.92 | .961038 | 4.23 | .038962 | 34 |
| 27 | .829269 | 2.30 | .867978 | 1.93 | .961292 | 4.22 | .038708 | 33 |
| 28 | .829407 | 2.30 | .867862 | 1.92 | .961545 | 4.23 | .038455 | 32 |
| 29 | .829545 | 2.30 | .867747 | 1.93 | .961799 | 4.23 | .038201 | 31 |
| 30 | .829683 | 2.30 | .867631 | 1.93 | .962052 | 4.22 | .037948 | 30 |
| 31 | 9.829821 | 2.30 | 9.867515 | 1.93 | 9.962306 | 4.22 | 10.037694 | 29 |
| 32 | .829959 | 2.30 | .867399 | 1.93 | .962560 | 4.22 | .037440 | 28 |
| 33 | .830097 | 2.28 | .867283 | 1.93 | .962813 | 4.23 | .037187 | 27 |
| 34 | .830234 | 2.30 | .867167 | 1.93 | .963067 | 4.22 | .036933 | 26 |
| 35 | .830372 | 2.28 | .867051 | 1.93 | .963320 | 4.23 | .036680 | 25 |
| 36 | .830509 | 2.28 | .866935 | 1.93 | .963574 | 4.23 | .036426 | 24 |
| 37 | .830646 | 2.30 | .866819 | 1.93 | .963828 | 4.22 | .036172 | 23 |
| 38 | .830784 | 2.28 | .866703 | 1.93 | .964081 | 4.22 | .035919 | 22 |
| 39 | .830921 | 2.28 | .866586 | 1.92 | .964335 | 4.22 | .035665 | 21 |
| 40 | .831058 | 2.28 | .866470 | 1.95 | .964588 | 4.23 | .035412 | 20 |
| 41 | 9.831195 | 2.28 | 9.866353 | 1.93 | 9.964842 | 4.22 | 10.035158 | 19 |
| 42 | .831332 | 2.28 | .866237 | 1.95 | .965095 | 4.23 | .034905 | 18 |
| 43 | .831469 | 2.28 | .866120 | 1.93 | .965349 | 4.22 | .034651 | 17 |
| 44 | .831606 | 2.27 | .866004 | 1.95 | .965602 | 4.23 | .034398 | 16 |
| 45 | .831742 | 2.28 | .865887 | 1.95 | .965855 | 4.23 | .034145 | 15 |
| 46 | .831879 | 2.27 | .865770 | 1.95 | .966109 | 4.22 | .033891 | 14 |
| 47 | .832015 | 2.28 | .865653 | 1.95 | .966362 | 4.23 | .033638 | 13 |
| 48 | .832152 | 2.27 | .865536 | 1.95 | .966616 | 4.22 | .033384 | 12 |
| 49 | .832288 | 2.28 | .865419 | 1.95 | .966869 | 4.23 | .033131 | 11 |
| 50 | .832425 | 2.27 | .865302 | 1.95 | .967123 | 4.22 | .032877 | 10 |
| 51 | 9.832561 | 2.27 | 9.865185 | 1.95 | 9.967376 | 4.23 | 10.032624 | 9 |
| 52 | .832697 | 2.27 | .865068 | 1.97 | .967629 | 4.23 | .032371 | 8 |
| 53 | .832833 | 2.27 | .864950 | 1.95 | .967883 | 4.22 | .032117 | 7 |
| 54 | .832969 | 2.27 | .864833 | 1.95 | .968136 | 4.23 | .031864 | 6 |
| 55 | .833105 | 2.27 | .864716 | 1.97 | .968389 | 4.23 | .031611 | 5 |
| 56 | .833241 | 2.27 | .864598 | 1.95 | .968643 | 4.22 | .031357 | 4 |
| 57 | .833377 | 2.25 | .864481 | 1.97 | .968896 | 4.22 | .031104 | 3 |
| 58 | .833513 | 2.27 | .864363 | 1.97 | .969149 | 4.23 | .030851 | 2 |
| 59 | .833648 | 2.25 | .864245 | 1.97 | .969402 | 4.23 | .030597 | 1 |
| 60 | 9.833783 | | 9.864127 | | 9.969656 | | 10.030344 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| 0 | 9.833783 | | 9.864127 | | 9.969656 | | 10.030344 | 60 |
| 1 | .833919 | 2.27 | .864010 | 1.95 | .969909 | 4.22 | .030091 | 59 |
| 2 | .834054 | 2.25 | .863892 | 1.97 | .970162 | 4.22 | .029838 | 58 |
| 3 | .834189 | 2.25 | .863774 | 1.97 | .970416 | 4.22 | .029584 | 57 |
| 4 | .834325 | 2.27 | .863656 | 1.97 | .970669 | 4.22 | .029331 | 56 |
| 5 | .834460 | 2.25 | .863538 | 1.97 | .970922 | 4.22 | .029078 | 55 |
| 6 | .834595 | 2.25 | .863419 | 1.98 | .971175 | 4.22 | .028825 | 54 |
| 7 | .834730 | 2.25 | .863301 | 1.97 | .971429 | 4.22 | .028571 | 53 |
| 8 | .834865 | 2.25 | .863183 | 1.97 | .971682 | 4.22 | .028318 | 52 |
| 9 | .834999 | 2.23 | .863064 | 1.98 | .971935 | 4.22 | .028065 | 51 |
| 10 | .835134 | 2.25 | .862946 | 1.97 | .972188 | 4.22 | .027812 | 50 |
| 11 | 9.835269 | 2.23 | 9.862827 | 1.97 | 9.972441 | 4.22 | 10.027559 | 49 |
| 12 | .835403 | 2.25 | .862709 | 1.98 | .972695 | 4.22 | .027305 | 48 |
| 13 | .835538 | 2.23 | .862590 | 1.98 | .972948 | 4.22 | .027052 | 47 |
| 14 | .835672 | 2.23 | .862471 | 1.98 | .973201 | 4.22 | .026799 | 46 |
| 15 | .835807 | 2.25 | .862353 | 1.97 | .973454 | 4.22 | .026546 | 45 |
| 16 | .835941 | 2.23 | .862234 | 1.98 | .973707 | 4.22 | .026293 | 44 |
| 17 | .836075 | 2.23 | .862115 | 1.98 | .973960 | 4.22 | .026040 | 43 |
| 18 | .836209 | 2.23 | .861996 | 1.98 | .974213 | 4.22 | .025787 | 42 |
| 19 | .836343 | 2.23 | .861877 | 1.98 | .974466 | 4.23 | .025534 | 41 |
| 20 | .836477 | 2.23 | .861758 | 1.98 | .974720 | 4.22 | .025280 | 40 |
| 21 | 9.836611 | 2.23 | 9.861638 | 1.98 | 9.974973 | 4.22 | 10.025027 | 39 |
| 22 | .836745 | 2.22 | .861519 | 1.98 | .975226 | 4.22 | .024774 | 38 |
| 23 | .836878 | 2.23 | .861400 | 1.98 | .975479 | 4.22 | .024521 | 37 |
| 24 | .837012 | 2.23 | .861280 | 2.00 | .975732 | 4.22 | .024268 | 36 |
| 25 | .837146 | 2.23 | .861161 | 1.98 | .975985 | 4.22 | .024015 | 35 |
| 26 | .837279 | 2.22 | .861041 | 2.00 | .976238 | 4.22 | .023762 | 34 |
| 27 | .837413 | 2.23 | .860922 | 1.98 | .976491 | 4.22 | .023509 | 33 |
| 28 | .837546 | 2.22 | .860802 | 2.00 | .976744 | 4.22 | .023256 | 32 |
| 29 | .837679 | 2.22 | .860682 | 2.00 | .976997 | 4.22 | .023003 | 31 |
| 30 | .837812 | 2.22 | .860562 | 2.00 | .977250 | 4.22 | .022750 | 30 |
| 31 | 9.837945 | 2.22 | 9.860442 | 2.00 | 9.977503 | 4.22 | 10.022497 | 29 |
| 32 | .838078 | 2.22 | .860322 | 2.00 | .977756 | 4.22 | .022244 | 28 |
| 33 | .838211 | 2.22 | .860202 | 2.00 | .978009 | 4.22 | .021991 | 27 |
| 34 | .838344 | 2.22 | .860082 | 2.00 | .978262 | 4.22 | .021738 | 26 |
| 35 | .838477 | 2.22 | .859962 | 2.00 | .978515 | 4.22 | .021485 | 25 |
| 36 | .838610 | 2.20 | .859842 | 2.02 | .978768 | 4.22 | .021232 | 24 |
| 37 | .838742 | 2.22 | .859721 | 2.00 | .979021 | 4.22 | .020979 | 23 |
| 38 | .838875 | 2.20 | .859601 | 2.02 | .979274 | 4.22 | .020726 | 22 |
| 39 | .839007 | 2.22 | .859480 | 2.00 | .979527 | 4.22 | .020473 | 21 |
| 40 | .839140 | 2.20 | .859360 | 2.02 | .979780 | 4.22 | .020220 | 20 |
| 41 | 9.839272 | 2.20 | 9.859239 | 2.00 | 9.980033 | 4.22 | 10.019967 | 19 |
| 42 | .839404 | 2.20 | .859119 | 2.02 | .980286 | 4.20 | .019714 | 18 |
| 43 | .839536 | 2.20 | .858998 | 2.02 | .980538 | 4.22 | .019462 | 17 |
| 44 | .839668 | 2.20 | .858877 | 2.02 | .980791 | 4.22 | .019209 | 16 |
| 45 | .839800 | 2.20 | .858756 | 2.02 | .981044 | 4.22 | .018956 | 15 |
| 46 | .839932 | 2.20 | .858635 | 2.02 | .981297 | 4.22 | .018703 | 14 |
| 47 | .840064 | 2.20 | .858514 | 2.02 | .981550 | 4.22 | .018450 | 13 |
| 48 | .840196 | 2.20 | .858393 | 2.02 | .981803 | 4.22 | .018197 | 12 |
| 49 | .840328 | 2.18 | .858272 | 2.02 | .982056 | 4.22 | .017944 | 11 |
| 50 | .840459 | 2.20 | .858151 | 2.03 | .982309 | 4.22 | .017691 | 10 |
| 51 | 9.840591 | 2.18 | 9.858029 | 2.02 | 9.982562 | 4.20 | 10.017438 | 9 |
| 52 | .840722 | 2.20 | .857908 | 2.03 | .982814 | 4.22 | .017186 | 8 |
| 53 | .840854 | 2.18 | .857786 | 2.02 | .983067 | 4.22 | .016933 | 7 |
| 54 | .840985 | 2.18 | .857665 | 2.03 | .983320 | 4.22 | .016680 | 6 |
| 55 | .841116 | 2.18 | .857543 | 2.03 | .983573 | 4.22 | .016427 | 5 |
| 56 | .841247 | 2.18 | .857422 | 2.02 | .983826 | 4.22 | .016174 | 4 |
| 57 | .841378 | 2.18 | .857300 | 2.03 | .984079 | 4.22 | .015921 | 3 |
| 58 | .841509 | 2.18 | .857178 | 2.03 | .984332 | 4.20 | .015668 | 2 |
| 59 | .841640 | 2.18 | .857056 | 2.03 | .984584 | 4.22 | .015416 | 1 |
| 60 | 9.841771 | | 9.856934 | | 9.984837 | | 10.015163 | 0 |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

| ′ | Seno. | D. 1″. | Coseno. | D. 1″. | Tang. | D. 1″. | Cotang. | ′ |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | 10.015163 | 60 |
| 0 | 9.841771 | 2.18 | 9.856934 | 2.03 | 9.984837 | 4.22 | .014910 | 59 |
| 1 | .841902 | 2.18 | .856812 | 2.03 | .985090 | 4.22 | .014657 | 58 |
| 2 | .842033 | 2.17 | .856690 | 2.03 | .985343 | 4.22 | .014404 | 57 |
| 3 | .842163 | 2.18 | .856568 | 2.03 | .985596 | 4.20 | .014152 | 56 |
| 4 | .842294 | 2.17 | .856446 | 2.05 | .985848 | 4.22 | .013899 | 55 |
| 5 | .842424 | 2.18 | .856323 | 2.03 | .986101 | 4.22 | .013646 | 54 |
| 6 | .842555 | 2.17 | .856201 | 2.05 | .986354 | 4.22 | .013393 | 53 |
| 7 | .842685 | 2.17 | .856078 | 2.03 | .986607 | 4.22 | .013140 | 52 |
| 8 | .842815 | 2.18 | .855956 | 2.05 | .986860 | 4.20 | .012888 | 51 |
| 9 | .842946 | 2.17 | .855833 | 2.03 | .987112 | 4.22 | .012635 | 50 |
| 10 | .843076 | 2.17 | .855711 | 2.05 | .987365 | 4.22 | | |
| | | | | | | | 10.012382 | 49 |
| 11 | 9.843206 | 2.17 | 9.855588 | 2.05 | 9.987618 | 4.22 | .012129 | 48 |
| 12 | .843336 | 2.17 | .855465 | 2.05 | .987871 | 4.20 | .011877 | 47 |
| 13 | .843466 | 2.15 | .855342 | 2.05 | .988123 | 4.22 | .011624 | 46 |
| 14 | .843595 | 2.17 | .855219 | 2.05 | .988376 | 4.22 | .011371 | 45 |
| 15 | .843725 | 2.17 | .855096 | 2.05 | .988629 | 4.22 | .011118 | 44 |
| 16 | .843855 | 2.15 | .854973 | 2.05 | .988882 | 4.20 | .010866 | 43 |
| 17 | .843984 | 2.17 | .854850 | 2.05 | .989134 | 4.22 | .010613 | 42 |
| 18 | .844114 | 2.15 | .854727 | 2.07 | .989387 | 4.22 | .010360 | 41 |
| 19 | .844243 | 2.15 | .854603 | 2.05 | .989640 | 4.22 | .010107 | 40 |
| 20 | .844372 | 2.17 | .854480 | 2.07 | .989893 | 4.20 | | |
| | | | | | | | 10.009855 | 39 |
| 21 | 9.844502 | 2.15 | 9.854356 | 2.05 | 9.990145 | 4.22 | .009602 | 38 |
| 22 | .844631 | 2.15 | .854233 | 2.07 | .990398 | 4.22 | .009349 | 37 |
| 23 | .844760 | 2.15 | .854109 | 2.05 | .990651 | 4.20 | .009097 | 36 |
| 24 | .844889 | 2.15 | .853986 | 2.07 | .990903 | 4.22 | .008844 | 35 |
| 25 | .845018 | 2.15 | .853862 | 2.07 | .991156 | 4.22 | .008591 | 34 |
| 26 | .845147 | 2.15 | .853738 | 2.07 | .991409 | 4.22 | .008338 | 33 |
| 27 | .845276 | 2.15 | .853614 | 2.07 | .991662 | 4.20 | .008086 | 32 |
| 28 | .845405 | 2.13 | .853490 | 2.07 | .991914 | 4.22 | .007833 | 31 |
| 29 | .845533 | 2.15 | .853366 | 2.07 | .992167 | 4.22 | .007580 | 30 |
| 30 | .845662 | 2.13 | .853242 | 2.07 | .992420 | 4.20 | | |
| | | | | | | | 10.007328 | 29 |
| 31 | 9.845790 | 2.15 | 9.853118 | 2.07 | 9.992672 | 4.22 | .007075 | 28 |
| 32 | .845919 | 2.13 | .852994 | 2.08 | .992925 | 4.22 | .006822 | 27 |
| 33 | .846047 | 2.13 | .852869 | 2.07 | .993178 | 4.22 | .006569 | 26 |
| 34 | .846175 | 2.15 | .852745 | 2.08 | .993431 | 4.20 | .006317 | 25 |
| 35 | .846304 | 2.13 | .852620 | 2.07 | .993683 | 4.22 | .006064 | 24 |
| 36 | .846432 | 2.13 | .852496 | 2.08 | .993936 | 4.22 | .005811 | 23 |
| 37 | .846560 | 2.13 | .852371 | 2.07 | .994189 | 4.20 | .005559 | 22 |
| 38 | .846688 | 2.13 | .852247 | 2.08 | .994441 | 4.22 | .005306 | 21 |
| 39 | .846816 | 2.13 | .852122 | 2.08 | .994694 | 4.22 | .005053 | 20 |
| 40 | .846944 | 2.13 | .851997 | 2.08 | .994947 | 4.20 | | |
| | | | | | | | 10.004801 | 19 |
| 41 | 9.847071 | 2.13 | 9.851872 | 2.08 | 9.995199 | 4.22 | .004548 | 18 |
| 42 | .847199 | 2.13 | .851747 | 2.08 | .995452 | 4.22 | .004295 | 17 |
| 43 | .847327 | 2.13 | .851622 | 2.08 | .995705 | 4.20 | .004043 | 16 |
| 44 | .847454 | 2.13 | .851497 | 2.08 | .995957 | 4.22 | .003790 | 15 |
| 45 | .847582 | 2.13 | .851372 | 2.10 | .996210 | 4.22 | .003537 | 14 |
| 46 | .847709 | 2.12 | .851246 | 2.08 | .996463 | 4.20 | .003285 | 13 |
| 47 | .847836 | 2.12 | .851121 | 2.08 | .996715 | 4.22 | .003032 | 12 |
| 48 | .847964 | 2.13 | .850996 | 2.10 | .996968 | 4.22 | .002779 | 11 |
| 49 | .848091 | 2.12 | .850870 | 2.08 | .997221 | 4.20 | .002527 | 10 |
| 50 | .848218 | 2.12 | .850745 | 2.10 | .997473 | 4.22 | | |
| | | | | | | | 10.002274 | 9 |
| 51 | 9.848345 | 2.12 | 9.850619 | 2.10 | 9.997726 | 4.22 | .002021 | 8 |
| 52 | .848472 | 2.12 | .850493 | 2.08 | .997979 | 4.20 | .001769 | 7 |
| 53 | .848599 | 2.12 | .850368 | 2.10 | .998231 | 4.22 | .001516 | 6 |
| 54 | .848726 | 2.12 | .850242 | 2.10 | .998484 | 4.22 | .001263 | 5 |
| 55 | .848852 | 2.12 | .850116 | 2.10 | .998737 | 4.20 | .001011 | 4 |
| 56 | .848979 | 2.12 | .849990 | 2.10 | .998989 | 4.22 | .000758 | 3 |
| 57 | .849106 | 2.10 | .849864 | 2.10 | .999242 | 4.22 | .000505 | 2 |
| 58 | .849232 | 2.12 | .849738 | 2.12 | .999495 | 4.20 | .000253 | 1 |
| 59 | .849359 | 2.10 | .849611 | 2.10 | .999747 | 4.22 | 10.000000 | 0 |
| 60 | 9.849485 | | 9.849485 | | 10.000000 | | | |
| ′ | Coseno. | D. 1″. | Seno. | D. 1″. | Cotang. | D. 1″. | Tang. | ′ |

**La tabla de las cuerdas** que sigue nos permite trazar ángulos sobre el papel más exactamente que con un transportador ordinario. Para hacer esto, después de haber trazado y medido el primer lado (digamos *ac*) de la figura que será delineada; desde su extremo *c* como centro se describe un arco de círculo *n* y de suficiente longitud. El radio *cn* del arco, debe ser lo más grande posible y se debe suponer igual á la unidad. Divídaselo y subdivídaselo decimalmente para usarlo como escala para representar las cuerdas que se tomen de la tabla, en la cual las longitudes de éstas están dadas en partes de dicho radio 1. Habiendo descripto el arco,




búsquese en la tabla la longitud de la cuerda *nt* que corresponde al ángulo *nct*. Supongamos que este ángulo sea de 45° y encontramos entonces que la cuerda de la tabla es .7654 de nuestro radio 1. Por tanto, desde *n* trazamos la cuerda *nt* igual á .7654 del radio elegido y la línea *cs* tirada por el punto *t* formará el ángulo requerido *act* de 45°. Así para cada ángulo. El grado de exactitud que se obtenga dependerá evidentemente de la longitud del radio y de la precisión de los instrumentos. Este método se hace preferible al del transportador á proporción que la longitud de los lados de los ángulos exceda el radio del transportador. Con un trasportador de 10 á 15 centímetros de radio, y si los lados de los ángulos no exceden en mucho á los mismos límites del transportador, será generalmente preferible éste. Los compases de división de las cajas de instrumentos son rara vez adecuados para trazar arcos exactos de más de 15 cm de diámetro. En la práctica no es necesario describir efectivamente todo el arco sino simplemente la parte próxima á *t*. Así evitamos el frecuente uso de la goma y también gastarle la punta al lápiz. Para radios mayores podemos prescindir del compás y usar una tira de papel estrecha con la longitud del radio marcada en una orilla, colocándola de *c* hacia *s* y colocando al mismo tiempo otra tira (con una orilla dividida en partes del radio) de *n* hacia *t*, podemos por tanteo encontrar el punto exacto de intersección en *t*. En semejantes casos la práctica y un poco de ingenio son muy esenciales para obtener resultados satisfactorios. No podemos dedicar mayor espacio á este asunto.

# CUERDAS PARA UN RADIO 1

| M. | 0° | 1° | 2° | 3° | 4° | 5° | 6° | 7° | 8° | 9° | 10° | M. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0' | .0000 | .0175 | .0349 | .0524 | .0698 | .0872 | .1047 | .1221 | .1395 | .1569 | .1743 | 0 |
| 2 | .0006 | .0180 | .0355 | .0529 | .0704 | .0878 | .1053 | .1227 | .1401 | .1576 | .1749 | 2 |
| 4 | .0012 | .0186 | .0361 | .0535 | .0716 | .0884 | .1058 | .1233 | .1407 | .1581 | .1755 | 4 |
| 6 | .0017 | .0192 | .0366 | .0541 | .0715 | .0890 | .1064 | .1238 | .1413 | .1587 | .1761 | 6 |
| 8 | .0023 | .0198 | .0372 | .0547 | .0721 | .0896 | .1070 | .1244 | .1418 | .1592 | .1768 | 8 |
| 10 | .0029 | .0204 | .0378 | .0553 | .0727 | .0901 | .1076 | .1250 | .1424 | .1598 | .1772 | 10 |
| 12 | .0035 | .0209 | .0384 | .0558 | .0733 | .0907 | .1082 | .1256 | .1430 | .1604 | .1778 | 12 |
| 14 | .0041 | .0215 | .0390 | .0564 | .0739 | .0913 | .1087 | .1262 | .1436 | .1610 | .1784 | 14 |
| 16 | .0047 | .0221 | .0396 | .0570 | .0745 | .0919 | .1093 | .1267 | .1442 | .1616 | .1789 | 16 |
| 18 | .0052 | .0227 | .0401 | .0576 | .0750 | .0925 | .1099 | .1273 | .1447 | .1621 | .1795 | 18 |
| 20 | .0058 | .0233 | .0407 | .0582 | .0756 | .0931 | .1105 | .1279 | .1453 | .1627 | .1801 | 20 |
| 22 | .0064 | .0239 | .0413 | .0588 | .0762 | .0936 | .1111 | .1285 | .1459 | .1633 | .1807 | 22 |
| 24 | .0070 | .0244 | .0419 | .0593 | .0768 | .0942 | .1116 | .1291 | .1465 | .1639 | .1813 | 24 |
| 26 | .0076 | .0250 | .0425 | .0599 | .0774 | .0948 | .1122 | .1296 | .1471 | .1645 | .1818 | 26 |
| 28 | .0081 | .0256 | .0430 | .0605 | .0779 | .0954 | .1128 | .1302 | .1476 | .1650 | .1824 | 28 |
| 30 | .0087 | .0262 | .0436 | .0611 | .0785 | .0960 | .1134 | .1308 | .1482 | .1656 | .1830 | 30 |
| 32 | .0093 | .0268 | .0442 | .0617 | .0791 | .0965 | .1140 | .1314 | .1488 | .1662 | .1836 | 32 |
| 34 | .0099 | .0273 | .0448 | .0622 | .0797 | .0971 | .1145 | .1320 | .1494 | .1668 | .1842 | 34 |
| 36 | .0105 | .0279 | .0454 | .0628 | .0803 | .0977 | .1151 | .1325 | .1500 | .1674 | .1847 | 36 |
| 38 | .0111 | .0285 | .0460 | .0634 | .0809 | .0983 | .1157 | .1331 | .1505 | .1679 | .1853 | 38 |
| 40 | .0116 | .0291 | .0465 | .0640 | .0814 | .0989 | .1163 | .1337 | .1511 | .1685 | .1859 | 40 |
| 42 | .0122 | .0297 | .0471 | .0646 | .0820 | .0994 | .1169 | .1343 | .1517 | .1691 | .1865 | 42 |
| 44 | .0128 | .0303 | .0477 | .0651 | .0826 | .1000 | .1175 | .1349 | .1523 | .1697 | .1871 | 44 |
| 46 | .0134 | .0308 | .0483 | .0657 | .0832 | .1006 | .1180 | .1355 | .1529 | .1703 | .1876 | 46 |
| 48 | .0140 | .0314 | .0489 | .0663 | .0838 | .1012 | .1186 | .1360 | .1534 | .1708 | .1882 | 48 |
| 50 | .0145 | .0320 | .0494 | .0669 | .0843 | .1018 | .1192 | .1366 | .1540 | .1714 | .1888 | 50 |
| 52 | .0151 | .0326 | .0500 | .0675 | .0849 | .1023 | .1198 | .1372 | .1546 | .1720 | .1894 | 52 |
| 54 | .0157 | .0332 | .0506 | .0681 | .0855 | .1029 | .1204 | .1378 | .1552 | .1726 | .1900 | 54 |
| 56 | .0163 | .0337 | .0512 | .0686 | .0861 | .1035 | .1209 | .1384 | .1558 | .1732 | .1905 | 56 |
| 58 | .0169 | .0343 | .0518 | .0692 | .0867 | .1041 | .1215 | .1389 | .1563 | .1737 | .1911 | 58 |
| 60 | .0175 | .0349 | .0524 | .0698 | .0872 | .1047 | .1221 | .1395 | .1569 | .1743 | .1977 | 60 |

| M. | 11° | 12° | 13° | 14° | 15° | 16° | 17° | 18° | 19° | 20° | M. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0' | .1917 | .2091 | .2264 | .2437 | .2611 | .2783 | .2956 | .3129 | .3301 | .3473 | 0' |
| 2 | .1923 | .2096 | .2270 | .2443 | .2616 | .2789 | .2962 | .3134 | .3307 | .3479 | 2 |
| 4 | .1928 | .2102 | .2276 | .2449 | .2622 | .2795 | .2968 | .3140 | .3312 | .3484 | 4 |
| 6 | .1934 | .2108 | .2281 | .2455 | .2628 | .2801 | .2973 | .3146 | .3318 | .3490 | 6 |
| 8 | .1940 | .2114 | .2287 | .2460 | .2634 | .2807 | .2979 | .3152 | .3324 | .3496 | 8 |
| 10 | .1946 | .2119 | .2293 | .2466 | .2639 | .2812 | .2985 | .3157 | .3330 | .3502 | 10 |
| 12 | .1952 | .2125 | .2299 | .2472 | .2645 | .2818 | .2991 | .3163 | .3335 | .3507 | 12 |
| 14 | .1957 | .2131 | .2305 | .2478 | .2651 | .2824 | .2996 | .3169 | .3341 | .3513 | 14 |
| 16 | .1963 | .2137 | .2310 | .2484 | .2657 | .2830 | .3002 | .3175 | .3347 | .3519 | 16 |
| 18 | .1969 | .2143 | .2316 | .2489 | .2662 | .2835 | .3008 | .3180 | .3353 | .3525 | 18 |
| 20 | .1975 | .2148 | .2322 | .2495 | .2668 | .2841 | .3014 | .3186 | .3358 | .3530 | 20 |
| 22 | .1981 | .2154 | .2328 | .2501 | .2674 | .2847 | .3019 | .3192 | .3364 | .3536 | 22 |
| 24 | .1986 | .2160 | .2333 | .2507 | .2680 | .2853 | .3025 | .3198 | .3370 | .3542 | 24 |
| 26 | .1992 | .2166 | .2339 | .2512 | .2685 | .2858 | .3031 | .3203 | .3376 | .3547 | 26 |
| 28 | .1998 | .2172 | .2345 | .2518 | .2691 | .2864 | .3037 | .3209 | .3381 | .3553 | 28 |
| 30 | .2004 | .2177 | .2351 | .2524 | .2697 | .2870 | .3042 | .3215 | .3387 | .3559 | 30 |
| 32 | .2010 | .2183 | .2357 | .2530 | .2703 | .2876 | .3048 | .3221 | .3393 | .3565 | 32 |
| 34 | .2015 | .2189 | .2362 | .2536 | .2709 | .2881 | .3054 | .3226 | .3398 | .3570 | 34 |
| 36 | .2021 | .2195 | .2368 | .2541 | .2714 | .2887 | .3060 | .3232 | .3404 | .3576 | 36 |
| 38 | .2027 | .2200 | .2374 | .2547 | .2720 | .2893 | .3065 | .3238 | .3410 | .3582 | 38 |
| 40 | .2033 | .2206 | .2380 | .2553 | .2726 | .2899 | .3071 | .3244 | .3416 | .3587 | 40 |
| 42 | .2038 | .2212 | .2385 | .2559 | .2732 | .2904 | .3077 | .3249 | .3421 | .3593 | 42 |
| 44 | .2044 | .2218 | .2391 | .2564 | .2737 | .2910 | .3083 | .3255 | .3427 | .3599 | 44 |
| 46 | .2050 | .2224 | .2397 | .2570 | .2743 | .2916 | .3088 | .3261 | .3433 | .3605 | 46 |
| 48 | .2056 | .2229 | .2403 | .2576 | .2749 | .2922 | .3094 | .3267 | .3439 | .3610 | 48 |
| 50 | .2062 | .2235 | .2409 | .2582 | .2755 | .2927 | .3100 | .3272 | .3444 | .3616 | 50 |
| 52 | .2067 | .2241 | .2414 | .2587 | .2760 | .2933 | .3106 | .3278 | .3450 | .3622 | 52 |
| 54 | .2073 | .2247 | .2420 | .2593 | .2766 | .2939 | .3111 | .3284 | .3456 | .3628 | 54 |
| 56 | .2079 | .2253 | .2426 | .2599 | .2772 | .2945 | .3117 | .3289 | .3462 | .3633 | 56 |
| 58 | .2085 | .2258 | .2432 | .2605 | .2778 | .2950 | .3123 | .3295 | .3467 | .3639 | 58 |
| 60 | .2091 | .2264 | .2437 | .2611 | .2783 | .2956 | .3129 | .3301 | .3473 | .3645 | 60 |

## Cuerdas para un radio 1. (Continuación.)

| M. | 21° | 22° | 23° | 24° | 25° | 26° | 27° | 28° | 29° | 30° | M. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0' | .3645 | .3816 | .3987 | .4158 | .4329 | .4499 | .4668 | .4838 | .5008 | .5176 | 0' |
| 2 | .3650 | .3822 | .3993 | .4164 | .4334 | .4505 | .4675 | .4844 | .5013 | .5182 | 2 |
| 4 | .3656 | .3828 | .3999 | .4170 | .4340 | .4510 | .4680 | .4850 | .5019 | .5188 | 4 |
| 6 | .3662 | .3833 | .4004 | .4175 | .4346 | .4516 | .4686 | .4855 | .5024 | .5193 | 6 |
| 8 | .3668 | .3839 | .4010 | .4181 | .4352 | .4522 | .4692 | .4861 | .5030 | .5199 | 8 |
| 10' | .3673 | .3845 | .4016 | .4187 | .4357 | .4527 | .4697 | .4867 | .5036 | .5204 | 10 |
| 12 | .3679 | .3850 | .4022 | .4192 | .4363 | .4533 | .4703 | .4872 | .5041 | .5210 | 12 |
| 14 | .3685 | .3856 | .4027 | .4198 | .4369 | .4539 | .4708 | .4878 | .5047 | .5216 | 14 |
| 16 | .3690 | .3862 | .4033 | .4204 | .4374 | .4544 | .4714 | .4884 | .5053 | .5221 | 16 |
| 18 | .3696 | .3868 | .4039 | .4209 | .4380 | .4550 | .4720 | .4889 | .5058 | .5227 | 18 |
| 20 | .3702 | .3873 | .4044 | .4215 | .4386 | .4556 | .4725 | .4895 | .5064 | .5233 | 20 |
| 22 | .3708 | .3879 | .4050 | .4221 | .4391 | .4561 | .4731 | .4901 | .5070 | .5238 | 22 |
| 24 | .3713 | .3885 | .4056 | .4226 | .4397 | .4567 | .4737 | .4906 | .5075 | .5244 | 24 |
| 26 | .3719 | .3890 | .4061 | .4232 | .4403 | .4573 | .4742 | .4912 | .5081 | .5249 | 26 |
| 28 | .3725 | .3896 | .4067 | .4238 | .4408 | .4578 | .4748 | .4917 | .5086 | .5255 | 28 |
| 30 | .3730 | .3902 | .4073 | .4244 | .4414 | .4584 | .4754 | .4923 | .5092 | .5261 | 30 |
| 32 | .3736 | .3908 | .4079 | .4249 | .4420 | .4590 | .4759 | .4929 | .5098 | .5266 | 32 |
| 34 | .3742 | .3913 | .4084 | .4255 | .4425 | .4595 | .4765 | .4935 | .5103 | .5272 | 34 |
| 36 | .3748 | .3919 | .4090 | .4261 | .4431 | .4601 | .4771 | .4940 | .5109 | .5277 | 36 |
| 38 | .3753 | .3925 | .4096 | .4266 | .4437 | .4607 | .4776 | .4946 | .5115 | .5283 | 38 |
| 40 | .3759 | .3930 | .4101 | .4272 | .4442 | .4612 | .4782 | .4951 | .5120 | .5289 | 40 |
| 42 | .3765 | .3936 | .4107 | .4278 | .4448 | .4618 | .4788 | .4957 | .5126 | .5294 | 42 |
| 44 | .3770 | .3942 | .4113 | .4283 | .4454 | .4624 | .4793 | .4963 | .5131 | .5300 | 44 |
| 46 | .3776 | .3947 | .4118 | .4289 | .4459 | .4629 | .4799 | .4968 | .5137 | .5306 | 46 |
| 48 | .3782 | .3953 | .4124 | .4295 | .4465 | .4635 | .4805 | .4974 | .5143 | .5311 | 48 |
| 50 | .3788 | .3959 | .4130 | .4300 | .4471 | .4641 | .4810 | .4979 | .5148 | .5317 | 50 |
| 52 | .3793 | .3965 | .4135 | .4306 | .4476 | .4646 | .4816 | .4985 | .5154 | .5322 | 52 |
| 54 | .3799 | .3970 | .4141 | .4312 | .4482 | .4652 | .4822 | .4991 | .5160 | .5328 | 54 |
| 56 | .3805 | .3976 | .4147 | .4317 | .4488 | .4658 | .4827 | .4996 | .5165 | .5334 | 56 |
| 58 | .3810 | .3982 | .4153 | .4323 | .4493 | .4663 | .4833 | .5002 | .5171 | .5339 | 58 |
| 60 | .3816 | .3987 | .4158 | .4329 | .4499 | .4669 | .4838 | .5008 | .5176 | .5345 | 60 |

| M. | 31° | 32° | 33° | 34° | 35° | 36° | 37° | 38° | 39° | 40° | M |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0' | .5345 | .5513 | .5680 | .5847 | .6014 | .6180 | .6346 | .6511 | .6676 | .6840 | 0' |
| 2 | .5350 | .5518 | .5686 | .5853 | .6020 | .6186 | .6352 | .6517 | .6682 | .6846 | 2 |
| 4 | .5356 | .5524 | .5691 | .5859 | .6025 | .6191 | .6357 | .6522 | .6687 | .6851 | 4 |
| 6 | .5362 | .5530 | .5697 | .5864 | .6031 | .6197 | .6363 | .6528 | .6693 | .6857 | 6 |
| 8 | .5367 | .5535 | .5703 | .5870 | .6036 | .6202 | .6368 | .6533 | .6698 | .6862 | 8 |
| 10 | .5373 | .5541 | .5708 | .5875 | .6042 | .6208 | .6374 | .6539 | .6704 | .6868 | 10 |
| 12 | .5378 | .5546 | .5714 | .5881 | .6047 | .6214 | .6379 | .6544 | .6709 | .6873 | 12 |
| 14 | .5384 | .5552 | .5719 | .5886 | .6053 | .6219 | .6385 | .6550 | .6715 | .6879 | 14 |
| 16 | .5390 | .5557 | .5725 | .5892 | .6058 | .6225 | .6390 | .6555 | .6720 | .6884 | 16 |
| 18 | .5395 | .5563 | .5730 | .5897 | .6064 | .6230 | .6396 | .6561 | .6725 | .6890 | 18 |
| 20 | .5401 | .5569 | .5736 | .5903 | .6070 | .6236 | .6401 | .6566 | .6731 | .6895 | 20 |
| 22 | .5406 | .5574 | .5742 | .5909 | .6075 | .6241 | .6407 | .6572 | .6736 | .6901 | 22 |
| 24 | .5412 | .5580 | .5747 | .5914 | .6081 | .6247 | .6412 | .6577 | .6742 | .6906 | 24 |
| 26 | .5418 | .5585 | .5753 | .5920 | .6086 | .6252 | .6418 | .6583 | .6747 | .6911 | 26 |
| 28 | .5423 | .5591 | .5758 | .5925 | .6092 | .6258 | .6423 | .6588 | .6753 | .6917 | 28 |
| 30 | .5429 | .5597 | .5764 | .5931 | .6097 | .6263 | .6429 | .6594 | .6758 | .6922 | 30 |
| 32 | .5434 | .5602 | .5769 | .5936 | .6103 | .6269 | .6434 | .6599 | .6764 | .6928 | 32 |
| 34 | .5440 | .5608 | .5775 | .5942 | .6108 | .6274 | .6440 | .6605 | .6769 | .6933 | 34 |
| 36 | .5446 | .5613 | .5781 | .5947 | .6114 | .6280 | .6445 | .6610 | .6775 | .6939 | 36 |
| 38 | .5451 | .5619 | .5786 | .5953 | .6119 | .6285 | .6451 | .6616 | .6780 | .6944 | 38 |
| 40 | .5457 | .5625 | .5792 | .5959 | .6125 | .6291 | .6456 | .6621 | .6786 | .6950 | 40 |
| 42 | .5462 | .5630 | .5797 | .5964 | .6130 | .6296 | .6462 | .6627 | .6791 | .6955 | 42 |
| 44 | .5468 | .5636 | .5803 | .5970 | .6136 | .6302 | .6467 | .6632 | .6797 | .6961 | 44 |
| 46 | .5474 | .5641 | .5808 | .5975 | .6142 | .6307 | .6473 | .6638 | .6802 | .6966 | 46 |
| 48 | .5479 | .5647 | .5814 | .5981 | .6147 | .6313 | .6478 | .6643 | .6808 | .6971 | 48 |
| 50 | .5485 | .5652 | .5820 | .5986 | .6153 | .6318 | .6484 | .6649 | .6813 | .6977 | 50 |
| 52 | .5490 | .5658 | .5825 | .5992 | .6158 | .6324 | .6489 | .6654 | .6819 | .6982 | 52 |
| 54 | .5496 | .5664 | .5831 | .5997 | .6164 | .6330 | .6495 | .6660 | .6824 | .6988 | 54 |
| 56 | .5502 | .5669 | .5836 | .6003 | .6169 | .6335 | .6500 | .6665 | .6829 | .6993 | 56 |
| 58 | .5507 | .5675 | .5842 | .6009 | .6175 | .6341 | .6506 | .6671 | .6835 | .6999 | 58 |
| 60 | .5513 | .5680 | .5847 | .6014 | .6180 | .6346 | .6511 | .6676 | .6840 | .7004 | 60 |

## Cuerdas para un radio 1. (Continuación.)

| m. | 41° | 42° | 43° | 44° | 45° | 46° | 47° | 48° | 49° | 50° | M. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0' | .7004 | .7167 | .7330 | .7492 | .7654 | .7815 | .7975 | .8135 | .8294 | .8452 | 0' |
| 2 | .7010 | .7173 | .7335 | .7498 | .7659 | .7820 | .7980 | .8140 | .8299 | .8458 | 2 |
| 4 | .7015 | .7178 | .7341 | .7503 | .7664 | .7825 | .7986 | .8145 | .8304 | .8463 | 4 |
| 6 | .7020 | .7184 | .7346 | .7508 | .7670 | .7831 | .7991 | .8151 | .8310 | .8468 | 6 |
| 8 | .7026 | .7189 | .7352 | .7514 | .7675 | .7836 | .7996 | .8156 | .8315 | .8473 | 8 |
| 10 | .7031 | .7195 | .7357 | .7519 | .7681 | .7841 | .8002 | .8161 | .8320 | .8479 | 10 |
| 12 | .7037 | .7200 | .7362 | .7524 | .7686 | .7847 | .8007 | .8167 | .8326 | .8484 | 12 |
| 14 | .7042 | .7205 | .7368 | .7530 | .7691 | .7852 | .8013 | .8172 | .8331 | .8489 | 14 |
| 16 | .7048 | .7211 | .7373 | .7535 | .7697 | .7857 | .8018 | .8177 | .8336 | .8495 | 16 |
| 18 | .7053 | .7216 | .7379 | .7541 | .7702 | .7863 | .8023 | .8183 | .8341 | .8500 | 18 |
| 20 | .7059 | .7222 | .7384 | .7546 | .7707 | .7868 | .8028 | .8188 | .8347 | .8505 | 20 |
| 22 | .7064 | .7227 | .7390 | .7551 | .7713 | .7873 | .8034 | .8193 | .8352 | .8510 | 22 |
| 24 | .7069 | .7232 | .7395 | .7557 | .7718 | .7879 | .8039 | .8198 | .8357 | .8516 | 24 |
| 26 | .7075 | .7238 | .7400 | .7562 | .7723 | .7884 | .8044 | .8204 | .8363 | .8521 | 26 |
| 28 | .7080 | .7243 | .7406 | .7568 | .7729 | .7890 | .8050 | .8209 | .8368 | .8526 | 28 |
| 30 | .7086 | .7248 | .7411 | .7573 | .7734 | .7895 | .8055 | .8214 | .8373 | .8531 | 30 |
| 32 | .7091 | .7254 | .7417 | .7578 | .7740 | .7900 | .8060 | .8220 | .8378 | .8537 | 32 |
| 34 | .7097 | .7260 | .7422 | .7584 | .7745 | .7906 | .8066 | .8225 | .8384 | .8542 | 34 |
| 36 | .7102 | .7265 | .7427 | .7589 | .7750 | .7911 | .8071 | .8230 | .8389 | .8547 | 36 |
| 38 | .7108 | .7270 | .7433 | .7595 | .7756 | .7916 | .8076 | .8236 | .8394 | .8552 | 38 |
| 40 | .7113 | .7276 | .7438 | .7600 | .7761 | .7922 | .8082 | .8241 | .8400 | .8558 | 40 |
| 42 | .7118 | .7281 | .7443 | .7605 | .7766 | .7927 | .8087 | .8246 | .8405 | .8563 | 42 |
| 44 | .7124 | .7287 | .7449 | .7611 | .7772 | .7932 | .8092 | .8251 | .8410 | .8568 | 44 |
| 46 | .7129 | .7292 | .7454 | .7616 | .7777 | .7938 | .8098 | .8257 | .8415 | .8573 | 46 |
| 48 | .7135 | .7298 | .7460 | .7621 | .7782 | .7943 | .8103 | .8262 | .8421 | .8579 | 48 |
| 50 | .7140 | .7303 | .7465 | .7627 | .7788 | .7948 | .8108 | .8267 | .8426 | .8584 | 50 |
| 52 | .7146 | .7308 | .7471 | .7632 | .7793 | .7954 | .8113 | .8273 | .8431 | .8589 | 52 |
| 54 | .7151 | .7314 | .7476 | .7638 | .7799 | .7959 | .8119 | .8278 | .8437 | .8594 | 54 |
| 56 | .7156 | .7319 | .7481 | .7643 | .7804 | .7964 | .8124 | .8283 | .8442 | .8600 | 56 |
| 58 | .7162 | .7325 | .7487 | .7648 | .7809 | .7970 | .8129 | .8289 | .8447 | .8605 | 58 |
| 60 | .7167 | .7330 | .7492 | .7654 | .7815 | .7975 | .8135 | .8294 | .8452 | .8610 | 60 |

| M. | 51° | 52° | 53° | 54° | 55° | 56° | 57° | 58° | 59° | 60° | M. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0' | .8610 | .8767 | .8924 | .9080 | .9235 | .9389 | .9543 | .9696 | .9848 | 1.0000 | 0' |
| 2 | .8615 | .8773 | .8929 | .9085 | .9240 | .9395 | .9548 | .9701 | .9854 | 1.0005 | 2 |
| 4 | .8621 | .8778 | .8934 | .9090 | .9245 | .9400 | .9553 | .9706 | .9859 | 1.0010 | 4 |
| 6 | .8626 | .8783 | .8940 | .9095 | .9250 | .9405 | .9559 | .9711 | .9864 | 1.0015 | 6 |
| 8 | .8631 | .8788 | .8945 | .9101 | .9256 | .9410 | .9564 | .9717 | .9869 | 1.0020 | 8 |
| 10 | .8636 | .8794 | .8950 | .9106 | .9261 | .9415 | .9569 | .9722 | .9874 | 1.0025 | 10 |
| 12 | .8642 | .8799 | .8955 | .9111 | .9266 | .9420 | .9574 | .9727 | .9879 | 1.0030 | 12 |
| 14 | .8647 | .8804 | .8960 | .9116 | .9271 | .9425 | .9579 | .9732 | .9884 | 1.0035 | 14 |
| 16 | .8652 | .8809 | .8966 | .9121 | .9276 | .9430 | .9584 | .9737 | .9889 | 1.0040 | 16 |
| 18 | .8657 | .8814 | .8971 | .9126 | .9281 | .9436 | .9589 | .9742 | .9894 | 1.0045 | 18 |
| 20 | .8663 | .8820 | .8976 | .9132 | .9287 | .9441 | .9594 | .9747 | .9899 | 1.0050 | 20 |
| 22 | .8668 | .8825 | .8981 | .9137 | .9292 | .9446 | .9599 | .9752 | .9904 | 1.0055 | 22 |
| 24 | .8673 | .8830 | .8986 | .9142 | .9297 | .9451 | .9604 | .9757 | .9909 | 1.0060 | 24 |
| 26 | .8678 | .8835 | .8992 | .9147 | .9302 | .9456 | .9610 | .9762 | .9914 | 1.0065 | 26 |
| 28 | .8684 | .8841 | .8997 | .9152 | .9307 | .9461 | .9615 | .9767 | .9919 | 1.0070 | 28 |
| 30 | .8689 | .8846 | .9002 | .9157 | .9312 | .9466 | .9620 | .9772 | .9924 | 1.0075 | 30 |
| 32 | .8694 | .8851 | .9007 | .9163 | .9317 | .9472 | .9625 | .9778 | .9929 | 1.0080 | 32 |
| 34 | .8699 | .8856 | .9012 | .9168 | .9323 | .9477 | .9630 | .9783 | .9934 | 1.0086 | 34 |
| 36 | .8705 | .8861 | .9018 | .9173 | .9328 | .9482 | .9635 | .9788 | .9939 | 1.0091 | 36 |
| 38 | .8710 | .8867 | .9023 | .9178 | .9333 | .9487 | .9640 | .9793 | .9945 | 1.0096 | 38 |
| 40 | .8715 | .8872 | .9028 | .9183 | .9338 | .9492 | .9645 | .9798 | .9950 | 1.0101 | 40 |
| 42 | .8720 | .8877 | .9033 | .9188 | .9343 | .9497 | .9650 | .9803 | .9955 | 1.0106 | 42 |
| 44 | .8726 | .8882 | .9038 | .9194 | .9348 | .9502 | .9655 | .9808 | .9960 | 1.0111 | 44 |
| 46 | .8731 | .8887 | .9044 | .9199 | .9353 | .9507 | .9661 | .9814 | .9965 | 1.0116 | 46 |
| 48 | .8736 | .8893 | .9049 | .9204 | .9359 | .9512 | .9666 | .9818 | .9970 | 1.0121 | 48 |
| 50 | .8741 | .8898 | .9054 | .9209 | .9364 | .9518 | .9671 | .9823 | .9975 | 1.0125 | 50 |
| 52 | .8747 | .8903 | .9059 | .9214 | .9369 | .9523 | .9676 | .9828 | .9980 | 1.0131 | 52 |
| 54 | .8752 | .8908 | .9064 | .9219 | .9374 | .9528 | .9681 | .9833 | .9985 | 1.0136 | 54 |
| 56 | .8757 | .8914 | .9069 | .9225 | .9379 | .9533 | .9686 | .9838 | .9990 | 1.0141 | 56 |
| 58 | .8762 | .8919 | .9075 | .9230 | .9384 | .9538 | .9691 | .9843 | .9995 | 1.0146 | 58 |
| 60 | .8767 | .8924 | .9080 | .9235 | .9389 | .9543 | .9696 | .9848 | 1.0000 | 1.0151 | 60 |

## Cuerdas para un radio 1. (Continuación.)

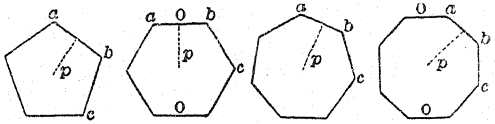
| M. | 61° | 62° | 63° | 64° | 65° | 66° | 67° | 68° | 69° | 70° | M. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0' | 1.0151 | 1.0301 | 1.0450 | 1.0598 | 1.0746 | 1.0893 | 1.1039 | 1.1184 | 1.1328 | 1.1472 | 0' |
| 2 | 1.0156 | 1.0306 | 1.0455 | 1.0603 | 1.0751 | 1.0898 | 1.1044 | 1.1189 | 1.1333 | 1.1476 | 2 |
| 4 | 1.0161 | 1.0311 | 1.0460 | 1.0608 | 1.0756 | 1.0903 | 1.1048 | 1.1194 | 1.1338 | 1.1481 | 4 |
| 6 | 1.0166 | 1.0316 | 1.0465 | 1.0613 | 1.0761 | 1.0907 | 1.1053 | 1.1198 | 1.1342 | 1.1486 | 6 |
| 8 | 1.0171 | 1.0321 | 1.0470 | 1.0618 | 1.0766 | 1.0912 | 1.1058 | 1.1203 | 1.1347 | 1.1491 | 8 |
| 10 | 1.0176 | 1.0326 | 1.0475 | 1.0623 | 1.0771 | 1.0917 | 1.1063 | 1.1208 | 1.1352 | 1.1495 | 10 |
| 12 | 1.0181 | 1.0331 | 1.0480 | 1.0628 | 1.0775 | 1.0922 | 1.1068 | 1.1213 | 1.1357 | 1.1500 | 12 |
| 14 | 1.0186 | 1.0336 | 1.0485 | 1.0633 | 1.0780 | 1.0927 | 1.1073 | 1.1218 | 1.1362 | 1.1505 | 14 |
| 16 | 1.0191 | 1.0341 | 1.0490 | 1.0638 | 1.0785 | 1.0932 | 1.1078 | 1.1222 | 1.1366 | 1.1510 | 16 |
| 18 | 1.0196 | 1.0346 | 1.0495 | 1.0643 | 1.0790 | 1.0937 | 1.1082 | 1.1227 | 1.1371 | 1.1514 | 18 |
| 20 | 1.0201 | 1.0351 | 1.0500 | 1.0648 | 1.0795 | 1.0942 | 1.1087 | 1.1232 | 1.1376 | 1.1519 | 20 |
| 22 | 1.0206 | 1.0356 | 1.0504 | 1.0653 | 1.0800 | 1.0946 | 1.1092 | 1.1237 | 1.1381 | 1.1524 | 22 |
| 24 | 1.0211 | 1.0361 | 1.0509 | 1.0658 | 1.0805 | 1.0951 | 1.1097 | 1.1242 | 1.1386 | 1.1529 | 24 |
| 26 | 1.0216 | 1.0366 | 1.0514 | 1.0662 | 1.0810 | 1.0956 | 1.1102 | 1.1246 | 1.1390 | 1.1533 | 26 |
| 28 | 1.0221 | 1.0370 | 1.0519 | 1.0667 | 1.0815 | 1.0961 | 1.1107 | 1.1251 | 1.1395 | 1.1538 | 28 |
| 30 | 1.0226 | 1.0375 | 1.0524 | 1.0672 | 1.0820 | 1.0966 | 1.1111 | 1.1256 | 1.1400 | 1.1543 | 30 |
| 32 | 1.0231 | 1.0380 | 1.0529 | 1.0677 | 1.0824 | 1.0971 | 1.1116 | 1.1261 | 1.1405 | 1.1548 | 32 |
| 34 | 1.0236 | 1.0385 | 1.0534 | 1.0682 | 1.0829 | 1.0976 | 1.1121 | 1.1266 | 1.1409 | 1.1552 | 34 |
| 36 | 1.0241 | 1.0390 | 1.0539 | 1.0687 | 1.0834 | 1.0980 | 1.1126 | 1.1271 | 1.1414 | 1.1557 | 36 |
| 38 | 1.0246 | 1.0395 | 1.0544 | 1.0692 | 1.0839 | 1.0985 | 1.1131 | 1.1275 | 1.1419 | 1.1562 | 38 |
| 40 | 1.0251 | 1.0400 | 1.0549 | 1.0697 | 1.0844 | 1.0990 | 1.1136 | 1.1280 | 1.1424 | 1.1567 | 40 |
| 42 | 1.0256 | 1.0405 | 1.0554 | 1.0702 | 1.0849 | 1.0995 | 1.1140 | 1.1285 | 1.1429 | 1.1571 | 42 |
| 44 | 1.0261 | 1.0410 | 1.0559 | 1.0707 | 1.0854 | 1.1000 | 1.1145 | 1.1290 | 1.1433 | 1.1576 | 44 |
| 46 | 1.0266 | 1.0415 | 1.0564 | 1.0712 | 1.0859 | 1.1005 | 1.1150 | 1.1295 | 1.1438 | 1.1581 | 46 |
| 48 | 1.0271 | 1.0420 | 1.0569 | 1.0717 | 1.0863 | 1.1010 | 1.1155 | 1.1299 | 1.1443 | 1.1586 | 48 |
| 50 | 1.0276 | 1.0425 | 1.0574 | 1.0721 | 1.0868 | 1.1014 | 1.1160 | 1.1304 | 1.1448 | 1.1590 | 50 |
| 52 | 1.0281 | 1.0430 | 1.0579 | 1.0726 | 1.0873 | 1.1019 | 1.1165 | 1.1309 | 1.1452 | 1.1595 | 52 |
| 54 | 1.0286 | 1.0435 | 1.0584 | 1.0731 | 1.0878 | 1.1024 | 1.1169 | 1.1314 | 1.1457 | 1.1600 | 54 |
| 56 | 1.0291 | 1.0440 | 1.0589 | 1.0736 | 1.0883 | 1.1029 | 1.1174 | 1.1319 | 1.1462 | 1.1605 | 56 |
| 58 | 1.0296 | 1.0445 | 1.0593 | 1.0741 | 1.0888 | 1.1034 | 1.1179 | 1.1323 | 1.1467 | 1.1609 | 58 |
| 60 | 1.0301 | 1.0450 | 1.0598 | 1.0746 | 1.0893 | 1.1039 | 1.1184 | 1.1328 | 1.1472 | 1.1614 | 60 |

| M. | 71° | 72° | 73° | 74° | 75° | 76° | 77° | 78° | 79° | 80° | M. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0' | 1.1614 | 1.1756 | 1.1896 | 1.2036 | 1.2175 | 1.2313 | 1.2450 | 1.2586 | 1.2722 | 1.2856 | 0 |
| 2 | 1.1619 | 1.1760 | 1.1901 | 1.2041 | 1.2180 | 1.2318 | 1.2455 | 1.2591 | 1.2726 | 1.2860 | 2 |
| 4 | 1.1624 | 1.1765 | 1.1906 | 1.2046 | 1.2184 | 1.2322 | 1.2459 | 1.2595 | 1.2731 | 1.2865 | 4 |
| 6 | 1.1628 | 1.1770 | 1.1910 | 1.2050 | 1.2189 | 1.2327 | 1.2464 | 1.2600 | 1.2735 | 1.2869 | 6 |
| 8 | 1.1633 | 1.1775 | 1.1915 | 1.2055 | 1.2194 | 1.2332 | 1.2468 | 1.2604 | 1.2740 | 1.2874 | 8 |
| 10 | 1.1638 | 1.1779 | 1.1920 | 1.2060 | 1.2198 | 1.2336 | 1.2473 | 1.2609 | 1.2744 | 1.2878 | 10 |
| 12 | 1.1642 | 1.1784 | 1.1924 | 1.2064 | 1.2203 | 1.2341 | 1.2478 | 1.2614 | 1.2748 | 1.2882 | 12 |
| 14 | 1.1647 | 1.1789 | 1.1929 | 1.2069 | 1.2208 | 1.2345 | 1.2482 | 1.2618 | 1.2753 | 1.2887 | 14 |
| 16 | 1.1652 | 1.1793 | 1.1934 | 1.2073 | 1.2212 | 1.2350 | 1.2487 | 1.2623 | 1.2757 | 1.2891 | 16 |
| 18 | 1.1657 | 1.1798 | 1.1938 | 1.2078 | 1.2217 | 1.2354 | 1.2491 | 1.2627 | 1.2762 | 1.2896 | 18 |
| 20 | 1.1661 | 1.1803 | 1.1943 | 1.2083 | 1.2221 | 1.2359 | 1.2496 | 1.2632 | 1.2766 | 1.2900 | 20 |
| 22 | 1.1666 | 1.1807 | 1.1948 | 1.2087 | 1.2226 | 1.2364 | 1.2500 | 1.2636 | 1.2771 | 1.2905 | 22 |
| 24 | 1.1671 | 1.1812 | 1.1952 | 1.2092 | 1.2231 | 1.2368 | 1.2505 | 1.2641 | 1.2775 | 1.2909 | 24 |
| 26 | 1.1676 | 1.1817 | 1.1957 | 1.2097 | 1.2235 | 1.2373 | 1.2509 | 1.2645 | 1.2780 | 1.2914 | 26 |
| 28 | 1.1680 | 1.1821 | 1.1962 | 1.2101 | 1.2240 | 1.2377 | 1.2514 | 1.2650 | 1.2784 | 1.2918 | 28 |
| 30 | 1.1685 | 1.1826 | 1.1966 | 1.2106 | 1.2244 | 1.2382 | 1.2518 | 1.2654 | 1.2789 | 1.2922 | 30 |
| 32 | 1.1690 | 1.1831 | 1.1971 | 1.2111 | 1.2249 | 1.2386 | 1.2523 | 1.2659 | 1.2793 | 1.2927 | 32 |
| 34 | 1.1694 | 1.1836 | 1.1976 | 1.2115 | 1.2254 | 1.2391 | 1.2528 | 1.2663 | 1.2798 | 1.2931 | 34 |
| 36 | 1.1699 | 1.1840 | 1.1980 | 1.2120 | 1.2258 | 1.2396 | 1.2532 | 1.2668 | 1.2802 | 1.2936 | 36 |
| 38 | 1.1704 | 1.1845 | 1.1985 | 1.2124 | 1.2263 | 1.2400 | 1.2537 | 1.2672 | 1.2807 | 1.2940 | 38 |
| 40 | 1.1709 | 1.1850 | 1.1990 | 1.2129 | 1.2267 | 1.2405 | 1.2541 | 1.2677 | 1.2811 | 1.2945 | 40 |
| 42 | 1.1713 | 1.1854 | 1.1994 | 1.2134 | 1.2272 | 1.2409 | 1.2546 | 1.2681 | 1.2816 | 1.2949 | 42 |
| 44 | 1.1718 | 1.1859 | 1.1999 | 1.2138 | 1.2277 | 1.2414 | 1.2550 | 1.2686 | 1.2820 | 1.2954 | 44 |
| 46 | 1.1723 | 1.1864 | 1.2004 | 1.2143 | 1.2281 | 1.2418 | 1.2555 | 1.2690 | 1.2825 | 1.2958 | 46 |
| 48 | 1.1727 | 1.1868 | 1.2008 | 1.2148 | 1.2286 | 1.2423 | 1.2559 | 1.2695 | 1.2829 | 1.2962 | 48 |
| 50 | 1.1732 | 1.1873 | 1.2013 | 1.2152 | 1.2290 | 1.2428 | 1.2564 | 1.2699 | 1.2833 | 1.2967 | 50 |
| 52 | 1.1737 | 1.1878 | 1.2018 | 1.2157 | 1.2295 | 1.2432 | 1.2568 | 1.2704 | 1.2838 | 1.2971 | 52 |
| 54 | 1.1742 | 1.1882 | 1.2022 | 1.2161 | 1.2299 | 1.2437 | 1.2573 | 1.2708 | 1.2842 | 1.2976 | 54 |
| 56 | 1.1746 | 1.1887 | 1.2027 | 1.2166 | 1.2304 | 1.2441 | 1.2577 | 1.2713 | 1.2847 | 1.2980 | 56 |
| 58 | 1.1751 | 1.1892 | 1.2032 | 1.2171 | 1.2309 | 1.2446 | 1.2582 | 1.2717 | 1.2851 | 1.2985 | 58 |
| 60 | 1.1756 | 1.1896 | 1.2036 | 1.2175 | 1.2313 | 1.2450 | 1.2586 | 1.2722 | 1.2856 | 1.2989 | 60 |

**Cuerdas para un radio 1.** (Continuación.)

| ′ | 81° | 82° | 83° | 84° | 85° | 86° | 87° | 88° | 89° | M. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0′ | 1.2989 | 1.3121 | 1.3252 | 1.3383 | 1.3512 | 1.3640 | 1.3767 | 1.3893 | 1.4018 | 0′ |
| 2 | 1.2993 | 1.3126 | 1.3257 | 1.3387 | 1.3516 | 1.3644 | 1.3771 | 1.3897 | 1.4022 | 2 |
| 4 | 1.2998 | 1.3130 | 1.3261 | 1.3391 | 1.3520 | 1.3648 | 1.3776 | 1.3902 | 1.4026 | 4 |
| 6 | 1.3002 | 1.3134 | 1.3265 | 1.3396 | 1.3525 | 1.3653 | 1.3780 | 1.3906 | 1.4031 | 6 |
| 8 | 1.3007 | 1.3139 | 1.3270 | 1.3400 | 1.3529 | 1.3657 | 1.3784 | 1.3910 | 1.4035 | 8 |
| 10 | 1.3011 | 1.3143 | 1.3274 | 1.3404 | 1.3533 | 1.3661 | 1.3788 | 1.3914 | 1.4039 | 10 |
| 12 | 1.3015 | 1.3147 | 1.3279 | 1.3409 | 1.3538 | 1.3665 | 1.3792 | 1.3918 | 1.4043 | 12 |
| 14 | 1.3020 | 1.3152 | 1.3283 | 1.3413 | 1.3542 | 1.3670 | 1.3797 | 1.3922 | 1.4047 | 14 |
| 16 | 1.3024 | 1.3156 | 1.3287 | 1.3417 | 1.3546 | 1.3674 | 1.3801 | 1.3927 | 1.4051 | 16 |
| 18 | 1.3029 | 1.3161 | 1.3292 | 1.3421 | 1.3550 | 1.3678 | 1.3805 | 1.3931 | 1.4055 | 18 |
| 20 | 1.3033 | 1.3165 | 1.3296 | 1.3426 | 1.3555 | 1.3682 | 1.3809 | 1.3935 | 1.4060 | 20 |
| 22 | 1.3038 | 1.3169 | 1.3300 | 1.3430 | 1.3559 | 1.3687 | 1.3813 | 1.3939 | 1.4064 | 22 |
| 24 | 1.3042 | 1.3174 | 1.3305 | 1.3434 | 1.3563 | 1.3691 | 1.3818 | 1.3943 | 1.4068 | 24 |
| 26 | 1.3046 | 1.3178 | 1.3309 | 1.3439 | 1.3567 | 1.3695 | 1.3822 | 1.3947 | 1.4072 | 26 |
| 28 | 1.3051 | 1.3183 | 1.3313 | 1.3443 | 1.3572 | 1.3699 | 1.3826 | 1.3952 | 1.4076 | 28 |
| 30 | 1.3055 | 1.3187 | 1.3318 | 1.3447 | 1.3576 | 1.3704 | 1.3830 | 1.3956 | 1.4080 | 30 |
| 32 | 1.3060 | 1.3191 | 1.3322 | 1.3452 | 1.3580 | 1.3708 | 1.3834 | 1.3960 | 1.4084 | 32 |
| 34 | 1.3064 | 1.3196 | 1.3326 | 1.3456 | 1.3585 | 1.3712 | 1.3839 | 1.3964 | 1.4089 | 34 |
| 36 | 1.3068 | 1.3200 | 1.3331 | 1.3460 | 1.3589 | 1.3716 | 1.3843 | 1.3968 | 1.4093 | 36 |
| 38 | 1.3073 | 1.3204 | 1.3335 | 1.3465 | 1.3593 | 1.3721 | 1.3847 | 1.3972 | 1.4097 | 38 |
| 40 | 1.3077 | 1.3209 | 1.3339 | 1.3469 | 1.3597 | 1.3725 | 1.3851 | 1.3977 | 1.4101 | 40 |
| 42 | 1.3082 | 1.3213 | 1.3344 | 1.3473 | 1.3602 | 1.3729 | 1.3855 | 1.3981 | 1.4105 | 42 |
| 44 | 1.3086 | 1.3218 | 1.3348 | 1.3477 | 1.3606 | 1.3733 | 1.3860 | 1.3985 | 1.4109 | 44 |
| 46 | 1.3090 | 1.3222 | 1.3352 | 1.3482 | 1.3610 | 1.3738 | 1.3864 | 1.3989 | 1.4113 | 46 |
| 48 | 1.3095 | 1.3226 | 1.3357 | 1.3486 | 1.3614 | 1.3742 | 1.3868 | 1.3993 | 1.4117 | 48 |
| 50 | 1.3099 | 1.3231 | 1.3361 | 1.3490 | 1.3619 | 1.3746 | 1.3872 | 1.3997 | 1.4122 | 50 |
| 52 | 1.3104 | 1.3235 | 1.3365 | 1.3495 | 1.3623 | 1.3750 | 1.3876 | 1.4002 | 1.4126 | 52 |
| 54 | 1.3108 | 1.3239 | 1.3370 | 1.3499 | 1.3627 | 1.3754 | 1.3881 | 1.4006 | 1.4130 | 54 |
| 56 | 1.3112 | 1.3244 | 1.3374 | 1.3503 | 1.3631 | 1.3759 | 1.3885 | 1.4010 | 1.4134 | 56 |
| 58 | 1.3117 | 1.3248 | 1.3378 | 1.3508 | 1.3636 | 1.3763 | 1.3889 | 1.4014 | 1.4138 | 58 |
| 60 | 1.3121 | 1.3252 | 1.3383 | 1.3512 | 1.3640 | 1.3761 | 1.3893 | 1.4018 | 1.4142 | 60 |

## POLÍGONOS




**Pentágono.** 5 *lados.* **Exágono.** 6 *lados.* **Heptágono.** 7 *lados.* **Octógono.** 8 *lados.*

Polígono es toda figura plana terminada por líneas rectas. Si todos los lados y ángulos son iguales es un polígono **regular;** si no, es **irregular.** Por supuesto, el número de los polígonos es infinito.

**Tabla de polígonos regulares.**

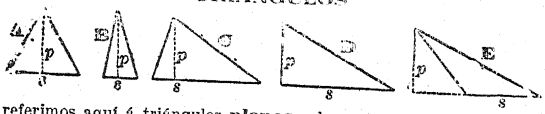
| Número de lados. | Nombre del polígono. | **Área** = (cuadrado de un lado) multiplicado por | **Radio** del círculo circunscrito = lado multiplicado por | **Ángulo** interior *abc*. | **Ángulo** del centro *apb*. |
|---|---|---|---|---|---|
| 3 | Triángulo equilátero. | 0.433013 | 0.577350 | 60° | 120° |
| 4 | Cuadrado. | 1.000000 | 0.707107 | 90° | 90° |
| 5 | Pentágono. | 1.720477 | 0.850651 | 108° | 72° |
| 6 | Exágono. | 2.598076 | 1.000000 | 120° | 60° |
| 7 | Eptágono. | 3.633912 | 1.152382 | 128° 34.2857 | 51° 25.7143 |
| 8 | Octógono. | 4.828427 | 1.306563 | 135° | 45° |
| 9 | Nonágono. | 6.181824 | 1.461902 | 140° | 40° |
| 10 | Decágono. | 7.694209 | 1.618034 | 144° | 36° |
| 11 | Endecágono. | 9.365640 | 1.774733 | 147° 16.3636 | 32° 43.6364 |
| 12 | Dodecágono. | 11.196152 | 1.931854 | 150° | 30° |

**Área de cualquier polígono regular** = longitud de lado *ab* × perpendicular *p* trazada del centro de la figura al centro del lado × la mitad del número de los lados.

**La suma de los ángulos interiores** *abc*, etc., de cualquier polígono regular ó irregular es = 180° × (número de lados—2).

**El ángulo del centro subtendido por un lado en cualquier polígono regular** = 360 ÷ número de lados.

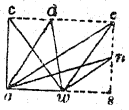
TRIÁNGULOS




Nos referimos aquí á triángulos **planos** solamente, es decir, á aquellos cuyos lados son **rectos.**

**Un triángulo es equilátero** cuando sus lados son iguales, como A; **isósceles**, cuando sólo dos lados son iguales, B; **escaleno**, cuando sus lados son desiguales como C, D y E; **acutángulo**, cuando todos sus ángulos son agudos, es decir, menores de 90°, como A, B y C; **rectángulo**, cuando contiene un ángulo recto, D; **obtusángulo**, cuando contiene un ángulo obtuso, es decir, mayor de 90°, como E.

**La suma de los tres ángulos de cualquier triángulo es igual á dos ángulos rectos** ó 180°; por tanto, si conocemos dos de ellos podemos encontrar el tercero restando la suma de ellos de 180°. **Todos los triángulos que tienen iguales bases** é iguales alturas tienen también iguales áreas, así




las áreas de *awc*, *awd*, *awe*, son iguales entre sí. **El área de cualquier triángulo** es igual á la mitad de la de cualquier paralelogramo que tenga la misma base y altura. **Las áreas de triángulos que tienen iguales bases** pero diferentes alturas, están entre sí en proporción á sus alturas. Así el triángulo *awn*, con una altura *sn* mitad de *se*, es la mitad de cualquiera de los otros tres triángulos que tienen la misma base *aw* pero doble altura *se*.

**El área de un triángulo cualquiera** como los de las figuras A, B, C, D, F = la mitad de la base *s* × la altura ó distancia *p* del vértice opuesto á la base. Cualquier lado puede tomarse como la base de un triángulo; pero la altura debe

medirse siempre desde el vértice opuesto bajando una perpendicular al lado elegido como base, para lo cual algunas veces debe prolongarse la base como en la figura E, pero la prolongación no debe considerarse como parte de la base.

El área de cualquier triángulo equilátero = .433013 × cuadrado de un lado.

**Encontrar el área de un triángulo conociendo los tres lados.** Súmense juntos; divídase la suma por 2; de esta mitad réstese cada lado separadamente; multipliquese entre sí la mitad de la suma y los tres residuos así hallados y tómese del producto total la raíz cuadrada.

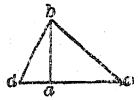
**Ej.** : Sean los tres lados = 20, 30, 40, pies ó metros, $20+30+40 = 90$ y $\frac{90}{2} = 45$ y $45-20 = 25$; $45-30 = 15$ y $45-40 = 5$ y $45\times25\times15\times5 = 84{,}375$ y la raíz cuadrada de 84,375 es 290.47 pies ó metros cuadrados es el área requerida.

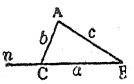
**Para encontrar el área teniendo un lado y los dos ángulos adyacentes.** Súmense los dos ángulos, réstese la suma de 180°, la diferencia será el ángulo opuesto al lado dado; búsquese el seno de este ángulo así como también los senos de los otros ángulos y multipliquense entre sí estos dos últimos. Entonces, lo que el seno del ángulo sencillo es al producto de los senos de los otros dos ángulos será el *cuadrado* del lado dado al *doble* del área requerida.

**Para encontrar el área, teniendo dos lados y el ángulo comprendido.** Multipliquense entre sí los dos lados y el seno del ángulo comprendido y divídase por dos el producto.

**Ej.** : Lados 650 y 980 pies ó metros, ángulo comprendido 69° 20′. Por la tabla encontramos que el seno de 69° 20′ es .9356, por tanto $\frac{650\times980\times.9356}{2} = 297988.6$ pies ó metros cuadrados (según la medida elegida) es el área del triángulo.

**Para encontrar el área teniendo los tres ángulos y la altura** *ab*. Búsquense los senos de los tres ángulos, multipliquense entre sí los senos de los ángulos *d* y *o*; divídase por este producto el seno del ángulo *b*, multipliquese el cociente por el cuadrado de la altura *ab* y divídase por 2.




**Para encontrar cualquier lado, como** *do*, **teniendo los tres ángulos** *d*, *b*, *o*, **y el área** : (sen *d* × sen *o*) : sen *b* : : el doble del área : cuadrado de *do*.

**La altura de un triángulo equilátero** es igual á un lado × .866025. Por consiguiente, uno de sus lados es igual á la altura dividida por .866025, ó la altura × 1.1547.

**También para encontrar un lado** se multiplica la raíz cuadrada de su área por 1.51967. El lado de un triángulo equilátero multiplicado por .658037 = al lado de un cuadrado de la misma área; ó multiplicado por .742517 da el diámetro de un círculo de igual área.

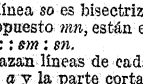
Las siguientes reglas se aplican á cualquier triángulo plano :

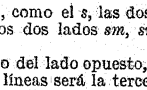
**1.** Los tres ángulos sumados dan 180°; ó dos ángulos rectos.

**2.** Cualquier ángulo exterior como A*Cn* es igual á los dos internos opuestos A, B.

**3.** El mayor lado está opuesto al ángulo mayor.

**4.** Los lados están entre sí como los senos de los ángulos opuestos. Así el lado *a* es al lado *b* como el seno de A es al seno de B.




**5.** Si una línea *so* es bisectriz de cualquier ángulo, como el *s*, las dos partes *mo*, *on* del lado opuesto *mn*, están entre sí como los otros dos lados *sm*, *sn*; es decir, que *mo* : *on* : : *sm* : *sn*.

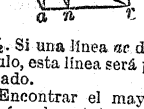
**6.** Si se trazan líneas de cada ángulo *rst* al centro del lado opuesto, se cruzarán en un punto *a* y la parte corta de cada una de las líneas será la tercera parte de la línea total. También *a* **será el centro de gravedad** del triángulo.

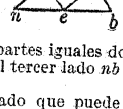
**7.** Si se trazan líneas bisectrices de los tres ángulos, se encontrarán en un punto equidistante de cada lado y, por consiguiente, ese punto será **centro del círculo** máximo que puede trazarse en ese triángulo.

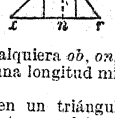
**8.** Si se traza una línea *sn* paralela á cualquier lado *ca*, los dos triángulos *rsn*, *rca* serán semejantes.

**9.** Para dividir cualquier triángulo *acr*, en dos partes iguales con una línea *sn* paralela á cualquiera de sus lados, *ca* se procederá así: sobre cualquiera de los otros lados, *ar*, como diámetro, descríbase una semicircunferencia *aor* y búsquese su punto medio *o*. De *r* (opuesto á *ca*), con un radio *ro*, descríbase el arco *on*. De *n* trácese *ns* paralela á *ca*.

**10.** Para encontrar el mayor paralelogramo que puede trazarse en cualquier triángulo dado, *onb*. Córtense en la mitad los tres lados en *a*, *c*, *e*, y unánse *ac*, *ae*, *ec*. Entonces cualquiera de los paralelogramos *aebc*, *aeco*, *accn*, cada uno igual á la mitad del triángulo, será el paralelogramo que se requiere. Cualquiera de estos paralelogramos puede convertirse fácilmente en un rectángulo de igual área que también será el mayor que pueda ser trazado en el triángulo.




**10½.** Si una línea *nc* divide en partes iguales dos lados cualquiera *ob*, *on*, de un triángulo, esta línea será paralela al tercer lado *nb* y tendrá una longitud mitad de dicho lado.

**11.** Encontrar el mayor cuadrado que puede trazarse en un triángulo *axr*. De un ángulo *a* trácese una perpendicular *an* al lado opuesto *xr* y búsquese su longitud. Entonces *on* ó un lado *vt* del cuadrado será $= \frac{xr \times an}{xr + an}$.

**Advertancia.** Si el triángulo es tal que se pueden trazar dos ó tres perpendiculare análogas, pueden encontrarse entonces dos ó tres cuadrados iguales.

## Triángulos rectángulos.

Todo lo anterior se aplica también á los triángulos rectángulos, pero lo que sigue se aplica solamente á ellos.

Llamemos al ángulo recto A y á los otros B, C. Llamemos los lados respectivamente opuestos á los ángulos A, B. C; *a*, *b*, *c*. Entonces tendremos

$$a = \frac{c}{\text{sen C}} = c \times \sec B = \frac{b}{\cos C} = b \times \sec C = \sqrt{b2 + c2}$$

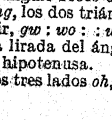
$b = a \times \text{sen B} = a \times \cos C = c \times \cot C = c \times \text{tang B}.$

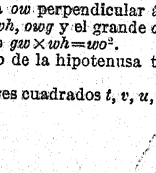
$c = a \times \text{sen C} = a \times \cos B = b \times \text{tang C}.$

También $\text{sen C} = \frac{c}{a}$; $\cos C = \frac{b}{a}$; $\text{tang C} = \frac{c}{b}$;

y $\text{sen B} = \frac{b}{a}$; $\cos B = \frac{c}{a}$; $\text{tang B} = \frac{b}{c}$.

Sen A ó 90° = 1; cos A = 0; tang A = el infinito; sec A = infinito.




**1.** Si por el ángulo recto *o* se traza una línea *ow* perpendicular á la hipotenusa ó lado mayor *hg*, los dos triángulos pequeños, *owh*, *owg* y el grande *ohg* serán semejantes; es decir, $gw : wo :: wo : wh$; y por tanto $gw \times wh = wo^2$.

**2.** Una línea lirada del ángulo recto al centro de la hipotenusa tendrá de largo la mitad de la hipotenusa.

**3.** Si sobre los tres lados *oh*, *og*, *gh*, trazamos tres cuadrados *t*, *v*, *u*, ó tres círculos,

ó tres triángulos ó cualesquiera otras tres figuras que sean semejantes : el área de la más grande es igual á la suma de las áreas de las otras dos.

**4.** En un triángulo cuyos lados estén como 3, 4, y 5 (como son los del triángulo A, B, C) los ángulos son muy aproximadamente 90°; 53° 7′ 48″, 38 y 36° 52′ 11 62″. Sus senos 1, 8 y 6. Sus tangentes, el infinito; 1.3333 y 0.75.

**5.** Uno cuyos lados estén como 7, 7, y 9, tiene muy aproximadamente un ángulo de 90° y dos de 45° cada uno. Estas cantidades son suficientemente aproximadas para casi todos los casos prácticos.

# TRIGONOMETRÍA PLANA

La trigonometría plana enseña á encontrar elementos desconocidos de los triángulos planos por medio de otros conocidos permitiéndonos por este medio medir distancias inaccesibles, etc. Un triángulo tiene seis elementos, á saber : tres lados y tres ángulos, y si conocemos 3 cualesquiera de sus elementos, con tal que entre un lado por lo menos en los 3 conocidos, podemos encontrar los otros tres. Los cuatro casos siguientes abrazan todo el asunto y deben los estudiantes aprenderlos de memoria.

**1er Caso. Teniendo dos ángulos y un lado encontrar los otros dos lados y el otro ángulo.**

Súmense los dos ángulos y réstese la suma de 180, la diferencia será el tercer ángulo, y para los lados procédase así :

Seno del ángulo opuesto al lado dado : Seno del ángulo opuesto al lado que se busca = El lado dado : Lado que se busca.

Úsese ahora, para encontrar el otro lado que falta, al lado que se ha encontrado como el lado dado y de la misma manera búsquese el tercer lado.

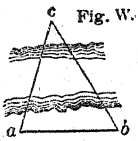


Fig. W.

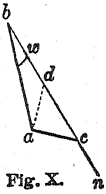


Fig. X.

**2.° Caso. Teniendo dos lados** *ba, ac,* **fig. X, y el ángulo** *abc* **opuesto á uno de ellos,** encontrar el otro lado y los otros ángulos.

El lado *ac* opuesto al ángulo dado *abc* : El otro lado dado *ba* = Seno del ángulo dado *abc* : Seno del ángulo *bda*, ó *bca* opuesto al otro lado dado *ba*.

Habiendo encontrado el seno tómese el ángulo correspondiente en la tabla de los senos naturales; pero al hacer esto, si el lado *ac* opuesto al ángulo es más pequeño que el otro lado dado *ba,* recuérdese que un ángulo y su suplemento tienen el mismo seno. Así en la fig. X el seno según se encontró de la tabla pertenece tanto al ángulo *bca* como al *bda* que es suplemento de *acb* si se hace *ad* igual *ac* formando un triángulo isóseles *dac*. Hasta que no se determine el otro lado *bc* no se sabe si el triángulo resuelto es el *adb* ó el *acb*. No sucede así si *ac* es tan grande ó mayor que *ba,* pues entonces no puede haber duda, porque en ese caso no puede trazarse el lado *ac* hacia *b* como en *ad* sino sólo hacia *n* y el ángulo *bca* será el único á que corresponde el seno hallado en la tabla.

Cuando se conozcan los dos ángulos *abc, bca,* búsquese el lado que falta como en el caso 1.°

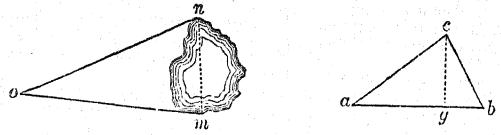
Para encontrar el ángulo que falta *bac* súmense el ángulo *abc* que se dió primero y el *bca* encontrado como se dijo arriba y réstese su suma de 180°.

**3.er Caso. Teniendo dos lados y el ángulo comprendido entre ellos.** Réstese el ángulo conocido de 180°, la diferencia será la suma de los dos ángulos desconocidos; divídase esta suma por 2 y búsquese la tangente del cociente. Entonces

La suma de los dos lados dados : á su diferencia = Tangente de la semisuma de los dos ángulos desconocidos : Tangente de la semidiferencia de estos mismos ángulos.

Despéjese el cuarto término y tómese en la tabla de las tangentes el ángulo opuesto á esta última tangente, agréguese este ángulo á la mitad de la suma de los dos ángulos desconocidos y dará el ángulo opuesto al lado dado más largo; y para el ángulo opuesto al lado dado más corto, réstese aquel de la misma mitad de la suma de los dos ángulos. Habiendo encontrado así los ángulos, búsquese el tercer lado, aplicando el 1.er caso.

Como un ejemplo práctico del uso del caso 3.° podemos determinar la distancia, *nm*, en un estanque ó lago, etc., midiendo dos líneas *mo*, *no*, y el ángulo *nom*. Con este dato podemos calcular *nm*; también trazando los dos lados y el ángulo sobre el papel por medio de una escala podemos medir después *nm* sobre el dibujo en partes de la escala.




**4.° Caso. Conociendo los tres lados** encontrar los tres ángulos.

En un lado *ab*, como base, trácese (ó supóngase trazada) una perpendicular *cg* bajada del ángulo opuesto *c*; búsquese la diferencia entre los otros dos lados *ac* *cb*, y también la suma de ellos; y dígase :

La base : La suma de los otros dos lados = La diferencia de los otros dos lados : La diferencia de las dos partes *ag*, *bg* de la base

Súmese la mitad de esta diferencia de las partes con la mitad de la base *ab*; la suma será la parte más larga *ag*, la que restada de la base total *ab* da la parte más corta *gb*. Por estos medios obtenemos en cada uno de los pequeños triángulos *acg*, *cgb*, dos lados (á saber : *ac*, *ag*; *cb*, *gb*), y un ángulo (el ángulo recto *cga*, ó *cgb*) opuesto á uno de los lados dados. Luego empléese el método del 2.° caso para buscar los ángulos *a*, *b*. Hecho esto réstese de 180° la suma de ellos y se tendrá el ángulo *acb*.

**2.° Modo.** Llámese **la mitad** de la suma de los tres lados *s*; y llámense los dos lados que forman cualquiera de los dos ángulos, *m*, *n*; entonces el seno de la mitad de ese ángulo será igual á $\sqrt{\frac{(s-m)\times(s-n)}{m\times n}}$.

**Ej. 1.° Encontrar la distancia, de *a*, á un objeto inaccesible *c*.**

Mídase una línea *ab* y desde sus extremos mídanse los ángulos *cab*, *cba*. Habiendo encontrado así un lado y dos ángulos del triángulo *abc*, calcúlese *ac* por el caso 1.°

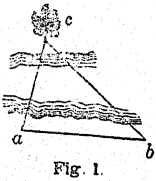



Fig. 1.

Si no se necesita gran exactitud trácese la línea *ab* sobre el papel en una escala conveniente y por medio de un transportador colóquense los ángulos *cab*, *cba*: trácense *ac*, *cb* y mídase entonces en el papel *ac*, en la misma escala.

**Ej. 2.° Encontrar la altura de un objeto vertical** *na.*
Colóquese el instrumento de medir ángulos en un punto conveniente *o;* mídase a distancia *oa;* ó si *oa* no puede medirse á consecuencia de algún obstáculo,

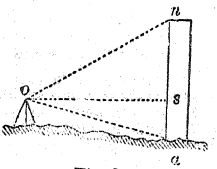



Fig. 2.

calcúlese por medio del mismo procedimiento que *ac* en la figura 1. Entonces, dirigiendo primero horizontalmente el instrumento *, como en *s*, mídase el ángulo de depresión *son*, supongámoslo de 12°, también el ángulo *son* supongámoslo de 30°. Estos dos ángulos unidos dan el ángulo *aon*=42°. Ahora, en el triángulo pequeño *osa*, teníamos el ángulo *osa* igual á 90° porque *an* es vertical y *os* horizontal, y como los tres ángulos de cualquier triángulo son iguales á 180°, si restamos los ángulos *osa* (90°) y *soa* (12°) de 180° la diferencia (78°) será el ángulo *oas*, ó *oan*. Por tanto en el triángulo *ona* tenemos un lado *oa* y dos ángulos *aon*, *oan* para calcular el lado *an* según el caso 1.°

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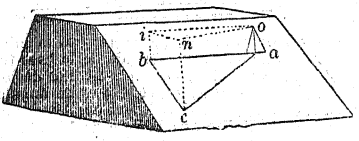
* **Los ángulos y las distancias, en un terreno inclinado, deben ser medidos horizontalmente.** El círculo horizontal graduado de un instrumento mide evidentemente el ángulo entre dos objetos horizontales ó el de sus proyecciones en un plano horizontal, no importa lo más alto que esté el uno respecto al otro. Algunas veces es necesario dirigir un anteojo del instrumento hacia arriba y el otro hacia abajo. Por consiguiente los lados de los triángulos que descansan sobre terrenos inclinados deben medirse también horizontalmente porque de lo contrario no puede haber la relación que se busca entre ellos. Así, en el triángulo *abc* situado sobre un terreno inclinado,




el instrumento en *o*, mide el ángulo horizontal *ion*, y no el ángulo *bac*. Por tanto el lado que le corresponde á este ángulo horizontal *ion* es la distancia horizontal *in* y no la distancia inclinada *bc*. En otras palabras, cuando los lados y ángulos de un triángulo están en terreno inclinado no buscamos sus medidas reales sino las horizontales de sus proyecciones. Esta advertencia corresponde á todas las medidas de granjas, ferrocarriles, triangulaciones de campos, etc., etc.; y la falta de una estricta atención en este punto es una causa de errores casi inevitables (afortunadamente de insignificantes consecuencias en la práctica) que ocurren en casi todas las operaciones ordinarias sobre el terreno.

**Cuando se usa un sextante**, los ángulos entre objetos á diferentes alturas, como *pq* (figura que sigue), pueden medirse horizontalmente, colocando primero dos reglas verticales *o*, *s* en línea recta con los objetos y tomando entonces el ángulo horizontal subtendido por las reglas.

**Los ángulos pueden medirse sin ningún instrumento**, así:
Mídanse 100 pies ó metros desde cada objeto y clávense estacas; mídase la distancia de una estaca á otra, la mitad de esta distancia será el seno de la mitad del ángulo para un radio de 100; y si dividimos esta cantidad por 100 tendremos el seno de esta mitad del ángulo para un radio 1 como está en la tabla. Así, supongamos que la distancia sea 80.64, entonces 40.32 es el seno de la mitad del ángulo y .4032 será el seno que, buscado en la tabla de senos le corresponderá en la columna de los ángulos el ángulo 23°47′ que multiplicado por 2 da 47°34 que será el ángulo deseado. Si algún obstáculo impide medir hacia los objetos, podemos medir directamente de ellos hacia el punto y prolongar ambas líneas, porque cuando dos líneas se cortan, los ángulos opuestos por el vértice son iguales. Se puede hacer una medida aproximada clavando tres alfileres verticales, separados algunas pulgadas, en un pedazo pequeño de cartón, fijo horizontalmente en la punta de un bastón. Los alfileres ocuparán las posiciones *nos* de la última figura.
Trácense entonces líneas de lápiz para unir los huecos de los alfileres y se medirá el ángulo con un transportador. Clavando un pedazo de cartón verticalmente sobre un árbol y trazando sobre aquél una línea horizontal pequeña, por medio de un nivel

**Advertencia.** Si como en la figura 3, fuese necesario medir la altura vertical *an* desde un punto *o*, más alto que *n*, ambos ángulos medidos en *o*, á saber: *son* y *soa* serán ángulos de depresión, es decir, más bajos que *o*. En este caso tenemos el lado *oa* como antes, el ángulo *noa* igual *soa* — *son*; y el ángulo *oan* = 180° — (*osa* (90°) y *soa*) para calcular *an* por medio del caso 1.°

O si como en la figura 4 deben ser tomados desde un punto *o* enteramente

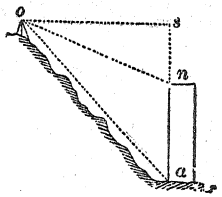



Fig. 3.

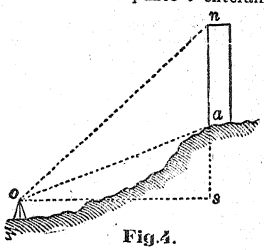



Fig. 4.

debajo del objeto *an* ambos ángulos *soa*, *son* serán ángulos de elevación, es decir por encima de la línea horizontal supuesta *os*. En este caso, tenemos en el triángulo *ona* el lado dado *oa*, como antes; el ángulo *aon* = *son*-*soa*, y el ángulo *ona* = 180° — (*osn* (90°) + *son*) para calcular *an* según el caso 1.°

Si el objeto *an*, como en la figura 5, en lugar de ser vertical está inclinado, y

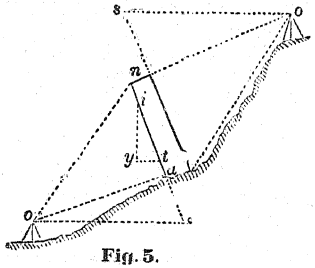



Fig. 5.

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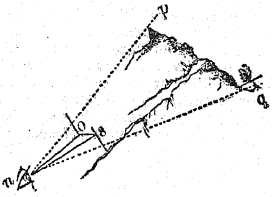
de aire, de carpintero, pueden medirse de la misma manera los ángulos verticales, de elevación y depresión. De esta manera el autor se ha aprovechado algunas veces de la puerta de una casa abriéndola hasta que su borde pase por el pico de alguna montaña cuya distancia conocía aproximadamente, pero sin tener idea de su altura. Respecto á las correcciones de refracción, etc., véase la tabla más próxima.




**Un triángulo cuyos lados están como los números** 3, 4 y 5, es rectángulo; y uno cuyos lados están como 7; 7; 9, 9 contiene un ángulo recto y dos ángulos de 45°, cada uno. Como es necesario frecuentemente trazar sobre el terreno ángulos de 45° y 90°, estas proposiciones se pueden usar con este fin, formando con un pedazo de cinta de agrimensor ó cadena uno de dichos triángulos y clavando una estaca en cada vértice

en lugar de su altura vertical deseamos saber su longitud *an*, debemos medir primero su ángulo *yti* de inclinación con la horizontal á cuyo ángulo cada uno de los ángulos *osn* (mídase de abajo ó de arriba) serán iguales. Para buscar este ángulo *yti*, suspéndase una plomada *iy* de cualquier longitud conocida en cualquier punto del objeto *an*, mídase *yt* horizontalmente y luego dígase:

$$yt : yi :: 1 : \text{tang } yti.$$

De la tabla de tangentes tómese el ángulo *yti* que corresponde á esta tangente y úsese este ángulo para los ángulos *osn* ú *osa*, en lugar del ángulo de 90° que

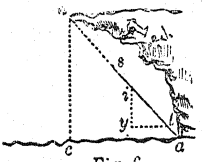


Fig. 6.

usamos en las figuras 3 y 4. Si el objeto, como la roca *an*, es curvo ó irregular, se debe colocar un bastón *as* inclinado en la dirección *an;* y mídase su ángulo *yt* de inclinación con la horizontal como antes y como antes también calcúlese la distancia *an*. Si se busca la altura vertical *cn*, debe encontrarse primero el punto *c* mirando hacia arriba á lo largo de una plomada sostenida sobre la cabeza.

**Ej. 3.° Encontrar la altura aproximada *sx* de una montaña** cuya cumbre es tan sólo visible porque se interponen árboles ú otros obstáculos y sólo se conoce la distancia *mi*. Para este caso, diríjase primero el instrumento horiz según *mh*, y mídase el ángulo *imx*. En el triángulo *imx* tenemos un lado *mi*, el ángulo medido *imx* y el ángulo *mix* que es recto, búsquese *ix* como en el caso 1.° Pero á este *ix* debemos agregarle *io*, igual á la altura *ym*, del instrumento sobre el

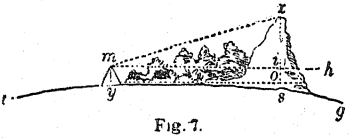


Fig. 7.

terreno y también *os*. Como *os* es debido solamente á la curvatura de la tierra, la cual es igual aproximadamente á cerca de 8 pulgadas, es decir, .667 de pies por cada milla (ó .126$^m$ por cada kilómetro) y aumenta como los cuadrados de las distancias siendo 4 veces 8 pulgadas en 2 millas (4 × .126$^m$ en dos kilómetros); 9 veces 8 pulgadas en 3 millas, etc. Esto lo disminuye en algo la refracción de la atmósfera que varía con la temperatura, humedad, etc.; y que tiende siempre á hacer aparecer el objeto *x* más alto de lo que realmente está. Como término medio esta ilusoria elevación alcanza como á una séptima parte del coeficiente de curvatura de la tierra ya citado, y también varía como ésta con los cuadrados de las distancias. Por consiguiente si sustraemos $^1/_7$ de 8 pulgadas ó de.667 de pies, tenemos el efecto combinado de la curvatura y refracción para una milla igual á 6.857 pulgadas ó .5714 de pies. (*N. del T.* Quitándole la séptima parte á .126$^m$ coeficiente de curvatura por kilómetro, resulta .108$^m$ por kilómetro para la corrección combinada de curvatura y refracción.) Para otras distancias tenemos la tabla siguiente, con cuyo uso nos evitamos la necesidad de hacer cálculos por separado, para la curvatura de la tierra y la refracción.

**Tabla de las cantidades que deben agregarse á las alturas por causa de la curvatura y refracción combinadas.**

| Dist. en yards. | Corrección en pies. | Dist. en millas. | Corrección en pies. | Dist. en millas. | Corrección en pies. | Dist. en millas. | Corrección en pies. |
|---|---|---|---|---|---|---|---|
| 100 | .002 | ¼ | .036 | 6 | 20.6 | 20 | 229 |
| 150 | .004 | ½ | .143 | 7 | 28.0 | 22 | 277 |
| 200 | .007 | ¾ | .321 | 8 | 36.6 | 25 | 357 |
| 300 | .017 | 1 | .572 | 9 | 46.3 | 30 | 514 |
| 400 | .030 | 1¼ | .893 | 10 | 57.2 | 35 | 700 |
| 500 | .046 | 1½ | 1.29 | 11 | 69.2 | 40 | 915 |
| 600 | .066 | 1¾ | 1.75 | 12 | 82.3 | 45 | 1158 |
| 700 | .090 | 2 | 2.29 | 13 | 96 6 | 50 | 1429 |
| 800 | .118 | 2½ | 3.57 | 14 | 112 | 55 | 1729 |
| 900 | .149 | 3 | 5.14 | 15 | 129 | 60 | 2058 |
| 1000 | .185 | 3½ | 7.00 | 16 | 146 | 70 | 2801 |
| 1200 | .266 | 4 | 9.15 | 17 | 165 | 80 | 3659 |
| 1500 | .415 | 4½ | 11.6 | 18 | 185 | 90 | 4631 |
| 2000 | .738 | 5 | 14.3 | 19 | 206 | 100 | 5717 |

**La misma tabla anterior en metros y kilómetros.** (*Del T.*)

| Dist. en metros. | Corrección en metros. | Dist. en kilómetros. | Corrección en metros. | Dist. en kilómetros. | Corrección en metros. | Dist. en kilómetros. | Corrección en metros. |
|---|---|---|---|---|---|---|---|
| 91.4 | .0006 | .402 | .011 | 9.656 | 6.283 | 32.187 | 69.797 |
| 137.1 | .0012 | .805 | .044 | 11.265 | 8.534 | 35.406 | 84.427 |
| 182.8 | .0021 | 1.207 | .098 | 12.875 | 11.156 | 40.224 | 108.810 |
| 274.2 | .0052 | 1.609 | .174 | 14.484 | 14.112 | 48.280 | 156.662 |
| 365.6 | .0092 | 2.012 | .272 | 16.093 | 17.434 | 56.327 | 213.353 |
| 457.0 | .0140 | 2.414 | .393 | 17.703 | 21.092 | 64.374 | 278 883 |
| 548.4 | .0201 | 2.816 | .533 | 19.312 | 25.084 | 72.421 | 355.952 |
| 639.8 | .0274 | 3.218 | .698 | 20.921 | 29.443 | 80.467 | 435.551 |
| 731.2 | .0359 | 4.023 | 1.088 | 22.531 | 34.137 | 88.514 | 526.989 |
| 822.6 | .0447 | 4.828 | 1.567 | 24.140 | 39.319 | 96.561 | 627.567 |
| 914.0 | .0564 | 5.633 | 2.134 | 25.750 | 44.500 | 112.654 | 853.730 |
| 1096.8 | .0810 | 6.437 | 2.789 | 27.359 | 50.291 | 128.748 | 1115.243 |
| 1371.0 | .1264 | 7.242 | 3.536 | 28.968 | 56.386 | 144.841 | 1412.499 |
| 1828.0 | .2249 | 8.047 | 4.361 | 30.578 | 62.830 | 160.935 | 1743.511 |

**Por consiguiente**, para una persona cuya vista está á 1,567 metros sobre el nivel del mar, y ve un objeto, precisamente en el horizonte del mar, ese objeto estará como á 4,828 kil de distancia de él. (Véase la tabla última.)

**Una línea horizontal no es una línea á nivel** porque las líneas rectas no pueden estar á nivel puesto que la superficie terrestre es curva. La curvatura de la tierra aparece muy clara en una extensión de aguas tranquilas y éstas si están á nivel. Si en la figura 7 suponemos que la línea curva representa la superficie del mar, los puntos *t*, *y*, *s*, *g* están al mismo nivel entre sí. No siempre los puntos á nivel equidistan del centro de la tierra, porque el mar en los polos está como 13 millas (20,921 metros) más cerca del centro de la tierra que en el ecuador, sin embargo su superficie está en todas partes á nivel.

Cuando digamos **sobre y debajo**, significará sobre y debajo del nivel del mar. **Á nivel** significará paralelo á la curvatura del mar, y **horizontal** significa tangente al nivel del mar ó superficie de la tierra.

**Ej. 4.° Si la altura vertical *cd*, figura 8, es inaccesible.**

Colóquese entonces el instrumento para medir ángulos en cualquier punto *n* y plántense dos bastones en el plano vertical de *nd* cuyos extremos superiores *o*, *i*, estén en línea recta con *n* aun cuando no estén en la horizontal de *n*. Mídase *no*:

desde *n* mídanse los ángulos *ond*, *onc*. Muévase entonces el instrumento al punto donde está el bastón *o* y colóquese aquél á la altura y en el mismo lugar del punto *o*. Desde *o* mídanse los ángulos *iod*, *doc*. Hecho esto réstese el ángulo *ioc* de 180°, la diferencia será el ángulo *con*. Por consiguiente, en el triángulo *noc* tenemos un lado *no*, y dos ángulos *cno*, *con* para encontrar por medio del caso 1.°

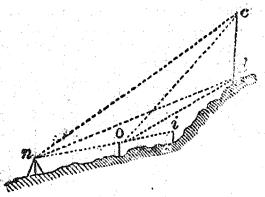



Fig. 8.

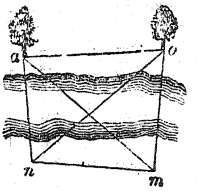



Fig. 9.

el lado *oc*. Además, réstese el ángulo *iod* de 180°, la diferencia será el ángulo *nod*; de modo que en el triángulo *dno* tenemos un lado *no* y los dos ángulos *dno*, *nod*, para encontrar aplicando el caso 1.° el lado *od*; y finalmente en el triángulo *cod* tenemos dos lados *co*, *od* y el ángulo comprendido *cod* para encontrar *cd* la altura vertical que se busca.

**Advertencia.** Si *cd* estuviera en un valle ó sobre un cerro y tuvieran que hacerse las observaciones desde un terreno alto ó bajo, las operaciones hubieran sido las mismas.

**Ej. 5.** Véase el ejemplo 10.

**Encontrar la distancia** *a*, *o*, fig. 9, **entre dos objetos inaccesibles.** Mídase un lado *mn*, en *n* mídanse los ángulos *anm* y *onn*; también en *m* mídanse los ángulos *omn*, *amn*. Hecho esto tenemos en el triángulo *anm*, un lado *mn*, fig. 9, y los ángulos *anm* y *nma*; entonces por el caso 1 podemos calcular el lado *an*. También en el triángulo *omn* tenemos un lado *mn* y los dos ángulos *omn*, *mno*; por el mismo caso 1 podemos calcular el lado *on*. Hecho esto tenemos en el triángulo *ano*, dos lados: *an*, *no*, y el ángulo comprendido *ano*; entonces por el caso 3 podemos calcular la distancia que buscamos. De paso podemos conocer la posición y dirección de la línea *ao* porque podemos saber el tamaño del ángulo *oan* en el triángulo *ano*, lo que nos permite hallar del lado acá una línea paralela á la línea *ao*. También se puede trazar en una escala conveniente la línea *mn* y los cuatro ángulos medidos y tomar luego en partes de la escala la distancia *ao*.

Si la posición de la línea inaccesible *cn*, fig. 10, fuere tal que se pudiera clavar una estaca *p* en la dirección *cn*, se procederá así: sitúese el instrumento en un punto conveniente *s* y mídanse los áng *psc*, *csn*; mídase también el áng *cps* y la dist *ps*. Entonces en el triángulo *psc*, encuéntrese la distancia *sc* por el caso 1.° Es claro que el áng externo *ncs* será igual á la suma de los internos opuestos *cps*, *psc* y tendremos en el triángulo *csn* un lado y dos ángulos para buscar el lado *cn* por el caso 1.°

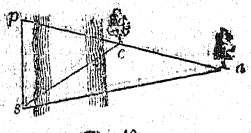



Fig. 10.

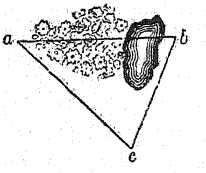



Fig. 11.

**Ej. 6. Medir una distancia** *ab* **cuyos extremos solamente son accesibles.** De *a* y *b* mídanse dos líneas cualesquiera *ac*, *bc* que se corten en *c*; y mídase también el áng *acb*. Tenemos entonces dos lados y el áng comprendido, resuélvase el triáng por el caso 3.°

**Ej. 7. Encontrar la altura vertical** *om* **de una montaña sobre un punto dado** *i*. Sitúese el instrumento en *i*, mídase *im*, póngase el instrumento

horizontal y mídase el áng *man*; como *anm* es de 90°, fig. 12, tenemos un lado *am* y dos áng *man*, *anm*, búsquese *mn* por el caso 1.° Agréguese la altura del instrumento *ai* igual *no*. Si la montaña es muy alta hágase la corrección combinada de curvatura y refracción como en el ejemplo 3, fig. 7.

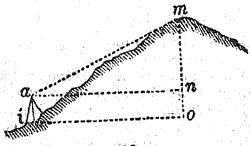



Fig. 12.

También puede colocarse el instrumento en el vértice *m* de la montaña y medir el áng *nma* de depresión en vez del áng *nam*.

**Advertencia 1.°** Si lo que conocemos es la altura *om* de la montaña, la medida del áng *nam* nos servirá para encontrar la distancia *ma* por el caso 1. Así en el ejemplo 2 si se conoce la altura *na*, los ángulos medidos en ese ejemplo nos facilitan el modo de deducir la distancia *ao*; lo mismo decimos para las figs. 3, 4, 5, 7, donde el procedimiento es tan claro que no se necesitan más explicaciones.

**Advertencia 2.° Para medir la altura de un objeto por su sombra.** Colóquese un bastón vertical en el suelo, mídase el largo de la sombra que arroja, también mídase el largo de la sombra del objeto. Entonces : el largo de la sombra del bastón es á la altura del bastón sobre el suelo, como la longitud de la sombra del objeto es á su altura. Si el objeto está inclinado, el bastón debe colocarse con la misma inclinación.

**Advertencia 3.° La altura de un objeto vertical** *mn*, fig. 12½, cuya distancia *rm* es conocida, puede encontrarse valiéndose de su **reflexión** en una vasija de agua, ó en un pedazo de espejo colocado perfectamente horizontal en *r*; porque *ra* es á la altura *ai*, de la vista sobre el reflector *r*, como *rm* es á la altura *mn* del objeto sobre *r*.

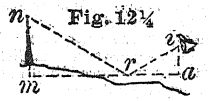



Fig. 12¼

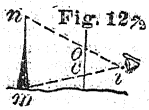



Fig. 12¾

**Advertencia 4.°** Supongamos que *oc*, fig. 12½, sea una estaca plantada ó una vara sostenida verticalmente por un asistente. Entonces, coloquémonos á una distancia propia detrás de ella y manteniendo la vista fija háganse marcas en *o*, *c*, donde las visuales *in*, *im*, cortan la vara ó bastón *oc* y entonces : *ic* es á *co* como *im* á *mn*.

Los ejemplos siguientes pueden sustituir á un procedimiento trigonométrico y serán útiles á veces en caso de no tener á la mano una tabla de senos para hacer los cálculos trigonométricos.

**Ej. 8. Encontrar la distancia** *ab* **en la cual sólo un extremo es acce-**

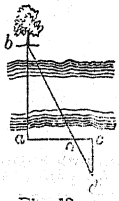



Fig. 13

**sible.** Clávese una estaca en cualquier punto conveniente *a*; en *a* trácese un ángulo cualquiera *bac*. En la línea *ac*, en cualquier punto conveniente *c* clávese una estaca

y en *c* trácese un ángulo *acd*, igual al *bac*. En la línea *cd* y en cualquier punto conveniente, *d*, clávese una estaca. Luego parándose en *d* y mirando hacia *b* colóquese una estaca *o* en línea con *db* y que quede situada al mismo tiempo en línea *ac*, mídanse las distancias *ao*, *oc*, *cd*; y entonces según el principio de los triángulos semejantes

$$oc : cd :: ao : ab.$$

También puede procederse de este otro modo, fig. 14 : siendo la distancia que se ha de medir *nh*, colóquese una estaca en *n*; y trácese el ángulo $hnm = 90°$; á cualquiera distancia conveniente *nm* colóquese una estaca *m*; hágase el áng $hmy = 90°$ y colóquese una estaca en *y* en línea con *hn*. Mídase *my*, *nm*, y entonces según el principio de los triángulos semejantes tenemos $ny : nm = nm : nh$.

**Otro modo** : fig. 14. Trácese el ángulo $hnm = 90°$ colocando una estaca *m* á cualquier distancia conveniente *nm*. Mídase *nm* y también el ángulo *nmh*. Búsquese la tangente de *nmh* por medio de la tabla y multiplíquese esta tangente natural por *nm*; el producto será *nh*.

**Otro modo** : Trácese el ángulo $hnm = 90°$, desde *m* mídase el ángulo *nmh*, y trácese el ángulo *nmy* igual al ángulo *nmh*; colóquese una estaca en *y* en línea con *hn*. Entonces $ny = nh$.

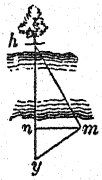



Fig. 14.

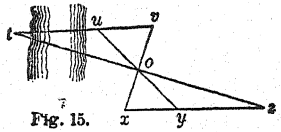



Fig. 15.

**Ó sin medir ningún ángulo, así** : Siendo la distancia *tu* la que quiere medirse tómese *uv* de cualquier longitud en línea recta, con *tu* mídanse *vo* y *ox* iguales. Mídanse *ou*; y *o y* iguales también entre sí. Colóquese una estaca *z*, en línea recta con *xy*, y *to* : entonces *yz* será igual á *tu* y paralela.

**También puede hacerse sin medir ningún ángulo, así** : Clávense dos estacas *t* y *u* en línea con el objeto *s*, fig. 16. Desde *t* márquese una distancia *tx* en cualquiera dirección; desde *u* trácese *uw* paralela á *tx* colocando el punto *w* en línea recta con *xs*. Hágase *uv* igual *tx*, mídase *wv*, *vx*, *tx* y entonces

$$wv : vx :: xt : ts.$$

**Ó así** : * trácese desde un punto *o* la línea *oc* en ángulo recto con *oa* y búsquese en esa línea *oc* un punto *c* desde donde el ángulo *oca* sea de 84° 17′ de manera que el ángulo *oac* resulta 5° 43′; mídase ahora *oc* y *ao* será $= 10 \times oc$ con un error en exceso de 1.06 por 1,000.

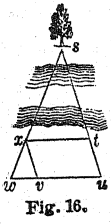



Fig. 16.

**Ej. 9. Encontrar la distancia *ab* cuyos extremos sólo son accesibles**. Trácese desde *a* el ángulo *bac* y desde *b* el ángulo *abd*, cada uno de 90°.

*N. del T. — Aquí discrepamos del autor, pues siendo inaccesible uno de los extremos de la línea, en este caso el *a*, todas las operaciones deben hacerse del lado *oc* y por eso hemos variado las operaciones en este supuesto sin cambiar el sistema de entrar con un ángulo fijo; lo que no nos parece práctico.

Háganse *ac*, *bd*, iguales entre sí. Entonces *cd*=*ab*. También puede considerarse *ab* como la distancia á través del río en las figs. 13, 15 ó 14; y puede encontrarse por los procedimientos ya empleados en dichos casos. O de otro modo, mídase cualquier distancia *ao*, fig. 17, y constrúyase *on* igual y en línea recta con *ao*. Mídase también *bo* y hágase *om* igual y en línea recta de *ob*. Entonces *mn* será paralela é igual *ab*.

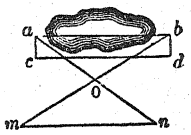



Fig. 17.

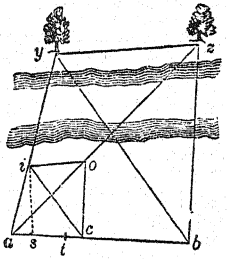



Fig. 18.

**Ej. 10.** Véase el ejemplo 4.º

**Encontrar la distancia y dirección de la línea enteramente inaccesible** *yz*. En dos puntos convenientes *ab*, desde donde los puntos *yz* puedan verse, clávense estacas. Tenemos entonces los cuatro ángulos de una figura de cuatro lados en la cual están dadas las direcciones de tres de sus lados y de sus dos diagonales. Estos datos nos facilitan el trazar sobre el terreno una pequeña figura de cuatro lados semejante á la grande. Así, en la línea *ab* colóquese una estaca *c* y hágase *co* paralela á *bz* situando el punto *o* al mismo tiempo en la diagonal *az*. Desde *c* hágase *ci* paralela á *by*; situando el punto *i* al mismo tiempo en la línea *ay*, entonces *io* será paralela á *yz*. Mídase *ac*, *ab*, *io*, y entonces por el principio de las figuras semejantes

$$ac \ : \ ab \ : : \ io \ : \ yz.$$

Si *yz* fuere una línea visible tal como una cerca ó camino, podríamos desde *a* dividirla en cualquier número de partes. Por ejemplo, si deseamos colocar una estaca en la mitad de *yz*, colóquese primero una en la mitad de *io*; y parándose en *a* por medio de señales diríjase por la estaca situada en el medio de *io* una visual, y donde ésta corte la línea *yz* será el punto medio de *yz* que desde *a* queda determinado así.

Para encontrar sobre *ab* un punto *t* de una perpendicular á la línea *yz* levantada en *y*; hágase primero *ois*=90º y mídase *as*. Entonces

$$oi \ : \ as \ : : \ yz \ : \ at.$$

**Ej. 11. Encontrar la posición de un punto** *n*, **fig. 19, por medio** de dos ángulos *anb*, *bnc*, tomadas desde él á los tres objetos *a*, *b*, *c*, cuyas posición y distancias entre sí son conocidas.

El uso de este problema es más frecuente en cartas marinas que en la mensura

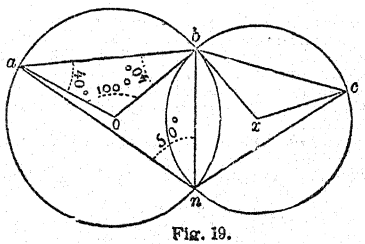



Fig. 19.

de terrenos. Se emplea principalmente para determinar la posición *n* de un bote desde el cual se han hecho sondas á lo largo de la costa. Cada vez que el bote se

mueve de un puesto á otro para sondear nuevamente se hace necesario practicar nuevas observaciones en cada punto con el objeto de fijar su posición en la carta. Cada observación consiste en la medida, por medio de un sextante, de los dos ángulos *anb*, *bnc*, á las señales *a*, *b*, *c*, fijadas de antemano en la costa. Cuando sea practicable tomar las observaciones dirigiendo visuales al bote al mismo instante por medio de dos observadores en la costa, situados en dos de las estaciones, debe preferirse este método. El bote hará entonces una señal en el momento que deban hacerse las observaciones. El método más expedito para fijar el punto *n* sobre el mapa, es trazar tres líneas formando los dos ángulos y prolongarlas indefinidamente en un pedazo de papel transparente. Colóquese el papel sobre el mapa y muévase éste hasta que las tres líneas pasen por las tres estaciones *a*, *b*, *c*; márquese entonces el punto *n* dondequiera que quede.

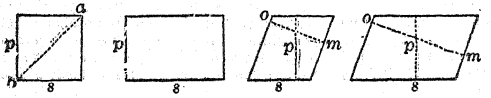
En lugar del papel transparente se puede usar el instrumento llamado marcador de estaciones, cuando hay muchos puntos que marcar.

Pero la posición del punto puede encontrarse, quizás con más exactitud, describiendo dos círculos, como en la fig. 19, que pase cada uno por el punto *n* y por dos de los otros tres puntos; encontrando luego los centros, *o*, *x*, de dichos dos círculos. Esto es muy sencillo. Sabemos que el ángulo *aob* en el centro de un círculo es dos veces mayor que cualquier ángulo *anb* en la circunferencia del mismo círculo cuando éstos están subtendidos por la misma cuerda *ab*. Por consiguiente, si el ángulo *anb* observado desde el bote es de 50°, el ángulo *aob* debe ser de 100° y como la suma de los tres ángulos de un triángulo es 180° los dos ángulos *oab*, *oba*, valen juntos $180° - 100° = \frac{80°}{2}$, y como los dos lados *ao*, *bo*, son iguales (siendo radios del mismo círculo), los ángulos *oab*, *oba* son iguales entre sí y cada uno igual á $80°/_2 = 40°$. Por consiguiente, tenemos solamente que trazar sobre el mapa, en *a* y *b*, dos ángulos de 40°, el punto *o* de intersección será el centro. Procédase de la misma manera con el ángulo *bnc* para encontrar el centro.

La intersección de los dos círculos en *n* será el punto que se busca.

## PARALELOGRAMOS

**Cuadrado. Rectángulo. Losango. Paralelogramo.**




Un paralelogramo ó rombo es una figura plana encerrada por cuatro líneas rectas y paralelas las opuestas. No hay sino los cuatro indicados en las figuras anteriores. En el cuadrado y el *losango* los cuatro lados son iguales; en el *paralelogramo rectángulo* sólo los lados opuestos son iguales. En cualquier paralelogramo los cuatro ángulos suman cuatro ángulos rectos ó 360° y los dos ángulos diagonales opuestos son iguales entre sí; por consiguiente, teniendo un ángulo pueden encontrarse fácilmente los otros tres. En un cuadrado ó un rectángulo cada diagonal divide á cada uno de los ángulos en dos partes iguales, pero en los otros dos paralelogramos no sucede así.

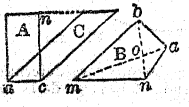
**Para encontrar el área de cualquier paralelogramo.** Multiplíquese cualquier lado, *s*, por la altura *p* tomada perpendicularmente al lado *s* ó multiplíquense entre sí dos lados y el seno del ángulo comprendido.

**La diagonal *ab* de cualquier cuadrado** es igual á un lado multiplicado por 1.41421, y un lado es igual á la diagonal multiplicada por .707107.

**El lado de un cuadrado igual en superficie á un círculo dado** es igual al diámetro × .886227.

**El lado del mayor cuadrado** que puede inscribirse en un círculo dado es igual al diámetro × .707107.

El lado de un cuadrado multiplicado por 1.51967 da el lado de **un triángulo**



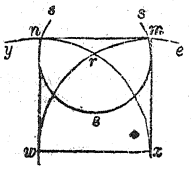
**equilátero** de la misma área. **Todos los paralelogramos** como A, C, **que tienen iguales bases** *ac*, é igual altura *nc*, tienen iguales áreas y el área de cada uno es dos veces la de un triángulo que tenga la misma base y la misma altura. **El área de un cuadrado inscrito en un círculo** es igual á dos veces el cuadrado del radio.

**En todo paralelogramo** los cuatro cuadrados trazados sobre sus lados tienen una área igual á los de los dos cuadrados trazados en sus 2 diagonales. Si se traza **un cuadrado grande** en la diagonal *ab* de un cuadrado más pequeño su área será dos veces la del cuadrado pequeño. **Cualquiera de las diagonales de cualquier paralelogramo** lo dividen en dos triángulos iguales y las 2 diagonales lo dividen en 4 triángulos de iguales áreas. **Las dos diagonales de cualquier paralelogramo** se dividen entre sí en dos partes iguales. **Cualquiera línea recta trazada por el centro** del paralelogramo lo divide en dos partes iguales.

**Advertencia 1.ª El área de cualquiera figura, como B formada por cuatro líneas rectas,** puede encontrarse así: Multipliquense las dos diagonales *am*, *nb*, y el seno del ángulo menor *aob*, ó *nom* formado por la intersección de ellas. Divídase el producto por 2. Esto es útil en la mensura de terrenos cuando los obstáculos, como á menudo sucede, hacen difícil la medida de los lados de la figura, mientras que es fácil medir las diagonales y después encontrar su punto de intersección *o* y medir el ángulo que se requiere. **Pero si se tiene que trazar la figura, las partes** *oa*, *ob*, *on*, *om* de las diagonales deben ser medidas también.

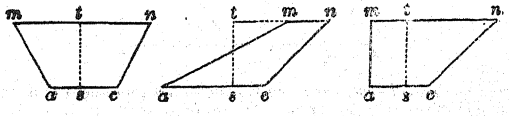
**Advertencia 2.ª Los lados de un paralelogramo ó triángulo y los de muchas otras figuras pueden encontrarse cuando sólo se conoce el área y los ángulos,** así: Supongamos que uno de los lados sea 1 y calcúlese cuál sería su área para el lado 1 y dígase: la raíz cuadrada del área así encontrada es á este lado 1 como la raíz cuadrada del área dada es al lado correspondiente de la figura que se busca.

**En una línea dada** *wx* **trazar un cuadrado** *wxnm*. De *w*, y de *x*, con radio *wx* descríbanse los arcos *xry*, *wre*. Desde su intersección *r*, y con un radio igual á ½




*wx*, descríbase *sss*. Desde *wx* trácese *wn*, *xm* tangente á *sss*, hasta llegar á los otros arcos y únase *n* con *m*.

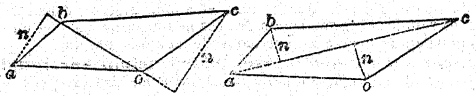
## TRAPECIOS




Un trapecio *acnm* es un cuadrilátero que tiene dos lados paralelos *ac*, *nm*.

**Para encontrar el área de cualquier trapecio.** Súmense los dos lados *ac*, *nm*, multipliquese la suma por la distancia *st* y divídase el producto por 2. Véanse las reglas siguientes para los cuadriláteros en general que son todas aplicables á los trapecios y véanse también las advertencias después de los paralelogramos.

**Un** cuadrilátero *abco* es cualquiera figura terminada por cuatro líneas rectas.



**Para encontrar el área de un cuadrilátero conociendo la diagonal** *bo* ó la *ac* **que une dos ángulos opuestos y también las dos perpendiculares** *nn* **bajadas desde los otros dos ángulos** : súmense estas dos perpendiculares, multiplíquese la suma por la diagonal y divídase el producto por 2.

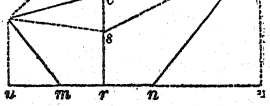
**Teniendo los cuatro lados y uno de los dos pares de ángulos opuestos como** *abc, aoc*, ó bien *bao, bco*, considérese el cuadrilátero dividido en dos triángulos en cada uno de los cuales se conocen dos lados y el ángulo comprendido. Búsquese el área de cada uno de estos triángulos como se ha indicado ya en el capítulo « Triángulos » y súmense.

**Teniendo los cuatro ángulos y un par de lado supuestos.** Supongamos que se conoce un lado y los dos ángulos de sus extremos. Si la suma de estos ángulos excede de 180° réstese cada uno de ellos de 180° y hágase uso de estas diferencias en lugar de los ángulos mismos. Considérense entonces este lado y sus dos ángulos adyacentes (ó las dos diferencias, según sea el caso) como los de un triángulo y búsquese su área como se ha indicado en el capítulo « Triángulos ». Hágase lo mismo con el otro lado dado y sus dos ángulos adyacentes (ó sus diferencias); réstese la menor de las áreas encontradas así de la mayor y la diferencia será el área que se busca.

**Teniendo tres lados y los dos ángulos comprendidos.** Multiplíquese el lado intermedio por uno de los lados adyacentes; multiplíquese el producto por el seno del ángulo comprendido entre estos dos lados, llámese el resultado *a*. Hágase lo mismo con el mismo lado intermedio, el otro lado adyacente y el seno del otro ángulo comprendido y llámese *b* al resultado. Súmense los dos ángulos, búsquese la diferencia entre esa suma y 180°, sea mayor ó menor, búsquese el seno de esa diferencia, multiplíquense los dos lados dados opuestos el uno al otro; multiplíquese el producto por el seno que se acaba de encontrar y llámase *c* el resultado. Súmense los resultados *a, b*, si la suma de los dos ángulos dados es menor de 180°, réstese *c* de la suma $a+b$ y la mitad de la diferencia será el área del cuadrilátero. Pero si la suma de los dos ángulos dados es mayor que 180° súmense los tres resultados *a, b, c* y la mitad de dicha suma será el área.

**Teniendo las dos diagonales y uno de los dos ángulos formados por la intersección de ellas.** Véanse las advertencias después del párrafo paralelogramos.

**En las medidas de banqueos y terraplenes en los ferrocariles** se presenta muy frecuentemente la medida del cuadrilátero *lmno* y también la de las




dos figuras de 5 lados *lmnot, lmnos*, en las cuales *nm* representa el ancho de vía, *rs, rc, rt*, lo que hay que banquear en el centro; *lu, ov* la altura del talud inferior y superior sobre la vía; y *lm, no* los dos taludes.

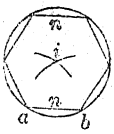
La misma regla general para medir las áreas se seguirá en estas tres figuras; á saber : multiplíquese la anchura del extremo horizontal *uv* por la mitad de la altura en el centro *rs, rc*, ó *rt*, según sea el caso. Multiplíquese también una cuarta parte del ancho de vía *mn* por la suma de las dos alturas *lu, ov*, súmense los dos productos y la suma será el área que se busca. Esta regla es aplicable cuando tengan, ó no, los dos taludes *lm, no*, la misma inclinación.

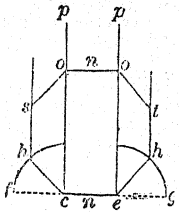
**En los trabajos de ferrocarril, etc.**, las dimensiones que acabamos de ver de la sección media del prismoide; como el ancho medio horizontal, altura del centro y de los taludes,es la semisuma de esas mismas dimensiones en los extremos del sólido y pueden obtenerse sin necesidad de medidas directas.

**Para trazar un exágono cuyos lados deban ser iguales á una línea dada,** *ab*. De *ab* con un radio *ab* descríbanse dos arcos; desde su inter-

sección *i*, con el mismo radio descríbase un círculo; alrededor y sobre su circunferencia váyase colocando el mismo radio.

Lado de un exágono $= n \times .57735$.

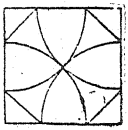


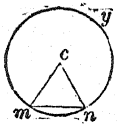




**Para trazar un octógono cuyos lados sean iguales á una línea dada, *ce*.** De *c* y de *e* trácense dos perpendiculares *cp*, *ep*, prolónguese también *c*, *e* hacia *f*, *g* y de *c*, *e* con un radio igual *ce* trácense dos cuadrantes y búsquense sus centros *hh*; únanse *ch*, *eh*; trácense *hs*, *ht* paralelas á *cp* é iguales á *ce*; háganse *co*, *eo*, iguales cada una á *hh* únanse *co*, *os*, *ot*.

Lado de un octógono $= n \times .41421354$.

**Para trazar un octógono en un cuadrado dado** : desde cada ángulo del cuadrado y con un radio igual á la mitad de su diagonal, descríbanse cuatro arcos y únanse los puntos en los cuales cortan los lados del cuadrado.






**Para trazar cualquier polígono regular de lados iguales á *mn*** ; divídase á 360° por el número de lados, réstese el cociente de 180°, divídase la diferencia por 2. Esto dará el ángulo *cmn*, ó *cnm*. En *nm* trácense estos ángulos por medio de un transportador, los lados de estos ángulos se encontrarán en un punto *c* desde el cual se describirá el círculo *mny*, sobre su circunferencia se irán colocando distancias iguales á *mn*.

**En cualquier círculo *mny* trazar cualquier polígono regular.** Divídase á 360° por el número de lados, el cociente será el ángulo *mcn* del centro. Trácese el ángulo por medio de un transportador y su cuerda *mn* será un lado que se va colocando alrededor de la circunferencia.

**Para reducir cualquier polígono como *abcdefu*, á un triángulo de la misma área.** Prolongamos el lado *fa* hacia *w* y trazamos *bg* paralela á *ca*

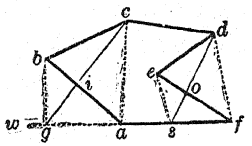



Fig. 1.

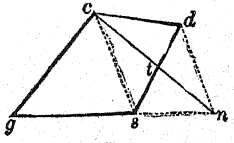
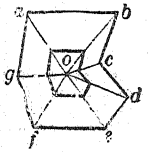



Fig. 2.

y trazamos *ge*; obtenemos 2 triángulos *abc*, *age*, ambos de la misma base *ca* y de la misma altura, puesto que sus vértices están entre las dos paralelas *ac*, *gb*. Pero la parte *aci* es común á ambos; por tanto las partes restantes *icb* de uno *iga* del otro serán iguales entre sí. Ahora si la parte *icb* se la quita al polígono y se le pone la parte *iga*, el polígono *gfedcig* tendrá la misma área que *afedcba*, pero no tendrá sino cinco lados mientras que el otro tiene seis. Además, si *se* se traza paralela á *df* y se une *i* con *s* tenemos sobre la misma base *es*, y entre las mismas paralelas *es*, *df* los dos triángulos iguales *esd*, *esf* con la parte *eos* común á ambos y por consiguiente la parte restante *cod* de uno, *osf* del otro serán iguales. Por tanto si se quita *osf* del polígono y se agrega *eod*, el nuevo polígono *gsdc*, fig. 2,

tendrá la misma área que *gfedc;* pero no tiene sino cuatro lados, mientras que el otro tiene cinco. Finalmente, si se prolonga *gs*, fig. 2, hacia *n*, se traza *dn* paralela á *cs*, y se une *c* con *n* tenemos en la misma base *cs*, y entre las mismas paralelas *cs*, *dn* los triángulos equivalentes *csn*, *csd*, con la parte *cst* común á ambos; por tanto si quitamos á *cdt* del polígono y le agregamos *stn* tenemos el triángulo *gnc* igual al polígono *gsdcg*, fig. 2, ó á *afedcba*, fig. 1. Este método es aplicable á polígonos de cualquier número de lados.

**Reducir una figura grande *abcdefg* á una semejante más pequeña.** Desde cualquier punto interior *o* (será mejor mientras más cerca del centro esté) trácense líneas á todos los ángulos *abc*, etc. y únanse estas líneas con otras para-

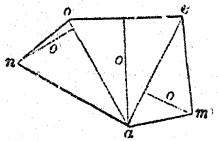


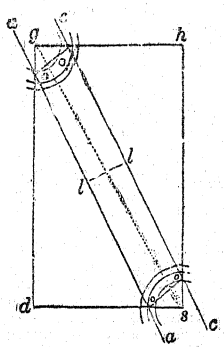

lelas á los lados de la figura. Si por el contrario se quiere aumentar una figura pequeña, trácense de cualquier punto *o* dentro de ella, líneas que se prolonguen más allá de sus ángulos y únanse estas líneas por otras paralelas á los lados de la figura pequeña.

**Para reducir un mapa á una escala menor.** El mejor método es dividir el mapa grande en cuadrados por medio de líneas ligeras trazadas con lápiz muy suave y trazar luego el mapa reducido sobre un papel dividido en cuadrados menores. El compás de división ayudará mucho al fijar puntos situados dentro de los cuadrados. Si el mapa grande ha sufrido por el dibujo y el continuo borrar, se pueden tender hilos para formar los cuadros.

**En una figura rectangular** *ghsd*, encontrar los puntos *oooo* sobre sus lados y á igual distancia de los ángulos *gs;* para inscribir un rectángulo diagonal *oooo* de una anchura dada *ll*. Desde *g*, *s*, como centros descríbanse varios arcos concéntricos, como están en la figura. Trácense sobre papel transparente dos líneas *aa*, *cc* separadas una distancia *ll*, y colocando estas líneas sobre el tablero muévanse hasta que se vea por los arcos que las cuatro distancias *go*, *go*, *so*, *so*, son iguales. En lugar del papel transparente se puede usar una tira de papel común del ancho *ll*.

**Advertencia.** Muchos problemas que de otro modo serían muy difíciles pueden resolverse de modo análogo con una exactitud suficiente para casos prácticos **por medio del papel transparente.**

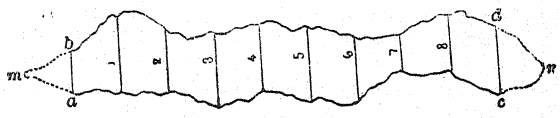



**Para encontrar el área de cualquier polígono irregular** *anbcm*. Divídase éste en triángulos como *anb*, *amc*, *abc*, búsquense las alturas *o*, *o*, *o*. de cada uno de ellos entre su base *ab*, *ac*, *bc* y el ángulo opuesto *n*, *m;* multipli-

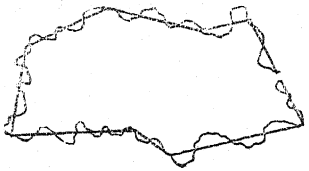
quese cada base por la altura respectiva, súmense todos los productos juntos y divídase por 2.

**Para encontrar aproximadamente el área de una figura muy larga, irregular como** *abcd*. Entre sus extremos *abcd* márquense iguales distancias (mientras más cortas sean más exacto será el resultado), trácense líneas paralelas intermedias 1, 2, 3, etc. Mídanse las longitudes de estas líneas




intermedias, súmense; á esta suma agréguese la semí suma de los dos anchos *ab*, *cd* en los extremos; multiplíquese la suma total por uno de los espacios iguales entre las líneas paralelas y el producto será el área que se busca. Esta regla se aplica del mismo modo si ambos, ó uno de los extremos, terminan en punta como *m*, *n*. En el último de estos casos los anchos *ab*, *cd*, entrarán á sumarse con las líneas intermedias y es claro que la mitad de la anchura en los dos extremos será *o*.

**Encontrar el área de una figura cuyo perfil es extremadamente irregular.** Trácense líneas alrededor de ella que le agreguen á la figura (tan aproximadamente como pueda juzgarse á la vista) tanto espacio como el que le quiten á la misma figura. El área de la figura nueva que se forma se hace de



este modo igual á la figura irregular y puede calcularse dividiéndola en triángulos, etc. Usando un pedazo de hilo fino puede encontrarse la adecuada posición para las nuevas líneas limítrofes antes de trazarlas. Para medir áreas pequeñas é irregulares en un dibujo se colocará sobre él un pedazo de papel transparente dividido cuidadosamente en pequeños cuadros cada uno de una área dada, digamos de 10, 20 ó 100 pies ó decímetros, etc., cuadrados y luego se contarán todos los cuadrados enteros y se agregarán las fracciones de cuadrados apreciadas al ojo.

## CÍRCULOS

**Un círculo** es la superficie comprendida por una curva reentrante llamada circunferencia cuyos puntos equidistan de otro interior llamado **centro**. Toda línea recta que pasa por el centro y termina por ambos lados en la circunf se llama **diámetro** del círculo. Toda línea recta que va del centro á la circunf se llama **radio**.

La relación de la circunf al diám ó de la semicircunf al radio, circunf ÷ diám se llama $\pi$ (**pi**).

$$\pi = 4\,(1 - 1/3 + 1/5 - 1/7 + 1/9 - \text{etc.})$$

$\pi$ es inconmensurable, no tiene expresión aritmética exacta. Ha sido determinado hasta varios cientos de cifras decimales.

**Valores aproximados de $\pi$; $\pi = x + a/a$:**

| $x$ | $a$ | $x$ | $a$ |
|---|---|---|---|
| 3.141 592 653 6. | — 307,788,000,000 | 355 ÷ 113 .. | — 11,776,700 |
| 3.141 592 65.... | ... + 875,085,000 | 377 ÷ 120 .. | ... — 42,447 |
| 3.141 593..... | ..... — 9,069,000 | 366 ÷ 116.6.. | ... + 13,576 |
| 3.141 6..... | ..... — 427,637 | 22 ÷ .. | .... — 2,485 |

## Funciones de $\pi$.

| | Logarit. | | Logarit. |
|---|---|---|---|
| $\pi = 3.14159265$ | 0.497 1499 | $2/\pi = 0.636620$ | $\bar{1}.803\ 8801$ |
| $\pi/2 = 1.57079632$ | 0.196 1199 | $3/\pi = 0.954930$ | $\bar{1}.979\ 9714$ |
| $\pi/3 = 1.04719755$ | 0.020 0286 | $\pi^2 = 9.869604$ | 0.994 2997 |
| $\pi\sqrt{2} = 4.442883$ | 0.647 6649 | $\pi^3 = 31.00628$ | 1.491 4496 |
| $\pi/\sqrt{2} = 2.221442$ | 0.346 6349 | $\sqrt{\pi} = 1.772454$ | 0.248 5749 |
| $1/\pi = 0.318310$ | $\bar{1}.502\ 8501$ | $1/\sqrt{\pi} = 0.564190$ | $\bar{1}.741\ 4251$ |

**Para múltiplos de** $\pi$, véase « Circunf » en las tablas de los *Círculos* que siguen.

**Radio,** $R$; **Diámetro,** $D$; **Circunf,** $C$; **Área,** $A$.

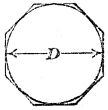
$$R = \frac{D}{2} = \frac{C}{2\pi} = \sqrt{\frac{A}{\pi}}; \qquad D = 2R = \frac{C}{\pi} = 2\sqrt{\frac{A}{\pi}};$$

$$C = 2\pi R = \pi D = 2\sqrt{\pi A}; \qquad A = \pi R^2 = \frac{\pi}{4} D^2 = \frac{C^2}{4\pi} = \frac{DC}{4}.$$

## RELACIONES DEL CÍRCULO CON OTRAS FIGURAS

**Polígono circunscrito,** regular ó irregular.

$$\frac{\text{Area del circ}}{\text{Area del políg}} = \frac{\text{circunf del círc}}{\text{perímetro del políg}}.$$




**Triángulo equilátero de igual área.** D=diám del círc.

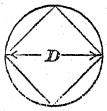
Lado del triáng $= D\sqrt{\pi} \div \sqrt[4]{3} = 1.34677\ D.$

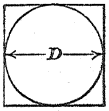
**Cuadrado de la misma área.**

Lado del cuad= .88623 × diám del círc.
Diám del círc =1.12838 × lado del cuad.

**Cuadrado inscrito.** D=diám del círc.

Lado del cuad $= D\sqrt{\frac{1}{2}} = .7071\ D.$
Area del cuad = 2 × radio² (del círculo)
Radio del círc = ½ × diagonal del cuad
$= \sqrt{\frac{1}{2}} \times$ lado del cuad = .7071 × lado.




**Cuadrado circunscrito.**

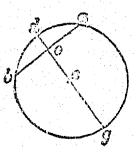
A=área del cuad; $a$=área del círc.

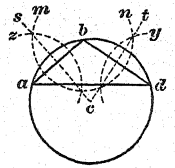
$$A = \frac{4}{\pi} a = 1.2732\ a; \qquad a = \frac{\pi}{4} A = .7854\ A.$$

## PROBLEMAS RELATIVOS Á LOS CÍRCULOS

**Encontrar el diám *dg* y el centro *c* de un círculo dado.**

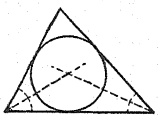
Trácese una cuerda cualquiera *ab* y en su centro *o* levántese un diám perpendicular *dg*. Encuéntrese el centro *c* de este diám. O véase más abajo.

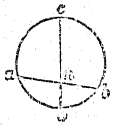


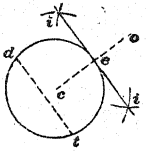




**Describir una circunf que pase por tres puntos *abc* que no están en línea recta, por ej. : los tres vértices de un triángulo.** Unanse los puntos por líneas *ab*, *bd*. Desde *b* con cualquier radio conveniente, se traza el arco *mn* y de *a*, y, *d* con el mismo radio trácense los arcos *y*, *z*; entonces las dos líneas *cs*, *ct* que unen las intersecciones de estos arcos, se cortarán en el centro *c* del círculo.

**Inscribir un círculo en un triángulo.** Trácense dos líneas *bisectrices* de dos cualesquiera de los ángulos; donde estas dos líneas se cortan está el centro del círculo.



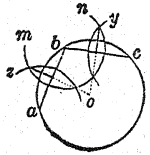


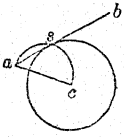



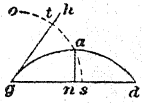

En **dos cuerdas** cualesquiera *ab*, *oc*, que se cortan en *n* se tiene que *on* : *nb* :: *an* : *nc*; por tanto, *nb*×*an*=*on*×*nc*. Es decir, el producto de los dos segmentos de una de las cuerdas=al producto de los dos segmentos de la otra.

**Tangentes.** Aquí la tang es simplemente una línea indefinida que toca á la circunf en un punto; no es la tang trigonométrica.

**Para trazar una tang *iei* á un circ por un punto dado *e* de su circunf**; trácese el radio *ce* á ese punto, hágase *eo*=*ec* de *c* y *o* con cualquier radio mayor que la mitad de *co*, descríbanse dos pares de arcos *i*, *i*, y únanse sus intersecciones *i*, *i*. O bien de *e*, trácense dos distancias cualesquiera iguales *ed*, *et*, y trácese *i*, *i*, paralela á *dt*.




**Para trazar una tang *asb* á un circ desde un punto afuera del circ**; únase *a* con *c*, centro del circ. Sobre *ac* trácese una semicircunf y por la intersección *s* trácese *asb*.

**Para trazar una tang *gh* á un arco circular *gad* cuya flecha es *na*.** Con el radio *ga*, descríbase un arco *sao*, hágase *ta*=*sa*; por *t* tírese *gh*.

## TABLA DE CÍRCULOS

**Los diámetros se dan en unidadades y décimos.**

| Diám. | Circf. | Area. | Diám. | Circf. | Area. | Diám. | Circf. | Area. |
|---|---|---|---|---|---|---|---|---|
| 0.1 | .314159 | .007854 | 6.3 | 19.79203 | 31.17245 | 12.5 | 39.26991 | 122.7185 |
| .2 | .628319 | .031416 | .4 | 20.10619 | 32.16991 | .6 | 39.58407 | 124.6398 |
| .3 | .942478 | .070686 | .5 | 20.42035 | 33.18307 | .7 | 39.89823 | 126.6769 |
| .4 | 1.256637 | .125664 | .6 | 20.73451 | 34.21194 | .8 | 40.21239 | 128.6796 |
| .5 | 1.570796 | .196350 | .7 | 21.04867 | 35.25652 | .9 | 40.52655 | 130.6981 |
| .6 | 1.884956 | .282743 | .8 | 21.36283 | 36.31681 | 13.0 | 40.84070 | 132.7323 |
| .7 | 2.199115 | .384845 | .9 | 21.67699 | 37.39281 | .1 | 41.15486 | 134.7822 |
| .8 | 2.513274 | .502655 | 7.0 | 21.99115 | 38.48451 | .2 | 41.46902 | 136.8478 |
| .9 | 2.827433 | .636173 | .1 | 22.30531 | 39.59192 | .3 | 41.78318 | 138.9291 |
| 1.0 | 3.141593 | .785398 | .2 | 22.61947 | 40.71504 | .4 | 42.09734 | 141.0261 |
| .1 | 3.455752 | .950332 | .3 | 22.93363 | 41.85387 | .5 | 42.41150 | 143.1388 |
| .2 | 3.769911 | 1.13097 | .4 | 23.24779 | 43.00840 | .6 | 42.72566 | 145.2672 |
| .3 | 4.084070 | 1.32732 | .5 | 23.56194 | 44.17865 | .7 | 43.03982 | 147.4114 |
| .4 | 4.398230 | 1.53938 | .6 | 23.87610 | 45.36460 | .8 | 43.35398 | 149.5712 |
| .5 | 4.712389 | 1.76715 | .7 | 24.19026 | 46.56626 | .9 | 43.66814 | 151.7468 |
| .6 | 5.026548 | 2.01062 | .8 | 24.50442 | 47.78362 | 14.0 | 43.98230 | 153.9380 |
| .7 | 5.340708 | 2.26980 | .9 | 24.81858 | 49.01670 | .1 | 44.29646 | 156.1450 |
| .8 | 5.654867 | 2.54469 | 8.0 | 25.13274 | 50.26548 | .2 | 44.61062 | 158.3677 |
| .9 | 5.969026 | 2.83529 | .1 | 25.44690 | 51.52997 | .3 | 44.92477 | 160.6061 |
| 2.0 | 6.283185 | 3.14159 | .2 | 25.76106 | 52.81017 | .4 | 45.23893 | 162.8602 |
| .1 | 6.597345 | 3.46361 | .3 | 26.07522 | 54.10608 | .5 | 45.55309 | 165.1300 |
| .2 | 6.911504 | 3.80133 | .4 | 26.38938 | 55.41769 | .6 | 45.86725 | 167.4155 |
| .3 | 7.225663 | 4.15476 | .5 | 26.70354 | 56.74502 | .7 | 46.18141 | 169.7167 |
| .4 | 7.539822 | 4.52389 | .6 | 27.01770 | 58.08805 | .8 | 46.49557 | 172.0336 |
| .5 | 7.853982 | 4.90874 | .7 | 27.33186 | 59.44679 | .9 | 46.80973 | 174.3662 |
| .6 | 8.168141 | 5.30929 | .8 | 27.64602 | 60.82123 | 15.0 | 47.12389 | 176.7146 |
| .7 | 8.482300 | 5.72555 | .9 | 27.96017 | 62.21139 | .1 | 47.43805 | 179.0786 |
| .8 | 8.796459 | 6.15752 | 9.0 | 28.27433 | 63.61725 | .2 | 47.75221 | 181.4584 |
| .9 | 9.110619 | 6.60520 | .1 | 28.58849 | 65.03882 | .3 | 48.06637 | 183.8539 |
| 4.0 | 9.424778 | 7.06858 | .2 | 28.90265 | 66.47610 | .4 | 48.38053 | 186.2650 |
| .1 | 9.738937 | 7.54768 | .3 | 29.21681 | 67.92909 | .5 | 48.69469 | 188.6919 |
| .2 | 10.05310 | 8.04248 | .4 | 29.53097 | 69.39778 | .6 | 49.00885 | 191.1345 |
| .3 | 10.36726 | 8.55299 | .5 | 29.84513 | 70.88218 | .7 | 49.32300 | 193.5928 |
| .4 | 10.68142 | 9.07920 | .6 | 30.15929 | 72.38229 | .8 | 49.63716 | 196.0668 |
| .5 | 10.99557 | 9.62113 | .7 | 30.47345 | 73.89811 | .9 | 49.95132 | 198.5565 |
| .6 | 11.30973 | 10.17876 | .8 | 30.78761 | 75.42964 | 16.0 | 50.26548 | 201.0619 |
| .7 | 11.62389 | 10.75210 | .9 | 31.10177 | 76.97687 | .1 | 50.57964 | 203.5831 |
| .8 | 11.93805 | 11.34115 | 10.0 | 31.41593 | 78.53982 | .2 | 50.89380 | 206.1199 |
| .9 | 12.25221 | 11.94591 | .1 | 31.73009 | 80.11847 | .3 | 51.20796 | 208.6724 |
| 4.0 | 12.56637 | 12.56637 | .2 | 32.04425 | 81.71282 | .4 | 51.52212 | 211.2407 |
| .1 | 12.88053 | 13.20254 | .3 | 32.35840 | 83.32289 | .5 | 51.83628 | 213.8246 |
| .2 | 13.19469 | 13.85442 | .4 | 32.67256 | 84.94867 | .6 | 52.15044 | 216.4243 |
| .3 | 13.50885 | 14.52201 | .5 | 32.98672 | 86.59015 | .7 | 52.46460 | 219.0397 |
| .4 | 13.82301 | 15.20531 | .6 | 33.30088 | 88.24734 | .8 | 52.77876 | 221.6708 |
| .5 | 14.13717 | 15.90431 | .7 | 33.61504 | 89.92024 | .9 | 53.09292 | 224.3176 |
| .6 | 14.45133 | 16.61903 | .8 | 33.92920 | 91.60884 | 17.0 | 53.40708 | 226.9801 |
| .7 | 14.76549 | 17.34945 | .9 | 34.24336 | 93.31316 | .1 | 53.72123 | 229.6583 |
| .8 | 15.07964 | 18.09557 | 11.0 | 34.55752 | 95.03318 | .2 | 54.03539 | 232.3522 |
| .9 | 15.39380 | 18.85741 | .1 | 34.87168 | 96.76891 | .3 | 54.34955 | 235.0618 |
| 5.0 | 15.70796 | 19.63495 | .2 | 35.18584 | 98.52035 | .4 | 54.66371 | 237.7871 |
| .1 | 16.02212 | 20.42821 | .3 | 35.50000 | 100.2875 | .5 | 54.97787 | 240.5282 |
| .2 | 16.33628 | 21.23717 | .4 | 35.81416 | 102.0703 | .6 | 55.29203 | 243.2849 |
| .3 | 16.65044 | 22.06183 | .5 | 36.12832 | 103.8689 | .7 | 55.60619 | 246.0574 |
| .4 | 16.96460 | 22.90221 | .6 | 36.44247 | 105.6832 | .8 | 55.92035 | 248.8456 |
| .5 | 17.27876 | 23.75829 | .7 | 36.75663 | 107.5132 | .9 | 56.23451 | 251.6494 |
| .6 | 17.59292 | 24.63009 | .8 | 37.07079 | 109.3588 | 18.0 | 56.54867 | 254.4690 |
| .7 | 17.90708 | 25.51759 | .9 | 37.38495 | 111.2202 | .1 | 56.86283 | 257.3043 |
| .8 | 18.22124 | 26.42079 | 12.0 | 37.69911 | 113.0973 | .2 | 57.17699 | 260.1553 |
| .9 | 18.53540 | 27.33971 | .1 | 38.01327 | 114.9901 | .3 | 57.49115 | 263.0220 |
| 6.0 | 18.84956 | 28.27433 | .2 | 38.32743 | 116.8987 | .4 | 57.80530 | 265.9044 |
| .1 | 19.16372 | 29.22467 | .3 | 38.64159 | 118.8229 | .5 | 58.11946 | 268.8025 |
| .2 | 19.47787 | 30.19071 | .4 | 38.95575 | 120.7628 | .6 | 58.43362 | 271.7163 |

## TABLA DE CÍRCULOS. (Continuación.)

**Los diámetros se dan en unidades y décimos.**

| Diám. | Circf. | Area. | Diám. | Circf. | Area. | Diám. | Circf. | Area. |
|---|---|---|---|---|---|---|---|---|
| 18.7 | 58.74778 | 274.6459 | 24.9 | 78.22566 | 486.9547 | 31.1 | 97.70353 | 759.6450 |
| 8 | 59.06194 | 277.5911 | 25.0 | 78.53982 | 490.8739 | .2 | 98.01769 | 764.5380 |
| 9 | 59.37610 | 280.5521 | .1 | 78.85398 | 494.8087 | .3 | 98.33185 | 769.4467 |
| 19.0 | 59.69026 | 283.5287 | .2 | 79.16813 | 498.7592 | .4 | 98.64601 | 774.3712 |
| 1 | 60.00442 | 286.5211 | .3 | 79.48229 | 502.7255 | .5 | 98.96017 | 779.3113 |
| .2 | 60.31858 | 289.5292 | .4 | 79.79645 | 506.7075 | .6 | 99.27433 | 784.2672 |
| .3 | 60.63274 | 292.5530 | .5 | 80.11061 | 510.7052 | .7 | 99.58849 | 789.2388 |
| .4 | 60.94690 | 295.5925 | .6 | 80.42477 | 514.7185 | .8 | 99.90265 | 794.2260 |
| .5 | 61.26106 | 298.6477 | .7 | 80.73893 | 518.7476 | .9 | 100.2168 | 799.2290 |
| .6 | 61.57522 | 301.7186 | .8 | 81.05309 | 522.7924 | 32.0 | 100.5310 | 804.2477 |
| .7 | 61.88938 | 304.8052 | .9 | 81.36725 | 526.8529 | .1 | 100.8451 | 809.2821 |
| .8 | 62.20353 | 307.9075 | 26.0 | 81.68141 | 530.9292 | .2 | 101.1593 | 814.3322 |
| .9 | 62.51769 | 311.0255 | .1 | 81.99557 | 535.0211 | .3 | 101.4734 | 819.3980 |
| 20.0 | 62.83185 | 314.1593 | .2 | 82.30973 | 539.1287 | .4 | 101.7876 | 824.4796 |
| .1 | 63.14601 | 317.3087 | .3 | 82.62389 | 543.2521 | .5 | 102.1018 | 829.5768 |
| .2 | 63.46017 | 320.4739 | .4 | 82.93805 | 547.3911 | .6 | 102.4159 | 834.6898 |
| .3 | 63.77433 | 323.6547 | .5 | 83.25221 | 551.5459 | .7 | 102.7301 | 839.8184 |
| .4 | 64.08849 | 326.8513 | .6 | 83.56636 | 555.7163 | .8 | 103.0442 | 844.9628 |
| .5 | 64.40265 | 330.0636 | .7 | 83.88052 | 559.9025 | .9 | 103.3584 | 850.1228 |
| .6 | 64.71681 | 333.2916 | .8 | 84.19468 | 564.1044 | 33.0 | 103.6726 | 855.2986 |
| .7 | 65.03097 | 336.5353 | .9 | 84.50884 | 568.3220 | .1 | 103.9867 | 860.4901 |
| .8 | 65.34513 | 339.7947 | 27.0 | 84.82300 | 572.5553 | .2 | 104.3009 | 865.6973 |
| .9 | 65.65929 | 343.0698 | .1 | 85.13716 | 576.8043 | .3 | 104.6150 | 870.9202 |
| 21.0 | 65.97345 | 346.3606 | .2 | 85.45132 | 581.0690 | .4 | 104.9292 | 876.1588 |
| 1 | 66.28760 | 349.6671 | .3 | 85.76548 | 585.3494 | .5 | 105.2434 | 881.4131 |
| .2 | 66.60176 | 352.9894 | .4 | 86.07964 | 589.6455 | .6 | 105.5575 | 886.6831 |
| .3 | 66.91592 | 356.3273 | .5 | 86.39380 | 593.9574 | .7 | 105.8717 | 891.9688 |
| .4 | 67.23008 | 359.6809 | .6 | 86.70796 | 598.2849 | .8 | 106.1858 | 897.2703 |
| .5 | 67.54424 | 363.0503 | .7 | 87.02212 | 602.6282 | .9 | 106.5000 | 902.5874 |
| .6 | 67.85840 | 366.4354 | .8 | 87.33628 | 606.9871 | 34.0 | 106.8142 | 907.9203 |
| .7 | 68.17256 | 369.8361 | .9 | 87.65044 | 611.3618 | .1 | 107.1283 | 913.2688 |
| .8 | 68.48672 | 373.2526 | 28.0 | 87.96459 | 615.7522 | .2 | 107.4425 | 918.6331 |
| .9 | 68.80088 | 376.6848 | .1 | 88.27875 | 620.1582 | .3 | 107.7566 | 924.0131 |
| 22.0 | 69.11504 | 380.1327 | .2 | 88.59291 | 624.5800 | .4 | 108.0708 | 929.4088 |
| 1 | 69.42920 | 383.5963 | .3 | 88.90707 | 629.0175 | .5 | 108.3849 | 934.8202 |
| .2 | 69.74336 | 387.0756 | .4 | 89.22123 | 633.4707 | .6 | 108.6991 | 940.2473 |
| .3 | 70.05752 | 390.5707 | .5 | 89.53539 | 637.9397 | .7 | 109.0133 | 945.6901 |
| .4 | 70.37168 | 394.0814 | .6 | 89.84955 | 642.4243 | .8 | 109.3274 | 951.1486 |
| .5 | 70.68583 | 397.6078 | .7 | 90.16371 | 646.9246 | .9 | 109.6416 | 956.6228 |
| .6 | 70.99999 | 401.1500 | .8 | 90.47787 | 651.4407 | 35.0 | 109.9557 | 962.1128 |
| .7 | 71.31415 | 404.7078 | .9 | 90.79203 | 655.9724 | .1 | 110.2699 | 967.6184 |
| .8 | 71.62831 | 408.2814 | 29.0 | 91.10619 | 660.5199 | .2 | 110.5841 | 973.1397 |
| .9 | 71.94247 | 411.8707 | .1 | 91.42035 | 665.0830 | .3 | 110.8982 | 978.6768 |
| 23.0 | 72.25663 | 415.4756 | .2 | 91.73451 | 669.6619 | .4 | 111.2124 | 984.2296 |
| .1 | 72.57079 | 419.0963 | .3 | 92.04866 | 674.2565 | .5 | 111.5265 | 989.7980 |
| .2 | 72.88495 | 422.7327 | .4 | 92.36282 | 678.8668 | .6 | 111.8407 | 995.3822 |
| .3 | 73.19911 | 426.3848 | .5 | 92.67698 | 683.4928 | .7 | 112.1549 | 1000.9821 |
| .4 | 73.51327 | 430.0526 | .6 | 92.99114 | 688.1345 | .8 | 112.4690 | 1006.5977 |
| .5 | 73.82743 | 433.7361 | .7 | 93.30530 | 692.7919 | .9 | 112.7832 | 1012.2290 |
| .6 | 74.14159 | 437.4354 | .8 | 93.61946 | 697.4650 | 36.0 | 113.0973 | 1017.8760 |
| .7 | 74.45575 | 441.1503 | .9 | 93.93362 | 702.1538 | .1 | 113.4115 | 1023.5387 |
| .8 | 74.76991 | 444.8809 | 30.0 | 94.24778 | 706.8583 | .2 | 113.7257 | 1029.2172 |
| .9 | 75.08406 | 448.6273 | .1 | 94.56194 | 711.5786 | .3 | 114.0398 | 1034.9113 |
| 24.0 | 75.39822 | 452.3893 | .2 | 94.87610 | 716.3145 | .4 | 114.3540 | 1040.6212 |
| 1 | 75.71238 | 456.1671 | .3 | 95.19026 | 721.0662 | .5 | 114.6681 | 1046.3467 |
| 2 | 76.02654 | 459.9606 | .4 | 95.50442 | 725.8336 | .6 | 114.9823 | 1052.0880 |
| .3 | 76.34070 | 463.7698 | .5 | 95.81858 | 730.6166 | .7 | 115.2965 | 1057.8449 |
| .4 | 76 65486 | 467.5947 | .6 | 96.13274 | 735.4154 | .8 | 115.6106 | 1063.6176 |
| .5 | 76.96902 | 471.4352 | .7 | 96.44689 | 740.2299 | .9 | 115 9248 | 1069.4060 |
| .6 | 77.28318 | 475.2916 | .8 | 96.76105 | 745.0601 | 37.0 | 116.2389 | 1075.2101 |
| .7 | 77 59734 | 479.1636 | .9 | 97 07521 | 749.9060 | .1 | 116.5531 | 1081.0299 |
| .8 | 77.91150 | 483.0513 | 31.0 | 97.38937 | 754.7676 | .2 | 116.8672 | 1086.8654 |

## TABLA DE CÍRCULOS. (Continuación.)

**Los diámetros se dan en unidades y décimos.**

| Diám. | Circf. | Area. | Diám. | Circf. | Area. | Diám. | Circf. | Area. |
|---|---|---|---|---|---|---|---|---|
| **37.3** | 117.1814 | 1092.7166 | **43.5** | 136.6593 | 1486.1697 | **49.7** | 156.1372 | 1940.0041 |
| .4 | 117.4956 | 1098.5835 | .6 | 136.9734 | 1493.0105 | .8 | 156.4513 | 1947.8189 |
| .5 | 117.8097 | 1104.4662 | .7 | 137.2876 | 1499.8670 | .9 | 156.7655 | 1955.6493 |
| .6 | 118.1239 | 1110.3645 | .8 | 137.6018 | 1506.7393 | **50.0** | 157.0796 | 1963.4954 |
| .7 | 118.4380 | 1116.2786 | .9 | 137.9159 | 1513.6272 | .1 | 157.3938 | 1971.3572 |
| .8 | 118.7522 | 1122.2083 | **44.0** | 138.2301 | 1520.5308 | .2 | 157.7080 | 1979.2348 |
| .9 | 119.0664 | 1128.1538 | .1 | 138.5442 | 1527.4502 | .3 | 158.0221 | 1987.1280 |
| **38.0** | 119.3805 | 1134.1149 | .2 | 138.8584 | 1534.3853 | .4 | 158.3363 | 1995.0370 |
| .1 | 119.6947 | 1140.0918 | .3 | 139.1726 | 1541.3360 | .5 | 158.6504 | 2002.9617 |
| .2 | 120.0088 | 1146.0844 | .4 | 139.4867 | 1548.3025 | .6 | 158.9646 | 2010.9020 |
| .3 | 120.3230 | 1152.0927 | .5 | 139.8009 | 1555.2847 | .7 | 159.2787 | 2018.8581 |
| .4 | 120.6372 | 1158.1167 | .6 | 140.1150 | 1562.2826 | .8 | 159.5929 | 2026.8299 |
| .5 | 120.9513 | 1164.1564 | .7 | 140.4292 | 1569.2962 | .9 | 159.9071 | 2034.8174 |
| .6 | 121.2655 | 1170.2118 | .8 | 140.7434 | 1576.3255 | **51.0** | 160.2212 | 2042.8206 |
| .7 | 121.5796 | 1176.2830 | .9 | 141.0575 | 1583.3706 | .1 | 160.5354 | 2050.8395 |
| .8 | 121.8938 | 1182.3698 | **45.0** | 141.3717 | 1590.4313 | .2 | 160.8495 | 2058.8742 |
| .9 | 122.2080 | 1188.4724 | .1 | 141.6858 | 1597.5077 | .3 | 161.1637 | 2066.9245 |
| **39.0** | 122.5221 | 1194.5906 | .2 | 142.0000 | 1604.5999 | .4 | 161.4779 | 2074.9905 |
| .1 | 122.8363 | 1200.7246 | .3 | 142.3141 | 1611.7077 | .5 | 161.7920 | 2083.0723 |
| .2 | 123.1504 | 1206.8742 | .4 | 142.6283 | 1618.8313 | .6 | 162.1062 | 2091.1697 |
| .3 | 123.4646 | 1213.0396 | .5 | 142.9425 | 1625.9705 | .7 | 162.4203 | 2099.2829 |
| .4 | 123.7788 | 1219.2207 | .6 | 143.2566 | 1633.1255 | .8 | 162.7345 | 2107.4118 |
| .5 | 124.0929 | 1225.4175 | .7 | 143.5708 | 1640.2962 | .9 | 163.0487 | 2115.5563 |
| .6 | 124.4071 | 1231.6300 | .8 | 143.8849 | 1647.4826 | **52.0** | 163.3628 | 2123.7166 |
| .7 | 124.7212 | 1237.8582 | .9 | 144.1991 | 1654.6847 | .1 | 163.6770 | 2131.8926 |
| .8 | 125.0354 | 1244.1021 | **46.0** | 144.5133 | 1661.9025 | .2 | 163.9911 | 2140.0843 |
| .9 | 125.3495 | 1250.3617 | .1 | 144.8274 | 1669.1360 | .3 | 164.3053 | 2148.2917 |
| **40.0** | 125.6637 | 1256.6371 | .2 | 145.1416 | 1676.3853 | .4 | 164.6195 | 2156.5149 |
| .1 | 125.9779 | 1262.9281 | .3 | 145.4557 | 1683.6502 | .5 | 164.9336 | 2164.7537 |
| .2 | 126.2920 | 1269.2348 | .4 | 145.7699 | 1690.9308 | .6 | 165.2478 | 2173.0082 |
| .3 | 126.6062 | 1275.5573 | .5 | 146.0841 | 1698.2272 | .7 | 165.5619 | 2181.2785 |
| .4 | 126.9203 | 1281.8955 | .6 | 146.3982 | 1705.5392 | .8 | 165.8761 | 2189.5644 |
| .5 | 127.2345 | 1288.2493 | .7 | 146.7124 | 1712.8670 | .9 | 166.1903 | 2197.8661 |
| 6 | 127.5487 | 1294.6189 | .8 | 147.0265 | 1720.2105 | **53.0** | 166.5044 | 2206.1834 |
| 7 | 127.8628 | 1301.0042 | .9 | 147.3407 | 1727.5697 | .1 | 166.8186 | 2214.5165 |
| .8 | 128.1770 | 1307.4052 | **47.0** | 147.6549 | 1734.9445 | .2 | 167.1327 | 2222.8653 |
| .9 | 128.4911 | 1313.8219 | .1 | 147.9690 | 1742.3351 | .3 | 167.4469 | 2231.2298 |
| **41.0** | 128.8053 | 1320.2543 | .2 | 148.2832 | 1749.7414 | .4 | 167.7610 | 2239.6100 |
| .1 | 129.1195 | 1326.7024 | .3 | 148.5973 | 1757.1635 | .5 | 168.0752 | 2248.0059 |
| .2 | 129.4336 | 1333.1663 | .4 | 148.9115 | 1764.6012 | .6 | 168.3894 | 2256.4175 |
| .3 | 129.7478 | 1339.6458 | .5 | 149.2257 | 1772.0546 | .7 | 168.7035 | 2264.8448 |
| .4 | 130.0619 | 1346.1410 | .6 | 149.5398 | 1779.5237 | .8 | 169.0177 | 2273.2879 |
| .5 | 130.3761 | 1352.6520 | .7 | 149.8540 | 1787.0086 | .9 | 169.3318 | 2281.7466 |
| .6 | 130.6903 | 1359.1786 | .8 | 150.1681 | 1794.5091 | **54.0** | 169.6460 | 2290.2210 |
| .7 | 131.0044 | 1365.7210 | .9 | 150.4823 | 1802.0254 | .1 | 169.9602 | 2298.7112 |
| .8 | 131.3186 | 1372.2791 | **48.0** | 150.7964 | 1809.5574 | .2 | 170.2743 | 2307.2171 |
| .9 | 131.6327 | 1378.8529 | .1 | 151.1106 | 1817.1050 | .3 | 170.5885 | 2315.7386 |
| **42.0** | 131.9469 | 1385.4424 | .2 | 151.4248 | 1824.6684 | .4 | 170.9026 | 2324.2759 |
| .1 | 132.2611 | 1392.0476 | .3 | 151.7389 | 1832.2475 | .5 | 171.2168 | 2332.8289 |
| .2 | 132.5752 | 1398.6685 | .4 | 152.0531 | 1839.8423 | .6 | 171.5310 | 2341.3976 |
| .3 | 132.8894 | 1405.3051 | .5 | 152.3672 | 1847.4528 | .7 | 171.8451 | 2349.9820 |
| .4 | 133.2035 | 1411.9574 | .6 | 152.6814 | 1855.0790 | .8 | 172.1593 | 2358.5821 |
| .5 | 133.5177 | 1418.6254 | .7 | 152.9956 | 1862.7210 | .9 | 172.4734 | 2367.1979 |
| .6 | 133.8318 | 1425.3092 | .8 | 153.3097 | 1870.3786 | **55.0** | 172.7876 | 2375.8294 |
| .7 | 134.1460 | 1432.0086 | .9 | 153.6239 | 1878.0519 | .1 | 173.1018 | 2384.4767 |
| .8 | 134.4602 | 1438.7238 | **49.0** | 153.9380 | 1885.7410 | .2 | 173.4159 | 2393.1396 |
| .9 | 134.7743 | 1445.4546 | .1 | 154.2522 | 1893.4457 | .3 | 173.7301 | 2401.8183 |
| **43.0** | 135.0885 | 1452.2012 | .2 | 154.5664 | 1901.1662 | .4 | 174.0442 | 2410.5126 |
| .1 | 135.4026 | 1458.9635 | .3 | 154.8805 | 1908.9024 | .5 | 174.3584 | 2419.2227 |
| .2 | 135.7168 | 1465.7415 | .4 | 155.1947 | 1916.6543 | .6 | 174.6726 | 2427.9485 |
| .3 | 136.0310 | 1472.5352 | .5 | 155.5088 | 1924.4218 | .7 | 174.9867 | 2436.6899 |
| .4 | 136.3451 | 1479.3446 | .6 | 155.8230 | 1932.2051 | .8 | 175.3009 | 2445.4471 |

## TABLA DE CÍRCULOS. (Continuación.

**Los diámetros se dan en unidades y décimos.**

| Diám. | Circf. | Area. | Diám. | Circf. | Area. | Diám. | Circf. | Area. |
|---|---|---|---|---|---|---|---|---|
| 55.9 | 175.6150 | 2454.2200 | 62.1 | 195.0929 | 3028.8173 | 68.3 | 214.5708 | 3663.7960 |
| 56.0 | 175.9292 | 2463.0086 | .2 | 195.4071 | 3038.5798 | .4 | 214.8849 | 3674.5324 |
| .1 | 176.2433 | 2471.8130 | .3 | 195.7212 | 3048.3580 | .5 | 215.1991 | 3685.2845 |
| .2 | 176.5575 | 2480.6330 | .4 | 196.0354 | 3058.1520 | .6 | 215.5133 | 3696.0523 |
| .3 | 176.8717 | 2489.4687 | .5 | 196.3495 | 3067.9616 | .7 | 215.8274 | 3706.8359 |
| .4 | 177.1858 | 2498.3201 | .6 | 196.6637 | 3077.7869 | .8 | 216.1416 | 3717.6351 |
| .5 | 177.5000 | 2507.1873 | .7 | 196.9779 | 3087.6279 | .9 | 216.4557 | 3728.4500 |
| .6 | 177.8141 | 2516.0701 | .8 | 197.2920 | 3097.4847 | 69.0 | 216.7699 | 3739.2807 |
| .7 | 178.1283 | 2524.9687 | .9 | 197.6062 | 3107.3571 | .1 | 217.0841 | 3750.1270 |
| .8 | 178.4425 | 2533.8830 | 63.0 | 197.9203 | 3117.2453 | .2 | 217.3982 | 3760.9891 |
| .9 | 178.7566 | 2542.8129 | .1 | 198.2345 | 3127.1492 | .3 | 217.7124 | 3771.8668 |
| 57.0 | 179.0708 | 2551.7586 | .2 | 198.5487 | 3137.0688 | .4 | 218.0265 | 3782.7603 |
| .1 | 179.3849 | 2560.7200 | .3 | 198.8628 | 3147.0040 | .5 | 218.3407 | 3793.6695 |
| .2 | 179.6991 | 2569.6971 | .4 | 199.1770 | 3156.9550 | .6 | 218.6548 | 3804.5944 |
| .3 | 180.0133 | 2578.6899 | .5 | 199.4911 | 3166.9217 | .7 | 218.9690 | 3815.5350 |
| .4 | 180.3274 | 2587.6985 | .6 | 199.8053 | 3176.9042 | .8 | 219.2832 | 3826.4913 |
| .5 | 180.6416 | 2596.7227 | .7 | 200.1195 | 3186.9023 | .9 | 219.5973 | 3837.4633 |
| .6 | 180.9557 | 2605.7626 | .8 | 200.4336 | 3196.9161 | 70.0 | 219.9115 | 3848.4510 |
| .7 | 181.2699 | 2614.8183 | .9 | 200.7478 | 3206.9456 | .1 | 220.2256 | 3859.4544 |
| .8 | 181.5841 | 2623.8896 | 64.0 | 201.0619 | 3216.9909 | .2 | 220.5398 | 3870.4736 |
| .9 | 181.8982 | 2632.9767 | .1 | 201.3761 | 3227.0518 | .3 | 220.8540 | 3881.5084 |
| 58.0 | 182.2124 | 2642.0794 | .2 | 201.6902 | 3237.1285 | .4 | 221.1681 | 3892.5590 |
| .1 | 182.5265 | 2651.1979 | .3 | 202.0044 | 3247.2209 | .5 | 221.4823 | 3903.6252 |
| .2 | 182.8407 | 2660.3321 | .4 | 202.3185 | 3257.3289 | .6 | 221.7964 | 3914.7072 |
| .3 | 183.1549 | 2669.4820 | .5 | 202.6327 | 3267.4527 | .7 | 222.1106 | 3925.8049 |
| .4 | 183.4690 | 2678.6476 | .6 | 202.9469 | 3277.5922 | .8 | 222.4248 | 3936.9182 |
| .5 | 183.7832 | 2687.8289 | .7 | 203.2610 | 3287.7474 | .9 | 222.7389 | 3948.0473 |
| .6 | 184.0973 | 2697.0259 | .8 | 203.5752 | 3297.9183 | 71.0 | 223.0531 | 3959.1921 |
| .7 | 184.4115 | 2706.2386 | .9 | 203.8894 | 3308.1049 | .1 | 223.3672 | 3970.3526 |
| .8 | 184.7256 | 2715.4670 | 65.0 | 204.2035 | 3318.3072 | .2 | 223.6814 | 3981.5289 |
| .9 | 185.0398 | 2724.7112 | .1 | 204.5177 | 3328.5253 | .3 | 223.9956 | 3992.7208 |
| 59.0 | 185.3540 | 2733.9710 | .2 | 204.8318 | 3338.7590 | .4 | 224.3097 | 4003.9284 |
| .1 | 185.6681 | 2743.2466 | .3 | 205.1460 | 3349.0085 | .5 | 224.6239 | 4015.1518 |
| .2 | 185.9823 | 2752.5378 | .4 | 205.4602 | 3359.2736 | .6 | 224.9380 | 4026.3908 |
| .3 | 186.2964 | 2761.8448 | .5 | 205.7743 | 3369.5545 | .7 | 225.2522 | 4037.6456 |
| .4 | 186.6106 | 2771.1675 | .6 | 206.0885 | 3379.8510 | .8 | 225.5664 | 4048.9160 |
| .5 | 186.9248 | 2780.5058 | .7 | 206.4026 | 3390.1633 | .9 | 225.8805 | 4060.2022 |
| .6 | 187.2389 | 2789.8599 | .8 | 206.7168 | 3400.4913 | 72.0 | 226.1947 | 4071.5041 |
| .7 | 187.5531 | 2799.2297 | .9 | 207.0310 | 3410.8350 | .1 | 226.5088 | 4082.8217 |
| .8 | 187.8672 | 2808.6152 | 66.0 | 207.3451 | 3421.1944 | .2 | 226.8230 | 4094.1550 |
| .9 | 188.1814 | 2818.0165 | .1 | 207.6593 | 3431.5695 | .3 | 227.1371 | 4105.5040 |
| 60.0 | 188.4956 | 2827.4334 | .2 | 207.9734 | 3441.9603 | .4 | 227.4513 | 4116.8687 |
| .1 | 188.8097 | 2836.8660 | .3 | 208.2876 | 3452.3669 | .5 | 227.7655 | 4128.2491 |
| .2 | 189.1239 | 2846.3144 | .4 | 208.6018 | 3462.7891 | .6 | 228.0796 | 4139.6452 |
| .3 | 189.4380 | 2855.7784 | .5 | 208.9159 | 3473.2270 | .7 | 228.3938 | 4151.0571 |
| .4 | 189.7522 | 2865.2582 | .6 | 209.2301 | 3483.6807 | .8 | 228.7079 | 4162.4846 |
| .5 | 190.0664 | 2874.7536 | .7 | 209.5442 | 3494.1500 | .9 | 229.0221 | 4173.9279 |
| .6 | 190.3805 | 2884.2648 | .8 | 209.8584 | 3504.6351 | 73.0 | 229.3363 | 4185.3868 |
| .7 | 190.6947 | 2893.7917 | .9 | 210.1725 | 3515.1359 | .1 | 229.6504 | 4196.8615 |
| .8 | 191.0088 | 2903.3343 | 67.0 | 210.4867 | 3525.6524 | .2 | 229.9646 | 4208.3519 |
| .9 | 191.3230 | 2912.8926 | .1 | 210.8009 | 3536.1845 | .3 | 230.2787 | 4219.8579 |
| 61.0 | 191.6372 | 2922.4666 | .2 | 211.1150 | 3546.7324 | .4 | 230.5929 | 4231.3797 |
| .1 | 191.9513 | 2932.0563 | .3 | 211.4292 | 3557.2960 | .5 | 230.9071 | 4242.9172 |
| .2 | 192.2655 | 2941.6617 | .4 | 211.7433 | 3567.8754 | .6 | 231.2212 | 4254.4704 |
| .3 | 192.5796 | 2951.2828 | .5 | 212.0575 | 3578.4704 | .7 | 231.5354 | 4266.0394 |
| .4 | 192.8938 | 2960.9197 | .6 | 212.3717 | 3589.0811 | .8 | 231.8495 | 4277.6240 |
| .5 | 193.2079 | 2970.5722 | .7 | 212.6858 | 3599.7075 | .9 | 232.1637 | 4289.2243 |
| .6 | 193.5221 | 2980.2405 | .8 | 213.0000 | 3610.3497 | 74.0 | 232.4779 | 4300.8403 |
| .7 | 193.8363 | 2989.9244 | .9 | 213.3141 | 3621.0075 | .1 | 232.7920 | 4312.4721 |
| .8 | 194.1504 | 2999.6241 | 68.0 | 213.6283 | 3631.6811 | .2 | 233.1062 | 4324.1195 |
| .9 | 194.4646 | 3009.3395 | .1 | 213.9425 | 3642.3704 | .3 | 233.4203 | 4335.7827 |
| 62.0 | 194.7787 | 3019.0705 | .2 | 214.2566 | 3653.0754 | .4 | 233.7345 | 4347.4616 |

## TABLA DE CÍRCULOS. (Continuación.)

**Los diámetros se dan en unidades y décimos.**

| Diám. | Circf. | Area. | Diám. | Circf. | Area. | Diám. | Circf. | Area. |
|---|---|---|---|---|---|---|---|---|
| 74.5 | 234.0487 | 4359.1562 | 80.7 | 253.5265 | 5114.8977 | 86.9 | 273.0044 | 5931.0206 |
| .6 | 234.3628 | 4370.8664 | .8 | 253.8407 | 5127.5819 | 87.0 | 273.3186 | 5944.6787 |
| .7 | 234.6770 | 4382.5924 | .9 | 254.1548 | 5140.2818 | .1 | 273.6327 | 5958.3525 |
| .8 | 234.9911 | 4394.3341 | 81.0 | 254.4690 | 5152.9974 | .2 | 273.9469 | 5972.0420 |
| .9 | 235.3053 | 4406.0916 | .1 | 254.7832 | 5165.7287 | .3 | 274.2610 | 5985.7472 |
| 75.0 | 235.6194 | 4417.8647 | .2 | 255.0972 | 5178.4757 | .4 | 274.5752 | 5999 4681 |
| .1 | 235.9336 | 4429.6535 | .3 | 255.4115 | 5191.2384 | .5 | 274.8894 | 6013.2047 |
| .2 | 236.2478 | 4441.4580 | .4 | 255.7256 | 5204.0168 | .6 | 275.2035 | 6026.9570 |
| .3 | 236.5619 | 4453.2783 | .5 | 256.0398 | 5216.8110 | .7 | 275.5177 | 6040.7250 |
| .4 | 236.8761 | 4465.1142 | .6 | 256.3540 | 5229.6208 | .8 | 275.8318 | 6054.5088 |
| .5 | 237.1902 | 4476.9659 | .7 | 256.6681 | 5242.4463 | .9 | 276.1460 | 6068.3082 |
| .6 | 237.5044 | 4488.8332 | .8 | 256.9823 | 5255.2876 | 88.0 | 276.4602 | 6082.1234 |
| .7 | 237.8186 | 4500.7163 | .9 | 257.2964 | 5268.1446 | .1 | 276.7743 | 6095.9542 |
| .8 | 238.1327 | 4512.6151 | 82.0 | 257.6106 | 5281.0173 | .2 | 277.0885 | 6109.8008 |
| .9 | 238.4469 | 4524.5296 | .1 | 257.9248 | 5293.9056 | .3 | 277.4026 | 6123.6631 |
| 76.0 | 238.7610 | 4536.4598 | .2 | 258.2389 | 5306.8097 | .4 | 277.7168 | 6137.5411 |
| .1 | 239.0752 | 4548.4057 | .3 | 258.5531 | 5319.7295 | .5 | 278.0309 | 6151.4348 |
| .2 | 239.3894 | 4560.3673 | .4 | 258.8672 | 5332.6650 | .6 | 278.3451 | 6165.3442 |
| .3 | 239.7035 | 4572.3446 | .5 | 259.1814 | 5345.6162 | .7 | 278.6593 | 6179.2693 |
| .4 | 240.0177 | 4584.3377 | .6 | 259.4956 | 5358.5832 | .8 | 278.9734 | 6193.2101 |
| .5 | 240.3318 | 4596.3464 | .7 | 259.8097 | 5371.5653 | .9 | 279.2876 | 6207.1666 |
| .6 | 240.6460 | 4608.3708 | .8 | 260.1239 | 5384.5641 | 89.0 | 279.6017 | 6221.1389 |
| .7 | 240.9602 | 4620.4110 | .9 | 260.4380 | 5397.5782 | .1 | 279.9159 | 6235.1268 |
| .8 | 241.2743 | 4632.4669 | 83.0 | 260.7522 | 5410.6079 | .2 | 280.2301 | 6249.1304 |
| .9 | 241.5885 | 4644.5384 | .1 | 261.0663 | 5423.6534 | .3 | 280.5442 | 6263.1498 |
| 77.0 | 241.9026 | 4656.6257 | .2 | 261.3805 | 5436.7146 | .4 | 280.8584 | 6277.1849 |
| .1 | 242.2168 | 4668.7287 | .3 | 261.6947 | 5449.7915 | .5 | 281.1725 | 6291.2356 |
| .2 | 242.5310 | 4680.8474 | .4 | 262.0088 | 5462.8840 | .6 | 281.4867 | 6305.3021 |
| .3 | 242.8451 | 4692.9818 | .5 | 262.3230 | 5475.9923 | .7 | 281.8009 | 6319.3843 |
| .4 | 243.1593 | 4705.1319 | .6 | 262.6371 | 5489.1163 | .8 | 282.1150 | 6333.4822 |
| .5 | 243.4734 | 4717.2977 | .7 | 262.9513 | 5502.2561 | .9 | 282.4292 | 6347.5958 |
| .6 | 243.7876 | 4729.4792 | .8 | 263.2655 | 5515.4115 | 90.0 | 282.7433 | 6361.7251 |
| .7 | 244.1017 | 4741.6765 | .9 | 263.5796 | 5528.5826 | .1 | 283.0575 | 6375.8701 |
| .8 | 244.4159 | 4753.8894 | 84.0 | 263.8938 | 5541.7694 | .2 | 283.3717 | 6390 0309 |
| .9 | 244.7301 | 4766.1181 | .1 | 264.2079 | 5554.9720 | .3 | 283.6858 | 6404.2073 |
| 78.0 | 245.0442 | 4778.3624 | .2 | 264.5221 | 5568.1902 | .4 | 284.0000 | 6418 3995 |
| .1 | 245.3584 | 4790.6225 | .3 | 264.8363 | 5581.4242 | .5 | 284.3141 | 6432.6073 |
| .2 | 245.6725 | 4802.8983 | .4 | 265.1504 | 5594.6739 | .6 | 284.6283 | 6446 8309 |
| .3 | 245.9867 | 4815.1897 | .5 | 265.4646 | 5607.9392 | .7 | 284.9425 | 6461.0701 |
| .4 | 246.3009 | 4827.4969 | .6 | 265.7787 | 5621.2203 | .8 | 285.2566 | 6475.3251 |
| .5 | 246.6150 | 4839.8198 | .7 | 266.0929 | 5634.5171 | .9 | 285.5708 | 6489.5958 |
| .6 | 246.9292 | 4852.1584 | .8 | 266.4071 | 5647.8296 | 91.0 | 285.8849 | 6503.8822 |
| .7 | 247.2433 | 4864.5128 | .9 | 266.7212 | 5661.1578 | .1 | 286.1991 | 6518.1843 |
| .8 | 247.5575 | 4876.8828 | 85.0 | 267 0354 | 5674.5017 | .2 | 286.5133 | 6532.5021 |
| .9 | 247.8717 | 4889.2685 | .1 | 267 3495 | 5687.8614 | .3 | 286.8274 | 6546.8356 |
| 79.0 | 248.1858 | 4901.6699 | .2 | 267 6637 | 5701.2367 | .4 | 287.1416 | 6561.1848 |
| .1 | 248.5000 | 4914.0871 | .3 | 267 9779 | 5714.6277 | .5 | 287.4557 | 6575.5498 |
| .2 | 248.8141 | 4926.5199 | .4 | 268.2920 | 5728.0345 | .6 | 287 7699 | 6589.9304 |
| .3 | 249.1283 | 4938.9685 | .5 | 268.6062 | 5741.4569 | .7 | 288 0840 | 6604.3268 |
| .4 | 249.4425 | 4951.4328 | .6 | 268.9203 | 5754.8951 | .8 | 288.3982 | 6618.7388 |
| .5 | 249.7566 | 4963.9127 | .7 | 269.2345 | 5768.3490 | .9 | 288.7124 | 6633.1666 |
| .6 | 250.0708 | 4976 4084 | .8 | 269.5486 | 5781.8185 | 92.0 | 289 0265 | 6647.6101 |
| .7 | 250 3849 | 4988.9198 | .9 | 269.8628 | 5795.3038 | .1 | 289.3407 | 6662.0692 |
| .8 | 250.6991 | 5001.4469 | 86.0 | 270.1770 | 5808.8048 | .2 | 289 6548 | 6676.5441 |
| 9 | 251.0133 | 5013.9897 | .1 | 270.4911 | 5822.3215 | .3 | 289.9690 | 6691 0347 |
| 80.0 | 251.3274 | 5026.5482 | .2 | 270.8053 | 5835.8539 | .4 | 290.2832 | 6705.5410 |
| .1 | 251.6416 | 5039.1225 | .3 | 271.1194 | 5849 4020 | .5 | 290.5973 | 6720.0630 |
| .2 | 251.9557 | 5051.7124 | .4 | 271.4336 | 5862.9659 | .6 | 290.9115 | 6734.6008 |
| .3 | 252.2699 | 5064.3180 | .5 | 271.7478 | 5876.5454 | .7 | 291.2256 | 6749.1542 |
| .4 | 252.5840 | 5076.9394 | .6 | 272.0619 | 5890.1407 | .8 | 291.5398 | 6763.7233 |
| .5 | 252.8982 | 5089.5764 | .7 | 272.3761 | 5903.7516 | .9 | 291.8540 | 6778.3082 |
| .6 | 253.2124 | 5102.2292 | .8 | 272.6902 | 5917.3783 | 93.0 | 292.1681 | 6792.9087 |

## TABLA DE CÍRCULOS. (Continuación.)

**Los diámetros se dan en unidades y décimos.**

| Dia. | Circumf. | Area. | Dia. | Circumf. | Area. | Dia. | Circumf. | Area. |
|---|---|---|---|---|---|---|---|---|
| 93.1 | 292.4823 | 6807.5250 | 95.5 | 300.0221 | 7163.0276 | 97.8 | 307.2478 | 7512.2078 |
| .2 | 292.7964 | 6822.1569 | .6 | 300.3363 | 7178.0366 | .9 | 307.5619 | 7527.5780 |
| .3 | 293.1106 | 6836.8046 | .7 | 300.6504 | 7193.0612 | 98.0 | 307.8761 | 7542.9640 |
| .4 | 293.4248 | 6851.4680 | .8 | 300.9646 | 7208.1016 | .1 | 308.1902 | 7558.3656 |
| .5 | 293.7389 | 6866.1471 | .9 | 301.2787 | 7223.1577 | .2 | 308.5044 | 7573.7830 |
| .6 | 294.0531 | 6880.8419 | 96.0 | 301.5929 | 7238.2295 | .3 | 308.8186 | 7589.2161 |
| .7 | 294.3672 | 6895.5524 | .1 | 301.9071 | 7253.3170 | .4 | 309.1327 | 7604.6648 |
| .8 | 294.6814 | 6910.2786 | .2 | 302.2212 | 7268.4202 | .5 | 309.4469 | 7620.1293 |
| .9 | 294.9956 | 6925.0205 | .3 | 302.5354 | 7283.5391 | .6 | 309.7610 | 7635.6095 |
| 94.0 | 295.3097 | 6939.7782 | .4 | 302.8495 | 7298.6737 | .7 | 310.0752 | 7651.1054 |
| .1 | 295.6239 | 6954.5515 | .5 | 303.1637 | 7313.8240 | .8 | 310.3894 | 7666.6170 |
| .2 | 295.9380 | 6969.3406 | .6 | 303.4779 | 7328.9901 | .9 | 310.7035 | 7682.1444 |
| .3 | 296.2522 | 6984.1453 | .7 | 303.7920 | 7344.1718 | 99.0 | 311.0177 | 7697.6874 |
| .4 | 296.5663 | 6998.9658 | .8 | 304.1062 | 7359.3693 | .1 | 311.3318 | 7713.2461 |
| .5 | 296.8805 | 7013.8019 | .9 | 304.4203 | 7374.5824 | .2 | 311.6460 | 7728.8206 |
| .6 | 297.1947 | 7028.6538 | 97.0 | 304.7345 | 7389.8113 | .3 | 311.9602 | 7744.4107 |
| .7 | 297.5088 | 7043.5214 | .1 | 305.0486 | 7405.0559 | .4 | 312.2743 | 7760.0166 |
| .8 | 297.8230 | 7058.4047 | .2 | 305.3628 | 7420.3162 | .5 | 312.5885 | 7775.6382 |
| .9 | 298.1371 | 7073.3037 | .3 | 305.6770 | 7435.5922 | .6 | 312.9026 | 7791.2754 |
| 95.0 | 298.4513 | 7088.2184 | .4 | 305.9911 | 7450.8839 | .7 | 313.2168 | 7806.9284 |
| .1 | 298.7655 | 7103.1488 | .5 | 306.3053 | 7466.1913 | .8 | 313.5309 | 7822.5971 |
| .2 | 299.0796 | 7118.0950 | .6 | 306.6194 | 7481.5144 | .9 | 313.8451 | 7838.2815 |
| .3 | 299.3938 | 7133.0568 | .7 | 306.9336 | 7496.8532 | 100.0 | 314.1593 | 7853.9816 |
| .4 | 299.7079 | 7148.0343 | | | | | | |

## Circunferencias cuando el diámetro tiene más de una cifra decimal.

| Diám. | Circf. | Diám. | Circf. | Diám. | Circf. | Diám. | Circf. | Diám. | Circf. |
|---|---|---|---|---|---|---|---|---|---|
| .1 | .314159 | .01 | .031416 | .001 | .003142 | .0001 | .000314 | .00001 | .000031 |
| .2 | .628319 | .02 | .062832 | .002 | .006283 | .0002 | .000628 | .00002 | .000063 |
| .3 | .942478 | .03 | .094248 | .003 | .009425 | .0003 | .000942 | .00003 | .000094 |
| .4 | 1.256637 | .04 | .125664 | .004 | .012566 | .0004 | .001257 | .00004 | .000126 |
| .5 | 1.570796 | .05 | .157080 | .005 | .015708 | .0005 | .001571 | .00005 | .000157 |
| .6 | 1.884956 | .06 | .188496 | .006 | .018850 | .0006 | .001885 | .00006 | .000188 |
| .7 | 2.199115 | .07 | .219911 | .007 | .021991 | .0007 | .002199 | .00007 | .000220 |
| .8 | 2.513274 | .08 | .251327 | .008 | .025133 | .0008 | .002513 | .00008 | .000251 |
| .9 | 2.827433 | .09 | .282743 | .009 | .028274 | .0009 | .002827 | .00009 | .000283 |

## Ejemplos.

**Diám**=3.12699
**Circunf**= Suma de
Circ para el diám de 3.1 = 9.738937
— .02 = .062832
— .006 = .018850
— .0000 = .002827
— .00009= .000283
9.823729

**Circunf**=9.823729
**Diám** = Suma de
Diám p' la circ. de 9.738937=3.1
.084792
— .062832= .02
.021960
— .018850= .006
.003110
— .002827= .0009
.000283
— .000283= .00009
3.12699

## ARCOS CIRCULARES

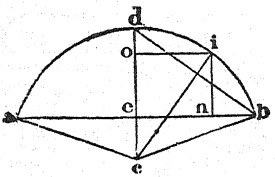



Fig. 1

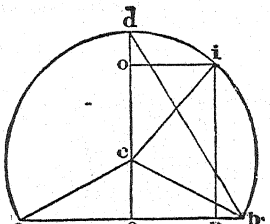



Fig. 2.

Las reglas establecidas para la fig. 1 se aplican á todos los arcos iguales ó *menores* que una semicircunferencia.

Las reglas para la fig. 2 se aplican á todos los arcos iguales ó *mayores* que una semicircunferencia.

### Cuerda, *ab*, de todo arco *adb*.

$= 2 \times \sqrt{\text{radio}^2 - (\text{radio} - \text{flecha})^2}$. Fig. 1.

$= 2 \times \sqrt{\text{radio}^2 - (\text{flecha} - \text{radio})^2}$. Figs. 2.

$= 2 \times \sqrt{\text{flecha} \times (2 \times \text{radio} - \text{flecha})}$. Figs. 1 y 2.

$= 2 \times \text{radio} \times \text{sen } \frac{1}{2}\ acb$. Figs. 1 y 2.

$= 2 \times \dfrac{\text{flecha}}{\text{tangente } abd^*}$. Figs. 1 y 2.

$= 2 \times db\,\S \times \cos abd^*$. Figs. 1 y 2.

$= 2 \times \sqrt{db^{\S\,2} - \text{flecha}^2}$. Figs. 1 y 2 **.

= aproximadamente $8 \times db^{**} - 3 \times$ longitud del arco *adb* fig. 1.

### Longitud del arco *adb*.

$= 2\pi$ radio $\times \dfrac{\text{arco } adb \text{ en grados}}{360}$. Figs. 1 y 2.

= .01745 × radio × arco *adb* en grados. Figs. 1 y 2.

= Circunferencia del círculo — longitud del arco pequeño que subtende el ángulo *acb*, fig. 2.

= aproximadamente $\dfrac{8 \times db\,\S - \text{cuerda } ab^{***}}{3}$. Fig. 1.

---

* *abd* = 1/4 ángulo *acb* subtendido por el arco. En la fig. 2 el ángulo último excede á 180°.

** *db* = cuerda *dib* ó la mitad de *adb* es = $\sqrt{\text{flecha}^2 + (\frac{1}{2}\ ab)^2}$. Figs. 1 y 2.

| § si la flecha = | multiplíquese el resultado por | Si la flecha = | multiplíquese el resultado por |
|---|---|---|---|
| .5 cuerda | 1.036 | .25 cuerda | 1.0044 |
| .4 — | 1.0196 | .2 — | 1.0021 |
| .333 — | 1.0114 | .125 — | 1.00036 |
| .3 — | 1.0083 | .1 — | 1.00015 |

| *** si la flecha = | multiplíquese el resultado por | Si la flecha = | multiplíquese el resultado por |
|---|---|---|---|
| .5 cuerda | 1.012 | .25 cuerda | 1.0015 |
| .4 — | 1.0065 | .2 — | 1.0007 |
| .333 — | 1.0038 | .125 — | 1.00012 |
| .3 — | 1.0028 | .1 — | 1.00005 |

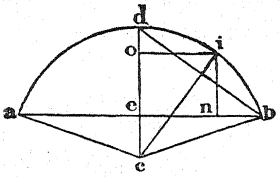



Fig. 1.

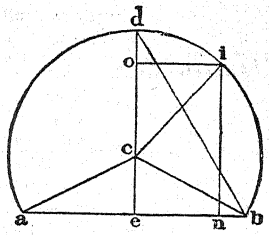



Fig. 2.

Las reglas para la figura 1 se aplican á todos los arcos iguales ó *menores* que una semicircunferencia.

Las reglas para la figura 2 se aplican á todos los arcos iguales ó *mayores* que una semicircunferencia.

**Radios** *ca, cd, ci* ó *cb*.

$$= \frac{(\frac{1}{2}\,ab)^2 + \text{flecha}^2}{2 \times \text{flecha}}. \text{ Figs. 1 y 2.}$$

$$= \frac{db^2\,\S}{2 \times \text{flecha}}. \text{ Figs. 1 y 2.}$$

$$= \frac{\frac{1}{2}\,ab}{\text{sen de } \frac{1}{2}\,acb}. \text{ Fig. 1 y 2.} \qquad = \frac{\frac{1}{2}\,db\,\S}{\text{sen } \frac{1}{2}\,bcd\,\|}. \text{ Figs. 1 y 2.}$$

$$= \frac{\text{flecha } de}{1 - \text{cos de } \frac{1}{2}\,acb}. \text{ Fig. 1.} \qquad = \frac{\text{flecha } de}{1 + \cos \frac{1}{2}\,acb\,\dagger}. \text{ Figs. 2.}$$

**Flecha ú ordenada en el centro** *de*.

$$= \text{radio} - \sqrt{\text{radio}^2 - (\tfrac{1}{2}\,ab)^2}. \text{ Fig. 1.}$$

$$= \text{radio} + \sqrt{\text{radio}^2 - (\tfrac{1}{2}\,ab)^2}. \text{ Fig. 2.}$$

$$= \text{radio} \times (1 - \cos bcd\,\|\,). \text{ Fig. 1.}$$

$$= \text{radio} \times (1 + \cos bcd\,\|\,)\dagger. \text{ Fig. 2.}$$

$$= \frac{db^2\,\S}{2 \times \text{radio}}. \text{ Figs. 1 y 2.}$$

$$= \tfrac{1}{2}\,ab \times \text{tang } abd\,(*). \text{ Figs. 1 y 2.}$$

$$= \text{aproximadamente } \frac{(\frac{1}{2}\,ab)^2}{2 \times \text{radio}}. \text{ Fig. 1.}$$

Cuando el radio es=cuerda *ab* el resultado es 6.7 por ciento más pequeño
Cuando es=3 × cuerda *ab* el resultado es 0.7 por ciento más pequeño.

**Ordenada lateral** como *ni*.

$$= \sqrt{\text{radio}^2 - en^2} + \text{flecha} - \text{radio. Figs. 1 y 2.}$$

$$= \text{aproximadamente } \frac{an \times nb}{2 \times \text{radio}}. \text{ Fig. 1.}$$

---

(*) *abd* es = 1/4 del ángulo *acb* subtendido por el arco.

† En verdad que esto debería leerse 1 *menos* coseno; pero los cosenos de ángulos entre 90° y 270° deben entonces ser considerados como negativos. Nuestra regla por tanto nos conduce al mismo resultado.

§ *db* = cuerda de *dib* ó la mitad de *adb* es = $\sqrt{\text{flecha}^2 + (\frac{1}{2}\,ab)^2}$. Figs. 1 y 2.

‖ *bcd* = la mitad del ángulo *acb* subtendido por el arco. En la figura 2 el último ángulo excede de 180°.

### Ángulo *acb* subtendido por el arco *adb*.

Un ángulo y su suplemento (como *bce* y *bcd*, fig. 2) tienen el mismo seno, el mismo coseno y la misma tangente.

**Advertencia.** Los senos, etc., siguientes son sólo de la mitad de *acb*.

$$\text{sen } \tfrac{1}{2}\, acb = \frac{\tfrac{1}{2}\, ab}{\text{radio}}. \text{ Figs. 1 y 2.}$$

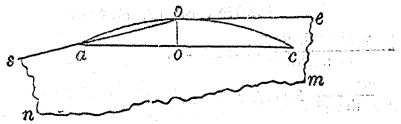
$$\cos \tfrac{1}{2}\, acb = \frac{\text{radio} - \text{flecha}}{\text{radio}}. \text{ Fig. 1;} \qquad = \frac{\text{flecha} - \text{radio}}{\text{radio}}. \text{ Fig. 2.}$$

$$\text{tang } \tfrac{1}{2}\, acb = \frac{\tfrac{1}{2}\, ab}{\text{radio} - \text{flecha}}. \text{ Fig. 1;} \qquad = \frac{\tfrac{1}{2}\, ab}{\text{flecha} - \text{radio}}. \text{ Fig. 2.}$$

$$\text{seno verso } \tfrac{1}{2}\, acb = \frac{\text{flecha}}{\text{radio}}. \text{ Figs. 1 y 2.}$$

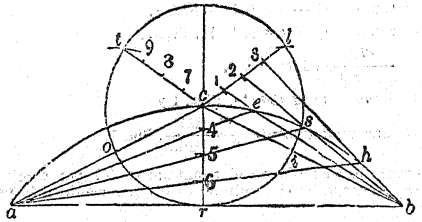
### Describir el arco de un círculo demasiado grande.

**Método 1.°** Sea *ac* la cuerda y *ob* la flecha del arco que se requiere. En una tira separada de papel *semn*, trácese *ac*, *ob*, *ab*, y también *be* paralela á la cuerda *ac*




será bueno hacer *bs*, *be*, un poco más largas que *ab*. Entonces córtese el papel cuidadosamente, á lo largo de las líneas *sb*, *be*, para que sólo quede la tira *sabemn*. Ahora, si los lados rectos *sb*, *be* se aplican al dibujo de modo que cualquier parte de ellos pase al mismo tiempo por los puntos *a* y *b* ó bien *b* y *c*, el punto *b* sobre la tira estará en la circunferencia del arco y se marcará con un punto. Así pueden encontrarse muchos puntos sobre el arco y unirse después para formar la curva.

**Método 2.°** Trácense la cuerda del arco *ab*, la flecha *rc*, y *ac*, *bc*. Descríbase un círculo desde *c* con un radio *cr*. Háganse cada uno de los arcos *ot*, *il*, iguales á *ro* ó *ri*, y trácese *st*, *cl*. Divídanse *ct*, *cl*, *cr*, en la mitad del número de partes



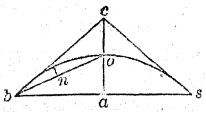

iguales en que se quiera dividir la curva. Trácense las líneas *b* 1, *b* 2, *a* 4, *a* 5, *a* 6, prolongadas hasta que se corten en *e*, *s*, *h*. Entonces *e*, *s*, *h*, serán puntos de la curva; y para la otra mitad trácense líneas semejantes desde *a* hasta 7, 8, 9; y otras desde *b* hasta que se encuentren como antes. Trácese á la mano la curva, por los puntos así determinados.

**Advertencia.** Puede con frecuencia ser útil recordar que en cualquier arco *bos* que no exceda de 29°, ó de otro modo, cuya cuerda *bs* sea por lo menos **diez y seis veces** su flecha *ao;* ésta será aproximadamente la mitad de *ae* siendo *bc*, *sc*, tangentes al arco. † Y viceversa, si en arcos análogos á este

† Para arcos de 29° resultará el valor de *oc* así obtenido demasiado corto en $\frac{3}{100}$.

hacemos *oc* igual *ao; c* será aproximadamente el punto de encuentro de las tangentes *bc, sc* del arco. También **la ordenada media ó flecha *n* de la mitad del arco** *ob* ú *os* será aproximadamente $^1/_4$ de *ao* flecha del arco entero.

En verdad, esta última observación se aplica con bastante precisión en muchos




casos hasta si el arco es mayor de 45°, porque si en ese caso tomamos $^1/_4$ de *os* para valor de la flecha *n*, el error será sólo de $\frac{1}{103}$, en menos, y por tanto despreciable en algunos dibujos y muy útil para encontrar los puntos de una curva de radio muy grande para ser trazada con el compás de división. De la misma manera $^1/_4$ de *n* será la flecha para el arco *nb* ó *no* y asimismo de los demás. Más abajo se dá **una tabla para encontrar la flecha de un semiarco** con mayor exactitud.

## ARCOS CIRCULARES DE USO FRECUENTE

La quinta columna sirve para encontrar puntos y trazar arcos, según el principio dado arriba, cuando son demasiado grandes para el compás. Casi nunca será necesario usar más de las tres primeras decimales de la quinta columna y después que el arco se ha subdividido en partes menores de 35° las primeras dos decimales serán suficientes. Original.

| Flecha en parte de cuerda. | Número de grados del arco entero. | | Para radio multip. la cuerda por. | Para longitud arco multip cuerda por. | Para flecha semiarco multip. flecha por. | Flecha en parte de cuerda. | Número de grados del arco entero. | | Para radio multip. la cuerda por. | Para longitud arco multip. cuerda por. | Para flecha semiarco multip flecha por |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | ° | ′ | | | | | ° | ′ | | | |
| 1-50 | 9 | 9.75 | 313. | 1.00107 | .2501 | 1/4 | 56 | 8.70 | 8.5 | 1.04116 | .2538 |
| 1-45 | 10 | 10.75 | 253.625 | 1.00132 | .2501 | 1-7 | 63 | 46.90 | 6.625 | 1.05356 | .2549 |
| 1-40 | 11 | 26.98 | 200.5 | 1.00167 | .2502 | .155 | 68 | 53.63 | 5.70291 | 1.06288 | .2557 |
| 1-35 | 13 | 4.92 | 153.625 | 1.00219 | .2502 | 1-6 | 73 | 44 39 | 5. | 1.07256 | .2565 |
| 1-30 | 15 | 15.38 | 113. | 1.00296 | .2503 | .18 | 79 | 11.73 | 4.35803 | 1.08428 | .2576 |
| 1-25 | 18 | 17.74 | 78.625 | 1.00426 | .2504 | 1-5 | 87 | 12.34 | 3.625 | 1.10347 | .2593 |
| 1-20 | 22 | 50.54 | 50.5 | 1.00665 | .2506 | 207107 | 90 | | 3.41422 | 1.11072 | .2599 |
| 1-19 | 24 | 2.16 | 45.625 | 1.00737 | .2507 | .225 | 96 | 54.67 | 2.96913 | 1.12997 | .2615 |
| 1-18 | 25 | 21.65 | 41. | 1.00821 | .2508 | 1/4 | 106 | 15 61 | 2.5 | 1.15912 | .2639 |
| 1-17 | 26 | 50.36 | 36.625 | 1.00920 | .2509 | .275 | 115 | 14.59 | 2.15289 | 1.19082 | .2665 |
| 1-16 | 28 | 30.00 | 32.5 | 1.01038 | .2510 | .3 | 123 | 51 30 | 1.88889 | 1.22495 | .2692 |
| 1-15 | 30 | 22.71 | 28.625 | 1.01181 | .2511 | 1/4 | 134 | 45.62 | 1.625 | 1.27401 | .2729 |
| 1-14 | 32 | 31.22 | 25. | 1.01355 | .2513 | .365 | 144 | 30.98 | 1.43827 | 1.32413 | .2766 |
| 1-13 | 34 | 59.08 | 21.625 | 1.01571 | .2515 | .4 | 154 | 38.35 | 1.28125 | 1.38322 | .2808 |
| 1-12 | 37 | 50.96 | 18.5 | 1.01842 | .2517 | .425 | 161 | 27.52 | 1.19204 | 1.42764 | .2838 |
| 1-11 | 41 | 13.16 | 15.625 | 1.02189 | .2520 | .45 | 167 | 56.93 | 1.11728 | 1.47377 | .2868 |
| 1-10 | 45 | 14.38 | 13. | 1.02646 | .2525 | .475 | 174 | 7.49 | 1.05402 | 1.52152 | .2899 |
| 1-9 | 50 | 6.91 | 10.625 | 1.03260 | .2530 | .5 | 180 | | 1. | 1.57080 | .2929 |

Para **rectificación de arcos circulares**, véase pág. 192.

**Si el radio=1 milla,** 1° = 92.1534 pies = 28.491$^m$
1′ = 1.5859 — — = .479$^m$
1″ = 0256 — — = .0076$^m$

( *N. del T.*)

**Si el radio=1 kilómetro,** 1° = 17.453m
1′ = .291m
1″ = .0048m

**Desarrollo de los arcos circulares. Si el arco excede á una semicircunferencia.** Véase pág. 192.

Conociendo la cuerda y la altura del arco ó flecha, divídase la altura por la cuerda; búsquese en la columna de las alturas (tabla pag. 191) el número igual á este cociente. Tómese el número correspondiente en la columna de las longitudes y multipliquese este último número por la longitud de la cuerda dada. (*N. del T.* — Esta tabla sirve para cualquier unidad de medida que se use.)

# TABLA DE ARCOS CIRCULARES

[Sin errores.

| Altura. | Longit. | Altura. | Longit. | Altura. | Longit. | Altura. | Longit. | Altura. | Longit. |
|---|---|---|---|---|---|---|---|---|---|
| .001 | 1.00002 | .076 | 1.01533 | .151 | 1.05973 | .226 | 1.13108 | .301 | 1.22636 |
| .002 | 1.00002 | .077 | 1.01573 | .152 | 1.06051 | .227 | 1.13219 | .302 | 1.22778 |
| .003 | 1.00003 | .078 | 1.01614 | .153 | 1.06130 | .228 | 1.13331 | .303 | 1.22920 |
| .004 | 1.00004 | .079 | 1.01656 | .154 | 1.06209 | .229 | 1.13444 | .304 | 1.23063 |
| .005 | 1.00007 | .080 | 1.01698 | .155 | 1.06288 | .230 | 1.13557 | .305 | 1.23206 |
| .006 | 1.00010 | .081 | 1.01741 | .156 | 1.06368 | .231 | 1.13671 | .306 | 1.23349 |
| .007 | 1.00013 | .082 | 1.01784 | .157 | 1.06449 | .232 | 1.13785 | .307 | 1.23492 |
| .008 | 1.00017 | .083 | 1.01828 | .158 | 1.06530 | .233 | 1.13900 | .308 | 1.23636 |
| .009 | 1.00022 | .084 | 1.01872 | .159 | 1.06611 | .234 | 1.14015 | .309 | 1.23781 |
| .010 | 1.00027 | .085 | 1.01916 | .160 | 1.06693 | .235 | 1.14131 | .310 | 1.23926 |
| .011 | 1.00032 | .086 | 1.01961 | .161 | 1.06775 | .236 | 1.14247 | .311 | 1.24070 |
| .012 | 1.00038 | .087 | 1.02006 | .162 | 1.06858 | .237 | 1.14363 | .312 | 1.24216 |
| .013 | 1.00045 | .088 | 1.02052 | .163 | 1.06941 | .238 | 1.14480 | .313 | 1.24361 |
| .014 | 1.00053 | .089 | 1.02098 | .164 | 1.07025 | .239 | 1.14597 | .314 | 1.24507 |
| .015 | 1.00061 | .090 | 1.02146 | .165 | 1.07109 | .240 | 1.14714 | .315 | 1.24654 |
| .016 | 1.00069 | .091 | 1.02192 | .166 | 1.07194 | .241 | 1.14832 | .316 | 1.24801 |
| 017 | 1.00078 | .092 | 1.02240 | .167 | 1.07279 | .242 | 1.14951 | .317 | 1.24948 |
| .018 | 1.00087 | .093 | 1.02289 | .168 | 1.07365 | .243 | 1.15070 | .318 | 1.25095 |
| .019 | 1.00097 | .094 | 1.02339 | .169 | 1.07451 | .244 | 1.15189 | .319 | 1.25243 |
| .020 | 1.00107 | .095 | 1.02389 | .170 | 1.07537 | .245 | 1.15308 | .320 | 1.25391 |
| .021 | 1.00117 | .096 | 1.02440 | .171 | 1.07624 | .246 | 1.15428 | .321 | 1.25540 |
| .022 | 1.00128 | 097 | 1.02491 | .172 | 1.07711 | .247 | 1.15549 | .322 | 1.25689 |
| .023 | 1.00140 | .098 | 1.02542 | .173 | 1.07799 | .248 | 1.15670 | .323 | 1.25838 |
| .024 | 1.00153 | .099 | 1.02593 | .174 | 1.07888 | .249 | 1.15791 | .324 | 1.25988 |
| .025 | 1.00167 | .100 | 1.02646 | .175 | 1.07977 | .250 | 1.15912 | .325 | 1.26138 |
| .026 | 1.00182 | .101 | 1.02698 | .176 | 1.08066 | .251 | 1.16034 | .326 | 1.26288 |
| .027 | 1.00196 | .102 | 1.02752 | .177 | 1.08156 | .252 | 1.16156 | .327 | 1.26437 |
| .028 | 1.00210 | .103 | 1.02806 | .178 | 1.08246 | .253 | 1.16279 | .328 | 1.26588 |
| .029 | 1.00225 | .104 | 1.02860 | .179 | 1.08337 | .254 | 1.16402 | .329 | 1.26740 |
| .030 | 1.00240 | .105 | 1.02914 | .180 | 1.08428 | .255 | 1.16526 | .330 | 1.26892 |
| .031 | 1.00256 | .106 | 1.02970 | .181 | 1.08519 | .256 | 1.16650 | .331 | 1.27044 |
| .032 | 1.00272 | .107 | 1.03026 | .182 | 1.08611 | .257 | 1.16774 | .332 | 1.27196 |
| .033 | 1.00289 | .108 | 1.03082 | .183 | 1.08704 | .258 | 1.16899 | .333 | 1.27349 |
| .034 | 1.00307 | .109 | 1.03139 | .184 | 1.08797 | .259 | 1.17024 | .334 | 1.27502 |
| .035 | 1.00327 | .110 | 1.03196 | .185 | 1.08890 | .260 | 1.17150 | .335 | 1.27656 |
| .036 | 1.00345 | .111 | 1.03254 | .186 | 1.08984 | .261 | 1.17276 | .336 | 1.27810 |
| .037 | 1.00364 | .112 | 1.03312 | .187 | 1.09079 | .262 | 1.17403 | .337 | 1.27964 |
| .038 | 1.00384 | .113 | 1.03371 | .188 | 1.09174 | .263 | 1.17530 | .338 | 1.28118 |
| .039 | 1.00405 | .114 | 1.03430 | .189 | 1.09269 | .264 | 1.17657 | .339 | 1.28273 |
| .040 | 1.00426 | .115 | 1.03490 | .190 | 1.09365 | .265 | 1.17784 | .340 | 1.28428 |
| .041 | 1.00447 | .116 | 1.03551 | .191 | 1.09461 | .266 | 1.17912 | .341 | 1.28583 |
| .042 | 1.00469 | .117 | 1.03611 | .192 | 1.09557 | .267 | 1.18040 | .342 | 1.28739 |
| .043 | 1.00492 | .118 | 1.03672 | .193 | 1.09654 | .268 | 1.18169 | .343 | 1.28895 |
| .044 | 1.00515 | .119 | 1.03734 | .194 | 1.09752 | .269 | 1.18299 | .344 | 1.29052 |
| .045 | 1.00539 | .120 | 1.03797 | .195 | 1.09850 | .270 | 1.18429 | .345 | 1.29209 |
| .046 | 1.00563 | .121 | 1.03860 | .196 | 1.09949 | .271 | 1.18559 | .346 | 1.29366 |
| .047 | 1.00587 | .122 | 1.03923 | .197 | 1.10048 | .272 | 1.18689 | .347 | 1.29523 |
| .048 | 1.00612 | .123 | 1.03987 | .198 | 1.10147 | .273 | 1.18820 | .348 | 1.29681 |
| .049 | 1.00638 | .124 | 1.04051 | .199 | 1.10247 | .274 | 1.18951 | .349 | 1.29839 |
| .050 | 1.00665 | .125 | 1.04116 | .200 | 1.10347 | .275 | 1.19082 | .350 | 1.29997 |
| .051 | 1.00692 | .126 | 1.04181 | .201 | 1.10447 | .276 | 1.19214 | .351 | 1.30156 |
| .052 | 1.00720 | .127 | 1.04247 | .202 | 1.10548 | .277 | 1.19346 | .352 | 1.30315 |
| 053 | 1.00748 | .128 | 1.04313 | .203 | 1.10650 | .278 | 1.19479 | .353 | 1.30474 |
| .054 | 1.00776 | .129 | 1.04380 | .204 | 1.10752 | .279 | 1.19612 | .354 | 1.30634 |
| .055 | 1.00805 | .130 | 1.04447 | .205 | 1.10855 | .280 | 1.19746 | .355 | 1.30794 |
| .056 | 1.00834 | .131 | 1.04515 | .206 | 1.10958 | .281 | 1.19880 | .356 | 1.30954 |
| .057 | 1.00864 | .132 | 1.04584 | .207 | 1.11062 | .282 | 1.20014 | .357 | 1.31115 |
| .058 | 1.00895 | .133 | 1.04652 | .208 | 1.11165 | .283 | 1.20149 | .358 | 1.31276 |
| .059 | 1.00926 | .134 | 1.04722 | .209 | 1.11269 | .284 | 1.20284 | .359 | 1.31437 |
| .060 | 1.00957 | .135 | 1.04792 | .210 | 1.11374 | .285 | 1.20419 | .360 | 1.31599 |
| .061 | 1.00989 | .136 | 1.04862 | .211 | 1.11479 | .286 | 1.20555 | .361 | 1.31761 |
| .062 | 1.01021 | .137 | 1.04932 | .212 | 1.11584 | .287 | 1.20691 | .362 | 1.31923 |
| .063 | 1.01054 | .138 | 1.05003 | .213 | 1.11690 | .288 | 1.20827 | .363 | 1.32086 |
| .064 | 1.01088 | .139 | 1.05075 | .214 | 1.11796 | .289 | 1.20964 | .364 | 1.32249 |
| .065 | 1.01123 | .140 | 1.05147 | .215 | 1.11904 | .290 | 1.21102 | .365 | 1.32413 |
| .066 | 1.01158 | .141 | 1.05220 | .216 | 1.12011 | .291 | 1.21239 | .366 | 1.32577 |
| .067 | 1.01193 | .142 | 1.05293 | .217 | 1.12118 | .292 | 1.21377 | .367 | 1.32741 |
| .068 | 1.01228 | .143 | 1.05367 | .218 | 1.12225 | .293 | 1.21515 | .368 | 1.32905 |
| .069 | 1.01264 | .144 | 1.05441 | .219 | 1.12334 | .294 | 1.21654 | .369 | 1.33069 |
| .070 | 1.01302 | .145 | 1.05516 | .220 | 1.12444 | .295 | 1.21794 | .370 | 1.33234 |
| .071 | 1.01338 | .146 | 1.05591 | .221 | 1.12554 | .296 | 1.21933 | .371 | 1.33399 |
| .072 | 1.01376 | .147 | 1.05667 | .222 | 1.12664 | .297 | 1.22073 | .372 | 1.33564 |
| .073 | 1.01414 | .148 | 1.05743 | .223 | 1.12774 | .298 | 1.22213 | 373 | 1.33730 |
| .074 | 1.01453 | .149 | 1.05819 | .224 | 1.12885 | .299 | 1.22354 | .374 | 1.33896 |
| .075 | 1.01493 | .150 | 1.05896 | .225 | 1.12997 | .300 | 1.22495 | .375 | 1.34063 |

## TABLA DE ARCOS CIRCULARES. (Continuación.)

| Altura. | Longit. | Altura. | Longit. | Altura. | Longit. | Altura. | Longit. | Altura. | Longit. |
|---|---|---|---|---|---|---|---|---|---|
| .373 | 1.34229 | .401 | 1.38496 | .426 | 1.42945 | .451 | 1.47565 | .476 | 1.52346 |
| .377 | 1.34396 | .402 | 1.38671 | .427 | 1.43127 | .452 | 1.47753 | .477 | 1.52541 |
| .378 | 1.34563 | .403 | 1.38846 | .428 | 1.43309 | .453 | 1.47942 | .478 | 1.52736 |
| .379 | 1.34731 | .404 | 1.39021 | .429 | 1.43491 | .454 | 1.48131 | .479 | 1.52931 |
| .380 | 1.34899 | .405 | 1.39196 | .430 | 1.43673 | .455 | 1.48320 | .480 | 1.53126 |
| .381 | 1.35068 | .406 | 1.39372 | .431 | 1.43856 | .456 | 1.48509 | .481 | 1.53322 |
| .382 | 1.35237 | .407 | 1.39548 | .432 | 1.44039 | .457 | 1.48699 | .482 | 1.53518 |
| .383 | 1.35406 | .408 | 1.39724 | .433 | 1.44222 | .458 | 1.48889 | .483 | 1.53714 |
| .384 | 1.35575 | .409 | 1.39900 | .434 | 1.44405 | .459 | 1.49079 | .484 | 1.53910 |
| .385 | 1.35744 | .410 | 1.40077 | .435 | 1.44589 | .460 | 1.49269 | .485 | 1.54106 |
| .386 | 1.35914 | .411 | 1.40254 | .436 | 1.44773 | .461 | 1.49460 | .486 | 1.54302 |
| .387 | 1.36084 | .412 | 1.40432 | .437 | 1.44957 | .462 | 1.49651 | .487 | 1.54499 |
| .388 | 1.36254 | .413 | 1.40610 | .438 | 1.45142 | .463 | 1.49842 | .488 | 1.54696 |
| .389 | 1.36425 | .414 | 1.40788 | .439 | 1.45327 | .464 | 1.50033 | .489 | 1.54893 |
| .390 | 1.36596 | .415 | 1.40966 | .440 | 1.45512 | .465 | 1.50224 | .490 | 1.55091 |
| .391 | 1.36767 | .416 | 1.41145 | .441 | 1.45697 | .466 | 1.50416 | .491 | 1.55289 |
| .392 | 1.36939 | .417 | 1.41324 | .442 | 1.45883 | .467 | 1.50608 | .492 | 1.55487 |
| .393 | 1.37111 | .418 | 1.41503 | .443 | 1.46069 | .468 | 1.50800 | .493 | 1.55685 |
| .394 | 1.37283 | .419 | 1.41682 | .444 | 1.46255 | .469 | 1.50992 | .494 | 1.55884 |
| .395 | 1.37455 | .420 | 1.41861 | .445 | 1.46441 | .470 | 1.51185 | .495 | 1.56083 |
| .396 | 1.37628 | .421 | 1.42041 | .446 | 1.46628 | .471 | 1.51378 | .496 | 1.56282 |
| .397 | 1.37801 | .422 | 1.42221 | .447 | 1.46815 | .472 | 1.51571 | .497 | 1.56481 |
| .398 | 1.37974 | .423 | 1.42402 | .448 | 1.47002 | .473 | 1.51764 | .498 | 1.56681 |
| .399 | 1.38148 | .424 | 1.42583 | .449 | 1.47189 | .474 | 1.51958 | .499 | 1.56881 |
| .400 | 1.38322 | .425 | 1.42764 | .450 | 1.47377 | .475 | 1.52152 | .500 | 1.57080 |

Si el arco A es mayor que la semicircunf, sea H=su altura, $a$=circunf del círculo — A, y $h$=altura del arco $a$=(semicuerda)$^2$ ÷ H. Encuéntrese $a$ por la tabla como se dijo. Entonces A=circunf — $a$.

### Rectificación de un arco circular*.

(a) Fig. A. **Para encontrar aproximadamente* la longitud de un arco** GBC. De un extremo C del arco tírese una tang CE al arco. (Véase pág. 179.) Proiónguese la cuerda GFC, hacia D, y hágase CD=FC=$^1/_2$ GFC. De D con un radio=DG, trácese el arco GE. Entonces CE=arco CBG * aproximadamente.

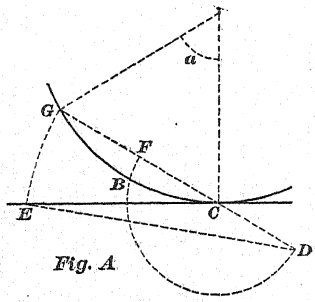



*Fig. A*

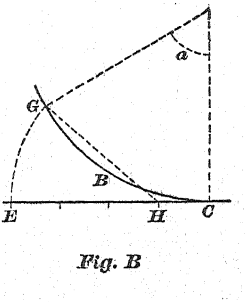



*Fig. B*

(b) Fig. B. En un círculo dado, **encontrar un arco** GC, aproximadamente * **igual á una recta** CE. Por el punto C en el círculo tírese una tang CE de la longitud dada. Hágase CH=$^1/_4$ CE. De H con radio=HE trácese arc EG. Entonces arco CBG=CE aproximadamente.

(c) En un círculo dado **trazar un arco igual en longitud á otro arco** de otro círculo: *primero* por (a) encuéntrese una línea recta igual al arco dado y por (b) búsquese el arco igual á esa línea recta.

**Para encontrar el desarrollo de un arco circular por medio de**

* El error crece con la 4.ª potencia del ángulo. En un arco de 60° es como de 4 minutos.

**la tabla siguiente.** Conociendo el radio del círculo y la medida del arco en grados, minutos, etc., se observará la siguiente :

**Regla.** Súmense las longitudes encontradas respectivamente frente á los grados, minutos, etc., del arco en la tabla y multiplíquese la suma por el radio del círculo.

Ej. : En un círculo de 12.23 m de radio hay un arco de 13 grados, 27 minutos y 8 segundos. ¿De qué tamaño es el arco?

Frente á 13 grados en la tabla encontramos .2268928
— á 27 minutos encontramos .0078540
— á 8 segundos encontramos .0000388
Suma= .2347856

Y .2347856 × 12.43 = 2.918385 m es la longitud que tiene el arco.

## Longitud de los arcos circulares para un radio 1.

*N. del T.* Esta tabla sirve para pies, metros ó cualquiera otra unidad que se use.

| Gr. | Long. | Gr. | Long. | Gr. | Long. | Min. | Long. | Seg. | Long. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | .0174533 | 61 | 1.0646508 | 121 | 2.1118484 | 1 | .0002909 | 1 | .0000048 |
| 2 | .0349066 | 62 | 1.0821041 | 122 | 2.1293017 | 2 | .0005818 | 2 | .0000097 |
| 3 | .0523599 | 63 | 1.0995574 | 123 | 2.1467550 | 3 | .0008727 | 3 | .0000145 |
| 4 | .0698132 | 64 | 1.1170107 | 124 | 2.1642083 | 4 | .0011636 | 4 | .0000194 |
| 5 | .0872665 | 65 | 1.1344640 | 125 | 2.1816616 | 5 | .0014544 | 5 | .0000242 |
| 6 | .1047198 | 66 | 1.1519173 | 126 | 2.1991149 | 6 | .0017453 | 6 | .0000291 |
| 7 | .1221730 | 67 | 1.1693705 | 127 | 2.2165682 | 7 | .0020362 | 7 | .0000339 |
| 8 | .1396263 | 68 | 1.1868239 | 128 | 2.2340214 | 8 | .0023271 | 8 | .0000388 |
| 9 | .1570796 | 69 | 1.2042772 | 129 | 2.2514747 | 9 | .0026180 | 9 | .0000436 |
| 10 | .1745329 | 70 | 1.2217305 | 130 | 2.2689280 | 10 | .0029089 | 10 | .0000485 |
| 11 | .1919862 | 71 | 1.2391838 | 131 | 2.2863813 | 11 | .0031998 | 11 | .0000533 |
| 12 | .2094395 | 72 | 1.2566371 | 132 | 2.3038346 | 12 | .0034907 | 12 | .0000582 |
| 13 | .2268928 | 73 | 1.2740904 | 133 | 2.3212879 | 13 | .0037815 | 13 | .0000630 |
| 14 | .2443461 | 74 | 1.2915436 | 134 | 2.3387412 | 14 | .0040724 | 14 | .0000679 |
| 15 | .2617994 | 75 | 1.3089969 | 135 | 2.3561945 | 15 | .0043633 | 15 | .0000727 |
| 16 | .2792527 | 76 | 1.3264502 | 136 | 2.3736478 | 16 | .0046542 | 16 | .0000776 |
| 17 | .2967060 | 77 | 1.3439035 | 137 | 2.3911011 | 17 | .0049451 | 17 | .0000824 |
| 18 | .3141593 | 78 | 1.3613568 | 138 | 2.4085544 | 18 | .0052360 | 18 | .0000873 |
| 19 | .3316126 | 79 | 1.3788101 | 139 | 2.4260077 | 19 | .0055269 | 19 | .0000921 |
| 20 | .3490659 | 80 | 1.3962634 | 140 | 2.4434610 | 20 | .0058178 | 20 | .0000970 |
| 21 | .3665191 | 81 | 1.4137167 | 141 | 2.4609142 | 21 | .0061087 | 21 | .0001018 |
| 22 | .3839724 | 82 | 1.4311700 | 142 | 2.4783675 | 22 | .0063995 | 22 | .0001067 |
| 23 | .4014257 | 83 | 1.4486233 | 143 | 2.4958208 | 23 | .0066904 | 23 | .0001115 |
| 24 | .4188790 | 84 | 1.4660766 | 144 | 2.5132741 | 24 | .0069813 | 24 | .0001164 |
| 25 | .4363323 | 85 | 1.4835299 | 145 | 2.5307274 | 25 | .0072722 | 25 | .0001212 |
| 26 | .4537856 | 86 | 1.5009832 | 146 | 2.5481807 | 26 | .0075631 | 26 | .0001261 |
| 27 | .4712389 | 87 | 1.5184364 | 147 | 2.5656340 | 27 | .0078540 | 27 | .0001309 |
| 28 | .4886922 | 88 | 1.5358897 | 148 | 2.5830873 | 28 | .0081449 | 28 | .0001357 |
| 29 | .5061455 | 89 | 1.5533430 | 149 | 2.6005406 | 29 | .0084358 | 29 | .0001406 |
| 30 | .5235988 | 90 | 1.5707963 | 150 | 2.6179939 | 30 | .0087266 | 30 | .0001454 |
| 31 | .5410521 | 91 | 1.5882496 | 151 | 2.6354472 | 31 | .0090175 | 31 | .0001503 |
| 32 | .5585054 | 92 | 1.6057029 | 152 | 2.6529005 | 32 | .0093084 | 32 | .0001551 |
| 33 | .5759587 | 93 | 1.6231562 | 153 | 2.6703538 | 33 | .0095993 | 33 | .0001600 |
| 34 | .5934119 | 94 | 1.6406095 | 154 | 2.6878070 | 34 | .0098902 | 34 | .0001648 |
| 35 | .6108652 | 95 | 1.6580628 | 155 | 2.7052603 | 35 | .0101811 | 35 | .0001697 |
| 36 | .6283185 | 96 | 1.6755161 | 156 | 2.7227136 | 36 | .0104720 | 36 | .0001745 |
| 37 | .6457718 | 97 | 1.6929694 | 157 | 2.7401669 | 37 | .0107629 | 37 | .0001794 |
| 38 | .6632251 | 98 | 1.7104227 | 158 | 2.7576202 | 38 | .0110538 | 38 | .0001842 |
| 39 | .6806784 | 99 | 1.7278760 | 159 | 2.7750735 | 39 | .0113446 | 39 | .0001891 |
| 40 | .6981317 | 100 | 1.7453293 | 160 | 2.7925268 | 40 | .0116355 | 40 | .0001939 |
| 41 | .7155850 | 101 | 1.7627825 | 161 | 2.8099801 | 41 | .0119264 | 41 | .0001988 |
| 42 | .7330383 | 102 | 1.7802358 | 162 | 2.8274334 | 42 | .0122173 | 42 | .0002036 |
| 43 | .7504916 | 103 | 1.7976891 | 163 | 2.8448867 | 43 | .0125082 | 43 | .0002085 |
| 44 | .7679449 | 104 | 1.8151424 | 164 | 2.8623400 | 44 | .0127991 | 44 | .0002133 |
| 45 | .7853982 | 105 | 1.8325957 | 165 | 2.8797933 | 45 | .0130900 | 45 | .0002182 |
| 46 | .8028515 | 106 | 1.8500490 | 166 | 2.8972466 | 46 | .0133809 | 46 | .0002230 |
| 47 | .8203047 | 107 | 1.8675023 | 167 | 2.9146999 | 47 | .0136717 | 47 | .0002279 |
| 48 | .8377580 | 108 | 1.8849556 | 168 | 2.9321531 | 48 | .0139626 | 48 | .0002327 |
| 49 | .8552113 | 109 | 1.9024089 | 169 | 2.9496064 | 49 | .0142535 | 49 | .0002376 |
| 50 | .8726646 | 110 | 1.9198622 | 170 | 2.9670597 | 50 | .0145444 | 50 | .0002424 |
| 51 | .8901179 | 111 | 1.9373155 | 171 | 2.9845130 | 51 | .0148353 | 51 | .0002473 |
| 52 | .9075712 | 112 | 1.9547688 | 172 | 3.0019663 | 52 | .0151262 | 52 | .0002521 |
| 53 | .9250245 | 113 | 1.9722221 | 173 | 3.0194196 | 53 | .0154171 | 53 | .0002570 |
| 54 | .9424778 | 114 | 1.9896753 | 174 | 3.0368729 | 54 | .0157080 | 54 | .0002618 |
| 55 | .9599311 | 115 | 2.0071286 | 175 | 3.0543262 | 55 | .0159989 | 55 | .0002666 |
| 56 | .9773844 | 116 | 2.0245819 | 176 | 3.0717795 | 56 | .0162897 | 56 | .0002715 |
| 57 | .9948377 | 117 | 2.0420352 | 177 | 3.0892328 | 57 | .0165806 | 57 | .0002763 |
| 58 | 1.0122910 | 118 | 2.0594885 | 178 | 3.1066861 | 58 | .0168715 | 58 | .0002812 |
| 59 | 1.0297443 | 119 | 2.0769418 | 179 | 3.1241394 | 59 | .0171624 | 59 | .0002860 |
| 60 | 1.0471976 | 120 | 2.0943951 | 180 | 3.1415927 | 60 | .0174533 | 60 | .0002909 |

## SECTORES CIRCULARES, ANILLOS, SEGMENTOS, ETC.

**Área de un sector circular** *adbc*, fig. A.

$$= \frac{\text{arco } adb}{2} \times \text{radio } ca.$$

$$= \text{Area del círculo entero} \times \frac{\text{arco } adb \text{ en grados}}{360}.$$

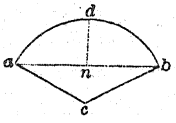



Fig. A.

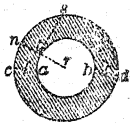



Fig. B.

**Área de un anillo circular**, fig. B.

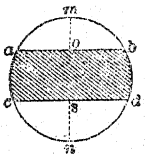
= Area del círculo mayor *cd* — área del menor *ab*.
= .7854 × (suma de diámetros *(cd + ab)* × (diferencia de los diámetros *cd* — *ab)*.
= 1.5708 × espesor *ca* × suma de diám *cd*, y *ab*.

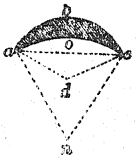
**Para encontrar el radio de un círculo que deba tener la misma área que un anillo circular dado** *csdab*, fig. B.

Trácese cualquier radio *nr* del círculo exterior y de donde dicho radio corte el círculo interior en *t* trácese *ts* perpendicular á él : *ts* es el radio que se busca.

**La anchura** *cd* = *bd*, fig. B.

= ½ diferencia de los diámetros *cd*, *ab*.
= ½ (diámetro *cd* — $\sqrt{1.2732}$ área del círculo *ab)*.




**Área de una zona circular** *abcd* = área del círculo *mn* — áreas de los segmentos *amb*, *cnd*. (Para áreas de segmentos véase más abajo.)

**Una luna circular** es una figura comprendida entre dos arcos *abc*, *aoc* de círculos de diferentes radios *ad*, *an* cuyas concavidades están de un mismo lado. Area de una luna circular *abco* = área del segmento *abc* — área del segmento *aoc*. (Para áreas de segmentos, véase abajo.)

Fig. C.

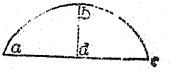



Fig. D.

**Para encontrar el área de un segmento circular** *abcd*, figs. C, D.

**El área del segmento** *adbn*, fig. A (al empezar la pág.), es :

= área del sector *adbc* — área del triángulo *abc*.
= ½ (arco *adb* × radio *ac* — *cn* × cuerda *ab)*.

**Teniendo el área de un segmento perteneciente á un círculo dado, encontrar su cuerda y flecha.**

Divídase el área por el cuadrado del diámetro del círculo, búsquese el cociente en la columna de las tablas siguientes, tómese de la tabla el número correspondiente en la columna de las flechas. Multiplíquese este número por el diámetro. El producto será la flecha que se busca y entonces la cuerda

$$= 2 \times \sqrt{(\text{diám} - \text{flecha}) \times \text{flecha}}.$$

**Tabla de áreas de segmentos circulares,** figs. C, D.

**Si el segmento excede á un semicírculo** su área es=área del círculo — área de un segmento cuya flecha es=(diámetro del círculo — flecha del segmento dado).

**El diámetro del círculo** es=(cuadrado de la mitad de la cuerda — flecha) ÷ flecha, sea que el segmento exceda al semicírculo ó no.

(*N. del T.* — Esta tabla sirve para cualquier unidad que se adopte : pies, metros, etc.)

| Flecha dividida por diám. | Area = cuadrado diam. mult. por | Flecha dividida por diám. | Area = cuadrado diam. mult. por | Flecha dividida por diám. | Area = cuadrado diam. mult. por | Flecha dividida por diám. | Area = cuadrado diam. mult. por | Flecha dividida por diám. | Area = cuadrado diam. mult. por |
|---|---|---|---|---|---|---|---|---|---|
| .001 | .000042 | .064 | .021168 | .127 | .057991 | .190 | .103900 | .253 | .156149 |
| .002 | .000119 | .065 | .021660 | .128 | .058658 | .191 | .104686 | .254 | .157019 |
| .003 | .000219 | .066 | .022155 | .129 | .059328 | .192 | .105472 | .255 | .157891 |
| .004 | .000337 | .067 | .022653 | .130 | .059999 | .193 | .106261 | .256 | .158763 |
| .005 | .000471 | .068 | .023155 | .131 | .060673 | .194 | .107051 | .257 | .159636 |
| .006 | .000619 | .069 | .023660 | .132 | .061349 | .195 | .107843 | .258 | .160511 |
| .007 | .000779 | .070 | .024168 | .133 | .062027 | .196 | .108636 | .259 | .161386 |
| .008 | .000952 | .071 | .024680 | .134 | .062707 | .197 | .109431 | .260 | .162263 |
| .009 | .001135 | .072 | .025196 | .135 | .063389 | .198 | .110227 | .261 | .163141 |
| .010 | .001329 | .073 | .025714 | .136 | .064074 | .199 | .111025 | .262 | .164020 |
| .011 | .001533 | .074 | .026236 | .137 | .064761 | .200 | .111824 | .263 | .164900 |
| .012 | .001746 | .075 | .026761 | .138 | .065449 | .201 | .112625 | .264 | .165781 |
| .013 | .001969 | .076 | .027290 | .139 | .066140 | .202 | .113427 | .265 | .166663 |
| .014 | .002199 | .077 | .027821 | .140 | .066833 | .203 | .114231 | .266 | .167546 |
| .015 | .002438 | .078 | .028356 | .141 | .067528 | .204 | .115036 | .267 | .168431 |
| .016 | .002685 | .079 | .028894 | .142 | .068225 | .205 | .115842 | .268 | .169316 |
| .017 | .002940 | .080 | .029435 | .143 | .068924 | .206 | .116651 | .269 | .170202 |
| .018 | .003202 | .081 | .029979 | .144 | .069626 | .207 | .117460 | .270 | .171090 |
| .019 | .003472 | .082 | .030526 | .145 | .070329 | .208 | .118271 | .271 | .171978 |
| .020 | .003749 | .083 | .031077 | .146 | .071034 | .209 | .119084 | .272 | .172868 |
| .021 | .004032 | .084 | .031630 | .147 | .071741 | .210 | .119898 | .273 | .173758 |
| .022 | .004322 | .085 | .032186 | .148 | .072450 | .211 | .120713 | .274 | .174650 |
| .023 | .004619 | .086 | .032746 | .149 | .073162 | .212 | .121530 | .275 | .175542 |
| .024 | .004922 | .087 | .033308 | .150 | .073875 | .213 | .122348 | .276 | .176436 |
| .025 | .005231 | .088 | .033873 | .151 | .074590 | .214 | .123167 | .277 | .177330 |
| .026 | .005546 | .089 | .034441 | .152 | .075307 | .215 | .123988 | .278 | .178225 |
| .027 | .005867 | .090 | .035012 | .153 | .076026 | .216 | .124811 | .279 | .179122 |
| .028 | .006194 | .091 | .035586 | .154 | .076747 | .217 | .125634 | .280 | .180020 |
| .029 | .006527 | .092 | .036162 | .155 | .077470 | .218 | .126459 | .281 | .180918 |
| .030 | .006866 | .093 | .036742 | .156 | .078194 | .219 | .127286 | .282 | .181818 |
| .031 | .007209 | .094 | .037324 | .157 | .078921 | .220 | .128114 | .283 | .182718 |
| .032 | .007559 | .095 | .037909 | .158 | .079650 | .221 | .128943 | .284 | .183619 |
| .033 | .007913 | .096 | .038497 | .159 | .080380 | .222 | .129773 | .285 | .184522 |
| .034 | .008273 | .097 | .039087 | .160 | .081112 | .223 | .130605 | .286 | .185425 |
| .035 | .008638 | .098 | .039681 | .161 | .081847 | .224 | .131438 | .287 | .186329 |
| .036 | .009008 | .099 | .040277 | .162 | .082582 | .225 | .132273 | .288 | .187235 |
| .037 | .009383 | .100 | .040875 | .163 | .083320 | .226 | .133109 | .289 | .188141 |
| .038 | .009764 | .101 | .041477 | .164 | .084060 | .227 | .133946 | .290 | .189048 |
| .039 | .010148 | .102 | .042081 | .165 | .084801 | .228 | .134784 | .291 | .189956 |
| .040 | .010538 | .103 | .042687 | .166 | .085545 | .229 | .135624 | .292 | .190865 |
| .041 | .010932 | .104 | .043296 | .167 | .086290 | .230 | .136465 | .293 | .191774 |
| .042 | .011331 | .105 | .043908 | .168 | .087037 | .231 | .137307 | .294 | .192685 |
| .043 | .011734 | .106 | .044523 | .169 | .087785 | .232 | .138151 | .295 | .193597 |
| .044 | .012142 | .107 | .045140 | .170 | .088536 | .233 | .138996 | .296 | .194509 |
| .045 | .012555 | .108 | .045759 | .171 | .089288 | .234 | .139842 | .297 | .195423 |
| .046 | .012971 | .109 | .046381 | .172 | .090042 | .235 | .140689 | .298 | .196337 |
| .047 | .013393 | .110 | .047006 | .173 | .090797 | .236 | .141538 | .299 | .197252 |
| .048 | .013818 | .111 | .047633 | .174 | .091555 | .237 | .142388 | .300 | .198168 |
| .049 | .014248 | .112 | .048262 | .175 | .092314 | .238 | .143239 | .301 | .199085 |
| .050 | .014681 | .113 | .048894 | .176 | .093074 | .239 | .144091 | .302 | .200003 |
| .051 | .015119 | .114 | .049529 | .177 | .093837 | .240 | .144945 | .303 | .200922 |
| .052 | .015561 | .115 | .050165 | .178 | .094601 | .241 | .145800 | .304 | .201841 |
| .053 | .016008 | .116 | .050805 | .179 | .095367 | .242 | .146656 | .305 | .202762 |
| .054 | .016458 | .117 | .051446 | .180 | .096135 | .243 | .147513 | .306 | .203683 |
| .055 | .016912 | .118 | .052090 | .181 | .096904 | .244 | .148371 | .307 | .204605 |
| .056 | .017369 | .119 | .052737 | .182 | .097675 | .245 | .149231 | .308 | .205528 |
| .057 | .017831 | .120 | .053385 | .183 | .098447 | .246 | .150091 | .309 | .206452 |
| .058 | .018297 | .121 | .054037 | .184 | .099221 | .247 | .150953 | .310 | .207376 |
| .059 | .018766 | .122 | .054690 | .185 | .099997 | .248 | .151816 | .311 | .208302 |
| .060 | .019239 | .123 | .055346 | .186 | .100774 | .249 | .152681 | .312 | .209228 |
| .061 | .019716 | .124 | .056004 | .187 | .101553 | .250 | .153546 | .313 | .210155 |
| .062 | .020197 | .125 | .056664 | .188 | .102334 | .251 | .154413 | .314 | .211083 |
| .063 | .020681 | .126 | .057327 | .189 | .103116 | .252 | .155281 | .315 | .212011 |

| Flecha dividida por diám. | Area = cuadrado diám. mult. por | Flecha dividida por diám. | Area = cuadrado diám. mult. por | Flecha dividida por diám. | Area = cuadrado diám. mult. por | Flecha dividida por diám. | Area = cuadrado diám. mult. por | Flecha dividida por diám. | Area = cuadrado diám. mult. por |
|---|---|---|---|---|---|---|---|---|---|
| .316 | .212941 | .353 | .247845 | .390 | .283593 | .427 | .319959 | .464 | .356730 |
| .317 | .213871 | .354 | .248801 | .391 | .284569 | .428 | .320949 | .465 | .357728 |
| .318 | .214802 | .355 | .249758 | .392 | .285545 | .429 | .321938 | .466 | .358725 |
| .319 | .215734 | .356 | .250715 | .393 | .286521 | .430 | .322928 | .467 | .359723 |
| .320 | .216666 | .357 | .251673 | .394 | .287499 | .431 | .323919 | .468 | .360721 |
| .321 | .217600 | .358 | .252632 | .395 | .288476 | .432 | .324909 | .469 | .361719 |
| .322 | .218534 | .359 | .253591 | .396 | .289454 | .433 | .325900 | .470 | .362717 |
| .323 | .219469 | .360 | .254551 | .397 | .290432 | .434 | .326891 | .471 | .363715 |
| .324 | .220404 | .361 | .255511 | .398 | .291411 | .435 | .327883 | .472 | .364714 |
| .325 | .221341 | .362 | .256472 | .399 | .292390 | .436 | .328874 | .473 | .365712 |
| .326 | .222278 | .363 | .257433 | .400 | .293370 | .437 | .329866 | .474 | .366711 |
| .327 | .223216 | .364 | .258395 | .401 | .294350 | .438 | .330858 | .475 | .367710 |
| .328 | .224154 | .365 | .259358 | .402 | .295330 | .439 | .331851 | .476 | .368708 |
| .329 | .225094 | .366 | .260321 | .403 | .296311 | .440 | .332843 | .477 | .369707 |
| .330 | .226034 | .367 | .261285 | .404 | .297292 | .441 | .333836 | .478 | .370706 |
| .331 | .226974 | .368 | .262249 | .405 | .298274 | .442 | .334829 | .479 | .371705 |
| .332 | .227916 | .369 | .263214 | .406 | .299256 | .443 | .335823 | .480 | .372704 |
| .333 | .228858 | .370 | .264179 | .407 | .300238 | .444 | .336816 | .481 | .373704 |
| .334 | .229801 | .371 | .265145 | .408 | .301221 | .445 | .337810 | .482 | .374703 |
| .335 | .230745 | .372 | .266111 | .409 | .302204 | .446 | .338804 | .483 | .375702 |
| .336 | .231689 | .373 | .267078 | .410 | .303187 | .447 | .339799 | .484 | .376702 |
| .337 | .232634 | .374 | .268046 | .411 | .304171 | .448 | .340793 | .485 | .377701 |
| .338 | .233580 | .375 | .269014 | .412 | .305156 | .449 | .341788 | .486 | .378701 |
| .339 | .234526 | .376 | .269982 | .413 | .306140 | .450 | .342783 | .487 | .379701 |
| .340 | .235473 | .377 | .270951 | .414 | .307125 | .451 | .343778 | .488 | .380700 |
| .341 | .236421 | .378 | .271921 | .415 | .308110 | .452 | .344773 | .489 | .381700 |
| .342 | .237369 | .379 | .272891 | .416 | .309096 | .453 | .345768 | .490 | .382700 |
| .343 | .238319 | .380 | .273861 | .417 | .310082 | .454 | .346764 | .491 | .383700 |
| .344 | .239268 | .381 | .274832 | .418 | .311068 | .455 | .347760 | .492 | .384699 |
| .345 | .240219 | .382 | .275804 | .419 | .312055 | .456 | .348756 | .493 | .385699 |
| .346 | .241170 | .383 | .276776 | .420 | .313042 | .457 | .349752 | .494 | .386699 |
| .347 | .242122 | .384 | .277748 | .421 | .314029 | .458 | .350749 | .495 | .387699 |
| .348 | .243074 | .385 | .278721 | .422 | .315017 | .459 | .351745 | .496 | .388699 |
| .349 | .244027 | .386 | .279695 | .423 | .316005 | .460 | .352742 | .497 | .389699 |
| .350 | .244980 | .387 | .280669 | .424 | .316993 | .461 | .353739 | .498 | .390699 |
| .351 | .245935 | .388 | .281643 | .425 | .317981 | .462 | .354736 | .499 | .391699 |
| .352 | .246890 | .389 | .282618 | .426 | .318970 | .463 | .355733 | .500 | .392699 |

## ELIPSE (véase la pág. siguiente).

Distancia entre los focos $= fg = 2\sqrt{co^2 - bo^2}$; *

$$cf = gw = \frac{cw - fg}{2} = \frac{cw}{2} - \sqrt{co^2 - bo^2}.$$

* Porque $co = \frac{cw}{2} = \frac{fc + cg}{2} = \frac{fb + bg}{2} = fb.$

## LA ELIPSE

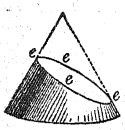



Fig. 1.

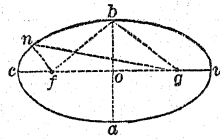



Fig. 2.

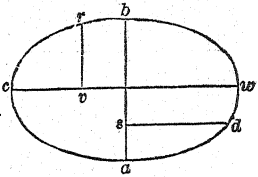



Fig. 3.

Una elipse es una curva *eeee*, fig. 1, formada por una sección oblicua hecha en un cono ó un cilindro sin cortar la base.

Tiene la propiedad de que, trazando dos líneas *nf*, *ng*, de cualquier punto *n* de su periferia á dos puntos especiales *f*, *g* de su eje mayor *c w* (llamados focos de la elipse) su suma será igual á la de otras dos líneas cualesquiera como *bf*, *bg* trazadas desde cualquier otro punto *b* de la periferia á los mismos focos *f*, *g* y además la suma de cualesquiera de dichas dos líneas será igual al eje mayor *cw*. La línea *cw* que divide la elipse en su mayor longitud en dos partes iguales se llama eje mayor; y la línea *ab* perpendicular al eje mayor que la divide igualmente en dos partes iguales se llama eje menor. Para encontrar la posición del foco de una elipse: desde cualquiera de los dos extremos *b* del eje menor mídanse las distancias *bf*, *bg*, fig. 2, cada una igual *oc*, ó la mitad del eje mayor.

El parámetro de una elipse es la longitud obtenida así: el eje mayor : al eje menor : : el eje menor: al parámetro. Cualesquiera línea *rv* ó *sd*, fig. 3, trazada desde la periferia y en ángulos rectos ó uno de los dos diámetros se llama ordenada; y las partes *cv*, *vw*, *bs*, *sa* de un diámetro entre la ordenada y la periferia se llaman abscisas.

**Para encontrar la longitud de cualquiera ordenada** *rv* ó *sd* **sobre cualquiera de los diámetros** *cw* ó *ba;* conociendo las abscisas *cv* ó *sa* y los dos diámetros *cw*, *ba;*

$$cw^2 : ba^2 :: cv \times vw : rv^2 \qquad ba^2 : cw^2 :: bs \times sa : sd^2$$

**Encontrar la periferia de una elipse.** Los matemáticos no les han dado á los hombres prácticos una regla simple para conseguir este objeto. Las llamadas fórmulas aproximadas no merecen este nombre. Son como siguen. Llamando D el eje mayor y *d* el menor:

$$\text{Regla 1. Periferia} = 3.1416\,\frac{D+d}{2}; \quad \text{Regla 2. } 3.1416\,\sqrt{\left(\frac{D^2+d^2}{2}\right)};$$

$$\text{Regla 3. } 2.2215\,\sqrt{D^2+d^2}.$$

Esta es la misma que la regla 2.ª pero con diferente forma.

$$\text{Regla 4. } 2\times\sqrt{D^2+1.4674\,d^2}.$$

En una elipse cuyos ejes sean 10 y 2, la periferia es muy aproximadamente 21: pero la fórmula 1.ª da para este caso=18.85, la fórmula 2.ª ó 3.ª=22.65 y la fórmula 4.ª=20.58. Si los diámetros son 10 y 6 la periferia realmente es = 25.59 y la fórmula 4.ª da 24.72. Estos ejemplos demuestran que ninguna de las fórmulas generalmente dadas son dignas de confianza. La siguiente fórmula del autor es suficientemente exacta para casos ordinarios y los errores que produce no llegan probablemente á más de una milésima. Cuando D no es más de 5 veces mayor que *d*.

$$\text{Periferia} = 3.1416\,\sqrt{\frac{D^2+d^2}{2}-\frac{(D-d)^2}{8.8}}.$$

**Si D es más de cinco veces** *d*, entonces en lugar de dividir $(D-d)^2$ por 8.8, divídase por el número que la corresponde en esta tabla.

| D ÷ *d* > | 5 = 6 | 7 | 8 | 9 | 10 | 12 | 14 | 16 | 18 | 20 | 25 | 30 | 40 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *c* = | 8.8 | 9.0 | 9.2 | 9.3 | 9.35 | 9.4 | 9.5 | 9.6 | 9.68 | 9.75 | 9.80 | 9.87 | 9.92 | 9.98 |

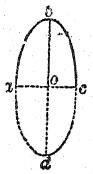
| D ÷ *d* = | 50 | 60 | 70 | 80 | 100 |
|---|---|---|---|---|---|
| *c* = | 10.04 | 10.10 | 10.17 | 10.23 | 10.35 |

La fórmula siguiente, original del señor Arnoldo Pears,

$$\text{Periferia} = 3.1416\,d + 2[(D - d) - \frac{d\,(D - d)}{\sqrt{(D + d) \times (D + 2d)}}.$$

**La tabla que sigue de arcos semielípticos** fué construída por nuestra fórmula.

Para usar esta tabla divídase la altura ó flecha del arco por su luz ó cuerda. El cociente será la altura de un arco cuya cuerda es 1. Búsquese este cociente en la




columna de las flechas ó alturas y tómese el número correspondiente en la columna de las longitudes, multiplíquese este número por la luz ó abertura real, el producto será la longitud que se busca. Cuando la altura viene á ser .500 de la cuerda (como al fin de la tabla), la elipse se transforma en un círculo. Cuando la altura excede de .500 de la cuerda, como en *abc*, tómese entonces *ao* ó la mitad de la cuerda como la elevación y divídase esta elevación por el eje mayor *bd* para buscar el cociente en la columna de alturas y multiplicarlo por el eje mayor, y obtenemos así la longitud del arco *bad* que es evidentemente igual á *abc*.

## TABLA DE LONGITUDES Ó DESARROLLOS DE SEMIARCOS ELÍPTICOS

(*N. del T.* — Esta tabla sirve para cualquier unidad que se elija: metros pies, etc.).

(Original.)

| Altura ÷ abertura | Longitud = abertura × por | Altura ÷ abertura | Longitud = abertura × por | Altura ÷ abertura | Longitud = abertura × por | Altura ÷ abertura | Longitud = abertura × por |
|---|---|---|---|---|---|---|---|
| .005 | 1.000 | .130 | 1.079 | .255 | 1.219 | .380 | 1.390 |
| .01 | 1.001 | .135 | 1.084 | .260 | 1.226 | .385 | 1.397 |
| .015 | 1.002 | .140 | 1.089 | .265 | 1.233 | .390 | 1.404 |
| .02 | 1.003 | .145 | 1.094 | .270 | 1.239 | .395 | 1.412 |
| .025 | 1.004 | .150 | 1.099 | .275 | 1.245 | .400 | 1.419 |
| .03 | 1.006 | .155 | 1.104 | .280 | 1.252 | .405 | 1.426 |
| .035 | 1.008 | .160 | 1.109 | .285 | 1.259 | .410 | 1.434 |
| .04 | 1.011 | .165 | 1.115 | .290 | 1.265 | .415 | 1.441 |
| .045 | 1.014 | .170 | 1.120 | .295 | 1.272 | .420 | 1.449 |
| .05 | 1.017 | .175 | 1.125 | .300 | 1.279 | .425 | 1.456 |
| .055 | 1.020 | .180 | 1.131 | .305 | 1.285 | .430 | 1.464 |
| .06 | 1.023 | .185 | 1.137 | .310 | 1.292 | .435 | 1.471 |
| .065 | 1.026 | .190 | 1.142 | .315 | 1.298 | .440 | 1.479 |
| .07 | 1.029 | .195 | 1 147 | .320 | 1.305 | .445 | 1.486 |
| .075 | 1.032 | .200 | 1.153 | .325 | 1.312 | .450 | 1.494 |
| .08 | 1.036 | .205 | 1.159 | .330 | 1.319 | .455 | 1.501 |
| .085 | 1.039 | .210 | 1.165 | .335 | 1.325 | .460 | 1.509 |
| .09 | 1.043 | .215 | 1.171 | .340 | 1.332 | .465 | 1.517 |
| .095 | 1.046 | .220 | 1.177 | .345 | 1.339 | .470 | 1.524 |
| .100 | 1.051 | .225 | 1.183 | .350 | 1.346 | .475 | 1.532 |
| .105 | 1.055 | .230 | 1.189 | .355 | 1.353 | .480 | 1.540 |
| .110 | 1.059 | .235 | 1.196 | .360 | 1.361 | .485 | 1.547 |
| .115 | 1.064 | .240 | 1.202 | .365 | 1.368 | .490 | 1.555 |
| .120 | 1.069 | .245 | 1.207 | .370 | 1.375 | .495 | 1.563 |
| .125 | 1.074 | .250 | 1.213 | .375 | 1.382 | .500 | 1.571 |

**El área de una elipse**=producto de los diámetros×.7854. Ej. : D=10; d=6.

Entonces 10×6×.7854=47.124. El área de una elipse es media proporcional entre las áreas de dos círculos descriptos sobre sus dos diámetros, por tanto puede encontrarse multiplicando las áreas de esos dos círculos y tomando la raíz cuadrada del producto. El área de una elipse es, por tanto, siempre mayor que la de la sección circular del cilindro de que se deriva la elipse.

**El diámetro de un círculo de la misma área que una elipse dada,**

$$\text{es} = \sqrt{\text{eje mayor} \times \text{eje menor}}.$$

**Para encontrar el área de un segmento elíptico cuya base es paralela á uno de los dos ejes.** Divídase la altura del segmento por el diámetro de que forma parte dicha altura ó flecha. De la tabla de segmentos circulares tómese el área opuesta al cociente y multipliquense entre sí, esta área, el eje mayor y el menor.

**Para trazar una elipse** teniendo sus ejes mayor y menor *ab*, *cd*, fig. 4.

Regla 1.ª De cualquiera de los dos extremos del diámetro corto, como *c*, trácense las distancias *cf*, *cf'*, cada una igual *ae*, es decir, á la mitad del eje mayor. Los puntos *ff* son los focos de la elipse. Prepárese una cuerda *fnf* ó bien *f'gf* con un lazo en cada extremo haciendo la longitud total de la cuerda, de extremo á extremo del lazo, igual al eje mayor. Colóquense alfileres en *f* y *f'* y colocando los lazos sobre

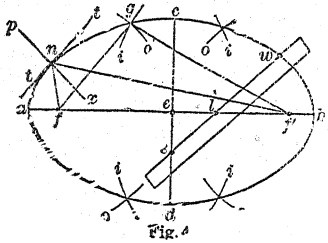


Fig. 4

ellos trácese la curva con un lápiz, el cual en cualquier posición *n* ó *g* mantenga la cuerda *fnf*, *f'gf* igualmente tensa.

Nota : Debido á la dificultad de mantener la cuerda igualmente estirada este método no es tan exacto como el siguiente :

Regla 2.ª En la orilla de una tira de papel *ws* márquese *wl* igual á la mitad del eje menor y *ws* igual á la mitad del eje mayor. Entonces en cualquier posición que se coloque esta tira, manteniendo el punto *l* sobre el eje mayor y el punto *s* sobre el eje menor, *w* marcará un punto en la periferia de la elipse y podremos obtener de este modo tantos puntos cuantos deseemos y trazar luego á mano la curva por sobre ellos.

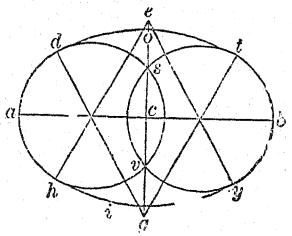
Regla 3.ª Desde los dos focos *f*, *f'*, fig. 4, con un radio igual á una parte cualquiera del diámetro mayor, descríbanse 4 arcos cortos *o*, *o*, *o*, *o*, y con un radio igual á la parte restante del diámetro mayor, descríbanse 4 arcos *i*, *i*, *i*, *i*. Las intersecciones de estos cuatro pares de arcos darán cuatro puntos de la periferia. De esta manera se puede encontrar cualquier número de puntos y trazar la curva **á mano.**

**Para trazar una tangente *tt* en cualquier punto *n* de una elipse** Trácese *nf*, *nf'*, á los focos; divídase el ángulo *fnf'* por medio de la línea *xp*, en partes iguales y trácese *tnt* perpendicular á *xp*.

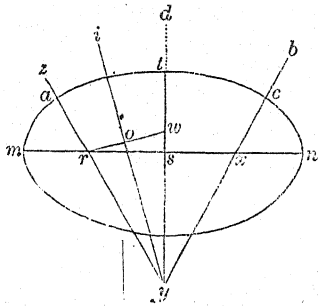
**Para trazar una normal *np* á un arco elíptico en cualquier punto *n* del arco.** Procédase como en la regla anterior para una tangente omitiendo sólo *á tt; np* será la normal.

**Para trazar un óvalo ó falsa elipse** cuando sólo se ha dado el diámetro mayor *ab*, úsese el siguiente método que dará curvas en las cuales la abertura *ab* no excederá en más de tres veces la flecha *co*. Sobre *ab* descríbanse dos círculos de cualquier radio que se intercepten; por sus intersecciones *sr* trácese *eg*, hágase *sg*, *ve*, cada una igual al diámetro de uno de los círculos. A través de los centros de los círculos trácense *ey*, *eh*, *gd*, *gt;* desde *e* descríbanse *hiy*, y desde *g* descríbase *dot*.

**Cuando se dan la abertura** *mn* **y la elevación** *st*. Háganse 2 distancias cua-



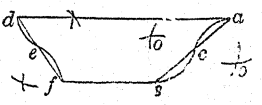

lesquiera *tw*, *mr*, iguales entre sí, pero ambas menores que *ts*. Trácese *rw* y por su centro *o* trácese la perpendicular *ioy*. Trácese *yrz*. Hágase *nx* igual *mr* y trácese *yxb*. Desde *x*, *r*, descríbanse *ne*, *ma*, y desde *y* descríbase *atc*. Haciendo *sd* igual *sy* obtenemos el centro para el otro lado del óvalo.




La belleza de la curva dependerá de la parte de *ts* que se toma para *mr* = *tw*. Cuando un óvalo es muy chato se necesitan más de tres centros para trazar una curva elegante; pero el encontrar estos centros da más trabajo que trazar una elipse.

**Sobre la línea dada** *as* **trazar el cimacio recto** *acs*. Búsquese el centro *c* de *as*.

Desde *a*, *c* y *s*, con la mitad de *as*, como radio, trácense los cuatro arcos *o*, *o*.




Las intersecciones *o*, *o*, son los centros para trazar el cimacio con dicho radio. Invirtiendo la posición de los arcos, obtendremos el *cimacio* inverso *def*.

## LA PARÁBOLA

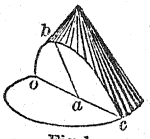


Fig. 1.

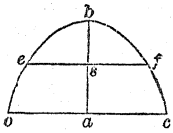


Fig. 2.

**La parábola común ó cónica** *obc*, es una curva formada por la sección de un cono en la dirección *ba* paralela á uno de sus lados. La misma línea curva *obe* se llama *perímetro* de la parábola; la línea *oc* es la *base*; *ba*, su altura ó eje; *b*, el vértice; una línea cualquiera *oa* ó *es*, fig. 2, tirada de la curva al eje perpendicular á éste es una *ordenada;* y las partes *sb*, *ab* del eje, entre la ordenada y el vértice *b*, son *abscisas*. El *foco* de una parábola es el punto del eje, en donde la abscisa *bs* es igual á la mitad de la ordenada *es*. La distancia del vértice al foco se llama *distancia focal*. La distancia al vértice se encuentra así: elévese al cuadrado cualquiera ordenada *oa;* divídase este cuadrado por la abscisa *ba* de esta ordenada y divídase el cociente por 4. La naturaleza de la parábola es tal que sus abscisas, *bs*, *ba*, etc., están entre sí como los cuadrados de sus respectivas ordenadas *es*, *oa*, etc.; esto es, $bs : ba :: es^2 : oa^2$; ó $bs : es^2 :: ba : oa^2$. Si se divide el cuadrado de cualquiera ordenada por su abscisa, el cociente será una cantidad constante, esto es, será igual al cuadrado de cualquiera otra ordenada dividida por su abscisa. Este cociente ó cantidad constante se llama *parámetro* de la parábola. Por tanto el parámetro se encuentra elevando al cuadrado á *es* ú *oa* (mitad de la base) y dividiendo dicho cuadrado por la altura *bs* ó *ba*, según el caso. Si se divide el cuadrado de cualquiera ordenada por el parámetro, el cociente será la abscisa de esa ordenada.

**Buscar el desarrollo de una curva parabólica.** La regla aproximada que dan varias carteras, es como sigue:

$$\text{Longitud} = 2 \times \sqrt{(\tfrac{1}{2}\ \text{base})^2 + 1\ \tfrac{1}{3}\ (\text{altura})^2}.$$

Cuando la altura no excede de $^1/_{10}$ de la base, esta fórmula puede considerarse en la práctica como exacta para usos ordinarios. Para altura $= {}^1/_3$ base da un error por exceso de ½ %; para altura = ½ base, como de $3\ {}^1/_3$ %; para altura = base como $8\ {}^1/_3$ %; $h = 2$ bases, como $12\ {}^1/_3$ %; $h = 10 \times$ base, ó más, como 15 ½ %.

**La regla que sigue dada por el autor es correcta**, para cualquier caso, con una aproximación de 1 en 800, y por tanto satisface á muchas aplicaciones.

Sea *adb*, fig. 3, ó *nad*, fig. 4, la parábola, en la cual se da la base *ab* ó *nd*, y la altura *cd* ó *ca*. Imaginemos trazada la figura completa *adbs* ó *nadb*, y en uno ú

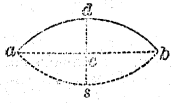


Fig. 3.

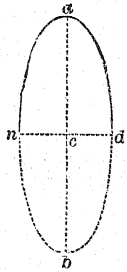


Fig. 4.

otro caso, supongamos su diámetro *mayor ab* la cuerda ó base, y la mitad del diámetro *cd*, la flecha de un arco de círculo. Búsquese la longitud de este arco de

círculo por medio de la regla y tabla dadas con este objeto. Luego divídase la cuerda ó base *ab* ó *nd* de la parábola por su altura *cd* ó *ca*. Búsquese el cociente en la columna base de la tabla que sigue y tómese de ella el correspondiente multiplicador. Multiplíquese por éste la longitud del arco de círculo, el producto será la longitud del arco *adb* ó *nad*, según el caso.

Para bases de parábolas menores de .05 de la altura ó diez veces mayores que aquélla, el multiplicador es muy aproximadamente 1, ó en otras palabras, la parábola será casi exactamente de la longitud del arco de círculo.

**Encontrar el área de una parábola** *manb*. Multiplíquese su base *mn*, fig. 5, por su altura *ab*, y tómense $^2/_3$ del producto.

El área de cualquier segmento, *ubv*, cuya base *uv* es paralela á *mn*, se encuentra de la misma manera, tomando *uv* y *sb* en lugar de *mn* y *ab*.

**Encontrar el área de una zona parabólica** *mnuv*. Regla 1.ª Búsquese

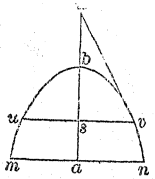


Fig. 5.

primero por la regla anterior el área de la parábola entera *mbn*, luego la del segmento *ubv;* y réstese la última de la primera.

Regla 2. Del cubo de *mn* réstese el cubo de *uv;* llámese *c* la diferencia. Del cuadrado de *mn*, réstese el cuadrado de *uv*, llámese *s* la diferencia; divídase *c* por *s*. Multiplíquese el cociente por $^2/_3$ de la altura *as*.

**Tabla de la longitud de las curvas parabólicas.** (Original.)

| Base. | Mult. | Base. | Mult. | Base. | Mult. | Base. | Mult. |
|---|---|---|---|---|---|---|---|
| .05 | 1.000 | 1.10 | .999 | 2.15 | .949 | 3.20 | .983 |
| .10 | 1.001 | 1.15 | .997 | 2.20 | .951 | 3.30 | .984 |
| .15 | 1.002 | 1.20 | .995 | 2.25 | .954 | 3.40 | .985 |
| .20 | 1.004 | 1.25 | .993 | 2.30 | .956 | 3.50 | .986 |
| .25 | 1.006 | 1.30 | .990 | 2.35 | .958 | 3.60 | .987 |
| .30 | 1.007 | 1.35 | .987 | 2.40 | .960 | 3.70 | .988 |
| .35 | 1.007 | 1.40 | .984 | 2.45 | .962 | 3.80 | .989 |
| .40 | 1.008 | 1.45 | .980 | 2.50 | .963 | 3.90 | .990 |
| .45 | 1.009 | 1.50 | .977 | 2.55 | .965 | 4.00 | .991 |
| .50 | 1.010 | 1.55 | .974 | 2.60 | .967 | 4.25 | .992 |
| .55 | 1.010 | 1.60 | .970 | 2.65 | .969 | 4.50 | .993 |
| .60 | 1.010 | 1.65 | .966 | 2.70 | .970 | 4.75 | .994 |
| .65 | 1.011 | 1.70 | .963 | 2.75 | .972 | 5.00 | .995 |
| .70 | 1.011 | 1.75 | .960 | 2.80 | .973 | 5.25 | .996 |
| .75 | 1.010 | 1.80 | .957 | 2.85 | .975 | 5.50 | .997 |
| .80 | 1.009 | 1.85 | .953 | 2.90 | .976 | 5.75 | .998 |
| .85 | 1.008 | 1.90 | .950 | 2.95 | .978 | 6.00 | .998 |
| .90 | 1.006 | 1.95 | .946 | 3.00 | .979 | 7.00 | .999 |
| .95 | 1.004 | 2.00 | .942 | 3.05 | .980 | 8.00 | 1.000 |
| 1.00 | 1.002 | 2.05 | .944 | 3.10 | .981 | 10.00 | 1.000 |
| 1.05 | 1.001 | 2.10 | .946 | 3.15 | .982 | | |

**Trazar una parábola** *cos* de base *cs* y altura *eo*, fig. 6. Hágase *ot* igual á la altura *eo*. Trácense *tc* y *st*, divídanse cada una de ellas en un número cualquiera de partes iguales, y numérense como en la figura. Unanse 1.1; 2.2; 3.3; etc.; luego trácese la curva á mano. Se observará que las intersecciones de las líneas 1.1; 2.2; etc., no dan punto de curva; pero una parte de cada una de aquellas líneas forma una tangente á la curva. Aumentando el número de divisiones de *ct* y *st*, se

forma una curva casi perfecta, que apenas se necesita corregirla á la mano. En la práctica es mejor dibujar primero la parte central de las dos líneas que se cortan en *o*, y de aquí seguir hacia abajo, trazando efectivamente sólo aquella parte de cada línea inferior sucesiva que sea necesaria para indicar la curva.

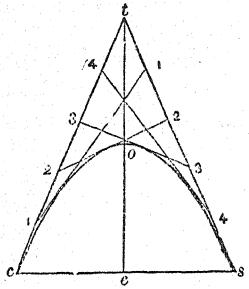



Fig. 6.

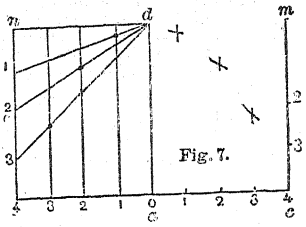



Fig. 7.

**También puede trazarse la parábola como sigue** : Sea *bc*, fig. 7, la base, y *ad* la altura.

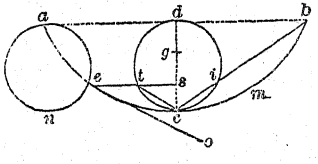
Trácese el rectángulo *bnmc;* divídase cada mitad de la base en cualquier número de partes iguales y numérense del centro hacia ambos lados. Divídase *nb*, mc en el mismo número de partes iguales; y numérense de arriba hacia abajo. Desde los puntos de división de la línea *bc* trácense líneas verticales y desde los puntos de los lados tírense líneas al punto *d*. Las intersecciones de las líneas 1.1, 2.2, etc., serán puntos de la parábola. Como en el caso anterior, no es necesario trazar las líneas enteras, sino simplemente porciones de ellas, como se indica entre *d* y *c*.

También puede trazarse una parábola dividiendo primero la altura *ab*, fig. 5, en cualquier número de partes, iguales ó desiguales, y luego calcular las ordenadas *us*, etc.; así : altura *ab* : cuadrado de la semibase *am* : : cualquiera abscisa *bs* : cuadrado de su ordenada *us*.

*Observación.* Cuando la altura de una parábola no es mayor de $^1/_{10}$ de su base, la curva coincide tan próximamente con un arco de círculo, que en la preparación de los dibujos de puentes de suspensión puede emplearse el arco de círculo si no se necesita gran precisión. También puede usarse el círculo aun cuando la altura llegue á un octavo de la base.

**Trazar una tangente** *wv*, **fig. 5, á una parábola desde un punto cualquiera** *v*. Trácese *vs* perpendicular al eje *ab*, prolónguese *ab* hasta que *bw* sea igual *sb*. Unase *wv*.

**La cicloide** *acb* es la curva descripta por un punto *a* de la circunferencia de




un círculo *an* durante una revolución completa de éste á lo largo de una línea recta *ab*, que se llama base de la cicloide.

**El vértice** de la cicloide está en *c*.

**La base** *ab* = circunferencia del círculo generador *an*,
= diámetro *cd* del círculo generador $\times \pi = 3.1416 \times cd$.

**Eje ó altura** $cd = an$.

**Longitud** $acb = 4cd$.

**Área** *acbd*=3 veces el área del círculo generador

$$= \frac{3\ cd^2\pi}{4} = cd^2 \times {}^3/_4\,\pi = cd^2 \times 2.3562.$$

**Centro de gravedad de la superficie en** *g*; $cg = {}^7/_{12}\,cd$. Centro de gravedad de la curva (línea *acb*) está en el eje *cd*, en un punto *s* distante $^4/_3$ *cd* del punto *c*.

**Trazar una tangente** *eo*, desde un punto cualquiera *e* de una cicloide. Trácese *es* en ángulo recto al eje *cd*; sobre *cd* describase el círculo generador *dct*; únase *tc*; desde *e* trácese *eo* paralela á *tc*. **La curva de más rápido descenso** es la cicloide; de manera que un cuerpo podría caer de *b* á *c* por la curva *bmc* en menor tiempo que por el plano inclinado *bic*, ó por otra línea cualquiera.

## SÓLIDOS. CUERPOS REGULARES

**Poliedro regular** es el que tiene todos sus lados y ángulos sólidos iguales. Sólo hay cinco, á saber :

| Nombre | Limitados por | Superficie (= suma de las superficies de todas las caras). Multip. por el cuadrado de la longitud de una arista por | Volumen = cubo de la longitud de una arista ó lado por |
|---|---|---|---|
| Tetraedro..... | 4 triángulos equiláteros. | 1.7320 | .1178 |
| Exaedro ó cubo. | 6 cuadrados. | 6. | 1. |
| Octaedro..... | 8 triángulos equiláteros. | 3.4641 | .4714 |
| Dodecaedro..... | 12 pentágonos equiláteros. | 20.6458 | 7.6631 |
| Icosaedro..... | 20 triángulos equiláteros. | 8.6602 | 2.1817 |

**Teorema de Guldinus.** Para encontrar el **volumen de cualquier cuerpo** (como la masa irregular *abcm*, fig. A, ó el anillo *abcm*, fig. B), engendrado

Fig. A.

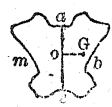
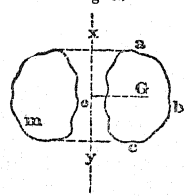



Fig. B.




por una revolución completa ó parcial de una figura cualquiera (*abca*) alrededor de uno de sus lados (*ac*, fig. A), ó alrededor de un eje cualquiera *xy*, fig. B.

**Volumen** = superficie *abca* × longitud del arco descripto por su centro de gravedad G.

Si la revolución es completa, el arco descripto es=circunferencia=radio 0 G *× $2\pi$=radio 0 G *×6.283186; y *Volumen*=superficie *abca*×radio 0 G *×6.283186.

Si la revolución es incompleta,

$$\text{Revolución completa} : \text{Revolución incompleta} = \text{Circunferencia hallada arriba} : \text{Arco descripto}$$

(*) Medido perpendicularmente al eje de revolución.

## PARALELEPÍPEDOS

Fig. 1.

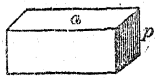


Fig. 2.

Fig. 3

Fig. 4.

Paralelepípedo es un sólido limitado por seis paralelogramos y de caras opuestas paralelas. Sólo mencionamos cuatro, á saber : el *cubo*, fig. 1, cuyas caras son cuadrados iguales y sus ángulos rectos; el *paralelepípedo rectángulo*, fig. 2, que tiene todos sus ángulos rectos y sólo las caras opuestas iguales; el *romboedro*, fig. 3, cuyas caras son todas rombos iguales; y el *prisma romboédrico*, fig. 4, cuyas caras son rombos, pero sólo iguales entre sí las opuestas. Todos los paralelepípedos son prismas.

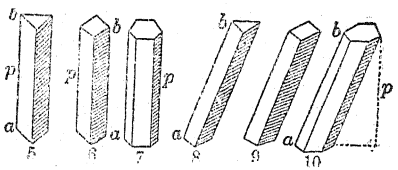
(*N. del T.* — Nos han parecido más sencillas estas definiciones.)

**Volumen de un paralelepípedo**=área de cualquier cara $a$ × distancia $p$ á la cara opuesta.

**Volumen de un cubo**=cubo de la longitud de un lado.
=1.90985 × volumen de la esfera inscrita.
=1.27324 × — de cilindro. inscrito
=3.81972 × — de cono. inscrito

**Diagonal de un cubo**=diámetro de la esfera circunscrita.
=1.7320508 × longitud de un lado del cubo.

## PRISMAS




Un prisma es un sólido de *bases* paralelas é iguales, cuyas caras laterales son paralelogramos, figs. 5 á 10.

Por tanto, los paralelepípedos precedentes son prismas. Prisma *recto* es aquel que tiene las caras perpendiculares á sus bases, figs. 5, 6 y 7, de no ser así, es *oblicuo*, figs. 8, 9 y 10. Si los polígonos de las bases son polígonos regulares, se dice que el prisma es *regular*, de cualquier otro modo es *irregular*.

**Volumen de un prisma cualquiera** (regular ó irregular, recto ú oblicuo) =área de una de sus bases × su altura $p$=área de la sección transversal perpendicular á las caras × longitud $ab$, figs. 5 á 10=3 × volumen de la pirámide de base y altura iguales á las del prisma.

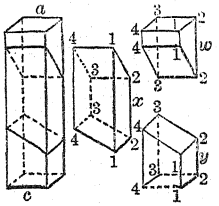
**Encontrar el volumen de un tronco de prisma**, cuya sección transversal perpendicular á sus caras puede ser triángulo, ó paralelogramo, ó un cuadrado (como en la fig. 10[1]/.), ó un polígono regular de cualquier número de lados, y cualquiera que sea la inclinación de una de las bases del tronco con respecto á la otra, sea una de ellas ó ninguna paralela á la base del prisma original.

$$\text{Volumen del tronco} = \frac{\text{Suma de las longitudes de las aristas paralelas}}{\text{número de dichas aristas (4 en la fig. } 10^1/_4)} \times \text{área de la sección transversa } \textit{perpendicular} \text{ á dichas aristas}$$

Esta regla puede aplicarse para calcular de antemano la cantidad de tierra que se ha de extraer de una excavación. Primero se divide con estacas, en cuadrados,

la superficie irregular del terreno. (Al medir los lados debe colocarse la cinta perfectamente *horizontal.*)

Los cuadros deben ser de tal dimensión que sin diferencias apreciables puedan considerarse como superficies planas horizontales ó inclinadas. Determinada la profundidad del fondo horizontal de la fosa ó excavación, y encontrados los




Figs. $10^1/_4$.

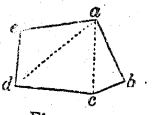



Fig. $10^1/_2$.

vértices de los cuadrados, determinamos la longitud de las cuatro aristas verticales paralelas de cada uno de los prismas truncados que resultan en la excavación. En la fig. $10^1/_4$ podemos representar por *y* uno de estos prismas truncados.

Si el prisma truncado es un tetraedro *irregular,* ó prisma poligonal, divídase primero en triángulos la sección transversal perpendicular á sus caras por medio de líneas tiradas desde uno cualquiera de sus ángulos, *a*, fig. $10^1/_2$. Calcúlese el área de cada uno de estos triángulos separadamente, luego considérese el prisma truncado entero formado de otros tantos prismas triangulares; calcúlese el volumen de cada uno de ellos por la regla precedente y súmese para obtener el volumen de todo el prisma truncado.

**Volumen de cualquier cilindro truncado.** Consideremos una cualquiera de sus bases y busquemos el área. Búsquese también el centro de gravedad *c* de la otra base y la distancia vertical *nc* de la base á dicho centro de gravedad.

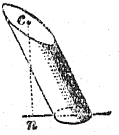



Fig. $10^3/_4$.

Tenemos **volumen del tronco** = área de la base × *nc* (fig. $10^3/_4$). La base inclinada *c* es una elipse. Su área es mayor que la de la base circular.

**Superficie de un prisma cualquiera,** figs. 5 á 10, recto ú oblicuo, regular ó irregular

$$= \left(\begin{array}{l}\text{perímetro de la sección}\\ \text{perpendicular á las caras}\end{array} \times \text{longitud } ab\right) + \begin{array}{l}\text{suma de las áreas}\\ \text{de las dos bases.}\end{array}$$

## CILINDROS

Cilindro es un sólido de bases paralelas, semejantes y limitadas por líneas de igual curvatura, cuyas secciones, pararelas á las bases, son iguales á éstas. De aquí que hayan cilindros circulares, elípticos y otros muchos; pero cuando no se

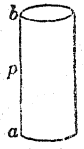


Fig. 11

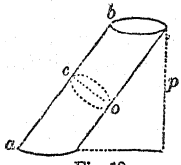


Fig. 12.

expresa á qué clase de cilindro nos referimos, se subentiende que se trata de cilindros circulares.

Cilindro *recto* es el que tiene las bases perpendiculares á sus lados, como en la fig. 11; de cualquiera otro modo es *oblicuo*, como en la fig. 12.

Si se cortan los extremos de un cilindro recto circular de manera que se transformen en oblicuas sus bases, se convierte aquél en cilindro elíptico; porque entonces ambas bases y todas las secciones á ellas paralelas son elipses.

Rara vez se presenta un cilindro circular oblicuo; puede concebírsele imaginando que las dos bases de la fig. 12 son círculos unidos por líneas rectas que forman la superficie lateral del cilindro.

Un cilindro es un prisma de infinito número de caras.

**Volumen de cualquier cilindro** (Circular, elíptico, etc., recto ú oblicuo).

=área de una de las bases × distancia *p* á la otra base

=área de la sección transversal medida perpendicularmente á las caras × longitud *ab*, figs. 11 y 12

=3 × volumen de un cono cuya base y altura sean iguales á las del cilindro.

**Superficie de un cilindro cualquiera** (Circular, elíptico, etc., recto ú oblicuo).

$$= \begin{pmatrix}\text{Circunferencia medida perpendicu-}\\ \text{larmente á los lados, como } co \text{ fig. 12}\end{pmatrix} \times \text{longitud } ab \Big) + \begin{matrix}\text{suma de las áreas}\\ \text{de las dos bases.}\end{matrix}$$

### Cilindro circular recto de altura=al diámetro.

| | |
|---|---|
| Volumen | =1 ½ volumen de la esfera inscrita. |
| Superficie curva | =superficie de la esfera inscrita. |
| Area de una de las bases | =$^1/_4$ de la superficie de la esfera inscrita=$^1/_4$ de la superficie curva. |
| Superficie entera | =1½ × superficie de la esfera inscrita=1½ × superficie curva. |

## UÑAS CIRCULARES CILÍNDRICAS

**I. Cuando el plano secante no corta la base.** Figs. 13 y 14.

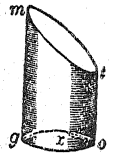



FIG. 13.

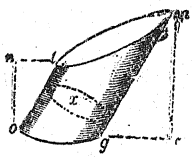



FIG. 14.

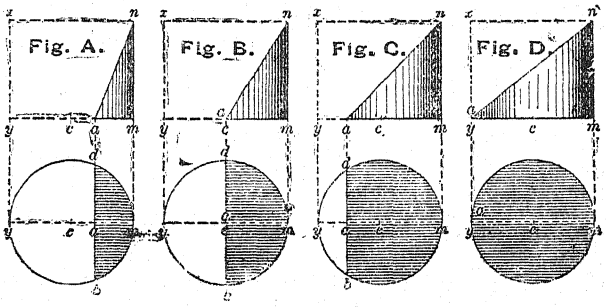
**Volumen de la uña** = área de la base $og \times \frac{1}{2}$ suma de las alturas *on*, *em*.
= área de la sección media *x* perpend á los lados $\times \frac{1}{2}$ suma de los lados *gm*, *ot*.

**Área de la superf curva** = circunf medida perp á los lados como en *x* $\times \frac{1}{2}$ suma de los lados *gm*, *ot*.

**Agréguense las supfs de los extremos** si se desea.

**Para las áreas de las secciones** perpendiculares á los lados, véase *Círculos*, y para las oblicuas, *La Elipse*.

**II. Cuando el plano secante toca la base.** Figs. A á D.




**Volumen** (recto ú oblicuo)

Fig. **A** $= (\frac{2}{3}\ ab^3 - ac \times$ área *admb* de la base$)\ \frac{mn}{am}$.

Fig. **B** $= \frac{2}{3}\ cb^2 \times mn$.

Fig. **C** $= (\frac{2}{3}\ ab^3 + ac \times$ área *admb* de la base$)\ \frac{mn}{am}$.

($\frac{mn}{am}$ = *mn* medido perpend á la base)

Fig. **D** $= \frac{1}{2}$ área del círculo $ym \times mn$.
$= \frac{1}{2}$ volumen del cilindro *xymn*.

**Superficie curva.** (uña *recta* solamente)

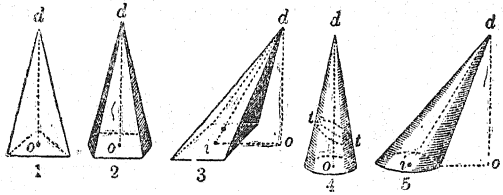
Fig. **A** $= (ab \times my - ac \times$ longitud arco *dmb*$)\ \frac{mn}{am}$.

Fig. **B** $= my \times mn$.

Fig. **C** $= (ab \times my + ac \times$ longitud arco *dmb*$)\ \frac{mn}{am}$.

Fig. **D** $= \frac{1}{2}$ circunf. de la base $my \times mn$.
$= \frac{1}{2}$ superf curva del cilindro *xymn*.

## PIRÁMIDES Y CONOS




**Una pirámide**, figs. 1, 2, 3, es un sólido que tiene por base una figura plana de cualquier número de lados, y por caras, triángulos que terminan todos en un punto, *d*, llamado *vértice*. Cuando la base es un polígono *regular*, la pirámide es regular; si no, es *irregular*.

**Un cono**, figs. 4 y 5, es un sólido cuya base es un plano de perímetro curvilíneo, y que se puede considerar engendrado por una línea recta, que apoya uno de sus extremos en un punto fijo *d*, que se llama el vértice, mientras el otro se mueve sobre el perímetro de la base. También se puede considerar el cono como una pirámide de infinito número de lados.

El eje de una pirámide ó cono, es la línea recta, *do*, en las figs. 1, 2, 4; y, *di* en las figs. 3 y 5, que une el vértice, *d*, con el centro de gravedad de la base Cuando el eje es perpendicular á la base, como en las figs. 1, 2, 4, se dice que la pirámide ó el cono, son *rectos;* cuando no, son *oblicuos*. Cuando se dice sólo *el cono* se entiende el *recto*. Si se corta un cono recto, con un plano *tt*, fig. 4, inclinado á la base, la nueva base, *tt*, es una *elipse* y el cono nuevo, *dtt*, es un cono *elíptico*.

**Volumen de la pirámide ó del cono**, regular ó irregular, recto ú oblicuo.

**Volumen** $= {}^1/_3$ del área de la base × la altura perpendicular *h*, ó bien *do*, figs. 1 á 5.

$= {}^1/_3$ del volumen del prisma ó cilindro que tienen la misma área en la base y la misma altura.

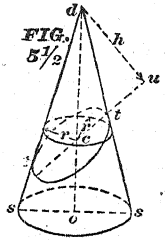
$= {}^1/_2$ del volumen del hemisferio de la misma base y altura.

O bien los volúmenes de un cono, un hemisferio y un cilindro, de la misma base y altura, están como 1, 2, 3.

**Área de las caras**, de la pirámide, recta, regular y del cono circular recto.

**Área** = semicircunf de la base × el lado*
En el cono esto equivale á
**Área** $= \dfrac{\text{área de la base}}{\text{radio de la base}} \times$ lado.

} Si se requiere agréguese el área de la base.




**Área de la superf del cono elíptico oblicuo**, *dtt*, fig. 5½; cortado de un cono circular recto *dss*. Del punto *c* donde el eje *do* del cono circular recto

* En la pirámide el lado se mide por la perpendicular bajada desde el vértice á uno de los lados del polígono de la base.

corta la base elíptica, *tt*, mídase una perpendicular, *r*, en cualquiera dirección, á la superf curva del cono. Sea, *v*=el volumen del cono oblicuo, *dtt;* sea *a*=área de su base elíptica, *tt*, y *h* la altura, *du*, medida perpendicularmente á dicha base. Entonces

$$\textbf{La superf curva} = \frac{ah}{r} = \frac{3v}{r}.$$

Si se requiere, agréguese el área de la base.

No se ha encontrado ninguna medida para la superficie del **cono circular oblicuo.**

**El área de la superficie de una pirámide irregular** recta ú oblicua = la suma de las áreas de las caras, medida cada una como un triángulo. Si se requiere, agréguese el área de la base.

## TRONCOS DE PIRÁMIDES Y CONOS

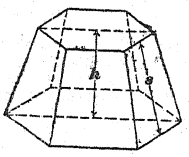



Fig. 6.

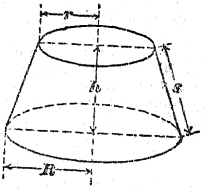



Fig. 7.

*a*=área de la base superior; *c*=circunf base sup.
A= — de la base; *C*= — inf.
M= — de la sección media paralela á las bases;
*h*=altura del tronco; *s*=lado del tronco *.

**En la fig. 7.**

*r*=radio de la base superior; R=radio de la base inf.

### Volúmenes.

**Tronco de pirámide** (fig. 6) **ó de cono** (fig. 7), regular ó irregular, recto ú oblicuo, de bases paralelas.

$$\textbf{Volumen} = \frac{h}{3}(a+\mathrm{A}+\sqrt{a\mathrm{A}}) = \frac{h}{6}(a+\mathrm{A}+4\mathrm{M}).$$

**Tronco de cono circular recto ú oblicuo.** Fig. 7.

$$\textbf{Volumen} = \frac{h}{3}\pi\,(r^2+\mathrm{R}^2+r\mathrm{R});\quad \pi = 3.1416.$$

**Área de las caras.** (Si se requiere, agréguense las bases.)

**Tronco de pirámide ó cono, recto, regular,**
de bases paralelas. Figs. 6 y 7.

$$\textbf{Área} = \frac{s}{2}(c+\mathrm{C})\ *.$$

**Para el tronco de cono recto** circular :

$$\textbf{Área} = \pi s\,(r+\mathrm{R});*\quad \pi = 3.1416.$$

**Tronco de pirámide irregular ú oblicua :**

**Área** = suma de las superficies de las caras.

Cada lado debe tratarse como un cuadrilátero. **Agréguense las áreas de las bases, si se desea.**

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* En el tronco de pirámide, fig. 6, *s*, debe medirse á lo largo de la mitad de una de las caras, no sobre las aristas.

## PRISMOIDES

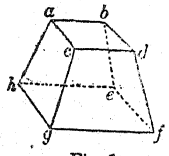



Fig. 1.

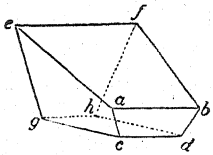



Fig. 2.

**Un prismoide** se define á veces como un sólido que tiene por bases dos figuras planas, unidas por superficies, en las cuales puede trazarse una línea recta entre sus lados paralelos. Estas figuras planas pueden ser paralelogramos ó no, y también pueden ó no ser paralelas entre sí.

Esta definición puede comprender al cubo ó cualquier otro paralelepípedo; el prisma; el cilindro (considerado como un prisma de infinito número de caras); la pirámide y el cono (en que una de las bases, la que forma el vértice, se considera infinitamente pequeña) y sus troncos de bases paralelas, y la cuña.

Pero el término prismoide, se restringe frecuentemente, al sólido de seis caras, cuyas bases paralelas son cuadriláteros desiguales y las caras laterales, trapezoides, como en las figs. 1 y 2, y, por algunos escritores, al caso en que las bases son *rectángulos*.

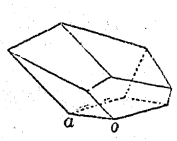
La **fórmula prismoidal** que vamos á dar se aplica á todos los sólidos que siguen y á otros como lo indicamos después.

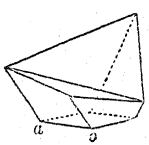
Sean A y *a*, las áreas de las bases paralelas;
M, una sección media paralela á ellas;
L, la distancia perpendicular entre las bases.

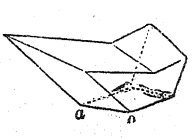
Entonces : $$\textbf{Volumen} = L \times \frac{A+a+4M}{6}$$

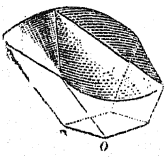
$$= L \times \text{área de la sección media.}$$

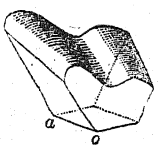
Las seis figuras que siguen representan algunos de los sólidos, comprendidos en la amplia definición que hemos dado del **prismoide** y á los que se aplica la fórmula. Deben considerarse como partes de una cortada en los trabajos de ferrocarriles, siendo *ao* la distancia horizontal entre las dos caras verticales y paralelas.

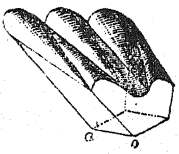


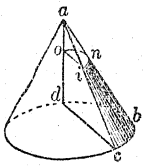



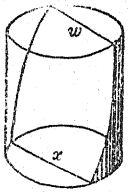




**La fórmula prismoidal se aplica también** á la esfera, hemisferio y otros segmentos esféricos; también á cualquier sección, como *abcd; onidbc*, del cono, en que los lados *ad*, *ac*, ó bien, *od*, *ic*, son *rectos;* como lo son solamente cuando el plano secante, *adc*, pasa por el vértice *a*. También al cilindro, cuando

el plano, *paralelo á los lados*, corta las bases, pero no, si como él, *wx*, es *oblicuo* (véase la figura). **En este último caso** se prolonga el plano hasta que corte los lados del cilindro y se mide el volumen de la *uña* así formada. Si se quiere




medir el tronco de uña, *wx*, se mide el vol de la pequeña uña que está sobre *w* y se resta de la grande.

Esta fórmula prismoidal, tan extensa y aplicable, fué encontrada por primera vez por Ellwood Morris, Ing. C. de Filadelfia, 1840.

## CUÑAS

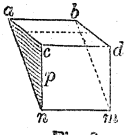



Fig. 8.

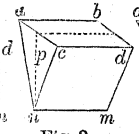



Fig. 9.

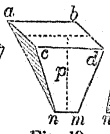



Fig. 10.

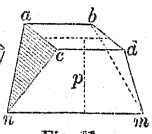



Fig. 11.

Se las define generalmente como un sólido engendrado, figs. 8 y 9, por un triángulo plano, *anc*, que se mueve, paralelamente á sí mismo, en línea recta. Esta definición requiere que los dos extremos triangulares de la cuña sean paralelos; pero una cuña puede tener la forma de las figs. 10 ú 11. Proponemos la siguiente definición que comprende todos los casos. Un sólido de cinco caras planas; una de las cuales es un paralelogramo, *abcd;* dos de cuyos lados, como *ac, bd*, están unidos por dos caras triangulares *acn, bdm*, á una arista ó línea *nm*, paralela á los otros lados opuestos *ab, cd*. El paralelogramo, *abcd*, puede ser rectángulo ó no, y lo mismo las otras dos caras. La regla siguiente se aplica á todas las cuñas :

$$\textbf{Volumen} = \frac{1}{6} \times \begin{matrix}\text{suma longitud de}\\ \text{las 3 aristas}\\ ab + cd + nm\end{matrix} \times \begin{matrix}\text{altura perp de la}\\ \text{arista sobre la base}\end{matrix} \times \begin{matrix}\text{ancho de la base}\\ (abcd)\\ \text{medido perp á } ab\end{matrix}$$

## ESFERAS

**La esfera** es un sólido engendrado por la revolución de un semicírculo alrededor de su diámetro. Todos los puntos de la superficie de la esfera equidistan de uno interior llamado centro. Cualquier línea que una dos puntos de la superficie pasando por el centro, es un diámetro ó *eje* de la esfera. Cualquier círculo descripto en la superficie de la esfera que tenga por centro el de la esfera, es un círculo máximo. La esfera, á igualdad de superficie exterior, encierra mayor volumen que cualquier otro sólido. La intersección de un plano con la esfera es un círculo.

**Volumen** de la esfera :

$= \frac{4}{3}\ \pi\ \text{radio}^3 \qquad = 4.1888\ \text{radio}^3$

$= \frac{1}{6}\ \pi\ \text{diámetro}^3 \qquad = .5236\ \text{diám}^3$

$= \frac{1}{6}\ \frac{\text{circunferencia}^3}{\pi^2} \qquad = .01689\ \text{circunf}^3$

$= \frac{1}{6}$ diám × área superficie.

$= \frac{2}{3}$ diám × área del círc° máximo

$= \frac{2}{3}$ volumen del cilindro circunscripto

= .5236 volumen del cubo circunscripto.

**Área de la superficie** de la esfera :

$= 4\ \pi\ \text{radio}^2 \qquad = 12.5664\ \text{radio}^2$

$= \pi\ \text{diám}^2 \qquad = 3.1416\ \text{diám}^2$

$= \frac{\text{circunf}^2}{\pi} \qquad = .3183\ \text{circunf}^2$

= diám × circunf

= 4 × área círc° máximo

= superficie curva del cilindro circunscripto

$= \frac{6 \times \text{volumen}}{\text{diám}}$

**Radio** de la esfera :

$= \sqrt[3]{\frac{3}{4}\ \frac{\text{volumen}}{\pi}} \qquad = .62035\ \sqrt[3]{\text{volumen}}$

$= \sqrt{\frac{\text{área superf}}{4\pi}} \qquad = \sqrt{.07958 \times \text{área superf}}$

**Circunferencia** de la esfera :

$= \sqrt[3]{6\ \pi^2\ \text{volumen}} \qquad = \sqrt[3]{59.2176\ \text{volumen}}$

$= \sqrt{\pi\ \text{área superf}} \qquad = \sqrt{3.1416\ \text{área superf}}$

$= \frac{\text{área superf}}{\text{diámetro}}$

# ESFERAS

| Diám. | Superf. | Solidez. | Diám. | Superf. | Solidez. | Diám. | Superf. | Solidez. | Diám. | Superf. | Solidez. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1-64 | .00077 | .00002 | 13-32 | 18.190 | 7.2949 | 5/8 | 170.87 | 210.93 | 1/8 | 921.33 | 2629.6 |
| 1-32 | .00307 | .00002 | 7-16 | 18.866 | 7.5829 | 3/4 | 176.71 | 220.89 | 1/4 | 934.83 | 2687.6 |
| 3-64 | .00690 | .00005 | 15-32 | 19.547 | 7.8763 | 5/8 | 182.66 | 232.13 | 3/8 | 948.43 | 2746.5 |
| 1 16 | .01227 | .00013 | 1/2 | 19.635 | 8.1813 | 7/8 | 188.69 | 243.73 | 1/2 | 962.12 | 2806.2 |
| 5-32 | .02761 | .00042 | 17-32 | 20.129 | 8.4910 | 1/8 | 194.83 | 255.72 | 5/8 | 975.91 | 2866.8 |
| 1/8 | .04909 | .00102 | 9-16 | 20.629 | 8.8103 | 8. | 201.06 | 268.08 | 3/4 | 989.80 | 2928.2 |
| 7-32 | .07670 | .00200 | 19-32 | 21.135 | 9.1366 | 1/4 | 207.39 | 280.85 | 7/8 | 1003.8 | 2990.5 |
| 3-16 | .11045 | .00345 | 5/8 | 21.648 | 9.4708 | 3/8 | 213.82 | 294.01 | 18. | 1017.9 | 3053.6 |
| 7-32 | .15033 | .00548 | 21-32 | 22.165 | 9.8131 | 1/2 | 220.36 | 307.58 | 1/8 | 1032.1 | 3117.7 |
| 1/4 | .19635 | .00818 | 11-16 | 22.691 | 10.164 | 5/8 | 226.99 | 321.56 | 1/4 | 1046.4 | 3182.6 |
| 9-32 | .24851 | .01165 | 23-32 | 23.222 | 10.522 | 3/4 | 233.71 | 335.95 | 3/8 | 1060.8 | 3248.5 |
| 5-16 | .30680 | .01598 | 3/4 | 23.758 | 10.889 | 7/8 | 240.53 | 350.77 | 1/2 | 1075.2 | 3315.3 |
| 11-32 | .37123 | .02127 | 25-32 | 24.302 | 11.257 | 1/8 | 247.45 | 366.02 | 5/8 | 1089.8 | 3382.9 |
| 3/8 | .44179 | .02761 | 13-16 | 24.850 | 11.640 | 9. | 254.47 | 381 70 | 3/4 | 1104.5 | 3451.5 |
| 13-32 | .51848 | .03511 | 27-32 | 25.405 | 12.034 | 1/4 | 261.59 | 397.83 | 7/8 | 1119.3 | 3521.0 |
| 7-16 | .60132 | .04385 | 1/2 | 25.967 | 12.443 | 1/2 | 268.81 | 414.41 | 19. | 1134.1 | 3591.4 |
| 15-32 | .69028 | .05393 | 29-32 | 26.535 | 12.866 | 3/4 | 276.12 | 431.44 | 1/8 | 1149.1 | 3662.8 |
| 1/2 | .78540 | .06545 | 15-16 | 27.109 | 13.273 | 1/8 | 283.53 | 448.92 | 1/4 | 1164.2 | 3735.0 |
| 17-32 | .88664 | .07850 | 31-32 | 27.688 | 13.706 | 1/4 | 291.04 | 466.87 | 3/8 | 1179.3 | 3808.2 |
| 9-16 | .99402 | .09319 | 3. | 28.274 | 14.137 | 1/2 | 298.65 | 485.31 | 1/2 | 1194.6 | 3882.5 |
| 19-32 | 1.1075 | .10966 | 1-16 | 28.865 | 14.589 | 3/4 | 306.36 | 504.21 | 5/8 | 1210.0 | 3957.6 |
| 5/8 | 1.2272 | .12783 | 1/8 | 30.080 | 15.279 | 10. | 314.16 | 523.60 | 3/4 | 1225.4 | 4033.7 |
| 21-32 | 1.3530 | .14793 | 3-16 | 31.919 | 16.957 | 1/4 | 322.06 | 543.48 | 7/8 | 1241.0 | 4110.8 |
| 11-16 | 1.4849 | .17014 | 1/4 | 33.183 | 17.974 | 1/2 | 330.06 | 563.86 | 20. | 1256.7 | 4188.8 |
| 23-33 | 1.6230 | .19442 | 5/16 | 34.472 | 19.031 | 3/4 | 338.16 | 584.74 | 1/8 | 1272.4 | 4267.8 |
| 3/4 | 1.7671 | .22089 | 3/8 | 35.784 | 20.129 | 5/8 | 346.35 | 606.13 | 1/4 | 1288.3 | 4347.8 |
| 25-32 | 1.9175 | .24967 | 7-16 | 37.122 | 21.268 | 1/8 | 354.65 | 628.04 | 3/8 | 1304.2 | 4428.8 |
| 13-16 | 2.0739 | .28084 | 1/2 | 38.484 | 22.449 | 3/4 | 363.05 | 650.46 | 1/2 | 1320.2 | 4510.9 |
| 27-32 | 2.2365 | .31451 | 9-16 | 39.872 | 23.674 | 7/8 | 371.54 | 673.42 | 5/8 | 1336.4 | 4593.9 |
| 1/8 | 2.4053 | .35077 | 5/8 | 41.283 | 24.942 | 11. | 380.13 | 696.91 | 3/4 | 1352.7 | 4677.9 |
| 29-32 | 2.5802 | .38971 | 11-16 | 42.719 | 26.254 | 1/8 | 388.83 | 720.95 | 7/8 | 1369.0 | 4763.0 |
| 15-16 | 2.7611 | .43143 | 3/4 | 44.179 | 27.611 | 1/4 | 397.61 | 745.51 | 21. | 1385.5 | 4849.1 |
| 31-32 | 2.9483 | .47603 | 13-16 | 45.664 | 29.016 | 3/8 | 406.49 | 770.64 | 1/4 | 1402.0 | 4936.2 |
| 1. | 3.1416 | .52360 | 7/8 | 47.173 | 30.466 | 1/2 | 415.48 | 796.33 | 1/2 | 1418.6 | 5024.3 |
| 1-32 | 3.3410 | .57424 | 15-16 | 48.708 | 31.965 | 5/8 | 424.56 | 822.58 | 3/4 | 1435.4 | 5113.5 |
| 1-16 | 3.5466 | .62804 | 4. | 50.265 | 33.510 | 3/4 | 433.73 | 849.40 | 7/8 | 1452.2 | 5203.7 |
| 3-32 | 3.7583 | .68511 | 1-16 | 51.849 | 35.106 | 7/8 | 443.01 | 876.78 | 22. | 1469.2 | 5295.1 |
| 1/8 | 3.9761 | .74551 | 1/8 | 53.456 | 36.751 | 12. | 452.39 | 904.78 | 1/4 | 1486.2 | 5387.4 |
| 5-32 | 4.2000 | .80939 | 3-16 | 55.089 | 38.448 | 1/4 | 461.87 | 933.34 | 1/2 | 1503.3 | 5480.8 |
| 3-16 | 4.4301 | .87681 | 1/4 | 56.745 | 40.195 | 1/2 | 471.44 | 962.52 | 22. | 1520.5 | 5575.3 |
| 7-32 | 4.6664 | .94788 | 5-16 | 58.427 | 41.994 | 3/4 | 481.11 | 992.28 | 1/4 | 1537 9 | 5670.8 |
| 1/4 | 4.9068 | 1.0227 | 3/8 | 60.133 | 43.847 | 5/8 | 490.87 | 1022.7 | 1/2 | 1555.3 | 5767.6 |
| 9-32 | 5.1573 | 1.1013 | 7-16 | 61.863 | 45.752 | 3/4 | 500.73 | 1053.6 | 3/4 | 1572.8 | 5865.2 |
| 5-16 | 5.4119 | 1.1839 | 1/2 | 63.617 | 47.713 | 7/8 | 510.71 | 1085.3 | 23. | 1590.4 | 5964.1 |
| 11-32 | 5.6728 | 1.2704 | 9-16 | 65.397 | 49.729 | 1/8 | 520.77 | 1117.5 | 1/8 | 1608.2 | 6064.1 |
| 3/8 | 5.9396 | 1.3611 | 5/8 | 67.201 | 51.801 | 13. | 530.93 | 1150.3 | 1/4 | 1626.0 | 6165.2 |
| 13-32 | 6.2126 | 1.4561 | 11-16 | 69.030 | 53.929 | 1/4 | 541.19 | 1183.8 | 3/8 | 1643.9 | 6267.3 |
| 7-16 | 6.4919 | 1.5553 | 3/4 | 70.885 | 56.116 | 1/2 | 551.55 | 1218.0 | 1/2 | 1661.9 | 6370.5 |
| 15-32 | 6.7771 | 1.6590 | 13-16 | 72.759 | 58.359 | 3/4 | 562.00 | 1252.7 | 5/8 | 1680.0 | 6475.0 |
| 1/2 | 7.0686 | 1.7671 | 7/8 | 74.663 | 60.663 | 1/8 | 572.55 | 1288.3 | 3/4 | 1698.2 | 6580.6 |
| 17-32 | 7.3663 | 1.8799 | 15-16 | 76.589 | 63.026 | 1/4 | 583.20 | 1324.4 | 7/8 | 1716.5 | 6687.3 |
| 9-16 | 7.6699 | 1.9974 | 5. | 78.540 | 65.450 | 3/8 | 593.95 | 1361.2 | 24. | 1735.0 | 6795.2 |
| 19-32 | 7.9798 | 2.1195 | 1-16 | 80.516 | 67.935 | 1/2 | 604.80 | 1398.6 | 1/4 | 1753.5 | 6904.2 |
| 5/8 | 8.2957 | 2.2468 | 1/8 | 82.516 | 70.482 | 14. | 615.75 | 1436.8 | 1/2 | 1772.1 | 7014.3 |
| 21-32 | 8.6180 | 2.3789 | 3-16 | 84.541 | 73.092 | 1/4 | 626.80 | 1475.6 | 3/4 | 1790.8 | 7125.6 |
| 11-16 | 8.9461 | 2.5161 | 1/4 | 86.591 | 75.767 | 1/2 | 637.95 | 1515.1 | 25. | 1809.6 | 7238.2 |
| 23-32 | 9.2805 | 2.6586 | 5-16 | 88.664 | 78.505 | 3/4 | 649.17 | 1555.3 | 1/8 | 1828.5 | 7351.9 |
| 3/4 | 9.6211 | 2.8062 | 3/8 | 90.763 | 81.308 | 1/8 | 660.52 | 1596.3 | 1/4 | 1847.5 | 7465.7 |
| 25-32 | 9.9678 | 2.9592 | 7-16 | 92.887 | 84.178 | 1/4 | 671.95 | 1637.9 | 3/8 | 1866.6 | 7583.0 |
| 13-16 | 10.321 | 3.1177 | 1/2 | 95.033 | 87.113 | 1/2 | 683.49 | 1680.3 | 1/2 | 1885.8 | 7700.1 |
| 27-32 | 10.680 | 3.2818 | 9-16 | 97.205 | 90.118 | 3/4 | 695.13 | 1723.3 | 5/8 | 1905.1 | 7818.6 |
| 1/8 | 11.044 | 3.4514 | 5/8 | 99.401 | 93.189 | 15. | 706.85 | 1767.2 | 3/4 | 1924.4 | 7938.3 |
| 29-32 | 11.416 | 3.6270 | 11-16 | 101.62 | 96.331 | 1/8 | 718.69 | 1811.7 | 7/8 | 1943.9 | 8059.2 |
| 15-16 | 11.793 | 3.8083 | 3/4 | 103.87 | 99.541 | 1/4 | 730.63 | 1857.0 | 25. | 1963.5 | 8181.3 |
| 31-32 | 12.177 | 3.9956 | 13-16 | 106.14 | 102.82 | 3/8 | 742.65 | 1903.0 | 1/8 | 1983.2 | 8304.7 |
| 2. | 12.566 | 4.1888 | 7/8 | 108.44 | 106.18 | 1/2 | 754.77 | 1949.8 | 1/4 | 2002.9 | 8429.2 |
| 1-32 | 12.962 | 4.3882 | 15-16 | 110.75 | 109.60 | 5/8 | 767.00 | 1997.4 | 3/8 | 2022.9 | 8554.9 |
| 1-16 | 13.364 | 4.5929 | 6. | 113.10 | 113.10 | 3/4 | 779.32 | 2045.7 | 1/2 | 2042.8 | 8682.0 |
| 3-32 | 13.772 | 4.8036 | 1/8 | 117.87 | 120.31 | 7/8 | 791.73 | 2094.8 | 5/8 | 2062.9 | 8810.3 |
| 1/4 | 14.186 | 5.0243 | 1/4 | 122.72 | 127.33 | 16. | 804.25 | 2144.7 | 3/4 | 2083.0 | 8939.9 |
| 5-32 | 14.607 | 5.2493 | 3/8 | 127.68 | 135.66 | 1/4 | 816.85 | 2195.3 | 7/8 | 2103.4 | 9070.6 |
| 3-16 | 15.033 | 5.4809 | 1/2 | 132.73 | 143.79 | 1/2 | 829.57 | 2246.8 | 26. | 2123.7 | 9202.8 |
| 7-32 | 15.466 | 5.7190 | 5/8 | 137.89 | 152.25 | 3/4 | 842.40 | 2299.1 | 1/4 | 2144.2 | 9336.2 |
| 1/4 | 15.904 | 5.9641 | 3/4 | 143.14 | 161.03 | 7/8 | 855.29 | 2352.4 | 1/2 | 2164.7 | 9470.8 |
| 9-32 | 16.349 | 6.2161 | 7/8 | 148.49 | 170.14 | 17. | 868.31 | 2406.0 | 3/4 | 2185.5 | 9606.7 |
| 5-16 | 16.800 | 6.4751 | 7. | 153.94 | 179.59 | 1/4 | 881.42 | 2460.6 | 27. | 2206.2 | 9744.0 |
| 11-32 | 17.258 | 6.7412 | 1/4 | 159.49 | 189.39 | 1/2 | 894.62 | 2516.1 | 1/4 | 2227.1 | 9882.5 |
| 3/8 | 17.721 | 7.0144 | 1/2 | 165.13 | 199.53 | 3/4 | 907.93 | 2572.4 | 1/2 | 2249.0 | 10023 |

## ESFERAS (Continuación.)

| Diám. | Superf. | Solidez. | Diám. | Superf. | Solidez. | Diám. | Superf. | Solidez. | Diám. | Superf. | Solidez. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2269.1 | 10164 | | 4214.1 | 25724 | | 6756.5 | 52222 | | 9896.0 | 92570 |
| 27. | 2290.2 | 10306 | | 4243.0 | 25988 | | 6792.9 | 52645 | | 9940.2 | 93190 |
| | 2311.5 | 10450 | | 4271.8 | 26254 | | 6829.5 | 53071 | | 9984.4 | 93812 |
| | 2332.8 | 10595 | 37. | 4300.9 | 26522 | | 6866.1 | 53499 | 58. | 10029 | 94438 |
| | 2354.3 | 10741 | | 4330.0 | 26792 | | 6902.9 | 53929 | | 10073 | 95066 |
| | 2375.8 | 10889 | | 4359.2 | 27063 | 47. | 6939.9 | 54362 | | 10118 | 95697 |
| | 2397.5 | 11038 | | 4388.5 | 27337 | | 6976.8 | 54797 | | 10163 | 96330 |
| | 2419.2 | 11189 | | 4417.9 | 27612 | | 7013.9 | 55234 | 57. | 10207 | 96967 |
| | 2441.1 | 11341 | | 4447.5 | 27889 | | 7050.9 | 55674 | | 10252 | 97606 |
| 28. | 2463.0 | 11494 | | 4477.1 | 28168 | | 7088.3 | 56115 | | 10297 | 98248 |
| | 2485.1 | 11649 | | 4506.8 | 28449 | | 7125.6 | 56559 | | 10343 | 98893 |
| | 2507.2 | 11805 | 38. | 4536.5 | 28731 | | 7163.1 | 57006 | | 10387 | 99541 |
| | 2529.5 | 11962 | | 4566.5 | 29016 | | 7200.7 | 57455 | | 10432 | 100191 |
| | 2551.8 | 12121 | | 4596.4 | 29302 | 48. | 7238.3 | 57906 | | 10478 | 100845 |
| | 2574.3 | 12281 | | 4626.5 | 29590 | | 7276.0 | 58360 | | 10523 | 101501 |
| | 2596.7 | 12443 | | 4656.7 | 29880 | | 7313.9 | 58815 | 58. | 10568 | 102161 |
| | 2619.4 | 12606 | | 4686.9 | 30173 | | 7351.9 | 59274 | | 10614 | 102823 |
| 29. | 2642.1 | 12770 | | 4717.3 | 30466 | | 7389.9 | 59734 | | 10660 | 103488 |
| | 2665.0 | 12936 | | 4747.9 | 30762 | | 7428.0 | 60197 | | 10706 | 104155 |
| | 2687.8 | 13103 | 39. | 4778.4 | 31059 | | 7466.3 | 60663 | | 10751 | 104826 |
| | 2710.9 | 13272 | | 4809.0 | 31359 | | 7504.5 | 61131 | | 10798 | 105499 |
| | 2734.0 | 13442 | | 4839.9 | 31661 | 49. | 7543.1 | 61601 | | 10844 | 106175 |
| | 2757.3 | 13614 | | 4870.8 | 31964 | | 7581.6 | 62074 | 59. | 10890 | 106854 |
| | 2780.5 | 13787 | | 4901.7 | 32270 | | 7620.1 | 62549 | | 10936 | 107536 |
| | 2804.0 | 13961 | | 4932.7 | 32577 | | 7658.9 | 63026 | | 10983 | 108221 |
| 30. | 2827.4 | 14137 | | 4964.0 | 32886 | | 7697.7 | 63506 | | 11029 | 108909 |
| | 2851.1 | 14315 | | 4995.3 | 33197 | | 7736.7 | 63989 | | 11076 | 109600 |
| | 2874.8 | 14494 | 40. | 5026.5 | 33510 | | 7775.7 | 64474 | | 11122 | 110294 |
| | 2898.7 | 14674 | | 5058.1 | 33826 | | 7814.8 | 64961 | | 11169 | 110990 |
| | 2922.5 | 14856 | | 5089.6 | 34143 | 50. | 7854.0 | 65450 | | 11216 | 111690 |
| | 2946.6 | 15039 | | 5121.3 | 34462 | | 7893.3 | 65941 | | 11265 | 112392 |
| | 2970.6 | 15224 | | 5153.1 | 34783 | | 7932.8 | 66436 | 60. | 11310 | 113098 |
| 31. | 2994.9 | 15411 | | 5184.9 | 35106 | | 7972.2 | 66934 | | 11357 | 113806 |
| | 3019.1 | 15599 | | 5216.8 | 35431 | | 8011.8 | 67433 | | 11404 | 114518 |
| | 3043.6 | 15788 | | 5248.9 | 35758 | | 8051.6 | 67936 | | 11452 | 115233 |
| | 3068.0 | 15979 | 41. | 5281.1 | 36087 | | 8091.4 | 68439 | | 11499 | 115949 |
| | 3092.7 | 16172 | | 5313.3 | 36418 | | 8131.3 | 68946 | | 11547 | 116669 |
| | 3117.3 | 16366 | | 5345.6 | 36751 | 51. | 8171.2 | 69456 | | 11595 | 117392 |
| | 3142.1 | 16561 | | 5378.1 | 37086 | | 8211.4 | 69967 | | 11642 | 118118 |
| | 3166.9 | 16758 | | 5410.7 | 37423 | | 8251.6 | 70482 | 61. | 11690 | 118847 |
| | 3192.0 | 16957 | | 5443.3 | 37763 | | 8292.0 | 70999 | | 11738 | 119579 |
| 32. | 3217.0 | 17157 | | 5476.0 | 38104 | | 8332.3 | 71519 | | 11786 | 120315 |
| | 3242.2 | 17359 | | 5508.9 | 38448 | | 8372.8 | 72040 | | 11834 | 121053 |
| | 3267.4 | 17563 | 42. | 5541.9 | 38792 | | 8413.4 | 72565 | | 11882 | 121794 |
| | 3292.9 | 17768 | | 5574.9 | 39140 | | 8454.1 | 73092 | | 11931 | 122538 |
| | 3318.3 | 17974 | | 5608.0 | 39490 | 52. | 8494.8 | 73622 | | 11980 | 123286 |
| | 3343.9 | 18182 | | 5641.3 | 39841 | | 8535.8 | 74154 | | 12028 | 124036 |
| | 3369.6 | 18392 | | 5674.5 | 40194 | | 8576.8 | 74689 | 62. | 12076 | 124789 |
| | 3395.4 | 18604 | | 5708.0 | 40551 | | 8617.8 | 75226 | | 12126 | 125545 |
| 33. | 3421.2 | 18817 | | 5741.5 | 40908 | | 8658.9 | 75767 | | 12174 | 126305 |
| | 3447.3 | 19032 | | 5775.2 | 41268 | | 8700.4 | 76309 | | 12223 | 127067 |
| | 3473.3 | 19248 | 43. | 5808.8 | 41630 | | 8741.7 | 76854 | | 12272 | 127832 |
| | 3499.5 | 19466 | | 5842.7 | 41994 | | 8783.2 | 77401 | | 12322 | 128601 |
| | 3525.7 | 19685 | | 5876.5 | 42360 | 53. | 8824.8 | 77952 | | 12371 | 129373 |
| | 3552.1 | 19907 | | 5910.7 | 42729 | | 8866.4 | 78505 | | 12420 | 130147 |
| | 3578.5 | 20129 | | 5944.7 | 43099 | | 8908.2 | 79060 | 63. | 12469 | 130925 |
| | 3605.1 | 20354 | | 5978.9 | 43472 | | 8950.1 | 79617 | | 12519 | 131706 |
| 34. | 3631.7 | 20580 | | 6013.2 | 43846 | | 8992.0 | 80178 | | 12568 | 132490 |
| | 3658.5 | 20808 | | 6047.7 | 44224 | | 9034.1 | 80741 | | 12618 | 133277 |
| | 3685.3 | 21037 | 44. | 6082.1 | 44602 | | 9076.4 | 81308 | | 12668 | 134067 |
| | 3712.3 | 21268 | | 6116.8 | 44984 | | 9118.5 | 81876 | | 12718 | 134860 |
| | 5739.3 | 21501 | | 6151.5 | 45367 | 54. | 9160.8 | 82448 | | 12768 | 135657 |
| | 3766.5 | 21736 | | 6186.3 | 45753 | | 9203.3 | 83021 | | 12818 | 136456 |
| | 3793.7 | 21972 | | 6221.2 | 46141 | | 9246.0 | 83598 | 64. | 12868 | 137259 |
| | 3821.1 | 22210 | | 6256.1 | 46530 | | 9288.5 | 84177 | | 12918 | 138065 |
| 35. | 3848.5 | 22449 | | 6291.2 | 46922 | | 9331.2 | 84760 | | 12969 | 138874 |
| | 3876.1 | 22691 | | 6326.5 | 47317 | | 9374.1 | 85344 | | 13019 | 139686 |
| | 3903.7 | 22934 | 45. | 6361.7 | 47713 | | 9417.2 | 85931 | | 13070 | 140501 |
| | 3931.5 | 23179 | | 6397.2 | 48112 | | 9460.2 | 86521 | | 13121 | 141320 |
| | 3959.2 | 23425 | | 6432.7 | 48513 | 55. | 9503.2 | 87114 | | 13172 | 142142 |
| | 3987.2 | 23674 | | 6468.3 | 48916 | | 9546.5 | 87709 | | 13222 | 142966 |
| | 4015.2 | 23924 | | 6503.9 | 49321 | | 9590.0 | 88307 | 65. | 13273 | 143794 |
| | 4043.3 | 24176 | | 6539.7 | 49729 | | 9633.3 | 88908 | | 13324 | 144625 |
| 36. | 4071.5 | 24429 | | 6575.5 | 50139 | | 9676.8 | 89511 | | 13376 | 145460 |
| | 4099.9 | 24685 | | 6611.6 | 50551 | | 9720.6 | 90117 | | 13427 | 146297 |
| | 4128.3 | 24942 | 46. | 6647.6 | 50965 | | 9764.4 | 90726 | | 13478 | 147138 |
| | 4156.9 | 25201 | | 6683.7 | 51382 | | 9808.1 | 91338 | | 13490 | 147982 |
| | 4185.5 | 25461 | | 6720.0 | 51801 | 56. | 9852.0 | 91953 | | 13582 | 148827 |

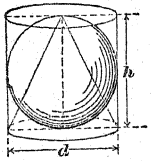
## ESFERAS (Continuación.)

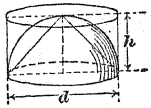
| Diám. | Superf. | Solidez. | Diám. | Superf. | Solidez. | Diám. | Superf. | Solidez. | Diám. | Superf. | Solidez. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 13633 | 149680 | | 17437 | 216505 | | 21708 | 300743 | | 26446 | 404406 |
| | 13685 | 150533 | | 17496 | 217597 | | 21773 | 302100 | | 26518 | 406060 |
| | 13737 | 151390 | | 17554 | 218693 | | 21839 | 303463 | 92. | 26590 | 407721 |
| | 13789 | 152251 | | 17613 | 219792 | | 21904 | 304831 | | 26663 | 409384 |
| | 13841 | 153114 | 75. | 17672 | 220894 | | 21970 | 306201 | | 26735 | 411054 |
| | 13893 | 153980 | | 17731 | 222001 | | 22036 | 307576 | | 26808 | 412726 |
| | 13946 | 154850 | | 17790 | 223111 | | 22102 | 308957 | | 26880 | 414405 |
| | 13998 | 155724 | | 17849 | 224224 | 84. | 22167 | 310340 | | 26953 | 416086 |
| | 14050 | 156600 | | 17908 | 225341 | | 22234 | 311728 | | 27026 | 417774 |
| 67. | 14103 | 157480 | | 17968 | 226463 | | 22300 | 318118 | | 27099 | 419464 |
| | 14156 | 158363 | | 18027 | 227588 | | 22366 | 314514 | 93. | 27172 | 421161 |
| | 14208 | 159250 | | 18087 | 228716 | | 22432 | 315915 | | 27245 | 422862 |
| | 14261 | 160139 | 76. | 18146 | 229848 | | 22499 | 317318 | | 27318 | 424567 |
| | 14314 | 161032 | | 18206 | 230984 | | 22565 | 318726 | | 27391 | 426277 |
| | 14367 | 161927 | | 18266 | 232124 | | 22632 | 320140 | | 27464 | 427991 |
| | 14420 | 162827 | | 18326 | 233267 | 85. | 22698 | 321556 | | 27538 | 429710 |
| | 14474 | 163731 | | 18386 | 234414 | | 22765 | 322977 | | 27612 | 431433 |
| 68. | 14527 | 164637 | | 18446 | 235566 | | 22832 | 324402 | | 27686 | 433160 |
| | 14580 | 165547 | | 18506 | 236719 | | 22899 | 325831 | 94. | 27759 | 434894 |
| | 14634 | 166460 | | 18566 | 237879 | | 22966 | 327264 | | 27833 | 436630 |
| | 14688 | 167376 | 77. | 18626 | 239041 | | 23034 | 328702 | | 27907 | 438373 |
| | 14741 | 168295 | | 18687 | 240206 | | 23101 | 330142 | | 27981 | 440118 |
| | 14795 | 169218 | | 18748 | 241376 | | 23168 | 331588 | | 28055 | 441871 |
| | 14849 | 170145 | | 18809 | 242551 | 86. | 23235 | 333039 | | 28130 | 443625 |
| | 14903 | 171074 | | 18869 | 243728 | | 23303 | 334492 | | 28204 | 445387 |
| 69. | 14957 | 172007 | | 18930 | 244908 | | 23371 | 335951 | | 28278 | 447151 |
| | 15012 | 172944 | | 18992 | 246093 | | 23439 | 337414 | 95. | 28353 | 448920 |
| | 15066 | 173883 | | 19053 | 247283 | | 23506 | 338882 | | 28428 | 450695 |
| | 15120 | 174828 | 78. | 19114 | 248475 | | 23575 | 340352 | | 28503 | 452475 |
| | 15175 | 175774 | | 19175 | 249672 | | 23643 | 341829 | | 28577 | 454259 |
| | 15230 | 176723 | | 19237 | 250873 | | 23711 | 343307 | | 28652 | 456047 |
| | 15284 | 177677 | | 19298 | 252077 | 87. | 23779 | 344792 | | 28727 | 457839 |
| | 15339 | 178635 | | 19360 | 253284 | | 23847 | 346281 | | 28802 | 459638 |
| 70. | 15394 | 179595 | | 19422 | 254496 | | 23916 | 347772 | | 28878 | 461439 |
| | 15449 | 180559 | | 19483 | 255713 | | 23984 | 349269 | 96. | 28953 | 463248 |
| | 15504 | 181525 | | 19545 | 256932 | | 24053 | 350771 | | 29028 | 465059 |
| | 15560 | 182497 | 79. | 19607 | 258155 | | 24122 | 352277 | | 29104 | 466875 |
| | 15615 | 183471 | | 19669 | 259383 | | 24191 | 353785 | | 29180 | 468697 |
| | 15670 | 184449 | | 19732 | 260613 | | 24260 | 355301 | | 29255 | 470524 |
| | 15726 | 185430 | | 19794 | 261848 | 88. | 24328 | 356819 | | 29331 | 472354 |
| | 15782 | 186414 | | 19856 | 263088 | | 24398 | 358342 | | 29407 | 474189 |
| 71. | 15837 | 187402 | | 19919 | 264330 | | 24467 | 359869 | | 29483 | 476029 |
| | 15893 | 188394 | | 19981 | 265577 | | 24536 | 361400 | 97. | 29559 | 477874 |
| | 15949 | 189389 | | 20044 | 266829 | | 24606 | 362935 | | 29636 | 479725 |
| | 16005 | 190387 | 80 | 20106 | 268083 | | 24676 | 364476 | | 29712 | 481579 |
| | 16061 | 191389 | | 20170 | 269342 | | 24745 | 366019 | | 29788 | 483438 |
| | 16117 | 192395 | | 20232 | 270604 | | 24815 | 367568 | | 29865 | 485302 |
| | 16174 | 193404 | | 20296 | 271871 | 89 | 24885 | 369122 | | 29942 | 487171 |
| | 16230 | 194417 | | 20358 | 273141 | | 24955 | 370678 | | 30018 | 489045 |
| 72. | 16286 | 195433 | | 20422 | 274416 | | 25025 | 372240 | | 30095 | 490924 |
| | 16343 | 196453 | | 20485 | 275694 | | 25095 | 373806 | 98. | 30172 | 492808 |
| | 16400 | 197476 | | 20549 | 276977 | | 25165 | 375378 | | 30249 | 494695 |
| | 16456 | 198502 | 81 | 20612 | 278263 | | 25236 | 376954 | | 30326 | 496588 |
| | 16513 | 199532 | | 20676 | 279553 | | 25306 | 378531 | | 30404 | 498486 |
| | 16570 | 200566 | | 20740 | 280847 | | 25376 | 380115 | | 30481 | 500388 |
| | 16628 | 201604 | | 20804 | 282145 | 90 | 25447 | 381704 | | 30558 | 502296 |
| | 16685 | 202645 | | 20867 | 283447 | | 25518 | 383297 | | 30636 | 504208 |
| 73. | 16742 | 203689 | | 20932 | 284754 | | 25589 | 384894 | | 30713 | 506125 |
| | 16799 | 204737 | | 20996 | 286064 | | 25660 | 386496 | 99. | 30791 | 508047 |
| | 16857 | 205789 | | 21060 | 287378 | | 25730 | 388102 | | 30869 | 509975 |
| | 16914 | 206844 | 82. | 21124 | 288696 | | 25802 | 389711 | | 30947 | 511906 |
| | 16972 | 207903 | | 21189 | 290019 | | 25873 | 391327 | | 31025 | 513843 |
| | 17030 | 208966 | | 21253 | 291345 | | 25944 | 392945 | | 31103 | 515785 |
| | 17088 | 210032 | | 21318 | 292674 | 91. | 26016 | 394570 | | 31181 | 517730 |
| | 17146 | 211102 | | 21382 | 294010 | | 26087 | 396197 | | 31259 | 519682 |
| | 17204 | 212175 | | 21448 | 295347 | | 26159 | 397831 | | 31338 | 521638 |
| | 17262 | 213252 | | 21512 | 296691 | | 26230 | 399468 | 100. | 31416 | 523598 |
| | 17320 | 214333 | | 21578 | 298092 | | 26302 | 401117 | | | |
| | 17379 | 215417 | 83. | 21642 | 299388 | | 26374 | 402756 | | | |

**Esfera, S; cono, C; y cilindro, Y;** de igual diámetro $d$ y de igual altura $h$. $(d = h)$.

$$\textbf{Volumen.}\quad Y = \frac{3}{2}S = 3C.$$

$$\textbf{Superficie curva.}\quad Y = S = \frac{2C}{\sqrt{1.25}}.$$



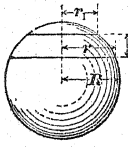

**Hemisferio, H; cono, C; y cilindro, Y;** de igual diámetro $d$, y de igual altura $h$. $(d = 2h)$.

$$\textbf{Volumen.}\quad Y = \frac{3}{2}H = 3C.$$

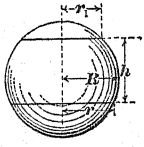
$$\textbf{Superficie curva.}\quad Y = H = C\sqrt{2}.$$

## Zonas esféricas y segmentos.

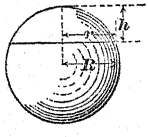
Sea R = radio de la esfera;
$r$ = radio de la base del segmento
= radio de cualquiera de las bases de la zona;
$r_1$ = radio de la otra base de la zona;
$h$ = altura de la zona ó segmento
$\pi = \frac{\text{circunf.}}{\text{diám.}} = 3.14159.$




Zona.




Zona.




Segmento.

Cáscara.

Entonces **Volumen de la zona** $= \frac{\pi}{2}\left(r^2 + r_1^2 + \frac{h^2}{3}\right)h.$

**Volumen del segmento** $= \frac{\pi}{2}\left(r^2 + \frac{h^2}{3}\right)h = \pi\left(R - \frac{h}{3}\right)h^2.$

**Superf curva de la zona ó segmento** $= \frac{h}{2R} \times$ superf de la esfera $= \frac{h}{2R} \times 4\pi R^2 = 2\pi Rh.$

En el segmento, $2R = \frac{r^2}{h} + h.$

## Cáscara esférica.

**Volumen** = volumen de la esfera $ab$ — volumen de la esf $cd$.

## Huso circular.

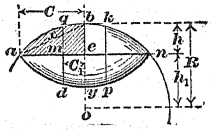
Es el sólido *abnya*, engendrado por la revolución de un segmento circular *abnea*, alrededor de su cuerda, *an*. Sea

$C = ae = \frac{1}{2}$ cuerda del segmento; $x = \sqrt{C^2 + h^2}$.
$h = eb =$ altura — = cuerda de la mitad del segmento;
$R = ob =$ radio del círculo. L = longitud del arco, *abn*;
$= \frac{C^2 + h^2}{2h}$; S = área *abe* $= \frac{LR}{4} - \frac{Ch_1}{2}$;
$h_1 = oe = R - h$ = distancia del centro, *o*, del círculo, al centro, *e*, de la cuerda.

Entonces **Volumen** $= 4\pi\left(\frac{C^2}{3} - Sh_1\right)$.

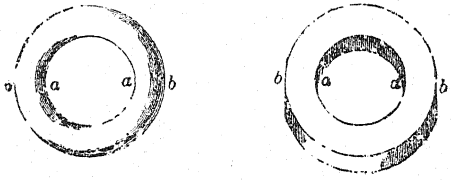
**Superf** $= 2\pi\ (2\ CR - Lh_1)$.

**Zona media** *qdkp*; $(C_1 = me)$.




$$\textbf{Volumen} = 2\pi\left[\left(C^2 - \frac{C_1^2}{3}\right)C_1 - h_1\ S_1\right], \text{ aquí } S_1 = \frac{\text{área, } qdkp}{2}.$$

## Anillo circular.




Sea
*a*=área de la sección *ab* del anillo;
*c*=circunf ó periferia de esta sección;
*d*=semisuma de los diámetros *aa* y *bb*.
$\pi$=3.141593.

Entonces **Volumen**$=\pi ad$; **Superficie**$=\pi cd$.

## Elipsoide ó esferoide.

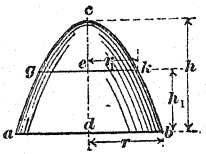
El elipsoide ó esferoide es el sólido engendrado por una elipse que gira alrededor de uno de sus diámetros. Si sobre su diám mayor resulta un esferoide *alargado*, si sobre el diám menor resulta *achatado*.

En ambos casos, **Volumen** $= \frac{\pi}{6} \times$ diámetro fijo $\times$ (diámetro de revolución)$^2$.

## Paraboloide.

Paraboloide es el sólido engendrado por la revolución de una parábola, *acb* alrededor de su eje, *cd*.

$r$=radio *db* de la base; $h$=altura *dc*.




**Paraboloide.**

$$\textbf{Volumen} = \frac{\pi r^2 h}{2} = \frac{h \times \text{área de la base}}{2};$$

$$\textbf{Superficie curva} = \left(\frac{2\pi r}{12\,h^2}\right)\left[(r^2 + 4h^2)^{\frac{3}{2}} - r^3\right].$$

## Tronco de paraboloide.

Extremos *gk* y *ab*, perpendiculares al eje *dc*. $r_1$=radio *ek* de la base *gk*. $h_1$=altura *de* del tronco.

$$\textbf{Volumen} = \text{paraboloide } acb - \text{paraboloide } gck = \frac{\pi}{2} h_1 \,(r^2 + r_1^2).$$

**Superficie curva**=superf *acb* — superf *gck*.

# PESO ESPECÍFICO

**1. El peso específico** ó **densidad relativa, D** *, de una substancia, es la relación entre el peso, W, de cualquier volumen de dicha substancia y el peso A del mismo volumen de otra substancia elegida como término de comparación. Por tanto : $D = \frac{W}{A}$.

**2.** Para los gases, el término de comparación es el aire seco á la temperatura de 0° Centígrado=32 Fahr, y á la presión barométrica de 760 mm=29.922 pulgadas.

**3.** Los sólidos y líquidos se comparan con el agua destilada en su máximum de densidad, á la temperatura de 4° C=39.2° F.

**4.** Para casi todos los propósitos del ingeniero, sirve cualquier agua fresca á cualquier temperatura ordinaria; con agua á 30° C=86° F, el error es de 4 por mil en exceso.

**5.** Cuando se sumerge un cuerpo en el agua, la fuerza de **empuje** ejercida por el agua sobre el cuerpo obra de abajo hacia arriba y le hace perder **una parte de su peso**, igual al peso del agua **desalojada** por el cuerpo; es decir, que si :

W=al peso del cuerpo en el aire,
$w$=su peso dentro del agua,
D=su peso específico,
A=al peso del agua desalojada;

entonces $A = W - w$; $D = \frac{W}{A} = \frac{W}{W - w}$.

**6.** Como el volumen V de un cuerpo de peso W, está en razón inversa de su densidad ó peso específico D, su densidad será igual también á la relación entre el volumen $V_s$ de la substancia elegida por término de comparación y el volumen $V_1$ de la substancia del cuerpo que pese lo mismo que aquélla; ó bien $D = \frac{V_s}{V_1}$.

**7.** El peso específico de las substancias más pesadas que el agua, se determina generalmente, pesando primero una masa de la substancia en el aire (W) y después pesándola completamente sumergida en el agua (obteniendo así su peso $w$ disminuído). Entonces (véase § **5**) $D = \frac{W}{W - w}$.

**8. Si el cuerpo pesa menos que el agua**, debe ser completamente sumergido, obrando contra su tendencia á flotar. Su peso $w$ en el agua, ó su tendencia á subir es entonces una cantidad *negativa* y debe idearse el modo de medirla; como, por ej., haciéndola obrar contra el platillo de la balanza hidrostática. Se tiene entonces, $A = W - (-w) = W + w$, es decir :

Pérdida debida á la inmersión=peso del cuerpo en el aire más su fuerza ascensional.

**9.** O bien : déjese primero flotar el cuerpo sobre el agua y anótese el desplazamiento del agua, por el nivel á que sube en un vaso angosto. Luego sumérjase todo el cuerpo y anótese también el desplazamiento V. Ahora el volumen $v$ desplazado por el cuerpo cuando flotaba, y V, volumen desplazado estando completamente sumergido, son proporcionales, respectivamente, al peso W del cuerpo y al peso $W - w$ de la masa de agua de volumen igual al del cuerpo. Así, $D = \frac{W}{W - w} = \frac{v}{V}$.

**10.** O bien átese al cuerpo liviano $b$, un cuerpo pesado, á manera de plomada ó lastre, S, de tal densidad y masa que ambos cuerpos se sumerjan en el agua. Sea W el peso del cuerpo liviano $b$ en el aire; Q el peso de ambos cuerpos en el aire y $q$ el de ambos también en el agua. Entonces $Q - q$ es el peso de una masa de agua de igual volumen que los dos cuerpos y $Q - W$=al peso S del cuerpo (que sirve de lastre) en el aire. Sumergiendo el lastre solo encuéntrese el peso $k$, del agua de igual volumen al del lastre, igual á la pérdida de peso del lastre debida á la inmersión. Entonces, para el peso A del agua igual en volumen a

* Estrictamente hablando, el «peso específico» se refiere al *peso* y la densidad relativa á la *masa;* pero como ambos son *numéricamente* iguales, se les considera indistintamente.

cuerpo liviano $b$ ó por la pérdida del peso de $b$ debida á la inmersión, se tiene $A = Q - q - k$; y para el peso específico D, del cuerpo liviano $b$, se tiene $D = \frac{W}{Q - q - k} = \frac{W}{W - w}$ en que $w$ es la fuerza ascensional desconocida de $b$, ó su peso *negativo* dentro del agua.

**11. Un cuerpo granular** ó **pulverulento** es una masa como el aserrín, la arena, el cemento, etc., ó un cuerpo poroso como un pedazo de madera, de concreto, etc., que tiene una parte sólida y otra parte de aire en sus intersticios. Así un pie cúbico de arena de cuarzo pesa como 100 lbs, mientras que un pie cúbico de piedra de cuarzo pesa como 160 lbs.

**12. El peso específico de las substancias porosas** se toma por el del compuesto de sólido y aire. Así una madera que pese (con su contenido de aire) 62.5 lbs por pie cúb, es decir, lo mismo que el agua, se dice que tiene un peso específico 1. Cuando á estos cuerpos hay que sumergirlos en agua para determinar su densidad se evita la absorción del agua por sus poros, dándole una ligera capa de barniz.

**13. La densidad de las substancias granulares** se establece algunas veces tomando la de la parte sólida que contienen. Así el cemento Portland ordinario, pesa (en el aire) de 75 á 90 lbs por pie cúb ó sea de 1.20 á 1.44 kilogramo por litro, es decir, que tiene por densidad de 1.20 á 1.44 (puesto que un litro de agua pesa *un kilogramo;* pero la parte sólida tiene una densidad de 3.00 á 3.25 y son estas cifras las que se toman generalmente para expresar su densidad.

**14.** Para determinar la densidad de las substancias (como el cemento) que **son solubles en el agua** ó afectadas por ésta, se pesan en líquidos (como la bencina, trementina, alcohol, etc.) que no los afecte, y el resultado que se obtenga se multiplica por la relación entre la densidad del líquido empleado y la del agua.

**15. La densidad de un líquido se determina** pesando iguales volúmenes del líquido y del agua.

**16.** O bien pésese en el líquido un cuerpo cuyo peso W en el aire y cuya densidad $d$ sean conocidos. Sea $w'$ su peso en el líquido. Entonces para la densidad D del líquido se tiene

$$W : W - w' = d : D; \text{ ó } D = \frac{d\ (W - w')}{W}.$$

**17.** O supongamos que el cuerpo **(16)** (que pesa W en el aire) pese $w$ en agua y que, como antes, pese $w'$ en el líquido en cuestión. Entonces, como la densidad del agua es=1, se tiene :

$$W - w : W - w' = 1 : D; \quad D = \frac{W - w'}{W - w}.$$

**18.** La densidad de los líquidos se determina generalmente observando la profundidad á que se sumerge en ellos un instrumento flotante llamado **aréometro.** Cuanto más se sumerge, menos denso es el líquido. En el **aréometro de Beaumé,** la profundidad de la inmersión está marcada en una escala arbitraria que tiene el aparato. Para líquidos más pesados que el agua el cero corresponde á la densidad 1 y 76° á la densidad 2. Para los más livianos, 10° corresponde á la densidad 1; y 60° á 0.745.

**19.** En el **aréometro de Twaddell** para líquidos más pesados que el agua,

$$\text{la densidad} = \frac{5 \times \text{N.° de grados} + 1{,}000}{1{,}000}.$$

Así, si la lectura es de 90°,

$$\text{la densidad} = \frac{5 \times 90 + 1{,}000}{1{,}000} = \frac{1{,}450}{1{,}000} = 1.45.$$

**20. En el aréometro de Nicolson,** muy usado también para sólidos, la densidad se deduce del peso requerido para hacerlo sumergir hasta un punto determinado fijo. Este aréometro consiste en un cuerpo metálico hueco, del que sube una varilla que sostiene un platillo que siempre queda sobre el agua. Del flotador hueco está suspendido abajo como lastre, otroplatillo pesado que siempre está sumergido como el flotador. En la varilla que sostiene el platillo superior está marcado el punto fijo, al que se obliga siempre en las experiencias á situarse al nivel del líquido. La densidad se determina por medio de los pesos en los dos platillos respectivamente.

**21.** La determinación de la **densidad de los gases** requiere la intervención y conocimientos de químicos expertos.

## Tabla de pesos específicos.

En esta tabla el peso específico del aire y el de los gases, están también comparados con el del agua en lugar de estarlo con el del aire. Así se usa últimamente.

| El peso específico de cualquier substancia es=**su peso en gramos por centímetro cúbico.** | Peso específico medio. |
|---|---|
| *N. del T.* — El peso de un litro (un decímetro cúbico) de cualquier substancia es un número *de kilogramos*=á su peso específico. | |
| Aire atmosférico á 60° Fahr (15°.6 C) y bajo la presión de una atmósfera pesa $^1/_{813}$ parte de lo que pesa el agua á 60°F = 15° .6.C | .00123 |
| Alcohol puro | .793 |
| — de comercio | .834 |
| — absoluto | .916 |
| Alabastro llamado así falsamente pero realmente mármol | 2.70 |
| — real..... término medio. | 2.31 |
| Aluminio | 2.6 |
| Antimonio fundido de 6.66 á 6.74..... término medio. | 6.70 |
| — nativo..... — — | 6.67 |
| Antracita. (Véase carbón abajo.) | |
| Asfalto de 1 á 1.8..... — — | 1.4 |
| Aceites: de ballena; de oliva..... — — | .92 |
| Aceite de trementina | .87 |
| Arena de cuarzo puro perfectamente seca y suelta | * |
| Si se la mueve ligeramente se compacta como de 2 á 3% y apisonada como 12% cuando está seca. | |
| Arena perfectamente húmeda, con intersticios llenos de agua | 1.89 á 2.06 |
| Arena angular, cuarzo puro con granos secos muy grandes y muy pequeños puede pesar | 1.87 |
| Si cualquiera arena ordinaria pura y natural se separa en dos ó tres ó más porciones de granos de diferentes tamaños, una medida de cualquiera de estas porciones pesará considerablemente menos que una medida igual de la arena original. Así una arena que pese (1.57 kilog el litro) puede dar otras que no pesen más que .98 á 1.20. A (1.57 kilog el litro) un volumen de cuarzo puro ha producido 1.68 de arena, en el cual el sólido ocupa .6; y el vacío .4. Pero si esta misma arena se compacta á 1.76 kilog el litro, entonces una medida de cuarzo sólido hace 1 $^1/_2$ medidas de arena, de la cual $^2/_3$ son sólidos y $^1/_3$ vacío. | |
| La arena conserva mucho la humedad y en grandes volúmenes rara vez está seca como se supone en esta tabla. Pero con su natural humedad y flojedad es más liviana que cuando está seca, no excediendo entonces el término medio de su peso de 1.36 á 1.44 kilog el litro. | |
| Areniscas propias para construcción, secas, 2.1 á 2.73 | 2.41 |
| — de cantera amontonadas. | |
| (Una medida sólida hace como 1 $^3/_4$ apiladas.) | |
| Arcilla esquistosa colorada ó negra 2.4 á 2.8..... término medio. | 2.6 |
| — — de cantera en pilas..... — — | 1.47 |
| Acero 7.7 á 7.9. El más pesado contiene menos carbón | 7.85 |
| El acero no es más pesado que el hierro del cual se hace; á menos que el hierro tenga impurezas que bote durante su conversión en acero. | |
| Abeto perfectamente seco (nota al pie de caoba española) | .4 |
| Mil pies (2.36 m cub), etc., pesan .930 toneladas (medida de tablas.). | |

* El peso específico de la arena pura de cuarzo, encontrado como se indicó al comenzar este capítulo de pesos específicos, es por supuesto el mismo que el del cuarzo puro (2.65); pero un volumen de arena seca pesa sólo de 1.44 á 1.7 veces más que igual cantidad de agua. La mayor parte de los autores dan 1.5 de peso específico.

## Tabla de pesos específicos.

| El peso específico de cualquier substancia es=**su peso en gramos por centímetro cúbico.** | Peso específico medio. |
|---|---|
| Alquitrán..... término medio. | 1. |
| Agua de lluvia pura ó destilada á 32° Fahr (0° C) barómetro á 760 mm..... | .9986 |
| Agua de lluvia pura ó destilada á 62° Fahr (16°7 C) barómetro á 760 mm..... | .99768 |
| Agua de lluvia pura ó destilada á 212° Fahr (100° C) barómetro á 760 mm..... | .955 |
| Agua de mar, 1.026 á 1.030..... término medio. Aun cuando se supone generalmente el peso del agua fresca como .998 kilog el litro, sin embargo .997 kilog el litro es más cercano á la verdad en temperaturas ordinarias como de 21° C. | 1.028 |
| Barro seco compacto..... | 1.28 á 1.76 |
| — húmedo comprimido medianamente..... | 1.76 á 2.08 |
| — — fluido..... | 1.66 á 1.92 |
| Basalto..... | 2.9 |
| Bismuto fundido. También nativo..... término medio. | 9.74 |
| Betún sólido. Véase asfalto. | |
| Bronce : cobre 8 partes; estaño 1 (metal de cañón) 8.4 á 8.6. término medio. | 8.5 |
| Boj seco..... — — | .96 |
| Calcita transparente..... | 2.722 |
| Cok de buen carbón (pedazos sueltos)..... | .37 á .51 |
| — un *bushel* colmado (*bushel*=36.34766 litros) 35 á 42 lib (15.87 á 19.05 kilog)..... | |
| Cok, 1 tonelada ocupa de 80 á 97 pies cúbicos (de 2.265 á 2.747 met. cúb.)..... término medio. Quemando el carbón aumenta en volumen de 25 á 50 por ciento. Igual peso de cok y carbón evaporan casi iguales pesos de agua y cada uno como el doble del mismo peso de madera seca. | |
| Corindón puro de 3.8 á 4..... | 3.9 |
| Cerezo perfectamente seco..... término medio. Mil pies corridos de tablas 12 por 1 pulgada pesa 1.562 toneladas. | .672 |
| Carbón antracita 1.3 á 1.7 generalmente ..... | 1.5 |
| — — en pedazos de cualquier tamaño. término medio. | .83 á .90 |
| — — — regulares..... | .90 á .96 |
| — — *bushel* colmado de 77 á 83 lib (más ó menos 1 kilog el litro)..... | |
| Un metro cúbico sólido tiene por término medio como 1.75 metros cúbicos cuando está en pedazos del tamaño que se acostumbra en el mercado. 1 tonelada en pedazos sueltos tiene más ó menos de 40 á 43 pies cúbicos (de 1,132 á 1,218 litros). A 54 lib por pie cúbico, una yarda cúbica pesa 1,458 lib=.651 toneladas. El metro cúbico pesa 824 kilogramos. | |
| Carbón bituminoso de 1.2 á 1.5..... | 1.35 |
| — — en pedazos de cualquier tamaño; sueltos..... | .75 á .83 |
| — — ligeramente quebrado..... | .82 á .90 |
| Carbón betuminoso un *bushel* (36,348 litros) colmado, suelto de 70 a 78 lib (de 32 á 35 kilog : más ó menos). | |
| Carbón betuminoso 1 tonelada ocupa de 43 á 48 pies cúbicos, es decir, de 1,271 á 1,359 litros. Un metro cúbico sólido tiene más ó menos 1.75 metros cúbicos partidos en pedazos sueltos como se usa para el mercado. | |
| Cemento hidráulico americano Rosendale en polvo suelto. tér. m°. | .90 |

## Tabla de pesos específicos *.

| El peso específico de cualquier substancia es=**su peso en gramos por centímetro cúbico.** | Peso específico medio. |
|---|---|
| Cemento hidráulico americano Louisville (en polvo)..... | .79 |
| — — inglés de Portland, (en polvo)..... | 1.30 á 1.63 |
| Cemento hidráulico inglés de Portland, un barril de 400 á 430 lib (de 181 á 195 kilog). | |
| Cemento hidráulico francés de Boulogne Portland..... | 1.22 á 1.41 |
| Ocurren á menudo diferencias de 40 á 50 kilog por tonelada más ó menos en el cemento de la misma fábrica, debido no solamente á la dificultad de medirlo con exactitud, sino á la falta de homogeneidad en la composición de la piedra, grado de calcinación, molida, sequedad, etc. | |
| Cobre fundido..... 8.6 á 8.8..... | 8.7 |
| — laminado..... 8.8 á 9.0..... | 8.9 |
| Cristal. Cuarzo puro. (Véase cuarzo.) | |
| Corcho..... | .25 |
| Cascajo casi lo mismo que arena. (Véase arena.) | |
| Cicuta perfectamente seca. (Véase nota en caoba española.)..... | .4 |
| Mil pies corridos de tabla de 12 pies 1 pulg pesa .930 toneladas. | |
| Caucho..... término medio. | .93 |
| Cal viva de piedra caliza ordinaria y mármol 1,470 á 1,570 kilog el metro cúbico..... | 1.50 |
| Cal viva en pequeños pedazos; ó molida, suelta, 800 á 900 kilogramos el metro cúbico..... | .85 |
| En cualquiera de los dos casos una medida sólida hace como 1.8 medida suelta; y entonces .555 de la masa es sólida y .445 vacíos. | |
| Para medir correctamente, ninguno de los pedazos debe exceder de $^1/_8$ ó $^1/_{10}$ de la más pequeña dimensión de los envases usados para medir. | |
| Cal viva molida suelta..... | .85 |
| — — — bien movida..... | 1.02 |
| — — — perfectamente triturada..... | 1.20 |
| Caoba española seca **..... término medio. | .85 |
| — de Honduras..... — — | .56 |
| Cuarzo ordinario puro 2.64 á 2.67..... | 2.65 |
| — — bien pulverizado..... | 1.50 |
| — — finamente pulverizado, bien empaquetado..... | 1.79 |
| — de cantera, suelto. Una medida sólida hace 1 $^3/_4$ en pedazos y apilados..... | 1.40 |
| Cera de abejas..... término medio. | .97 |
| Circón, 4.0 á 4.9..... | 4.45 |
| Diamante, generalmente 3.51 á 3.55..... | 3.53 |
| Diorita 2.8 á 3.2..... | 3. |
| — de canteras en montonces sueltos..... | 1.71 |
| Durazno..... | 1.15 |
| Eter..... | .716 |
| Ebano seco..... término medio | 1.22 |
| Esmeralda 2.63 á 2.76..... | 2.7 |
| Esteatita 2.65 á 2.8..... término medio. | 2.73 |
| Estaño fundido 7.2 á 7.5..... | 7.35 |
| Fresno, perfectamente seco. (Véase la nota puesta en la caoba española)..... | .752 |
| Mil pies corridos de tabla de 12 pulg por 1 pulg pesan 1,748 toneladas. | |

* *N. del T.* — Hemos tomado la lista de pesos específicos de la 16ª edición.

**Las *maderas verdes* pesan generalmente desde 1/5 hasta cerca de la mitad más que las secas; y las maderas ordinarias de construcción cuando están tolerablemente secas como 1/6 más que las perfectamente secas.

## Tabla de pesos específicos y pesos.

| El peso específico de cualquier substancia es=**su peso en gramos por centímetro cúbico.** | | | Peso específico medio. |
|---|---|---|---|
| Fresno blanco americano seco | | | .61 |
| Mil pies corridos de 12 × 1 pulgada pesan de 1,414 toneladas. | | | |
| Feldespato 2.5 á 2.8 | término medio. | | 2.65 |
| Gas ácido carbónico es 1 $^1/_2$ veces más pesado que el aire. | término medio. | | .00187 |
| Greda seca de alfarería 1.8 á 2.1 | — | — | 1.9 |
| — — en terrones | — | — | 1.008 |
| Grasa | — | — | .93 |
| Granate 3.5 á 4.3; precioso 4.1 á 4.3 | — | — | 4.2 |
| Granito 2.56 á 2.88. (Véase piedra caliza.) | | | 2.72 |
| Gneis ordinario 2.62 á 2.76 | término medio. | | 2.69 |
| — en pilas sueltas | — | — | 1.54 |
| Gutapercha | — | — | .98 |
| Guayaco seco | — | — | 1.33 |
| Gas nitrógeno es como $^1/_{35}$ partes más ligero que el aire | | | .0012 |
| Hornablenda negra 3.1 á 3.4 | término medio. | | 3.25 |
| Hidrógeno, es 14 ½ veces más liviano que el aire, y 16 veces más liviano que el oxígeno | término medio. | | .000084 |
| Hielo .917 á .922 | — | — | .92 |
| Hierro fundido 6.9 á 7.4 | — | — | 7.15 |
| — — supuesto generalmente á * | — | — | 7.21 |
| *Metal para cañones* | | | 7.48 |
| Hierro forjado 7.6 á 7.9; el más puro tiene el mayor peso específico | término medio. | | 7.77 |
| Hierro en grandes barras cilíndricas. | | | |
| — supuesto generalmente á | | | 7.6 |
| Hierro en láminas | término medio. | | 7.76 |
| El hierro liviano es señal de que está impuro. | | | |
| Ladrillo bien prensado | término medio. | | 2.40 |
| — duros ordinarios | — | — | 2.00 |
| — blando inferior | — | — | 1.60 |
| Latón (cobre y zinc) fundidos, 7.8 á 8.4 | — | — | 8.1 |
| Marfil | — | — | 1.82 |
| Manteca de puerco | — | — | .95 |
| Mármoles. (Véase piedra caliza). | | | |
| Mampostería de granito ó piedra caliza bien preparada | | | 2.64 |
| Mampostería de granito (bien desbastada ó emparejando las piedras de morrillos y teniendo como $^1/_3$ de la masa de mortero. | | | 2.46 |
| Mampostería de granito bien desbastado, piedra bruta seca | | | 2.21 |
| Mampostería de granito toscamente desbastado, de piedra bruta y con $^1/_4$ á $^1/_3$ de mortero | | | 2.40 |
| Mampostería de piedra arenosa; como $^1/_8$ parte menos que la anterior. | | | |
| Mampostería de ladrillo comprimido de juntas finas. | término medio. | | 2.24 |
| — — mediana calidad | | | 2.00 |
| — — ordinario; ladrillos inferiores blandos. | término medio. | | 1.60 |
| Mercurio á 32° Fahr=0° C | | | 13.62 |
| — 60° F=15° 6 C | | | 13.58 |
| — 212° F=100° C | | | 13.38 |
| Mica 2.75 á 3.1 | | | 2.93 |
| Mortero fraguado 1.4 á 1.9 | | | 1.65 |
| Nogal perfectamente seco | | | .66 |
| Nafta | | | .848 |
| Nieve, acabada de caer | | | .08 á .19 |
| — húmeda y compactada por la lluvia | | | .24 á .80 |

(*) Véanse las tablas del peso del hierro fundido más adelante.

## Tabla de pesos específicos y pesos.

| El peso específico de cualquier substancia es = **su peso en gramos por centímetro cúbico.** | Peso específico medio. |
|---|---|
| Nogal, negro perfectamente seco. (Véase nota en caoba española.) | .61 |
| Mil pies, etc. pesan 1,414 toneladas. | |
| Nogal de América perfectamente seco | .85 |
| Olmo perfectamente seco. (Véase la nota puesta en caoba española) término medio. | .56 |
| Mil pies corridos de tabla de 12 por 1 pulgada pesan 1.302 toneladas. | |
| **Oro** fundido puro, ó de 24 quilates | 19.258 |
| — nativo puro 19.3 á 19.34 término medio. | 19.32 |
| — que contiene plata frecuentemente 15.6 á 19.3. | |
| — puro forjado 19.4 á 19.6 | 19.5 |
| Oolitas 1.9 á 2.5 término medio. | 2.2 |
| Piedra de chispa — — | 2.6 |
| Plomo de comercio 11.30 á 11.47 en barras ó fundido | 11.38 |
| Piedra caliza y mármoles 2.4 á 2.86 | 2.6 |
| — — — ordinariamente como | 2.7 |
| Piedra caliza y mármoles de cantera en fragmentos irregulares: un metro cúbico sólido hace como 1.9 metros cúbicos perfectamente suelto; ó como 1 $^{3}/_{4}$ metros amontonados. En este último caso, .571 del montón es sólido, y las restantes .429 partes de él están vacías amontonado. | 1.54 |
| Petróleo | .878 |
| Pino blanco perfectamente seco .35 á .45 * | .40 |
| 1,000 pies ó 305 metros corridos de tablas de 12 por 1 pulgada pesan .930 toneladas *. | |
| Pino amarillo del Norte .48 á .62 | .55 |
| Mil pies corridos de tablas de 12 por 1 pulgada pesan 1.276 toneladas *. | |
| Pino amarillo del Sur 64 á 80 | .72 |
| Mil pies, etc , pesan 1.67 toneladas *. | |
| Pino de corazón, del amarillo, del Sur (verde). (Nota al pie de caoba española.) | 1.04 |
| Mil pies, etc., pesan 2.418 toneladas. | |
| Pólvora | 1.0 |
| Pórfido 2.66 á 2.8 | 2.73 |
| Platino 21 á 22 | 21.5 |
| — nativo en granos 16 á 19 | 17.5 |
| Pizarra 2.7 á 2.9 término medio. | 2.8 |
| Plata | 10.5 |
| Rubí y zafiro 3.8 á 4.0 | 3.9 |
| Resina | 1.1 |
| Roble fuerte perfectamente seco, .88 á 1.02 * término medio. | .95 |
| — blanco perfectamente seco, .66 á .88 | .77 |
| — rojo negro, etc. * término medio. | .51 á .72 |
| Sal ordinaria; Siracusa, Nueva York | .72 |
| — — Islas Turcas, Cádiz, Lisboa | .99 |
| — — San Barts | 1.12 |
| — — muy seca de las Indias Occidentales | 1.18 |
| — — Liverpool | .67 |
| — fina de Liverpool para uso de mesa e | .78 |
| Serpentina 2.5 á 2.65 | 2.6 |
| Sicomoro, perfectamente seco. (Véase nota al pied . caoba española.) | .59 |
| Mil pies, etc., pesan 1.376 toneladas. | |
| Sulfuro término medio. | 2. |
| Sebo — — | .94 |
| Tiza 2.2 á 2.8. (Véase piedra caliza de cantera.)... — — | 2.5 |

* Véase la nota ** pág. 225.

## Tabla de pesos específicos y pesos.

| El peso específico de cualquier substancia es = **su peso en gramos por centímetro cúbico.** | Peso específico medio. |
|---|---|
| Tierra común perfectamente seca y suelta | 1.15 á 1.28 |
| — — — — movida | 1.31 á 1.47 |
| Tierra común perfectamente seca moderadamente pisada | 1.44 á 1.60 |
| — suelta y apenas húmeda | 1.12 á 1.22 |
| — — más — | 1.06 á 1.09 |
| — — — — movida | 1.20 á 1.44 |
| — — — — moderadamente compacta | 1.44 á 1.60 |
| Tierra común, como un fango blando líquido | 1.66 á 1.79 |
| — — como un fango blando líquido bien comprimida dentro de una caja | 1.76 á 1.92 |
| Turba seca sin comprimir | .32 á .48 |
| Topacio 3.45 á 3.65 | 3.55 |
| Vidrio 2.5 á 3.45 término medio. | 2.98 |
| — ordinario de ventanas — — | 2.52 |
| — grueso de *Millville, Nueva Jersey,* para piso | 2.53 |
| Vinos, .993 á 1.04 | .998 |
| Yeso 2.24 á 2.30 | 2.27 |
| — en pedazos irregulares | 1.31 |
| — molido suelto | .90 |
| — — bien resquebrajado | 1.02 |
| — — calcinado suelto | .83 á .96 |
| Zinc del comercio 6.8 á 7.2 término medio. | 7. |

# PESOS Y MEDIDAS

**Las medidas de longitud y peso de los Estados Unidos y de Inglaterra,** de la misma denominación, pueden ser consideradas como iguales para todas las aplicaciones ordinarias; pero en otra clase de medidas, como las de líquidos, de la misma denominación difieren notablemente en los dos países. **La medida modelo de longitud** de ambos países es teóricamente la de un péndulo de segundos en el vacío al nivel del mar á la latitud de Londres y á la temperatura de 62° Fahr (16°7 C). La longitud de dicho péndulo se supone dividida en 39.1393 partes iguales, llamadas pulgadas; y 36 de estas pulgadas fueron adoptadas como la yarda modelo de ambos países. Pero habiendo sido destruído por el fuego en 1834 el modelo, se encontró que era imposible reponerlo por medio de la medida de un péndulo. La yarda actual inglesa como está determinada á una temperatura de 62° Fahr (16°7 C) (por el modelo conservado en el Parlamento), es el modelo empleado en el levantamiento geodésico y litoral de los E. U. *(The U. S. Coast and Geodetic Survey)* y está reconocido como modelo en todo el país y departamentos del Gobierno, aun cuando no ha sido declarado así por ningún Acto del Congreso.

**El modelo legal de pesos** de los Estados Unidos es la **libra Troy del cuño de Filadelfia.** Este modelo que contiene 5,760 granos, es una copia exacta de la **libra Troy imperial de la Gran Bretaña.** El *avoirdupois* ó libra comercial de los Estados Unidos que contiene 7000 granos, y se deriva de la libra Troy, modelo del cuño, se ha encontrado de acuerdo, con una aproximación de una milésima de grano, con la libra *avoirdupois* británica. « The U. S. Coast Survey » declara, por tanto, los pesos de los dos países iguales.

**La Tonelada.** En las ordenanzas revisadas de los E. E. W. 2.ª edición, 1878, dice : « Dondequiera que se use la palabra tonelada en este capít, refiriéndose á pesos, se considerará como que significa 20 quintales y cada quintal con un peso de 112 libras *avoirdupois.* » Parece que es ésta la única reglamentación del Gobierno de los E. E. W. sobre la materia.

La ton de 2,240 lbs, con frecuencia llamada tonelada bruta (**gross**), se usa generalmente en el comercio de minerales de hierro, hierro nativo, rieles y otras manufacturas de hierro y acero. El **cok** y otros artículos se comercian por tonelada neta de 2,000 libs. La *(bloom)* tonelada tenía 2.464 lbs=2,240 lbs+2 quintales de 112 lbs; y la de hierro nativo tenía 2,268 lbs=2,240 lbs+un *arenaie (sandage)* de 28 lbs por la arena que quedaba adherida al hierro nativo; pero muchos fundidores sólo concedían 14 lbs. En trabajos de tracción eléctrica la ton representa 2.000 lbs.

**Como volumen** la tonelada se considera conteniendo 252 galones (952.56 litros); para medir maderas en bruto de 40 pies cúb (1,132.60 litros) y para madera labrada, de 50 pies cúb (1,415.77 litros). 252 galones (de los E. E. W.) de agua pesan como 2,100 lbs (952.55 kilogramos); 252 gals Imperiales (ingleses) pesan como 2,500 lbs (1,133.98 kilog). *

**El sistema métrico fué legalizado en los Estados Unidos** en 1866, pero no se ha hecho obligatorio.

El Gobierno ha suministrado luego muy exactos modelos métricos á los diferentes Estados. El uso del sistema métrico se permitió en la Gran Bretaña el 6 de agosto de 1897 y en Rusia en 1900. Su uso es ahora por lo menos permitido en todos los países civilizados. La unidad métrica de longitud es el metro, el cual se supuso ser una diez millonésima $\left(\frac{1}{10,000,000}\right)$ del cuadrante de la tierra, esto es, de la parte de un meridiano, comprendida entre cualquier polo y el ecuador. Esta extensión fué medida y fueron preparados una serie de modelos de pesos

*El sistema métrico comparado con el inglés tiene las mismas ventajas y desventajas que las de nuestro sistema americano decimal de monedas, comparados con el sistema monetario inglés de libras, chelines y peniques. Facilita enormemente todos los cálculos; pero como toda innovación causará necesariamente algunos inconvenientes mientras se efectúa el cambio. El sistema métrico tiene también la lejana y gran ventaja de que tiende á hacerse universal en todas las naciones civilizadas.

y medidas, de acuerdo con el resultado, y depositados en los archivos de Francia, en París (metro de archivos, kilogramos de archivos, etc.). Se ha descubierto desde entonces que ocurrieron errores en los cálculos hechos para determinar la longitud del cuadrante; pero los modelos permanecen sin embargo como se prepararon en su origen.

**Las medidas métricas de superficie y de capacidad** son los cuadrados y cubos del metro y de sus fracciones decimales y múltiples.

**La unidad métrica del peso es el gramo**, que es el peso de un mililitro ó de un centímetro cúbico * de agua pura á la temperatura de su mayor densidad como de 4.5° centígrados ó 40° Fahr.

Por medio de la acción unida de los principales gobiernos del mundo, se ha establecido, con su asiento cerca de París, **una oficina internacional de pesos y medidas**. Se han preparado dos barras puras de platino-iridio y se ha hecho de una de ellas un número de kilogramos (1,000 gramos) modelo y de la otra un número de metros modelo, derivados ambos de los modelos de los Archivos de Francia. De estos algunos fueron seleccionadas como modelos internacionales y los otros distribuídos entre los diferentes gobiernos. Los enviados á los Estados Unidos están en poder de la *U. S. Coast Survey*.

La determinación del equivalente del metro en medida inglesa es asunto muy difícil. El *metro* modelo es medido de extremo á extremo de una barra de platino ó 0° C, mientras que la *yarda* modelo se mide *entre dos líneas* trazadas en una escala de *plata* incrustada en una barra de bronce y á 62° F (16°7 C) *The Unities States Coast Survey*† adopta por longitud del metro á 62° Fahr (16°7 C) el valor determinado por el capitán *A. R. Clarke* y el coronel *Sir Henry James*, en 1866. á saber: 39.370432 pulgadas (=3.2808666 pies=1.0936222 yarda); pero el equivalente legal establecido por el Congreso es 39.37 pulgadas (=3.28083 pies = 1.093611 yardas). Este valor es tan exacto como puede deducirse con los datos de que se dispone.

**El gramo pesa** según la determinación del profesor *W. H. Miller* ‡ 15.43234874 granos. Un examen hecho en la oficina internacional de pesos y medidas en 1884 lo hace igual á 15.43235639 granos. El **valor legal** en los Estados Unidos es 15.432 granos.

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* Un centímetro = $\frac{1}{100}$ metro = .3937 pulgadas. Un mililitro ($\frac{1}{1000}$ litro) ó centímetro cúbico = .061 pulgadas cúbicas.

† Apéndice número 22 al informe de 1876, pág. 6.

‡ Transacciones filosóficas, 1856, págs. 893, etc.

## Valores aproximados de las monedas extranjeras en dinero de los Estados Unidos.

**Las referencias** (1, 2, 3 y 4) están en las notas al pie.

De la circular del Departamento del Tesoro, oficina del cuño de los Estados Unidos, enero 1.º de 1887, de la « Cuestión monetaria », por H. Costes, París, 1884, y de nuestra décima edición.

República Argentina. Peso=100 centavos, 96.5 cts[2-3]. Argentino=5 pesos, $4.82.

Austria. Florín=100 kreutzer, 47.7 cts[2], 35.9 cts[3]. Ducado, $2.29. María Teresa *Thaler* ó *Levantín*, 1780, $1.00[3]. *Rix Thaler* 97 cts[4]. Soberano, $3.57[4].

Bélgica. Franco=100 céntimos, 17.9 cts[2], 19.3 cts[3].

Bolivia. Boliviano=100 céntimos, 96.5 cts[2], 72.7 cts[3]. Onza, $14.95. Dólar 96 cts[4].

Brasil. Milreis=1,000 reis, 50.2 cts[2] 54.6 cts[3].

Canadá. Moneda de los Estados Unidos é inglesa. También la libra, $4[4].

Centro América[4]. Doblón $14.50 á 15.65. Real, término medio, 5 3/4 cts. (Véase Honduras.)

Ceylán. *Rupee*, lo mismo que en la India.

Chile. Peso=10 dineros ó décimos=100 cts, 96.5 cts[2], 91.2 cts[3]. Condor=dos doblones=5 escudos=10 pesos. Dólar, 93 cts[4].

Cuba. Peso 93.2 cts[3]. Doblón, $5.02.

Dinamarca. Corona=100 oro, 25.7 cts[2], 26.8 cts[3]. Ducado, $1.81[4]. *Skilling* 3/4 cts[4].

Ecuador. Sucre, 72.7 cts[3]. Doblón, $3.86. Condor, $9.65. Dólar, 93 cts[4]. Real 9 cts[4].

Egipto. Libra=100 *piastres*=4,000 *paras*, $4.94,33[3].

Finlandia. *Markka*=100 *penni* 19.1 cts[3] 10 *markkaa*, $1.93.

Bulgaria (Principado de). 5 *leva*=96.5 cts[3]. 1 *lew*=17.95 cts[3]. 50 stotinkis=8.88 cts[3].

Francia. Franco=100 céntimos, 17.9 cts[2], 19.3 cts[3]. Napoleón $3.84[4]. Libra 18.5 cts[4]. Centavo 1 cts[4].

Alemania. Marco=100 pfennigs, 21.4 cts[2], 23.8 cts[3]. Augusto (Sajonia) $3.98[4]. Carolina (Baviera), $4.93[4]. Corona (Baden, Baviera, N. Germania), $1.06[4]. Ducado (*Hamburg Hanover*), $2.28[4]. Florín (Prusia, Hanover), 55 cts[4]. *Groschen*, 2.4 cts[4]. *Kreutzer* (Prusia), 0.7 cts. Maximiliano (Baviera), $3.30[4]. *Rix Thaler*, (Hamburg Hanover), $1.10[4] (Baden Brunswick), $1.00[4] (Prusia *N. Germany*, Bremen, Sajonia, Hanover), 69 cts[4].

Gran Bretaña. Libra esterlina ó Soberano (£)=20 chelines=240 peniques, $4.86.65[3]. Guinea=21 chelines. Corona=5 chelines. Chelin(s) 22.4 cts[2], 24.3 cts (1/20 de libra esterlina). Penique (d), 2 cts.

Grecia. *Drachma*=100 *lepta*, 17 cts[2], 19.3 cts[3].

Haití. Gourde de 100 cts 96.5 cts[2,3].

Honduras. Fuerte ó Peso de 100 centavos $1.01. (Véase Centro América.)

India. *Rupee*=16 *annas*, 45.9 cts[2], 34.6 cts[3]. *Mohur*=15 *rupees*, $7.10. *Star pagoda* (Madras), $1.81[4].

Italia, etc.[4] Lira=100 *centésimos*, 17.9 cts[2], 19.3 cts[3]. *Carlin* (Cerdeña), $8.21[4]. Corona (Sicilia), 96 cts[4]. Libra (Cerdeña), 18.5 cts[4].(Toscana, Venecia), 16 cts[4]. Onza (Sicilia), $2.50[4]. Paolo (Roma), 10 cts[4]. Pistola (Roma), $3.37[4]. Escudo[4] (Piamonte), $1.36. (Génova) 1.28, (Roma) $1.00 (Nápoles, Sicilia) 95 cts. (Cerdeña), 92 cts. Teston (Roma), 30 cts[4]. *Zecchino* (Roma), $2.27[4].

Japón. *Yen*=100 *sen* (oro) 99.7 cts[1] (plata) $1.04[2], 78.4 cts[3].

Liberia. Dólar $1.0034.

México. Dólar ó peso=100 centavos (oro) 98.3 cts, (plata), $1.05[3], 79 cts[3]. Onza ó doblón=16 pesos $15.74.

Países Bajos. Florín de 100 cts, 40.5 cts[2], 40.2 cts[3]. *Ducatoon*, $1.32[4]. *Guilder*, 40 cts[4]. *Rix dólar*, $1.05[4]. *Stiver*, 2 cts[4].

Nueva Granada. Doblón $15.34[4].

Noruega. Corona=100 *(ore)*=30 *skillings*, 25.7 cts[2], 26.8 cts[3].

Paraguay. Peso=8 reales, 90 cts.

Persia. *Thoman*=5 *sachib* — *kerans*=10 *banabats*=25 *abassis*=100 *scahis*, $2.29.

Perú. Sol=10 *dineros*=100 cts 96.5 cts[2]. 72.7 cts[3]. Dólars, 93 cts[4].

Portugal. *Milreis*=10 *tostones*=1,000 *reis*, $1.08[3]. Corona=10 *milreis*. *Moidore* $6.50[4].

Rusia. Rublo=2 poltinniks=4 tchetvertaks=5 abassis=10 griviniks=20 pie.

taks=100 kopecks, 77 cts[2], 58.2 cts[3]. Imperial=10 rublos, $7.72. Ducado=3 rublos, $2.39.

Islas de Sandwich. Dólar, $1.00[2].

Sicilia. (Véase Italia.)

España. Peseta=100 céntimos, 17.9 cts[2], 19.3 cts[3]. Doblón (nuevo)=10 escudos=100 reales, $5.02. Duro=2 escudos[2], $1.00[2]. Doblón (viejo), $15.65[4]. (Casi todo este sistema ha desaparecido.) Pistola=2 coronas, $3.90[4]. Peso, $1.04[4]. Real de plata 10 centavos[4]. Real de vellón 5 cts[4].

Suecia. Corona=100 ore, 25.7 cts[2], 26.8 cts[3]. Ducado, $2.20[4]. *Rix Dólars*, $1.05[4].

Suiza. [1] Franco=100 céntimos, 17.9 cts[2], 19.3 cts[3].

Trípoli. Mahbub=20 pesos, 65.6 cts[3].

Túnez. Peso=16 *karobos*, 12 cts[3]. 10 pesos, $1.16.6.

Turquía. Peso, 40 *paras*, 4.4 cts[3]. *Zecchin*, 1.40[1].

Colombia. Peso=10 décimos=100 centavos, 96.5 cts[2], 72.7 cts[3]. Condor=10 pesos, $9.65. Dólar, 93.5[4] cts.

Uruguay. Peso=100 centavos ó centésimos (oro), $1.03 (plata), 96.5 cts[2].]

Venezuela. Bolívar=2 reales, 17.9 cts[2], 19.3 cts[3].

1. Francia, Bélgica, Italia, Suiza y Grecia forman la unión latina. Sus monedas son parecidas en diámetro, peso y finura.

2. 19.3 veces el valor de una moneda sencilla en francos, según la de Costes.

3. Cambio á la par ó valor equivalente en términos de dólars de oro de los Estados Unidos (Circular del Tesoro).

4. De nuestra décima edición.

**El oro perfectamente** puro vale $1, los 23.22 grs=$20.67183 la onza troy=$18.84151 la onza *avoir*. El **Standard** (moneda de los E. U.) vale $18.60465 por onza troy=$16.95736 por onza *avoir*. Consta de 9 partes de peso de oro puro y 1 parte de aleación. Su valor es el del oro puro solamente; porque han sido pagados por el gobierno los gastos de la liga y de la acuñación.

El oro puro se llama fino ú oro de 24 *quilates;* y cuando se liga; la liga se supone dividida en 24 partes en peso, y según sean 10, 15 ó 20, etc., de estas partes de oro puro, se dice que la liga es de 10, 15 ó 20, etc., *quilates*.

**El término medio de la pureza del oro nativo de California**, por algunos millares de ensayos hechos en el cuño de los Estados Unidos en Filadelfia, es 88.5 partes de oro 11.5 de plata. Para algunos de Georgia, 99 % oro.

**La plata pura** fluctúa en valor; así, durante 1878, 1879 fluctuó entre $1.5 y $1.18 la onza *troy* ó $.957 y $1,076 por onza *avoir*.

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*N. del T.* — Aunque muchas de las medidas que van á continuación son exclusivamente inglesas, como las relaciones entre los países que usan este sistema con los que usan el métrico, aumentan día por día, hemos querido dejarlas tal como están en el texto.

### Peso troy de los E. U. y de la Gran Bretaña.

| | |
|---|---|
| 24 granos..... | 1 *pennyweight*, dwt. |
| 20 pennyweights..... | 1 onza=480 granos. |
| 12 onzas..... | 1 libra=240 *pennyweights*=5,760 granos. |

**El peso troy se usa para oro y plata. Un quilate** de los joyeros, para piedras preciosas es, en los Estados Unidos=3.2 granos; en Londres 3.17 grns; en París 3.18 grns, dividido entre 4 granos de joyeros. En los pesos **troy**, los de boticarios y en el **avoirdupois, el grano es el mismo.**

### Peso americano y británico de los boticarios.

| | |
|---|---|
| 20 granos..... | 1 escrúpulo. |
| 3 escrúpulos.... | 1 dracma=60 granos. |
| 8 dracmas..... | 1 onza=24 escrúpulos=480 granos. |
| 12 onzas..... | 1 libra=96 dracmas=288 escrúpulos=5,760 granos. |

En peso troy y de boticario, el grano, onza y libra son los mismos.

## Peso avoirdupois ó comercial de los Estados Unidos y de la Gran Bretaña.

| | |
|---|---|
| 27.34375 granos..... | 1 dracma. |
| 16 dracmas..... | 1 onza=437½ granos. |
| 16 onzas..... | 1 lib=256 dracmas=7,000 granos. |
| 28 libras..... | 1 cuarto=448 onzas. |
| 4 cuartos..... | 1 quintal=112 lib. |
| 20 quintales.... | 1 tonelada=80 cuartos=2,240 lib. |

**1 stone**=14 libras; **1 quintal**=100 libras **avoir.**

**El modelo de la libra avoirdupois**, que es la que está en uso comercial común, es el peso de 27.7015 pulgadas cúbicas de agua destilada pura á su mayor densidad á 39º2 Fahr (4º C) en la latitud de Londres, al nivel del mar y á la presión de 760 mm (ó sean=.453593 kilogramos). Pero esto adolece de error como de 1 en 1,362.

**Una libra troy**=.82286 de lib **avoir**; 1 libra avoir=1.21528 libra troy ó de boticario.

**Una onza troy**=1.09714 onza avoir; 1 onza avoir=.911458 onza troy ó de boticario.

## Medida de longitud de los Estados Unidos y Gran Bretaña*.

| | |
|---|---|
| 12 pulgadas. | 1 pie=.3047973 de metro. |
| 3 pies..... | 1 yarda=36 pulg=.9143919 de metro. |
| 5½ yardas. | 1 pértica ó percha=16½ pies=198 pulg=5.02 mets. |
| 40 pértica.. | 1 estadio=220 yardas=660 pies=201.17 mets. |
| 8 estadios.. | 1 milla leg ingl=320 pértic=1,760 yard =5,280 pies = 63,360 pulg. |
| 3 millas.... | 1 legua=24 estadios=960 pértica=5,280 yards =15,840 pies = 4828.03 metros. |

**1 punto**=$^1/_{72}$ de pulg; **1 línea**=6 puntos= $^1/_{12}$ de pulg; **1 mano**=4 pulg; **1 palmo**=9 pulgadas.

**1 brazada**=6 pies. **1 cable**=120 brazadas=720 pies; **1 cadena** de agrimensor de **Gunter** tiene de largo 66 pies ó 4 pértica. Tiene 100 eslabones de 7.92 pulgadas de largo. Ochenta cadenas de **Gunter**=1 milla. (*N. del T.* — La pulgada tiene .0253954 metros.)

**1 milla marina geográlica ó nudo** se define diversamente como = á la longitud de

| | metros. | pies. | milla legal. |
|---|---|---|---|
| 1 minuto de *longitud* en el Ecuador = | 1,855.345 | 6,087.15 | 1.15287 |
| 1 — de *latitud* en el Ecuador = | 1,842.787 | 6,045.95 | 1.14507 |
| 1 — — en el polo = | 1,861.655 | 6,107.85 | 1.15679 |
| 1 — — á la latitud 45º = | 1,852.181 | 6,076.76 | 1.15090 |
| 1 — de un círculo máximo de una verdadera esfera cuya área sea igual á la de la tierra. = | Valor adoptado por **U. S. Coast and Geodetic Survey** 1,853.248 | 6,080.27 | 1.15157 |
| Milla ó nudo del **Almirantazgo británico.** = | 1,853.169 | 6,080.00 | 1.15152 |

Las anteriores longitudes de minutos, en metros y pies, son las publicadas por The U. S. Coast and Geodetic Survey en el apéndice n.º 12 del informe de 1881 y están calculados del esferoide de Clarke que es ahora el modelo en aquel cuerpo.

En el ecuador, **1º de latitud es**=68.70 millas terrestres á la latitud de 20º=68.78; á 40º=69.00; á 60º=69.23; á 80º=69.39; á 90º=69.41.

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* *N. del T.* — Hemos agregado las equivalencias métricas.

**Largo de 1. grado de longitud en diferentes latitudes y al nivel del mar.** Estos largos son en millas terrestres comunes de 5,280 pies (=1,609.35 metros). Como la figura de la tierra no se ha determinado nunca exactamente, no son sino aproximaciones. Las intermedias pueden encontrarse correctamente por una simple proporción; 1º de longitud corresponde á 4 minutos de tiempo y 1 minuto de longitud á 4 segundos.

| Gr. de lat. | Millas. | Gr. de lat. | Millas. | Gr. de lat. | Millas. | Gr. de lat. | Millas. | Gr. de lat. | Millas. | Gr. de lat. | Millas |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 69.16 | 14 | 67.12 | 28 | 61.11 | 42 | 51.47 | 56 | 38.76 | 70 | 23.72 |
| 2 | 69.12 | 16 | 66.50 | 30 | 59.94 | 44 | 49.83 | 58 | 36.74 | 72 | 21.43 |
| 4 | 68.99 | 18 | 65.80 | 32 | 58.70 | 46 | 48.12 | 60 | 34.67 | 74 | 19.12 |
| 6 | 68.78 | 20 | 65.02 | 34 | 57.39 | 48 | 46.36 | 62 | 32.55 | 76 | 16.78 |
| 8 | 68.49 | 22 | 64.15 | 36 | 56.01 | 50 | 44.54 | 64 | 30.40 | 78 | 14.42 |
| 10 | 68.12 | 24 | 63.21 | 38 | 54.56 | 52 | 42.67 | 66 | 28.21 | 80 | 12.05 |
| 12 | 67.66 | 26 | 62.20 | 40 | 53.05 | 54 | 40.74 | 68 | 25.98 | 82 | 9.66 |

**(N. del T. La misma tabla anterior en kilómetros.)**

| Gr. de lat. | Kilómetros. | Gr. de lat. | Kilómetros. | Gr. de lat. | Kilómetros. | Gr. de lat. | Kilómetros. | Gr. de lat. | Kilómetros. | Gr. de lat. | Kilómetr. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 111.300 | 14 | 108.019 | 28 | 98.347 | 42 | 82.833 | 56 | 62.378 | 70 | 38.174 |
| 2 | 111.238 | 16 | 107.021 | 30 | 96.464 | 44 | 80.194 | 58 | 59.127 | 72 | 34.488 |
| 4 | 111.029 | 18 | 105.895 | 32 | 94.468 | 46 | 77.442 | 60 | 55.795 | 74 | 30.771 |
| 6 | 110.691 | 20 | 104.639 | 34 | 92.360 | 48 | 74.609 | 62 | 52.384 | 76 | 27.005 |
| 8 | 110.224 | 22 | 103.239 | 36 | 90.139 | 50 | 71.680 | 64 | 48.924 | 78 | 23.207 |
| 10 | 109.628 | 24 | 101.727 | 38 | 87.806 | 52 | 68.670 | 66 | 45.400 | 80 | 19.392 |
| 12 | 108.888 | 26 | 100.101 | 40 | 85.378 | 54 | 65.565 | 68 | 41.811 | 82 | 15.546 |

### Medidas de superficie ó medidas agrarias británicas y de los Estados Unidos.

144 pulgadas cuadradas=1 pie cuadrado (=.0929 mets. cuad.), (cien (100) pies cuadrados=**1 Square**) (=9.289968 met. cuad.). (*Obs. del T.* — No hay en español medida usada que equivalga al **Square**.)

9 pies cuadrados=1 yarda cuadrada=1,296 pulg cuad. (=.836.126 m cuad).

30 $^{1}/_{4}$ yardas cuadradas=1 pértica cuadrada=272 $^{1}/_{4}$ pies cuad. (=25.29 met. cuad.).

40 pérticas cuadradas=1 *rood* ($^{1}/_{4}$ de acre cuadrada)=1,210 yards cuad.

4 rood=1 acre =160 pérticas=4,840 yards cuad.=43,560 pies cuadrados.

640 acres=1 milla cuadrada.

**Los terrenos** públicos de los E. E. W. están divididos en **townships** (*N. del T.* — No tiene equivalente en español. Son cuadrados de 6 millas (9,656 mts) por lado, limitados por paralelos y meridianos.) Nominalmente cada **township** contiene 36 **secciones** de una milla cuadrada cada una y éstas á su vez cuatro **cuartos de sección** de .5 milla en cuadro cada una. Por **la ley**, los errores, incluyendo los de la convergencia de los meridianos, deben cargarse á los cuartos de sección al norte y al oeste del **township**.

| | Logaritmos. |
|---|---|
| **1 pulgada circular**=círculo de 1 pulg de diám= .785398..... pulg cuadrada.....n | 1.895 0899 |
| **1 pulgada cuadrada**= 1.27324..... pulg circular..... | .104 9101 |

### Medidas sólidas de los E. E. W. y Gran Bretaña*.

1,728 pulg cúb=1 pie cúb=28.315 litros; 27 pies cúb=1 yarda cúbica=.765 met cúb; 1 pulg cúb=16.39 cent cúb.

* *N. del T.* — Hemos agregado las equivalencias métricas.

**1 pértica (Rod) inglesa de albañil**=16.5 pies cuad (1.54 mets cuad) en un muro de 14 pulg (.m35) de espesor.
**1 pértica (Rod) de ingeniero**=306 pies cúb=8.66 mets cúb.
**1 toesa**=261.5 pies cúb=7.40 met cúb.
**1 chaldron** (medida de carbón)=58.64 pies cúb=1.66 met cúb.

| | | Logaritmos. |
|---|---|---|
| **1 pie cúbico**= | | |
| 3,300.24..... | pulgs esféricas..... | 3.518 5451 |
| 1.90986..... | pies esféricos..... | .281 0014 |
| .803564..... | *bushels* E. E. W.....n | 1.905 0204 |
| .267855..... | barril de harina de 3 *bushels*.....n | 1.427 8991 |
| .237477..... | barril de líquid de 31½ gals.....n | 1.375 6211 |
| **1 pulgada cúbica**= | | |
| 1.90986..... | pulg esférica..... | .281 0014 |
| **1 yarda cúbica**= | | |
| 201.974..... | galones (E. E. W.)..... | 2.305 2955 |
| 7.23207..... | barril de harina de 3 *bushels*..... | .859 2629 |
| 21.6962..... | *bushel*..... | 1.336 3842 |
| **1 pie esférico**=á una esfera de 1 pie de diám= | | |
| .523599..... | pie cúb.....n | 1.718 9986 |
| 904.779..... | pulgs cúb..... | 2.956 5423 |
| 14.8268..... | litros..... | 1.171 0461 |
| **1 pulgada esférica**=esfera de 1 pulg de diám= | | |
| .523599..... | pulg cúb.....n | 1.718 9986 |
| 8.58030..... | centíms cúb..... | .933 5024 |

### Medidas imperiales británicas para líquidos y para áridos.

Este sistema está establecido en toda la Gran Bretaña con exclusión de las antiguas. Su base es el galón imperial de 277.274 pulgs cúbs (4.543 lit) ó sean 10 lbs avoir (4.535927 kilog) de agua pura á la temperatura de 62º F (16º6 C) cuando el barómetro está á 30 pulgs (762 mm). **Esta base tiene un error** como de 1 en 1836, porque 10 lbs (4.535927 kilog) de agua equivalen solamente á 277.123 pulgs cúbs.

(*N. del T.* — En esta tabla hemos agregado las equivalencias en sistema métrico.)

| | | Libras avoir de agua. | Peso en kil. á 62 ° F. ó sean 16 66 C y barómetro á 30 pulg. | Pulg. cúb. | Litros. | Pies cúb. | Lado del cubo de igual capacidad. — Pulgs. | Lado del cubo de igual capacidad. — Centím. |
|---|---|---|---|---|---|---|---|---|
| 4 *gills* | 1 pinta... | 1.25 | .567 | 34.6592 | .568 | ..... | 3.2605 | 8.2816 |
| 2 pintas | 1 *quart*... | 2.50 | 1.134 | 69.3185 | 1.136 | ..... | 4.1079 | 10.4338 |
| 2 *quarts* | 1 *pottle*... | 5. | 2.267 | 138.637 | 2.272 | ..... | 5.1756 | 13.1458 |
| 2 *pottles* | 1 galón... | 10. | 4.536 | 277.274 | 4.543 | ..... | 6.5208 | 16.5625 |
| 2 galones | 1 *peck*.... | 20. | 9.072 | 554.548 | 9.087 | ..... | 8.2157 | 20.8675 |
| 4 *pecks* | 1 *bushel*.. | 80. | 36.287 | 2218.192 | 36.348 | 1.2837 | 13.0417 | 33.1253 |
| 4 *bushels* | 1 *coomb* .. | 320. | 145.150 | 8872.768 | 145.390 | 5.1347 | ..... | ..... |
| 2 *coombs* | 1 *quarter*.. | 640. | 290.299 | 17745.536 | 290.782 | 10.2694 | ..... | ..... |

Medidas de áridos

*N. del T.* — El peso del agua á la temperatura de 62º F (16º6 centg) con el barómetro á 30 pulgs (762 mm) es = 62.355 lbs el pie cúb = .99889 kilgms el litro; á la temperatura de 4º centg, sobre cero, con barómetro á 76 cent, es = 1 kilogramo el lit.
El galón imperial = .16046 pies cúb.

n. significa característica negativa.

(*N. del T.* — Hemos agregado las equivalencias métricas que siguen.)

| | Litros. |
|---|---|
| *Gill* | .14206 |
| Pinta | .56824 |
| *Quart* | 1.13649 |
| Galón | 4.54596 |
| *Peck* | 9.09192 |
| *Bushel* | 36.3677 |
| Quarter | 290.9416 |

| | Gramos. |
|---|---|
| Dragma *avoir dupois* | 1.771836 |
| Onza | 28.349375 |
| Libra | 453.59265 |
| *Hundred weight* | 50,802.35 |
| Tonelada | 1,016,047.0 |
| Grano Troy | .06479 |
| *Penny weight* | 1.5552 |
| Onza | 31.10346 |
| Libra | 373.2419 |

El peso del agua nos ofrece un medio sencillo de encontrar el volumen de un envase. Pésese primero el envase solo y después lleno de agua. La diferencia será el peso del agua, y éste dividido por el número de kilogramos por metro cúbico correspondiente á la temperatura del agua que indicamos en la tabla siguiente, será la capacidad en metros cúbs de dicho envase (estando a nivel del mar).

| Temp. Fahr. | Temp. Centíg. | Kilog por $m^3$ |
|---|---|---|
| 32° | 0° | 999.879 |
| 40° | 4°4 | 999.975 |
| 50° | 10° | 999.751 |
| 60° | 15°6 | 999.078 |
| 70° | 21°1 | 998.037 |
| 80° | 26°7 | 996.691 |
| 90° | 32°2 | 995.106 |
| 212° | 100° | 956.355 |

Cuando no se requiera tan extremada precisión puede calcularse el peso de metro cúbico de agua en 1,000 kilogs que es el que corresponde á la temperatura de 4° C.

## Medidas métricas de longitud según modelos de los Estados Unidos é ingleses.

| | Pulgadas. | Pies. | Yardas. | Millas. |
|---|---|---|---|---|
| Milímetro | .039370 | .003281 | ..... | ..... |
| Centímetro | .39370428 | .032809 | ..... | ..... |
| Decímetro | 3.9370428 | .3280869 | .1093623 | ..... |
| Metro | 39.370428 | 3.280869 | 1.093623 | ..... |
| Decámetro | 393.70428 | 32.80869 | 10.93623 | ..... |
| Hectómetro | Medidas de caminos. | 328.0869 | 109.3623 | .0621375 |
| Kilómetro | | 3280.869 | 1093.623 | .6213750 |
| Miriámetro | | 32808.69 | 10936.23 | 6.213750 |
| | 1 pulg = .m0254 | 1 pie = .m304794 | 1 yard = .m914383 | 1 milla = 1.6093 kilom. |

## Medidas métricas de superficie según modelo de los Estados Unidos é Inglaterra.

| | Pulgs cuads. | Pies cuads. | Yardas cuads. | Acres. |
|---|---|---|---|---|
| Milímetro cuadr. | .001550 | .00001076 | .0000012 | ..... |
| Centímetro — . | .155003 | .00107641 | .0001196 | ..... |
| Decímetro — . | 15.5003 | .10764101 | .0119601 | ..... |
| Metro — . | 1550.03 | 10.764101 | 1.196011 | .000247 |
| Decámetro — . | 155003 | 1076.4101 | 119.6011 | .024711 |
| Decárea (desuso). | ..... | 10764.101 | 1196.011 | .247110 |
| Hectárea..... | ..... | 107641.01 | 11960.11 | 2.47110 |
| Kilómetro cuadr. | .3861090 mill. | 10764101 | 1196011. | 247.110 |
| Miriámetro — . | 38.61090 cuad | ..... | ..... | 24711.0 |
| | 1 pulg. cuad. = 6.45159 cent. cuad. | 1 pie cuadr. = .0929 met. cuad. | 1 yarda cuad. = .83613 met. cuad. | 1 acre = .404671 hectáreas. |

## Medidas métricas cúbicas ó de capacidad según modelo

Sólo las marcadas « *Brit* » son británicas.

| | Pulgs cúbs. | | |
|---|---|---|---|
| Mililitro ó centímetro cúbico.. | .0610254 | Para líquido. | .0084537 *gill.* |
| | | — . | .0070428 *brit gill.* |
| | | Para árido. | .0018162 pinta de árido. |
| Centilitro..... | .610254 | Para líquido. | .084537 *gill.* |
| | | — . | .070428 *brit gill.* |
| | | Para árido. | .018162 pinta de árido. |
| Decilitro..... | 6.10254 | Para líquido. | .84537 *gill* = .21134 pinta. |
| | | — . | .70428 (*brit*) *gill* = .17607 *brit* pinta. |
| | | Para árido. | .18162 pinta. |
| Litro ó decímetro cúbico.... | 61.0254 | Para líquido. | 1.05671 cuarto = 2.1134 pintas. |
| | | — . | .88036 (*brit*) cuarto = 1.7607 (*brit*) pinta. |
| | | Para árido. | .11351 *peck* = .9081 pinta. |
| Decalitro..... | 610.254 | Para líquido. | 2.64179 galón líquid. de los E.U. |
| | **Pies cúbs.** | — . | 2.20090 galón *brit.* |
| | .353156 | Para árido. | .283783 *bushel* = 1.1351 *peck* = 9.081 cuartos. |
| Hectolitro..... | 3.53156 | Para líquido. | 26.4179 galón líquido de los E.U. |
| | | — . | 22.0090 galón *brit.* |
| | | Para árido. | 2.83783 *bushel.* |
| Kilolitro, metro cúbico ó estero..... | 35.3156 | Para líquido. | 264.179 galón líq. } yard. cúb. |
| | | — . | 220.090 *brit.* } 1.3080. |
| | | Para árido. | 28.3783 *bushel.* } |
| Mirialitro..... | 353.156 | Para líquido. | 2641.79 galón líq. } yards cúb. |
| | | Para árido. | 283.783 *bushel.* } 13.080. |

**Pesos métricos reducidos á peso comercial ordinario ó Avoir de 1 libra=16 onzas ó 7,000 granos.**

| | Granos. |
|---|---|
| Miligramos | .015432 |
| Centigramos | .15432 |
| Decigramos | 1.5432 |
| Gramo | 15.432 |
| | Libras avoir. |
| Decagramo | .022046 |
| Hectogramo | .22046 |
| Kilogramo | 2.2046 |
| Miriagramo | 22.046 |
| Quintal * | 220.46 |
| Tonelada | 2,204.6 |

El gramo es la base de los pesos franceses; y es el peso de un centímetro cúbico de agua destilada á su mayor densidad, al nivel del mar, á la latitud de París, barómetro 29.922 pulgadas ($^{m}0.76$).

## Rusas.

(*N. del T.* — Siempre introducimos el sistema métrico.)

**El pie**, lo mismo que el pie de los E. U. ó pie británico. **Sachine**=7 pies=$=2^{m}.13356$. **Verst**=500 sachine=3,500 pies=1,166 $^{2}/_{3}$ yardas=.6629 de milla=1,066.78 metros. **Pood**=36,114 libras *avoirdupois*=(16,381 kilogramos)

## Españolas.

**El castellano** de España y Nueva Granada, para pesar oro es diversamente estimado entre 71.07 á 71.04 granos (4.605 á 4.603 gramos). A 71.055 granos (4.604 gramos) (término medio entre los dos) 1 *avoirdupois* ú onza comercial ordinaria contiene 6.1572 castellanos; y 1 libra *avoirdupois* contiene 98.515. También una onza troy es=6.7553 castellanos y 1 libra troy es=81.064 castellanos. Tres dólares de oro de los E. U. pesan como 1.1 castellanos.

**El marco español** para metales preciosos, debe tomarse en la práctica, en Sud América, por .5065 de 1 libra *avoirdupois* (.2297 kilog). En España, 0.5076 de la mayor parte de ellos están sin embargo entre .5 y .54 de 1 libra *avoirdupois* (.227 y .243 kilog). El .5065 de 1 libra (.2297 kilog) es=3545½ granos; y .5076 de libra (.230 kilog) es=3553.2 granos. Un marco=50 castellanos=400 *tomines*=4800 granos de oro español.

**La arroba** tiene varios valores en diferentes partes de España. La de Castilla ó Madrid es 25.4025 *libras avoirdupois* (11.522 kilog); **la tonelada** de Castilla es=2032.2 libras *avoirdupois* (921.79 kilog); **el quintal** es=101.61 libras *avoirdupois* (46.089 kilog); **la libra** es=1.0161 libras *avoirdupois* (0.4608 kilog); **la cántara** de vino etc., de Castilla es=4.263 galones de los E. U. (16.11 litros) la de la Habana es=4.1 galones (15.50 litros).

**La vara** de Castilla ($^{m}$83591) es=3.8748 pulgs ó casi 32 $^{7}/_{8}$ pulgs justas; ó 2 pies 8 $^{7}/_{8}$ pulgs. **La fanegada** de terreno desde el año de 1801 es=1.5871 acres=69134.08 pies cuad (.64225 hectáreas). **La fanega** de granos, etc., es=1.59914 *bushel* raso de los E. U. En California **la vara legal** es=33.372 pulgs de los E. U. ($0^{m}847633$); y **la legua** es=5,000 varas, ó 2.6335 millas de los E. U.

# TABLAS DE CONVERSIÓN PARA UNIDADES DE MEDIDAS, PESOS, etc. (Introducción).

En las tablas que siguen las cifras decimales que se repiten están indicadas por un *. Así : 1.2*01=1.201010101...

**Los valores exactos** están indicados por un †. Así : 12† pulg=1 pie; 1 metro=.001† kilómetro.

**Las características negativas** se indican por la letra n. Así: n 1.2855573=$\overline{1}$.2855573=.2855573 — 1=9.2855573 — 10. Véase pág. 74 para el uso de los logaritmos.

La cadena y eslabones de Gunter y de Ingenieros, se distinguen por una G, y una I. Así : una cadena G=66† pies; 1 eslabón I=1† pie.

La tabla A contiene ecuaciones que tienen por base las equivalencias de las tablas de 1 á 36.

En la tabla C se dan números y equivalentes de uso común para transformar las ecuaciones como se explica abajo.

Cada tabla ha sido calculada independientemente por dos personas, á lo menos, y sus resultados comparados. Uno de estos trabajos se dió como copia al compositor y las pruebas fueron corregidas por el otro.

Los cálculos comprenden muchas veces el número de las ecuaciones que aquí se dan, y de haber publicado una considerable proporción de estos cálculos, se habría hecho una tabla demasiado voluminosa para un formulario de bolsillo.

Con poca práctica, el lector puede, no obstante, encontrar muchas otras equivalencias por medio de estas que se dan. Al hacer esto, los logaritmos dados en la tabla C serán muy útiles.

Supongamos que se desee la longitud del metro en *rods*. En la tabla 1 (longitud) ó en la C (equivalentes de uso común) se encuentra.

1 met=3.28083 pies, Log .515 9842

y en la tabla 1 se encuentra :

1 pie=.*0606 *rod*, Log n 2.782 5161

Sumando los logaritmos se encuentra :

1 metro=.19884 *rod*, Log n 1.298 5003

Otros ejemplos : 1 met por seg=? millas por día.

Tabla 18 (longitud por tiempo ó velocidad) da :

1 metro por seg=2.23693 millas por hora. Log .349 6528

y la tabla 5 (tiempo) ó la C dan :

1 día 24 horas Log 1.380 2112

Sumando logs se tiene :

1 met por seg=53.686 millas por día. 1.729 8640

En algunos casos de uso frecuente damos dos tablas, conteniendo cada una el *inverso* de los valores de la otra; así damos « Pesos de los volúmenes » (como 1 pie cúb=? lbs) y « Volúmenes de los Pesos » (como 1 lb=? pies cúbs) de agua. Cuando sólo uno de los dos inversos está dado, el otro se deduce muy fácilmente del conocido. (Véase la primera *Advertencia.*)

Log 1 .000 0000

1 pie por seg=720 pulgs por min Log 2.857 3325

1 seg por pie=.0013*888 min por pulg. Log n 3.142 6675

**Advertencia**. Debe observarse que aunque un cuerpo moviéndose 1 pie por seg (=720 pulgs por minuto) necesita 1 seg para recorrer un pie (.0013*888 min para 1 pulg) y aunque 1 pie por seg y 1 seg por pie son numéricamente iguales, no son en realidad idénticos sino inversos. Así, también,

1 real por dólar=1 : 10

pero, 1 dólar por real=10 : 1

Seleccionando las equivalencias para publicarlas, hemos preferido, en general, aquellas que no pueden fácilmente encontrarse por otras de la misma tabla ó con tablas muy simples. Así al dar en la tabla 12 (Peso por superficie) el equivalente de kilogramo por centím cuad, en lbs por pulgs cuad, se omitió el equivalente en lbs por pie cuad que se encuentra usando la tabla 2 (Superficies) del equivalente dado en libras por pulgs cuad.

Lo mismo el equivalente de una presión de 1 lb por pulg cuad en onzas por pulg cuad ó en lbs por pie cuad, no se dió en la tabla 12 (Pesos por superficie); porque se deducen de 1 lb por pulg cuad por

* Decimales que se repiten. † Valores exactos. n, Característica negativa. I, Ingenieros. G, Gunter.

medio de la tabla 4 (Pesos) y de la tabla 2 (Superficies) respectivamente.

Como las relaciones entre los valores de las monedas de distintas naciones cambian con frecuencia, los equivalentes de las tablas 25 á 30 inclusive, (Valores) pueden ser algunas veces simplemente aproximadas.

Sus logs se dan solamente hasta la tercera decimal, no obstante habérselos calculado hasta la 7.ª Cuando se quiera más aproximación úsense los logs de 4 decimales, págs 80 y 81.

### Lista de las tablas (índice).


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## Tabla A.

### Equivalencias fundamentales.

(Véanse las notas abajo.)

| | | | Logaritmos. |
|---|---|---|---|
| 1 metro= | 39.37 | pulgs (a) | 1.595 1654 |
| 1 gramo= | 15.43235 | grns (a) | 1.188 4320 |
| 1 cent cúb de agua pesa | 1 | gram (a) | .000 0000 |
| 1 metro cúb de aire pesa | 1.293052 | kgs (b) | .111 6159 |
| 1 pulg de mercurio pesa | 13.5956 | pulgs agua (c) | 1.133 4112 |
| 1 atmósfera= | 760 | mm de mercurio | 2.880 8136 |
| 1 libra Fahr= | 778 | pies-lb (d) | 2.890 9796 |
| $g$=aceleración debida á la grav= | 9.81 | metros por segundo por segundo (e) | .991 6690 |
| 1 £= | $4.86 | (f) | .686 6363 |
| 1 franco= | $ .193 | (f) | n 1.285 5573 |
| 1 marco= | $ .238 | (f) | n 1.376 5770 |

(a) Véase pág. 229.

(b) Véase Densidad del aire, pág. 335.

★ Decimales que se repiten. ‡ Valores exactos. n, Característica negativa. I, Ingenieros. G, Gunter.

(c) 1 pulgada de mercurio=13.5956 de agua. Valor adoptado por la « Oficina internacional de pesos y medidas ». Lat 45°. A nivel del mar. El mercurio á 0° C=32° Fahr.

(d) 1 lb Fahr=778 pies-lbs. De acuerdo con la determinación muy cuidadosa hecha por el profesor Henry A. Rowland, aceptada generalmente como tipo.

(e) $g$=9 m. 81 por seg por seg; generalmente aceptada como término medio. A la lat de 45° y á nivel del mar, $g$=9.806056 m.

Para otras latitudes L y alturas $h$, en centímetros;

$g$ en cent por seg por seg=aproximadamente 980.6056 — 2.5028 cos 2 L — .000003 h.

(f) De acuerdo con la circular del Departamento de la Tesorería de los E. U., enero 1 de 1887, £ 1=$4.8665 oro; 1 franco=.193; 1 marco=$.238.

## Tabla B.

### Abreviaciones.

| | | | |
|---|---|---|---|
| a | acre | grn | granos |
| bu | bushel | gal | galón |
| C | centígrado | hec | hectárea |
| c | centésimo | h | hora |
| cc | centímetro cúb | hg | hectogramo |
| cg | centígramo | hl | hectolitro |
| cad | cadena | hm | hectómetro |
| cl | centilitro | HP | caballo de vapor. |
| cm | centímetro | Imp | Imperial |
| ct | centavo | pulg | pulgada |
| cúb | cúbico | j | joule |
| dól | dólar | kg | kilogramo |
| d | día | kgm | kilogrametro |
| dcg | decigramo | kl | kilolitro |
| dcm | decímetro | km | kilómetro |
| dcl | decalitro | kw | kilowatt |
| F | Fahrenheit | £ | libra (moneda) |
| g | aceleración de la gravedad | lit | litro |
| grm | gramos | lib | libra (peso) |
| liq | líquido | qq | quinta |
| m | metro | sh | chelín |
| m² | metro cuadrado | s | segundo |
| m³ | metro cúbico | sec | sección |
| min | minuto | cuad | cuadrado |
| mlil | mililitro | st | esterio |
| mm | milímetro. | w | watt |
| p | pie | y | yarda |
| pen | penique | $ | peso (dólar) |

## Tabla C.

### Equivalencias y números de uso común.

*(Ordenadas de menor á mayor.)*

| | | | Logaritmos. |
|---|---|---|---|
| 1 cc de agua pesa | 1 | gram.* | .000 0000 |
| 1 met | 1.093611 | yarda | .038 8629 |
| 1 kilogramo | 2.20462 | libras | .343 3340 |
| 1 yarda | 3 | pies | .477 1213 |
| 1 circunferencia | $\pi$ | diámetros | .497 1499 |
| 1 metro | 3.280833 | pies | .515 9842 |
| 1 galón | 4 | cuartos | .602 0600 |
| 1 eslabón G | 7.92 | pulgs | .898 7252 |
| 1 yarda cuad | 9 | pies cuad | .954 2425 |
| $g$=aceleración de la gravedad | 9.81 | metros por seg por seg | .991 6690 |
| 1 metro | 10 | decímetros | 1.000 0000 |
| 1 pie | 12 | pulgs | 1.079 1812 |
| 1 gramo | 15.43235 | granos | 1.188 4320 |
| 1 libra | 16 | onzas | 1.204 1200 |
| 1 rod | 16.5 | pies | 1.217 4839 |
| 1 día | 24 | horas | 1.380 2112 |
| 1 yarda cúb | 27 | pies cúb | 1.431 3638 |
| 1 yarda | 36 | pulgs | 1.556 3025 |
| 1 metro | 39.37 | pulgs | 1.595 1654 |
| 1 hora | 60 | minutos | 1.778 1513 |

* A su máxima densidad, = 4°C, = 39,°2 F.

| | | | Logaritmos. |
|---|---|---|---|
| 1 cadena G..... | 66 | pies..... | 1.819 5439 |
| 1 pie cuad..... | 144 | pulg cuad..... | 2.158 3625 |
| 1 galón (E. U.)... | 231 | pulg cúb..... | 2.363 6120 |
| 1 onza..... | 437.5 | granos..... | 2.640 9781 |
| 1 caballo vapor... | 550 | pies-lbs por seg.... | 2.740 3627 |
| 1 milla cuad..... | 640 | acres..... | 2.806 1800 |
| 1 yarda cuad..... | 1,296 | pulg cuad..... | 3.112 6050 |
| 1 día..... | 1,440 | minutos..... | 3.158 3625 |
| 1 pie cúb..... | 1,728 | pulgs cúbicas..... | 3.237 5437 |
| 1 milla..... | 1,760 | yardas..... | 3.245 5127 |
| 1 tonelada..... | 2,240 | libras..... | 3.350 2480 |
| 1 hora..... | 3,600 | segundos..... | 3.556 3025 |
| 1 acre..... | 4,840 | yardas cuad..... | 3.684 8454 |
| 1 milla..... | 5,280 | pies..... | 3.722 6339 |
| 1 libra..... | 7,000 | granos..... | 3.845 0980 |
| 1 caballo de vapor | 33,000 | pies-libras por minuto..... | 4.518 5139 |
| 1 acre..... | 43,560 | pies cuad..... | 4.639 0879 |
| 1 yarda cúbica... | 46,656 | pulgs cúb..... | 4.668 9075 |
| 1 milla..... | 63,360 | pulgs..... | 4.801 8152 |
| 1 día..... | 86,400 | segundos..... | 4.936 5137 |

## Tabla D.

### Prefijos usados en el sistema métrico.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 Miria- | metro, gramo, etc. = | 10,000 | mets, grm, etc. | | | 4.000 | 0000 |
| 1 Kilo- | — — — = | 1,000 | — — — | | | 3.000 | 0000 |
| 1 Hecto- | — — — = | 100 | — — — | | | 2.000 | 0000 |
| 1 Deca- | — — — = | 10 | — — — | | | 1.000 | 0000 |
| 1 | metro, gramo, etc. = | 1 | met, grm, etc. | | | .000 | 0000 |
| 1 Deci- | — — — = | .1 | — — — | — | n | 1.000 | 0000 |
| 1 Centi- | — — — = | .01 | — — — | — | n | 2.000 | 0000 |
| 1 Mili- | — — — = | .001 | — — — | — | n | 3.000 | 0000 |

## (1) LONGITUDES

| | | | Logaritmos. |
|---|---|---|---|
| **1 Cadena I=** | | | |
| 100†..... | pies..... | | 2.000 0000 |
| 6.*0606..... | rods..... | | .782 5161 |
| 1.*5151..... | cadenas G..... | | .180 4561 |
| .018*93..... | milla..... | n | 2.277 3661 |
| 30.4801..... | metros..... | | 1.484 0158 |
| **1 Cadena G=** | | | |
| 66†..... | pies..... | | 1.819 5439 |
| 4†..... | pértica (*rods*)..... | | .602 0600 |
| .66†..... | cadena I..... | n | 1.819 5439 |
| .0125†..... | milla..... | n | 2.096 9100 |
| 20.1168..... | metros..... | | 1.303 5597 |
| **1 Brazada=** | | | |
| 6†..... | pies..... | | .778 1513 |
| **1 Pie=** | | | |
| 12†..... | pulgadas..... | | 1.079 1812 |
| .*33333..... | yard..... | n | 1.522 8787 |
| .*0606..... | *rod*..... | n | 2.782 5161 |
| .0*1515..... | cadena G..... | n | 2.180 4561 |
| .00018*93... | milla..... | n | 4.277 3661 |
| .30480..... | metro..... | n | 1.484 0158 |
| **1 Estadio=** | | | |
| .125†..... | milla..... | n | 1.096 9100 |
| **1 Pulgada=** | | | |
| .083333..... | pie..... | n | 2.920 8188 |
| .02*77..... | yarda..... | n | 2.443 6975 |
| 2.540005..... | centímetros..... | | .404 8346 |
| **1 Eslabón G=** | | | |
| 7.92†..... | pulgadas..... | | .898 7252 |
| .66†..... | pie..... | n | 1.819 5439 |

* Decimales que se repiten. † Valor exacto. n, Característica negativa. I, Ingenieros. G, Gunter.

| | | Logaritmos. |
|---|---|---|
| **Milla=** | | |
| 63,360†..... | pulgadas..... | 4.801 8152 |
| 8,000†..... | eslabones G..... | 3.903 0900 |
| 5,280†..... | pies..... | 3.722 6339 |
| 1,760†..... | yardas..... | 3.245 5127 |
| 320†..... | *rods*..... | 2.505 1500 |
| 80†..... | cadenas G..... | 1.903 0900 |
| 52.8†..... | cadenas I..... | 1.722 6339 |
| 8.†..... | estadios..... | .903 0900 |
| 1.60935..... | kilómetros..... | .206 6497 |
| **1 Pértica** (*rod*)= | | |
| 25†..... | eslabón G..... | 1.397 9400 |
| 16.5†..... | pies..... | 1.217 4839 |
| .25†..... | cadena G.....n | 1.397 9400 |
| .003125†.... | milla.....n | 3.494 8500 |
| **1 Yarda=** | | |
| 36†..... | pulgadas..... | 1.556 3025 |
| 3†..... | pies..... | .477 1213 |
| .00056*81... | milla.....n | 4.754 4873 |
| .914402..... | metros.....n | 1.961 1371 |
| **1 Micron=** | | |
| .001†..... | milímetro.....n | 3.000 0000 |
| **1 Milímetro=** | | |
| .03937..... | pulgadas.....n | 2.595 1654 |
| 1,000†..... | micron..... | 3.000 0000 |
| .001†..... | metro.....n | 3.000 0000 |
| **Centímetro=** | | |
| .3937..... | pulgadas.....n | 1.595 1654 |
| .0328083.... | pie.....n | 2.515 9842 |
| 10†..... | milímetros..... | 1.000 0000 |
| .01†..... | metro.....n | 2.000 0000 |
| **1 Decímetro=** | | |
| 3.937..... | pulgadas..... | .595 1654 |
| .1†..... | metro.....n | 1.000 0000 |
| **1 Metro=** | | |
| 39.37..... | pulgadas..... | 1.595 1654 |
| 3.28083..... | pies..... | .515 9842 |
| 1.09361..... | yardas..... | .038 8629 |
| 1,000†..... | milímetros..... | 3.000 0000 |
| 100†..... | centímetros..... | 2.000 0000 |
| 10†..... | decímetros..... | 1.000 0000 |
| .001†..... | kilómetro.....n | 3.000 0000 |
| **1 Decámetro=** | | |
| 10†..... | metros..... | 1.000 0000 |
| **1 Hectómetro=** | | |
| 100†..... | metros..... | 2.000 0000 |
| **1 Kilómetro=** | | |
| 3,280.83..... | pies..... | 3.515 9842 |
| 1,093.61..... | yardas..... | 3.038 8629 |
| .62137..... | milla.....n | 1.793 3503 |
| 1,000†..... | metros..... | 3.000 0000 |

## (2) SUPERFICIE

| | | |
|---|---|---|
| **1 Acre=** | | |
| 100,000†..... | eslabones G cuad..... | 5.000 0000 |
| 43,560†..... | pies cuad..... | 4.639 0879 |
| 4,840†..... | yardas cuad..... | 3.684 8454 |
| 208.710..... | pies en cuadro..... | 2.319 5440 |
| 160†..... | pérticas cuad..... | 2.204 1200 |
| 10†..... | cadenas G cuad..... | 1.000 0000 |
| 3.16228..... | cadenas G, en cuadro..... | .500 0000 |
| .0015625†... | millas cuad.....n | 3.193 8200 |
| 4,046.87..... | metros cuad..... | 3.607 1196 |
| .404687..... | hectárea.....n | 1.607 1196 |

★ Decimales que se repiten. † Valor exacto. n, Característica negativa. I, Ingenieros. G, Gunter.

| | | Logaritmos. |
|---|---|---|
| **1 Pie cuadrado=** | | |
| 144†..... | pulgadas cuad ..... | 2.158 3625 |
| .*111..... | yarda cuad.....n | 1.045 7575 |
| .0929034.... | metro cuad.....n | 2.968 0316 |
| **1 Pulgada cuadrada=** | | |
| .0069 7444... | pie cuad.....n | 3.841 6375 |
| 6.45163..... | cm cuad..... | .809 6692 |
| **1 Milla cuadrada=** | | |
| 27,878,400†..... | pies cuadrados..... | 7.445 2678 |
| 3,097,600†..... | yardas cuad..... | 6.491 0253 |
| 640†..... | acres..... | 2.806 1800 |
| 1†..... | sección..... | .000 0000 |
| 259.000..... | hectáreas..... | 2.413 2996 |
| 2.59000..... | kilómetros cuad.....n | .413 2996 |
| **1 Yarda cuadrada=** | | |
| 1,296†..... | pulgadas cuads..... | 3.112 6050 |
| 9†..... | pies cuads..... | .954 2425 |
| .836131..... | metro cuad.....n | 1.922 2742 |
| **1 Milímetro cuadrado=** | | |
| .001550..... | pulgada cuad.....n | 3.190 3308 |
| .01†..... | cm cuad.....n | 2.000 0000 |
| **1 Centímetro cuadrado=** | | |
| .15500..... | pulgada cuad.....n | 1.190 3308 |
| .0001†..... | metro cuad.....n | 4.000 0000 |
| **1 Metro cuadrado=** | | |
| 1,550.0..... | pulgadas cuads..... | 3.190 3308 |
| 10.7639..... | pies cuads..... | 1.031 9683 |
| 1.19598..... | yardas cuads..... | .077 7258 |
| 10,000†..... | cm cuads..... | 4.000 0000 |
| **1 Hectómetro cuadrado=** | | |
| 1..... | hectárea. (Véase hectárea.) | |

| | | Logaritmos. |
|---|---|---|
| **1 Kilómetro cuadrado=** | | |
| 247.105..... | acres..... | 2.392 8804 |
| .386101..... | milla cuad.....n | 1.586 7005 |
| 1,000,000†..... | metros cuads..... | 6.000 0000 |
| 100†..... | hectáreas..... | 2.000 0000 |
| **1 Hectárea=** | | |
| 107,639..... | pies cuads..... | 5.031 9683 |
| 2.47104..... | acres..... | .392 8804 |
| .003861..... | milla cuad.....n | 3.586 7005 |
| 10,000..... | metros cuads..... | 4.000 0000 |

## (3) VOLÚMENES

| | | |
|---|---|---|
| **1 Bushel (E. U.)=** | | |
| 1.24446..... | pies cúbs..... | .094 9796 |
| **1 Bushel británico=** | | |
| 1.28368..... | pies cúbs..... | .108 4555 |
| **1 Cuerda de Madera=** | | |
| 4×4×8†..... | pies cúbs | |
| 128†..... | pies cúbs..... | 2.107 2100 |
| **1 Pie cúbico=** | | |
| 1728/231 = 1,728†..... | pulgs cúbs..... | 3.237 5437 |
| 7.48052..... | gals líquid E U..... | .873 9317 |
| 6.23210..... | gals Imper..... | .794 6345 |
| .*037..... | yard cúb.....n | 2.568 6362 |
| 28.317..... | litros..... | 1.452 0475 |
| .028317..... | metro cúb.....n | 2.452 0475 |
| **1 Pulgada cúbica=** | | |
| 16.3872..... | cm. cúbs..... | 1.214 5038 |
| **1 Yarda cúbica=** | | |
| 46,656†..... | pulgs cúbs..... | 4.668 9075 |
| 27†..... | pies cúbs..... | 1.431 3638 |
| .00061983... | pie-acre.....n | 4.792 2759 |
| .76456..... | metro cúb.....n | 1.883 4113 |

* Decimales que se repiten; ej.: .0*1515 = .01515151515. † Valores exactos. n, Característica negativa.

| | | Logaritmos. |
|---|---|---|
| **1 Galón E. U.=** | | |
| 231† | pulgs cúbs | .363 6120 |
| 8† | pintas liqd E. U | .903 0900 |
| .833111 | gals Imp | n 1.920 7028 |
| 231/1728 = .133681 | pie cúb | n 1.126 0683 |
| 3.78543 | litros | .578 1158 |
| .003785 | metro cúb | n 3.578 1158 |
| **1 Galón Imperial=** | | |
| 277.274 | pulgs cúbs | 2.442 9092 |
| 1.20032 | galones liqd E. U | .079 2972 |
| .160459 | pie cúb | n 1.205 3655 |
| 4.54374 | litros | .657 4130 |
| **1 Onza, líquida=** | | |
| 1.8047 | pulgs cúbs | .256 4050 |
| 29.5739 | cm cúbs | 1.470 9088 |
| **1 Milímetro cúbico=** | | |
| .00006102 | pulg cúb | n 5.785 4962 |
| .001† | cm cúb | n 3.000 0000 |
| **1 Centímetro cúb ó mililitro=** | | |
| .0610234 | pulg cúb | n 2.785 4962 |
| 1,000† | milímetros cúbs | 3.000 0000 |
| .001† | litro | n 3.000 0000 |
| **1 Centilitro=** | | |
| 10† | mililitros | 1.000 0000 |
| .01† | litro | n 2.000 0000 |
| **1 Decilitro=** | | |
| .1† | litro | n 1.000 0000 |
| **1 Litro ó decímetro cúbico=** | | |
| 61.0234 | pulgs cúbs | 1.785 4962 |
| .26417 | galón líquid E. U | n 1.421 8842 |
| .220083 | galón Imp | n 1.342 5870 |
| .0353145 | pies cúbs | n 2.547 9525 |
| 1,000† | cm cúbs | 3.000 0000 |
| .001† | metro cúb | n 3.000 0000 |

| | | Logaritmos. |
|---|---|---|
| **Decalitro=** | | |
| 10† | litros | 1.000 0000 |
| .01† | metro cúb | n 2.000 0000 |
| **1 Hectolitro=** | | |
| 100† | litros | 2.000 0000 |
| **1 Metro cúb ó kilolitro=** | | |
| 61,023.4 | pulgs cúbs | 4.785 4962 |
| 264.170 | galones líquid E. U | 2.421 8842 |
| 220.083 | galones británicos | 2.342 5870 |
| 35.3145 | pies cúbs | 1.547 9525 |
| 1.30794 | yardas cúbs | .116 5887 |
| .00081070 | pie-acre | n 4.908 8646 |
| 1,000† | litros | 3.000 0000 |
| **1 Pie-acre=** | | |
| 325,851 | galones líquid E. U | 5.513 0196 |
| 271,472 | galones Imp | 5.433 7224 |
| 43,560† | pies cúbs | 4.639 0879 |
| 1,613.333 | yardas cúbs | 3.207 7241 |
| 1,233.49 | metros cúbs | 3.091 1354 |

## (4) PESO (FUERZA)

Sistema métrico y Avoirdupois.

| | | Logaritmos. |
|---|---|---|
| **1 Grano=** | | |
| .0647989 | gramo | n 2.811 5680 |
| **1 Onza=** | | |
| 437.5† | granos | 2.640 9781 |
| .0625† | libra | n 2.705 8800 |
| 28.3496 | gramos | 1.452 5461 |
| **1 Libra=** | | |
| 7,000† | granos | 3.845 0980 |
| 16† | onzas | 1.204 1200 |
| 453.593 | gramos | 2.656 6660 |
| .453593 | kilogramo | n 1.656 6660 |

* Decimales que se repiten. † Valores exactos. n, Característica negativa.

### Avoirdupois y Métrico.

| | | Logaritmos. |
|---|---|---|
| **1 Tonelada** (2,240 libras)= | | |
| 2,240†..... | libras..... | 3.350 2480 |
| 1,016.05..... | kilgs..... | 3.006 9141 |
| 1.01605..... | tons métricas..... | .006 9141 |
| **1 Miligramo**= | | |
| .001†..... | gramo.....n | 3.000 0000 |
| **1 Centigramo**= | | |
| .01†..... | gramo.....n | 2.000 0000 |
| **1 Decigramo**= | | |
| .1†..... | gramo.....n | 1.000 0000 |
| **1 Gramo**= | | |
| 15.43235..... | granos..... | 1.188 4320 |
| .001†..... | kilogr.....n | 3.000 0000 |
| **1 Decagramo**= | | |
| 10†..... | gramos..... | 1.000 0000 |
| **1 Hectogramo**= | | |
| 100†..... | gramos..... | 2.000 0000 |
| **1 Kilogramo**= | | |
| 2.20462..... | libras..... | .343 3340 |
| 1,000†..... | gramos..... | 3.000 0000 |
| **1 Miriagramo**= | | |
| 10†..... | kilogramos..... | 1.000 0000 |
| **1 Quintal** (métrico)= | | |
| 100†..... | kilogramos..... | 2.000 0000 |
| **1 Tonelada métrica**= | | |
| 2,204.62..... | libras..... | 3.343 3340 |
| 1.10231..... | ton (2,000 lbs)..... | .042 3040 |
| .984206..... | ton (2,240 lbs).....n | 1.993 0860 |
| 1,000†..... | kilogs..... | 3.000 0000 |

### (5) TIEMPO

1 año=365¼ días. 1 año bisiesto=366¼ días.

| | | Logaritmos. |
|---|---|---|
| **1 año bisiesto**= | | |
| 8,784†..... | horas..... | 3.943 6923 |
| 366†..... | días..... | 2.563 4811 |
| **1 año**= | | |
| 8,760†..... | horas..... | 3.942 5041 |
| 365†..... | días..... | 2.562 2929 |
| **1 Día**= | | |
| 86,400†..... | segundos..... | 4.936 5137 |
| 1,440†..... | minutos..... | 3.158 3625 |
| 24†..... | horas..... | 1.380 2112 |
| .142857..... | semana.....n | 1.154 9020 |
| .00273973... | año.....n | 3.437 7071 |
| .00273224... | año bisiesto.....n | 3.436 5189 |
| **1 Hora**= | | |
| 3,600†..... | segundos..... | 3.556 3025 |
| 60†..... | minutos..... | 1.778 1513 |
| .041666..... | día.....n | 2.619 7888 |
| .000114155.. | año.....n | 4.057 4959 |
| **1 Minuto**= | | |
| 60†..... | segundos..... | 1.778 1513 |
| .016666..... | hora.....n | 2.221 8487 |
| .00069444... | día.....n | 4.841 6375 |
| **1 Segundo**= | | |
| .016666..... | minuto.....n | 2.221 8487 |
| .000277777.. | hora.....n | 4.443 6975 |
| .000011574.. | día.....n | 5.063 4863 |
| **1 Semana**= | | |
| 168†..... | horas..... | 2.225 3093 |
| 7†..... | días..... | .845 0980 |

† Valores exactos. n, Característica negativa.

## (6) TRABAJO Y CALOR

Para Potencia, véase tabla (22).

| **Pie-libra=** | | Logaritmos. |
|---|---|---|
| 13,825.5..... | gram-centím..... | 4.140 6818 |
| 1.3562844.... | joules..... | .132 3508 |
| .138255..... | kilográmetro.....n | 1.140 6818 |
| .00184340... | caballo-seg.....n | 3.265 6205 |
| .00037 6746.. | watt-hor†0 ∠38ᵍ·9.·ᵍ·····ᵗ···n | |

| **1 HP-hora** (caballo-hora)= | | |
|---|---|---|
| 1,980,000†..... | pie-libras..... | 6.296 6652 |
| 2,685,443..... | joules..... | 6.429 0160 |
| 273,746..... | kilogrametros..... | 5.437 3470 |

| **1 Pulgada-libra=** | | |
|---|---|---|
| 1,152.13..... | gramo-centím..... | 3.061 5006 |

| **1 joule=** | | |
|---|---|---|
| .737308..... | pie-libra.....n | 1.867 6492 |
| .101937..... | kilogrametro.....n | 1.008 3310 |

| **1 Kilográmetro=** | | |
|---|---|---|
| 7.23300..... | pie-libra..... | .859 3182 |
| .0131509.... | caballo-seg.....n | 2.118 9555 |
| 9.81..... | joules..... | .991 6690 |
| .01333..... | caballo métrico-seg.....n | 2.124 9387 |
| .002725..... | watt-hora.....n | 3.435 3665 |

| **1 Caloria.** (Equivale á un trabajo de.) | | |
|---|---|---|
| 3,087.35..... | pie-libras..... | 3.489 5861 |
| 426.843..... | kilográmetros..... | 2.630 2679 |

| **1 Watt-hora=** | | |
|---|---|---|
| 2,654.31..... | pie-libras..... | 3.423 9517 |
| 3,600†..... | joules..... | 3.556 3025 |
| 366.972..... | kilográmetros..... | 2.564 6335 |
| .00135916... | caballo métrico-hora.....n | 3.133 2697 |

| **1 Caballo métrico-hora=** | | Logaritmos. |
|---|---|---|
| 1,952,910..... | pie-libras..... | 6.290 6820 |
| 2,648,700..... | joules..... | 6.423 0328 |
| 270,000†..... | kilográmetros..... | 5.431 3638 |

## (7) LONGITUD POR LONGITUD (PENDIENTES)

| **1 Pie por milla**=1 : 5,280†= | | |
|---|---|---|
| .000189393.. | .....n | 4.277 3661 |
| .0022727.... | pulg por pie.....n | 3.356 5473 |

| **1 Pulgada por pie**=1 · 12= | | |
|---|---|---|
| .083333 (tang 4º46′=0.083386)..... | .....n | 2.920 8188 |
| .00694444... | pie por pulg.....n | 3.841 6376 |
| 440†..... | pies por milla..... | 2.643 4527 |

| **1 Pulgada por milla=** | | |
|---|---|---|
| .00157828... | pie por 100 pies.....n | 3.198 1848 |

| **1 Milla por pulgada=** | | |
|---|---|---|
| 63,360†..... | ..... | 4.801 8152 |
| 21,120†..... | yarda por pie..... | 4.324 6939 |

## (8) SUPERFICIE POR LONGITUD (ANCHOS)

| **1 Acre por cadena I=** | | |
|---|---|---|
| 435.6†..... | pies..... | 2.639 0879 |
| .0825†..... | milla cuad por milla.....n | 2.916 4539 |
| .132771..... | km cuad por km.....n | 1.123 1038 |

| **1 Acre por cadena G =** | | |
|---|---|---|
| 660†..... | pies..... | 2.819 5439 |
| 66,000†..... | pies cuad por 100 pies..... | 4.819 5439 |
| .125†..... | milla cuad por milla.....n | 1.096 9100 |
| .201168..... | km cuad por km.....n | 1.303 5599 |

† Valores exactos. n, Característica negativa.

| | | Logaritmos. |
|---|---|---|
| **1 Acre por pie=** | | |
| 43.560† | pies | 4.639 0879 |
| 4,356.000† | pies cuad por 100 pies | 6.639 0879 |
| 8.25† | millas cuad por milla | .916 4539 |
| 13.2771 | km cuad por km | 1.123 1038 |
| **1 Acre por milla=** | | |
| 8.25† | pies | .916 4539 |
| 825† | pies cuad por 100 pies | 2.916 4539 |
| 1,188† | pulgs cuad por pie | 3.074 8165 |
| .251461 | hectárea por km.....n | 1.400 4699 |
| **1 Pie cuad por milla=** | | |
| .002272727 | pulg=1 440 pulg.....n | 3.356 5473 |
| .00018939393 | pie=1 5280 pie.....n | 4.277 3661 |
| .0577274 | metro cuad por km.....n | 2.761 3819 |
| **1 Pulgada cuad por pie=** | | |
| 36.6666 | pie cuad por milla | 1.564 2714 |
| 2.11667 | mets cuad por km | .325 6534 |
| **1 Pulgada cuad por pulgada=** | | |
| .0833333 | pie cuad por pie.....n | 2.920 8187 |
| .00694444 | pie cuad por pulgada.....n | 3.841 6375 |
| .010101 | acre por milla.....n | 2.004 3648 |
| 25.4001 | metros cuad por km | 1.404 8346 |
| **1 Milla cuad por milla=** | | |
| .121212 | acre por pie.....n | 1.083 5461 |
| 1.60935 | km cuad por km | .206 6499 |
| **1 Milla cuad por cadena I=** | | |
| 440† | pie cuad por pulg | 2.643 4527 |
| **1 Metro cuad por kilómetro=** | | |
| 17.3228 | pie cuad por milla | 1.238 6180 |
| .47244 | pulg cuad por pie.....n | 1.674 3466 |
| **1 Hectárea por kilm=** | | |
| 10† | metros | 1.000 0000 |
| 3.97677 | acres por milla | .599 5301 |

## (9) VOLUMEN POR LONGITUD (SUPERFICIES)

| | | Logaritmos. |
|---|---|---|
| **1 Pie cúb por pie=** | | |
| 1† | pie cuad | .000 0000 |
| 195.5555 | yarda cúb por milla | 2.291 2701 |
| 92.9034 | litros por metro | 1.968 0317 |
| **1 Pie cúb por pulgada=** | | |
| 12† | pies cuads | 1.079 1812 |
| .444 | yarda cúb por pie.....n | 1.647 8174 |
| 1,114.84 | litros por metro | 3.047 2129 |
| **1 Pie cúb por milla=** | | |
| .000189393 | pie cuad=1 5280† pie cuad n | 4.277 3661 |
| .0272727 | pulg c. = 144/5280† pulg c.n | 2.435 7286 |
| .3272727 | pulg cúb por pie.....n | 1.514 9098 |
| .0175953 | litro por metro.....n | 2.245 3978 |
| **1 Pulgada cúb por pie=** | | |
| .0833333 | pulg cuad=1/12† pulg cuad n | 2.920 8188 |
| 3.05555 | pie cúb por milla | .485 0902 |
| .0537637 | litros por metro.....n | 2.730 4880 |
| **1 Pulgada cúb por pulgada=** | | |
| 1† | pulgada cuad | .000 0000 |
| 36.666666 | pie cúb por milla | 1.564 2715 |
| .645163 | litro por metro.....n | 1.809 6692 |
| **1 Yarda cúb por pie=** | | |
| 3† | yardas cuad | .477 1213 |
| 2.25† | pies cúb por pulgada | .352 1826 |
| 2,508.39 | litros por metro | 3.399 3955 |
| **1 Yarda cúb por pulgada=** | | |
| 46,656† | pulgadas cuads | 4.668 9075 |
| **1 Yarda cúb por milla=** | | |
| .00056818 | yarda cuad =1/1760† yard cuad.....n | 4.754 4873 |
| 8.8363636 | pulg cúb por pie | .946 2736 |
| .47507 | litro por metro.....n | 1.676 7616 |

† Valores exactos. n, Característica negativa.

| | | Logaritmos. |
|---|---|---|
| **1 Galón (E. U.) por pie=** | | |
| 19.25† | pulgs cuad | 1.284 4307 |
| 705.833 | pies cúb por milla | 2.848 7022 |
| 12.4194 | litros por metro | 1.094 1000 |
| **1 Galón (E. U.) por pulgada=** | | |
| 231† | pulgada cuad | 2.363 6120 |
| 1.60417 | pies cúbs por pie | .205 2495 |
| 149.033 | litros por metro | 2.173 2812 |
| **1 Litro por metro=** | | |
| .001† | metro cuad n | 3.000 0000 |
| 18.6 | pulgs cúb por pie | 1.269 5120 |
| 56.8332 | pies cúb por milla | 1.754 6022 |

## (10) PESO POR LONGITUD

| | | |
|---|---|---|
| **1 Libra por pie=** | | |
| 1.48816 | kg por metro | .172 6502 |
| **1 Libra por pulgada=** | | |
| 17.8579 | kg por metro | 1.251 8314 |
| **1 Libra por milla=** | | |
| .281849 | gram por metro n | 1.450 0163 |
| **1 Gramo por metro=** | | |
| 3.5480 | libra por milla | .549 9837 |
| **1 Kilogramo por metro=** | | |
| .67197 | libra por pie n | 1.827 3498 |
| **1 Kilogramo por kilómetro=** | | |
| 3.548 | libras por milla | .549 9837 |

## (11) VOLUMEN POR SUPERFICIE (ESPESORES)

| | | |
|---|---|---|
| **1 Pie cúb por acre=** | | |
| .000275482 | pulg n | 4.440 0938 |
| 23.7037 | yardas cúbs por milla cuad. | 1.374 8162 |
| .0699726 | metro cúb por hectárea n | 2.844 9279 |
| **1 Pie cúb por pie cuad=** | | |
| 1† | pie | .000 0000 |
| 1,613.33333 | yarda cúb por acre | 3.207 7241 |
| .05195 | galón (E. U.) por pulg cuad n | 2.715 5692 |
| **1 Pie cúb por milla cuad=** | | |
| 2.7† | pulgs cúbs por acre | .431 3638 |
| .0109332 | metro cúb por km cuad n | 2.038 7479 |
| **1 Pulgada cúb por pie cuad=** | | |
| 25.2083 | pie cúb por acre | 1.401 5442 |
| 597.531 | yarda cúb por milla cuad | 2.776 3603 |
| .176389 | litro por metro cuad n | 1.246 4722 |
| **1 Pulgada cúb por pulgada cuad=** | | |
| 1† | pulgada | .000 0000 |
| 3,630† | pies cúbs por acre | 3.559 9067 |
| .02540 | metro cúb por metro cuad n | 2.404 8346 |
| **1 Yarda cúb por acre=** | | |
| 1.07107 | pulgs cubs por pie cuad | .029 8196 |
| .188926 | litro por metro cuad n | 1.276 2917 |
| **1 Yarda cúb por pie cuad=** | | |
| 27† | pies | 1.431 3638 |
| 8.22962 | metros cúb por metro cuad. | .915 3797 |
| **1 Yarda cúb por milla cuad=** | | |
| 72.9† | pulgs cúbs por acre | 1.862 7275 |
| .295197 | metros cúbs por km cuad n | 1.470 1117 |
| **1 Galón (E. U.) por acre=** | | |
| 85.555555 | pies cúbs por milla cuad | 1.932 2483 |
| .935397 | metro cúb por km cuad n | 1.970 9962 |
| **1 Galón (E. U.) por pie cuad=** | | |
| 5,823.13 | pies cúbs por acre | 3.765 1562 |
| 40.7459 | litros por metro cuad | 1.610 0842 |

† Valores exactos. n, Característica negativa

| | | Logaritmos. |
|---|---|---|
| **1 Litro por metro cuad=** | | |
| .001† | metro | n 3.000 0000 |
| 142.913 | pies cúbs por acre | 2.155 0721 |
| .0245423 | galón (E. U.) por pie cuad. | n 2.389 9159 |
| **1 Litro por hectárea=** | | |
| 9.14644 | pies cúbs por milla cuad. | .961 2521 |
| .106906 | galón (E. U.) por acre | n 1.029 0038 |
| **1 Metro cúb por metro cuad=** | | |
| 1† | metro | .000 0000 |
| 24.5423 | galones (E. U.) por pie cuad. | 1.389 9158 |
| **1 Metro cúb por kilómetro cuad=** | | |
| .001† | mm | n 3.000 0000 |
| 91.4644 | pies cúbs por milla cuad. | 1.961 2521 |
| 1.06906 | galones (E. U.) por acre | .029 0038 |

## (12) PESO POR SUPERFICIE (UNIDADES DE PRESIÓN)

Avoirdupois y métricas.

| | | |
|---|---|---|
| **1 Atmósfera=** | | |
| 760 | mm de mercurio | 2.880 8136 |
| 29.9212 | pulgs de mercurio | 1.475 9790 |
| 33.9007 | pies de agua | 1.530 2090 |
| 14.697 | libs por pulg cuad | 1.167 2280 |
| 1.033296 | kilogm por cm cuad | .014 2248 |
| 1.033296 | atmósferas métricas | .014 2248 |
| **1 Atmósfera métrica=** | | |
| 1 | kgrm por cm cuad | .000 0000 |
| .96777 | atmósfera | n 1.985 7752 |
| **1 Libra por pulgada cuad=** | | |
| 2.30665 | pies de agua | .362 9810 |
| .0680412 | atmósfera | n 2.832 7720 |
| .0703067 | kgr por cm cuad | n 2.846 9968 |
| 2.03589 | pulgs de mercurio | .308 7510 |
| **1 Libra por pie cuad=** | | |
| .0141380 | pulgs de mercurio | n 2.150 3885 |
| .0160184 | pie de agua | n 2.204 6185 |
| **1 Kilogramo por centímetro cuad=** | | |
| 1 | atmósf métrica | .000 0000 |
| 14.2234 | lbs por pulg cuad | 1.153 0032 |
| .96777 | atmósf | n 1.985 7752 |
| 32.8083 | pies de agua | 1.515 9842 |
| 28.9570 | pulgs de mercurio | 1.461 7542 |
| **1 Pie de agua=** | | |
| .0294979 | atmósfera | n 2.469 7910 |
| .882612 | pulg de mercurio | n 1.945 7700 |
| .433530 | libra por pulg cuad | n 1.637 0190 |
| 30.4801 | gramos por cm cuad | 1.484 0158 |
| **1 Pie de mercurio=** | | |
| .401053 | atmósf | n 1.603 2022 |
| .414407 | kg por cm cuad | n 1.617 4270 |
| **1 Pulgada de agua=** | | |
| 2.54001 | grams por cm cuad | .404 8346 |
| **1 Pulgada de mercurio=** | | |
| .03342 | atmósfera | n 2.524 0210 |
| 1.133 | pies de agua | .054 2299 |
| .491189 | libs por pulg cuad | n 1.691 2490 |
| 34.5339 | grams por cm cuad | 1.538 2458 |

## (13) PESO POR VOLUMEN (PESO ESPECÍFICO)

Avoirdupois y métrico.

| | | |
|---|---|---|
| **1 Libra por pulgada cúb=** | | |
| 27.6797 | grams por cm cúb | 1.442 1621 |
| **1 Libra por galón (E. U.)=** | | |
| .119826 | kg por litro | n 1.078 5501 |

† Valores exactos. n, Característica negativa.

| | | Logaritmos. |
|---|---|---|
| **1 Libra por galón Imp=** | | |
| .0998281.... | kg por litro.....n | 2.999 2530 |
| **1 Libra por pie cúb=** | | |
| 16.0184..... | kg por metro cúb..... | 1.204 6184 |
| **1 Libra por yarda cúb=** | | |
| .593273..... | kg por metro cúb.....n | 1.773 2546 |
| **1 Kilogramo por metro cúb=** | | |
| .0624283.... | libra por pie cúb.....n | 2.795 3816 |
| **1 Gramo por centímetro cúb=** | | |
| 62.4283..... | libras por pie cúb..... | 1.795 3816 |

## (14) PESO DE VOLÚMENES DE AGUA

Avoirdupois y métricos.
Agua al máximum de densidad=1 gramo por cm cúb.

| | | | Logaritmos. |
|---|---|---|---|
| **1 Pulgada cúb pesa** | | | |
| .578040.... | onzas.....n | | 1.761 9578 |
| .0361275.... | libras.....n | | 2.557 8378 |
| 16.3872..... | gramos..... | | 1.214 5038 |
| **1 Galón líquido de (E. U.) pesa** | | | |
| 8.34545..... | libras..... | | .921 4498 |
| 3.78543..... | kilogramos..... | | .578 1158 |
| **1 Galón Imp pesa** | | | |
| 10.0172..... | libras..... | | 1.000 7469 |
| 4.54373..... | kilogramos..... | | .657 4129 |
| **1 Pie cúb pesa** | | | |
| 62.4283..... | libras..... | | 1.795 3815 |
| 28.3170..... | kilogramos..... | | 1.452 0475 |
| **1 Yarda cúb pesa** | | | |
| 1,685.56..... | libras..... | | 3.226 7453 |
| 764.559..... | kilogramos..... | | 2.883 4113 |
| **1 Centímetro cúb ó mililitro pesa** | | | |
| .0352739.... | onza.....n | | 2.547 4540 |
| 1†..... | gramo..... | | .000 0000 |
| **1 Litro ó decímetro cúb pesa** | | | |
| 2.20462..... | libras..... | | .343 3340 |
| 1†..... | kilogramo..... | | .000 0000 |
| **1 Metro cúb ó kilolitro pesa** | | | |
| 2,204.62..... | libras..... | | 3.343 3340 |
| 1,000†..... | kilogramos..... | | 3.000 0000 |

## (15) VOLÚMENES DE PESOS DE AGUA

Avoirdupois y métrico.
Agua en su máximum de densidad=1 gramo por cm cúb.

| | | | Logaritmos. |
|---|---|---|---|
| **1 Libra mide** | | | |
| 27.6798..... | pulgs cúbs..... | | 1.442 1621 |
| .119825..... | galón (E. U.).....n | | 1.078 5501 |
| .0998281.... | galón Imp.....n | | 2.999 2530 |
| .0160184.... | pie cúb.....n | | 2.204 6184 |
| .453592..... | litro.....n | | 1.656 6660 |
| **1 Gramo mide** | | | |
| .0610233.... | pulgada cúb.....n | | 2.785 4962 |
| 1†..... | cm cúb..... | | .000 0000 |
| **1 Kilogramo mide** | | | |
| 61.0234..... | pulgadas cúb..... | | 1.785 4962 |
| .264170..... | galón (E. U.).....n | | 1.421 8842 |
| .220083..... | galón Imp.....n | | 1.342 5871 |
| .0353144.... | pie cúb.....n | | 2.547 9525 |
| 1†..... | litro..... | | .000 0000 |

## (16) PESO DE VOLÚMENES DE AIRE

Avoirdupois y métrico. (Véase más adelante cap Aire-Atmósfera.)

| | | | Logaritmos. |
|---|---|---|---|
| **1 Pulgada cúb pesa** | | | |
| .327003..... | grano.....n | | 1.514 5518 |
| 21.1895..... | miligramos..... | | 1.326 1198 |

† Valores exactos. n, Característica negativa.

| | | Logaritmos. |
|---|---|---|
| **1 Pie cúb pesa** | | |
| 565.061..... | granos..... | 2.752 0955 |
| .0807230.... | libra.....n | 2.906 9075 |
| 36.6154..... | gramos..... | 1.563 6635 |
| **1 Yarda cúb pesa** | | |
| 2.17952..... | libras..... | .338 3613 |
| 988.615..... | gramos..... | 2.995 0273 |
| **1 Metro cúb pesa** | | |
| 2.85069..... | libras..... | .454 9500 |
| 1,293052.... | kg..... | .111 6159 |

## (17) VOLÚMENES DE PESOS DE AIRE

Avoirdupois y métrico. (Véase adelante cap Aire-Atmósfera.)

| | | |
|---|---|---|
| **1 Grano mide** | | |
| 3.05808..... | pulg cúbs..... | .485 4482 |
| .0501132.... | litro.....n | 2.699 9520 |
| **1 Libra mide** | | |
| 21,406.5..... | pulgs cúbs..... | 4.330 546- |
| 12.3880..... | pies cúbs..... | 1.093 002- |
| .458816..... | yarda cúb.....n | 1.661 638 |
| .350792..... | metro cúb.....n | 1.545 050 |
| **1 Kilogramo mide** | | |
| 47,193.3..... | pulgs cúbs..... | 4.673 8802 |
| 27.3109..... | pies cúbs..... | 1.436 3365 |
| 1.01152..... | yardas cúbs..... | .004 9727 |
| .773364..... | metro cúb.....n | 1.888 3840 |

## (18) LONGITUD POR TIEMPO (VELOCIDAD)

| | | |
|---|---|---|
| **1 Pie por segundo=** | | |
| .6$\frac{8}{18}$18181... | milla por hora.....n | 1.833 6686 |
| 1.09728..... | kilómetros por hora..... | .040 3183 |
| **1 Pulgada por segundo=** | | |
| 300†..... | pies por hora..... | 2.477 1213 |
| 152.400..... | cm por minuto..... | 2.182 9859 |
| **1 Milla por hora=** | | |
| 1.4$\frac{6}{66}$666666... | pies por segundo..... | .166 3314 |
| .447041..... | metro por seg.....n | 1.650 3472 |
| **1 Milla por minuto=** | | |
| 88†..... | pies por seg..... | 1.944 4826 |
| 96.5608..... | km por hora..... | 1.984 8010 |
| **1 Milla por segundo=** | | |
| 96.5608..... | km por minuto..... | 1.984 8010 |
| **1 Metro por segundo=** | | |
| 196.85..... | pies por minuto..... | 2.294 1354 |
| 2.23693..... | millas por hora..... | .349 6528 |
| **1 Kilómetro por día=** | | |
| .03797 26.... | pie por seg.....n | 2.579 4705 |
| .0258904.... | milla por hora.....n | 2.413 1391 |
| **1 Kilómetro por hora=** | | |
| .911343..... | pie por seg.....n | 1.959 6817 |
| 14.9129..... | millas por día..... | 1.173 5615 |
| **1 Centímetro por segundo=** | | |
| 1.96850..... | pies por minuto..... | .294 1355 |

## (19) SUPERFICIE POR TIEMPO

| | | |
|---|---|---|
| **1 Acre por día=** | | |
| 1,815†..... | pies cuad por hora..... | 3.258 8766 |
| **1 Acre por hora=** | | |
| 12.1†..... | pies cuad por seg..... | 1.082 7854 |
| .03750†..... | milla cuad por día.....n | 2.574 0313 |
| 1.12413..... | metros cuad por seg..... | .050 8170 |
| **1 Acre por segundo=** | | |
| 5.625†..... | millas cuad por hora..... | .750 1225 |
| 14,5687..... | km cuad por hora..... | 1.163 4222 |
| **1 Pie cuad por hora=** | | |
| 2.22968..... | metros cuad por día..... | .348 2429 |

† Valores exactos. n, Característica negativa.

| | | Logaritmos. |
|---|---|---|
| **1 Pie cuad por segundo=** | | |
| .0826446.... | acre por hora.....n | 2.917 2147 |
| .00309917... | milla cuad por día.....n | 3.491 2460 |
| 334.452..... | metros cuad por hora..... | 2.524 3341 |
| **1 Pulgada cuad por segundo=** | | |
| 25†..... | pies cuad por hora..... | 1.397 9400 |
| .0137741.... | acre por día.....n | 2.139 0633 |
| 2.32259..... | metros cuad por hora..... | .365 9718 |
| **1 Milla cuad por día=** | | |
| 322$\frac{8}{1}$66666..... | pies cuad por seg..... | 2.508 7541 |
| 26$\frac{1}{1}$6666666.... | acres por hora..... | 1.425 9688 |
| .107917..... | km cuad por hora.....n | 1.033 0883 |
| **1 Milla cuad por hora=** | | |
| 7,744†..... | pies cuad por seg..... | 3.888 9653 |
| 15,360†..... | acres por día..... | 4.186 3912 |
| 719.444..... | metros cuad por seg..... | 2.856 9970 |
| **1 Metro cuad por segundo=** | | |
| 21.3498..... | acres por día..... | 1.329 3942 |
| **1 Kilómetro cuad por día=** | | |
| 124.582..... | pies cuad por seg..... | 2.095 4545 |
| 10.2960..... | acres por hora..... | 1.012 6692 |
| **1 Kilómetro cuad por segundo=** | | |
| 1,389,96..... | millas cuad por hora..... | 3.143 0030 |
| **1 Centímetro cuad por segundo=** | | |
| 3.87499..... | pies cuad por hora..... | .588 2708 |

## (20) VOLUMEN POR TIEMPO (DESCARGAS)

| | | |
|---|---|---|
| **1 Pie cúb por día=** | | |
| . 311688..... | galón (E. U.) por hora.....n | 1.493 7205 |
| 1.17987..... | litros por hora..... | .071 8363 |
| **1 Pie cúb por hora=** | | |
| 179.532..... | galones (E. U.) por día.... | 2.254 1429 |
| .679608..... | metro cúb por día.....n | 1.832 2587 |
| **1 Pie cúb por minuto=** | | |
| 448.831..... | galones (E. U.) por hora... | 2.652 0830 |
| 1.69902..... | metros cúb por hora..... | .230 1988 |
| **1 Pie cúb por segundo=** | | |
| 646,317..... | galones (E. U.) por día.... | 5.810 4453 |
| 448.831..... | galones (E. U.) por minuto. | 2.652 0828 |
| 1.98347..... | pies-acres por día..... | .297 4258 |
| 101.941..... | metros cúb por hora..... | 2.008 3500 |
| como 40..... | pulgadas de mineros..... | 1.602 0600 |
| **1 Pulgada cúb por segundo=** | | |
| 50†..... | pies cúb por día..... | 1.698 9700 |
| 15.5844..... | galones (E. U.) por hora.... | 1.192 6905 |
| 58.9038..... | litros por hora..... | 1.770 8063 |
| **1 Yarda cúb por hora=** | | |
| 18.3494..... | metros cúb por día..... | 1.263 6225 |
| **1 Yarda cúb por segundo=** | | |
| 53.5537..... | pies-acres por día..... | 1.728 7896 |
| **1,000,600 gal (E. U.) por día=** | | |
| 1.54723..... | pies cúbs por seg..... | .189 5546 |
| 43.813..... | litros por seg..... | 1.641 6021 |
| **1 Galón (E. U.) por hora=** | | |
| 3.208$\frac{3}{1}$33333.. | pies cúbs por día..... | .506 2795 |
| .0908504.... | metro cúb por día.....n | 2.958 3270 |
| **1 Galón (E. U.) por minuto=** | | |
| 8.02084..... | pies cúbs por hora..... | .904 2196 |
| .002228..... | pie cúb por seg.....n | 3.347 9171 |
| .00441919... | pie-acre por día.....n | 3.645 3429 |
| **1 Galón (E. U.) por segundo=** | | |
| 481.25†..... | pies cúbs por hora..... | 2.682 3708 |
| .26$\frac{5}{1}$515151.. | pie-acre por día.....n | 1.423 4943 |
| 13.6276..... | metros cúbs por hora..... | 1.134 4183 |
| **1 Pulgada de mineros=** | | |
| como .025..... | pie cúb por segundo.....n | 2.397 9400 |

† Valores exactos. n, Característica negativa.

| | | Logaritmos. |
|---|---|---|
| **1 Centímetro cúb por segundo=** | | |
| 3.05117..... | pies cúbs por día..... | .484 4662 |
| **1 Litro por segundo=** | | |
| 127.132..... | pies cúbs por hora..... | 2.104 2550 |
| **1 Metro cúb por día=** | | |
| 1.47144..... | pies cúbs por hora..... | .167 7413 |
| **1 Metro cúb por hora=** | | |
| 6,340.09..... | galones (E. U.) por día.... | 3.802 0954 |
| .00980957... | pie cúb por seg.....n | 3.991 6500 |
| **1 Metro cúb por segundo=** | | |
| 2.91855..... | pies-acre por hora..... | .465 1671 |
| **1 Pie-acre por día=** | | |
| 1,815†..... | pies cúbs por hora..... | 3.258 8767 |
| 3.77143..... | galones (E. U.) por seg..... | .576 5059 |
| .0142765.... | metro cúb por seg.....n | 2.154 6217 |
| **1 Pie-acre por hora=** | | |
| 726†..... | pies cúbs por minuto..... | 2.860 9366 |
| 90.5143..... | galones (E. U.) por seg..... | 1.956 7171 |
| .342636..... | metro cúb por seg.....n | 1.534 8329 |

## (21) PESO POR TIEMPO

Avoirdupois y métrico.

| | | |
|---|---|---|
| **1 Libra por segundo=** | | |
| 27.2156..... | kilogr por minuto..... | 1.434 8173 |
| **1 Kilogramo por hora=** | | |
| 52.9109..... | libras por día..... | 1.723 5452 |
| **1 Kilogramo por segundo=** | | |
| 7,936.6..... | libras por hora..... | 3.899 6365 |

## (22) TRABAJO POR TIEMPO (POTENC)

Para Trabajo. (Véase la tabla 6.)

| | | Logaritmos. |
|---|---|---|
| **1 HP (caballo-vapor)=** | | |
| 5,694,120..... | pie-galón por día..... | 6.755 4265 |
| 33,000†..... | pie-lbs por minuto..... | 4.518 5139 |
| 550†..... | pie-lbs por seg..... | 2.740 3627 |
| 745.956..... | joules por seg..... | 2.872 7134 |
| 1.01387..... | HP métrico..... | .005 9832 |
| **1 Pie-libra por segundo=** | | |
| .00181818... | HP.....n | 3.259 6373 |
| .00184340... | HP métrico.....n | 3.265 6205 |
| **1 Millón galones-pie agua por día=** | | |
| .175620..... | HP.....n | 1.244 5734 |
| **1 Watt=** | | |
| 1†..... | joule por seg..... | .000 0000 |
| .737308..... | pie-lb por seg.....n | 1.867 6492 |
| .00134056... | HP.....n | 3.127 2865 |
| **1 Kilowatt=** | | |
| 1,000†..... | watts. (Véase watt.) | |
| **1 HP Métrico=** | | |
| 735.75..... | watts..... | 2.866 7303 |
| 542.475..... | pie-lbs por seg..... | 2.734 3795 |
| 75†..... | kilogr por seg..... | 1.875 0613 |
| .986318..... | HP.....n | 1.994 0168 |
| **1 joule por segundo=** | | |
| .00134056... | HP.....n | 3.127 2865 |
| .00135916... | HP métrico.....n | 3.133 2697 |
| 366.973..... | kilogr por hora..... | 2.564 6336 |
| **1 Kilográmetro por segundo=** | | |
| .0131509.... | HP.....n | 2.118 9555 |
| .0133333.... | HP métrico.....n | 2.124 9387 |

† Valores exactos. n, Característica negativa.

## (23) TRABAJO POR VOLUMEN

| | | Logaritmos. |
|---|---|---|
| **1 Pie-libra por pie cúb=** | | |
| .000171826.. | libra F por galón (E. U.)..n | 4.235 0886 |
| 4.88241..... | kgm por metro cúb..... | .688 6343 |
| **1 Pie-libra por pulg cúb=** | | |
| 8.4368..... | kgm por litro..... | .926 1780 |
| **1 Pie-libra por galón (E. U.)=** | | |
| 36.5230..... | kgm por metro cúb..... | 1.562 5660 |
| **1 HP hora por pie cúb=** | | |
| 340.215..... | libras F por galón (E. U.).. | 2.531 7538 |
| 22.6481..... | kilg C por litro..... | 1.355 0316 |
| **1 HP hora por yarda cúb=** | | |
| 73,333.33333..... | pie-lbs por pie cúb..... | 4.865 3014 |
| 12.6006..... | libras F por gal (E. U.).... | 1.100 3901 |
| 358.044..... | kgs por litro..... | 2.553 9358 |
| **1 HP hora por gal (E. U.)=** | | |
| .267835..... | HP metric hora por litro..n | 1.427 8674 |
| **1 Libra F por pie cúb=** | | |
| .0106091.... | HP hora por yarda cúb...n | 2.025 6782 |
| 104.003..... | pie-lbs por gal (E. U.)..... | 2.017 0478 |
| 8.89909..... | kg C por metro cúb..... | .949 3459 |
| **1 Libra F por galón (E. U.)=** | | |
| .0793615.... | HP hora por yarda cúb...n | 2.899 6099 |
| 66.5699..... | kg C por metro cúb..... | 1.823 2776 |
| **1 Kilogramo C por litro=** | | |
| 50.5929..... | pie-lbs por pulgada cúb.... | 1.704 0899 |
| .0441538.... | HP hora por pie cúb.....n | 2.644 9684 |
| **1 Kilogramo C por metro cúb=** | | |
| 11.6870..... | pie-lbs por galón (E. U.)... | 1.067 7019 |
| **1 Kilográmetro por litro=** | | Logaritmos. |
| 204.817..... | pie-lbs por pie cúb..... | 2.311 3657 |
| .00279296... | HP hora por yarda cúb...n | 3.446 0643 |
| .0351929.... | libra F por gal (E. U.)....n | 2.546 4544 |
| **1 Kilográmetro por metro cúb=** | | |
| .027380..... | pie-lbs por galón (E. U.)..n | 2.437 4340 |

## (24) TRABAJO POR PESO

| | | |
|---|---|---|
| **1 Kilográmetro por gramo=** | | |
| 5.96515..... | HP seg por libra..... | .775 6215 |
| **1 Pie-libra por libra=** | | |
| .304801..... | kilográmetro por kilogramo n | 1.484 0158 |
| **1 Libra F por libra=** | | |
| .237135..... | kilográmetro por gramo...n | 1.374 9954 |

## (25) VALOR POR LONGITUD

| | | |
|---|---|---|
| **1 Dólar por pie=** | | |
| 12.35..... | chelines por yarda..... | 1.092 |
| .1379..... | marcos por cm.....n | 1.139 |
| .170..... | francos por cm.....n | 1.230 |
| **1 Penique por pie=** | | |
| .0607..... | dólar por yarda.....n | 2.783 |
| 106.92..... | dólares por milla..... | 2.029 |
| 440†..... | chelines por milla..... | 2.643 |
| .344..... | franco por metro.....n | 1.537 |
| .279..... | marco por metro.....n | 1.446 |
| **1 Céntimo por pulgada=** | | |
| .12†..... | dólar por pie.....n | 1.079 |
| .494..... | chelín por pie.....n | 1.694 |
| 2.04..... | franco por metro..... | .310 |
| 1.654..... | marcos por metro..... | .219 |

† Valores exactos. n, Característica negativa.

| | | Logaritmos. |
|---|---|---|
| **1 Penique por pulgada=** | | |
| .7290..... | dólar por yarda.....n | 1.863 |
| 4.13..... | céntimos por cm..... | .616 |
| 3.35..... | pfenig por cm ..... | .525 |
| **1 Céntimo por yarda=** | | |
| 17.60†..... | dólares por milla..... | 1.246 |
| 72.43..... | chelines por milla..... | 1.860 |
| .0566..... | franco por metro.....n | 2.753 |
| .0459..... | marco por metro.....n | 2.662 |
| **1 Penique por yarda=** | | |
| 35.64..... | dólares por milla..... | 1.552 |
| 7.333..... | libras por milla..... | .865 |
| .1148..... | franco por metro.....n | 1.060 |
| .09305..... | marcos por metro.....n | 2.969 |
| **1 Chelín por yarda=** | | |
| 8.1..... | centavos por pie..... | .908 |
| 1.377..... | francos por metro..... | .139 |
| 1.117..... | marcos por metro..... | .048 |
| **1 Dólar por milla=** | | |
| .018939393.. | centavo por pie.....n | 2.277 |
| .009353..... | penique por pie.....n | 3.971 |
| .0007795.... | chelín por pie.....n | 4.892 |
| 3.22..... | francos por km..... | .508 |
| 2.61..... | marcos por km..... | .417 |
| **1 Franco por metro=** | | |
| .0588..... | dólar por pie.....n | 2.770 |
| 2.905..... | peniques por pie..... | .463 |
| .242..... | chelín por pie.....n | 1.384 |
| 310.60..... | dólares por milla ..... | 2.492 |
| 1,278..... | chelines por mill ..... | 3.107 |

| | | Logaritmos. |
|---|---|---|
| **1 Marco por metro=** | | |
| .0726..... | dólar por pie.....n | 2.861 |
| 3.58..... | peniques por pie..... | .554 |
| .2985..... | chelín por pie.....n | 1.475 |
| 383..... | dólars por milla..... | 2.583 |
| 1,576..... | chelines por milla..... | 3.198 |
| **1 Franco por kilómetro=** | | |
| .3106..... | dólar por milla.....n | 1.492 |
| 15.34..... | peniques por milla..... | 1.186 |
| 1.28..... | chelines por milla..... | .107 |
| **1 Marco por kilómetro=** | | |
| .383..... | dólar por milla.....n | 1.583 |
| 18.92..... | peniques por milla..... | 1.277 |
| 1.576..... | chelines por milla..... | .198 |

## (26) VALOR POR SUPERFICIE

| | | |
|---|---|---|
| **1 Dólar por acre=** | | |
| .002295..... | centavo por pie cuad.....n | 3.361 |
| .00113..... | penique por pie cuad.....n | 3.054 |
| .000850..... | chelín por yarda cuad.....n | 4.930 |
| 12.8..... | francos por hectárea..... | 1.107 |
| 10.38..... | marcos por hectárea..... | 1.016 |
| **1 Centavo por pie cuad=** | | |
| .09†..... | dólar por yarda cuad.....n | 2.954 |
| 4.44..... | peniques por yarda cuad..... | .648 |
| .557..... | francos por m cuad.....n | 1.746 |
| .452..... | marcos por m cuad.....n | 1.655 |
| **1 Chelín por pie cuad=** | | |
| 10,585..... | dólares por acre..... | 4.025 |
| 108†..... | peniques por yarda cuad..... | 2.033 |
| 13.55..... | francos por m cuad..... | 1.132 |
| 11.0..... | marcos por m cuad..... | 1.041 |

† Valores exactos. n, Característica negativa.

| | | Logaritmos. |
|---|---|---|
| **1 Centavo por pulgada cuad=** | | |
| 12.96† | dólar por yarda cuad. | 1.113 |
| 71.1 | peniques por pie cuad. | 1.852 |
| 53.3 | chelines por yarda cuad. | 1.727 |
| 80.3 | francos por m cuad. | 1.905 |
| 65.1 | marcos por m cuad. | 1.814 |
| **1 Dólar por yarda cuad=** | | |
| .457 | chelín por pie cuad. n | 1.660 |
| 996 | libras por acre | 2.998 |
| 6.20 | francos por m cuad. | .792 |
| 5.02 | marcos por m cuad. | .701 |
| **1 Penique por yarda cuad=** | | |
| 98.01 | dólares por acre | 1.991 |
| .125 | franco por m cuad. n | 1.099 |
| .102 | marco por m cuad. n | 1.008 |
| **1 Chelín por yarda cuad=** | | |
| 1,176 | dólares por acre | 3.070 |
| 1.506 | francos por m cuad. | .178 |
| 1.22 | marcos por m cuad. | .087 |
| **1 Franco por metro cuad=** | | |
| 1.79 | cénts por pie cuad. | .254 |
| .885 | penique por pie cuad. n | 1.947 |
| .664 | ch por yarda cuad. n | 1.822 |
| **1 Marco por metro cuad=** | | |
| 2.21 | cénts por pie cuad. | .345 |
| 1.09 | peniques por pie cuad. | .038 |
| .819 | chelín por yarda cuad. n | 1.913 |
| **1 Franco por hectárea=** | | |
| .0781 | dólar por acre. n | 2.893 |
| .321 | chelín por acre. n | 1.507 |
| **1 Marco por hectárea=** | | |
| .0963 | dólar por acre. n | 2.984 |
| .396 | chelín por acre. n | 1.598 |

## (27) VALOR POR VOLUMEN

| | | Logaritmos. |
|---|---|---|
| **1 Dólar por pie cúb=** | | |
| 111.1 | chelines por yarda cúb. | 2.046 |
| 13.36 | cénts por galón (E. U.) | 1.126 |
| .133 | franco por litro. n | 1.262 |
| .1483 | marco por litro. n | 1.171 |
| **1 Chelín por pie cúb=** | | |
| 6.56 | dólares por yarda cúb. | .817 |
| 324† | peniques por yarda cúb. | 2.511 |
| .0445 | franco por litro. n | 2.648 |
| .03605 | marco por litro. n | 2.557 |
| **1 Centavo por pulgada cúb=** | | |
| 17.28† | dólares por pie cúb. | 1.238 |
| 71.11 | chelines por pie cúb. | 1.852 |
| .316 | céntimo por cc. n | 1.500 |
| .2565 | pfenig por cc. n | 1.409 |
| **1 Penique por pulgada cúb=** | | |
| 944.78 | dólares por yarda cúb. | 2.975 |
| 3,888† | chelines por yarda cúb. | 3.590 |
| 6.40 | francos por litro. | .806 |
| 5.19 | marcos por litro. | .715 |
| **1 Chelín por pulgada cúb=** | | |
| 56.13 | dólares por galón (E. U.) | 1.749 |
| .07683 | franco por cc. n | 2.886 |
| .0624 | marco por cc. n | 2.795 |
| **1 Dólar por yarda cúb=** | | |
| 3.703703704 | centavos por pie cúb. | .569 |
| .1523 | chelín por pie cúb. n | 1.183 |
| 6.777 | francos por m cúb. | .831 |
| 5.495 | marcos por m cúb. | .740 |

† Valores exactos. n, Característica negativa.

**1 Chelín por yarda cúb=** Logaritmos.

.9000..... centavos por pie cúb.....n 1.954
.444444..... penique por pie cúb.....n 1.648
.165..... céntimo por litro.....n 1.217
.1336..... pfenig por litro.....n 1.126

**1 Penique por galón Imp=**

1.687..... centavos por galón (E. U.)..... .227
2.309..... céntimos por litro..... .363
1.87..... pfenig por litro..... .272

**1 Chelín por galón Imp=**

.2025..... dólar por gal (E. U.).....n 1.306
.277..... franco por litro.....n 1.443
.225..... marco por litro.....n 1.352

**1 Dólar por galón (E. U.)=**

4.04..... chelines por galón Imp..... .694
1.369..... francos por litro..... .136
1.11..... marcos por litro..... .045

**1 Franco por metro cúb=**

.1477..... dólar por yarda cúb.....n 1.169
7.287..... peniques por yarda cúb..... .863
.607..... chelín por yarda cúb.....n 1.783

**1 Marco por metro cúb=**

.182..... dólar por yarda cúb.....n 1.260
8.985..... peniques por yarda cúb..... .954
.7488..... chelín por yarda cúb.....n 1.874

## (28) VALOR POR PESO

Avoirdupois y métrico.

**1 Dólar por libra=**

11.48..... francos por kg..... 1.058
9.263..... marcos por kg..... .967

**1 Penique por libra=** Logaritmos.

.231..... franco por kg.....n 1.364
.1875..... marco por kg.....n 1.273

**1 Chelín por libra=**

2.775..... francos por kg..... .443
2.25..... marcos por kg..... .352

Avoirdupois y Métrico.

**1 Franco por kilogramo=**

.08754..... dólar por lib.....n 2.942
.3603..... chelín por lib.....n 1.557

**1 Marco por kilogramo=**

.108..... dólar por lib.....n 1.033
.444..... chelín por lib.....n 1.648

## (29) VALOR POR TIEMPO

**1 Dólar por día=**

4.1666666.... centavos por hora..... .620
2.058..... peniques por hora..... .313
.216..... franco por hora.....n 1.334
.175..... marco por hora.....n 1.243

**1 Penique por día=**

.08437..... centavo por hora.....n 2.926
.437..... cént por hora.....n 1.641
.355..... pfenig por hora.....n 1.550

**1 Chelín por día=**

1.012..... centavos por hora..... .005
.5†..... penique por hora.....n 1.699
.0525..... franco por hora.....n 2.720
.0426..... marco por hora.....n 2.629

**1 Franco por día=**

.0338..... marco por hora.....n 2.529
.8042..... cent por hora.....n 1.905
.397..... penique por hora.....n 1.599

† Valores exactos, n, Característica negativa.

| | | Logaritmos. |
|---|---|---|
| **1 Marco por día=** | | |
| .0514..... | franco por hora.....n | 2.711 |
| .9917..... | centavo por hora.....n | 1.996 |
| .49..... | penique por hora.....n | 1.690 |
| **1 Centavo por hora=** | | |
| .9877..... | chelín por día.....n | 1.995 |
| 1.244..... | francos por día..... | .095 |
| 1.008..... | marcos por día..... | .004 |
| **1 Dólar por hora=** | | |
| .0137..... | penique por seg.....n | 2.137 |
| .144..... | cént por seg.....n | 1.158 |
| .1168..... | pfenig por seg.....n | 1.067 |
| **1 Penique por hora=** | | |
| .486..... | dólar por día.....n | 1.687 |
| 2†..... | chelines por día..... | .301 |
| 2.04..... | marcos por día..... | .310 |
| **1 Chelín por hora=** | | |
| .035..... | cént por seg.....n | 2.544 |
| .0284..... | pfenig por seg.....n | 2.453 |
| 5.83..... | dólares por día..... | .766 |
| **1 Franco por hora=** | | |
| 19.46..... | marcos por día..... | 1.289 |
| 4.63..... | dólares por día..... | .666 |
| 19.06..... | chelines por día..... | 1.280 |
| **1 Marco por hora=** | | |
| 29.6 ..... | francos por día..... | 1.471 |
| 5.712..... | dólares por día..... | .757 |
| 23.5.. ..... | chelines por día..... | 1.371 |
| **1 Centavo por segundo=** | | |
| 36†..... | dólares por hora ..... | 1.556 |
| 1,778..... | peniques por hora..... | 3.250 |
| 148.2..... | chelines por hora..... | 2.171 |
| 186.5..... | francos por hora..... | 2.271 |
| 151.3..... | marcos por hora..... | 2.180 |

| | | Logaritmos. |
|---|---|---|
| **1 Penique por segundo=** | | |
| 72.90..... | dólares por hora..... | 1.863 |
| 300†..... | chelines por hora..... | 2.477 |
| 378..... | francos por hora..... | 2.577 |
| 306..... | marcos por hora..... | 2.486 |
| **1 Céntimo por segundo=** | | |
| 36†..... | francos por hora..... | 1.556 |
| 29.2..... | marcos por hora..... | 1.465 |
| 6.95..... | dólares por hora..... | .842 |
| 28.6..... | chelines por hora..... | 1.456 |
| **1 Pfennig por segundo=** | | |
| 44.4..... | francos por hora..... | 1.647 |
| 36.†..... | marcos por hora..... | 1.556 |
| 8.568..... | dólares por hora..... | .933 |
| 35.26..... | chelines por hora..... | 1.547 |

## (30) VALOR DE TRABAJO

| | | |
|---|---|---|
| **1 Centavo por pie-libra=** | | |
| .375..... | francos por kilográmetro.....n | 1.574 |
| .304 ..... | marco por kgm.....n | 1.483 |
| **1 Penique por pie-libra=** | | |
| .7589..... | francos por kgm.....n | 1.880 |
| .615..... | marcos por kgm.....n | 1.789 |
| **1 Chelin por pie-libra=** | | |
| 9.107..... | francos por kgm..... | .959 |
| 7.38..... | marcos por kgm..... | .868 |
| **1 Franco por kilográmetro=** | | |
| 2.668..... | centavos por pie-lbs..... | .426 |
| 1.318..... | peniques por pie-lbs..... | .120 |
| **1 Marco por kilográmetro=** | | |
| 3.29..... | centavos por pie-lbs..... | .517 |
| 1.625..... | peniques por pie-lbs..... | .211 |

†Valores exactos. n, Característica negativa.

## (31) LONGITUD EN TIEMPO POR TIEMPO (ACELERACIÓN)

| Aceleración de la gravedad=g= | | Logaritmos. |
|---|---|---|
| 32.1850..... | pies en 1 seg por seg... ... | 1.507 6532 |
| 9.81..... | mets en 1 seg por seg.... . | .991 6690 |
| **en 1 seg por seg=** | | |
| .0310704.... | g en pies en un seg por seg.n | 2.492 3468 |
| **1 Metro en un seg por seg=** | | |
| .101937..... | g en cm en un seg por seg..n | 1.008 3310 |

## (32) VOLUMEN POR SUPERFICIE, DADOS LOS ESPESORES

| | | |
|---|---|---|
| **1 Pulgada de espesor da:** | | |
| 27,154.3..... | gal (E. U.) por acre..... | 4.433 8384 |
| 3,630†..... | pies cúbs por acre..... | 3.559 9067 |
| .623376..... | gal (E. U.) por pie cuad...n | 1.794 7505 |
| 254.0005..... | mets cúbs por hectárea..... | 2.404 8346 |
| **1 Pie de espesor da:** | | |
| 43,560..... | pies cúbs por acre..... | 4.639 0879 |
| 67.3247..... | gal (E. U.) por yarda cuad.. | 1.828 1742 |
| 3,048.006..... | mets cúbs por hect..... | 3.484 0158 |
| **1 Centímetro de espesor da:** | | |
| 10†..... | litros por m cuad,..... | 1.000 0000 |
| 1,429.13..... | pies cúbs por acre..... | 3.155 0721 |
| .245423..... | gal (E. U.) por pie cuad...n | 1.389 9159 |
| **1 Metro de espesor da:** | | |
| 1,000†..... | litros por m cuad..... | 3.000 0000 |
| 142,913..... | pies cúbs por acre..... | 5.155 0721 |
| 24.5423..... | gal (E. U.) por pie cuad.... | 1.389 9159 |

## (33) ESPESORES DADOS LOS VOLÚMENES POR SUPERFICIE

| | | |
|---|---|---|
| **1 Pie cúb por acre=** | | Logaritmos. |
| .000275482.. | pulg de espesor.....n | 4.440 0933 |
| .000699726.. | cm de espesor.....n | 4.844 9279 |
| **1 Pulgada cúb por pie cuad=** | | |
| .0069444.... | pulg de espesor.....n | 3.841 6375 |
| .0176389.... | cm de espesor.....n | 2.246 4721 |
| **1 Yarda cúb por acre=** | | |
| .00743802... | pulg de espesor.....n | 3.871 4571 |
| .0188926.... | cm de espesor.....n | 2.276 2917 |
| **1 Galón (E. U.) por pie cuad=** | | |
| 1.60417..... | pulgs de espesor..... | .205 2495 |
| 4.07459..... | cm de espesor..... | .610 0841 |
| **1 Litro por metro cuad=** | | |
| 1†..... | mm de espesor..... | .000 0000 |
| .03937..... | pulg de espesor.....n | 2.595 1654 |

## (34) VOLUMEN POR SUPERFICIE Y POR TIEMPO

| | | |
|---|---|---|
| **1 Pie cúb por acre y por día=** | | |
| .00292..... | m cúb por hect y por hora.n | 3.464 7167 |
| **1 Pie cúb por acre y por hora=** | | |
| 1.32987..... | gal (E. U.) por mill cuad y por seg..... | .123 8092 |
| 1.67934..... | mets cúbs por hect y por día. | .225 1391 |
| **1 Pie cúb por acre y por segundo=** | | |
| .618225..... | gal (E. U.) por pie cuad y por hora.....n | 1.791 1463 |
| 251.901..... | m cúbs por hect y por hora.. | 2.401 2304 |
| **1 Pie cúb por milla cuad y por día=** | | |
| .000487013.. | gal (E. U.) por acre y por hora.....n | 4.687 5405 |
| .00455551... | litro por hect y por hora..n | 3.658 5367 |

† Valores exactos. n, Característica negativa.

| | Logaritmos. |
|---|---|
| **1 Pie cúb por milla cuad y por hora=** | |
| .280519..... gal (E. U.) por acre y por día.....n | 1.447 9629 |
| .262397..... m cúb por km cuad y por día.....n | 1.418 9591 |
| **1 Pie cúb por milla cuad y por segundo=** | |
| 42.0779..... gal (E. U.) por acre y por hora..... | 1.624 0542 |
| 944.63..... mets cúbs por km cuad y por día..... | 2.975 2616 |
| **1 Galón (E. U.) por acre y por día=** | |
| 3.56482..... pies cúbs por milla cuad y por hora..... | .552 0371 |
| .0389749.... m cúb por km cuad y por hora.....n | 2.590 7850 |
| **1 Galón (E. U.) por acre y por hora=** | |
| .0237654.... pie cúb por milla cuad y por seg.....n | 2.375 9458 |
| 22.4495..... mets cúbs por km cuad y por día..... | 1.351 2074 |
| **1 Galón (E. U.) por acre y por segundo=** | |
| .0110480.... pie cúb por pie cuad y por hora.....n | 2.043 2829 |
| 56.1239..... mets cúbs por km cuad y por min..... | 1.749 1475 |
| **1 Galón (E. U.) por pie cuad y por día=** | |
| 242.630..... pies cúbs por acre y por hora..... | 2.384 9450 |
| 1.69775..... litros por m cuad y por hora. | .229 8730 |
| **1 Galón (E. U.) por pie cuad y por hora=** | |
| 1.61754..... pies cúbs por acre y por seg. | .208 8537 |
| .0113183.... litro por m cuad y por seg. n | 2.053 7817 |

| | Logaritmos. |
|---|---|
| **1 Galón (E. U.) por pie cuad y por segundo=** | |
| 146.685..... mets cúb por m cuad y por hora..... | 2.166 3867 |
| **1 Galón (E. U). por pulgada cuad y por día=** | |
| .802084..... pie cúb por pie cuad y por hora.....n | 1.904 2196 |
| .244476..... m cúb por m cuad y por hora.....n | 1.388 2354 |
| **1 Galón (E. U.) por pulgada cuad y por hora=** | |
| 462†..... pies cúbs por pie cuad y por día..... | 2.664 6420 |
| 140.818..... mets cúbs por m cuad y por día..... | 2.148 6578 |
| **1 Litro por metro cuad y por segundo=** | |
| 283.464..... pies cúbs por pie cuad y por día..... | 2.452 4978 |
| 88.3524..... galones (E. U.) por pie cuad y por hora..... | 1.946 2183 |
| **1 Metro cúb por metro cuad y por día=** | |
| .000284055.. gal (E. U.) por pie cuad y por seg.....n | 4.453 4021 |
| **1 Metro cúb por hectárea y por día=** | |
| .595471..... pie cúb por acre y pr. hora n | 1.774 8609 |
| **1 Metro cúb por hectárea y por hora=** | |
| 342.991..... pies cúbs por acre y por día | 2.535 2833 |
| **1 Metro cúb por hectárea y per seg=** | |
| 51,448.7..... pies cúbs por acre y por hora..... | 4.711 3746 |
| **1 Metro cúb por kilómetro cuad y por día=** | |
| 3.81102..... pies cúbs por milla cuad y por hora..... | .581 0409 |

† Valores exactos. n, Característica negativa.

| **1 Metro cúb por kilómetro cuad y por hora=** | | Logaritmos. |
|---|---|---|
| 2,195.15..... | pies cúbs por milla cuad y por día..... | 3.341 4633 |
| 25.6575..... | gal (E. U.) por acre y por día..... | 1.409 2150 |
| **1 Metro cúb por kilómetro cuad y por seg=** | | |
| 329,272..... | pies cúbs por milla cuad y por hora..... | 5.517 5546 |
| 64.1439..... | gal (E. U.) por acre y por minuto..... | 1.807 1551 |

## (35) VELOCIDAD DADOS LOS VOLÚMENES POR SUPERF Y POR TIEMPO

| | | |
|---|---|---|
| **1,000,000 de pies cúb por acre y por día=** | | |
| .000265704.. | pies por segundo.....n | 4.424 3984 |
| .00809868... | cm por seg.....n | 3.908 4142 |
| **1,000,000 pies cúbs por acre y por hora=** | | |
| .0063769.... | pies por seg.....n | 3.804 6096 |
| .194368..... | cm por seg.....n | 1.288 6254 |
| **1 Pie cúb por acre y por segundo=** | | |
| 1.98347..... | pies por día..... | .297 4258 |
| .604563..... | m por día.....n | 1.781 4416 |
| **1,000,000 pies cúb por milla cuad y por seg=** | | |
| 129.132..... | pies por hora..... | 2.111 0346 |
| 944.63..... | mets por día..... | 2.975 2616 |
| **1 Pulgada cúb por pie cuad y por seg=** | | |
| 25†..... | pulgs por hora..... | 1.397 9400 |
| 50†..... | pies por día..... | 1.698 9700 |
| 15.240..... | m por día..... | 1.182 9858 |
| **1,000,000 Galón (E. U.) por milla cuad y por día=** | | |
| .00239757... | pulg por hora.....n | 3.379 7704 |
| **1,000,000 Galón (E. U.) por milla cuad y por hora=** | | Log |
| .115083..... | pie por día.....n | 1.061 0116 |
| .0350774.... | m por día.....n | 2.545 0274 |
| **1,000,000 Galón (E. U.) por milla cuad y por seg=** | | |
| 5.26161..... | m por hora..... | .721 1187 |
| **1,000,000 Galón (E. U.) por acre y por día=** | | |
| .000035519.. | pie por seg.....n | 5.550 4667 |
| .00108264... | cm por seg.....n | 3.034 4825 |
| **1,000,000 Galón (E. U.) por acre y por hora=** | | |
| .000852468.. | pie por seg.....n | 4.930 6779 |
| .0259833.... | cm por seg.....n | 2.414 6937 |
| **1 Galón (E. U.) por acre y por seg=** | | |
| .132576..... | pulg por hora.....n | 1.122 4641 |
| .265152..... | pie por día.....n | 1.423 4941 |
| 8.08183..... | mets por día..... | .907 5099 |
| **1,000,000 Galón (E. U.) por pie cuad y por día=** | | |
| 1.54723..... | pies por seg..... | .189 5546 |
| 47.1596..... | cm por seg..... | 1.673 5705 |
| **1 Galón (E. U.) por pie cuad y por hora=** | | |
| 38.5†..... | pulgadas por día..... | 1.585 4607 |
| .977902..... | m por día.....n | 1.990 2954 |
| **1 Galón (E. U.) por pie cuad y por seg=** | | |
| 146.685..... | mets por hora..... | 2.166 3867 |
| **1 Metro cúb por metro cuad y por seg=** | | |
| 11,811..... | pies por hora..... | 4.072 2867 |
| **1 Metro cúb por hectárea y por seg=** | | |
| 340.157..... | pulgadas por día..... | 2.531 6791 |

## (36) VOLUMEN POR SUPERFICIE Y POR TIEMPO DADA LA VELOCIDAD

| | | |
|---|---|---|
| **1 Pulgada por seg=** | | |
| 2,244.16..... | gal (E. U.) por pie cuad y por hora..... | 3.351 0531 |

† Valores exactos. n, Característica negativa.

| | | |
|---|---|---|
| 91.4402..... | mets cúbs por m cuad y por hora..... | 1.961 1372 |
| **1 Pulgada por hora=** | | Logaritmos. |
| 87.120†..... | pies cúbs por acre por día.. | 4.940 1179 |
| 645.33333..... | pies cúbs por milla cuad por seg..... | 2.809 7841 |
| 14.9610..... | gal (E. U.) por pie cuad y por día..... | 1.174 9617 |
| 7.54286..... | gal (E. U.) por acre por seg. | .877 5358 |
| 6,096.01..... | mets cúb por hectárea por día..... | 3.785 0458 |
| **1 Pie por segundo=** | | |
| 26,929.9..... | gal (E. U.) por pie cuad por hora..... | 4.430 2343 |
| 1,097.28... .... | m cúb por m cuad por hora. | 3.040 3184 |
| **1 Pie por hora=** | | Logaritmos. |
| 7,744.0†..... | pies cúbs por milla cuad por seg..... | 3.888 9653 |
| 179.532..... | gal (E. U.) por pie cuad por día..... | 2.254 1429 |
| 90.5143..... | gal (E. U.) por acre por seg. | 1.956 7171 |
| 12.10†..... | pies cúbs por acre y por seg.. | 1.082 7854 |
| .84667..... | m cúb por hect y por seg...n | 1.927 7133 |
| **1 Centímetro por segundo=** | | |
| 118.11..... | pies cúbs por pie cuad por hora..... | 2.072 2867 |
| **1 Metro por hora=** | | |
| 39.6981..... | pies cúbs por acre por seg... | 1.598 7695 |
| 589.016..... | gal (E. U.) por pie cuad por día..... | 2.770 1271 |

† Valores exactos. n, Característica negativa.

## Pendientes.

**Véanse las tablas y figs.** de las páginas que siguen.

**Altura** del plano=$v$; **longitud** del plano=$s$; **distancia horizontal** ó base del plano=$h$; ángulo del plano con la horizontal=A (que lo seguiremos llamando *ángulo del plano*), sen A=$v/s$; cos A=$h/s$; tang A=$v/h$; cot A=$h/v$; sec A = $s/h$; cosec A=$s/v$.

Para ángulos pequeños.

$v = s$ . sen A = $h$ . tang A = $h \div$ cotang A = $s \div$ cosec A | $v =$ .01745 $s$ A°
$s = v \div$ sen A = $h \div$ cos A = $h$ . sec A = $v$ . cosec A | $s =$ 57.30 $v \div$ A°
$h = s$ . cos A = $v \div$ tang A = $v$ . cot A = $s \div$ sec A | $h =$ 57.29 $v \div$ A°

**La relación entre los 3 lados del plano inclinado para un ángulo de 1°**, están indicadas en las 3 figs. después de la tabla que sigue en que el ángulo de 1° está necesariamente muy exagerado. **Para ángulos pequeños**, puede suponerse que la altura del plano varía en proporción con el ángulo; y la longitud y distancia horizontal para una altura dada, en razón inversa del ángulo.

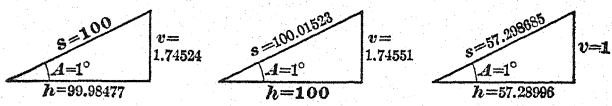
### Tabla de alturas del plano en metros para 100 metros, horiz correspondientes á diferentes ángulos de inclinación.

| Gr. Min. | Metros por 100 mets. | Gr. Min. | Metros por 100 mets. | Gr. Min. | Metros por 100 mets. | Gr. Min. | Metros por 100 mets. |
|---|---|---|---|---|---|---|---|
| 0 1 | .0291 | 0 45 | 1.3090 | 1 58 | 3.4341 | 3 26 | 5.9994 |
| 2 | .0582 | 46 | 1.3381 | 2 | 3.4924 | 28 | 6.0579 |
| 3 | .0873 | 47 | 1.3672 | 2 | 3.5506 | 30 | 6.1163 |
| 4 | .1164 | 48 | 1.3963 | 4 | 3.6087 | 32 | 6.1747 |
| 5 | .1455 | 49 | 1.4254 | 6 | 3.6669 | 34 | 6.2330 |
| 6 | .1746 | 50 | 1.4545 | 8 | 3.7250 | 36 | 6.2914 |
| 7 | .2037 | 51 | 1.4837 | 10 | 3.7833 | 38 | 6.3498 |
| 8 | .2328 | 52 | 1.5128 | 12 | 3.8416 | 40 | 6.4083 |
| 9 | .2619 | 53 | 1.5419 | 14 | 3.8999 | 42 | 6.4664 |
| 10 | .2909 | 54 | 1.5710 | 16 | 3.9581 | 44 | 6.5246 |
| 11 | .3200 | 55 | 1.6000 | 18 | 4.0163 | 46 | 6.5832 |
| 12 | .3491 | 56 | 1.6291 | 20 | 4.0746 | 48 | 6.6418 |
| 13 | .3782 | 57 | 1.6583 | 22 | 4.1329 | 50 | 6.7004 |
| 14 | .4073 | 58 | 1.6873 | 24 | 4.1911 | 52 | 6.7583 |
| 15 | .4364 | 59 | 1.7164 | 26 | 4.2494 | 54 | 6.8163 |
| 16 | .4655 | 1 | 1.7455 | 28 | 4.3076 | 56 | 6.8751 |
| 17 | .4946 | 2 | 1.8038 | 30 | 4.3659 | 58 | 6.9339 |
| 18 | .5237 | 4 | 1.8620 | 32 | 4.4242 | 4 | 6.9926 |
| 19 | .5528 | 6 | 1.9202 | 34 | 4.4826 | 5 | 7.1384 |
| 20 | .5818 | 8 | 1.9784 | 36 | 4.5409 | 10 | 7.2842 |
| 21 | .6109 | 10 | 2.0366 | 38 | 4.5993 | 15 | 7.4300 |
| 22 | .6400 | 12 | 2.0948 | 40 | 4.6576 | 20 | 7.5767 |
| 23 | .6691 | 14 | 2.1530 | 42 | 4.7159 | 25 | 7.7234 |
| 24 | .6982 | 16 | 2.2112 | 44 | 4.7742 | 30 | 7.8701 |
| 25 | .7273 | 18 | 2.2694 | 46 | 4.8325 | 35 | 8.0163 |
| 26 | .7564 | 20 | 2.3277 | 48 | 4.8908 | 40 | 8.1625 |
| 27 | .7855 | 22 | 2.3859 | 50 | 4.9492 | 45 | 8.3087 |
| 28 | .8146 | 24 | 2.4441 | 52 | 5.0075 | 50 | 8.4554 |
| 29 | .8436 | 26 | 2.5023 | 54 | 5.0658 | 55 | 8.6021 |
| 30 | .8727 | 28 | 2.5604 | 56 | 5.1241 | 5 | 8.7489 |
| 31 | .9018 | 30 | 2.6186 | 58 | 5.1824 | 5 | 8.8951 |
| 32 | .9309 | 32 | 2.6768 | 3 | 5.2407 | 10 | 9.0413 |
| 33 | .9600 | 34 | 2.7350 | 2 | 5.2990 | 15 | 9.1875 |
| 34 | .9891 | 36 | 2.7932 | 4 | 5.3573 | 20 | 9.3347 |
| 35 | 1.0182 | 38 | 2.8514 | 6 | 5.4158 | 25 | 9.4819 |
| 36 | 1.0472 | 40 | 2.9097 | 8 | 5.4742 | 30 | 9.6292 |
| 37 | 1.0763 | 42 | 2.9679 | 10 | 5.5326 | 35 | 9.7755 |
| 38 | 1.1054 | 44 | 3.0262 | 12 | 5.5909 | 40 | 9.9218 |
| 39 | 1.1345 | 46 | 3.0844 | 14 | 5.6493 | 45 | 10.068 |
| 40 | 1.1636 | 48 | 3.1427 | 16 | 5.7077 | 50 | 10.215 |
| 41 | 1.1927 | 50 | 3.2010 | 18 | 5.7660 | 55 | 10.362 |
| 42 | 1.2218 | 52 | 3.2592 | 20 | 5.8244 | 6 | 10.510 |
| 43 | 1.2509 | 54 | 3.3175 | 22 | 5.8827 | | |
| 44 | 1.2800 | 56 | 3.3758 | 24 | 5.9410 | | |

**Hasta en pendientes** de 1°38′=1 en 35=2.85% (véase tabla) pueden bajar los caballos trotando rápidamente con seguridad. En caminos de montaña se llega hasta 3° (=5.24 por ciento) y 5° (=8.75 por ciento). No se debe pasar de 2½° (=4.36 por ciento), á menos que sea absolutamente necesario.

**Cualquier dist horizontal** =longitud del plano × cos A
— **longitud del plano**=distancia horizontal ÷ cos A
— **altura del plano** =distancia horizontal × tang A
ó bien=longitud del plano × sen A.

A una pediente de *n* pies ó metros de altura para 100 pies ó metros, respectivamente, de dist horizontal, se la llama **pendiente de *n* por ciento.**




## Dimensiones relativas de los lados del plano inclinado para una pendiente de 1°.

## PENDIENTES EN METROS PARA 100 METROS HORIZONTALES

Las fracciones de minutos se dan solamente hasta 34 mets para 100.

Un **eclímetro** graduado por la 3.ª columna y numerado por la primera, dará á la vista las pendientes en mets para 100 mets. No hay errores. Original del autor.

| Altura en mets para 100 mets. | Longitud de la pendiente para 100 horiz. | Angulo de la pendiente. | | Altura en mets para 100 mets. | Longitud de la pendiente para 100 horiz. | Angulo de la pendiente. | | Altura en mets para 100 mets. | Longitud de la pendiente para 100 horiz. | Angulo de la pendiente. | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mets. | Gr. | Min. | | Mets. | Gr. | Min. | | Mets. | Gr. | Min. |
| 1 | 100.005 | 0 | 34.4 | 35 | 105.948 | 19 | 17 | 69 | 121.495 | 34 | 36 |
| 2 | 100.020 | 1 | 8.7 | 36 | 106.283 | 19 | 48 | 70 | 122.066 | 35 | 0 |
| 3 | 100.045 | 1 | 43.1 | 37 | 106.626 | 20 | 18 | 71 | 122.642 | 35 | 23 |
| 4 | 100.080 | 2 | 17.5 | 38 | 106.977 | 20 | 48 | 72 | 123.223 | 35 | 45 |
| 5 | 100.125 | 2 | 51.8 | 39 | 107.336 | 21 | 18 | 73 | 123.810 | 36 | 8 |
| 6 | 100.180 | 3 | 26.0 | 40 | 107.703 | 21 | 48 | 74 | 124.403 | 36 | 30 |
| 7 | 100.245 | 4 | 0.3 | 41 | 108.079 | 22 | 18 | 75 | 125.000 | 36 | 52 |
| 8 | 100.319 | 4 | 34.4 | 42 | 108.462 | 22 | 47 | 76 | 125.603 | 37 | 14 |
| 9 | 100.404 | 5 | 8.6 | 43 | 108.853 | 23 | 16 | 77 | 126.210 | 37 | 36 |
| 10 | 100.499 | 5 | 42.6 | 44 | 109.252 | 23 | 45 | 78 | 126.823 | 37 | 57 |
| 11 | 100.603 | 6 | 16.6 | 45 | 109.659 | 24 | 14 | 79 | 127.440 | 38 | 19 |
| 12 | 100.717 | 6 | 50.6 | 46 | 110.073 | 24 | 42 | 80 | 128.062 | 38 | 40 |
| 13 | 100.841 | 7 | 24.4 | 47 | 110.494 | 25 | 10 | 81 | 128.690 | 39 | 1 |
| 14 | 100.975 | 7 | 58.2 | 48 | 110.923 | 25 | 38 | 82 | 129.321 | 39 | 21 |
| 15 | 101.119 | 8 | 31.9 | 49 | 111.359 | 26 | 6 | 83 | 129.958 | 39 | 42 |
| 16 | 101.272 | 9 | 5.4 | 50 | 111.803 | 26 | 34 | 84 | 130.599 | 40 | 2 |
| 17 | 101.435 | 9 | 38.9 | 51 | 112.254 | 27 | 1 | 85 | 131.244 | 40 | 22 |
| 18 | 101.607 | 10 | 12.2 | 52 | 112.712 | 27 | 28 | 86 | 131.894 | 40 | 42 |
| 19 | 101.789 | 10 | 45.5 | 53 | 113.177 | 27 | 55 | 87 | 132.548 | 41 | 1 |
| 20 | 101.980 | 11 | 18.6 | 54 | 113.649 | 28 | 22 | 88 | 133.207 | 41 | 21 |
| 21 | 102.181 | 11 | 51.6 | 55 | 114.127 | 28 | 49 | 89 | 133.869 | 41 | 40 |
| 22 | 102.391 | 12 | 24.5 | 56 | 114.612 | 29 | 15 | 90 | 134.536 | 41 | 59 |
| 23 | 102.611 | 12 | 57.2 | 57 | 115.104 | 29 | 41 | 91 | 135.207 | 42 | 18 |
| 24 | 102.840 | 13 | 29.8 | 58 | 115.603 | 30 | 7 | 92 | 135.882 | 42 | 37 |
| 25 | 103.078 | 14 | 2.2 | 59 | 116.108 | 30 | 32 | 93 | 136.561 | 42 | 55 |
| 26 | 103.325 | 14 | 34.5 | 60 | 116.619 | 30 | 58 | 94 | 137.244 | 43 | 14 |
| 27 | 103.581 | 15 | 6.6 | 61 | 117.137 | 31 | 23 | 95 | 137.931 | 43 | 32 |
| 28 | 103.846 | 15 | 38.5 | 62 | 117.661 | 31 | 48 | 96 | 138.622 | 43 | 50 |
| 29 | 104.120 | 16 | 10.3 | 63 | 118.191 | 32 | 13 | 97 | 139.316 | 44 | 8 |
| 30 | 104.403 | 16 | 42.0 | 64 | 118.727 | 32 | 37 | 98 | 140.014 | 44 | 25 |
| 31 | 104.695 | 17 | 13.4 | 65 | 119.269 | 33 | 1 | 99 | 140.716 | 44 | 43 |
| 32 | 104.995 | 17 | 44.7 | 66 | 119.817 | 33 | 25 | 100 | 141.421 | 45 | 00 |
| 33 | 105.304 | 18 | 15.8 | 67 | 120.370 | 33 | 49 | 101 | 142.130 | 45 | 17 |
| 34 | 105.622 | 18 | 46.7 | 68 | 120.930 | 34 | 13 | 102 | 142.843 | 45 | 34 |

Tratándose de las **pendientes en los ferrocarriles** se acostumbra, como en nuestra tabla, determinarla por la relación entre la altura A y la distancia

horizontal B. Así se tiene $\frac{A}{B}$ = tang $a$ (ángulo que forma el plano con la horizontal). Cuando se establece la relación entre la altura A y la longitud del plano C se tiene $\frac{A}{C}$ = sen $a$; y esta fracción es proporcional á la componente S del peso W en la dirección de la pendiente. Así en una pendiente en que la altura A = .1 × longitud C, se tendrá que sen $a = \frac{A}{C} = .1$; y S = .1 W. La tang $a$ es sólo aproximadamente proporcional á S; pero las pendientes recorridas con tracción sola-

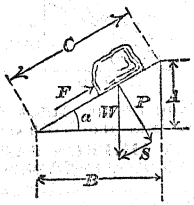



Fig. 78.

mente, aun en los tranvías eléctricos, rara vez exceden á 13 ó 15 por ciento, y en estos casos el error debido á usar tang $a$ en lugar de sen $a$, es menor que una diferencia de .2 por ciento en la pendiente y como de 1 por ciento en el verdadero valor de S. Para pendientes muy fuertes como las de los ferrocarriles de cremallera, debe especificarse siempre si se trata del seno ó de la tang del ángulo $a$.

Las **pendientes transversales**, como las de los movimientos de tierra, se dan, á veces, como en los ferrocarriles; así : $\frac{\text{altura vert}}{\text{dist horizontal}}$ $\frac{A}{B}$, pero usualmente se da invertida $\frac{\text{dist horizontal}}{\text{altura vert}}$, $\frac{B}{A}$. En este caso $\frac{B}{A}$ = cotang del ang $a$ con la horizontal ó tang del ángulo con la vertical que es = (90°-$a$).

## Tabla de cargas ó alturas de agua correspondientes á presiones dadas.

El agua en su mayor densidad, 62.425 libras por pie cúbico = 1 gramo por centímetro cúbico; corresponde á una temperatura de 4° centígs = 39°.2 Fahr.

Carga en pies = 2.306768 × presión en libs por pulg cuadrada.
— — = .0160192 × presión en libs por pie cuadrado.

*Obs. del T.* — N.° 1. En sistema métrico tenemos : Presión en kilogramos por metro cuadrado = carga en metros × 1,000, de donde se deduce que la **carga en mets es** = .001 × *presión en kilgms por met cuad*, con cuya fórmula se hace innecesaria la construcción de una tabla, por la sencillez del cálculo. Ej. : ¿A qué carga corresponde una presión de 105,464.96 kilogms por met cuad. Tendremos que la carga en met es la milésima parte de esta presión ó sea 105.465 mets.

N.° 2. Para algún caso en que se quiera relacionar *algunos de estos datos en el sistema métrico con los de* medidas inglesas ó *viceversa*, recuérdese que para convertir *libras por pulgadas cuadradas en kilogramos por centímetro cuad*, se multiplican aquéllas por .07039. Para convertir *pies en metros* se multiplican aquéllos por .30479.

Para las conversiones contrarias : *kilogramos por centímetro cuad* en *libras por pulg cuad*, se multiplican aquéllos por 14.2228; y para convertir *metros en pies*, se multiplican aquéllos por 3.2809.

## TABLA DE PRESIONES CORRESPONDIENTES Á CARGAS DE AGUA DADAS

El agua en su mayor densidad pesa á razón de 62.425 lbs por pie cúb = 1 gramo por cent cúb = 1 kilogramo el litro; á la temperatura de 4° centígrados = 39°2 Fahrenheit.

Presión en lbs por pulg cuad = .433507 × carga en pies.
Presión en lbs por pie cuad = 62.425 × carga en pies.

*N. del T.* Presión en kilgms por met cuad = 1,000 × carga en mets.
Presión en kilgms por cent cuad = .1 × carga en mets.

Las presiones correspondientes á cargas no dadas en la tabla pueden encontrarse por medio de estas fórmulas, ó deducirse de dicha tabla por una simple proporción.

| Carga. | Presión. | | Carga. | Presión. | |
|---|---|---|---|---|---|
| Pulgs. | Libs por pulg². | Libs por pie². | Pulgs. | Libs por pulg². | Libs por pie². |
| 1 | 0.036126 | 5.202083 | 7 | 0.252879 | 36.414583 |
| 2 | 0.072251 | 10.404167 | 8 | 0.289005 | 41.616667 |
| 3 | 0.108377 | 15.606250 | 9 | 0.325130 | 46.818750 |
| 4 | 0.144502 | 20.808333 | 10 | 0.361256 | 52.020833 |
| 5 | 0.180628 | 26.010417 | 11 | 0.397381 | 57.222917 |
| 6 | 0.216753 | 31.212500 | 12 | 0.433507 | 62.425000 |

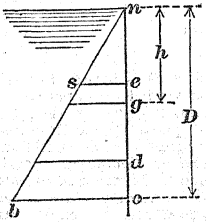
*N. del T.* — Véanse « Obs. del T. » pag 266.

*Obs. del T.* — De igual modo que para hallar la carga en unidades métricas, tampoco es necesario una tabla para encontrar la presión, pues basta para hallar la presión en kilogramos por metro cuadrado multiplicar por 1,000 la carga en metros, ó mutiplicándola por .1 la obtenemos en kilogramos por centímetro cuadrado. Ej. : ¿Qué presión por metro cuadrado produce una carga de 11.2774 met? Según lo dicho, tendremos : 11.4774 × 1,000 = 11277.4 kilogms por met cuad.

Otro ej. : ¿Qué presión por centím cuad nos dará una carga de .304794 met (12 pulgs)? Conforme á la fórmula, tendremos :

.304794 × .1 = .0304794 kilogms por cm cuad.

**Presión total P** contra un plano vertical, *no*, de un ancho igual á la unidad, *y*, perpendicular al papel y de profundidad D = *no*, á contar del nivel del agua *n*.




Area de la superficie plana = *a* = D = 2 *h*; *h* = distancia de la superficie *n* al centro de gravedad, *g*, del plano *no*. El agua en su máximum de densidad, ó peso *w* = 1 gramo por cm cúb = 62.425 lbs por pie cúb á la temperatura de 4° C = 39°2 F.

Sea *bo* = *w*D = á la unidad de presión á la profundidad D. Entonces $P = \frac{wD}{2} \times D =$ *wha*. Por tanto, P está representado por el área del triángulo, *nbo*.

Con D en metros; si *y* = 1 metro, P en kilogramos = 500 D².

La presión **sobre una parte** *ed*, está representada por la presión sobre el trapezoide *ds* y se encuentra así :

Presión total en *ed* = presión total en *nd* — presión total en *ne*.

*Obs. del T.* — Para reemplazar las del autor damos la siguiente tabla de la presión total en gramos ejercida contra un **plano vertical** de 1 cent de ancho introducido desde la superficie del agua hasta las profundidades que indica la primera columna.

La presión total es igual á la mitad del cuad de la profundidad.

| Profd. en centímt. | Presión total en grams. | Profd. en centímt. | Presión total en grams. | Profd. en centímt. | Presión total en grams. | Profd. en centímt. | Presión total en grams. |
|---|---|---|---|---|---|---|---|
| 1 | .5 | 17 | 144.5 | 33 | 544.5 | 49 | 1200.5 |
| 2 | 2 | 18 | 162 | 34 | 578 | 50 | 1250 |
| 3 | 4.5 | 19 | 180.5 | 35 | 612.5 | 51 | 1300.5 |
| 4 | 8 | 20 | 200 | 36 | 648 | 52 | 1352 |
| 5 | 12.5 | 21 | 220.5 | 37 | 684.5 | 53 | 1404.5 |
| 6 | 18 | 22 | 242 | 38 | 722 | 54 | 1458 |
| 7 | 24.5 | 23 | 264.5 | 39 | 760.5 | 55 | 1512.5 |
| 8 | 32 | 24 | 288 | 40 | 800 | 60 | 1800 |
| 9 | 40.5 | 25 | 312.5 | 41 | 840.5 | 65 | 2122.5 |
| 10 | 50 | 26 | 338 | 42 | 882 | 70 | 2450 |
| 11 | 60.5 | 27 | 364.5 | 43 | 924.5 | 75 | 2812.5 |
| 12 | 72 | 28 | 392 | 44 | 968 | 80 | 3200 |
| 13 | 84.5 | 29 | 420.5 | 45 | 1012.5 | 85 | 3612.5 |
| 14 | 98 | 30 | 450 | 46 | 1058 | 90 | 4050 |
| 15 | 112.5 | 31 | 480.5 | 47 | 1104.5 | 95 | 4512.5 |
| 16 | 128 | 32 | 512 | 48 | 1152 | 100 | 5000 |

La presión total en kilogramos (para $y=1$ m) es igual al cuadrado de la profundidad en metros, multiplicado por 500. $P=500\ D^2$.

## TIEMPO

60 segundos, $\overset{*}{\prime}$ marcados $s=1$ minuto.
60 minutos, $\overset{\dagger}{\prime}$ — $m=3{,}600$ segundos.
24 horas, — $h=1$ día $=1{,}440$ m $=86{,}400$ s.
7 días, — $d=1$ semana $=168$ h $=10{,}080$ m.

| Arco. | Tiempo. |
|---|---|
| $1^o=4$ | minutos. |
| $1'=4$ | segundos. |
| $1'=$ | .066 segundos. |

| Tiempo. | Arco. |
|---|---|
| 24 horas | $=360^o$ |
| 1 hora | $=15^o$ |
| 1 minuto | $=0^o15'$ |
| 1 segundo | $=0^o0'15''$ |

**Métodos para calcular el tiempo.** Los astrónomos distinguen el tiempo solar medio, el tiempo solar verdadero ó aparente y el tiempo sideral.

Para un meridiano normal (pag 270) el tiempo solar medio es el tiempo que marcan los relojes. Para otro punto no situado en el meridiano normal, el tiempo **normal** es el tiempo solar medio local del meridiano adoptado como normal para dicho punto; y el tiempo *local*=al tiempo en el meridiano normal *más* la corrección por longitud de ese meridiano si el lugar está situado al *este* del meridiano y viceversa. Para el valor de esta corrección véase la segunda tabla arriba. **Un día solar verdadero ó aparente** es el tiempo transcurrido entre dos culminaciones sucesivas del sol por el meridiano de un mismo lugar; pero como estos intervalos son desiguales, no se corresponden con el movimiento uniforme de los relojes. Se supone un sol ficticio llamado « sol medio », moviéndose en el ecuador celeste con movimiento uniforme de tal manera que el intervalo entre sus culminaciones sea constante. Este intervalo se llama día solar medio y es el término

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*El segundo se dividía antes en 60 partes iguales llamadas *terceros* (marcados $''$), pero ahora se lo divide por el sistema decimal.

† La antigua y confusa práctica de designar los minutos y segundos y terceros de tiempo así: $'$, $''$, $'''$, ya no está en boga. Ahora los días, horas, minutos y segundos se designan con las letras *d*, *h*, *m*, *s*, respectivamente; así: 2d (2 días), 20h (20 horas), 48m (48 minutos), 53s (53 segundos), y los símbolos $'$ y $''$ para los minutos y segundos *de arco*.

medio de todos los días solares *aparentes* en un año. El tiempo medio y el aparente concuerdan cuatro veces en el año, á saber : hacia mediados de abril y de junio, en septiembre 1 y diciembre 24. El sol está á veces « retardado » y á veces « adelantado » respecto al sol medio. En invierno está « retardado » y el máximo es hacia febrero 11, que pasa por el meridiano como 14 m 28 s *después* del medio día con respecto á cualquier reloj correcto. El sol está « adelantado » respecto á los relojes en mayo y después llegando á un máximum como de 16 m 20 s hacia noviembre 2.

La diferencia entre el día solar verdadero y el día solar medio se llama **ecuación del tiempo**. Se la encuentra en el Almanaque Náutico (ó en el Conocimiento de los Tiempos) ó aproximadamente tomando el término medio entre la salida y puesta del sol, dada por casi todos los almanaques.

Así como se mide el tiempo solar, por el movimiento aparente, diario del *sol*, así se mide el tiempo *sideral* por el de las estrellas fijas ó, hablando con más precisión, por el movimiento del equinoccio de primavera que es el punto en que el sol corta el ecuador en esa época.

**Un día sideral** es el intervalo de tiempo transcurrido entre dos pasos sucesivos del equinoccio de primavera (ó prácticamente de cualquier estrella) por el meridiano de un lugar. Es prácticamente el tiempo requerido para una revolución completa de la tierra alrededor de su eje. El día sideral es de 23 h, 56 m, 4.09 s, de día solar medio. Con otras palabras, una estrella aparece en un punto determinado del cielo 3 m, 55.91 s, antes de la hora en que apareció la anterior en el mismo punto midiendo el tiempo en un buen reloj. Por tanto, hay un día más sideral en un año que días solares medios.

El día sideral, como el solar, se divide en 24 horas y las horas siderales son por tanto más cortas que las solares en la misma proporción que los respectivos días. Se cuentan de 0 á 24 comenzando por el instante en que el equinoccio de primavera pasa por el meridiano.

**El día civil** (=24 horas de los relojes ó de tiempo solar medio) comienza á media noche y **el día solar astronómico** al mediodía del día civil de la misma fecha. Así, para un meridiano normal, el jueves, mayo 9, á las 2 A. M. tiempo civil, es miércoles mayo 8, 14 h tiempo astronómico; pero jueves mayo 9, 2 P. M., tiempo civil, es jueves mayo 9, 2 h tiempo astronómico.

**El mes civil** es el mes ordinario y arbitrario de los almanaques, que varía entre 28 y 31 días solares medios.

**El mes sideral** es el tiempo en que la Luna hace, con referencia á las estrellas, una revolución completa y su duración media en tiempo solar medio es como de 27 d, 7 h, 43 m, 12 s.

**El mes lunar** es el tiempo transcurrido entre dos lunas nuevas sucesivas; su duración media es como de 29 d, 12 h, 44 m, 3 s.

**El año natural ó tropical** es el tiempo transcurrido entre dos pasos sucesivos de la Tierra por un mismo equinoccio; su duración media en tiempo solar medio es ahora como de 365 d, 5 h, 48 m, 49 s.

**El año sideral** es el tiempo en que la Tierra describe su órbita completa respecto á las estrellas. Su duración media en tiempo solar medio es como 365 d 6 h, 9 m, 10 s.

**El año civil** es el arbitrario ó convencional y variable del tiempo comprendido entre el 1.º de enero y el 31 de diciembre, inclusives. Contiene generalmente 365 días solares medios de 24 horas; pero los años cuyos números son divisibles por 4, contienen 366 y se llaman **bisiestos**. Se exceptúan los años que terminan en dos ceros (00) y que no son divisibles por **400**. Estos no son bisiestos.

**Arreglo de un reloj por las estrellas.** El autor, después de haber arreglado su cronómetro durante un año por este método solamente, encontró diferencias de muy pocos segundos, con el tiempo calculado por cuidadosas observaciones solares. Escójase una ventana que vea hacia el **oeste** si es posible y que domine la vista de un caballete de techo ú otra línea horizontal fija, con preferencia hacia los 40º de altura sobre el horizonte, para evitar las perturbaciones debidas á la refracción, y distante 20 ms ó más. Anótese la hora en que cualquier estrella fija, brillante (no un planeta), pase por el alineamiento entre el techo y cualquiera línea fija y horizontal en el marco de la ventana, como por ejemplo un alfiler colocado en el marco. El punto de mira en la ventana, y el reloj deben estar iluminados. La estrella pasará cada noche por la línea dicha 3 m, 55.91 s más temprano que la anterior. Las estrellas próximas al Ecuador se mueven más rápidamente y son por tanto las mejores para estas observaciones. Si la primera observación se hace con una estrella hacia la media noche, esta misma estrella

servirá durante 3 meses, al fin de los cuales pasará ya con la luz del día. Antes que esto suceda, transfiérase el tiempo á otra estrella que salga más tarde. Formando tablas de observaciones durante el año con una media docena de estrellas que se sigan á intervalos de tiempo más ó menos iguales, se tendrá un regulador ó guía para regularizar la marcha de los relojes en cualquier noche clara. Haciendo observaciones el autor con dos de los mejores **cronómetros**, encontró variaciones de 3 á 8 segundos por día.

Un hombre normal da **dos pasos** (uno á la derecha, otro á la izquierda) **por segundo**. Por esto, la música marcial tiene un segundo por medida ó compás. Los relojes modernos dan cinco golpes (tic) por segundo, y los de pared, uno, dos ó cuatro.

## MODELO DE HORA DE LOS FERROCARRILES, ADOPTADO EN 1883*

La disposición siguiente, adoptada acerca de la hora modelo, fué recomendada por las Convenciones generales de la hora de los ferrocarriles de los E. U. y del Canadá, reunidas respectivamente en San Luis, Mo; en Nueva York en abril de 1883, y en Chicago Ill, y Nueva York en octubre de 1883, puesto en vigencia en casi todos los ferrocarriles de los E. U. y Canadá el 18 de noviembre de 1883. La mayor parte de las principales ciudades de los E. U. han hecho que sus respectivas horas locales correspondan con ella. Este sistema fué propuesto par W. F. Allen, Secretario de la « Convenciones del Tiempo » (« Time Conventions »); y su adopción fué debida en gran parte á sus esfuerzos. Nosotros debemos á Allen los documentos de donde se ha extractado lo siguiente. Cinco modelos de hora, ó cinco « horas » han sido adoptados por los E. U. y Canadá. Éstas son respectivamente la hora media de los meridianos 60, 75, 90, 105 y 120 al oeste de Greenwich (Inglaterra). Como cada uno de estos meridianos, en el orden anterior está 15° al oeste del que le precede, tiene una hora de atraso. Así, cuando es medio día en el meridiano 90, es la 1 P. M. en el 75, y las 11 A. M. en el 105. La tabla siguiente da el nombre adoptado de la hora modelo de cada meridiano y los colores convencionales elegidos y aceptados uniformemente por Allen, con el objeto de consignarlos juntamente con la hora y su tiempo, etc., en los mapas publicados bajo sus auspicios.

| Longitud al oeste de Greenwich. | Nombre de la hora modelo. | Color convencional, |
|---|---|---|
| 60° | Intercolonial. | Castaño. |
| 75° | Oriental. | Rojo. |
| 90° | Central. | Azul. |
| 105° | Montaña. | Verde. |
| 120° | Pacífico. | Amarillo. |

Teóricamente, se puede decir que cada meridiano da la hora para una faja de terreno de 15° de ancho, y con el meridiano por centro. Así, en el meridiano en el cual se supone que tiene lugar el cambio de hora de dos meridianos modelos, está situado en el medio de ellos. Pero no sería práctico para los ferrocarriles el uso de una línea imaginaria al pasar de una hora modelo á otra. Los cambios se hacen en las estaciones principales que constituyen el término de dos ó más líneas; ó en el caso de las grandes vías del Pacífico, al final de las secciones. Hasta donde se ha podido, en la práctica, los puntos en los cuales se habían efectuado cambios previos de hora, fueron elegidos por puntos de cambio bajo el nuevo sistema. *Detroit, Michigán; Pittsburgh, Pensylvania : Wheeling* y *Parkersburg, W. Va*, y *Augusta, Ga*, aun cuando no están situados sobre el mismo meridiano, son puntos de cambio entre los modelos de hora *orientales* y *centrales*. Un tren que llega á Pittsburg del Este á mediodía, y que sale para el Oeste 10 minutos después de su llegada, sale (según las indicaciones de su itinerario y reloj del conductor) no á las 12 h y 10 m, sino á las 11 h y 10.

La necesidad de efectuar cambios de hora en los puntos principales, en lugar

* *N. del T.* — Aunque esto no tiene importancia sino para los E. U., lo hemos dejado porque da muy clara idea de lo que puede hacerse en otras naciones.

de hacerlo en el meridiano verdadero, exige también alguna modificación de la hora ó de sus colores convencionales en el mapa. Así, en la mayor parte de las vías entre Búffalo y Detroit, al *norte* del lago Erie rige la hora « oriental » ó « roja »; mientras que en las del *sur* del lago, entre Búffalo y Toledo, inmediata y directamente opuestas al sur de ellos, rige la hora « central » ó « azul ».

Si los cambios de hora fueran hechos en los meridianos intermedios entre los meridianos modelos, no necesitaría ninguna ciudad variar su hora en más de 30 m. Como se hace actualmente, tienen que efectuarse mayores cambios en algunos puntos. De este modo, la hora modelo en Detroit tiene 32 minutos de adelanto, y en Savannah 36 minutos de atraso, con respecto al tiempo medio local.

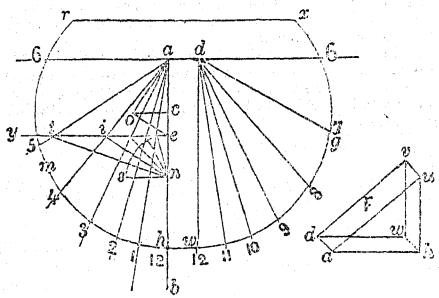
En la mayor parte de los casos se hacían los cambios de hora en los ferrocarriles adelantando ó atrasando los relojes el número necesario de minutos, sin hacer modificación alguna en sus itinerarios locales.

Halifax y algunas ciudades adyacentes usan la hora del meridiano sexagésimo por ser el más cercano; pero los *ferrocarriles* del mismo distrito han adoptado el meridiano 75 ú hora « oriental »; de modo que para los usos del ferrocarril, no se ha puesto en ejecución la hora intercolonial.

En 1873 habían 71 modelos de hora en uso en los ferrocarriles de los Estados Unidos y el Canadá. Para la época de la adopción de este sistema, este número había sido reducido á 53. Por su adopción, el número disminuyó á 5, ó practicamente, á 4, debido á la adopción de la hora « oriental » para las vías intercoloniales, como se ha explicado antes.

# GNOMÓNICA

**Para hacer un reloj de sol horizontal** trácese una línea *ab;* y en ángulo recto á ella trácese 66. Desde un punto conveniente *c*, en *ab*, trácese la perpendicular *co*. Hágase el ángulo *cao* igual á la latitud del lugar; también el ángulo *coe* igual á la misma latitud; trácese *oe*. Hágase *en* igual á *oe*, y desde *n* como centro. con el radio *en* descríbase un cuadrante *es*, y divídase éste en 6 partes iguales. Trácese *ey* paralela á 66; y desde *n* por los 5 puntos del cuadrante, trácense líneas




*nt, ni*, etc., que terminen en *ey*. Desde *a* trácense líneas *a*5, *a*4, etc., que pasen por *t*, *i*, etc. Desde cualquier punto conveniente *c*, descríbase un arco *rmh*, especie de límite ó borde de la mitad del reloj.

Ahora pueden borrarse todas las líneas, excepto las horarias *a*6, *a*5, *a*4, etc, hasta *a*12 ó *ah*, á menos que, como ocurre generalmente, haya que dividir el reloj en cuartos de hora. En este caso cada una de las divisiones del cuadrante *es* debe ser subdividida en cuatro partes iguales; y las líneas trazadas desde *n*, por los puntos de subdivisión terminarlas en *ey*. Las líneas de cuarto de hora deben ser trazadas desde *a* como lo fueron las líneas horarias. Pueden hacerse subdivisiones de 5 minutos de la misma manera; pero éstas, así como los minutos, pueden marcarse al ojo, alrededor del borde. Para un reloj ordinario úsese como 8 ó 10 veces el tamaño de la figura. Para trazar la otra mitad de la figura, hágase *ad* igual al espesor que se le quiere dar al estilo, del reloj; y trácese *d*12 paralela é igual á *a*12, y el arco *xgw* precisamente semejante al arco *rmh*. Entre *x* y *w* en el arco *xgw* tómense divisiones iguales á las del arco *rmh*, y numérense las horas como en la figura. El estilo F de metal ó piedra (la madera es muy fácil de torcerse) será triangular, su grueso igual en toda su extensión á *ad*, ó *hw;* su base debe cubrir el espacio *adhw;* su extremo debe estar en *ad;* y su altura *hu*, perpendicular á *hw*, debe ser tal que las líneas *vd*, *ua*, trazadas desde su parte superior hacia *ad*, hagan los ángulos *uah*, *vdw*, iguales cada uno á la latitud del lugar. Su espesor, si es de metal, puede ser de (3 á 6 mm); y si es de piedra 1 ó 2 pulgs (25 á 50 mm) ó más, según el tamaño del reloj. Generalmente, por elegancia, la parte de atrás *huvw* del estilo es hueca interiormente. Los bordes superiores *ua*, *vd* que producen las sombras, deben ser afilados y rectos. El reloj debe fijarse horizontal, ó perfectamente á nivel; *ha* y *dw* deben ser colocados exactamente de norte á sur; *ad* al sur, y *hw* al norte. El cuadrante indica solamente horas solares ó tiempo solar; pero se puede conseguir la hora de los relojes por medio del « retardo ó adelanto del sol» (ecuación del tiempo). Si, según el almanaque, el sol está adelantado 5 minutos, etc., el cuadrante estará lo mismo, y para que el reloj esté correcto debe estar 5 minutos más atrasado que él y viceversa.

**Para hacer un reloj de sol vertical.** Procédase como se ha indicado arriba excepto que los ángulos *cao* y *coe* del trazado, y el ángulo *uah* ó *vdw*, del estilo,

deben ser iguales á la *co*-latitud del lugar (=90°—latitud) y las horas deben numerarse en sentido inverso á las de la figura anterior, esto es : desde *h* hacia *y*, los números 12, 11, 10, 9, 8, 7 y, desde *w* hacia *g* los 12, 1, 2, 3, 4, 5. La estera ó muestra del cuadrante debe ser colocada verticalmente, en la posición que da la figura con el frente exactamente al sur y *ah* y *dw* verticales.

# MEDIDA DE TABLAS DE MADERA*

**Advertencia sobre la siguiente tabla.** Esta tabla suministra la medida ó el número de pies cuadrados de tabla que contiene el pie corrido de piezas de madera hasta de 12 pulgs de espesor por 24 pulgs de ancho; pero es fácil calcular el número de pies contenidos en piezas de mayores dimensiones. Por ej. : el número de pies contenidos en una pieza de 19×22 será el doble del de una de 19×11 ó sean (véase la tabla) 17.42×2=34.84 pies cuadrados de tabla; el de una de $19^1/_2 \times 22$ será el de una de $10^1/_2 \times 22$ sumado con el de una de 9×22 ó sean 18.79+16.50= =35.29. Se entiende aquí por un pie cuadrado de tabla ó simplemente un pie de tabla, una tabla de 1 pie en cuadro por 1 pulgada de espesor ó sean 144 pulgadas cúbicas. Por consiguiente, 1 pie cúb=á 12 pies de tabla.

* *N. del T.* — Aunque estas tablas están en sistema inglés, es tan frecuente en las compras y empleo de las maderas el uso de aquellas medidas, que hemos creído conveniente dejarlas.
El m cúb contiene 423.777 pies de tabla.
Un pie de tabla = 2359.72 cm cúb = 0.00235972 m cúb.

## Medida de tablas de madera.

Número de pies cuadrados de tabla contenidos en un pie lineal de madera en piezas de diferentes dimensiones (Original.)
1,000 pies de tabla=83 1/3 pies cúbs.

| Ancho en pulgs. | Espesor en pulgadas. 1 | 1¼ | 1½ | 1¾ | 2 | 2¼ | 2½ | 2¾ | 3 | Ancho en pulgs. |
|---|---|---|---|---|---|---|---|---|---|---|
| ¼ | .0208 | .0260 | .0313 | .0365 | .0417 | .0469 | .0521 | .0573 | .0625 | ¼ |
| ½ | .0417 | .0521 | .0625 | .0729 | .0833 | .0938 | .1042 | .1146 | .1250 | ½ |
| ¾ | .0625 | .0781 | .0938 | .1094 | .1250 | .1406 | .1563 | .1719 | .1875 | ¾ |
| 1. | .0833 | .1042 | .1250 | .1458 | .1667 | .1875 | .2083 | .2292 | .2500 | 1. |
| ¼ | .1042 | .1302 | .1563 | .1823 | .2083 | .2344 | .2604 | .2865 | .3125 | ¼ |
| ½ | .1250 | .1563 | .1875 | .2188 | .2500 | .2813 | .3125 | .3438 | .3750 | ½ |
| ¾ | .1458 | .1823 | .2187 | .2552 | .2917 | .3281 | .3646 | .4010 | .4375 | ¾ |
| 2. | .1667 | .2083 | .2500 | .2917 | .3333 | .3750 | .4166 | .4583 | .5000 | 2. |
| ¼ | .1875 | .2344 | .2813 | .3281 | .3750 | .4219 | .4688 | .5156 | .5625 | ¼ |
| ½ | .2083 | .2604 | .3125 | .3646 | .4167 | .4688 | .5208 | .5729 | .6250 | ½ |
| ¾ | .2292 | .2865 | .3438 | .4010 | .4583 | .5156 | .5729 | .6302 | .6875 | ¾ |
| 3. | .2500 | .3125 | .3750 | .4375 | .5000 | .5625 | .6250 | .6875 | .7500 | 3. |
| ¼ | .2708 | .3385 | .4063 | .4739 | .5416 | .6094 | .6771 | .7448 | .8125 | ¼ |
| ½ | .2917 | .3616 | .4375 | .5104 | .5833 | .6563 | .7292 | .8021 | .8750 | ½ |
| ¾ | .3125 | .3906 | .4689 | .5469 | .6250 | .7031 | .7813 | .8594 | .9375 | ¾ |
| 4. | .3333 | .4167 | .5000 | .5833 | .6667 | .7500 | .8333 | .9167 | 1.000 | 4. |
| ¼ | .3542 | .4427 | .5313 | .6198 | .7083 | .7969 | .8854 | .9740 | 1.063 | ¼ |
| ½ | .3750 | .4688 | .5625 | .6563 | .7500 | .8438 | .9375 | 1.031 | 1.125 | ½ |
| ¾ | .3958 | .4948 | .5938 | .6927 | .7917 | .8906 | .9896 | 1.086 | 1.188 | ¾ |
| 5. | .4167 | .5208 | .6250 | .7292 | .8333 | .9375 | 1.042 | 1.146 | 1.250 | 5. |
| ¼ | .4375 | .5469 | .6563 | .7656 | .8750 | .9844 | 1.094 | 1.203 | 1.313 | ¼ |
| ½ | .4583 | .5729 | .6875 | .8020 | .9167 | 1 031 | 1.146 | 1.260 | 1.375 | ½ |
| ¾ | .4792 | .5990 | .7188 | .8385 | .9583 | 1.078 | 1.198 | 1.318 | 1.438 | ¾ |
| 6. | .5000 | .6250 | .7500 | .8750 | 1.000 | 1.125 | 1.250 | 1.375 | 1.500 | 6. |
| ¼ | .5208 | .6510 | .7813 | .9115 | 1.042 | 1.172 | 1.302 | 1.432 | 1.563 | ¼ |
| ½ | .5417 | .6771 | .8125 | .9479 | 1.083 | 1.219 | 1.354 | 1.490 | 1.625 | ½ |
| ¾ | .5625 | .7031 | .8438 | .9844 | 1.125 | 1.266 | 1.406 | 1.547 | 1.688 | ¾ |
| 7. | .5833 | .7292 | .8750 | 1.021 | 1.167 | 1.312 | 1.458 | 1.604 | 1.750 | 7. |
| ¼ | .6042 | .7552 | .9063 | 1.057 | 1.208 | 1.359 | 1.510 | 1.661 | 1.813 | ¼ |
| ½ | .6250 | .7813 | .9375 | 1.094 | 1.250 | 1.406 | 1.563 | 1.719 | 1.875 | ½ |
| ¾ | .6458 | .8073 | .9688 | 1.130 | 1.292 | 1.453 | 1.615 | 1.776 | 1.938 | ¾ |
| 8. | .6667 | .8333 | 1.000 | 1.167 | 1.333 | 1.500 | 1.667 | 1.833 | 2.000 | 8. |
| ¼ | .6875 | .8594 | 1.031 | 1.203 | 1.375 | 1.547 | 1.719 | 1.891 | 2.063 | ¼ |
| ½ | .7083 | .8854 | 1.063 | 1.240 | 1.417 | 1.594 | 1.771 | 1.948 | 2.125 | ½ |
| ¾ | .7292 | .9114 | 1.094 | 1.276 | 1.458 | 1.641 | 1.823 | 2.005 | 2.188 | ¾ |
| 9. | .7500 | .9375 | 1.125 | 1.313 | 1.500 | 1.688 | 1.875 | 2.062 | 2.250 | 9. |
| ¼ | .7708 | .9635 | 1.156 | 1.349 | 1.542 | 1.734 | 1.927 | 2.120 | 2.313 | ¼ |
| ½ | .7917 | .9896 | 1.188 | 1.385 | 1.583 | 1.781 | 1.979 | 2.177 | 2.375 | ½ |
| ¾ | .8125 | 1.016 | 1.219 | 1.422 | 1.625 | 1.828 | 2.031 | 2.234 | 2.438 | ¾ |
| 10. | .8333 | 1.042 | 1.250 | 1.458 | 1.667 | 1.875 | 2.083 | 2.292 | 2.500 | 10. |
| ¼ | .8542 | 1.068 | 1.281 | 1.495 | 1.708 | 1.922 | 2.135 | 2.349 | 2.563 | ¼ |
| ½ | .8750 | 1.094 | 1.313 | 1.531 | 1.750 | 1.969 | 2.188 | 2.406 | 2.625 | ½ |
| ¾ | .8958 | 1.120 | 1.344 | 1.568 | 1.792 | 2.016 | 2.240 | 2.463 | 2.688 | ¾ |
| 11. | .9167 | 1.146 | 1.375 | 1.604 | 1.833 | 2.063 | 2.292 | 2.521 | 2.750 | 11. |
| ¼ | .9375 | 1.172 | 1.406 | 1.641 | 1.875 | 2.109 | 2.344 | 2.578 | 2.813 | ¼ |
| ½ | .9583 | 1.198 | 1.438 | 1.677 | 1.917 | 2.156 | 2.396 | 2.635 | 2.875 | ½ |
| ¾ | .9792 | 1.224 | 1.469 | 1.714 | 1 958 | 2.203 | 2.448 | 2.693 | 2.938 | ¾ |
| 12. | 1.000 | 1.250 | 1.500 | 1.750 | 2.000 | 2.250 | 2.500 | 2.750 | 3.000 | 12. |
| ½ | 1.042 | 1.302 | 1.563 | 1.823 | 2.083 | 2.344 | 2.604 | 2.865 | 3.125 | ½ |
| 13. | 1.083 | 1.355 | 1.625 | 1.896 | 2.167 | 2.438 | 2.708 | 2.979 | 3.250 | 13. |
| ½ | 1.125 | 1.406 | 1.688 | 1.969 | 2.250 | 2.531 | 2.813 | 3.094 | 3.375 | ½ |
| 14. | 1.167 | 1.458 | 1.750 | 2.042 | 2.333 | 2.625 | 2.917 | 3.208 | 3.500 | 14. |
| ½ | 1.208 | 1.510 | 1.813 | 2.115 | 2.417 | 2.719 | 3.021 | 3.322 | 3.625 | ½ |
| 15. | 1.250 | 1.563 | 1.875 | 2.188 | 2.500 | 2.813 | 3.125 | 3.438 | 3.750 | 15. |
| ½ | 1.292 | 1.615 | 1.938 | 2.260 | 2.583 | 2.906 | 3.229 | 3.552 | 3.875 | ½ |
| 16. | 1.333 | 1.667 | 2.000 | 2.333 | 2.667 | 3.000 | 3.333 | 3.667 | 4.000 | 16. |
| ½ | 1.375 | 1.719 | 2.063 | 2.406 | 2.750 | 3.094 | 3.438 | 3.781 | 4.125 | ½ |
| 17. | 1.417 | 1.771 | 2.125 | 2.479 | 2.833 | 3.188 | 3.542 | 3.896 | 4.250 | 17. |
| ½ | 1.458 | 1.823 | 2.187 | 2.552 | 2.917 | 3.281 | 3.646 | 4.010 | 4.375 | ½ |
| 18. | 1.500 | 1.875 | 2.250 | 2.625 | 3.000 | 3.375 | 3.750 | 4.125 | 4.500 | 18. |
| 19. | 1.583 | 1.979 | 2.375 | 2.771 | 3.167 | 3.563 | 3.958 | 4.354 | 4.750 | 19. |
| 20. | 1.667 | 2.083 | 2.500 | 2.917 | 3.333 | 3.750 | 4.167 | 4.583 | 5.000 | 20. |
| 21. | 1.750 | 2.188 | 2.625 | 3.063 | 3.500 | 3.938 | 4.375 | 4 812 | 5.250 | 21. |
| 22. | 1.833 | 2.292 | 2.750 | 3.208 | 3.667 | 4.125 | 4.583 | 5.042 | 5.500 | 22. |
| 23. | 1.917 | 2.396 | 2.875 | 3.354 | 3.833 | 4.313 | 4.792 | 5.270 | 5.750 | 23. |
| 24. | 2.000 | 2.500 | 3.000 | 3.500 | 4.000 | 4.500 | 5.000 | 5.500 | 6.000 | 24. |

## Medida de tablas de madera. (Continuación.)

| Ancho en pulgadas. | Número de pies cuadrados de tabla contenidos en un pie lineal de madera en piezas de diferentes dimensiones. (Original.) Espesor en pulgadas. | | | | | | | | | Ancho en pulgadas. |
|---|---|---|---|---|---|---|---|---|---|---|
| | 3¼ | 3½ | 3¾ | 4 | 4¼ | 4½ | 4¾ | 5 | 5¼ | |
| ¼ | .0677 | .0729 | .0781 | .0833 | .0885 | .0938 | .0990 | .1042 | .1094 | ¼ |
| ⅜ | .1354 | .1457 | .1562 | .1667 | .1770 | .1875 | .1979 | .2083 | .2188 | ⅜ |
| ½ | .2031 | .2187 | .2344 | .2500 | .2656 | .2813 | .2969 | .3125 | .3282 | ½ |
| 1. | .2708 | .2917 | .3125 | .3333 | .3542 | .3750 | .3958 | .4167 | .4375 | 1. |
| ¼ | .3385 | .3646 | .3906 | .4167 | .4427 | .4688 | .4948 | .5208 | .5469 | ¼ |
| ⅜ | .4063 | .4375 | .4688 | .5000 | .5313 | .5625 | .5938 | .6250 | .6563 | ⅜ |
| ½ | .4740 | .5104 | .5469 | .5833 | .6198 | .6563 | .6927 | .7292 | .7656 | ½ |
| 2. | .5417 | .5833 | .6250 | .6667 | .7083 | .7500 | .7917 | .8333 | .8750 | 2. |
| ¼ | .6094 | .6563 | .7031 | .7500 | .7969 | .8438 | .8906 | .9375 | .9844 | ¼ |
| ⅜ | .6771 | .7292 | .7813 | .8333 | .8854 | .9375 | .9896 | 1.042 | 1.094 | ⅜ |
| ½ | .7448 | .8021 | .8594 | .9167 | .9740 | 1.031 | 1.089 | 1.146 | 1.203 | ½ |
| 3. | .8125 | .8750 | .9375 | 1.000 | 1.062 | 1.125 | 1.188 | 1.250 | 1.313 | 3. |
| ¼ | .8802 | .9479 | 1.016 | 1.083 | 1.151 | 1.219 | 1.286 | 1.354 | 1.422 | ¼ |
| ⅜ | .9479 | 1.021 | 1.094 | 1.167 | 1.240 | 1.313 | 1.385 | 1.458 | 1.531 | ⅜ |
| ½ | 1.016 | 1.094 | 1.172 | 1.250 | 1.327 | 1.406 | 1.484 | 1.563 | 1.641 | ½ |
| 4. | 1.083 | 1.167 | 1.250 | 1.333 | 1.416 | 1.500 | 1.583 | 1.667 | 1.750 | 4. |
| ¼ | 1.151 | 1.240 | 1.328 | 1.417 | 1.504 | 1.594 | 1.682 | 1.771 | 1.859 | ¼ |
| ⅜ | 1.219 | 1.313 | 1.406 | 1.500 | 1.593 | 1.688 | 1.781 | 1.875 | 1.969 | ⅜ |
| ½ | 1.286 | 1.384 | 1.484 | 1.583 | 1.681 | 1.781 | 1.880 | 1.979 | 2.078 | ½ |
| 5. | 1.354 | 1.457 | 1.566 | 1.666 | 1.770 | 1.875 | 1.979 | 2.083 | 2.188 | 5. |
| ¼ | 1.422 | 1.530 | 1.644 | 1.750 | 1.858 | 1.969 | 2.078 | 2.188 | 2.297 | ¼ |
| ⅜ | 1.490 | 1.603 | 1.722 | 1.833 | 1.947 | 2.063 | 2.177 | 2.292 | 2.406 | ⅜ |
| ½ | 1.557 | 1.676 | 1.800 | 1.917 | 2.035 | 2.156 | 2.276 | 2.396 | 2.516 | ½ |
| 6. | 1.625 | 1.750 | 1.875 | 2.000 | 2.125 | 2.250 | 2.375 | 2.500 | 2.625 | 6. |
| ¼ | 1.693 | 1.823 | 1.953 | 2.083 | 2.214 | 2.344 | 2.474 | 2.604 | 2.734 | ¼ |
| ⅜ | 1.760 | 1.896 | 2.031 | 2.167 | 2.302 | 2.438 | 2.573 | 2.708 | 2.843 | ⅜ |
| ½ | 1.828 | 1.969 | 2.109 | 2.250 | 2.391 | 2.531 | 2.672 | 2.813 | 2.953 | ½ |
| 7. | 1.896 | 2.042 | 2.188 | 2.333 | 2.479 | 2.625 | 2.771 | 2.917 | 3.063 | 7. |
| ¼ | 1.964 | 2.115 | 2.266 | 2.416 | 2.568 | 2.719 | 2.870 | 3.021 | 3.172 | ¼ |
| ⅜ | 2.031 | 2.187 | 2.344 | 2.500 | 2.656 | 2.813 | 2.969 | 3.125 | 3.281 | ⅜ |
| ½ | 2.099 | 2.260 | 2.422 | 2.583 | 2.745 | 2.906 | 3.068 | 3.229 | 3.391 | ½ |
| 8. | 2.167 | 2.333 | 2.500 | 2.667 | 2.833 | 3.000 | 3.167 | 3.333 | 3.500 | 8. |
| ¼ | 2.234 | 2.406 | 2.578 | 2.750 | 2.922 | 3.094 | 3.266 | 3.438 | 3.609 | ¼ |
| ⅜ | 2.302 | 2.479 | 2.656 | 2.833 | 3.010 | 3.188 | 3.365 | 3.542 | 3.718 | ⅜ |
| ½ | 2.370 | 2.552 | 2.734 | 2.916 | 3.099 | 3.281 | 3.464 | 3.646 | 3.828 | ½ |
| 9. | 2.438 | 2.625 | 2.813 | 3.000 | 3.187 | 3.375 | 3.563 | 3.750 | 3.938 | 9. |
| ¼ | 2.505 | 2.698 | 2.891 | 3.083 | 3.276 | 3.469 | 3.661 | 3.854 | 4.047 | ¼ |
| ⅜ | 2.573 | 2.771 | 2.969 | 3.167 | 3.365 | 3.563 | 3.760 | 3.958 | 4.156 | ⅜ |
| ½ | 2.641 | 2.844 | 3.047 | 3.250 | 3.453 | 3.656 | 3.859 | 4.063 | 4.266 | ½ |
| 10. | 2.708 | 2.917 | 3.125 | 3.333 | 3.542 | 3.750 | 3.958 | 4.167 | 4.375 | 10. |
| ¼ | 2.776 | 2.990 | 3.203 | 3.416 | 3.630 | 3.844 | 4.057 | 4.271 | 4.484 | ¼ |
| ⅜ | 2.844 | 3.063 | 3.281 | 3.500 | 3.719 | 3.938 | 4.156 | 4.375 | 4.594 | ⅜ |
| ½ | 2.911 | 3.135 | 3.359 | 3.583 | 3.807 | 4.031 | 4.255 | 4.479 | 4.703 | ½ |
| 11. | 2.979 | 3.208 | 3.438 | 3.666 | 3.896 | 4.125 | 4.354 | 4.583 | 4.813 | 11. |
| ¼ | 3.047 | 3.281 | 3.516 | 3.750 | 3.984 | 4.219 | 4.453 | 4.688 | 4.922 | ¼ |
| ⅜ | 3.115 | 3.354 | 3.594 | 3.833 | 4.073 | 4.313 | 4.552 | 4.792 | 5.031 | ⅜ |
| ½ | 3.182 | 3.427 | 3.672 | 3.916 | 4.161 | 4.406 | 4.651 | 4.896 | 5.141 | ½ |
| 12. | 3.250 | 3.500 | 3.750 | 4.000 | 4.250 | 4.500 | 4.750 | 5.000 | 5.250 | 12. |
| ½ | 3.385 | 3.646 | 3.906 | 4.167 | 4.427 | 4.688 | 4.948 | 5.208 | 5.469 | ½ |
| 13. | 3.521 | 3.792 | 4.063 | 4.333 | 4.604 | 4.875 | 5.146 | 5.417 | 5.688 | 13. |
| ½ | 3.656 | 3.938 | 4.219 | 4.500 | 4.781 | 5.063 | 5.344 | 5.625 | 5.906 | ½ |
| 14. | 3.792 | 4.083 | 4.375 | 4.667 | 4.958 | 5.250 | 5.542 | 5.833 | 6.125 | 14. |
| ½ | 3.927 | 4.229 | 4.531 | 4.833 | 5.135 | 5.438 | 5.740 | 6.042 | 6.344 | ½ |
| 15. | 4.063 | 4.375 | 4.688 | 5.000 | 5.313 | 5.625 | 5.938 | 6.250 | 6.563 | 15. |
| ½ | 4.198 | 4.521 | 4.844 | 5.166 | 5.490 | 5.813 | 6.135 | 6.458 | 6.781 | ½ |
| 16. | 4.333 | 4.667 | 5.000 | 5.333 | 5.667 | 6.000 | 6.333 | 6.667 | 7.000 | 16. |
| ½ | 4.469 | 4.813 | 5.156 | 5.500 | 5.844 | 6.188 | 6.531 | 6.875 | 7.219 | ½ |
| 17. | 4.604 | 4.958 | 5.313 | 5.667 | 6.021 | 6.375 | 6.729 | 7.083 | 7.438 | 17. |
| ½ | 4.740 | 5.104 | 5.469 | 5.833 | 6.198 | 6.563 | 6.927 | 7.292 | 7.656 | ½ |
| 18. | 4.875 | 5.250 | 5.625 | 6.000 | 6.375 | 6.750 | 7.125 | 7.500 | 7.875 | 18. |
| 19. | 5.146 | 5.542 | 5.938 | 6.333 | 6.729 | 7.125 | 7.521 | 7.917 | 8.313 | 19. |
| 20. | 5.417 | 5.833 | 6.250 | 6.667 | 7.083 | 7.500 | 7.917 | 8.333 | 8.750 | 20. |
| 21. | 5.688 | 6.125 | 6.563 | 7.000 | 7.438 | 7.875 | 8.313 | 8.750 | 9.188 | 21. |
| 22. | 5.958 | 6.417 | 6.875 | 7.333 | 7.792 | 8.250 | 8.708 | 9.167 | 9.625 | 22. |
| 23. | 6.229 | 6.708 | 7.188 | 7.667 | 8.145 | 8.625 | 9.104 | 9.583 | 10.06 | 23. |
| 24. | 6.500 | 7.000 | 7.500 | 8.000 | 8.500 | 9.000 | 9.500 | 10.00 | 10.50 | 24. |

## Medida de tablas de madera. (Continuación.)

| Ancho en pulgadas. | Número de pies cuadrados de tabla contenidos en un pie lineal de madera en piezas de diferentes dimensiones. (Original.) Espesor en pulgadas. | | | | | | | | | Ancho en pulgadas. |
|---|---|---|---|---|---|---|---|---|---|---|
| | 5½ | 5¾ | 6 | 6¼ | 6½ | 6¾ | 7 | 7¼ | 7½ | |
| ¼ | .1146 | .1198 | .1250 | .1302 | .1354 | .1406 | .1458 | .1510 | .1563 | ¼ |
| ½ | .2292 | .2396 | .2500 | .2604 | .2708 | .2813 | .2917 | .3021 | .3125 | ½ |
| ¾ | .3438 | .3594 | .3750 | .3906 | .4063 | .4219 | .4375 | .4531 | .4688 | ¾ |
| 1. | .4583 | .4792 | .5000 | .5208 | .5417 | .5625 | .5833 | .6042 | .6250 | 1. |
| ¼ | .5729 | .5990 | .6250 | .6510 | .6771 | .7031 | .7292 | .7552 | .7813 | ¼ |
| ½ | .6875 | .7188 | .7500 | .7812 | .8125 | .8438 | .8750 | .9062 | .9375 | ½ |
| ¾ | .8021 | .8385 | .8750 | .9115 | .9479 | .9844 | 1.020 | 1.057 | 1.094 | ¾ |
| 2. | .9167 | .9583 | 1.000 | 1.042 | 1.083 | 1.125 | 1.167 | 1.208 | 1.250 | 2. |
| ¼ | 1.031 | 1.078 | 1.125 | 1.172 | 1.219 | 1.266 | 1.313 | 1.359 | 1.406 | ¼ |
| ½ | 1.146 | 1.198 | 1.250 | 1.302 | 1.354 | 1.406 | 1.458 | 1.510 | 1.563 | ½ |
| ¾ | 1.260 | 1.318 | 1.375 | 1.432 | 1.490 | 1.547 | 1.604 | 1.661 | 1.719 | ¾ |
| 3. | 1.375 | 1.438 | 1.500 | 1.562 | 1.625 | 1.688 | 1.750 | 1.813 | 1.875 | 3. |
| ¼ | 1.490 | 1.557 | 1.625 | 1.693 | 1.760 | 1.828 | 1.896 | 1.964 | 2.031 | ¼ |
| ½ | 1.604 | 1.677 | 1.750 | 1.823 | 1.896 | 1.969 | 2.042 | 2.115 | 2.188 | ½ |
| ¾ | 1.719 | 1.797 | 1.875 | 1.953 | 2.031 | 2.109 | 2.188 | 2.266 | 2.344 | ¾ |
| 4. | 1.833 | 1.917 | 2.000 | 2.083 | 2.167 | 2.250 | 2.333 | 2.417 | 2 500 | 4. |
| ¼ | 1.948 | 2.036 | 2.125 | 2.214 | 2.302 | 2.391 | 2.479 | 2.568 | 2.656 | ¼ |
| ½ | 2.063 | 2.156 | 2.250 | 2.344 | 2.438 | 2.531 | 2.625 | 2.719 | 2.813 | ½ |
| ¾ | 2.177 | 2.276 | 2.375 | 2.474 | 2.573 | 2.672 | 2.771 | 2.870 | 2.969 | ¾ |
| 5. | 2.292 | 2.396 | 2.500 | 2.604 | 2.708 | 2.813 | 2.917 | 3.021 | 3.125 | 5. |
| ¼ | 2.406 | 2.516 | 2.625 | 2.734 | 2 844 | 2.953 | 3.063 | 3.172 | 3.281 | ¼ |
| ½ | 2.521 | 2.635 | 2.750 | 2.865 | 2.979 | 3.094 | 3.208 | 3.323 | 3.438 | ½ |
| ¾ | 2.635 | 2.755 | 2.875 | 2.995 | 3.115 | 3.234 | 3.354 | 3.474 | 3.594 | ¾ |
| 6. | 2.750 | 2.875 | 3.000 | 3.125 | 3.250 | 3.375 | 3.500 | 3.625 | 3.750 | 6. |
| ¼ | 2.865 | 2.995 | 3.125 | 3.255 | 3.385 | 3.516 | 3.646 | 3.776 | 3.906 | ¼ |
| ½ | 2.979 | 3.115 | 3.250 | 3.385 | 3.521 | 3.656 | 3.792 | 3.927 | 4.063 | ½ |
| ¾ | 3.094 | 3.234 | 3.375 | 3.516 | 3.656 | 3.797 | 3.938 | 4.078 | 4.219 | ¾ |
| 7. | 3.208 | 3.354 | 3.500 | 3.646 | 3.792 | 3.938 | 4.083 | 4.229 | 4.375 | 7. |
| ¼ | 3.323 | 3.474 | 3.625 | 3.776 | 3.927 | 4.078 | 4.229 | 4.380 | 4.531 | ¼ |
| ½ | 3.438 | 3.594 | 3.750 | 3.906 | 4.063 | 4.219 | 4.375 | 4.531 | 4.688 | ½ |
| ¾ | 3.552 | 3.714 | 3.875 | 4.036 | 4.198 | 4.359 | 4.521 | 4.682 | 4.844 | ¾ |
| 8. | 3.667 | 3.833 | 4.000 | 4.167 | 4.333 | 4.500 | 4.667 | 4.833 | 5.000 | 8. |
| ¼ | 3.781 | 3.953 | 4.125 | 4.297 | 4.469 | 4.641 | 4.813 | 4.984 | 5.156 | ¼ |
| ½ | 3.896 | 4.073 | 4.250 | 4.427 | 4.604 | 4.781 | 4.957 | 5.135 | 5.313 | ½ |
| ¾ | 4.010 | 4.193 | 4.375 | 4.557 | 4.740 | 4.922 | 5.103 | 5.286 | 5.469 | ¾ |
| 9. | 4.125 | 4.313 | 4.500 | 4.687 | 4.875 | 5.063 | 5.249 | 5.438 | 5.625 | 9. |
| ¼ | 4.240 | 4.432 | 4.625 | 4.818 | 5.010 | 5.203 | 5.395 | 5.589 | 5.781 | ¼ |
| ½ | 4.354 | 4.552 | 4.750 | 4.948 | 5.146 | 5.344 | 5.541 | 5.740 | 5.938 | ½ |
| ¾ | 4.469 | 4.672 | 4.875 | 5.078 | 5.281 | 5.484 | 5.687 | 5.891 | 6.094 | ¾ |
| 10. | 4.583 | 4.792 | 5.000 | 5.208 | 5.417 | 5.625 | 5.833 | 6.042 | 6.250 | 10. |
| ¼ | 4.698 | 4.911 | 5.125 | 5.339 | 5.552 | 5.766 | 5.979 | 6.193 | 6.406 | ¼ |
| ½ | 4.813 | 5.031 | 5.250 | 5.469 | 5.688 | 5.906 | 6.125 | 6.344 | 6.563 | ½ |
| ¾ | 4.927 | 5.151 | 5.375 | 5.599 | 5.823 | 6.047 | 6.271 | 6.495 | 6.719 | ¾ |
| 11. | 5.042 | 5.271 | 5 500 | 5.729 | 5.958 | 6.188 | 6.417 | 6.646 | 6.876 | 11. |
| ¼ | 5.156 | 5.391 | 5.625 | 5.859 | 6.094 | 6.328 | 6.563 | 6.797 | 7.031 | ¼ |
| ½ | 5.271 | 5.510 | 5.750 | 5.990 | 6.229 | 6.469 | 6.708 | 6.948 | 7.188 | ½ |
| ¾ | 5.385 | 5.630 | 5.875 | 6.120 | 6.365 | 6.609 | 6.854 | 7.099 | 7.344 | ¾ |
| 12. | 5.500 | 5.750 | 6.000 | 6.250 | 6.500 | 6.750 | 7.000 | 7.250 | 7.500 | 12. |
| ¼ | 5.729 | 5.990 | 6.250 | 6.510 | 6.771 | 7.031 | 7.292 | 7.552 | 7.813 | ½ |
| 13. | 5.958 | 6.229 | 6.500 | 6.771 | 7.042 | 7.313 | 7.583 | 7.854 | 8.125 | 13. |
| ½ | 6.188 | 6.469 | 6.750 | 7.031 | 7.313 | 7.594 | 7.875 | 8.156 | 8.438 | ½ |
| 14. | 6.417 | 6.708 | 7.000 | 7.292 | 7.583 | 7.875 | 8.167 | 8.458 | 8.750 | 14. |
| ½ | 6.646 | 6.948 | 7.250 | 7.552 | 7.854 | 8.156 | 8.458 | 8.760 | 9.063 | ½ |
| 15. | 6.875 | 7.188 | 7.500 | 7.812 | 8.125 | 8.438 | 8.750 | 9.063 | 9.375 | 15. |
| ½ | 7.104 | 7.427 | 7.750 | 8.073 | 8.396 | 8.719 | 9.042 | 9.365 | 9.688 | ½ |
| 16. | 7.333 | 7.667 | 8.000 | 8.333 | 8.667 | 9.000 | 9.333 | 9.667 | 10.00 | 16. |
| ½ | 7.563 | 7.906 | 8.250 | 8.594 | 8.938 | 9.281 | 9.625 | 9.969 | 10.31 | ½ |
| 17. | 7.792 | 8.146 | 8.500 | 8.854 | 9.208 | 9.563 | 9.917 | 10.27 | 10.63 | 17. |
| ½ | 8.021 | 8.385 | 8.750 | 9.115 | 9.479 | 9.844 | 10.21 | 10.57 | 10.94 | ½ |
| 18. | 8.250 | 8.625 | 9.000 | 9.375 | 9.750 | 10.13 | 10.50 | 10.88 | 11.25 | 18. |
| 19. | 8.708 | 9.104 | 9.500 | 9.896 | 10.29 | 10.69 | 11.08 | 11.48 | 11.88 | 19. |
| 20. | 9.167 | 9.583 | 10.00 | 10.42 | 10.83 | 11.25 | 11.67 | 12.08 | 12.50 | 20. |
| 21. | 9.625 | 10.06 | 10.50 | 10.94 | 11.38 | 11.81 | 12.25 | 12.69 | 13.13 | 21. |
| 22. | 10.08 | 10.54 | 11.00 | 11.46 | 11.92 | 12.38 | 12.83 | 13.29 | 13.75 | 22. |
| 23. | 10.54 | 11.02 | 11.50 | 11.98 | 12.46 | 12.94 | 13.42 | 13.90 | 14.38 | 23. |
| 24. | 11.00 | 11.50 | 12.00 | 12.50 | 13.00 | 13.50 | 14.00 | 14.50 | 15.00 | 24. |

## Medida de tablas de madera. (Continuación.)

Número de pies cuadrados de tabla contenidos en un pie lineal de madera en piezas de diferentes dimensiones. (Original.)

| Ancho en pulgadas. | Espesor en pulgadas. | | | | | | | | | Ancho en pulgadas. |
|---|---|---|---|---|---|---|---|---|---|---|
| | 7¾ | 8 | 8¼ | 8½ | 8¾ | 9 | 9¼ | 9½ | 9¾ | |
| ¼ | .1615 | .1667 | .1719 | .1771 | .1823 | .1875 | .1927 | .1979 | .2031 | ¼ |
| ½ | .3229 | .3333 | .3438 | .3542 | .3646 | .3750 | .3854 | .3958 | .4063 | ½ |
| ¾ | .4844 | .5000 | .5156 | .5313 | .5467 | .5625 | .5781 | .5938 | .6094 | ¾ |
| 1. | .6458 | .6667 | .6875 | .7083 | .7292 | .7500 | .7708 | .7917 | .8125 | 1. |
| ¼ | .8073 | .8333 | .8594 | .8854 | .9115 | .9375 | .9635 | .9896 | 1.016 | ¼ |
| ½ | .9688 | 1.000 | 1.031 | 1.063 | 1.094 | 1.125 | 1.156 | 1.188 | 1.219 | ½ |
| ¾ | 1.130 | 1.167 | 1.203 | 1.240 | 1.276 | 1.313 | 1.349 | 1.385 | 1.422 | ¾ |
| 2. | 1.292 | 1.333 | 1.375 | 1.417 | 1.458 | 1.500 | 1.542 | 1.583 | 1.625 | 2. |
| ¼ | 1.453 | 1.500 | 1.547 | 1.594 | 1.641 | 1.688 | 1.734 | 1.781 | 1.828 | ¼ |
| ½ | 1.615 | 1.667 | 1.719 | 1.771 | 1.822 | 1.875 | 1.927 | 1.979 | 2.031 | ½ |
| ¾ | 1.776 | 1.833 | 1.891 | 1.948 | 2.005 | 2.063 | 2.120 | 2.177 | 2.234 | ¾ |
| 3. | 1.938 | 2.000 | 2.063 | 2.125 | 2.188 | 2.250 | 2.313 | 2.375 | 2.438 | 3. |
| ¼ | 2.099 | 2.167 | 2.234 | 2.302 | 2.370 | 2.438 | 2.505 | 2.573 | 2.641 | ¼ |
| ½ | 2.260 | 2.333 | 2.406 | 2.479 | 2.552 | 2.625 | 2.698 | 2.771 | 2.844 | ½ |
| ¾ | 2.422 | 2.500 | 2.578 | 2.656 | 2.734 | 2.813 | 2.891 | 2.969 | 3.047 | ¾ |
| 4. | 2.583 | 2.667 | 2.750 | 2.833 | 2.917 | 3.000 | 3.083 | 3.167 | 3.250 | 4. |
| ¼ | 2.745 | 2.833 | 2.922 | 3.010 | 3.099 | 3.188 | 3.276 | 3.365 | 3.453 | ¼ |
| ½ | 2.906 | 3.000 | 3.094 | 3.188 | 3.281 | 3.375 | 3.469 | 3.563 | 3.656 | ½ |
| ¾ | 3.068 | 3.167 | 3.266 | 3.365 | 3.464 | 3.563 | 3.661 | 3.760 | 3.859 | ¾ |
| 5. | 3.229 | 3.333 | 3.438 | 3.542 | 3.646 | 3.750 | 3.854 | 3.958 | 4.063 | 5. |
| ¼ | 3.391 | 3.500 | 3.609 | 3.719 | 3.828 | 3.938 | 4.047 | 4.156 | 4.266 | ¼ |
| ½ | 3.552 | 3.667 | 3.781 | 3.895 | 4.010 | 4.125 | 4.240 | 4.354 | 4.469 | ½ |
| ¾ | 3.714 | 3.833 | 3.953 | 4.073 | 4.193 | 4.313 | 4.432 | 4.552 | 4.672 | ¾ |
| 6. | 3.875 | 4.000 | 4.125 | 4.250 | 4.375 | 4.500 | 4.625 | 4.750 | 4.875 | 6. |
| ¼ | 4.036 | 4.167 | 4.297 | 4.427 | 4.557 | 4.688 | 4.818 | 4.948 | 5.078 | ¼ |
| ½ | 4.198 | 4.333 | 4.469 | 4.604 | 4.740 | 4.875 | 5.010 | 5.146 | 5.281 | ½ |
| ¾ | 4.359 | 4.500 | 4.641 | 4.781 | 4.922 | 5.063 | 5.203 | 5.344 | 5.484 | ¾ |
| 7. | 4.521 | 4.667 | 4.813 | 4.958 | 5.104 | 5.250 | 5.360 | 5.542 | 5.688 | 7. |
| ¼ | 4.682 | 4.833 | 4.984 | 5.135 | 5.286 | 5.438 | 5.590 | 5.740 | 5.891 | ¼ |
| ½ | 4.844 | 5.000 | 5.156 | 5.313 | 5.469 | 5.625 | 5.782 | 5.938 | 6.094 | ½ |
| ¾ | 5.005 | 5.167 | 5.328 | 5.490 | 5.651 | 5.813 | 5.975 | 6.135 | 6.297 | ¾ |
| 8. | 5.167 | 5.333 | 5.500 | 5.667 | 5.833 | 6.000 | 6.167 | 6.333 | 6.500 | 8. |
| ¼ | 5.328 | 5.500 | 5.672 | 5.844 | 6.016 | 6.188 | 6.359 | 6.531 | 6.703 | ¼ |
| ½ | 5.490 | 5.667 | 5.844 | 6.021 | 6.198 | 6.375 | 6.552 | 6.729 | 6.906 | ½ |
| ¾ | 5.651 | 5.833 | 6.016 | 6.198 | 6.380 | 6.563 | 6.745 | 6.927 | 7.109 | ¾ |
| 9. | 5.813 | 6.000 | 6.188 | 6.375 | 6.563 | 6.750 | 6.938 | 7.125 | 7.313 | 9. |
| ¼ | 5.974 | 6.167 | 6.359 | 6.552 | 6.745 | 6.938 | 7.130 | 7.323 | 7.516 | ¼ |
| ½ | 6.135 | 6.333 | 6.531 | 6.729 | 6.927 | 7.125 | 7.323 | 7.521 | 7.719 | ½ |
| ¾ | 6.297 | 6.500 | 6.703 | 6.906 | 7.109 | 7.313 | 7.516 | 7.719 | 7.922 | ¾ |
| 10. | 6.458 | 6.667 | 6.875 | 7.083 | 7.292 | 7.500 | 7.708 | 7.917 | 8.125 | 10. |
| ¼ | 6.620 | 6.833 | 7.047 | 7.260 | 7.474 | 7.688 | 7.901 | 8.115 | 8.328 | ¼ |
| ½ | 6.781 | 7.000 | 7.219 | 7.438 | 7.656 | 7.875 | 8.094 | 8.313 | 8.531 | ½ |
| ¾ | 6.943 | 7.167 | 7.391 | 7.615 | 7.839 | 8.063 | 8.286 | 8.510 | 8.734 | ¾ |
| 11. | 7.104 | 7.333 | 7.563 | 7.792 | 8.021 | 8.250 | 8.479 | 8.708 | 8.938 | 11. |
| ¼ | 7.266 | 7.500 | 7.735 | 7.969 | 8.203 | 8.438 | 8.672 | 8.906 | 9.141 | ¼ |
| ½ | 7.427 | 7.667 | 7.906 | 8.146 | 8.386 | 8.625 | 8.865 | 9.104 | 9.344 | ½ |
| ¾ | 7.589 | 7.833 | 8.078 | 8.323 | 8.568 | 8.813 | 9.057 | 9.302 | 9.547 | ¾ |
| 12. | 7.750 | 8.000 | 8.250 | 8.500 | 8.750 | 9.000 | 9.250 | 9.500 | 9.750 | 12. |
| ½ | 8.073 | 8.333 | 8.594 | 8.854 | 9.115 | 9.375 | 9.635 | 9.896 | 10.16 | ½ |
| 13. | 8.396 | 8.666 | 8.938 | 9.208 | 9.479 | 9.750 | 10.02 | 10.29 | 10.56 | 13. |
| ½ | 8.719 | 9.000 | 9.281 | 9.563 | 9.844 | 10.13 | 10.41 | 10.69 | 10.97 | ½ |
| 14. | 9.042 | 9.333 | 9.625 | 9.917 | 10.21 | 10.50 | 10.79 | 11.08 | 11.38 | 14. |
| ½ | 9.365 | 9.666 | 9.969 | 10.27 | 10.57 | 10.88 | 11.18 | 11.48 | 11.78 | ½ |
| 15. | 9.688 | 10.000 | 10.31 | 10.63 | 10.94 | 11.25 | 11.56 | 11.88 | 12.19 | 15. |
| ½ | 10.01 | 10.33 | 10.66 | 10.98 | 11.30 | 11.63 | 11.95 | 12.27 | 12.59 | ½ |
| 16. | 10.33 | 10.67 | 11.00 | 11.33 | 11.67 | 12.00 | 12.33 | 12.67 | 13.00 | 16. |
| ½ | 10.66 | 11.00 | 11.34 | 11.69 | 12.03 | 12.38 | 12.72 | 13.06 | 13.41 | ½ |
| 17. | 10.98 | 11.33 | 11.69 | 12.04 | 12.40 | 12.75 | 13.10 | 13.46 | 13.81 | 17. |
| ½ | 11.30 | 11.66 | 12.03 | 12.40 | 12.76 | 13.13 | 13.49 | 13.85 | 14.22 | ½ |
| 18. | 11.63 | 12.00 | 12.38 | 12.75 | 13.13 | 13.50 | 13.88 | 14.25 | 14.63 | 18. |
| 19. | 12.27 | 12.67 | 13.06 | 13.46 | 13.85 | 14.25 | 14.65 | 15.04 | 15.44 | 19. |
| 20. | 12.92 | 13.33 | 13.75 | 14.17 | 14.58 | 15.00 | 15.42 | 15.83 | 16.25 | 20. |
| 21. | 13.56 | 14.00 | 14.44 | 14.88 | 15.31 | 15.75 | 16.19 | 16.63 | 17.06 | 21. |
| 22. | 14.21 | 14.66 | 15.13 | 15.58 | 16.04 | 16.50 | 16.96 | 17.42 | 17.88 | 22. |
| 23. | 14.85 | 15.33 | 15.81 | 16.29 | 16.77 | 17.25 | 17.73 | 18.21 | 18.69 | 23. |
| 24. | 15.50 | 16.00 | 16.50 | 17.00 | 17.50 | 18.00 | 18.50 | 19.00 | 19.50 | 24. |

## Medida de tablas de madera. (Continuación.)

| Ancho en pulgadas. | Número de pies cuadrados de tabla contenidos en un pie lineal de madera en piezas de diferentes dimensiones. (Original.) | | | | | | | | | Ancho en pulgadas. |
|---|---|---|---|---|---|---|---|---|---|---|
| | Espesor en pulgadas. | | | | | | | | | |
| | 10 | 10¼ | 10½ | 10¾ | 11 | 11¼ | 11½ | 11¾ | 12 | |
| ¼ | .2083 | .2135 | .2188 | .2240 | .2292 | .2344 | .2396 | .2448 | .2500 | ¼ |
| ½ | .4167 | .4271 | .4375 | .4479 | .4583 | .4688 | .4792 | .4896 | .5000 | ½ |
| ¾ | .6250 | .6406 | .6563 | .6719 | .6875 | .7031 | .7188 | .7344 | .7500 | ¾ |
| 1. | .8333 | .8542 | .8750 | .8958 | .9167 | .9375 | .9583 | .9792 | 1.000 | 1. |
| ¼ | 1.042 | 1.068 | 1.094 | 1.120 | 1.146 | 1.172 | 1.198 | 1.224 | 1.250 | ¼ |
| ½ | 1.250 | 1.281 | 1.313 | 1.344 | 1.375 | 1.406 | 1.438 | 1.469 | 1.500 | ½ |
| ¾ | 1.458 | 1.495 | 1.531 | 1.568 | 1.604 | 1.641 | 1.677 | 1.714 | 1.750 | ¾ |
| 2. | 1.667 | 1.708 | 1.750 | 1.792 | 1.833 | 1.875 | 1.917 | 1.958 | 2.000 | 2. |
| ¼ | 1.875 | 1.922 | 1.969 | 2.016 | 2.063 | 2.109 | 2.156 | 2.203 | 2.250 | ¼ |
| ½ | 2.083 | 2.135 | 2.188 | 2.240 | 2.292 | 2.344 | 2.396 | 2.448 | 2.500 | ½ |
| ¾ | 2.292 | 2.349 | 2.406 | 2.464 | 2.521 | 2.578 | 2.635 | 2.693 | 2.750 | ¾ |
| 3. | 2.500 | 2.563 | 2.625 | 2.688 | 2.750 | 2.813 | 2.875 | 2.938 | 3.000 | 3. |
| ¼ | 2.708 | 2.776 | 2.844 | 2.911 | 2.979 | 3.047 | 3.115 | 3.182 | 3.250 | ¼ |
| ½ | 2.917 | 2.990 | 3.063 | 3.135 | 3.208 | 3.281 | 3.354 | 3.427 | 3.500 | ½ |
| ¾ | 3.125 | 3.203 | 3.281 | 3.359 | 3.438 | 3.516 | 3.594 | 3.672 | 3.750 | ¾ |
| 4. | 3.333 | 3.417 | 3.500 | 3.583 | 3.667 | 3.750 | 3.833 | 3.917 | 4.000 | 4. |
| ¼ | 3.542 | 3.630 | 3.719 | 3.807 | 3.896 | 3.984 | 4.073 | 4.161 | 4.250 | ¼ |
| ½ | 3.750 | 3.844 | 3.938 | 4.031 | 4.125 | 4.219 | 4.313 | 4.406 | 4.500 | ½ |
| ¾ | 3.958 | 4.057 | 4.156 | 4.255 | 4.354 | 4.453 | 4.552 | 4.651 | 4.750 | ¾ |
| 5. | 4.167 | 4.271 | 4.375 | 4.479 | 4.583 | 4.688 | 4.791 | 4.896 | 5.000 | 5. |
| ¼ | 4.375 | 4.484 | 4.594 | 4.703 | 4.813 | 4.922 | 5.031 | 5.141 | 5.250 | ¼ |
| ½ | 4.583 | 4.698 | 4.813 | 4.927 | 5.042 | 5.156 | 5.270 | 5.385 | 5.500 | ½ |
| ¾ | 4.792 | 4.911 | 5.031 | 5.151 | 5.271 | 5.391 | 5.510 | 5.630 | 5.750 | ¾ |
| 6. | 5.000 | 5.125 | 5.250 | 5.375 | 5.500 | 5.625 | 5.750 | 5.875 | 6.000 | 6. |
| ¼ | 5.208 | 5.339 | 5.469 | 5.599 | 5.729 | 5.859 | 5.990 | 6.120 | 6.250 | ¼ |
| ½ | 5.417 | 5.552 | 5.688 | 5.823 | 5.958 | 6.094 | 6.229 | 6.365 | 6.500 | ½ |
| ¾ | 5.625 | 5.766 | 5.906 | 6.047 | 6.188 | 6.328 | 6.469 | 6.609 | 6.750 | ¾ |
| 7. | 5.833 | 5.979 | 6.125 | 6.271 | 6.417 | 6.563 | 6.708 | 6.854 | 7.000 | 7. |
| ¼ | 6.042 | 6.193 | 6.344 | 6.495 | 6.646 | 6.797 | 6.948 | 7.099 | 7.250 | ¼ |
| ½ | 6.250 | 6.406 | 6.563 | 6.719 | 6.875 | 7.031 | 7.188 | 7.344 | 7.500 | ½ |
| ¾ | 6.458 | 6.620 | 6.781 | 6.943 | 7.104 | 7.266 | 7.427 | 7.589 | 7.750 | ¾ |
| 8. | 6.667 | 6.833 | 7.000 | 7.167 | 7.333 | 7.500 | 7.667 | 7.833 | 8.000 | 8. |
| ¼ | 6.875 | 7.047 | 7.219 | 7.391 | 7.563 | 7.734 | 7.906 | 8.078 | 8.250 | ¼ |
| ½ | 7.083 | 7.260 | 7.438 | 7.615 | 7.792 | 7.969 | 8.146 | 8.323 | 8.500 | ½ |
| ¾ | 7.292 | 7.474 | 7.656 | 7.839 | 8.021 | 8.203 | 8.385 | 8.568 | 8.750 | ¾ |
| 9. | 7.500 | 7.688 | 7.875 | 8.063 | 8.250 | 8.438 | 8.625 | 8.813 | 9.000 | 9. |
| ¼ | 7.708 | 7.901 | 8.094 | 8.286 | 8.479 | 8.672 | 8.865 | 9.057 | 9.250 | ¼ |
| ½ | 7.917 | 8.115 | 8.313 | 8.510 | 8.709 | 8.906 | 9.104 | 9.302 | 9.500 | ½ |
| ¾ | 8.125 | 8.328 | 8.531 | 8.734 | 8.939 | 9.141 | 9.344 | 9.547 | 9.750 | ¾ |
| 10. | 8.333 | 8.542 | 8.750 | 8.958 | 9.167 | 9.375 | 9.583 | 9.792 | 10.00 | 10. |
| ¼ | 8.542 | 8.755 | 8.969 | 9.182 | 9.396 | 9.609 | 9.823 | 10.04 | 10.25 | ¼ |
| ½ | 8.750 | 8.969 | 9.188 | 9.406 | 9.625 | 9.844 | 10.06 | 10.28 | 10.50 | ½ |
| ¾ | 8.958 | 9.182 | 9.406 | 9.630 | 9.854 | 10.08 | 10.30 | 10.53 | 10.75 | ¾ |
| 11. | 9.167 | 9.396 | 9.625 | 9.854 | 10.08 | 10 31 | 10.54 | 10.77 | 11.00 | 11. |
| ¼ | 9.375 | 9.609 | 9.844 | 10 08 | 10.31 | 10.55 | 10.78 | 11.02 | 11.25 | ¼ |
| ½ | 9.583 | 9.823 | 10.06 | 10.30 | 10.54 | 10.78 | 11.02 | 11.26 | 11.50 | ½ |
| ¾ | 9.792 | 10.04 | 10.28 | 10.53 | 10.77 | 11.02 | 11.26 | 11.51 | 11.75 | ¾ |
| 12. | 10.00 | 10.25 | 10.50 | 10.75 | 11.00 | 11.25 | 11.50 | 11.75 | 12.00 | 12. |
| ½ | 10.42 | 10.68 | 10.94 | 11.20 | 11.46 | 11.72 | 11.98 | 12.24 | 12.50 | ½ |
| 13. | 10.83 | 11.10 | 11.38 | 11.65 | 11.92 | 12.19 | 12.46 | 12.73 | 13.00 | 13. |
| ½ | 11.25 | 11.53 | 11.81 | 12.09 | 12.38 | 12.66 | 12.94 | 13.22 | 13.50 | ½ |
| 14. | 11.67 | 11.96 | 12.25 | 12.54 | 12.83 | 13.13 | 13.42 | 13.71 | 14.00 | 14. |
| ½ | 12.08 | 12.39 | 12.69 | 12.99 | 13.29 | 13.59 | 13.90 | 14.20 | 14.50 | ½ |
| 15. | 12.50 | 12.81 | 13.13 | 13.44 | 13.75 | 14.06 | 14.38 | 14.69 | 15.00 | 15. |
| ½ | 12.92 | 13 24 | 13.56 | 13.89 | 14.21 | 14.53 | 14.85 | 15.18 | 15.50 | ½ |
| 16. | 13.33 | 13.67 | 14.00 | 14.33 | 14.67 | 15.00 | 15.33 | 15.67 | 16.00 | 16. |
| ½ | 13.75 | 14.09 | 14.44 | 14.78 | 15.13 | 15.47 | 15.81 | 16.16 | 16.50 | ½ |
| 17. | 14.17 | 14.52 | 14.88 | 15.23 | 15.58 | 15.94 | 16.29 | 16.65 | 17.00 | 17. |
| ½ | 14.58 | 14.95 | 15.31 | 15.77 | 16.04 | 16.41 | 16.77 | 17.14 | 17.50 | ½ |
| 18. | 15.00 | 15.38 | 15.75 | 16.13 | 16.50 | 16.88 | 17.25 | 17.63 | 18.00 | 18. |
| 19. | 15.83 | 16.23 | 16.63 | 17.02 | 17.42 | 17.81 | 18.21 | 18.60 | 19 00 | 19. |
| 20. | 16.67 | 17.08 | 17.50 | 17.92 | 18.33 | 18.75 | 19.17 | 19.58 | 20.00 | 20. |
| 21. | 17.50 | 17.94 | 18.38 | 18.81 | 19.25 | 19.69 | 20.13 | 20.56 | 21.00 | 21. |
| 22. | 18.33 | 18.79 | 19.25 | 19.71 | 20.17 | 20.63 | 21.08 | 21.54 | 22.00 | 22. |
| 23. | 19.17 | 19.65 | 20.13 | 20.60 | 21.08 | 21.56 | 22.04 | 22.52 | 23.00 | 23. |
| 24. | 20.00 | 20.50 | 21.00 | 21.50 | 22.00 | 22.50 | 23.00 | 23.50 | 24.00 | 24 |

# LEVANTAMIENTO DE PLANOS

Al levantar una porción de terreno, los lados que formen el perímetro se designan por números en el orden que se van levantando. En cada lado el extremo que se encuentra primero en el curso del levantamiento, puede denominarse *extremo inmediato*, y el otro, *extremo distante*. El número de cada lado se coloca en el extremo distante. Así, fig. 1, suponiendo que el levantamiento comienza en el vértice **6** y continúa en el sentido de las flechas, el primer lado es 6, 1; y su número se coloca en el extremo distante 1 y así de los demás. Sea N-S una meridiana, es decir, una línea norte sur; y E-O una línea este oeste. En un lado cualquiera que se incline hacia el Norte, ya sea hacia el propio Norte, ó hacia el noreste, como los lados 5 y 1, la distancia del extremo inmediato al distante, tomada en dirección norte, se llama

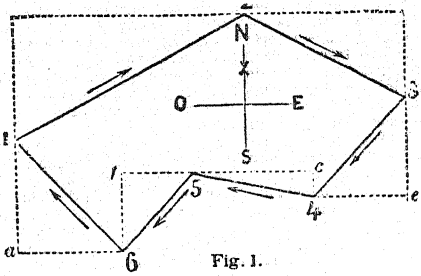



Fig. 1.

latitud norte; así, *a*1 es la dif de lat N del lado 1; 1*b* la dif de lat N del lado 2; 4*c* la del lado 5. De igual manera, si un lado cualquiera se dirige hacia el Sur sea exactamente en dirección Sur, ó con desviación al Este, como el lado 3, ó al Oeste, como los lados 4 y 6, la dist medida en la dirección Sur, del extremo inmediato al distante, se llama dif de lat S de dicho lado; así, *d*3 es la dif de lat S del lado 3; 3*e*, la del lado 4; *f*6, la del lado 6. Ambas dif están comprendidas bajo el término general de *Diferencias de latitud* de un lado, ó como se dice más comúnmente, pero erróneamente, su *latitud*. La dist entre los extremos de un lado cuando éste se inclina al Este ó al Oeste, se llama, de igual manera, dif de meridiano al Este ó al Oeste según el caso, de modo que 6*a* es la dif de mer O del lado 1; 5*f*, la del lado 6; *c*5, la del 5; *e*4 la del 4, y *b*2 la dif de mer E del lado 2; 2*d* la del lado 3. Cuando se habla sólo de la *diferencia de meridiano* (*departure*) de un lado, se entiende, indiferentemente al Este ó al Oeste.

No se puede llegar á una completa exactitud en las operaciones que tienen por objeto la medida de ángulos y distancias *. Hecha una operación con una exactitud suficiente, no se puede obtener mayor precisión sino haciendo mayores gastos. El autor cree que las diferencias en la precisión con que se efectúa un buen levantamiento, viene á traducirse siempre en dólares y céntimos. En la compra y venta de porciones de terreno, de labranzas, etc., una dif de 1 en 200 en la superficie, y de consiguiente en el precio, es muy probable que nunca sea motivo á que se desista del negocio, y basados en esto suponemos que una mensura hecha

* Una cadena de 100 pies de largo, por el solo cambio de temperatura del invierno al verano puede producir un error de 5 pies en la medida de una milla en longitud. En la medida de superficies será de 1 en 533 **.

** *N. del T.*— No sabemos por qué fija la longitud de la cadena para determinar el valor del error en longitud de 5 pies por milla, á causa de la dilatación ó contracción producidas por los cambios de temperatura extremos entre el invierno y el verano, toda vez que esto depende de la naturaleza del metal que la forma (que aunque no lo indica, lo suponemos de acero como se usa generalmente), pero no de la longitud de la cadena, pues cualquiera que ella sea producirá siempre el mismo error por milla. Hablando en metros resultará un error por dicha causa y con cadenas del metal que generalmente se usa de .95 m por cada kilómetro que se mida. El error en superficie será siempre de 1 en 533 cualquiera que sea la unidad que se adopte.

entre estos límites de error, merece generalmente que se la considere como exacta. No hay dificultad en llegar á este límite de exactitud, y cuando no se logra, es porque se ha trabajado mal. Muchas circunstancias se combinan para producir errores de poca significación y absolutamente inevitables, los cuales aparecen al trasladar al papel las anotaciones tomadas en el terreno, y desde luego afectan al plano y al cálculo de la superficie. Entonces hacemos lo que comúnmente se llama *corregir* los errores; pero que en realidad es simplemente disimularlos. Esto sucederá siempre por más esmero científico que se haya tenido en el procedimiento. Dos métodos se usan con este fin, ambos basados en el mismo principio, uno de ellos, mecánico, por medio del trazado; el otro, por el cálculo, más exacto, pero mucho más enojoso. Describiremos ambos métodos; pero haremos la observación de que proporcionando una escala para el plano como lo indicamos en seguida, el método mecánico empleado por un dibujante hábil es bastante exacto para los usos ordinarios. Súmense todos los lados en pies y divídase la suma por el número de lados para obtener su longitud media. Divídase esta longitud media por 8; el cociente será la escala propia en pies por pulgada. En otras palabras, representemos por 8 pulgadas cada lado medio †*. Suponemos que ningún ingeniero considere exacta la medida de los ángulos con una aproximación de un cuarto de grado, que es la práctica usual entre los agrimensores. Pueden medirse por medio del tránsito de ingeniero, actualmente de uso universal en nuestras obras públicas, con uno ó dos minutos de aproximación, y siendo mucho más exactos que los rumbos tomados con la brújula (los cuales no pueden llevarse á tanta aproximación, y están por otra parte sujetos á muchas causas de errores), sirven para corregir éstos en la oficina.

La anotación de los rumbos, sin embargo, no debe limitarse á cuartos de grado, deben apreciarse los minutos del ángulo tan próximamente como sea posible al observador; también debe tomarse el rumbo á la espalda en cada vértice, como comprobación adicional. Es bueno, al tomar las indicaciones de la brújula, adoptar como regla que se dirija siempre el norte de la caja de la brújula hacia el objeto cuyo ángulo se va á tomar; y hacer la lectura desde la punta N de la aguja. Los que usen indiferentemente el N y el S de la caja de la aguja estarán muy expuestos á cometer errores. También es mejor medir el ángulo más pequeño marcado con líneas de puntos, fig. 2) en los vértices, sea exterior como en el

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† *N. del T.* — En el sistema métrico, procédase así: Súmense todos los lados medidos en metros, y divídase la suma por el número de lados para obtener su longitud media. Multiplíquese ésta por 5 y el producto será el dominador de la escala cuyo numerador es 1. Ej.: supongamos que la suma de los lados da 3660 metros y que son 10 lados; $\frac{3660}{10} = 366$ metros será el largo medio de ellos y multiplicado por $5 = 366 \times 5 = 1830$ da el denominador de la escala, que será $\frac{1}{1830}$. Siempre se escogerá para escala una fracción próxima más adecuada. En este caso $\frac{1}{2000}$, por ejemplo.

★Los jóvenes ayudantes deberían practicar delineando ó trazando con datos perfectamente exactos. Este es el único modo de aprender lo que se entiende por trabajo exacto. El transportador semicircular debe ser de 9 á 12 pulgs de diám (25 á 30 cm) y graduado á 10 minutos. La regla y escuadra deben ser de metal; preferimos la plata alemana que no se oxida como el acero, y deben estar hechas con *escrupulosa exactitud* por un hábil fabricante de instrumentos. Una aguja muy fina, con cabeza de lacre manejada verticalmente, debe emplearse para marcar las dist y los ángs, y el ojo del dibujante debe colocarse directamente sobre ella. El lápiz debe ser duro (Faber nº 4 es bueno para transportar) y conservarle la punta aguda frotándola en una lima fina después de tajar la madera con la navaja. La escala debe ser por lo menos de la longitud del lado más largo del plano, y deberá trazársela en el borde de una tira del mismo papel en que se dibuja. Esto evitará, en gran parte, los errores que nacen por la contracción y dilatación del papel. Desgraciadamente, un pliego de papel no se contrae y dilata en la misma proporción, á lo largo que al través, impidiendo así que la escala hecha en la tira de papel, como lo hemos indicado, proporcione un medio eficaz de corregir el error. En los planos de levantamientos comunes de granjas, haciendas, etc., pueden despreciarse los errores de esta procedencia. Para tales planos, como han de transportarse, dividirse y calcularse en un tiempo demasiado corto para admitir cambios apreciables, las escalas ordinarias de madera, marfil ó metal, pueden emplearse; pero no se pueden obtener con ellas resultados satisfactoriamente exactos en planos que exijan muchos días, y más si el aire está alternativamente húmedo y seco, ó sujeto á variaciones considerables de temperatura. El papel llamado de *pergamino* (parchment) es peor á este respecto que el buen papel de dibujo.

Con las precedentes precauciones, se puede hacer un dibujo con tanta exactitud como a que se alcanza en las operaciones del terreno.

vértice 5, ó interior como en todos los otros; porque así es siempre menor que 180°; de modo que hay menos peligro, de hacer una lectura equivocada, que cuando excede de 180°, esto es, en el caso de que el tránsito esté graduado á un lado y á otro de 0° á 180°; si tiene una sola graduación de 0° á 360°, la precaución es

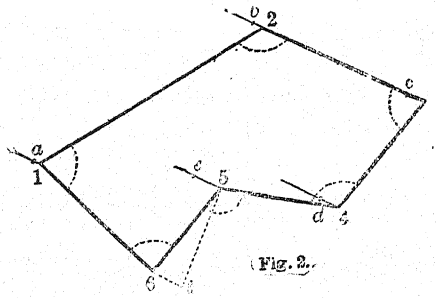



Fig. 2.

inútil. Cuando el ángulo exterior es pequeño, réstese de 360° para obtener el interior.

Suponiendo concluído el trabajo sobre el terreno, y que deseamos un plano que nos dé la superficie dividiéndola en triángulos (cuyas bases y alturas puedan medirse á la escala y sus áreas calcularse una por una) debemos desde luego transportar las anotaciones del terreno, sea usando los *ángulos* ó calculando primero las orientaciones en un solo sentido, por medio de los ángulos, y luego aplicándolas. El último modo es preferible, porque en el primero el transportador puede ser movido para cada ángulo, mientras que en el último permanece fijo en tanto que se marcan los rumbos. Cada movimiento de aquél aumenta la probabilidad de error. El modo de corregir los errores se explica en la página siguiente.

En uno ú otro caso, el polígono transportado de seguro que no cerrará exactamente, no sólo como consecuencia de los errores por las operaciones del terreno, sino también al transportarlas al dibujo. Así, el último lado, n.° 6, fig. 2, en lugar, de cerrar en el vértice 6, terminará en cualquiera otra parte, por ejemplo en *t*, siendo la distancia 6 *t* el error correspondiente, el cual, sin embargo, tal como está representando en la fig. 2, es más de diez veces mayor en relación al tamaño del plano, que lo que se admite en la práctica. Para distribuir este error, trácese por cada vértice una pequeña línea paralela á *t* 6 y siempre en la dirección *t* 6. Súmense todos los lados, y mídase *t* 6 en la escala del plano. Luego empezando en el vértice 1, digamos :

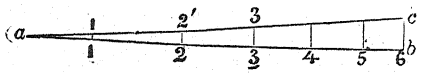
$$\text{Suma de todos los lados} : \text{Error total de cierre } t\,6 :: \text{Lado 1} : \text{Error corresp. al lado 1.}$$

Llévese este error de 1 á *a*. Entonces en el vértice 2 tenemos :

$$\text{Suma de todos los lados} : \text{Error total de cierre } t\ 6 :: \text{Suma de los lados 1 y 2} : \text{Error corresp. al lado 2.}$$

Dicho error colóquese de 2 á *b*, y así en cada vértice; siempre usando, como tercer término, la suma de los lados entre el punto de que se trata y el error de cierre dado. Finalmente, únanse *a*, *b*, *c*, *d*, *e*, 6, y el alineamiento está concluído. La corrección ha cambiado evidentemente la longitud de cada lado, alargando á unos y acortando á otros. También han cambiado los ángulos. Las nuevas longitudes y los nuevos ángulos pueden con exactitud tolerable hallarse por medio de la escala y el transportador, y ser marcados en el trazado en lugar de los anteriores. Cuando el plano tiene muchos lados, el cálculo del error para cada uno de ellos se hace fastidioso, y como en una mensura bien practicada y bien transportada, el error total es muy pequeño (no excederá de $\frac{1}{400}$ del perímetro), puede entonces

dividirse entre los lados, simplemente distribuyendo $^1/_4$, $^1/_2$ y $^3/_4$ del error en los vértices situados á distancia más ó menos de $^1/_4$, $^1/_2$ y $^3/_4$ del contorno del perímetro; y en los vértices intermedios, distribúyaselos al ojo. Puede también evitarse todo cálculo así: trazando una línea *ab* de longitud igual á la longitud unida de todos los lados, y dividiéndola en dist *a* 1, 1 2, etc., iguales á los lados res-




pectivos. Hágase *bc* igual al error total, únase *ac* y trácese 1.1'; 2,2', etc., que darán el error en cada vértice.

Cuando el dibujo está terminado así, debe dividirse con líneas finas de lápiz en triángulos cuyas bases y alturas se miden por la escala con el fin de calcular el área. Poniendo cuidado, tanto en el levantamiento como en el trazado, el error no excederá de $^1/_{200}$ del área verdadera. Para evitar equivocaciones deben trazarse y calcularse, por lo menos, dos series distintas de triángulos, y si el área que resulta difiere en más de $^1/_{200}$, es señal de que no han sido preparadas con suficiente cuidado.

El error por el cual no cierra el perímetro debido á la imperfección de trabajo sobre el terreno debe calcularse exactamente como lo indicamos, y hacerse constar en el papel antes de comenzar el dibujo del plano, logrando así una prueba perfecta de la exactitud del dibujo, que si se hace correctamente, no cerrará en el punto de partida, sino en el indicado por el error. El cálculo de éste, con un poco de más trabajo, suministra igualmente el modo de distribuirlo dentro de los diferentes lados y la manera de trazar desde luego el plano correctamente sin el uso del transportador, poniéndonos en capacidad de hacer con más certeza las medidas subsiguientes y el cálculo de los triángulos.

Describiremos ahora este procedimiento; pero recomendamos que aun cuando se emplee, especialmente en levantamientos complicados, se haga primero un croquis, y se corrija por medio del primer método mecánico á que hemos hecho alusión. Se verá que es de gran utilidad al usar el método del cálculo. Al mismo tiempo que da práctica para evitar á la simple vista equivocaciones desagradables, que de otro modo suelen ocurrir, sus principios son por demás sencillos y fácilmente se recuerdan una vez comprendidos. Los continuos cambios en las direcciones de los lados, nos harán, si no se tiene gran cuidado, tomar diferencias de lat. N por dif. de lat S y dif de mer al E por O.

Suponemos, pues, que dicho croquis ha sido preparado, y que los ángulos, direcciones y distancias, figuran en él al lapiz como han sido tomadas del libro de memorias.

Súmense los ángulos interiores formados en todos los vértices; llámese esa suma *a*, multipliquese el número de lados por 180°, réstese del producto, 360°, si la diferencia es igual á la suma *a*, es prueba de que los ángulos han sido medidos correctamente *. Esto, sin embargo, ocurrirá rara vez, siempre habrá discrepancias; pero si el trabajo en el terreno ha sido ejecutado con regular cuidado, no excederán los errores de dos minutos, más ó menos, por cada ángulo. En este caso distribúyase el exceso por partes iguales entre todos los ángulos, sumando ó restando según sea el caso; excepto que corresponda á menos de dos minutos para cada ángulo, pues esto debe despreciarse en levantamientos ordinarios. Los ángulos corregidos pueden trazarse entonces con tinta en el plano y borrarse el trazado de lápiz. Supongamos los ángulos ya corregidos como se ven en la figura 3. Luego, por medio de estos ángulos corregidos, corríjanse también las direcciones; así: fig. 3. Escójase un lado (mientras más largo mejor). Sea el lado 2 el elegido, su orientación N 75°32' E, la supondremos correcta por ser tomada sobre el terreno. Trácese por sus extremos la meridiana y llévense paralelas á ella por los demás vértices del polígono. Ahora bien, siendo la dirección del lado 1, 2, N 75°32' E y necesitándose conocer la del lado 3, es claro que la dirección ú orientación inversa del primero desde el vértice 2, es S 75°32' O † y que por tanto el ángulo 1, 2, *m*, es de 75°32'. De consiguiente, si tomamos 75°32 del ángulo entero corregido 1, 2, 3, ó de 144°57',

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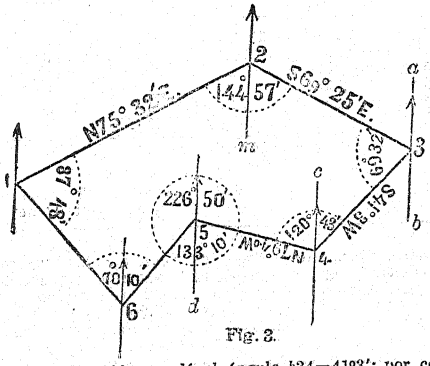
* Porque la suma de los ángulos internos de un polígono es igual á tantas veces dos rectos como lados tiene menos 4 ángulos rectos ó 360°.

† *N. del T.* — En esta figura la letra W indica el Oeste (West).

la diferencia de 69°25′ será el ángulo *m*, 2, 3; por tanto la dirección del lado 3 debe ser S 69°25′ E. Para hallar la del lado 4, tenemos ahora el ángulo 2, 3, *a*, orientación inversa del lado 2, 3, también igual á 69°25′, y si agregamos este ángulo al entero corregido 2, 3, 4, ó á 69°32′, tendremos el ángulo *a*3, 4=69°25′+69°32′ =




138°57′, que restado de 180° nos dá el ángulo *b*34=41°3′; por consiguiente, la dirección del lado 4 debe ser S 41°3′ O.

Para la dirección de lado 5 tenemos ahora el ángulo 3, 4, *c*=41°3′, que restado del ángulo corregido 3, 4, 5 ó 120°43′ queda el ángulo *c*, 4, 5=79°40′; por tanto la dirección del lado 5 debe ser N 79°40′ O. En el vértice 5 para la dirección del lado 6, tenemos el ángulo 4, 5, *d*=79°40′ que tomados de 133°10′ queda el ángulo *d*5 6=53°30′; por consiguiente, la dirección del lado 5, 6 debe ser S 53°30′ O. Así en cada uno de los lados, sólo se necesita una observación cuidadosa para ver cómo deben aplicarse los diferentes ángulos en cada vértice.

Algunas veces se han dado reglas con este objeto; pero si no se usan frecuentemente, se olvidan pronto. El plano previamente preparado evita la necesidad de dichas reglas, tanto más cuanto que el principio del procedimento aludido se reduce únicamente á un asunto de simple vista y tiende en gran manera á impedir los errores provenientes de erradas direcciones, en tanto que el transportador revela inmediatamente cualquiera equivocación sería cometida en los ángulos, impidiendo así su propagación. Después de haber obtenido todas las direcciones corregidas, deben trazarse en el plano, en lugar de las tomadas en el terreno. Sin embargo, requerirán todavía una nueva corrección desde luego que estarán afectadas al cerrarse el polígono del error correspondiente.

Ahora procedamos al cálculo de este error, *t*6, fig. 2, el cual está basado en el principio de que en una medida correcta, las diferencias de latitud norte son iguales á las de latitud sur, y las diferencias de meridiano del este á las del oeste. Prepárese una tabla de siete columnas como se ve á continuación, y en las primeras tres columnas colóquense los números de los lados y sus direcciones corregidas : también las distancias ó longitudes de los lados según fueron medidas en el croquis, si se ha hecho uno, pero si no, según fueron medidas en el terreno. A saber

| Lado. | Dirección. | Dist. en m. | Diferencias de latitudes. | | Diferencias de meridianos. | |
|---|---|---|---|---|---|---|
| | | | N. | S. | E. | O. |
| 1 | N. 16° 40′ O. | 1060 | 1015.5 | | | 304. |
| 2 | N. 75° 32′ E. | 1202 | 300.3 | | 1163.9 | |
| 3 | S. 69° 25′ E. | 1110 | | 390.2 | 1039.2 | |
| 4 | S. 41° 3′ O. | 850 | | 641. | | 558.2 |
| 5 | N. 79° 40′ O. | 802 | 143.9 | | | 789. |
| 6 | S. 53° 30′ O. | 705 | | 419.3 | | 566.7 |
| | | | 1459.7 | 1450.5 | 2203.1 | 2217.9 |
| | | | 1450.5 | | | 2203.1 |
| | | | 9.2 | Error en lat. | Error en las dif. de mer. | 14.8 |

Búsquense ahora por medio de la Tabla de senos, etc., las proyecciones N, S, E, O, de los diferentes lados y colóquense en la que le corresponde de las cuatro columnas. Así: para el lado 1, que tiene 1,060 m con rumbo N 16 40 ; cos 16°40′ = .9580; sen 16 40′ = .2868. Entonces N = 1,060 × .9580; O = 1,060 × .2868 = 304 Procédase así con todos. Súmense las cuatro columnas; búsquese la diferencia entre las columnas N y S y entre las columnas E O. En este caso encontramos que las dif de lat N son 9.2 mayores que las dif de lat S y que las dif de mer O exceden en 14.8 á las del E. En otras palabras, hay un error al cerrar el polígono que haría que al transportar fielmente las primeras tres columnas se terminase 9.2 m hacia el N del punto de partida y con una desviación de 14.8 al O del mismo.

De modo que habiendo anotado este error sobre el papel antes de comenzar á transportar, habríamos tenido un medio de comprobar su exactitud; pero como se ha advertido anteriormente, un poquito de más trabajo nos facilita la división del error entre todos los lados y nos pone en disposición de trazar el plano de una vez correctamente.

Para distribuir los errores prepárese una tabla precisamente igual á la anterior con la excepción de que los espacios horizontales han de estar más separados, y en la cual se omiten las adiciones de las columnas N, S, E, O. Las adiciones de que hemos hecho mención se hacen inmediatamente después.

La nueva tabla es la inmediata siguiente.

**Observación.** La **orientación de una línea** en un extremo **no tiene exactamente el mismo valor angular** que la inversa en el otro extremo, la diferencia varía con la latitud y con el largo de la línea; pero no en la misma proporción. Sin embargo, la diferencia es generalmente tan pequeña, que no puede ser apreciada por la aguja. Según Gummere, alcanza sólo á $^{3}/_{4}$ de minuto en una milla (1,609 m), á la latitud de 40°; en latitudes más altas, es mayor, y menor en las más bajas, lo que se debe á que los meridianos ó líneas N-S no son verdaderamente paralelos entre sí, pues se cortan en los polos.

De aquí que **el único rumbo que puede seguirse en línea recta con toda exactitud** es la verdadera N-S, excepto en el Ecuador mismo donde una línea E-O es también una línea recta. Puede trazarse una verdadera E-O en cualquiera latitud, con bastante exactitud para los propósitos del agrimensor, así: habiendo obtenido primero, por medio de la estrella Polar, pág. 296, una verdadera N-S; en el punto de partida tómense 90° á contar de la N-S para obtener la verdadera dirección E-O en ese punto. Esta dirección E-O será tangente á la curva E-O del paralelo respectivo. Sígase esta tangente cuidadosamente y á intervalos (digamos al fin de cada milla) desviese desde ella (hacia el N si estamos en latitud N ó á la inversa si en latitud S) una distancia cuya longitud en pies sea igual á la que le corresponda según la latitud en los datos siguientes, multiplicando la desviación indicada abajo por el cuadrado de la distancia en millas desde el punto de partida. Estas dist marcarán puntos de la verdadera curva E-O.

**Latitud N ó S**

5° 10° 15° 20° 25° 30° 35° 40° 45° 50° 55° 60° 65°

**Desviación en pies para una milla á contar del punto de partida.**

.058 .118 .179 .243 .311 .385. 467 .559 .667 .795 .952 1.15 1.43

*N. del T.* — **Desviación en centímetros para un kilómetro á contar del punto de partida.**

| 5° | 10° | 15° | 20° | 25° | 30° | 35° | 40° | 45° | 50° | 55° | 60° | 65° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.10 | 2.20 | 3.39 | 4.60 | 5.89 | 6.80 | 8.84 | 10.59 | 12.63 | 15.06 | 18.03 | 21.78 | 27.8 |

**Cualquiera desviación en pies** es=.6666×(dist total en millas)²×tang de la latitud.

*N. del T.* — Esta fórmula para el sistema métrico es así :

**Cualquiera desviación en metros**=.0782447×(dist total en kilómetros)²×tang de la latitud.

**Rumbo** es una línea cualquiera que corta al meridiano oblicuamente, es decir, que no es ni N-S ni E-O.

| Lados. | Rumbos. | Dist. en m. | Diferencias de latitudes. | | Diferencias de meridianos. | |
|---|---|---|---|---|---|---|
| | | | N. | S. | E. | O. |
| 1 | N. 16° 40′ O. | 1060 | 1015.5<br>1.7 | | | 304.0<br>2.7 |
| | | | 1013.8 | ..... | ..... | 301.3 |
| 2 | N. 75° 32′ E. | 1202 | 300.3<br>1.9 | | 1163.9<br>3.1 | |
| | | | 298.4 | ..... | 1167.0 | |
| 3 | S. 69° 25′ E. | 1110 | | 390.2<br>1.8 | 1039.2<br>2.9 | |
| | | | | 392 | 1042.1 | |
| 4 | S. 41° 3′ O. | 850 | | 641.0<br>1.3 | | 558.2<br>2.2 |
| | | | | 642.3 | ..... | 556.0 |
| 5 | N. 79° 40′ O. | 802 | 143.9<br>1.3 | | | 789.0<br>2.1 |
| | | | 142.6 | ..... | ..... | 786.9 |
| 6 | S. 53° 30′ O. | 705 | | 419.3<br>1.1 | | 566.7<br>1.8 |
| | | | | 420.4 | ..... | 564.9 |
| | | 5729<br>Suma de los lados. | 1454.8<br>Coordenad. N. | 1454.7<br>Coordenad. S. | 2209.1<br>Coordenad. E. | 2209.1<br>Coordenad. O. |

Ahora hemos encontrado por la tabla vieja que las difs de lat N y las difs de mer al oeste son demasiado largas y por consiguiente deben acortarse, mientras que las al S y al E deben alargarse; todo en las proporciones siguientes :

Suma de todos los lados : Cualquier lado dado :: Error total de lat ó dif de meridiano : Error de lat ó de dif de meridiano del lado dado.

Así, comenzando por la lat del lado 1 tendremos :

Suma de todos los lados. 5,729 : Lado 1 1,060 :: Error total de latitud 9.2 : Error de lat del lado 1. es decir 1.7.

Ahora, como la lat del lado 1 es Norte, debe ser acortada y viene á ser por consiguiente 1,015.5 — 1.7 = 1,013.8 como está en la nueva tabla.

También tenemos por dif de meridiano del lado 1.

Suma de todos los lados 5,729 : Lado 1 1,060 :: Error total en la dif de mer 14.8 : Error en la dif de mer del lado 1 ó sea 2.7.

Ahora bien, como la dif de meridiano del lado 1 es Oeste, debe acortarse; de consiguiente se convierte en 304 — 2.7 = 301.3 como figura fuera en la nueva tabla*

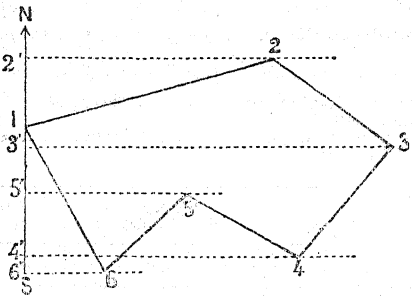



Fig. 4.

Procediendo así con cada lado obtendremos todas las lats y difs de meridianos correctas, como se ve en la nueva tabla donde están unidas á sus lados respectivos por medio de líneas de puntos; pero en la práctica es mejor tachar las originales cuando se ha terminado y probado el cálculo. Si sumamos ahora las 4 columnas corregidas N S E O, encontramos que las difs de lat N son=las difs de lat S y las difs de mer E=las difs de mer O, probando esto que está correcto el trabajo. Hay en verdad una discrepancia de .1 de m entre las difs de lat N y las difs de lat S; pero esto es debido á que hemos apreciado las correcciones con una sola cifra decimal y es demasiado pequeña para ser considerada. Ocurrirán algunas veces, por esta causa, discrepancias de 10 ó 12 cm, pero pueden ser despreciadas. Las lats y difs de meridiano corregidas deben evidentemente cambiar los rumbos y distancias de todos los lados; pero sin conocer ninguna de estas dos, podemos ahora transportar las mensuras por medio de sólo las lats y difs de meridiano corregidas. El sistema se explica evidentemente por sí mismo. Trácese primero una línea meridiana N-S, fig. 4, y sobre ella fíjese un punto 1 para representar el vértice extremo oeste* de la mensura.

Entonces desde el punto 1 indíquese con puntos en partes de la escala hacia el norte la distancia 1.2=la dif de lat norte corregida 298.4 del lado 2, tomada de la tabla última; desde 2′ hacia el sur indíquese del mismo modo la distancia 2′, 3′, dif de lat sur corregida 392 del lado 3; de 3′ hacia el sur la 3′, 4′, =dif en la lat sur 642.3 del lado 4; desde 4′ hacia el norte la 4′, 5′, =dif de lat norte 142.6 del lado 5; desde 5′ indíquese hacia el sur 5′, 6′, =dif. de lat sur del lado 6†. Entonces desde los puntos 2′, 3′, 4′, 5′, 6′, trácense líneas indefinidas con dirección hacia el este, es decir, en ángulos rectos á la meridiana. Hágase por la escala á 2′, 2=dif de mer corregida del lado 2, y únanse 1 y 2. Hágase 3′ 3=dif de mer del lado 2+dif de mer del lado 3, y únanse 2 y 3; hágase 4′, 4=3′, 3 — dif de mer del lado 4, y únanse 3 y 4; hágase 5′, 5=4′, 4 — dif de mer del lado 5, y únanse 4 y 5;

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* El vértice del extremo este servirá también con una pequeña variación en la operación subsiguiente como se comprende á la simple vista.

† En lugar de indicar con puntos estas dif de lat norte y sur sucesivamente, será más correcto en la práctica, preparar primero una tabla demostrando á qué distancia está cada uno de los puntos 2′, 3′, etc. del punto 1 hacia el norte ó el sur. Habiendo hecho esto, los puntos pueden indicarse de aquel modo hacia el norte ó hacia el sur desde el punto 1, sin tener que mover la escala en cada vez y, además, con mayor exactitud. Dicha tabla ha sido ya formada. Ráyese como sigue, y en las primeras tres columnas colóquense los números de los lados (comenzando por el lado 2 extremo del lado 1, 2) y sus respectivas diferencias de latitudes norte y sur corregidas. La formación de la

hágase 6', 6=5', 5 — dif de mer del lado 6, y únanse 5 y 6 ‡, y finalmente únanse 6 y 1 y estará completo el polígono. Si no se requiere mucha exactitud, la superficie puede encontrarse por medio de triángulos; las direcciones por medio del trans-

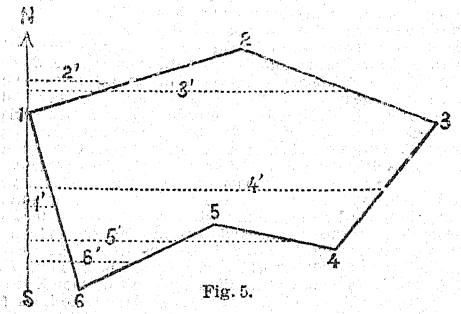


Fig. 5.

portador; y las longitudes de los lados por medio de la escala; todo con una aproximación suficiente para los fines ordinarios y quizá casi tan aproximada como por el cálculo, cuando como es costumbre los rumbos se toman con un cuarto de grado de aproximación. Hemos dicho ya que con una escala de pies por pulgadas $= \frac{\text{término medio de la longitud de los lados}}{8}$, el error en el área no excederá de $\frac{1}{200}$.

(*N. del T.* — Como dijimos también en el mismo párrafo en que el autor dió

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columnas 4.ª y 5.ª por medio de la 3.ª y 4.ª se explica á la simple vista. La exactitud está probada por el resultado final que es 0.

| Lado. | Lat. N. | Lat. S. | Dist. N. ó S. desde el punto 1. | |
|---|---|---|---|---|
| | | | N. | S. |
| 2 | 298.4 | | 298.4 | |
| 3 | | 392. | | 93.6 |
| 4 | | 642.3 | | 735.9 |
| 5 | 142.6 | | | 593.3 |
| 6 | | 420.4 | | 1013.7 |
| 1 | 1013.8 | | 000.0 | 000.0 |

‡ Debería de antemano prepararse una tabla semejante para las distancias de los puntos, 2, 3, 4, etc. al este de la meridiana. Esta se hace de la misma manera; pero necesita una columna menos, pues todas las líneas están del mismo lado de la meridiana. Así, comenzando desde el punto 1 con el lado 2.

| Lado. | Dif. de mer. E. | Dif. de mer. O. | Dist. este desde la línea meridiana. |
|---|---|---|---|
| 2 | 1167.0 | | 1167.0 |
| 3 | 1042.1 | | 2209.1 |
| 4 | | 556.0 | 1653.1 |
| 5 | | 786.9 | 866.2 |
| 6 | | 564.9 | 301.3 |
| 1 | | 301.3 | 000.0 |

Este trabajo se comprueba él mismo porque su resultado final debe ser 0.

esta regla : « En el sistema métrico » la escala para este límite de error es $= \frac{1}{\text{término medio de la longitud de los lados} \times 5)}$.

Pero si se necesita calcular el área del plano corregido con rigurosa exactitud, se puede hacer según el principio siguiente. (Véase fig. 5.)

Supongamos trazada una línea meridiana N-S por el vértice 1 del extremo oeste del polígono y líneas (llamadas *distancias medias*) trazadas (como las indicadas con puntos en la figura) en ángulos rectos á dicho meridiano, desde el centro de cada lado del polígono. Entonces, si cada una de las distancias medias de dichos lados, que tienen diferencia de latitud norte, se multiplica por la dif de lat, N corregida de su lado correspondiente; y si cada una de las distancias medias de dichos lados que tengan diferencias de lat S se multiplica por la dif corregida de lat S de su lado correspondiente; si sumamos separadamente todos los productos provenientes del factor dif de lat norte y todos los productos provenientes de la dif de lat S, y sustraemos la menor de estas sumas de la mayor, el residuo será el área del polígono *. Hemos encontrado ya las difs de lat norte y sur corregidas, así como también las difs de mer este y oeste. Las dist medias se encuentran por medio de las últimas, empleando sus mitades; sumando la mitad de las difs de mer E y sustrayendo la mitad de las difs de mer O. De este modo es evidente que la distancia media 2′ del lado 2 es igual á la mitad de la dif de mer E del lado 2. A ésta agréguese la otra mitad de la dif de mer E del lado 2 y la mitad de la dif de mer E del lado 3, y la suma es claramente igual á la distancia media 3′ del lado 3. A esto agréguese la otra mitad de la dif de mer E, del lado 3 y réstese la mitad de la dif de mer O del lado 4 para obtener la distancia media 4′ del lado 4. De ésta réstese la otra mitad de la dif de mer O del lado 4 y la mitad de la dif de mer O del lado 5 para obtener la distancia media 5′, del lado 5 y así de los demás. Este cálculo debe hacerse así :

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* *Prueba.* Para ilustrar el principio sobre el cual está basada esta regla, represéntese por *ab*, *bc*, *ca*, fig. 6, los tres lados de un polígono triangular de una mensura y por *df* una meridiana trazada por el vértice del extremo oeste *a*. Trácense líneas *bd* y *cf*, perpendiculares á la línea meridiana desde cada vértice y desde el medio de cada lado trácense líneas *we*, *mn*, *so*, perpendiculares también al meridiano y que

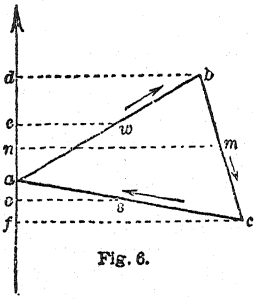


Fig. 6.

representen las dists medias de los lados. Considerando los lados en el orden, *ab*, *bc*, *ca*, es claro que *ad*, representa la dif de lat N del lado *ab*; *fa*, la dif de lat N de *ca*, y *df* la dif de lat S de *bc*. Ahora bien, si multiplicamos la dif de lat N *ad*, del lado *ab* por su dist media *ew*, el producto será el área del triángulo *abd*. De la misma manera la dif de lat N *fa* del lado *ca*, multiplicada por la distancia media *so*, dará el área del triángulo *acf*. Y también la dif de lat S *df* del lado *bc* multiplicada por la distancia media *mn*, dará el área encerrada por la figura entera *dbcfd*. Si de esta área sustraemos las áreas de los dos triángulos *abd*, *acf*, la dif es evidentemente el área del triángulo *abc*, y así con cualquiera otro polígono por complicado que sea.

Mitad de la dif de mer E del lado 2 = $\frac{1,167}{2}$ = 583.5 E = dist media del lado 2.

583.5 E

Mitad de la dif de mer E del lado 3 = $\frac{1,042.1}{2}$ = 1,167.0 E / 521.0 E

1,688.0 E = dist media del lado 3.

521.0 E

Mitad de la dif de mer O del lado 4 = $\frac{556}{2}$ = 2,209.0 E / 278.0 O

1,931.0 E = dist media del lado 4.

278.0 O

Mitad de la dif de mer O del lado 5 = $\frac{786.9}{2}$ = 1,653.0 E / 393.5 O

1,259.5 E = dist media del lado 5.

393.5 O

Mitad de la dif de mer O del lado 6 = $\frac{564.9}{2}$ = 866.0 E / 282.4 O

583.6 E = dist media del lado 6.

282.4 O

Mitad de la dif de mer O del lado 1 = $\frac{301.3}{2}$ = 301.2 E / 150.6 O

150.6 E = dist media del lado 1.

El trabajo se comprueba siempre por sí mismo cuando los dos últimos resultados son iguales.

Hágase luego una tabla en cuyas cuatro primeras columnas se colocan los números, las distancias medias y las difs de lat N y S. Multiplíquese cada distancia media por su correspondiente dif de lat N ó S y colóquese el producto en su propia columna. Súmese cada columna y réstese la suma menor de la mayor.

| Lado. | Dist. media. | Dif. de lat. N. | Dif. de lat. S. | Producto proveniente del factor dif. de lat. N. | Producto proveniente del factor dif. de lat. S. |
|---|---|---|---|---|---|
| 1 | 150.6 | 1013.8 | | 152678 | |
| 2 | 583.5 | 298.4 | | 174116 | |
| 3 | 1688 | | 392 | | 661696 |
| 4 | 1931 | | 642.3 | | 1240281 |
| 5 | 1259.5 | 142.6 | | 179605 | |
| 6 | 583.6 | | 420.4 | | 245345 |
| | | | | 506399 | 2147322 |
| | | | | | 506399 |
| | | | | 43560)1640923)37.67 Acres. | |

El residuo será el área de la mensura en pies cuadrados, la cual dividida por 43,560 (número de pies cuadrados que tiene un acre) será el área en acres. En este ejemplo tenemos 37.67 acres. (*N. del T.* — Todo esto se aplica á cualquiera unidad de medida, pies, metros, etc. Si se toman las medidas en metros, el residuo será metros cuad.)

Sólo falta ahora calcular los rumbos corregidos y las longitudes de los lados, las cuales están necesariamente cambiadas por la adopción de las diferencias corregidas de latitud y de meridianos. Para encontrar la dirección de cualquier lado, divídase su dif de mer (E ú O) por su lat (N ó S), búsquese el cociente en la tabla de tangentes naturales; y el ángulo opuesto á él será el ángulo de la dirección ó rumbo que se desea. De este modo tenemos por dirección del lado 1 $\frac{301.3\ O}{1,013.8\ N}$ = =.2972=tangente natural, opuesta en la tabla al ángulo que se busca de 16°33'. La dirección por tanto es N 16°33' O.

Para tener la magnitud de cualquier lado : en la tabla de cosenos naturales, tómese el coseno opuesto al ángulo de la dirección corregido y divídase la lat corregida (N ó S) del lado por el coseno. Así, para tener la magnitud del lado 1 encontramos opuesto á 16°33′ el coseno .9586 y

Lat. Cos.
1,013.8 ÷ .9586 = 1,057.6 distancia requerida.

La tabla siguiente contiene todas la correcciones de la medida anterior; por

| Lado. | Dirección ó rumbos. | Distancia. |
|---|---|---|
| 1 | N 16° 33′ O | 1057.6 |
| 2 | N 75° 39′ E | 1204.0 |
| 3 | S 69° 23′ E | 1113.3 |
| 4 | S 40° 53′ O | 849.6 |
| 5 | N 79° 44′ O | 800.1 |
| 6 | S 53° 21′ O | 704.3 |

consiguiente, si las direcciones y distancias están correctamente trazadas, cerrarán perfectamente. Se aconseja al ayudante que practique haciendo esto; y también dividiendo el trazado en triángulos y calculando el área; de este modo comprenderá pronto el gran cuidado que se requiere para obtener resultados exactos.

Las apuntaciones siguientes pueden ser útiles á menudo : 1.° Evítese tomar rumbos y distancias á lo largo de una línea limítrofe tortuosa como *abc*, fig. 7;

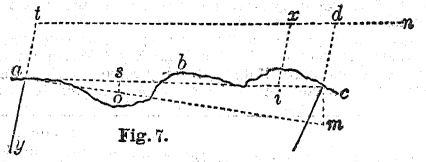

Fig. 7.

evántese ésta por medio de perpendiculares á una línea recta *ac* que se trazará en lugar de aquélla. 2.° Cuando se desee medir una línea recta desde *a* hasta *c* y no se pueda dirigir el instrumento precisamente hacia *c*, debido á la interposición de árboles ú otros obstáculos, tírese primero una línea de ensayo *am* tan próxima y en la misma dirección de la línea *ac* como se pueda; mídase *mc* y hágase la siguiente proporción ; *am* : *mc* : : 100 : *x*. Trácese *ao* = 100 pies ó metros y *os* igual á *x* y córrase la línea final *asc*. Si *mc* es muy pequeña, calcúlense perpendiculares *os* para cada 100 pies ó de 30 en 30 metros á lo largo de *am*, evitando así la necesidad de seguir una segunda línea. 3.° Cuando *c* es visible desde *a*, pero el terreno inter-

Fig. 8.

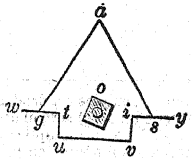

Fig. 9.

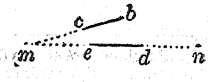

Fig. 10.

puesto hace dificultosa su medida directamente por los pantanos, etc., prolónguese el lado *y a* hasta el terreno bueno en *t*, y luego haciendo el ángulo *ytd* igual á *yac*, trácese la línea *tn* hasta un punto *d* en el cual el ángulo *ndc* se encuentre por tanteo igual al ángulo *atd*. Rara vez es necesario hacer más de un tanteo para encontrar este punto *d*; porque suponiendo hecho un ensayo en *x* y no en *d*, véase dónde encuentra á la línea *ac* en *i*, mídase entonces *ic*, y sígase desde *x*, haciendo la línea *xd* = *ic*. 4.° Caso de que nos encontremos con un pedazo de terreno ó lago fig. 8, muy irregulares, rodéese por líneas rectas, mídanse éstas, y levántese el

borde por perpendiculares á aquellas líneas. 5.° Si al trazar una línea recta de *w* hacia *y*, fig. 9, encontramos un obstáculo *o*, trácese un ángulo recto *wtu*, mídase cualquiera distancia *tu*, hágase *tuv* = 90°; mídase *uv*; hágase *uvi* = 90°; hágase *vi* = *tu* y hágase *viy* = 90°. Entonces *ti* será igual á *uv* y la línea *iy* será prolongación de *wt*. Con menos trabajo, en *g* hágase *tga* = 60°; mídase cualquier distancia *ga;* hágase *gas* = 60° y *as* = *ga;* hágase *a s i* = 60°. Entonces *gs* es = *ga*, ó *as* y la línea *is* prolongada hacia *y* estará en línea recta con *wt*. 6.° Estando situado entre dos objetos *m* y *n* y deseando colocarme en la línea recta que los une, acuesto un bastón *cb* en el terreno y lo pongo en la dirección de uno de los objetos *m;* luego yendo hacia el extremo *c*, encuentro que no se halla en la dirección del otro objeto. Por medio de tanteos sucesivos se encuentra la posición *ed* que señala á ambos objetos y en consecuencia se hallará en su mismo alineamiento.

## CADENAS Y CINTAS

**Cadena.** Los ingenieros (americanos) han abandonado la cadena de Gunter de 66 pies (20.11 mets) dividida en 100 eslabones de 7.92 pulgs (.2011 mets) cada uno y úsanlas de 100 pies (30.479 mets) con 100 eslabones de 1 pie (.3047 mets) cada uno, y calculan las áreas en pies cuad, cuyo número dividido por 43560 los convierte en acre y sus decimales. El Gobierno de los E. U. emplea la cadena de Gunter en las mensuras.

Las cadenas se construyen de alambre de hierro ó acero. Cada eslabón está doblado en sus extremidades formando un anillo de conexión con el adyacente, ya directamente como en la cadena patentada de Grumman, ó más comúnmente por pequeños eslabones. El desgaste de estos anillos es causa de errores, pues aumentan el largo de la cadena. De aquí la necesidad de compararla de vez en cuando con una medida fija normal.

**Cintas.** Debido á las actuales facilidades en la fabricación de cintas de acero, se está abandonando el uso de la cadena. Siendo la cinta más liviana requiere menos tensión, y no teniendo eslabones que se desgasten, su tamaño es más constante. Hasta cierto punto reemplaza la percha para la mensura de las bases en los trabajos geodésicos. Las cintas de acero se hacen en piezas enterizas hasta de 500, 600 y aun de 1,000 pies (152.40; 182.88; 304.79 metros); pero las de 100 pies (30.48 mets) son las más usadas. Las cintas muy largas se quiebran más fácilmente con el uso. Estas roturas son difíciles de soldar, mientras que la torcedura de un eslabón apenas altera temporalmente la longitud de una cadena. El paso de un carro puede hasta quebrar una cinta de acero. Sin embargo, la ligereza, limpieza y seguridad de la cinta de acero compensan aquellas desventajas; las cuales, además, pronto aprende á evitarlas el agrimensor.

Las cintas empleadas en las mensuras de campos son, por lo general, angostas (3 á 6 mm) y su espesor de .3 á .6 mm, y están graduadas por medio de pequeños remaches de cobre, separados, generalmente, por espacios de 5 pies y de 1 pie en los 10 pies de cada extremo. *

Las que se emplean en la ciudad son más anchas, de .25 á .5 pulgs (6 á 12 mm) y de menos grueso, de .007 á .010 pulgs (.17 á .25 mm), divididas generalmente en décimos de pie.

**Agujas.** Son generalmente de alambre, agudas en un extremo, y el otro en forma de anillo; se clavan con facilidad en terrenos no muy pedregosos. Cada aguja lleva un pedazo de franela roja en el anillo para hacerla visible entre la yerba.

**Correcciones de la cinta por seno y alargamiento.** (*Obs. del T.* — Hemos construído este diagrama, reemplazando el que trae el autor en medidas inglesas, transformando de igual manera los ejemplos anexos.) †

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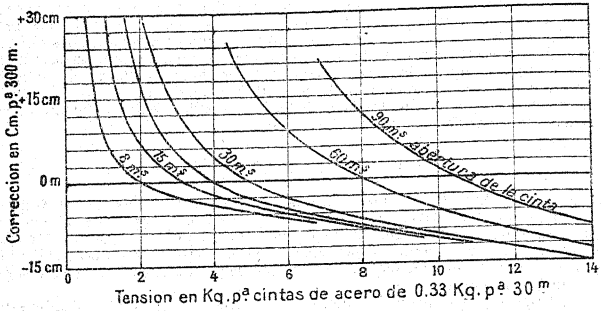
* El tamaño de la cinta varía mucho con los diversos fabricantes, y por tanto para aplicarles las correciones se debe conocer muy bien el espesor y su peso por unidad de longitud.

† Deducido de los diagramas construídos por Mr. J. O. Clark de los trabajos del Club de Ingenieros de Filadelfia, abril 1901, vol. XVIII, n.° 2, y de la fórmula : Alargamiento en pies $= \frac{PS}{EA}$ en que

P = tensión en lbs.
S = longitud empleada de la cinta en pies.
E = módulo de elasticidad del acero = 27,500,000 lbs por pulg cuad.
A = área de la sección transversal de la cinta con peso de .75 lbs por 100 pies = .0022 pulgs cuads; y de la ecuación de la parábola se deduce que el acortamiento

**Diagrama en Sistema Métrico.** Este diagrama demuestra que una abertura de 60 m de cinta, con el peso de .33 kg por 30 m, se requiere una tensión de 8 kg para reducir á 0 (cero) la corrección; otra abertura de 30 m, 4.8 kg; para



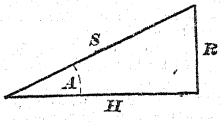

8 m, 2 kg, etc. Con estas tensiones (que se llaman normales), los efectos opuestos, por el seno que hace la cinta y la tensión, se compensan. Con mayores tensiones el alargamiento supera á la reducción producida por el seno, y viceversa.

**Las cintas de otros pesos** necesitan tensiones proporcionales á sus pesos. Así en una cinta de cualquier tamaño que pese 1 kg para 30 m (llamando $y$ la tensión para una corrección dada en una cinta normal que pese .33 kg para 30 m) tendremos $x : y = 1 : .33 = 100 : 33$; de donde $x = \frac{100}{3}\, y = 3\, y$.

Ahora, al contrario, *búsquese la corrección*, dada una tensión de 4 kg para una cinta de 15 m que pese .22 kg por 30 m.

Para producir el mismo error en la cinta que pesa .33 por 30 m se necesita una tensión $y = 4 \frac{.33}{.22} = 6$ kg. Buscando en el diagrama de la columna de 6 kg y en la curva de 15 m, se encuentra la corrección $= -6$ cm.

**Correcciones debidas á la temperatura.** Las cintas se gradúan generalmente calculando su longitud normal á 62° F=16°7 C. Para cintas corrientes de acero, la corrección por temperatura es como de .0000037 m por metro y por grado C.




Las correcciones de temperaturas no son exactas puesto que la temperatura de la cinta no se puede determinar con exactitud. Las mensuras que requieren gran exactitud deben hacerse, por consiguiente, con tiempo nublado ó de noche, manteniendo la cinta y el termómetro separados del suelo.

Cuando se hacen mensuras en **terrenos inclinados**, la cadena ó la cinta

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debido al seno que forma la cinta es en pies $= \frac{W^2 S^3}{24 P^2}$ en que W es el peso de la cinta en lbs por pie. Excepto para tensiones muy suaves, esta última fórmula da prácticamente los mismos resultados que la ecuación de la catenaria, la que es exacta pero mucho más complicada.

*N. del T.* — La primera fórmula en el sistema métrico es la siguiente: Alargamiento en metros $= \frac{P \times S}{E \times A}$; donde P se da en kilogramos; S en metros; E = 1,925,000 kg por cm cuadrado = módulo de elasticidad del acero (equivalente al del autor); A = área de la sección transversal de la cinta = 0.01419 cm cuad. Pesa. 337 kg por 30 metros.

deben mantenerse lo más horizontalmente posible, proyectando en el terreno por medio de una plomada el extremo en alto de la cinta. O bien, se mide con la cinta paralela al terreno inclinado y se corrige la medida con las siguientes fórmulas :

$$\frac{H}{S} = \cos A; \quad H = S \cos A; \quad S = \frac{H}{\cos A} = H \sec A;$$

$$\frac{R}{H} = \text{tang } A; \quad R = H \text{ tang } A;$$

$$\frac{R}{S} = \text{sen } A; \quad R = S \text{ sen } A.$$

## TRAZADO DE LA MERIDIANA

### Por medio de una estrella circumpolar.

(1) Vista desde un punto O de la tierra (figs. 1 y 2), una estrella circumpolar *e* (estrella próxima al polo P) parece describir, diariamente * y en sentido opuesto al movimiento de las manillas de un reloj, una pequeña circunf *euwl*, alrededor del polo. El ángulo POe, POu, etc., subtendido por el radio Pe, Pu, etc., de este círculo, ó sea la distancia aparente de la estrella al polo, se llama **distancia polar.** Las distancias polares de las estrellas varían muy poco de año en año (véase tabla 3). Varían menos **durante** el año. En el caso de la estrella Polar, esta última variación llega á unos 50 segundos de arco.

(2) El ángulo NOP, que mide la **altura** del polo sobre el horizonte NESW, es= á la **latitud** del punto O de observación. La **declinación** es igual á su distancia angular norte ó sur del ecuador celeste. Por eso la declinación del polo es=90° y la de cualquier estrella es=á 90° — su distancia polar.

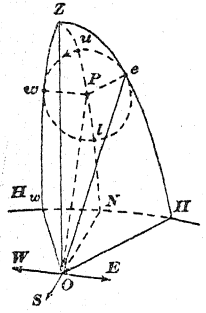


Fig. 1.

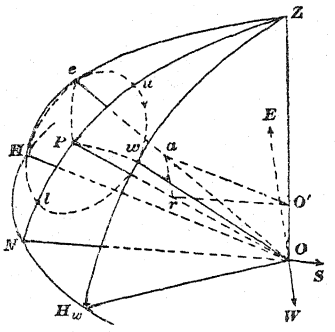


Fig. 2.

(3) Sea ZeH un arco de círculo vertical † que pasa por una estrella circumpolar *e* y sea H el punto en que este arco encuentra al horizonte NESW. El ángulo NZH cuyo vértice está en el zénit Z, ó el NOH en el punto de observación O, formado por el meridiano NZO y el plano HZO del círculo vertical de la estrella ó sea el arco NH se llama el **acimut** ‡ de la estrella. Si este ángulo NOH se traza desde OH en el terreno, la línea ON estará en el plano del **meridiano** NZS y será la **línea norte-sur** ‖.

(4) Cuando una estrella se encuentra en el meridiano ZN del observador, arriba ó debajo del polo P como en *u* ó *l*, se dice que está, respectivamente, en su **paso superior ó culminación** ó en su **paso inferior.** Su acimut es entonces **=cero**; la línea HO coincide con el meridiano ON.

(5) Cuando la estrella llega á su mayor distancia al Este ó al Oeste del polo, como en *e* ó *w*, se dice que está **en su máxima elongación Este ú Oeste** §.

---

* En 23 h. 56.1 m.

† Un círculo máximo es la sección en la esfera hecha por un plano que pasa por su centro. Un círculo vertical es un círculo máximo que pasa por el zénit Z.

‡ Los astrónomos toman generalmente el acimut, contando desde el extremo sur hacia los puntos oeste, norte y este y luego al sur; pero para nuestro objeto es mucho más conveniente medirlo desde el norte, hacia el este ó el oeste, según sea el caso.

‖ El punto N en el horizonte, se llama el *Norte* y no se debe confundir con el *polo* norte P.

§ Vista desde el ecuador una estrella, en cualquiera de estas dos posiciones, está como el mismo polo, en el horizonte; y las dos líneas Pe, Pw la unen con el polo y forman una perpendicular al meridiano. El acimut de la estrella será entonces igual á su distancia polar. Pero en otras latitudes Pe y Pw forman ángulos agudos con el meridiano, como se puede ver, y estos ángulos decrecen, y el acimut de la estrella aumenta más con su elongación á medida que aumenta la latitud.

**(6) El ángulo horal** de una estrella cualquiera, en un momento dado, es el tiempo transcurrido desde que alcanzó su culminación *.

**(7)** Es evidente que el acimut de una estrella cambia constantemente. En las estrellas circumpolares varía de 0° al máximo de su elongación y retrocede á 0° dos veces diariamente, pues la estrella parece girar alrededor del polo; pero cuando la estrella se encuentra cerca de cualquiera de sus elongaciones, el cambio de acimut se efectúa tan lentamente que, por algunos minutos, es apenas perceptible y parece que la estrella se mueve verticalmente.

**(8)** Tenemos para cualquiera estrella cuya declinación (=90° — su distancia polar) exceda la latitud del punto de observación †:

$$\left.\begin{array}{l}\text{seno del acimut de la estrella}\\ \text{en su elongación}\end{array}\right\} = \frac{\text{seno de la distancia polar de la estrella}}{\text{cos de latitud del punto de observación}}$$

ó véase **(11)** y tabla 3. Cuando la latitud > que la declinación, el seno acimut > 1, y en este caso no se aplica esta fórmula.

**(9)** Las siguientes estrellas circumpolares son útiles para determinar el meridiano. (Véase fig. 3.)

| Constelaciones. | Letras. | Nombres. |
|---|---|---|
| Osa menor | $\alpha$ (alfa) | Polar |
| — mayor | $\varepsilon$ (epsilon) | Aliot |
| — — | $\zeta$ (zeta) | Mizar |
| Casiopea | $\delta$ (delta) | Delta ‡ |

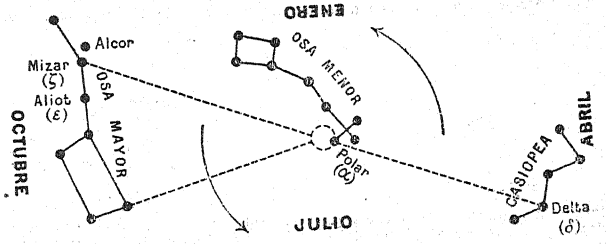


**Fig. 3.**

**(10)** La Polar se encuentra muy bien situada para la determinación del meridiano, pues su distancia polar es más ó menos de 1 $^1/_4$ grado. En la fig. 3 se ven las estrellas circumpolares, en la posición que ocupan á media noche en julio; invertidas quedan como se ven en enero, con el lado izquierdo hacia arriba como se ven en abril y para octubre con el lado derecho hacia arriba ‖.

**(11)** La tabla 3 da las distancias polares de la Polar y los logaritmos de sus

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* En la latitud 40° N el ángulo horal ZP$e$ = ZP$w$ de la polar en su elongación, ó sea igual á 5 h 55 m del tiempo solar. *Advertencia.* Nótese que, sólo para el observador situado en el ecuador, la elongación *no* ocurre á los 90° del meridiano.

† En el triángulo esférico ZP$e$, tenemos

$$\frac{\text{seno } e\text{ZP}}{\text{seno ZeP}} = \frac{\text{seno Pe}}{\text{seno PZ}}$$

pero como ZeP = 90°, sen ZeP = 1. También sen PZ = cos(90° — PZ) y $e$ZP = al acimut $e$ De donde seno acimut de $e$ = $\frac{\text{sen Pe}}{\text{sen PZ}} = \frac{\text{sen dista polar PO}e}{\text{cos latitud}}$.

‡ La δ de Casiopea la llamamos Delta por brevedad.

‖ *La Polar* se encuentra fácilmente por medio de dos conocidas estrellas llamadas *punteras*, en el trapecio de la Osa mayor, fig. 3, que forman la parte posterior de esta constelación. La línea que une estas dos estrellas prolongada pasa cerca de la Polar. La Osa mayor tiene una cola formada por tres estrellas y, de modo análogo, la Osa menor tiene otra cerca de cuyo extremo están la Polar y sus vecinas. La Polar está á mitad de línea que une á Delta y Mizar. La Polar se encuentra al extremo del brazo derecho de una cruz que forma ella con tres de menor brillo. En la fig. 3 esta cruz está invertidv

senos para el 1.° de enero, cada 3 años, de 1900 á 1930 inclusive, el log de los cosenos de cada cinco grados de latitud, de 20° á 50°, y los correspondientes acimutes de la Polar en su máxima elongación. Los valores intermedios se encuentran por interpolación.

(12) **Por la observación de la Polar en su máxima elongación.** Este método tiene la ventaja de que durante y cerca de su máxima elongación, aquella estrella parece que se mueve sobre una vertical, durante algunos minutos, y su acimut durante este tiempo no cambia apreciablemente; pero en algunas épocas (véase tabla 1) dicha elongación se efectúa de día, y este método no se puede emplear entonces. Véase (18), (19) y (22). Tampoco se puede emplear en ningún tiempo, en lugares situados al sur ó á menos de 4° lat N, porque en éstos no es visible la Polar.

(13) La tabla 1 da las horas aproximadas de la máxima elongación de la Polar en ciertos días de 1900 é instrucciones para calcular las horas en otros días. O bien obsérvese la Polar en conexión con cualquiera de aquellas estrellas que se encuentran aproximadamente en línea con ella y el Polo, como Delta, Mizar y Aliot (véase fig. 3). La hora de la elongación es bastante aproximada para determinar el acimut, por la cesación del movimiento horizontal durante la observación.

(14) Quince ó veinte minutos antes de la hora de la elongación colóquese el tránsito (véase 21) bien centrado sobre un punto marcado en una estaca, clavada previamente en el suelo.

(15) El instrumento debe tener lo necesario para iluminar los pelos del retículo. Esto se puede lograr por medio de una linterna sorda colocada de modo que no arroje luz sobre los ojos del observador, ó mejor aún, con un pedazo de hojalata, cortada y perforada como en la fig. 4, doblado en ángulo de 45° como en la fig. 5

Fig. 4.

Fig. 5.

y pintado de blanco en la superficie que ve al anteojo. El anillo que se forme con la tira larga se coloca alrededor del objetivo. Una luz, cubierta para el observador, se sitúa á un lado del instrumento, de modo que sus rayos caigan sobre la superficie inclinada y blanqueada de la hojalata y entren reflejados al anteojo.

(16) Hágase que el pelo vertical corte la Polar y por medio del tornillo tangente sígase á la estrella en su aparente curso hacia *la derecha*, si se acerca á la elongación *oriental* y *viceversa*, manteniendo el pelo sobre la estrella todo lo posible. A medida que se acerca á su máxima elongación, la estrella, parece moverse con mayor lentitud. Cuando parece que se mueve verticalmente á lo largo del pelo es porque ha llegado, prácticamente, á su máxima elongación, y el plano vertical del tránsito, *con el pelo cortando la estrella*, se encontrará en el mismo plano vertical de la estrella. Bájese el anteojo y fíjese un punto en la dirección de la vertical á unos cien metros ó más del tránsito*. Hágase girar éste inmediatamente (en sentido horizontal, 180°), vuélvase á ver la estrella, vuélvase á bajar el anteojo, y, si la visual coincide exactamente con la primera marca hecha, ambas se encontrarán en el plano vertical de la estrella. De no ser así, márquese la dirección de la visual y fíjese una tercera marca entre las dos anteriores. La visual dirigida á esta tercera marca se encontrará en el plano requerido, desde el cual debe trazarse el acimut, como en el (8), hacia el meridiano, ya sea á la izquierda de la elongación oriental ó á la derecha de la occidental.

(17) Para distinguir la estaca á mayor distancia y para señalarla de noche, se puede hacer uso de una placa ó mira.

(18) **Por observación de la estrella Polar en su culminación.** Debido

* La estaca debe estar iluminada, lo que se consigue alumbrando la cara de la estaca que ve al instrumento, ó mejor, colocando una hoja de papel detrás de la estaca y una luz detrás de la hoja; de este modo, el pelo del anteojo y la punta del lápiz con que el ayudante marca sobre la estaca la proyección del pelo, se destacan como sombras en el papel.

á sus dificultades este método se emplea sólo cuando no es posible hacer uso del método de elongación. Consiste aquél en observar la Polar en conexión con otras estrellas circumpolares (como Mizar * ó Delta) hasta que la Polar se vea en el mismo plano vertical con la otra, y luego esperando un corto espacio de tiempo conocido T, como se dice después, † hasta que la Polar llegue á su culminación en donde se observa ésta. La visual está entonces en el meridiano. En sus culminaciones Mizar y Delta están demasiado cerca del zenit, para ser observadas con exactitud en las latitudes nortes de 25° y 30° respectivamente. En sus pasos inferiores se encuentran demasiado cercanas al horizonte para servirse de ellas en lugares mucho más bajos de 38° de latitud norte. Generalmente, Delta se observa bien en su paso inferior de febrero á agosto y Mizar durante el resto de año.

| | Mizar T= | Delta T= |
|---|---|---|
| En 1900 | 2.6 min | 3.4 min |
| En 1910 | 6.5 — | 7.2 — |
| Aumento medio anual, 1900-1910 | .39 min | .38 min |

(19) **Por la observación de la Polar en cualquier punto de su curso.** La tabla 1 da la hora media solar de la culminación superior de la Polar para el 1.° de cada mes en 1900, é instrucciones para conocer las horas en las otras fechas; y la tabla 2 da el acimut de la Polar correspondiente á los diversos valores de su ángulo horal en la hora media solar común para diversas latitudes, desde 30° á 50° para los años 1901 y 1906. Para los ángulos horales y latitudes intermedias de los contenidos en las tablas, se pueden obtener los acimutes por interpolación. Véase la advertencia y fórmula pág. 300.

(20) La hora *local*,‡ de la observación debe conocerse con exactitud y se deducirá de ella la hora de la anterior culminación (como se obtiene en la tabla 1). La diferencia es el ángulo horal. Si éste, así encontrado, es de 11 h 58 min ó menos, la estrella se encuentra al oeste del meridiano. Si es mayor de 11 h 58 min, la estrella se encuentra al este del meridiano. En este caso réstese el ángulo horal de 23 h 56 min y éntrese en la tabla con el residuo como ángulo horal. Véase fig. 1.

(21) Cuando no se requiere gran exactitud, puede observarse la Polar á la simple vista por medio de una plomada. Un ladrillo ú otro objeto pesado puede servir para hacer una plomada. Ésta debe colgar sobre un balde de agua y se puede usar una pínula de brújula ó cualquier aparato con una ranura exactamente recta, como de 1½ mm de ancho. La pínula debe mantenerse exactamente vertical, pero debe poderse ver por ella horizontalmente á algunos metros al este y al oeste. La plomada y la pínula deben estar separadas unos 4 ó 5 metros por lo menos y colocadas de manera que por la pínula se vean la estrella y la plomada. La cuerda de la plomada debe estar iluminada. Sería conveniente hacer estos preparativos la noche antes de la observación. Cuando la estrella alcanza su máxima elongación, la vista debe colocarse en la misma línea que la plomada y la estrella. Desde la línea, así obtenida, trácese el acimut, hacia el oeste para la elongación oriental y viceversa.

(22) **Por estrellas á igual altura.** Este método, aplicable tanto en las latitudes sur como á las septentrionales, consiste en observar una estrella cuando se encuentra á alturas iguales sobre el horizonte al este ú oeste del meridiano, fijando así en el horizonte dos puntos de igual y opuesto acimut. El meridiano pasará por todo el medio entre los dos puntos.

(23) **Por sombras solares simétricas (aproximado),** fig. 6. Durante los solsticios (hacia junio 21 y diciembre 21), el rastro *abcd* recorrido antes y después de mediodía por el extremo de la sombra solar *Oa*, etc., de un objeto vertical O, ó por la sombra de un nudo hecho en la cuerda de una plomada suspendida en O, cortará un arco de círculo *aNd*, descripto alrededor de O á iguales distancias *am*, *md*, del meridiano ON. Las observaciones deben hacerse durante dos horas antes y después del mediodía. En el equinoccio de estío (marzo 21) la línea así determinada se encontrará algo al oeste, y en el equinoccio de otoño (sep-

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* Mizar se reconoce por la pequeña estrella Alcor, situada cerca de ella.

† Deducido de los valores calculados en tiempo astronómico (pág. 269) por las Mensuras geodésicas y de las costas de los EE.

‡ La hora local corresponde con la hora normal (pág. 270) tan sólo en los meridianos normales. Para otros puntos agréguese á la hora normal 4 minutos por cada grado de longitud al este de un meridiano normal y *viceversa*.

tiembre 21) algo al este, del meridiano exacto y á menos de 2½ minutos de arco. Para fechas intermedias el error es casi proporcional al período de tiempo trans-

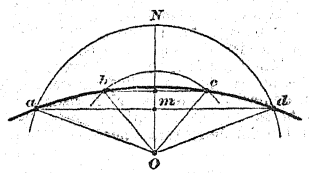



**Fig. 6.**

currido desde las citadas fechas. Es conveniente trazar varios arcos de diversos radios O*a*, O*b*, etc., marcando los dos puntos en que el paso de la sombra intersecte cada arco, y tomando el promedio de todos los resultados. Se puede emplear en vez del extremo de la sombra un pequeño pedazo de hoja de lata agujerado en el centro, colocándolo verticalmente sobre O; el punto brillante formado por la luz que pasa por el agujero sustituye á la sombra dicha.

## Tabla 1.

**Hora común aproximada de la elongación y culminación de la estrella Polar** en lat 40º N, long 90º O de Greenwich para el 1.º de cada mes en 1900. La hora de esta tabla es la solar media. **Las horas P. M.** (entre mediodía y media noche) aparecen en tipo negro.

En la latitud 25º la elongación oeste ocurre más tarde y la este más temprano ..... } cerca
En la latitud 50º las elongaciones oeste ocurren más temprano y las del este más tarde ..... } de 2 minutos.

La corrección de longitudes es de poco menos de un minuto de tiempo en cualquier parte de los E.U.

Para otros días del mes dedúzcase 3.94 min por cada día subsiguiente.

Por lo común las horas van retardando un poco cada año. En 1904 estarán unos 5½ minutos en retardo; pero en 1905 estarán sólo 3 minutos más en retardo que en 1900. Esta discrepancia se debe á que el año 1904 es bisiesto.

Como esta tabla sirve principalmente para alertar al observador y como éste debe estar en su puesto 15 ó 30 minutos antes de aquellas horas, el aumento gradual de las horas es de poca consecuencia. La posición de la estrella en su elongación máxima se determina por observación.

Al observar la estrella Polar en su culminación, cuando el cambio de acimut es más rápido, un error de dos minutos en tiempo se traduce por un error como de un minuto en el acimut.

| En su elongación un error de tiempo de | produce un error en el acimut menor de |
|---|---|
| 20 minutos | 30 segundos |
| 10 — | — de 6 — |
| 5 — | — de 2 — |
| 1 minuto | como .06 segundo. |

En todas las latitudes hay 11 h 58 m (tiempo solar medio) entre dos pasos sucesivos de una estrella por el meridiano. El tiempo entre las elongaciones sucesivas varía con las latitudes.

**Elongaciones.** (E, este; O, oeste.) **1900.**

| En 1. O. | Feb. 1. O. | Marzo 1. O. | Abr. 1. O. | Mayo 1. E. | Jun. 1. E. |
|---|---|---|---|---|---|
| 12.31 A. M. | **10.30 P. M.** | **8.40 P. M.** | **6.38 P. M.** | 4.50 A. M. | 2.49 A. M. |
| Jul. 1. E. | Agt. 1. E. | Sep. 1. E. | Oct. 1. E. | Nov. 1. O. | Dic. 1. O. |
| 12.51 A. M. | **10.46 P. M.** | **8.45 P. M.** | **6.47 P. M.** | 4.33 A. M. | 2.35 A. M. |

**Culminaciones.** (C, culminaciones; D, pasos inferiores.) **1900.**

| En. 1. C. | Feb. 1. D. | Marzo 1. D. | Abr. 1. D. | Mayo 1. D. | Jun. 1. D. |
|---|---|---|---|---|---|
| **6.38 P. M.** | 4.38 A. M. | 2.47 A. M. | 12.45 A. M. | **10.43 P. M.** | **8.42 P. M.** |
| Jul. 1. D. | Agt. 1. C. | Sep. 1. C. | Oct. 1. C. | Nov. 1. C. | Dic. 1. C. |
| **6.44 P. M.** | 4.45 A. M. | 2.43 A. M. | 12.46 A. M. | **10.40 P. M.** | **8.42 P. M** |

## Tabla 2.

### ÁNGULOS ACIMUTALES DE LA POLAR

| Angulo horal. Hora solar media. | | Acimut para la latitud de | | | | |
|---|---|---|---|---|---|---|
| 1901 | 1906 | 30° | 35° | 40° | 45° | 50° |
| h m | h m | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 0 4 | 0 4 | 0 2 | 0 2 | 0 2 | 0 2 | 0 2 |
| 0 8 | 0 9 | 0 3 | 0 3 | 0 4 | 0 4 | 0 4 |
| 0 13 | 0 13 | 0 5 | 0 5 | 0 5 | 0 6 | 0 6 |
| 0 17 | 0 17 | 0 6 | 0 7 | 0 7 | 0 8 | 0 8 |
| 0 21 | 0 21 | 0 8 | 0 8 | 0 9 | 0 10 | 0 11 |
| 0 25 | 0 26 | 0 9 | 0 10 | 0 11 | 0 12 | 0 13 |
| 0 29 | 0 30 | 0 11 | 0 12 | 0 12 | 0 14 | 0 15 |
| 0 33 | 0 34 | 0 12 | 0 13 | 0 14 | 0 16 | 0 17 |
| 0 38 | 0 38 | 0 14 | 0 15 | 0 16 | 0 18 | 0 19 |
| 0 42 | 0 43 | 0 15 | 0 17 | 0 18 | 0 20 | 0 21 |
| 0 46 | 0 47 | 0 17 | 0 18 | 0 19 | 0 21 | 0 23 |
| 0 50 | 0 51 | 0 19 | 0 20 | 0 21 | 0 23 | 0 25 |
| 0 54 | 0 55 | 0 20 | 0 22 | 0 23 | 0 25 | 0 27 |
| 0 59 | 1 0 | 0 22 | 0 23 | 0 25 | 0 27 | 0 29 |
| 1 3 | 1 4 | 0 23 | 0 25 | 0 26 | 0 29 | 0 32 |
| 1 8 | 1 9 | 0 25 | 0 26 | 0 28 | 0 31 | 0 34 |
| 1 13 | 1 15 | 0 27 | 0 29 | 0 31 | 0 34 | 0 37 |
| 1 19 | 1 20 | 0 29 | 0 31 | 0 33 | 0 36 | 0 39 |
| 1 24 | 1 25 | 0 31 | 0 33 | 0 35 | 0 38 | 0 42 |
| 1 29 | 1 31 | 0 32 | 0 35 | 0 37 | 0 40 | 0 44 |
| 1 34 | 1 37 | 0 34 | 0 37 | 0 39 | 0 43 | 0 47 |
| 1 40 | 1 42 | 0 36 | 0 39 | 0 41 | 0 45 | 0 49 |
| 1 45 | 1 47 | 0 38 | 0 41 | 0 43 | 0 47 | 0 51 |
| 1 50 | 1 53 | 0 39 | 0 42 | 0 45 | 0 49 | 0 54 |
| 1 56 | 1 58 | 0 41 | 0 44 | 0 47 | 0 51 | 0 56 |
| 2 1 | 2 3 | 0 43 | 0 46 | 0 49 | 0 53 | 0 59 |
| 2 6 | 2 9 | 0 45 | 0 48 | 0 51 | 0 55 | 1 1 |
| 2 11 | 2 14 | 0 46 | 0 50 | 0 53 | 0 57 | 1 3 |
| 2 17 | 2 20 | 0 48 | 0 51 | 0 54 | 0 59 | 1 5 |
| 2 22 | 2 25 | 0 50 | 0 53 | 0 56 | 1 1 | 1 8 |
| 2 27 | 2 31 | 0 51 | 0 54 | 0 58 | 1 3 | 1 10 |
| 2 33 | 2 36 | 0 53 | 0 56 | 1 0 | 1 5 | 1 12 |
| 2 38 | 2 42 | 0 54 | 0 58 | 1 2 | 1 7 | 1 14 |
| 2 43 | 2 47 | 0 56 | 0 59 | 1 3 | 1 9 | 1 16 |
| 2 49 | 2 53 | 0 57 | 1 1 | 1 5 | 1 11 | 1 18 |
| 2 54 | 2 59 | 0 59 | 1 3 | 1 7 | 1 13 | 1 20 |
| 3 0 | 3 5 | 1 0 | 1 4 | 1 8 | 1 15 | 1 22 |
| 3 5 | 3 10 | 1 2 | 1 6 | 1 10 | 1 16 | 1 24 |
| 3 11 | 3 16 | 1 3 | 1 7 | 1 12 | 1 18 | 1 26 |
| 3 18 | 3 23 | 1 5 | 1 9 | 1 13 | 1 20 | 1 28 |
| 3 24 | 3 30 | 1 6 | 1 10 | 1 15 | 1 22 | 1 30 |
| 3 31 | 3 37 | 1 8 | 1 12 | 1 17 | 1 24 | 1 32 |
| 3 38 | 3 45 | 1 9 | 1 14 | 1 19 | 1 26 | 1 34 |
| 3 45 | 3 52 | 1 11 | 1 15 | 1 20 | 1 27 | 1 36 |
| 3 53 | 4 1 | 1 12 | 1 17 | 1 22 | 1 29 | 1 38 |
| 4 1 | 4 11 | 1 14 | 1 18 | 1 24 | 1 31 | 1 40 |
| 4 10 | 4 20 | 1 15 | 1 20 | 1 25 | 1 33 | 1 42 |
| 4 20 | 4 33 | 1 17 | 1 22 | 1 27 | 1 35 | 1 44 |
| 4 33 | 4 49 | 1 19 | 1 24 | 1 29 | 1 37 | 1 47 |
| 4 46 | 5 6 | 1 20 | 1 25 | 1 31 | 1 39 | 1 49 |
| 5 1 | 5 31 | 1 22 | 1 27 | 1 33 | 1 41 | 1 51 |

| Angulo horal. Hora solar media. | | Acimut para la latitud de | | | | |
|---|---|---|---|---|---|---|
| 1901 | 1906 | 30° | 35° | 40° | 45° | 50° |
| h m | h m | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 6 27 | 6 27 | 1 21 | 1 26 | 1 32 | 1 39 | 1 49 |
| 7 12 | 6 52 | 1 20 | 1 24 | 1 30 | 1 37 | 1 47 |
| 7 25 | 7 9 | 1 18 | 1 22 | 1 28 | 1 35 | 1 44 |
| 7 38 | 7 25 | 1 16 | 1 20 | 1 26 | 1 33 | 1 42 |
| 7 48 | 7 38 | 1 14 | 1 19 | 1 24 | 1 31 | 1 39 |
| 7 57 | 7 47 | 1 13 | 1 17 | 1 22 | 1 29 | 1 37 |
| 8 5 | 7 57 | 1 11 | 1 15 | 1 20 | 1 27 | 1 35 |
| 8 13 | 8 6 | 1 10 | 1 14 | 1 18 | 1 25 | 1 33 |
| 8 20 | 8 13 | 1 8 | 1 12 | 1 17 | 1 23 | 1 31 |
| 8 27 | 8 21 | 1 7 | 1 11 | 1 15 | 1 21 | 1 29 |
| 8 34 | 8 28 | 1 5 | 1 9 | 1 13 | 1 19 | 1 27 |
| 8 40 | 8 35 | 1 3 | 1 7 | 1 12 | 1 18 | 1 25 |
| 8 47 | 8 42 | 1 2 | 1 6 | 1 10 | 1 16 | 1 23 |
| 8 53 | 8 48 | 1 0 | 1 4 | 1 8 | 1 14 | 1 21 |
| 8 58 | 8 53 | 0 59 | 1 3 | 1 7 | 1 12 | 1 19 |
| 9 4 | 8 59 | 0 58 | 1 1 | 1 5 | 1 10 | 1 17 |
| 9 9 | 9 5 | 0 56 | 1 0 | 1 3 | 1 8 | 1 15 |
| 9 15 | 9 11 | 0 55 | 0 58 | 1 2 | 1 7 | 1 13 |
| 9 20 | 9 16 | 0 53 | 0 56 | 1 0 | 1 5 | 1 11 |
| 9 25 | 9 22 | 0 51 | 0 54 | 0 58 | 1 3 | 1 9 |
| 9 31 | 9 27 | 0 50 | 0 53 | 0 56 | 1 1 | 1 7 |
| 9 36 | 9 33 | 0 49 | 0 52 | 0 55 | 0 59 | 1 5 |
| 9 41 | 9 38 | 0 47 | 0 50 | 0 53 | 0 57 | 1 2 |
| 9 47 | 9 44 | 0 45 | 0 48 | 0 51 | 0 55 | 1 0 |
| 9 52 | 9 49 | 0 44 | 0 46 | 0 49 | 0 53 | 0 58 |
| 9 57 | 9 55 | 0 42 | 0 44 | 0 47 | 0 51 | 0 56 |
| 10 2 | 10 0 | 0 40 | 0 42 | 0 45 | 0 49 | 0 54 |
| 10 8 | 10 5 | 0 39 | 0 41 | 0 43 | 0 47 | 0 51 |
| 10 13 | 10 11 | 0 37 | 0 39 | 0 41 | 0 45 | 0 49 |
| 10 18 | 10 16 | 0 35 | 0 37 | 0 40 | 0 43 | 0 47 |
| 10 24 | 10 21 | 0 33 | 0 35 | 0 38 | 0 41 | 0 44 |
| 10 29 | 10 27 | 0 32 | 0 34 | 0 36 | 0 39 | 0 42 |
| 10 34 | 10 33 | 0 30 | 0 32 | 0 34 | 0 37 | 0 40 |
| 10 39 | 10 38 | 0 28 | 0 29 | 0 31 | 0 34 | 0 37 |
| 10 45 | 10 43 | 0 26 | 0 27 | 0 29 | 0 32 | 0 35 |
| 10 50 | 10 49 | 0 24 | 0 25 | 0 27 | 0 29 | 0 32 |
| 10 55 | 10 54 | 0 23 | 0 24 | 0 25 | 0 27 | 0 30 |
| 10 59 | 10 58 | 0 21 | 0 22 | 0 24 | 0 26 | 0 28 |
| 11 4 | 11 3 | 0 20 | 0 21 | 0 22 | 0 24 | 0 26 |
| 11 8 | 11 7 | 0 18 | 0 19 | 0 20 | 0 22 | 0 25 |
| 11 12 | 11 11 | 0 17 | 0 18 | 0 19 | 0 20 | 0 22 |
| 11 16 | 11 15 | 0 15 | 0 16 | 0 17 | 0 18 | 0 20 |
| 11 20 | 11 20 | 0 14 | 0 14 | 0 15 | 0 16 | 0 18 |
| 11 25 | 11 24 | 0 12 | 0 13 | 0 14 | 0 15 | 0 16 |
| 11 29 | 11 28 | 0 11 | 0 11 | 0 12 | 0 13 | 0 14 |
| 11 33 | 11 32 | 0 9 | 0 9 | 0 10 | 0 11 | 0 12 |
| 11 37 | 11 37 | 0 8 | 0 8 | 0 8 | 0 9 | 0 10 |
| 11 41 | 11 41 | 0 6 | 0 6 | 0 7 | 0 7 | 0 8 |
| 11 45 | 11 45 | 0 5 | 0 5 | 0 5 | 0 6 | 0 6 |
| 11 50 | 11 49 | 0 3 | 0 3 | 0 3 | 0 4 | 0 4 |
| 11 54 | 11 54 | 0 1 | 0 1 | 0 2 | 0 2 | 0 2 |

Cuando la estrella está cerca de su máx elong (áng horales entre 5 y 7 horas) á pequeñas variaciones del acimut corresponden grandes variaciones en el ángulo horal.

## Tabla 3.

### ESTRELLA POLAR. DISTANCIAS POLARES Y ACIMUTES EN LAS ELONGACIONES

| Año. | Dist polar de la Polar. | Log del seno dist pol. | Acimutes en las elongaciones en las latitudes de: 20° | 25° | 30° | 35° | 40° | 45° | 50° |
|---|---|---|---|---|---|---|---|---|---|
| | ° ′ ″ | | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° |
| 1900 | 1 13 33 | 8.33 027 | 1 18.3 | 1 21.1 | 1 24.9 | 1 29.8 | 1 36.1 | 1 44.1 | 1 54.4 |
| 1903 | 1 12 37 | 8.32 472 | 1 17.3 | 1 20.1 | 1 23.8 | 1 28.7 | 1 34.8 | 1 42.7 | 1 53.0 |
| 1906 | 1 11 41 | 8.31 910 | 1 16.3 | 1 19.1 | 1 22.8 | 1 27.6 | 1 33.6 | 1 41.4 | 1 51.5 |
| 1909 | 1 10 45 | 8.31 341 | 1 15.3 | 1 18.1 | 1 21.7 | 1 26.4 | 1 32.3 | 1 40.1 | 1 50.1 |
| 1912 | 1 9 49 | 8.30 765 | 1 14.3 | 1 17.0 | 1 20.6 | 1 25.2 | 1 31.1 | 1 38.7 | 1 48.6 |
| 1915 | 1 8 53 | 8.30 181 | 1 13.3 | 1 16.0 | 1 19.5 | 1 24.1 | 1 29.9 | 1 37.5 | 1 47.2 |
| 1918 | 1 7 58 | 8.29 594 | 1 12.3 | 1 15.0 | 1 18.5 | 1 23.0 | 1 28.7 | 1 36.4 | 1 45.7 |
| 1921 | 1 7 2 | 8.28 999 | 1 11.4 | 1 14.0 | 1 17.4 | 1 21.9 | 1 27.5 | 1 34.8 | 1 44.3 |
| 1924 | 1 6 7 | 8.28 401 | 1 10.4 | 1 13.0 | 1 16.3 | 1 20.7 | 1 26.3 | 1 33.5 | 1 42.9 |
| 1927 | 1 5 12 | 8.27 794 | 1 9.4 | 1 11.9 | 1 15.3 | 1 19.6 | 1 25.1 | 1 32.2 | 1 41.4 |
| 1930 | 1 4 16 | 8.27 169 | 1 8.4 | 1 10.9 | 1 14.2 | 1 18.5 | 1 23.9 | 1 30.9 | 1 40.7 |
| Log cos lat | | | 9.97 299 | 9.95 728 | 9.93 753 | 9.91 337 | 9.88 425 | 9.84 949 | 9.80 800 |

Debido á los cambios de la Polar durante el año, las posiciones dadas en la tabla pueden á veces variar hasta en un minuto. El error es mayor en las altas latitudes.

Teniendo la dist polar norte, $p$, de una estrella, y la latitud, L, del punto de observación, la declinación de la estrella será $=d=90-p$; y el **acimut** $a$ de la estrella, correspondiente á cualquier ángulo horal, $h$, se **puede** encontrar por las fórmulas siguientes:

$$\text{Tang M} = \frac{\cot p}{\cos h} = \frac{\text{tang } d}{\cos h}. \quad \text{Entonces} \quad \text{Tang } a = \frac{\cos \text{M tang } h}{\cos (\text{M} - \text{L})}.$$

Las declins, $d$, de la Polar se publican en las efemérides. En éstas se encuentran las dist polares con más exactitud que en la tabla 3.

**Advertencia.** Cuando se quiere determinar la meridiana con la aproximación de un minuto de arco, deben emplearse varios métodos de observación y comparar sus resultados. Por ej.: obsérvese la Polar tanto al Este como al Oeste y otra estrella al sur del zénit y á iguales alturas.

*Nota.* Si la Polar se observa en uno de los crepúsculos, se puede trabajar sin necesidad de iluminar artificialmente los pelos del anteojo. Para las horas de máx elongs véase la tabla 1.

### Conversión de los arcos en tiempo, y viceversa.

| Arco. Tiempo. | Tiempo. Arco. |
|---|---|
| 1° = 4 minutos. | 24 horas = 360° |
| 1′ = 4 segundos. | 1 hora = 15° |
| 1″ = .0666 segundo. | 1 minuto = 0°15′ |
| | 1 segundo = 0°0′15″ |

# TRÁNSITO DE INGENIERO

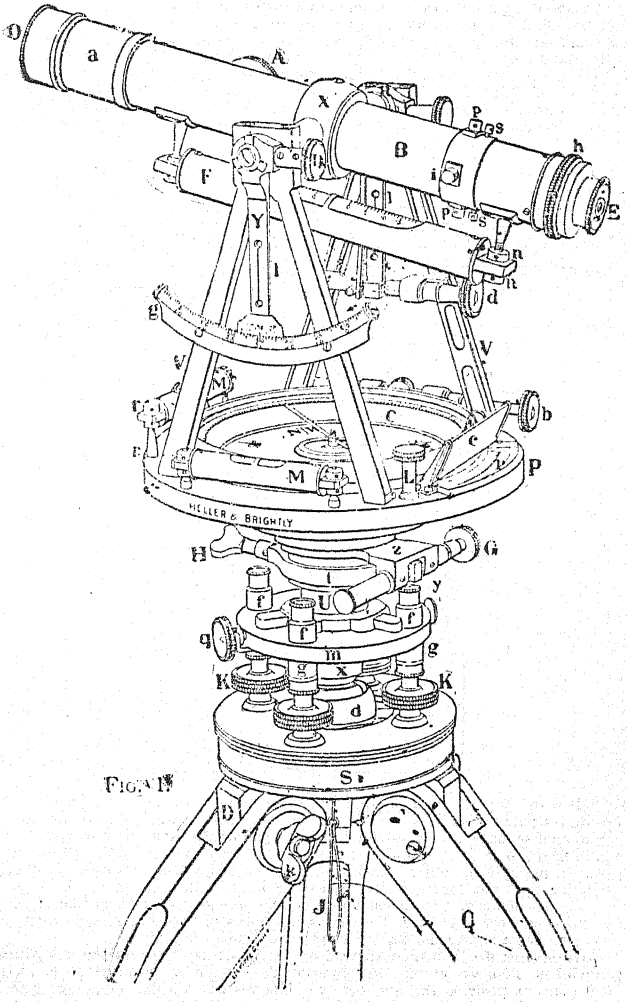



FIGURA 19

Los detalles del tránsito lo mismo que los del nivel, varían según los diferentes fabricantes y el uso á que se destinan. Lo describiremos en su forma moderna como los hacen Heller y Brightly, de Filadelfia. Sin el **nivel largo de burbuja** F, fig. 1, que está debajo del anteojo, y sin el **arco graduado** *g*, la figura representa un **tránsito común**; pero con estos accesorios, ó más bien con un **círculo** graduado en lugar del **arco**, se tiene un **teodolito completo**.

BDD, **fig. 2**, es **la cabeza de la trípode**. Las roscas del tornillo en *v* reciben el tornillo de una tapa de madera que sirve de protección á la cabeza de la trípode cuando ésta no está en uso. SSA es la **placa paralela inferior**. Después de haberse puesto el tránsito muy próximamente en el centro de una estaca, la placa movible *ddcc* nos permite, aflojando ligeramente los **tornillos, de nivelar** K, mover un poco la parte superior del instrumento horizontalmente, y de este modo colocar la plomada exactamente sobre el centro de la estaca con más

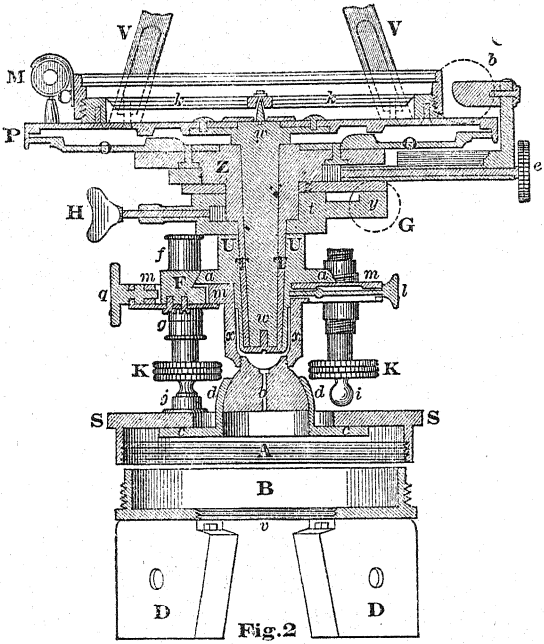


Fig.2

facilidad que por el método antiguo de enterrar un poco más en el suelo una ó dos de las patas ó abrirlas más ó menos. Hecho esto se aprietan los tornillos K, con lo cual se empuja hacia arriba la **placa paralela superior** *mmmxx*, y con ella la media bola *b;* apretando así á *cc* contra la cara inferior de S. El hilo de la plomada pasa por el agujero vertical *b*. Los tornillos niveladores están protegidos del polvo por casquetes *f*, *g*. Las patas *i* de los tornillos, se mueven en cubos ó muñoneras sueltas *j*, que tienen plana la cara que se apoya en la placa S, para que no se gaste con el roce. Las piezas descriptas hasta ahora se dejan generalmente unidas á la trípode. La fig. 1 muestra el modo de unirlas.

**Colocación de la parte superior del instrumento sobre las placas paralelas.** Póngase el extremo inferior de UU en *xx*, sosteniendo el instrumento en una posición tal, que los tres bloques *mm* (de los cuales el que se muestra en F es movible) puedan entrar en las tres escopladuras correspon

dientes *a*, permitiendo de este modo que éstas se apoyen perfectamente en *m*, donde descansa entonces la parte superior del instrumento. (La punta interior del resorte de presión *l*, penetra mientras tanto en una ranura que está alrededor de U, justamente debajo de *a*, é impide que se caiga la parte superior cuando el instrumento se transporta al hombro.) Comuníquesele á dicha parte superior un movimiento horizontal muy pequeño en una ú otra dirección hasta que se detenga al tropezar una pequeña pestaña ó uña que tiene en *a*, contra uno de los bloques F. Las cavidades ó escopladuras *a*, están ahora libres de los bloques. Apriétese á *q* con lo cual se empuja hacia adentro el bloque movible F, que aprisiona la pestaña en bisel *a* entre él y los dos bloques fijos en *mm* y sujeta al eje U á las placas paralelas fijas. Permanece dicha pestaña sujeta de este modo durante el uso del instrumento.

**Para separar la parte superior del instrumento de las placas paralelas.** Aflójese á *q*, llévense las cavidades ó escopladuras *a* frente á los bloques F. Reténgase á *l* y levántese la parte superior del instrumento, que se halla ahora sujeta por la cabeza ancha del tornillo colocado entre el pie del eje *w*.

TT es el **eje exterior giratorio** fundido en una sola pieza con la **placa de soporte** ZZ, á la cual está fijo el **limbo graduado** OO. Este limbo sobresale á la caja de la brújula, permitiendo así graduaciones mayores; *ww* es el **eje giratorio interior.** En su parte superior tiene un ancho reborde al cual está fijo el **nonio** P. A aquél está sujeta **la caja de la brújula** C, los **tubos de burbujas** MM y los **soportes** VV que sostienen el anteojo, etc. Cada tubo de burbuja está sostenido y ajustado por dos tornillos situados uno en cada extremo. La tira doblada que se encorva sobre el tubo protege el vidrio.

**Movimiento del limbo graduado** OO **y del disco del nonio** P. El tornillo tangencial G y un resorte espiral (que no se ve) opuesto á él, están fijos al limbo graduado OO y tienen entre ellos una proyección *y* del collar *t*, que está así ligado al limbo y se mueve con él. El tornillo H pasa por el collar *t* y comprime la pieccita que se ve en su extremo en el interior. Cuando se aprieta H, esta pieccita comprime el eje UU, al cual queda así fijado el limbo graduado. Sólo se le puede dar un pequeño movimiento al limbo por medio del tornillo tangencial G.

El movimiento del nonio P sobre el disco graduado OO se da de un modo análogo por el tornillo tangencial *b* y su resorte espiral (no visible) fijo al disco del nonio P y por el tornillo de presión *e* que pasa por el collar Z y comprime la pieccita que se ve en el interior y en su extremo de modo análogo al H. En los instrumentos de Heller y Brightly, el tornillo *b* está provisto de los medios de recuperar el movimiento perdido, ó en otros términos, llegado al fin de su carrera, puede recomenzarla.

En el instrumento hay dos nonios; uno se ve en *p*, fig. 1. Ambos se deben leer, y tomarse el término medio de su lectura cuando se necesita una gran exactitud. Los reflectores de marfil *c* facilitan su lectura. Antes de mover el instrumento de un lugar á otro, la **aguja de la brújula** *k*, fig. 2, debe siempre apretarse contra la cubierta de vidrio de la caja de la brújula por medio del tornillo vertical de cabeza acordonada que se ve en la placa del nonio, fig. 1, justamente á la derecha del pie del soporte. La punta del estilo está así á cubierto de cualquier daño.

R, en la fig. 1, es un anillo provisto de un gancho ó abrazadera (este último no se ve) para sostener el anteojo en cualquiera posición que se desee. Es mejor dejar que la extremidad *ocular* E del anteojo gire hacia *abajo*, porque de lo contrario la tapa en O puede caerse si está puesta. El tornillo tangencial *d* mueve un brazo vertical fijo en R, y se usa para variar ligeramente la elevación del anteojo. En el brazo tapado en la figura hay una rendija semejante á la que se ve en el brazo del nonio *l*.

Cuando se coloca el 0° del nonio en la división que señala 30° del arco *g* y se sitúa el índice del brazo opuesto sobre una pequeña ranura que tiene la abrazadera horizontal de los soportes (que no se ve en nuestras figuras), las dos rendijas estarán opuestas la una á la otra, y pueden usarse para fijar puntos en ángulo recto á la línea visual, sea como referencias ó con cualquier otro objeto. Un extremo R del eje del anteojo descansa en una caja movible debajo de la cual hay un tornillo. Por medio de este tornillo, la caja puede subirse ó bajarse, y ajustarse al eje del anteojo en caso de que haya cualquier pequeño desarreglo en los apoyos. Con respecto á E, B, O y A (véase Nivel, pág. 318), *a* es un estuche donde se mueve el objetivo.

**Pelos de la Estadia.** Inmediatamente detrás del tornillo *p*, fig. 1, [illegible]

otro tornillo más pequeño. Éste, y otro semejante que está del lado opuesto del anteojo, se mueven en un anillo en el interior de dicho anteojo, llamado retículo. y mantienen este anillo en su posición. A través del anillo están colocados otros dos pelos horizontales, llamados pelos de la Estadia, puestos á una distancia vertical tal, el uno del otro, que pueden abarcar 10 divisiones de una mira graduada colocada á 100 metros (por ej.) de distancia del instrumento, 15 divisiones, á 150, etc. Se usan para medir distancias horizontales é inclinadas.

(*Obs. del T.*— En el sistema métrico es muy práctico separar los pelos de manera que á 100 metros de distancia abarquen un metro en la mira y así en proporción : un decímetro para 10 metros, etc.)

**El tubo largo de burbuja** FF, fig. 1, nos permite emplear el tránsito como nivel, aunque no se presta tan bien como este último para tal uso *.

## Corrección de un tránsito común.

Cuando se compra un nivel ó un tránsito, es una buena precaución (pero que el autor nunca ha visto mencionada) llevar á su lugar el objetivo por medio de su tornillo hasta que esté bien apretado; y hacer entonces una corta raya continua en el anillo del objetivo y sobre la corredera del lente; de tal modo que se pueda ver en cualquier momento, cuando se está trabajando, si el lente se conserva en la misma posición con respecto á su corredera. Porque, si después de terminar todas las correcciones del instrumento, la posición del lente cambia (como es fácil que suceda si se desatornilla y luego no se atornilla de nuevo exactamente como antes lo estaba), entonces pueden ser inútiles todas las correcciones, especialmente si el lente del objetivo es excéntrico, ó si no está hecho con toda exactitud, como resulta muchas veces. El fabricante debe preparar dichas marcas. Al hacer las correcciones, así como al hacer uso de un tránsito ó de un nivel, téngase cuidado de que el ocular y el objetivo estén en una posición tal, *que no haya paralaje.* (*N. del T.* — Es decir, sin que se note un movimiento aparente alrededor de los pelos, cuando se mueve el ojo un poco hacia arriba ó hacia abajo ó lateralmente.) El ocular debe sacarse primeramente lo necesario para ver con mucha claridad los pelos que se cruzan; conseguido esto, no debe moverse sino el objetivo para ver á diferentes distancias.

**Primero. Averiguar si los niveles de aire, MM, son paralelos á la placa del nonio,** y *si en consecuencia, cuando ambas burbujas están en el centro de sus tubos, el eje del instrumento está vertical.* Por medio de los cuatro tornillos de nivelar, K, tráiganse ambas burbujas á los centros de sus tubos respectivos en una posición cualquiera del instrumento, luego désele *media vuelta* á la parte superior del instrumento. Si las burbujas no permanecen en el centro, corríjase la *mitad* del error por medio de los dos tornillos *rr* y la otra mitad por medio de los tornillos de nivelar K. Repítase el ensayo hasta que ambas burbujas se queden en el centro, mientras se le da una vuelta entera al instrumento sobre su eje.

**Segundo. Ver si los parales ó soportes han sufrido algún desarreglo;** es decir, si tienen igual altura y están colocados perpendicularmente á la placa del nonio, como lo están siempre cuando salen de la fábrica.

Póngase el instrumento perfectamente á nivel, luego diríjase la intersección de los pelos á cualquier punto de un objeto elevado (como la punta de una torre) que no esté muy distante; fíjese el instrumento por medio de los tornillos H y *e* y bájese el anteojo hasta que la intersección encuentre algún punto de un objeto bajo (si no hay ninguno, clávese una estaca ó aguja). Luego afílojese á H ó á *e* y désele media vuelta á la parte superior del instrumento; llévese otra vez la intersección sobre el punto alto; fíjese el instrumento y llévese el anteojo al punto bajo.

Si la intersección de los pelos encuentra otra vez al punto bajo, los parales ó soportes estarán bien. Si no, corríjase *la mitad* de la diferencia por medio de los bloques ó cojinetes de ajuste y del tornillo R que está en el extremo del eje de suspensión del anteojo, fig. 1, y repítase el ensayo de nuevo, cambiando el lugar de la estaca ó de la aguja en cada ensayo. Si el eje del instrumento carece de bloque de ajuste, devuélvase al fabricante para que corrija cualquier desarreglo que tengan los soportes.

* *N. del T.* — Esta minuciosa descripción no sirve sin duda (como lo dijo el autor para tránsitos de otros fabricantes, porque de unos á otros hay notables diferencias de estructura; sin embargo hemos dejado ésta, como un ejemplo que da á conocer esta clase de instrumentos.

El tránsito puede usarse *para trazar líneas rectas*, aun cuando los soportes estuviesen ligeramente doblados, por el procedimiento descrito al fin de la cuarta corrección.

**Tercero. Observar si los pelos transversales permanecen verdaderamente verticales y horizontales** cuando el instrumento está á nivel. Cuando el anteojo da imágenes invertidas, los pelos transversales están más cerca del ocular que cuando da las imágenes en su posición real. El fabricante tiene cuidado de colocar los pelos en ángulo recto en el anillo ó diafragma llamado retículo; y generalmente coloca el anillo en el anteojo, de manera que estando éste á nivel, aquéllos estén vertical uno y horizontal el otro. Algunas veces, sin embargo, se descuidan en este particular, ó el retículo puede girar un poco por cualquier accidente. Para cerciorarse de que uno de los pelos está vertical (en cuyo caso el otro debe estar, por construcción, horizontal), después de haber ajustado los niveles de aire póngase el instrumento cuidadosamente á nivel y diríjase una visual con el anteojo á una plomada ú otra línea vertical. Si el pelo vertical coincide con este objeto, estará bien, pero si no coincide, entonces aflójense *ligeramente* dos tornillos adyacentes *solamente*, de los cuatro *ppii*, fig. 1, y con una navaja, llave ó cualquier pequeño instrumento, golpéese suavemente contra las cabezas de los tornillos para hacer girar un poco el retículo entre el anteojo, hasta que el pelo esté vertical. Cuando se haya hecho esto, apriétense los tornillos.

Cuando no se tenga una plomada, ó regla vertical, diríjase el pelo á un punto claro muy pequeño, y véase si el pelo continúa sobre el mismo punto, bajando ó alzando el anteojo alrededor de su eje transversal.

El modo de hacer lo dicho se comprenderá prontamente con esta figura que representa una sección transversal de la parte superior del anteojo con los pelos transversales; *a* es el anillo de los pelos ,ó retículo; *v* el pelo vertical; *g* el tubo

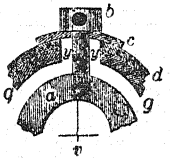


Fig. 3.

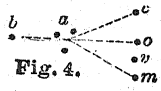


Fig. 4.

del anteojo; *d* el anillo exterior del tubo del anteojo; *b* es la cabeza de uno de los cuatro tornillos que mantienen en su puesto al retículo *a* y que sirven también para ajustarlo. Las puntas interiores de estos tornillos obran en el espesor del retículo, de modo que cuando se los afloja un poco siempre sostienen el anillo. Debajo de la cabeza *b* de los tornillos hay unas arandelas pequeñas sueltas *c*. Se deja un espacio *yy* alrededor de cada tornillo, en donde atraviesan el tubo del anteojo, para permitir que se muevan un poco, transversalmente, los tornillos y el anillo, cuando se aflojan ligeramente aquéllos.

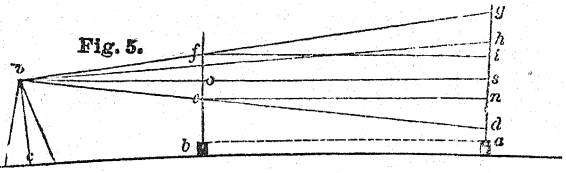
**Cuarto. Ver si el pelo vertical está en el eje de colimación.** Colóquese la trípode firmemente en el suelo como en *a*, fig. 4. Póngase el instrumento á nivel, sujétese y diríjase el pelo vertical por medio de los tornillos tangenciales G (figs. 1 y 2) hacia un objeto conveniente *b*, y si no existe ninguno, clávese una estaca delgada ó una aguja. Luego, haciendo girar el anteojo verticalmente sobre su eje, obsérvese un objeto cualquiera *c*. que encuentre el pelo vertical, y si no lo hubiere, colóquese una segunda aguja. Aflójese el instrumento por medio del tornillo H, y hágase girar toda su parte superior hasta que el pelo vertical encuentre otra vez á *b*. Fíjese el instrumento nuevamente y hágase girar otra vez el anteojo verticalmente sobre su eje. Si el pelo vertical encuentra ahora á *c*, como antes, es prueba de que *c* está verdaderamente en *o*, y de que *b*, *a*, *c* **están en la misma línea recta** y por consiguiente en este sentido está bien el instrumento. En el caso contrario, obsérvese donde encuentra por ejemplo á *m* (tomándose la distancia *am* igual á *ac*), y colóquese allí también una aguja. Mídase *mc*, y colóquese una aguja en *v*, en la línea *mc*, haciendo $mv = \frac{1}{4} mc$. Colóquese también una aguja en *o* entre *m* y *c*, en dirección de *ab*. Por medio de los dos tornillos horizontales que mueven el retículo ajústese el pelo vertical

hasta que encuentre á *v*. Repítase de nuevo *toda* la operación, y prosígase hasta que el anteojo después de haberse dirigido hacia *b* encuentre al mismo objeto *o* cuando el anteojo se haga girar sobre su eje. Obsérvese si el movimiento del retículo ha cambiado la verticalidad del pelo en esta cuarta corrección, y si así fuere, repítase la corrección tercera. Después repítase otra vez la cuarta si fuere necesario, y sígase haciendo así hasta que ambas correcciones estén perfectas al mismo tiempo. De este modo puede trazarse una línea recta aun cuando los pelos no estuviesen corregidos; pero con mucho más trabajo. Porque en cada estación como *a*, habrá que dirigir dos visuales, *ac*, *am*, y tomar el medio entre *c* y *m* que estará en línea recta con *ab*. Entonces se puede trasladar el instrumento á *o* y así continuar.

Los *ángulos* medidos por el tránsito, sean horizontales ó verticales, no resultarán afectados por la no corrección de los pelos con tal que haya un pelo vertical, ó que usemos la *intersección* de los pelos al medirlos.

**Las correcciones que anteceden son las únicas que se requieren**, á menos que el instrumento se necesite para nivelar; porque en este caso debe atenderse á lo que sigue.

**Para corregir el tubo largo del nivel de aire** FF, fig. 1, primeramente colocaremos la visual del anteojo horizontal, y luego haremos que el tubo del nivel esté también horizontal, de manera que los dos estén paralelos. Al efecto




clávense dos estacas *a*, *b*, fig. 5, con sus cabezas exactamente á nivel y por lo menos á una distancia como de cien metros una de otra.

Colóquese el instrumento firmemente en la dirección de dichas estacas, como en *c*, haciendo á *bc*, una parte alícuota de *ab*, tan corta como lo permita el anteojo. No es necesario que el instrumento esté á nivel, sino que tal como lo instalemos dirijamos una visual que supondremos pasar por *e* y *d*. Tómense las lecturas *b* y *ad*. Su diferencia es : $be - ad = an - ad = dn$; y $ab : ac :: dn : ds$; siendo *s* la altura de la tablilla ó corredera de la mira en *a*, cuando las lecturas (*as*, *bo*) hechas sobre las dos estacas son iguales. $as = ad + ds = ad + \frac{dn \times ac}{ab}$.

Si la lectura de la mira en *a* *excede* á la de la mira en *b* (como cuando la visual es *vfg*), la diferencia de lectura será $= ag - bf = ag - ai = gi$; y $as = ag - gs = ag - \frac{gi \times ac}{ab}$.

Conociendo á $as = ob$ diríjase la visual á *s*, la cual quedará situada horizontalmente, y llévese la burbuja al centro del tubo por medio de las dos tuercas pequeñas *nn* situadas en un extremo del tubo, fig. 1, y tendremos el anteojo y el tubo paralelos *.

Los ceros del círculo vertical y de su nonio, pueden ahora arreglarse, si fuere necesario, aflojando los tornillos del nonio y moviéndolos luego hasta que coincidan los dos ceros.

**Nota.** Si no hay á la mano un nivel para nivelar las dos estacas, puede hacerse

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* Aquí no se toma en cuenta el pequeñísimo error debido á la curvatura de la tierra porque la línea *horizontal* en *v* es *vh*, y representa la tangente á la superficie curva (ó «nivel») del agua en equilibrio en *v*; mientras que *vs* representa la tangente á la superficie del agua en un punto intermedio entre *a* y *b*. Por consiguiente si el anteojo en *v* se dirige á *s* no será paralelo el eje del nivel. Tomando en cuenta esto y la refracción del aire *que disminuye* el error, súbase la corredera ó tablilla á un punto *h* más arriba de *s*, de modo que *hs* será $= .0000000205 \times$ cuadrado de *ac* en pies, pero cuando *ac* sea igual á 650 pies, *hs* será solamente como un décimo de pulgada, y apenas cubrirá el espesor aparente del pelo transversal del anteojo.

** *N. del T.* —Para usar esta fórmula en el sistema métrico : *hs* en m $= .0000000672 \times ac$ elevado al cuadrado (en metros). Para una distancia *ac* de 200 metros, *hs* será solamente de 2.7 mm.

con el mismo tránsito de la manera siguiente : Póngase cuidadosamente á nivel los dos niveles pequeños de aire, por medio de los tornillos de nivelar K. Clávese una estaca *m* á una distancia de 30 á 100 metros del instrumento *o*. Colocando

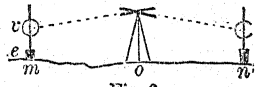
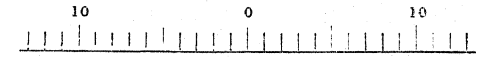



**Fig. 6.**

entonces en *m* una mira de corredera ó tablilla, fíjese ésta á cualquiera altura *ev*, en que la intersecte el pelo horizontal; no importa que el anteojo esté á nivel ó no, aunque sería mejor ponerlo lo más horizontal posible á la vista. Fíjese el anteojo por medio del anillo R, fig. 1. Hágase girar el instrumento alrededor de su eje vertical un espacio considerable, como, por ejemplo, media vuelta ó cerca de media vuelta; colóquese otra estaca *n*, *á la misma distancia precisamente que está m* del instrumento y sígase clavando la estaca, hasta que el pelo horizontal intersecte la tablilla de la mira puesta sobre ella y fija todavía á dicha mira á misma altura que tenía en *m*. Hecho esto, las cabezas de las estacas estarán á nivel, y en disposición de usarlas como anteriormente. Cuando se trata de usar un tránsito para deslindar ó levantar planos de haciendas, etc., ó para trazar de nuevo las líneas de una mensura antigua, es muy útil arreglar la brújula de modo que se puedan apreciar las variaciones de declinación del meridiano magnético durante el tiempo transcurrido entre los dos levantamientos. Para este objeto, se agrega **un nonio de declinación** á dicho tránsito, y también á la brújula Cuando la graduación del tránsito está numerada de tal manera que se puede leer á ambos lados del cero, como se ve en la figura, se hace el nonio doble




también; es decir, que se gradúa y numera de modo, que pueda leerse del cero á derecha é izquierda de éste. En este caso, si se mide el ángulo del cero á la derecha, la lectura debe hacerse en la parte del nonio que está á la derecha del cero, y viceversa. Si la numeración es sencilla ó en una dirección solamente, de cero á 360°, entonces no se necesita sino un nonio sencillo, porque los ángulos se miden solamente en la dirección de la numeración. Los ingenieros tienen diferentes opiniones sobre la preferencia que se debe dar á estos modos de numerar las graduaciones. El autor prefiere la numeración de cero á 180° en ambos sentidos con dos nonios dobles.

**Reposición de los pelos en un nivel ó tránsito.** Sáquese el tubo de la extremidad ocular del anteojo. Obsérvese y anótese cuál lado del retículo está hacia el extremo ocular. Luego aflójense los cuatro tornillos que sostienen el diafragma ó retículo de modo que puedan sacarse del anteojo. Fíjense nuevos pelos con cera, barniz, cola ó goma, etc. Esta operación requiere mucho cuidado. Para volver el diafragma á su lugar, colóquese firmemente, en uno de los agujeros de los tornillos que están en la circunferencia del mismo diafragma, la punta de una astillita de madera, de suficiente largo para que llegue fácilmente hasta dentro del anteojo al lugar que ocupa el diafragma. Por medio de esta astilla á manera de mango, colóquese el diafragma de canto en su puesto en el anteojo y sosténgase así hasta que los dos tornillos *opuestos* se hallen colocados en su lugar y estén atornillados. Sáquese entonces la punta ó astilla del agujero del diafragma y con el mismo hágase girar éste hasta que la misma cara, que estaba antes hacia el ocular, vuelva á estar de la misma manera; y después colóquense los otros dos tornillos.

Los así llamados pelos cruzados son de tela de araña, tan finos que apenas son visibles á la simple vista. Los señores Heller y Brightly usan hilos de platino muy finos, que son mucho mejores. El cabello es demasiado grueso.

**Reposición del tubo de vidrio de un nivel de aire.** Quítese el nivel del instrumento; sáquense sus extremos de correderas; empújese hacia fuera el tubo de vidrio roto y la substancia con que está adherido, colóquese el tubo nuevo con el lado que corresponde hacia arriba (que siempre está marcado con una lima por el fabricante), envolviendo sus extremos con papel, si quedan flojos. Finalmente, póngase un poco de pasta ó cera derretida en las puntas del tubo de vidrio para evitar que se mueva en el tubo del nivel.

Al comprar instrumentos, especialmente cuando han de usarse lejos de la fábrica, es conveniente proveerse de repuestos de las piezas que puedan romperse ó perderse fácilmente, tales como las tapas de vidrio de la brújula, agujas, llaves para apretar los tornillos, tubos de burbujas, lentes de aumento, etc.

Las correcciones de un **teodolito** se hacen lo mismo que las del nivel y de tránsito :

1.° La de los pelos que se cruzan; lo mismo que en el nivel.

2.° La del nivel del anteojo; también como en el nivel.

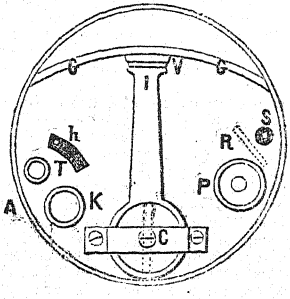
3.° La de los dos niveles pequeños; lo mismo que en el tránsito.

4.° El nonio del círculo vertical; lo mismo que en el tránsito de círculo vertical.

5.° Para ver si el pelo vertical se mueve verticalmente lo mismo que en la cuarta corrección del tránsito. Algunos teodolitos no están provistos de medios para hacer esta corrección; pero á los grandes se les provee de tornillos debajo de los pies de los apoyos. Algunas veces se agrega un segundo anteojo, bajo el limbo horizontal, con sus abrazaderas y tornillo tangencial propios. Se emplea para averiguar si el cero del limbo graduado se ha movido durante la medida de los ángulos horizontales. Cuando antes de principiar la medida el cero y el anteojo superior están dirigidos hacia el primer punto, diríjase el anteojo inferior hacia cualquier objeto pequeño distante y fíjesele. Durante las medidas subsiguientes, obsérvese de vez en cuando por él, para asegurarse de si todavía intersecta el mismo objeto, probando de este modo que no ha ocurrido movimiento alguno.

# SEXTANTE DE CAJA O DE BOLSILLO

La facilidad con que se lleva un sextante de bolsillo, y el hecho de que se aprecien en él hasta minutos, lo hace algunas veces muy útil al ingeniero. Con él podemos medir ángulos desde un bote ó á caballo; y en muchos casos donde no es posible el empleo de un tránsito. Por medio de un horizonte artificial es muy



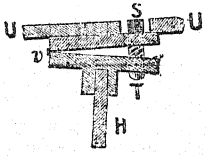

fácil obtener latitudes. Cerrado, parece una caja cilíndrica de cobre de tres pulgadas de diámetro (8 cm) más ó menos y de 1½ pulg (4 cm) de alto. La caja se compone de dos partes que desatornillándolas é invirtiendo una de ellas y atornillándola otra vez, la parte inferior sirve de mango para agarrar el instrumento. Mirándola de arriba hacia abajo después de arreglada así, observamos, como en la figura, un brazo movible IC, llamado **índice**, que gira alrededor de un centro C, y tiene un nonio V en el otro extremo; GG es un arco graduado ó **limbo**. El arco tiene una amplitud de 73°; pero está dividido en 146°. Su cero está en una extremidad. No se ven las graduaciones en la figura.

Adherida al índice hay una pequeña lente movible (que no se ve en la figura), la cual gira también alrededor de C, para leer las graduaciones pequeñas del limbo. Para medir un ángulo, se mueve el índice dándole vueltas á la cabeza acordonada del piñón P, que actúa en una cremallera colocada en el interior de la caja. El ojo

se aplica á una abertura circular que está á un lado de la caja cerca de A. Un pequeño anteojo, de 8 cm más ó menos de largo, acompaña al instrumento, pero generalmente no es necesario. Cuando se usa sin el anteojo, el hueco destinado al ojo en A, debe cerrarse parcialmente por medio de una corredera, que tiene un agujero muy pequeño para aplicar el ojo y que se mueve por medio de una clavija *h* en una ranura curva. Otra corredera ó anillo movible, que se halla á un lado de la caja, está provisto de un vidrio obscuro para cuando se observa el sol. Cuando se usa el anteojo se fija por medio del tornillo de cabeza acordonada T. La parte superior que se muestra en nuestra figura, puede separarse de la parte cilíndrica, sacando 3 ó 4 tornillitos que tiene en el borde; y entonces puede examinarse el interior y limpiarse si se desea. Como los sextantes náuticos y otros, éste tiene dos vidrios principales, ambos azogados ó reflectores. El uno, el **espejo central**, está fijo á la parte inferior del índice en C cuyo borde superior está representado por las líneas de puntos.

El otro, llamado **espejo de horizonte** (porque cuando se miden ángulos verticales de cuerpos celestes se dirige hacia el horizonte), se halla también en el interior de la caja, y la posición de su borde superior está indicada por la línea de puntos en R. El espejo del horizonte está azogado solamente en su mitad superior; de manera que uno de los objetos observados puede verse directamente á través de su parte inferior, mientras que la imágen del otro objeto se ve en la parte superior, reflejada por el espejo central ó espejo del índice. Para que el instrumento esté correcto y en disposición de usarlo, estos dos espejos deben ser perpendiculares al plano del instrumento; esto es, á la cara á que están fijos y paralelos entre sí cuando el cero del limbo coincide con el cero del nonio.

El espejo central ha sido fijado de antemano y de modo permanente por el fabricante y no requiere ninguna otra corrección; pero al horizontal hay que hacerle dos correcciones, por medio de una llave semejante á una llave de reloj, de cabeza cordonada. Está atornillada en la tapa de la caja, para tenerla siempre á la mano. Cuando es necesario, se desatornilla. Esta llave ajusta en dos cabezas cuadradas pequeñas (como algunas llaves de relojes), una de las cuales se ve en S, mientras que la otra está cerca de S, pero al lado de la caja. Estos cuadrados son las cabezas de dos tornillos pequeños. Si el espejo del horizonte H no estuviese como en el



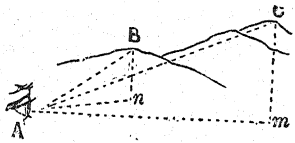

diseño (donde se ve de canto), en ángulo recto á la tapa UU de la caja, se puede arreglar dándole vuelta á la cabeza cuadrada S del tornillo ST; y si después de estar rectificado así no está aún paralelo al espejo central, cuando coinciden los ceros, entonces se le mueve un poco hacia adelante ó hacia atrás por medio de la cabeza cuadrada del tornillo del lado.

**Para corregir un sextante de bolsillo**; háganse coincidir exactamente los dos ceros; luego mírese por el agujero destinado al ojo y por la parte baja ó sin azogue del espejo del horizonte, un objeto distante. Si el instrumento está arreglado, el objeto visto así directamente coincidirá exactamente con su imagen reflejada vista al mismo tiempo y en el mismo lugar. Pero si no lo está, los dos objetos aparecerán separados horizontal ó verticalmente, ó de ambos modos, así: ⁂; en cuyo caso aplíquese la llave K á la cabeza cuadrada S; y haciéndola girar suavemente en cualquier dirección que sea necesario (*viéndose siempre el objeto y su imagen*) llévense á ambos á una posición horizontal, ó al mismo nivel, así: **. Luego aplíquese la llave á la cabeza cuadrada del lado de la caja y, dándole vueltas suavemente, háganse coincidir las dos imágenes perfectamente. Entonces el instrumento quedará corregido.

En algunos instrumentos, el espejo horizontal tiene una *visagra* en *v*, que le permite cierto juego durante el ajuste con el solo tornillo ST; pero otros no tienen esta visagra, sino *dos* tornillos semejantes á S en la parte superior de la caja además del que tienen al lado.

Si se emplea el sextante para medir ángulos verticales por medio de un **hori-**

**zonte artificial**, la altura verdadera no es sino la mitad de la que se lee en el limbo; porque entonces leemos al mismo tiempo el ángulo real y el ángulo reflejado. El gran inconveniente que presenta el sextante para los trabajos de ingeniería, es que no mide los ángulos horizontalmente, como lo hace el tránsito, á menos que el observador y los dos objetos se hallen en el mismo plano horizontal. De este modo, situado un observador con un sextante en A, para medir el ángulo




que forma este punto con la cima de los cerros B y C, deben mantener el plano graduado del sextante en el plano ABC; y realmente medir el ángulo BAC, en tanto que lo que necesita es el ángulo horizontal *nAm* proyección del BAC. Este ángulo es mayor que el BAC, porque las dist *An* y *Am* son más cortas que AB y AC. El tránsito nos da el ángulo horizontal *nAm*, porque el círculo graduado se pone de antemano horizontal con los tornillos de nivelar; y la medida del ángulo no se afecta porque se dirija la visual hacia arriba, sobre B y C. Para más detalles sobre esta materia, véase la nota del ejemplo 2, caso 4 de la « Trigonometría ».

**El sextante náutico**, usado á bordo, está construído sobre el mismo principio del sextante de bolsillo; y sus correcciones son muy semejantes. En él el espejo central ó espejo del índice está también fijo, una vez por todas, por el fabricante; y el espejo horizontal tiene las dos correcciones del sextante de bolsillo. También tiene vidrios obscuros para mirar el sol y un pequeño agujero para dirigir visuales cuando no se emplea el anteojo.

# LA BRÚJULA

### Corrección de una brújula.

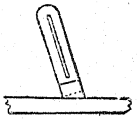
**La primera corrección** es la de los niveles. Colóquese firmemente el instrumento en el suelo y póngasele horizontal en cualquier posición, es decir, llévense las burbujas á los centros de sus tubos respectivos. Luego, désele media vuelta al instrumento. Si las burbujas permanecen en el centro, están correctas, pero si no, corríjase la *mitad* de la dif de cada burbuja por medio de los tornillos de los tubos. Póngase el instrumento otra vez horizontal, désele media vuelta; y si las burbujas no permanecen aún en el centro, hay que mover otra vez un poco los tornillos hasta rectificar la mitad de la dif que haya todavía. Generalmente tienen que hacerse varios tanteos de esta manera, hasta que las burbujas se mantengan en el centro, al darle una vuelta entera á la brújula.

**Segunda corrección.** Póngase horizontal la brújula y véase luego si la aguja está horizontal, y si no lo está, póngasele horizontal por medio de un pedacito de alambre fino que se arrolla alrededor de ella, y se desliza hacia su punta elevada. Una aguja arreglada de este modo, es decir, horizontal en un lugar, no se mantiene así si se la transporta hacia el norte ó hacia el sur de dicho lugar. Si se la lleva hacia el norte, la punta norte bajará; si hacia el sur, descenderá la punta sur. El alambre se usa para contrarresta reste movimiento.

**Tercera corrección.** Las pínulas por donde se dirigen las visuales se colocan en ángulo recto á la plancha de la brújula, de manera que cuando ésta está horizontal las pínulas están verticales. El fabricante siempre las fija correctamente. Para cerciorarse si esto es así, cuélguese una plomada, y habiendo nivelado la brújula, diríjasele una visual y obsérvese si las pínulas coinciden con el hilo de la plomada. Si una ó ambas hendiduras no coinciden, como se observa en este diseño exagerado, deben desatornillarse de la brújula y gastar un poco el pie por medio de una línea ó piedra de amolar como lo indica la línea de puntos. Como medio

provisional, puede ponerse una cuña pequeña debajo de la punta más baja, para levantarla.

**Cuarta corrección.** Para enderezar la aguja, si se hubiere doblado. Estando la brújula á nivel y la aguja horizontal floja en el estilo, obsérvese si los dos extremos continúan marcando graduaciones exactamente opuestas (es decir, gra-

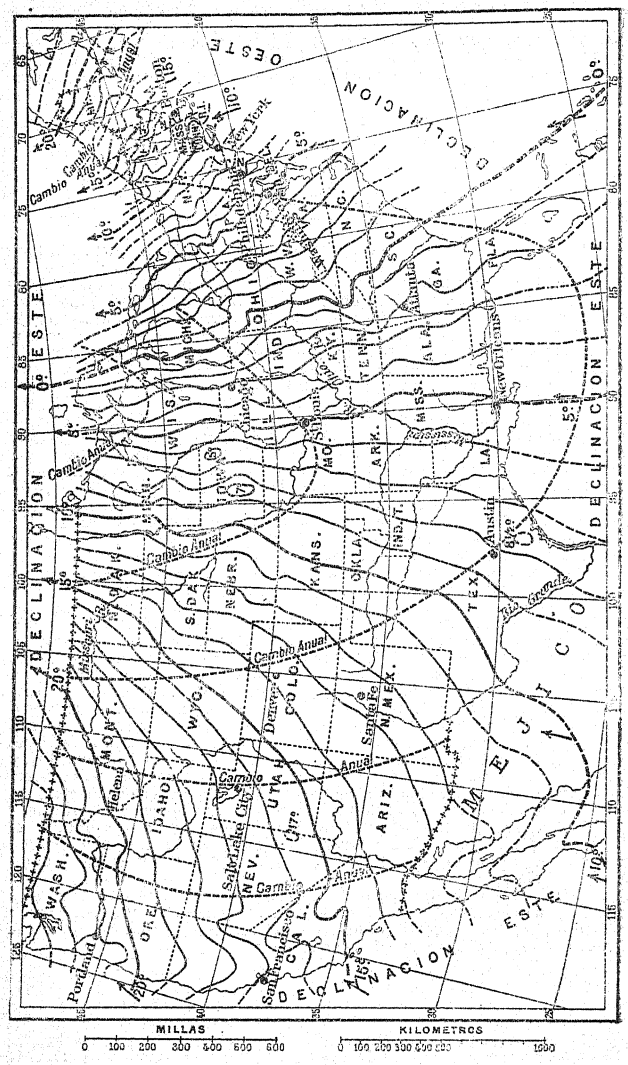


duaciones con 180° de diferencia cuando se le ha dado una vuelta entera á la brújula). Si la aguja lo hace así, estará derecha, y el estilo en que se apoya estará en el centro del círculo graduado; pero si no, uno de los dos, ó ambos, necesitan corrección. Póngase primeramente el instrumento horizontal, luego hágasele girar hasta que una graduación cualquiera (por ejemplo 90°) coincida exactamente con la punta norte de la aguja; si la punta sur no señala ahora exactamente la graduación opuesta á 90°, levántese la aguja, y dóblese *la punta del estilo* hasta que aquélla se dirija al punto deseado, teniendo presente que, cada vez que se doble la punta, se debe hacer girar un poco el instrumento para conservar siempre la punta *norte* en los 90°. Entonces désele media vuelta á la brújula hasta que los 90° *opuestos* lleguen exactamente á la punta *norte* de la aguja. Hágase una marca fina de lápiz en el lugar donde está la punta sur de la aguja actualmente. Luego, levántese ésta, y dóblese hasta que la punta sur señale exactamente la *mitad* de la distancia comprendida entre los 90° y la marca de lápiz, cuando su punta norte se mantiene en los 90° moviendo la brújula un poquito (el grueso de un pelo). Entonces la aguja estará derecha; y no debe alterarse al hacer la corrección que sigue, aunque todavía no marque con toda exactitud los grados opuestos.

**Quinta corrección.** *Corrección del estilo.* Después de estar cierto de que la aguja está derecha, hágase girar la brújula hasta que se llegue á un punto en que los extremos de la aguja marquen divisiones diametralmente opuestas; luego hágase girar la brújula 90°. Si la aguja aún señala divisiones opuestas, la punta del estilo estará bien; pero si la aguja no señala dichas divisiones, dóblese la *punta del estilo* hasta lograrlo. Repítase la operación hasta que la aguja marque siempre grados diametralmente opuestos cuando se le dé una vuelta entera á la brújula.

Se necesita cuidado y esmero en las observaciones para hacer las correcciones con exactitud; porque todo error es generalmente una cantidad mínima, y el novicio es propenso á hacer más penosa la corrección por no saber usar el lente ó microscopio al observar la punta de la aguja y las graduaciones correspondientes. Debe tenerse el microscopio siempre con *su centro directamente sobre* el punto que se examina, y paralelo al círculo graduado. De lo contrario se cometerán errores perjudiciales hasta de algunos minutos en una sola observación; y la acumulación de dos ó tres de estos errores, producidos por una causa desconocida para él, pueden hacerlo abandonar las correcciones desesperadamente. Esta observación se aplica también á la lectura de ángulos tomados con el tránsito, etc., aunque los errores entonces no resultarán tan grandes como con la brújula. Al comprar un microscopio para una brújula, obsérvese que ninguna de las piezas, como visagras, remaches, sean de hierro, porque éstos influyen sobre la dirección de la aguja.

Si las pínulas de la brújula no están dispuestas por el fabricante de manera que estén en una misma línea con los ceros opuestos, el ingeniero no puede corregir este defecto. Para conocerlo se puede pasar un hilo fino por las hendeduras y observar si éste pasa naturalmente sobre los ceros.


DECLINACION OESTE
DECLINACION ESTE
Cambio Anual
Cambio Anual
Cambio Anual
Cambio Anual
Cambio Anual
Cambio Anual
Cambio Anual
Cambio Anual
0°
5°
10°
15°
20°
25°
30°
WASH.
OREG.
CALIF.
NEV.
IDAHO
MONT.
WYO.
UTAH
ARIZ.
COLO.
N. MEX.
DAK.
NEBR.
KANS.
OKLA.
TEX.
MINN.
IOWA
MO.
ARK.
LA.
WIS.
ILL.
MICH.
IND.
TENN.
MISS.
ALA.
GA.
FLA.
S. C.
N. C.
OHIO
KY.
W. Va.
Va.
Pa.
N. Y.
Portland
San Francisco
Salt Lake City
Denver
Santa Fe
Austin
Nueva Orleans
Washington
Filadelfia
Nueva York
Boston
MILLAS
0 100 200 300 400 500 600
KILOMETROS
0 100 200 300 400 500 1000

(*Advertencia del Traductor.*— Aunque este mapa, y lo que se dice en seguida sobre la Declinación Magnética y sus variaciones, se refieren exclusivamente á los Estados Unidos, pueden servir algunas veces de enseñanza á los agrimensores ó ingenieros, y siempre de advertencia, para que indaguen y recojan en cada localidad donde hagan un levantamiento Topográfico ó Geodésico, los datos análogos, tan indispensables para lograr un plano ó carta que merezca fe. Además, la circunstancia de que éste, como otros muchos capítulos de grandísima utilidad, no están tratados en los otros formularios ó lo están rara vez y muy someramente, nos ha inducido á dejarlo.)

## DECLINACIONES Y VARIACIONES MAGNÉTICAS

Generalmente la aguja magnética no marca exactamente el meridiano astronómico; es decir, no está sobre la Norte-Sur. El ángulo que ella forma con esta línea se llama **declinación magnética.** La declinación magnética es distinta (á una hora dada) en los diversos lugares, y en un mismo lugar varía con el tiempo. Estos cambios se llaman **variación magnética.**

### Variaciones según los lugares.

Además de las perturbaciones locales, como las debidas á la influencia del hierro en el suelo, ó en las minas, el cascajo ferruginoso, las piedras volcánicas, etc., la declinación varía, de lugar á lugar, de acuerdo con las leyes generales del magnetismo terrestre. Así en los E. U. (véase al frente la carta * de líneas isogónicas) existe una línea donde no hay declinación, marcada 0°, que actualmente empieza en Michigan y termina en la Carolina del Sur. En puntos situados al Este de esta línea, la declinación es hacia el Oeste y viceversa; el valor de la declinación aumenta con la dist del punto á la línea; actualmente llega á 21° al Oeste en Maine, y 23° Este en el Estado de Wáshington. Las líneas que unen los puntos donde la declinación es la misma se llaman **líneas isogónicas.** Los lugares donde no se ha podido determinar con exactitud esta línea están indicados con puntos suspensivos.

### Variaciones con el tiempo.

**Variaciones seculares†.** Actualmente, en cualquier punto de los E. U. la declinación va aumentando hacia el Oeste (las declinaciones al Oeste aumentan y las al Este decrecen) á razón de 1 á 5 minutos por año. Ésta se llama **variacion secular,** y su valor en un año es la **variación anual.** La carta señala con líneas en contorno y de puntos los lugares de igual **variación anual,** de 1′ á 5′, y éstas generalmente cortan á las líneas isogónicas casi en ángulo recto. Más allá de los límites de la carta, á ambos lados, se encuentran las líneas donde no hay variación anual. A lo largo de esta línea, la declin oeste (en el Atlántico) y la declin este (en el Pacífico) han alcanzado su máximum y están retrogradando hacia el Este y el Oeste respectivamente. La variación secular imprime á la línea de declinación 0 (cero) y á la línea donde la variación anual es nula (ó sea de máxima declinación) un movimiento hacia el Oeste de unos 3 ó 4 minutos de longitud por año.

**Variación diurna.** En cualquier punto dado la aguja oscila, en un día alrededor de su posición media en un arco que llega á veces hasta 12′ ó 15′, alcanzando su distancia máxima al Este, de 8 á 8-30 A.M.; y la del Oeste de 1 h 15 m á 1 h 30 m P.M.; á la posición media, ó la normal diurna, llega generalmente de 10 h á 11 h 30 m A.M. y de 7 h á 8 h P.M.

Su máximum hacia el Este lo alcanza con media hora de anticipación en verano y media hora más tarde en invierno, y la del Oeste como $^1/_4$ de hora más temprano en verano y $^1/_4$ más tarde de invierno que en la época media.

El arco descrito entre las dos posiciones extremas Este y Oeste, ó sea de la mañana á la tarde, es la **variación diurna.** En Filadelfia, ésta varía de 5′ á mitad del invierno á 12′ en mitad del verano; en Key-West, de 3′ á 8′; en los Angeles de 4′ á 9′. En los años de menor actividad en las manchas solares, la variación es un 20% menor y de 30 % mayor en épocas de mucha actividad; todo esto con respecto al promedio. Pero la variación diurna es muy irregular, especialmente en las estaciones de mínimos.

**Otras variaciones.** La aguja también está sometida á pequeñas variaciones mensuales y anuales y á la influencia de las tempestades eléctricas, las que causan perturbaciones de 15′ y más en la posición normal de la aguja.

**Electricidad.** La electricidad, ya sea atmosférica ó producida por fricción en el cristal de la caja de la brújula, produce perturbaciones, las que se evitan tocando el vidrio con la lengua ó el dedo húmedos.

* Esta carta es la reducción de un gran mapa para 1902, publicado por la Oficina de estudios Geodésicos y de las costas de los E. U., basado en 4.700 observaciones, hechas y escogidas por los mejores observadores. En las regiones en que no se pudo determinar satisfactoriamente la posición de las líneas por aquellos medios, se usaron gran cantidad de datos compilados por H. Gannett.

† Secularó que comprende un dilatado período de tiempo.

### DESMAGNETIZACIÓN

Si la aguja está hecha de un metal blando, pierde á veces parte de su magnetismo y no funciona bien. Se la puede restaurar, friccionándola 10 ó 12 veces de su centro á la punta sur con el polo norte de un imán (en herradura ó recto). Después de cada pasada retírese el imán á unos 40 cm de la aguja y vuélvase á pasar comenzando siempre en el centro. Cada mitad de la aguja, á su turno, mientras se la está tratando así, debe mantenerse á plano sobre una superficie lisa y dura. La mayor parte de las dificultades que se observan en el funcionamiento de la aguja, se deben en gran parte al deterioro de la punta del estilo que la sostiene. La remagnetización desequilibra la aguja y esto se remedia con un anillito de alambre corredizo en la mitad levantada.

Para evitar errores por contar los ángulos, á veces desde un extremo de la aguja, á veces desde el otro, siempre se debe colocar el punto N de la caja hacia el objeto cuya direccion se determina, y luego leer desde el extremo norte de la aguja hacia un mismo lado siempre. Esto es también más exacto.

## CURVAS DE NIVEL

**Curva de nivel** es una línea curva horizontal cuyos puntos están al mismo nivel †.

Así pues, cada una de las curvas de nivel 88*c*, 91*c*, 94*c*, fig. 1, indica que cada uno de los puntos por donde están trazadas se hallan al mismo nivel y á la altura de 88, 91 ó 94 metros sobre otro plano á nivel *c* llamado plano de comparación, y al cual se refieren.

A menudo se supone el punto de partida con la cota O (*cero*) é imaginamos que pasa por él el plano de *comparación*, y si estamos seguros de no encontrar puntos más bajos, nos servirá para todas las operaciones; pero si hay la probabilidad de que hallemos otros más bajos aún, es mejor atribuirle al punto de partida una cota dada sobre un plano de comparación situado en un punto más bajo que aquéllos. El único objeto es evitar los errores que se pueden cometer al trabajar con cotas negativas y positivas al mismo tiempo. Para lo cual podemos suponer que el punto de partida tiene una cota de 10, 100, 1,000 metros sobre el plano indicado, según las circunstancias. Conforme á la definición, la dist vertical entre cada dos curvas de nivel se supone igual, y en levantamientos para ferrocarriles en terrenos bien conocidos, en que el ingeniero sabe que el trazado de su línea verdadera no ha de variar mucho, dicha dist puede ser de 1 metro solamente, no siendo necesario trazar curvas de nivel á más de 40 ó 50 metros á cada lado de las estacas de centro; pero en regiones cuya topografía es relativamente desconocida, y donde por esta razón pueden presentarse obstáculos insuperables, que exijan variaciones importantes desde atrás en el trazado y á considerable distancia, los estudios deben ser más amplios y la equidistancia de las curvas de 1 á 5 metros según la conformación del terreno. Cuando el levantamiento se hace para la formación del mapa topográfico de un Estado ó de un departamento, son suficientes equidistancias de 2 á 3 metros. Supongamos que la línea AB, fig. 1, que parte de O, represente tres estaciones (S 1, S 2, S 3) del trazado del eje de una línea férrea, y que los números 100, 103, 101, 104 indicados en ella, señalan las altúras de las estacas sobre el plano de comparación, tales como se determinaron en la nivelación.

Entonces el objeto de las curvas de nivel es estudiar en la oficina el efecto de la variación del eje del trazado AB, al mover cualquier parte de él hacia la derecha ó hacia la izquierda *. Así, por ejemplo, moviendo la línea AB 30 metros hacia la izquierda, el punto de partida O estaría en un punto del terreno más ó menos 6 metros más alto que ahora, porque su cota sería más ó menos 106 metros en lugar de 100. La estación 1 estaría más ó menos 7 metros más alta ó 110 metros en lugar de 103. La estación 2 estaría como á 7 metros más alta ó

† *N. del T.* — Se da el nombre de *curvas de nivel* á la intersección de la superficie del terreno con planos horizontales ó á nivel.

* Al usar las palabras derecha é izquierda, suponemos que tenemos el punto de partida del levantamiento á nuestra espalda. **La margen derecha de un río** es la que está á nuestra derecha, **descendiendo**, es decir, se supone que tenemos el lugar donde nace ó su origen á nuestra espalda.

108 metros en lugar de 101. Si la línea se tira hacia el lado derecho, cae en terreno más bajo.

Estas curvas de nivel se trazan algunas veces en el terreno con un nivel de aire, pero más frecuentemente con una regla recta graduada, de 4 metros de largo más ó menos y con un eclímetro.

En cada estación se coloca la regla en el suelo formando lo más posible al ojo ángulo recto con el eje AB, y colocando el eclímetro sobre la regla, se mide el

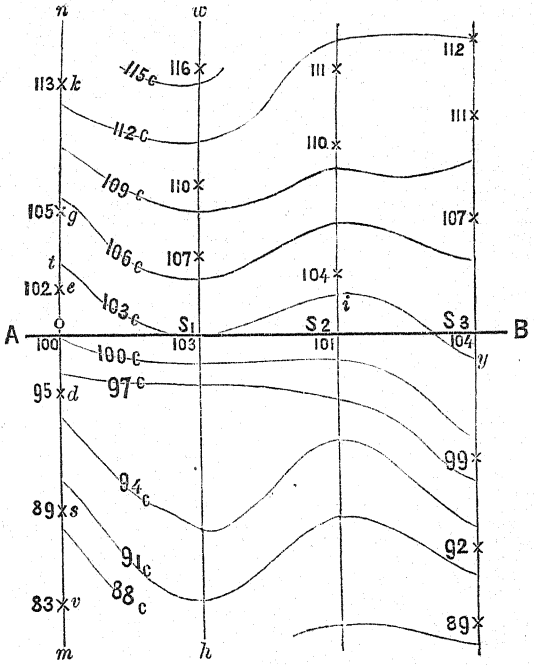



Fig. 1.

ángulo de inclinación del terreno con un cuarto de grado de aproximación. También se observa hasta qué distancia más allá de la regla continúa el terreno con la misma pendiente; y se mide esta distancia con ella. Luego poniendo la regla en este otro punto donde cambia la pendiente, se mide la inclinación que sigue lo mismo que su longitud; y se continúa así hasta donde se juzgue necesario. Estas notas se llevan en una cartera de apuntes (minuta), como se ve en la fig. 2; anotando los ángulos de inclinación sobre las líneas y las distancias debajo, todo acompañado de las observaciones que se ocurran, como indicar que hay bosques, rocas, arena, pantanos, sembrados jardines, ó bien donde se atraviesa un arroyo, etc., etc.

No es absolutamente necesario representar las pendientes en la libreta de apuntes, como en la fig. 2; porque usando el signo + para significar « ascenso », el — « descenso » y el = « al nivel », las pendientes pueden trazarse en línea recta, como en la figura 2½.

Habiendo recogido todos los datos y observaciones, el trazo de las curvas de nivel por medio de ellos es trabajo de oficina y se hace generalmente al mismo tiempo que el plano. Las observaciones del terreno, hechas en cada estación, se trazan luego

por separado por medio del transportador y en partes de la escala, como se muestra en la fig. 3, comenzado en el punto de partida O. La escala no debe ser menor de $^1/_{100}$, si se desea alguna exactitud. Suponiendo ahora que en dicha estación las pendientes

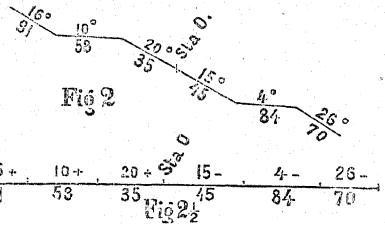



Fig 2

Fig 2½

hacia la derecha sean como en la fig. 2, de 15°, 4° y 26°, y las de la izquierda, de 20°, 10°, y 16°, y que las dist sean las de la misma figura. Trácese una línea horizontal *ho*, fig. 3, y supóngase que el centro sea la estaca donde se hizo estación. De este punto como centro, trácense estos ángulos con el transportador, como lo indican los arcos en la fig. 3. Luego principiando, por ejemplo, por el lado derecho, con una regla de trazar paralelas trácese la primera dist *ae* con su propia inclinación de 15.°, y su longitud por la escala. Luego lo mismo con *ey*, *yt*. Hágase del mismo modo con las inclinaciones y distancias del lado izquierdo.

Así tenemos una *sección transversal* del terreno en la estación O. Después trácese en el plano, como en la fig. 1, líneas *mn*, ó *hw*, en ángulo recto al eje del camino, y que pasen por las estacas donde se hizo estación. Sobre esta línea trácense las distancias horizontales *ad*, *ds*, *sv*, *ae*, *eg*, *gk*, marcándo as con un asterisco pequeño, como se hizo y se indicó con letras en la fig. 1 en la estación O.

Cuando se requiere mucha exactitud, estas distancias horizontales deben obtenerse por medidas hechas en la fig. 3; pero en general será suficiente aproximación, cuando las pendientes del terreno no exceden de 10°, suponer que son lo mismo que las distancias inclinadas medidas en el tereno. Ahora averíguese qué cota corresponde á cada uno de los puntos *c*, *y*, *t*, *l*, *n*, *i*, sobre el plano de comparación. Así: Mídase por la escala la dist vertical *cd*, fig. 3. Supongamos que sean 5 metros, ó en otras palabras, que *c* esté 5 metros *debajo* de la estaca O. Como la cota de O es 100 metros, la de *c* será 5 metros *menos*, ó 100—5=95 metros, la cual puede indicarse ligeramente con un lápiz en el plano, como en *d*, fig. 1. Ahora bien, con respecto al punto *y*, suponiendo que hemos encontrado para *sy* 11 metros, ó que *y* está á 11 metros *debajo* de la estaca O, su altura sobre el plano de comparación será 100—11=89; que marcaremos también con lápiz, como en *s*. Procédase de la misma manera con *t*. Luego pasando al lado izquierdo de la estaca encontramos

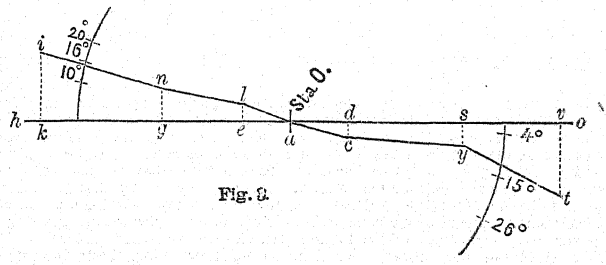



Fig. 3.

que *el*, por ejemplo, son 2 metros; pero como *l* está *sobre* el nivel de dicha estaca, su altura sobre el plano de comparación será, de consiguiente, 100+2=102 metros, como está indicado en *e* en el plano. Supongamos que *ng* sean 5 metros; entonces *n* estará á 100+5=105 metros sobre el plano de comparación, como se indica en *g*, y así en cada estación.

Cuando esto se haya hecho en varias estaciones, podremos trazar las curvas de

nivel de aquella parte á la mano así. Suponiendo que ellas han de representar equidistancias verticales de 3 metros. Principiando á trazar en el punto O, fig. 1 (cuya cota es de 100 metros sobre el plano de comparación), una curva de nivel de 103 metros de cota, vemos desde luego que la altura de 103 metros corresponde al punto *t*, situado á $^1/_3$ de la dist de *e* á *g*. Hágase una marca ligera con lápiz en *t*, y sígase á la próxima estación 1, donde vemos que la altura de 103 metros coincide con la misma estaca; márquese allí otro punto con lápiz y sígase á la estación 2. La cota de esta estaca es 101 metros; por lo tanto, la curva de nivel de 103 metros de cota está 2 metros más alta, ó en *i*, á $^2/_3$ partes de la distancia comprendida entre la estaca 2 y +104; márquese un punto con lápiz en *i*.

Sígase á la estación 3. Siendo la cota de este punto 104 metros, la curva de nivel de 103 metros de cota debe pasar por *y* ó á $^1/_3$ de la distancia de la estación 3 á +99; márquese un punto con lápiz en *y*. Finalmente, trácese á la mano una curva por *t*, S1, *i*, *y*, y quedará trazada la curva 103. Todas las otras se trazan de la misma manera, una por una. La cota de cada una de ellas debe anotarse en el mapa á distancias cortas, como en 103*c*, 106*c*, etc. O, en lugar de poner primero los puntos así marcados sobre el plano, para indicar las distancias inclinadas verdaderas, medidas en el terreno, podemos de una vez, y con menos trabajo, encontrar y fijar solamente las que representan los puntos *t*, S1, *i*, *y*, etc., de las curvas de nivel mismas. Así: Sea 104, por ejemplo, la cota de cualquier estaca, fig. 4; que la sección transversal *cs* haya sido dibujada como antes, y que necesitamos las distancias horizontales de la estaca á las curvas de nivel de 94, 97, 100 metros, etc., con cotas de 3 metros de equidistancias verticales.

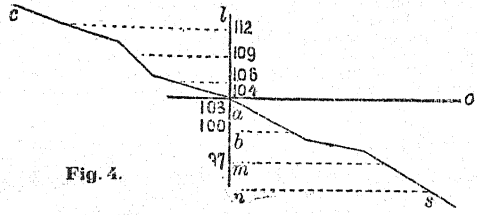



**Fig. 4.**

Tírese una línea vertical, *vl*, por la estaca y en ella márquense por la escala las cotas 94, 97, 100, etc. Esto se puede hacer fácilmente, puesto que tenemos la cota, dada 104 de la estaca. Por estos puntos trácense las líneas horizontales *a*, *b*, *m*, *n*, etc. á la línea de pendiente del terreno; entonces estas líneas horizontales, medidas por la escala, nos darán evidentemente las distancias requeridas.

Cuando el terreno es muy irregular ó quebrado transversalmente, las secciones transversales deben tomarse sobre el terreno á distancias unas de otras de menos de 30 metros. El trazado de las curvas de nivel se facilita mucho con el uso del papel cuadriculado de no menos de 2 milímetros por lado que hace también muy fácil el trazado de las secciones transversales.

Cuando el terreno es muy escarpado, se acostumbra sombrear el plano para representar las elevaciones.

Mientras más próximas queden en el dibujo las curvas de nivel, tanto más escarpado será, por supuesto, el terreno comprendido entre ellas; y la sombra debe ser relativamente más obscura en dicha porción del plano.

Pero en los planos de *trabajo* es mejor omitir la sombra.

En levantamientos de comarcas extensas se usa el tránsito de círculo ó arco graduado vertical *g*, pág. 301, para medir los ángulos de inclinación, en lugar del eclímetro ordinario.

En muchos casos, en los levantamientos para ferrocarriles, se reemplazan las curvas de nivel con simples notas como la siguiente:

| | | |
|---|---|---|
| Est 60 | $-3.1$ D | $+2.1$ Y |
| 61 | $+2.2$ D | $-1.3$ Y |
| 62 | $=1.$ D | $+4.2$ Y |
| 63 | — | — |

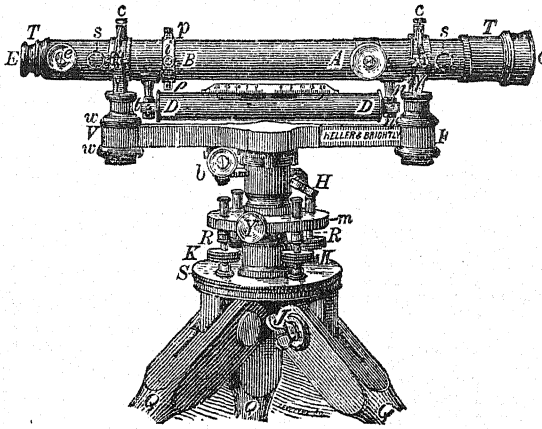
Lo que quiere decir que en la estación 60, la pendiente del terreno determinada aproximadamente á la vista ó con un nivel de mano, va en descenso hacia el lado derecho á razón de 3 metros en la dist de una cadena ó 30 metros, y hacia la

izquierda, sube como 2 metros por cadena. En la 61, sube 2 metros, en dos cadenas á la derecha, y baja 1 metro en 3 cadenas á la izquierda. En la 62, el terreno está á nivel hasta una cadena á la derecha, y asciende 4 metros en dos cadenas á la izquierda. En la 63, lo mismo que en la 62. En algunos puntos bastaría solamente agregar un croquis (de la sección transversal) semejante á la fig. 2 en lugar de los ángulos, y tomar los metros de ascenso ó descenso, para indicar las pendientes, determinadas á la simple vista ó con un nivel de mano. De este modo el resultado tendrá un pequeño error en cada estación; pero estos errores tan pequeños casi se compensan unos á otros, de manera que el resultado total puede considerarse suficientemente correcto para hacer los cálculos preliminares del costo de una vía. Cuando se colocan las estacas finales para guía de los trabajadores, entonces deben determinarse las pendientes con exactitud, para calcular el movimiento de tierra y hacer los pagos.

# EL NIVEL

Aunque los niveles de los diferentes fabricantes varían algo en sus detalles, las partes principales que los constituyen pueden comprenderse por medio de la figura siguiente. El anteojo TT descansa sobre dos soportes YY, llamados *ies*, de los cuales se puede levantar sacando las clavijas *ss*, que aseguran las dos *abrazaderas* semicirculares *cc*, y luego abriendo éstas. Las clavijas *ss* deben atarse á los soportes con una cuerdita para evitar que se pierdan.

El *tubo movible* que contiene el objetivo O, se desliza hacia adelante y hacia atrás por medio de un tornillo de cremallera, con la cabeza acordonada A. El del *ocular* E se mueve de la misma manera por medio del tornillo acordonado *e*. Casi todo nivel




tiene un tubo cilíndrico de bronce, llamado *pantalla*, que se corre sobre la parte del anteojo que contiene el objetivo, para evitar los reflejos del sol sobre aquél cuando éste está bajo. B es un anillo exterior que envuelve el anteojo y tiene 4 tornillos pequeños, dos de ellos, *pp*, están en las partes superior é inferior, mientras que los otros dos, de los cuales *i* es uno, se hallan en los lados del tubo y en ángulo recto á *pp*. Del lado interior de este anillo exterior y en el inferior del anteojo hay otro anillo, que tiene atravesadas dos hebras de tela de araña, llamadas generalmente *pelos transversales*. Son mucho más delgadas de lo que parecen, porque están aumentadas considerablemente por la lente. Estos pelos están en ángulo recto uno con otro; y al nivelar se mantiene uno vertical y el otro horizontal.

Algunas veces están expuestos á salirse de su posición, cuando se transporta el nivel al hombro de una parte á otra, ó cuando se coloca bruscamente la trípode en el suelo; pero como al nivelar se dirige la *intersección* de los pelos hacia la mira, este desarreglo no afecta la exactitud del trabajo. Sin embargo es mejor conservarlos próximamente verticales y horizontales por medio del nivel de aire DD.

Esto le facilita al nivelador ver si el que lleva la mira tiene ésta casi vertical, lo que es absolutamente esencial para una nivelación correcta.

Si se desea la perfecta verticalidad de la mira, como es necesario cuando se quieren colocar con mucha precisión estacas para el trabajo, esto se puede conseguir (*si el instrumento está perfectamente corregido y á nivel*) dirigiendo la vista á una plomada ú otro objeto vertical, y haciendo girar entonces el anteojo un poco en sus *ies* hasta que coincida el pelo con la línea vertical. Hecho esto, puede hacerse una marca ligera y continua, en el anteojo y el soporte, para evitar rectificaciones en lo sucesivo.

*Advertencia del Traductor.* — Así se sabe si la mira no está inclinada, hacia la derecha ó izquierda del observador, respecto á la verticalidad *en el plano perpendicular á la visual;* pero, la falta de verticalidad de la mira *en el plano mismo de la visual*, porque su extremo superior esté inclinado hacia el observador ó en sentido contrario, no se puede apreciar bien desde el anteojo, ni por el anteojo; es necesario que el peón portamira tenga una plomada que lo guíe al plantar la mira. Para precaverse de los descuidos del portamira, en este sentido, es lo más práctico ordenarle un pequeño movimiento oscilatorio de la mira en el plano de la visual y tomar *la menor lectura que marque el pelo horizontal* (ó mejor la intersección de los pelos) al aparecer éste, moviéndose de arriba hacia abajo y viceversa sobre las divisiones de la mira, á causa del movimiento oscilatorio de ésta.

Los señores Heller Brightly dotan los niveles que fabrican de un pequeño resalto ó parte saliente en las *ies*, y otro correspondiente en el anteojo, cuyo contacto asegura la verticalidad del pelo. Si se rompen los pelos por un accidente, pueden reponerse como se explica más adelante.

Los pequeños agujeros que se encuentran alrededor de los cuatro tornillos *ip* aludidos, son para introducir el extremo de una pequeña clavija de acero, con la cual se les hace girar. Si se afloja primero el tornillo superior *p* y se aprieta el más bajo, el anillo interior descenderá y con él el pelo horizontal; pero al mirar por el anteojo *parece* que han subido. Aflojando primero el tornillo inferior y apretando el superior, el pelo horizontal sube realmente; pero visto por el anteojo, baja aparentemente. Resulta esto, porque las lentes del ocular E invierten la posición aparente de los objetos en el *interior* del anteojo, cuyo efecto se evita, para los objetos *exteriores*, por medio del objetivo O. Debe tenerse presente esta circunstancia a, arreglar los pelos, porque cuando un pelo parece intersectar demasiado alto, debe aún elevarse más, y si el pelo vertical parece demasiado á la derecha ó á la izquierda para corregirlo debe moverse más en el mismo sentido.

Esta observación no se aplica á los anteojos que invierten los objetos.

No hay peligro de que se dañen los pelos con estos movimientos, porque los 4 tornillos no actúan sino sobre el anillo, y no llegan á tocar los pelos mismos.

Debajo del anteojo se halla el *nivel de aire* DD. Un extremo de este nivel puede subirse y bajarse ligeramente por medio de las dos tuercas de tornillo *nn*, una de las cuales debe aflojarse antes de apretar la otra.

En la parte superior del tubo hay unas rayas para indicar cuándo la burbuja está en el centro. A menudo se hacen las rayas en una tira de bronce colocada sobre el tubo, como se observa en la figura. De estas rayas hay varias, para abarcar la burbuja en sus dilataciones y contracciones ocasionadas por los cambios de temperatura. En la otra extremidad del nivel hay dos tornillos pequeños colocados horizontalmente en los lados opuestos. La cabeza circular de uno de esos tornillos se ve cerca de *t*. Por medio de ellos puede moverse el extremo mencionado del nivel, un poco horizontalmente hacia la derecha ó hacia la izquierda. Debajo del nivel de aire está la barra VF, que en una de sus puntas, V, tiene dos tuercas grandes *w*, *w*, que obran sobre un tornillo fuerte interior, que forma una prolongación del soporte Y. Los agujeros de estas tuercas son más grandes porque necesitan de una palanca más grande para darles vuelta.

Si se afloja la tuerca inferior y se aprieta la superior, el soporte Y sube y la extremidad correspondiente del anteojo se aleja de la barra y viceversa. Algunos fabricantes colocan tornillos y tuercas semejantes debajo de ambos soportes, mientras que otros no ponen tuercas sino una cabeza grande cilíndrica acordonada, que se le da vuelta con los dedos. Ésta está expuesta sin embargo á alteraciones accidentales que deben evitarse.

Cuando la porción del instrumento de *m* hacia arriba se coloca sobre *m*, y se as

gura con el tornillo Y„puede hacerse girar horizontalmente aflojando el **tornillo de presión** H; ó apretando éste, impedir todo movimiento.

Sucede frecuentemente que después que el anteojo se ha dirigido muy cerca de un objeto, y está fijo con el tornillo H, deseamos hacer coincidir los pelos transversales con el objeto más exactamente de lo que se puede hacer, dándole vuelta al instrumento *con la mano;* y en este caso usamos el **tornillo tangencial** *b*, por cuyo medio podemos dar al instrumento un movimiento pequeño, pero regular, después de fijo. Para más detalles sobre los tornillos de presión y tangenciales, véase « Tránsito ».

**Las placas paralelas** *m* y S se mueven por medio de los cuatro **tornillos de nivelar**, de los cuales se ven tres, KK, en la figura.

Estos tornillos trabajan en encajes R, los que al igual de los tornillos se prolongan hasta la placa superior.

Cuando se instala el instrumento en el suelo para nivelar, es conveniente colocarlo de manera que la placa paralela inferior S esté casi horizontal á la simple vista, para evitar en cuanto sea posible trabajar mucho con los tornillos de nivelación KK, al poner horizontal la placa superior *m*. La placa inferior S y las piezas de bronce en conexión con ella, se llaman **cabeza de la trípode**; y junto con las tres patas de madera, QQQ, constituyen la trípode. En la figura se ven las tuercas de aletas J que fijan las patas á la cabeza de la trípode.

Debajo del centro de ésta debe haber siempre una argolla pequeña para colgar la plomada. Esto no se necesita en una nivelación ordinaria, pero sí al colocar estacas de centro y otros trabajos de precisión.

## Corrección del nivel.

Esta es una operación muy sencilla, pero que requiere un poco de paciencia. Téngase cuidado de *no forzar* ninguno de los tornillos. Las grandes tuercas *ww* de las Y necesitan algunas veces de un poco de fuerza para *principiar á moverlas;* pero esta fuerza debe aplicarse por presión paulatina y no por golpes. Antes de principiar las correcciones, hágase con el objetivo lo que se dijo al empezar el párrafo titulado « Corrección de un tránsito común ».

Un nivel necesita tres correcciones *que deben hacerse en el orden siguiente :*

**Primero, la de los pelos transversales.** Cerciorarse si su cruzamiento continúa intersectando siempre el mismo punto de un objeto distante durante una vuelta completa del anteojo en sus apoyos *. Esto se llama corregir el **eje de colimación**, ó algunas veces la *línea visual;* pero no es estrictamente la línea visual hasta no *concluir* todas las correcciones, pues hasta entonces la línea ó eje de colimación no servirá para dirigir visuales de nivelación.

**Si se rompen los pelos transversales**, véase la pág. 307.

**Segundo, la del nivel de aire** DD. Hacer que éste sea paralelo al eje ó línea de colimación ya corregido, de manera que estando la burbuja en el centro de su tubo y, por consiguiente, indicando que está horizontal, sepamos que la visual dirigida por el anteojo es horizontal. Para reponer **un nivel de aire roto**, véase pág. 308.

**Tercero, la de los apoyos** YY que sostienen el anteojo y el tubo del nivel de aire. Hacer que el eje del nivel y la visual dirigida sean perpendiculares al eje vertical del instrumento, para que de esta manera *permanezca* horizontal cuando se dirija el anteojo hacia objetos situados en diferentes direcciones, como cuando se dirigen visuales atrás y adelante.

**Para hacer la primera corrección**, es decir, la de los pelos transversales, póngase la trípode *firmemente* en el suelo. En esta corrección no es necesario poner el instrumento á nivel. Abranse las abrazaderas de los apoyos; sáquese el ocular E, hasta que los pelos transversales se vean *perfectamente claros;* diríjase la visual hacia un punto claro de algún objeto distante ó, mejor todavía, hacia una línea recta vertical ó no. Por medio de la cabeza acordonada A, muévase el objetivo O hasta que se vea *claro* el objeto, **y sin paralaje**; esto es, sin que se note un movimiento aparente alrededor de los pelos, al mover el ojo un poco hacia arriba ó hacia abajo, ó lateralmente. Para conseguir esto, el objetivo solo es el que debe moverse cuando se quiere graduar el instrumento para diferentes distancias; el ocular no se mueve después de fijo y de que se ven los pelos con claridad. La falta de atención en la paraluje es origen de frecuentes errores en la nivelación. Fíjese

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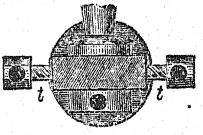
* *N. del T.* — Es decir, que el punto de intersección de los pelos se halle en el eje óptico del anteojo, lo que se verifica si coincide siempre con el mismo punto de un objeto lejano durante una vuelta completa del anteojo alrededor de sus YY.

el anteojo con el tornillo de presión, y con el tornillo tangencial hágase coincidir *exactamente* á cualquiera de los pelos con el objeto. Luego, con suavidad y sin trepidación, désele media vuelta al anteojo en los apoyos. Hecho esto, si el pelo todavía coincide exactamente con el objeto, el instrumento estará correcto y procedemos á probar el otro pelo. Pero si no coincide, por medio de los 4 tornillos *i p*, muévase el retículo que contiene los pelos, hasta rectificar lo más que se pueda á la simple vista, *la mitad* del error solamente; recordando que el retículo debe moverse en dirección opuesta á la en que parece estar, á menos que el anteojo invierta los objetos.

Entonces, désele vuelta al anteojo á la inversa hasta llevarlo á su posición anterior y hágase coincidir otra vez el pelo con el objeto por medio del tornillo tangencial. Ahora hágase girar el anteojo como antes, y se encontrará el pelo casi en la posición deseada, aunque probablemente no con toda exactitud, porque es muy difícil estimar á la vista con precisión la mitad del error.

Por lo tanto debe hacerse otra pequeña alteración al retículo y puede que sea necesario repetir la operación varias veces antes de que se llegue á la perfecta corrección. Por último, trátese el otro pelo de la misma manera. Cuando ambos pelos estén corregidos, su intersección marcará con exactitud el mismo punto durante una vuelta *entera* del anteojo en los apoyos. Esto debe comprobarse antes que demos por terminada la corrección, porque algunas veces la del segundo pelo desarregla el primero; especialmente si, al principiar, ambos tenían un error notable

**Para hacer la segunda corrección** ó para poner el tubo de burbujas paralelo al eje del anteojo se hacen dos correcciones separadas, una vertical y otra horizontal. La primera de éstas se efectúa por medio de las dos tuercas *nn*. que están sobre el tornillo vertical en un extremo del nivel; y la segunda con los dos tornillos horizontales de la otra punta del mismo nivel. Viendo el tubo del nivel de punta en *t*, en esta figura inmediata, sus dos tornillos de corrección *tt* se ven como en la figura. La tuerca más grande de *abajo* nada tiene que hacer con las correcciones, sólo mantiene la punta del tubo en su puesto. Para hacer




las correcciones verticales del nivel de aire por medio de las dos tuercas *nn*, póngase el anteojo sobre dos de los tornillos diagonales KK de nivelación y fíjese. Abranse las abrazaderas de las YY, y por medio de los tornillos de nivelación llévese la burbuja al centro del tubo. Levántese suavemente el anteojo é inviértase su posición, poniendo el objetivo donde estaba el ocular y colóquese de nuevo en sus apoyos así invertido. Hecho esto, si la burbuja permanece todavía en el centro, estará correcto; pero si se mueve hacia uno de los extremos, dicho extremo estará demasiado alto y debe bajarse éste ó levantarse la otra punta. Primero, corríjase la *mitad* del error por medio de los tornillos de nivelación KK, y la otra mitad con las tuercas ó tornillos *nn* del tubo de la burbuja. Para *elevar* la punta *n* aflójese primero la tuerca superior y apriétese la inferior, y para efectuar esto, désele vuelta á cada tuerca, de manera que el lado *más próximo* de cada uno se mueva hacia la *derecha*. Para *bajarlo*, aflójese primero la tuerca inferior y apriétese la superior, moviendo el lado *más próximo* de ellas hacia *la izquierda*. Habiéndose llevado la burbuja de este modo otra vez al centro, levántese otra vez el anteojo de las YY, é inviértase y colóquese de nuevo en su lugar. La burbuja se quedará ahora más cerca del centro que antes; pero probablemente necesitará aún otros tanteos. Si fuese así, corríjase la *mitad* del error restante por medio de los tornillos de nivelación, y la otra mitad por medio de las tuercas del tubo como antes, y continúese repitiendo así la operación hasta que la burbuja permanezca en el centro en ambas posiciones. Con respecto á otros métodos, véase « Para corregir el tubo largo del nivel de aire », pág. 306. La corrección horizontal del nivel de aire es para ver si su eje está en el mismo plano que el eje del anteojo, como lo está generalmente en los instrumentos nuevos. El desarreglo en este sentido no se produce fácilmente sino por golpes fuertes. Colóquese el nivel de aire debajo del anteojo lo más que se pueda en la dirección de éste, ó sobre el eje de la barra VF. Llévese el anteojo sobre dos de los tornillos de nivelación KK.

fíjesele allí y llévese la burbuja á su centro por medio de dichos tornillos; hágase girar alrededor de su eje al anteojo en sus *Yes* como seis milímetros, llevando el nivel de burbuja fuera del eje de la barra, primero hacia un lado y después hacia el otro. Si la burbuja permanece en el centro durante la operación, no hay corrección que hacer. Si dicha burbuja, durante el movimiento de vaivén ú oscilatorio comunicádole hacia los lados opuestos al eje, corre hacia los extremos *opuestos* del tubo, muévase entonces la punta *t* por medio de los tornillos horizontales *tt* hasta que la burbuja se quede en su centro durante el desalojamiento del nivel hacia ambos lados del eje mencionado. Si la burbuja corre hacia la misma punta del tubo *en ambos lados*, es porque el tubo no es verdaderamente cilíndrico, sino ligeramente cónico *, de modo que si se hace girar el anteojo en sus soportes, la burbuja se saldrá de su centro, aunque la corrección horizontal esté bien hecha. En los tubos de esta especie se conoce, que la corrección horizontal está perfecta, en que la burbuja recorre desde el centro la misma cantidad cuando se desvía dicho tubo la misma dist hacia ambos lados. Hecha la corrección horizontal, vuélvase atrás el anteojo en sus Yes hasta que el nivel de burbuja esté directamente sobre el centro de la barra. Repítase la corrección *vertical*, pág. 321, que puede haberse desarreglado mientras se hacía la horizontal. Continúese hasta que ambas correcciones resulten bien hechas al mismo tiempo.

**Tercera corrección ó corrección de la altura de las Yes,** hasta hacer que la línea de colimación sea paralela á la barra VF ó perpendicular al eje vertical del instrumento. Después de haber hecho las otras correcciones, asegúrense las abrazaderas de las Yes. Póngase el instrumento casi horizontal por medio de los cuatro tornillos K. Colóquese el anteojo sobre dos de los tornillos diagonales de nivelar y déjesele en dicha posición sin trancarlo, luego llévese la burbuja al centro del tubo por medio de los dos tornillos de nivelar. Désele media vuelta á la parte superior del instrumento, de manera que el anteojo quede de nuevo sobre los mismos tornillos; pero con el ocular donde estaba el objetivo.

Hecho esto, si la burbuja se sale del centro, hágasele retroceder la *mitad* por medio de las tuercas grandes *w, w*, y la otra mitad por medio de los tornillos K. Recuérdese que para elevar ó hacer subir el soporte Y y el extremo del nivel de aire sobre *w w*, la *w* inferior debe aflojarse y la superior apretarse y viceversa. Ahora colóquese el anteojo sobre los otros dos tornillos diagonales y repítase toda la operación con ellos. Habiéndola concluído, hágase la prueba otra vez con el primer par de tornillos, y sígase así hasta que la burbuja se quede en el centro de su tubo, en cualquiera posición del anteojo. Puede nivelarse correctamente aun cuando no se haya hecho ninguna de las correcciones mencionadas, con tal que *cada visual ó golpe de nivel hacia adelante se haga á la misma dist del instrumento que la visual ó golpe de nivel hacia atrás;* pero un buen nivelador tendrá siempre su instrumento corregido y lo rectificará por lo menos una vez por día, cuando esté trabajando †.

Mucho depende la exactitud de una nivelación del que carga la mira. Un hombre que sea descuidado en mantener la mira vertical, ó que no lea las graduaciones con exactitud, debe despedirse sin consideración alguna **.

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* Este defecto puede solamente remediarse sacando el tubo y colocando otro de forma correcta, lo cual será mejor hecho por un fabricante de instrumentos; pero, á pesar de todo, puede efectuarse un trabajo perfecto de la manera siguiente: Háganse todas las correcciones tan aproximadamente exactas como se pueda; póngase á nivel el instrumento. Girando el anteojo en sus apoyos (YY), hágase coincidir el pelo vertical con una plomada ú otra línea vertical y practíquese con un cortaplumas una raya continua en el anillo más próximo al objetivo y prolónguese esta marca hasta el apoyo adyacente. Levántese el anteojo de las Y é inviértase de modo que el objetivo ocupe el lugar del ocular; colóquese otra vez en sus apoyos; póngase nuevamente vertical el pelo del anteojo, y en el otro apoyo hágase una raya que coincida con lo que se hizo en el anillo inmediato al objetivo; luego al nivelar ó corregir, véase siempre si la raya del anillo coincide con la del apoyo adyacente cuando el nivel de aire está debajo del anteojo.

† *N. del T.* — Siempre será conveniente corregir el instrumento lo más á menudo posible; pero como en nivelaciones de carácter urgente las tres correcciones del nivel quitarían mucho tiempo al hacerlas diariamente, creemos muy suficiente en estos casos limitarse á efectuar diariamente sólo la primera de las dos partes de que consta la *segunda corrección*.

** *N. del T.* — Esto será en el caso en que se usen *miras de corredera* que ofrecen el inconveniente de que no puede leerlas el observador sin trasladarse de la estación al punto de mira ó proveerse de un peón portamira que haga la lectura; esto se usa poco,

Los tornillos de nivelar se ponen muy apretados con el polvo y es difícil darles vueltas. Límpieselos con agua y un cepillo de dientes. No se use aceite en los instrumentos de campo.

**Forma de las minutas de nivelación.** Cuando la dist es corta y no se requieran dos juegos de libretas, la forma siguiente de minuta es tan buena quizá como cualquiera otra.

| N.° de la Estación. | Visual á la espalda. | Visual al frente. | Difer. | Cotas. | Pendiente. | Banqueo. | Terraplén. |
|---|---|---|---|---|---|---|---|

Pero en las Obras públicas no se usan sino las primeras cinco columnas. Después de haberse determinado la pendiente por medio del perfil trazado, se anotarán los resultados en otra libreta que tiene solamente la primera y las cuatro columnas últimas. En ambos casos la página del lado derecho se deja para las observaciones. El autor cree que sería mejor considerar el término «Estación» tanto en las operaciones que se hagan con el nivel como en las que se efectúen con el tránsito aplicado á la dist total comprendida entre dos estacas consecutivas; y que su número se escriba sobre la última estaca. Así, con el tránsito, la estación 6 quiere decir la dist de la estaca 5 á la estaca 6, y que tiene tal ó cual dirección y que su extensión es tal ó cual. Y con el nivel, la estación 6 indica la dist de la estaca 5 á la estaca 6, habiéndose hecho la visual á la espalda en la estaca 5 y la visual adelante en la estaca 6, y que la cota, la pendiente y el banqueo ó terraplén se refieran á la estaca 6. El punto de partida de un levantamiento se llamará 0 (cero), ya sea una estaca ú otra cosa cualquiera **.

| Estación. | Visual al frente (aditiva). | Altura del ojo. | Visuales á la espalda (sustractivas). | Cotas. |
|---|---|---|---|---|
| 1 | 1.50 | 101.50 | | 100 |
| 2 | | | 2.25 | 99.25 |
| 3 | | | 2.50 | 99.00 |
| 4 | 3.20 | 101.20 | 3.50 | 98.00 |
| 5 | | | 4.10 | |

(*Explicación.* En el punto de partida, que llamaremos estación 1, cuya cota suponemos ser 100, tomaremos la primera lectura de mira, por ejemplo 1.50, que como visual al frente es aditiva. Sumada esta altura de mira con la cota 100 de la estación 1, obtendremos 101.50=la altura del eje óptico del instrumento ú ojo del observador. Trasladada la mira sucesivamente á las estaciones 2, 3, 4, etc., tomaremos nuevas lecturas 2.25, 2.50, 3.50, etc., *sin cambiar el instrumento*, las que restadas (como sustractivas que son) de la altura de ojo dicha, darán las cotas 99.25, 99.00, 98.00, etc., de aquellos puntos. Al cambiar el instrumento para continuar la nivelación, dejaremos la mira en la última estación 4, cuya cota es 98; y se toma sobre ella, después de cambiar el instrumento, una nueva lectura de mira que viene á ser *visual al frente* en la nueva posición del instrumento. Supongámosla igual á 3.20; como es aditiva, la sumaremos con la cota 98 de la estación 4, donde se dejó la mira, obtendremos 101.20, altura del ojo en la nueva posición del instrumento. Mudando la mira á otras estaciones siguientes, verbigracia 5, sin cambiar el instrumento leeremos en ella, por ejemplo 4.10, que restado de 101.20 dará 97.10, cota de la estación 5, y así sucesivamente.)

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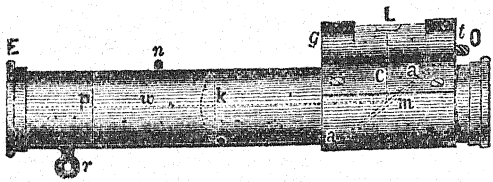
aunque puede ser de gran utilidad cuando se hacen observaciones á grandes dist. Es más expedito emplear la *mira*, de 4 m 20 á 5 m de largo, dividida en decímetros, centímetros y á veces en milímetros, la cual nos excusamos de describir por ser muy conocida. Para mantener su verticalidad puede agregársele una plomada convenientemente dispuesta; y para evitar hundimientos en terrenos blandos, colocar dicha mira sobre un *galapaguito* de hierro, con las tres puntas ó esquinas dobladas igualmente para apoyarlo en el suelo.

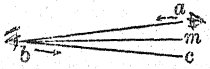
** *N. del T.* — Recomendamos esta minuta que nos parece muy práctica.

## NIVEL DE MANO




Este pequeño y muy útil instrumento, tal como lo ha dispuesto el profesor Locke, de Cincinnati, es más ó menos de 12½ á 15 centímetros de largo. Sosteniéndolo simplemente en una mano y viendo por él en cualquiera dirección, podemos determinar desde luego, muy aproximadamente, los objetos que se hallan al mismo nivel del ojo. E, es el ocular y O el objetivo; L es un nivelito colocado dentro de una especie de caja de bronce, *tg*, cuyo fondo está abierto, y hay debajo de él, en la parte superior del tubo principal EO, una abertura correspondiente. Inmediato al fondo del nivel L hay un hilo colocado transversalmente á dicha abertura y fijo sobre una placa pequeña, la que puede empujarse un poco hacia atrás para ajustar el alambre, apretando el tornillo *t*, ó empujarse hacia adelante por medio de un pequeño resorte que hay entre la caja cerca de *g*, cuando se afloja el tornillo *t*. En *m* hay un espejito semicircular *aa*, azogado en su dorso *m*, inclinado á 45º y ocupa la mitad del ancho del tubo EO. A través de las aberturas arriba mencionadas se reflejan las imágenes del hilo transversal y de la burbuja del nivel sobre la cara no azogada *aa* del espejo, y de aquí hacia el ojo, como se indica por las líneas de puntos *c* y *w*, y cuando el instrumento está arreglado y mantenido horizontalmente, el hilo parece estar en el centro de la burbuja. En K se halla la mitad de una lente plano-convexa, en el extremo interior de un tubo corto *pk*, que puede moverse hacia adelante ó hacia atrás por medio de una clavija *n* que sobresale a través de una pequeña ranura hecha en el tubo principal. Por este medio se llega á ver clara la imagen del hilo transversal; y la semilente debe moverse hasta que, al ver el objeto, el hilo no manifieste paralaje, sino que aparezca fijo sobre el objeto, cuando se mueva el ojo ligeramente hacia arriba ó hacia abajo. En cada punta del tubo EO hay una pieza circular de vidrio plano para evitar que entre el polvo.

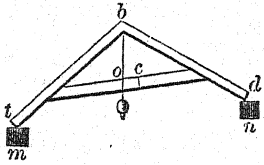
**Corrección del nivel de mano.** Fíjense primeramente dos puntos exactamente á nivel como de 20 á 100 metros de distancia uno del otro. Hecho esto, colóquese el instrumento en uno de ellos y diríjase la visual al otro. Si entonces el hilo no parece estar exactamente en el centro de la burbuja, muévasele ligeramente hacia adelante ó hacia atrás, según el caso, por medio del tornillo *t*, hasta que lo




parezca. Los dos puntos á nivel pueden fijarse por medio del nivel de mano mismo, aunque no esté corregido; así: Primero, por medio de la clavija *n* arréglese la semilente *k* hasta que se vea claro el hilo y sin paralaje. Luego, teniendo el nivel fijo en cualquier objeto *a* de manera que el hilo parezca intersectar el centro de la burbuja, obsérvese donde intersecta á cualquier otro objeto conveniente *b*. Luego váyase á *b* y diríjase desde allá del mismo modo, visual atrás hacia *a*. Si el instrumento está correcto, el hilo intersectará á *a;* pero si no, á cualquier otro punto *c* situado más abajo ó más arriba. En uno ú otro caso hágase una marca *m*, en medio de *c* y *a*, y entonces *b* y *m* serán los puntos á nivel requeridos. Hechas desde luego estas correcciones con cuidado, permanecerá el instrumento en buen estado por algunos años. Generalmente el instrumento está provisto de un pequeño anillo ó ganchito *r* para cargarlo colgando del cuello. No es propio para un trabajo de mucha exactitud, pero es admirable para explorar un camino. La altura de un cerro desnudo ó sin vegetación puede hallarse principiando en el pie, y dirigiendo el nivel á cualquier objeto que el hilo intersecte, como una piedrita, etc., luego avanzando nos colocamos sobre este objeto y se fija el hilo sobre otro que se halle más allá, y así hasta llegar á la cima. En cada observación es claro que subimos una altura igual á la de

nuestro ojo, por ejemplo $1^m75$, ó lo que fuere. Ya sea subiendo ó bajando, si el cerro está cubierto de paja, arbustos, etc., hágase uso de una mira de corredera para tomar las visuales adelante, y la altura constante del ojo puede considerarse como la visual atrás en cada estación. Puede proveerse el instrumento de un empalme para atornillarlo al extremo de una caña ó de un bastón largo armado de una articulación esférica, ó de rodillo cuando se necesita más exactitud.

**Corrección del nivel de albañil ó de perpendículo** *tbd.* Colóquesele sobre dos soportes *m* y *n*, y márquense en *o* donde toca la plomada, luego inviértase,




colocando el extremo *t* donde está *n* y *d* sobre *m*; márquese la dirección de la plomada y hágase una marca permanente en el medio de *o*, y *c*. Cuando el hilo de la plomada coincide con esta marca, los dos extremos *t* y *d* están á nivel.

**Corrección de un nivel de pendiente ó eclímetro.** Generalmente el nivel de aire está adherido á una barra movible por medio de un tornillo cerca de cada extremo, y la cabeza de uno de los tornillos entra en una cavidad practicada en la barra que permite al tornillo un ligero movimiento vertical cuando está flojo, y con él á la extremidad del tubo. Por tanto, para arreglar la burbuja se afloja primero este tornillo y se le mueve un poco hacia arriba ó hacia abajo, según se requiere, y entonces se vuelve á apretar.

## NIVELACIÓN POR MEDIO DEL BARÓMETRO

**1.** Muchas circunstancias se reunen para hacer que se desconfíe de los resultados de esta especie de nivelación cuando se requiere gran exactitud. El hecho de que tales circunstancias influyen desfavorablemente, está comprobado de manera concluyente por las observaciones hechas por el capitán T. J. Cram, de la U. S. Coat Survey. Véase el informe de la U. S. Coat Survey del año de 1854. Es difícil apreciar con un aneroide (clase de barómetro que generalmente se emplea en los trabajos de ingeniería) difs de nivel de 0.50 á 1½ ó 2 metros. La humedad ó sequedad del aire afectan los resultados. De igual modo, los vientos, la proximidad de las montañas y el flujo y reflujo atmosféricos, son causas de fluctuaciones irregulares é incesantes en el barómetro. Un barómetro colgado en un cuarto sin moverlo, varía á menudo 2½ milímetros en pocas horas, y esto representa una dif de nivel próximamente igual á 30 metros. Es imposible idear una fórmula que comprenda todas estas causas de error. Las variaciones que dependen de la temperatura, de la latitud, etc., se han tomado en cuenta hasta cierto grado de exactitud, de manera que, con instrumentos *muy delicados*, un observador hábil puede medir la dif de nivel entre dos puntos cercanos, como la base y el extremo de una torre, con una aproximación de 50 cents á un metro. Pero si transcurre un intervalo de tiempo corto, como de algunas horas, entre las dos observaciones, pueden ocurrir tales cambios en la condición de la atmósfera, que puede resultar de las observaciones la punta superior de la torre más baja que su base, ó por lo menos una incertidumbre de tres ó seis metros, y esto puede ocurrir sin cambios perceptibles en la atmósfera. Cada vez que sea posible, debe situarse una persona en cada estación para observar en ambos puntos al mismo tiempo. Las observaciones hechas por una sola persona, en puntos á muchos kilómetros de distancia, y en diferentes estados de la atmósfera, son de poco valor. El término medio de muchas observaciones hechas en varios días, semanas ó meses, practicadas cuando está el aire sin aparentes perturbaciones, darán aproximaciones tolerables. En los trópicos, la diferencia de presión atmosférica es mucho menor que en otras regiones, variando rara vez y á lo sumo 12 milímetros en cualquier lugar. En estas regiones este método es más regular y por tanto menos susceptible de producir errores. Sin embargo, el barómetro, especialmente si es un aneroide, puede serle muy útil al ingeniero civil en casos donde no se requiere una gran exactitud. Cami-

nando á la ligera de un punto á otro, y repitiendo las operaciones, puede juzgarse cuál de dos cimas de cerros es la más elevada. Un observador cuidadoso, yendo algunos kilómetros adelante de una partida de ingenieros que practiquen un estudio, puede disminuirles mucho el trabajo, especialmente en los terrenos quebrados, escogiendo con anticipación la zona que deben seguir. Las referencias hechas de que se han efectuado nivelaciones de montañas elevadas por diferentes observadores con diferentes aneroides, y han concordado hasta algunos centímetros de aproximación, y de que se ha determinado la pendiente exacta de una vía férrea haciendo lecturas de barómetro dentro de un coche, no es creíble sino por personas ignorantes de la materia. Estos resultados se obtienen solamente por casualidad.

Si es posible, las observaciones en diferentes lugares deben hacerse á la misma hora del día para disminuir los efectos de los flujos y reflujos atmosféricos diarios; y en casos importantes, debe tomarse un memorándum del año, mes, día y hora, lo mismo que del estado del tiempo, dirección del viento, latitud del lugar, etc.

**Los efectos de la latitud** no están incluídos en ninguna de nuestras fórmulas. Si se necesitan pueden hallarse en la Regla 2, pág. 328. Deben hacerse algunas otras correcciones cuando se desea una exactitud grande; pero éstas requieren tablas extensas.

Al hacer exploraciones rápidas para un ferrocarril puede prescindirse de estos detalles, porque no se espera alcanzar tanta exactitud, sino que, por el contrario, á menudo ocurrirán errores de 25 centímetros á 3, ó más metros en una altura de 30 ó más metros.

**Como término medio aproximado** podemos suponer que el barómetro desciende 2½ milímetros por cada 27 metros de elevación sobre el nivel del mar hasta la altura de 300 metros; pero es el hecho que la proporción en que desciende disminuye continuamente á medida que se sube, de tal manera que á la altura de 1,609 metros el barómetro baja 2½ milímetros por cada 62m más ó menos. La tabla 2 indica la proporción verdadera.

## Determinación de la diferencia de altura entre dos puntos *.

*Regla* 1.ª Tómense las lecturas del barómetro y del termómetro (Fahrenheit) **en la sombra** en ambas estaciones, súmense las dos lecturas del barómetro y divídase la suma por 2 para obtener el término medio que llamamos *b*. Hágase lo mismo con las dos lecturas del termómetro y llámese *t* el término medio. Réstese la lectura menor del barómetro de la mayor y llámese la dif *d*, luego multiplíquese entre sí esta dif *d*; el número de la próxima tabla n.º 1 opuesto á *t* y el número constante 30. Divídase el producto por *b*, es decir:

$$\text{Altura en pies} = \frac{\text{Dif } (d) \text{ del barómetro} \times \text{Número tabular opuesto al término medio } (t) \text{ del termómetro} \times \text{Número constante } 30}{\text{Término medio } (b) \text{ del barómetro.}}$$

**Ej.**: Lectura del barómetro en la estación baja 26.64 pulgs; y en la estación alta 20.82. Del termómetro en la estación baja 70°, en la alta 40°.

¿Cuál es la diferencia de la altura entre las dos estaciones?

| | | | |
|---|---|---|---|
| Barómetro, | 26.64 | Termómetro, | 70° |
| — | 20.82 | — | 40° |
| | 2)47.46 | | 2)110° |
| | 23.73 = término medio del barómetro ó *b*. | | 55° = término medio del termómetro = *t*. |

El número tabular opuesto á 55° es 917.2.

Ahora, 26.64 — 20.82 = 5.82, diferencia de baróm; ó *d*. Se tiene entonces:

$$\text{Altura en pies} = \frac{5.82 \times 917.2 \times 30}{23.73 \text{ (ó } b)} = \frac{160143.12}{23.73} = \text{pies } 6{,}748.5$$

Cuando se quiera más exactitud hágase la corrección por medio de la tabla que sigue después, titulada **corrección por latitud.**

**El tornillo que está en la parte posterior** de los aneroides sirve para corregir el índice por medio de un barómetro normal. Después de haber hecho esto,

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* *N. del T.* — Después de ésta damos las fórmulas y tablas en sistema métrico.

no se las debe tocar más. En algunos instrumentos mandados á hacer especialmente con este objeto, este tornillo puede emplearse también para volver á su lugar el índice, después de hacer subido á una elevación tan grande, que el índice haya llegado al límite extremo de la graduación. Después de haberlo vuelto á su lugar, las indicaciones que de nuevo dé para mayores elevaciones deben agregarse á las obtenidas antes de restituirlo á su puesto.

**TABLA 1.ª para la Regla 1.ª**

| Término medio del termómetro | Número. | Término medio del termómetro | Número. | Término medio del termómetro | Número. | Término medio del termómetro | Número. |
|---|---|---|---|---|---|---|---|
| 0° | 801.1 | 30° | 864.4 | 60° | 927.7 | 90° | 991.0 |
| 1 | 803.2 | 31 | 866.5 | 61 | 929.8 | 91 | 993.1 |
| 2 | 805.3 | 32 | 868.6 | 62 | 931.9 | 92 | 995.2 |
| 3 | 807.4 | 33 | 870.7 | 63 | 934.0 | 93 | 997.3 |
| 4 | 809.5 | 34 | 872.8 | 64 | 936.1 | 94 | 999.4 |
| 5 | 811.7 | 35 | 874.9 | 65 | 938.2 | 95 | 1001.6 |
| 6 | 813.8 | 36 | 877 0 | 66 | 940.3 | 96 | 1003.7 |
| 7 | 815.9 | 37 | 879.2 | 67 | 942.4 | 97 | 1005.8 |
| 8 | 818.0 | 38 | 881 3 | 68 | 944.5 | 98 | 1007.9 |
| 9 | 820.1 | 39 | 883.4 | 69 | 946.7 | 99 | 1010.0 |
| 10 | 822 2 | 40 | 885 4 | 70 | 948.8 | 100 | 1012.1 |
| 11 | 824.3 | 41 | 887.5 | 71 | 950.9 | 101 | 1014.2 |
| 12 | 826.4 | 42 | 889.6 | 72 | 953.0 | 102 | 1016.3 |
| 13 | 828.5 | 43 | 891.7 | 73 | 955.1 | 103 | 1018.4 |
| 14 | 830 6 | 44 | 893.8 | 74 | 957.2 | 104 | 1020.5 |
| 15 | 832.8 | 45 | 896.0 | 75 | 959.3 | 105 | 1022.7 |
| 16 | 834.9 | 46 | 898.1 | 76 | 961.4 | 106 | 1024.8 |
| 17 | 837.0 | 47 | 900.2 | 77 | 963.5 | 107 | 1026.9 |
| 18 | 839.1 | 48 | 902.3 | 78 | 965.6 | 108 | 1029.0 |
| 19 | 841.2 | 49 | 904.5 | 79 | 967 7 | 109 | 1031.1 |
| 20 | 843.3 | 50 | 906.6 | 80 | 969 9 | 110 | 1033.2 |
| 21 | 845.4 | 51 | 908.7 | 81 | 972.0 | 111 | 1035.3 |
| 22 | 847.5 | 52 | 910.8 | 82 | 974 1 | 112 | 1037.4 |
| 23 | 849.6 | 53 | 913 0 | 83 | 976.2 | 113 | 1039.5 |
| 24 | 851.8 | 54 | 915.1 | 84 | 978.3 | 114 | 1041.6 |
| 25 | 853.9 | 55 | 917.2 | 85 | 980.4 | 115 | 1043.8 |
| 26 | 856.0 | 56 | 919.3 | 86 | 982.6 | 116 | 1045.9 |
| 27 | 858.1 | 57 | 921 4 | 87 | 984.7 | 117 | 1048.0 |
| 28 | 860.2 | 58 | 923.5 | 88 | 986.8 | 118 | 1050.1 |
| 29 | 862.3 | 59 | 925.6 | 89 | 988.9 | 119 | 1052.2 |

(*Advertencia del T.* — La fórmula análoga á la usada en la Regla 1.ª, y que debe emplearse cuando se usen las medidas barométricas en milímetros y las temperaturas en grados centígrados, es la siguiente:

$$\text{Altura en metros} = \frac{\text{diferencia } (d) \text{ del barom en milímetros} \times \text{número opuesto á la media temperatura en grados C} \times \text{número constante } 9.1437}{\text{media } (b) \text{ barométrica en milímetros.}}$$

Para buscar el número opuesto á la temperatura *en centígrados* se usará la tabla siguiente 1*a*.)

**TABLA 1ª para la advertencia del traductor á la Regla 1.ª**

| Media del termómetro. | Número. | Media del termómetro. | Número. | Media del termómetro. | Número. |
|---|---|---|---|---|---|
| — 16 | 807.8 | 5 | 887.5 | 26 | 967.3 |
| — 15 | 811.6 | 6 | 891.3 | 27 | 971.1 |
| — 14 | 815.4 | 7 | 895.1 | 28 | 974.8 |
| — 13 | 819.1 | 8 | 898.8 | 29 | 978.6 |
| — 12 | 822.9 | 9 | 902.6 | 30 | 982.6 |
| — 11 | 826.8 | 10 | 906.6 | 31 | 986.4 |
| — 10 | 830.6 | 11 | 910.4 | 32 | 990.2 |
| — 9 | 834.5 | 12 | 914.2 | 33 | 993.9 |
| — 8 | 838.3 | 13 | 917.9 | 34 | 997.7 |
| — 7 | 842.0 | 14 | 921.7 | 35 | 1001.6 |
| — 6 | 845.8 | 15 | 925.6 | 36 | 1005.4 |
| — 5 | 849.6 | 16 | 929.4 | 37 | 1009.2 |
| — 4 | 853.4 | 17 | 933.2 | 38 | 1012.9 |
| — 3 | 857.2 | 18 | 936.9 | 39 | 1016.7 |
| — 2 | 860.9 | 19 | 940.7 | 40 | 1020.5 |
| — 1 | 864.7 | 20 | 944.5 | 41 | 1024.3 |
| 0 | 868.6 | 21 | 948.3 | 42 | 1028.1 |
| 1 | 872.4 | 22 | 952.1 | 43 | 1031.8 |
| 2 | 876.2 | 23 | 955.8 | 44 | 1035.6 |
| 3 | 879.9 | 24 | 959.6 | 45 | 1039.5 |
| 4 | 883.7 | 25 | 963.5 | 46 | 1043.3 |

*Regla* 2.ª **La corta y aproximada regla de Belville** es la que mejor se adapta á trabajos rápidos en el campo, á saber: súmense solamente las dos lecturas del barómetro; búsquese también la diferencia de las dos lecturas; y luego establézcase la proporción siguiente: **la suma de las dos lecturas es á su diferencia como 55,000 pies es á la altura buscada.**

(*Advertencia del T.* — La regla equivalente á ésta para usar el sistema métrico es la siguiente:

Suma de las alturas barométricas en milímetros : diferencia de las mismas en mm = 16759 : á la altura que se busca en metros.

Pero este procedimiento da errores de alguna magnitud.)

**Corrección por latitud.** Ésta se omite generalmente cuando no se requiere una exactitud grande. Para hacer esta corrección, búsquese primero la diferencia de nivel por las reglas anteriores, luego divídase ésta por el número correspondiente de la tabla siguiente que se halla opuesto á la latitud del lugar. (Si las dos estaciones están á diferentes latitudes, tómese el término medio de ellas.) *Agréguese* el cociente á la dif de altura si la latitud es *menor* de 45°. *Réstesela* si la latitud es *mayor* de 45°. Si la latitud es de 45° no se necesita corrección.

**Tabla de corrección por latitud.**

| Lat. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0° | 352 | 14° | 399 | 28° | 630 | 42° | 3367 | 54° | 1140 | 68° | 490 |
| 2 | 354 | 16 | 416 | 30 | 705 | 44 | 10101 | 56 | 941 | 70 | 460 |
| 4 | 356 | 18 | 436 | 32 | 804 | 45 | ∞ | 58 | 804 | 72 | 436 |
| 6 | 360 | 20 | 460 | 34 | 941 | 46 | 10101 | 60 | 705 | 74 | 416 |
| 8 | 367 | 22 | 490 | 36 | 1140 | 48 | 3367 | 62 | 630 | 76 | 399 |
| 10 | 375 | 24 | 527 | 38 | 1458 | 50 | 2028 | 64 | 572 | 78 | 386 |
| 12 | 386 | 26 | 572 | 40 | 2028 | 52 | 1458 | 66 | 527 | 80 | 375 |

(*Advertencia del T.* — Usando la fórmula en metros, como dice nuestra advertencia anterior, se usa esta misma tabla de latitudes; buscando primero la diferencia en metros por la fórmula, luego dividiendo este número de metros por el número correspondiente de la tabla y agregando el cociente á dicha altura en metros si la latitud es menor de 5°, y restándolo, si es mayor.)

## Nivelación por medio del barómetro, ó por la temperatura de ebullición del agua.

*Regla* 3.ª La tabla que sigue, n.º 2, nos pone en capacidad de medir alturas, sea por medio del agua hirviendo, ó por el barómetro. La tercera columna da las alturas aproximadas sobre el nivel del mar, correspondientes á diferentes alturas, ó lecturas del barómetro, y á los diferentes grados del termómetro C, á que el agua hierve al aire libre. Así, cuando el barómetro en una atmósfera tranquila está á 611.6 mm, ó cuando el agua pura de lluvia ó destilada hierve á la temperatura de 93°86, el lugar se halla más ó menos á 1,757 m sobre el nivel del mar, como lo dice la tabla. Es, por tanto, muy fácil encontrar la *diferencia* de altura de dos lugares. Para ello, tómense de la tabla próxima, n.º 2, las alturas opuestas á las dos temperaturas de ebullición, ó á las dos lecturas del barómetro; réstese la opuesta á la lectura más baja, de la opuesta á la lectura más alta, y el residuo será la dif de altura buscada, algo aproximada. Para hacerlo más exacto, súmense las dos lecturas del termómetro y divídase la suma por 2 para obtener el término medio. En la tabla de corrección por temperatura, pág. 329 *b*, tómese el número opuesto á este término medio. Multiplíquese la altura aproximada que se acaba de hallar por este número tabular, y hágase entonces la corrección por latitud si fuere necesario.

Ej. : Supongamos que en la estación baja marca el barómetro 676.6 mm y en la alta 536.9. El termómetro en la baja, 21°1 C; en la alta, 4°4. ¿Cuál es la dif de altura de las dos estaciones?

| | | Altura. |
|---|---|---|
| Las alturas tabulares son : | para 536.9..... | 2,798.0 m. |
| | para 676.6..... | 949.4 m. |
| | Aproximadamente... | 1,848.6 m. |

Para corregir ésta tenemos : $\frac{21.1+4.4}{2} = 12°7$, término medio, y, en la tabla, pág. 329 *b*, corresponde á 12°7... 1.048. Por tanto, $1,848.6 \times 1.048 = 1937.33$ m es la altura aproximada que se busca.

A 21° C el agua pura hierve á medio grado C menos para una altura, por término medio, de 160 m sobre el nivel del mar, hasta una altura de 800 m. A 1,600 m, medio grado en la temperatura de ebullición, corresponde á 164 m. En la tabla 2 el término medio de las dos temperaturas en las dos estaciones, se supone de 0° C, y para esta dif no se requiere corección, por eso el número tabular opuesto á 0° es 1. Esta dif en la *temperatura de ebullición* producida por el cambio de altura no debe confundirse con la de la atmósfera debida á la misma causa. El aire se enfría más á medida que se sube sobre el nivel del mar, y muy aproximadamente, á razón de un grado C por cada 110 m.

**La tabla que sigue** (en lo que se refiere á las alturas barométricas) fué deducida por el autor, del trabajo sobre el barómetro, del teniente coronel R. S. Williamson (U. S. army).

**Tabla 2.** (*Obs. del T.* — Convertida al sistema métrico.) **Nivelación barométrica ó termométrica.** Se supone la temperatura de 0° C. Si no, multip altura barométrica como indica la tabla próxima.

| Temp ebulli-ción. | Lectura baróme-tro. | Altura sobre el mar. | Temp ebulli-ción. | Lectura baróme-tro. | Altura sobre el mar. | Temp ebulli-ción. | Lectura baróme-tro. | Altura sobre el mar. | Temp ebulli-ción. | Lectura baróme-tro. | Altura sobre el mar. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ctigr. | mm. | met. | centíg. | mm. | met. | centíg. | mm. | met. | centíg. | mm. | metros. |
| 84.40 | 426.5 | 4639 | 88.46 | 499.4 | 3378 | 92.53 | 582.4 | 2148 | 96.59 | 675.4 | 964.4 |
| 84.45 | 427.5 | 4620 | 88.52 | 500.4 | 3362 | 92.58 | 583.7 | 2131 | 96.6 | 676.6 | 949.4 |
| 84.51 | 428.2 | 4606 | 88.57 | 501.4 | 3345 | 92.64 | 584.7 | 2117 | 96.65 | 677.9 | 934.5 |
| 84.56 | 429.3 | 4587 | 88.63 | 502.4 | 3329 | 92.69 | 586.0 | 2099 | 96.71 | 679.4 | 916.5 |
| 84.62 | 430.0 | 4573 | 88.68 | 503.4 | 3313 | 92.7 | 587.0 | 2086 | 96.76 | 680.7 | 901.6 |
| 84.67 | 431.0 | 4557 | 88.74 | 504.7 | 3293 | 92.75 | 588.3 | 2068 | 96.82 | 682.2 | 883.6 |
| 84.73 | 431.8 | 4540 | 88.79 | 506.0 | 3273 | 92.81 | 589.5 | 2051 | 96.87 | 683.5 | 868.7 |
| 84.78 | 432.8 | 4521 | 88.8 | 507.0 | 3257 | 92.86 | 590.8 | 2034 | 96.93 | 685.0 | 851.0 |
| 84.84 | 433.8 | 4502 | 88.85 | 508.0 | 3241 | 92.92 | 592.1 | 2017 | 96.98 | 686.3 | 836.0 |
| 84.89 | 434.8 | 4484 | 88.91 | 509.3 | 3221 | 92.97 | 593.3 | 1999 | 97.04 | 687.8 | 818.4 |
| 85.00 | 435.9 | 4465 | 88.96 | 510.5 | 3201 | 93.03 | 594.4 | 1986 | 97.09 | 689.1 | 803.8 |
| 85.05 | 436.9 | 4446 | 89.02 | 511.6 | 3185 | 93.08 | 595.6 | 1969 | 97.2 | 690.4 | 789.1 |
| 85.11 | 437.6 | 4433 | 89.07 | 512.6 | 3169 | 93.14 | 596.6 | 1955 | 97.25 | 691.6 | 774.2 |
| 85.16 | 438.6 | 4414 | 89.13 | 513.6 | 3153 | 93.19 | 597.9 | 1938 | 97.31 | 693.2 | 756.8 |
| 85.22 | 439.7 | 4395 | 89.18 | 514.9 | 3134 | 93.3 | 599.2 | 1921 | 97.36 | 694.4 | 742.2 |
| 85.27 | 440.7 | 4377 | 89.24 | 515.9 | 3118 | 93.35 | 600.5 | 1904 | 97.42 | 696.0 | 724.5 |
| 85.33 | 441.4 | 4363 | 89.29 | 516.9 | 3102 | 93.41 | 601.7 | 1888 | 97.47 | 697.2 | 709.9 |
| 85.38 | 442.5 | 4345 | 89.4 | 517.9 | 3087 | 93.46 | 603.0 | 1870 | 97.53 | 698.7 | 692.5 |
| 85.44 | 443.5 | 4327 | 89.45 | 518.9 | 3071 | 93.52 | 604.3 | 1854 | 97.58 | 700. | 677.9 |
| 85.49 | 444.5 | 4308 | 89.51 | 520.2 | 3051 | 93.57 | 605.5 | 1837 | 97.64 | 701.5 | 660.5 |
| 85.5 | 445.0 | 4290 | 89.56 | 521.5 | 3032 | 93.63 | 606.8 | 1820 | 97.69 | 702.8 | 646.2 |
| 85.55 | 446.5 | 4272 | 89.62 | 522.5 | 3016 | 93.68 | 608.1 | 1803 | 97.7 | 704.3 | 628.8 |
| 85.61 | 447.5 | 4254 | 89.67 | 523.5 | 3001 | 93.74 | 609.1 | 1790 | 97.75 | 705.6 | 614.5 |
| 85.66 | 448.6 | 4235 | 89.73 | 524.5 | 2985 | 93.79 | 610.4 | 1774 | 97.81 | 707.1 | 597.1 |
| 85.72 | 449.6 | 4217 | 89.78 | 525.5 | 2970 | 93.8 | 611.6 | 1757 | 97.86 | 708.4 | 582.8 |
| 85.77 | 450.6 | 4199 | 89.84 | 526.5 | 2954 | 93.85 | 612.9 | 1740 | 97.92 | 709.9 | 565.7 |
| 85.83 | 451.6 | 4181 | 89.89 | 527.5 | 2939 | 93.91 | 614.2 | 1724 | 97.97 | 711.2 | 551.4 |
| 85.88 | 452.6 | 4163 | 90.0 | 528.8 | 2920 | 93.96 | 615.4 | 1707 | 98.03 | 712.7 | 534.3 |
| 85.94 | 453.6 | 4145 | 90.05 | 530.1 | 2900 | 94.02 | 616.7 | 1691 | 98.08 | 714.0 | 520.0 |
| 85.99 | 454.7 | 4128 | 90.11 | 531.1 | 2885 | 94.07 | 618.0 | 1675 | 98.14 | 715.3 | 502.9 |
| 86.1 | 455.4 | 4114 | 90.16 | 532.4 | 2866 | 94.13 | 619.2 | 1658 | 98.19 | 717.0 | 486.2 |
| 86.15 | 456.4 | 4096 | 90.22 | 533.4 | 2851 | 94.18 | 620.5 | 1642 | 98.3 | 718.6 | 469.1 |
| 86.21 | 457.2 | 4083 | 90.27 | 534.7 | 2832 | 94.24 | 621.8 | 1625 | 98.35 | 720.1 | 452.0 |
| 86.26 | 458.2 | 4065 | 90.33 | 535.7 | 2817 | 94.29 | 623.1 | 1609 | 98.41 | 721.4 | 438.0 |
| 86.32 | 459.2 | 4047 | 90.38 | 536.9 | 2798 | 94.4 | 624.3 | 1593 | 98.46 | 722.6 | 424.0 |
| 86.37 | 460.2 | 4030 | 90.44 | 538.0 | 2783 | 94.45 | 625.6 | 1576 | 98.52 | 724.1 | 407.2 |
| 86.43 | 461.3 | 4012 | 90.49 | 539.0 | 2768 | 94.51 | 626.9 | 1560 | 98.57 | 725.4 | 393.2 |
| 86.48 | 462.3 | 3995 | 90.5 | 540. | 2752 | 94.56 | 628.1 | 1544 | 98.63 | 726.9 | 376.5 |
| 86.54 | 463.3 | 3977 | 90.55 | 541.3 | 2734 | 94.62 | 629.4 | 1528 | 98.68 | 728.2 | 362.4 |
| 86.59 | 464.3 | 3960 | 90.61 | 542.3 | 2719 | 94.67 | 630.7 | 1512 | 98.74 | 729.7 | 346.0 |
| 86.6 | 465.3 | 3942 | 90.66 | 543.6 | 2700 | 94.73 | 631.9 | 1496 | 98.79 | 731.3 | 328.9 |
| 86.65 | 466.3 | 3925 | 90.72 | 544.6 | 2685 | 94.78 | 633.2 | 1480 | 98.8 | 732.8 | 312.4 |
| 86.71 | 467.4 | 3907 | 90.77 | 545.8 | 2667 | 94.84 | 634.5 | 1464 | 98.85 | 734.3 | 295.6 |
| 86.76 | 468.4 | 3890 | 90.83 | 546.9 | 2652 | 94.89 | 635.8 | 1448 | 98.91 | 735.8 | 279.2 |
| 86.82 | 469.4 | 3873 | 90.88 | 548.1 | 2633 | 95.0 | 637.0 | 1432 | 98.96 | 737.3 | 262.8 |
| 86.87 | 470.4 | 3855 | 90.94 | 549.1 | 2618 | 95.05 | 638.3 | 1416 | 99.02 | 738.9 | 246.3 |
| 86.93 | 471.4 | 3838 | 90.99 | 550.4 | 2600 | 95.11 | 639.6 | 1400 | 99.07 | 740.4 | 229.8 |
| 86.98 | 472.4 | 3821 | 91.1 | 551.4 | 2585 | 95.16 | 640.8 | 1384 | 99.13 | 741.7 | 216.1 |
| 87.04 | 473.4 | 3804 | 91.15 | 552.7 | 2567 | 95.22 | 642.1 | 1368 | 99.18 | 742.9 | 202.4 |
| 87.09 | 474.5 | 3787 | 91.21 | 554.0 | 2549 | 95.27 | 643.4 | 1352 | 99.24 | 744.5 | 185.9 |
| 87.2 | 475.5 | 3769 | 91.26 | 555.2 | 2530 | 95.33 | 644.6 | 1337 | 99.29 | 745.7 | 172.2 |

**Tabla 2.** (Continuación.)

| Temp ebu-lli-ción. | Lec-tura ba-róme-tro. | Al-tura sobre el mar. | Temp ebu-lli-ción. | Lec-tura ba-róme-tro. | Al-tura sobre el mar. | Temp ebu-lli-ción. | Lec-tura ba-róme-tro. | Al-tura sobre el mar. | Temp ebu-lli-ción. | Lec-tura ba-róme-tro. | Al-tura sobre el mar. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ctígr. | mm. | met. | centíg. | mm. | met. | centíg. | mm. | met. | centíg. | mm. | metros. |
| 87.25 | 476.5 | 3752 | 91.32 | 556.3 | 2516 | 95.38 | 645.9 | 1321 | 99.4 | 747.3 | 156.0 |
| 87.31 | 477.5 | 3736 | 91.37 | 557.5 | 2497 | 95.44 | 647.4 | 1302 | 99.45 | 748.8 | 139.6 |
| 87.36 | 478.5 | 3719 | 91.43 | 558.5 | 2483 | 95.49 | 648.7 | 1286 | 99.51 | 750.3 | 123.4 |
| 87.42 | 479.5 | 3702 | 91.48 | 559.8 | 2465 | 95.5 | 650. | 1270 | 99.56 | 751.8 | 107.2 |
| 87.47 | 480.6 | 3685 | 91.54 | 560.8 | 2450 | 95.55 | 651.3 | 1255 | 99.62 | 753.1 | 93.8 |
| 87.53 | 481.6 | 3668 | 91.59 | 562.1 | 2432 | 95.61 | 652.8 | 1236 | 99.67 | 754.6 | 77.7 |
| 87.58 | 482.6 | 3651 | 91.6 | 563.1 | 2418 | 95.66 | 654.3 | 1218 | 99.73 | 756.2 | 61.5 |
| 87.64 | 483.6 | 3634 | 91.65 | 564.4 | 2400 | 95.72 | 655.6 | 1202 | 99.78 | 757.7 | 45.4 |
| 87.69 | 484.6 | 3617 | 91.71 | 565.6 | 2382 | 95.77 | 656.8 | 1187 | 99.84 | 758.9 | 32.0 |
| 87.7 | 485.9 | 3596 | 91.76 | 566.9 | 2364 | 95.83 | 658.1 | 1171 | 99.89 | 760.4 | 15.8 |
| 87.75 | 486.9 | 3580 | 91.82 | 567.9 | 2349 | 95.88 | 659.4 | 1156 | 100.0 | 762. | |
| 87.81 | 487.9 | 3563 | 91.87 | 569.2 | 2331 | 95.94 | 660.6 | 1140 | Bajo el nivel del mar. | | |
| 87.86 | 488.9 | 3546 | 91.93 | 570.2 | 2317 | 95.99 | 661.9 | 1125 | 100.05 | 763.5 | — 15.9 |
| 87.92 | 490. | 3530 | 91.98 | 571.5 | 2299 | 96.1 | 663.2 | 1110 | 100.11 | 765. | — 31.7 |
| 87.97 | 491.0 | 3513 | 92.04 | 572.5 | 2285 | 96.15 | 664.7 | 1091 | 100.16 | 766.6 | — 50.6 |
| 88.03 | 492.0 | 3497 | 92.09 | 573.8 | 2267 | 96.21 | 666.0 | 1076 | 100.22 | 768.1 | — 69.5 |
| 88.08 | 493.0 | 3480 | 92.2 | 575.0 | 2250 | 96.26 | 667.5 | 1058 | 100.27 | 769.6 | — 79.6 |
| 88.14 | 494.0 | 3464 | 92.25 | 576.3 | 2232 | 96.32 | 668.8 | 1043 | 100.33 | 770.9 | — 92.7 |
| 88.19 | 495.0 | 3447 | 92.31 | 577.6 | 2215 | 96.37 | 670. | 1027 | 100.38 | 772.4 | —117.6 |
| 88.3 | 496.3 | 3427 | 92.36 | 578.9 | 2197 | 96.43 | 671.3 | 1012 | 100.44 | 773.9 | —136.5 |
| 88.35 | 497.3 | 3411 | 92.42 | 580.1 | 2179 | 96.48 | 672.6 | 997 | 100.49 | 775.5 | —152.1 |
| 88.41 | 498.3 | 3394 | 92.47 | 581.4 | 2162 | 96.54 | 674.1 | 979 | 100.5 | 777.0 | —155.7 |

**Corrección de temperatura, en conexión con la Regla 3, cuando se requiere gran exactitud.**

| Tem-peratura media. | Multipli-cador. | Tem-peratura media. | Multipli-cador. | Tempe-ratura media. | Multipli-cador. | Tempe-ratura media. | Multipli cador. |
|---|---|---|---|---|---|---|---|
| — 16° | .935 | — 2° | .991 | 12° | 1.045 | 27° | 1.101 |
| — 15° | .939 | — 1° | .996 | 13° | 1.049 | 28° | 1.105 |
| — 14° | .944 | 0° | 1.000 | 14° | 1.052 | 29° | 1.108 |
| — 13° | .949 | 1° | 1.004 | 16° | 1.059 | 30° | 1.112 |
| — 12° | .953 | 2° | 1.007 | 17° | 1.063 | 31° | 1.117 |
| — 11° | .957 | 3° | 1.011 | 18° | 1.067 | 32° | 1.120 |
| — 10° | .962 | 4° | 1.014 | 19° | 1.071 | 33° | 1.124 |
| — 9° | .967 | 6° | 1.021 | 20° | 1.074 | 34° | 1.127 |
| — 8° | .971 | 7° | 1.025 | 21° | 1.079 | 36° | 1.134 |
| — 7° | .974 | 8° | 1.029 | 22° | 1.082 | 37° | 1.139 |
| — 6° | .977 | 9° | 1.032 | 23° | 1.086 | 38° | 1.143 |
| — 5° | .980 | 10° | 1.036 | 24° | 1.089 | 39° | 1.146 |
| — 3° | .988 | 11° | 1.041 | 26° | 1.098 | 40° | 1.150 |

# EL SONIDO

**La velocidad del sonido** al aire libre y tranquilo se ha determinado por medio de experimentos y es muy aproximadamente de 1,090 pies ó 332 metros por segundo, cuando la temperatura es de 0° centígrado ó 32° F. Para cada grado Fahr de aumento en la temperatura, la velocidad aumenta por segundo (de 15 á 38 cm) según los diferentes autores. Calculando el aumento en un pie por segundo para cada grado F ó sean 55 cm por cada grado centígrado (lo que está bastante de acuerdo con los cálculos teóricos), tenemos á

| | | | |
|---|---|---|---|
| —30° | Fahr | 1,030 | pies por seg |
| —20° | — | 1,040 | — — |
| —10° | — | 1,050 | — — |
| 0° | — | 1,060 | — — |
| 10° | — | 1,070 | — — |
| 20° | — | 1,080 | — — |
| 32° | — | 1,092 | — — |
| 40° | — | 1,100 | — — |
| 50° | — | 1,110 | — — |
| 60° | — | 1,120 | — — |
| 70° | — | 1,130 | — — |
| 80° | — | 1,140 | — — |
| 90° | — | 1,150 | — — |
| 100° | — | 1,160 | — — |
| 110° | — | 1,170 | — — |
| 120° | — | 1,180 | — — |

(*N. del T.* — La tabla siguiente es para temperaturas en C y velocidades en metros.)

| Temperatura del aire en grados C. | Metros por segundo. | Dif. |
|---|---|---|
| —34° | 313.71 | 3.21 |
| —30° | 316.92 | 2.74 |
| —24° | 319.66 | 3.53 |
| —18° | 323.19 | 3.05 |
| —12° | 326.24 | 3.27 |
| — 6° | 329.51 | 3.32 |
| 0° | 332.83 | 2.23 |
| 4° | 335.06 | 3.27 |
| 10° | 338.33 | 3.27 |
| 16° | 341.60 | 3.33 |
| 22° | 344.93 | 2.17 |
| 26° | 347.10 | 3.30 |
| 32° | 350.40 | 3.27 |
| 38° | 353.67 | 2.78 |
| 43° | 356.45 | |

Para las temperaturas intermedias se deduce la velocidad del sonido por una simple proporción. Ej. : ¿Cuál será la velocidad para 18 ½°? Se tendrá que, si para 6 grados de temperatura (22-16) hay una diferencia de 3.33 mets, para 2½ cuánto habrá. Es decir :

6 : 3.33 : : 2½ : $x$, de donde $x$=1.39; luego, 341.60+1.39=342.99 será la velocidad á 18 ½ grados C.

Cuando el aire está en calma la neblina ó la lluvia no afectan apreciablemente los resultados; pero sí los vientos. Los sonidos fuertes parece que andan algo más ligero que los débiles. El alerta de los centinelas se ha oído á través de aguas tranquilas en una noche de calma, á una distancia de 17 kilómetros, y un cañón á 32 kilómetros. Los sonidos producidos á intervalos de $^1/_{10}$ de segundo no se oyen separados sino como ligados unos á otros. Las distancias á que se puede oir un orador colocándose uno enfrente, á un lado ó detrás de él, están más ó menos como los números 4, 3 y 1.

El Dr. Carlos M. Cresson informó al autor que había hallado, por ensayos diferentes en Filadelfia, que en un tubo matriz de gas, de 50 centímetros de diámetro, y de 4.876 mets de largo, colocado en la tierra y cubierto, pero sin contener

gas, y con un codo horizontal de 90° y de 12 mets de radio, el sonido de un tiro de pistola anduvo más ó menos los 4,876 mets en 16 segundos, ó 305 mets por segundo. La llegada del sonido era casi imperceptible; pero se hizo sensible por el movimiento comunicado á un diafragma de papel de seda puesto en la extremidad del tubo.

**Dos botes anclados** á alguna distancia uno de otro pueden servir como línea de base en triangulaciones hechas con objetos situados á lo largo de la costa, hallando primero la distancia entre dichos botes por medio de disparos hechos á bordo de uno de ellos *.

**En el agua la velocidad** es más ó menos de 1,435 mets por segundo, ó como 4 veces la del aire. **En las maderas** es de 10 á 16 veces, y en **los metales** de 4 á 16 veces mayor que en el aire, según algunos autores.

## Dilatación aproximada de los sólidos por el calor; y su temperatura de fusión en F y C †.

| | Para 180° Fah. ó 100° centígrados. | | Temperaturas de fusión. | |
|---|---|---|---|---|
| | 1 8 de pulgada en 21.14 pies | 1 milím. en 2.15mets | Fah. | Centg. |
| Ladrillos refractarios | | | | |
| Granito de | 10.85 | 1.10 | | |
| Granito á | 13.20 | 1.34 | | |
| Varillas de vidrio | 12.81 | 1.30 | | |
| Tubo de vidrio | 12.40 | 1.26 | | |
| Crown glass | 12.24 | 1.24 | | |
| Vidrios planos | 12.13 | 1.23 | | |
| Platino | 12.08 | 1.23 | 4593 | 2536 |
| Mármol granular, blanco, seco | 10.00 | 1.02 | | |
| — — — húmedo. | 7.41 | 0.75 | | |
| — negro compacto | 23.44 | 2.38 | | |
| Antimonio | 9.63 | 0.98 | 955 | 513 |
| Hierro fundido | 9.38 | 0.95 | 1920 á | 1050 á |
| Pizarra | 10.00 | 1.02 | 2800 | 1928 |
| Acero | 8.75 | 0.89 | 2370 á | 1300 á |
| — cimentado | 9.25 | 0.94 | 2550 | 1400 |
| — sin templar | 9.69 | 0.98 | | |
| — templado amarillo | 7.60 | 0.77 | | |
| — duro | 8.50 | 0.86 | | |
| — destemplado | 8.54 | 0.87 | | |
| Hierro cilindrado | 8.68 | 0.88 | 3000 á | 1650 á |
| — blando forjado | 8.53 | 0.87 | 3500 | 1928 |
| Alambre de hierro | 8.55 | 0.87 | | |
| Bismuto | 7.50 | 0.76 | 506 | 273 |
| Oro destemplado | 7.12 | 0.72 | 2016 | 1103 |
| Cobre término medio. | 6.04 | 0.61 | 2000 | 1094 |
| Piedra arenisca ‡ | 5.98 | 0.61 | | |
| Bronce término medio. | 5.66 | 0.57 | 1873 | 1024 |
| Alambre de bronce | 5.45 | 0.55 | | |
| Plata | 5.50 | 0.56 | 1861 | 1017 |
| Estaño término medio. | 5.08 | 0.52 | 444 | 229 |
| Plomo — | 3.66 | 0.37 | 612 | 322 |
| Peltre | 4.56 | 0.46 | | |
| Cinc (más que todos los otros metales) | 3.58 | 0.36 | 680 á 772 | 360 á 411 |
| Pino blanco | 25.49 | 2.59 | | |

* *N. del T.* — Mejor es hacer disparos recíprocos y tomar el término medio de las dist obtenidas.

† **La temperatura de fusión** es muy incierta; damos el promedio de las mejores autoridades.

‡ La tabla enseña que las **contracciones** y **expansiones** de las piedras pueden producir grietas en el invierno, y en verano triturar la argamasa entre las piedras, en bloques gruesos y largos.

Llamando $a_c$, ó $a_f$, al **coeficiente de dilatación lineal** de una substancia = á la fracción de la longitud original en que aumenta ó disminuye una barra prismática de esa substancia para cada cambio en su temperatura de un grado C ó F respectivamente, se tendrá que, como cada grado centígrado vale $^9/_5$ de un grado F y cada F $^5/_9$ de un centígrado, el coeficiente de dilatación para un grado C será los $^9/_5$ del coeficiente para uno F, y á la inversa, el coeficiente para uno F será los $^5/_9$ de uno C; así : $a_c = {}^9/_5\, a_f$; $a_f = {}^5/_9\, a_c$.

El coeficiente de dilatación es prácticamente constante para las temperaturas ordinarias.

**La fuerza** ejercida longitudinalmente por semejante barra en su dilatación ó contracción, es $P = atEF$, donde $a$ = al coeficiente dicho; $t$ = cambio de temperatura en grados; E = al módulo de la elasticidad de la substancia (véase el párrafo 20 y algunos anteriores y posteriores pág. 479); F = área de la sección transversal. **El trabajo** hecho por esta fuerza, estirando ó contrayendo la barra de longitud original L á través de la longitud $l$, es $W = Pl = PLat = a^2t^2EFL$.

**El coeficiente de dilatación superficial** (relación entre el cambio de área de una superficie y su área original) = como á $2a$; y el coeficiente de dilatación cúbica = como á $3a$; suponiendo que el coeficiente de dilatación lineal es el mismo en todas direcciones.

**La madera** común al quemarse produce una temperatura de 427° á 616° C; el carbón vegetal como 1,200° C, y el de piedra como 1,310° C.

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# TERMÓMETROS

Sean C, R, F, la lectura dada en grados Celsius (centígrados), Reaumur, Fahrenheit, respectivamente. Entonces (véanse tablas 1, 2, 3, más abajo):

$$\mathbf{C} = \frac{5}{4}\,\mathrm{R} = \frac{5}{9}\,(\mathrm{F} - 32)\,;$$

$$\mathbf{R} = \frac{4}{5}\,\mathrm{C} = \frac{4}{9}\,(\mathrm{F} - 32)\,;$$

$$\mathbf{F} = \frac{9}{5}\,\mathrm{C} + 32 = \frac{9}{4}\,\mathrm{R} + 32.$$

Así, sea F = — 40. Entonces $\mathrm{C} = \frac{5}{9}\,(-40 - 32) = -40$. Para **coeficientes de dilatación**, véase pág. 331.

Bajo una temperatura más ó menos de — 37° C (= — 30 R = — 35 F, el termómetro y barómetro mercurial se hacen irregulares. El mercurio comienza á solidificarse más ó menos á — 40 C = — 32 R = — 40 F. Debajo de esta temperatura se usa el alcohol.

## TABLA 1. Fahrenheit, comparado con Centígrado y Réaumur.

| F. | C. | R. | F. | C. | R. | F. | C. | R. | F. | C. | R. | F. | C. | R. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ° | ° | ° | ° | ° | ° | ° | ° | ° | ° | ° | ° | ° | ° | ° |
| 212 | 100 | 80.0 | 158 | 70.0 | 56.0 | 104 | 40 0 | 32.0 | 50 | 10.0 | 8.0 | —8 | —19.4 | —15.6 |
| 211 | 99.4 | 79.6 | 157 | 69.4 | 55.6 | 103 | 39.4 | 31.6 | 49 | 9.4 | 7.6 | —4 | —20.0 | —16.0 |
| 210 | 98.9 | 79.1 | 156 | 68.9 | 55.1 | 102 | 38.9 | 31.1 | 48 | 8.9 | 7.1 | —5 | —20.6 | —16.4 |
| 209 | 98.3 | 78.7 | 155 | 68.3 | 54.7 | 101 | 38.3 | 30.7 | 47 | 8.3 | 6.7 | —6 | —21.1 | —16.9 |
| 208 | 97.8 | 78.2 | 154 | 67.8 | 54.2 | 100 | 37.8 | 30.2 | 46 | 7.8 | 6.2 | —7 | —21.7 | —17.3 |
| 207 | 97.2 | 77.8 | 153 | 67.2 | 53.8 | 99 | 37.2 | 29.8 | 45 | 7.2 | 5.8 | —8 | —22.2 | —17.8 |
| 206 | 96.7 | 77.3 | 152 | 66.7 | 53.3 | 98 | 36.7 | 29.3 | 44 | 6.7 | 5.3 | —9 | —22.8 | —18.2 |
| 205 | 96.1 | 76.9 | 151 | 66.1 | 52.9 | 97 | 36.1 | 28.9 | 43 | 6.1 | 4.9 | —10 | —23.3 | —18.7 |
| 204 | 95.6 | 76.4 | 150 | 65.6 | 52.4 | 96 | 35.6 | 28.4 | 42 | 5.6 | 4.4 | —11 | —23.9 | —19.1 |
| 203 | 95.0 | 76.0 | 149 | 65.0 | 52.0 | 95 | 35.0 | 28.0 | 41 | 5.0 | 4.0 | —12 | —24.4 | —19.5 |
| 202 | 94.4 | 75.6 | 148 | 64.4 | 51.6 | 94 | 34.4 | 27.6 | 40 | 4.4 | 3.6 | —13 | —25.0 | —20.0 |
| 201 | 93.9 | 75.1 | 147 | 63.9 | 51.1 | 93 | 33.9 | 27.1 | 39 | 3.9 | 3.1 | —14 | —25.6 | —20.4 |
| 200 | 93.3 | 74.7 | 146 | 63.3 | 50.7 | 92 | 33.3 | 26.7 | 38 | 3.3 | 2.7 | —15 | —26.1 | —20.9 |
| 199 | 92.8 | 74.2 | 145 | 62.8 | 50.2 | 91 | 32.8 | 26.2 | 37 | 2.8 | 2.2 | —16 | —26.7 | —21.3 |
| 198 | 92.2 | 73.8 | 144 | 62.2 | 49.8 | 90 | 32.2 | 25.8 | 36 | 2.2 | 1.8 | —17 | —27.2 | —21.8 |
| 197 | 91.7 | 73.3 | 143 | 61.7 | 49.3 | 89 | 31.7 | 25.3 | 35 | 1.7 | 1.3 | —18 | —27.8 | —22.2 |
| 196 | 91.1 | 72.9 | 142 | 61.1 | 48.9 | 88 | 31.1 | 24.9 | 34 | 1.1 | 0.9 | —19 | —28.3 | —22.7 |
| 195 | 90.6 | 72.4 | 141 | 60.6 | 48.4 | 87 | 30.6 | 24.4 | 33 | 0.6 | 0.4 | —20 | —28.9 | —23.1 |
| 194 | 90.0 | 72.0 | 140 | 60.0 | 48.0 | 86 | 30.0 | 24.0 | 32 | 0.0 | 0.0 | —21 | —29.4 | —23.6 |
| 193 | 89.4 | 71.6 | 139 | 59.4 | 47.6 | 85 | 29.4 | 23.6 | 31 | —0.6 | —0.4 | —22 | —30.0 | —24.0 |
| 192 | 88.9 | 71.1 | 138 | 58.9 | 47.1 | 84 | 28.9 | 23.1 | 30 | —1.1 | —0.9 | —23 | —30.6 | —24.4 |
| 191 | 88.3 | 70.7 | 137 | 58.3 | 46.7 | 83 | 28.3 | 22.7 | 29 | —1.7 | —1.3 | —24 | —31.1 | —24.9 |
| 190 | 87.8 | 70.2 | 136 | 57.8 | 46.2 | 82 | 27.8 | 22.2 | 28 | —2.2 | —1.8 | —25 | —31.7 | —25.3 |
| 189 | 87.2 | 69.8 | 135 | 57.2 | 45.8 | 81 | 27.2 | 21.8 | 27 | —2.8 | —2.2 | —26 | —32.2 | —25.8 |
| 188 | 86.7 | 69.3 | 134 | 56.7 | 45.3 | 80 | 26.7 | 21.3 | 26 | —3.3 | —2.7 | —27 | —32.8 | —26.2 |
| 187 | 86.1 | 68.9 | 133 | 56.1 | 44.9 | 79 | 26.1 | 20.9 | 25 | —3.9 | —3.1 | —28 | —33.3 | —26.7 |
| 186 | 85.6 | 68.4 | 132 | 55.6 | 44.4 | 78 | 25.6 | 20.4 | 24 | —4.4 | —3.6 | —29 | —33.9 | —27.1 |
| 185 | 85.0 | 68.0 | 131 | 55.0 | 44.0 | 77 | 25.0 | 20.0 | 23 | —5.0 | —4.0 | —30 | —34.4 | —27.6 |
| 184 | 84.4 | 67.6 | 130 | 54.4 | 43.6 | 76 | 24.4 | 19.6 | 22 | —5.6 | —4.4 | —31 | —35.0 | —28.0 |
| 183 | 83.9 | 67.1 | 129 | 53.9 | 43.1 | 75 | 23.9 | 19.1 | 21 | —6.1 | —4.9 | —32 | —35.6 | —28.4 |
| 182 | 83.3 | 66.7 | 128 | 53.3 | 42.7 | 74 | 23.3 | 18.7 | 20 | —6.7 | —5.3 | —33 | —36.1 | —28.9 |
| 181 | 82.8 | 66.2 | 127 | 52.8 | 42.2 | 73 | 22.8 | 18.2 | 19 | —7.2 | —5.8 | —34 | —36.7 | —29.3 |
| 180 | 82.2 | 65.8 | 126 | 52.2 | 41.8 | 72 | 22.2 | 17.8 | 18 | —7.8 | —6.2 | —35 | —37.2 | —29.8 |
| 179 | 81.7 | 65.3 | 125 | 51.7 | 41.3 | 71 | 21.7 | 17.3 | 17 | —8.3 | —6.7 | —36 | —37.8 | —30.2 |
| 178 | 81.1 | 64.9 | 124 | 51.1 | 40.9 | 70 | 21.1 | 16.9 | 16 | —8.9 | —7.1 | —37 | —38.3 | —30.7 |
| 177 | 80.6 | 64.4 | 123 | 50.6 | 40.4 | 69 | 20.6 | 16.4 | 15 | —9.4 | —7.6 | —38 | —38.9 | —31.1 |
| 176 | 80.0 | 64.0 | 122 | 50.0 | 40.0 | 68 | 20.0 | 16.0 | 14 | —10.0 | —8.0 | —39 | —39.4 | —31.6 |
| 175 | 79.4 | 63.6 | 121 | 49.4 | 39.6 | 67 | 19.4 | 15.6 | 13 | —10.6 | —8.4 | —40 | —40.0 | —32.0 |
| 174 | 78.9 | 63.1 | 120 | 48.9 | 39.1 | 66 | 18.9 | 15.1 | 12 | —11.1 | —8.9 | —41 | —40.6 | —32.4 |
| 173 | 78.3 | 62.7 | 119 | 48.3 | 38.7 | 65 | 18.3 | 14.7 | 11 | —11.7 | —9.3 | —42 | —41.1 | —32.9 |
| 172 | 77.8 | 62.2 | 118 | 47.8 | 38.2 | 64 | 17.8 | 14.2 | 10 | —12.2 | —9.8 | —43 | —41.7 | —33.3 |
| 171 | 77.2 | 61.8 | 117 | 47.2 | 37.8 | 63 | 17.2 | 13.8 | 9 | —12.8 | —10.2 | —44 | —42.2 | —33.8 |
| 170 | 76.7 | 61.3 | 116 | 46.7 | 37.3 | 62 | 16.7 | 13.3 | 8 | —13.3 | —10.7 | —45 | —42.8 | —34.2 |
| 169 | 76.1 | 60.9 | 115 | 46.1 | 36.9 | 61 | 16.1 | 12.9 | 7 | —13.9 | —11.1 | —46 | —43.3 | —34.7 |
| 168 | 75.6 | 60.4 | 114 | 45 6 | 36.4 | 60 | 15.6 | 12.4 | 6 | —14.4 | —11.6 | —47 | —43.9 | —35.1 |
| 167 | 75.0 | 60.0 | 113 | 45.0 | 36.0 | 59 | 15.0 | 12.0 | 5 | —15.0 | —12.0 | —48 | —44.4 | —35.6 |
| 166 | 74.4 | 59.6 | 112 | 44.4 | 35.6 | 58 | 14.4 | 11.6 | 4 | —15.6 | —12.4 | —49 | —45.0 | —36.0 |
| 165 | 73.9 | 59.1 | 111 | 43.9 | 35.1 | 57 | 13.9 | 11.1 | 3 | —16.1 | —12.9 | —50 | —45.6 | —36.4 |
| 164 | 73.3 | 58.7 | 110 | 43.3 | 34.7 | 56 | 13.3 | 10.7 | 2 | —16.7 | —13.3 | —51 | —46.1 | —36.9 |
| 163 | 72.8 | 58.2 | 109 | 42 8 | 34.2 | 55 | 12.8 | 10.2 | 1 | —17.2 | —13.8 | —52 | —46.7 | —37.3 |
| 162 | 72.2 | 57.8 | 108 | 42.2 | 33.8 | 54 | 12.2 | 9.8 | 0 | —17.8 | —14.2 | —53 | —47.2 | —37.8 |
| 161 | 71.7 | 57.3 | 107 | 41.7 | 33.3 | 53 | 11.7 | 9.3 | —1 | —18.3 | —14.7 | —54 | —47.8 | —38.2 |
| 160 | 71.1 | 56.9 | 106 | 41.1 | 32.9 | 52 | 11.1 | 8.9 | —2 | —18.9 | —15.1 | —55 | —48.3 | —38.7 |
| 159 | 70.6 | 56.4 | 105 | 40.6 | 32.4 | 51 | 10.6 | 8.4 | | | | | | |

## TABLA 2. Centígrado, comparado con Fahrenheit y Réaumur.

| C. | F. | R. | C. | F. | R. | C. | F. | R. | C. | F. | R. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ° | ° Exact. | ° Exact. | ° | ° Exact. | ° Exact. | ° | ° Exact. | ° Exact. | ° | ° Exact. | ° Exact. |
| 100 | 212.0 | 80.0 | 62 | 143.6 | 49.6 | 24 | 75.2 | 19.2 | —14 | 6.8 | —11.2 |
| 99 | 210.2 | 79.2 | 61 | 141.8 | 48.8 | 23 | 73.4 | 18.4 | —15 | 5.0 | —12.0 |
| 98 | 208.4 | 78.4 | 60 | 140.0 | 48.0 | 22 | 71.6 | 17.6 | —16 | 3.2 | —12.8 |
| 97 | 206.6 | 77.6 | 59 | 138.2 | 47.2 | 21 | 69.8 | 16.8 | —17 | 1.4 | —13.6 |
| 96 | 204.8 | 76.8 | 58 | 136.4 | 46.4 | 20 | 68.0 | 16.0 | —18 | —0.4 | —14.4 |
| 95 | 203.0 | 76.0 | 57 | 134.6 | 45.6 | 19 | 66.2 | 15.2 | —19 | —2.2 | —15.2 |
| 94 | 201.2 | 75.2 | 56 | 132.8 | 44.8 | 18 | 64.4 | 14.4 | —20 | —4.0 | —16.0 |
| 93 | 199.4 | 74.4 | 55 | 131.0 | 44.0 | 17 | 62.6 | 13.6 | —21 | —5.8 | —16.8 |
| 92 | 197.6 | 73.6 | 54 | 129.2 | 43.2 | 16 | 60.8 | 12.8 | —22 | —7.6 | —17.6 |
| 91 | 195.8 | 72.8 | 53 | 127.4 | 42.4 | 15 | 59.0 | 12.0 | —23 | —9.4 | —18.4 |
| 90 | 194.0 | 72.0 | 52 | 125.6 | 41.6 | 14 | 57.2 | 11.2 | —24 | —11.2 | —19.2 |
| 89 | 192.2 | 71.2 | 51 | 123.8 | 40.8 | 13 | 55.4 | 10.4 | —25 | —13.0 | —20.0 |
| 88 | 190.4 | 70.4 | 50 | 122.0 | 40.0 | 12 | 53.6 | 9.6 | —26 | —14.8 | —20.8 |
| 87 | 188.6 | 69.6 | 49 | 120.2 | 39.2 | 11 | 51.8 | 8.8 | —27 | —16.6 | —21.6 |
| 86 | 186.8 | 68.8 | 48 | 118.4 | 38.4 | 10 | 50.0 | 8.0 | —28 | —18.4 | —22.4 |
| 85 | 185.0 | 68.0 | 47 | 116.6 | 37.6 | 9 | 48.2 | 7.2 | —29 | —20.2 | —23.2 |
| 84 | 183.2 | 67.2 | 46 | 114.8 | 36.8 | 8 | 46.4 | 6.4 | —30 | —22.0 | —24.0 |
| 83 | 181.4 | 66.4 | 45 | 113.0 | 36.0 | 7 | 44.6 | 5.6 | —31 | —23.8 | —24.8 |
| 82 | 179.6 | 65.6 | 44 | 111.2 | 35.2 | 6 | 42.8 | 4.8 | —32 | —25.6 | —25.6 |
| 81 | 177.8 | 64.8 | 43 | 109.4 | 34.4 | 5 | 41.0 | 4.0 | —33 | —27.4 | —26.4 |
| 80 | 176.0 | 64.0 | 42 | 107.6 | 33.6 | 4 | 39.2 | 3.2 | —34 | —29.2 | —27.2 |
| 79 | 174.2 | 63.2 | 41 | 105.8 | 32.8 | 3 | 37.4 | 2.4 | —35 | —31.0 | —28.0 |
| 78 | 172.4 | 62.4 | 40 | 104.0 | 32.0 | 2 | 35.6 | 1.6 | —36 | —32.8 | —28.8 |
| 77 | 170.6 | 61.6 | 39 | 102.2 | 31.2 | 1 | 33.8 | 0.8 | —37 | —34.6 | —29.6 |
| 76 | 168.8 | 60.8 | 38 | 100.4 | 30.4 | 0 | 32.0 | 0.0 | —38 | —36.4 | —30.4 |
| 75 | 167.0 | 60.0 | 37 | 98.6 | 29.6 | —1 | 30.2 | —0.8 | —39 | —38.2 | —31.2 |
| 74 | 165.2 | 59.2 | 36 | 96.8 | 28.8 | —2 | 28.4 | —1.6 | —40 | —40.0 | —32.0 |
| 73 | 163.4 | 58.4 | 35 | 95.0 | 28.0 | —3 | 26.6 | —2.4 | —41 | —41.8 | —32.8 |
| 72 | 161.6 | 57.6 | 34 | 93.2 | 27.2 | —4 | 24.8 | —3.2 | —42 | —43.6 | —33.6 |
| 71 | 159.8 | 56.8 | 33 | 91.4 | 26.4 | —5 | 23.0 | —4.0 | —43 | —45.4 | —34.4 |
| 70 | 158.0 | 56.0 | 32 | 89.6 | 25.6 | —6 | 21.2 | —4.8 | —44 | —47.2 | —35.2 |
| 69 | 156.2 | 55.2 | 31 | 87.8 | 24.8 | —7 | 19.4 | —5.6 | —45 | —49.0 | —36.0 |
| 68 | 154.4 | 54.4 | 30 | 86.0 | 24.0 | —8 | 17.6 | —6.4 | —46 | —50.8 | —36.8 |
| 67 | 152.6 | 53.6 | 29 | 84.2 | 23.2 | —9 | 15.8 | —7.2 | —47 | —52.6 | —37.6 |
| 66 | 150.8 | 52.8 | 28 | 82.4 | 22.4 | —10 | 14.0 | —8.0 | —48 | —54.4 | —38.4 |
| 65 | 149.0 | 52.0 | 27 | 80.6 | 21.6 | —11 | 12.2 | —8.8 | —49 | —56.2 | —39.2 |
| 64 | 147.2 | 51.2 | 26 | 78.8 | 20.8 | —12 | 10.4 | —9.6 | —50 | —58.0 | —40.0 |
| 63 | 145.4 | 50.4 | 25 | 77.0 | 20.0 | —13 | 8.6 | —10.4 | | | |

## TABLA 3. Réaumur comparado con Fahrenheit y Centígrado.

| R. | F. | C. | R. | F. | C. | R. | F. | C. | R. | F. | C. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ° | ° Exact. | ° Exact. | ° | ° Exact. | ° Exact. | ° | ° Exact. | ° Exact. | ° | ° Exact. | ° Exact. |
| 80 | 212.00 | 100.00 | 49 | 142.25 | 61.25 | 19 | 74.75 | 23.75 | —11 | 7.25 | —13.75 |
| 79 | 209.75 | 98.75 | 48 | 140.00 | 60.00 | 18 | 72.50 | 22.50 | —12 | 5.00 | —15.00 |
| 78 | 207.50 | 97.50 | 47 | 137.75 | 58.75 | 17 | 70.25 | 21.25 | —13 | 2.75 | —16.25 |
| 77 | 205.25 | 96.25 | 46 | 135.50 | 57.50 | 16 | 68.00 | 20.00 | —14 | 0.50 | —17.50 |
| 76 | 203.00 | 95.00 | 45 | 133.25 | 56.25 | 15 | 65.75 | 18.75 | —15 | —1.75 | —18.75 |
| 75 | 200.75 | 93.75 | 44 | 131.00 | 55.00 | 14 | 63.50 | 17.50 | —16 | —4.00 | —20.00 |
| 74 | 198.50 | 92.50 | 43 | 128.75 | 53.75 | 13 | 61.25 | 16.25 | —17 | —6.25 | —21.25 |
| 73 | 196.25 | 91.25 | 42 | 126.50 | 52.50 | 12 | 59.00 | 15.00 | —18 | —8.50 | —22.50 |
| 72 | 194.00 | 90.00 | 41 | 124.25 | 51.25 | 11 | 56.75 | 13.75 | —19 | —10.75 | —23.75 |
| 71 | 191.75 | 88.75 | 40 | 122.00 | 50.00 | 10 | 54.50 | 12.50 | —20 | —13.00 | —25.00 |
| 70 | 189.50 | 87.50 | 39 | 119.75 | 48.75 | 9 | 52.25 | 11.25 | —21 | —15.25 | —26.25 |
| 69 | 187.25 | 86.25 | 38 | 117.50 | 47.50 | 8 | 50.00 | 10.00 | —22 | —17.50 | —27.50 |
| 68 | 185.00 | 85.00 | 37 | 115.25 | 46.25 | 7 | 47.75 | 8.75 | —23 | —19.75 | —28.75 |
| 67 | 182.75 | 83.75 | 36 | 113.00 | 45.00 | 6 | 45.50 | 7.50 | —24 | —22.00 | —30.00 |
| 66 | 180.50 | 82.50 | 35 | 110.75 | 43.75 | 5 | 43.25 | 6.25 | —25 | —24.25 | —31.25 |
| 65 | 178.25 | 81.25 | 34 | 108.50 | 42.50 | 4 | 41.00 | 5.00 | —26 | —26.50 | —32.50 |
| 64 | 176.00 | 80.00 | 33 | 106.25 | 41.25 | 3 | 38.75 | 3.75 | —27 | —28.75 | —33.75 |
| 63 | 173.75 | 78.75 | 32 | 104.00 | 40.00 | 2 | 36.50 | 2.50 | —28 | —31.00 | —35.00 |
| 62 | 171.50 | 77.50 | 31 | 101.75 | 38.75 | 1 | 34.25 | 1.25 | —29 | —33.25 | —36.25 |
| 61 | 169.25 | 76.25 | 30 | 99.50 | 37.50 | 0 | 32.00 | 0.00 | —30 | —35.50 | —37.50 |
| 60 | 167.00 | 75.00 | 29 | 97.25 | 36.25 | —1 | 29.75 | —1.25 | —31 | —37.75 | —38.75 |
| 59 | 164.75 | 73.75 | 28 | 95.00 | 35.00 | —2 | 27.50 | —2.50 | —32 | —40.00 | —40.00 |
| 58 | 162.50 | 72.50 | 27 | 92.75 | 33.75 | —3 | 25.25 | —3.75 | —33 | —42.25 | —41.25 |
| 57 | 160.25 | 71.25 | 26 | 90.50 | 32.50 | —4 | 23.00 | —5.00 | —34 | —44.50 | —42.50 |
| 56 | 158.00 | 70.00 | 25 | 88.25 | 31.25 | —5 | 20.75 | —6.25 | —35 | —46.75 | —43.75 |
| 55 | 155.75 | 68.75 | 24 | 86.00 | 30.00 | —6 | 18.50 | —7.50 | —36 | —49.00 | —45.00 |
| 54 | 153.50 | 67.50 | 23 | 83.75 | 28.75 | —7 | 16.25 | —8.75 | —37 | —51.25 | —46.25 |
| 53 | 151.25 | 66.25 | 22 | 81.50 | 27.50 | —8 | 14.00 | —10.00 | —38 | —53.50 | —47.50 |
| 52 | 149.00 | 65.00 | 21 | 79.25 | 26.25 | —9 | 11.75 | —11.25 | —39 | —55.75 | —48.75 |
| 51 | 146.75 | 63.75 | 20 | 77.00 | 25.00 | —10 | 9.50 | —12.50 | —40 | —58.00 | —50.00 |
| 50 | 144.50 | 62.50 | | | | | | | | | |

# AIRE. — ATMÓSFERA

Se cree que la atmósfera se extiende hasta una altura por lo menos de 72 kilómetros sobre la superficie de la tierra. Es una mezcla de cerca de 79 partes de gas nitrógeno y 21 de gas oxígeno, en volumen; en peso, 77 y 23. Generalmente contiene una traza de agua, de ácido carbónico, de hidrógeno carburado y aun de amoníaco.

**Densidad del aire.** En condiciones normales de 760 mm de presión, á la temperatura de 0° C=32 F y á 45° de latitud, 1.292673 kilogramos por metro cúbico.

Para otras latitudes y elevaciones :

$$\text{Densidad en kilog por metro cúb} = 1.292673 \times \frac{R}{R + 2h} \times (1 - .002837 \cos 2 \text{ latitud})$$

*, en que R es el radio medio de la tierra = 6,366,198 mets; $h$ = altura sobre el nivel del mar en metros. Para otras temperaturas, véase más abajo.

Bajo condiciones normales, pero con .04 partes de ácido carbónico ($CO_2$) en 100 partes de aire, la densidad es=1.293052 kilog por metro cúbico **.

**La presión atmosférica** puede variar en cualquier lugar como 5 cm y más, de un día para otro. La presión media á nivel del mar varía de 745 á 770 milímetros de mercurio, según la latitud y la localidad. Se aceptan 760 mm * como la presión media de la atmósfera y es lo que se entiende por **una atmósfera.** Por **atmósfera métrica** se entiende en toda la Europa Continental un kilog por centímetro cuadrado. La presión disminuye cuando la altura aumenta **. Esta es la razón de por qué una bomba situada en un punto elevado no levanta el agua á tanta altura como en otro más bajo. La presión del aire, como la del agua, es siempre igual en todas direcciones.

Se dice frecuentemente que la **temperatura de la atmósfera baja** (como 1° F para cada 300 pies de altura) **1° C para cada 165 metros** que nos levantamos sobre la superficie de la tierra; pero esto depende de muchas circunstancias y está influído por muchas causas locales. Las observaciones hechas en globo parecen indicar que, después de 300 metros, baja como 1° C por cada 110 m; á los 600 m baja como 1° C por cada 140 m; á 1,200 m, 1° C por cada 165 m, y á 1,600 m, 1° C por cada 192 m.

**Una persona en reposo** necesita para respirar bien de 7 á 10 litros de aire por minuto, los que, ya respirados, vician de 100 á 140 litros de aire. Caminando, ó en un trabajo fuerte, necesita y vicia doble ó triple cantidad. En invierno se necesitan como 140 litros de aire fresco por persona y por minuto para la perfecta ventilación de los cuartos : como 220 en verano. En los hospitales, de 1,000 á 2,000.

**Bajo la superficie** de la tierra, en regiones templadas, se encuentra una **temperatura** tolerable de 10° á 16° C, hasta una profundidad de 15 á 18 m; después aumenta 1° C por cada 30 m más ó menos; todo esto sometido, por supuesto, á considerables cambios por las diversas causas locales. En Hulleras de Rose Bridge, en Inglaterra, á la profundidad de 739 m, la temperatura del carbón es de 34° C; y en el fondo de una perforación hecha en Berlín á 1,271 metros, la temperatura es de 48°3.

**El aire es un conductor lento del calor**; por eso los muros huecos sirven para conservar el calor en las habitaciones y mantenerlas secas. **Se precipita en el vacío** al nivel del mar con una velocidad de 353 m por segundo; es decir, con más ó menos la velocidad del sonido en un aire tranquilo. Véase « Sonido ».

**Como todos los fluidos, el aire se dilata proporcionalmente** á la temperatura, y sus volúmenes varían en razón inversa de las **presiones**, hasta presiones de 52 kilog por cm cuadrado; es decir, hasta presiones como 50 veces mayores que la ordinaria : el aire, en este caso, ocupa un volumen como de $^1/_{50}$ de su volumen primitivo. Las substancias que siguen estas leyes se dice que son **perfectamente elásticas.** Bajo una presión de más ó menos (5½ toneladas por pulg cuad) 862 kilog por cm cuadrado, el aire se hace tan pesado como el agua. Como el aire en la superficie de la tierra está sometido á una presión que equivale al peso de una columna de agua de 10.33 m, se deduce que el aire á 10, 20,

* M. V. Regnault. Memorias de la Academia Real de Ciencias del Instituto de Francia, 1847, tomo XXI.

** Trabajos y Memorias de la Oficina Internacional de pesos y medidas.

30 metros de profundidad, **dentro del agua**, estará comprimido á $^1/_2$, $^1/_3$, $^1/_4$, de su volumen en la superficie.

**Los buzos**, después de algunas experiencias, pueden trabajar varias horas á una presión de 2½ atmósferas; y, sin temor de sufrir mucho, hasta una horas bajo presión de 3 $^1/_2$ atmósferas. No obstante, en el puente de San Luis se trabaj, á una profundidad de 33.70 metros.

**La temperatura del rocío** es aquella (variable) en que el aire deposita suó vapores. La máxima temperatura del aire, al sol, no ha pasado probablemente jamás de 63º C; ni el mayor frío durante la noche de — 59º C.

# VIENTO

**La relación entre la velocidad del viento, y su presión** contra un obstáculo colocado en ángulo recto ú oblicuamente á su dirección, no está bien determinada, y menos aún contra superficies curvas. Probablemente la presión sobre una superficie grande es relativamente mayor que sobre una pequeña. Se supone, generalmente, que varía como los cuadrados de las velocidades.

(*N. del T.* — Para reemplazar la tabla, en unidades inglesas del autor, damos las dos siguientes, tomadas del Manual de los señores Sorna y Castro.)

V = velocidad en m por seg
P = presión en kilogramos.

| V = | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| P = | 1.102 | 4.409 | 9.921 | 17.637 | 27.558 | 39.684 | 54.01 | 70.55 | 8 .29 | 110.23 | 133.38 | 158.75 |

## Velocidad y presión del viento.

| Designación del viento. | Velocidad metros por seg. | Presión en kilogs por 1m². | Designación del viento. | Velocidad metros por seg. | Presión en kilogs por 1m². |
|---|---|---|---|---|---|
| Viento poco sensible.. | 2.5 | .765 | Viento muy fuerte.... | 15 | 27.558 |
| — fresco..... | 4.7 | 2.706 | — tempestuoso... | 30 | 110.230 |
| — fuerte..... | 7 | 6.004 | — huracanado ... | 40 | 195.968 |

Se ha observado, en vientos extraordinarios, una presión hasta de 273 kgs.

**Tredgold recomienda que se estime como en 194 kg por metro cuadrado de techo** la presión del viento ejercida sobre éste; pero como los techos se construyen con declives y por consiguiente no soportan la fuerza total del viento, esto es evidentemente demasiado *. Además, generalmente sólo la mitad del techo está expuesta al viento, y algunas veces parcialmente. Es probable que en estos casos la fuerza varíe aproximadamente como los senos de los ángulos de inclinación del techo. Según observaciones hechas en Liverpool en 1860, un viento de 61 km por hora produjo una presión de 68 kg por metro cuadrado contra un objeto, colocado en ángulo recto á su dirección; y uno de 112.65 km por hora (el temporal más fuerte observado en aquella ciudad) una presión de 205 kg por metro cuadrado. En Escocia, durante un temporal violento, la fuerza, registrada por un anemómetro ó medidor de viento excelente, llegó á 219 kg por metro cuadrado. Se dice que en Glasgow se ha observado una presión hasta de 266 kg por metro cuadrado. Los vientos muy fuertes, á menudo *levantan* los techos.

El anemómetro del Colegio de Girard (Filadelfia) se rompió bajo una presión de 205 kg por metro cuadrado, en cuyo momento pasaba un tornado como á distancia de 400 metros.

* El autor cree que 8 lbs por pie cuadrado, ó sean como 39 kg por metro cuadrado para los *techos ordinarios de dos aguas*, ó 16 lbs (78 kg) para los de una sola, es suficiente para la presión del viento.

*N. del T.* — Si se tiene la velocidad del viento en kilómetros por hora, se puede obtener la presión en kg por metro cuadrado, dividiendo el cuadrado de la velocidad en kilómetros por 108. Esto es solamente aproximado.

# LLUVIA Y NIEVE

**La caída anual**, en cualquier lugar, varía mucho de un año á otro, y la relación entre el máximo y el mínimo llega á veces á ser hasta de 2: 1. **Téngase mucho cuidado de los promedios.** En una corriente, para prever sus efectos dañinos, calcúlese el *máximo*, y para explotarla como fuente de distribución, calcúlese el *mínimo* y téngase en cuenta la pérdida por evaporación y filtración.

Los máximos y mínimos deducidos de observaciones de 4 ó 5 años, no merecen fe; se requieren 15 ó 20 años para tener datos aceptables.

(*N. del T.*— La lluvia en mm de algunas ciudades, hoyas hidrográficas y zonas, es la siguiente: Nueva York, 926; Madrid, 490; Hoya del Rhin, 720; la del Sena, 631; la del Ródano, 956; en la ciudad de París, 495.

Dice Henri Mager, en su Hidrología Subterránea, que del Polo Norte al Ecuador se pueden distinguir 5 zonas de lluvias: la zona Polar Artica, limitada por los 60° de latitud norte en Siberia y sobre el continente americano, sus inviernos no son lluviosos.

Después al sur de 60°, limitado por los Pirineos, Alpes Marítimos, Balkanes Altaí y el lago Baikal, una zona en la que llueve en cada estación, pero moderadamente. Más allá y hasta el Trópico de Cáncer, una zona en que no llueve jamás en verano y las lluvias son poco abundantes el resto del año. A esta *zona seca* sigue la *zona de los Monzones*, que por el contrario recibe las lluvias más abundantes de la Tierra; sobre todo en la costa de Arakan, más allá del Ganges, en el fondo del golfo de Bengala, y sobre la costa de Malabar: sobre algunos puntos elevados de esta costa, caen más de 6 metros; en Cherra-Ponjec, á la altura de 1,360 m caen 15 metros de agua. En fin, la *zona de las lluvias periódicas* corresponde á la zona de las calmas ecuatoriales. Más allá del Ecuador, al sur, se repiten las mismas zonas en sentido inverso.

John Marray calcula que la lluvia que cae sobre la Tierra toda, anualmente, equivale á una capa de 970 mm de espesor, que la cubra por entero.)

En Filadelfia, en 1869, año de la mayor sequía conocida en 50 años, cayeron 1075 mm de agua; el 13 de agosto de 1873 cayeron en un día 183 mm; en julio de 1842 cayeron 150 mm en dos horas. En Génova, en Italia, en una ocasión, cayeron 800 mm en 24 horas. En Chicago, en septiembre de 1878, cayeron 24 mm en 7 minutos.

**Cerca de Londres** el promedio de varios años es de 575 mm. En una ocasión cayeron 150 mm en 1½ horas! En los distritos montañosos de los lagos, la cantidad de lluvia es enorme; alcanza algunos años de 4.50 á 6.75 metros; mientras que en los alrededores es apenas de 1 á 1½ metros. En Liverpool el promedio es de 850 mm; en Edimburgo, 750; Glasgow, 550; Irlanda, 900; Madrás, 1170 mm; Calcuta, 1500; Roma, 980. En las montañas al norte de Calcuta han llegado á caer, en los 6 meses de la estación lluviosa, 500 pulgs, ó sean 12m699! En otros distritos montañosos de la India son comunes fuertes caídas anuales de 3 á 6 m. Una lluvia moderada pero firme durante 24 horas alcanza á 25 mm.

**Como regla general, llueve más en los países cálidos que en los fríos** y más también en los lugares elevados que en los bajos. Sin embargo, algunas circunstancias locales producen á veces un efecto contrario y también causan grandes diferencias entre las cantidades caídas en lugares muy cercanos entre sí, como en los distritos de los lagos de Inglaterra arriba mencionados. Algunas veces es difícil encontrar la causa de estas variaciones. En algunas lagunas de Colombia, A. del S. ha sabido el autor que por espacio de algunos meses ocurrieron varias veces 3 ó 4 fuertes lluvias, y durante este tiempo, ni una sola gota cayó en unos cerros como de 305 metros de altura, situados á plena vista y á una dist de 16 kilms.

En otra localidad, lugar casi llano, cayó en la mañana las $^3/_4$ partes de la lluvia caída durante dos años en un lugar distante 3 kilms.

La relación entre la caída de agua y la que llega á **correr como raudal**, varía mucho por la existencia de bosques, por las talas, por la pendiente y naturaleza del terreno en la vertiente, sobre todo por la permeabilidad de aquél, según la época del año, las heladas, etc. El agua que corre puede suponerse que varía generalmente entre .2 y .8 de la lluvia. Las corrientes en regiones [illegible]áreas pierden

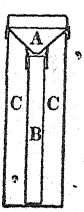
frecuentemente una gran parte de su volumen al atravesar cavernas subtérraneas. Suponiendo una caída de .61 m por año=8351 litros por hectárea y por día, ya que la mitad de la caída de lluvia es aprovechable para el servicio, y que sea suficiente un consumo de 113 litros por cabeza y por día; una milla cuadrada (=259 hectáreas) abastecerá á 19,095 personas, ó un cuadrado de 11.66 m por lado, abastecerá á una persona.

**Una pulgada de lluvia*** equivale á 254 toneladas por hectárea.

Las lluvias más destructoras son generalmente las que caen sobre nieve, debajo de la cual el terreno está helado y no puede absorber el agua.

Algunos ensayos hechos por el autor en diferentes épocas, demuestran que **el peso de la nieve recién caída** varía de 80 á 194 gramos el litro, y parece depender esta diferencia, principalmente, del grado de humedad del aire que atraviesa. En cierta ocasión que cayeron nieve y granizo mezclados hasta la altura de 6 pulgs (15 cent), encontré que su peso era igual á 31 lbs el pie cúbico (499 gramos el litro). Estaba muy seco é incoherente. Un litro de nieve dura, rociada suavemente con agua, puede convertirse como en medio litro de nieve derretida con peso de 650 gr, la cual no se **desliza** en un techo de madera de 30° de inclinación en tiempo de frío. Un bloque de nieve saturado de agua hasta que pese á razón de 45 lbs por pie cúb (721 gramos por litro) resbala justamente en una tabla sin acepillar con 45° de inclinación; en una lisa, acepillada, con 30°, y en pizarra con 18°, aproximadamente. Un prisma de nieve saturado hasta el peso 52 lbs por pie cúbico (833 gramos el litro) de una pulg cuadrada (6.45 cent cuad) de base por 4 pulgs (10 cent) de altura, **soportó un peso** de 7 lbs (3.17 kgs), el cual peso redujo primeramente el prisma en una cuarta parte de su altura. Los ingenieros europeos consideran suficiente **estimar el peso de la nieve** en 29.29 kgs por metro cuad de techo y en 8 lbs por pie cuad (39.06 kgs por metro cuad) la presión del viento, en todo 14 lbs por pie cuad (68.35 kgs por metro cuad). El autor opina que en los Estados Unidos no debe tomarse por peso de la nieve, *menos* de 12 lbs por pie cuad (58.59 kgs por m cuad), en todo 20 lbs por pie cuad (97.65 kgs por m cuad) por la nieve y el viento. No hay peligro en que la nieve se sature en un techo hasta el grado arriba mencionado, porque una lluvia que suministre la cantidad de agua necesaria para producir este efecto, arrastraría también la nieve en la violencia de su caída; pero no nos cabe duda de que en los Estados del Norte la presión de la nieve y del viento unidos alcanza algunas veces y hasta pasa de 20 lbs por pie cuadrado (97.65 kgs por metro cuadrado) de techo. **El límite de las nieves perpetuas** en el ecuador está á la altura de 4,800 m aproximadamente. A la latitud de 45° norte ó sur está más ó menos de la mitad de esta altura, mientras que cerca de los polos está al nivel del mar.

**Medidas de alturas de lluvias. Pluviómetros.** Los envases completamente cilíndricos no se prestan bien para las medidas de las lluvias suaves, aunque




suficientes para dar una apreciable cantidad en una área regular, no llegan á bastante altura para que se las mida con exactitud, á menos que se logre aumentar aquella altura para medirla. La inexactitud en la medida, que siempre es considerable, es á veces demasiado grande con relación á la altura de agua caída.

En su forma más sencilla y usual el pluviómetro (véase figura) consiste esencialmente en un embudo, A, que recibe la lluvia y la eleva á un tubo medidor, B, de menor sección transversal. El embudo deberá tener un borde vertical, y á fin de disminuir la pérdida por evaporización, estar pegado al tubo, y que su extremo inferior sea de pequeño diámetro.

* *N. del T.* — Un centímetro de lluvia equivale á diez millones de kgs por kilom cuadrado.

La profundidad del agua se averigua metiendo hasta el fondo del tubo una varilla de medir, de madera no pulida, en la que se verá hasta qué profundidad se ha mojado. La varilla puede estar graduada ó puede ser comparada con una escala ordinaria en cada observación. El tubo es generalmente de tal diámetro que el área de la sección transversal, menos la de la varilla, es un décimo del área de la boca del embudo. La altura de la lluvia caída será, pues, $^1/_{10}$ de la profundidad medida por la varilla.

*Dimensiones del* **pluviómetro** *oficial de observaciones metereológicas de los Estados Unidos.*

| | | | |
|---|---|---|---|
| A. receptor ó embudo. | | Diámetro 20 | cm |
| B. tubo medidor : | Altura 50 cm | — 6.3 | — |
| CC. aparato de derrame y medida de la nieve. | | — 20. | — |

Tales medidores, con los tubos cuidadosamente hechos de láminas de bronce sin soldadura, cuestan como cinco dólares cada uno; pero un latonero inteligente y cuidadoso, teniendo las dimensiones exactas, puede construir uno de hierro galvanizado, que servirá muy bien, por menos de un dólar.

La **exposición** del aparato tiene un efecto importante sobre los resultados obtenidos. El embudo debe elevarse como 1 m sobre el piso á fin de evitar que caiga en él la lluvia salpicada del mismo piso ó del techo. Si se coloca sobre los techos, éstos deben ser chatos y de 15 m de ancho ó más, y también situar el aparato lo más distante posible de la orilla, pues de lo contrario las corrientes de aire producidas al chocar el viento contra los lados del edificio, llevarán alguna lluvia sobre el medidor.

No debe haber objetos mucho más altos que el aparato cerca de él, pues producen corrientes de aire variables que pueden afectar seriamente sus indicaciones.

Debe tener un depósito C de derrame, para los casos en que el tubo se rebose.

El agua helada en la medida puede reventarla ó averiar el fondo, ó al menos deformarle hasta destruir su exactitud.

**Para medir la nieve** se quita el embudo y se reune la nieve en el aparato de derrame ó en otro envase cilíndrico bastante hondo (para impedir que se vuele), cuya sección transversal se conozca exactamente. La nieve luego se derrite poniéndola en lugar caliente, ó con menos pérdida por evaporización, agregándole una cantidad exactamente conocida de agua tibia. En este último caso dedúzcase de la medida el volumen de agua agregado.

**Equivalencia entre la lluvia y la nieve.** Se supone generalmente que 10 cm de nieve equivalen á uno de lluvia; pero, según varias autoridades, el equivalente puede variar entre 2½ y 34; es decir, como de 30 á 400 gr por litro.

**Los medidores automáticos**, de los que hay varios en el mercado, son bastante costosos, y muchos, aun comprados directamente á los fabricantes, rara vez resultan precisos. Los medidores que tienen un envase con punta registradora no son exactos para fuertes lluvias; los de flotante están limitados en cuanto á la profundidad total que pueden apreciar, y los que pesan el agua caída son afectados por el viento.

# AGUA

El agua pura (como la hervida y destilada) **se compone** de dos gases, hidrógeno y oxígeno, cuya proporción (en volumen) es de 2 á 1, y en peso de 1 de hidróg y 8 de oxíg. Ordinariamente contiene, no obstante, diversas substancias extrañas, como ácido carbónico y otros ácidos; minerales solubles ó substancias orgánicas. Cuando contiene mucha cal, es salobre, *cruda* y no hace mucha espuma con el jabón. **El aire** en su estado ordinario **contiene** (4 granos de agua por pie cúb) como 9 miligramos por litro.

Una columna de agua de 10.33 m equilibra la presión atmosférica á nivel del mar; esto equivale á una columna de mercurio de .76 m.

**Su peso** es como 815 veces el del aire, cuando los dos están á una temperatura

como de 17° C. Como á 4° C tiene su máxima densidad y pesa *un gramo* por centímetro cúbico. Damos en la tabla siguiente lo que pesa á diversas temperaturas.

(*N. del T.* — Hemos ampliado la tabla del autor, agregando los datos en sistema métrico.)

| Temp. Fahr. | Temp. Centíg. | Lbs por pie cúb. | Kg por litros. |
|---|---|---|---|
| 32° | 0° | 62.417 | .9998204 |
| 40° | 4°.44 | 62.423 | .9999165 |
| 50° | 10° | 62.409 | .9996923 |
| 60° | 15°.56 | 62.367 | .9990196 |
| 70° | 21°.11 | 62.302 | .9979785 |
| 80° | 26°.44 | 62.218 | .9966330 |
| 90° | 32°.22 | 62.119 | .9950408 |
| 212° | 100° | 59.700 | .9562984 |

**El peso del agua de mar** es de 64 á 64.27 lbs por pie cúbico ó sea de 1,025.556 klgs á 1,029.560 klgs por metro cúbico.

El agua tiene su **densidad máxima** cuando llega su temperatura un poco más arriba de 39° Fahr (3°89 C). Las mejores autoridades la fijan á los 39°2 F (4° C). A los 39° F ó 3°89 C se dilata tanto por la acción del frío como por la del calor. Cuando la temperatura de 32° (0° C) la reduce á **hielo**, su peso no es sino de 916 gramos por decímetro cúbico y su peso específico .9157 más ó menos según las investigaciones del Sr. L. Dufour. De aquí que, como el hielo se dilata una duodécima parte del volumen que tenía cuando líquido, **la fuerza de dilatación repentina** ejercida en el momento de congelarse es suficientemente grande para rajar los tubos de hierro para el agua; siendo la fuerza probablemente no menos de 2109 kg por cm cuadrado. Ha habido casos de reventarse pilares tubulares fundidos, en puentes de hierro, y de construcciones ordinarias, cuando se han llenado de agua completamente, y después se han congelado expuestos á la intemperie. También el hielo afloja y salta cantidades de roca, por cuyas aberturas ó juntas haya entrado el agua de lluvia ó de corrientes. Los muros de sostenimiento se vuelcan algunas veces, ó por lo menos adquieren combas á causa de la congelación del agua que penetra entre su paramento posterior y el relleno de tierra que sostienen; y los muros que no tienen cimientos de suficiente profundidad, á menudo se levantan por la misma causa.

Se dice que en **un tubo de vidrio de seis milímetros** de diámetro interior, el agua no se congela hasta que la temperatura no se reduzca á — 5° C, y en tubos de un diámetro interior menor que $^1/_4$ de milímetro hasta los 3° ó 4° F (— 16°1 ó — 15°5 C). Tampoco el agua se congela en los ríos de corriente rápida hasta que no esté á mucho menos de 0° C.

**El hielo de anclas**, hallado algunas veces á una profundidad como de 7.61 metros, se compone de una agregación de pequeños cristales ó agujas de hielo congeladas en la superficie del agua libre de corriente rápida, y que probablemente han sido llevadas hacia abajo por la fuerza de la corriente. No se forma debajo del agua helada.

**Como el hielo flota en el agua**, y como un cuerpo flotante desaloja un peso de líquido igual á su peso propio, resulta que un pie cúbico de hielo flotante con peso de 57.2 lbs debe desalojar 57.2 lbs de agua; pero un volumen de 57.2 lbs de agua de un pie cuadrado tiene 11 pulgs de altura, de consiguiente un volumen de hielo flotante de forma cúbica ó paralelepipédica tendrá $^{11}/_{12}$ debajo del agua y sólo $^1/_{12}$ sobre ella; y un pie cuadrado de hielo de cualquier grueso necesita un peso igual á $^1/_{12}$ parte de su peso propio, para sumergirlo hasta la superficie del agua. Sin embargo, en la práctica, esto debe considerarse solamente como una aproximación cercana, porque el peso del hielo varía según las burbujas de aire que contiene.

Se supone generalmente que el agua pura *entra en ebullición* á 100° C al aire libre, al nivel del mar; estando el barómetro á 76 centímetros y más ó menos 0°56 C menos por cada 158 metros sobre el nivel del mar hasta la altura de 1,609 metros. Pero de hecho su temperatura de ebullición varía lo mismo que su punto de congelación, con su estado de pureza, la densidad del aire, material de que esté hecho el envase, etc. En un vaso metálico, el agua puede entrar en ebullición á 210° Fahr (98°89 C), y en uno de vidrio, de 212° á 220° Fahr (100° á 104°44 C); y, si se extrae todo el aire previamente, se requieren 275° Fahr (135° C).

**El agua se evapora** á todas las temperaturas; **disuelve** más substancias que cualquier otro agente, y tiene una capacidad calorífica mayor que cualquiera otra substancia conocida.

**El agua se comprime** á razón de $^1/_{21740}$ partes de su volumen por cada atmósfera de presión (1.033 kg por cm cuad). Cuando se suprime la presión, su elasticidad la hace recuperar su volumen primitivo.

**Su efecto sobre los metales.** La sal contenida en muchas aguas forma depósitos en los tubos metálicos de agua, y en los canales de loza, ó de mampostería, especialmente si la corriente es lenta. Algunas otras substancias hacen lo mismo; obstruyendo la corriente del agua de tal manera, que siempre es conveniente usar tubos de diámetros mayores que los que hubieran sido necesarios sin esta circunstancia. La cal también forma **incrustaciones muy duras en el fondo de las calderas,** que impiden en mucho su eficacia y las hacen más propensas á reventar. Esta agua no sirve para las locomotoras. Hemos visto mencionado que la C.ª de T. C. Southewstern, Inglaterra, evita estos depósitos de cal, en las secciones calcáreas, por medio de la disolución de 1 onza de sal amoníaco en 90 galones de agua (409.91 litros). La sal del agua de mar forma depósitos semejantes en las calderas, como lo hacen también el fango y otros ingredientes impuros.

El agua, cuando está muy pura (como la de lluvia), ó cuando contiene ácido carbónico (como la mayor parte de las aguas), **produce carbonato de plomo** en los tubos de plomo, y como esta substancia es un veneno activo, esta clase de tubos no debiera usarse para tales aguas. Debían substituirse por tubos de plomo estañado.

Pero si también el sulfato de cal se halla en el agua, como sucede muchas veces, no se produce siempre este efecto, y varias otras substancias que generalmente se hallan en el agua, de ríos ó de manantiales, también lo disminuye en un grado más ó menos grande. **El agua dulce corroe más rápidamente al hierro forjado que al fundido**, pero lo contrario parece suceder con **el agua de mar**, aunque esto también afecta el hierro forjado muy ligeramente, de modo que se pueden separar de él con facilidad capas gruesas ó escamas. La corrosión del hierro ó del acero por el agua de mar aumenta con el carbono que contengan. Los cañones de hierro fundido de una embarcación que estuvo sumergida en el agua dulce del río Deleware por más de 40 años, estaban perfectamente libres de oxidación. El general Pasley, que examinó los metales encontrados en los barcos *Royal George* y *Edgar*, el primero de los cuales permaneció sumergido durante 62 años, y el último 133 años, dijo « que el hierro fundido generalmente se había puesto muy blando y en algunos casos parecía plombagina. Algunas balas se calentaron al exponerlas al aire y reventaron en muchos pedazos. El hierro forjado no estaba tan dañado, *excepto en las partes que habían estado en contacto con el cobre ó con el bronce de cañón.* Ninguno de estos cañones estaba muy dañado, con excepción de la parte en contacto con el hierro. Una parte del hierro forjado fué labrado otra vez por un herrero, quien hizo constar que era superior al hierro nuevo ». Cottam dice « que algunos de los cañones fueron transportados cuidadosamente en su estado blando á la Torre de Londres y recuperaron su dureza primitiva con el tiempo (como á los 4 años). Los cañones de bronce de la Mary Rose, que estuvieron sumergidos en el mar durante 292 años, estaban carcomidos considerablemente en algunas partes (donde quizás habían estado en contacto con el hierro). Los cañones viejos, hechos de barras de hierro forjado unidas con aros de hierro, estaban corroídos hasta una profundidad de $^1/_4$ de pulg (seis milímetros) más ó menos; pero probablemente habían estado protegidos por el fango. Las balas de hierro fundido se calentaron hasta el rojo al exponerlas al aire, y se volvieron pedazos como greda seca. »

» Las partes no protegidas de las válvulas de hierro fundido de las compuertas del Canal de Caledonia, se convirtieron en una masa blanda plombaginosa, hasta una profundidad de 18 milímetros en 4 años, pero en las partes que tenían una capa de alquitrán vegetal ordinario, no habían sufrido daño alguno. Este efecto de reblandecimiento del hierro fundido parece ser tan rápido, aun cuando el agua esté solamente algo salobre y aunque no obre sino á intervalos. El mismo efecto se produce en el hierro fundido enterrado en terreno que contenga sal. Algunos tubos de aguas colocados cerca de las dársenas en Liverpool, estaban a los 20 años tan blandos, que podían cortarse con un cuchillo, mientras que otros de la misma clase en terreno más elevado, fuera de la influencia del agua salada, estaban como nuevos, después de 50 años. »

Sin embargo, las observaciones han demostrado **que la rapidez de esta**

**acción depende en mucho de la calidad del hierro**; y que el hierro de un color obscuro, y que contenga mucho carbón combinado mecánicamente, se corroe más rápidamente, mientras que las fundiciones de hierro duro blanco, ó ligeramente gris, permanecen en buen estado mucho tiempo. Algunos pilares de hierro fundido, colocados en el mar, no mostraban deterioro á los 40 años.

**El contacto del hierro con el bronce ó cobre** se dice que produce una acción galvánica, que contribuye á la destrucción rápida, tanto en el agua dulce como en la salada. En algunos fusiles que se recogieron de un navío naufragado y sumergido en agua de mar por espacio de 70 años cerca de Nueva York, las partes de bronce estaban en perfecta condición, pero las partes de hierro habían desaparecido enteramente.

**La galvanización** (dándole una capa de zinc) obra como un preservativo del hierro; pero á expensas del zinc que desaparece pronto, y entonces se corroe aquél. Si se calienta el hierro bien y se le da entonces una capa de **alquitrán mineral caliente**, resistirá á la acción del agua, sea dulce ó salada, por muchos años. El muy importante que el alquitrán esté bien purificado. Una capa así de alquitrán ó de pintura, no impedirá que **algunos moluscos** se adhieran al hierro. El asfalto bien puro es tan bueno como el alquitrán mineral.

**El cobre y el bronce** son muy poco atacados por el agua de mar. *En los tubos de agua de Filadelfia* no se ha encontrado ninguna acción galvánica, en donde se han colocado sunchos de bronce.

**Lo que más perjudica al hierro**, lo mismo que á la madera, es su colocación en lugares donde se seca y se humedece alternativamente. En algunos puntos peligrosos de Long Island Sound, era costumbre enterrar barras redondas de hierro laminado de 10 cm de diámetro más ó menos, para sostener señales. Éstas se gastan muy repetidamente en los puntos donde las toca la alta y la baja marea; á razón de 2½ centímetros más ó menos en 20 años, en cuyo tiempo la barra de 10 cm se reduce á 5 cm en las partes mencionadas. Una capa delgada de barniz de brea mineral, aplicada con esmero, protege al hierro, del mismo modo que á los tubos de agua, etc., durante mucho tiempo, debajo del agua dulce especialmente, pero también enterrados en la tierra.

El ácido sulfúrico contenido en el agua de las minas de carbón corroe los tubos de hierro rápidamente.

**En el agua dulce de algunos canales**, las lanchas de hierro han estado en servicio constante de 20 á 40 años. **La madera permanece bien** durante siglos debajo del agua, sea dulce ó salada, si no está expuesta á ser gastada por la acción de las corrientes ó destruída por los insectos marítimos.

**El agua de mar difiere un poco en peso en diferentes lugares**, pero en el mismo lugar varía de modo inapreciable á diversas profundidades y puede suponerse generalmente de 64 lbs por pie cúbico (1.025 kilgs por decímetro cúbico). Su exceso de peso sobre el agua dulce es principalmente sal ordinaria.

**El agua de mar se congela á** 27° Fahr ( — 2°8 C). El hielo es dulce, pero (especialmente á bajas temperaturas) puede encontrarse salmuera en el hielo.

Una cucharilla de té, llena de alumbre en polvo, echada en un tobo de agua sucia, y bien revuelta después, la purifica generalmente en un par de horas lo suficiente para poderla beber. Si se cava un hoyo de 1 á 1½ m de profundidad en arena de playa de mar, el agua que se obtiene es por lo común suficientemente dulce para lavar en ella con jabón, y hasta se puede beber. Se dice también que el agua puede conservarse en buen estado por muchos años, echando en el envase que lo contiene 1 onza de óxido negro de maganeso por cada galón de agua (3.78 litros).

*Se dice que el agua depositada en estanques de zinc*, ó la que corre por tubos de hierro galvanizados interiormente, se hace venenosa rápidamente por la formación de sales de zinc solubles; y se recomienda dar una capa de barniz de asfalto á las superficies de zinc para evitar esto. Sin embargo, en la ciudad de Hartford (Conn.) se adoptaron tubos de hierro para el servicio en 1855, galvanizados por dentro y por fuera, recomendados por los comisionados de las aguas, y que han estado en uso hasta ahora. También se han usado bastante en Filadelfia y otras ciudades. En muchos hoteles y otros edificios de Boston, los tubos de latón (brass) (bronce) sin empates de la *American Tube Works en Boston*, han estado en uso por muchos años y han dado resultados satisfactorios. Se dice que el agua más potable puede guardarse durante años en envases de bronce sin que produzca ningún resultado dañino.

**La acción del plomo** sobre algunas aguas (hasta sobre aguas puras) es alta-

mente venenosa. Esto es, sin embargo, muy variable : un ingrediente nocivo puede estar acompañado por otro que neutralice su acción. Las materias orgánicas, sean vegetales ó animales, son dañinas. El ácido carbónico, no estando en exceso, es inofensivo.

**El hielo puede ser tan impuro** que sea peligroso beber el agua que de él se obtenga.

**La idea popular de que el agua caliente se congela con más rapidez que el agua fría** con aire á la misma temperatura, es errónea.

# MAREAS

**Las mareas** son las elevaciones y descensos muy conocidos de la superficie del mar y de algunos ríos, causados por la atracción del sol y de la luna. Hay dos elevaciones ó mareas altas y dos descensos ó mareas bajas cada 24 horas y 50 min (**un día lunar**); haciendo un término medio de 6 horas, 12½ minutos entre las mareas altas y bajas. Estos intervalos están, sin embargo, **sujetos á grandes variaciones**, como también las alturas de las mareas; y no solamente en diferentes lugares, sino en uno mismo. Tales irregularidades son debidas á la forma de la costa, la profundidad del agua, el viento y otras causas. Generalmente en la luna nueva y en la luna llena, ó más bien un día ó dos después (ó dos veces en cada mes lunar, á intervalos de dos semanas), la marea sube y baja más que en otro tiempo; ésta se llama **marea fuerte**. Un día ó dos después de los *cuartos de la luna*, sube y baja menos que en otro tiempo, y ésta se llama **marea muerta**. De la marea muerta á la marea fuerte, suben y bajan diariamente más, y viceversa. **La época de la marea alta** en cualquier lugar es generalmente dos ó tres horas después que la luna ha pasado por el meridiano superior ó inferior del lugar; y se llama el *establishment* del lugar, porque, cuando se establece este tiempo, la época de la alta marea en cualquier otro día puede hallarse por aquél en muchos casos. La altura total de la marea mayor es *generalmente* de 1½ á 2 veces la de la marea muerta. Cada marea alta ocurre como 24 minutos más tarde que la precedente. Lo mismo sucede con la baja.

# EVAPORACION, FILTRACION Y MERMA

**La evaporación total de la superficie de las aguas** expuestas á los efectos naturales del aire libre, es mayor en el verano que en el invierno; sin embargo, es muy perceptible aún en tiempo muy frío. La evaporación es mayor en aguas poco profundas que en hondos depósitos, aunque el sol penetre también hasta el fondo y lo caliente. Es mayor en el agua corriente que en la estancada; y mayor cuando hay viento que cuando hay calma. Es probable que la pérdida media, producida solamente por la evaporación en un estanque de una profundidad mediana, exceda rara vez de 7½ mm por día, durante los 3 meses cálidos (junio, julio y agosto) en cualquiera parte de los Estados Unidos. O 2½ mm durante los 9 meses más fríos, excepto en los Estados del Sur. Estos dos promedios darían un término medio diario de $3^{3}/_{4}$ mm ó una pérdida total de 1.39 m por año. Probablemente es de 1.066 á 1.219 m.

**El resultado de algunos ensayos hechos por el autor en países tropicales** demuestra que los estanques hechos con una greda dura y con el agua á 2 m 43 de profundidad, expuesta todo el día á un sol muy caliente, mermaron durante la estación seca exactamente 5 cent en 16 días ó $3^{1}/_{3}$ mm por día, mientras que la evaporación de un vaso de agua era de 6 mm por día. El aire de aquella

región está muy cargado de humedad y el sereno es muy fuerte. Todos los días, durante estos ensayos, las indicaciones del termómetro eran de 46° á 51° C en el sol. La evaporación total por año en algunas partes de Inglaterra y Escocia se dice que es de 588 á 965 mm; en París, 863 mm; en Boston Mass, 812 mm, y en muchos lugares de los E. U. de 761 á 914 mm. Esta última daría un término medio diario de 2½ mm para todo el año. Estos datos, sin embargo, son de poco valor si no están acompañados de referencias sobre las circunstancias en cada caso, tales como profundidad, superficie expuesta, tamaño y naturaleza del envase, ó estanque que contiene el agua, etc. Algunas veces la evaporación total anual de una sección de un país excede á la lluvia caída y viceversa.

**En los canales, estanques, etc.**, se acostumbra unir la pérdida por la evaporación á la de la filtración. Esta última es la absorbida por la tierra, y de la cual una parte pasa enteramente á través del relleno (si éste existe), y si es en pequeña cantidad, puede ser evaporada por el sol y el aire, tan luego como salga fuera, de tal manera que no se percibe; pero, si la cantidad filtrada es suficiente, se nota la *merma*.

**E. H. Gill, ingeniero civil, dice que la evaporación y filtración media de los canales Sandy y Beaver, Ohio**, de 11 m 58 de ancho en la superficie, 7 m 92 en el fondo y 1 m 219 de profundidad, no es sino de 228.8 lit por km y por minuto *durante una estación seca*. La superficie expuesta del agua es de 11580 m² por km; y por tanto, para perder con esta superficie 228.8 lit por km y por minuto, ó sean 329.443 m³ por km, y por día de 24 horas, la cantidad perdida debe ser de .028 m en profundidad por día. Además, con las dimensiones arriba dadas resulta que un kilómetro de canal contiene 11885 m³; por consiguiente, tendríamos $\frac{11885}{329.443} = 36$ días. Las observaciones hechas en tiempo cálido en 35.40 km del canal Chenango, N. Y., de 12.19, 8.53 y 1.21 m, dieron por resultado 1.152 m³ por km y por minuto ó cinco veces el resultado anterior. En esta proporción se vaciaría el canal en 8 días más ó menos. Además hubo una merma excesiva por las compuertas del canal que ascendió á 383 lit por km y por minuto. En otros acueductos y compuertas alcanzaron las pérdidas á 334 lit por km y por minuto. En el canal Chesapeake y Ohio, de 15.23, 9.75 y 1.82 mts, Fish, ingeniero civil, calculó la evaporación y filtración durante dos semanas de tiempo cálido equivalente á toda el agua del canal. **El profesor Rankine presume que es de 5 cm diarios la pérdida por evaporación y filtración del lecho del canal en los canales ingleses.** J. B. Jervis, I. C., estima la pérdida por evaporación, filtración y merma á través de las compuertas del canal primitivo de Erie (12.91, 8.53 y 1.21 m) en 1.758 m³ por km y por minuto, ó sean 2,533 m³ por día y por km. Siendo la superficie del agua de 12,190 m² en un kilómetro, la pérdida diaria será, por consiguiente, de $\frac{2533}{12190} = 207$ mm de profundidad más ó menos.

**En la división Delaware de los canales de Pensilvania**, cuando la provisión ordinaria está temporalmente en una gran extensión, el agua desciende de 101 á 203 mm por día. La filtración es, por supuesto, mayor en los terraplenes que en las cortadas. En algunos de nuestros canales, la profundidad del agua llega á hacerse muy considerable en los grandes rellenos ó terraplenes, cuando por motivos de economía no se echa la tierra del relleno bajo el lecho del canal en capas á nivel, sino que se le deja formar por sí misma su talud natural. En un lugar por lo menos del canal Ches y Ohio, en el cual está un lado formado con la superficie natural de roca vertical, esta profundidad es de 12 m 19. Estas profundidades aumentan mucho la filtración, especialmente cuando, como sucede en muchos casos, no se reviste el terraplén con arcilla; hay otras razones para no recomendar esta práctica.

**El término medio de las pérdidas totales en los estanques de profundidad moderada**, en caso de que se construyan cuidadosamente los diques de tierra y se dejen para que el tiempo los consolide bien, no excederá de 12 á 25 mm por día; pero en los estanques nuevos la pérdida es ordinariamente mayor.

**La pérdida en las zanjas ó canales de áreas pequeñas** es mucho mayor que en los canales navegables, de modo que en los grandes canales surtidores ó de alimentación, sólo se pierde una cantidad relativamente pequeña del agua que les entra.

# MECANICA. LA FUERZA APLICADA Á LOS CUERPOS RIGIDOS

En las páginas subsiguientes nos proponemos aclarar algunos principios elementales de mecánica. Los primeros artículos están dedicados principalmente al estudio de la **materia en movimiento**; pues aunque el conocimiento de ésta tal vez no sea absolutamente necesario para la adquisición del conocimiento *práctico* de aquellos principios de Estática de aplicación tan extensa para el ingeniero civil, sin embargo le serán un auxiliar importante para sus inteligentes apreciaciones.

**Art. 1 (a). La mecánica** es la ciencia que trata de los efectos de la fuerza sobre la materia.

Esta definición tan lata de la palabra « Mecánica » incluye no sólo la hidrostática, la hidráulica, la neumática, etc., sino también la electricidad, la óptica, la acústica y ciertamente todos los ramos de la Física; pero nosotros nos circunscribimos principalmente á considerar la acción de las fuerzas extrañas sobre los cuerpos que suponemos *rígidos*, es decir, que no cambian de forma.

**(b)** * La mecánica se divide en tres partes, á saber :

**La Cinemática** ó el estudio de los *movimientos* de los cuerpos, sin referencia á las causas del movimiento, y

**La Dinámica** ó el estudio de las fuerzas y sus efectos.

**La Estática**, que considera los casos en que fuerzas iguales y opuestas neutralizan sus efectos.

**Art. 2** (a). **Materia ó substancia** puede definirse « todo aquello que ocupa un espacio », como los metales, piedras, madera, agua, aire, vapor, gas, etc.

**(b) Cuerpo** es una cantidad cualquiera de materia que en realidad está más ó menos del todo separada de otra, ó que la consideramos como si lo estuviera.

Así pues, una piedra es un cuerpo, ya sea que se encuentre atravesando el aire en su caída, situada en el terreno ó formando parte de un muro. El muro es también un cuerpo, y podemos considerar que también lo es una parte cualquiera de él, como un pie cúbico, un metro cúbico, un centímetro cúbico, etc. La tierra y los otros planetas son cuerpos así como sus más pequeños átomos.

Una serie de carros puede considerarse como un cuerpo, del mismo modo que cada carro, cada rueda, eje ú otra parte cualquiera del carro; cada pasajero, etc., etc.

De la misma manera, el océano es un cuerpo, y podemos considerar como tal, á voluntad, cualquiera parte de él, como un metro cúbico, una bahía determinada, una gota, etc.

**(c)** Pero en lo que sigue consideraremos especialmeente (como ya se ha dicho) los cuerpos *rígidos*, es decir, aquellos que no están sujetos á cambios de formas : como la contracción, el alargamiento, la ruptura ó la penetración por otro cuerpo. Todos los cuerpos existentes están más ó menos sujetos á alguno de estos cambios de forma, es decir, que ningún cuerpo es en *realidad* absolutamente rígido; pero podemos, por conveniencia, suponer que existen semejantes cuerpos, porque muchos de ellos son tan rígidos que en las circunstancias ordinarias experimentan poco ó ningún cambio de forma, y porque estos cambios, cuando ocurren, deben estudiarse en un capítulo aparte titulado « Resistencia de Materiales ».

**(d)** Al mismo tiempo que los cuerpos son considerados como incapaces de cambios de *forma*, es igualmente importante que se los considere como *susceptibles* de cambiar de posición como un todo (no compuesto de partes). Así pues, pueden volcarse y hacerse girar horizontalmente y en cualquiera otra dirección; moverse en cualquiera línea recta ó curva, girando ó no, alrededor de un punto situado en el mismo cuerpo. En suma, son capaces de *movimiento* como un *todo*.

**Art. 3** (a). **El movimiento** de un cuerpo es su cambio de lugar con relación á otro cuerpo ó algún punto real ó imaginario que *consideramos* (por conveniencia) fijo ó en reposo. Así, cuando una piedra cae de un techo al suelo, su posición con

* *N. del T.* — El autor trae dos diversas acepciones para los términos *kinematics* y *kinetics* que no existen en español, así es que nos ha parecido mejor establecer la división de la mecánica como sigue.

relación al techo cambia constantemente, del mismo modo que con relación al suelo y á cualquier punto dado en la pared, y entonces decimos que la piedra está animada de un *movimiento relativo con respecto á uno de aquellos cuerpos* ó á cualquier punto considerado en ellos. Pero si dos piedras, A y B, caen del techo al mismo tiempo y llegan al suelo en el mismo instante, decimos que aun cuando cada una se mueve, con relación al techo y al suelo, no tienen sin embargo *movimiento relativo* entre sí, es decir, que se hallan en *reposo* la una con relación á la otra, pues sus posiciones relativas no cambian, es decir, que cualquiera que haya sido la dirección y la distancia á que se encontraba la piedra A de la piedra B en el momento de la salida, ha permanecido en una misma dirección y á la misma distancia de B, durante todo el tiempo de la caída. De la misma manera, el techo, la pared y el suelo están en reposo relativo entre sí, no obstante estar en movimiento con relación á la piedra que cae. Igualmente están en movimiento con relación al Sol, debido á la rotación diurna de la tierra alrededor de su eje y á su movimiento anual alrededor del Sol.

(b) Si un hombre camina hacia atrás en la parte superior de un tren de carga con la misma velocidad con que el tren se mueve hacia adelante, dicho hombre estará en *movimiento* con relación al tren; pero si lo consideramos como un todo separadamente del conjunto, se hallará en reposo con relación á los *edificios*, etc., cercanos, pues para un espectador que se encuentra parado á una pequeña distancia de la vía, lo verá constantemente frente á la misma parte del edificio, etc. Si el hombre cesa de caminar sobre el tren, entonces estará fijo ó en reposo con relación al *tren*, pero al mismo tiempo en *movimiento* con relación á los *edificios*, etc., que se encuentran á su alrededor, pues el espectador entonces lo verá moverse con el tren.

(c) Como no conocemos ningún punto absolutamente fijo en el espacio, no podemos decir cuál es el movimiento *absoluto* de un cuerpo. Por consiguiente, no sabemos de nada que esté en *reposo absoluto* y podemos decir con certeza que *todos* los cuerpos están con movimiento.

**Art. 4 (a). La velocidad** de un cuerpo en movimiento es la *relación* de su movimiento. Un cuerpo (como un tren de ferrocarril) se dice que se mueve con **velocidad constante**, cuando las *distancias* que recorre en *tiempos iguales* son *iguales entre sí*, por más pequeños que se tornen estos tiempos.

(b) La velocidad se expresa por la distancia recorrida *durante un tiempo dado*. Así, pues, si un tren de ferrocarril que se mueve con velocidad constante, recorre diez millas en media hora, podemos decir que su velocidad durante ese tiempo es de 20 millas (esto es, que se mueve á razón de 20 millas por hora ó 105,600 pies por hora ó 1,760 por minuto ó $29\frac{1}{3}$ pies por segundo. O, si se quiere, podemos decir que se mueve á razón de 10 millas cada *media* hora ú 88 pies en *tres* segundos, etc. Pero en general es más conveniente fijar la distancia que recorra en la *unidad* de tiempo, como en *un* día, *una* hora, *un* segundo, etc.

(c) Si de dos trenes, A y B, que se mueven con velocidad constante, A recorre 10 kilómetros en media hora, y B 10 kilómetros en un cuarto de hora, las velocidades serán

A, 20 kilómetros por hora;
B, 40 kilómetros por hora.

En otras palabras, la velocidad de un cuerpo (que puede definirse *la distancia* recorrida en un *tiempo* dado) está en *razón inversa* del tiempo que se emplea en recorrer una distancia *dada*.

(d) **Por unidad de velocidad** se entienden las que de común acuerdo se toman iguales á la *unidad* ó á *uno*. Donde se usan medidas inglesas, la unidad de velocidad generalmente adoptada es un pie por segundo.

(e) Cuando decimos que un cuerpo tiene una velocidad de 20 kilómetros por hora, 10 metros por segundo, etc., no es necesario que recorra precisamente 20 kilómetros ó 10 metros, pues puede no tener suficiente tiempo para ello. Queremos decir simplemente que se está moviendo á *razón* de 20 kilómetros por hora ó de 10 metros por segundo, etc., de manera que, si *continua* moviéndose con la misma velocidad durante una hora ó un segundo, recorrerá 20 kilómetros ó 10 metros.

(f) Cuando la velocidad *aumenta*, se dice que el movimiento se **acelera**. Cuando *disminuye*, se dice que se **retarda**. Si la aceleración ó el retardo es exactamente proporcional al tiempo, es decir, cuando durante cualquiera y en cada intervalo igual de tiempo tiene lugar el mismo cambio, el movimiento es *uniformemente* acelerado ó retardado. Cuando es de otro modo, se dice que es *variable*.

(g) Un cuerpo puede estar animado al mismo tiempo por **dos ó más veloci-**

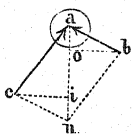
**dades independientes** que es necesario considerar; por ejemplo, una bala disparada verticalmente hacia arriba por un fusil y que luego cae de nuevo á tierra, está animada durante todo el tiempo de su elevación y descenso : (1.º) de la velocidad de *ascensión uniforme* con la cual parte de la boca; y (2.º) la velocidad continuamente *acelerada de descenso* dada por la gravedad que obra sobre ella durante todo el tiempo. Su velocidad *resultante* (ó aparente) en cualquier momento, es en este caso la diferencia de estas dos.

Así, inmediatamente después que parte del fusil, la velocidad de descenso comunicada por la gravedad es muy pequeña y la resultante de la velocidad es, por lo tanto, de ascensión y casi igual á la totalidad de la velocidad de ascenso producida por la pólvora. Pero después de un instante la velocidad descendente (aumentando constantemente) se hace igual á la velocidad ascendente; esto es, su diferencia ó la resultante de la velocidad desaparece y la bala se detiene en ese momento, pero su velocidad descendente continúa aumentando é inmediatamente se hace un poco mayor que la velocidad ascendente, luego se hace mayor cada vez, hasta que la bala cae á tierra. En este instante su velocidad resultante es:

$$= \left\{ \begin{array}{c} \text{La velocidad descendente que hubiera} \\ \text{adquirido al caer } \textbf{durante todo} \\ \textbf{el tiempo de su ascenso y descenso.} \end{array} \right\} - \left\{ \begin{array}{c} \text{La velocidad uniforme ascen-} \\ \text{dente comunicada por} \\ \text{la explosión de la pólvora.} \end{array} \right\}$$

Hemos hecho aquí omisión de la resistencia del aire, que de seguro retarda el ascenso y descenso de la bala.

(**h**) Para mayor claridad supongamos que *abnc* es una balsa impelida en la dirección *ca* ó *nb*. Que un hombre camina sobre dicha balsa con velocidad uniforme del extremo *n* al extremo *c* mientras que la balsa es arrastrada (con una velocidad uniforme un poco mayor que la del hombre) en la dirección *nb*. Por consiguiente, cuando el hombre llega al punto *c*, este punto se habrá trasladado al punto que




ocupaba *á* cuando él partió. La resultante del movimiento del hombre con relación al lecho del río ó á un punto de la ribera ha sido por consiguiente *na*. El movimiento en ángulo recto á *na* producido por su caminata en la balsa es *ic*, pero el debido al impulso de aquélla es *ob*. Estos dos son iguales y opuestos. Por consiguiente, la *resultante* de su movimiento en ángulo recto á *na* es nula; y por tanto, no se separa de la línea *na*. El paseo lo traslada á una distancia igual á *ni*, en la dirección *na*, y el impulso de la balsa lo hace recorrer una distancia igual á *ia*. La suma de estas dos es *na*.

(**i**) Todos los movimientos que les vemos comunicados á los cuerpos, no son sino *cambios* ó variaciones de sus movimientos *absolutos* desconocidos. Por conveniencia debemos limitar nuestra atención á uno ó más de estos cambios prescindiendo de los otros.

Así, en el caso de la bala disparada hacia arriba por un fusil (véase (**g**) atrás) podemos omitir su movimiento uniforme ascendente y considerar únicamente su movimiento acelerado constante descendente, producido por la acción de la gravedad; ó como más se acostumbra : considerar sólo el movimiento *resultante, aparente*, que primero se efectúa hacia arriba y después hacia abajo. En ambos casos prescindimos de los movimientos de la bala causados por los diferentes movimientos de la Tierra en el espacio.

**Art. 5 (a). La fuerza es « la causa del cambio del movimiento ».** Supongamos una bola perfectamente lisa colocada sobre una superficie perfectamente dura, sin frotamiento y á nivel, y supongamos que no existe la resistencia del aire. Para sólo *mover* la bola horizontalmente (esto es, para ponerla en movimiento ó para modificar su estado de movimiento) debe obrar alguna fuerza sobre ella. Si dicha bola estaba ya en movimiento, no podríamos retardarla, acelerarla, ni separarla de su trayectoria, sin aplicarle alguna fuerza, porque, como se establece en **la primera ley de Newton sobre el movimiento, « todo cuerpo continúa en su estado** de reposo, ó de movimiento en línea recta, á menos que obren sobre él otras fuerzas que cambien aquel estado».

Por otra parte, si una fuerza obra sobre un cuerpo, el movimiento de éste debe experimentar cambios.

**(b) Fuerza es la acción que entre dos cuerpos tiende á separarlos ó á unirlos.** Por ejemplo, cuando una piedra cae al suelo, explicamos el hecho diciendo que una fuerza (la gravedad) tiende á juntar la piedra y la tierra.

La atracción magnética y eléctrica y la fuerza adherente que obran entre las partículas de un cuerpo, son otros ejemplos de fuerzas *atractivas*.

**(c) Fuerzas aplicadas por contacto.** En la práctica le comunicamos fuerza á un cuerpo (B) por el contacto entre él y otro cuerpo (A) que tiene tendencia á moverse hacia B. Así se pone en acción una fuerza *repulsiva* entre los dos cuerpos (de un modo que no podemos comprender), y esta fuerza impele á B hacia adelante (ó en la dirección en que tiende á moverse A) y reacciona empujando á A hacia atrás, *disminuyendo* así su tendencia hacia adelante *.

Si, por ejemplo, se coloca una piedra sobre el suelo, ella tiende á moverse hacia abajo; pero no lo hace porque la tierra la rechaza ejerciendo una fuerza repulsiva precisamente igual á la fuerza con que la gravedad tiende á atraerlas.

De la misma manera, cuando tratamos de suspender con nuestra mano un peso, lo hacemos comunicándole á la mano una tendencia á moverse hacia arriba. Si la mano resbala sobre el peso, esto mueve la mano rápidamente hacia arriba antes de que nuestra voluntad pueda evitarlo. Pero puede suceder que la fuerza repulsiva engendrada por el contacto entre la mano (dirigiéndose hacia arriba) y el peso mueva á este último hacia arriba á pesar de la fuerza de la gravedad, é impulse la mano hacia abajo disminuyendo en mucho la velocidad ascensional que de otra manera tendría. Quizás del **esfuerzo**, del cual tenemos conciencia en dichos casos, es que derivamos nuestras nociones sobre la **fuerza**.

Cuando una bola de billar A, en movimiento, choca con otra B, que se halla en reposo, á la tendencia de A á continuar moviéndose, hacia adelante, opone resistencia una fuerza repulsiva que se ejerce entre A y B. Esta fuerza empuja á B hacia adelante y á A hacia atrás disminuyendo su anterior velocidad. Como se explica en el art. 23 (*a*), la fuerza repulsiva no existe en ninguno de los dos cuerpos antes de encontrarse.

(**d**) La fuerza repulsiva nacida así del contacto de dos cuerpos sólo continúa obrando mientras permanecen en contacto y sólo durante el tiempo que tienden (por alguna causa extrema) á unirse estrechamente. Pero está generalmente ó siempre acompañada de una fuerza repulsiva adicional debida á la *compresión* de las partículas de los cuerpos y á la tendencia que tienen á volver á su posición primitiva. Esta fuerza repulsiva *elástica* puede continuar obrando después que ha cesado la tendencia á la compresión.

(**e**) **Las fuerzas obran por atracción ó repulsión.** Cuando un peso está suspendido de un gancho atado al extremo de una cuerda, la gravedad *atrae* el peso; el peso *arrastra* el gancho, y el gancho *tira* de la cuerda, y cada una de estas acciones está acompañada de su correspondiente y opuesta « reacción ». Cuando dos cuerpos chocan, cada uno empuja al otro, generalmente por muy corto tiempo.

**(f) Igualdad de las acciones y reacciones.** Una fuerza ejerce siempre acciones *iguales* sobre dos cuerpos entre los cuales obra. Así, la fuerza (ó atracción) debida á la gravitación, cuando se ejerce entre la tierra y una piedra, atrae la tierra hacia arriba con la misma fuerza con que atrae la piedra hacia abajo, y la fuerza repulsiva que actúa entre una mesa y unapiedra que reposa sobre ella, impele la mesa y la tierra hacia abajo con la misma intensidad con que rechaza la piedra hacia arriba. Este es el hecho consignado por **Newton en su tercera ley del movimiento**, es decir, que « para cada acción hay siempre una reacción igual y contraria ». Con respecto á las medidas de las fuerzas, véanse los artículos 11, 12 y 13.

Si una bala de cañón en su carrera corta una hoja de un árbol, decimos que la hoja ha reaccionado contra la *bala*, con la misma fuerza precisamente con que la *bala* ha obrado contra la *hoja*. Esta fuerza era suficiente para cortar una hoja, pero no para detener la bala. Un buque de guerra al tropezar contra una canoa, ó el puño de un pugilista al pegarle á su contrario en la cara, recibe un golpe tan violento como el que da; pero el mismo golpe, que volvería ó sumergiría una canoa,

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* Generalmente expresamos todo esto diciendo simplemente que A empuja á B hacia adelante, y esto basta en la práctica; pero es bueno reconocer que es una expresión de mera conveniencia que no establece de una manera exacta los hechos y que cada fuerza *necesariamente* se compone de *dos* impulsos iguales y opuestos ejercidos entre dos cuerpos.

no afecta *apreciablemente* el movimiento de un buque, y el golpe que puede dañar seriamente la nariz, boca ú ojos, no produce efectos semejantes en los nudillos de las manos.

La resistencia que un estribo opone á la presión de un arco, ó un muro de sostenimiento á la presión de las tierras apoyadas detrás de él, no es mayor que las presiones mismas; pero el estribo y el muro se han hecho por razones de seguridad *capaces* de soportar presiones mucho mayores en caso de que circunstancias accidentales las produzcan.

(g) En la mayor parte de los casos prácticos **tenemos que considerar uno solo** de los dos cuerpos entre los cuales obra una fuerza. De aquí que por conveniencia comúnmente hablamos, como si la fuerza estuviese dividida en *dos* fuerzas iguales y opuestas, una para cada uno de los dos cuerpos, y limitamos nuestra atención á *uno* de los cuerpos y á la fuerza que obra sobre él sin ocuparnos del otro. Así, podremos hablar de la fuerza del vapor de una máquina como que obra sobre el *émbolo* y prescindir de la presión igual y opuesta, que obra contra la cabeza del cilindro ó cuerpo de bomba.

(h) El punto de un cuerpo al cual, teóricamente, se le aplica una fuerza, se llama **punto de aplicación**. En la práctica no se puede aplicar fuerza á un punto de acuerdo con el significado científico de esta palabra, sino que tenemos que aplicarla distribuída sobre una *área* (algunas veces muy grande) del cuerpo. Al presente vamos á suponer que la línea de acción de la fuerza pasa por el centro de gravedad del cuerpo y es perpendicular á la superficie en el punto de aplicación.

**Art. 7 (a). Aceleración.** Cuando una fuerza que no encuentra resistencia obra sobre un cuerpo y lo pone en movimiento (esto es, le comunica velocidad) en la dirección de la fuerza, esta velocidad *aumenta* mientras la fuerza continúa obrando; y en cada intervalo igual de tiempo (si la fuerza permanece constante) aumenta igual cantidad la velocidad.

Así, si se deja caer una piedra, la fuerza de la gravedad le comunica, en el primer intervalo de tiempo, una pequeña velocidad descendente; en el próximo intervalo igual de tiempo le comunica una segunda velocidad, igual á la primera, de manera que, al fin del segundo intervalo de tiempo, la velocidad de la piedra es el doble de la que tiene al fin del primero, y así sucesivamente. Podemos dividir el tiempo en intervalos iguales tan pequeños como querramos. En cada uno de dichos intervalos la fuerza constante * de la gravedad comunica á la piedra un incremento de velocidad igual.

Este incremento de velocidad se llama *aceleración*. Cuando se lanza un cuerpo verticalmente *hacia arriba*, la aceleración descendente de la gravedad obra como *fuerza retardatriz* del movimiento ascendente. Cuando una fuerza obra de esta manera, *contra* el movimiento de que se trata, su aceleración se hace *negativa*.

**Art. 8 (a). La aceleración** es el aumento de velocidad que tiene lugar en *un tiempo dado;* como *un* segundo, por ejemplo.

(b) **La unidad de aceleración** es el aumento de una unidad de velocidad en cada unidad de tiempo, como, por ejemplo, un pie por segundo en cada segundo, ó un metro por segundo en cada segundo (un metro por segundo, *por segundo*).

(c) Para una *aceleración dada*, las aceleraciones *totales* son, por supuesto, proporcionales á los tiempos durante los cuales la velocidad tiene dicha aceleración.

**Art. 9 (a). Leyes de las aceleraciones.** Supongamos dos barras de hierro: una (á la cual llamaremos A) de longitud doble que la otra (*a*), colocadas cada una sobre un plano perfectamente horizontal y sin frotamiento, de manera que, al moverlas horizontalmente, no encontremos oposición de ninguna fuerza que tienda á detenerlas. Ahora, apliquese á cada barra, por medio de una balanza de resorte, un empuje tal, que conserve la aguja de cada balanza siempre en la misma marca. Como, por ejemplo, en el n.º 2 en las dos balanzas. Tenemos así iguales fuerzas obrando sobre desiguales masas †. En este caso la aceleración comunicada á la masa *a* es doble de la comunicada á la A; pues cuando las **fuerzas son iguales, las aceleraciones están en razón inversa de las masas.**

En otras palabras, en un segundo (ó en cualquier otro tiempo dado), la pequeña barra de hierro *a* adquirirá doble aumento de velocidad que el que adquirirá A (de doble longitud), de modo que, si las dos barras parten al mismo tiempo de

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(*) Hablamos aquí de la fuerza de la gravedad ejercida en un lugar dado, como fuerza constante, porque esto basta para todos los fines prácticos. Hablando en un sentido estricto, aumenta muy poco á medida que la piedra se aproxima á la tierra.

† La masa de un cuerpo es la cantidad de materia que contiene.

su estado de reposo, la más pequeña, *a*, tendrá, al fin de cualquier tiempo dado, dos veces la velocidad de A, que tiene doble masa.

(b) Ahora, supongamos que las dos masas A y *a* sean iguales, pero hagamos que la *fuerza ejercida* sobre *a* sea doble de la que se ejerce sobre A; entonces la aceleración de *a* será (como antes) doble de la de A, porque **cuando las masas son iguales, las aceleraciones están en proporción de las fuerzas.**

(c) De esta manera llegamos á deducir el principio de que, en cualquier caso, **la aceleración es directamente proporciona lá la fuerza é invoersamente proporcional á las masas.**

(d) Así pues, si hacemos las dos fuerzas proporcionales á las dos masas, las aceleraciones serán iguales, ó **para una aceleración dada las fuerzas están en razón directa de las masas.**

(e) Por consiguiente, mayor fuerza se requiere para comunicar una velocidad dada á un cuerpo dado, en un tiempo corto, que para comunicarle la misma velocidad en mayor tiempo. Por ejemplo, los eslabones de unión de adelante en una larga serie de vagones se harían pedazos instantáneamente bajo una tracción suficiente á darle al tren en dos segundos una velocidad de 20 millas (32 km) por hora, suponiendo que existiera una locomotora suficientemente poderosa para ello. En muchos casos semejantes tenemos que contentarnos con una aceleración lenta en lugar de una rápida.

Una cuerda puede sostener con seguridad un peso de una libra suspendido de nuestra mano; pero si queremos comunicarle al peso una gran velocidad ascendente en un *tiempo muy corto*, podemos hacerlo, pero ejerciendo sobre él una gran fuerza ó, en otras palabras, tirando violentamente de la cuerda hacia arriba. Si la cuerda no tiene suficiente resistencia á la tensión para transmitir esta fuerza de nuestra mano al peso, se reventará. Podríamos con más seguridad darle al peso la velocidad deseada empleando una fuerza *menor* durante un *tiempo mayor*.

(f) Cuando una piedra cae, *la fuerza* que atrae la tierra hacia arriba es (como se ha dicho anteriormente) igual á la que atrae la piedra hacia abajo; pero la masa de la tierra es tantísimas veces mayor que la de la piedra, que su movimiento es totalmente imperceptible para nosotros, y lo mismo sería aun cuando no estuviera equilibrada por otros movimientos en otras direcciones y en otras partes de la tierra. Por consiguiente, tenemos razón, dada la insignificancia del fenómeno, aunque *no de un modo absoluto*, cuando decimos que la tierra permanece *en reposo* durante la caída de la piedra.

(g) Pero, en el caso de las dos bolas de billar (artículo 5*c*), podremos ver claramente el resultado de la acción de la fuerza sobre cada uno de los dos cuerpos, puesto que la segunda bola, B, que estaba en reposo se mueve ahora hacia adelante, mientras que la velocidad de avance de la primera, A, se ha disminuído ó destruído, apareciendo su movimiento de retroceso, como un *retardo* de su movimiento de avance; y (como la misma fuerza obra sobre ambas bolas) tenemos

$$\frac{\text{masa de A}}{\text{masa de B}} :: \frac{\text{la aceleración de B}}{\text{la aceleración } \textit{negativa} \text{ de A}}$$

ó (puesto que la fuerza obra durante el mismo tiempo sobre ambas bolas),

$$\frac{\text{masa de A}}{\text{masa de B}} :: \frac{\text{velocidad de avance de B}}{\text{pérdida de velocidad de A.}}$$

(h) *Observación.* Un hombre no puede *levantar* un peso de 20 toneladas; pero si se coloca éste sobre buenos rodillos, puede *moverlo* horizontalmente, como vemos en los puentes levadizos, placas giratorias, etc.; y si el frotamiento y la resistencia del aire pudieran suprimirse, lo podría mover con un soplo de la respiración y continuaría moviéndose después que el soplo hubiera cesado de obrar sobre él. Se movería, sin embargo, muy lentamente porque la fuerza muy débil de la simple respiración tendría que repartirse entre 20 toneladas de materia. Se puede mover si se coloca en un envase conveniente en el agua ó si se suspende de una larga cuerda. Una locomotora poderosa que pueda mover 2000 toneladas no podría suspender verticalmente 10.

Si imaginamos dos cuerpos, cada uno tan grande y pesado como la tierra, precisamente equilibrados en una balanza sin frotamiento, un simple grano de arena agregado á cualquiera de los dos platillos de la balanza les comunicaría movimiento á ambos cuerpos.

**Art. 10** (a). La fuerza constante de la gravedad es una fuerza aceleratriz uniforme, cuando obra sobre un cuerpo que cae librem ente, porque comunica á la velocidad un aumento uniforme, á razón de .0981 m durante cada

centésima parte de un segundo, ó de 9.81 m por segundo, durante cada segundo. También cuando obra sobre un cuerpo que se mueve hacia abajo en un plano inclinado; sin embargo de que, en este caso, el aumento no es tan rápido, porque es causado por sólo una parte de la gravedad, mientras que otra parte comprime el cuerpo contra el plano y otra tercera parte se emplea en vencer el frotamiento. Es una fuerza uniformemente retardada, cuando obra sobre un cuerpo lanzado hacia arriba verticalmente; porque sea cual fuere la velocidad del cuerpo, al lanzarlo hacia arriba disminuirá á razón de .098 m en cada centésima parte de un segundo, durante su elevación, ó 9.81 m por segundo durante cada segundo entero. Por lo menos, tal sería el caso si no fuera por las variadas resistencias que va presentando el aire en las diferentes velocidades que va adquiriendo el cuerpo. La gravedad es una fuerza uniforme de compresión ó de tensión, cuando hace que un cuerpo en reposo comprima á otro cuerpo ó cuando hace que tire de una cuerda á la cual está suspendido. Las expresiones anteriores, como cantidad de movimiento (momentum), compresión, tensión, impulso, empuje, levantamiento, trabajo, etc., no indican diferentes *clases* de fuerzas, sino simplemente diferentes clases de *efectos*, producidos por la gran causa principal llamada fuerza.

(**b**) Los 9.81 m por segundo mencionados se llaman **aceleración de la gravedad**, y está convencionalmente representada, por los escritores científicos, por medio de la letra **g**; ó, hablando más correctamente, no siendo la aceleración precisamente la misma en todos los puntos de la tierra, $g$ indica la aceleración por segundo, sea cual fuere el lugar de que se trate.

**Art. 11 (a). Relación entre la fuerza y la masa.** La masa de un cuerpo es la cantidad de materia que éste contiene. *Un* metro cúbico de agua tiene **doble** masa que *medio* metro cúbico; pero tiene *menos* masa que un metro cúbico de hierro. Así, pues, el *tamaño* de un cuerpo es la medida de la masa entre cuerpos de la *misma* substancia, pero no de la de cuerpos de *diferentes* substancias.

(**b**) Cuando se han dejado caer cuerpos libremente en el vacío, en un lugar dado, se ha visto que adquieren iguales velocidades en cualquier tiempo dado, sea cual fuere la substancia de que estén compuestos. Por esto sabemos (art. 9 *d*) que la fuerza que los hace descender, es decir, sus respectivos *pesos* en aquel lugar, deben ser proporcionales á sus *masas*.

Así, *en cualquier lugar dado*, el *peso* de un cuerpo es la medida exacta de su *masa*. Pero el *peso* de un cuerpo dado *cambia* cuando se traslada el cuerpo de una altura sobre el mar á otra, ó de una latitud á otra; mientras que la *masa* de un cuerpo permanece *la misma* en todos los lugares. Así, un pedazo de hierro que pesa un kilogramo al nivel del mar, pesará *menos* de un kilogramo pesándolo, por medio de una balanza de resorte, sobre una montaña cercana; porque la atracción entre la tierra y una masa dada, disminuye cuando la última se aleja del centro de la tierra. O, si el pedazo de hierro pesa un kilogramo cerca del polo norte ó del polo sur, pesará por la misma razón *menos* de un kilogramo en una balanza de resorte, si se pesa más próximo al ecuador y á la misma altura sobre el nivel del mar.

La diferencia del peso de un cuerpo en distintas localidades, es tan pequeña que no hay para qué tomarla en cuenta en los problemas de mecánica práctica *; pero la exactitud científica requiere una medida de la masa que dé la misma cantidad de materia para el contenido de un cuerpo dado, dondequiera que esté, y como el peso es un medio muy *conveniente* para llegar á la cantidad de materia que contiene un cuerpo, es de desearse que pudiéramos expresar la masa por los pesos. Ahora, cuando un cuerpo dado se traslada á un nivel más alto ó á una latitud más baja, su pérdida de peso es simplemente una disminución en la *fuerza* con la cual la gravedad lo atrae hacia abajo, y esta misma disminución produce también una disminución en la *velocidad* que el cuerpo adquiere al caer durante cualquier tiempo dado. El cambio de su velocidad, según el art. 9 (*b*), es necesariamente proporcional al cambio de su peso. Por lo tanto, si el peso de un cuerpo en cualquier lugar se divide por la velocidad en un segundo en el mismo lugar (llamada $g$ ó *aceleración de la gravedad* para aquel lugar), el *cociente* será el mismo en todos los lugares, y, por consiguiente, sirve como medida invariable de la masa.

(**c**) De común acuerdo, la *unidad de masa* en mecánica se dice que es la cantidad de materia á la cual una *unidad* de fuerza le comunica una unidad de ace-

* La mayor discrepancia que puede ocurrir en varias alturas y latitudes, por la adopción del peso como medida de cantidad, no excederá de 1 en 300; ó, en las circunstancias ordinarias, 1 en 1000.

leración. Esta unidad de aceleración, en paises donde se usan las medidas inglesas, es un pie por segundo, cada segundo. Resta entonces acordar las unidades de *fuerza* y de *masa*. Dos métodos (uno viejo y otro nuevo) están en uso para hacer esto. Nos referiremos á ellos llamándolos métodos A y B respectivamente.

(d) **En el método A**, usado todavía generalmente en cuestiones de *estática*, la **unidad de fuerza** está determinada por la fuerza que es igual al *peso de una libra* (un kilogramo en el sistema métrico) en un lugar determinado; esto es, la fuerza con la cual la tierra atrae, en aquel lugar, cierta medida de platino llamada una libra; y la unidad de *masa*, como se ha dicho en (c), la masa á la cual esta fuerza de una libra le comunica en un segundo una velocidad de un pie por segundo. Ahora bien, la fuerza de un kg de atracción de la tierra sobre una masa de un kg comunicará en un segundo á aquella masa una velocidad $=g$ ó digamos 10 metros; y como, art. 9 (*a*), para una fuerza dada las masas están en razón inversa de las velocidades comunicadas en un tiempo dado, para comunicarle en un segundo una velocidad de solo *un metro* por segundo (en lugar de $g$, ó como 10 m) el kg unidad de fuerza tendría que obrar sobre una masa $g$ veces mayor (10 veces) de la que pesa un kg.

Esto se podria demostrar con una máquina de Attwood, art. 16 (c), haciendo á cada uno de los dos pesos iguales á cuatro y medio kg y el peso adicional $=1$ kg.

Según el método A, por consiguiente, la unidad de masa es $g$ veces la masa de la pieza modelo, de metal, llamada un kg; es decir: un cuerpo que contiene una unidad de masa tal, pesa $g$ kg, ó por el **método A**,

$$\text{el peso de cualquier cuerpo dado, en kg} = g \times \text{la } \textit{masa} \text{ del cuerpo en unidades de masa.}$$

ó

$$\text{a } \textit{masa} \text{ del cuerpo en unidades de masa} = \frac{\text{el peso del cuerpo en kg}}{g}$$

Por ejemplo (*N. del T.* — Tomando á $g$ por 10, en lugar de su justo valor, para facilitar los ejemplos análogos á los que da el autor con las libras):

| En un cuerpo que pesa | La masa es como |
|---|---|
| ½ kg. | $\frac{1}{20}$ unidades de masa |
| 1 — | $\frac{1}{10}$ — |
| 2 — | $\frac{1}{5}$ — |
| 10 — | 1 — |
| 20 — | 2 — |

(e) **Por el método B**, la *masa* del kg modelo de platino se toma como la **unidad de masa** y se llama un **kilogramo**, y la fuerza que le comunica en un segundo una velocidad de un metro por segundo se toma como la unidad de **fuerza**. Esta unidad de fuerza se llama un **kilográmetro**.

Por consiguiente, **por el método B**,

$$\text{la masa de cualquier cuerpo dado en kg} = \frac{\text{el peso del cuerpo respectivamente en kg}}{g \text{ (en metros)}}$$

y

$$\text{el } \textit{peso} \text{ de un cuerpo} = g \times \text{la } \textit{masa} \text{ del cuerpo.}$$

(f) En el sistema **C. G. S.** (**centímetro-gramo-segundo**) una fuerza de una **dina**, obrando durante **un segundo**, sobre una masa de **1 gramo**, le comunica á ésta una velocidad de **1 centímetro por segundo**. Véase también el art. 17 (*b*).

**Art. 12** (a). El producto, fuerza × tiempo, se llama **impulso**. El producto masa × velocidad (cantidad de movimiento) ó **momentum** (texto inglés).

De acuerdo con **la segunda ley de Newton**,

$$\left.\begin{matrix}\text{Fuerza} \times \text{tiempo} = \text{masa} \times \text{velocidad} \\ \text{ó impulso} = \text{momentum}\end{matrix}\right\} \text{ ó bien, } ft = mv \ldots\ldots \quad (1)$$

$$\text{Fuerza} = \frac{\text{impulso}}{\text{tiempo}} = \frac{\text{masa} \times \text{vel}}{\text{tiempo}} = \frac{\text{momentum}}{\text{tiempo}}; \quad \text{ó bien } f = \frac{mv}{t} \ldots\ldots \quad (2)$$

$$\text{Tiempo} = \frac{\text{impulso}}{\text{fuerza}} = \frac{\text{masa} \times \text{vel}}{\text{fuerza}} = \frac{\text{momentum}}{\text{fuerza}}; \quad \text{ó bien } t = \frac{mv}{f} \ldots\ldots \quad (3)$$

$$\text{Masa} = \frac{\text{momentum}}{\text{velocidad}} = \frac{\text{fuerza} \times \text{tiempo}}{\text{velocidad}} = \frac{\text{impulso}}{\text{velocidad}}; \text{ ó bien } m = \frac{ft}{v} \ldots \quad (4)$$

$$\text{Velocidad} = \frac{\text{momentum}}{\text{masa}} = \frac{\text{fuerza} \times \text{tiempo}}{\text{masa}} = \frac{\text{impulso}}{\text{masa}}; \quad \text{ó bien } v = \frac{ft}{m} \ldots \quad (5)$$

(*Observación del T.* — Para la mejor comprensión de esta materia, en el sistema métrico, nos parece preferible reemplazar los últimos renglones de este párrafo (*f*) con la siguiente exposición tomada de un tratado elemental de física.)

**Unidades C. G. S. de masa y fuerza.** En el sistema C. G. S., la unidad de masa es una unidad fundamental, denominada *gramo* (masa), que es la *milésima* parte de la masa del kilogramo (masa), tipo que se guarda en los Archivos internacionales de Sèvres.

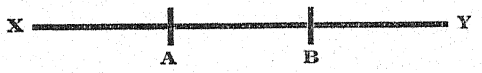
Este último debería representar la masa del decímetro cúbico de agua destilada, á la temperatura de 4º centígrados, pero difiere de ella una cierta cantidad muy pequeña, pero apreciable. Las medidas más precisas asignan, en efecto, al decímetro de agua pura á los 4º el valor de 1,000.013 gr. C. G. S., y, por consiguiente, al centímetro cúbico de agua pura á 4ºC, la masa 1.000 013 gr. C. G. S.

La relación $F = mj$ impone la elección de la unidad de fuerza. Si, en efecto, se hace $m = 1$ y $j = 1$, F debe ser también igual á 1. La unidad de fuerza es, pues, *la fuerza que actuando sobre la unidad de masa le imprime la unidad de aceleración.*

El peso de un gramo, producto de su masa igual á 1 por la aceleración 980.96 (en París) de la caída de los cuerpos, vale, pues, 980.86 *dinas* y la *dina* equivale, por tanto, *aproximadamente al peso de un miligramo.*

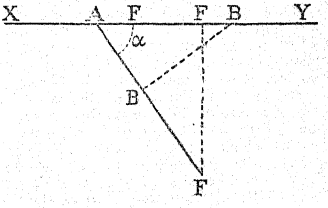
Es, pues, esta unidad una fuerza muy variable. Para expresar fuerzas un poco grandes se emplea frecuentemente una unidad secundaria, *la megadina*, que vale un millón de dinas (1 kilogramo aproximadamente).

**Trabajo mecánico de una fuerza constante en magnitud y en dirección. 1.º Caso en que el punto de aplicación se traslada en la dirección de la fuerza.** Se llama *trabajo de la fuerza* (para conducir su punto de aplicación desde A hasta B) *el producto del espacio*




AB = *e* (en unidades de longitud) *por la intensidad* F *de la fuerza* (en unidades de fuerza). Se tiene, pues, T = *Fe*.

Si la fuerza actúa en el sentido de la traslación XY, se dice que es *moviente* ó *motora* y que su trabajo es positivo; si la fuerza actúa en sentido contrario del camino recorrido, se dice que es *resistente* y que su trabajo es negativo: T = — (*Fe*).

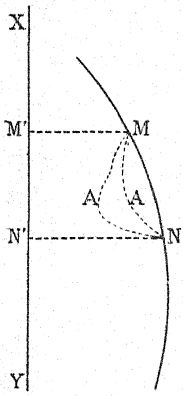



**2.º Caso de un traslado rectilíneo que no está en la dirección de la fuerza. El trabajo es entonces el producto de la fuerza por el camino recorrido, y por el coseno del ángulo de las dos direcciones**: $T = Fe \cos \alpha$. El trabajo es positivo ó negativo según que cos α es positivo ó negativo, es decir, según que α es un ángulo agudo ú obtuso. En el caso en que α es agudo, la proyección de la fuerza sobre la dirección del camino recorrido está dirigida en el mismo sentido que éste; se

puede decir que la fuerza proyectada es *motora;* lo contrario sucede en el caso en que $\alpha$ es obtuso; la proyección de la fuerza es *resistente.*

*Observación.* Se puede también considerar el producto $e \cos \alpha$, como la proyección AB' del camino sobre la dirección de la fuerza é incluir este caso en el precedente.

**3.° El punto de aplicación de la fuerza recorre una trayectoria curvilínea. El trabajo es entonces el producto de la fuerza por la proyección del arco de trayectoria MN ó MA₁N sobre la dirección constante XY de la fuerza:** $T = F \times M'N' = F \times MA_1N$.




**La intensidad (ó aceleración) de la gravedad,** ó la aceleración producida, en cualquier masa por su propio peso, se llama $g$, y á nivel del mar es como de 9.81 m (32.2 pies) por segundo en cada segundo. Véase el capítulo « Gravedad, caída de los cuerpos », pág. 364.

**Art. 13 (a).** Una fuerza se mide comúnmente por el valor de otra fuerza que ella puede destruir. Así, una balanza de resorte da por su escala en el resorte la intensidad de la tensión que justamente equilibra el peso de la masa suspendida. Así **las fuerzas se expresan convenientemente por pesos.**

**Una fuerza puede ser constante,** como la de la gravedad entre la tierra y una piedra colocada sobre ella. **Puede variar regularmente,** como la presión del aire comprimido por un émbolo que se mueva con una velocidad constante, **ó variar irregularmente,** como cuando el movimiento del émbolo dicho es irregular. Trataremos sólo de las fuerzas, suponiéndolas constantes.

(*N. del T.*— Se toma, en la práctica, como unidad de masa, la masa de un cuerpo que pese *un kilogramo* en el ecuador á nivel del mar y como unidad de velocidad la unidad de longitud ó sea un metro en la unidad de tiempo, un segundo. Tendremos que la unidad de fuerza será la que haga mover una masa de *un kilogramo* á *un metro* en un segundo; ésta se llama *kilogramo-metro* ó, abreviando, *kilogrametro* y se deduce también de la fórmula (1): en efecto, $f \times t = m \times v$; poniendo en lugar de $t$, $m$, $v$, sus valores 1 seg, 1 kilogramo, 1 metro, viene $f \times 1 = 1 \times 1$; de donde $f = 1$. La *unidad de fuerza*, ó la unidad de la *cantidad de movimiento*, ó la unidad de *momentum*, como la llama el autor, es en el sistema métrico *un kilogrametro por segundo.*)

**Art. 14 (a). Densidad.** Las *densidades* de las substancias son proporcionales á las *masas* contenidas en un *volumen* dado, como una pulgada ó un centímetro cúbico, ó en *razón inversa* del *volumen* requerido para contener una *masa* dada. Y como los pesos en un lugar dado son proporcionales á las masas, las densidades son proporcionales á los pesos por unidad de volumen (pesos específicos) de las materias. Así, un cuerpo que pesa á razón de 10 kilogramos el decímetro cúbico, tiene una densidad doble que uno que pesa solamente á razón de 5 kilogramos el decímetro cúbico en el mismo lugar.

**Art. 15 (a). Inercia.** Se llama inercia la incapacidad de la materia para ponerse en movimiento por sí sola, ó para modificar el grado ó dirección de su movimento. Cuando decimos que cierto cuerpo tiene doble inercia que otro, significamos con esto que se requiere *doble fuerza* para darle igual aceleración; y como toda fuerza, art. 5 (f), obra del mismo modo en ambas direcciones, experimentamos una *reacción* dos veces mayor (llamada « resistencia ») con el cuerpo más grande que con el más pequeño. La « inercia » de un cuerpo es, por consiguiente, una medida de la *fuerza* que se requiere para producir en él una aceleración dada, ó, lo que es lo mismo, es una medida de la *masa* del cuerpo. Por lo tanto, podemos considerar a « inercia » y la *masa* como idénticas.

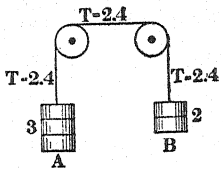
**(b)** Lo que se llama « resistencia de inercia » de un cuerpo, es simplemente la reacción (esto es, una de las dos acciones iguales y opuestas) de cualquiera fuerza que apliquemos al cuerpo. De aquí que su intensidad depende no solamente de la masa del cuerpo, sino también de la cantidad de aceleración que le comuniquemos. Por consiguiente, no podemos decir, por medio de la masa ó del peso solo de un cuerpo, cuál sea su « resistencia » en un caso dado.

**Art. 16 (a). Fuerzas en opuestas direcciones.** Cuando dos fuerzas iguales y opuestas obran sobre un cuerpo á un mismo tiempo y en la misma línea rectas decimos que sus tendencias mutuas á mover el cuerpo se destruyen y aquél permanece en reposo. Si dos fuerzas *desiguales* obran opuestamente, la fuerza ám-

pequeña y una parte de la mayor igual á aquélla se contrarían y destruyen mutuamente; pero lo que queda de la fuerza mayor, obra como una fuerza á la cual no se opone resistencia y mueve el cuerpo en su dirección propia, como lo haría si fuera la única fuerza que obrara sobre él.

Así, cuando en la práctica diaria movemos los cuerpos, no sólo tenemos que aplicar fuerzas para vencer la « resistencia de la inercia », sino también otras fuerzas que se nos oponen, como la fricción, la resistencia del aire y, muchas veces, parte ó la totalidad del *peso* del cuerpo. Cuando en lo sucesivo hablemos de « resistencias » nos referiremos siempre á estas fuerzas citadas sin incluir la « resistencia de la inercia ».

(b) Si separados los dos cuerpos A y B, de 3 y 2 kg respectivamente, deben caer con igual aceleración $g$, cada unidad de masa, $\frac{w}{g}$, estará solicitada por su solo peso. Conectados, como lo están, A se moverá hacia abajo y B hacia arriba, con una aceleración = solamente á $\frac{g}{3+2} = \frac{g}{5}$; ahora bien, una fuerza no equilibrada de sólo $3 - 2 = 1$ kg debe comunicarle aceleración á una masa de $\frac{3+2}{g} = \frac{5}{g}$. Pero, para darle á una masa B, de $\frac{2}{g}$, una aceleración de $\frac{g}{5}$, se requiere una fuerza de $\frac{2}{g} \cdot \frac{g}{5} = \frac{2}{5}$ kg $= .4$ kg. Esta, más 2 kg (requeridos para equilibrar el peso de B), es la tensión 2.4 kg que se ejerce por la cuerda. Ejercida en A, esta tensión equi-




libra 2.4 de los 3 kg que pesa A. El residuo ($3 - 2.4 = .6$ kg), obrando hacia abajo sobre la masa $\frac{3}{g}$ de A, le comunica la requerida aceleración de $\frac{g}{5}$; pues aquí $\frac{\text{fuerza}}{\text{masa}} = .6 \div \frac{3}{g} = \frac{.6g}{3} = .2g = \frac{g}{5}$. O bien, considérese la tensión total, 2.4 kg en la cuerda, en A, obrando sobre A y comunicándole una *aceleración hacia arriba, negativa* de $2.4 \div \frac{3}{g} = .8g$; la que deducida de $g$ (la aceleración que A tendría por sí), queda :

$$\text{Aceleración} = g - .8g = .2g = \frac{g}{5}.$$

Sea W = el peso de A;
$w$ = el peso de B;
F = fuerza que queda para comunicar aceleración = $W - w$;
M = masas combinadas de los dos cuerpos = $\frac{W+w}{g}$;
$m$ = masa de B = $\frac{w}{g}$;
$a$ = aceleración;
T = tensión en la cuerda.

Entonces : $$a = \frac{F}{M} = (W - w) \div \frac{W+w}{g} = \frac{g\,(W - w)}{W + w}$$

$$T = w + ma = w + \frac{w}{g}\,a = w + \frac{w}{g} \cdot \frac{g\,(W - w)}{W + w} = w\left(1 + \frac{W - w}{W + w}\right).$$

(c) Una **máquina de Atwood** consiste esencialmente en una polea, una cuerda flexible que pasa por sobre la polea, dos pesos iguales (suspendido cada

uno á un extremo de la cuerda) y un tercer peso, generalmente mucho más liviano que los otros dos. Los dos pesos iguales se equilibran por medio de la polea y la cuerda. El tercero se coloca sobre uno de los otros dos pesos. La fuerza de la gravedad obrando sobre el tercer peso, pone entonces las masas de los tres pesos en movimiento, á una pequeña pero constante velocidad creciente. Para lograr esto se debe vencer también el frotamiento de la polea con la cuerda, y la rigidez de la última; pero como la cuerda se hace tan delgada como sea posible, no se toman en cuenta. Se usa la máquina para estudiar las aceleraciones comunicadas á la materia inerte por medio de fuerzas no equilibradas, y constituye un excelente ejemplo de los dos distintos efectos que tiene que realizar generalmente una fuerza motriz, á saber : 1.º equilibrar las resistencias, y 2.º comunicar la aceleración.

(**d**) En el caso de una **locomotora que arrastre un tren sobre una línea horizontal**, el frotamiento y la resistencia del aire son las únicas resistencias que hay que vencer. pues el peso del tren no opone resistencia en este caso. Si la fuerza del vapor no es *más* que suficiente para equilibrar las resistencias, no puede mover el tren. Si excede á las resistencias, el exceso, por pequeño que sea, le comunica movimiento á la masa inerte del tren. Si en cualquier momento durante el movimiento del tren, la fuerza del vapor se hace *precisamente igual á las resistencias* (sea por un aumento de las últimas y por disminución de la fuerza), el tren continuará moviéndose con una velocidad *uniforme* igual á la que tenía en el momento en que la fuerza y resistencias estaban compensadas, y si se pudiese conservar siempre igual, se seguirá moviendo indefinidamente así.

Pero mientras el excesc de presión de vapor sobre las resistencias continúe abrando, se aumentará la velocidad á cada instante, porque durante cada uno de dichos instantes el exceso de fuerza agrega una pequeña velocidad á la que ya existe.

En un ferrocarril á nivel sea : (*N. del T.* — En este ejemplo hemos cambiado el cálculo, hecho en sistema inglés, por el métrico.)

P=la fuerza total de tracción de la locomotora=13 tons;
W=peso de la locomotora=50 tons;
V=peso del tren=336 tons;
R=resistencia de la locomotora (incluyendo fricciones interiores, etc.)=3 tons;
$r$=resistencia del tren=1 ton;
F=fuerza disponible para producir aceleración=P—R—$r$=9 tons;
M=masa de la máquina y del tren $= \frac{W+w}{g} = \frac{50+336}{g} = \frac{386}{9.81} = 39.35$;
$m$=masa del tren= $\frac{336}{9.81} = 34.25$;
$a$=aceleración;
T=tensión en la barra de enganche.

Entonces la aceleración $= a = \frac{F}{M} = \frac{9}{39.35} = .228$ m por segundo en cada segundo.

La tensión T, en la barra de enganche=á la resistencia del tren+la fuerza que produce la aceleración $a$, ó bien T=$r$+$ma$=1+34.25×.228=8.81 tons.

Esta tensión T, tirando de la locomotora hacia atrás, produce allí una *retardación ó aceleración negativa* de $\frac{T}{\text{masa de la locomotora}} = \frac{8.81\,g}{50} = 1.73$ mts por segundo, en cada segundo, y esto reduce, en dicha cantidad, la aceleración que la locomotura tendría y que hubiera sido $= \frac{(P—R)g}{50} = \frac{10 \times 9.81}{50} = 1$ m 96. Esta de 1 m 96—1.73=.23 mts por seg, en cada segundo=á la aceleración del tren

(**e**) Si la fuerza de tracción de una locomotora excede á las resistencias debidas á la fricción, al aire, á las pendientes, la velocidad se acelerará; pero entonces se hace más dificultoso mantener el exceso de fuerza, porque los émbolos tienen que moverse más rápidamente en los cilindros y la caldera no suple el vapor con suficiente rapidez para mantener la presión original en el cilindro; fuera de que algunas resistencias crecen con la velocidad. Así se llega á una velocidad en la cual, aunque la máquina desarrolle su fuerza máxima, no puede hacer más que equilibrar las resistencias. El tren se mueve entonces con una velocidad uniforme igual á la que tenía cuando se llegó á tales condiciones.

Cuando se hace necesario detenerse en alguna estación á cierta distancia, se cierra el vapor á fin de que su fuerza no equilibre por más tiempo las resistencias y además se aumenta el número de estas resistencias agregándoles la fricción de los frenos. Las resistencias así aumentadas son las únicas fuerzas que obran sobre el tren, y su aceleración es *negativa* ó de *retardo*. Por consiguiente, el tren se mueve más y más despacio y tiene al fin que pararse.

(*f*) **Advertencia.** Cuando dos fuerzas opuestas se hallan en equilibrio, un aumento hecho á una cualquiera de ellas, no siempre rompe su estado de equilibrio, pues en muchos casos *la otra fuerza aumenta igualmente* hasta cierto punto. Por ejemplo, cuando tratamos de levantar un peso W, su *resistencia* hacia abajo R, permance constantemente igual al esfuerzo paulatino hacia arriba P; sin embargo, P puede variar hasta que *exceda* á W, en cambio R no puede *exceder* nunca á W, pero puede ser mucho menos que él. Efectivamente, cuando se deja de empujar, R cesa á pesar de que W (la atracción entre la tierra y el peso) permanece la misma siempre. Tales variaciones de las fuerzas resistentes para hacer frente á las variaciones que ocurran, tienen efecto en todos aquellos casos innumerables en que las construcciones soportan pesos variables entre los límites de sus resistencias extremas.

**Art. 17 (a). Trabajo.** Cuando la fuerza mueve á un cuerpo * se dice que hace un trabajo. La totalidad del trabajo hecho por la fuerza, al mover el cuerpo á una distancia cualquiera, se mide multiplicando la *fuerza* por la distancia; ó *Trabajo* = = fuerza × distancia. Si la *fuerza* está tomada en *libras* y la *distancia* en *pies*, el producto (ó el trabajo hecho) está dado en *pies-libras* y (en *kilográmetros* si se toman kilogramos y metros respectivamente)†.

| Así, si una fuerza de | mueve un cuerpo á | su trabajo es de |
|---|---|---|
| 1 kilogramo | 10,000 metros | 10,000 kilográmetros |
| 100 kilogramos | 100 — | 10,000 — |
| 10,000 — | 1 metro | 10,000 — |

En cualquier caso, si la fuerza es de F libra ó F′ kilogramos, el trabajo total hecho al mover un cuerpo á lo largo de S pies ó S′ metros es F × S pies libras ó F′ × S′ kilográmetros.

(**b**) Con medidas inglesas la unidad ordinaria de **trabajo** es el **pie-libra**. **La unidad métrica de trabajo** es el **kilogramo-metro** ó, abreviando, **kilográmetro**. Véanse las tablas de conversión pág. 247.

**En el sistema C. G. S.** (Art. 11 *f*), 1 dina obrando en 1 centímetro de longitud produce **1 erg (una dina-centímetro)** de trabajo. 1 joule (pág. 247) = 10,000,000 ergs = .7373 pie-libra = .1019 kilográmetro. 1 kilográmetro = 9,812 joule.

(**c**) En la mayor parte de los casos, una porción por lo menos del **trabajo** hecho por una fuerza, **se emplea en vencer las resistencias.** Así, cuando una locomotora comienza á mover un tren, una parte de su fuerza trabaja contra los frotamientos ó para vencer una subida, mientras que la restante, obrando como fuerza *no* equilibrada sobre la masa inerte del tren, aumenta su velocidad.

Un esfuerzo hacia arriba de un kg no levantará un peso de un kg, sino que simplemente equilibrará la fuerza hacia abajo de la gravedad. Si aumentamos el impulso hacia arriba en un gramo, este gramo así agregado, y como fuerza no equilibrada, le comunicará movimiento á la masa y aumentará su velocidad hacia arriba todo el tiempo que continúe obrando.

Si reducimos ahora el empuje hacia arriba á un kg haciéndolo así exactamente igual á la fuerza de la gravedad hacia abajo, el cuerpo seguirá moviéndose hacia arriba con una velocidad uniforme; pero, si reducimos la fuerza hacia arriba á 999 gramos, habrá entonces una fuerza no equilibrada hacia abajo de 1 gramo, obrando sobre el cuerpo, y esta fuerza hacia abajo producirá en el cuerpo una velocidad hacia abajo, una aceleración *negativa*, ó *retardación*, y *destruirá* la velocidad hacia arriba en el mismo tiempo que necesitó el exceso *hacia arriba* de 1 gramo para *producirla*.

Durante cualquie tiempo, mientras los 1,001 gramos de « fuerza » ascendente obraban contra los 1,000 gramos de « resistencias » descendente, el producto de la fuerza total ascendente × la distancia debe ser *mayor* que el de la *resistencia* × la

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* Un hombre que esté parado, sin movimiento, no se le considera haciendo ningún trabajo; lo mismo se dice de un poste ó de una cuerda cuando sostienen una carga pesada; aunque aquél esté soportando una fuerte carga opresiva ó sosteniendo una carretilla de mano con toda su fuerza, porque ésta no está moviendo nada en ningún caso.

† Este producto no se debe confundir con el de los momentos = fuerza × brazo de palanca.

distancia. El exceso es el trabajo realizado para acelerar la velocidad, en virtud de la cual el cuerpo ha adquirido energía cinemática ó *capacidad para realizar un trabajo al volver al reposo*.

Por otra parte, mientras la velocidad ascendente se *retardaba*, el producto de la fuerza total ascendente × la distancia era *menor* que el de las *resistencias* × la distancia, siendo la diferencia de trabajo hecho por la energía cinemática contra la resistencia de la gravedad.

En la práctica, el término **trabajo** se emplea generalmente de modo restringido á aquella *parte* del trabajo que una fuerza ejecuta para equilibrar las *resistencias* que obran contra ella; en otras palabras, al trabajo ejecutado por una cantidad de fuerza igual á la resistencia.

Con esta restricción tenemos:

Trabajo = *fuerza* × distancia = *resistencia* × distancia.

Así, si la resistencia es un frotamiento de 4 kg vencida en todos los puntos en un distancia de 3 m, ó si es un peso de 4 kg, suspendido á 3 m de altura, entonces el trabajo ejecutado alcanza á 4 × 3 = 12 kilográmetros, suponiendo que las velocidades inicial y final sean iguales.

(d) En casos en **que la velocidad es uniforme**, como en una máquina giratoria sólida, la fuerza es necesariamente igual á la resistencia, y cuando las velocidades al principio y al fin de cualquier trabajo son iguales (como cuando la máquina parte del reposo y vuelve á él otra vez), la fuerza *media* es igual á la resistencia *media*. En dichos casos es claro que, los dos productos, fuerza media × distancia y resistencia media × distancia, son iguales, y tenemos como antes:

Trabajo = *fuerza* × distancia = *resistencia* × distancia.

(f) Al calcular el trabajo ejecutado por una maquinaria, etc., se debe tener en cuenta el gasto de una parte del trabajo en vencer las resistencias. Así, al extraer agua, una parte de la fuerza aplicada se necesita para equilibrar la fricción de las diferentes partes de la bomba, de manera que una fuerza de vapor ó de agua, de 100 kilogramos, que recorra 6 metros por segundo, no puede levantar 100 kilogramos de agua á una altura de 6 metros por segundo. Por tanto, las máquinas en lugar de *ganar fuerzas*, como se cree vulgarmente, en realidad, pierden. Al *poner en movimiento* una máquina, las fuerzas empleadas tienen: 1.° que equilibrar, destruir ú oponerse contra la fuerza resistente del frotamiento y la cohesión de la materia sobre la que obra; y 2.° comunicarle movimiento á las partes de la máquina á las cuales no se oponen resistencias y á las de las masas mismas después de haber vencido las resistencias que ellas presentaban. Pero después que se ha establecido la velocidad deseada, las fuerzas no tienen sino que equilibrar *las resistencias* á fin de que la velocidad continúe uniforme.

(g) La parte del trabajo de una máquina que se consume se llama algunas veces **trabajo perdido ó trabajo perjudicial**, mientras que se llama **trabajo útil** la parte que efectúa, que crea un servicio visible y tangible.

Así, al extraer agua, el trabajo ejecutado para vencer el frotamiento de la bomba y del agua, se dice que se pierde, y se llama perjudicial, mientras que el trabajo útil está representado por el producto del peso del agua salida × altura á la cual se ha obrado. Esta distinción, aunque artificial y de algún modo arbitraria, es á menudo muy conveniente; pero esa parte del trabajo no está realmente perdido, ni mucho menos es « perjudicial », porque no podría extraerse el agua sin vencer antes las resistencias. Un comerciante podría del mismo modo llamar dinero perdido el que gasta en salarios para sus dependientes, etc.

(h) Para una fuerza y *distancia* dadas, **el trabajo ejecutado es independiente del tiempo**, pues el producto de la fuerza × la distancia permanece el mismo; sea cual fuere el tiempo. Pero la distancia en la cual una *fuerza* dada trabaja con una velocidad dada, es por supuesto proporcional al tiempo durante el cual hace el trabajo. Así, para levantar á 10 metros, 50 kilogramos, un hombre debe hacer el mismo trabajo (= 500 kilográmetros), así lo haga en una hora ó en diez; pero si ejerce la *misma fuerza* constantemente, levantará 50 kg á 10 veces más altura en diez horas que en una, y efectuará de este modo diez veces más trabajo. Por tanto, para una fuerza dada, **el trabajo es proporcional al tiempo.**

**Art. 18 (a). Potencia.** La cantidad de cualquier trabajo puede valorarse, evidentemente, sin considerar el *tiempo* requerido para ejecutarlo; pero necesitamos á menudo conocer la *proporción* en la cual el trabajo se ejecuta, esto es, cuánto puede hacerse en cierto tiempo.

La proporción en la cual puede trabajar una máquina, etc., se llama su *potencia*. Así, al escoger una máquina de vapor es importante saber cuánto puede hacer por

*minuto, hora* ó *día.* Por lo tanto, estipulamos que debe ser de tantos *caballos de fuerza;* lo que significa, nada menos, que debe ser capaz de vencer fuerzas de resistencia á razón de tantas veces 4,500 kilográmetros por minuto, cuando tenga una velocidad constante, esto es, cuando fuerza × distancia = resistencia × distancia.

**(b)** El **caballo de fuerza** (33,000 pies-libras por minuto ó 550 pies-libras por segundo) es la **unidad de fuerza** ó **de proporción de trabajo,** comúnmente usada tratándose de máquinas. El **caballo de fuerza métrico** llamado « caballo de vapor » ó (en alemán) « Pferdekraft », es de 75 kilográmetros por segundo = 542.48 pies-libras por segundo = 32,549 pies-libras por minuto = .9863 caballos de fuerza. El caballo de fuerza = 1,0138 « force de cheval ». En mecánica teórica, el **pie-libra por segundo** se usa en las medidas inglesas y el **kilográmetro por segundo** en las medidas métricas.

1 pie-libra por segundo = .13825 kilográmetro por segundo.
1 kilográmetro por segundo = 7.2331 pies-libra por segundo.

**(c)** Hasta el momento en que la velocidad se hace uniforme, la **fuerza y proporción de trabajo** del tren, art. 16 **(d)**, es *variable, acelerándose* gradualmente. Pues en cada segundo vence sus resistencias (y mueve sus puntos de aplicación) *á mayor distancia* que durante el anterior segundo. También después que se suspende la entrada del vapor, la proporción de trabajo es variable, siendo *retartada* gradualmente. Cuando la fuerza del vapor equilibre exactamente las resistencias, la proporción de trabajo es uniforme.

**(d) Potencia = fuerza × velocidad.** Desde luego que la proporción de trabajo es igual al trabajo ejecutado *en un tiempo dado;* podemos encontrarlo dividiendo el trabajo (en pies-libras ó kilográmetros), ejecutado durante cualquier tiempo dado, por el número de segundos contenido en ese tiempo. Así :

$$\text{Potencia} = \text{proporción de trabajo} = \frac{\text{fuerza en kg} \times \text{distancia en metros}}{\text{número de segundos}}.$$

Pero esto es equivalente á :

$$\text{Potencia} = \text{proporción de trabajo} = \text{fuerza en kg} \times \frac{\text{distancia en mets}}{\text{tiempo en segundos}}$$

$$= \text{fuerza en kg} \times \text{velocidad en metros por segundo.}$$

O si nos ocupamos sólo del trabajo de la fuerza que vence *las resistencias,* ó de los casos en que la velocidad es enteramente uniforme, la misma al principio y fin del trabajo :

$$\underset{\text{por segundo}}{\text{Potencia en kilográmetros}} = \underset{\text{de trabajo}}{\text{proporción}} = \underset{\text{kilogramos}}{\text{resistencia en}} \times \underset{\text{por seg.}}{\text{velocidad en metros}}$$

Así pues, si la resistencia es de 100 kg y es vencida en distancias de 10 mets en cada minuto, ó si la resistencia es de 10 kg y es vencida en distancias de 100 mets por minuto, la proporción del trabajo es en ambos casos la misma, á saber: 1,000 kilográmetros por minuto, porque

kg velocidad kg velocidad.
$100 \times 10 = 10 \times 100 = 1{,}000$ kilográmetros por minuto.

**(e)** La misma « potencia » que vence á una resistencia dada, en una distancia dada, en un tiempo dado, vencerá también otra resistencia en cualquiera otra distancia en el mismo tiempo, siempre que la resistencia y la distancia multiplicadas den al misma cantidad que en el primer caso. Así, la fuerza que suspende 50 kgs á 10 mets en un segundo, elevará en un segundo 500 kgs á un metro ó 25 kgs á 20 mets ó 5000 kgs á $^1/_{10}$ de metro. En la práctica, lo que se acostumbra generalmente para vencer las diferentes resistencias son **ruedas dentadas, correas** ó **palancas.**

Por este medio, la máquina, rueda hidráulica, caballo ú otra fuerza motriz ejerciendo una fuerza dada y moviéndose con una velocidad dada, puede vencer pequeñas resistencias rápidamente ó grandes lentamente, según se desee.

**Art. 19 (a). Trabajo que un cuerpo puede efectuar en virtud de su movimiento** ó (lo que es lo mismo) **el trabajo necesario para poner al cuerpo en reposo. Energía cinemática ó fuerza viva.** Como ya se ha dicho, una fuerza igual al peso de cualquier cuerpo, en cualquier lugar, comunicará en un segundo, á la masa del cuerpo, una velocidad = *g;* que en la superficie de la tierra es como de 9.81 metros por segundo. O si se lanza un cuerpo hacia arriba con una velocidad = *g*, su peso lo hará detener en un segundo.

Desde luego que en el último caso las velocidades dadas al principio y al fin del segundo son, respectivamente, $g$ metros por segundo, y cero. La velocidad *media* del cuerpo es $\frac{g}{2}$ metros por segundo. Por tanto, durante el segundo aumentará $\frac{g}{2}$ metros, es decir, como 4.90 metros. En otras palabras, el trabajo que cualquier cuerpo puede ejecutar por haber sido lanzado hacia arriba con una velocidad inicial (velocidad en el momento de partir) de $g$ metros por segundo, es igual al producto de su peso multiplicado por $\frac{g}{2}$ metros; ó

$$\text{trabajo en kgm} = \text{peso} \times \frac{g}{2}\text{; (en kg y en metros, respectivamente).}$$

Nótese que en este caso (desde que la velocidad inicial $v$ es igual á $g$) $\frac{v}{g} = 1$.

Supongamos ahora que el mismo cuerpo es lanzado hacia arriba con una velocidad *doble* de la anterior; esto es, con una velocidad inicial igual á $2g$ ó 19.62 m por segundo. Desde que la gravedad requiere, art. 8 (c), .2 segundos para comunicar ó destruir esta velocdad, el cuerpo se moverá ahora hacia arriba durante 2 segundos ó dos veces mayor *tiempo* que antes. Pero ahora, su *velocidad* media es $g$ ó dos veces mayor que anteriormente. Por tanto, moviéndose por doble tiempo y con doble velocidad, irá *cuatro veces más* lejos venciendo la misma resistencia que antes; es decir, su propio peso.

Así pues, haciendo su velocidad inicial $v = 2g$, esto es, *doblando* su $\frac{v}{g}$, haciéndolo $= 2$ hemos puesto el cuerpo en estado de hacer *cuatro* veces el trabajo que podía hacer cuando su $\frac{v}{g}$ era 1; así es que el trabajo en el segundo caso es igual al producto del trabajo en el primer caso, multiplicado por el *cuadrado* de $\frac{v}{g}$; ó

$$\text{Trabajo} = \text{peso} \times \frac{g}{2} \times \left(\frac{v}{g}\right)^2$$

$$\text{—} = \text{peso} \times \frac{g}{2} \times \frac{v^2}{g^2}$$

$$\text{—} = \text{peso} \times \frac{v^2}{2g}.$$

Y claro es que será el mismo principio para cualquiera *otra* velocidad. Ahora la cantidad total del trabajo que el cuerpo puede ejecutar es independiente de la cantidad de resistencia contra la cual es ejecutado; pues si aumentamos la resistencia disminuimos la distancia, en la misma proporción, de modo que su producto ó la cantidad de trabajo, permanece el mismo. La fórmula anterior puede ser aplicada, por consiguiente, á todos los casos, esto es, la **cantidad total de trabajo**, en pies-libras ó en kilográmetros, que cualquiera cuerpo ejecutará contra cualquiera resistencia (en virtud de su movimiento únicamente), cuando llega al reposo es:

$$\textbf{Trabajo} = \text{peso del cuerpo en movimiento} \times \frac{\text{cuadrado de su velocidad}}{2g}$$

$$= \text{peso de cuerpo en movimiento} \times \text{caída requerida para comunicarle esta velocidad.}$$

$$= \frac{\text{peso del cuerpo en movimiento}}{g} \times \frac{\text{cuadrado de su velocidad}}{2}.$$

En estas ecuaciones, el *peso* es el que el cuerpo tiene en cualquier lugar dado y $g$ es la aceleración debida á la gravedad en ese mismo lugar.

(b) Desde luego que $\frac{\textit{el peso de un cuerpo}}{g} =$ á su masa (art. 11).

$$\textbf{Trabajo} = \text{masa del cuerpo} \times \frac{\text{cuadrado de su velocidad}}{2}.$$

(c) En las ecuaciones anteriores el lado *izquierdo* representa el *trabajo* (ó resistencia vencida á lo largo de una distancia) en cualquier caso dado, mientras que el lado *derecho* representa la **energía cinemática** del cuerpo, por la cual se pone

en estado de ejecutar aquel trabajo. (*N. del T.* — De la misma ecuación se deduce que el trabajo es igual á la mitad de la fuerza viva ($mv^2$).

(d) Como ejemplo de lo anteriormente dicho, tómese un tren que pesa 500,000 kg y moviéndose á razón de 22 mets por segundo; la *energía cinemática* de dicho tren es:

$$\text{energía} = \text{peso} \times \frac{\text{velocidad}^2}{2g};$$

$$500{,}000 \text{ kg} \times \frac{22^2}{19.62} = 12{,}335{,}000 \text{ kilográmetros.}$$

Esto es, si se impide la entrada del vapor el tren ejecutará un trabajo de 12,335,000 kgm para entrar en reposo. Así pues, si la suma de todas la resistencias (de frotamiento, aire, declive curvas, etc.) permanecen constantemente = 5,000 kg * el tren caminará

$$\frac{12{,}335{,}000 \text{ kgm}}{5{,}000 \text{ kg}} = 2{,}467 \text{ metros.}$$

(e) Vemos así que la cantidad total de trabajo que un cuerpo puede ejecutar en virtud de su movimiento solo y sin el auxilio de fuerzas extrañas, es proporcional al peso del cuerpo y al *cuadrado* de su velocidad cuando comienza á ejecutar el trabajo. Por ejemplo, supongamos que un tren en el momento en que se suprime la entrada del vapor, tiene una velocidad de 10 kilómetros por hora, y que la energía cinemática que le da la velocidad arrastre por sí el tren oponiéndose á las resistencias del camino, etc., hasta una distancia de un cuarto de kilómetro antes de detenerse. Sentado esto, cuando se suspenda el vapor moviéndose el tren á 5,20, 30 ó 40 kilómetros por hora (esto es, con ½,2, 3 ó 4 veces 10 kilómetros por hora), el tren hará $^1/_{16}$, 1, 2$^1/_4$ ó 4 kilómetros (es decir, $^1/_4$, 4, 9 ó 16 veces $^1/_4$ de kilómetro) antes de detenerse. Aquí se supone que la resistencia es uniforme.

Pero la relación del trabajo ejecutado es simplemente proporcional á la resistencia y á la *velocidad* (art. 18 (*d*)). Por tanto, la locomotora cuyo vapor se intercepta á 20, 30 ó 40 kilómetros por hora, no requerirá para correr los 4, 9 ó 16 cuartos de kilómetro sino 2, 3 ó 4 veces tantos segundos cuantos necesitaba á razón de 10 kilómetros por hora.

El mismo principio se emplea en todos los casos de aceleración y de retardación.

Por ejemplo, en el caso de un cuerpo que cae, la *distancia* á que debe caer para adquirir una velocidad dada, está como el *cuadrado* de esta velocidad; pero el *tiempo* requerido es simplemente proporcional á la *velocidad*. También si se arroja un cuerpo verticalmente hacia arriba con una velocidad dada, la *altura* á la cual se eleva hasta que la gravedad destruya esa velocidad, estará como el cuadrado de la velocidad, pero el tiempo será simplemente proporcional á la velocidad.

**Art. 20** (a). Cuando un cuerpo parte del reposo y se mueve bajo la acción de una fuerza constante, durante un tiempo $t$ adquiere una velocidad, $v$, su velocidad media durante el tiempo, $t$, es ½ $v$; y la distancia recorrida es $s = ½\ vt$. Como $mv = ft$ (art. 12), tenemos:

$$\frac{mv^2}{2} = mv \times \frac{v}{2} = ft\,\frac{v}{2} = fs.$$

(*N. del T.* — Es decir, que el trabajo es igual á la mitad de la fuerza viva.)

$$\text{Como } fs = k = \frac{mv^2}{2};$$

se tiene

$$\frac{fs}{t} = \frac{mv^2}{2t} = \frac{mv}{2} \times \frac{v}{t} = \frac{mv}{2} \times a = \frac{mv}{2} \times \frac{f}{m};$$

ó bien: **la cantidad de trabajo en la unidad de tiempo** ó la proporción del trabajo

= á la mitad de la cantidad de movimiento × aceleración

= á la mitad de la cantidad de movimiento × intensidad de la fuerza.

En otras palabras, si una masa de $m$ kg se mueve con velocidad de $v$ metros por seg, tiene una cantidad de movimiento de $mv$ kilográmetros por seg y la energía de $\frac{mv^2}{2} = fs$, es decir, á la cantidad de trabajo hecho con ella para darle la

* Este no es el caso en la práctica.

velocidad $v$; é igual cantidad de trabajo debe hacerse para traer el cuerpo al reposo.

Así, si $m=4$ kg, y $v=$velocidad inicial$=12$ ms por seg; tenemos $mv=4\times12=48$ kgms por segundo, y $fs=\frac{mv^2}{2}=mv\times\frac{v}{2}=48\times6=288$ kilográmetros.

Escogiendo diferentes fuerzas, $f$, para traer esta masa al reposo, se obtienen diferentes aceleraciones, diferentes proporciones de trabajo, etc.

**La cantidad de movimiento, $mv$, en cualquier momento**, es necesariamente doble de la cantidad de movimiento media, $\frac{mv}{2}$, que la masa tiene durante el tiempo, $t=\frac{v}{a}$ en que adquiere ó pierde la velocidad, $v$, bajo la acción de cualquier fuerza constante, $f$.

Sea el peso de un cuerpo que cae$=$W, y la aceleración de la gravedad$=g$. Entonces $W\times\frac{v}{2}\times\frac{v}{g}=$ peso × velocidad media × tiempo de caída = peso × distancia (altura) de caída $=W\times\frac{v^2}{2g}=$ al trabajo hecho $=\frac{W}{g}\times\frac{v^2}{2}=m\times\frac{v^2}{2}$ $=\frac{mv^2}{2}=$ energía cinética adquirida.

**Art. 21 (a). La energía es indestructible.** La energía empleada en el trabajo no se destruye. Se comunica á otros cuerpos ó se acumula en el cuerpo mismo, ó una parte se transmite y el resto se acumula. Pero aunque la energía no se pueda destruir, puede hacerse inútil. Así, un tren en movimiento al pararse en una vía á nivel cambia su energía cinética por otra energía de la misma especie, á saber; el calor inútil debido al frotaniento de los rieles, los frenos y cilindros; y este calor, aunque no está *destruído*, se disipa en la tierra y en el aire de modo que prácticamente es irrecuperable.

**Art. 22 (a). La energía potencial** ó energía posible, se puede definir como energía acumulada. Elevamos un cuerpo del peso de un kilogramo á un metro de altura consumiendo en él un kilográmetro de energía. Pero este kilográmetro se acumula en el « sistema » compuesto de la tierra y el cuerpo como un aumento á su provisión de energía potencial.

**(b)** La energía potencial de un « sistema » de cuerpos (tales como la tierra y un peso elevado sobre ella, ó las moléculas de una masa de pólvora, ó las de un resorte torcido) depende de la *posición* relativa de esos cuerpos y de sus *tendencias* á cambiar esas posiciones. La energía cinética de un sistema (como la tierra ó un tren de carros movible) depende de las masas de sus cuerpos y de sus movimientos relativos entre sí. Ejemplos comunes de la energía potencial son el peso ó resorte de un reloj cuando se le ha dado toda ó parte de la cuerda y cuando está andando ó no. El agua encerrada en un receptáculo, la presión del vapor en una caldera y la energía explosiva de la pólvora. Tenemos energía mecánica en el caso del peso, del resorte y el agua; energía calorífica en el caso del vapor, y energía química en el de la pólvora.

**(c)** En muchos casos podemos estimar la energía potencial total de un sistema. Así (sin ocuparnos de la resistencia del aire), la energía explosiva de una libra de pólvora es$=$al peso de cualquiera bala de cañón que se nos dé × la altura á la cual la fuerza de esta pólvora la podría arrojar$=$peso de la bala × (el cuadrado de la velocidad inicial que le comunica la explosión) ÷ $2g$. Pero en otros casos sólo nos interesa encontrar cierta parte *limitada* de la energía potencial *total*. Así, la energía potencial total de un reloj de peso * no se agotará hasta que el peso no llegue al centro de la tierra; pero nosotros sólo nos ocupamos de la parte de dicha energía acumulada en el reloj al darle cuerda, la cual será transformada en energía cinética al desarrollarse la cuerda y descender el peso. Esta parte es$=$al peso × la altura desde la cual tiene que caer$=$al peso × (el cuadrado de la velocidad que adquiriría al caer *libremente* desde esa altura) ÷ $2g$.

**(d)** Hay muchos casos, tratándose de energías en las cuales podemos titubear respecto á cuál de las palabras « cinética » ó « potencial » es más apropiada. Así, al considerar la presión del vapor en una caldera, se cree que es debida al movimento violento de las moléculas del vapor que golpean la superficie interior de la caldera, y desde este punto de vista deberíamos llamarla energía cinética del vapor. Pero al mismo tiempo, la caldera permanece estacionaria, y hasta que no se per-

* Por conveniencia hablamos de la energía de un sistema de cuerpos (la tierra y el peso del reloj) como residiendo sólo en uno de los cuerpos.

mita la salida del vapor de la caldera, no hay evidencia exterior de la energía en forma de trabajo. La energía permanece estacionada en la caldera dispuesta para ser aplicada. Desde este punto de vista podemos llamar la energía del vapor así encerrado energía *potencial*.

(c) Parece racional suponer que los conocimientos ulteriores acerca de la naturaleza de otras formas de energía, aparentemente potencial (como la del vapor), ponen de manifiesto que toda energía es finalmente cinética.

**Art. 23 (a).** Hay mucha **confusión de ideas** respecto á las acciones llamadas en mecánica **fuerza, energía, potencia**, etc. Esto viene de que en el lenguaje ordinario se usan indistintamente estos términos para expresar una misma idea. Así, comúnmente hablamos de la fuerza de una bala de cañón al atravesar el aire, queriendo significar, sin embargo, la fuerza repulsiva que se *ejercería* entre una bala y un edificio, etc., si se pusieran en contacto. Esta fuerza tendería á mover una parte del edificio en la dirección de la bala y movería á la bala hacia atrás (esto es, retardaría su movimiento hacia adelante); pero esta gran *fuerza* repulsiva no existe hasta que la bala choque con el edificio. En verdad, no podemos siquiera decir por la velocidad y peso de la bala cuál sería la intensidad de la fuerza, pues ésta depende de la resistencia, etc., del edificio. Si el edificio es de vidrio, la fuerza puede ser tan leve que apenas retarde el movimiento de la bala perceptiblemente, mientras que si el edificio ó la obra es un terraplén, la fuerza será mucho mayor y puede retardar el movimiento de la bala tan rápidamente que la pare antes de moverse unos centímetros más.

La bala en movimiento tiene una gran *energía* (cinética), pero la única *fuerza* que ejerce durante su carrera, es la comparativamente leve que se requiere para apartar las moléculas de aire.

La energía de la bala y, por lo tanto, el trabajo total que ella puede realizar son independientes de la naturaleza del obstáculo que encuentra; pero, desde que el trabajo es el producto de la resistencia que se presenta y de la distancia recorrida mientras se la destruye, esta distancia debe estar en razón inversa de la resistencia que se presenta ó, lo que es lo mismo, en razón inversa de la fuerza que necesita la bala para equilibrar esa resistencia. Como el trabajo en kilográmetros = fuerza en kg × distancia recorrida en metros, tenemos :

$$\text{fuerza en kgs} = \frac{\text{trabajo en kilográmetros}}{\text{distancia recorrida en metros}} = \frac{\text{cantidad de trabajo}}{\text{en kilográmetros } \textit{por metro.}}$$

**Art. 24 (a). Un choque,** golpe ó colisión tiene lugar cuando un cuerpo en movimiento tropieza con otro. Lo peculiar en dichos casos, es que el *tiempo de acción* de la fuerza repulsiva debido á la colisión, es tan *corto*, que, generalmente, es imposible medirlos, y, por consiguiente, no podemos calcular la fuerza por la cantidad de movimiento producida por ella en uno ú otro de los dos cuerpos; pero desde que ambos experimentan un gran cambio en su velocidad durante este corto tiempo, sabemos que la fuerza repulsiva, que actúa entre ellos, debe ser muy grande.

Consideraremos solamente los casos de *choques directos* ó choques en los cuales los centros de gravedad de los dos cuerpos se aproximan naturalmente en línea recta, y donde la naturaleza de la superficie de contacto es tal, que la fuerza repulsiva causada por el choque obra también de un extremo á otro de aquellos centros y en la línea en que se aproximan.

(b) Esta fuerza, obrando igualmente sobre los dos cuerpos (art. 5 *f*) durante el mismo tiempo (á saber, el tiempo que dure el contacto), necesariamente produce igual y opuestos cambios en sus cantidades de movimiento ó fuerzas (art. 12). Por consiguiente, la cantidad de movimiento ó impulso total (ó producto de la masa × velocidad) de los dos cuerpos es siempre la misma después del choque que antes.

(c) Pero el estado en que queden los dos cuerpos, después de la colisión, depende de su elasticidad. Si pudiesen ser absolutamente inelásticos, sus velocidades, después del choque, serían iguales. En otras palabras, se moverían juntos. Si pudieran ser perfectamente *elásticos*, se separarían el uno del otro, después de la colisión, con las mismas velocidades que tenían al aproximarse antes de la colisión.

(d) Entre estos dos extremos (los cuales jamás se realizan perfectamente en la práctica), hay todos los grados posibles de elasticidad, con correspondientes diferencias en el resultado del fenómeno, ó manera de conducirse los cuerpos. Esta materia, especialmente la del *choque indirecto*, es muy complicada, pero rara vez se presenta en la ingeniería práctica civil.

(e) « En algunos experimentos hechos con cuidado en el arsenal de Portsmouth

(Inglaterra), un hombre de mediana resistencia, golpeando con una maza de 8.10 kg de peso, cuyo mango era de 1.10 m de largo, apenas hizo penetrar un perno 3 mm en cada golpe, y se necesitó una presión tranquila de 107 toneladas simplemente apoyadas sobre el perno, para hacerlo entrar la misma cantidad; pero al agregarle un pequeño peso, la hizo entrar completamente. »

## GRAVEDAD, CAÍDA DE LOS CUERPOS

**Cuerpos que caen verticalmente.** Un cuerpo que cae libremente en el vacío al partir del reposo, adquiere, al fin del primer segundo, una velocidad como de 9.81 m por segundo, y en cada segundo subsiguiente una velocidad adicional ó aceleración como de 9.81 m por segundo. En otras palabras, la velocidad recibe *en cada segundo* una aceleración de 9.81 m por segundo. Esta proporción es generalmente llamada **aceleración de la gravedad** y se la designa por **g**. Aumenta, cada segundo, como 9.78 m en el ecuador y como 9.90 m en los polos. En la latitud de Londres es de 9.81 m. Estos son los cálculos al nivel del mar; pero á una altura de 8 kilómetros sobre ese nivel, disminuye 1 parte en 400. Para la mayor parte de los usos ordinarios se puede calcular en 9.81 m.

**Observación. A causa de la resistencia del aire** ninguna de las reglas que siguen dan resultados perfectamente exactos en la práctica, especialmente en grandes velocidades. Mientras mayor sea la densidad del cuerpo más exacto será el resultado. El aire opone resistencia á la elevación y á la caída de los cuerpos.

**Si se lanza un cuerpo verticalmente hacia arriba** con una velocidad dada, se elevará á la misma altura de la que debería caer para adquirir dicha velocidad, y en cada segundo se retardará su velocidad 9.81 m por segundo. Su velocidad media ascendente será la mitad de la de salida, como en todos los otros casos de velocidad uniformemente retardada. Cayendo adquirirá la misma velocidad con que salió y en el mismo tiempo. Véase la observación anterior.

### Velocidad adquirida

$$\left.\begin{array}{l}\text{en un tiempo dado}\\ \text{desde una altura de caída dada partiendo del reposo}\\ \text{en una altura de caída dada desde el reposo y un tiempo dado}\end{array}\right\} = \begin{cases} g \times \text{tiempo} \\ \sqrt{2g \times \text{altura de caída}} \\ \dfrac{2 \text{ veces altura de caída}}{\text{tiempo}} \end{cases}$$

### Tiempo requerido

$$\text{para adquirir una velocidad dada} = \frac{\text{velocidad dada}}{g}$$

$$\left.\begin{array}{l}\text{para caer de una altura dada partiendo del reposo}\\ \text{para bajar de una altura de caída dada partiendo del reposo ó no}\end{array}\right\} = \begin{cases} \sqrt{\dfrac{\text{altura de caída}}{\frac{1}{2}\,g}} = \dfrac{\text{altura de caída}}{\frac{1}{2} \text{ velocidad final}} \\ \dfrac{\text{altura de caída}}{\text{velocidad media}} = \dfrac{\text{altura de caída}}{\frac{1}{2}(\text{vel inicial} + \text{vel final})} \end{cases}$$

### Altura de caída

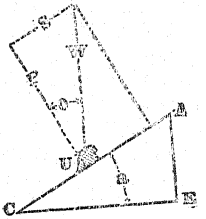
$$\left.\begin{array}{l}\text{recorrida en un tiempo dado (partiendo del reposo)}\\ \text{en tiempo dado (partiendo del reposo ó no)}\end{array}\right\} = \begin{cases} \text{tiempo} \times \tfrac{1}{2} \text{ velocidad final} = \text{tiempo}^2 \times \tfrac{1}{2}\, g \\ \text{tiempo} \times \text{velocidad media} = \\ = \text{tiempo} \times \dfrac{\text{velocidad inicial} + \text{veloc final}}{2} \end{cases}$$

$$\left.\begin{array}{l}\text{requerida para una velocidad dada}\\ \text{(partiendo del reposo)}\end{array}\right\} = \frac{\text{velocidad}^2}{2g}$$

$$\left.\begin{array}{l}\text{durante cualquier segundo dado}\\ \text{contado desde el origen del}\\ \text{movimiento, es decir, del reposo}\end{array}\right\} = g \times (\text{número del segundo (1, 2, etc.)} - \tfrac{1}{2}).$$

| Cuadro en *metros* calculando $g=9.81$ m. | Al fin del segundo. 1º | 2º | 3º | 4º | 5º | 6º | 7º | 8º | 9º | 10º |
|---|---|---|---|---|---|---|---|---|---|---|
| Velocidad adquirida al fin de cada segundo..... | 9.81 | 19.62 | 29.43 | 39.24 | 49.05 | 58.86 | 68.67 | 78.48 | 88.29 | 98.10 |
| Espacio recorrido durante cada segundo por separado..... | 4.90 | 14.70 | 24.50 | 34.30 | 44.10 | 53.90 | 63.70 | 73.50 | 83.30 | 93.10 |
| Espacio recorrido desde el origen del movimiento hasta el fin del segundo de que se trata..... | 4.90 | 19.60 | 44.10 | 78.40 | 122.50 | 176.40 | 240.10 | 313.60 | 396.90 | 490.00 |

**Descenso en planos inclinados.** Cuando se coloca un cuerpo U sobre un plano inclinado AC, todo su peso W no se empela en comunicarle velocidad (como cuando los cuerpos caen verticalmente), una porción de él P (= W × coseno de, $c$ = W × coseno de $a$ *) se emplea en ejercer una presión normal al plano; mientras que la componente S (= W × seno $o$ = W × seno $a$ *) solamente obra sobre U en dirección paralela á la superficie AC del plano y tiende á hacerlo descender por él




La aceleración comunicada á un cuerpo dado, en un tiempo dado, es proporciona á la fuerza que se ejerce sobre el cuerpo en la dirección de la aceleración. Por consiguiente, si representamos por W en escala, la aceleración $g$ (digamos 9.81 m por segundo), que la gravedad comunicaría á U en un segundo cayendo libremente, entonces la línea S representará en la misma escala la aceleración por segundo que la fuerza de deslizamiento S comunicará al cuerpo por el plano si no hubiera frotamiento. Tenemos, por consiguiente, que la aceleración *teórica* en el plano inclinado $= g$ sen $a$.

Por consiguiente nos basta con substituir $g$ sen $a$ en lugar de $g$ y la longitud del plano inclinado AC en lugar de la correspondiente altura vertical del plano AE en las últimas ecuaciones para obtener las velocidades, etc., como sigue para **un plano inclinado sin tener en cuenta el rozamiento.**

### Velocidad adquirida en el descenso

$$\text{en un tiempo dado} = \left.\begin{array}{c}\text{la velocidad adquirida en la caída vertical}\\ \text{durante el mismo tiempo}\end{array}\right\} \times \text{sen } a$$

$$= g \times \text{sen } a \times \text{tiempo.}$$

$$\left.\begin{array}{c}\text{en una longitud del plano dada}\\ \text{(como AC) á partir del reposo}\end{array}\right\} = \frac{\text{AC}}{\frac{1}{2}\text{ tiempo}}$$

$$= \left.\begin{array}{c}\text{velocidad adquirida ca-}\\ \text{yendo libremente por la}\\ \text{altura vertical AE co-}\\ \text{rrespondiente}\end{array}\right\} \sqrt{2g \times \text{AE}}$$

$$= \sqrt{2g \text{ sen } a \times \text{AC}}.$$

* Porque $o$ y $a$ son iguales.

### Tiempo necesario

$$\left.\begin{array}{l}\text{para adquirir una velocidad}\\ \text{dada en el plano}\end{array}\right\} = \frac{\text{velocidad dada}}{g \text{ sen } a}$$

$$\left.\begin{array}{l}\text{para recorrer una longitud dada}\\ \text{(como AC) partiendo del reposo}\end{array}\right\} = \frac{\text{longitud AC}}{\frac{1}{2} \text{ veloc final}} = \sqrt{\frac{\text{long AC}}{\frac{1}{2} g \text{ sen } a}}$$

$$= \frac{\text{tiempo necesario para caer por la altura vertical correspondiente AE}}{\text{sen } a}$$

$$\left.\begin{array}{l}\text{para recorrer una longitud del plano}\\ \text{dada, partiendo del reposo ó no}\end{array}\right\} = \frac{\text{longitud dada en el plano}}{\text{velocidad media por el plano}}$$

$$= \frac{\text{longitud dada del plano}}{\frac{1}{2} \text{ (vel inicial + vel final)}}$$

$$\textbf{Coseno a} = \frac{\text{base EC}}{\text{longitud AC}} = \frac{\text{proyección EC de la longitud AC}}{\text{longitud AC}} = \frac{\sqrt{\overline{AC}^2 - \overline{AE}^2}}{AC}$$

$$\textbf{Seno a} = \frac{\text{altura AE}}{\text{longitud AC}} = \frac{\text{altura correspondiente á cualquier otra longitud AC}}{\text{esa longitud AC}} =$$

$$= \frac{\sqrt{\overline{AC}^2 - \overline{EC}^2}}{AC}.$$

### Longitud AC

$$\left.\begin{array}{l}\text{recorrida en un tiempo dado}\\ \text{partiendo del reposo}\end{array}\right\} = \text{tiempo} \times \tfrac{1}{2} \text{ velocidad final por el plano}$$

$$= \text{tiempo}^2 \times \tfrac{1}{2}\, g \text{ sen } a.$$

$$\left.\begin{array}{l}\text{recorrida en un tiempo dado}\\ \text{partiendo del reposo ó no}\end{array}\right\} = \text{tiempo} \times \text{velocidad media por el plano}$$

$$= \text{tiempo} \times \tfrac{1}{2} \text{ (vel inicial + vel final)}$$

$$\left.\begin{array}{l}\text{que debe recorrer un cuerpo para adquirir}\\ \text{partiendo del reposo una velocidad dada}\end{array}\right\} = \frac{\text{velocidad dada}^2}{2g \text{ sen } a}.$$

**En la práctica, al movimiento de descenso se opone siempre el rozamiento; para tomar en cuenta su efecto** debe substituirse en las ecuaciones del plano inclinado en lugar de ($g$ sen $a$) el valor siguiente :

$g \times$ (sen $a$ — (cos $a \times$ coeficiente de rozamiento)). La razón es la siguiente :

**Rozamiento** = presión normal P $\times$ coeficiente de rozamiento
= peso W $\times$ cos $a \times$ coeficiente de rozamiento

y, por consiguiente, la aceleración $g$ debida á la gravedad se disminuirá por el rozamiento = $g$ cos $a \times$ coeficiente de rozamiento.

La **resultante** entre la fuerza S que hace deslizar el cuerpo y la de rozamiento que se opone, es :

= velocidad teórica (del descenso producida por la fuerza S) — efecto del rozamiento
= ($g$ sen $a$) — ($g$ cosen $a \times$ coeficiente de rozamiento)
= $g$ (sen $a$—cosen $a \times$ coeficiente de rozamiento).

# PÉNDULOS

El número de oscilaciones que diferentes péndulos hacen en cualquier lugar, en un tiempo dado, está en razón inversa de las raíces cuadradas de sus longitudes; así, si uno de ellos es 4, 9 ó 16 veces más largo que el otro, su raíz cuadrada será 2, 3 ó 4, y el número de oscilaciones será $^1/_2$, $^1/_3$ ó $^1/_4$ de las que hacía el péndulo de longitud 1. El tiempo en que diferentes péndulos hacen una oscilación, está en razón directa de la raíz cuadrada de sus longitudes. Así, si uno es 4, 9 ó 16 veces más largo que otro, su raíz cuadrada será 2, 3 ó 4 veces mayor y así será también el tiempo que emplea en hacer una oscilación. La longitud del péndulo que mide segundos al nivel del mar, en el vacío, á la latitud de Londres (51½° N) es 39.1393 pulgadas (.9941 m); y en la latitud de Nueva York (40 $^3/_4$° N) 39.1013 pulgadas (.9931 m). En el ecuador es como $^1/_{10}$ de pulgada (2½ mm) más corto y en los polos como $^1/_{10}$ de pulgada (2½ milímetros) más largo. Podemos decir que para experiencias de pocos segundos, la longitud del péndulo que mide segundos, en cualquier lugar y al aire libre, es, con una aproximación suficiente, de .990 m, y el que mide ½ segundo, de .2476 m; y podemos suponer que las oscilaciones largas y cortas del mismo péndulo son hechas en el mismo tiempo, como en efecto lo son con muy poca diferencia. Para medir profundidades ó distancias por medio del sonido, se puede hacer un péndulo de segundos suficientemente exacto con una piedra (un pedazo de metal en forma de disco es mejor) y un hilo suspendido de un alfiler ó un clavo delgado común. La longitud de .99 m debe medirse desde el centro de la piedra. Al comenzar las oscilaciones no debe impulsarse la *piedra ó disco;* simplemente *abandonarlo* á su propio peso después de separar la cuerda tesa de su posición vertical á una altura suficiente.

(*N. del T.* — **Para tener en milímetros** la longitud de un péndulo que haga un número de oscilaciones dadas en un minuto : divídase el número 3,571,875 por el cuadrado del número de oscilaciones, y el cociente es la longitud buscada del péndulo en milímetros. Ej. : supongamos un péndulo que haga 100 oscilaciones por minuto : dividiendo á 3,571,875 por el cuadrado de 100 que es 10,000, resultan 357.19 milímetros para la long del péndulo que se desea. Esta es la regla que da el autor convertida al sistema métrico.)

**Observación 1.ª** Practicando con un péndulo de segundos de un reloj, ó con uno preparado como se ha dicho, una persona aprenderá pronto á contar 5 en el tiempo de un segundo, y por algunos segundos sucesivos; y podrá también de este modo dividir el segundo en 5 partes iguales; ambas cosas pueden ser útiles algunas veces para cálculos aproximados cuando no se tiene péndulo á la mano.

## Centro de oscilación.

**Observación 2.ª** Cuando un péndulo ó cualquiera otro cuerpo suspendido oscila ligero ó despacio, es claro que las moléculas que están lejos del punto de suspensión se mueven más ligero que las que están cerca de él. Pero hay un punto tal en el péndulo, que, si *todas* las demás moléculas del péndulo se encontraran en él, y se movieran con la misma velocidad actual, ni el número de oscilaciones, ni su velocidad angular cambiarían. Este punto se llama *el centro de oscilación.* No es el mismo que *el centro de gravedad* y está siempre más lejos que él del punto de suspensión. La distancia entre este punto y el punto de suspensión se encuentra así : Supongamos al cuerpo dividido en muchas partes pequeñas (mientras mayor número y más pequeñas mejor). Búsquese el peso y el centro de gravedad de cada parte y la distancia de cada uno de esos centros de gravedad al punto de suspensión. Elévese al cuadrado cada una de estas distancias y multiplíquese cada cuadrado por el peso de la parte correspondiente del cuerpo. Súmense todos estos productos parciales y llámese *p* á esa suma; luego multiplíquese el peso del cuerpo entero por la distancia de su centro de gravedad al punto de suspensión. Llámese *g* el producto; divídase *p* por *g*. La suma *p* es el **momento de inercia** del cuerpo, y si se divide por su peso la raíz cuadrada del cociente será el **radio de giro** del cuerpo.

## Velocidad angular.

Cuando un cuerpo da vueltas alrededor de cualquier eje, las partes que se encuentran más distantes de ese eje se mueven con más velocidad que aquellas que están más cerca de él. Por lo tanto, no podemos asignar una velocidad lineal

fija en pies ó metros por segundo ó en millas ó kilómetros por hora y que pueda aplicarse á *cada una de sus partes*. Pero cada parte del cuerpo recorre una circunferencia entera ó un ángulo de 360° en el mismo tiempo. Por consiguiente todas las partes tienen *la misma velocidad angular* por segundo y dan el mismo número de *revoluciones* por seg alrededor del eje. Esto se llama la velocidad angular. Los autores científicos la miden por la longitud del arco descrito por cualquier punto en el cuerpo en un tiempo dado, como por ejemplo en un segundo; apreciando la longitud del arco por el número de veces que contiene la longitud de *su propio radio*. Cuando se mide de este modo tenemos

$$\text{La velocidad angular en radios por segundo} = \frac{\text{La velocidad lineal (en pies ó metros) por segundo.}}{\text{Longitud del radio (en pies ó metros).}}$$

En este caso, como queda dicho, la velocidad angular es la misma para todos los puntos del cuerpo, porque las velocidades de varios puntos están en razón directa de sus radios ó distancias al eje de revolución.

En cada revolución, cada punto describe la circunferencia del círculo en el cual gira $=2\pi r$ ($\pi=3.1416$, etc.; $r$=radio de dicho círculo); por consiguiente, si el cuerpo hace $n$ revoluciones por segundo, la longitud del arco descrito por cada punto en un segundo es $2\pi rn$; y la velocidad angular del cuerpo, ó la velocidad *lineal*, de cualquier punto *medido con su propio radio*, es

$$a = \frac{2\pi rn}{r} = 2\pi n = 6.2832 \times \text{el número de revoluciones por } \textit{segundo} =$$

$$.1047 \times \text{el número de revoluciones por } \textit{minuto}.$$

## Momento de inercia.

Supongamos un cuerpo girando alrededor de un eje, como una piedra de amolar, ú oscilando, como un péndulo. Supongamos que la distancia del eje de revolución (que en el péndulo es el punto de suspensión), á cada molécula en particular del cuerpo, ha sido medida, y que el cuadrado de cada una de dichas distancias ha sido multiplicado por el peso de sus respectivas moléculas.

La suma de todos estos productos es el momento de inercia del cuerpo. Esto

$$\textbf{Momento de inercia} = \left\{\begin{array}{c}\text{la suma para}\\ \text{todas las moléculas}\end{array}\right\} \text{de} \left\{\begin{array}{c}\text{peso}\\ \text{de}\\ \text{moléculas}\end{array}\right\} \times \begin{array}{c}\text{Cuadrado de las distancias de la molécula}\\ \text{al eje de revolución}\end{array}$$

ó, $I=\Sigma d^2 w$.

Los autores científicos, al calcular el momento de inercia usan frecuentemente la *masa* de cada molécula; que es $= \frac{\text{su peso}}{\text{aceleración } (g) \text{ de la gravedad} = 9.81 \text{ m}}$ en lugar de su peso.

En la práctica debemos suponer al cuerpo divido en partes de una pulgada cúbica ó un centímetro cúbico (ó cualquiera otro volumen pequeño) cada uno; y usar éstos en lugar de las partículas infinitamente pequeñas que supone la teoría. Mientras más pequeñas se tomen estas partes, más exacto será el resultado.

Cuando se quiere el momento de inercia de una simple *superficie* (en vez de la de un *cuerpo*), suponemos la superficie dividida en un número de *áreas* pequeñas, y empleamos éstas en lugar de los *pesos* de las moléculas del cuerpo.

$$\textbf{Momento de inercia} = \frac{\text{peso del cuerpo}}{\text{ó}}{\text{área de la superficie}} \times \begin{array}{c}\text{cuadrado del radio}\\ \text{de giro.}\end{array}$$

(*N. del T.* — El **radio de giro** de un cuerpo es la distancia á que sería preciso colocar un punto material de *una masa igual á la del cuerpo* para que tuviese *el mismo momento de inercia del cuerpo*.)

## Tabla de radios de giro.

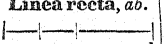
| Cuerpo. | Girando alrededor de | Radio de giro. |
|---|---|---|
| **Cualquier cuerpo ó figura.** | Cualquier eje dado. | $\sqrt{\frac{\text{Momento de inercia alrededor del eje dado}}{\text{Peso del cuerpo, ó área de su superficie}}}$ |
| **Cilindro sólido.** | Su eje longitudinal. | Radio del cilindro $\times \sqrt{\frac{1}{2}}$. |
| Ídem. | Un diám medio entre sus extremos. | = Radio del cilindro $\times$ .7071 (más ó menos). |
| Ídem infinitamente corto (superficie circular). | Un diámetro. | $\sqrt{\frac{\text{Longitud}^2}{12} + \frac{\text{radio}^2 \text{ del cilindro}}{4}}$. $\frac{\text{Radio del cilindro}}{2}$. |
| **Cilindro hueco.** | Su eje longitudinal. | $\sqrt{\frac{\text{Radio interior}^2 + \text{radio exterior}^2}{2}}$. |
| Ídem infinitamente delgado. | Ídem. | Radio del cilindro. |
| Ídem de cualquier espesor. | Un diám medio entre sus extremos. | $\sqrt{\frac{\text{Radio inter}^2 + \text{radio exter}^2}{4} + \frac{\text{longitud}^2}{12}}$. |
| Ídem infinitamente delgado. | Ídem. | $\sqrt{\frac{\text{Radio}^2 \text{ del cilindro}}{2} + \frac{\text{longitud}^2}{12}}$. |
| Ídem infinitamente delgado é infinitamente corto (circunferencia de un círculo). | Un diámetro. | Radio del cilindro $\times \sqrt{\frac{1}{2}}$. = Radio del cilindro $\times$ .7071 próximamente |
| **Esfera sólida.** | Un diámetro. | $\sqrt{\frac{\text{Radio}^2 \text{ de la esfera}}{2.5}}$ = radio de la esfera $\times \sqrt{.4}$. = radio de la esfera $\times$ .63246 próximamente. |
| **Esfera hueca** de cualquier espesor. | Un diámetro. | $\sqrt{\frac{2\ (\text{radio exterior}^5 - \text{radio interior}^5)}{5\ (\text{radio exterior}^3 - \text{radio interior}^3)}}$. |
| Ídem delgada. | Ídem. | Aproximado (radio exterior + radio interior) $\times$ .4085. |
| Ídem infinitamente delgada (superficie esférica). | Un diámetro. | Radio de esfera $\times \sqrt{\frac{2}{3}}$. = Radio de esfera $\times$ .8165 aproximadamente. |
| **Línea recta**, $ab$.   | Cualquier punto, $x$, en su longitud. | $\sqrt{\frac{ax^3 + xb^3}{3ab}}$. |
| | Cualquier extremo, $a$ ó $b$. | Longitud $ab \times \sqrt{\frac{1}{3}}$. = Longitud $ab \times$ .5775 aproximadamente. |
| | Su centro, $c$. | $ac \times \sqrt{\frac{1}{3}}$. = Longitud $ab \times$ .2887 aproximadamente. |

**Tabla de radios de giro.** (Continuación.)

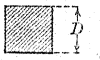
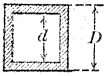
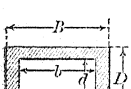
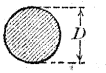
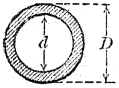
| Cuerpo. | Girando alrededor de | Radio de giro. |
|---|---|---|
| **Cono sólido.** | Su eje. | Radio de la base del cono $\times \sqrt{.3}$. = Radio de la base del cono $\times .5477$. |
| **Plancha circular, de sección** transversal rectangular. | Véase cilindro sólido. | Para el *espesor* de la plancha ó círculo medido perpendicularmente al plano de la circunferencia tómese la *longitud* del cilindro. |
| **Anillo circular** de sección transversal rectangular. | Véase cilindro hueco. | |
| **Cuadrados, rectángulos y otras superficies.** | Para los radios de giro *mínimos* alrededor de los ejes más *largos*, véanse páginas siguientes. | |

**Relaciones entre el radio menor de giro, $r$, y el lado menor, $D$.**

En una sección transversal cualquiera, sea :

$I$ = al menor momento de inercia;
$a$ = área;
$r = \sqrt{I/a}$ = radio menor de giro;
$D$ = diámetro ó lado menor externo;
$d$ = diámetro ó lado menor interno;
$B$ = diám ó lado mayor externo;
$b$ = diám ó lado mayor interno;
$t$ = espesor del macizo;
$c = D/t$; $1/12 = .0833$;
$m = B/D$; $\sqrt{12} = 3.4641$.

Entonces se tienen las siguientes relaciones :

| Sección transversal. | $r$ | $r^2$ | $\frac{D}{r}$ | $\left(\frac{D}{r}\right)^2$ |
|---|---|---|---|---|
|  Cuadrado sólido. | $\frac{D}{\sqrt{12}}$ | $\frac{D^2}{12}$ | $\sqrt{12}$ | 12 |
|  Cuadrado hueco * de espesor uniforme. | $\sqrt{\frac{D^2+d^2}{12}}$ | $\frac{D^2+d^2}{12}$ | $\sqrt{\frac{12\,D^2}{D^2+d^2}}$ * | $\frac{12\,D^2}{D^2+d^2}$ * |
| Rectángulo sólido. | $\frac{D}{\sqrt{12}}$ | $\frac{D^2}{12}$ | $\sqrt{12}$ | 12 |
|  Rectángulo hueco † de espesor uniforme. | $\sqrt{\frac{D^2B-d^2b}{12(DB-db)}}$ | $\frac{D^2B-d^2b}{12(DB-db)}$ | $\sqrt{\frac{12D^2DB-db}{D^2B-d^2b}}$ † | $12D^2\frac{DB-db}{D^2B-d^2b}$ † |
|  Círculo sólido. | $\frac{D}{4}$ | $\frac{D^2}{16}$ | 4 | 16 |
|  Círculo hueco ** de espesor uniforme. | $\sqrt{\frac{D^2+d^2}{16}}$ | $\frac{D^2+d^2}{16}$ | $4D\sqrt{\frac{1}{D^2+d^2}}$ ** | $\frac{16\,D^2}{D^2+d^2}$ ** |

★ Cuadrado hueco; $c = D/t$.

| Cuando $c$ = | 5 | 10 | 20 |
|---|---|---|---|
| $(D/r)^2$ = | 8.82 | 7.32 | 6.63 |

† Rectángulo hueco de espesores uniformes; $c = D/t$; $m = B/D$.

| Cuando $c$ = | 5 | | 10 | | 20 | |
|---|---|---|---|---|---|---|
| y, $m$ = | 1.5 | 2 | 1.5 | 2 | 1.5 | 2 |
| $(D/r)^2$ = | 7.98 | 7.54 | 6.62 | 6.23 | 6.01 | 5.66 |

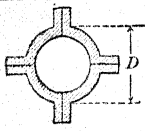
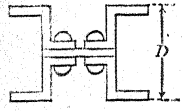
** Círculo hueco de espesor uniforme; $c = D/t$.

| Cuando $c$ = | 5 | 10 | 20 |
|---|---|---|---|
| $(D/r)^2$ = | 11.76 | 9.76 | 8.84 |

†† Ángulo de piernas desiguales; $m = B/D$. Véase pág. 372.

| Cuando $m$ = | 1.25 | 1.50 | 1.75 | 2.00 | 2.25 |
|---|---|---|---|---|---|
| $(D/r)^2$ = | 21.60 | 18.78 | 17.00 | 15.70 | 14.80 |

**Las ecuaciones de abajo son sólo aproximadas.**

| Sección transversal. | $r$ | $r^2$ | $\frac{D}{r}$ | $\left(\frac{D}{r}\right)^2$ |
|---|---|---|---|---|
|  Columna «Phoenix». | $0.3636\,D$ | $0.1322\,D^2$ | $\frac{1}{0.3636}=2.7527$ | $\frac{1}{0.1322}=7.5643$ |
|  Barra-Z. Columna Carnegie. | $0.295\,D$ | $0.087\,D^2$ | $\frac{1}{0.295}=3.3896$ | $\frac{1}{0.087}=11.4943$ |
| Viga I. | $\frac{D}{4.58}$ | $\frac{D^2}{21}$ | 4.58 | 21 |
| Viga en canal. | $\frac{D}{3.54}$ | $\frac{D^2}{12.5}$ | 3.54 | 12.50 |
| Viga de entrepiso. | $\frac{D}{6}$ | $\frac{D^2}{36.5}$ | 6 | 36.5 |
| Angulo de brazos iguales. | $\frac{D}{5}$ | $\frac{D^2}{25}$ | 5 | 25 |
| Angulo †† de brazos desiguales. | $\frac{BD}{2.6(B+D)}$ | $\frac{B^2D^2}{13(B^2+D^2)}$ | $\sqrt{13\left(\frac{D^2}{B^2}+1\right)}$ †† | $13\left(\frac{D^2}{B^2}+1\right)$ †† |
| T sencilla con $D=B$. | | | | |
| Cruz con $D=B$. | $\frac{D}{4.74}$ | $\frac{D^2}{22.5}$ | 4.74 | 22.5 |

## FUERZA CENTRÍFUGA

Cuando un cuerpo *a*, fig. 1, pág. 374, se mueve en una dirección circular *abd*, tiende en cada punto, *a* ó *b*, á moverse según la tangente *at* ó *bt'* del círculo, en aquel punto. Pero en cada punto, *a*, etc., de dicha dirección es desviado de la tangente por una fuerza que obra hacia el centro *c* del círculo. Esta fuerza puede ser la tensión de una cuerda, *ca*, ó la atracción entre un planeta en *c* y su satélite *a;* ó la presión de los rieles, *ab*, hacia el centro de una curva, etc., etc. Como toda fuerza, es una acción entre dos cuerpos que tiende á separarlos ó á unirlos y que actúa igualmente sobre ambos. (Véase **Mécanica**, art. 5 *b*.) En el caso de la cuerda, ésta tira del cuerpo *a* hacia el centro *c* del clavo ó mano, etc., y del centro *c* hacia el cuerpo en *a* ó *b*, etc. En el caso de un carro en una curva la fuerza tira al carro *fuera* del centro, y los rieles lo tiran *hacia* el centro. La atracción ó impulsión que tiende á lanzar el cuerpo giratorio *hacia* el centro se llama **fuerza centrípeta**, mientras que la impulsión que tiende á alejar del centro al cuerpo se llama **fuerza centrífuga**. Estas dos « fuerzas » constituyen meramente la acción y reacción de una misma fuerza, son por tanto necesariamente iguales y opuestas; sólo pueden existir juntas. En el momento en que la tensión de la cuerda excede á su resistencia ó fuerza de cohesión, se revienta aquélla, y fuerzas centrípeta y centrífuga cesan por consiguiente instantáneamente; el cuerpo deja de estar perturbado por la fuerza que lo desvía, y se sigue moviendo con una velocidad uniforme * en una tangente *at* ó *bt'*, etc., á su trayectoria circular, en ángulo recto con la dirección que la fuerza centrífuga tenía en el momento en que cesó

(a) **Cuerpo giratorio simple**, *a*, fig. 1. Sea

$f$ = á la fuerza centrífuga ó centrípeta, en libras (ó kgs).
W = al peso del cuerpo *a*.
R = al radio *ca* de la trayectoria del *centro de gravedad* del cuerpo *a*.
$v$ = la velocidad uniforme por segundo del cuerpo *a* en su dirección circular *abd*.
$n$ = al número de revoluciones por minuto.
$g$ = á la aceleración de la gravedad = 9.81 m por segundo; 900 $g$ = 8,829 m.
$\pi$ = circunferencia ÷ diámetro = 3.1416. $\pi^2$ = 9.8696.

Entonces, para la **fuerza centrífuga**, $f$, tenemos la fórmula siguiente :

Si conocemos la velocidad $v$, por segundo : $f = \mathrm{W}\dfrac{v^2}{\mathrm{R}g}$ †..... (1

ó si conocemos el número de revoluciones por minuto : $f = \mathrm{W}\dfrac{\pi^2 \mathrm{R} n^2}{900g}$ ‡..... (2)

Sustituyendo en lugar de $\pi^2$ y $900g$ sus valores, viene : $f = .0012\ \mathrm{W}\mathrm{R}n^2$ ..... (3

(b) **Ruedas, anillos y discos.** Supongamos una rueda cortada en muy pequeños pedazos como se ve (muy exagerado) en *a*, fig. 2. Para cada pedazo *a* tenemos por la fórmula (1) : $f$ = peso W del pedazo $\times \dfrac{v^2}{\mathrm{R}g}$ ;** y si cada uno estuviese unido

---

* Despreciando el rozamiento, la acción de la gravedad, la resistencia del aire, etc.

† Si *at*, fig. 1, representa la intensidad y dirección de la velocidad $v$ del cuerpo en *a* en un segundo, al fin de un segundo el cuerpo habrá alcanzado el punto *b* (habiéndose hecho el arco *ab* = *at*), y la intensidad y dirección de la velocidad en *b* estará entonces representada por la línea *bt'* = *at* en longitud; pero difiriendo en dirección. Trazando á *cu* y *cu'* por el centro, iguales y paralelas respectivamente á *at* y *bt'*, encontramos que el *cambio en la dirección* del movimiento (sea la aceleración hacia el centro durante el segundo de que se trata) está representada por el arco *uu'*; y como el ángulo *acb* = ángulo *ucu'*, tenemos la proporción, radio R ó *ac* : *ab* ó *at* = *cu* ó *at* : arco *uu'*. En otras palabras, la aceleración *uu'* en un segundo, es : $\dfrac{at^2}{\mathrm{R}} = \dfrac{v^2}{\mathrm{R}}$; y para la fuerza que produce esta aceleración tenemos $f$ = masa del cuerpo × valor de la aceleración = masa del cuerpo × $\dfrac{v^2}{\mathrm{R}} = \mathrm{W}\dfrac{v^2}{\mathrm{R}g}$.

‡ Por medio de la fórmula (1), $f = \mathrm{W}\dfrac{v^2}{\mathrm{R}g}$; pero $v = \dfrac{2\pi \mathrm{R}n}{60}$ ; y $v^2 = \dfrac{4\pi^2\mathrm{R}^2n^2}{3{,}600} = \dfrac{\pi^2\mathrm{R}^2n^2}{900}$ Por consiguiente, $f = \mathrm{W}\dfrac{\pi^2\mathrm{R}^2n^2}{900\ \mathrm{R}g} = \mathrm{W}\dfrac{\pi^2\mathrm{R}n^2}{900\ g}$.

** Si el disco es muy angosto en el sentido del radio con relación á su diámetro *mn*, podremos tomar el centro de gravedad de cada uno de los pedazos como si estuvieran

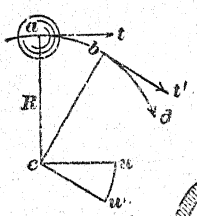


Fig. 1

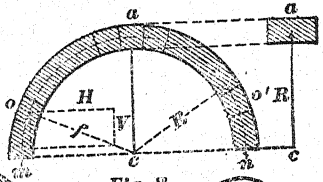


Fig. 2

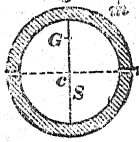


Fig. 3

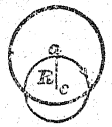


Fig. 4

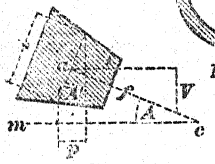


Fig. 5

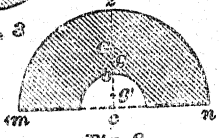


Fig. 6

Fig. 7

al centro por una cuerda, las sumas de las fuerzas en todas las cuerdas (despreciando el frotamiento de los pedazos adyacentes) sería :

F = suma de las fuerzas centrífugas de todos los pedazos ∴ = peso del disco $\times \frac{v^2}{Rg}$ ..... (4)

Pero el esfuerzo que generalmente nos interesa más en semejantes casos (p. ej., **la tensión en el anillo** en la dirección de una tangente á su circunferencia) es *mucho menor* que la intensidad teórica F obtenida por la fórmula (4), ya que en realidad es solamente $\frac{1}{6.2832}$ de ella. Porque se considera primero * que el disco se corta solamente en dos partes opuestas *m* y *n*, fig. 3, y que sus dos mitades están sujetas tan sólo por la cuerda S.

Y considerando así las cosas, tendremos entonces * :

semicircunferencia *mzn* : diámetro $mn = \frac{F}{2}$ : tensión de la cuerda S,

de manera que :

**Tensión de la cuerda** S = mitad del peso del anillo $\times \frac{v^2}{Rg} \times \frac{2}{\pi} =$ peso del anillo $\times \frac{v^2}{Rg\pi} = \frac{F}{\pi} = \frac{F}{3.1416}$ (5)

---

en una circunferencia intermedia que pasa entre los bordes interior y exterior del anillo, de manera que $R = \frac{\text{radio interior} + \text{radio exterior}}{2}$. En un anillo de espesor apreciable, este no será el caso; porque cada pedazo es un poco más ancho en su borde exterior que en el interior. (Véase la fig. 5.) Por consiguiente su centro de gravedad está un poquito fuera de la curva *mn*, fig. 2.

∴ En un anillo perfectamente en equilibrio (como en un anillo cuyo centro de gravedad coincida con su centro de rotación, como en la fig. 3), las fuerzas centrífugas de las moléculas de un lado *c* equilibran las del lado opuesto. También, tenemos aquí R = 0. Por consiguiente, considerado *como un todo*, dicho anillo no posee *fuerza centrífuga;* no hay fuerza que obre sobre el centro en ninguna dirección, por causa de la rotación. Pero si los dos centros *no coinciden* (fig. 4), entonces el anillo es un simple cuerpo rotatorio, y su fuerza centrífuga es *f* = peso del anillo entero $\times \frac{v^2}{Rg}$; siendo R la distancia entre los dos centros, y *v* la velocidad del centro de gravedad *a*. La fuerza *f* obra en la línea que une los dos centros.

* En la fig. 2, supongamos que la fuerza centrífuga de cualquier pedazo, *o*, esté representada por la diagonal *f* de un rectángulo, cuyos lados H y V, son respectivamente paralelos y perpendiculares al diámetro *mn;* H y V representan los componentes de *f*

y si el anillo se completa ahora uniendo los extremos $m$ y $n$, y si se suprime la cuerda S, entonces la tensión de la cuerda según la fórmula (5) estará dividida por igual entre $m$ y $n$. Por consiguiente cada sección transversal, $m$ ó $n$ del disco, resistirá una tensión igual á la mitad de la tensión de la cuerda; ó

$$\textbf{Tensión del anillo} = \frac{F}{2\pi} = \frac{F}{6.2832} = \frac{\text{peso del anillo} \times v^2}{6.2832\,Rg} \quad \ldots\ldots\ldots\ldots (6)$$

La fuerza centrípeta $f$, fig. 2, que sujeta una parte cualquiera $o$, del anillo, es la resultante de las dos tensiones iguales que actúan en los extremos de aquella parte.

Para apreciar la *fuerza por unidad de superficie* de las secciones transversales del anillo, tenemos:

$$\textbf{Esfuerzo por unidad} = \frac{\text{tensión en el anillo}}{\text{área A de las secciones transversales del anillo}}$$

$$= \frac{F}{6.2832\,A} = \frac{\text{peso del anillo} \times v^2}{6.2832\,ARg} \quad \ldots\ldots\ldots\ldots (7)$$

Llegaremos al mismo resultado si reflexionamos que la tensión de la cuerda S ó la *suma* de las dos tensiones en $m$ y $n$, es igual á la fuerza centrífuga $f$, de cualquiera de las dos *mitades* del anillo girando como un solo cuerpo alrededor del centro $c$. Búsquese el centro de gravedad G del medio anillo, y en la fórmula (1) úsese la velocidad de aquel punto y el radio $cG$ en lugar de la velocidad en $z$ y del radio $cz$ respectivamente; así pues :

$$\text{tensión de la cuerda} = f = \frac{\text{fuerza centrífuga}}{\text{del medio anillo}} = \frac{\text{peso del medio}}{\text{anillo}} \times \frac{(\text{velocidad en G})^2}{cG \times g}$$

y la mitad de esto es la tensión en cada sección transversal del anillo *.

Si el anillo fuera infinitamente angosto $cG$, fig. 3, sería .6366 $cz$.

Si su espesor debe tomarse en consideración, y si es de sección transversal rectangular, búsquense los centros de gravedad $g$ y $g'$, fig. 6, de todo el segmento semicircular $cz$, y del pequeño segmento $cb$ respectivamente ($cg$ = .4244 $cz$, y $cg'$ = = .4244 $cb$.

Tendremos : $$g'G = gg' \times \frac{\text{área del segmento entero } cz}{\text{área de la mitad del anillo}}.$$

Para anillos de secciones transversales, no rectangulares, empléense las fórmulas (4) (5) y (6).

**En un disco, tal como una piedra de amolar,** la tensión en cada sección transversal completa $mn$, fig. 7, es igual á la fuerza centrífuga $f$ de la *mitad* del disco. Sea W = al peso del medio disco. La distancia $cG$ del centro $c$ al centro de gravedad G del medio disco, es $cG = .4244 \times cz$; y la

$$\textbf{Tensión en } mn = f = W\frac{(\text{vel en G})^2}{\text{rad } cG \times g} = W\frac{.4244^2\ (\text{vel en } z)^2}{.4244 cz \times g}$$

$$= W\frac{.4244\ (\text{vel en } z)^2}{cz \times g} \quad \ldots\ldots\ldots\ldots (8)$$

$$= W\frac{.4244\ \pi^2 n^2 cz}{900\ g} \quad \ldots\ldots\ldots\ldots (9)$$

---

Las componentes horizontales, iguales y opuestas H de $o$, y del correspondiente pedazo $o'$, siendo paralelas á $mn$ no tienen tendencia á separar el disco en $m$ ó $n$. Por consiguiente, la tensión de una cuerda S, fig. 3, perpendicular á $mn$, es la suma de los componentes V de todos los pedazos. Para cada pedazo, fig 5 (demasiado exagerada), tenemos (siendo ángulo A = ángulo A') :

$$\text{Longitud } l \text{ del pedazo} : \text{su proyección horizontal, } p = \text{fuerza centrífuga } f, \text{ del pedazo} : \text{su componente vertical V}$$

Por consiguiente, para la mitad entera del anillo $mn$, fig. 3 (formado de dichos pedazos ó tajadas), tenemos :

$$\text{Longitud } mzn \text{ del medio disco} : \text{su proyección horizontal } mn = \text{la suma de las fuerzas centrífugas de todas las tajadas del medio disco} : \text{la suma de los componentes verticales V para todas aquellas tajadas;}$$

lo cual es idéntico á la proporción que sigue.

*Los *discos* ó *llantas* de las ruedas giratorias se hacen suficientemente fuertes para que resistan la tensión debida á la fuerza centrífuga sin ayuda de los *rayos*, los cuales tienen simplemente que soportar el *peso* de la rueda. Pero si se rompe la llanta, las fuerzas centrífugas de sus fragmentos obran totalmente sobre los rayos; y como la ruptura es siempre irregular, algunos de los rayos soportarán más de lo que les corresponde.

El valor de la fuerza por unidad de superficie en la sección $mn$ es

$$= \frac{\text{tensión en } mn}{\text{área de la sección transversal}}.$$

$$= W \frac{.4244\ (\text{velocidad en } z)^2}{\text{diám } mn \times \text{grueso del anillo} \times cz \times g} \quad \ldots\ldots\ldots (10$$

$$= W \frac{.4244\ \pi^2 n^2 cz}{\text{diám } mn \times \text{grueso ó espesor} \times 900\ g} \quad \ldots\ldots\ldots (11$$

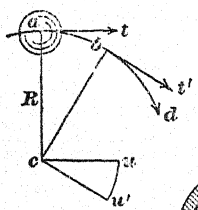


Fig. 1

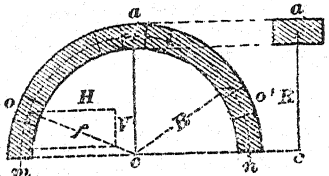


Fig. 2

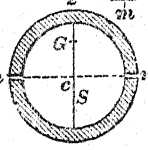


Fig. 3

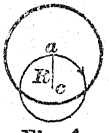


Fig. 4

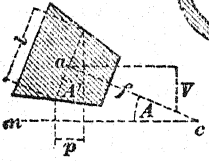


Fig. 5

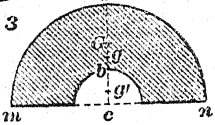


Fig. 6

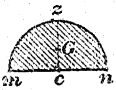


Fig. 7

$f$=á la fuerza centrípeta que actúa sobre un cuerpo giratorio simple *a*, figs. 1, 2, 4 y 5, ó sobre el semianillo ó semidisco, figs. 3, 6 y 7=á la fuerza centrífuga ejercida por dicho cuerpo.

**F**=la suma de las fuerzas centrífugas $f$ de todas las moléculas del anillo, fig. 3

**W**=peso del cuerpo.

**R**=radio *ca*, figs. 1, 4 y 5, de la trayectoria que describe el *centro de gravedad* del cuerpo.

$v$=la velocidad uniforme por segundo de que está animado el cuerpo en su trayectoria circular.

$n$=número de revoluciones por minuto.

$g$=la aceleración de la gravedad.

$\pi = \frac{\text{circunferencia}}{\text{diámetro}} = 3.1416.$

$\pi^2 = 9.8696$ próximamente.

**En una rueda en movimiento,** apoyada la llanta sobre el suelo, cada punto de la llanta está fijo *con respecto á la tierra,* mientras dura el contacto con el terreno; pero cada molécula está animada de una misma velocidad *alrededor del centro* de la rueda, considerando á éste como fijo, y por consiguiente la fuerza centrífuga no tiene efecto sobre el peso.

# ESTÁTICA

## FUERZAS

**1. Definición de la estática.** La estática, ó ciencia del equilibrio de las fuerzas, estudia los numerosos casos en que las fuerzas, que se consideran están en equilibrio, es decir, se destruyen. Por tanto, abarca todos los casos en que se dice que los cuerpos están « en reposo ».

**2. En los problemas que generalmente se presentan** á los ingenieros, ciertas fuerzas aplicadas á un cuerpo en reposo (como un puente ó un edificio) tienden á producir movimiento, ya sea en todo el cuerpo ó en una ó más de sus partes; y se necesita encontrar y aplicar otras fuerzas que destruyan la tendencia al movimiento, logrando así que el cuerpo y sus partes se mantengan en reposo. (Véase § 33, más adelante.)

**3. Equilibrio.** Supongamos un cuerpo sobre el que actúan varias fuerzas. Estas fuerzas se dicen en equilibrio, cuando combinadas no alteran ni el reposo ni el movimiento del cuerpo, ya sea considerado en su conjunto con respecto á su movimiento de traslación, ó con respecto á un movimiento de rotación referido á un punto del cuerpo ó fuera de él. (Véase § 84, más adelante.)

*4.* Un cuerpo puede estar en equilibrio con respecto á las fuerzas que se estudian, aunque no lo esté con respecto á otras. Así, una piedra sostenida entre el índice y el pulgar, está en equilibrio con respecto á estas dos presiones iguales, aun cuando se suba y baje la piedra por un exceso de fuerza muscular del brazo sobre la acción de la pesantez en la piedra. De manera semejante, en un ferrocarril á nivel, un carro se encuentra en equilibrio respecto á la pesantez y á la resistencia hacia arriba de los rieles, aun cuando el tiro de la locomotora exceda á la resistencia de la tracción.

**5. Acción molecular.** Toda fuerza aplicada á un cuerpo es prácticamente un sistema de fuerzas, á menudo paralelas, aplicado á las varias partículas del cuerpo. Así la acción ejercida por la tierra sobre un grano de arena ó sobre la luna, es, en verdad, un conjunto de fuerzas casi paralelas, ejercidas sobre las varias partículas de aquellos cuerpos; pero para mayor claridad, y tan sólo en lo que se refiere á su tendencia, para mover al cuerpo en su conjunto, concebimos aquellas fuerzas como sustituídas por una sola, igual á su suma, y obrando en una sola dirección. Al considerar así las fuerzas, suponemos que los cuerpos sean enteramente rígidos, de manera que cada una de ellas obra independiente como una « partícula » ó un « punto material ».

**6. Transmisión de fuerzas.** La presión hacia arriba del suelo sobre una piedra que reposa en él, obra directamente tan sólo sobre aquellas partículas que están más cerca del suelo. Éstas, á su vez, ejercen (prácticamente) una fuerza hacia arriba sobre las que se encuentran inmediatamente encima, y así sucesivamente, y en esta forma se transmite por toda la piedra.

**7. Cuerpos rígidos.** Al considerar rígidos á los cuerpos suponemos que las fuerzas intermoleculares mantienen las diversas partículas absolutamente en sus posiciones originales relativas.

No es *la materia* la que resiste á la ruptura, sino las fuerzas que mantienen sus partículas en posición. Así, un pedazo de hielo puede soportar una gran presión; pero sus partículas se separan fácilmente cuando una temperatura de deshielo destruye sus fuerzas de cohesión.

**8. Unidades de fuerza.** Las unidades de fuerza generalmente usadas en estática, son la de peso, como la libra ó el kilogramo. (Véanse las tablas de conversión, pág. 245.)

En estática no tenemos necesidad de considerar las masas de los cuerpos (excepto en los casos en que éstas determinan sus pesos ó la fuerza de la pesantez ejercida sobre ellos), pues los cuerpos se consideran tan sólo como el medio por donde, y á través del cual, obran las fuerzas que se consideran. Por eso en estática no se requiere la unidad de masa; como los cuerpos se consideran como « en reposo », tampoco se necesitan unidades de tiempo, velocidad, aceleración, impulso ó energía.

**9. Fuerzas; su determinación.** Una fuerza se encuentra completamente determinada cuando conocemos (1) su valor (en kg ú otra unidad de peso), (2) su dirección, (3) su sentido (véase § 10), y (4) su posición ó su punto de aplicación.

**10. Cuando una fuerza está representada por una línea**, puede hacerse que su longitud represente, por escala, la cantidad de la fuerza; y su dirección y posición pueden servir á menudo para indicar las de la fuerza, mientras que el sentido de la fuerza se puede indicar por flechas ó letras sobre la línea ó por los signos + y —.

Así, la *dirección* de las fuerzas representadas por las líneas *a* y *b*, fig. 1, son verticales, y aquellas de las *c* y *d* son horizontales. El sentido de *a* es hacia arriba, de *b* hacia abajo, de *c* hacia la derecha, de *d* hacia la izquierda. De modo que *a* y *b* son de la misma dirección, pero en sentido opuesto; lo mismo *c* y *d*. Al ocuparnos de fuerzas verticales ú horizontales, las llamamos generalmente hacia arriba ó á la derecha: **positivas**; y hacia abajo ó á la izquierda: **negativas**, como lo indican los signos + y —, en la fig. 1. Cuando se designa una fuerza por dos letras fijas á cada extremo de la línea que la representa, el sentido de la fuerza se puede indicar por el orden en que se han puesto las letras. Así, en la fig. 1, siguiendo la dirección de las flechas, tenemos las líneas *ef*, *hg*, *kl*, *nm*.

**11. Línea de acción, etc.** El punto (véase § 5) donde se supone aplicada una fuerza P (fig. 2), se llama su *punto de aplicación;* pero las fuerzas se transmiten á todo el cuerpo por medio de sus partículas (véase § 6), y el efecto de la fuerza, en lo que se refiere al cuerpo como un todo, no cambia si se la supone aplicada en un

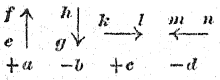



Fig. 1.

Fig. 2.

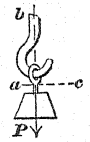



Fig. 3.

punto cualquiera *b* de su prolongación, y por tanto cualquier punto de esa línea se puede considerar como punto de aplicación. Por ejemplo, la tendencia á mover la piedra no cambia (fig. 2) aplicando la fuerza en *a* ó en *b;* y en cuanto al peso P, fig. 3, ejercerá la misma tendencia para mover el garfio, colocado arriba de aquél en *b* ó colgando en *a*. No se puede aplicar ninguna fuerza á un cuerpo en puntos fuera de su masa, como sucede en las partes vacías de la línea *ab* del garfio; no obstante, estas partes vacías pertenecen á la línea de aplicación de la fuerza. La fuerza vertical ejercida por el peso, P, se transmite al punto *b*, por medio de los momentos inclinados de la parte curva del gancho.

**12.** Aplicándole por contacto fuerzas opuestas á un cuerpo (véase art. 5 *c*, pág. 348), entra á obrar en la masa del cuerpo la fuerza intermolecular, tendiendo, ó á separarlas, y entonces se dice que las fuerzas obran por *tensión*, ó á juntarlas, se llama *compresión*. **El esfuerzo** (*tensión ó compresión*) **debido á dos fuerzas opuestas es igual á una de ellas.**

**Tensión y compresión. Tirantes, puntales, etc.** Si la acción de las fuerzas tiende á separar más las partículas del cuerpo sobre las que actúan, se dice que hay *tensión*. Si tiende á juntarlas, se dice que hay *compresión*. Una pieza larga, débil, sometida á tensión, se llama en general *cuerda*. La que trabaja por compresión se llama *poste, puntal*, etc. La que está sometida á veces á tensión y otras á compresión es una *cuerda-poste* ó *poste-cuerda*.

## MOMENTOS

**13. Momentos.** Si de cualquier punto *o* ú *o'*, fig. 4, se baja una perpendicular *oc* ú *o's* á la línea *nm*, dirección de la fuerza $P_1$, aun cuando el punto *o* ú *o'* se encuentre dentro ó fuera del cuerpo sobre el que obra la fuerza $P_1$, dicha línea *oc* ú *o's* se llama el *brazo de palanca* de la fuerza en aquel punto, y así la intensidad de la fuerza (en unidades de peso, libras ó kilogramos) se multiplica por el brazo de la palanca (en pies ó metros); el producto (pies-libras, kilográmetros) es el momento de la fuerza en aquel punto *. El momento representa la tendencia total de la fuerza á producir rotación alrededor de un punto dado. El momento de una fuerza en los puntos de su dirección es cero.

**14. Dirección de los momentos.** Desde que el *momento* de $P_1$ respecto á *o*, fig. 4, tienda á engendrar rotación (alrededor de aquel punto), en la misma dirección que las agujas de un reloj visto de frente, ó sea de izquierda á derecha, como lo indica la flecha en el círculo, *o*, el momento es **positivo** (+), pero, el momento de la misma fuerza en *o'* tiende á producir rotación en sentido contrario al del reloj y es **negativo** ( — ).

**15. El plano de un momento** es aquel en que se encuentran la dirección y el brazo de palanca de la fuerza.

**16. La resultante** ó tendencia combinada de dos ó más momentos en un mismo plano, es igual á la suma algebraica de todos los momentos.

Así, fig. 4, si las fuerzas $P_1$, $P_2$ y $P_3$ representan respectivamente 6, 5 y 3 kg, y si sus brazos de palanca *oc*, *oy*, *oe*, de sus momentos respecto *o*, fuesen de 7, 6 y 3 m, tenemos :

$$\begin{aligned} & P_1 \times oc - P_2 \times oy + P_3 \times oe \\ = \ & 6 \times 7 \ - \ 5 \times 6 \ + \ 3 \times 3 \\ = \ & 42 \ - \ 30 \ + \ 9 \ = 21 \text{ kilográmetros.} \end{aligned}$$

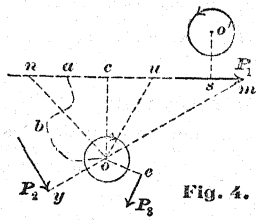



Fig. 4.

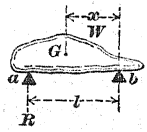



Fig. 5.

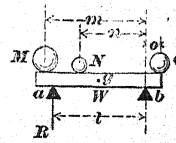



Fig. 6.

**17.** Si la suma algebraica de los momentos es cero, están en **equilibrio** y no tienden á hacer rotar el cuerpo alrededor del punto dado.

Así, en la figura 5, si W es el peso y G el centro de gravedad del cuerpo; y R la reacción hacia arriba del apoyo izquierdo *a*, tomando los momentos respecto del apoyo de la derecha, *b*, tenemos $Rl - Wx = 0$; ó bien : $Rl = Wx$. De donde $R = \frac{Wx}{l}$.

De modo análogo en la figura 6, donde W es el peso de la viga sola, y *g* el centro de gravedad de W, aquél está en el centro del espacio *l*, de manera que el brazo de palanca *bg* del peso de la viga respecto *b*, es $= \frac{l}{2}$, se toman los momentos respecto al punto *b*, así:

$$R \times l + Oo - W\frac{l}{2} - Mm - Nn = \text{cero; ó sea}$$

$$R = \frac{Mm + Nn + W\frac{l}{2} - Oo}{l}.$$

* Nótese que una fuerza muy pequeña puede tener un gran momento en un punto, mientras que una fuerza mucho mayor, pasando más cerca del mismo punto, puede tener un momento menor respecto á él, ó pasar por el punto con momento *cero*.

En la figura 7, en la que W es el peso de la viga misma, y $w$ su palanca, tomando los momentos respecto á $b$, tenemos:

$$+Rl+Oo-Nn-Ww+Mm=0;$$

$$\text{De donde, la reacción en } a=R=\frac{Ww+Nn-Mm-Oo}{l}.$$

En todo caso, si W es el peso combinado; G el centro común de gravedad de la viga y sus cargas, y $x$ la distancia horizontal de aquel centro al apoyo $b$ de la derecha; y si $l$ es la abertura; R la reacción del soporte izquierdo, $a$, y R' la del soporte $b$ de la derecha, tenemos:

$$R=\frac{Wx}{l};\qquad R'=W-R.\qquad \text{Si } x=\frac{l}{2},\qquad R=\frac{W}{2}=R'.$$

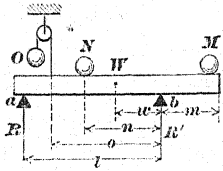



**Fig. 7.**

Nótese que los momentos de dos ó más fuerzas respecto á un punto dado, pueden estar en equilibrio, aun cuando las fuerzas mismas no lo estén. (Véase § 84, más adelante.)

**18. Centro de los momentos.** En lo que se relaciona con el equilibrio de los momentos, es indiferente el punto que se escoja como centro, pero conviene generalmente escogerlo en la dirección de una (ó más, si hay fuerzas concurrentes, véase § 19) de las fuerzas desconocidas, pues así eliminamos dicha fuerza, ó fuerzas, de la ecuación.

## CLASIFICACIÓN DE LAS FUERZAS

**19. Clasificación de las fuerzas. Fuerzas concurrentes, colineales, coplanas, y paralelas.** Se llaman concurrentes cuando sus direcciones se encuentran en un punto, como $a$, $b$, $c$, $d$, $e$, $f$, ó bien $f$ y $g$, fig. 8. No concurrentes,

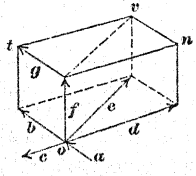



**Fig. 8.**

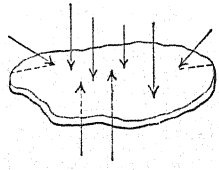


**Fig. 9.**

cuando no se encuentran así, como $c$ y $g$; colineales, cuando sus direcciones coinciden como $a$ y $b$, ó $c$ y $d$; no colineales, cuando no coinciden como $b$, y $f$; *coplanas* cuando sus direcciones están en un mismo * plano como $a$, $b$, $c$, $d$, $e$, ó bien $b$, $f$, y $g$; *no coplanas*, como $c$ y $g$, ó bien $b$, $f$, $d$, cuando están en planos diferentes; paralelas, cuando sus direcciones lo son, como $b$ y $g$; no paralelas, cuando no lo son sus direcciones, como $b$ y $f$.

Dos fuerzas paralelas cualesquiera están siempre en el mismo plano; tres ó más pueden estar ó no en el mismo plano. Dos fuerzas concurrentes están en un mismo plano, tres ó más pueden estar ó no. Dos fuerzas coplanas tienen que ser concurrentes ó paralelas.

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* No debe confundirse obrar *contra* un plano (una fuerza, como en la fig. 9, á obrar *en* un plano, como en las figs. 70, etc.

## COMPOSICIÓN Y DESCOMPOSICIÓN DE LAS FUERZAS

**20. Resultante.** Cuando una sola fuerza puede producir sobre un cuerpo, considerado como un todo, el mismo efecto que dos ó más combinadas, se llama la resultante de ellas.

Así en la figura 10 (*b*) la presión hacia abajo, G = *w* + W es la resultante de las presiones hacia abajo *w* y W; y en la figura 11 (*b*) la presión hacia abajo = W — *w*, es la resultante de la presión hacia abajo W y del empuje hacia arriba de la cuerda del lado izquierdo.

**21. Componentes.** Dos ó más fuerzas que producen juntas, sobre un cuerpo considerado como un todo, el mismo efecto de una sola fuerza, se llaman **componentes** de dicha fuerza, y ésta es la resultante de aquéllas.

Así en la figura 10 (*b*) W y *w* son las componentes de la fuerza total, G = W + *w*. En la figura 11 (*b*), + W(= 5) y *w* (= — 3) son las componentes de G.

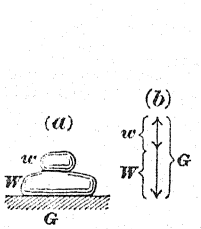



Fig. 10.

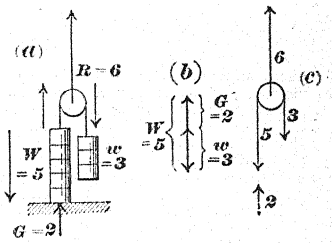



Fig. 11.

**22.** Cuando se considera la acción de la resultante de varias fuerzas, éstas (las componentes) dejan de considerarse, en lo que se refiere á su acción sobre el cuerpo como un todo, aun cuando se las tenga que apreciar por sus efectos sobre las partículas del cuerpo. Viceversa, consideradas las componentes, no se toma en cuenta la resultante.

**23. Antiresultante.** La antiresultante de una ó más fuerzas es una fuerza sola capaz de producir sobre cualquier cuerpo ó sistema de cuerpos considerado como un todo, el mismo efecto, pero en sentido contrario, que la resultante. En otras palabras : la antiresultante es la que destruye el efecto de las otras, manteniendo el equilibrio. Así, en la fig. 10 (*b*) la reacción del suelo hacia arriba G es la antiresultante de *w* y W obrando hacia abajo; y viceversa W + *w*, efecto hacia abajo, es la antiresultante de G. En la fig. 11 (*b*), G (obrando hacia arriba) es la antiresultante de W (hacia abajo) y *w* (obrando hacia arriba por intermedio de la cuerda izquierda). De modo análogo, el empuje hacia arriba de *w* es la antiresultante de W y G.

**24.** En cualquier grupo de fuerzas en equilibrio, una cualquiera de ellas es la antiresultante de las otras, y la resultante de dos ó más de ellas es la antiresultante de las demás. En tal sistema, la resultante y antiresultante de las fuerzas en equilibrio es cero.

**25. La anticomponente** de una fuerza ó de un sistema de fuerzas, son dos ó más fuerzas cuya resultante es la antiresultante de la fuerza ó sistema de fuerzas considerado.

**26. Composición y descomposición de las fuerzas.** La operación de encontrar la resultante de un sistema dado de fuerzas, se llama **composición** de fuerzas. Al contrario, encontrar las componentes de una fuerza dada, es la **descomposición** de la fuerza.

## Fuerzas colineales.

**27.** Supongamos que la línea vertical *w*, fig. 10 (*b*), representa, en una escala conveniente, el peso de la piedra superior en la fig. 10 (*a*) y W el de la piedra inferior. Entonces $w+W,=G,=$á la suma de las dos líneas, y dará, en la misma escala,

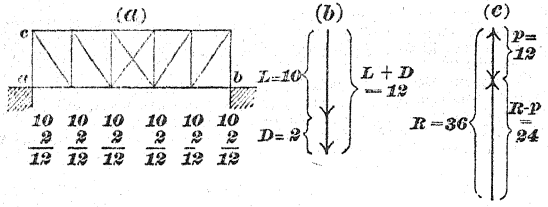



**Fig. 12.**

el peso combinado de las dos piedras, y la vertical G, que coincide con la línea de los pesos, $=$á su suma, y marcando su dirección hacia arriba, representará la anti-resultante, ó sea la reacción del suelo.

**28.** De modo análogo si en cada punto extremo de las secciones de la cuerda inferior de la armadura de un puente, fig. 12 (*a*), se supone una carga muerta de 2 toneladas (peso del puente, piso *, etc.), y 10 toneladas de carga viva (tren, vehículos, pasajeros, etc.), la longitud, sumadas las dos líneas, fig. 12 (*b*), dará L=10 y D=2, ó sea un total de peso, por sección, de 12 toneladas.

**29.** En la fig. 11, la presión de 5 kg de W sobre el suelo, está disminuída en 3 kg por el empuje hacia arriba del peso menor *w*, transmitido por la cuerda, y queda un residuo de dos kg de presión hacia arriba ejercido por el suelo, para mantener el equilibrio. La reacción hacia arriba R, de la polea es $=w+W-G=3+5-2=6$. Esto está gráficamente representado en la fig. 11 (*c*).

**30.** En la armadura de la fig. 12 (*a*) el total de las cargas muertas y móvil es $=6\times12=72$ toneladas, y la mitad móvil de este peso total (36 tons) descansa sobre cada estribo del puente. De aquí que, para conservar el quilibrio, cada estribo debe ejercer una reacción hacia arriba de 36 toneladas; pero, á fin de saber qué parte de las 36 toneladas se transmite por el *poste final, ac*, debemos deducir de ellas las doce toneladas que hemos calculado para la carga muerta y móvil, correspondiente al punto *a* de una sección, pues esta porción del peso es evidente que no se transmite por *ac*. Por tanto, en la fig. 12 (*c*), trazamos hacia arriba la línea R por escala $=36$ toneladas, y de su extremo superior hacia abajo trazamos $p=12$ toneladas. Lo que queda es $=R-p=36-12=24$ toneladas que es la presión transmitida por *ac*.

**31.** Las fuerzas colineales pueden estar en una misma ó en dirección contraria. Lo mismo sucede con las resultantes.

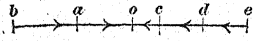



**Fig. 13.**

**32. Para que varias fuerzas colineales estén en equilibrio** se necesita que la suma de las fuerzas que obran en un sentido sea igual á la suma de las que obran en sentido opuesto, ó, de otro modo, que la suma algebraica de todas sea $=$cero.

Así, en la figura 13, si las fuerzas están en equilibrio, debe tenerse $ba+ao=ed+dc+co$; ó bien considerando las que obran hacia la derecha, *positivas*, y *negativas* las que obran hacia la izquierda, como se dijo en el § 10, debe tenerse :

$$b\ a+ao-oc-cd-de=0.$$

**33. Cuando dos fuerzas iguales y contrarias obran sobre un cuerpo,** se dice que lo mantienen en reposo; pero, hablando con exactitud, lo que hace cada fuerza es impedir que la otra lo mueva; mas no pueden evitar que otra ter-

* En realidad, la carga muerta no está nunca concentrada sobre una cuerda, como se indica aquí, pero por conveniencia suele suponérsela así.

cera, por pequeña que sea, y en cualquiera dirección que obre, lo ponga en movimiento. El cuerpo, por sí mismo, no tiende á moverse.

**34. Fuerzas desiguales opuestas.** Si dos fuerzas opuestas y desiguales obran sobre un cuerpo, la menor, y una parte de la mayor igual á ésta, obran en contra, sin producir efecto alguno sobre el cuerpo considerado como un todo, mientras que el residuo, ó sea la resultante, mueve el cuerpo en su propia dirección.

## Fuerzas coplanas concurrentes. Paralelogramo de las fuerzas.

**35. Composición.** Supongamos que las dos líneas *ao*, *bo*, en cualquiera de los dibujos de la fig. 14, representan en magnitud y dirección fuerzas concurrentes en el punto *o*. En el paralelogramo *acbo*, formado sobre las líneas *ao*, *bo*, la resultante de estas dos fuerzas está representada, en dirección y magnitud, por la diagonal R, que pasa por el punto *o*, donde concurren. El paralelogramo *acbo* se llama *paralelogramo de las fuerzas*.

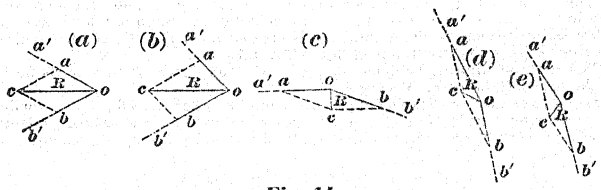



**Fig. 14.**

**36. Descomposición.** A la inversa, para encontrar las componentes de una fuerza dada *oc*, fig. 14, en dos direcciones dadas, *oa*, *ob*, trácese sobre esas direcciones las líneas *oa'*, *ob'*, y sobre éstas con la diagonal R=*oc*, constrúyase el paralelogramo de las fuerzas *abco*. Los lados *oa*, *ob* del paralelogramo que resulta representan las componentes en intensidad y dirección.

**37. Advertencia.** Las dos fuerzas *ao* y *bo*, fig. 14, pueden obrar las dos, ó empujando ó tirando del punto *o*, pero las líneas que las representan en el paralelogramo, y que se encuentran en *o*, deben ser trazadas lo mismo, y la resultante indicará una fuerza que empuja ó tira del punto *o*, según empujen ó tiren las componentes.

**38.** Así, en la fig. 15 (*a*), el poste final inclinado de la construcción empuja oblicuamente del punto *o* hacia abajo, con una fuerza representada por *a'o*, mientras el cordón inferior, tira de *o* hacia la derecha con una fuerza represen-

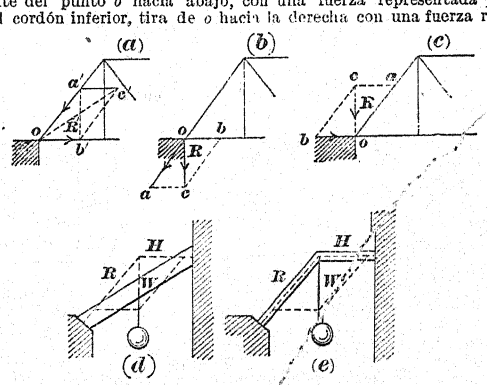



**Fig. 15.**

tada por *ob'*. Si construyésemos, como en la fig. 15 (*a*), el paralelogramo *oa'c'b'*, obtendríamos la diagonal *oc'* que no representa la verdadera resultante. Como

una de las fuerzas obra hacia el punto *o*, y la otra *desde* el punto *o*, no podríamos determinar en qué sentido estaría la flecha, aunque R' fuese la dirección de la resultante. Debemos suponer primero, que el empuje *a'o* en el poste extremo hacia *o*, pasa de *o* de manera que venga a obrar como una tensión *oa*, fig. 15 (*b*), quedando así ambas fuerzas consideradas como de tensión respecto al punto *o;* ó también podemos suponer que la tensión *ob'*, fig. 15 (*a*), de la cuerda se transforma en el empuje *bo*, fig. 15 (*c*), quedando consideradas ambas fuerzas como de empuje. En ambos casos se obtiene la verdadera resultante R (=*a'b'*, fig. 15 *a*); la cual, en este caso, representa la presión vertical hacia abajo del extremo de la construcción sobre el apoyo.

**Advertencia.** La fuerza de tensión, ejercida en un extremo de un tirante flexible, obra necesariamente en la dirección del tirante; pero, en general, la presión ejercida al extremo de un puntal sólo obra en la dirección del eje de aquél, cuando todas las fuerzas que la engendran están aplicadas al otro extremo. Así, en la figura 15 (*d*), las componentes R y H del peso, W, no coinciden con el eje de la viga que soporta la carga; pero en la fig. 15 (*e*), en donde el peso obra sobre la intersección de dos puntales, sus componentes R y H, sí coinciden con los ejes de éstos. (Véanse también figs. 143 y 145 *b*.)

**39. Demostración.** La demostración teórica del paralelogramo de las fuerzas se da en los tratados de Mecánica.

Se puede demostrar experimentalmente, como se indica en la fig. 16, en donde *co* representa por escala la tensión que marca el resorte C de la balanza, mientras que las fuerzas *oa*, *ob* están medidas por los resortes A y B respectivamente.

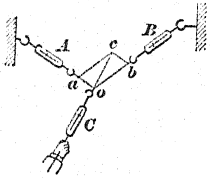



**Fig. 16.**

**40. Ecuaciones para las componentes y resultantes.** Dadas las intensidades de las fuerzas *a* y *c*, ó bien de la resultante R, y los ángulos por ellas formados, fig. 17 (*a*), tenemos * :

$$R = c\,\frac{\operatorname{sen}(x+y)}{\operatorname{sen} x} \quad = \quad a\,\frac{\operatorname{sen}(x+y)}{\operatorname{sen} y}$$

$$c = R\,\frac{\operatorname{sen} x}{\operatorname{sen}(x+y)}\,; \quad a = R\,\frac{\operatorname{sen} y}{\operatorname{sen}(x+y)}.$$

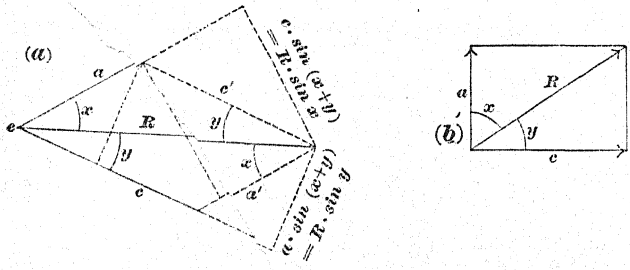



**Fig. 17.**

* Véase la línea de puntos, fig. 17 (*a*), y nótese que $c' = c \times \operatorname{sen}(x+y) = R.\ \operatorname{sen} x$; y $\operatorname{sen}(x+y) = R.\ \operatorname{sen} y$.

Si el ángulo entre las dos fuerzas es de 90°, fig. 17 (*b*) estas fórmulas se convierten en :

$$R = \frac{c}{\cos y} = \frac{a}{\cos x}; \qquad c = R \cos y; \qquad a = R \cos x.$$

**41. Posición y dirección de la resultante,** fig. 18. Si las líneas que representan las componentes se trazan de acuerdo con los §§ 37 y 38, y se traza una recta *mn*, ó bien *m'n'* por el punto *o* de aplicación, de modo que ambas fuerzas queden á un mismo lado de dicha línea, entonces la línea que representa la resultante queda del mismo lado que las componentes y entre ellas, y empujará ó tirará de la línea *mn* ó *m'n'* tal como lo hagan las componentes. La resultante está en el mismo plano que sus dos componentes.

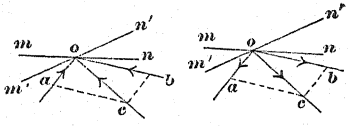



Fig. 18.

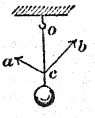



Fig. 19.

**42.** Si una de las componentes es colineal con la fuerza, entonces se confunden en una sola fuerza y puede considerarse la otra componente como *cero*. Es decir : una fuerza no puede descomponerse en dos fuerzas no colineales cuando una de ellas obra en la misma dirección de la fuerza. Así, la cuerda *oc*, fig. 19, puede recibir ayuda de *dos* cuerdas adicionales, que tiren según *ca* y *cb*, pues la resultante de sus tensiones puede coincidir con *co;* pero mientras *co* permanezca vertical, ninguna fuerza *sola* como *ca* ó *cb*, la puede reemplazar, á menos que actúe en su misma dirección *co*.

**43.** En la fig. 20, el peso P colocado en C está sostenido por completo por el miembro vertical BC, y no ejerce directamente ninguna tensión sobre el miembro horizontal CE. Ni tampoco ninguna tensión en este último ejercería efecto alguno sobre la fuerza que actúa en BC, mientras ésta permanezca vertical. Pero la tensión en BC, obrando en B, ejerce un empuje *oa* según BD, aun cuando ésta forma ángulo recto con BC. Como BC también encuentra allí la línea inclinada AB, la tensión *od* se descompone en *oa* y *ob* por BD y BA respectivamente.

El empuje horizontal *oa* en BD es en realidad la antiresultante de la componente horizontal *db*, del empuje oblicuo sobre el poste final BA, en su extremo B, cuyo empuje es=á la tensión en AE ejercida por P.

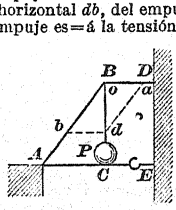



Fig. 20.

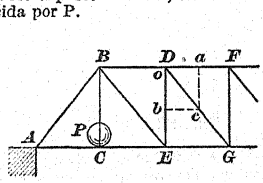



Fig. 21.

**44.** En la fig. 21, la tensión *oc* de la cuerda inclinada DG se descompone en D, en *oa* y *ob*, obrando una respecto á otra en ángulo recto, á lo largo de DF y DE respectivamente.

**45.** La resultante puede ser mayor ó menor que cualquiera de sus dos componentes oblicuas, pero siempre es menor que su suma. Si las componentes son iguales y el ángulo entre ellas es=120°, la resultante es=á una de ellas. Por tanto, con el peso que se rompe una cuerda vertical se rompen dos cuerdas iguales, formando cada una un ángulo de 60° con la vertical.

## Triángulo de las fuerzas.

**46. Triángulo de las fuerzas.** Como la diagonal de un paralelogramo lo divide en dos triángulos iguales, basta considerar uno de estos triángulos *aoc*, ó bien *boc*, figs. 14, 16, 18, en lugar de todo el paralelogramo.

**47.** Si tres fuerzas concurrentes, coplanas, se encuentran en equilibrio, las líneas que las representan formarán un triángulo, y las flechas que indican sus direcciones se siguen unas á otras alrededor del triángulo. Así, en la fig. 22 (*a*), tenemos, obrando en *o* y equilibradas, tres fuerzas, á saber : (1) la fuerza vertical hacia abajo *oc*, del peso, obrando como tensión por la cuerda *oc;* (2) el empuje horizontal *ao* por la viga *ao;* y (3) el empuje inclinado hacia arriba *bo* del puntal *ob*, las cuales obran en el sentido (*oc*, *ao*, *bo*), indicados por las flechas.

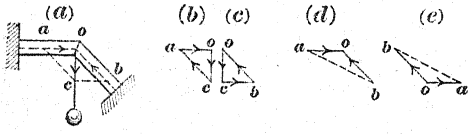



**Fig. 22.**

**48.** Cada una de las fuerzas de la fig. 22 (*b*) y (*c*) es la antiresultante de las otras dos del mismo triángulo, y si se invirtiese su dirección, se convertiría en la resultante. Así, *oc*, fig. 22 (*b*), es la antiresultante, y *co* la resultante de *ca* y *ao* : y *oc*, fig. 22 (*c*), es la antiresultante, y *co* la resultante de *cb* y *bo*, siendo *cb* paralela á *ao*, fig. (*b*), que representa el empuje ejercido por la viga horizontal en la unión *o*, fig. (*a*) *.

**49.** A la inversa, si los tres lados de un triángulo representan en dirección é intensidad tres fuerzas concurrentes, tales que las tres flechas que las representan, cada una en un lado, se siguen alrededor del triángulo, aquellas fuerzas estarán en equilibrio.

**50.** Las tres fuerzas, fig. 23, son respectivamente proporcionales á los senos de sus ángulos opuestos.

Así: fuerza *a* : fuerza *b* : fuerza *c*
= seno A · seno B : seno C.

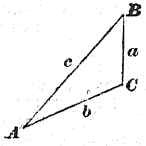



**Fig. 23.**

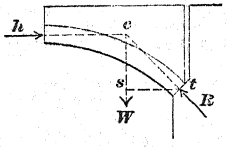



**Fig. 24.**

**51. Ej.** En la fig. 24, el semiarco y su riñón obrando como un solo cuerpo rígido, se presume que están en equilibrio por sus pesos combinados W, la presión horizontal *h* en la clave, y la reacción R del arranque oblicuo, la que se supone aplicada en su centro. En el triángulo de las fuerzas *cst*, obrando *cs* en el centro de gravedad del semiarco y su riñón representa el peso conocido W, y *st*, se traza horizontalmente, es decir, paralelo á la fuerza *h*.

En el punto *c*, donde la fuerza *h* prolongada encuentra la fuerza W, trácese *ct* por el centro del arranque, y entonces *st* y *ct* nos darán el valor de *h* y R respectivamente.

**52. Ej.** : Supongamos que la fig. 25 representa la armadura de un techo, que descansa en dos muros y que soporta tres pesos, indicados por las flechas. Trácese la vertical *a*R para representar los pesos cargados por el muro *a*, ó, lo que

---

* Fig. 22 (*d*) y (*e*), que representan las mismas dos fuerzas *ao* y *bo* de la fig. 22 (*a*) demuestra la resultante errónea (*ab*) que se obtiene si se trazan las fuerzas con sus flechas *hacia*, ó *del* punto de intersección de las líneas. (Véanse §§ 37 y 38.)

es lo mismo, la reacción vertical del apoyo hacia arriba. Luego trácese Rc paralela á *ad*, que es la cuerda de aquella parte de la armadura, hasta encontrar la arma-

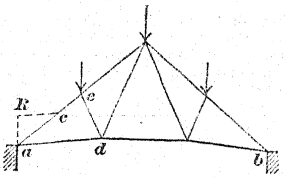



**Fig. 25.**

dura en *c*, y tendremos para los esfuerzos en *ae* y *ad* debidos á los tres pesos :

Esfuerzo en $ae = ac$

— $ad = Rc$.

**53.** Mientras que dos ó más fuerzas dadas como *ob* y *bc*, fig. 26 (*a*) (con sus flechas invertidas), ó bien *ob'* y *b'c*, ó bien *oa* y *ac*, ó bien *oa'* y *a'c*, pueden tener sólo

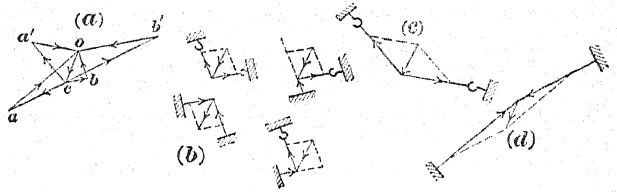



**Fig. 26.**

una resultante *oc;* una sola fuerza como *oc* puede descomponerse en dos ó más componentes, concurrentes en cualesquiera direcciones que se deseen. Es decir, que hay un infinito número de sistemas posibles de fuerzas concurrentes que tienen á *oc* por resultante.

## Componentes rectangulares.

**54.** Un ejemplo muy común de descomposición de fuerzas es aquel en que una fuerza, como la presión *cn*, del poste, fig. 27, se descompone en componentes que formen entre sí ángulo recto, como son las componentes vertical y horizontal *ct* y *tn* fig. 27 (*a*). Estas dos componentes, consideradas juntas, se llaman las componentes rectangulares de las fuerzas. El apoyo *od*, fig. 27 (*a*), se encuentra debidamente colocado perpendicular á *cn*, pero el apoyo *cib*. fig. 27 (*b*), prevé cualquier cambio accidental en la dirección *cn*. En la fig. 27 (*b*), las superficies *ci*, *ib*, deben trazarse, con preferencia, proporcionales respectivamente á las componentes *ct* y *tn*, fig. 27 (*a*), por la semejanza de los triángulos *cib* y *ctn*.

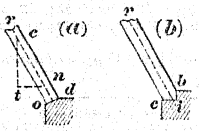



**Fig. 27.**

**55. Ej.** : En las armaduras de puentes y de techos, es preciso á menudo encontrar las componentes verticales y horizontales del esfuerzo en un elemento inclinado de aquéllas, es decir, determinar el esfuerzo que resulta sobre el elemento por la

acción de un esfuerzo vertical ú horizontal aplicado en uno de sus extremos en combinación con otro (cuya intensidad sea ó no conocida), y que obra en ángulo recto con aquél.

Así, en la fig. 28, la tensión *Cp*, en la diagonal *Cd*, se descompone en una presión *ep* á lo largo de la cuerda superior CD *, y en otra presión *Ce* sobre el poste *Ce* *. Agregándole á *Ce* el peso en *e*, y representando su suma por *fe*, tenemos la tensión *fg* en la parte de la cuerda *cd*, y la tensión *eg* en la diagonal *Be*. Haciendo *Bh*=*eg*,

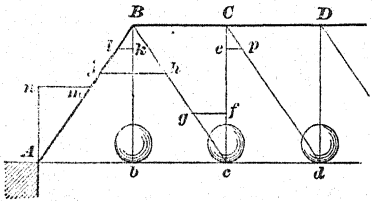



**Fig. 28.**

tenemos *jh*, compresión en BC; y *Bj*, compresión en el poste final inclinado BA. Pero la carga en *b* también transmite á B, por intermedio de la cuerda vertical *Bb*, una carga ó tensión igual á ella. Representando ésta por *Bk*, tenemos *lk* como su componente á lo largo de la porción de cuerda superior BC; y *Bl* como su componente á lo largo del poste final BA. Ahora, haciendo *Am*=á la suma de *Bj*+*Bl*, encontramos que la componente rectangular vertical es *An*=á la parte de la reacción vertical del apoyo que corresponde solamente á los tres pesos considerados, y la componente horizontal *mn*=al esfuerzo horizontal correspondiente á la parte *Ac* de la cuerda inferior.

**56. Ej. Plano inclinado.** Además, supongamos que en la fig. 29 se busquen las

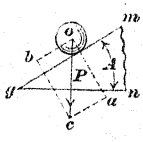



**Fig. 29.**

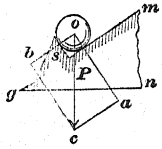



**Fig. 30.**

dos componentes rectangulares del peso P, respectivamente paralela y perpendicular al plano inclinado.

La primera es la tendencia del peso á rodar por el plano y se llama **componente tangencial**. La otra es la presión ejercida por el peso contra el plano y se llama **componente normal**.

Ahora sólo nos resta construir el triángulo de las fuerzas *oac*** trazando *oc*=P= peso del cuerpo; *oa* y *ac* en las direcciones requeridas. Entonces *oa* y *ac* dan respectivamente las componentes, normal y tangencial, de la fuerza P †.

**57.** Si suponemos que el plano inclinado *gm*, fig. 29, no produce fricción, y si al cuerpo *o* se le impide resbalar por él, empleado una fuerza aplicada paralela al plano, esta fuerza es=*ca*.

Así en la fig. 30, suponiendo que no haya fricción, tenemos *ac*= á la presión ejercida contra el obstáculo S.

---

* El esfuerzo encontrado, no es precisamente el *total* en el miembro. La compresión en *Ce* (despreciando su propio peso y el de la cuerda superior) se debe en su totalidad á la tensión *Cp* en *Cd* que obra en su parte superior, y de aquí que *Ce* represente la compresión total en *Ce;* pero *ep* es sólo *una parte* de la compresión soportada por CD, pues BC también contribuye.

** Ó bien *obc*. Si se trazan ambos triángulos tendremos el paralelogramo de las fuerzas *oacb*.

† La línea *ac* (ó bien *ca*) es la *proyección* de *oc* sobre el plano inclinado; y *oa* (ó bien *ao*) es la proyección de *oc* sobre *una normal* al plano inclinado.

**58.** Tabla de componentes normales y tangenciales para diferentes ángulos de inclinación:

| Inclinación del plano ó pendiente. La longitud de la pendiente = altura vertical / seno columna 4) | | | Presión sobre el plano dada en partes del peso ó coseno del ángulo del plano. | Fuerza de descenso en partes del peso, ó seno del ángulo del plano. |
|---|---|---|---|---|
| Vert. | Horiz. | Grad Min. | | |
| 1 en | 3. | 18 26 | .9487 | .3162 |
| 1 en | 4. | 14 2 | .9702 | .2425 |
| 1 en | 5. | 11 19 | .9806 | .1962 |
| 1 en | 6. | 9 28 | .9864 | .1645 |
| 1 en | 8. | 7 8 | .9923 | .1242 |
| 1 en | 9. | 6 20 | .9939 | .1103 |
| 1 en | 10. | 5 43 | .9950 | .0996 |
| 1 en | 11.4 | 5 00 | .9962 | .0872 |
| 1 en | 12. | 4 46 | .9965 | .0831 |
| 1 en | 14.3 | 4 00 | .9976 | .0698 |
| 1 en | 15. | 3 49 | .9978 | .0666 |
| 1 en | 19.1 | 3 00 | .9986 | .0523 |
| 1 en | 20. | 2 52 | .9987 | .0500 |
| 1 en | 23.1 | 2 30 | .9990 | .0436 |
| 1 en | 25. | 2 17 | .9992 | .0398 |
| 1 en | 28.6 | 2 00 | .9994 | .0349 |
| 1 en | 30. | 1 55 | » | .0334 |
| 1 en | 32.7 | 1 45 | .9995 | .0305 |
| 1 en | 35. | 1 38 | .9996 | .0285 |
| 1 en | 38.2 | 1 30 | .9997 | .0262 |
| 1 en | 40. | 1 26 | » | .0250 |
| 1 en | 45.8 | 1 15 | » | .0218 |
| 1 en | 50. | 1 9 | .9998 | .0201 |
| 1 en | 57.3 | 1 0 | » | .0175 |
| 1 en | 60. | 0 57½ | .9999 | .0167 |
| 1 en | 70. | 0 49 | » | .0143 |
| 1 en | 76.4 | 0 45 | » | .0131 |
| 1 en | 80. | 0 43 | » | .0125 |
| 1 en | 90. | 0 38 | » | .0111 |
| 1 en | 100. | 0 34 | 1.0000 | .0100 |
| 1 en | 114.6 | 0 30 | » | .0087 |
| 1 en | 125. | 0 27⅔ | » | .0080 |
| 1 en | 150. | 0 23 | » | .0067 |
| 1 en | 175. | 0 19⅔ | » | .0057 |
| 1 en | 200. | 0 17 | » | .0050 |
| 1 en | 229.2 | 0 15 | » | .0044 |
| 1 en | 250. | 0 14 | » | .0041 |
| 1 en | 300. | 0 11½ | » | .0033 |
| 1 en | 343.9 | 0 10 | » | .0029 |
| 1 en | 400. | 0 8⅔ | » | .0025 |
| 1 en | 500. | 0 7 | » | .0020 |
| 1 en | 600. | 0 6 | » | .0017 |
| 1 en | 800. | 0 4¾ | » | .0013 |
| 1 en | 1000. | 0 3¾ | » | .0010 |
| 1 en | 3437. | 0 1 | » | .0003 |
| Horiz. | | 0 0 | » | .0000 |

**59. Ecuaciones.** En la fig. 29,

$$oa = P \times \cos\ c\ o\ a$$
$$ac = P \times \operatorname{sen}\ c\ o\ a$$

y siendo el ángulo *coa*, entre la vertical *oc* y la componente normal *oa* = ángulo A de la inclinación del plano *gm* con la horizontal *gn*, tenemos :

Componente normal $oa = P \cdot \cos A$.
Componente tangencial $ac = P \cdot \operatorname{sen} A$.

**60.** Cuando una fuerza se descompone en fuerzas rectangulares como en las figs. 29 y 30, cada componente representa el *esfuerzo total* ó *tendencia* que aquella fuerza sola ejerce *en dicha dirección*.

Así, en la fig. 31, el máximum de fuerza que sólo el peso *oc* puede ejercer perpendicularmente contra el plano, está representado por la componente *oa*, pues si para impedir que el cuerpo se deslice por el plano aplicamos una fuerza en otro

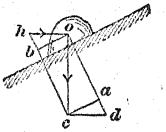



**Fig. 31.**

sentido, como por ejemplo la horizontal *ho*, en lugar de la tangencial *bo*, y encontramos las componentes de *oc* en las direcciones *ho*, *oa*, veremos que la componente normal *od* ha aumentado; pero el aumento *ad* se debe enteramente á la componente normal *hb*, de la fuerza horizontal *ho*. Así, el único efecto logrado sobre el cuerpo *o* y sobre el plano al sustituir *ho* por *bo*, es el de agregar la componente normal, *hb*, de la primera, á la componente normal (*oa*) de *oc*.

## Componentes.

**61. Componentes.** En la fig. 32 supongamos que *ao* y *bo* sean dos fuerzas cualesquiera y *co* su resultante. Desde *a* y *b* trácense *aa'* y *bb'* perpendiculares á la diagonal *oc* del paralelogramo de las fuerzas *aobc*, y constrúyanse los subparalelogramos (rectángulos) *oa'aa''* y *ob'bb''*. Cada una de las componentes originales *oa*, *ob*, quedan así descompuestas en dos subcomponentes, en ángulo recto, una de las cuales es también perpendicular á la resultante *oc*, mientras que la otra coincide con *oc* en posición y en dirección. Como las perpendiculares bajadas del vértice de los ángulos opuestos de un paralelogramo sobre su diagonal son iguales, resulta

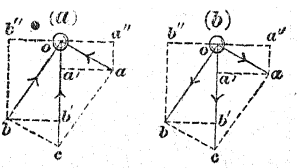



**Fig. 32.**

que las dos fuerzas colineales *oa''* y *ob''* son iguales y opuestas (aunque las líneas *a a* y *b'b* que las representan no son diametralmente opuestas; por tanto, están en equilibrio, y su único efecto sobre el cuerpo es de compresión en la fig. 32 (*a*), y de tensión en la fig. 32 (*b*). Las otras dos, *oa'* y *ob'*, se combinan para formar la resultante *oc*, que es igual á su suma, y que tiende á mover el cuerpo *o* en su dirección.

**62.** Las dos grandes fuerzas *oa*, *ob*, fig. 33 (*b*), tienen la misma resultante *oc* = *oc'* que las dos fuerzas pequeñas *oa'*, *ob'*, fig. 33 (*a*), aun cuando sus componentes $a''a = b''b$ son mucho mayores.

**63.** Sucede á menudo que una de las componentes es normal á la resultante. Así, en la fig. 22, donde *oc* es vertical, su componente *oa* es horizontal, y la perpendicular, tirada de *a* sobre *oc*, representa su anticomponente horizontal, *ao*. Aquí la viga horizontal y la inclinada soportan iguales presiones horizontales, por la presión vertical *oc*, = al peso W, está sostenida por la viga inclinada.

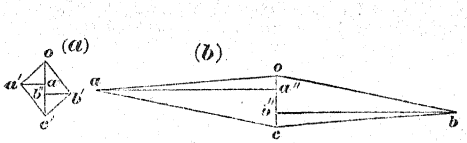



Fig. 33.

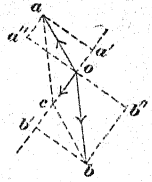



Fig. 34.

**64.** Cuando, como en la fig. 34, la resultante *oc* forma un ángulo *aoc* mayor de 90° con una de las componentes originales, las perpendiculares *aa'* y *bb'* de *a* y *b*, deben caer sobre la línea de la resultante prolongada. Aquí, sin embargo, como antes, las dos subcomponentes iguales y opuestas *oa''*, *ob''* están en equilibrio en *o*, mientras que las otras dos subcomponentes *ob'* y *oa'* van á formar la resultante *oc*, la cual, sin embargo (puesto que *ob'* y *oa'* obran aquí en sentido opuesto), es igual á su diferencia y no á su suma como en la fig. 32.

La fig. 34 demuestra que una fuerza *oc*, que obra hacia abajo, puede descomponerse de tal manera, que una de sus componentes *oa*, obrando *hacia arriba*, sea mayor que la fuerza original *hacia abajo*, y que la presión *ob* tenga una componente *ob'*, ó bien *b''b* paralela á *oc* y mayor que esta misma, pues $b''b = ob' = oc + cb'$.

## Aplicación de las fuerzas.

**65.** En la fig. 29, la bola está libre para correr por el plano, y aunque todo le peso P del cuerpo se aplica al plano *gmn*, sólo la *componente* normal *oa* obra sobre él ejerciendo una presión en la dirección *oa*. Pero, en la fig. 30, el plano *gmn* recibe y soporta no sólo la acción de la componente normal *oa*, sino también, por medio del obstáculo S, la componente tangencial *ob;* de manera que toda la fuerza P, ó bien, *oc*, obra sobre el plano *gmn* comprimiéndolo en la dirección *oc*.

## Composición y descomposición de las fuerzas concurrentes por medio de coordenadas.

**66.** Supongamos que, en la fig. 35 (*a*), las tres fuerzas coplanas E, F, G, obran en el punto *x*. Trácense dos líneas HH, VV en ángulo recto, cortándose en *o*, fig. 35 (*b*)*. Estas líneas se llaman coordenadas rectangulares. Desde el punto *o* trácense las líneas *o*E, *Fo* y *Go* paralelas á E*x*, F*x*, G*x* de la figura 35 (*a*) é iguales respectivamente á las fuerzas E, F y G usando una escala conveniente. Descompóngase cada una de estas fuerzas, fig. 35 (*b*), en dos componentes, paralelas á HH y VV respectivamente. Así *Eo* queda descompuesta en *to* y *no;* F*o*, en *ou* y *oe;* o*G*, en *oi* y *om*. Luego, sumando las componentes, tenemos

$$\text{Suma de las horizontales} = ou - oi - ot = -os,$$
$$\text{Suma de las verticales} = on + oe - om = oa;$$

y $-os$ y $oa$ serán las componentes de la resultante R de las tres fuerzas E, F y G.

**67.** Cuando un sistema de fuerzas concurrentes se encuentra en equilibrio, a suma algebraica** de las componentes de todas las fuerzas sobre cualquiera de las dos coordenadas, es cero.

Así, fig. 35 (*b*) ó 36, si la dirección de R es tal que venga á obrar como la antiresultante de las otras tres fuerzas E, F, G, sus componentes, *os* ó bien *oa*, sobre cualquiera de las coordenadas, compensará las de las otras fuerzas en la misma coordenada.

De aquí deducimos la importante proposición siguiente: cuando un sistema de

---

* Para más facilidad se trazan generalmente las coordenadas en ángulo recto, como en la fig. 35, mas se pueden hallar formando cualquier otro ángulo, como en la fig. 36; pero en todo caso las fuerzas deben, naturalmente, descomponerse en componentes *paralelas á las coordenadas*, cualquiera que sea la dirección de éstas.

** Las componentes se toman con signo + ó —, de acuerdo con la dirección de cada una.

fuerzas coplanas concurrentes está en equilibrio, las sumas algebraicas de sus componentes, en cualquiera de las dos direcciones, son iguales á cero.

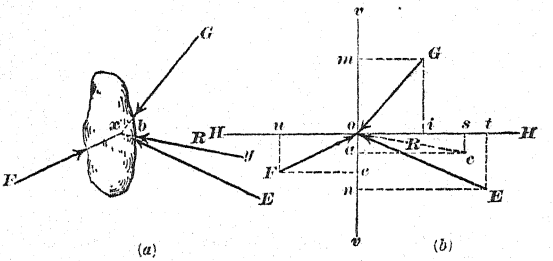



**Fig. 35.**

**68.** A la inversa, en un sistema de fuerzas concurrentes, si las sumas algebraicas de las componentes, en cualquiera de las dos direcciones, son iguales á cero, las fuerzas están en equilibrio.

Si la suma de las componentes en una cualquiera de las dos direcciones no es igual á cero, las fuerzas no pueden estar en equilibrio. Así, en la fig. 35 (*b*) ó 36 (*b*),

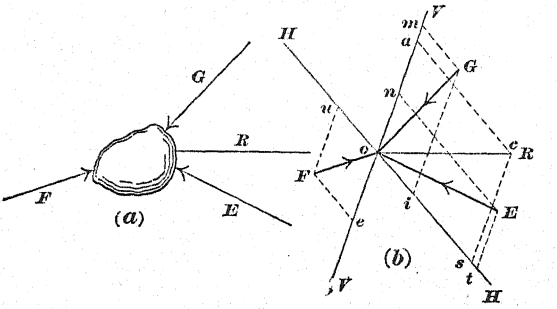



**Fig. 36.**

la suma de las componentes, á lo largo de una de las dos coordenadas (como VV), puede ser cero, y todavía si la suma de las fuerzas, á lo largo de la otra coordenada, no es cero, su resultante moverá el cuerpo sobre el que obran en la dirección de esa resultante.

**69.** Con coordenadas verticales y horizontales las condiciones de equilibrio * vienen á ser :

La suma de las componentes horizontales debe ser igual cero;
La suma de las componentes verticales debe ser igual cero;

ó más brevemente:

$$\Sigma \text{ componentes horizontales} = 0$$
$$\Sigma \text{ componentes verticales} = 0$$

A la inversa, si estas condiciones están satisfechas, las fuerzas están en equilibrio.

**70. Resultante de más de dos fuerzas coplanas.** Para encontrar la resultante de dos ó más fuerzas concurrentes coplanas como en la fig. 37, se encuentra primero la resultante $R_1$ de dos cualesquiera de ellas, como $P_1$ y $P_2$;

* Con fuerzas no concurrentes debe satisfacerse otra condición. (Véase § 83.)

entonces la resultante $R_2$ de $R_1$ y la tercera fuerza $P_3$; y así hasta obtener la resultante R de todas las fuerzas. Esta resultante es, evidentemente, concurrente y coplana de las fuerzas dadas.

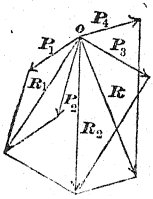



Fig. 37.

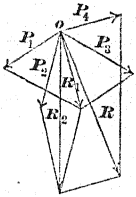



Fig. 38.

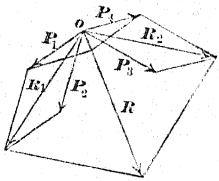



Fig. 39.

**71. El orden en que se van tomando las fuerzas** no tiene importancia. Así se puede componer primero, como en la fig. 38, la $P_1$ y la $P_2$, y entonces su resultante $R_1$ con $P_3$ obteniendo á $R_2$, y finalmente á $R_3$ con $P_4$ obteniendo R; ó, como en la fig. 39, se pueden combinar primero dos cualesquiera de las fuerzas como $P_1$ y $P_2$ obteniendo su resultante $R_1$; luego, procediendo con cualesquiera otras dos fuerzas como $P_3$ y $P_4$, obteniendo la resultante $R_2$, y finalmente combinando las dos resultantes $R_1$ y $R_2$ para obtener la resultante R.

## Polígono de las fuerzas.

**72. El polígono de las fuerzas.** Comparando las figs. 37 y 38 con las figs. 40 y 41, se ve que se puede llegar á la misma resultante R trazando simplemente, como en la fig. 41, líneas que representen las diversas fuerzas en cualquier orden, pero siguiéndose unas á otras, de acuerdo con su dirección. Adviértase que esto es solamente una abreviación del procedimiento de trazar los diversos paralelogramos de las fuerzas.

**73. Resultante y antiresultante.** La línea, — R, requerida para completar el polígono, representa la antiresultante de las otras fuerzas si su sentido es tal que las siga alrededor del polígono, como en la fig. 40. Si su dirección es opuesta á ellas, como en la fig. 41, entonces ella es su resultante R.

**74.** En otras palabras, si cualquier número de fuerzas concurrentes como $P_1$, $P_2$, $P_3$, $P_4$ y R, figs. 37 y 38 †, están en equilibrio, las líneas que las representan, trazadas en cualquier orden, pero de modo que en sus direcciones se sigan unas á otras, formarán un polígono cerrado, como en la fig. 40 (ó como en la fig. 41, si la dirección de R es contraria).

**75. A la inversa,** si las líneas que representan cualquier sistema de fuerzas concurrentes coplanas, cuando se las traza con su dirección, siguiéndose unas á

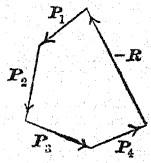



Fig. 40.

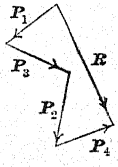



Fig. 41.

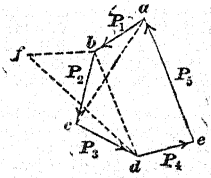



Fig. 42.

otras, forman un polígono cerrado, como en la fig. 40, esas fuerzas están en equilibrio.

Se observará que el triángulo de las fuerzas y la línea recta que representa un

† Aquí se considera que R obra hacia *arriba*, de manera de formar la antiresultante de las otras fuerzas.

sistema de fuerzas colineales, figs. 10 y 11, § 20, etc., ó un sistema de fuerzas paralelas, figs. 55, etc., § 111, etc., son simplemente casos especiales del polígono de las fuerzas.

**76.** En el polígono de las fuerzas, fig. 42, cualquiera de las fuerzas es la antiresultante de todas las demás. Dos ó más de cualquiera de las fuerzas equilibran á la demás, ó su resultante es la antiresultante de ellas.

Si se traza una línea *ac* ó *bd*, fig. 42, uniendo dos vértices cualesquiera de un polígono de fuerzas, esa línea representa la resultante ó la antiresultante (según vaya la flecha) de todas las fuerzas á cada lado de ellas; así:

| | | | | |
|---|---|---|---|---|
| *ac* es la resultante de | $P_1P_2$ | y la antiresultante de | $P_3P_4P_5$ | |
| *ca* | — | $P_1P_5P_4$ | — | $P_1P_3$ |
| *bd* | — | $P_2P_3$ | — | $P_1P_5P_4$ |
| *db* | — | $P_4P_5P_1$ | — | $P_2P_3$ |

**77.** Conociendo la *dirección* de *todas* las fuerzas de un sistema como $P_1 \ldots P_5$, fig. 42, y las intensidades de todas ellas, menos dos, como $P_2$ y $P_5$, podemos encontrar las intensidades de estas dos trazando primero las otras $P_4$, $P_5$ y $P_1$ como en la figura. Luego, dos líneas *bc* y *cd*, trazadas en las direcciones de las otras dos, y cerrando el polígono, darán sus intensidades.

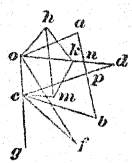



**Fig. 43.**

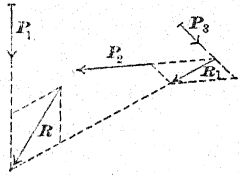



**Fig. 44.**

**78.** Si se toman dos puntos cualesquiera, como *o* y *c*, fig. 43, entonces la fuerza ó fuerzas representadas por cualquiera línea ó sistema de líneas que unan esos dos puntos, será equivalente á *oc*. Así, $oc = oabc = odc = onpc = ohkmc = ofc = ogc$, etc. Del mismo modo, en la fig. 42, el polígono de las fuerzas *abcdea* es equivalente al *abfdea* y al triángulo de las fuerzas *abca*, siendo cada cual = cero.

## Fuerzas coplanas no concurrentes.

**79. Fuerzas coplanas no concurrentes**, fig. 44. El procedimiento para encontrar la resultante de tres ó más fuerzas coplanas, pero no concurrentes, es el mismo que si fueran concurrentes. Así, supongamos que $P_1$, $P_2$ y $P_3$, representan tres de dichas fuerzas *. Puede encontrarse primero la resultante $R_1$ de dos cualesquiera de ellas, como $P_2$ y $P_3$; y luego, combinando $R_1$ con la fuerza restante $P_1$, se encuentra la resultante R de las tres. Aquí la línea R representa la resultante no sólo en intensidad y en dirección, sino también en posición. Es decir, la línea de acción de la resultante coincide con R.

**80. La resultante** R es la misma en intensidad y en dirección que si las fuerzas fueran concurrentes y tiene la misma posición que tendría si su punto de concurrencia estuviera en la línea de R. Para más de tres fuerzas se procede del mismo modo.

**81. Á la inversa**, la resultante R, ó cualquiera otra fuerza, puede descomponerse en un sistema de cualquier número de fuerzas coplanas, concurrentes ó no, en cualquiera dirección. Así podemos primero descomponer á R, en $P_1$ y $R_1$; luego á cualquiera de estas dos en otras dos fuerzas, como por ej.: $R_1$ en $P_2$ y $P_3$, etc.

**82.** Si un sistema de fuerzas coplanas no concurrentes está en equilibrio, permanecerán así situadas como si fueran concurrentes, con tal de que sus direcciones é intensidades no cambien; pero de aquí no se deduce que un sistema de fuerzas que está en equilibrio como concurrente, permanezca en equilibrio cuando esté colocado como no concurrente.

Así, las cinco fuerzas $P_1 \ldots P_5$, fig. 45 (*a*), pueden estar colocadas de tal manera

* Dos fuerzas coplanas, no paralelas, como $P_1$ y $P_2$ ó $P_2$ y $P_3$, son necesariamente concurrentes (véase § 19), pero pueden no concurrir en un solo punto las tres.

(como en la fig. 45 *b*), que la resultante *ac* de $P_1$ y $P_2$ no coincide con la resultante *ca* de $P_3$, $P_4$ y $P_5$, sino que le es paralela. Estas dos resultantes formarán entonces un par. (Véanse §§ 155, etc.)

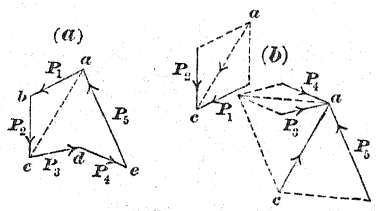



**Fig. 45.**

**83. Tercera condición de equilibrio.** De aquí que las condiciones de equilibrio para las fuerzas concurrentes, dichas en el § 69, son:

$$\Sigma \text{ de las componentes verticales} = 0$$
$$\Sigma \text{ de las componentes horizontales} = 0$$

no bastan para fuerzas no concurrentes, y debe agregarse una tercera condición, á saber:

$$\Sigma \text{ de los momentos} = 0;$$

es decir: los momentos de las fuerzas tomados con relación á cualquier punto deben estar en equilibrio.

Un sistema de fuerzas en equilibrio no tiene resultante; de aquí que no tiene momento con respecto á ningún punto. En otras palabras, los momentos de las fuerzas, así como las fuerzas mismas, están en equilibrio.

**84.** La resultante de un sistema de fuerzas no concurrentes ni en equilibrio, obrando sobre un cuerpo, puede ser:

(1) Una sola fuerza obrando por el centro de gravedad del cuerpo; ó

(2) Un par, es decir, dos fuerzas iguales, paralelas y contrarias (véanse § 155, etc.)

(3) O bien, (*a*) una sola fuerza obrando por el centro de gravedad del cuerpo, y un par; ó bien (*b*) una sola fuerza obrando fuera del centro de gravedad del cuerpo.

En el caso (3) las dos resultantes alternativas son cambiables entre sí; es decir, una sola fuerza obrando fuera del centro de gravedad del cuerpo, puede siempre ser reemplazada por una combinación equivalente, que consiste en una fuerza igual paralela, obrando por el centro de gravedad del cuerpo, y un par, y viceversa. (Véanse §§ 161, etc.)

La resultante en el caso (1) mueve el cuerpo en línea recta, sin rotación; en el caso (2) rotación sin traslación, y en el caso (3) ambos, traslación y rotación.

**85. Polígono de las fuerzas,** § 72, figs. 40, etc., y el método por coordenadas, § 66, fig. 35, nos dan por consiguiente solamente la intensidad, dirección y sentido de las fuerzas resultantes no concurrentes y *no* su posición. Para encontrar la posición de la resultante de fuerzas no concurrentes podemos recurrir á una figura como la fig. 44 en que las fuerzas están representadas en sus verdaderas posiciones; ó al polígono, §§ 86, etc., fig. 46.

## Polígono de cuerdas.

**86. En el triángulo de las fuerzas** dos cualesquiera de sus líneas pueden considerarse como representando las posiciones de dos miembros según sus direcciones (dos puntales ó dos cuerdas ó un puntal y una cuerda) de longitud indefinida y resistiendo á la tercera fuerza; al paso que sus longitudes dan las intensidades de las fuerzas que aquellos miembros deben ejercer para mantener el equilibrio.

**87.** Así, en la fig. 26 (*b*) tenemos cuatro diferentes sistemas de dos miembros cada uno, inclinados respectivamente como las fuerzas *cb* y *bo* en la fig. 26 (*a*) y equilibrando la tercera fuerza *oc*. Los miembros que obran como puntales están representados en la fig. 26 (*b*) como obrando contra superficies planas, al paso que los

que obran como cuerdas están representados como atados á ganchos de los cuales tiran.

En la fig. 26 (*c*) y (*d*) están indicados sistemas inclinados como las fuerzas *ca'*

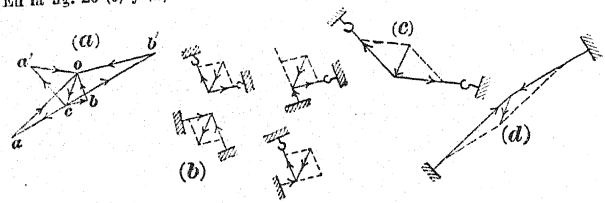



**Fig. 26 (repetida).**

y *a'o* y *ca* y *ao*, respectivamente, de la fig. 26 (*a*), por las cuales la tercera fuerza *oc* pudiera ser sostenida.

**88. En el polígono de la fuerza** *abcdea*, fig. 46 (*b*), que representa las cuatro fuerzas P P P P$_1$ de la fig. 46 (*a*), si escogemos á voluntad cualquier punto *o* (llamado **polo**) y trazamos desde él una serie de líneas rectas *oa*, *ob*, etc. (llamadas **radios polares**) hacia las extremidades *a*, *b*, *c*, etc., de las líneas P$_1$, P$_2$, etc., que representan las fuerzas, formaremos una serie de triángulos de fuerzas, *aob*, *boc*, etc. Así, en el triángulo *abo* tenemos la fuerza P$_1$, ó *ab*, equilibrada por las dos fuerzas *oa*, y *bo;* en el triángulo *bco*, la fuerza P$_2$, ó *bc* equilibrada por las dos fuerzas *ob* y *co;* y así de seguida.

**89. El polígono de cuerdas.** Si ahora, en la fig. 46 (*a*), tiramos las líneas *a* y *b*, paralelas respectivamente á los radios *oa* y *ob*, fig. 46 (*b*), que se encuentran en

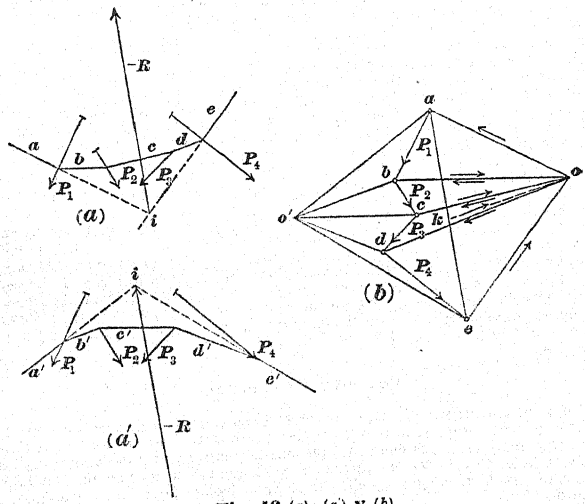



**Fig. 46** (*a*), (*a'*) **y** (*b*).

la línea que representa la fuerza P$_1$, aquéllas representarán las posiciones de las fuerzas de tensión de largo indefinido, que equilibrará la fuerza P$_1$, ejerciendo fuerzas representadas en intensidad y en dirección por los radios *oa* y *bo*, fig. 46 (*b*).

Del mismo modo, tomando el polo *o'*, fig. 46 (*b*), en vez de *o*, tenemos *a'* y *b'* fig. 46 (*a'*), paralelas respectivamente á los radios *o'a* y *o'b*, los que representarán un par de compresiones que hacen el mismo efecto.

**90.** Análogamente las líneas *b* y *c*, fig. 46 (*a*), paralelas respectivamente á los radios *ob* y *oc*, representan dos tensiones, las cuales, con fuerzas, ig uales respectivamente á *ob* y *co*, fig. 46 (*b*), equilibran la fuerza $P_2$.

**91.** Así obtenemos por fin un sistema de cinco fuerzas de tensión, *abcde*, fig. 46 (*a*), las cuales, si se las asegura debidamente en las extremidades *a* y *e*, equilibrarán, ejerciendo fuerzas representadas respectivamente por los radios *oa*, *ob*, *oc*, etc., fig. 46 (*b*), las cuatro fuerzas dadas $P_1$, $P_2$, $P_3$, $P_4$.

**92.** La figura *abcde*, fig. 46 (*a*), se llama *polígono de cuerdas, polígono funicular* ó *polígono de equilibrio*.

**93. Resultante, antiresultante. Intensidad y dirección.** En el polígono de las fuerzas, fig. 46 (*b*) ó (*d*), la línea *ea* que une el extremo de la última línea de fuerza *de* con el principio de la primera *ab*, se representa la antiresultante del sistema dado de cuatro fuerzas, y *ae* su resultante. Evidentemente pues, los rayos *ao* y *oe*, que representan dos componentes de *ae*, representan también, en dirección y en intensidad, dos fuerzas que equilibrarían á *ea*, ó que serían equivalentes al sistema dado de (cuatro) fuerzas.

**94. Posición de la resultante.** De aquí que, en el polígono de cuerdas, fig. 46 (*a*), la intersección *i* de las cuerdas *a* y *e*, paralelas respectivamente á los radios *oa* y *eo*, es un punto en la línea de acción de la resultante R; y si suponemos que *ai* y *ei* son piezas rígidas, y aplicamos en *i* una fuerza, —R, igual y paralela á *ae*, pero en dirección contraria, esa fuerza será la antiresultante de las (cuatro) fuerzas dadas, y tendremos un perímetro de cuerdas y piezas mantenido en equilibrio por las cinco fuerzas $P_1P_2P_3P_4$ y —R.

**95.** La elección del punto para situar el polo *o* en el polígono de las fuerzas, fig. 46 (*b*), no afecta á la resultante R; pero sí afecta á la forma del polígono funicular, fig. 46 (*a*) ó (*a'*). Así, con las fuerzas trazadas en el orden indicado en (*b*), y con el polo *á la derecha*, obtenemos, como en la fig. (*a*), una serie de fuerzas de

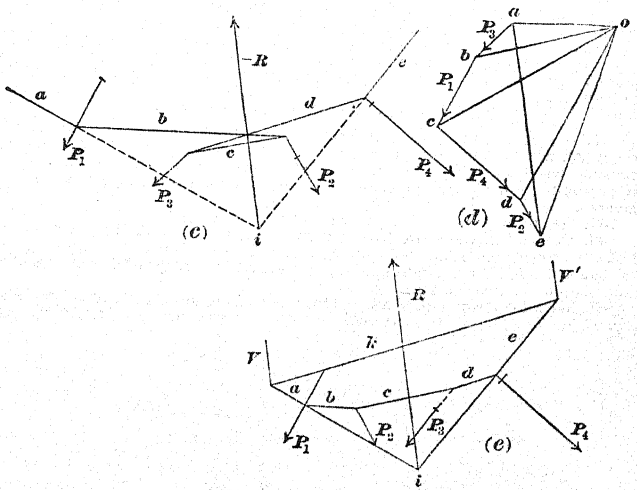



**Fig. 46** (*c*), (*d*) y (*e*).

tensión *a*, *b*, *c*, etc.; pero con el polo *á la izquierda* (fuerzas trazadas como antes) obtenemos una serie de fuerzas de compresión *a'*, *b'*, *c'*, etc. (fig. (*a'*).

**96.** Al construir el polígono funicular, fig. 46 (*a*), (*a'*), (*c*) y (*e*), debe tenerse

cuidado de trazar las fuerzas en sus correspondientes lugares; y para esto basta recordar que los dos radios pertenecientes á cualquiera línea de fuerza particular en el polígono de las fuerzas, fig. 46 (*b*), representan aquellos miembros que, en el polígono funicular, fig. 46 (*a*), toman las componentes de esa fuerza.

Así, *oa* y *bo*, fig. 46 (*b*), pertenecen á la fuerza $P_1$; *ob* y *co*, á la fuerza $P_2$. De aquí que, en la fig. 46 (*a*) ó (*c*), hagamos *a* y *b* (paralelas respectivamente á *oa* y *bo*) encontrarse en la línea de acción de $P_2$, etc.

**97.** Cada radio en el polígono de las fuerzas, fig. 46 (*b*), incluyendo los de afuera, se ven así pertenecer á dos fuerzas, y cada fuerza tiene dos radios. Las dos cuerdas paralelas respectivamente á los dos radios de cualquiera fuerza, deben ser trazadas de manera que se encuentren en la línea de la fuerza, y cada cuerda debe unir las líneas de acción de las dos fuerzas á que pertenezca su radio paralelo. Las líneas *a*, *b*, *c*, etc., en el polígono de cuerdas, fig. 46 (*a*) y (*c*), dan simplemente las *inclinaciones* de fuerzas que, de acuerdo con esa distribución, equilibrarían las fuerzas dadas. Las longitudes de estas líneas no tienen nada que hacer con las intensidades de las fuerzas. Éstas están dadas por las longitudes de los *radios* correspondientes en el polígono de las fuerzas, fig. 46 (*b*).

**98.** Si la antiresultante — R no está aplicada, puede suponerse que las cuerdas *a* y *e* estén fijadas á soportes firmes, contra los cuales ejercen fuerzas representadas, en intensidad y dirección, por los radios *ao* y *oe* respectivamente; pero es claro que las resistencias de aquellos dos soportes son iguales y opuestas respectivamente á *oa* y *eo*. De aquí que su resultante es la antiresultante — R de las cuatro fuerzas primitivas.

**99.** Si en la fig. 46 (*e*), las dos fuerzas extremas *a* y *e* estuvieran meramente sujetas á dos cuerdas, V y V', paralelas á la antiresultante — R, tirarían las extremidades de esas cuerdas para adentro, la una hacia la otra. Para impedir esto, colóquese el poste K con longitud tal, que las cuerdas V y V' permanezcan paralelas á —R, y trácese *ok*, fig. 46 (*b*), paralela á *k*. Entonces *ak* y *ke* representan las fuerzas en V y V' respectivamente.

**100.** Si la antiresultante — R, encontrada por medio del polígono de las fuerzas se aplica en una línea pasando por la intersección de los miembros exteriores (inicial y final) en el polígono funicular, todas las fuerzas, incluyendo por supuesto la antiresultante, estarán en equilibrio. En otras palabras, fuerzas coplanas están en equilibrio si se las puede trazar formando un polígono cerrado, y si se puede trazar entre ellas un polígono funicular. Pero si la antiresultante se aplica en otra parte, tendremos un par, compuesto de la antiresultante — R y de la resultante R de las fuerzas.

## Fuerzas concurrentes no coplanas.

**101.** Dos cualesquiera de las fuerzas concurrentes, como *oa* y *oc*, fig. 47 (*a*) e (*b*), son necesariamente coplanas. Encuéntrese su resultante *or*, que debe ser coplana con ellas y con una tercera fuerza *ob*. Entonces la resultante R, de *or* y *ob*, es la resultante de las tres fuerzas. Si hay otras fuerzas procédase de la misma manera.

**102.** Tres fuerzas no coplanas, concurrentes ó no, no pueden estar en equilibrio.

**103. Paralelepípedo de las fuerzas.** La resultante de tres fuerzas cualesquiera concurrentes, no coplanas, *oa*, *ob*, *oc*, fig. 47, estarán representadas por la diagonal *o*R, de un paralelepípedo, en el cual las tres aristas convergentes representan las tres fuerzas.

**104. Métodos por modelos. (a) Para tres fuerzas.** Constrúyase una caja, fig. 47 (*a*) ó (*b*), cuyas tres aristas convergentes representen las tres fuerzas en

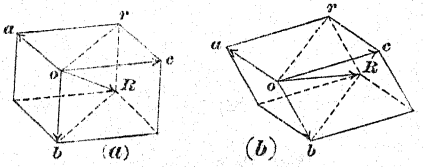
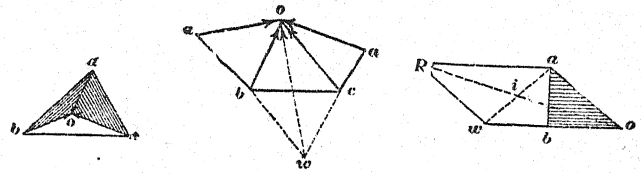



**Fig. 47.**

posición é intensidad. Entonces una cabulla *o*R que una los ángulos opuestos representa la resultante.

O bien, supongamos que las tres fuerzas *ao*, *bo*, *co*, fig. 48 (*a*), se encuentran en *o*. Trácense sobre un cartón las fuerzas *ao*, *bo*, *co*, como en la fig. 48 (*b*), con sus verdaderos ángulos *aob*, *boc*, *coa*, y encuéntrese la resultante *wo* de las dos del medio, *bo* y *co*. Córtese con nitidez toda la figura *aoacwba*. Háganse rayas profundas con un filo á lo largo de *ob*, *oc*, de manera que los dos triángulos exteriores puedan doblarse más fácilmente, en ángulos sobre el del medio, voltéense hasta que las dos aristas *oa*, *oa* se encuentren y luego péguese un pedacito de papel delgado á lo largo de la línea de unión para fijarlas. Párese el modelo sobre su lado *obwc* como base y ten-




(*a*) (*b*) (*c*)

**Fig. 48.**

dremos la forma de *chinela*, *aobw*, fig. 48 (*c*); siendo *ow* la suela y *aob* la cavidad para el pie. En el modelo la fuerza *ao* y la resultante *wo* de las otras dos fuerzas, se encuentran ahora en sus verdaderas posiciones relativas. Para encontrar su resultante córtese otro pedazo de cartón *Raow* con *Ra* y *Rw*, paralelas respectivamente á *wo* y *ao*. Trácese sobre cada lado de él la diagonal *Ro*, péguese este pedazo dentro del modelo con su arista inferior *wo* sobre la línea *wo*, fig. 48 (*b*), su arista *ao* en el ángulo *ao*. Hecho esto, *Ro* representa la resultante de *ao*, *bo*, *co*, fig. 48 (*a*), en su verdadera posición respecto á ellas.

**105. (b) Para cuatro fuerzas**, como *ao*, *bo*, *co*, *do*, en la fig. 49, se las traza, como en la fig. 49 (*a*), con sus ángulos *aob*, *boc*, etc. Trácense también las resultantes *vo*, de *co* y *bo*; y *wo* de *co* y *do*. Luego córtese toda la figura, como se hizo antes, y péguense las dos aristas *ao*, *ao*. Sosténgase el modelo de tal manera que dos de sus *planos* (como *aob* y *boc*) formen entre sí el mismo ángulo, como sucede con los dos planos correspondientes entre las fuerzas. Entonces tenemos las dos

(*a*) **Fig. 49.** (*b*)

resultantes *bo*, *wo*, fig. 49 (*b*), en sus *verdaderas relativas posiciones*. Córtese otro pedazo de cartón *Rvow*, fig. 49 (*b*), trácese la diagonal *Ro* de cada lado de él, y péguese dentro del modelo con *ov* y *ow*, sobre las líneas correspondientes del modelo. Luego *Ro* representará la resultante de las cuatro fuerzas *ao*, *bo*, *co*, *do* en su verdadera relativa posición respecto de ellas.

El modelo puede ser de madera, cortando separadamente los triángulos *aob*, *boc*, etc., biselando los filos de unión, y luego pegándolos con cola.

## Fuerzas no concurrentes y no coplanas.

**106. Fuerzas no concurrentes y no coplanas**, fig. 50 (*a*). (Para fuerzas paralelas no coplanas, véanse §§ 110, etc.) Descompóngase cada fuerza en dos rectangulares, una normal á un plano supuesto y la otra coincidiendo con el plano. Encuéntrese la resultante de las componentes (coplanas) que coinciden con el plano, por los métodos ya dados, y la de las componentes normales (paralelas) por los §§ 110, etc. Si estas dos resultantes son coplanas, serán también concu-

rrentes, y su resultante (que es la resultante de todo el sistema) se encuentra fácilmente.

**107.** Si no, supongamos que V, fig. 50 (*b*), sea la resultante normal al plano, y H, la resultante situada en el plano.

Por el § 162, sustitúyase la fuerza H por la H', igual y paralela, que encuentra

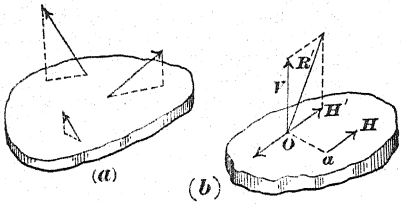



**Fig. 50.**

en O á V y el par $H \times Oa$; y búsquese la resultante R' de V y H'. El sistema de fuerzas queda así reducido á la única fuerza R' y al par $H \times Oa$. (Sobre pares, véase el § 155.)

**108. Momentos de fuerzas no coplanas.** La acción del peso W de la pared, fig. 51 (*a*), y de las fuerzas no coplanas $P_1$ $P_2$ pueden ser representadas, como en la fig. 51 (*b*), donde el eje *a'c'* representa la arista *ac* sobre la cual la pared tiende á girar, al paso que las perpendiculares de las fuerzas á la arista representan sus brazos de palanca. En cuanto a lo que se refiere á la estabilidad del muro, considerado como un cuerpo rígido y como capaz sólo de girar alrededor de la arista *ac*, no tiene importancia que una fuerza extraña como $P_1$ sea aplicada en *p* ó en *q*; pero sí tiene importancia si se considera la tendencia á hacer girar la pared horizontalmente, ó á romperla; ó en lo que se refiere á presiones (y fricciones con siguientes) entre el eje *a'c'* y sus puntos de apoyo.

Para el equilibrio $P_1 m = P_2 h + W \frac{b}{2}$. Aquí se ejerce en el eje una fuerza de torsión y se modifican más ó menos las presiones de sus extremidades en los puntos de apoyo; pero considerando sólo el equilibrio de los momentos, podemos suponer

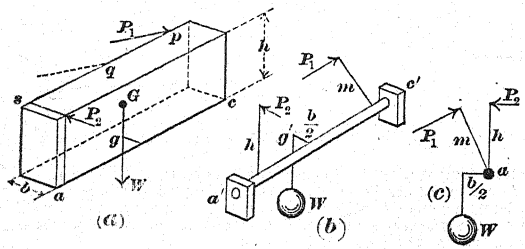



**Fig. 51.**

todas las fuerzas y sus momentos proyectados en el mismo plano, como en la fig. 51 (*c*).

**109.** En casos como el de la fig. 51, se acostumbra, por conveniencia, limitarnos á una supuesta sección vertical *s*, de un metro de grueso, y á considerar que las fuerzas obran sobre esa sección; suponiendo que el peso de la sección esté concentrado en su centro de gravedad y las fuerzas extrañas aplicadas en el mismo plano vertical del centro de gravedad. En efecto, así consideramos una sección indefinidamente delgada, pero cuyo peso es el de la sección de un metro.

## FUERZAS PARALELAS

**110. La resultante de cualquier número de fuerzas paralelas**, estén ó no en el mismo plano, y tengan ó no la misma dirección, es paralela á ellas y es = á su suma algebraica.

### Fuerzas paralelas coplanas.

**111. La resultante de cualquier número de fuerzas paralelas coplanas** está en el mismo plano que ellas, aunque las fuerzas sean del mismo sentido ó contrario; y los brazos de palanca de tales fuerzas y de su resultante, con respecto á cualquier punto del mismo plano, están en línea recta. Así, en la fig. 56 (*a*), donde las cinco fuerzas *a*, *b*, *c*, *d*, *e*, están en un plano, su resultante R está en ese mismo plano, y los brazos de palanca de las fuerzas y de R respecto á cualquier punto, como *b* ó *v*, del mismo plano, están en la línea recta R*v*.

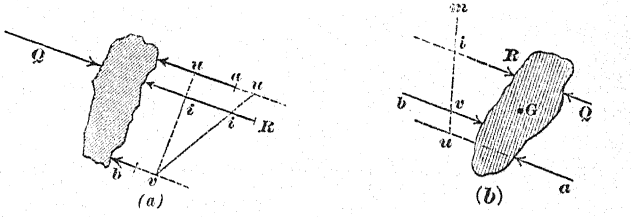



Fig. 52.

**112.** La resultante R ó la antiresultante Q, fig. 52, **de dos fuerzas paralelas** *a* y *b*, corta á cualquier línea recta, *uv*, que una las direcciones de las dos fuerzas; de aquí que, si tres fuerzas paralelas están en equilibrio, se hallan en el mismo plano. En la fig. 52 (*a*), las dos fuerzas *a* y *b* están en el mismo sentido. R está entonces entre *a* y *b*, y R = *b* + *a*. En la fig. 52 (*b*), *a* y *b* están en sentido opuesto. R no está entonces entre *a* y *b*, y R = *b* — *a*.

**113.** Para encontrar la posición de la resultante, trácese y mídase cualquier línea recta *uv*, uniendo las líneas de acción de las fuerzas. No importa que *uv* quede ó no perpendicular á dichas direcciones. La línea que representa la resultante corta á *uv* y su posición se encuentra así:

$$ui = uv \times \frac{b}{R}; \qquad \text{y } vi = uv \times \frac{a}{R}.$$

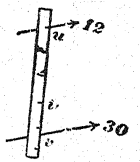



Fig. 53.

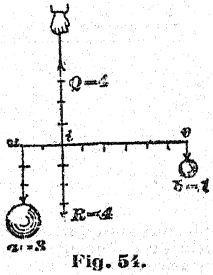
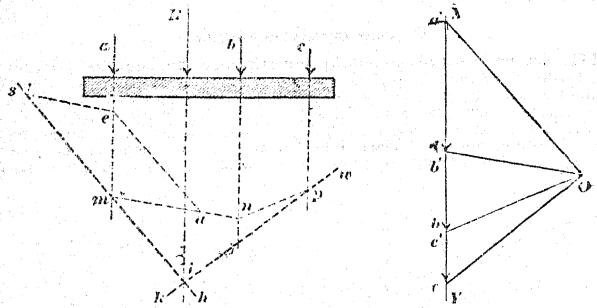



Fig. 54.

**114.** Esto se hace fácilmente trazando *uv* igual, en cualquier escala conveniente, á la suma de las fuerzas, como en la fig. 53, donde *uv* = 42. Luego hágase *ui igual*, por la misma escala, á la fuerza en *v*, ó *vi* = á la fuerza en *u*. Luego una línea R, fig. 52 (*a*), trazada por *i* paralela á *a* y b, da la posición y dirección de

su resultante, y su intensidad es=á la suma de $a$ y $b$; ó $R=a+b$. En otras palabras, si una fuerza Q, paralela á las fuerzas $a$ y $b$, é igual á su suma, pero en sentido contrario, se aplica al cuerpo en cualquier parte de una línea que pase por $i$, equilibrará á $a$ y $b$; será su antiresultante.



(a) **Fig. 55.** (b)

**115.** La posición de la resultante encontrada así, satisface la condición de equilibrio de los momentos; por tanto, $b \times vi - a \times ui = \text{cero}$. Si las dos fuerzas son iguales, su resultante R se encuentra evidentemente en el medio, entre ellas.

**116.** En la romana, fig. 54, las dos fuerzas $a$ y $b$ de la fig. 52 ($a$) están representadas por dos pesos, $a=3$ kg en $u$, y $b=1$ kg en $v$, con palancas $ui$ y $vi$ respectivamente, como 2 : 6 ó como 1 : 3.

Se observará en la fig. 56 ($a$) que la resultante R, debido á las posiciones é intensidades de las varias fuerzas, cae fuera del sistema de las fuerzas dadas.

**117.** Las figs. 55 hasta 58 ilustran la aplicación del polígono funicular (§§ 86 á 100) á las fuernas paralelas coplanas. Aquí el polígono de las fuerzas es, necesariamente, una línea recta.

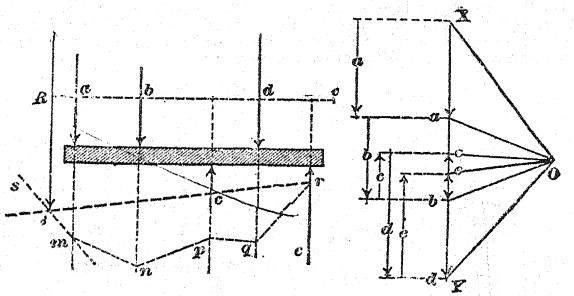
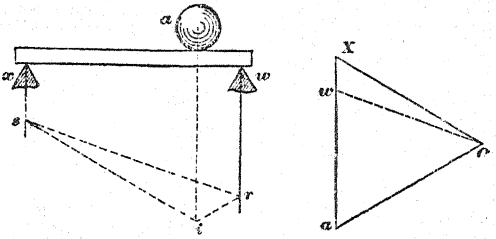


**Fig. 56.** (b)

**118. Descomposición de las fuerzas.** Supongamos que la fig. 57 ($a$ **representa una viga que sostiene una sola carga concentrada,** $a$, situada fuera de su centro; y que se quiere encontrar la presión en cada uno de los dos apoyos $w$ y $x$.

Trácese X$a$, fig. 57 ($b$), representando la carga $a$ en escala, y radios XO, $a$O, á cualquier punto O fuera de la línea X$a$. En la fig. ($a$), de cualquier punto $i$ de la vertical por el punto $a$ donde está aplicada la carga, trácese $is$ e $ir$, paralelas respectivamente á OX y O$a$. Únase $rs$, y en la fig. ($b$) trácese O$w$ paralela á $rs$

Entonces los dos segmentos $wa$ y $Xw$, de $Xa$, dan por la escala las presiones sobre los dos soportes $w$ y $x$ respectivamente.




(a) **Fig. 57.** (b)

La mayor presión estará, por supuesto, sobre el soporte que queda más cerca de la carga; pero también sirve de guía recordar que el segmento adyacente á la línea radial OX, en la fig. (b), representa la presión sobre ese soporte $x$, fig. (a), que pertenece á la línea $is$ paralela á OX; y viceversa.

**119.** La fig. 58 representa un caso en que hay varias cargas sobre la viga. La intersección $i$, de las líneas $hs$ y $kr$, fig. (a), halladas respectivamente paralelas á OX y $c$O, fig. (b), muestra la posición de la resultante de las tres cargas. Aquí, como en la página 57, unimos $rs$, fig. (a), y trazamos O$w$, fig. (b), paralela á $rs$. Entonces $Xw$, fig. (b), da la presión en $x$, y $wc$ la de $w$.

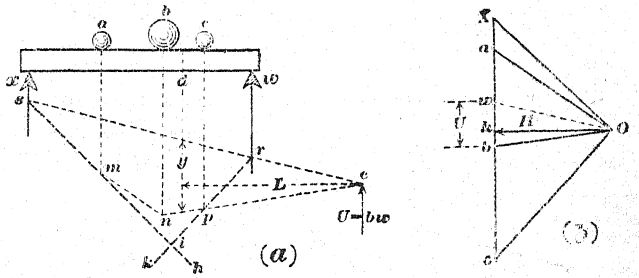



**Fig. 58.**

## Fuerzas paralelas no coplanas.

**120. Fuerzas paralelas no coplanas**, fig. 59 (a). Entre las líneas de acción de dos cualesquiera de las fuerzas, como $a$ y $b$, trácese cualquier línea recta $uv$, y se tendrá :

$$ui = uv \times \frac{b}{a+b}; \qquad vi = uv \times \frac{a}{a+b}.$$

Por $i$ trácese R′, paralela á $a$ y $b$, é igual á su suma. R′ es la resultante de $a$ y $b$. Entonces, de cualquier punto $i$, en la línea de acción de R′, trácese $iz$ á cualquier punto $z$ de la línea de acción de $c$, y hágase

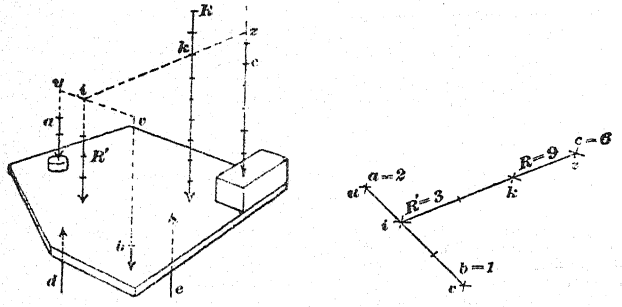
$$ik = iz \times \frac{c}{c+\mathrm{R}'}; \text{ ó } zk = iz \times \frac{\mathrm{R}}{c+\mathrm{R}'}.$$

Por $k$ trácese R paralela á $a$, $b$ y $c$, é igual á su suma y R será la resultante de $a$, $b$, $c$. Si hay otras fuerzas, procédase del mismo modo con ellas.

**121.** En la fig. 59 (a) colocamos á las fuerzas $a$ y $c$ obrando sobre superficies levantadas sobre el plano general, con el único objeto de demostrar que no es necesario que las fuerzas obren sobre una superficie plana.

**122.** También la fig. 59 (a) enseña el modo de encontrar la resultante de fuerzas

paralelas no coplanas, aunque no da todavía la verdadera posición relativa de las fuerzas y de su resultante, porque están dibujadas en una especie de perspectiva. y por tanto todas las partes no están sometidas á escala. La verdadera posición relativa puede, por supuesto, representarse en un plano, por las cinco cruceeitas $a$, $b$, $c$, $i$, $k$, fig. 59 ($b$), que corresponden á los puntos en que las fuerzas




(a)' **Fig. 59.** (b)

y la resultante cortan el plano supuesto. Pero ahora es imposible representar las fuerzas mismas por líneas; debe representárselas por números como se ha hecho aquí. Entonces es muy fácil encontrar, como antes, la posición de la resultante.

**123.** Si existen también fuerzas **que obran en dirección opuesta**, como $d$ y $e$, fig. 59 ($a$), encuéntrese su resultante separadamente.

Así se obtienen, finalmente, dos resultantes de sentido opuesto, Estas resultantes pueden ser iguales ó desiguales, colineales ó no. Si no son colineales, véase § 84, y *Pares*, §§ 155, etc.

**124. Método por proyecciones**, fig. 60. Encuéntrense, primero, las proyecciones $a'$, $b'$, $c'$ de las fuerzas $a$, $b$, $c$ sobre un plano cualquiera $xy$ paralelo á ellas, y luego sus proyecciones $a''$, $b''$, $c''$, sobre un segundo plano $xv$, paralelo á ellas y normal al primero. Encuéntrese la posición de la resultante R', de $a'$, $b'$, $c'$, en el plano $xy$ y la R'' de $a''$, $b''$, $c''$, en el plano $xv$. Ahora, como las líneas $a'$, $b'$ $c'$, y

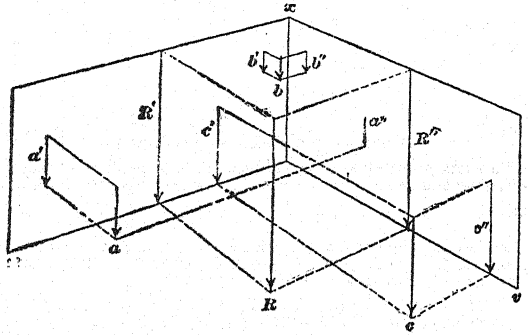



**Fig. 60.**

$a'$, $b''$, $c''$ son proyecciones de las fuerzas $a$, $b$, $c$, así R', R'' son proyecciones de la resultante R de las fuerzas. La posición de R está, por tanto, en la intersección de dos planos RR' y RR'' perpendiculares á los planos $xy$ y $xv$, y colocados sobre las proyecciones R', y R'', de la resultante R. $R=a+b+c$.

## CENTRO DE GRAVEDAD

**125.** Si un cuerpo, fig. 1 *, ó un sistema de cuerpos, fig. 2, se sostiene sucesivamente en diferentes posiciones, (*a*) y (*b*), la resultante de las fuerzas paralelas de la gravedad obrando sobre sus partículas, como lo indican las flechas en la figura, ocupará distintas posiciones con relación á la figura del cuerpo ó sistema. El punto donde todas aquellas líneas se encuentran, se llama el centro de gra-

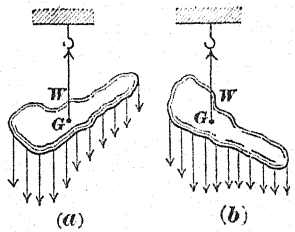



Fig. 1.

vedad del cuerpo ó sistema. Así, pues, si colocamos un cilindro recto homogéneo verticalmente sobre cualquiera de sus bases, la línea de gravedad coincidirá con el eje del cilindro; pero si luego se acuesta el cilindro sobre su lado, la vertical que pasa por el centro de gravedad dividirá al eje en dos partes iguales en ángulos rectos. De aquí que, en el cilindro, el centro de gravedad se encuentra en el centro del eje.

**126.** Respecto del centro de gravedad, todos los momentos debidos á la acción de la gravedad están en equilibrio, cualquiera que sea la posición del cuerpo ó sistema. De aquí que, si sobre el cuerpo ó sistema obra sólo la gravedad, y se los suspende por dicho centro, quedará en equilibrio; es decir, si está en reposo permanecerá así, ó si se le pone en movimiento, haciéndolo girar sobre su centro de gravedad, y luego se le deja, continuará girando indefinidamente, con velocidad angular uniforme. O si se le suspende libremente de cualquier punto, oscilará hasta que el centro de gravedad quede en reposo en la vertical del punto de suspensión.

**127.** En algunos cuerpos, tales como el cubo, ú otros paralelepípedos, la esfera, etc., el centro de *gravedad* es también el centro de la masa del cuerpo; pero con frecuencia no es así. En un cuerpo *ab*, fig. 2, por ejemplo, con su centro de gravedad en G, hay más masa (más peso) del lado *a*G que del lado G*b*.

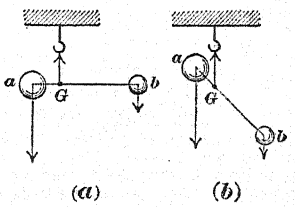



Fig. 2.

### Equilibrio estable, instable é indiferente.

**128.** Se dice que un cuerpo está en equilibrio estable cuando, como el péndulo, se le tiene suspendido de tal manera, que si se le mueve un poco hacia un lado,

* Las figs. 1 á 45 que se refieren al centro de gravedad están numeradas independientemente del resto de las series de figuras que se refieren á la Estática.

oscila hasta volver á su primitiva posición, es decir, con su centro de gravedad en la vertical y debajo del punto de suspensión.

**129.** Está en equilibrio instable, cuando, como en el caso de un huevo parado sobre su punta, se apoya de tal manera que si se le mueve sobre un lado y se le abandona, se separa de la vertical y cae.

**130.** Está en equilibrio indiferente, cuando, como sucede en una piedra de amolar sostenida por su eje horizontal, ó de una esfera que descansa sobre una mesa horizontal, está de tal modo suspendido ó apoyado, que si se le hace girar alrededor de su centro de gravedad y se le abandona, permanecerá en reposo ó con el movimiento angular transmitido.

## Reglas generales.

**131.** Las reglas generales que siguen, (1) á (6), forman la base de las reglas especiales, (7) á (39). Cuando hablamos del centro de gravedad de uno ó más cuerpos, suponemos para más facilidad que son homogéneos, es decir, de la misma densidad en todas sus partes y comparados unos con otros. El centro de *gravedad* es entonces el mismo que el centro de *figura*, y podemos usar los *volúmenes* en lugar de los *pesos*. Y, extendiendo esta regla á las **superficies** y líneas, usar las *áreas* de las superficies, y, en las *líneas*, sus *longitudes*.

En todas las reglas y figuras en que tratamos de esta materia, **G representa** el centro de gravedad, excepto cuando se establece otra cosa.

(1) **Dos cuerpos cualesquiera**, fig. 3, en los que se han encontrado los centros de gravedad, $g$, $g'$ de cada uno por medio de las reglas que se dan á continuación; **G** estará en la línea que une á $g$ con $g'$ y

$$g\mathbf{G} = gg' \times \frac{\text{peso en } g'}{\text{suma de los pesos } (g + g')}$$

$$g'\mathbf{G} = gg' \times \frac{\text{peso en } g}{\text{suma de los pesos } (g + g')}.$$

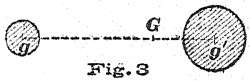


Fig. 3

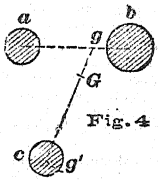


Fig. 4

(2) **Para cualquier número de cuerpos** como $a$, $b$, $c$, fig. 4, estén sus centros de gravedad en el mismo plano ó no, se procede así: Primero por la regla (1) búsquese el centro de gravedad $g$, de dos de ellos como $a$ y $b$. Entonces el centro de gravedad **G** de los tres cuerpos $a$, $b$, $c$ está en la línea $gg$ que une $g$ con el centro de gravedad $g'$ de $c$; y

$$g\mathbf{G} = gg' \times \frac{\text{peso de } c}{\text{suma de los pesos de } a,\ b,\ c};$$

$$g'\mathbf{G} = gg' \times \frac{\text{suma de los pesos de } a \text{ y } b}{\text{suma de los pesos de } a,\ b,\ c};$$

y así se sigue si hay otros cuerpos.

(3) En muchos casos un **cuerpo complejo** puede suponerse divido en partes cuyos varios centros de gravedad pueden encontrarse fácilmente.

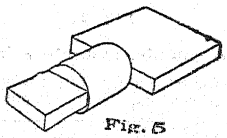


Fig. 5

Estonces el centro de gravedad del todo puede encontrarse por medio de las

reglas anteriores y las que siguen. Así, en el caso de la fig. 5 podemos encontrar separadamente los centros de gravedad de los dos paralelepípedos y del cilindro situado entre ellos, y en la fig. 6, los centros de gravedad del prisma cuadrado y la pirámide cuadrada (la última por medio de la regla (36); y entonces, conociendo en cualquiera de los dos casos los pesos de las diferentes partes se encuentra su centro común de gravedad como se ha indicado en las reglas (1) y (2).

Fig. 6

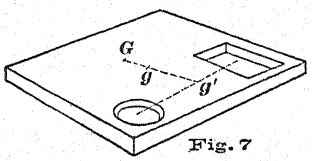



Fig. 7

(4) **Para cualquier cuerpo hueco,** ó que contiene uno ó más huecos, fig. 7, búsquese el centro de gravedad $g'$ de los huecos por medio de las reglas (1) ó (2), y el centro de gravedad $g$ de la figura entera como si no tuviera hueco. Entonces G está en la prolongación de la línea $gg'$, y

$$gG = gg' \times \frac{\text{suma de los volúmenes de los huecos}}{\text{volumen del cuerpo entero} - \text{volúmenes de los huecos}}$$

$$g'G = gg' \times \frac{\text{volumen del cuerpo entero}}{\text{volumen del cuerpo entero} - \text{volúmenes de los huecos}}.$$

**Advertencia.** Por conveniencia hemos mostrado los diferentes centros de gravedad $g$, $g'$, G, sobre la superficie de la figura. En el sólido real (que se supone que sea homogéneo) estarán en el centro de su espesor debajo de las posiciones indicadas en la figura.

(5) En cualquiera línea, figura ó cuerpo, ó en cualquier sistema de líneas, figuras ó cuerpos, cualquier plano que pase por el centro de gravedad se llama un « **plano de gravedad** » * para dicha línea ó sistema de líneas, etc. La intersección de dos planos de gravedad se llama « **línea de gravedad** ». El centro de gravedad es : (1.º) la intersección de dos líneas de gravedad; (2.º) la intersección de tres planos de gravedad, ó (3.º) la intersección de un plano de gravedad con una línea de gravedad que no está contenida en dicho plano.

Si una figura ó cuerpo tiene un eje ó plano de **simetría,** es decir (una línea ó plano que lo divida en dos partes equivalentes y semejantes), dicho eje ó plano es una línea ó plano de gravedad. Si una figura ó cuerpo homogéneo tiene un punto central, dicho punto es el centro de gravedad. (*N. del T.* — El texto dice así : « *Si una figura ó cuerpo tiene un punto central, dicho punto es el centro de gravedad* »; pero esto no es cierto sino cuando el cuerpo es homogéneo, por eso hemos agregado al traducir la frase *homogéneo* como condición.)

En la fig. 1, la cuerda representa una línea de gravedad y cualquier plano con el cual coincida la cuerda es un plano de gravedad. Así G puede encontrarse fácilmente, sobre todo en el caso de un cuerpo chato que permita colgarlo de una cuerda atada alternativamente por diferentes ángulos, ó sosteniéndolo en equilibrio en dos ó más posiciones sobre el filo de un cuchillo y buscando á G por medio de la intersección de las líneas ó planos de gravedad encontrados así.

(6) El '**método gráfico** de buscar la resultante de fuerzas paralelas puede usarse con ventaja para buscar el centro de gravedad de un cuerpo ó figura compuesta, ó de un sistema de cuerpos ó figuras cuando son conocidos los centros de gravedad de las diferentes partes.

Así, en la fig. 8, supongamos que $a$, $b$, $c$ representen tres figuras ó cuerpos cuyos centros de gravedad están en un plano. Trácense líneas verticales por dichos centros y fórmese el polígono de las fuerzas $xabc$, fig. 9, haciendo las líneas $xa$, $ab$, etc., proporcionales á los pesos de $a$, $b$, $c$; y trácense de cualquier punto conveniente O, líneas radiales $Ox$, $Oa$, etc. En la fig. 8 trácense $mh$, $mn$, $np$, $pk$, paralelas respec-

* *N. del T.* — No se usa en español esta frase ni la que sigue *línea de gravedad*, sin embargo las adoptamos porque son útiles y abrevian explicaciones.

tivamente á Ox, Oa, Ob, Oc. Entonces una línea vertical *i*G, trazada por la intersección *i*, de *mh* y *pk*, es una línea de gravedad del sistema ó figura. Si el cuerpo ó figura es *simétrico* como la sección transversal en forma de T de un carril ó de una viga en I, el eje de simetría que divide la figura en dos partes iguales y semejantes es también una línea de gravedad, y su intersección con la línea *i*G ya encontrada es el centro de gravedad G que se busca. En dichos casos es casi siempre más conveniente trazar las líneas á través de los diferentes centros de gravedad *perpendiculares* á los ejes de simetría, de modo que la línea de gravedad encontrada sea también perpendicular á ellos.

Pero si, como en la fig. 8, el cuerpo ó figura, etc., no es simétrico, debemos buscar una segunda línea de gravedad cuya intersección con la primera dé el centro de gravedad G. Para hacer esto repítase el procedimiento trazando otro juego de líneas paralelas á través de los diferentes centros de gravedad, fig. 8. Será más conveniente trazarlas horizontalmente ó en *ángulo recto* á las ya trazadas, como suponemos que se hace en las siguientes instrucciones.

Trácese entonces un segundo polígono funicular *m'n'p i'*, fig. 8, haciendo las líneas *m'n'*, etc., *perpendiculares* (en lugar de paralelas) á las líneas radiales Ox, etc., fig. 9; y trácese la segunda línea de gravedad *i'*G perpendicular á la primera. Entonces G está en la intersección de las dos líneas de gravedad.

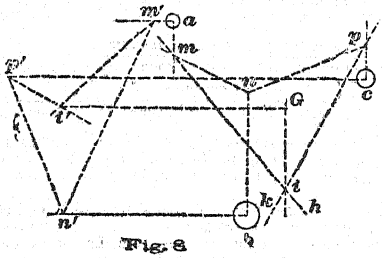


Fig. 8 Fig. 9

El dibujo del segundo polígono funicular es á menudo menos sencillo que el del primero, porque en el segundo las líneas paralelas á través de los diferentes centros de gravedad no se suceden necesariamente las unas á las otras en el mismo orden que en el primero. Debemos tener presente en este caso que las dos líneas (como *n'p'*, *n'm'*) que se encuentran en una línea paralela (como *bn'*), perteneciente á cualquier parte dada *b* de la figura, debe ser perpendicular respectivamente á las líneas radiales (*Oa*, *Ob*) que encuentran los extremos de la línea *ab* que representa esa misma parte.

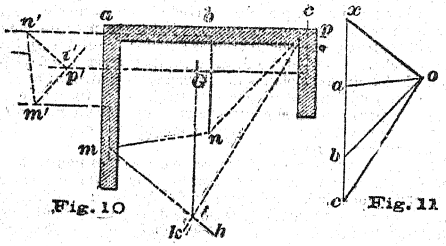


Fig. 10 Fig. 11

Las figs. 10 y 11 demuestran la aplicación del mismo procedimiento á una figura irregular compuesta de tres rectángulos *a*, *b*, *c*. Las letras son las mismas que

en las figs. 8 y 9; pero sucede en la fig. 10 que $i'$ y $p'$ del segundo polígono funicular cae sobre el mismo punto.

Si los centros de gravedad de varios cuerpos ó de varias partes del cuerpo, etc., están en más de un plano, debemos buscar sus proyecciones sobre determinados planos y aplicar el procedimiento á sus proyecciones.

## Casos particulares.

**132. Casos particulares** derivados de las reglas generales (1) á (6).

### Líneas.

(7) **En la línea recta**, G está en la línea y en el medio de su longitud.

(8) **Arco circular** * *aob*, figs. 12 y 13 (centro del círculo en *c*). G está en la línea *co* que une el centro del círculo con el medio del arco, y

$$cG = \text{radio } ac \times \frac{\text{cuerda } ab}{\text{longitud del arco } aob}.$$

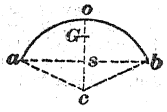


Fig. 12

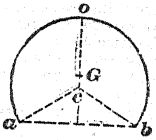


Fig. 13

(8*a*) Si el arco es una **semicircunferencia**, *

$$cG = \text{radio } ac \times \frac{2}{\pi} = \text{radio } ac \times .6366.$$

(8*b*) Reglas aproximadas para encontrar la distancia *s*G, fig. 12, de la cuerda al centro de gravedad.

| | | | | |
|---|---|---|---|---|
| Si la flecha | $so = .01$ | de la cuerda | $ab$; | $sG = .666 \times so$ |
| — | $= .10$ | — | — | $= .665 \times so$ |
| — | $= .15$ | — | — | $= .663 \times so$ |
| — | $= .20$ | — | — | $= .660 \times so$ |
| — | $= .25$ | — | — | $= .657 \times so$ |
| — | $= .30$ | — | — | $= .653 \times so$ |
| — | $= .35$ | — | — | $= .649 \times so$ |
| — | $= .40$ | — | — | $= .645 \times so$ |
| — | $= .45$ | — | — | $= .641 \times so$ |
| — | $= .50$ | — | — | $= .637 \times so$ |

(9) Para un **triángulo** *abc*, fig. 14, el centro de gravedad G de sus tres lados * es el centro del círculo inscrito en un triángulo, cuyos ángulos están en los centros de los lados del triángulo dado.

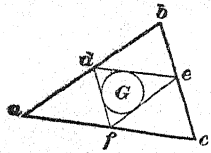


Fig. 14

(10) Para un **paralelogramo** (cuadrado, rectángulo, rombo ó romboide) el centro de gravedad de los cuatro lados * está en la intersección de las diagonales.

* Estamos tratando ahora solamente de líneas, no de las superficies encerradas por ellas. Respecto á superficies, véanse las Reglas (13), etc., etc.

(11) En el caso de un **círculo, elipse** ó **polígono regular**, el centro de gravedad de la circunferencia ó perímetro * es el centro de la figura.

(12) Para un **prisma regular** recto ú oblicuo y **pirámide regular** recta ó un **tronco de pirámide**, el centro de gravedad de sus aristas * es el centro de los ejes. En el *prisma* la posición de G no se afecta porque incluyamos ó no los lados de los dos polígonos que forman las bases.

(12*a*) **Cicloide** *. Véase pág. 204.

## Superficies.

### A. Superficies planas.

Tratemos ahora de los centros de gravedad de las *superficies* planas que pueden ser consideradas como cuerpos chatos infinitamente delgados. Las reglas para las superficies pueden también usarse para cuerpos chatos, no obstante que en éstos el centro de gravedad está en el medio del espesor inmediatamente bajo los puntos encontrados por las reglas establecidas para las superficies.

(13) **Paralelogramos** (cuadrado, rectángulo, rombo ó romboide), **círculo elipse** ó **polígono regular**. G está en el centro de la figura, ó en la intersección de dos diámetros cualesquiera, ó en el centro de cualquier diámetro. En un **paralelogramo**, G está en la intersección de las dos diagonales.

(14) **Triángulo**, fig. 15. G está en la intersección de líneas (como *ae* y *cd*) trazadas desde dos ángulos cualesquiera *a*, *c*, á los centros *e*, *d*, de los lados *bc*, *ab*, respectivamente opuestos á dichos ángulos. Dichas líneas se llaman « líneas medias ».

$$eG = {}^1/_3\, ae;\ dG = {}^1/_3\, cd;\ fG = {}^1/_3\, bf \text{ (siendo } f \text{ el centro de } ac).$$

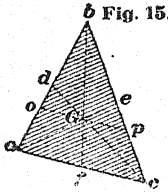


Fig. 15.

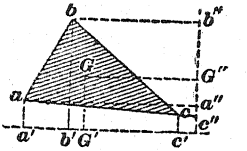


Fig. 16

(14*a*) Fig. 15. O en cualquiera de los dos lados (como *ab*), desde cualquier ángulo *a* hágase $ao = {}^1/_3\ ab$. Trácese *op* paralela al otro lado *ac*. Tómese $oG = {}^1/_2\ op$, y G estará en la intersección de *op* con cualquier línea media como *ae*, etc., etc.

(14*b*) Fig. 16. Si *aa'*, *bb'*, *cc'*, GG' son las distancias de los tres vértices y de G á cualquier línea recta ó plano *a'c'*, se tendrá que :

$$GG' = {}^1/_3\ (aa' + bb' + cc').$$

Esto nos da la posición de la línea de gravedad GG''. De la misma manera buscamos la distancia GG'' de G á cualquiera segunda línea ó plano *b''c''*. Esto nos da la posición de una segunda línea de gravedad GG' G está entonces en la intersección de GG' y GG'.

(14*c*) Fig. 17. La distancia G*n* de G á cualquier lado en cualquiera dirección

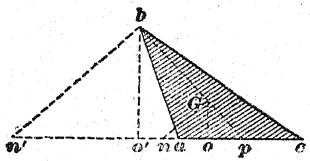


Fig. 17

como *ac* (prolongado si es necesario) es $= {}^1/_3$ de la distancia *n'b* medida en una dirección paralela desde el mismo lado al ángulo opuesto *b*.

* Véase nota página anterior.

Se deduce de esto que la distancia más *corta*, **G***o*, de **G** á cualquier lado (como *ac*) es $= {}^1/_3$ de la distancia más corta *o'b*, del mismo lado á su ángulo opuesto *b*.

Se deduce también que *p***G** es$={}^1/_3$ *pb* como en la Regla (14).

(**15**) **Trapecio ó trapezoide,** fig. 18. Para el trapezoide, véase también la Regla (16). Trácense las dos diagonales *ac*, *bd*. Divídase cualquiera de las dos como *ac*, en dos partes iguales, *am*, *cm*. Del punto *b* en *bd* hágase *bn*=*ds* (ó de *d* hágase *dn*=*bs*). Únanse *mn*. **G** estará en *mn*, y se tendrá

$$m\mathbf{G} = {}^1/_3\ mn.$$ (**G** es el centro de gravedad del triángulo *acn*.)

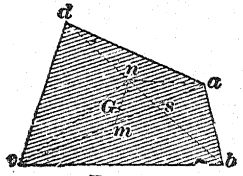


Fig. 18

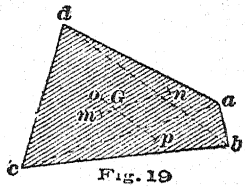


Fig. 19

(**15***a*) Fig. 19. O, de otro modo, búsquese primero los centros de gravedad *m* y *n* de los dos triángulos *cbd* y *abd* en los cuales está dividido el trapecio por una de sus diagonales *bd*. Únase *mn*. Búsquese los centros de gravedad *o* y *p* de los dos triángulos *dac* y *bac*, en los que está dividido el trapecio por su *otra* diagonal *ac*. Únase *op*. Entonces **G** estará en la intersección de *mn* con *op*.

(**16**) **Trapezoide solamente,** fig. 20. **G** está en la línea *ef* que une los centros *e*, *f* de los lados paralelos *ab*, *cd*. Para buscar su posición en dicha línea, prolónguese cualquiera de los lados paralelos, como *ab* en cualquiera dirección, digamos hacia *i*; y hágase *bi* igual al lado opuesto *cd*. Prolónguese entonces dicho lado opuesto *cd* en la dirección opuesta haciendo *dh*=*ab*. Únase *hi*. Entonces **G** estará en la intersección de *hi* con *ef*. Y se tendrá que

$$f\mathbf{G} = \frac{ef}{3} \times \frac{2ab + cd}{ab + cd};\quad \text{ó}\quad o\mathbf{G} = \frac{en}{3} \times \frac{2ab + cd}{ab + cd}.$$

Fig. 20

(**17**) **Polígonos regulares.** **G** es el *centro* de figura del polígono.

(**17***a*) **Polígonos irregulares.** Si el polígono se divide en dos partes cualesquiera, por cualquiera diagonal, **G** debe estar en la línea de gravedad que une los centros de gravedad de aquellas dos partes. Si dividimos otra vez el polígono entero en otras dos partes por otra diagonal, y unimos los centros de gravedad de esas dos partes, **G** estará en la intersección de las dos líneas de gravedad.

(**17***b*) O, de otro modo, podemos dividir el polígono en **triángulos buscando el** centro de gravedad de cada triángulo por medio de las Reglas (14), etc. y buscar entonces el centro **G** por la Regla general (1) (2) ó (6).

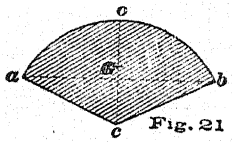


Fig. 21

(**18**) **Sector circular,** *aobc*, fig. 21. (Centro del círculo en *c*.)

$$c\mathbf{G} = \frac{2}{3}\text{radio } ac \times \frac{\text{cuerda } ab}{\text{arco } aob} = \frac{\text{radio}^2 \times \text{cuerda}}{3 \times \text{área}}.$$

Para encontrar la longitud del arco, véanse págs. 186, etc.

(18*a*) Si el sector es un **sextante** (sexta parte del círculo)

$$cG = \text{radio} \times \frac{2}{\pi} = \text{radio} \times .6366.$$

(18*b*) Si el sector es un **cuadrante**, fig. 22,

$$cG = \frac{4}{3}\,\text{radio} \times \frac{\sqrt{2}}{\pi} = \text{radio} \times .6002.$$

$$cx = xG = \frac{4}{3}\,\text{radio} \times \frac{1}{\pi}.$$

(18*c*) Si el sector es un **semicírculo**,

$$cG = \frac{4}{3}\,\text{radio} \times \frac{1}{\pi} = \text{radio} \times .4244$$

$$= \text{(aproximadamente) radio} \times \frac{14}{33}.$$

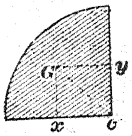


Fig. 22

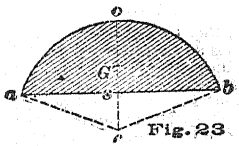


Fig. 23

(19) **Segmento circular**, *aobs*, fig. 23. (Centro de círculo en *c*.)

$$cG = \frac{\text{cubo de la cuerda } ab}{12 \times \text{área del segmento}}.$$

(19*a*) Si el segmento es un **semicírculo**,

$$cG = \frac{4}{3}\,\text{radio} \times \frac{1}{\pi} = \text{radio} \times .4244$$

$$= \text{(aproximadamente) radio} \times \frac{14}{33}.$$

(20) **Cicloide**, fig. 24. (Con el vértice en *v*.)

$$vG = \frac{7}{12}\,vl.$$

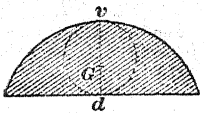
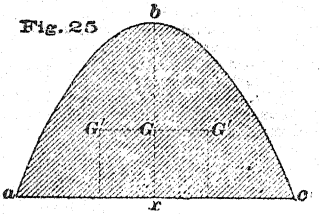


Fig. 24

Fig. 25




(21) **Parábola**, *abc*, fig. 25. *ac* es a base; *ax*, *cx*, ordenadas; y la altura ó eje *bx*, abscisa. Centro de gravedad en **G**, en el eje *xb*, y

$$xG = \frac{2}{5}\,xb.$$

(21*a*) **Semiparábola**, *abx* ó *cbx*. Centro de gravedad en **G'**; y

$$xG = \frac{2}{5}\,xb; \quad GG' = \frac{3}{8}\,ax = \frac{3}{8}\,cx.$$

(22) **Elipse,** *mnop*, fig. 26. El centro de gravedad, *c*, de toda la elipse, está en el centro de la figura.

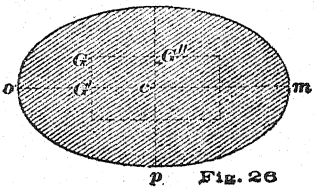



Fig. 26

**G** es el centro de gravedad de la cuarta parte de la elipse, *onc.*
**G'** — — mitad — *nop.*
**G'** — — — — *mno.*

$$cG' = \frac{4}{3}\,oc \times \frac{1}{\pi} = .4244\,oc = \text{(aproximadamente)}\ \frac{14}{33} \times oc.$$

$$cG = G'G = \frac{4}{3}\,cn \times \frac{1}{\pi} = .4244\ cn = \text{(aproximadamente)}\ \frac{14}{33}\,cn.$$

(23) **Para cualquier figura plana,** trácese la figura por escala en cartón grueso. Córtese y equilíbrese el cartón en dos ó más posiciones sobre la orilla de una mesa ó en el filo de un cuchillo; márquese en ella las diferentes posiciones de las líneas de sostén. Donde se intersecten éstas es el centro de gravedad. Se necesita, por supuesto, mucho cuidado para obtener resultados muy exactos por medio de este método. Antes de balancear el cartón, sus bordes superiores deben ser divididos en pequeñas partes iguales. De otra manera sería difícil fijar las posiciones de las líneas de sostén. El papel sobre el cual se prepara la figura debe ser suficientemente rígido para que la figura no se doble cuando se la sostiene en el filo del cuchillo. Véase Regla (5).

## B. Superficies de sólidos *.

(24) En la superficie curva * de la **esfera** ó **esferoide** (ó **elipsoide**), **G** está en el centro de la figura.

(25) En la superficie curva * de cualquier **zona esférica,** ó **segmento esférico, hemisferio,** etc., fig. 27, **G** es el centro del eje ó altura, *ao* †.

En el **hemisferio** $oG = \frac{1}{2}$ radio †.

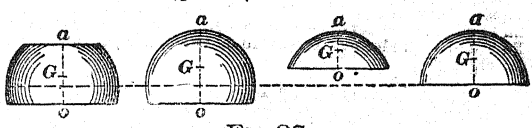



Fig. 27

(26) En el **prisma** recto ú oblicuo cuyos extremos son ó figuras regulares, ó paralelogramos (esto incluye el **cubo** y otros **paralelepípedos**); y en el **cilindro** recto ú oblicuo (circular ó elíptico), el centro **G** de la *superficie* * (incluyendo ó excluyendo ambos de los dos extremos paralelos) es el centro del eje ó línea que une los centros de los dos extremos paralelos.

(27) En la superficie curva *† de un **cono** recto, fig. 28 (circular ó elíptico) ó en las superficies *† inclinadas de **pirámides** rectas regulares, fig. 29, **G** está en el eje *oa* (línea que une el vértice con el centro de la base); y

$$oG = {}^1/_3\ oa.$$

En un cono *oblicuo* ó pirámide, la distancia perpendicular de **G** * contada desde

---

★ Tratamos ahora de las *superficies* de los sólidos y no de sus volúmenes. Para éstos, véanse las Reglas (29), etc.

† Si se han de incluir las **bases**, véanse las Reglas (1) y (2).

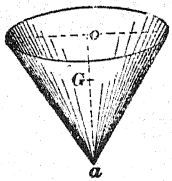



Fig. 28

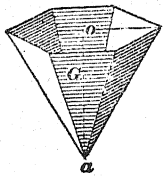



Fig. 29

la base es una tercera parte de la altura perpendicular como en el cono recto y pirámide; *pero no está en los ejes.*

(28) Superficies de troncos de pirámides y conos, de bases paralelas, figs. 30 y 31. En las superficies *+ curvas de los troncos de conos rectos (circulares ó elípticos), ó superficies *+ inclinadas de troncos de pirámides regulares rectas, G está en el eje *oa* (línea que une los centros de las dos bases paralelas), y

$$oG = \frac{1}{3}\,oa \times \frac{\text{circunferencia de } o + 2 \text{ circunferencia de } a}{\text{circunferencia de } o + \text{circunferencia de } a}.$$

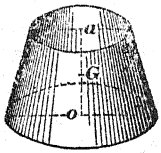



Fig. 30

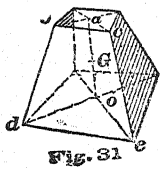



Fig. 31

En el **cono truncado**, fig. 30, podemos usar del *radio* de las dos bases; y en un **tronco de pirámide regular**, fig. 31, cualquier *lado* de las bases (como *be* y *de*) en lugar de las circunferencias.

## Sólidos.

En las Reglas siguientes para determinar el centro de gravedad de los sólidos, el sólido se supone *homógeneo*, es decir, de densidad uniforme en todas sus partes, de modo que el *centro de gravedad* es el *centro de figura.*

(29) En la **esfera** y **esferoide** (ó **elipsoide**), G es el centro del cuerpo.

(30) En el **hemisferio**, fig. 32. (Centro de esfera en *c*.) Altura $cT$ = radio $cb$. G está en el eje $cT$, y

$$cG = \frac{3}{8}\,cT = \frac{3}{8}\,\text{radio } cb.$$

(31) En el **sector esférico**, fig. 33. (Centro de esfera en *c*.)

$$cG = \frac{3}{4}\left(\text{radio } cb - \frac{h}{2}\right).$$

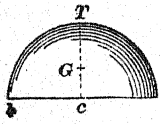



Fig. 32.

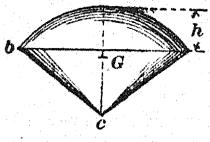



Fig. 33.

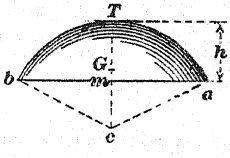



Fig. 34.

*+ Véanse estas notas al pie de la página anterior.

(32) **Segmento esférico** *amb*T, fig. 34. Centro de esfera en *c*. Centro de la base en *m*. Altura del segmento ó flecha $= mT = h$. G está en el eje *m*T, y

$$cG = \frac{3}{4} \times \frac{(2 \text{ radios } cb \text{ de la esfera} - \text{altura}, h)^2}{3 \text{ radios } cb \text{ de la esfera} - \text{altura}, h}$$

$$mG = \frac{\text{altura}, h}{2} \times \frac{2 (\text{radios } mb \text{ de la base})^2 + (\text{altura}, h)^2}{3 (\text{radios } mb \text{ de la base})^2 + (\text{altura}, h)^2}$$

$$= \frac{\text{altura}, h}{4} \times \frac{4 \times \text{radio } cb \text{ de la esfera} - \text{altura}, h}{3 \times \text{radio } cb \text{ de la esfera} - \text{altura}, h}.$$

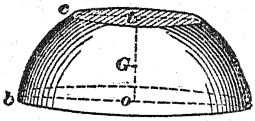



Fig. 35

(33) **Zona esférica**, fig. 35.

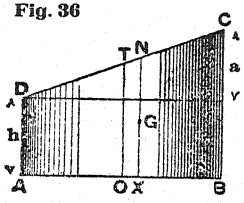
$$oG = \frac{ot}{2} \times \frac{2 (\text{radio } ob \text{ de la base inferior})^2 + 4 (\text{radio } te \text{ de la base sup})^2 + (\text{altura}, ot)^2}{3 \text{ radio } ob \text{ de la base inf})^2 + 3 (\text{radio } te \text{ de la base sup})^2 + (\text{altura}, ot)^2}$$

(34) **Prismas** regulares ó irregulares, rectos ú oblicuos (incluyendo el **cubo** y otros **paralelepípedos**, y **cilindros** circulares ó elípticos, etc., regulares ó irregulares, rectos ú oblicuos. G es el centro de los ejes que une los centros de gravedad de las dos bases.

(34*a*) En un cilindro ó prisma de muy **poca altura**, tal como una plancha de hierro, etc., búsquese el centro de gravedad de su *superficie*. El centro de gravedad del cuerpo está en el medio de su espesor, debajo del punto encontrado.

(35) **Uña de un cilindro**, circular ó elíptico (cuando uno de los ejes de la elipse coincide con el plano secante oblicuo), recto ú oblicuo, figs. 36 y 37.

Fig. 36



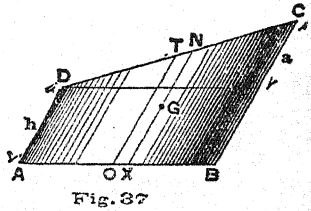




Fig. 37

Sea OT el eje (que une los centros de gravedad de las bases), y XN una línea trazada paralela al eje en el plano ABCD, que pasa por el eje y por los puntos superiores é inferiores C y D del plano secante oblicuo. La posición de G en el plano ABCD se encuentra así :

$$OX = \frac{OB}{4} \times \frac{a}{2h + a};$$

$$XG = \frac{XN}{2} = \frac{1}{4}\left(2h + a + \frac{1}{4}\frac{a^2}{2h + a}\right).$$

(35*a*) Figs. 38 y 39. Si el plano oblicuo CD *encuentra* la base AB, en A, del modo que $h = 0$, mientras CD permanece siendo una elipse ó círculo completo, éstas se transforman en

$$OX = \frac{OB}{4}; \quad XG = \frac{XN}{2} = \frac{5}{16}a.$$

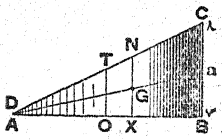


Fig. 38

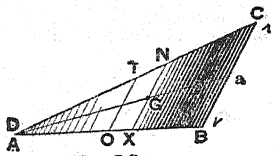


Fig. 39

(36) **Conos**, figs. 40 y 41, circulares, elípticos, etc., rectos ú oblicuos; ó **pirámides**, regulares ó irregulares, rectas ú oblicuas. El centro de gravedad G está en el eje OT, trazado del vértice T al centro de gravedad O de la base, y

$$OG = \frac{OT}{4}.$$

Fig. 40

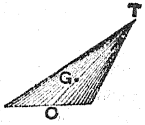


Fig. 41

(37) **Tronco de cono**, figs. 42 y 43, circular ó elíptico, recto ú oblicuo ó **tronco de pirámide**, regular ó irregular, recta ú oblicua; de bases AB, CD paralelas.

Llámese el área de la base grande A, y el de la pequeña *a;* y *h* la altura OZ del tronco, *medido á lo largo de su eje.* Entonces

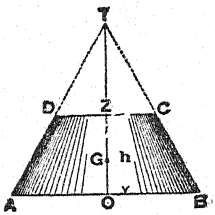


Fig. 42

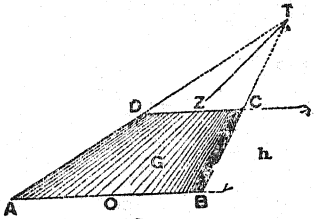


Fig. 43

G está en el eje OZ que une los centros de gravedad O y Z de las dos bases; y su distancia desde la base AB, medida á lo largo del eje,

$$OG = \frac{h}{4} \times \frac{A + 2\sqrt{Aa} + 3a}{A + \sqrt{Aa} + a}.$$

(37*a*) En un *tronco de cono circular*, recto ú oblicuo, de bases paralelas, tenemos:

$$OG = \frac{h}{4} \times \frac{R^2 + 2Rr + 3r^2}{R^2 + Rr + r^2},$$

en que R y *r* son los radios de las bases inferior y superior respectivamente.

(38) Fig. 44 y 45. **Tronco de cono**, ABCD, circular, elíptico, etc., recto ú oblicuo; ó **tronco de pirámide** regular ó irregular, recta ú oblicua, de bases paralelas ó no. Por la Regla (36) búsquese el centro de gravedad N de la pirámide total (ó del cono, según el caso) ABT, cuyo tronco forma la parte inferior; y el centro de gravedad S de la pirámide ó cono menor DCT. Búsquese también el volumen de cada uno; así:

El volumen de una pirámide ó cono $= \frac{\text{área de base} \times \text{altura perpendicular}}{3}$

y

$$\begin{matrix}\text{volumen}\\ \text{del tronco}\\ \text{ABCD}\end{matrix} = \begin{matrix}\text{volumen}\\ \text{de la pirámide entera}\\ \text{ó cono ABT}\end{matrix} - \begin{matrix}\text{volumen}\\ \text{de la pequeña}\\ \text{DCT.}\end{matrix}$$

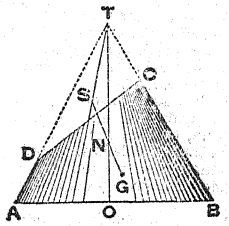


Fig. 44

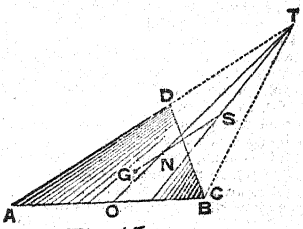


Fig. 45

El centro de gravedad G del tronco ABCD está entonces en la prolongación de la línea SN; y

$$NG = \frac{SN \times \text{volumen de la pequeña pirámide ó cono DCT}}{\text{volumen del tronco ABCD}}$$

(39) **Paraboloide.** G está en el eje y á un tercio de su longitud desde la base.

## LÍNEA DE PRESIÓN
## CENTROS DE FUERZAS Ó DE PRESIÓN

### Posición de la resultante.

**133.** En los §§ 133 á 154 discutiremos la posición de la resultante, ó línea de presión de un sistema de fuerzas paralelas, obrando contra una superficie. Para los cambios en esa posición, dentro de una construcción, debidos á la acción de fuerzas no paralelas, véanse *Arcos*, *Presas*, etc., §§ 251, etc.

**134.** En un sistema de fuerzas paralelas, que obren contra una superficie, la línea de presión es la posición de la resultante de las fuerzas; y el centro de las fuerzas ó centro de presión es el punto en que la línea de presión corta aquella superficie, contra la cual obran las fuerzas.

**135.** Si se toman las longitudes de las líneas que representan las fuerzas, como representando pesos equivalentes, y en una escala dada, la posición de la línea de presión es la que pasa por el centro de gravedad correspondiente á esos pesos. Véase n.° 5, § 131.

**136.** Así pues, en la fig. 55 (*a*), § 117, si las 3 fuerzas *a*, *b*, *c*, se toman como pesos representados en una escala por las flechas *a*, *b* y *c* respectivamente, entonces la resultante R de las 3 fuerzas ocupa la posición de la línea de gravedad de los 3 pesos.

**137.** Digamos más : en una masa de arena, fig. 61*, con una superficie irregular

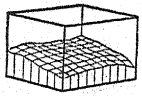


**Fig. 61.**

podemos suponer que la masa está formada por innumerables columnas de arena

* Siguiendo la fig. 60, de las fuerzas paralelas, § 124. Desde la fig. 1 hasta la 45 que tratan del centro de gravedad, están numeradas independientemente del resto de las series de figuras que se refieren á la Estática.

verticales de diferentes alturas y ejerciendo presiones proporcionales á esas alturas. Aquí, también la línea de presión es la línea vertical de gravedad de la masa, y el centro de presión contra la base de la caja es el punto donde dicha línea de presión encuentra á esa base.

**138.** Aunque generalmente consideramos las fuerzas que obran contra superficies, de manera que las líneas que las representan forman un sólido, y no meramente una superficie, sin embargo en la mayoría de los casos que ocurren en ingeniería civil, podemos por conveniencia considerar las fuerzas como concentradas en un solo plano, y por tanto como obrando simplemente contra una línea.

**139.** Así, en el caso de un arco, que ejerce presión contra el arranque, la presión está generalmente distribuída sobre la superficie de apoyo, ó sobre una parte considerable de ella; pero podemos, por conveniencia considerarla como concentrada en un solo plano, situado en el medio y paralelo á las dos caras del arco.

**140.** De manera análoga en el caso de la presión del agua, contra la parte posterior de una presa (ó compuerta, ó contra una pequeña sección de ella, que se extienda desde la superficie del agua hasta el fondo, ó hasta cualquiera profundidad), el agua, por supuesto, ejerce presión sobre la totalidad de la superficie de tal sección; pero podemos, por conveniencia, considerar que la presión está concentrada en un plano vertical normal á la parte posterior de la presa y que la encuentra en el eje vertical de la sección supuesta.

**141.** Acabamos de ver (§§ 138 á 140) que, cuando un sistema de presiones paralelas obra contra una superficie, se puede frecuentemente suponer que obran en un plano contra una sola línea; á saber: la intersección de ese plano con la superficie. También sucede, comúnmente, que tales fuerzas están de tal manera distribuídas á lo largo de esa línea, que las líneas que representan las fuerzas, son ó de longitud igual ó de longitudes que aumentan uniformemente desde un extremo de la línea hasta el otro.

**142.** Así pues, en el caso de que el agua repose sobre una superficie horizontal, fig. 62, la presión está distribuída uniformemente, y el diagrama, fig. (*b*). que

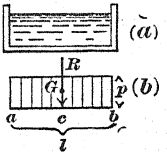



**Fig. 62.**

representa las presiones, es un rectángulo, limitado por una línea horizontal, y su centro de la gravedad G, está en el centro de la figura. De aquí que el centro de presión *c* se halle en el centro de la línea *ab*, en *l*.

Aquí la presión, *p*, es uniforme, y $R = pl$.

**143.** Pero cuando el agua ejerce presión horizontalmente contra una superficie vertical ó inclinada, *ab*, fig. 63, la presión aumenta uniformemente desde *o* en la superficie del agua *b*, hasta un máximo en el fondo *a*; y las presiones horizontales están representadas, en la fig. (*b*), por las ordenadas del triángulo $b'a'd$.

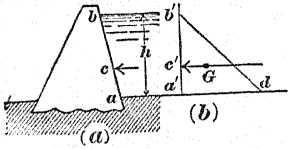



**Fig. 63.**

Puesto que la resultante pasa por el centro de gravedad, G, del triángulo, e centro de presión, *c*, está á tal profundidad, que $ca = {}^{1}/_{3}\,ab$, y $c'a' = {}^{1}/_{3}\,h$. Véase la regla (**14** *c*) del capítulo Centro de Gravedad.

Aquí la presión media horizontal $p$, es la mitad de la presión horizontal máxima en $a$, y la presión horizontal total $= ph$.

**144.** Si de nuevo consideramos sólo las presiones del agua contra cierta *parte*, $ab$, fig. 64, de la superficie posterior de una presa, el diagrama, fig. (*b*), que representa las presiones horiz, se convierte en un trapezoide, compuesto de un paralelogramo

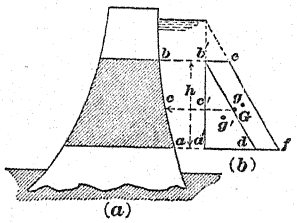



**Fig. 64.**

$b\,f$ y de un triángulo $b\,a'd$, con sus centros de gravedad respectivamente en $g$ y $g'$; y el centro de presión $c$, sobre $ab$, está frente á su centro de gravedad común (centro de gravedad del trapezoide) G. Si $h$ es la profundidad vertical de la porción considerada, entonces

$$a'c' = \frac{h}{3} \times \frac{2b'e + a'f}{b'e + a'f}.$$

Véase Regla 16 de capítulo Centro de Gravedad. Véase también el Centro de presión, en el capítulo de Hidrostática.

## Distribución de la presión.

**145.** A la inversa, si dos superficies, como las de una junta de mampostería, se hallan en un contacto tal que la presión esté regularmente distribuída, ó pueda considerarse así, y si se conoce la posición de la resultante, puede trazarse la figura rectilínea que representa la distribución de la presión, por medio de los principios que acabamos de explicar.

**146.** En las figuras 65 á 68 inclusive, supongamos

$o$ = al centro de la unión $ab$ de las dos superficies;
R = á la presión total = á la resultante de todas las presiones;
$c$ = al punto de aplicación de la resultante R;
$l = ab$ = á la longitud de la unión;
$x = oc$ = á la distancia del centro de presión al centro de la unión;
$y = \frac{l}{2} - x = ac$ = á la distancia del centro de presión al extremo más próximo á la unión;
$p$ = á la presión media $= \frac{\mathrm{R}}{l}$;
$p_a$ = á la presión máxima;
$p'$ = á la presión mínima.

Los §§ 147 á 154 se aplican igualmente si la superficie es horizontal, vertical ó inclinada, y si las fuerzas son normales ó inclinadas respecto á ella.

**147.** Si $x$ no es mayor que $\frac{l}{6}$, ó, en cualquier caso, si la unión puede soportar tensión, tan bien como compresión, tenemos:

$$\text{presión máxima} = p_a = p\left(1 + \frac{6x}{l}\right);$$

$$\text{presión mínima} = p_b = p\left(1 - \frac{6x}{l}\right);$$

Si $x$ excede á $\frac{l}{6}$, y si la unión no puede resistir tensión, véanse §§ 151, 152, 154.

**148. Demostración.** En la fig. 66, en que el paralelogramo $a'd$ representa la presión total R como si estuviera distribuída uniformemente á lo largo de $l$, vemos

que el momento de R, respecto de *o*, que cambia el paralelogramo *a'd* en el trapezoide *a'b'nm*, es equivalente á un par (véase *Pares*, §§ 155, etc.) compuesto de

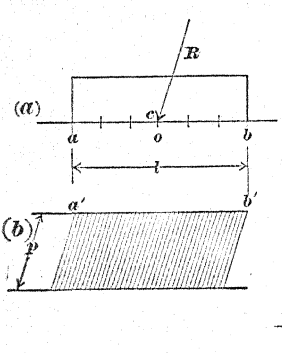



**Fig. 65.**

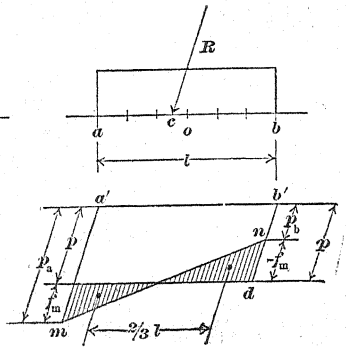



**Fig. 66 (repetida).**

dos fuerzas, á saber: una presión, *f* (no mostrada), distribuída sobre *oa* y representada por el triángulo sombreado á la izquierda, y una tensión, —*f*, ó disminución de presión, distribuída sobre *ob* y representada por el triángulo á la derecha. Las fuerzas, *f* y —*f*, obran por los centros de gravedad de estos dos triángulos, respectivamente; y la distancia de cada uno de estos centros de gravedad al centro, *o*, de la unión, medida paralelamente á la unión, es $= \frac{2}{3} \times \frac{l}{2}$. De aquí que la distancia entre los dos centros de gravedad, medida paralela á la unión, es $= \frac{2l}{3}$. Supongamos que *x* sea la excentricidad, *co*, de R, medida á lo largo de la unión, y supongamos que Aa y Ac (que no figuran) sean las palancas de R y del par, respecto al centro *o*, de la unión. Entonces, puesto que R es paralelo á *f*, y —*f*, Aa á Ac y *x* á *l*, tenemos

$$\text{Aa} : \text{Ac} = x : \frac{2l}{3}.$$

Si R es normal á la unión, entonces: $\text{AR} = x$; y $\text{AC} = \frac{2l}{3}$.

Ahora $$f = \frac{\text{momento de R}}{\text{palanca del par}} = \frac{\text{R} \times \text{Aa}}{\text{Ac}}.$$

Por consiguiente, $$f = \frac{\text{R} \cdot x}{\frac{2l}{3}} = \frac{\text{R}}{l} \times \frac{3x}{2} = p \times \frac{3x}{2}.$$

La presión adicional *media* sobre *oa* (ó tensión media sobre *ob*) es $= \frac{f}{\frac{1}{2}l}$ y la correspondiente presión máxima adicional es

$$f_m = 2\frac{f}{\frac{1}{2}l} = \frac{4}{l} f = \frac{4}{l} p \frac{3x}{2} = p \frac{6x}{l}.$$

Ahora $$p_a = p + f_m = p + p\frac{6x}{l} = p\left(1 + \frac{6x}{l}\right)$$

y $$p_b = p - f_m = p - p\frac{6x}{l} = p\left(1 - \frac{6x}{l}\right).$$

**149.** Si, como sucede en la fig. 65, el centro de presión, *c*, se halla en el centro, *o*, de la superficie, tenemos $x = oc = \text{cero}$, y la presión, R, está uniformemente distribuída sobre la superficie.

**150. « La tercera media ».** Si, como en la fig. 67, $x = \frac{l}{6}$, es decir, si la resultante, R, de todas las fuerzas, corta la superficie en la arista de la tercera media de esa superficie, entonces $p_a = 2\,p$; y $p_b = 0$. Véanse §§ 143 y 148.

**151.** Cuando, como en la fig. 68 (*a*), $x$ excede $\frac{l}{6}$, es decir, cuando el centro de presión, *c*, cae más allá de la tercera media de la superficie de presión, una parte, *sb*, de la superficie, se halla en tensión, siendo la tensión máxima, $p_b$, fig. 68 (*b*), $= p\left(1 - \frac{6x}{l}\right)$ como se ve arriba; la presión máxima $= p\left(1 + \frac{6x}{l}\right)$, y la presión total sobre $as = \frac{p_a \times as}{2} =$ R *más* la tensión en *sb*; pero si, como generalmente sucede en la mampostería, no son capaces las superficies de resistir tensión, la presión total, R, está simplemente concentrada sobre una porción *av*, fig. 68 (*a*), de la superficie siendo $av = 3y$.

Entonces la presión media sobre $av = \frac{R}{av} = \frac{R}{3y}$;

$$p_a = 2\frac{R}{3y} = 2\frac{pl}{3y}.$$

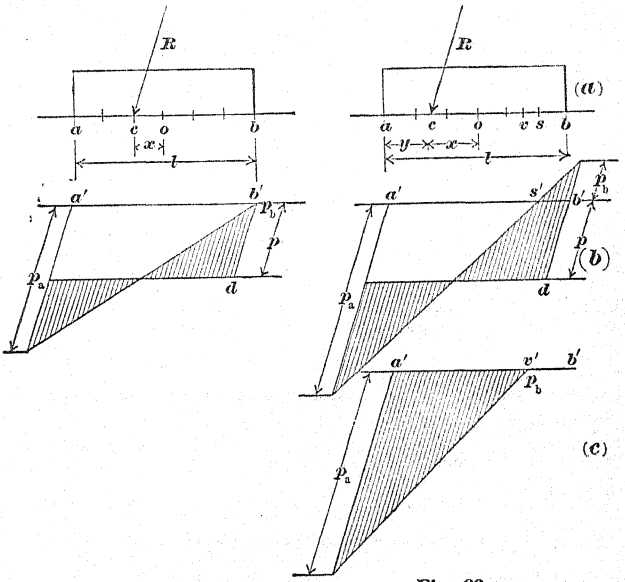


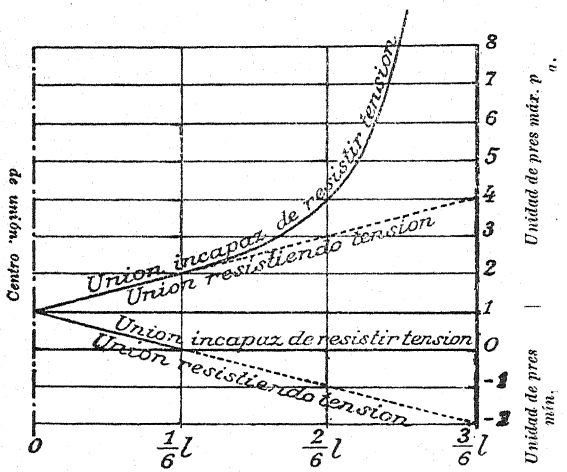

Fig. 67. Fig. 68.

**152.** Por tanto, en una unión que no pueda soportar tensión, fig. 68 (c), $P = 2\frac{R}{3y}$ es el esfuerzo máximo permitido, la distancia *ac*, del centro de presión, al extremo más próximo de la unión, no debe ser menor de $y = \frac{2R}{3p_a}$.

**153.** Si la unión es capaz de resistir tensión, fig. 68 (*b*), sustituímos, en la ecuación, $p_a = p\left(1 + \frac{6x}{l}\right)$, el valor de $x = \frac{l}{2} - y$, y, despejando $y$, tenemos:

$$y = \frac{2}{3}l - \frac{p_a l}{6p}.$$

**154.** Los diagramas de influencias, fig. 69 (véanse §§ 339, etc., y Armaduras §§ 79, etc.), muestran los cambios en las presiones máxima y mínima, $p_a$ y $p$ cuando el centro de presión, *c*, retrocede del centro de la unión *o*. Los diagrama están construídos para una presión media, *p*, de 1. Si las superficies de la unión pue-




*Valores de $x$ = oc = distancia del centro de la unión al centro de presión.*

**Fig. 69.**

den soportar tensión, toda la arista de unión soporta siempre, ó presión ó tensión; y (véanse líneas punteadas fig. 69) la presión máxima, $p_a = p\left(1 + \frac{6x}{l}\right)$, véase § 146, aumenta proporcionalmente con $x$; llegando á ser $= 4p = \frac{4R}{l}$ cuando *c* alcanza el extremo, *a*, de la unión, y cuando $\frac{x}{l} = \frac{1}{2}$. La tensión máxima, *p*, es entonces $= 2\,p = \frac{2R}{l}$. Pero si las superficies no pueden soportar tensión (véanse líneas continuas, fig. 69), el aumento de $p_a$ es proporcional á $x$ sólo mientras $x < \frac{l}{6}$; — es decir, mientras la resultante de todas las presiones cae dentro de la tercera media de la base *ab*. Excedido ese límite, la presión máxima $p_a$, empieza á aumentar más rápidamente que la distancia, *x*, de *c*, al centro *o* de la unión, volviéndose el diagrama una hipérbola rectangular; de manera que, si la resultante pudiera, de hecho, ser aplicada en la arista precisamente de la unión, la presión en aquel punto se volvería infinita.

## PARES

**155. Pares.** Dos fuerzas paralelas iguales, $p$ y $q$, ó $p'$ y $q'$, fig. 70 *, de sentido contrario, se llaman un par. Un par no tiene tendencia á mover el cuerpo†, como un todo en ninguna línea recta. En otras palabras: las dos fuerzas, formando un par, no pueden tener resultante. Su única tendencia es hacer al cuerpo girar sobre su centro de gravedad, G, y en el **plano del par**, es decir, el plano en que están las dos fuerzas. Un cuerpo que tiene un eje fijo puede girar solamente en un plano normal á ese eje.

El verdadero plano de rotación de un cuerpo libre depende de la distribución de la masa en el cuerpo, y no es necesariamente el plano del par.

**156. El momento de un par** es igual al producto de una de las dos fuerzas, $p$ ó $q$, por la distancia, $d$, entre las dos fuezas. Es decir, momento del par $= p.d = q.d$.

**157. Representación gráfica de los pares.** Un par, M ó N, fig. 70, se indica en intensidad, dirección y sentido, por una línea, L ó L' normal al plano del par, colocada de tal manera que, mirando á lo largo de ella hacia aquel plano, el par parece positivo ó que gira de izquierda á derecha, y de tal longitud que represente, en la escala, el momento del par. En la fig. 70, los dos pares M y N son de sentido contrario. De aquí que las líneas L y L', que los representan, estén en direcciones opuestas.

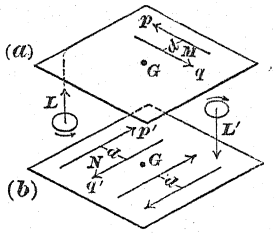



Fig. 70.

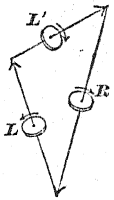



Fig. 71.

**158. Composición de los pares.** Si las líneas L y L', fig. 71, representan dos pares, de acuerdo con el § 157, la línea R, que completa el triángulo, representará de la misma manera su resultante ó antiresultanté. Trazada como está, con su flecha siguiendo las de los otros dos lados, representa su antiresultante. Para indicar su resultante, la flecha de R, y la que indica la dirección de la rotación, deben ser invertidas.

**159. Igualdad de los pares.** Dos pares, M y N, en el mismo plano, fig. 72 ó fig. 73, ó en planos paralelos, fig. 70, son iguales si sus momentos son iguales, sean, ó no, las fuerzas de uno de los pares iguales ó paralelas á las del otro. En la fig. 73, los dos pares, M y N, son de igual sentido; en las figs. 70 y 72, de sentido contrario.

**160.** Puesto que un par no tiene resultante (§ 155), no puede tener antiresultante; es decir, una sola fuerza no puede contrapesar á un par y mantener así

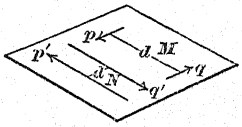



Fig. 72.

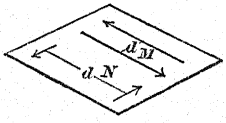



Fig. 73.

el equilibrio. (Pero véase § 168.) Para hacer esto se requiere un par igual y con-

* De las figs. 70 á 75 están dibujadas en perspectiva.
† Véase la nota (*) del párrafo 1.

trario. De este modo, en la fig. 72, se contrapesa el par M por el par igual y contrario N. Si, como en la fig. 72, los dos pares se hallan en un mismo plano, y si encontramos primero la resultante de una de las dos fuerzas no paralelas, como $p$ y $p'$, y luego las de las otras dos, $q$ y $q'$, encontraremos que estas resultantes son iguales y contrarias y que mantienen el equilibrio.

**161. Cualquier par**, como M, fig. 73, **puede ser reemplazado** por cualquier otro par igual, N, en el mismo plano ó en un plano paralelo, y de sentido igual.

**162. Si á una fuerza**, P, fig. 74 ($a$), **agregamos un par**, M, fig. 74 ($b$), en el mismo plano de la fuerza, podríamos reemplazar el par M, con un par igual y equivalente, N, fig. ($c$), compuesto de las fuerzas, — P y P', cada una = P, colocando a — P frente á P, según está indicado. Entonces P y — P se contrapesan, y nos queda sólo P', igual y paralela á P; y, puesto que $Pd=M$, tenemos

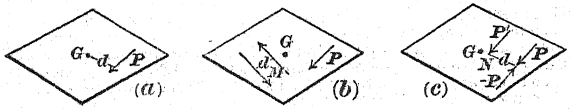



Fig. 74.

$d = \frac{M}{p}$. En otras palabras, el efecto de agregar el par M, fig. ($b$), á la fuerza, P, no es otro sino desviar la acción de P, paralela á sí misma, á una distancia, $d$. Si el par M es izquierdo, como se ve en la figura, P será desviada hacia la derecha (mirando en su propia dirección), y viceversa.

**163.** A la inversa, la fuerza, P, fig. ($c$), es equivalente á la combinación de la fuerza P y del par M, fig. ($b$).

**164.** Por otra parte, teniendo sólo la fuerza P', fig. ($c$), si aplicamos á cierta distancia, $d$, de P', las dos fuerzas opuestas, P y — P, cada una igual y paralela á P', así sustituiremos por P', á la fuerza igual y paralela, P, y un par $=Pd=M$.

**165.** De aquí que también la combinación de la fuerza P y del par M, fig. ($b$), sea equivalente á la combinación de la fuerza P y del par N, fig. ($c$).

**166.** Si el momento del par, M, fig. ($b$), ó N, fig. ($c$), es igual y contrario al momento de la fuerza P respecto al centro de gravedad, G, de un cuerpo, tenemos $d = \frac{M}{P}$ = á la distancia de P á G. En otras palabras, el efecto de tal par es desviar la fuerza P paralelamente á sí misma hasta una línea que pase por el centro de gravedad G.

**167.** Por consiguiente, el efecto de una fuerza, P, fig. ($a$), aplicado á un cuerpo á una distancia, $d$, de su centro de gravedad, G, es equivalente al efecto combinado de una fuerza igual y paralela, P', fig. ($c$), aplicada en el centro de gravedad, y un par (como M, fig. $b$) $=Pd$, y del mismo sentido aplicado á cualquier parte del cuerpo en un plano paralelo á P y P'.

**168.** Se verá que aunque (§ 160) ninguna fuerza por sí sola puede equilibrar á un par, sin embargo, si una fuerza, P, se aplica de tal manera que su momento $Pd$, respecto al centro de gravedad, G, del cuerpo, sea igual y contrario al momento del par, equilibrará la tendencia á rotar, debida al par y la sustituirá con un movimiento de traslación solamente.

**169.** Así, en la fig. 75, donde la fuerza, $p$, obra por el centro de gravedad, G, del cuerpo, supongamos que una fuerza, $-q$, igual y contraria á $q$, sea apli-

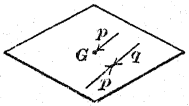



Fig. 75.

cada en la misma línea que aquélla. Se impedirá entonces la rotación y el cuerpo se moverá * bajo la acción de $p$ (=á la resultante de las tres fuerzas), que obra por el centro de gravedad, G, del cuerpo. Se impedirá del mismo modo la rota-

* Véase nota al pie del § 1.

ción si una fuerza *menor* que $q$ es aplicada *más allá* de G, de lo que se encuentra $q$; ó si una fuerza *mayor* que $q$ es aplicada *más cerca* de G, de lo que $q$ se encuentre; siempre que el momento de dicha tercera fuerza respecto á G, sea igual y contrario al del par $pq$. Pero en el primer caso la resultante de las tres fuerzas (que siempre es igual á la tercera fuerza) será menor, y en el segundo caso, mayor que $p$.

**170.** Si á un par se agregara una tercera fuerza, colineal con una de las fuerzas del par, tendríamos el caso de dos fuerzas desiguales y paralelas de sentido contrario. Véase § 112 del capítulo Fuerzas paralelas.

**170*a*. Momentos de los pares.** Las dos fuerzas iguales y paralelas, P, y P', fig. A, en direcciones opuestas, forman un par que gira como las agujas de un reloj = PL = P'L, donde L = á la distancia entre las líneas de acción de las fuerzas. Tomando los momentos de las fuerzas respecto á cualquier punto, digamos A, B, C, ó D, y considerando como *positivos* los momentos de los pares que se mueven como las agujas de un reloj, tenemos :

$$Pm - P'n = P\ (m - n) = P's - Pr = P\ (s - r)$$
$$= Pt + P'u = P\ (t + u) = P \times O + P'L = PL = P'L.$$

En otras palabras, mientras que los momentos, $Pm$, $Pr$, etc., de las *fuerzas son diferentes* para distintos puntos, A, B, etc., el momento $PL = P'L$ *del par* es el *mismo para todos* los puntos.

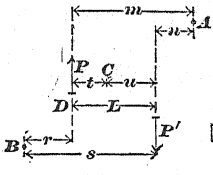



**Fig. A.**

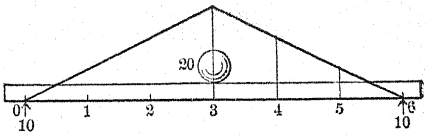



**Fig. B.**

Así pues, supongamos que la viga, fig. B, está dividida por una sección vertica en el número 4. Una fuerza descendente obra sobre la porción de la derecha, en la sección 4 = carga — reacción de la izquierda = 20 — 10 = 10; y en el número 6 obra una fuerza ascendente = reacción de la derecha = 10. Estas fuerzas forman un par de tendencia hacia la izquierda (izquierdo) con momento = 2 × 10 = 20. Obrah sobre la porción de la izquierda, en 0, la reacción de la izquierda = 10; en el número 3, la carga = 20; y, en la sección 4, la reacción de la derecha = 10. Combinando las dos reacciones, encontramos su resultante, = 10 + 10 = 20, en el número 2 (á medio camino entre 0 y 4), formando, con la carga, 20, un par de tendencia hacia la derecha, con momento = 1 × 20 = 20.

En la sección 5, por un procedimiento análogo, encontraremos pares con momentos = 10, y en la sección 3, pares con momentos = 30. Así pues, aunque el momento de un par es el mismo, cualquiera que sea el punto donde sean tomados los momentos, sin embargo, en una viga bajo cierto sistema de fuerzas arreglado de cierta manera, el momento varía de un punto á otro, porque en ese caso los pares son diferentes.

Como se ve, la tendencia rotatoria, en cualquiera sección, es causada por dos pares iguales y contrarios que obran, respectivamente, sobre los dos segmentos; resulta que la tendencia á rotar está contrariada por los pares iguales y opuestos debidos á las fuerzas internas del material, y que obran en la sección, sobre los dos segmentos respectivamente.

## ROZAMIENTO*

**171.** Cuando un cuerpo en bruto descansa sobre otro, las rugosidades y depresiones que forman **las asperezas** de sus superficies en contacto, **se penetran** más ó menos, y para resbalar el uno sobre el otro, tenemos que consumir parte

* « Rozamiento ». Esta palabra para significar el roce ó fricción de una superficie con otra envuelve un error porque implica que debe tener lugar el movimiento necesario al roce ó fricción mientras que encontramo esta resistencia, no solo durante el movimiento de los *cuerpos*, sino tambien antes de que el movimiento tenga lugar. « Resistencias debidas á las asperezas » expresaria mejor el fenómeno.

de la fuerza de rozamiento : ya en *separar* los cuerpos (tal como suspender el de encima) lo suficiente para salvar las asperezas, ya, para destruir algunas, ya para salvar otras.

**172.** La superficie plana mejor pulida como la *xy*, fig. 76, *no* es (como aparece á la vista) un *plano;* es de hecho una superficie dentada, como puede verse con un microscopio suficientemente poderoso, de manera que la fuerza *ab* en lugar de formar el ángulo aparente *abx* con una superficie lisa, *xy*, en realidad se descompone en una serie de fuerzas paralelas, *c*, *d*, *e*, que forman otros ángulos con las superficies, *mm*, *nn*, etc., de aplicación, é inclinadas éstas (á menudo en diversas direcciones) con la superficie general ó media *xy*, como está indicado. Entre estas superficies pueden haber algunas como *mm*, en ángulo recto con la fuerza aplicada, y la fuerza *e* obrará sobre ella en su dirección original, aunque esté aplicada *oblicuamente* á la *aparente* superficie, *xy*. En el caso de dos fuerzas *d* y *e*, aplicadas á las superficies *nn* y *ss*, si la tendencia al resbalamiento es igual en las dos superficies y obran *opuestamente* entre sí, la resistencia *combinada* de las dos superficies *nn*, *ss*, es directamente opuesta á las fuerzas, y equivaldrá á la de una sola superficie en ángulo recto con las fuerzas.

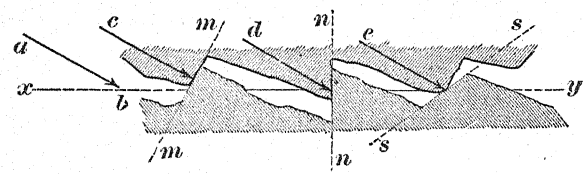



**Fig. 76.**

**173.** Por supuesto que no se trata de investigar la resistencia exacta, en un caso dado, de cada una de esas microscópicas proyecciones. En vez de esto, se encuentra experimentalmente las resistencias *combinadas* que todas las proyecciones presentan al resbalamiento, y á aquella resistencia se llama **rozamiento.**

**174.** El rozamiento siempre tiende á *impedir el movimiento relativo de dos cuerpos entre los cuales obra*, esto es, el movimiento de uno de los cuerpos con relación al otro, no obstante, tiende igualmente á *producir* movimiento * relativo entre cada uno de ellos y *un tercer cuerpo ó cuerpo exterior*. Así, el rozamiento entre una correa y la polea *movida por ella*, tiende á impedir el deslizamiento entre ellas; pero tiende así á hacer que ruede la correa sobre la polea en *movimiento* y pone en movimiento á ésta y á su árbol, *con relación al cojinete* ó *chumacera* en la cual gira el árbol. Este movimiento es contrariado por el rozamiento entre el eje y el *cojinete*, y este frotamiento, á su vez, tiende igualmente á hacer que el *cojinete* gire con el árbol por reacción y hace que la correa se deslice en la polea giratoria.

**175.** El frotamiento entre dos cuerpos que se hallan *en reposo* relativo el uno respecto al otro, se llama **rozamiento estático** ó rozamiento de reposo. El que se ejerce entre dos cuerpos que se encuentran *en movimiento relativo*, se denomina **rozamiento cinemático.**

**176. El rozamiento estático máximo** que se ejerce entre dos cuerpos U y L, fig. 77 (ó la mayor resistencia producida por el rozamiento á una fuerza cualquiera de resbalamiento estando en reposo), es igual á una fuerza (tal como el peso F) que está á punto de hacer que U comience á resbalar sobre L **. De manera que el rozamiento, como las otras fuerzas, puede expresarse por *pesos*, como libras ó kilogramos.

**177.** Una resistencia no puede exceder á la fuerza á la cual se opone ó resiste ***. Por consiguiente, si F es menor que el rozamiento estático máximo existente entre U y L, *la resistencia al rozamiento* realmente ejercida por ellos es también menor.

Cuando F es = al rozamiento máximo (y U está, por tanto, á punto de deslizarse),

---

* Véase nota (*) al pie del § 1.

** Despreciamos aquí el rozamiento de la cuerda y la polea y suponemos que toda la fuerza del peso F se transmite á U por medio de la cuerda.

*** Si una *fuerza* de resistencia excede á la fuerza á que se opone, el exceso ya no es resistencia, sino fuerza motriz.

a resistencia real es=al rozamiento estático. Si F excede a rozamiento estático máximo, el excedente comunica movimiento á U.

**178.** Si á un cuerpo en movimiento se le suprimen ó equilibran en un momento dado todas las fuerzas extrañas ó las resistencias que actúan sobre él, dicho cuerpo se moverá con velocidad uniforme. Por consiguiente, si la fuerza F, fig. 77, es justamente igual al rozamiento cinemático máximo desarrollado entre U y L, la velocidad de ellos será uniforme. Si F excede á aquél, el exceso *acelerará* la velocidad. Si el rozamiento cinemático máximo excede á F, el exceso *retardará* la velocidad. Así pues, la **resistencia real debida al rozamiento** que se ejerce entre dos cuerpos *en movimiento relativo* es=**á su rozamiento cinemático máximo**=á una fuerza (tal como F) que pueda conservarles justamente su velocidad uniforme relativa.

**179.** Por consiguiente, si la superficie horizontal S, sobre la cual descansa L, pudiera hacerse completamente lisa, es decir, sin que produzca rozamiento la presión de L contra el tope *m* (la cual siempre sería entonces igual á la resistencia *real* del rozamiento desarrollada entre U y L), sería también igual á su rozamiento *máximo*, mientras U continúe moviéndose sobre L, y podría ser, por tanto, mayor, menor ó igual á F; pero, cuando U se hallaba en reposo, la presión contra *m* era igual á F y menor (ó á lo sumo=) al rozamiento máximo.

## Coeficiente de rozamiento ó de fricción.

**180.** Desde luego que ninguna superficie puede hacerse *absolutamente lisa*, alguna separación de los dos cuerpos debe en todo caso tener lugar, á fin de evitar las asperezas que existen. Por consiguiente, el rozamiento se produce siempre más ó menos, según la intensidad de la presión perpendicular que tiende á juntar las superficies.

**181.** La *razón* entre el rozamiento máximo, en un caso dado, y la presión perpendicular, se llama **coeficiente de rozamiento** para aquel caso ó

$$\text{Coeficiente de rozamiento} = \frac{\text{rozamiento máximo}}{\text{presión perpendicular}}$$

y rozamiento máximo=presión perpendicular × coeficiente de rozamiento.

Así, si una fuerza F, fig. 77, de 10 kg justamente equilibra el frotamiento máximo desarrollado entre U y L; y si el peso de U (que es la presión perpendicular en este caso, puesto que la superficie entre U y L es *horizontal*) es de 50 kg, el coeficiente de rozamiento entre U y L es entonces $= \frac{10}{50} = .2$.

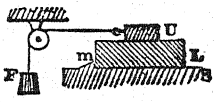



Fig. 77.

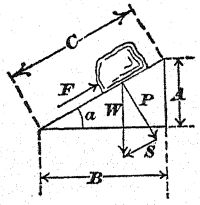



Fig. 78.

**182. El coeficiente se expresa generalmente** en decimales ó por una fracción ordinaria; pero, algunas veces, como en el caso de carros de vías férreas y máquinas, en kg por toneladas (de presión perpendicular), ó por medio del « ángulo de rozamiento ó fricción », en grados y minutos.

**183. Ángulo de rozamiento.** En la fig. 78, sea W=al peso del cuerpo, P, su presión normal sobre el plano, y S=la componente que tiende á hacer resbalar el cuerpo en el plano y hacia abajo. Cuando el ángulo $a$ es tal, que el cuerpo está á punto de resbalar, se llama ángulo de rozamiento ó de fricción. La fricción F y la fuerza S son entonces iguales.

Pero $\frac{S}{P} = \frac{F}{P} = \frac{A}{B}$ = coeficiente de fricción ó rozamiento = tang $a$, y por tanto, $= P \text{ tang } a = W \cos a . \text{ tang } a$.

**184.** Sea R, fig. 79, la resultante de todas las fuerzas que comprimen un cuerpo contra un plano, y N una normal al plano. Si el ángulo $i$ entre R y N excede al ángulo de rozamiento ($a$, fig. 78) entre las dos superficies en contacto, el cuerpo resbalará. Si $i$ no excede á dicho ángulo, la resultante entera R se aplicará al plano y en su propia dirección, y no solamente su componente normal V, como sucedería en el caso en que no hubiera fricción entre las superficies.

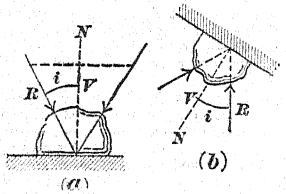



Fig. 79.

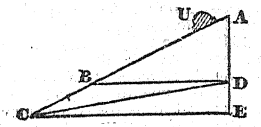



Fig. 80.

**185. Para buscar el coeficiente de rozamiento cinemático,** deslícese uno de los cuerpos U, fig. 80, hacia abajo por un plano inclinado AC, formado del otro cuerpo y que tenga una inclinación conocida ACE mayor que el ángulo de rozamiento (§ 183). Anótese la distancia vertical AE de donde desciende U al deslizarse por AC (AE = AC × sen ACE); también su velocidad real de resbalamiento al llegar á C. Calcúlese la distancia *vertical* AD de la cual tendría que descender *á lo largo del plano* (de A á B) para adquirir aquella velocidad *si no hubiera rozamiento.*

$$\left(\text{AD} = \frac{\text{velocidad}^2}{\text{el doble de la aceleración de la gravedad } g\,^*}\right).$$

Búsquese DE (= AE − AD) y la distancia horizontal EC correspondiente á AC (EC = AC × cos ACE = $\sqrt{\text{AC}^2 - \text{AE}^2}$). Y tenemos entonces :

$$\text{Coeficiente de rozamiento medio al deslizarse de A á C} = \frac{\text{DE}}{\text{EC}},$$

porque si se representa por AE la fuerza *total* de resbalamiento consumida (en mover á U de A á C y vencer el frotamiento), entonces AD representará la parte de AE gastada en *velocidad*, y DE la gastada en *rozamiento*, y como CE representa la presión perpendicular (§ 183),

$$\frac{\text{DE}}{\text{EC}} = \frac{\text{rozamiento}}{\text{presión perpendicular}} = \text{coeficiente.}$$

**186.** O búsquese el seno y la tangente de ACE; y la distancia AC (= tiempo² en segs × ½ $g$ * × sen ACE) que *recorrería* U en un tiempo dado *si no hubiera fricción.* Mídase la dist AB, sobre la que *realmente resbala* en ese tiempo; y encuéntrese BC = AC − AB. Entonces :

$$\left.\begin{matrix}\text{coef del rozamiento medio,}\\ \text{resbalando de A á B}\end{matrix}\right\} = \text{tang DCE} = \text{tang ACE} \times \frac{\text{BC}}{\text{AC}}$$

porque

$$\text{(Primero) AC : AB : BC} = \text{AE : AD : DE}$$

$$\left.\begin{matrix}\text{la velocidad teórica debida á}\\ \text{= la fuerza total de resbalamiento}\end{matrix}\right. : \text{la velocidad real} : \left.\begin{matrix}\text{al retardo por el rozamiento =}\\ \text{a fuerza total de resbal}\end{matrix}\right. : \left.\begin{matrix}\text{la fuerza de resbalamiento em-}\\ \text{pleada en comunicar la velo-}\\ \text{cidad real.}\end{matrix}\right. : \left.\begin{matrix}\text{al rozamiento ó la fuerza de resba-}\\ \text{lamiento requerida para equilibrar}\\ \text{el rozamiento.}\end{matrix}\right.$$

Y si AE es = la fuerza total de resbalamiento, entonces EC es = la presión perpendicular, y

$$\frac{\text{DE}}{\text{EC}} = \text{el coef de rozamiento} = \text{tang DCE.}$$

* $g$ = 32.2 pies = 9 m 81.

(Segundo.) Debido á la semejanza de los dos triángulos ABD; ACE tenemos

$$AC : BC = AE : DE = \frac{AE}{EC} : \frac{DE}{EC} = \text{tang ACE} : \text{tang DCE}.$$

**187.** De 1831 á 1834, el **general Arturo Morin** * hizo experimentos con presiones que no excedían de 30 libs por pulg cuad (2.109 kg por cm cuad) y llegó á las conclusiones siguientes respecto al rozamiento debido al resbalamiento, donde la presión perpendicular es considerablemente menor que la que sería necesaria para gastar apreciablemente las superficies. Éstas fueron generalmente y por largo tiempo consideradas como **tres leyes fundamentales del frotamiento.**

**1.º** El rozamiento máximo entre dos cuerpos es proporcional á la fuerza *total* y normal que los comprime. Por consiguiente :

**2.º** Para cualquier presión *total* perpendicular dada, **el coeficiente es independiente del área de las superficies en contacto.**

Si sobre un apoyo horizontal colocamos un ladrillo que mida (20×10×5) centímetros, y lo colocamos primero sobre su canto (20×5 cm), y luego sobre su cara (20×10 cm), es claro que ahora doblamos el área de contacto, sin que haya variado la presión total (=al peso del ladrillo); pero sí *hemos reducido la presión por cm cuad* á la mitad, puesto que ahora á doble superficie le toca la misma presión. Por consiguiente (permaneciendo el coeficiente de rozamiento prácticamente el mismo), tenemos sólo la mitad del *rozamiento por cm cuad.* Pero tenemos doble número de cm cuad de contacto, y por tanto el mismo rozamiento *total.*

Ahora, si aumentamos ó disminuímos el área de contacto sin variar la presión por cm cuad, la presión total variará por supuesto como el área, y el rozamiento total variará en la misma proporción, porque el coeficiente permanece el mismo. Así, si colocamos dos hojas de papel iguales entre las hojas de un libro (teniendo cuidado de no colocarlas entre las mismas dos hojas) y luego apretamos el libro en una prensa de copiar, necesitaremos como el doble de la fuerza para sacar las dos hojas de la que necesitaríamos para sacar una sola de ellas.

**3.º** Aun cuando el coeficiente del rozamiento *estático* entre dos cuerpos es á menudo mucho mayor que su coeficiente de rozamiento cinemático, sin embargo **el coeficiente de rozamiento cinemático es independiente de la velocidad.**

Esto se aplica también (aproximadamente) al *rozamiento,* y por consiguiente a *trabajo* (en kg por m) que representa el rozamiento vencido en una *distancia* dada, porque entonces el trabajo (=resistencia × distancia) es independiente de la velocidad. Pero cuando se trata de *tiempo dado,* como la distancia varía con la velocidad, es claro que el trabajo también varía como la velocidad.

**188 (a).** Ciertas especies de superficies parece que adhieren mucho mejor sus asperezas cuando están en reposo relativo entre sí, que cuando están en movimiento, aunque éste sea muy lento; y en algunos casos el grado de adherencia parece aumentar con la duración del contacto. De aquí que á menudo hay gran diferencia de intensidad entre el rozamiento en reposo y el rozamiento en movimiento; así el general Morin encontró que en el roble sobre roble, con las fibras en ángulos rectos, la resistencia al resbalamiento estando todavía en reposo y después de haber estado por « algún tiempo en contacto », resultó como un octavo mayor que cuando los pedazos tenían una velocidad relativa de .30 m á 1.50 m por seg.

**(b)** Pero la experiencia demuestra que hasta un choque muy leve es suficiente para eliminar esta diferencia; y como toda construcción hasta las más pesadas están sujetas á choques eventuales (como un puente cercano á un edificio, ó una colina durante el paso de un tren; ó una gran fábrica por el movimiento de sus máquinas, y en numerosos casos por la acción del viento), es conveniente no contar con el rozamiento para la *estabilidad* de las construcciones, más allá de lo que valga su coeficiente en el caso de *movimiento* de los cuerpos entre sí. Cuando deba considerarse como una resistencia, que debemos vencer con fuerzas, debe considerársele bastante *mayor* que la que da la tabla.

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* Véanse sus « Fundamental Ideas of Mechanics », traducidas por José Bennet D. Appleton y C.ª, Nueva York, 1860.

**Tabla de frotamiento con movimiento de superficies planas, perfectamente lisas, limpias y secas,** las de Morin principalmente.

| Materias con las cuales se ha experimentado. | Coeficiente de rozamiento ó relación del rozamiento á la presión. | Ángulo de rozamiento. Grad. | Min. |
|---|---|---|---|
| Roble sobre roble; todas las fibras paralelas al movimiento | .48 | 25 | 38 |
| — — — fibras en movimiento en ángulos rectos á las otras y al movimiento | .32 | 17 | 45 |
| — — — todas las fibras en ángulos rectos al movimiento | .34 | 18 | 47 |
| — — — fibras en movimiento en el extremo; fibras en reposo paralelas al movimiento | .19 | 10 | 46 |
| — — hierro fundido, fibras en ángulos rectos al movimiento | .37 | 20 | 19 |
| Olmo — roble, fibras todas paralelas al movimiento | .43 | 23 | 17 |
| Roble — olmo — — — — | .25 | 14 | 3 |
| Olmo — roble — en movimiento en ángulo recto á las otras | .45 | 24 | 16 |
| Fresno — — — todas paralelas al movimiento | .40 | 21 | 49 |
| Abeto — — — — — | .36 | 19 | 48 |
| Haya — — — — — | .36 | 19 | 48 |
| Hierro forjado sobre roble, fibras paralelas al movimiento | .62 | 31 | 47 |
| — — — olmo, — — — | .25 | 14 | 3 |
| — — — hierro, fundido — — | .19 | 10 | 46 |
| — — — — forjado — — | .14 | 7 | 58 |
| — — — latón | .17 | 9 | 39 |
| — — — piedra caliza blanda, bien preparada | .49 | 26 | 6 |
| — — — — dura — | .24 | 13 | 30 |
| — — — — — — y húmeda. | .30 | 16 | 42 |
| — — ó acero sobre mármol duro aserrado. (Por el autor) como | .17 | 9 | 39 |
| — — — sobre caoba lisa y acepillada; fibras paralelas al movimiento | .18 | 10 | 12 |
| — — — — pino blanco aplanado y liso | .16 | 9 | 6 |
| — fundido — roble, fibras paralelas al movimiento | .49 | 26 | 6 |
| — — — olmo — — — | .20 | 11 | 19 |
| — — — hierro fundido | .15 | 8 | 32 |
| — — — latón | .15 | 8 | 32 |
| Acero sobre hierro fundido | .20 | 11 | 19 |
| — — acero. (Por el autor) | .14 | 7 | 59 |
| — — latón | .15 | 8 | 32 |
| — — vidrio pulido. (Por el autor) | .11 | 6 | 17 |
| — perfectamente liso, pero no pulido, sobre pino perfectamente seco y aplanado; fibras paralelas al movimiento como | .16 | 9 | 6 |
| — perfectamente liso, pero no pulido, sobre caoba perfectamente seca, aplanada y acepillada; fibras paralelas al movimiento como | .18 | 10 | 1 |
| Cobre amarillo sobre hierro fundido | .19 | 10 | 46 |
| — — — roble | .62 | 31 | 48 |
| Latón sobre hierro fundido | .22 | 12 | 25 |
| — — hierro forjado; fibras paralelas al movimiento | .16 | 9 | 6 |
| — — latón | .20 | 11 | 19 |
| — — pino blanco perfectamente seco y aplanado; fibras paralelas al movimiento como | .19 | 10 | 46 |
| — — caoba perfectamente seca, aplanada y pulida; fibras paralelas al movimiento como | .24 | 13 | 30 |
| Mármol bruñido sobre mármol bruñido. (Por el autor). tér. me. | .16 | 9 | 6 |
| — — ladrillo común | .44 | 23 | 45 |
| Ladrillo común sobre ladrillo | .64 | 32 | 38 |
| Piedra caliza blanda, bien preparada sobre la misma | .64 | 32 | 38 |
| Ladrillo común sobre piedra caliza blanda bien preparada | .65 | 33 | 2 |
| — — dura — | .60 | 31 | 00 |
| Roble á través de la fibra sobre piedra caliza blanda, bien preparada | .38 | 20 | 48 |
| Roble á través de la fibra sobre piedra caliza dura bien preparada | .38 | 20 | 48 |

**Tabla de frotamiento con movimiento de superficies planas, perfectamente lisas, limpias y secas,** las de Morin principalmente. (Continuación.)

| Materias con las cuales se ha experimentado. | Coeficiente de rozamiento ó relación del rozamiento á la presión. | Ángulo de rozamiento |
|---|---|---|
| | | Grad. Min |
| Piedra caliza dura sobre piedra caliza dura, ambas bien preparadas | .38 | 20 48 |
| Piedra caliza dura sobre piedra caliza blanda, ambas bien preparadas | .67 | 33 50 |
| Piedra caliza blanda sobre piedra caliza dura, ambas bien preparadas | .65 | 33 2 |
| Madera sobre metal generalmente de .2 á .62...término medio | .41 | 22 18 |
| — *muy lisa*. sobre metal, generalmente de .25 á .50 término medio | .38 | 20 48 |
| Metal sobre metal *muy liso*, seco generalmente, de .15 á .22 término medio | .18 | 10 12 |
| Mampostería ordinaria y mampostería de ladrillos secos, generalmente de .6 á .7 término medio | .65 | 33 2 |
| Mampostería ordinaria y mampostería de ladrillos con la mezcla húmeda...aproximadamente | .47 | 25 30 |
| Mampostería ordinaria con mampostería de ladrillos con mortero un poco húmedo...aproximadamente | .74 | 36 30 |
| Mampostería sobre arcilla seca... — | .51 | 27 00 |
| — — húmeda... — | .33 | 18 15 |
| Mármol aserrado, sobre el mismo, ambos secos. (Por el autor)*...término medio | .4 | 21 9 |
| Mármol aserrado, sobre el mismo, ambos húmedos (Por el autor)*...término medio | .55 | 28 59 |
| Mármol aserrado, sobre pino blanco, perfectamente seco y aplanado (Por el autor)*...término medio | .45 | 24 14 |
| Mármol aserrado, sobre pino blanco, plano y húmedo. (Por el autor.)*...término medio | .6 | 31 00 |
| Mármol bruñido sobre pino blanco, perfectamente seco y aplanado. (Por el autor)...término medio | .26 | 14 35 |
| Pino blanco, perfectamente seco; aplanado; sobre el mismo; todas las fibras paralelas al movimiento...más ó menos | .4 | 21 48 |
| Pino blanco húmedo, aplanado; sobre el mismo... — | .6 | 31 00 |

**189.** Experiencias recientes, con mucho mayores variaciones de presión y velocidad y con aparatos más delicados para apreciar ligeros cambios en el coeficiente, y, aun cuando han dado resultados contradictorios**, demuestran que **las tres leyes en el § 187**, están lejos de ser exactas para superficies que se mueven con grandes velocidades y bajo grandes presiones y que aquéllas **son aproximadamente exactas sólo** para velocidades y presiones ordinarias; pues se ha encontrado que el coeficiente varía con la intensidad de la presión, con la velocidad y también con la temperatura *. Pero en los casos más frecuentes, para el ingeniero,

* Pero después de algunos ensayos las superficies se hacen más suaves hasta reducirse los ángulos de 2° á 5°; los bloques destinados á deslizar tienen peso de 13 kg cada uno, más ó menos.

** Esto no es extraño en vista del grado en que está afectado el coeficiente por la naturaleza de la superficie. Si la forma de las pequeñas asperezas es tal que se penetran entre sí tan bien bajo pequeñas como bajo grandes presiones, y si son demasiado fuertes para que las destruya la presión aplicada, el coeficiente, como se establece en la primera ley, debe ser independiente de la presión. Pero si una presión comprime las asperezas de un cuerpo más estrechamente contra las del otro, el coeficiente debe aumentar bajo semejante presión. Así como si la presión superior destruye las asperezas, mientras que la inferior no puede hacerlo, el coeficiente debe disminuir bajo la presión superior. Las partículas destruídas pueden obrar ó como un lubricante y entonces *reducir* todavía más el rozamiento y su coeficiente; pero si resultan angulosas y duras, lo tienen que *aumentar*. El cambio del área en contacto bajo una presión dada, como produce cambios en el valor de la presión por unidad de superficie, puede producir cambios en el coeficiente, semejantes á los que acabamos de mencionar.

En grandes velocidades las asperezas no tienen tiempo de penetrarse tan bien como en

algunas pequeñas diferencias en la disposición de las superficies, ó en la humedad de aire, causan á menudo mayores variaciones en los coeficientes que las debidas á cualquier cambio probable en la presión, velocidad ó temperatura: de modo que entre los límites del mayor ó menor pulimento podemos generalmente tomar las reglas de Morin como suficientemente exactas.

**190. El profesor A. S. Kimbal,** del Instituto de Ciencias Industriales de Worcester (Massachusets), ha hecho algunos experimentos muy minuciosos respecto del rozamiento entre superficies de madera de pino *. Damos los resultados en la fig. 3, para demostrar simplemente, cómo varía el coeficiente con la velocidad y presión. Nuestra tabla da un coeficiente de .4 para pino sobre pino.

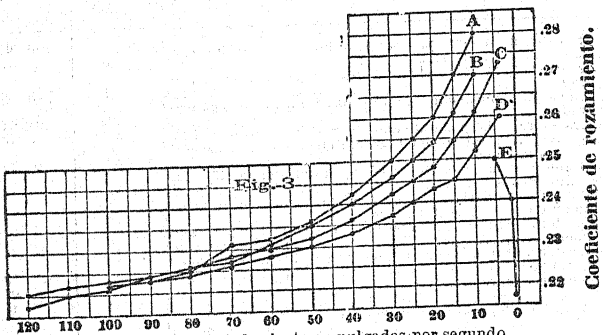

Fig. 3.

La línea A muestra los coeficientes á diferentes velocidades bajo una presión de 1.58 libs por pulg cuad (.11109 kg por cent cuad).

La línea B muestra los coeficientes á diferentes velocidades bajo una presión de 1.59 libs por pulg cuad (.11179 kg por cent cuad).

La línea C muestra los coeficientes á diferentes velocidades bajo una presión de 1.60 libs por pulg cuad (.11249 kg por cent cuad).

La línea D muestra los coeficientes á diferentes velocidades bajo una presión de 1.61 libs por pulg cuad (.11320 kg por cent cuad).

La línea E muestra los coeficientes á diferentes velocidades bajo una presión de 4.17 libs por pulg cuad (.29319 kg por cent cuad).

Se habrá visto que á baja velocidad el coeficiente disminuye cuando se aumenta casi imperceptiblemente la presión por unidad de superficie; pero esta diferencia desapareció á proporción que aumentó la velocidad. A velocidades de .10 á 3 m por segundo el coeficiente disminuye generalmente á proporción que aumenta la velocidad; rápidamente al principio, pero más despacio luego. Esto está de acuerdo con otros experimentos recientes. Pero á muy baja velocidad 2 á 125 mm por seg. El profesor Kimball encontró que el coeficiente (línea E) *aumenta muy rápidamente con la velocidad.*

Hemos hecho grande la escala de coeficientes con el fin de demostrar sus variaciones, la cuales son tan leves que de otro modo serían apenas perceptibles. Experimentos menos precisos no habría sido posible apreciarlos en el diseño.

**191 (a).** En 1878 el **capitán Douglas Galton** y el **Sr. George Westinghouse, hijo,** hicieron cuidadosos experimentos en Inglaterra para determinar

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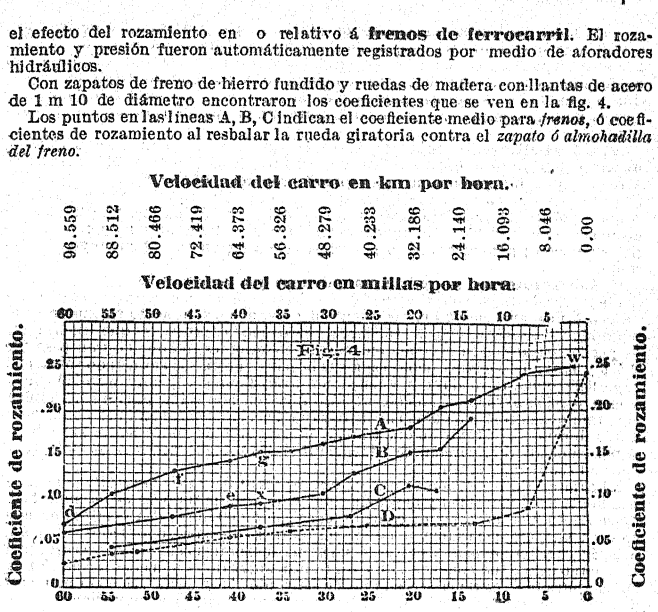
las bajas; por consiguiente debemos esperar un coeficiente menor en aquéllas. Pero las altas velocidades generalmente aumentan el número de asperezas destruídas, y esto puede ó aumentar ó disminuir el coeficiente como se ha explicado arriba. Las grandes velocidades á menudo lo afectan *indirectamente*, aumentando la *temperatura*.

* El Diario de Silliman (Diario Americano de Ciencias), marzo de 1876 y mayo de 1877.

el efecto del rozamiento en o relativo á **frenos de ferrocarril.** El rozamiento y presión fueron automáticamente registrados por medio de aforadores hidráulicos.

Con zapatos de freno de hierro fundido y ruedas de madera con llantas de acero de 1 m 10 de diámetro encontraron los coeficientes que se ven en la fig. 4.

Los puntos en las líneas A, B, C indican el coeficiente medio para *frenos,* ó coeficientes de rozamiento al resbalar la rueda giratoria contra el *zapato ó almohadilla del freno.*




La línea A indica coeficientes obtenidos inmediatamente después de la aplicación del freno.

La línea B indica los coeficientes obtenidos 5 segundos después de la aplicación del freno.

La línea C indica los coeficientes obtenidos 15 segundos después de la aplicación del freno.

La línea D indica los coeficientes de rozamiento contra el riel resbalando una rueda sujeta fuertemente por el freno.

(b) De las líneas A, B y C aparece que el **coeficiente** para el freno que se obtuvo en una cantidad de tiempo dada después de la aplicación del freno, es generalmente mayor en **velocidades pequeñas que en grandes.** Pero cuando la velocidad se mantenía uniforme, **el coeficiente disminuía á proporción que el freno y la rueda permanecían mayor tiempo en contacto.** Así, las líneas A y B muestran que á 37½ millas (60.337 km) por hora, el coeficiente era de .154 cuando el freno fué aplicado primero (punto *g*); pero descendió á .096 en 5 segundos (*x*). La línea A (inmediatamente después de aplicado el freno) muestra un coeficiente mayor (.132 en *f*) á 47½ millas (76.427 km) que la línea B (5 segundos después de la aplicación) muestra á (60.337 km) 37½ millas (.096 en *x*).

La disminución del coeficiente de rozamiento contra el riel con algún tiempo de aplicación del freno fué apenas perceptible.

(c) Cuando el rozamiento del freno (por causa de la disminución de la velocidad y el consiguiente aumento del coeficiente) se hace=á la « adherencia » ó rozamiento estático entre el riel y la llanta de la rueda, la velocidad de rotación decae rápidamente hasta ser menor que la debida á la velocidad del carro; esto es, **la rueda** comienza á patinar ó á *resbalar* á lo largo del riel, y entre .75 y 3 segundos la rotación de la rueda cesa enteramente.

(d) **El coeficiente del riel,** línea D, **es generalmente mucho menor que el coeficiente de los frenos,** líneas A, B y C. La presión sobre el riel

(=peso sobre una rueda) era como de 352 kilog por cm cuad, es decir, excedía en mucho al límite del desgaste. La presión en el freno era como de 14 kilog por cent cuad. Se hicieron algunos experimentos con zapatos de freno que no tenían sino $\frac{1}{3}$ de la superficie usual de contacto, y por tanto el triple de la presión por unidad de superficie bajo la presión dada, y no consiguieron demostrar de una manera concluyente que esto causara ningún cambio notable en el coeficiente.

(e) **El coeficiente del riel**, línea D, así como el coeficiente del freno, **aumenta á proporción que la velocidad disminuye**, y el aumento es lento al principio; pero mucho más rápido á proporción que se hace menor la velocidad; hasta que en el momento de detenerse llega aun á ser mayor que el coeficiente del freno inmediatamente antes de que la rueda patine. Con llantas de acero sobre rieles de hierro á grandes velocidades fué algún tanto mayor que sobre rieles de acero; pero esta diferencia desapareció á medida que disminuyó la velocidad.

(f) **Locomotoras.** La resistencia que tiene que vencer es de $\frac{1}{5}$ á $\frac{1}{3}$ ó más, del peso que descansa sobre las ruedas motrices, es decir, que tienen un coeficiente de .33 ó más aun cuando el coeficiente del acero sobre el acero en movimiento á presión baja es sólo como de .15. Pero los casos son tan diferentes que difícilmente se puede contar con la armonía entre estos coeficientes, porque el gran peso (de 2 á 6 y hasta 7 toneladas sobre cada rueda motriz) está reconcentrado sobre una superficie (donde la rueda toca el riel) tan sólo de 2 pulgs × ½ pulg (6.25 cm cuad), y esta presión excede en mucho, no sólo á las en que están basadas las tablas, sino también al límite de desgaste de la superficie. Además, cualquier punto de la llanta durante el instante en que trabaja, como punto de apoyo por la presión del vapor en el cilindro, permanece estacionario ó fijo sobre el riel, y el rozamiento se hace en ese tiempo estático.

El capitán Galton encontró que **el coeficiente de « adherencia »** era independiente de la velocidad y dependía solamente de la naturaleza de las superficies en contacto. Con un carro de cuatro ruedas con 5,000 libras (2,268 kilog) de peso en cada una, era generalmente de .20 en rieles secos; en algunos casos .25 y aun más. Sobre rieles húmedos ó grasosos, sin arena descendió hasta .15 en un caso; pero el término medio fué como de .18. **Con arena** sobre rieles húmedos fué de .20. La arena aplicada á los rieles secos antes de salir, dió .35 y hasta .40 en el momento de salir y un término medio de .28 próximamente durante el movimiento, y con la arena aplicada á los rieles secos durante el movimiento del carro, éste estuvo á punto de ser arrojado fuera de los rieles por el movimiento producido por las ruedas y el carro.

(g) Debido á la constnacia del coeficiente de « adherencia », bajo condiciones dadas de la llanta y riel, el rozamiento del freno necesario para hacer patinar las ruedas en cualquier caso, ha resultado también prácticamente constante para todas las velocidades. Pero en grandes velocidades, debido al coeficiente inferior del freno, se necesita una presión de freno más alta para producir esta intensidad fija de su rozamiento. El patinamiento también se alcanza en un tiempo mayor que á pequeñas velocidades.

**192.** Si la presión es suficiente para producir desgaste, el rozamiento varía mucho; pero no ha sido descubierta todavía ninguna ley precisa para apreciar las variaciones. Rennie da **la tabla siguiente de coeficientes de rozamiento de superficies secas, bajo presiones gradualmente aumentadas hasta los límites del desgaste.** Se notará que **en esta tabla el coeficiente aumenta generalmente** con la intensidad de la presión.

**Coeficientes de frotamiento de superficies secas, bajo presiones aumentadas gradualmente hasta los límites del desgaste.** (Por G. Rennie, I. C.)

| Presión en kilog por centímetro cuadrado. | Presión en libs por pulgada cuadrada. | Hierro forjado sobre hierro forjado. | Hierro forjado sobre hierro fundido. | Acero sobre hierro fundido. | Latón sobre hierro fundido. |
|---|---|---|---|---|---|
| 2.29 | 32.5 | .140 | .174 | .166 | .157 |
| 13.08 | 186 | .250 | .275 | .300 | .225 |
| 15.75 | 224 | .271 | .292 | .333 | .219 |
| 23.62 | 336 | .312 | .333 | .347 | .215 |
| 31.49 | 448 | .376 | .365 | .354 | .208 |
| 39.37 | 560 | .409 | .397 | .358 | .231 |
| 47.25 | 672 | .... | .376 | .403 | .253 |
| 40.85 | 709 | .... | .424 | .... | .234 |
| 55.12 | 784 | .... | .... | .... | .232 |
| 57.72 | 821 | .... | .... | .... | .273 |

**193 (a). Frotamiento rodando** es el que se produce entre la circunferencia de un cuerpo que rueda y la superficie sobre la cual rueda; es algo parecida al de un piñón sobre una cremallera. Al desengranar las asperezas intermedias ó al levantar la rueda sobre un obstáculo *o*, figs. 5 y 6, la fuerza motriz F, en lugar de hacer rodar la una sobre la otra como en la fig. 76, § 172, obra en el extremo de una palanca quebrada FRW, figs. 5 y 6, cuyo otro extremo W obra en dirección perpendicular á la superficie de contacto. En general, cuando se trata de cuerpos rodantes, el brazo de palanta RW, del peso resistente de la rueda y su carga, es siempre mucho menor en proporción al de (FR) que tiene la fuerza F en nuestras exageradas figuras. De aquí que, la fuerza F, requerida para hacer *rodar* una rueda, etc., es de ordinario muchísimo menor que la que se necesitaría para hacerla *resbalar*.

(b) Hay comúnmente **dos maneras de aplicar la fuerza** para vencer el rozamiento rodando : 1.ª (fig. 5) en el eje del cuerpo rodante, como sucede con la fuerza de un caballo aplicada en el eje de una rueda de carro ó la de un hombre en

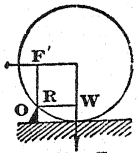


Fig. 5

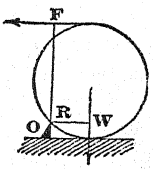


Fig. 6

el eje de una carretilla; 2.ª (fig. 6) en la circunferencia, como cuando los obreros empujan una troza de madera pesada colocada sobre dos ó más rodillos; ó como cuando la armazón de un puente de hierro se desliza hacia atrás y hacia adelante (por contracción y dilatación debida á la temperatura) sobre rodillos ó bolas metálicas. Cuando se aplica la fuerza como en la fig. 5, tenemos además del rozamiento rodante de la circunferencia de la rueda el rozamiento resbalando de su eje en la chumacera. En la fig. 6 tenemos solamente rozamiento rodante, pero en la parte superior é inferior de la rueda.

(c) Cuando los obstáculos *o* son muy pequeños, como en el caso de ruedas de carro sobre caminos planos resistentes ó ruedas de vagón sobre carriles de hierro ó acero, el brazo de palanca (FR) de F viene á ser prácticamente el *radio* en la fig. 5, y en la fig. 6, el *diámetro* de la rueda; mientras que el (RW) de la resistencia es muy pequeño. Por consiguiente, despreciando el rozamiento del eje en la fig. 5, la fuerza F requerida para vencer el rozamiento de rotación en dichos casos es directamente proporcional al peso W de la rueda y su carga, é inversamente al diámetro de la rueda.

Los pocos experimentos que se han hecho respecto al coeficiente de rozamiento por rotación, aparte del rozamiento del eje, son demasiado incompletos para que sirvan de base á reglas prácticas.

(d) **La adherencia** entre la rueda y el riel que pone á una locomotora en capacidad de moverse y mover el tren, ó que tiende á hacer girar una rueda de carro á pesar de la presión del freno, es una resistencia al resbalamiento de la rueda sobre

el carril y no es por tanto rozamiento de rotación, sino de resbalamiento; estático, cuando las ruedas permanecen en reposo ó dan vuelta perfectamente sobre los carriles, y cinemático, cuando « resbalan » ó *patinan*.

**194. El rozamiento de los líquidos** en movimiento en contacto con cuerpos sólidos, **es independiente de la presión**; porque el resbalamiento de las moléculas del fluido por sobre las asperezas de la superficie del cuerpo sólido, está ayudado por la presión de las otras moléculas que las rodean, las cuales tienden á ocupar los lugares de aquellas que han pasado. Por consiguiente, no tenemos para los líquidos coeficiente de rozamiento que corresponda á los de (=*resistencia dividida por presión*) los sólidos. Se cree que la resistencia está en razón directa del área de la superficie de contacto. Recientes investigaciones indican que la resistencia es =un coeficiente × área de superficie × vel $n$, en que $n$ y el coeficiente dependen de la velocidad y de la clase de superficie; y además á pequeñas velocidades, $n=1$; pero á determinada velocidad « crítica » (que varía con las circunstancias) $n$ de repente se hace=2, debido al cambio de la corriente, en determinadas contracorrientes ó remolinos. La resistencia del frotamiento del fluido nace principalmente de las contracorrientes puestas en movimiento.

**195. Tabla de los coeficientes de rozamiento en movimiento de las superficies planas y lisas, cuando se conservan perfectamente lubricadas.** (Morin.)

| Substanc | Jabón seco. | Aceite de olivas. | Sebo. | Manteca de puerco. | Manteca y plombagina. |
|---|---|---|---|---|---|
| Roble sobre roble; fibras paralelas al movimiento... | .164 | .... | .075 | .067 | |
| — — — fibras paralelas al movimiento.... | .... | .... | .083 | .072 | |
| — — olmo; fibras paralelas al movimiento .... | .136 | .... | .073 | .066 | |
| — — hierro fundido; fibras paralelas al movimiento ..... | .... | .... | .080 | | |
| — — hierro forjado; fibras paralelas al movimiento ..... | .... | .... | .098 | | |
| Haya — roble; fibras paralelas al movimiento .... | .... | .... | .055 | | |
| Olmo — roble; — — — ..... | .137 | .... | .070 | .060 | |
| — — olmo; — — — ..... | .139 | .... | | | |
| — — hierro fundido; fibras paralelas al movimiento..... | .... | .... | .066 | | |
| Hierro forjado sobre roble; fibras paralelas engrasadas y húmedas. .256..... | | .... | | | |
| Hierro forjado sobre roble; fibras paralelas al movimiento ..... | .214 | .... | .085 | | |
| Hierro forjado sobre olmo; fibras paralelas al movimiento ..... | .... | .055 | .078 | .076 | |
| Hierro forjado sobre hierro fundido; fibras paralelas al movimiento..... | .... | .066 | .103 | .076 | |
| Hierro forjado sobre hierro forjado; fibras paralelas al movimiento..... | .... | .070 | .082 | .081 | |
| Hierro en bruto sobre latón; fibras paralelas al movimiento..... | .... | .078 | .103 | .075 | |
| Hierro fundido sobre roble; fibras paralelas al movimiento..... | .189 | | | | |
| Hierro fundido sobre roble; fibras paralelas al movimiento engrasadas y húmedas, 218..... | .... | .075 | .078 | .075 | |
| Hierro fundido sobre olmo; fibras paralelas al movimiento..... | .... | .061 | .077 | .... | .01 |
| Hierro fundido sobre hierro fundido con agua .344... | .197 | .064 | .100 | .070 | .055 |
| — — — latón ..... | .... | .078 | .103 | .075 | |
| Cobre sobre roble; fibras paralelas al movimiento... | .... | .... | .069 | | |
| — amarillo sobre hierro fundido ..... | .... | .066 | .072 | .068 | |
| Latón — — — ..... | .... | .077 | .086 | | |
| — — hierro forjado..... | .... | .072 | .084 | .... | .089 |
| — — latón ..... | .... | .058 | | | |
| Acero — hierro fundido..... | .... | .079 | .105 | .084 | |
| — — — forjado ..... | .... | .... | .093 | .076 | |
| — — latón..... | .... | .053 | .056 | .... | .067 |
| Cuero curtido sobre hierro colado engrasado y muy mojado .365..... | .... | .133 | .159 | | |
| Cuero curtido sobre latón..... | .... | .144 | .241 | | |
| — — — roble con agua .29..... | | | | | |

**El rozamiento ocasionado al botar al agua** la fragata de madera *Princeton* fué calculado por el comité del Instituto Franklin en 1844, con un término medio aproximado de .067 ó un quinzavo ($^1/_{15}$) de la presión durante los primeros .75 de segundo y .022 ó un cuarenta y cincoavo en los 4 segundos siguientes de su movimiento; la inclinación de los durmientes era de 1 en 13, ó 4 grados 24 minutos. Fueron muy bien engrasados con sebo. La presión sobre ellos era = más ó menos 1.12 kg por cm cuad; en los primeros .75 segundo, el buque se deslizó 6.25 cm; en los 4 segundos siguientes, 4 m. 74; total para 4.75 segundos, 4 m 80.

**196. El rozamiento de las superficies lubricadas** varía mucho con la especie de superficie, con el lubricante y con el modo de aplicación. Si el lubricante es de mala calidad y está aplicado escasa y desigualmente bajo gran presión, puede destruirse en ciertos lugares y dejar partes de la superficie seca en contacto. Las condiciones entonces se aproximan á las de superficies no lubricadas. Pero si se usan los mejores lubricantes que se emplean con este objeto, y se aplican regularmente y en cantidad suficiente á fin de conservar las superficies siempre perfectamente separadas, el caso viene á ser prácticamente como un caso de rozamiento de líquidos y la resistencia es muy pequeña. Entre estos dos extremos hay una extensa serie de variaciones (véase la tabla, § 197 *d*), porque el coeficiente varía con el menor cambio en las condiciones. Donde se requiera cierto grado de exactitud, remitimos al lector á los resultados experimentales dados en la muy completa * obra del profesor Thurston, dedicada exclusivamente á este intrincado asunto.

**197 (a).** Los experimentos hechos por el Sr. Arthur M. Wéllington sobre **el rozamiento de los muñones lubricados** †, dieron un aumento gradual y continuo del coeficiente á medida que la velocidad de revolución disminuía desde 5.48 m por segundo (=la velocidad de un carro de 19.308 km por hora) hasta pararse. El aumento fué muy leve para grandes velocidades; pero mucho más rápido para las pequeñas; como en las figs. 3 y 4. En velocidades de .60 m á 5.50 por segundo, el coef fué mucho menor á altas presiones que á bajas; pero al comenzar el movimiento hubo poca diferencia á este respecto. El coef aumentó rápidamente á proporción que se elevó la temperatura de 38° á 66° C.

**(b)** El profesor Thurston, experimentando también con **muñones lubricados** ‡, encontró que al comienzo el coeficiente subió con el aumento de presión, lo que sucedió también con el movimiento cuando la presión excedía en gran manera al máximum (digamos de 34 á 40 kg por cm cuad) admisible en máquinas.

Encontró también que á una gran velocidad el coeficiente *aumentaba* muy despacio (en lugar de continuar disminuyendo) á proporción que aumentaba la velocidad.

**(c)** El profesor Thurston da las siguientes **fórmulas aproximadas para el rozamiento de muñones** á temperaturas, presiones y velocidades ordinarias, y estando el muñón y la chumacera en buenas condiciones y bien lubricados.

(*N. del T.* — Hemos agregado las equivalentes en medidas métricas.)

**Coeficiente para comenzar**: usando medidas inglesas $= (.015 \text{ á } .02) \times \sqrt[3]{\text{presión en libs por pulg cuad.}}$

**Coeficiente** para comenzar: usando medidas métricas $= (.015 \text{ á } .02) \times \sqrt[3]{14.223 \times \text{presión en kg por cm cuad.}}$

**Coeficiente** cuando el eje está **girando**: usando medidas inglesas $= (.02 \text{ á } .03) \times \frac{\sqrt[5]{\text{de velocidad en pies por } \textit{minuto}}}{\sqrt[2]{\text{de presión en libs por pulg cuad.}}}$

**Coeficiente** cuando el eje está **girando** : usando medidas métricas =

$$= (.02 \text{ á } .03) \times \frac{\sqrt[5]{3.2809 \times \text{velocidad en metros por } \textit{minuto}}}{\sqrt[2]{14.223 \times \text{presión en kg por cent cuad.}}}$$

* Friction and Lost Work in Machinery and Mill Work. John Wiley and Sons, New York, 1885.

† Trans Amer Soc of Civil Engrs, New York, Dec. 1884.

‡ Journal of the Franklin Institute, nov. 1878.

A presión como de 14 kilog por cent cuad :

$$\text{Temperatura del rozamiento mínima en grad Fahr} = 15 \times \sqrt[3]{\text{velocidad en pies por } \textit{minuto}.}$$

$$\text{En centígrados y sistema métrico} = (15 \times \sqrt[3]{3.2809 \times \text{velocidad en metros por } \textit{minuto}} - 32)\,\frac{5}{9}.$$

**Advertencia. El brazo de palanca,** con el cual el rozamiento del muñón resiste al movimiento, **aumenta con el diámetro** del muñón.

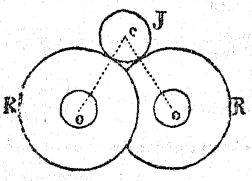
(d) Las cifras siguientes, elegidas de una tabla de resultados experimentales dados por el profesor Thurston, **demuestran la influencia de la presión, velocidad y temperatura en el coeficiente de rozamiento del muñón;** y de aquí el peligro en que se incurre en semejantes casos al aplicar con toda rigidez las reglas generales. En estos experimentos la naturaleza del muñón y chumacera, el lubricante y su modo de aplicarlo se suponen invariables. En los casos en que éstos varíen pueden ocurrir variaciones en el coeficiente mucho mayores todavía.

**Muñón de acero en chumacera de bronce, lubricado con aceite de ballena.** (*N. del T.* — Hemos agregado las medidas métricas, dejando las inglesas que pueden ser de utilidad.)

| | Velocidad de revolución. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 30 pies (9 m. 14) por *minuto*. | | | 100 pies (30 m. 48) por *minuto*. | | | 1.200 p.(365m75) por *minuto*. | | 500 p. (152m39) por *minuto*. | |
| | Presiones. | | | | | | | | | |
| Temperatura. | Kilog. p. cent cuad. | | | Kilog. p. cent cuad. | | | K. p. cent cuad. | | K. p. cent cuad | |
| | 14.06 | 7.03 | .28 | 14.06 | 7.03 | .28 | 14.06 | 7.03 | 14.06 | 7.03 |
| | Libs por pulg. cuad. | | | Libs por pulg. cuad. | | | Lib. p. pulg. c. | | Lib. p. pulg. c. | |
| | 200 | 100 | 4 | 200 | 100 | 4 | 200 | 100 | 200 | 100 |
| | Coeficientes. | | | Coeficientes. | | | Coeficientes. | | Coeficientes. | |
| 54.4 C — 130° F.. | .0160 | .0044 | .125 | .0087 | .0019 | .0630 | .0053 | .0037 | .0065 | .0075 |
| 32.2 C — 90° F.. | .0055 | .0031 | .094 | .0040 | .0019 | .0630 | .0075 | .0061 | .0100 | .0150 |

(e) **Cuando se aplica la fuerza primero en un lado del muñón y luego en el lado opuesto,** como en los muñones de manubrio, el rozamiento es menor que cuando la presión resultante está siempre sobre un lado, como en los ejes de los volantes; porque en el primer caso el aceite tiene tiempo para extenderse sobre ambos lados del muñón.

(f) **Rodillos de fricción.** Si un muñón J, en lugar de girar sobre chumaceras ordinarias está soportado por rodillos de fricción R, R, la fuerza que se requiere




para hacer girar á J se reducirá casi en la misma proporción que el diámetro del eje *o*, de los rodillos, sea menor que el diámetro de los rodillos mismos.

Wellington experimentó con una chumacera de rodillo basada en este principio, patentada é inventada por A. Higley. Diámetro de los rodillos R, R, 8 pulgs (.20 m); de sus ejes, *oo*, 1 ¾ pulgs (.044 m); del muñón *c*, 3 ½ pulgs (.088 m). De este modo tenemos teóricamente,

rozamiento del muñón de patente = rozamiento del muñón de 3½ pulgs (.088 m) $\times \frac{\text{diámetro de los ejes } oo}{\text{diámetro de los rodillos R, R}} = \frac{0.044}{0.20}$

ó, como de 1 á 4.6. Bajo una carga de 19.60 kg por cm cuad, el Sr. Wéllington encontró la relación de 1 á 4 al partir del reposo, y de 1 á 2 á la velocidad de un carro de 10 millas (16 km) por hora.

**198 (a). Resistencia del material rodante de ferrocarril.** Se compone del rozamiento rodante entre las llantas de las ruedas y los rieles (las llantas *resbalan* también algunas veces sobre los rieles cuando doblan una curva); del frotamiento de resbalamiento entre los muñones y sus chumaceras, y entre las pestañas ó rebordes de las ruedas y las cabezas de los rieles; de la resistencia del aire; y de las oscilaciones y choques que consumen fuerza motriz por sus movimientos laterales y verticales y que aumentan también los rozamientos de las ruedas y los muñones.

Su intensidad depende en gran parte de la condición de la vía y los rieles (como balasto, alineamiento, superficie, espacio entre las puntas de los rieles, sequedad, etc.); de la condición del material rodante (como el peso que se conduce, calidad de los resortes que se usan, especie y cantidad del lubricante, condiciones y dimensiones de las ruedas, ejes, etc.); de las pendientes y curvas; de la dirección y fuerza del viento, y de muchas consideraciones menores. Los experimentos dan resultados muy opuestos.

**(b)** Durante el verano de 1878, el **Sr. Wéllington** hizo experimentos con carros de plataforma y carros cubiertos, cargados y vacíos; carros de pasajeros y carros dormitorios á velocidades variables desde 0 hasta 56 km por hora. Los carros salieron rodando (por su propio peso) por una pendiente uniforme, de .7 por ciento, y de una extensión de 1,950 m. Sus resistencias fueron calculadas como en el § 185. « Los rieles eran de hierro, del peso de 30 kilog por met, y la vía estaba bien provista de balasto, bien recta y presentaba muy buena superficie; pero no era estrictamente de primera clase. » Los números aproximados siguientes se dedujeron de los experimentos hechos por el Sr. Wéllington sobre carros provistos de muñones ordinarios *. (*N. del T.* — Hemos agregado los datos en sistema métrico.)

**Resistencia en libras, y también en kilog por tonelada de (2,240 libs), 1,016 kilog, de peso del tren,** en vía recta y á nivel, en buenas condiciones.

| Velocidad del tren en kilómetros por hora. | Velocidad del tren en millas por hora. | Carros vacíos. | | | | | | | | Carros cargados. | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Eje, llanta y pestaña. | | Oscilación y choque. | | Aire. | | Total. | | Eje, llanta y pestaña. | | Oscilación y choque. | | Total. | | Aire. | |
| | | L. | K. | L. | K. | L. | K. | L. | K. | L. | K. | L. | K. | L. | K. | L. | K. |
| 0 | 0 | 14 | 6.35 | 0 | 0 | 0 | 0 | 14 | 6.35 | 18 | 8.16 | 0 | 0 | 0 | 0 | 18 | 8.16 |
| 16.093 | 10 | 6 | 2.72 | .6 | .27 | .4 | .18 | 7 | 3.17 | 4 | 1.81 | .6 | .27 | 0.4 | .18 | 5 | 2.27 |
| 32.186 | 20 | 6 | 2.72 | 2.7 | 1.22 | 1.3 | .54 | 10 | 4.54 | 4 | 1.81 | 2.0 | .91 | 1.0 | .45 | 7 | 3.17 |
| 48.279 | 30 | 6 | 2.72 | 5.3 | 2.40 | 2.7 | 1.22 | 14 | 6.35 | 4 | 1.81 | 4.7 | 2.13 | 2.3 | 1.04 | 11 | 4.99 |

**(c)** Con el muñón de rodillo de patente Higley, la resistencia al partir no fué sino como de 2 kg por tonelada.

**(d)** Como á la mitad del camino en que se hizo el experimento, había una **curva** de un ángulo de desviación de 1° (1,746.47 m de radio) de 914 m de largo, con su riel exterior elevado de 7½ á 10 cm como 152 m antes de llegar á la curva. En los primeros 152 m de curva, la resistencia fué mayor que la que se encontró inmediatamente antes de llegar á ella de .27 kilog á .95 kilog (término medio .50 kilog) por tonelada. En los últimos 152 m este exceso había disminuído de .09 kilog á .41 kilog (término medio .25 kilog) por tonelada. Debido á la continuidad de la pendiente de descenso en la curva, la velocidad aumentó á proporción que el tren la recorría; pero no se vió claramente si la disminución en

* Transactions, American Society of Civil Engineers, Feb. 1879.

la resistencia de la curva fué debida al aumento en velocidad ó al hecho de que las oscilaciones producidas por la entrada á ella cesaron gradualmente á proporción que el tren avanzó.

(c) **El Sr. P. H. Dudley,** haciendo experimentos con su « **dinamógrafo** » * obtuvo resultados de los que se han deducido los siguientes :

**Resistencia de un tren en libs y en kilog por tonelada (de 2,240 libs = =1,016 kilog) de peso del tren (incluyendo pendientes).** (*N. del T.* — Hemos agregado los datos en sistema métrico.)

| Descripción del tren. | | | Viaje. | Término medio de velocidad por hora. | | Término medio de resistencia. | |
|---|---|---|---|---|---|---|---|
| Carros cargados. | Carros vacíos. | Peso en toneladas 2,240 libs =1,016 k. | | Millas | Km. | libras | Kilog. |
| 20 | 2 | 526 | Toledo á Cleveland : 95 millas (152.8 km) | 20 | 32.18 | 8.34 | 3.77 |
| 37 | 0 | 633 | Cleveland á Erie : 95.5 millas (153.6 km) | 20 | 32.18 | 7.67 | 3.47 |
| 25 | 2 | 458 | Erie á Búffalo : 88 millas (141.6 km) | 20 | 32.18 | 8.89 | 4.02 |

Con los trenes largos y pesados del ferrocarril L. S. y M. S. Ry, de 600 á 650 toneladas, se requería menos combustible con la misma máquina para correr trenes de 30 á 32 km que para los de 16 á 19 km, debido al hecho de que en las grandes velocidades se usó mucho más el vapor por expansión; por consiguiente se logró mayor economía.

**199. El trabajo que se necesita para vencer el rozamiento** á cualquiera distancia, es = al rozamiento en kilogs × la distancia en metros. Un cuerpo que haya salido resbalando ó rodando por un plano horizontal, y que luego se le abandone á sí mismo, rodará ó resbalará una distancia dada, efectuando el trabajo que acabamos de apreciar, y su energía cinemática será entonces (= su peso en kilogs × su velocidad$^2$ en mets por segundo ÷ $2g$ †). Este valor debe ser igual al primero. Por tanto, **la distancia en metros** recorrida por el cuerpo deslizándose ó rodando sobre un plano horizontal, es : (*N. del T.* — Usando sólo el sistema métrico.)

$$= \frac{\text{su energía cinemática medida en kilográmetros al salir}}{\text{rozamiento en kilogs}}$$

$$= \frac{\text{peso del cuerpo en kilogs} \times \text{velocidad inicial}^2 \text{ en mets por segundo}}{\text{peso del cuerpo en kilogs} \times \text{coeficiente de rozamiento} \times 2g \dagger}$$

$$= \frac{\text{velocidad inicial}^2 \text{ en metros por segundo}}{\text{coeficiente de rozamiento} \times 2g \dagger}$$

**El tiempo requerido,** en segundos, es :

$$= \frac{\text{distancia en mets encontrada}}{\text{velocidad media en mets por segundo}} = \frac{\text{distancia en mets}}{\frac{1}{2} \text{ veloc. inicial en mets por segundo}}$$

Supongamos dos locomotoras semejantes, A y B, tirando cada una de un tren sobre una vía á nivel y recta; A á 10 km y B á 20 km por hora. La resistencia total de cada máquina y tren (que por conveniencia suponemos ser independiente de la velocidad) es de 1,000 kilogs. De aquí que la fuerza ó presión total del vapor en los dos cilindros que se requiere para equilibrar el rozamiento y mantener de este modo la velocidad, es la misma en cada máquina. Al hacer *diez km* esta fuerza efectúa la misma cantidad de trabajo (1,000 kilogs × 10 km = 10 millones de kilográmetros) en cada máquina y con el mismo gasto de vapor en cada una; aun cuando B debe suministrar vapor á sus cilindros *dos veces más ligero* que A, á fin

* Un instrumento para medir el esfuerzo de la barra de tracción de una locomotora, la fuerza que ésta ejerce sobre el tren.

† $g$ = aceleración de la gravedad = 9.81 m.

de *mantener* en ellos la misma presión. En *una hora*, la fuerza en A hace 10 millones de kilográmetros como antes; pero la fuerza en B hace (1,000 kilogs × 20 km =) 20 millones de kilográmetros, con doble gasto de vapor que A.

Pero, en verdad, la resistencia de un tren dado es mucho mayor á grandes velocidades. Véase la tabla § 198 (*b*). Y si suponemos aún que la resistencia es igual en ambas velocidades, B debe ejercer más fuerza para *adquirir* una velocidad de 20 km por hora, que A para adquirir una de 10.

**200. Talud natural.** Cuando se depositan lentamente las substancias granuladas, como : arena, tierra, granos, etc., como cuando son paleadas, en una cortada ó de un carro, el ángulo formado por la superficie inclinada del montón del material con un plano horizontal, se llama talud natural. Este ángulo depende de la fricción y adherencia entre las partículas del material y varía, á menudo, transcurrido algún tiempo, en el mismo material, con las condiciones atmosféricas, etc., y especialmente con la humedad.

**201.** Cualquier fuerza, *p*, fig. 85, obrando sobre un cuerpo, B, basta para mover el cuerpo (véase nota *, § 1) con tal de que exceda á la suma, S, de todas las

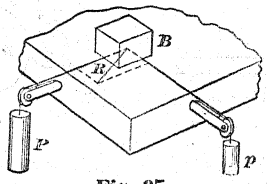



**Fig. 85.**

resistencias, incluyendo la fricción entre B y la superficie sobre que reposa B, ó si produce, con cualquiera otra fuerza ó fuerzas, P, una resultante, R, mayor que S.

Si antes de ser aplicada *p*, el cuerpo está en movimiento uniforme, P = S; cualquier fuerza, *p*, por pequeña que sea, es suficiente para cambiar la dirección del movimiento. Esto explica la facilidad con que un eje de revolución puede resbalar longitudinalmente en sus chumaceras y, el hecho de que es más fácil sacar un corcho de una botella si se le comunica antes un movimiento de torsión en el cuello de la botella.

## PALANCAS

**202. Géneros de palancas,** figs. 86. Las palancas se clasifican de acuerdo

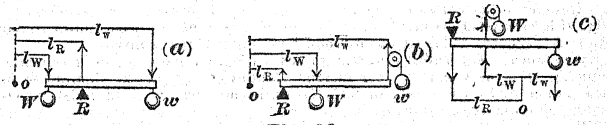



**Fig. 86.**

con la posición relativa de su « potencia », « resistencia » y « punto de apoyo »; así * :

Fig. (*a*), 1.er género. Punto R entre potencia *w* y resist W;
— (*b*), 2.º — Resist W entre potencia *w* y punto R;
— (*c*), 3.er — Poten W entre resist *w* y punto R.

En el 2.º género, el brazo de palanca de la potencia es necesariamente mayor que el de la resistencia. En el 3.º, viceversa.

**203.** En la fig. 86, tomando los momentos de las fuerzas, respecto á un punto cualquiera como *o*, se tiene, para el equilibrio :

Fig. (*a*), $W \cdot l_W - Rl_R + w \cdot l_W = 0$;
— (*b*), $W \cdot l_W - Rl_R - w \cdot l_W = 0$;
— (*c*), $W \cdot l_W - Rl_R + w \cdot l_W = 0$.

* Cuando las palancas se usan para levantar pesos, ó para vencer otras resistencias, la fuerza aplicada se llama « potencia » y la fuerza ó fuerzas que se van á vencer, « resistencia ».

**204. Palancas combinadas**, fig. 87. Se usan cuando no hay espacio para un solo brazo de palanca suficientemente largo. En un sistema de palancas,

$$\frac{\text{resist}}{\text{poten}} = \frac{\text{producto brazos palanca de potens}}{\text{prod brazos palanca de resists}} = \frac{8\times10\times2}{2\times\ 1\times4} = 20.$$

Las tres palancas de la fig. 87, tomadas separadamente, y empezando por la potencia extrema, da :

$$\frac{\text{resist}}{\text{poten}} = \frac{8}{2} = 4; \quad \frac{10}{1} = 10; \quad \frac{2}{4} = \frac{1}{2}; \quad \text{y } 4\times10\times\frac{1}{2} = 20.$$

como antes.

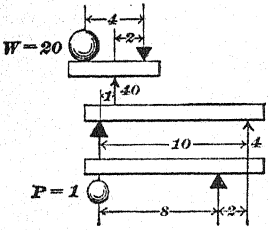



Fig. 87.

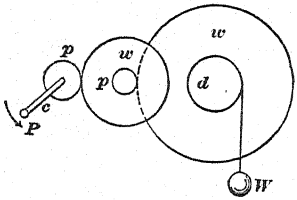



Fig. 88.

**205. Engranajes dentados. Ruedas y piñones**, fig. 88. Éstos son una serie de palancas compuestas. La potencia se aplica generalmente donde va el manubrio en *c* y la resistencia al cilindro *d*. La rueda más ancha se llama « la rueda »; la menor, *p*, el piñón. Sea *c*=radio del manubrio, *d*=el del cilindro, *m*=al producto de los radios de los piñones, *n*=al producto de los radios de las ruedas. Entonces, despreciando la fricción :

$$\frac{\text{resist}}{\text{poten}} = \frac{c\ .\ n}{m\ .\ d}.$$

En lugar de los diversos radios, se puede, por supuesto, usar los diámetros ó las circunferencias correspondientes, y como los dientes tienen necesariamente entre sí igual « paso » (longitud medida sobre la circunf), se acostumbra tomar el número de dientes de las ruedas ó piñones en lugar de los radios.

Las palancas combinadas y los engranajes se emplean para convertir velocidades pequeñas en altas y para vencer fuerzas grandes con pequeñas. Cuando se usan para aumentar la velocidad, la posición de la potencia y de la resistencia son contrarias á las que se indican en las figs. 87 y 88.

**206.** Siempre que la poten y resist se equilibren, ya sea en una palanca simple, ó en un sistema de palancas, si suponemos que ambos se mueven alrededor del punto de apoyo, sus respectivas velocidades estarán en la misma porporción que sus brazos de palanca; es decir, que si la palanca de la potencia es 2, 5 ó 50 veces mayor que la de la resist, la poten se moverá 2, 5 ó 50 veces más ligero que la resist. Por tanto, observando estas velocidades, se determina la proporción de sus brazos de palanca. La resist y la poten están entre sí en razón inversa de sus velocidades y también en razón inversa de sus brazos de palanca.

**207.** No se obtiene ninguna ventaja mecánica con sólo aumentar la *longitud* de una palanca, como por ejemplo, encorvándola, como en *abo*, fig. 4, § 13, ó dándole una inclinación hacia la línea de acción de la potencia, P, como en *om*, *ou*, *on*.

**208.** Así, en la fig. 89, que representa una palanca encorvada, *afb*, la longitud de la *palanca* ó de cualquiera de sus miembros como *fb*, no debe confundirse con el brazo de palanca de la fuerza. Éstos pueden, ó no, ser iguales. Por ejemplo, la parte *fb* es mucho más larga que *fa;* sin embargo, si los brazos de palanca *fa* y *fc*, de las fuerzas ó pesos, son iguales, los pesos *m* y *n* deben también ser iguales para que haya equilibrio.

**209.** Si se quita el peso $m$, una fuerza, $c$, ó $s$, ó $y$, ó $d$, con palanca $= c'$, $s'$, $y'$, $d'$, respectivamente, puede ser aplicada en cualquier punto, como $b$, para equilibrar el momento de $n$. En todo caso, esta fuerza debe ser tal que

$$\text{fuerza} \times \text{su brazo de palanca} = n \,.\, af.$$

Por tanto,

$$\text{fuerza} = \frac{n \,.\, af}{\text{brazo de palanca de la fuerza}}.$$

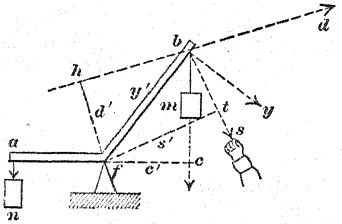



**Fig. 89.**

**210.** Por consiguiente, también la fuerza requerida es *menor*, como cuando en $y$, es *perpendicular* á la longitud de $fb$, pues la palanca (que evidentemente no puede exceder á $fb$) es entonces la *máxima*. La fuerza requerida aumenta cuando se desvía en una ú otra dirección de la línea $by$ (perpendicular á $fb$) y se aproxima más á $fb$; pues su brazo de palanca disminuye constantemente. Ninguna fuerza, por más grande que fuera, podría compensar el momento de $n$ respecto de $f$, si fuera aplicada en la dirección $fb$ ó $bf$, pues tal fuerza no tendría momento respecto á $f$.

**211.** De modo análogo, en la fig. 90, el momento respecto al punto $a$, de una carga W, colocada en $b$, es $=$ W . $ac$, y lo mismo sería colocada en $c$.

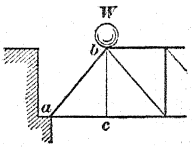



**Fig. 90.**

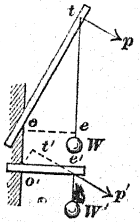



**Fig. 91.**

**212.** En la fig. 91, también los momentos W . $oe$ y W . $o'e'$, de los pesos iguales W y W', son iguales. Pero si las fuerzas $p$ y $p'$ son aplicadas en direcciones perpendiculares á la viga más larga, $ot$, la palanca $ot$ de $p$ llega á ser como 6 veces la de ($o't'$) de $p'$. Por tanto, una fuerza $p$ aplicada en $t$, tiene más ó menos el mismo momento inclinado que una fuerza paralela $= 6\,p$, aplicada en $e'$.

## ESTABILIDAD

**213.** Figs. 92. Si la resultante R (fig. *a*) de la fuerza P y del peso W se sale de la base, como está indicado, entonces el momento P de su tendencia á volcarlo, fig. 92 (*b*), respecto al pie *n*, excederá al momento de estabilidad del peso W en el mismo punto, y el cuerpo se volcará alrededor de *n*. Si no, quedará en pie.

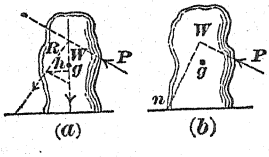



Fig. 92.

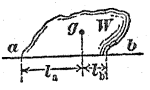



Fig. 93.

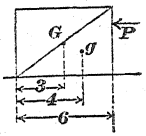



Fig. 94.

**214.** Suponiendo estabilidad suficiente para evitar el volcamiento, el cuerpo se deslizará si la componente horizontal *h* de R, fig. 92 (*a*), excede á la resistencia del frotamiento y á las demás que existan.

**215.** En la práctica, el borde *n*, ó el piso debajo de éste, pudieran ceder, si la piedra girará sobre aquél ó si R cayese cerca de *n* (véanse §§ 145, etc.); pero esto pertenece a la resistencia de los materiales. Poniendo concreto, etc., entre la base y el peso, se añadiría una tercera fuerza y cambiaría el problema.

**216.** Debido al mayor brazo de palanca, $l_a$ fig. 93, de W, respecto al punto *a*, el momento de estabilidad es mucho mayor respecto á éste que respecto al punto *b*.

**217.** En la fig. 94, sea G=2 kg; *g*=1 kg; los brazos de palanca=3, 4 y 6 m, como está indicado. Así, el momento de estabilidad del cuerpo rectangular G opuesto á una fuerza horizontal P, es=3 G=3×2=6 kilográmetros, y el momento del triángulo bajo, *g*, es=4*g*=4×1=4 kgm; así, aunque el cuerpo grande pesa el doble del triángulo bajo, sólo tiene 1.5 veces su estabilidad.

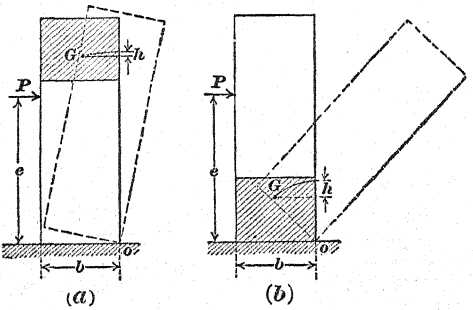



Fig. 95.

**218. Volcamiento.** Supongamos que, en la fig. 95 (*a*) y (*b*), la parte sombreada sea de plomo y el resto de madera, y, además, que el centro de gravedad de todo el cuerpo esté en cada caso en G. Entonces, como el peso W es el mismo en ambos casos, como también la palanca de su estabilidad respecto al punto $O = \frac{b}{2}$, el momento de estabilidad $= \frac{1}{2} b.W$, es el mismo en ambos casos, como lo es también la fuerza P, requerida para equilibrar dicho momento, cuando se aplica á una altura dada *e*. En los que se refiere al volcamiento, el peso, W, queda constante; el brazo de palanca, el momento de estabilidad, y el requerido de volcamiento, decrecen, llegando á ser=0 cuando el cuerpo alcanza la

posición que indican las líneas de puntos. Si la elevación, $e$, queda constante, la fuerza, P, necesaria para volcarlo, decrece en la misma proporción que el brazo de palanca.

**219.** Pero, cuando tratamos del volcamiento por la sola fuerza de la gravedad, deben llevarse los cuerpos á la posición indicada por las líneas de punto. Esto equivale á levantar el peso del cuerpo á una altura = á la distancia $h$, á que sube el centro de gravedad G. Así

$$\text{trabajo de volcamiento} = W.\ h.$$

Como $h$ es mayor en la fig. ($b$), este trabajo es mayor en este caso. En ingeniería civil se considera generalmente el valor de la fuerza para *comenzar* el volcamiento, con preferencia á la requerida para *completarlo*.

**220.** Es claro que la estabilidad contra el volcamiento está afectada y puede ser aumentada por otras fuerzas que no son el peso solo. Así, la estabilidad de una pila de puente se aumenta de ordinario con el peso del puente si éste se le aplica simétricamente. De otro modo el peso del puente aumentará ó disminuirá la estabilidad de la pila según las circunstancias.

**221.** El coeficiente de estabilidad es, en cualquier caso, la relación entre los momentos de estabilidad y de volcamiento; es decir:

$$\text{Coeficiente de estabilidad} = \frac{\text{momento de estabilidad}}{\text{momento de volcamiento}}.$$

**222.** Sea, en la fig. 96, el peso W de la piedra = 10 kg; G, su centro de gravedad, y $og = 2$ m. Entonces, el momento de estabilidad, respecto á $o$, es $10 \times 2 = 20$ kgm, si $on$ es = 5 m, una fuerza $P = \frac{20}{5} = 4$ kg, tendrá precisamente en equilibrio el momento del peso W, y excepto en el borde $o$, sobre la base $om$ no se ejercerá ninguna presión, aunque la piedra esté en contacto con la base. Si la fuerza P excede de 4 kg, la piedra empezará á girar alrededor de $o$, y si es menor, ejercerá una presión sobre la base $om$.

Supongamos que la piedra está sostenida en $o$ y en $m$ solamente. El brazo de palanca de la fuerza sustentadora R, en $m$, es $= om = l$. Sea P = 1 y la base $om$ = 4.5 m. Entonces, para el equilibrio, se tendrá:

$$W.\ og - P.\ on - R.\ om = 0;$$

ó bien $$20 \text{ kgm} - 1 \times 5 = 4.5 \times R;$$

de donde $$R = \frac{20 - 5}{4.5} = 3.33\ldots \text{ kg}.$$

En otras palabras: una fuerza vertical hacia arriba, R, de 3.33 kg en $m$, mantendrá el equilibrio.

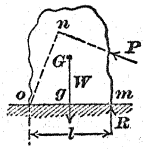



**Fig. 96.**

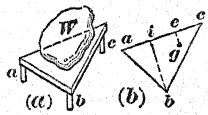



**Fig. 97.**

**223.** Sea, en la fig. 97 ($b$), $g$ el centro de gravedad, del peso W y de la mesa combinados, fig. 97 ($a$). Entonces, la reacción hacia arriba de $b = \frac{W.eg}{ib}$. Las de $a$ y $c$ se encuentran del mismo modo.

**224.** En la fig. 98, sea $h$ la fuerza horizontal ejercida por la parte izquierda del semiarco, sobre la clave del otro semiarco de la fig., y sea $e$ su brazo de palanca respecto al punto $o$. Sea W el peso del semiarco y de la enjuta obrando como un

solo cuerpo rígido, y $l$ su brazo de palanca respecto al punto $o$. Entonces, para e equilibrio, se tendrá:

$$h \cdot e = W \cdot l; \qquad h = \frac{W \cdot l}{e}.$$

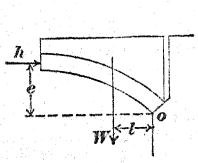



Fig. 98.

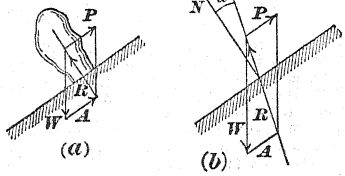



Fig. 99.

## Estabilidad en los planos inclinados.

**225.** Estabilidad en los planos inclinados, fig. 99. Aquí, como en el § 213, si la resultante R, de la fuerza P y el peso W, se sale de la base, es decir, si el momento de l volcamiento excede al de estabilidad, el cuerpo se volcará; si no, se sostendrá.

La fuerza, P, requerida para evitar el volcamiento, es=á la antiresultante A del peso W y de la reacción R; y la reacción R=á la antiresultante de P y W.

**226.** Despreciando la fricción, como en la fig. 99 ($a$), R será normal al plano. Tomando en cuenta la fricción, fig. 99 ($b$), R formará con la normal, N, al plano un ángulo, $a$, que no excederá el ángulo de fricción entre el cuerpo y el plano. R puede estar hacia arriba ó hacia abajo respecto á N.

**227.** En la fig. 100, el cuerpo B opone menos estabilidad al volcamiento, alrededor de $a$, que su semejante A, cuando la fuerza $n$ lo empuja hacia abajo; pero opone mayor estabilidad al volcamiento alrededor de su arista $c$ bajo la acción de una fuerza que lo empuje hacia arriba.

**228.** El cuerpo C, que se volcaría sobre una base horizontal, tendrá estabilidad contra el volcamiento, situándolo en un plano inclinado, como en D. Suponiendo $ao = tc$, cualquier fuerza vertical hacia arriba tendrá el mismo momento, para el

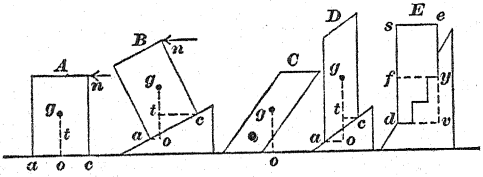



Fig. 100.

volcamiento, apliquésela en $a$ ó en $c$. Pero cualquier fuerza horizontal, aplicada á cualquier altura, como en $g$, tiene mayor palanca, $go$, empujando hacia abajo, que cuando empuja hacia arriba. En este caso su palanca es sólo $gt$.

**229.** Las construcciones fabricadas sobre pendientes tienen tendencia á resbalar. Esto se evita dividiendo la pendiente en escalones horizontales como en $dy$, fig. E; pero las caras verticales de los escalones dividen la trabazón de la mampostería, y, además, en el lado más alto, $sd$, las juntas son más numerosas y hay más argamasa que en la cara corta de arriba, $ey$, quedando así la tendencia á una desigual estabilidad y la parte baja más alta tiende á separarse de la de arriba, como sucedería en una fundación que tuviera un piso firme en una parte y compresible en otra. Así, cuando las circunstancias lo permitan, es preferible poner horizontal toda la base como en $dv$; ó si la construcción tiene que soportar presiones hacia abajo, $v$ puede estar más bajo que $d$ y las hiladas de mamposterías, colocadas con la correspondiente inclinación.

## LA CUERDA

**230. La cuerda**, figs. 101 (*a*) y (*b*), y 102 (*a*) y (*b*). En los §§ 230 á 239 consideramos las cuerdas como completamente flexibles, inextensibles, sin fricción ni peso é infinitamente delgadas.

**231.** Sea P la fuerza exterior aplicada en el nudo, ó gancho *o*, y sea R la resultante de los esfuerzos $s_1$ y $s_2$, ó bien *oa* y *ob*, en los dos segmentos *om* y *on* de la cuerda. Entonces, para el equilibrio, R debe ser igual y colineal con P.

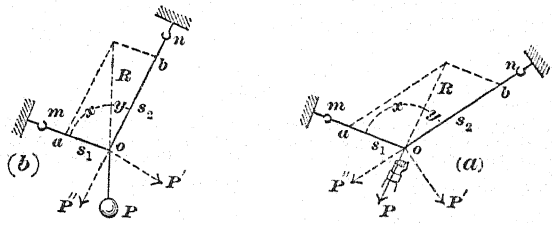



**Fig. 101.**

**232.** Conociendo el valor de P (=R), las tensiones $s_1$ y $s_2$ se encontrarán por el § 36; y viceversa dadas $s_1$ y $s_2$ se encontrará R (=P) por el § 35. O véase el §.40.

**233.** Si, como en las figs 101 (*a*) y (*b*), la fuerza P se aplica á la cuerda, en *o*, por medio de un nudo fijo incapaz de resbalar á lo largo de ella, de manera que los segmentos *om* y *on* sean de longitud fija, y el ángulo, $x+y$, entre ellos, también lo sea, entonces la fuerza puede ser aplicada en cualquier dirección como P, ó P', pasando entre los dos segmentos de la cuerda; y las componentes $s_1$ y $s_2$ serán iguales solamente cuando R forme ángulos iguales, $x$ é $y$, con los dos pedazos de la cuerda. Si la dirección de la fuerza, como P'', coincide con uno de los segmentos, como *on*, éste transmitirá toda la fuerza, P'', y el otro nada.

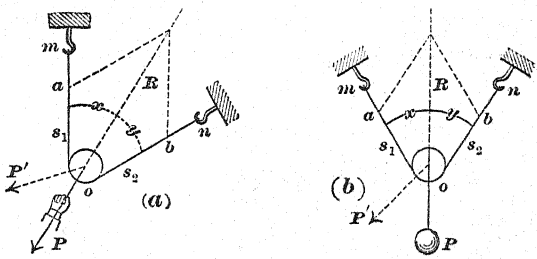



**Fig. 102.**

**234.** Pero si, como en las figs 102 y 103, la fuerza P se aplica á la cuerda, por medio de un anillo, polea ó gancho, etc., resbaladizo, entonces para el equilibrio las dos tensiones $s_1$ y $s_2$ deben ser iguales y también los dos ángulos $x$, $y$; y si suponemos que la dirección de la fuerza P cambia á la posición P', el gancho ó polea y la cuerda se ajustarán de nuevo, como lo indica la línea de puntos, fig. 103, hasta que el gancho ó polea llegue al equilibrio en el punto *o'* en que los ángulos $x'$, $y'$, sean iguales y también las tensiones $s_1'$ y $s_2'$.

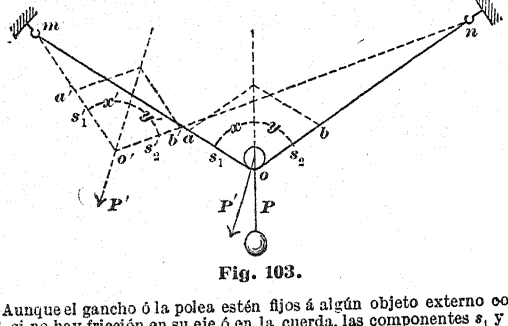



**Fig. 103.**

**235.** Aunque el gancho ó la polea estén fijos á algún objeto externo como en *o*, fig. 104, si no hay fricción en su eje ó en la cuerda, las componentes $s_1$ y $s_2$, serán iguales y su resultante R dividirá en dos partes iguales el ángulo $x+y$, entre ellas.

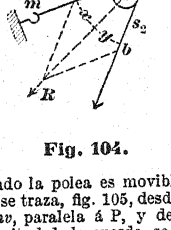



**Fig. 104.**

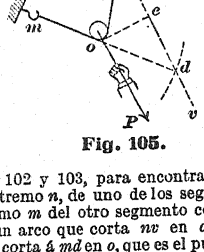



**Fig. 105.**

**236.** Cuando la polea es movible, figs. 102 y 103, para encontrar la posición que tomará, se traza, fig. 105, desde un extremo *n*, de uno de los segmentos *on* de la cuerda, *nv*, paralela á P, y del extremo *m* del otro segmento con el radio = *mo* + *on* = longitud de la cuerda, se traza un arco que corta *nv* en *d*. Tómese el medio *e* de *nd*; trácese *oe* normal á *nd*, que corta á *md* en *o*, que es el punto deseado.

**237.** Ya sea *o* un nudo fijo ó polea movible, estará siempre situado en la periferia de una elipse, cuyos focos son los extremos, *m*, *n*, de la cuerda.

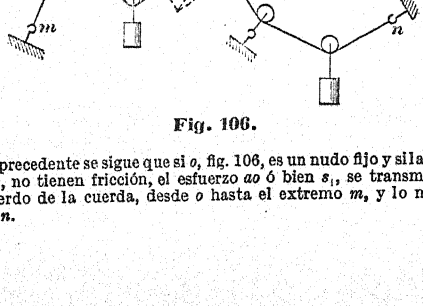



**Fig. 106.**

**238.** De lo precedente se sigue que si *o*, fig. 106, es un nudo fijo y si las otras poleas, ganchos, etc., no tienen fricción, el esfuerzo *ao* ó bien $s_1$, se transmitirá por todo el lado izquierdo de la cuerda, desde *o* hasta el extremo *m*, y lo mismo la *bo* = $s_2$, de *o* hacia *n*.

**239.** Advertencia. Obsérvese que, en la fig. 107 (*b*), la tensión en todas las cuerdas es doble de la tensión en las cuerdas correspondientes de la fig. 107 (*a*), aunque en cada fig. está suspendido de la polea un peso=4. Así, si el peso fuera el de un hombre colgado de la cuerda, y si la cuerda en la fig. (*a*) tuviese *exactamente* la fuerza necesaria para sostenerlo, ésta se reventaría, si él le da un extremo de la cuerda á otro para que la sostenga ó si amarra el extremo, como en la fig. (*b*).

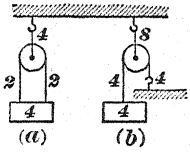



**Fig. 107.**

## La máquina funicular.

**240.** Cuando los ángulos $x$ é $y$, figs 101, etc., son muy grandes, una fuerza muy pequeña, P, equilibra una gran tensión, $s_1$ ó $s_2$, en la cuerda. Cuando $x=y=90°$, tenemos cos $x$=cos y=0, y $s_1=s_2$=infinito, por muy pequeña que sea P. Si la línea *mn*, que une los extremos de la cuerda, es horizontal ó inclinada, el peso de la cuerda solo obra como una fuerza P. Por tanto ; « No hay fuerza, por grande que sea, que pueda estirar una cuerda horizontal hasta ponerla en línea recta. »

**241.** En la máquina funicular se aprovecha la ventaja de que, cuando el ángulo total $x+y$, entre los dos segmentos de la cuerda, se aproxima á 180°, una pequeña fuerza P equilibrará grandes tensiones $s_1$ y $s_2$. Así, supongamos, en la fig. 108, que W representa un bote muy pesado (todo visto de plan) que debe ser acercado á tierra. Un extremo de la cuerda se ata á la proa, la cuerda se pasa, por un poste liso *n*, á otro *m*, alrededor del cual se le dan una ó más vueltas; y un hombre, de pie en *e*, sostiene la cuerda con alguna flojedad, mientras otros, con una fuerza P, tiran de la cuerda entre *m* y *n*, trayéndola á la posición *mon*. Si los ángulos $x$ é $y$ son iguales, la componente en el segmento *on* excede á P, tanto como el ángulo $x$ excede á 60°, y una tracción igual á esta componente (deducida, por supuesto, la rigidez de la cuerda y la fricción con el poste) acercará el bote un poco hacia tierra. Entonces, cobrando cuerda en *e*, se vuelve á poner el cable recto en *mn* y se repite la operación cuantas veces sea necesario.

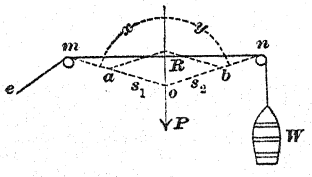



**Fig. 108.**

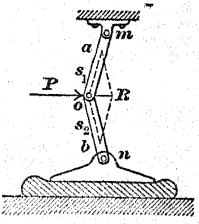



**Fig. 109.**

## La junta de codillo.

**242.** Esta es, simplemente, la inversa de la máquina funicular, con un nudo fijo y la fuerza P y las componentes $s_1$ y $s_2$ obrando por compresión y no por tensión. Como la unión no puede moverse á lo largo de los brazos, la fuerza P se puede aplicar en la dirección que se quiera; pero se acostumbra aplicarla de modo que forme aproximadamente ángulos iguales con los dos brazos.

## La polea.

**243.** Las figs. 110 muestran las relaciones de las tensiones y pesos en diferentes disposiciones de poleas fijas y movibles. Así, en (*a*), 1 kg equilibra 1 kg, en (*b*) $2_p$ en (*c*) y en (*d*) 4 kg. En cada caso, si los cuerpos se ponen en movimiento, sus velocidades estarán en razón inversa de sus masas. Véase § 206.

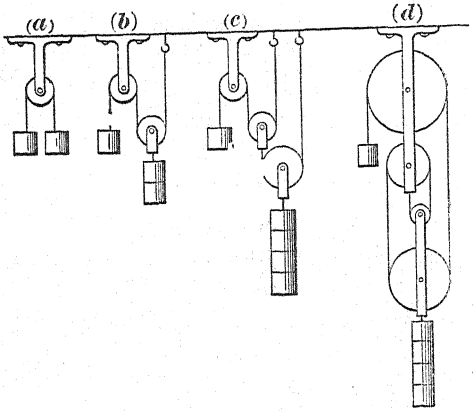



**Fig. 110.**

**244.** La polea simple, fig. 110 (*a*), se usa simplemente para cambiar la dirección de la fuerza, porque éstas en los dos extremos de la cuerda son iguales; pero en la polea compuesta, figs. 110 (*b*), (*c*), (*d*), una fuerza pequeña (la potencia), moviéndose rápidamente, en una parte de la cuerda, equilibra fuerzas mayores (la resistencia), moviéndose en la otra parte. Así, la polea compuesta se usa para vencer lentamente grandes resistencias, por medio de pequeñas fuerzas que se mueven con rapidez.

**245.** Para poner tal sistema en movimiento * (para levantar el peso) se necesita romper el equilibrio, haciendo que la « potencia » exceda á la tensión en la cuerda, debida al peso. Pero, ya engendrado el movimiento, continuará indefinidamente si la potencia es suficientemente mayor que el peso para vencer las resistencias de la fricción, etc.

## La cuerda ó cadena cargada.

**246.** En las figs 111, el principio del polígono de las cuerdas, §§ 86, etc., se aplica al caso de una cuerda ó cadena flexible, que sostiene cuatro pesos, $p_1 \ldots p_4$, en puntos fijos, ejerciendo una tensión horizontal†, H, en su extremo bajo, y una inclinada, R, en su extremo alto. Los pesos $p_1 \ldots p_4$, se representan por la línea vertical 0-4, fig. 111 (*a*); la tensión horizontal, H, por 0-*c*; la intensidad y dirección de la tensión inclinada, R, en la parte alta de la cuerda, por 4-*c*, y la tensión en los segmentos, 1-2, 2-3 y 3-4, por los radios, 1-*c*, 2-*c*, y 3-*c*, respectivamente.

**247.** La tensión horizontal, H (=á la componente horizontal de la tensión en cada segmento), es uniforme á lo largo de la cuerda; pero, la componente vertical de la tensión en cualquier segmento, es igual á la suma de los pesos entre ese segmento y la polea, *m*. Así, la componente vertical (0-2, fig. *a*) de la tensión *c*-2, en el segmento 2-3, es $=p_1+p_2$; en el segmento 3-4, es $0\text{-}3=p_1+p_2+p_3$, etc.

**248.** Si todos los pesos, incluyendo W, se aumentan en la misma proporción, como se indica por las líneas de puntos en la fig. 111 (*a*), ó se disminuyen en la misma proporción, los nuevos triángulos, *c'*, 4', 0, etc., fig. (*a*), serán semejantes al

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* Véase la nota (*) al pie del § 1.
† Véase la nota (*) al pie del § 249.

viejo, y el perfil de la cuerda, fig. (*b*), quedará inalterable, aunque las tensiones en los segmentos es claro que aumentarán ó disminuirán en la misma proporción.

**249.** En la fig. 111 se ha hecho el peso, W, que es necesariamente igual á la tensión horizontal H * (véase la Cuerda, §§ 230, etc.), igual también á la suma de los

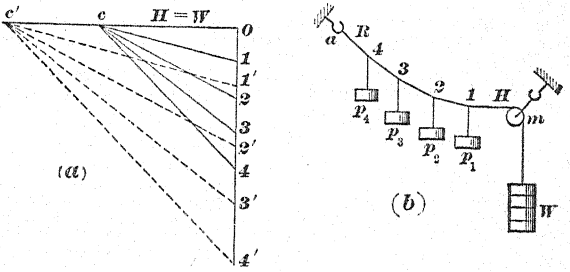


Fig. 111.

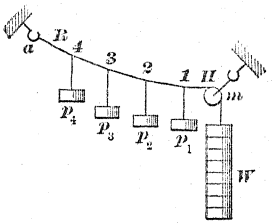


Fig. 112.

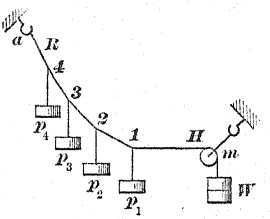


Fig. 113.

pesos, $p_1 \ldots p_4$. Cuando es así, el segmento de cuerda, *a*–4, próximo al soporte, *a*, y la línea correspondiente, *c*–4, fig. (*a*), formarán 45° con la vertical.

**250.** Pero si, quedando los pesos $p_1 \ldots p_4$ invariables, se levanta la polea *m*, de manera que H sea horizontal *, se obtendrá una curva suave, achatada, como en la fig. 112; y, para el equilibrio, H (=W) debe hacerse *mayor* que la suma de $p_1 \ldots p_4$. Por otra parte, si se coloca la polea, *m*, más baja que en la fig. 111 (siendo H siempre horizontal), se obtiene una curva más ahondada, como en la fig. 113; y H (=W) debe hacerse *menor* que la suma de $p_1 \ldots p_4$.

* En las figs. 111, 112 y 113 se supone el peso, W, y la posición de la polea, *m*, de tal manera dispuestos respecto al soporte *a*, que la tensión, H, quede horizontal.

## ARCOS, PRESAS, ETC., LÍNEAS DE EMPUJE Y DE RESISTENCIA

### El arco.

En los §§ 251 á 257 se dan los elementos de la teoría comúnmente aceptada del arco. Para las consideraciones prácticas, véanse los §§ 258 á 266 y Puentes de piedra.

**251** Si las figs. 111, 112 y 113 se invierten, los segmentos de cuerda representarán puntales que resisten la compresión, como lo hacen las piedras de un arco de mampostería, fig. 114.

### Línea de empuje. Línea de resistencia.

**252.** En el caso de un arco, fig. 114, suponiendo * que el empuje horizontal, H, en la clave, *m*, y la reacción, R, del arranque, *a*, obra en el centro (ó en algún otro punto definido) de la clave y del arranque, respectivamente, sus intensidades y la dirección de la reacción, R, pueden encontrarse por medio del Triángulo de las fuerzas, § 51, ó por los Momentos, § 224. (Véase § 257.) Suponemos entonces que el semiarco y su enjuta están divididos por planos verticales *, fig. 114 (*b*),

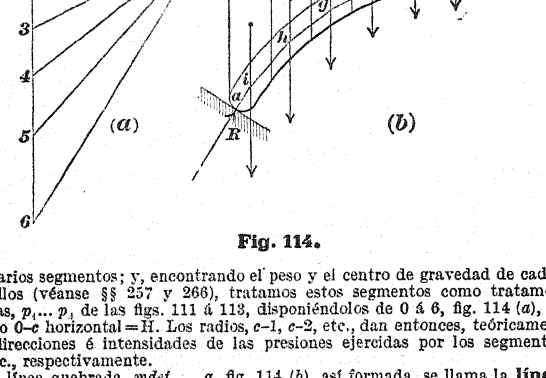


**Fig. 114.**

en varios segmentos; y, encontrando el peso y el centro de gravedad de cada uno de ellos (véanse §§ 257 y 266), tratamos estos segmentos como tratamos las cargas, $p_1 \ldots p_1$ de las figs. 111 á 113, disponiéndolos de 0 á 6, fig. 114 (*a*), y trazando 0–*c* horizontal = H. Los radios, *c*–1, *c*–2, etc., dan entonces, teóricamente *, las direcciones é intensidades de las presiones ejercidas por los segmentos, 1, 2, etc., respectivamente.

La línea quebrada, *mdef*. . . . *a*, fig. 114 (*b*), así formada, se llama la **línea de empuje**, ó línea de las resultantes. Ella corresponde á los polígonos de cuerda de las figs. 111 (*b*), etc. *

**253. La línea de resistencia** es una línea quebrada que une los puntos en donde las varias resultantes, que forman la línea de empuje, cortan las respectivas juntas entre las piedras del arco.

**254.** Cuando los planos, por los cuales se supone dividido el arco en segmentos, son verticales *, como en la fig. 114 (*b*) y como sucede en la mayor parte de los arcos reales, las líneas de empuje y de resistencia, fig. 115, coinciden prácticamente; pero, si estos planos están lejos de ser verticales, como en la fig. 115, las dos líneas se separan, siendo siempre la línea de resistencia la exterior.

Así, en la fig. 115 (donde la línea de empuje es la continua, y la línea de resistencia la puntuada), notando dónde la resultante *a* corta la junta A, dónde la resultante *b* corta la junta B, etc., se verá que las dos líneas prácticamente coinciden hasta la junta C, donde empiezan á divergir.

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* Véase Consideraciones prácticas, §§ 258, etc.

**255.** En el § 252 supusimos que el arco y su enjuta están divididos en segmentos verticales que, en este caso, ejercen presiones verticales. La línea de resistencia teórica, así obtenida, puede pasar, en algunos lugares y especialmente en arcos rebajados, del espesor del arco; de modo que, si no obrasen otras fuerzas,

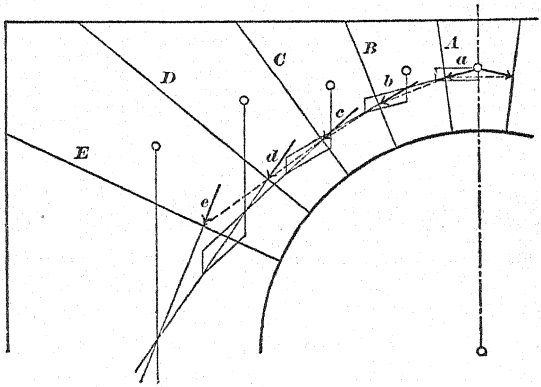


**Fig. 115.**

el arco se abriría en tales lugares; en el intradós, cuando la línea de resistencia corta el trasdós, y viceversa; pero tal ruptura se impide ordinariamente con otras fuerzas, tales como las presiones horizontales ó inclinadas de la enjuta. La verdadera línea de resistencia se reduce así al interior del espesor del arco. En general, la verdadera línea de resistencia, fig. 116, se acerca al trasdós en la clave y al intradós en los riñones, de modo que el arco tiende á hundirse en la clave (abriéndose allí en el intradós) y á levantarse en los riñones (abriéndose en éstos en el trasdós), como lo indica la figura.

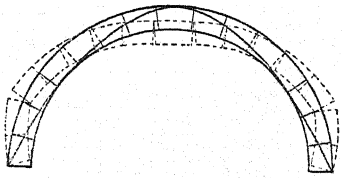

**Fig. 116.**

**256.** A fin de evitar cualquier tendencia de las juntas á abrirse en uno ú otro lado, debe trazarse de tal modo el arco que la línea de verdadera resistencia esté en todas partes dentro del tercio medio (véanse los §§ 145, etc.) del espesor del arco.

**257.** En general, el trazado de un arco se obtiene por una serie de aproximaciones. Así, debe suponerse de antemano una forma de arco y de enjuta, á fin de encontrar su centro común de gravedad y poder determinar el empuje horizontal, H, y la reacción del arranque R, como en el § 252; y si después es necesario modificar la forma adoptada al principio, á fin de satisfacer las exigencias del § 256, ó por otras razones, tendremos que recomputar H y R, modificando otra vez el diseño, y así se continúa.

## Consideraciones prácticas.

**258.** Si bien las líneas de empuje y resistencia, basadas en las suposiciones precedentes, se hallan fácilmente, existe mucha incertidumbre en cuanto á las posiciones de dichas líneas en un arco de mampostería.

**259.** En primer lugar, no sabemos por qué puntos en la clave y en el arranque, respectivamente, pasan las resultantes H y R.

**260.** Además, hemos supuesto que los pesos en el arco, como en la cuerda, figs. 111 á 113, son incapaces de obrar de otro modo que verticalmente; mientras que los muros de arranque y los rellenos que forman una gran parte del peso en un arco de mampostería pueden ofrecer resistencias que obren en otras direcciones. Si la carga fuera un líquido, como por ej. agua, sus presiones sobre el arco serían radiales, como las de las partículas de vapor, en una caldera, sobre los tubos de la caldera, y esta condición está probablemente y aproximadamente satisfecha en el caso de una carga de arena limpia, seca; y, menos exactamente, en el caso de un relleno de tierra. Por tanto, aunque la determinación de las líneas teóricas de empuje y de resistencia en un arco se facilita por la suposición de que el arco está exactamente representado por la cuerda invertida, la diferencia entre los dos casos debe tenerse presente cuando se deduzcan conclusiones prácticas de las líneas así encontradas.

**261.** En muchos casos, las líneas teóricas de empuje y de resistencia cortan los intradós ó los trasdós en algunos lugares saliéndose del arco; de modo que éste caería inevitablemente (véase § 255), si no fuera por las resistencias horizontales ó inclinadas ejercidas por las partes superiores de los pies derechos y por los muros y rellenos.

**262.** Por tanto, á fin de determinar la línea de resistencia real, no sólo debemos saber por qué puntos, en la clave y en el arranque, respectivamente, pasan las resultantes H y R, sino que también habremos de averiguar y tomar en cuenta las posibles resistencias horizontales é inclinadas de los muros y relleno. Pero, como esto es de ordinario impracticable, nos contentamos, ó con determinar las líneas teóricas de empuje y de resistencia, como se dispone arriba, y entonces apreciamos, tan bien como se pueda, las resistencias de los arranques; ó bien razonando por analogía con lo que sucede en las construcciones reales. Véase Puentes de piedra.

**263.** Si la cuerda invertida representara exactamente la verdadera línea de empuje en un arco de mampostería, las claves, en arcos elípticos ó muy rebajados, habría que hacerlas anchas fuera de toda regla, á fin de que la línea de resistencia no dejara de pasar en ninguna parte por el tercio medio de su espesor (véanse §§ 145, etc.); y quizás sería entonces racional hacer corresponder aproximadamente el perfil del arco con la línea de empuje, que habitualmente se acerca á una parábola. Pero, debido á las resistencias del arranque, la línea real de empuje, aun en arcos semicirculares, raras veces excede en mucho al tercio medio.

**264.** Cuando hay un muro ó un profundo relleno continuo, sobre un arco, el muro ó el relleno formarán sobre él un arco natural, como lo indican las líneas

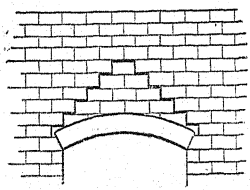

**Fig. 117.**

quebradas en la fig. 117; y sólo esa porción debajo de ese arco caería si se suprime el arco. Por tanto, sólo á esta porción se la considera con propiedad, ejerciendo presión sobre el arco.

**265.** Despreciando la fuerza de la mezcla, la inclinación de cada junta entre dos piedras de arco debe naturalmente ser tal, que el ángulo, entre el empuje, en cualquier junta y una normal á esa junta, sea menor que el ángulo de frotamiento. Véanse §§ 183, 184.

**266.** Acontece á menudo que los arranques ó el relleno son de menor densidad que el arco. En tales casos, á fin de facilitar el modo de encontrar los centros de gravedad de los segmentos, podemos, antes de dividir el semiarco y sus arranques en segmentos verticales (§ 252), darle á la parte más liviana del relleno una altura menor, tal que suponiéndola de igual densidad que el arco, pese lo mismo que pesaba el relleno. Las áreas de los varios segmentos (como se ven en el dibujo) así reducidos, pueden entonces tomarse como representando sus pesos. Así, en

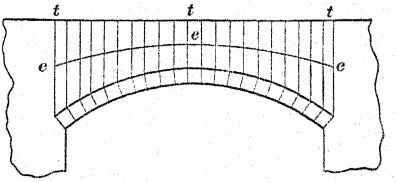


**Fig. 118.**

la fig. 118, *ttt* representa el término del relleno, y la línea curva *eee* representa el término de un relleno de peso igual al anterior por metro corrido y de la misma densidad que el arco. Cuando, como en la fig 119, la carga consiste en una serie de arcos, podemos suponer que el arco principal soporta una serie de pesos concentrados en los pilares de los arcos de arriba.

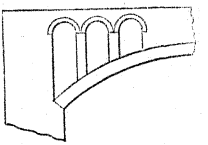

**Fig. 119.**

## La presa de mampostería.

**267.** Una presa debe ser incapaz de resbalar, en su base ó en cualquier plano dentro del cuerpo de la presa, incapaz de volcarse ni de demolerse por el peso del material. No debe tampoco agrietarse en ninguna de sus caras.

**268.** La presa estará segura contra resbalamientos si la resultante de todas la presiones, sobre cualquier superficie, forma, con una normal á esa superficie, un ángulo menor que el ángulo de rozamiento de esa superficie. Véanse §§ 183, etc. En la práctica, la base de la presa se baja bien hasta el cimiento de roca, como se indica en la fig. 122 (*a*), y se evita la continuidad de las juntas, colocando las piedras de modo que alternen las juntas. El ángulo de fricción llega á ser así, en efecto, de 90° y el resbalamiento no puede ocurrir sin cortar las piedras mismas.

**269.** Si el material es suficientemente fuerte para resistir á la trituración, bajo las fuerzas que obran sobre él, y si la resultante de todas las fuerzas que obran sobre cualquier sección cae dentro del cuerpo de la presa, ésta no se volcará. Pero véase § 270.

**270.** Para una presión total dada sobre cualquier sección, la máxima presión por unidad de superficie en la sección será menor cuando la resultante pase por el punto medio de la sección. Véase Centro de presión, §§ 133, etc. Generalmente es imposible conseguir esto, pero la presa debe trazarse de tal suerte, que, bajo la máxima presión, el material no resista más de lo que indique su coeficiente de seguridad para trituración. Si esto se hace y si para todo caso se mantiene el centro de presión dentro de la tercera media (§ 150) de cada sección horizontal en toda la presa, no habrá tendencia á que se raje ninguna de las caras de la presa.

**271.** Sea la fig. 120 un bloque de piedra (que descansa en un cimiento sólido) destinado á sostener de un lado la presión, *p*, del agua tranquila. Por el centro de

gravedad, $g$, del bloque, trácese $o'N$ verticalmente, y representando el peso W del bloque. Entonces, el punto, $s$, donde $o'N$ encuentra el cimiento, es el centro de presión del bloque sólo, esto es : sin el agua.

**272.** Sea $h$ la profundidad del agua detrás del bloque y supongamos el bloque de un m de largo, medido normalmente al papel. Entonces la cantidad, en kg, de la presión del agua, contra la cara posterior vertical, $ab$, es $p = 1{,}000h \times \frac{1}{2} h$ (un m cúb de agua pesa 1,000 kg) y su centro de presión está á una profundidad, $d = \frac{2}{3} h$ debajo de la superficie del agua,

**273.** Componiendo $p$ con W (§ 35), obtenemos R como resultante de ellas, y $r$ como centro de presión sobre el cimiento cuando el bloque está sosteniendo la presión del agua.

**274.** Supongamos que la fig. 121 represente varios bloques semejantes superpuestos. Sea

$g_1$ = centro de grav; $p_1$ = centro de pres de agua, para bloque 1;
$g_2$ = — — $p_2$ = — — — bloques 1 y 2 combinados;
$g_3$ = — — $p_3$ = — — — 1, 2 y 3 comb., etc.,

Entonces, hallando $r_1$ y $s_1$; $r_2$ y $s_2$; $r_3$ y $s_3$, etc., para las juntas 1–2, 2–3, 3–4, etc., como antes, tenemos los puntos $r_1$, $r_2$, $r_3$, etc., en la línea de resistencia para la presa llena y los puntos $s_1$, $s_2$, $s_3$, etc., en la línea de resistencia para la presa vacía.

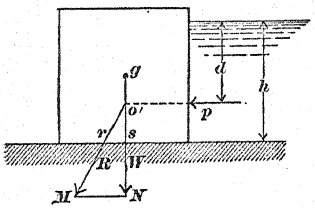


Fig. 120.

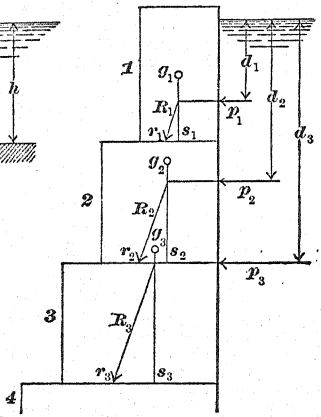


Fig. 121.

**275.** En la fig. 122 ($a$) las curvas $uu$ y $dd$, indican los límites, aguas arriba y aguas abajo, respectivamente, de la tercera media del plano que separa cada dos bloques, suponiendo el perfil vertical atrás; y los puntos $r_1 \ldots r_7$ y $s_1 \ldots s_7$ son puntos en las correspondientes líneas de resistencia para la presa llena y la presa vacía respectivamente.

**276.** Si bien la teoría requeriría que la sección transversal de la presa terminara en ángulo agudo arriba, siempre se hace con cierto espesor, como se indica en la fig. 122 ($a$).

**277.** Para la junta 2-3 podemos suponer que el bloque 2 se había trazado, al principio, rectangular, como en la línea de puntos $ca$, fig. 122 ($c$) (que muestra los bloques 1 y 2 en mayor escala); pero esto hace que, el centro de presión, $r'$, para la presa llena, caiga fuera de la tercera media de la base estrecha, $ab$. Nosotros estudiamos por tanto la forma trapezoidal $cib$, con su base más ancha $ib$, y encontramos que, con ésta, el centro de presión, $r_2$, aunque más lejos de la cara interior que antes, cae dentro de la tercera media de dicha base más ancha. El resto del perfil se determina por ensayos semejantes.

**278. Método gráfico.** Supóngase que la sección transversal se divida, por sec-

ciones horizontales, en numerosos bloques, 1, 2, 3, 4, etc., de una profundidad aproximada=al ancho superior de la presa. En la fig. 122 (*b*) trácese 0–1, 1–2, 2–3, 3–4, etc., verticalmente, para representar, en escala, los pesos de los varios bloques, respectivamente, y 0–1′, 1′–2′, 2′–3′, 3′–4′, etc., horizontalmente, para representar las presiones del agua contra dichos bloques, respectivamente; y trácese 1′–1, 2′–2, 3′–3, 4′–4, etc., representando, en intensidad y en dirección, as presiones totales inclinadas sobre las juntas 1–2, 2–3, 3–4, etc., y sobre la base, respectivamente. Así, 2′–2 representa la resultante de la presión del agua (véase Hidrostática) sobre los bloques 1 y 2 y el peso combinado de esos bloques. Desde los puntos $o_1$, $o_2$, etc., fig. 122 (*c*), donde se encuentran esas dos fuerzas en cada caso, trácense $o_1r_1$, $o_2r_2$, etc., paralelas, respectivamente, á 1′–1, 2′–2, etc., fig. 122 (*b*), hasta la junta correspondiente. Así obtenemos puntos, $r_1 \ldots r_7$, de la línea de resistencia para el caso en que la presa se llene á la profundidad supuesta. Lo que precede se refiere al diagrama para una presa ya diseñada. Al diseñar, empezamos por supuesto por arriba y trazamos las líneas 0–1, 0–1′ y 1′–1 en la fig. (*b*) para el primer bloque; luego las líneas 1–2, 1′–2′, y 2′–2, para el segundo bloque, y así sucesivamente; haciendo los cambios necesarios, como en el § 277.

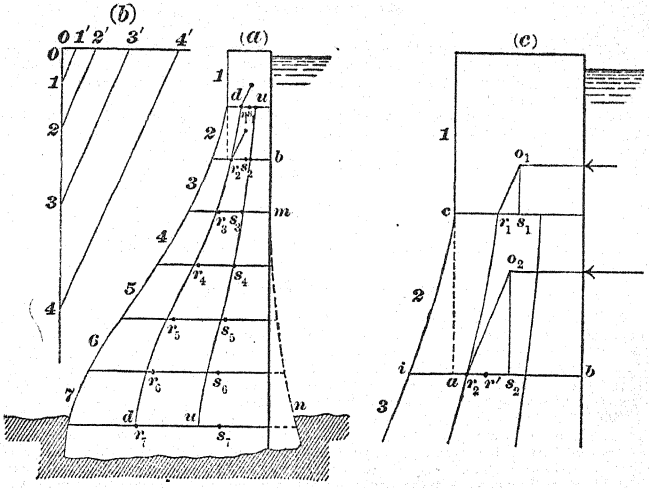


**Fig. 122.**

**279.** A fin de que la línea de resistencia, $r_1 \ldots r_7$ para la presa llena, pueda caer bien dentro de la tercera media es necesario á veces adoptar una sección transversal algo pesada; pero en vista del inminente peligro que implica la más pequeña apertura del lado de las aguas (véase § 281), es bueno aquí inclinarse del lado seguro.

**280.** A medida que este procedimiento se lleva más hacia abajo, el ángulo formado entre la cara exterior y la vertical, se hace considerable; y la tercera media, en cada una de las juntas inferiores, se va alejando más del paramento interior. Los centros de presión, $s_1 \ldots s_7$, para presa vacía, pueden pues caer más acá de la tercera media, del lado del agua, como se indica en las juntas 5–6 y 6–7. Para obviar esto, se le da á veces un perfil curvo á la cara interior, como en *mn*.

## Advertencias prácticas.

**281. La adopción de condiciones ideales** es particularmente peligrosa en el caso de presas de mampostería. Así, cualquier compresión del material sobre la cara exterior puede abrir grietas en la cara interior; y el agua, entrando en estas

grietas, ejercerá una acción semejante á la de una cuña, empujando la línea de resistencia hacia afuera, aumentando así la tendencia á la trituración del lado afuera y la formación de nuevas grietas en la cara interior. Además, si se han dejado algunas juntas relativamente blandas, el agua, penetrando así en la presa ó debajo de ella, aumenta la tendencia á resbalar, disminuyendo no sólo el peso efectivo de las porciones superiores, sino también obrando como lubricante en la junta donde penetra.

Hase insinuado que algunos defectos de las presas pueden haber sido ocasionados, en parte, á lo menos, por el vacío formado enfrente de la cara exterior entre la lámina de agua que cae y dicha cara.

**282.** Teóricamente, las curvaturas de arcos, presas, y otras construcciones compuestas de bloques, pueden hallarse por medio de las fórmulas de los §§ 162–167 del capítulo Armaduras; pero, á causa de la incertidumbre en cuanto á los valores de los coeficientes de elasticidad, E, de las piedras de construcción y de la mezcla, y á causa de la relativa inexactitud del acabado en obras de albañilería, las fórmulas son de poco valor práctico en tales casos.

## EL TORNILLO

**283.** El tornillo es un plano en espiral. « La potencia » describe una espiral, al extremo de un brazo de palanca, mientras la resistencia (ó « peso ») se mueve á lo largo del eje del tornillo. Durante el tiempo en que la fuerza hace una revolución, la resistencia recorre el « paso » del tornillo.

**284.** En consecuencia, si P=potencia, $w$=peso, $d$=paso, $l$=brazo de palanca, $v$=velocidad rectilínea del peso, y V=velocidad circular de la potencia, tenemos, teóricamente *:

$$\frac{w}{\mathrm{P}} = \frac{\mathrm{V}}{v} = \frac{2\pi l}{d}.$$

## FUERZAS QUE OBRAN SOBRE VIGAS Y ARMADURAS

### Condiciones de equilibrio.

**285.** Para el equilibrio, en vigas y armaduras es necesario y suficiente que las fuerzas resistentes, ejercidas por el material de la construcción y los momentos de esas fuerzas contrarresten las fuerzas externas ó destructivas y sus momentos. Aquí estudiamos principalmente las fuerzas destructivas. Para las fuerzas resistentes, véanse Resistencias, en capítulo Armaduras; y Vigas ó Resistencia transversal, en capítulo Resistencia de Materiales.

**286.** Las fuerzas destructivas son (1) los pesos sobre la construcción, inclusive su propio peso, cargas « vivas » ó movibles, como viento, etc., y (2) las reacciones de los apoyos. Aquí discutiremos la acción de los pesos verticales solamente, inclusive (*a*) el peso muerto, ó el peso de la construcción misma; junto con el de la calzada, etc., y (*b*) el peso vivo movible ó extraño de vehículos, trenes, personas, etc. La acción de los pesos ó cargas horizontales (viento, fuerza centrífuga, etc.), se rige por leyes semejantes y se discute en Armaduras.

**287.** Sea la fig. 123 (*a*) un *cantilever* ** que descanse en un opoyo, *b*, y lleve un peso, W, en su extremo exterior *a*. Se impide que el *cantilever* gire alrededor de *b* por la tensión T, de una cadena horizontal y por la compresión C, en un puntal horizontal***. Despreciando el peso del cantilever mismo éste está solicitado por cuatro

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* Despreciando la fricción, que, sin embargo, modifica mucho el resultado.

** Conservamos el término *cantilever*, porque, además de no tener equivalente en español, ya está muy usado en construcción. Los términos *can*, *canecillo*, ó consola, no lo traducen como lo han creído algunos. (*N. del T.*).

*** En las figs. 123 á 127 inclusive, y las figs. 132 y 133, que muestran *cantilevers*, vigas y partes de vigas solicitadas por pesos, por reacciones, por tracciones de cadenas y empujes de postes, las flechas indican fuerzas que obran *en el cantilever ó viga ó en sus segmentos*, y no fuerzas que obran sobre el peso, los soportes, las cadenas conectadoras ó postes. Así, la tensión en una cadena tiende á *juntar* los dos cuerpos que une. En consecuencia, en estos casos, las correspondientes flechas se señalan una á otra. Por otra parte, la compresión en un puntal tiende á separar los dos cuerpos entre los cuales obra. De aquí que sus dos echas aparezcan alejándose una de otra.

fuerzas externas, que forman dos pares; un par que consta de dos fuerzas verticales á saber: el peso, W, y la reacción, R', del apoyo; el otro par que consta de dos fuerzas horizontales, á saber: la tensión T, cerca del tope, y la compresión C, cerca

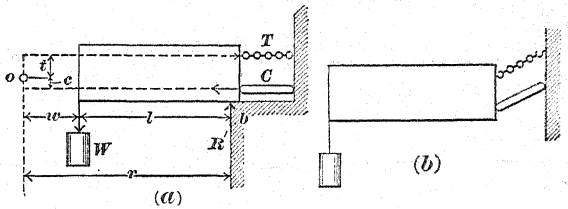


**Fig. 123.**

de la parte inferior. Si no fuera por la reacción, R', del soporte, *b*, el peso, W, empujaría el cantilever hacia abajo, como se indica en la figura 123 (*b*).

**288.** En la fig. 123 (*a*) tenemos:

Suma algebraica de fuerzas verticales $= R' - W = 0$;
— — — horizontales $= T - C = 0$;
— — momentos respecto de cualquier punto, como *o*,

$$W \cdot w - R' \cdot r + T \cdot t + C \cdot c = 0.$$

**289.** Si, como en la fig. 124, las fuerzas horizontales son ejercidas en el extremo distante del apoyo, y á la misma distancia que antes, sus intensidades y sentidos deben permanecer respectivamente los mismos; pero ahora tenemos la compre-

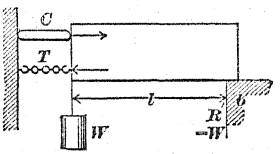


**Fig. 124.**

sión C, en lo alto, y la tensión T por debajo. Ahora, si la fig. 123 se invierte; R' obra como el peso, y W, como la reacción hacia arriba; y tenemos, como en la fig. 124, compresión, C, arriba, y tensión T abajo. Así, la fig. 124 es prácticamente la fig. 123 invertida.

**290.** La condición descrita en el § 289, fig. 124, representa también la condición en cada segmento, A, B, de una viga, figs. 125 (*a*) y (*b*) ó figs. 126 (*a*) y (*b*), sostenida en ambos extremos y llevando un peso concentrado W+W, fig. 125, ó W+*w*, fig. 126.

**291.** Supongamos la viga, fig. 125 (*a*), ó fig. 126 (*a*), dividida en dos *cantilevers* ó vigas parciales, como en la fig. 125 (*b*) ó la fig. 126 (*b*), sosteniendo cada parte

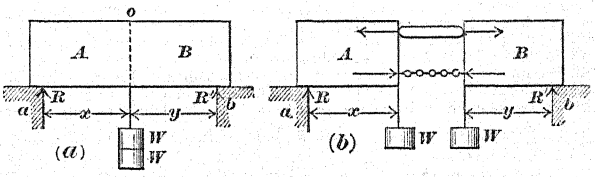


**Fig. 125.**

en su extremo, una parte del peso original. (Véase § 292.) Los esfuerzos en el pun-

tal y la cadena, figs. (*b*), reemplazan los esfuerzos del material (situado en la línea de puntos) del *cantilever* ó viga, figs, (*a*) En una armadura, estas fuerzas son ejercidas por las cuerdas; en una viga, por las particulas ó fibras en toda la sección.

**292.** Si, como en la fig. 125 (*a*), el peso está en el centro de la abertura, las longitudes, $x$ é $y$, de los *cantilevers*, fig. 125 (*b*), son iguales, como lo son también los pesos W = W, que sostienen. Pero si, como en la fig, 126 (*a*), la carga, W + *w*,

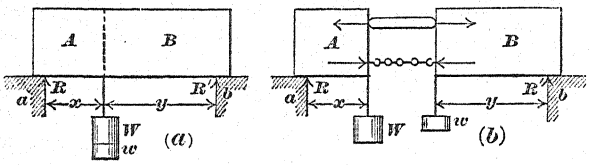


**Fig. 126.**

en la viga no está en el centro de la luz, los pesos parciales W y *w*, que se suponen sostenidos en los extremos de las dos vigas parciales, respectivamente, fig. 126 (*b*), son desiguales, é inversamente proporcionales á los brazos de palanca de sus respectivos apoyos. De aquí que los momentos de los dos pares opuestos son iguales. La reacción de cada apoyo es igual al peso cargado por el *cantilever* que descansa sobre aquél.

## Reacciones de los extremos.

**293.** Si en un *cantilever*, fig. 127, no hay sino un apoyo vertical, la reacción, R', de ese apoyo, es = la suma de todos los pesos, inclusive el peso del *cantilever* mismo; y la reacción debida á cada peso parcial es = á dicho peso parcial. Así si B = peso del *cantilever*,

$$R = W + w + B.$$

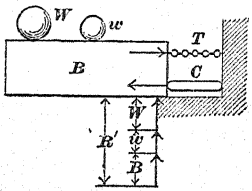


**Fig. 127.**

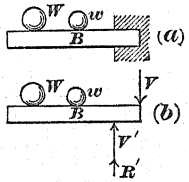


**Fig. 128.**

**294.** La reacción, R', no debe confundirse con otras fuerzas verticales. Así, un *cantilever* está á menudo sostenido como en la fig. 128 (*a*). El par, compuesto de dos fuerzas horizontales T y C, fig. 127, está reemplazado entonces por un par compuesto de dos fuerzas verticales, V y V', fig. 128 (*b*); una de las cuales, V', coincide con la reacción, R'. Aquí, R' + V', que obran hacia arriba forman la antiresultante de W, *w*, B y V que obran hacia abajo.

**295.** En una viga, fig. 129, la suma de las dos reacciones de los extremos es = la suma de todos los pesos, inclusive el peso de la viga misma.

**296.** La reacción, R, del apoyo izquierdo, *a*, fig. 129, debida al peso W, solo es $R = W \cdot \frac{y}{l}$ (véase § 17), y la reacción, R', del soporte derecho, *b*, es $= W - R = W \cdot \frac{x}{l}$. Si el peso es central, $\frac{x}{l} = \frac{y}{l} = \frac{1}{2}$. y $R = R' = \frac{W}{2}$.

**297.** Gráficamente, la fig. 156 supone un peso concentrado, W (no mostrado), que ha de colocarse en la viga en cualquier punto, como *c*. Trácese *a'a''* y *b'b''*,

verticales y cada una = W. Unanse $a''b'$, y, $a'b''$, y trácese $gh$, verticalmente por $c$. Entonces la ordenada, $c'g$, á la línea superior, $a''b'$, y la ordenada, $c'h$, á la línea

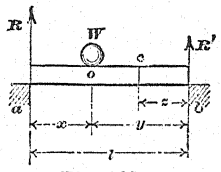


Fig 129.

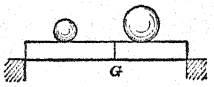


Fig. 130.

inferior, $a'b''$, dan las reacciones del extremo de la izquierda y del de la derecha, R y R', respectivamente.

**298.** Cuando, como en la fig. 130, hay dos ó más pesos (en que el peso de la viga puede ó no estar incluído). las reacciones debidas á cada peso ó carga pueden obtenerse separadamente, dando las sumas de estas reacciones, las reacciones totales; ó encuéntrese primero el centro común de gravedad, G, de todos los pesos (véanse §§ 125, etc.), y luego calcúlese la reacción como para un solo peso, W, fig. 129; todos los pesos elementales combinados cuyo centro de gravedad está en G, se suponen concentrados allí.

**299.** En una viga, fig. 131, sometida á un peso, W, uniformemente distribuído sobre cualquier parte de la luz, sea G el centro de gravedad del peso, y sean $x$ é $y$ los segmentos de la luz, $l$, á la izquierda y derecha de G respectivamente.

Luego, despreciando el peso de la viga, $R = W\frac{y}{l}$; y $R' = W - R = W\frac{x}{l}$.

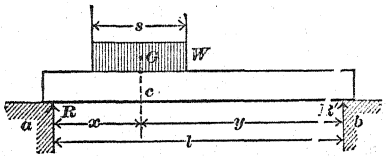


Fig. 131.

**300.** Si el peso está uniformemente distribuído sobre toda la abertura ó luz su centro de gravedad está en el centro de la abertura y tenemos :

$$\frac{x}{l} = \frac{y}{l} = \frac{1}{2}; \quad \text{y } R = R' = \frac{W}{2}.$$

## Momentos y esfuerzo cortante.

**301.** A fin de determinar qué fuerzas internas se requieren, en cualquier punto de la abertura ó luz, para el equilibrio, podemos suponer que el *cantilever* ó la viga está cortado en dos por una sección, *cc*, figs. 132 ó 133, en dicho punto, é inquirir qué fuerzas deben aplicarse, en la sección, para mantener en equilibrio los dos segmentos, E y F, en que la sección, *cc*, divide la luz, fig. 132, ó esa parte de la abertura entre el peso y un soporte, fig. 133. Las fuerzas, así determinadas, son con evidencia equivalentes á las realmente ejercidas con el mismo fin, por la viga misma

**302.** En las figs. 132 y 133, los momentos de los pesos y de las reacciones, ó los momentos externos ó de flexión, están indicados por flechas, *por debajo* del *cantilever* y de la viga respectivamente, mientras que los momentos resistentes de las fuerzas internas están indicados por flechas *dentro* de dichos cuerpos.

**303.** *En el cantilever*, fig. 132, el peso, $w = 4$ lbs, distante 6 p de la sección, *cc*, produce allí un momento izquierdo ó negativo de $6 w = 6 \times 4 = 24$ p-lbs. Por tanto, para el equilibrio, el puntal horizontal y la cadena, en *cc*, deben ejercer un momento

resistente á la derecha ó positivo, de 24 p-lbs; y, estando á 2 pies de distancia, deben ejercer una tensión, T, y una compresión, C, de $\frac{24}{2} = 12$ lbs cada uno. En el apoyo, el momento del peso $= 9\ w = 9 \times 4 = 36$ p-lbs; y $T' = C' = \frac{36}{2} = 18$ lbs *.

**304.** Pero, considerando sólo las fuerzas hasta ahora discutidas, encontramos en el segmento derecho, F, obrando en *cc*, un par izquierdo, $= d \times T = d \times C = 2 \times 12 = 24$ p-lbs; y en el apoyo, un par derecho, $= d \times T' = d \times C' = 2 \times 18 = 36$ p-lbs.

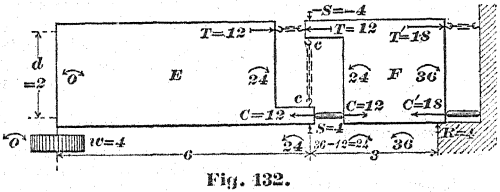


Fig. 132.

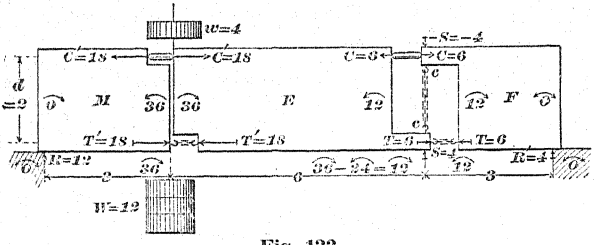


Fig. 133.

En otros términos, habría un exceso no equilibrado de momento derecho, $= 36 - 24 = 3$ R′$= 3 \times 4 = 12$ p-lbs, obrando sobre F. F recibe también, en el apoyo, la reacción hacia arriba R′, $= 4$ lbs, de ese apoyo. De manera semejante, el par, $d \times T = d \times C$, en *cc*, ejerce, en el segmento izquierdo, E, un momento derecho aparentemente no equilibrado de $2 \times 12 = 6\ w = 6 \times 4 = 24$ p-lbs, y E recibe, del peso *w*, un empuje hacia abajo $= 4$ lbs.

**305.** Para el equilibrio, por tanto, la cadena vertical en *cc* debe ejercer una tensión $= S = w = R' = 4$ lbs, que empuje á F hacia abajo y á E hacia arriba. La tensión hacia abajo — S, obrando sobre F en *cc*, forma, con la reacción, R′, del soporte, un par izquierdo $= 3$ R′$= 3 \times 4 = 12$ p-lbs, equilibrando el exceso de momento derecho que obra sobre F; mientras que la tensión hacia arriba, +S, que obra sobre E en *cc*, forma, con el peso, *w*, un par izquierdo $= 6w = 6 \times 4 = 24$ p-lbs, equilibrando el exceso de momento derecho, que obra sobre E.

**306.** De modo semejante, si suponemos el *cantilever* cortado por una sección en cualquier otro punto, hallaremos que se requiere una fuerza vertical $= S = w = R'$, obrando hacia arriba sobre el segmento izquierdo y hacia abajo sobre el segmento derecho, á fin de mantener el equilibrio y transmitir el peso, *w*, al apoyo, de modo que los dos segmentos puedan obrar unidos como un solo *cantilever*. Esta fuerza, S es llamada esfuerzo cortante. Véanse §§ 325, etc. Sin ella, la sección E caería, como en la fig. 123 (*b*).

**307. En la viga**, fig. 133, el peso total es 16 lbs; y sus distancias, 3 p y 9 p, desde el soporte izquierdo y derecho respectivamente, estando como 1 á 3, las

* *N. del T.* — Empleando el kilogramo como unidad de peso, y el metro para las distancias, los momentos vendrían en kilogramos × metros; es decir, en *kilogrametros* y no en pies-lbs. Pero como para la comprensión de la teoría en nada influye este cambio hemos dejado las medidas inglesas. Así vamos de acuerdo con los números (interiores) de los clisés.

reacciones de los extremos (§§ 293, etc.) estarán como 3 á 1; ó, $R = 16 \times \frac{3}{4} = 12$ lbs; $R' = 16 \times \frac{1}{4} = 4$ lbs. Nosotros consideramos, por tanto, la viga como cortada por una sección donde está el peso (lo mismo que en *cc*), y el peso total de 16 lbs como dividido en dos porciones; una, $W = R = 12$ lbs, inherente al segmento del extremo M; y la otra $w = R' = 4$ lbs sostenida por el segmento medio, E. Aquí, como en la fig. 132, los segmentos E y F juntos forman un *cantilever* de 9 p de largo, cargado con un peso, *w*, de 4 lbs, en su extremo; pero, en la fig. 133, las fuerzas resistentes horizontales, T' y C', que sostienen el *cantilever* entero (E+F), están ejercidas, no en el apoyo, como en la fig. 132, sino en el extremo más distante del apoyo.

**308.** Tenemos, en consecuencia, en la fig. 133, momentos positivos * (*como el del minutero*) y negativos (contra el del minutero), obrando sobre E, cerca del extremo inferior de *cc*, así :

$$dC' - 6w - dC = 2 \times 18 - 6 \times 4 - 2 \times 6 = 0.$$

Aquí, $dC' - 6w = 36 - 24 = 12$, es momento de flexión, y $(-dC, = -12)$, es momento resistente.

Tenemos, también, en *cc* el esfuerzo cortante, $S = w = R' = 4$ lbs.

**309.** En la fig. 132 ó en la fig. 133, considerando que el segmento se extienda desde el peso á uno ú otro apoyo (en la fig. 132 no hay sino un solo segmento), se verá que, en el extremo libre de cualquier segmento semejante, las fuerzas horizontales son cero, y que ellas aumentan uniformemente hasta un máximum en el otro extremo del segmento. Así, en la fig. 132, aumentan uniformemente desde 0, en el extremo cargado ó libre, hasta 18 lbs en el apoyo; mientras, que en la fig. 133, aumentan uniformemente desde 0, en cada apoyo, ó extremo libre, hasta 18 lbs, en la carga.

**Momentos en los cantilevers.** (Véanse también §§ 285, etc., pág. 458.)

**310.** En un *cantilever*, fig. 134, cada peso ejerce, sobre cualquier punto situado entre él y el apoyo, un momento = su peso × la distancia horizontal de su centro de gravedad á dicho punto; y el momento total, respecto á cualquier punto, es la suma de los momentos de los varios pesos respecto á ese punto. Así, despreciando el peso del *cantilever* mismo, tenemos :

respecto al punto *b*, momento $= Ax + By$; respecto á *d*, momento $= Az$; para *c*, momento $= Am + Bn$; para A, ó cualquier punto más allá de A, momento $= 0$.

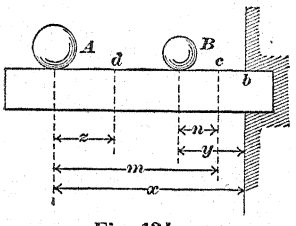


**Fig. 134.**

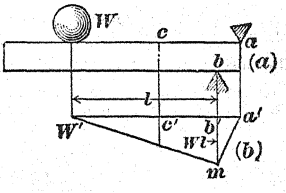


**Fig. 135.**

**311.** En un *cantilever*, fig. 135, la palanca máxima de cualquier peso W, es evidentemente su distancia, *l*, al apoyo, *b*. De aquí, que el momento de flexión

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* *N. del T.* — Cuando el movimiento rotatorio engendrado por un par en un plano visto de frente, se efectúa de *izquierda á derecha pasando por arriba*, precisamente como se mueven las manecillas de un reloj, lo llaman los ingleses *clockwise*; es decir: « á modo del reloj »; y al movimiento contrario, *counterclockwise*; es decir: « contra el modo del reloj ». Además extienden estos nombres á la sola tendencia del movimiento en dichos sentidos y aun al par y á su momento; considerando además á los primeros, como positivos, y á los segundos, como negativos. En el deseo de enriquecer la terminología española en el ramo de ingeniería, donde no abundan los términos de la sintética claridad y expresión de éstos, hemos resuelto emplear en lugar del *clockwise*, « *como el del minutero* », y en lugar del *counterclockwise*, « *contra el minutero* », ya que sería muy largo decir: « en el sentido del de las agujas de un reloj » y « en sentido contrario al de las agujas de un reloj ».

máximo de cualquier peso en un *cantilever* está en el apoyo, y es=W*l*. De este máximo, el momento disminuye uniformemente hasta 0 en el peso y en *a*.

En la fig. 134, el extremo, *ba*, está uniformemente cargado, y la disminución del momento, entre *b* y *a* sigue las ordenadas de una parábola, como en *mk'*, fig. 137.

**311 a.** En los *cantilevers* y vigas, figs. 129 y 134, los momentos (como el del minutero) * de las fuerzas á *la izquierda* de un punto dado, *c*, se consideran *positivos*, y viceversa. Los momentos *positivos extienden* las fibras *inferiores* y *comprimen* las *superiores*, y viceversa.

**312.** En la fig. 135 (*b*), (*cantilever* con un solo peso, W), trácese por una escala *b'm*, =mom máx y trácense *m*W' y *ma'*. Entonces, para cualquier punto, *c*, el momento=á la ordenada en *c'* respecto á la línea *m*W'.

**313.** En un *cantilever*, fig. 136 (*a*), con dos ó más pesos concentrados, W y *w*, sean :

*b'm*, fig. 136 (*b*), =momento de W, en el apoyo;
*b'm'*, fig. 136 (*b*), =momento de *w*, en el apoyo.

Entonces, para ambos pesos, W y *w*, (despreciando el peso de la viga),

en *d*, momento= de W solo, =ordenada en *d'*;
en *c*, momento=suma de los momentos de W y *w*,
=suma de las dos ordenadas, *c'n* y *c'n'* en *c'*.

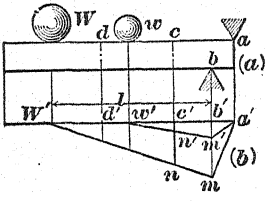


**Fig. 136.**

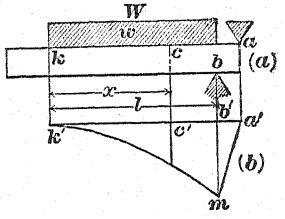


**Fig. 137.**

**314.** En un *cantilever*, fig. 137 (*a*), bajo un peso, W, uniformemente distribuído sobre una longitud, *l*, que empieza en el apoyo, *b*, el momento máximo, en el soporte *b*, es=W*l*/2.

En la fig. 137 (*b*) hágase *b'm*=dicho momento máximo y trácese una semiparábola *mk'*, con el vértice en *k'*. Luego, en cualquier otra sección, *c*, el momento está representado por la ordenada *c'* de dicha parábola, y es=$w \times \frac{x}{2}$, donde *w*=el peso de esa parte de W más allá de *c*, y *x*=el largo de dicha parte.

En *k*, ó en cualquier punto más allá de *k*, el momento=0.

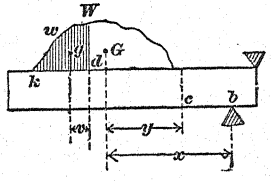


**Fig. 138.**

**315.** En la fig. 138, despreciando el peso del *cantilever* mismo, sea W el peso de toda la carga, y *w* el de la parte sombreada, concentrados en sus respectivos centros de gravedad G y *g*. Entonces,

respecto al punto *b*, el momento=W . *x*;
— — *c*, — =W . *y*;
— — *d*, — =*w* . *v*;
— — *k*, ó cualquier punto más allá de *k*, el momento=0.

* *N. del T.* — Véase la *N. del T.* al pie de la página 463.

## Momentos en las vigas.

**316.** En una viga, fig. 129, la reacción hacia arriba que cada apoyo ejerce, da para cualquier punto un momento = reacción × distancia de dicho punto al apoyo; pero cualquier peso, entre el punto elegido y el apoyo, ejerce un momento contrario = peso × distancia del peso al punto. Así,

$$\text{para } c, \text{ momento} = R' \,.\, z = R\,(l - z) - W\,(y - z).$$

En cada apoyo, el momento es 0.

**317.** En una viga, fig. 129, cargada con un solo peso concentrado, W, el momento, $R' \,.\, z$, en cualquier punto, $c$, es $= R' \,.\, z = W\,\frac{x}{l} \,.\, z = R\,(l - z) - W\,(y - z) = W \,.\, \frac{y}{l}\,(l - z) - W\,(y - z)$. En un punto, como $c$, que no está bajo el peso, el momento, $R'z$, es evidentemente menor que el momento, $R' \,.\, y$, para el punto $o$, bajo el peso. En otras palabras, el momento máximo está en el punto, $o$, debajo del peso.

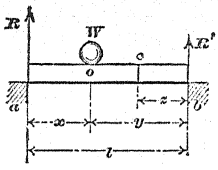


**Fig. 129 (repetida).**

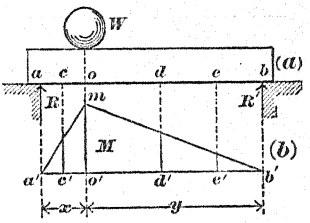


**Fig. 139.**

**318.** Desde el punto, $o'$, fig. 139 ($b$), correspondiente al punto, $o$, fig. ($a$), donde se aplica el peso, levántese una ordenada, $o'm$, igual por escala al momento (máximo), $= R' \,.\, y = R \,.\, x$, en ese punto. Unase $a'mb'$. Entonces las ordenadas hacia $a'm$, ó hacia $mb'$, en cualesquiera puntos, $c'$, $d'$, $e'$, etc., representan por escala el momento para el punto correspondiente, $c$, $d$, $e$, etc., en la luz.

**319.** Cuando el peso, W, está en el centro de la abertura, $l$, fig. 140, la reacción en cada extremo es $= \frac{W}{2}$. En consecuencia, el momento $s'$, en cualquier punto, $s$; distante $y$ de un apoyo, $b$, es $= \frac{W}{2} \,.\, y$.

En el centro de la luz (es decir, en el punto, bajo el peso central, W), tenemos:

$$\text{momento máximo, } M, = \frac{W}{2} \cdot \frac{l}{2} = \frac{W \,.\, l}{4}$$

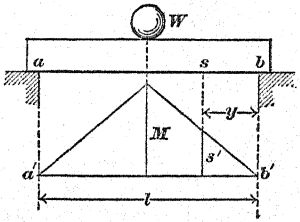


**Fig. 140.**

A fin de que el momento máximo (en $o$, fig. 139) debido á un peso excéntrico, W,

sea igual al momento máximo (en el centro de la luz, $l$) debido á un peso dado C en el centro debemos tener

$$W\frac{x}{l}\cdot y = C\cdot\frac{l}{4}; \quad \text{ó bien } W = C\frac{l}{4}\cdot\frac{l}{xy} = C\frac{\left(\frac{l}{2}\right)^2}{xy}$$

**320.** Cuando hay dos ó más pesos concentrados, $c$, $d$, $e$, fig. 141, trátese cada peso como en la fig. 139, haciendo que cada ordenada corta $m$, $m'$, $m''$, represente el momento máximo de su peso, $c$, $d$, ó $e$ solo. Háganse las ordenadas largas, M, M' y M''=las sumas de los momentos separados, medidos en $c'$, en $d'$ y en $e'$, respectivamente. Luego las ordenadas hacia $a'$ MM'M''$b'$, en cualquier punto, representan el momento total en ese punto, debido á los varios pesos combinados.

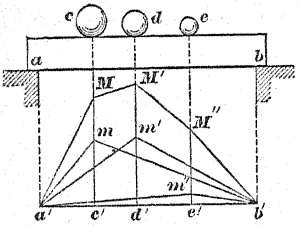


**Fig. 141.**

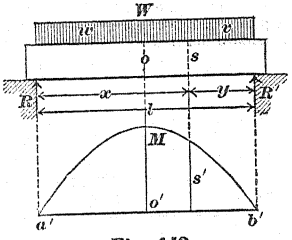


**Fig. 142.**

**321.** En una viga, fig. 142, bajo un peso uniforme, W, que cubra la luz, $l$, el momento máximo está en el centro, y es=

$$R\cdot\frac{l}{2} - \frac{W}{2}\cdot\frac{l}{4} = \frac{W}{2}\cdot\frac{l}{2} - \frac{W}{2}\cdot\frac{l}{4} = \frac{W}{2}\cdot\frac{l}{4} = \frac{W.l}{8}.$$

Hágase o'M, por escala=al momento máximo; y trácese la parábola $a'Mb'$, con el vértice en M. Entonces el momento, en cualquier sección, como $s$, está representado por la ordenada correspondiente, $s'$, á esa parábola.

Sean $w$ y $v$=á las partes de W á la izquierda y derecha de $s$, respectivamente. Entonces, momento en $s = \frac{w}{2}y^* = \frac{v}{2}x^* = \frac{1}{2}$ del momento debido á todo el peso W, concentrado en $s$*.

En uno ú otro apoyo el momento=0.

En la fig. 131, en un punto, $c$, bajo el centro de gravedad de un peso W, uniformemente distribuído sobre una porción, $s$, de la luz, despreciando el peso de la viga, se tiene :

$$\text{momento} = R\cdot x - \frac{W}{2}\cdot\frac{s}{4} = R\cdot x - \frac{W\cdot s}{8} = R'\cdot y - \frac{W\cdot s}{8}.$$

---

* Momento en $s = R'y - v\frac{y}{2} = \frac{W}{2}y - \frac{v}{2}y = \frac{W-v}{2}y = \frac{w}{2}y$; $= Rx - w\frac{x}{2} = \frac{W}{2}x - \frac{w}{2}x = \frac{W-w}{2}x = \frac{v}{2}x$.

Con W concentrado en $s$,

$$\text{momento en } s = W\frac{x}{l}y = wy = W\frac{y}{l}x = vx.$$

**322.** Sean W = al peso total, concentrado ó uniforme, y $l$=la luz. Entonces el momento máximo, M, es el siguiente:

| | | | | |
|---|---|---|---|---|
| *Cantilever.* | Peso, W, en el extremo. | M en el apoyo | | $M = Wl$; |
| — | — uniforme. | — | — | $M = \frac{Wl}{2}$; |
| Viga apoyada †. | — en el centro. | M en el centro | | $M = \frac{Wl}{4}$; |
| — — | — uniforme. | — | — | $M = \frac{Wl}{8}$; |
| Viga fija ‡. | — en el centro. | — | — ó apoyo | $M = \frac{Wl}{8}$; |
| — — | — uniforme. | — | apoyo | $M = \frac{Wl}{12}$. |

**323.** En la viga inclinada, fig. 143, para encontrar las reacciones pueden usarse las distancias inclinadas en vez de las distancias horizontales. Así,

$$\text{Reacción } R' = W \cdot \frac{x}{l} = W \cdot \frac{x'}{l'}.$$

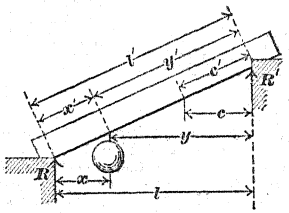


**Fig. 143.**

Pero, para encontrar los momentos de las fuerzas verticales, debemos por supuesto usar las distancias horizontales, no las inclinadas. Así, para $c$ el momento es R'$c$; no, R'$c'$.

**324.** En las vigas curvas se aplican los mismos principios que en las vigas rectas. Así, fig. 144, en $s$, el momento=W . $l$. En la fig. 145 (*a*), la reacción $R' = \frac{W \cdot x}{l}$, y en $s$, momento=R' . $y$. O, como en la fig. 145 (*b*), desde $o$, donde

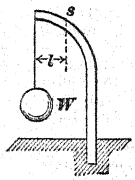


**Fig. 144.**

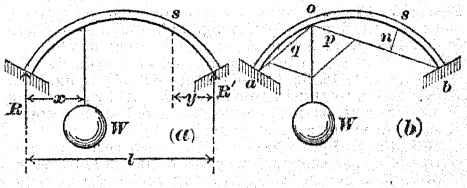


**Fig. 145.**

se aplica el peso, trácese *oa* y *ob* hasta los dos apoyos respectivamente, y, por medio del paralelogramo de las fuerzas, encuéntrense las componentes, $p$ y $q$, de W. Entonces el momento en $s$=$p$ . $n$.

† Viga apoyada en cada extremo, pero fija.
‡ Viga fijada en cada extremo.

## Esfuerzo cortante.

**325.** En la viga, *ab*, fig. 146 (*a*), considérense los segmentos, *ac* y *cb*, á la izquierda y á la derecha respectivamente del plano *nn*. Además de las fuerzas horizontales que obran al través del plano *nn*, hemos visto (§ 305) que necesitamos también, para el equilibrio, una fuerza vertical = á la reacción del extremo izquierdo, R, que obre hacia abajo sobre el segmento izquierdo *ac* y forme un par con R; y que, al mismo tiempo, obre hacia arriba sobre el segmento derecho, *cb*, y sea = al peso, W, menos la reacción del extremo derecho, R'. Esta fuerza es el esfuerzo cortante, S, en la sección *nn*. Puede considerársela como la transmisión de las fuerzas verticales de los pesos á los apoyos ó viceversa.

**326.** Los dos segmentos, *ac* y *cb*, tienden así á separarse verticalmente por la sección *nn*, tendiendo el segmento derecho, *cb*, á bajar, á causa de la preponderancia del peso, W, sobre la reacción del extremo derecho, R'; y á esta tendencia la resiste el esfuerzo cortante, *s*, que es = á la reacción del extremo izquierdo, R. La misma tendencia existe uniformemente entre W y *a*, y en todas partes la equilibra el esfuerzo cortante = S = R.

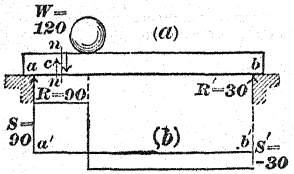


Fig. 146.

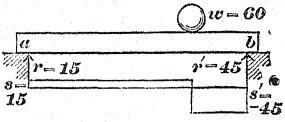


Fig. 147.

**327.** Entre el peso, W, y el apoyo derecho, *b*, también existe esfuerzo cortante uniforme; pero aquí el esf cort S' = á la reacción del extremo derecho, R' = R — W; y, mientras que dicho esfuerzo cortante, S, á la *izquierda* del peso era *derecho, como el del minutero* (véase *N. del T.*, pág. 463) (recibiendo la porción á la *derecha* de cualquier sección, *nn*, la fuerza *hacia abajo*), se le considera *positivo*, ó +; el esfuerzo á la derecha del peso es *izquierdo, contra el minutero* (recibiendo la porción á la *izquierda* de cualquier sección la fuerza *hacia abajo*), y se le considera *negativo*, ó —.

**328.** Los esfuerzos cortantes, S y S', á la izquierda y á la derecha del peso, W, están representados, en la fig. 146 (*b*), donde S, á la izquierda del peso, está trazado por encima de la línea cero que es la *a'b'*, para indicar un esfuerzo cortante *positivo*, y viceversa.

**329.** Comparando las figs. 146, 147 y 148, nótese que entre el apoyo izquierdo, *a*, y el peso W, fig. 146, tenemos esfuerzos cortantes positivos, S = 90, fig. 146, y *s* = 15, fig. 147; de modo que, en la fig. 148, donde *ambos* pesos, W y *w*, están sobre la misma viga, tenemos, entre *a* y W, un esfuerzo positivo total de S + *s* = 90 + 15 = 105. Entre el apoyo derecho, *b*, y el peso *w*, fig. 147, tenemos esfuerzo negativo S' = — 30, fig. 146, y *s'* = — 45, fig. 147; de modo que, en la fig. 148, entre *b* y *w*, tenemos un esfuerzo total negativo = S' + *s'* = — 30 — 45 = — 75. Pero, entre los puntos de aplicación de W y *w*, tenemos S' = — 30, fig. 146, y *s* = + 15, fig. 147; dejando, entre W y *w*, fig. 148, *s* + S' = 15 — 30 = — 15. Si la reacción total del extremo derecho, R' + *r'*, excede *w*, como aquí suponemos, el esfuerzo cort, en cualquier punto entre los dos pesos, W y *w*, fig. 148, es negativo, como se indica, y viceversa.

**330.** En cualquier sección el esfuerzo cortante = á la reacción en cualquier extremo, menos cualesquiera pesos entre ese extremo y la sección dada.

**331.** Si, como en la fig. 149, la reacción del extremo derecho, R' + *r'*, es = al peso, *w*, entonces, la reacción del extremo izquierdo, R + *r*, es = al peso, W; y no hay esfuerzo cortante en ningún punto entre los dos pesos. En otros términos, si se corta la viga en una sección cualquiera entre W y *w*, las fuerzas horizontales solas conservarán el equilibrio, no requiriéndose fuerzas verticales, desde que los dos segmentos no tienen tendencia á moverse verticalmente alejándose el uno del otro.

**332.** Condición semejante existe en cualquier sección en donde el signo del esfuerzo cortante cambia de + á — ó viceversa. Así, si se corta la viga en una

sección inmediatamente bajo W, fig. 146 ó 148, ó bajo *w*, fig. 147, las fuerzas horizontales equivalentes á las resistencias de las fibras en la viga, bastarán para conservar el equilibrio, sin fuerza vertical, ó esfuerzo cortante; no habiendo tendencia de los dos segmentos á moverse verticalmente separándose uno del otro. También, cuando, como en la fig. 149, bajo W y *w*, el esfuerzo cort cambia de valor, en un lado de una sección, para hacerse cero en el otro lado. El esfuerzo cortante en la sección misma es=0.

**333.** Pero cuando, como en la sección bajo *w*, fig. 148, el esfuerzo cort cambia en intensidad sin cambiar de signo, hay un esfuerzo=al *menor* de los dos en los lados opuestos de la sección, pues éste es la intensidad del esfuerzo cortante al través de la sección; ó dicho de otro modo, la tendencia de los dos segmentos á moverse allí verticalmente alejándose uno del otro.

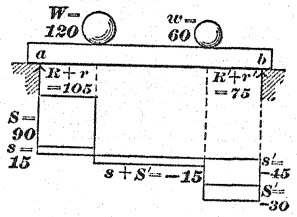


Fig. 148.

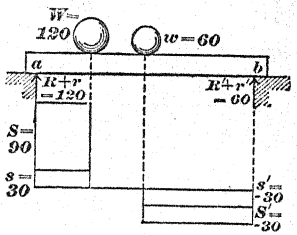


Fig. 149.

**334.** Con cualquier número de pesos, si llamamos X á la porción del peso total á la izquierda de cualquier sección, y á la porción á la derecha de la misma sección la llamamos Y, encontraremos que el esfuerzo cortante en la sección es igual á la diferencia entre la parte de X que va al apoyo derecho, *b*, y la parte de Y que va al apoyo izquierdo, *a*.

**335.** Con un peso, W, fig. 150 (*a*), uniformemente distribuído sobre la abertura toda, el esfuerzo cort máximo, $= R = R' = \frac{W}{2}$, obra en cada apoyo *a* y *b*. El mínimo esf cort=0 está, en el centro, *c*, de la abertura, centro donde está también el máximo del momento de flexión. Véanse § 321 y fig. 142.

En cualquier punto, *d*, el esfuerzo cortante está dado por la ordenada correspondiente *d'*, fig. 150 (*b*). Véase Relación entre «el momento y el esfuerzo cortante» §§ 359, etc.

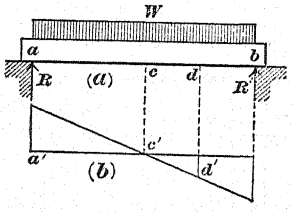


Fig. 150.

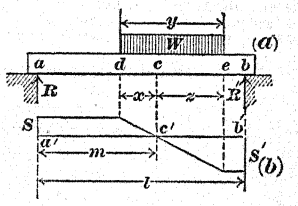


Fig. 151.

**336.** Con un peso, W, uniformemente distribuído sobre cualquier parte, *y*, de la luz, fig. 151 (*a*), encuéntrense las reacciones de los extremos, R y R', como en el § 299. Luego

entre *a* y *d*, el esfuerzo cortante=S =R;
— *e* y *b* — — =S'=R';
en *c*, — — =0;

$$x = dc = y.\frac{R}{W};\quad z = y - x = ce = y.\frac{R}{W}.$$

**337.** Cuando la parte cargada, $y$, de la luz, empieza en uno de los apoyos, $b$ fig. 152 ($a$), entonces, puesto que $R = W\frac{\frac{1}{2}y}{l} = W\frac{y}{2l}$, tenemos

$$x = dc = y.\frac{R}{W} = y\,\frac{W\frac{y}{2l}}{W} = y\,\frac{y}{2l} = \frac{y^2}{2l}.$$

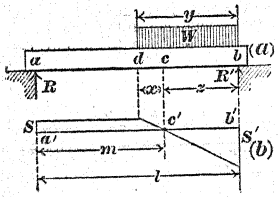


Fig. 152.

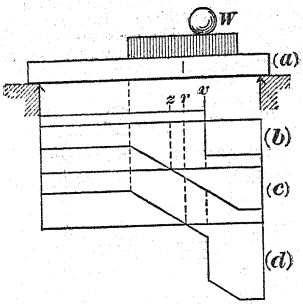


Fig. 153.

**338.** Cuando se agrega un peso concentrado, W, fig. 153, á un peso uniformemente distribuído sobre la luz ó abertura entera, ó sobre parte de ella, cada peso produce los mismos esfuerzos cortantes que si estuviese solo. Los debidos á W están representados en la fig. 153 ($b$), mientras los debidos al peso uniforme están representados en la fig. 153 ($c$). El esfuerzo cortante resultante, debido á ambos pesos combinados, está representado en la fig. 153 ($d$). Nótese que, entre $v$ y $r$, la adición de W, con su esfuerzo cortante positivo, *reduce* el *negativo* debido al peso uniforme, y que, entre $r$ y $z$, la adición de W invierte el esfuerzo negativo; también que cambia el punto cero de $z$ á $r$.

**Para vigas continuas,** véase Vigas en el capítulo « Resistencia de Materiales ».

## Diagramas de influencia.

**339.** Las reacciones de los extremos, debidas á un peso dado, y por consiguiente los momentos, efectos cortantes y tensiones, producidos, en cualquier punto dado de una abertura ó luz, por dicho peso, varían con la posición del peso respecto á los apoyos. Llámase diagrama ó línea de influencia, un diagrama, figs. 154 ($b$), 155 ($b$), 156 ($b$), que muestra los cambios así producidos para cualquier punto dado.

### Diagrama de influencia para los momentos.

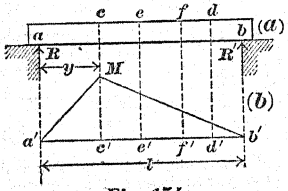


Fig. 154.

**340.** Así, en la fig. 154 ($b$), $a'Mb'$ es el diagrama de influencia de los momentos para el punto, $c$, bajo un solo peso concentrado, W *.

* Como el peso, en esta discusión, ocupa diferentes posiciones en diferentes tiempos, no se indica en la figura.

**341.** En la fig. 154, sean $l$ la luz, $x$ la distancia variable del peso, W, *, desde el apoyo derecho, $b$, é $y$ la distancia constante, $ac$, de un punto dado, $c$, al apoyo izquierdo, $a$. Entonces, para cualquier posición de W, la reacción del extremo izquierdo, R, es $= W.\frac{x}{l}$; y el momento de esa reacción para $c$ $= R.y, = W.\frac{x}{l}.y$.

La reacción del extremo derecho es $R' = W\frac{l-x}{l}$, y su momento, respecto á $c$, es

$$R'(l-y) = W\frac{l-x}{l}(l-y).$$

Mientras W esté entre $b$ y $c$, el momento, en $c$, es $= R.y = W.\frac{x}{l}y$.

**342.** Puesto que W, $y$, $l$ son constantes, el momento, en $c$, mientras W esté entre $b$ y $c$, es proporcional á la distancia variable, $x$, del peso al punto $b$. Aquél aumenta por tanto uniformemente, desde 0, cuando W está en $b$, hasta su valor máximo, M, cuando W está en $c$ mismo. Véase § 317. Por tanto, si se hace la ordenada, $c'$ M, igual, por escala, al momento máximo, M, entonces el momento, para $c$, en cualquier posición $d$, $e$, $f$, de W, entre $c$ y $b$, lo da la correspondiente ordenada, $d'$, $e'$, $f'$, á la línea $b'$M. Semejantemente, los momentos, en $c$, para cualesquiera posiciones de W entre $c$ y $a$, los dan las ordenadas á la línea $a'$M.

**343.** Para el momento, en $c$, para cualquier número de pesos, en cualesquiera posiciones, encuéntrese como arriba el momento, en $c$, para cada peso separadamente, y tómese la suma de ellos.

**344.** Es costumbre construir el diagrama de influencia para los momentos de un peso, W = unidad (1 tonelada, 1 libra, 1 mil kilogramos, etc.). Cada ordenada debe entonces multiplicarse por su correspondiente peso, medida con la correspondiente unidad, á fin de obtener el momento requerido.

**345.** Cuando W está en el punto $c$, tenemos $x = l - y$. En consecuencia, la ordenada, $c'$M = momento máximo, $= W.\frac{l-y}{l}y$; ó, si W = 1, $C'M = \frac{l-y}{l}.y$.

El área del diagrama, $a'Mb'$, es $= \frac{l}{2}.c'M = \frac{l}{2}.\frac{l-y}{l}.y = \frac{(l-y)y}{2}$.

**346.** Si un peso = 1, se distribuye sobre una longitud = 1, en $c$, fig. 155 ($a$), el momento resultante, en $c$, puede ser representado por el área del rectángulo que se halla en $c'$, fig. ($b$), siendo la altura de dicho rectángulo la ordenada, $c'$M, y su largo = 1. Así mismo, el momento, en $c$, debido á un peso uniformemente distribuído, $ef$, de 1 por unidad de longitud, fig. ($a$), puede ser representado por la suma de las áreas de los rectángulos entre $e'$ y $f'$, fig. ($b$); y, si suponemos el peso, $ef$, fig. ($a$), de 1 por unidad de long, dividido en muchas fajas verticales muy angostas, el momento resultante, en $c$, puede suponerse representado por el área del trapezoide sombreado sobre $e'f'$, fig. 155 ($b$). El momento, en $c$, debido á un peso de $p$ (lbs, tone-

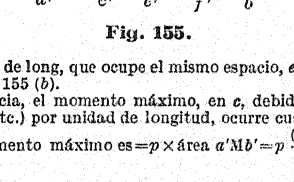


**Fig. 155.**

ladas, etc.) por unidad de long, que ocupe el mismo espacio, $ef$, es $= p \times$ área del trapezoide sobre $e'f'$, fig. 155 ($b$).

**347.** En consecuencia, el momento máximo, en $c$, debido á un peso uniforme de $p$ (lbs, toneladas, etc.) por unidad de longitud, ocurre cuando ese peso cubre la luz entera. Este momento máximo es $= p \times$ área $a'Mb' = p\frac{(l-y)y}{2}$. Véase §345.

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* Véase nota pág. 470.

## Diagrama de influencia para el esfuerzo cortante.

**348.** Bajo un solo peso concentrado, este esfuerzo, en cualquier punto entre el peso y uno de los apoyos, es = la reacción del apoyo. Véanse §§ 326 y 327.

**349.** En el diagrama de influencia del esfuerzo cortante, fig. 156, como en el diagrama de influencia del momento, fig. 154, sea $l$ la luz; $x$ la distancia variable del peso, W, al apoyo derecho, $b$; $y$, la distancia constante, $ac$, de un punto dado, $c$, al apoyo izquierdo, $a$. Entonces, para cualquier posición de W, la reacción del extremo izquierdo, R, ó el esfuerzo cortante, S, en cualquier punto entre el peso y el soporte izquierdo, es $= W.\frac{x}{l}$; y la reacción derecha, R', ó el esfuerzo cort S', en cualquier punto entre el peso y el soporte derecho, es $= W.\frac{l-x}{l}$.

**350.** La línea de influencia del esf cort como la de los momentos, § 344, se construye de ordinario para un peso = á la unidad, de modo que $S = R = \frac{x}{l}$; y $S' = R' = \frac{l-x}{l}$. Cada ordenada del diagrama del esf cort debe entonces multiplicarse por W, para obtener el esfuerzo cortante requerido.

**351.** Puesto que W (=1) y $l$ son constantes, R y S varían directamente (y R' y S' inversamente *) con $x$. Así, cuando W (=1) está en $b$, tenemos $x=0$; $S=R=0$, y $S'=R'=W=1$. Cuando W (=1) está en $a$, tenemos $x=l$; $S=R=1$, y $S'=R'=0$. Trácese $a'a''$ y $b'b''$, cada una = W (=1), y júntense $a''b'$ y $a'b''$. Luego, con W en $c$, el esfuerzo cortante (positivo), S, en cada punto, como $f$, entre $c$ y $a$, está dado por la ordenada, $c'g$, á la línea $a''b'$; mientras la ordenada, $c'h$, á la línea $a'b''$; da el esfuerzo (negativo) en cada punto, como $e$, entre $c$ y $b$.

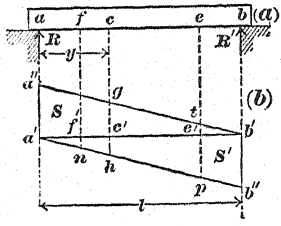


**Fig. 156.**

**352.** De modo semejante, con W en $e$, la ordenada, $e't$, da el esf cort (positivo) en cada punto, como $c$, entre $e$ y $a$; mientras que $e'p$ da el (negativo) en cada punto entre $e$ y $b$.

**353.** Se notará que, á medida que el peso, W, pasa de un lado al otro de cualquier punto, como $c$, el esf cort en este punto se invierte, siendo el cambio total del esf cort $= a'a'' + c'g = hg =$ al peso, W.

**354.** Con un peso, W (=1), en $b$, el esf cort en $c$, es = 0. Véase § 351. A medida que el peso avanza de $b$ hacia $a$, el esf cort positivo, $S = R = \frac{x}{l}$, en $c$, crece en proporción á las ordenadas á la línea $b'g$, haciéndose $= c'g = \frac{l-y}{l}$, cuando W esté justamente á la derecha de $c$. Con W justamente á la izquierda de $c$, tenemos el esf cort negativo en $c = S' = R' = c'h = \frac{y}{l}$. Pero, á medida que W avanza de $c$ hacia $a$ este esf cort negativo, en $c$, decrece en proporción á las ordenadas á la línea $ha'$, haciéndose 0 cuando W llega á $a$. Así, $a'hgb'$ es el diagrama de influencia del esf cort para el punto, $c$. Del mismo modo $a'ptb'$ es el del punto $e$, etc.

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*N. del T.* — Así dice el texto inglés, pero R' y S' *no* varían en razón inversa de $x$, pues $x$ es sólo sustraendo en el numerador. Es cierto que disminuyen aumentando $x$, y, á la inversa, aumentan cuando $x$ disminuye, pero no *en razón inversa*.

**355.** Si una serie de pesos concentrados casi uniformes y equidistantes, tales como los pesos de ruedas de una locomotora y de un tren, entran en la luz por el apoyo *b* y avanzan hacia *a*, el esf cort en *c* aumenta evidentemente hasta que el primer peso llega al punto *c*. Disminúyese luego de repente en una cantidad = al primer peso á medida que ese peso pasa de *c*. Continúa luego disminuyendo, á medida que cada rueda pasa sobre *c*, pero más lentamente, hasta que el primer peso llegue al punto *a*. Véase § 358.

**356.** Con un peso *uniformemente distribuído*, igual á la unidad, por unidad de longitud, moviéndose como en el § 355, los esf cort en *c* (véase § 346) están representados por las áreas de aquellas porciones del diagrama, *a'hgb'*, sucesivamente cubiertas por el peso, tomándose como negativas las porciones del diagrama por debajo de la línea cero, *x'b'*. Así, cuando el principio del peso llega á *e*, el esf cort positivo en *c* está dado por el área del triángulo *b'e't*. Cuando llega el principio del peso al punto *c*, el esf cort en *c* llega á su máximo y está dado por el área del triángulo, *b'c'g*. Cuando el principio del peso llega al punto *f*, el esf cort en *c* es = área *b'c'g* — área *f'c'hn*.

**357.** De modo semejante, los esf cort en *e* están dados por las áreas de las porciones del diagrama, *a'ptb'*.

**358.** La fig. 157 * muestra el diagrama de influencia, 0 *dz* 14 16 para los esf cort en el punto 6, para un peso dado distribuído uniformemente, y tan largo

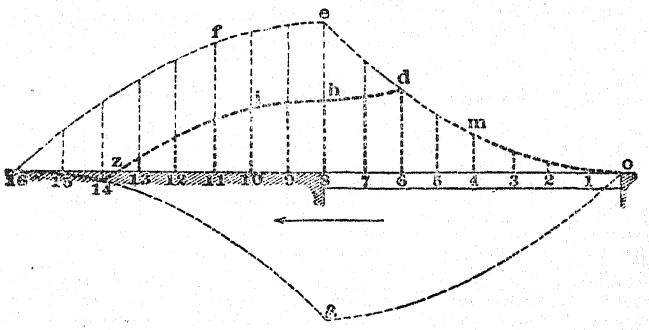


**Fig. 157.**

á lo menos como la abertura, llegando á ella en el punto 0, pasando al través y dejándola en el punto 8; también el correspondiente diagrama, 0, *e*, 16, para el apoyo izquierdo, 8; y el 0, *g*, 16 para el apoyo derecho, 0.

Para la acción de las resistencias internas en vigas y armaduras, véase « Resistencia transversal » en el capítulo « Resistencia de Materiales », y « Esfuerzos » en « Armaduras ».

## Relación entre el momento y el esfuerzo cortante.

**359.** El esf cort, en cualquier punto de la luz ó abertura, es simplemente la proporción en que va cambiando el momento de flexión en ese punto.

**360.** Así, en la fig. 158, el momento M, fig. (*b*), en el apoyo, *b*, debido al peso concentrado, W, de 6 lbs **, es = Wl = 6 × 4 = 24 p-lbs; pero, entre el apoyo y el peso, el momento va decreciendo en la proporción *uniforme* de 6 p-libs por cada pie de *x*, ó 6 pies-lbs por pie = 6 lbs; y estas 6 lbs son el esfuerzo cortante uniforme, V, fig. (*c*), en toda la viga. En consecuencia, el diagrama del esf cort, fig. (*c*), es una línea *horizontal;* esto es : sus ordenadas son de *igual* longitud.

**361.** Otra vez, en la fig. 159, las ordenadas del diagrama del esf cort entre *a"* y *o"*, fig. (*c*), son *positivas*, y demuestran el *aumento* (algebraico) del momento de flexión, M, fig. (*b*), á medida que pasamos del apoyo izquierdo, *a*, hacia el centro, *o*, de la luz; mientras que, las ordenadas negativas del diagrama del

* Por primera vez publicada en nuestra 9.ª edición de 1885.
** Véase *N. del T.*, pág. 462.

esfuerzo cortante, entre $o''$ y $b''$, muestran la disminución (algebraica) del momento de flexión, á medida que pasamos del centro, $o$, hacia el apoyo derecho, $b$. En el centro, $o$, la proporción del cambio del momento de flexión es cero, como lo es también el esf cort vertical.

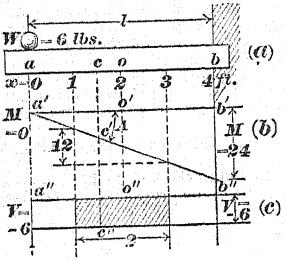


Fig. 158.

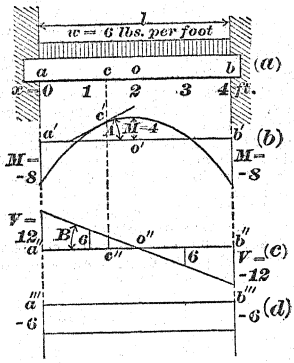


Fig. 159.

**362**. Tanto en la fig. 158 como en la 159, el momento de flexión, M, cambia constantemente; pero en la fig. 158 su *tipo de cambio* (=6 p-libs por pie de luz) es *constante**. En consecuencia, el diagrama del *momento* es una línea *recta inclinada*, y el diagrama del esfuerzo cortante es una línea *horizontal;* mientras que en la fig. 159 el tipo de cambio del momento de flexión está variando constantemente, siendo =12 p-lbs por pie de luz (el esfuerzo cortante=12 lbs) en el apoyo, y disminuyendo hasta cero en el centro, $o$. En consecuencia, en la fig. 159, el diagrama del momento, fig. ($b$), no es ya recto, sino *curvo;* y el diagrama del esf cort, fig. ($c$), no es ya horizontal, sino *inclinado*.

**363**. Pero, en la fig. 159 ($c$), el esfuerzo cortante, V, ó el *tipo* de cambio del momento de flexión (aunque no ya *constante*, como lo era en la fig. 158 ($c$), sin embargo *disminuye uniformemente*, á medida que pasamos de $a$ hacia $b$. Así, en el punto, 1, fig. 159, en el medio entre $a$ y $o$, el momento de flexión está cambiando al tipo de 6 p-lbs por pie, ó á la mitad de la proporción que tenía en $a$. Así, el diagrama del esfuerzo cort, aunque no es ya una línea *horizontal*, es todavía una línea *recta;* y la disminución *uniforme* (=6 p-lbs por pie por pie) en el tipo de cambio del momento de flexión, ó la disminución *uniforme* (=6 lbs por pie) en el esf cort, está indicado por el diagrama *horizontal* en la fig. 159 ($d$).

**364**. En una ú otra fig. trácese una línea recta, tangencial al diagrama del momento ($b$), en cualquier punto, $c'$, y formando con la línea horizontal $a'b'$ que indica el cero, un ángulo, A. (En la fig. 158 esta línea coincide con el diagrama del momento.) Entonces, la tangente de A está dada por la ordenada del diagrama del esf cort, $c''$, correspondiente al punto $c$, ó, para cualquier punto, V=tang A.

**365**. En la fig. 158, donde este ángulo A, fig. ($b$), es *constante*, las ordenadas del esf cort, fig. ($c$), son de longitud *constante*. En otros términos, el diagrama del esf cort, fig. 158, es una línea *horizontal*.

**366**. Puesto que las ordenadas del diagrama del esf cort representan *fuerzas* (como lbs, kgs, etc.) y las abscisas representan *distancias* (como pies, metros, etc.), el producto de la distancia entre cualesquiera dos ordenadas del esfuerzo cortante multiplicada por el término medio de esas ordenadas, es un *área*, que representa un *momento* (en kgm ó p-lbs, etc.). Este momento es=á la diferencia entre los dos momentos representados por las correspondientes ordenadas en el diagrama de los momentos.

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* Véase *N. del t.*, pág. 462.

**367.** Así, en la fig. 158 (*b*), el aumento en el momento (negativo) de la flexión entre los puntos 1 y 3, es=18 — 6=12 p-libs; y, en la fig. 158 (*c*), el momento representado por el área (sombreado), entre los mismos dos puntos, es=2 p×6 lbs=12 p-lbs. En la fig. 159, el aumento (algebraico) del momento de flexión, fig. (*b*), entre el apoyo izquierdo, *a*, y el centro, *o*, de la luz, es=8+4=12 p-lbs; y el momento, representado por el área (triángulo) del diagrama del esf cort, entre los mismos dos puntos, fig. (*c*), es

$$= \frac{1}{2} \text{V} \frac{l}{2} = \frac{12 \times 2}{2} = 12 \text{ p-lbs.}$$

**368.** Otra vez, en la fig. 159, en cualesquiera dos puntos igualmente distantes del centro, *o*, de la abertura, los momentos son iguales; ó la diferencia de los momentos=cero; y, puesto que las ordenadas del esf cort por debajo de la línea de cero, $a''b''$, fig. (*c*), se consideran como *negativas*, la suma algebraica de los dos triángulos del esfuerzo cortante correspondientes, fig. (*c*), es también=cero.

De modo semejante, las áreas en la fig. 159 (*d*) corresponden á las diferencias de ordenadas en la fig. 159 (*c*).

## Diagrama para reacciones, esfuerzos cortantes y momentos.

**369.** En la fig. 58 (*b*), p. 403, *cw* y *w*X dan respectivamente las reacciones de los apoyos derecho é izquierdo, *w* y *x*, fig. 58 (*a*).

**El esfuerzo cortante**, §§ 325, etc., es constante para todas las secciones entre cualesquiera dos pesos. Para el esf cort en cualquiera sección así situada, como *d*, encuéntrense, en el polígono de equilibrio, fig. 58 (*b*), las líneas, *cw* y *bc*, que representan las fuerzas, *w* y *c*, á la derecha, ó las *w*X, X*a*, *ab*, que representan las fuerzas *x*, *a*, *b*, á la izquierda, de la sección. Entonces la línea, *bw*, = *cw* — *bc*, ó *wb*=*w*X — X*a* — *ab*, representa la resultante de uno ú otro juego de fuerzas, y así representa el esfuerzo cortante en la sección, *d*.

**Para encontrar el momento de flexión** en cualquier sección, como *d*, fig. 58 (*a*); desde *d* trácese una vertical, cortando las líneas *sr* y *np* del polígono de las cuerdas, y sea *y*=al largo de la ordenada, comprendida, entre aquellas dos líneas.

Prolónguense *sr* y *np* de modo que se corten en *e*. Entonces *e* es el punto de aplicación de la resultante, U=*bw*, fig. 58 (*b*), de las dos fuerzas á la derecha de *d*, á saber: del peso *c* representado en la fig. 58 (*b*) por *bc*, y la reacción en *w*, representada por *cw;* porque si el peso, *c*, y la reacción en *w* se movieran, necesitaríamos, para el equilibrio, una reacción vertical, *bw*=*cw* — *bc;* y las líneas correspondientes, *b*O y *w*O, en el diagrama de fuerzas, son respectivamente paralelas á *np* y *sr* en el polígono de cuerda.

En otros términos, eliminando el peso, *c*, y la reacción en *w*, y sustituyendo la resultante de ellos, U=*bw*, sustituímos también un nuevo polígono *smnes*, fig. 58 (*a*), con U (=*bw*) aplicado en *e*.

En la sección *d*, sea L el brazo de palanca de esta resultante, U,=la distancia horizontal desde *e* hasta *y*. En la fig. 58 (*b*), trácese O*h* horizontal, representando la componente horizontal de cada una de las fuerzas XO, *a*O, etc. Sea H=á la longitud de O*h*. Entonces el momento de flexión, en la sección *d*, es M=H*y;* pues M=UL; y, por los triángulos semejantes, L : *y*=H : U; ó UL=H*y*.

**Escala de los momentos.** Puesto que M=H*y*, podemos, escogiendo la posición de O * á distancia conveniente de X*c*, obtener la escala de momentos que se quiera para medir los momentos directamente, sobre *y*. Así, supongamos que se ha usado una escala de 1 pulgada=2 pies en la fig. 58 (*a*), y que se desea tener una escala para los momentos de 1 pulgada=50 p-lbs. Entonces necesitamos solamente escoger de tal manera la posición de O *, que su distancia, H, de X*c*, represente 50 p-lbs ÷ 2 pies=25 lbs, en la escala de la fig. 58 (*b*). Entonces 1 pulgada de longitud de *y* corresponderá á 2 p×25 lbs=50 p-lbs †.

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* Véase § 95, pág. 397.

† *N. del T.* — No hemos creído necesario cambiar el sistema de medidas inglesas en estos §§, pues aquí es tan claro como el métrico para la exposición de la teoría.

# RESISTENCIA DE MATERIALES

## PRINCIPIOS GENERALES

**1.** Las fuerzas que obran sobre un cuerpo, pueden hacerlo de tal manera, que sus partículas tiendan á moverse simultáneamente, con diferentes velocidades ó en diferentes direcciones; para hacer esto, las partículas deben cambiar sus posiciones relativas. Esto ocurre, por ejemplo, cuando un cuerpo está situado de tal modo que se opone al movimiento relativo de otros dos cuerpos; como cuando un bloque está situado entre un peso y una mesa horizontal. Aquí los dos cuerpos (peso y mesa) tienden á juntarse; pero no lo pueden hacer sin deformación del cuerpo, y éste resiste á la deformación con *fuerzas internas* que obran entre las partículas del bloque y tienden á mantenerlas en sus posiciones relativas originales. La acción de estas fuerzas internas se llama *resistencia* del cuerpo ó de la materia que lo forma.

**2.** De modo análogo, si un cuerpo está suspendido por una cuerda larga y se le empuja ó se tira de él hacia un lado, las partículas sobre las que obra la fuerza tienden á moverse primero y la transmisión de esta tendencia, á las otras partículas, produce resistencia dentro del cuerpo.

**3.** Para el equilibrio interior, **las resistencias internas deben equilibrar las fuerzas externas.** Por eso se acostumbra á veces, llamar « esfuerzo » á cualquiera de las dos.

**4.** Supongamos las dos fuerzas *a* y *b*, figs. A, B, obrando sobre el cuerpo, *o*, bajo un ángulo *aob*. Las dos componentes, iguales y contrarias, *a"o*, y *b"o*, producen compresión ó tensión en el cuerpo, *o*, como en el § 1; en tanto que las otras dos componentes, *a'o* y *b'o*, se unen para formar la resultante *co* que, si no está equilibrada por otras fuerzas, moverá el cuerpo, *o*, en su propia dirección, produciendo, como dice el § 2, otra compresión, fig. A, ó tensión, fig. B.

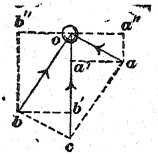


**Fig. A.**

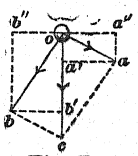


**Fig. B.**

**5. Sobre cualquier plano,** dentro de un cuerpo, puede obrar una fuerza (**1**) **normal**, (**2**) **tangencial**, (**3**) ú **oblicua**. Si obra oblicuamente, se descompondrá en dos (véase Estática, § 65), una obrando normal, y la otra, tangencial al plano.

**6.** Considerando las dos porciones en que queda dividido el cuerpo, por dicho plano, las fuerzas (**1**) obrando normalmente al plano, producen **tensión** (ó **compresión** en el plano, tendiendo á separar las dos porciones, ó á comprimirlas entre sí); y (**2**) las fuerzas que obran **tangencialmente** al plano, producen esfuerzo cortante (ó **torsión**) en el plano, tendiendo á hacer resbalar las dos porciones, una sobre la otra, en línea recta (ó con un movimiento de torsión). **La torsión** ocurre en planos paralelos y entre **dos pares contrarios**, como en las secciones transversales del eje ó árbol de un freno de mano al aplicarlo.

**7.** Así, si una barra de hierro es estirada (ó comprimida) en el sentido de su longitud, sus secciones transversales soportan tensiones (ó compresiones) normales. Si es cizallada transversalmente (ó torcida), las secciones transversales, entre, y paralelas á las dos fuerzas de cizallamiento ó de torsión, resisten á ellas; es decir, al esfuerzo cortante.

**8.** En cualquier punto de la trayectoria circular de un esfuerzo de torsión, se pueden considerar las tangentes á la trayectoria, como representando esfuerzos cortantes. **La torsión** es por consiguiente un simple esf cort que cambia de dirección en cada punto.

**9. Esfuerzo transversal.** En la fig. 124, pág. 459, las dos fuerzas, iguales y paralelas, W y R, en opuestas direcciones, producen un esfuerzo tangencial = W = R, en el plano vertical entre sus líneas de acción; pero W y R como un *par*, tienen un *momento*, que debe, para el equilibrio, ser destruído por el momento igual de otro par, como C y T; y la oposición de estos dos pares produce esfuerzos normales (compresión y tensión) en los mismos planos verticales entre W y R y paralelo á ellas.

**10.** La mayor tendencia de la acción de fuerzas, exteriores y opuestas, es romper el cuerpo, separando sus partículas. Aun bajo compresión, la ruptura ocurre sólo por separación de partes.

## Alargamiento.

**11.** Cuando las resistencias interiores y las fuerzas exteriores están en equilibrio, no hay deformación; pero al empezar la acción opuesta de las fuerzas externas, cuando aún no se han desarrollado las resistencias interiores, comienzan las deformaciones bajo la acción de aquéllas, aún no equilibradas. (Véanse §§ 35, etc.) Pero las resistencias entran en acción por las deformaciones y crecen con ella; y si las fuerzas exteriores no pasan el límite de elasticidad (§ 26), las resistencias acaban por igualarla y evitan ulteriores deformaciones.

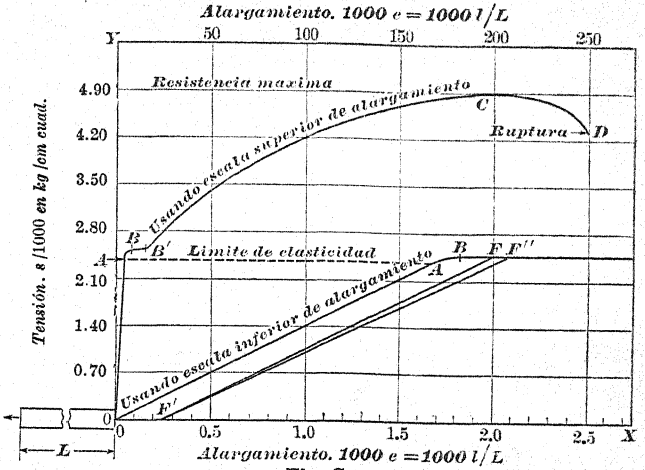


**Fig. C.**

## Conducta bajo una tracción normal.

**12.** La fig. C **demuestra cómo se conduce un material típico** (acero blando) **bajo tracción.** De 0 á A, es decir, bajo el límite de elasticidad (§ 26), digamos 34,000 lbs por pulg (=23.9 kg por milímetro cuad), el estiramiento crece, proporcionalmente á la tracción, como lo indica la línea recta, 0A. El comienzo del proceso, está representado en el diagrama *bajo* en que la escala de alargamientos es 100 veces mayor que la del diagrama superior. Después del punto A los alargamientos crecen más rápidamente que las tensiones; y, entre B y B', el alargamiento (en hierro y acero) crece, aumentando muy poco ó no aumentando la tensión; y, aunque ésta disminuya un poco *, B es el punto donde *cede* (**yield point**). Véase § 31. La escala del diagram abajo no pasa de B'. Más allá de B', (diagrama superior) el alargamiento aumenta mucho menos rápidamente que

* Véanse §§ 16 y 17.

entre B y B', y es por algún tiempo proporcional á la tracción * (aunque mucho mayor, relativamente al alargamiento, que en 0A); pero ahora los alargamientos van cada vez más rápidamente aumentando en relación con la tracción, hasta que ésta llegue á su **maximo** (digamos 70,000 lbs por pulg =49.2 kg por milímetro cuad) en C. Aquí, el alargamiento crece sin que crezca la tracción (línea horizontal); y más allá de C, el alargamiento continúa creciendo, aunque la tracción disminuya, hasta que finalmente en D viene la **ruptura.**

**13.** Si después de pasar el límite de elasticidad, se suspende el esfuerzo en la barra como en F, fig. C, diagrama bajo, la barra no vuelve á su primitiva longitud antes de sufrir la tracción. El alargamiento, 0F', con que queda, se llama **permanente.** La línea FF' es, en general, aproximadamente paralela á la línea 0A, de alargamientos proporcionales á la tensión. Cuando se vuelve á aplicar la misma tracción, el alargamiento es mayor que antes, en una pequeña cantidad, representada por la línea FF".

**14.** Cuando la tracción se hace **dentro del límite de elasticidad** (§ 26), la longitud recuperada al cesar la tensión, es tan próxima á la primitiva, que el alargamiento permanente no se puede indicar en la figura (§ 28).

**15.** Bajo la *tensión*, el **área de la sección** *disminuye*, y bajo la *compresión*, *aumenta*. En los materiales dúctiles, la reducción del área de la sección es muy notable, sobre todo en una parte relativamente pequeña de la longitud, generalmente cerca del medio, y la fractura ocurre normalmente en la sección más reducida.

**16.** En la fig. C, ambos diagramas, y en la fig. D, la curva continua, representan el **esfuerzo nominal por unidad de superficie**, ó sea el que se usa más. Este se encuentra dividiendo el esf total por el área *original* de la sección, como en el § 18.

**17.** Las curvas de puntos, fig. D, representan el esf **real por unidad de superf** encontrado, dividiendo el esf total por el *área verdadera de la sección*. Bajo *tensiones*, la tensión real por unidad es, por supuesto, *mayor*, y, bajo *compresiones*, la compresión real por unidad de superf es *menor* que las nominales correspondientes.

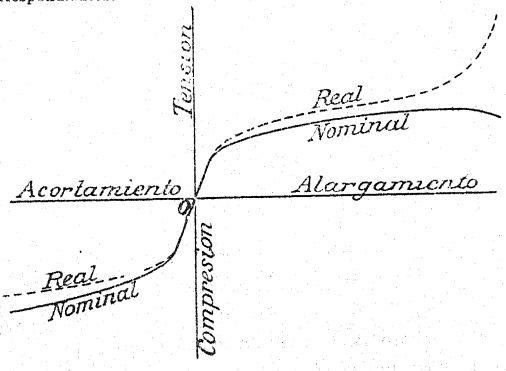


**Fig. D.**

**Coeficiente de elasticidad.**

**18.** Sea P=el **peso** (una de las dos fuerzas iguales y opuestas) que obra en uno de los extremos de la barra y en la prolongación de su eje; y sea $a$=al área de la sección transversal original de la barra. (Véanse §§ 16 y 17.) Entonces, $s = P/a$ es la tensión normal **por unidad de área** en la barra. Suponemos que, mientras las fuerzas obren en la dirección del eje de la barra, P está uniformemente distribuído sobre $a$, aunque esto sucede rara vez en la práctica.

**19.** Sea L=la longitud primitiva de la barra ó una porción determinada de ella, y $l$=el **alargamiento** * de la longitud L bajo la acción de una tensión, $s$, por

* Veanse §§ 16 y 17.

unidad de superficie. Entonces, $e=l/L$ es el alargamiento **por unidad de longitud** * correspondiente á la tensión, $s$, por unidad de superficie.

**20.** En muchos materiales, la tensión, $s$, por unidad de superficie, y el alargamiento, $e$, por unidad de longitud, crecen al principio proporcionalmente; y la relación $s/e$, ó la tensión por unidad de superficie ÷ por el alargamiento por unidad de longitud que es lo que se llama **coeficiente ó módulo de elasticidad** y se designa por E, permanece prácticamente constante; así

**Módulo ó coeficiente de elasticidad** = $E=s/e$ = tensión por unidad de sup ÷ alargamiento por unidad de long.

**20 a.** El módulo de elasticidad es así **proporcional á la tangente** del ángulo XOA, fig. C. La proporción depende de las escalas adoptadas.

**20 b.** El módulo de elasticidad, E, crece con la tensión por unidad de superficie requerida para producir una unidad dada de alargamiento. Por tanto, E es la medida de la *rigidez* del cuerpo, es decir, de su capacidad para resistir al cambio de forma. Por tanto, mejor le quedaría á este módulo el nombre de **módulo de rigidez.**

**20 c.** Si iguales tensiones adicionales pueden indefinidamente continuar produciendo iguales y adicionales alargamientos en una barra, suponiendo que no se pasa el límite de elasticidad, entonces una fuerza igual al módulo de elasticidad, aplicada como *tensión*, **doblaría el largo de la barra**, y como *compresión*, reduciría á *cero* su longitud.

**20 d.** Por ejemplo, dentro del límite de elasticidad, una barra de acero laminado de 6.25 cm cuad (1 pulg cuad) de sección se alargará ó acortará, término medio $\frac{1}{30{,}000}$ de su longitud por cada carga de 75 kg por cm cuad. Si ella se pudiera alargar ó acortar indefinidamente, en la proporción de $\frac{1}{30{,}000}$ de su longitud por cada 75 kg de peso adicional (siempre por cm cuad), entonces con 30,000 veces 75 kg, ó sean 2,250,000 kg (por cm cuad, que es más ó menos el módulo de elasticidad de tales barras), se podría ó alargar la barra al doble de su longitud ó reducir ésta á cero; según se la aplique.

**20 e.** Si fuerzas iguales é infinitesimales, aplicadas á una barra, producen siempre una relación constante en los cambios de longitud de la barra, alargamientos, si por *tensión* y acortamientos, si por *compresión;* entonces el mismo peso que *doblaría* la longitud original de la barra, aplicado como *tensión*, reduciría á la *mitad* su longitud original, aplicada como *compresión.*

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* Consideramos á los *acortamientos*, bajo compresión, como **alargamientos negativos.**

**21. En una barra prismática, sometida á tensión longitudinal ó compresión,** sea

$W$ = peso total;
$a$ = área de la sección transversal;
$s = \frac{W}{a}$ = esfuerzo por unidad de superficie;
$L$ = longitud original;
$l$ = alargamiento * total;
$e = l/L$ = alargamiento * por unidad de longitud original;
$E$ = módulo de elasticidad de la substancia;
$r = Ea$ = una medida de la resistencia de la barra.

Entonces

**Módulo de elasticidad** $= E = \frac{W}{a} \cdot \frac{L}{l} = s/e$ ..... (1)

**Carga total** $= W = Ea \cdot \frac{l}{L} = re$ ..... (2)

**Esfuerzo por unid de super** $= s = \frac{W}{a} = Ee$ ..... (3)

**Alargamiento total** * $= l = \frac{W}{a} \cdot \frac{L}{E}$ ..... (4)

$= s \cdot \frac{L}{E}$ ..... (5)

**Alarg. por unidad de long** * $= e = \frac{l}{L} = \frac{W}{aE} = \frac{s}{E}$ ..... (6)

**22. En una viga,** apoyada en ambos extremos y cargada en el centro, sea

$L$ = longitud de la viga en la luz ó abertura;
$w$ = peso — — —
$\Delta$ = flexión — — —
$b$ = ancho de la sección transversal de la viga;
$d$ = profundidad — — —
$I$ = momento de inercia — —

Entonces

$$E = \frac{(W + \frac{5}{8} w)\, L^3}{48\, \Delta\, I} \quad \ldots\ldots (7$$

Si la viga es rectangular, $I = \frac{bd^3}{12}$ (véase Momentos de inercia, pág. 491) y

$$E = \frac{12\,(W + \frac{5}{8} w)\, L^3}{48\, \Delta\, bd^3} = \frac{(W + \frac{5}{8} w)\, L^3}{4\, \Delta\, bd^3} \quad \ldots\ldots (8)$$

**Para vigas** (véanse también págs. 502 y 503).

**23. El inverso del módulo de elasticidad. El módulo de elasticidad** $= \frac{\text{tensión por unid de sup}}{\text{unidad de alargamiento}}$, indica la tensión requerida para producir cierto alargamiento.

**Su inverso,** $= \frac{\text{unidad alarg}}{\text{tensión por unid de sup}}$, indica cuanto debe deformarse una barra, etc., de un material dado para producir cierto esfuerzo. Esto puede ser de mucha importancia, especialmente en las construcciones de madera, cuyo módulo de elasticidad es bajo, relativamente al del acero; y en las que puede presentarse una gran deformación relativa antes de que la resistencia de seguridad máxima de las fibras entre en acción. Así, en el caso de un muelle, por ej., sostenido por largos pilotes de madera, pueden éstos estar sometidos á fuerzas laterales tan intensas que, produciéndoles fuertes flexiones, den á la carga que resisten

* El *acortamiento* por la compresión lo consideramos como alargamiento negativo.

tan largos y peligrosos brazos de palanca, horizontales, que resulten grandes y peligrosos momentos capaces de volcar y destruir la obra.

**24. Módulo de elasticidad variable.** Los experimentos sobre hormigones de cemento muestran un ejemplo (para ambos, tensión y compresión) de un material, en el cual el módulo de elasticidad, E, cambia constantemente; los alargamientos crecen más ligero que las tensiones.

**25.** Sin embargo, en el caso de materiales dúctiles, los alargamientos producidos por tensiones, dentro del límite de elasticidad (§ 26), son tan pequeños y tan irregulares, que no se puede llegar á un valor satisfactorio del término medio de dicho módulo sino comparando muchos experimentos. En el caso de materiales quebradizos, donde apenas se nota una insignificante deformación antes de la ruptura, es muy incierta la determinación de este módulo.

## Límite de elasticidad.

**26.** La tensión OA, fig. C, más allá de la cual los alargamientos de cualquier cuerpo crecen perceptiblemente con más rapidez que las tensiones, se llama el **límite de elasticidad.** Debido á la irregularidad en la conducta de diversas muestras de un mismo material y la extrema pequeñez de las deformaciones causadas en la mayor parte de los materiales por pesos moderados, y porque no podemos muchas veces saber en qué momento los alargamientos empiezan á crecer más rápidamente que las cargas, resulta que rara vez se puede determinar con exactitud el límite de elasticidad *. Pero, por medio de un gran número de experimentos sobre un material dado, se puede obtener un valor medio ó mínimo, muy útil para mantener en todos los casos prácticos las resistencias dentro de tales valores; puesto que, si se pasa del límite de elasticidad (por errores de cálculo ó debido á aumento de las fuerzas contra el material ó á disminución en la resistencia de éste), la construcción se derrumbará rápidamente. La tabla del § 30 da los términos medios aproximados de los límites de elasticidad de algunos materiales. El límite de elasticidad, como lo hemos explicado aquí, se llama algunas veces **el verdadero límite de elasticidad.**

**27.** Los materiales quebradizos, como piedras, cementos, ladrillos, etc., apenas puede decirse que tienen un límite de elasticidad, ó si lo tienen es casi imposible determinarlo, ya que la ruptura se efectúa en tales cuerpos antes que se pueda medir satisfactoriamente ninguna deformación.

**28.** Probablemente, siempre queda un pequeño alargamiento **permanente,** aun bajo pesos moderados; pero generalmente viene á hacerse perceptible, más ó menos, en el momento en que se pasa del límite de elasticidad. **El límite de elasticidad se define algunas veces** como la tensión bajo la cual se observa el primer alargamiento permanente.

**29. El cociente elástico** de un material es el cociente, $\frac{\text{límite de elasticidad}}{\text{la máxima resistencia}}$

Se expresa generalmente por una fracción decimal.

La carga de trabajo permitida para un material debe determinarse más bien por su límite de elasticidad que por su resistencia máxima. En igualdad de condiciones, un elevado cociente máximo de elasticidad es en general cualidad deseable; pero, por otra parte, es posible, modificando los procedimientos de manufactura, obtener materiales de elevados cocientes de elasticidad, pero de poca rigidez, es decir, de poca capacidad para resistir golpes ó choques, ó aplicaciones repentinas, ó fluctuaciones bruscas de fuerzas. Véanse §§ 34, 35, etc.

En la manufactura del acero, el cociente elástico se aumenta, aumentando la reducción del área en el martilleo ó la laminación, y la proporción del aumento del cociente elástico con las reducciones de áreas, aumenta rápidamente á medida que se hace muy grande la reducción. Kirkaldy encontró **

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| para | planchas | de | acero | de 25 mm | grueso, | cociente | elástico | medio = .53 |
| — | — | — | — | de 18 mm | — | — | — | = .53 |
| — | — | — | — | de 12 mm | — | — | — | = .54 |
| — | — | — | — | de 6 mm | — | — | — | = .61 |

* La oficina de los E. U., designada para ensayar el hierro, acero, etc., encontró una variación de cerca de 281 kg por cm cuad en el límite de elasticidad de barras de hierro laminado, preparadas con gran cuidado y teniendo muy uniforme resistencia á la tensión, y en otra de hierro, muy cuidadosamente hecha, una diferencia de 30 0/0 entre dos barras de las mismas dimensiones. Informe, 1881, vol. 1, pág. 34.

** Informe anual del secretario de la Marina, Washington, 1885, vol. I, pág. 499; y Experimentos de la marina mercante sobre el acero (Parlamentary Paper), C. 2897, Londres, 1881.

*N. del T.* — Creemos útil insertar, autorizados por su autor, esta tabla que los esfuerzos, etc., vienen todos en kg por milímetro cuadrado. Para tenerlos

**META-**

**TABLA DE LAS PRINCIPALES**

| MATERIALES | PESO del metro cúbico. — Kilogramos. | Coeficiente de elasticidad por extensión y compresión *E* Kilogs por mm². |
|---|---|---|
| **Hierro** | | |
| Palastros y barras de todas clases | 7790 á 8000 | 18000 á 20000 |
| Alambre | » | » |
| Idem galvanizado | » | » |
| **Acero** | | |
| Acero extradulce para construcciones | 7790 á 8000 | 20000 á 22000 |
| Alambre de acero | » | » |
| Acero moldeado, recocido | » | » |
| Idem templado al aceite y recocido | » | » |
| **Fundiciones** | | |
| Fundición ordinaria de 2.ª fusión | 7000 á 7500 | 9500 |
| Fundiciones tenaces | » | » |
| Fundiciones maleables | » | » |
| **Otros metales y aleaciones** | | |
| Cobre batido | 8900 | 12000 |
| Cobre fundido | 8600 | » |
| Alambre de cobre no recocido | » | » |
| Latón fundido | 7800 á 8400 | 6500 |
| Alambre no recocido | 8540 | 10000 |
| Plomo | 11400 | 500 |
| Cinc | 6800 á 7200 | » |
| Estaño fundido | 7300 á 7500 | » |
| Aluminio laminado | 2560 á 2670 | 7200 |
| Bronce de cañones | 8400 | 6000 |
| Idem de íd. comprimido, en el ánima | » | » |
| Idem de íd. en la parte exterior | » | » |
| Bronce fosforoso | » | » |
| Idem al manganeso | » | » |
| Metal Delta | 8400 | » |
| Bronce de aluminio | » | » |

trae en su « Mecánica Aplicada » el Sr. D. José Marvá y Mayer. Obsérvese que por cm cuad hay que multiplicarlos por 100.

## LES

### CONSTANTES ESPECÍFICAS

| COMPRESION | | | EXTENSION | | | ESFUERZO CORTANTE | | |
|---|---|---|---|---|---|---|---|---|
| Coeficiente de fractura. — Kilogs. por mm². | Límite de elasticidad. — Kilogs. por mm². | Coeficiente de trabajo. — Kilogs. por mm². | Coeficiente de fractura. — Kilogs. por mm². | Límite de elasticidad. — Kilogs. por mm². | Coeficiente trabajo. — Kilogs. por mm². | Coeficiente de elasticidad. $E'$ Kilogs. por mm². | Coeficiente de fractura. — Kilogs. por mm². | Coeficiente de trabajo. — Kilogs. por mm². |
| 30 á 40 | 15 á 20 | 5 á 9 | 30 á 40 | 15 á 20 | 5 á 9 | 8000 | 24 á 32 | 4 á 8 |
| » | » | » | 70 | » | 12 | » | » | » |
| » | » | » | 46 | » | 8 | » | » | » |
| 40 á 50 | 22 á 30 | 9 á 12 | 40 á 50 | 22 á 30 | 9 á 12 | 8800 | 30 á 38 | 7 á 9 |
| » | » | » | 100 á 200 | » | » | » | » | » |
| 60 á 80 | » | 10 á 13 | 45 á 65 | » | » | » | » | » |
| 400 | » | 66 | 45 á 65 | » | » | » | » | » |
| 50 á 110 | 14 á 24 | 6 á 12 | 9 á 15 | 4 á 8 | 2 á 3 | 4000 | 16 á 20 | » |
| » | 24 á 40 | » | 24 á 27 | 8 á 12 | 4 á 5 | » | » | » |
| » | » | » | 30 á 40 | » | 5 á 7 | » | » | » |
| 80 | 14 á 20 | 6 á 7 | 20 á 28 | 14 á 20 | 4 á 6 | 4400 | 15 á 20 | 3 á 5 |
| » | » | » | 13 á 14 | » | » | » | » | » |
| » | » | » | 44 | » | » | » | » | » |
| 7 á 8 | » | 1 á 2 | 19 á 32 | 7 á 10 | 3 á 5 | » | » | » |
| » | » | » | 56 | » | 9 á 10 | » | » | » |
| 5 | » | 0,8 á 1 | 2 á 3 | » | 0,3 á 0,5 | » | » | » |
| » | » | » | 5 á 6 | » | 0,8 á 1 | » | » | » |
| 10 á 11 | » | 1 á 2 | 3 á 4 | » | 0,5 á 0,7 | » | » | » |
| » | » | » | 27 | » | 4 | » | » | » |
| » | » | » | 14 á 25 | 3 á 4 | 2 á 3 | 1060 | » | » |
| » | » | » | 48 | 18 | 8 á 9 | » | » | » |
| » | » | » | 33 | 7 | 5 á 6 | » | » | » |
| » | » | » | 15 á 36 | 7 á 17 | 2,5 á 6 | » | » | » |
| » | » | » | 45 á 52 | 17 á 27 | 8 á 9 | » | » | » |
| 100 | » | 16 | 33 á 64 | 22 | 6 á 11 | » | » | » |
| » | » | » | 44 á 83 | » | 7 á 14 | » | » | » |

(*N. del T.* — Los datos y la tabla que siguen son los que corresponden, en el sistema métrico, á los que trae el autor en medidas inglesas.)

**30. Módulos ó coeficientes de elasticidad y límites de elasticidad.** Términos medios aproximados **.

$E$=**módulo de elasticidad** en millones de kilogramos por cm cuadrado;

$l$=**estiramiento ó compresión** en milímetro, para un metro de longitud bajo un peso de 73 kg por cm cuadrado;

$s_e$=**esfuerzo en el límite de elasticidad** en kg por cm cuadrado.

| | $E$ | $l$ | $s_e$ |
|---|---|---|---|
| Metales | | | |
| Hierro, fundido | .7 á 2.1 | .096 á .032 | 280 á 560 |
| — — ordinariamente | .84 á 1.05 | .080 á .064 | 420 á 500 |
| — forjado* | 1.89 á 2.17 | .032 | 1400 á 2800 |
| Acero de construcción * | » á » | » | 2400 á 2700 |
| Bronce, fundido | .56 á .7 | .120 á .096 | 355 á 511 |
| — (alambre) | .84 á 1.12 | .080 á .056 | 1000 á 1314 |
| Cobre, fundido | .7 á .98 | .096 á .072 | 438 á 512 |
| — (alambre) | » á » | » á » | 564 á 876 |
| Plomo | .056 á .07 | 1.200 á .960 | 70 á 88 |
| Estaño, fundido | .42 á .49 | .160 á .136 | 102 á 117 |
| Bronces | .91 á 1.05 | .072 á .064 | 1022 á 1095 |
| Piedras, etc. ** | .28 á .56 | .240 á .125 | 70 á 140 |
| Mamposterías ** | .035 á .14 | 1.12 á .480 | Art. 4 (*b*). |
| Madera *** | .105 á .14 | .640 á .480 | 365 á 511 |

**31. Punto cedente** (*Yield point*) §. **Límite elástico, comercial, relativo ó aparente.** Al probar muestras de hierro y acero encuéntrase comúnmente que, en un esfuerzo que excede ligeramente del verdadero límite elástico (§ 26), el alargamiento empieza á crecer *sin más aumento de peso*. Este punto se llama de ordinario « el punto cedente » ó « el límite de elasticidad » en las pruebas comerciales. La Comisión francesa de métodos de probar los materiales de construcción lo llamó « el límite de elasticidad aparente ». El finado profesor J. B. Johnson (« Los Materiales de Construcción », Nueva York, John Wiley and Sons, 1906, pág. 19), aplicó el término « límite elástico relativo ó aparente » á aquel punto en el diagrama de la fuerza en que la proporción de deformación es 50 por ciento mayor que en puntos que estén por debajo del verdadero límite de elasticidad.

* En el hierro laminado y el acero, el módulo de elasticidad es notablemente constante. En el hierro forjado, el límite de elasticidad depende principalmente del grado de reducción de la sección transversal, en el laminado; teniendo los tamaños más pequeños el límite más alto. En el acero este efecto es menos marcado.

** Véanse §§ 25, 26.

*** En la madera, « la fuerza de la fibra extrema en el verdadero límite de elasticidad (§ 26) de una viga es prácticamente idéntica á la fuerza compresiva perpendicular del material ». Véase cuadro del art. 1, **Resistencia á la compresión** de las maderas americanas, en Resistencia de las maderas, después de Explosivos modernos.

§ *N. del T.* — Este punto lo llaman los americanos « Yield point », y como hemos consultado muchos diccionarios y varios ingenieros notables sin encontrar como se expresa con brevedad su significado, hemos resuelto llamarlo « Punto cedente », porque es precisamente el punto en que la barra sigue alargándose *sin que se aumente el esfuerzo á que está sometida*, y puede considerarse como el tercer período elástico Marvá, Mecánica aplicada); caracterizado además porque el alargamiento se localiza y se « forma el huso » ó región de la gran contracción transversal ó *estrechamiento de sección*.

## Resistencia viva.

**32. La resistencia viva** de una barra, bajo un esfuerzo $s$, es el trabajo hecho sobre la barra, al producir esa fuerza, ó teóricamente, el trabajo que hará la barra, al recobrar su forma original, cuando deja de obrar el esfuerzo. Ordinariamente nos referimos á la **resistencia viva elástica**, ó sea la que corresponde al esfuerzo $s_e$ en el límite de elasticidad.

**33.** Sea

$s_e$ = al esfuerzo por unidad de superficie en el límite de elasticidad;

$a$ = área de la sección de la barra;

$P_e = as_e$ = la carga ó peso correspondiente á la tensión $s_e$;

L = al largo original de la barra;

$l$ = á su alargamiento, en el límite de elasticidad;

E = al coeficiente ó módulo de elasticidad.

El trabajo se ha hecho por la carga *media* $P_e/2 = as_e/2$, obrando en la distancia $l = L\, s_e/E$. En consecuencia:

$$\textbf{Resistencia viva} = K = P_e l/2 = as_e L s_e/2E = (s_e^2/2E)aL.$$

**34.** Aquí $s_e^2/2E$ es el **módulo de resistencia viva** = á la resist viva de una barra que tiene la unidad de área por sección y la unidad de longitud (Hágase en la fórmula, $a = 1$, $L = 1$.)

El módulo de resistencia viva de un material da la medida de su capacidad para resistir choques ó golpes.

## Pesos aplicados repentinamente.

**35.** Suspéndase un cuerpo, de peso, W, por una cuerda, y dispóngase de modo que no haga sino tocar el platillo de una balanza de resorte sin hacerlo bajar. Córtese entonces la cuerda con unas tijeras.

**36.** En el momento de cortar la cuerda, el resorte no se ha estirado; su esfuerzo resistente, S, es por tanto cero, y la fuerza de descenso neta ó resultante, que obra sobre el cuerpo, es $F = W - S = W - 0 = W$.

**37.** Bajo la acción de esta fuerza, el resorte se estira, y S aumenta proporcionalmente con el estiramiento. En consecuencia (quedando constante W) la fuerza del descenso, aceleratriz, F, que obra sobre el cuerpo, disminuye hasta que $S = W$, entonces $F = W - S = W - W = 0$.

**38.** El cuerpo, hasta entonces constantemente acelerado (por una fuerza decreciente, F), ha aumentado constantemente su velocidad. Sea $h$ = á la altura de donde ha caído ahora, y sea $x$ el punto alcanzado, al extremo de $h$.

**39.** Más allá de $x$ (quedando constante W, mientras S continúa aumentando), obra sobre el cuerpo una fuerza ascendente retardatriz constantemente creciente, $-F = W - S$, que lo lleva al reposo en un segundo punto, $z$, al fin de una segunda distancia = $h$. Su **caída total** es por tanto. = $2h$.

**40.** Sea S máx = al valor máximo de S, ó el que tiene al término $z$, de la caída, $2h$. Entonces, puesto que S ha aumentado proporcionalmente con $h$, su valor *medio* durante la caída, $2h$, fué S máx/2; y el trabajo hecho, durante la caída entera, $2h$, fué $2Wh = (S\ máx/2)\ 2h = S\ máx \times h$. En consecuencia,

## S máx = 2 W.

**41.** Al término, $z$, de la caída, $2h$, el cuerpo, al llegar al reposo, es movido por una fuerza ascendente, $-F = W - S$ máx $= W - 2W = -W$; y (despreciando la fricción) se repite ahora el mismo fenómeno, pero en dirección ascendente, y así indefinidamente.

**42. Pero las pérdidas de energía**, debidas á la resistencia del aire y á la fricción interna, hacen cada oscilación menor que su valor teórico; y el cuerpo en definitiva va á reposar en el punto, $x$, á mitad de la distancia de la caída $2h$.

**43.** Así (§ 40) dentro del límite de elasticidad, **un peso, aplicado de súbito** (aunque sin choque), **produce temporalmente un alargamiento** casi igual **á dos veces el que pudiera producir aplicado gradualmente**; esto es: dos veces el que puede sostener después de quedar en reposo; y desarrolla temporalmente, en el cuerpo alargado, **una resistencia = dos veces el peso.**

**44. Si el peso se agrega en pequeñas porciones**, cada una aplicada repentinamente, entonces cada porción produce un pequeño alargamiento temporal que después se reduce y queda con un valor mitad. Bajo la última pequeña porción

de peso, el resorte se estira temporalmente hasta una longitud mayor que la que puede mantener el peso total, por un valor igual á la mitad del pequeño estiramiento temporal producido por la aplicación súbita de la última pequeña porción de peso.

**Una sección puede debilitarse aumentando el ancho.** En las páginas 418, etc., consideramos el caso en que el ancho de la base era fijo y en que el punto de aplicación de la resultante de las fuerzas que obraban sobre ella cambiaba de posición en dicha base. Mencionaremos ahora el caso en que la resultante se aplica á una distancia constante de un extremo de la base, pero en que esta base es de ánch variable, de manera que la distancia constante pueda ser igual, mayor ó menor que la mitad de ella.

La fig. E representa una vista de lado de una barra de espesor uniforme = 1*, pero (como se indica) de ancho variable y sometida á presiones, cuya resultante P es = 1* y pasa por el centro de la sección *ab* de ancho igual á 1 * **.

Fig. E.

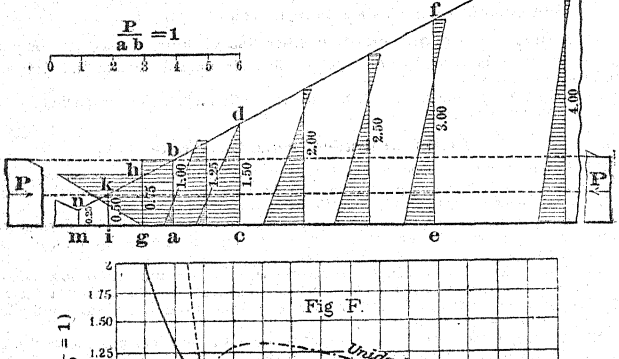
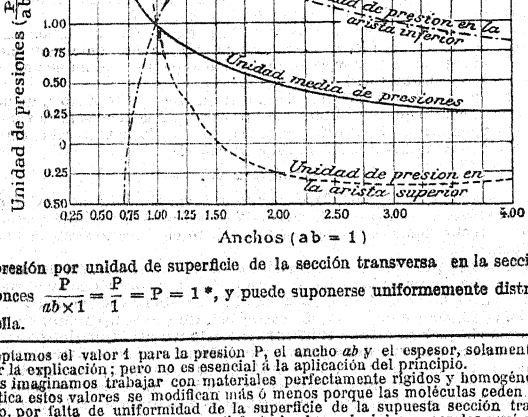



Fig F.




La presión por unidad de superficie de la sección transversa en la sección *ab*, es entonces $\frac{P}{ab \times 1} = \frac{P}{1} = P = 1$ *, y puede suponerse uniformemente distribuída sobre ella.

* Adoptamos el valor 1 para la presión P, el ancho *ab* y el espesor, solamente para facilitar la explicación; pero no es esencial á la aplicación del principio.

** Nos imaginamos trabajar con materiales perfectamente rígidos y homogéneos. En la práctica estos valores se modifican más ó menos porque las moléculas ceden bajo el esfuerzo, por falta de uniformidad de la superficie de la supuesta sección transversal, etc. Sin embargo, son bastante satisfactorios los principios generales aquí establecidos.

Pero en otras secciones de la barra, la resultante se halla más cerca de una arista que de la otra, y la fuerza por unidad de superf no puede entonces suponerse por más tiempo uniformemente distribuída sobre la sección transversal; pero, como se explicó en las págs. 418 á 422, dicho esfuerzo adquiere el valor máximo en la arista más próxima á la resultante, y disminuye gradual y uniformemente hasta llegar á un mínimo en la arista más distante.

Esto se indica por medio de triángulos sombreados, etc., en la fig. E, y por medio de curvas en la fig. F, que indican el esfuerzo ó presiones medias por unidad de superficie * de las diversas secciones de la fig. E, y los esfuerzos ó presiones por unidad de superficie ejercidos en las aristas superiores é inferiores, respectivamente, calculados por las reglas de las págs. 419 á 422.

En la tabla siguiente se dan también estas presiones.

**Presión por unidad de superficie en la fig. E;** tomándose la presión por unidad $\frac{P}{ab}$ en la sección *ab* igual á 1 **.

| Sección. | Ancho. | Presión por unidad de superficie de la sección transversal. | | |
|---|---|---|---|---|
| | | Medio. | En la arista inferior *me.* | En la arista superior *nf.* |
| | 4.00 | .25 | .8125 | — .3125 † |
| *ef* | 3.00 | $^1/_3$ | 1.00 | — $^1/_3$ |
| | 2.50 | .40 | 1.12 | — .32 |
| | 2.00 | .50 | 1.25 | — .25 |
| *cd* | 1.50 | $^2/_3$ | $1^1/_3$ | 0 |
| | 1.25 | .80 | 1.28 | .32 |
| *ab* | 1.00 | 1.00 | 1.00 | 1.00 |
| *gh* | .75 | $1^1/_3$ | 0 | $2^2/_3$ |
| *ik* ‡ | .50 | 2.00 | —4 † | 8.00 |
| *mn* ‡ | .25 | 4.00 | —32 † | 40.00 |

**Es de importancia observar** que para una fuerza dada P, y para anchos, menores que 3*ab*, la sección más fuerte de esta barra no es *la más ancha*, sino aquella (*ab*) en que la resultante P pasa por el centro de la sección. En otras palabras, **una barra puede debilitarse aumentando su sección transversal,** si este aumento es tal que obligue á la resultante de las presiones á pasar por cualquiera otra parte que no sea el centro de cualquiera sección transversal. Esto es enteramente independiente del *peso* de la porción añadida.

Entre las secciones más anchas que *ab*, la más débil es aquella (*cd*) cuyo ancho es $=1.5ab$. En esta sección la arista inferior ó más baja *me* tiene por presión ó esfuerzo máximo por unidad de superficie $\left(=1^1/_3 \times \frac{P}{ab}\right)$, mientras que en *d*, en la arista superior, no hay presión. Más allá de *cd* la arista superior *nf* está sometida á *tensión* † y la presión por unidad de superf á lo largo de *me* decrece haciéndose

* En el caso discutido en las págs. 418 á 422, la presión media $un = \frac{P}{uv}$, permanece constante en tanto que la superficie entera *uv* entraba en juego. Aquí, al contrario, el área de la sección varía. Por tanto, la presión media por unidad de superficie varía también, y en razón inversa del área.

** Véase la nota * de la página anterior.

† En la actual discusión, así como en la de las págs. 418 á 422, hemos supuesto casos de *compresión*; pero, el principio es el mismo cuando las fuerzas se aplican por *tensión*. En tales casos, sin embargo, los términos « presión » y « tensión » están invertidos.

‡ Los esfuerzos por unidad de superficie en las aristas de la sección *ik* son demasiado grandes para indicarlos convenientemente en cualquiera de las figuras; y los de la sección *mn* (como lo indica la tabla) exceden con mucho los límites de las figuras. La presión en *k* sería $\frac{P}{o} = \infty$ (infinita) si no fuera por las tensiones que se ejercen en la parte inferior de la sección.

de nuevo $= \frac{P}{ab}$ en *ej*, en que el ancho *ej* es $=3ab$; y decreciendo aún más con ulteriores incrementos del ancho. Cuando el ancho se hace *menor* que *ab*, como en *gh*, etc., la arista *superior* de la barra se acerca más á la resultante que la arista inferior, y por tanto recibe la presión máxima.

Cuando el ancho es $=\frac{3}{4}ab$ como en *gh*, la distancia de la resultante á la arista superior es $\frac{1}{3}$ del ancho de la sección. La presión en la arista inferior es entonces $=0$; la presión media en *gh* es $\frac{P}{ab} \times \frac{1}{.75} = 1\frac{1}{3} \times \frac{P}{ab}$, y la presión en la arista superior es el doble de la presión media que se ejerce en *gh*, ó $2\frac{2}{3} \times \frac{P}{ab}$.

Cuando el ancho se hace menor que $\frac{3}{4}\ ab$, como en *ik* y *mn*, la presión en la arista inferior *mn* se hace negativa, es decir, se convierte en tensión.

Así, cuando como en *ik*, el ancho es $=\frac{1}{2}ab$, y la resultante pasa por la arista superior, la presión por unidad de superficie en esta arista es $=8 \times \frac{P}{ab}$, mientras que la arista inferior soporta una *tensión* de $4 \times \frac{P}{ab}$; y, á medida que la sección sea posteriormente reducida, estas fuerzas aumentan más y más, muy rápidamente. La condición de las secciones (tal como *mn*) en que la resultante pasa por fuera de ellas, es semejante á la de la sección *mn* de un gancho que sostenga un peso, como en la fig. G.

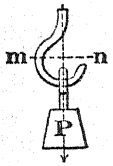



**Fig. G.**

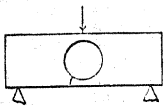


**Fig. H.**

La casa de William Sellers and Co., de Filadelfia, tuvo ocasión de ensayar un número de vigas de hierro fundido, provista cada una de una gran abertura circular como en la figura anexa. Estas vigas se rompieron, no por la sección más pequeña situada directamente debajo del centro de la abertura, sino un poquito hacia un lado, en donde la sección era más ancha, como se indica en la fig. H.

**Fatiga del material.** En los artículos siguientes sobre resistencia de materiales, la carga máxima ó carga de ruptura es aquella que, en su primera aplicación ó al obrar por primera vez, produce en un *corto* tiempo la ruptura de una pieza dada. Pero los experimentos de Wohler y Spangenberg muestran que una pieza puede quebrarse **aplicando sucesivamente** una carga mucho *menor* que aquélla y que mientras más á menudo se aplique la carga tanto menos se necesita para producir la ruptura. De este modo, el hierro forjado, que requiere una tensión de 3,726 kg por cm cuad para romperse después de obrar sobre él 800 veces, lo rompió una carga de 2,460 kg por cm cuad aplicada como 10 millones de veces sucesivas. El esfuerzo vuelve á cero después de cada aplicación en ambos casos.

**La diferencia** entre la tensión máxima y la tensión mínima de una pieza sometida solamente á tensión, ó entre la compresión máxima y la compresión mínima de una pieza sometida á compresión solamente, ó la suma de la tensión máxima y la compresión máxima de una pieza sometida alternativamente á ambos esfuerzos, la llamaremos **variación del esfuerzo** sobre la pieza. Cuando la dif de los esfuerzos extremos es menor que el límite de elasticidad, pueden repetirse las aplicaciones de la fuerza un número « enorme » de veces *.

* Esto no siempre es así en los casos en que el límite de elasticidad se ha aumentado artificialmente por medio de procedimientos de manufacturas, etc. Los esfuerzos á menudo repetidos, de tensión y compresión alternativa, inferiores á dicho límite, reducen éste al límite natural. Una ligera grieta puede ocasionar la ruptura por la acción de un esfuerzo aplicado relativamente pocas veces, de una intensidad ó *diferencia de esfuerzos extremos* sólo un poco mayor ó aún menor que el límite de elasticidad. El reposo entre dos esfuerzos aumenta la resistencia de la pieza. En muchos casos los esfuerzos un poco más allá del límite de elasticidad, aun si se los aplica á menudo,

Cuando se aplica un peso ó carga un número dado de veces, á una pieza, el peso que se requiere para producir la ruptura es el menor cuando la **dif de los esfuerzos extremos** es muy grande. Si los esfuerzos aplicados son alternativamente de tensión y de compresión, la ruptura tiene efecto más prontamente que si lo fueran siempre de tensión ó siempre de compresión. Esto es, tiene efecto con menores *variaciones de esfuerzos extremos* aplicados un número dado de veces, ó con menor número de aplicaciones de una **dif de esfuerzos extremos dada**. Para una **diferencia de esfuerzos extremos** dada y número dado de veces que ha de aplicarse, la circunstancia más desfavorable es cuando la tensión y la compresión son iguales.

Todo lo que hemos dicho se toma generalmente en consideración en el cálculo de las piezas de las construcciones de importancia sometidas á cargas movibles. Por ejemplo, Mr. Jos. M. Wilson, ingeniero civil, miembro del Instituto de ingenieros civiles de Londres, miembro de la Sociedad americana de ing. civs., emplea las fórmulas siguientes para determinar el « esfuerzo permitido » en los puentes de hierro, en kg por cm cuad, para calcularle á cada pieza el área de sección transversal que le corresponde.

Para piezas sometidas á **una clase de esfuerzos solamente** (todos de compresión ó todos de tensión) (*Obs. del T.* — Hemos cambiado los coeficientes de las notas para aplicarlas á medidas métricas) :

$$\mathbf{a} = u\left(1 + \frac{\text{esfuerzo mín aplicado á la pieza}}{\text{esfuerzo máximo aplicado á la pieza}}\right).$$

Para una pieza sometida **alternativamente á la tensión y á la compresión**, búsquese la compresión máxima y la tensión máxima ejercida sobre la pieza. Llámese el **más pequeño** de estos dos máximos « máximo menor » y el otro máximo más grande « máximo mayor ». Luego tendremos :

$$\mathbf{a} = u\left(1 - \frac{\text{máximo menor}}{2\ \text{máximo mayor}}\right).$$

Para una pieza cuya **comp máxima** y **tensión máxima** sean iguales, esta fórmula se convierte en

$$\mathbf{a} = u\left(1 - \frac{1}{2}\right) = \frac{u}{2}*.$$

La a de arriba es el esfuerzo de **tensión** permitido sobre cualquier pieza. Pero, el esfuerzo de **compresión** permitido se encuentra por la « fórmula de Gordon »**. Gordon dedujo de las experiencias de Hodgkinson las fórmulas siguientes :

Para **columnas cilíndricas macizas**. Carga de fractura :

$$P_o = \frac{5630\ w}{1 + H \times .0025\left(\frac{l}{d}\right)^2}.$$

Para la carga de trabajo ordinario, se usa la misma fórmula poniendo en el numerador 940 en lugar de 5630.

Para las **columnas huecas** la carga de fractura es

$$P = \frac{5630\ w}{1 + H \times .00125\left(\frac{l}{d}\right)^2}.$$

y la carga de trabajo ordinario se obtiene cambiando sólo, como se hizo antes, el factor 5630 por 940.

---

elevan dicho límite y la resistencia, pero hacen la pieza quebradiza y por tanto más propensa á la ruptura por medio de choques, y además un pequeño aumento de esfuerzo posterior disminuye rápidamente ó puede destruir enteramente la elasticidad. Un esfuerzo de *tensión* superior al límite de elasticidad hace descender notablemente ó puede aun destruir la elasticidad *compresiva*, y viceversa. Si un esfuerzo de tensión, a alargar una pieza, reduce su área de resistencia, puede así reducir su resistencia *total* aun cuando la resistencia *por cm cuad* haya aumentado. B. Baker encontró que el acero duro se fatiga mucho más ligero bajo la acción de cargas ó pesos repetidos que el acero dulce ó el hierro.

* Para el hierro laminado en compresión, $u = 455$ kg/cm cuad; para el mismo en tensión, $u = 490$.

** *N. del T.*— El autor cita aquí la fórmula de Gordon que no está ni en la página ni en la materia aludidas, pues esta edición, alterando una de las anteriores en que se hace la cita, ya no la trae. Para llenar este vacío la hemos tomado, con permiso de Marvá, de su « Mecánica », tomo 2.º, pág. 64.

En todas:

$w$ = área sección transversal en cm cuad;
$d$ = { diám columna maciza / — exterior columna hueca } en cm;
$l$ = longitud de la columna en cm;
P y $P_0$ = cargas en kg.

El coeficiente H del segundo término del denominador tiene los valores siguientes en relación con la disposición de los extremos de la pieza comprendida:

Para las dos bases planas, H=1.
Una base plana y otra redondeada ó articulada, H=2.
Dos bases redondeadas ó articuladas, H=4.

Los experimentos demuestran que el material puede fallar ó ceder sometido á un **esfuerzo muy prolongado** de mucha menos intensidad que la producida por la carga máxima ó carga de ruptura.

## RESISTENCIA TRANSVERSAL

**1.** En Estática, §§ 285, etc., discutimos la acción de las fuerzas externas ó destructivas sobre los cantilevers, vigas y armaduras. Aquí discutimos la reacción de las fuerzas (esfuerzos) internas ó resistentes en cantilevers y vigas sólidos, á fin de determinar sus pesos. Véanse también, §§ 104, etc.

**2.** A menos que de otro modo se diga ó aparezca, suponemos que las fuerzas en todas partes del cantilever ó viga, están dentro del límite de elasticidad.

### Condiciones de equilibrio.

**3.** Para el equilibrio, las fuerzas internas y sus momentos deben contrarrestar las fuerzas externas y sus momentos. En otros términos, si el cantilever ó la viga se suponen cortados por una sección en cualquier punto, debemos tener

(1) Σ fuerzas verticales =0
(2) Σ fuerzas horizontales=0
(3) Σ momentos =0

O:

(1) Suma algebraica de los esfuerzos verticales internos=suma algebraica de las *fuerzas* verticales externas en uno ú otro lado de la sección;

(2) Suma algebraica de las tensiones horizontales resistentes=suma de las *compresiones horizontales;* y

(3) Suma algebraica de los momentos de las *resistencias* internas=suma algebraica de los momentos de las *fuerzas* externas en uno ú otro lado de la sección.

**4.** Los cantilevers y las vigas de sección transversal uniforme tienen ordinariamente una superabundancia de resistencia contra el esfuerzo cortante. De aquí que la discusión de su resistencia verse principalmente sobre el equilibrio de los *momentos.* Para su resistencia al esfuerzo cortante vertical, véase Estática, §§ 325, etc., y pág. 528. Véase también Esfuerzo cortante horizontal, §§ 51½á 53, más abajo.

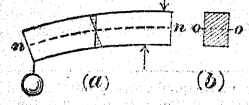



Fig. 1.

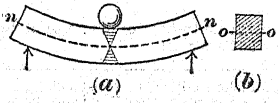



Fig. 2.

**5.** Para que haya equilibrio, el momento resistente, R (=la suma de los momentos resistentes, $r$, de todas las partículas en cualquier sección transversal del cantilever ó de la viga, fig. 1 ó 2) debe ser igual al momento de flexión, M, ó suma algebraica de los momentos de todas las fuerzas externas en cada lado de la sección.

### Reacciones de las fibras.

**6.** En una *armadura* ó viga *armada* (véase Armaduras) la resistencia de cada uno de sus dos cordones se considera obrando en una línea que pasa por los centros de

gravedad de sus secciones transversales; pero, en un cantilever sólido, fig. 1, ó viga, fig. 2, el momento resistente total es la suma de los momentos resistentes separados de las varias fibras de toda la sección transversal.

## Superficie neutra. Eje neutro.

**7.** Cuando un cantilever (ó viga) se cimbra, las fibras en la parte superior (ó inferior) de cada sección transversal se extienden mientras las de la parte inferior (ó superior) se comprimen (véanse figs. 1 y 2); siendo la extensión y la compresión mayores en el borde superior y en el inferior de la sección y decreciendo de all á uniformemente hacia una superficie interior *nn*, figs. (*a*), cerca del centro de la sección transversal. En esta superficie, que se llama la **superficie neutra**, las fibras ni se alargan ni se comprimen. La línea, *oo*, figs. (*b*), formada por la intersección de la superficie neutra con cualquier sección transversal del cantilever ó viga, se llama el **eje neutro** de esa sección.

**8.** A fin de que la suma algebraica de todas las fuerzas horizontales en la sección transversal sea cero, como se requiere para el equilibrio, el eje neutro debe pasar por el centro de gravedad de la sección. En consecuencia, la superficie neutra pasa por los centros de gravedad de todas las secciones transversales.

**9.** El eje neutro puede encontrarse equilibrando la sección (hecha en cartón) sobre el filo de un cuchillo. O véase Centro de gravedad, en Estática, §§ 125, etc. Toda sección tiene un número indefinido de ejes neutros, que pasan todos por el centro de gravedad de ella en otras tantas direcciones diferentes. El eje requerido, en cualquier caso dado, es aquel que es perpendicular al plano del momento de flexión que se estudia.

En la siguiente discusión, suponemos que el eje neutro de la sección es perpendicular á la línea de acción (ordinariamente vertical) del peso; así lo es generalmente.

## Momento de resistencia. Unidad del esfuerzo ó esfuerzo unitario.

**10.** Supónese que el alargamiento ó compresión de cada fibra, y por tanto la resistencia efectivamente ejercida por ella, es proporcional á su distancia vertical sobre ó debajo del eje neutro.

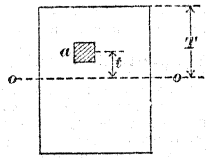



**Fig. 3.**

En la fig. 3, sea

T = la distancia del eje neutro, *oo*, á la fibra más distante, por encima ó por debajo del eje;
S = la tensión por unidad de superficie en dicha fibra más distante (llamada también *tensión unitaria*);
$t$ = la distancia desde el eje neutro hasta cualquier fibra dada;
$s$ = la tensión por unidad de superficie en dicha fibra dada;
$a$ = el área de dicha fibra;
F = el esfuerzo total en dicha fibra;
$r$ = el momento de resistencia de la fibra respecto al eje neutro;
M = momento de flexión en la sección transversal que se considera;
R = momento de resistencia de la sección transversal entera;
$= \Sigma r$ = la suma de los momentos de resistencia de todas las fibras;
I = el momento de inercia de la sección transversal. Véanse §§ 14, etc;
$= \Sigma t^2 a$ = la suma, para todas las fibras, de $t^2 a$;
X = el módulo de la sección (ó momento resistente *) $= \frac{I}{T} = \frac{R}{S}$. Véanse §§ 25, etc.

* *N. del T.* — Entre los ingenieros españoles es más conocido por *momento resistente*.

Entonces el esfuerzo por unidad de superficie en cualquier fibra dada, es $= s = S\frac{t}{T}$; su esfuerzo total, F, es $= as = Sa\frac{t}{T}$; y su momento resistente, $r$, es $= Ft = Sa\frac{t^2}{T}$.

En consecuencia, el momento resistente, R, de la sección entera, es

$$R = M = \Sigma r = \Sigma Sa\frac{t^2}{T} = \frac{S}{T}\Sigma t^2 a = \frac{S}{T}. \quad I.$$

En consecuencia también, $S = MT/I$; $I = MT/S = TX$; $T = SI/M$.

Puesto que $S/s = T/t$, tenemos $S/T = s/t$, y $R = M = SI/T = sI/t = SX$.

**En vigas de sección rectangular,** pág. 491, de ancho, B, y de altura, D, tenemos $I = BD^3/12$; y $T = D/2$. En consecuencia,

$$S = 12MT/BD^3 = 6M/BD^2; \text{ y}$$
$$R = M = SBD^3/12T = SBD^2/6.$$

Cuando se ensayan las resistencias de las vigas hasta su ruptura, el valor alcanzado por S se llama: el **módulo de ruptura.**

**11.** Se notará que las resistencias de vigas semejantes de cualquier forma, y las de vigas rectangulares, sean semejantes ó no, son directamente proporcionales al producto, ancho × cuadrado de altura. Véase § 63.

**12.** Cuando el esfuerzo, S, sobre las fibras extremas, es = al límite de elasticidad del material, el fracaso es inminente. El esfuerzo permitido por unidad de superficie es, como máximo, la mitad del que corresponde al límite de elasticidad, y la carga de seguridad es aquella bajo la cual S no excede de la unidad del esfuerzo permitido.

**13.** La misma cantidad de material que compone una viga sólida, fig. 2, presentaría mayor resistencia á la flexión ó ruptura, si estuviera cortada en dos longitudinalmente á lo largo de la superficie neutra, *nn*, y convertida en cordones superior é inferior de una viga armada; porque, primero, el brazo de palanca con que obra la resistencia se aumenta así considerablemente; y, segundo, las alturas de los cordones son tan pequeñas, comparadas con sus distancias al eje neutro, que puede suponerse que sus fibras obran *unidas* é *igualmente*. En consecuencia, prácticamente, *todas* las fibras en el cordón superior deben ser trituradas ó todas las del inferior desgarradas, *en el mismo instante*, antes de que la viga armada pueda romperse; mientras que, en la viga sólida, las fibras extremas superiores ó inferiores ceden primero; luego las que les siguen y así sucesivamente, unas después de otras.

## Momento de inercia.

**14.** A diferencia del momento de una fuerza, que es el producto de una fuerza y una distancia, el momento de inercia es una cantidad puramente *geométrica*, pues es la suma de los productos de las *áreas* de las fibras por los cuadrados de sus *distansias* al eje neutro. Así, el momento de inercia de una sección dada depende solamente de las dimensiones y forma de esa sección, y es independiente del material y de la abertura que cubre la viga y de la manera cómo está ésta sostenida ó cargada.

**Unidad del momento de inercia.** Siendo el momento de inercia de una figura el producto de un área por el cuadrado de una distancia, su unidad es la cuarta potencia de una unidad de longitud. Así, en un rectángulo de 3 cm de ancho y 4 de alto, $I = \frac{bd^3}{12} = \frac{3 \times 64}{12} = \frac{192}{12} = 16$ cm *cuartos** $= 16$ cm$^4$.

En un rectángulo de 1 cm de ancho y 6 cm de alto, $I = \frac{1 \times 6^3}{12} = 18$ cm$^4$.

**15.** Comparando secciones semejantes de cualquier forma, sus momentos de nercia son proporcionales al producto, ancho × cubo de altura. Compárese § 11.

**16 Los cuadros ilustrados** de las págs. 491 á 493, dan para varias figuras que se emplean con frecuencia:

(1) I = el momento de inercia $= \Sigma t^2 a$;

(2) T = la distancia desde el eje neutro á la fibra más distante;

(3) X = el módulo de la sección ó momento resistente $\frac{I}{T} = \frac{\Sigma t^2 a}{T} = \frac{R}{S} = \frac{M}{S}$;

(4) A = el área de la sección transversal.

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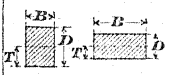
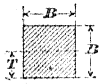
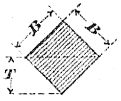
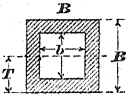
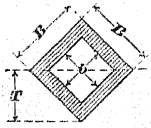
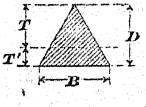
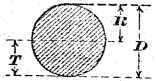
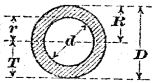
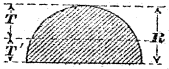
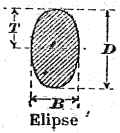
*N. del T.* — El momento de inercia tiene cuatro dimensiones. Muchos autores llaman a estas unidades del momento de inercia, centímetros *cuartos*, milímetros *cuartos*, etc.; según la unidad que se elija.

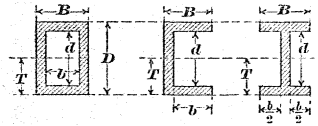
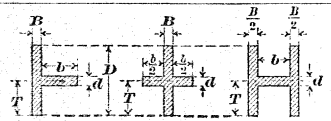
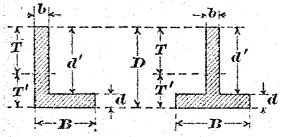
**Momentos de inercia, etc.**

| | | I<br>Momento de inercia. | T<br>Distancia del eje neutro á la fibra más distante. | $X = \frac{I}{T}$<br>Módulo de la sección, ó momento resistente. | A<br>Area de la sección. |
|---|---|---|---|---|---|
| 1 |  | $\frac{B D^3}{12}$ | $\frac{D}{2}$ | $\frac{B D^2}{6}$ | B D |
| 2 | | $\frac{B (D^3 - d^3)}{12}$ | $\frac{D}{2}$ | $\frac{B (D^3 - d^3)}{6 D}$ | B (D - d) |
| 3 |  | $\frac{B^4}{12}$ | $\frac{B}{2}$ | $\frac{B^3}{6}$ | $B^2$ |
| 4 |  | $\frac{B^4}{12}$ | $\frac{B}{\sqrt{2}}$ | $\frac{\sqrt{2}}{12} B^3$<br>$= 0.118\, B^3$ | $B^2$ |
| 5 |  | $\frac{B^4 - b^4}{12}$ | $\frac{B}{2}$ | $\frac{B^4 - b^4}{6B}$ | $B^2 - b^2$ |
| 6 |  | $\frac{B^4 - b^4}{12}$ | $\frac{B}{\sqrt{2}}$ | $\frac{\sqrt{2}}{12} \cdot \frac{B^4 - b^4}{B}$<br>$= 0.118 \frac{B^4 - b^4}{B}$ | $B^2 - b^2$ |
| 7 |  | $\frac{B D^3}{36}$ | $T = \frac{2}{3} D$<br>$T' = \frac{1}{3} D$ | $X = \frac{B D^2}{24}$<br>$X' = \frac{B D^2}{12}$ | $\frac{B D}{2}$ |

**Momentos de inercia, etc.** (Continuación.) ($\pi = 3.14159...$)

| | | I<br>Momento de inercia. | T<br>Distancia del eje neutro á la fibra más distante. | $X = \frac{I}{T}$<br>Módulo de la sección, ó momento resistente. | A<br>Area de la sección. |
|---|---|---|---|---|---|
| 8 |  <br>Círculo | $\frac{\pi R^4}{4} = 0.7854\ R^4$<br>$\frac{\pi D^4}{64} = 0.0491\ D^4$ | $R = \frac{D}{2}$ | $\frac{\pi R^3}{4} = \frac{\pi D^3}{32}$ | $\pi R^2 = \frac{\pi D^2}{4}$ |
| 9 |  <br>Círculo hueco | $\frac{\pi (R^4 - r^4)}{4} = \frac{\pi (D^4 - d^4)}{64}$ | $R = \frac{D}{2}$ | $\frac{\pi (R^4 - r^4)}{4R} = \frac{\pi (D^4 - d^4)}{32D}$ | $\pi (R^2 - r^2) = \frac{\pi (D^2 - d^2)}{4}$ |
| 10 |  <br>Sem círculo | $\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) R^4 = 0.1098\ R^4$ | $T = 0.5756\ R$<br>$T' = 0.4244\ R$ | $X = 0.1905\ R^3$<br>$X' = 0.2587\ R^3$ | $\frac{\pi R^2}{2} = \frac{\pi D^2}{8}$ |
| 11 |  <br>Elipse | $\frac{\pi B D^3}{64}$ | $\frac{D}{2}$ | $\frac{\pi B D^2}{32}$ | $\frac{\pi B D}{4}$ |

**Momentos de inercia, etc.** (Continuación.)

| | | I<br>Momento<br>de<br>inercia. | T<br>Distancia del eje<br>neutro á la<br>fibra más distante. | $X = \frac{I}{T}$<br>Módulo de la<br>sección. | A<br>Area<br>de<br>la sección. |
|---|---|---|---|---|---|
| 12 |  | $\frac{B D^3 - b d^3}{12}$ | $\frac{D}{2}$ | $\frac{B D^3 - b d^3}{6 D}$ | $B D - b d$ |
| 13 |  | $\frac{B D^3 + b d^3}{12}$ | $\frac{D}{2}$ | $\frac{B D^3 + b d^3}{6 D}$ | $B D + b d$ |
| 14 |  | $I = \frac{B\left[T'^3 - (T' - d)^3\right] + b\left[T^3 + (T - d)^3\right]}{3}$<br>$T' = \frac{B d^2 + b d' (D + d)}{2\left[B D - (B - b) d'\right]}$; $\quad X' = \frac{I}{T'}$;<br>$T = D - T'$; $\quad X = \frac{I}{T}$; | | | $B d + b d'$ |

**17.** En las secciones en que la distancia desde el eje neutro hasta la fibra más baja, y el correspondiente módulo de la sección difieren de los (T y X) pertenecientes á la fibra más alta, los correspondientes á la fibra más baja se distinguen con T' y X' respectivamente.

**18.** En cada figura el eje neutro está indicado por una línea horizontal que atraviesa la sección.

**19.** El momento de inercia de cualquier figura, respecto de su eje neutro, es la suma de los momentos de inercia de sus varias partes respecto de ese mismo eje.

**20.** Sea I=al momento de inercia de la figura entera respecto á su eje neutro, *oo*;
$i$=al momento de inercia de cualquier parte respecto del eje neutro de la figura entera;
$m$=al momento de inercia de esa parte respecto de su propio eje neutro;
$a$=al área de esa parte;
$t$=á la distancia de su centro de gravedad al eje neutro, *oo*, de la figura entera.

Entonces $I=\Sigma i$; además, $i=m+at^2$.

**21.** Así, en la fig. 4,

$$i_1 = \frac{BD^3}{12} + BDt_1^2;$$

$$i_2 = \frac{bd^3}{12} + bdt_2^2;$$

$I=i_1+i_2$.

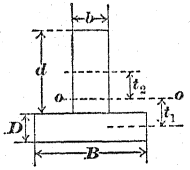



**Fig. 4.**

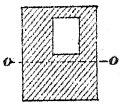



**Fig. 5.**

**22.** En consecuencia, en cualquier sección hueca, como en el rectángulo hueco, fig. 5, sea I'=al momento de inercia de toda la figura (el rectángulo sombreado y el no sombreado), $i$=al del rectángulo que falta ó no sombreado, é I=al de la porción sombreada; todos referidos al eje neutro, *oo*, de la *porción sombreada*. Tendremos : $I=I'-i$.

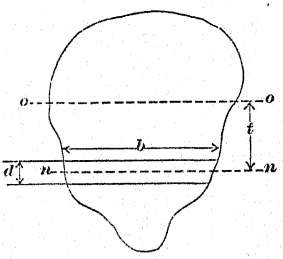



**Fig. 6.**

**23.** En el caso de una sección irregular, como la fig. 6, divídase la sección en numerosas fajas, paralelas al eje neutro y bastante angostas para poderlas considerar como rectángulos; y procédase como en los §§ 19 á 21.

**24.** Mientras más angostas se toman las fajas, más pequeño se hace $m$. Si las fajas se toman tan angostas (relativamente á la altura de la sección), que $m$ pueda despreciarse, entonces $I=\Sigma t^2 a$, como en el § 10. Las fajas no necesitan ser de ancho uniforme.

## El módulo de la sección (ó momento resistente).

**25. Definición.** Si el momento de resistencia, $R = \frac{S}{T}$. $\Sigma t^2 a$, se divide por el esfuerzo por unidad de superficie, S, en las fibras extremas, el cociente, $X = \frac{R}{S} = \frac{\Sigma t^2 a}{T} = \frac{I}{T}$, se llama momento resistente ó módulo de la sección. Éste, como el momento de inercia, §§ 14, etc., es una cantidad puramente *geométrica*, que depende solamente de las dimensiones y de la forma de la sección, y es independiente de la substancia, de la luz y del modo de estar la carga.

**26.** Teniendo el módulo de la sección, X, no tenemos más que multiplicarlo por el esfuerzo por unidad de superficie, S, en las fibras extremas, para obtener el momento de resistencia, R=SX.

**27.** Multiplicando el módulo de la sección, X, por la distancia, T, del eje neutro á las fibras más distantes, obtenemos el momento de inercia, I=TX.

**28.** El módulo de la sección ó momento resistente se da de ordinario en las tablas de vigas laminadas, canales, etc.

## Carga. Resistencia.

**29. El siguiente cuadro ilustrado** da (1) el momento máximo, M, correspondiente á un peso dado; y (2) el peso, W*, correspondiente á un esfuerzo dado, S, por unidad de superficie en diferentes condiciones de apoyo y de carga. En este cuadro,

M = momento máximo de flexión;
R = M = momento de resistencia de la sección transversal;
W = carga total extraña * sobre la viga, ya concentrada en un punto (como se ha mostrado), ya uniformemente distribuída sobre la luz;
$l$ = la abertura ó luz;
S = al esfuerzo por unidad de superficie en las fibras más distantes del eje neutro, debido al peso extraño, W *;
T = á la distancia del eje neutro á las fibras más distantes;
I = al momento de inercia.

En vigas rectangulares,

$b$ = ancho;
$d$ = alto;
I = momento de inercia = $\frac{bd^3}{12}$;
$n = \frac{Wl}{Sbd^2}$.

De los diagramas debajo de cada carga, el primero representa los momentos y el segundo los esfuerzos cortantes en las varias partes de la luz.

**30.** Si S=al esfuerzo permitido por unidad de superficie de la fibra, entonces, en las fórmulas que preceden, W=al peso extraño permitido *.

**31.** Se notará que las resistencias de vigas seme(antes son proporcionales á los valores de $\frac{bd^2}{l}$; esto es: las resistencias de vigas de secciones transversales semejantes son directamente proporcionales á susanchos, a los cuadrados de sus alturas; é inversamente proporcionales á sus luces.

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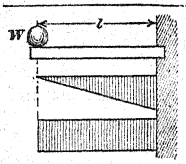
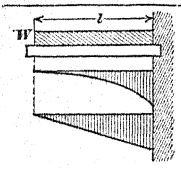
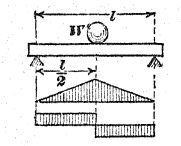
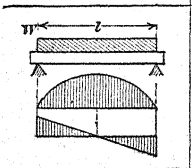
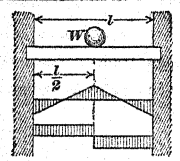
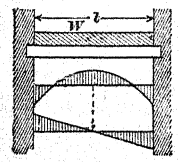
* Aquí se supone la viga sin peso. Veanse §§ 42, etc.

| **Para los símbolos,** véase la página que sigue. | M=R **Momento máximo de flexión.** | (Véase final § 10.) **Peso, W, y esfuerzo, S.** S=MT /I=RT /I. **En general.** | **En vigas rectangulares.** | **n** |
|---|---|---|---|---|
|  | En el apoyo. $M = Wl$ | $W = \frac{M}{l}$ <br> $= S \cdot \frac{I}{Tl}$ | $W = \frac{S}{12} \cdot \frac{bd^3}{Tl}$ <br> $= \frac{S}{6} \cdot \frac{bd^2}{l}$ | $\frac{1}{6}$ |
| | | $S = \frac{WTl}{I}$ | $S = \frac{12\,WTl}{bd^3}$ <br> $= \frac{6\,Wl}{bd^2}$ | |
|  | En el apoyo. $M = \frac{Wl}{2}$ | $W = 2\frac{M}{l}$ <br> $= 2S \cdot \frac{I}{Tl}$ | $W = \frac{S}{6} \cdot \frac{bd^3}{Tl}$ <br> $= \frac{S}{3} \cdot \frac{bd^2}{l}$ | $\frac{1}{3}$ |
| | | $S = \frac{WTl}{2I}$ | $S = \frac{6\,WTl}{bd^3}$ <br> $= \frac{3\,Wl}{bd^2}$ | |
|  | En el centro. $M = \frac{Wl}{4}$ | $W = 4\frac{M}{l}$ <br> $= 4S \cdot \frac{I}{Tl}$ | $W = \frac{S}{3} \cdot \frac{bd^3}{Tl}$ <br> $= \frac{2S}{3} \cdot \frac{bd^2}{l}$ | $\frac{2}{3}$ |
| | | $S = \frac{WTl}{4I}$ | $S = \frac{3\,WTl}{bd^3}$ <br> $= \frac{3\,Wl}{2bd^2}$ | |
|  | En el centro. $M = \frac{Wl}{8}$ | $W = 8\frac{M}{l}$ <br> $= 8S \cdot \frac{I}{Tl}$ | $W = \frac{2S}{3} \cdot \frac{bd^3}{Tl}$ <br> $= \frac{4S}{3} \cdot \frac{bd^2}{l}$ | $\frac{4}{3}$ |
|  | En el centro y en el apoyo. | $S = \frac{WTl}{8I}$ | $S = \frac{3\,WTl}{2bd^3}$ <br> $= \frac{3\,Wl}{4bd^2}$ | |
|  | En el apoyo. $M = \frac{Wl}{12}$ | $W = 12\frac{M}{l}$ <br> $= 12S \cdot \frac{I}{Tl}$ | $W = S \cdot \frac{bd^3}{Tl}$ <br> $= 2S \cdot \frac{bd^2}{l}$ | 2 |
| | | $S = \frac{WT}{12I}$ | $S = \frac{WTl}{bd^3}$ <br> $= \frac{Wl}{2bd^2}$ | |

## Símbolos en el cuadro enfrente.

**M**=momento máximo de flexión;
R=M=momento de resistencia de la sección transversal;
S=tensión por unidad de superficie debida á W, en las fibras extremas;
T=distancia del eje neutro á las fibras extremas;
I=momento de inercia; M=momento de flexión máximo;
W=carga; $l$=luz.

En vigas rectangulares $\begin{cases} b=\text{ancho}; \\ d=\text{alto}; \end{cases}$ $n=\dfrac{Wl}{Sbd^2}$; $W=nS\dfrac{bd^2}{l}$.

## Viga de 1 cm cuadrado y 1 decímetro de luz *.

**32.** En una viga, de un cm en cuadro y 1 dcm (10 cm) de abertura ó luz, sostenida en ambos extremos, tenemos, para el peso extraño en el centro ** :

$$W'=S\frac{2bd^2}{3l}=\frac{nSbd^2}{l}=S.\frac{2}{3\times10}=\frac{S}{15};\ \text{de donde } S=15W'.$$

**33.** Para cualquier otra viga rectangular, sea L=á la luz en m. Entonces, el peso extraño W en el centro **, requerido para producir la misma tensión por unidad. S, en las fibras extremas, es

$$W=W'\frac{bd^2}{L}.$$

**34.** Así, para pino amarillo, sea S=á la tensión permitida por unidad de superficie=1,620 lbs por pulg cuad ó sean como 114 kg por cm cuad. Entonces, para una viga de 1 cm en cuadro, 1 dcm de luz, sostenida en ambos extremos y cargada en el centro, el peso ** permitido, W', es

$$W'=\frac{nS}{10}=\frac{2S}{3\times10}=\frac{S}{15}=\frac{114}{15}=7.6\ \text{kg};$$

y para una viga de 8×30 cm y 6 metros de luz (= 60 dcm), la carga extraña** central permitida es

$$W=W'\frac{bd^2}{l}=7.6\frac{8\times(30)^2}{60}=7.6\frac{8\times900}{60}=912\ \text{kg};$$

y la carga extraña uniforme permitida es $2W=2\times912=1{,}824$ kg.

**35.** Si se dan el peso, W, la luz, L, y el coeficiente, W', tenemos $bd^2=\dfrac{WL}{W'}$

Así, en el caso de la viga de pino amarillo, mencionada en el § 34, de 6 m de luz con un peso extraño** uniforme, donde $2W=912\times2$, tenemos

$$bd^2=\frac{WL}{W'}=\frac{912\times60}{7.6}=7{,}200.$$

**36.** Entonces, si se da $b$ ó $d$, el otro se encuentra fácilmente. Si no, asignésele á uno ú otro de ellos un valor arbitrario. Así, si $b=6$, tenemos $d^2=\dfrac{7{,}200}{6}=1{,}200$;

y $d=\sqrt{1{,}200}$=digamos 35.

**37.** Con la regla de cálculo pueden hacerse estas operaciones.

**38.** Si la relación, S=15W' se considera más allá del límite de elasticidad, y si W'=al peso central de ruptura, en una viga de cualquier substancia, de un cm cuadrado y de 1 dcm de luz, sostenida en ambos extremos; entonces, para cualquier otra viga de la misma substancia, y de $b$ cm ancho y $d$ de alto y L dcm de luz, el peso central de ruptura sería $W=W'\dfrac{bd^2}{L}$.

**39.** A pesar de la base defectuosa de este método, aplicado á pesos fuera del límite de elasticidad, su sencillez lo hace muy conveniente y está muy en uso. Véase el siguiente cuadro de valores de W', y el ejemplo, § 40.

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* *N. del T.* — Hemos cambiado los ejemplos que trae el autor por los análogos en sistema métrico, y con los elementos adecuados, para que resulten también análogas las comparaciones en conexión con la tabla que sigue.

** La viga aquí se supone sin peso; para tomar éste en consideración véanse §§ 42, etc.

**Cargas centrales de ruptura, W', en kilogramos, para vigas de un centímetro cuadrado de sección transversal y de un decímetro de luz, apoyadas en ambos extremos *.**

En las vigas de madera deduzcase en la práctica una tercera parte por nudos, torceduras, etc.

| Maderas. | W' |
|---|---|
| Fresno, Inglés (fam Oleáceas) | 54.8 |
| Fresno, Amer blanco (autor) | 54.8 |
| Fresno, Ciénaga | 33.6 |
| — negro | 50.7 |
| Arbor vitæ (tuya), Amer. | 21.0 |
| Bálsamo, Canadá | 29.5 |
| Haya (fam Cupulíferas) | |
| — Amer | 71.6 |
| Abedul, Amer negro (fam Betuláceas) | 46.3 |
| Abedul, Amer amarillo | 71.6 |
| Cedro, Bermudas | 33.7 |
| — Guadalupe | 50.6 |
| — Amer blanco ó Arbor Vitæ | 21.1 |
| Castaño (fam Cupulíferas) | 38.0 |
| Olmo, Amer blanco | 54.8 |
| — Roca, Canadá | 67.4 |
| Pino del Canadá | 42.2 |
| Nogal, Amer | 67.5 |
| — — amargo | 67.5 |
| Quiebrahacha (gabí), Canadá | 50.6 |
| Algarrobo (fam Leguminosas) | 59.0 |
| Guayacán (fam Cigofíleas) | 54.8 |
| Alerce (fam Coníferas) | 33.7 |
| Caoba | 63.3 |
| Mangle, blanco | 54.8 |
| — negro | 46.3 |
| Arce, negro (fam Aceríneas) | 63.3 |
| Arce, blando | 63.3 |
| Roble, inglés | 46.3 |
| Roble, Amer blanco (por autor) | 50.7 |
| Roble, Amer rojo, negro | 71.6 |
| Roble vivo | 50.6 |
| Pino, Amer blanco (por autor) | 38.0 |
| — — amarillo | 42.2 |
| — — de tea | 46.3 |
| — Georgia | 71.6 |
| Alamo (fam Salicíneas) | 46.3 |
| Madera de Singapore (Poon) | 59.0 |
| Abeto (por autor) (fam Coníferas) | 38.0 |
| Abeto, negro | 46.3 |
| Sicomoro | 42.2 |
| Tamarack (fam Coníferas) | 33.7 |
| Teca (fam Verbenáceas) | 63.3 |
| Nogal | 46.3 |
| Sauce | 29.5 |

| Metales. | |
|---|---|
| Bronce | 71.6 |
| Hierro fundido, promedio | 177.2 |
| Hierro fundido fundiciones de lingote | 194.0 |
| Hierro fundido empleado en nuestras tablas | 170.8 |
| Hierro fundido para fundiciones de 6 ó 7 cm de grueso. | 151.9? |
| Hierro forjado, promedio | 189.8 |
| El hierro forjado no se *rompe;* pero cerca del promedio de 189.8 llega á su límite de elasticidad. | |
| Acero, batido ó laminado, la elasticidad se destruye entre 250 y 580 | 415.0 |
| Bajo cargas pesadas, el acero friable se quiebra como hierro fundido y el acero dulce se cimbra como hierro forjado. | |

| Piedras, etc. | |
|---|---|
| Piedra azul de enlosado (Río Hudson) | 10.52 |
| Ladrillo común, promedio | 1.68 |
| Ladrillo bueno americano, comprimido, promedio | 3.35 |
| Piedra de Caen | 2.08 |
| Concreto, véase artículo sobre concreto. | |
| Granito, 4.18 á 12.5, promedio | 8.38 |
| Granito Quincy | 8.38 |
| Vidrio, Millville, N. Jersey, para pavimento grueso (por el autor) | 14.30 |
| Mezcla, de cal sola, de 60 días. | 0.86 |
| Mezcla, 1 medida de cal apagada en polvo, 1 de arena | 0.68 |

* *N. del T.* — Esta tabla la trae el autor con la carga de ruptura en lbs para vigas de 1 pulg cuad de sección y un pie de luz; pero la hemos calculado, por medio de la fórmula $\frac{bd^2}{l}$, para vigas de 1 cm cuad de sección y un decímetro de longitud. Así nos sirve para los problemas análogos á los del autor, pero en medidas métricas. (Véase lo que dice el autor en el § 39, antes de la tabla.)

TABLA. (Continuación.)

| | | | |
|---|---|---|---|
| Mezcla, 1 medida de cal apagada en polvo, 2 de arena.. | 0.58 | Mármol Rutland, Vt, Gray (autor)..... | 5.90 |
| Mármol italiano, blanco (autor). | 9.14 | Mármol Glenn's Falls, N. Y. Black (autor)..... | 13.06 |
| Mármol Manchester, Vt, blanco (autor)..... | 8.02 | Mármol Baltimore, Md, blanco, ordinario..... | 8.62 |
| Mármol East Dorset, Vt, blanco (autor)..... | 9.34 | Oolitas..... | 2.95 |
| Mármol Lee, Mass, blanco (autor)..... | 7.26 | Piedra arenisca, 1.6 á 5.8, promedio..... | 3.81 |
| Mármol Montg'y Co. Pa. Gray, blanco (autor)..... | 8.66 | Piedra arenisca roja de Connecticut y New Jersey..... | 3.81 |
| Mármol Montg'y Co. Pa. Clouded (autor)..... | 11.97 | Pizarra, promedio..... | 27.39 |

**40. Ejemplo.** En la viga de pino amarillo, §§ 34 á 36, de 8×30 cm y de 6 m de luz, tenemos de la tabla última, W′=42.2 kg. En consecuencia :

Carga central de ruptura * $W = W' \frac{bd^2}{L} = \frac{42.2 \times S \times 900}{60} = 5{,}064$ kg, ó como 5.5 veces el peso permitido encontrado, por medio de la tensión permitida por unidad de superficie, en el § 34.

## Dimensiones.

**41.** Puesto que $W = nS \frac{bd^2}{l} = W' \frac{bd^2}{L}$, y $W' = \frac{nS}{10}$, tenemos *

$$\textbf{Ancho} = b = \frac{Wl}{nSd^2} = \frac{Wl}{W'd^2};$$

$$\textbf{Alto} = d = \sqrt{\frac{Wl}{nSb}} = \sqrt{\frac{WL}{W'b}};$$

donde W=carga extraña* requerida;

W′=carga extraña en vigas de 1 cm cuadrado y 1 dcm de luz;

$n$=coeficiente de la última columna de la tabla $= \frac{Wl}{Sbd^2}$;

S=tensión en la fibra por unidad de superficie;

$l$=luz en cm;

L=luz en dcm.

## Peso de la viga considerado como carga.

**42.** Por simplificar hemos considerado hasta ahora nuestros *cantilevers* y vigas sin peso propio, y, en vigas de dimensiones moderadas como se emplean de ordinario, su propio peso, $w$, es tan pequeño, en comparación con sus cargas, W, que puede casi siempre y sin riesgo ser despreciado; pero en las vigas grandes debe generalmente tomarse en cuenta. Los pesos, encontrados como se ha dicho, con S=la mayor tensión permitida por unidad de superficie, debe entonces comprender no sólo el peso extraño, W, sino también el peso, $w$, de la viga misma, para una longitud=á la luz.

**43.** Si la viga es de sección transversal uniforme, su peso, $w$, obra como uniformemente distribuído, y tenemos, para el peso, ó carga extraña, W, en el caso de un peso central concentrado en una viga, ó de un peso concentrado en el extremo de la longitud, $l$, en *cantilevers*,

$$W = \text{peso ó carga total} — \frac{w}{2};$$

en el caso de un peso uniformemente distribuído,

$$W = \text{peso ó carga total} — w.$$

* Se supone que la viga no tiene peso; para tomar éste en consideración, véanse §§ 42 etc.

**44. Al encontrar el ancho ó el alto** de una viga rectangular, que ha de llevar un peso dado con una luz dada y un esfuerzo dado por unidad de superficie, podemos tener en cuenta el peso con aproximaciones sucesivas. Así,

**45. Para encontrar el ancho,** $b$, requerido para una viga de alto dado, $d$; despréciese el peso, $w$, de la viga y encuéntrese el primer ancho aproximado, $b$, por las fórmulas del § 41, para el peso extraño, W. Luego, calcúlese el peso, $w$, del una viga con ancho, $b$, trátese dicho peso como distribuído uniformemente; y por las mismas fórmulas, encuéntrese el ancho adicional, $b'$, requerido para llevar este peso adicional, $w$. Entonces $b+b'$=un segundo ancho aproximado. Si fuere necesario, encuéntrese el peso, $w'$, de una viga de ancho, $b'$, y, de éste, un segundo ancho adicional, $b''$, requerido para sostenerlo. Entonces, $b+b'+b''$=un tercer ancho aproximado, y así sucesivamente.

**46. Para encontrar el alto,** $d$, que necesita una viga de ancho dado, $b$; encuéntrese un primer alto aproximado, $d$, por la fórmula, § 41, para el peso extraño W. Encuéntrese el peso, $w$, de una viga de ese alto; y aplíquese otra vez la fórmula, usando (en lugar de W) $W+w$, si W es peso uniforme, ó $W+\frac{w}{2}$, si W está concentrado. El alto, $d'$, así encontrado, es una segunda aproximación. Podemos otra vez aplicar la fórmula, como antes, usando el peso, $w'$, de la viga de alto $d'$; ó, más sencillamente, auméntese el ancho, como en el § 45.

**47.** En la práctica, las vigas de sección rectangular son casi siempre de madera; y tales vigas se consiguen á precios económicos solamente con ciertos tamaños comerciales. En consecuencia, basta generalmente la segunda aproximación.

## Fuerzas y pesos de vigas semejantes, de diferentes dimensiones. Comparación entre modelos y formas reales.

**48.** En cualquier viga dada, sea $W_1$=al peso que cause cualquier tensión dada, S, por unidad de superficie. Entonces, $W_1 = \frac{nSbd^2}{l}$ (para $n$, véase cuadro pág. 496); y, en cualquier viga semejante, de $a$ veces el ancho, altura y luz, el peso correspondiente, $W = \frac{nSaba\,d^2}{al}$. En consecuencia, la razón de sus *cargas* es $\frac{W}{W_1} = a^2$; ó $W=a^2W_1$; pero la razón de sus *pesos* es $\frac{w}{w_1} = \frac{abadal}{bdl} = a^3$; ó $w=a^3w_1$.

**49.** En otras palabras, comparando una viga con otra, de $a$ veces su ancho, alto y luz, sus resistencias están como los *cuadrados* de sus respectivas dimensiones; pero sus pesos están como los *cubos* de esas dimensiones.

**50.** En consecuencia, si un modelo de una viga se rompe precisamente bajo una carga uniforme (que incluya su propio peso, $w$)=2, 3 ó 4, etc., veces su propio peso, entonces una viga de semejante sección transversal, pero de 2, 3 ó 4, etc., veces su ancho, alto y luz, se romperá bajo su propio peso solamente.

## Esfuerzo cortante horizontal. Véanse también los §§ 119-122.

**51.** Cuando (figs. 7 y 8) ocurre flexión en un *cantilever* ó una viga compuesta de capas horizontales separadas, como una pila de tablas sueltas, las varias capas resbalan unas sobre otras; pero, si están firmemente unidas entre sí ó por otro

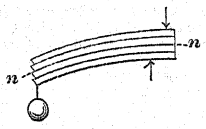



Fig. 7.

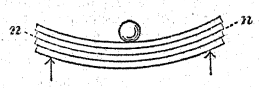



Fig. 8.

medio impedidas de resbalar, ejercen, una sobre otra, un esfuerzo cortante horizontal. En cualquier sección, esta fuerza disminuye desde un máximum en la superficie neutra, $nn$, hasta cero, en la parte superior y en la inferior.

**52.** En cualquier sección de una viga rectangular el esfuerzo cortante horizontal máximo, por unidad de superficie neutra, es

$$H = \frac{3V}{2bd};$$

donde V=al esfuerzo cortante vertical en la sección, y $b$ y $d$=el ancho y el alto de la sección.

Dicho de otro modo, la unidad del esfuerzo cortante horizontal, en cualquier punto, es directamente proporcional al vertical en ese punto. En consecuencia, el diagrama del esfuerzo cortante horizontal es semejante, en la forma, al diagrama del vertical; pero en sentido contrario, correspondiendo el esfuerzo cortante vertical positivo al horizontal negativo.

**53.** Si al esfuerzo cortante horizontal se le opone resistencia por medio de un borde ú obstáculo aplicado á un solo punto, dicho obstáculo debe hacerse suficientemente fuerte para resistir á la *suma de todos* los esfuerzos cortantes horizontales desde dicho punto hasta aquel en donde el esf cort es=0.

**54.** En la fig. 9, los diagramas (*b*) y (*c*) muestran respectivamente los momentos y los esfuerzos cortantes verticales debidos á pesos concentrados y distribuídos en una viga como indica la figura; y, en la fig. (*d*), cada ordenada representa la fuerza que debe aplicarse en dicho punto para resistir á la suma de todos los esfs corts horizontales entre ese punto y el punto en que aquéllos son cero. Las ordenadas por sobre una línea cero indican momentos ó esfs corts positivos y viceversa. En los momentos *positivos*, el segmento á la *izquierda* de una sección tiende á girar

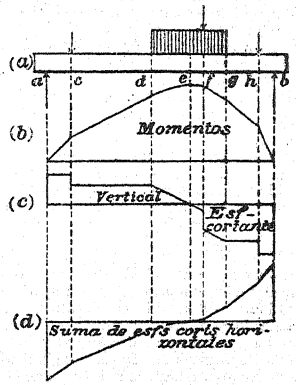



**Fig. 9.**

« como el minutero ». (Véase *N. del T.* pág. 463.) En los esfs corts *positivos*, el segmento de la *izquierda* tiende á *resbalar hacia arriba* ó el segmento *superior* á *resbalar* hacia la *derecha*. Entre *a* y *c*, entre *c* y *d*, entre *g* y *h*, y entre *h* y *b*, todos los diagramas son líneas rectas, siendo las figs. (*b*) y (*d*) inclinadas y la fig. (*c*) horizontal. En *c* y en *h*, las figs. (*b*) y (*d*), cambian su inclinación, y la fig. (*c*) cambia su posición. Entre *d* y *f* y entre *f* y *g* (esto es: bajo el peso distribuído), las figs. (*b*) y (*d*) son curvas parabólicas, y la fig. (*c*) muestra líneas rectas inclinadas. En *f*, las figs. (*b*) y (*d*) cambian de curvatura y la fig. (*c*) cambia su posición. En *e*, punto del momento máximo, las figs. (*c*) y (*d*) cambian de signo. Véase Relación entre el momento y el esfuerzo cortante, §§ 359 á 368 (Estática).

**55.** Atendiendo á que el esfuerzo cortante horizontal es una resistencia á la flexión, al no tomarlo en cuenta, en la teoría común de las vigas, como se ha explicado hasta ahora, contribuye en general á la seguridad. Pero, en vigas compuestas de capas horizontales, deben emplearse medios para transmitirlos de una capa á la inmediata.

**56.** Así, vigas de madera muy altas, fig. 10, se hacen frecuentemente de dos ó más vigas, una sobre otra. Para impedir la flexión, debido al resbalamiento de uno de estos maderos sobre el otro, se insertan bloques entre ellos á intervalos, como indica la figura, ó se ranuran los lados adyacentes de los maderos de modo que se compe-

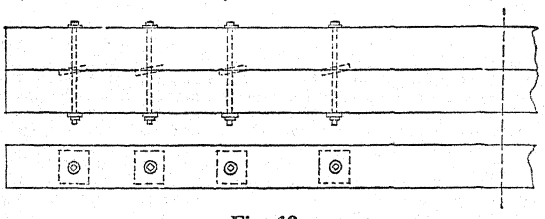


**Fig. 10.**

netren. En uno ú otro caso, los maderos están estrechamente ligados entre sí. Los bloques ó ranuras sirven entonces para transmitir el esf cort horizontal de un madero al otro. En la fig. 10 los bloques son más frecuentes cerca de los extremos, como lo exige el diagrama del esfuerzo cortante horizontal, fig. 9 (*d*).

## Flexiones.

**57. El cuadro del frente** da las **flexiones** dentro del límite de elasticidad de cualquier viga de sección transversal uniforme **bajo diferentes disposiciones de apoyo y de carga**; también da (en la última columna) el **peso** *extraño* **que producirá una flexión dada**, sin ayuda del peso de la viga misma. Todas las fórmulas están basadas en el supuesto de que el aumento de flexión es proporcional al aumento de peso.

Las letras tienen los significados que siguen :

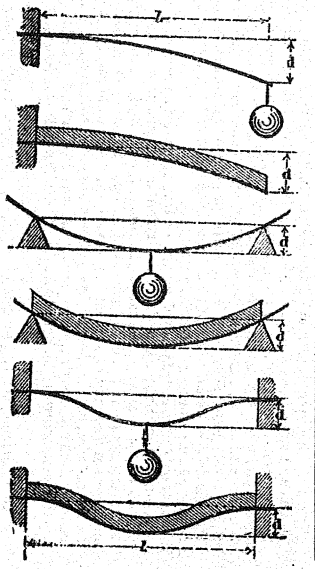
$d$ = flexión de la viga, en cm (véanse las figs.);
**W** = el peso extraño, en kg;
$w$ = peso de la viga en una longitud igual á la abertura ó luz, en kg;
$l$ = longitud de la viga sobre la luz, en cm (véanse las figs.);
**E** = módulo de elasticidad del material de la viga, en kg por cm cuadrado;
**I** = momento de inercia de la sección transversal de la viga, en cm (cuatro dimensiones).

Véase atrás Momento de inercia, § 14.

(*Obs. del T.*— Creemos oportuno advertir que, al hacer uso de estas fórmulas y de sus análogas, es necesario tener cuidado en que resulte homogénea la fórmula empleando las mismas unidades de medida en la valoración de las cantidades que entran en ellas. Si, por ejemplo, se toma á W y $w$ en kg y $l$ en metros, debe tomarse entonces el valor de E en kg por metro cuadrado y el valor de I (de cuatro dimensiones) se formará con el metro por unidad. El valor de la flexión, $d$, vendrá entonces en metros. Se procederá de modo análogo cuando se tome el milímetro por unidad.

También juzgamos oportuno agregar las fórmulas para la flexión cuando la viga (1.°) está *apoyada* en los extremos y cargada con un peso W *en cualquier punto* de su longitud; y (2.°) en las mismas condiciones, *pero empotrada* en ambos extremos. Llamando $m$ y $n$ las distancias de la carga á los apoyos, la flecha $d$, para el primer caso, será $= \frac{1}{3} \times \frac{Wm^2n^2}{EIl}$ ; para el segundo caso, $= \frac{1}{3} \frac{Wm^3n^3}{EIl}$).

**Flexión de vigas de sección transversal uniforme.** (Para el significado de las letras, véase pág. 502.)




| Si la viga está : | Flexión $d$, en cm, causada : por pesos extraños. | por peso de la viga. | por peso de la viga y carga. | Peso extraño para una flexión dada (se desprecia el peso de la viga). |
|---|---|---|---|---|
| Fija en un extremo y cargada en el otro. | $\frac{1}{3}\ \frac{l^3\ W}{E\ I}$ | $\frac{1}{8}\ \frac{l^3\ w}{E\ I}$ | $\frac{1}{3}\ \frac{l^3\ (W + \frac{3}{8}\ w)}{E\ I}$ | $3\ \frac{d\ E\ I}{l^3}$ |
| Fija en un extremo y cargada uniformemente. | $\frac{1}{8}$ " | $\frac{1}{8}$ " | $\frac{1}{8}\ \frac{l^3\ (W + w)}{E\ I}$ | 8 " |
| Apoyada en ambos extremos y cargada en el centro. | $\frac{1}{48}$ " | $\frac{5}{384}$ " | $\frac{1}{48}\ \frac{l^3\ (W + \frac{5}{8}\ w)}{E\ I}$ | 48 " |
| Apoyada en ambos extremos y cargada uniformemente. | $\frac{5}{384}$ " | $\frac{5}{384}$ " | $\frac{5}{384}\ \frac{l^3\ (W + w)}{E\ I}$ | $\frac{384}{5}$ " |
| Fija en ambos extremos y cargada en el centro. | $\frac{1}{192}$ " | $\frac{1}{384}$ " | $\frac{1}{192}\ \frac{l^3\ (W + \frac{1}{2}\ w)}{E\ I}$ | 192 " |
| Fija en ambos extremos y cargada uniformemente. | $\frac{1}{384}$ " | $\frac{1}{384}$ " | $\frac{1}{384}\ \frac{l^3\ (W + w)}{E\ I}$ | 384 " |

De los principios comprendidos en el cuadro anterior encontramos que en vigas de sección semejante y de la misma substancia y dentro del límite de elasticidad, el peso y las flexiones (despreciando el peso de la viga misma) están como sigue :

| Con igual | Las flexiones bajo un peso extraño dado varían en razón |
|---|---|
| luz | inversa á los anchos y como los cubos de los altos |
| — y ancho | — á los cubos de los altos |
| — y alto | — á los anchos |
| ancho y alto | directa de los cubos de las luces. |

| Con igual | Los pesos extraños para una flexión dada varían en razón |
|---|---|
| luz | directa á los anchos y como los cubos de los altos |
| — y ancho | — á los cubos de los altos |
| — y alto | — á los anchos |
| ancho y alto | inversa á los cubos de las luces. |

**Flexión en función de la tensión por unidad de superficie en la fibra extrema.** En el cuadro, pág. 496, el peso $W = kS\frac{I}{Tl}$; donde $k$=á un coeficiente, como se indica abajo; S=á la tensión por unidad de superficie en las fibras extremas; I=momento de inercia; T=distancia del eje neutro á la fibra extrema y $l$=á la luz. Del cuadro último, tenemos ·

$W = m\frac{dEI}{l^3}$; donde $m$ = un coeficiente, como se indica abajo; $d$ = flexión, y E=módulo de elasticidad. En consecuencia, $Sk\frac{I}{Tl} = m\frac{dEI}{l^3}$; y $d = \frac{k}{m}\cdot\frac{l^2S}{ET} = \frac{l^2S}{ETc}$

donde $c = \frac{m}{k}$.

| | | | |
|---|---|---|---|
| En un *cantilever*, cargado en el extremo, | $m=$ 3; | $k=$ 1; | $c=$ 3. |
| — — — uniformemente, | $m=$ 8; | $k=$ 2; | $c=$ 4. |
| En una viga apoyada, y cargada en el centro, | $m=$ 48; | $k=$ 4; | $c=$12. |
| — — — — uniformemente, | $m=$ 76.8; | $k=$ 8; | $c=$ 9.6. |
| — — empotrada, — en el centro, | $m=$192; | $k=$ 8; | $c=$24. |
| — — — — uniformemente, | $m=$384; | $k=$12; | $c=$32. |

## Límite de elasticidad.

**58.** Bajo cargas moderadas, las flexiones son prácticamente proporcionales al peso. Cuando ellas empiezan á aumentar perceptiblemente con más rapidez que el peso, se dice que el último ha llegado al límite elástico ó límite de elasticidad. Es generalmente en este punto donde la « **deformación permanente** » se hace por primera vez perceptible; esto es : después de removido el peso, la viga no vuelve á su estado original y permanece más ó menos cimbrada. Las flexiones empiezan entonces también á aumentar *irregularmente*, y continúan indefinidamente sin aumentos de peso. En breve, la viga está en peligro. En consecuencia el peso efectivo nunca *debe exceder* al límite de elasticidad; no debe pasar de un tercio á dos tercios de su valor, según las circunstancias.

**El límite de elasticidad de una viga** de cualquier forma ó material especial, **se determina experimentalmente** con una viga semejante, como en el caso de las constantes de cargas de ruptura, etc., de la manera siguiente : Cárguese una viga en el centro, por la cuidadosa adición gradual de pesos pequeños iguales; anótese cuidadosamente la flexión que ocurre dentro de algunos minutos (mientras más, mejor) después de haberse aplicado cada peso, á fin de averiguar cuándo empiezan las flexiones á aumentar más rápidamente que los pesos, porque, cuando esto ocurre, se ha llegado á la carga del límite de elasticidad *.

* Naturalmente, en la práctica es difícil con frecuencia averiguar con precisión

No son las flexiones de *toda* la viga las que han de anotarse, sino las de la luz solamente. Deben ensayarse varias vigas, á fin de obtener una constante *media*, pues aun en vigas de hierro laminadas del mismo modelo y del mismo hierro, hay diferencias muy apreciables de resistencias y flexiones.

Entonces, para obtener las constantes, usando el *peso total* aplicado durante las flexiones iguales, incluyendo la mitad del peso de la viga misma, tenemos :

$$\textbf{Constante para el límite de elasticidad} = \frac{\text{luz en m} \times \text{peso total en kg}}{\text{ancho en cm} \times \text{cuadrado del alto en cm}}.$$

**La constante, para vigas de madera,** puede obtenerse, con bastante aproximación para la práctica común, tomando un *tercio* de las constantes de *ruptura* en el cuadro del § 39.

(*Obs. del T.* — Creemos oportuno aclarar un poco más este punto. Esta constante experimental se encuentra cargando una viga en las condiciones dichas con el peso máximo que la lleva á su límite de elasticidad; luego, de acuerdo con la fórmula $\frac{bd^2}{l}$ ($b$=ancho en cm; $d$=alto en cm; $l$=luz en m), se encuentra que una viga de *un metro* de luz resistirá (llamando $c$ la carga dicha en kg), $l$ veces más carga, ó sea $c \times l$, y para un cm de ancho y uno de altura $\frac{c \times l}{bd^2}$, que es la fórmula dada en lenguaje vulgar. Si se la quiere para una viga de un decímetro de largo, el multiplicador de la carga en el numerador será el número de decímetros que tenga la viga.)

Dicha constante, así calculada, es pues el límite de elasticidad de una viga de la forma y material dados de 1 cm de ancho, 1 cm de alto y un m de luz sostenida, en ambos extremos y cargada en el centro. Para obtener ahora el límite de elasticidad de cualquier otra viga de la misma forma ‡ y del mismo material, sostenida y cargada de modo análogo, pero de otras dimensiones, tendremos :

$$\textbf{Límite de elasticidad} = \text{constante} \times \frac{\text{ancho en cm} \times \text{cuadrado del alto en cm}}{\text{luz en m.}}$$

| Si la viga está | | | | Multiplíquese el resultado por |
|---|---|---|---|---|
| sostenida en ambos extremos y cargada | | | en el centro, | 1 |
| — | — | — | — uniformemente, | 2 |
| empotrada † | — | — | — en el centro, | 2 |
| — | — | — | — uniformemente, | 3 |
| — | un extremo | | — en el otro extremo, | 1/4 |
| — | — | | — uniformemente, | 1/2 |

---

cuándo ó bajo qué peso empiezan efectivamente las flexiones á aumentar más rápidamente que los pesos sucesivos. Pues, aunque *por la teoría* las flexiones son prácticamente iguales para pesos iguales, hasta que se llega al límite de elasticidad, sinembargo, *de hecho*, están sujetas á más ó menos irregularidades; pues nunca el material de que se compone una viga es perfectamente uniforme en toda su extensión, en contextura y fuerza. En consecuencia, no siempre habrá aumentos regulares de flexión, sino que tendremos variaciones á veces más grandes y otras más pequeñas. Por consiguiente, se requiere un buen criterio para determinar el punto final. Es mejor, en caso de duda, inclinarse hacia el dato de más *seguridad*. Supónese siempre que el peso no está sujeto á movimientos ni vibraciones. Estos aumentarían las flexiones.

† Dícese que una viga está « empotrada » (bien fijada) en uno ú otro extremo cuando la tangente al eje longitudinal de la viga flexada en ese extremo permanece siempre horizontal.

‡ Las formas de las dos vigas no necesitan ser *semejantes*. Por ejemplo, la constante deducida de experimentos en cualquier viga rectangular es aplicable á cualquier otra viga rectangular, sea cuadrada ú oblonga.

## La curva de elasticidad.

**59.** Cuando un *cantilever*, fig. 1, ó una viga, fig. 2, sostenida ó empotrada de cualquier manera se flexa bajo la acción de cualquier peso, la superficie neutra, *nn*, forma una curva tal, que, en cualquier sección,

$$R = \frac{EI}{M} = \frac{IS}{Mk};$$

donde R = al radio de curvatura, en la sección;
M = el momento de flexión, en la sección;
I = el momento de inercia de la sección;
E = el coeficiente de elasticidad del material $= \frac{S}{k}$;
S = cualquier esfuerzo por unidad de superficie dentro del límite de elasticidad.
*k* = á la unidad de alargamiento ó acortamiento producido en el material por S como fuerza de tensión ó compresión.

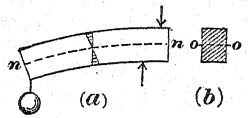



**Fig. 1 (repetida).**

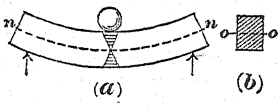



**Fig. 2 (repetida).**

## El coeficiente de flexión.

**60. Definición.** El coeficiente de flexión, para cualquier material dado, es la flexión, en cm, de una viga, de ese material, de un cm en cuadro y de 1 m de luz**, sostenida en cada extremo, y llevando en su centro un peso extraño de 1 kg — $^5/_8\,w'$, donde $w'$ = peso de la parte de viga que ocupa la luz en kg.

**61.** Sea *y* = el coeficiente de flexión para cualquier material dado. Entonces, en cualquier viga rectangular del mismo material, con peso central ó peso uniforme, sean

*b* = el ancho, en cm;
*d* = el alto, en cm;
L = la luz, en metros;
*w* = el peso, en kgs, de la parte de la viga misma que ocupa la luz;
W = peso central $+ \,^5/_8\,w$;
$= \,^5/_8$ (peso uniforme $+ w$).

Entonces, en la viga dada, tendremos:

$$\textbf{Flexión} = Y = y\,\frac{W.L^3}{b.d^3}; \quad \textbf{Ancho}\,* = b = W.\frac{L^3.y}{d^3.Y};$$

$$\textbf{Carga} = W = Y.\frac{b.d^3}{L^3.y}; \quad \textbf{Alto}\,* = d = L\sqrt[3]{\frac{Wy}{bY}}.$$

**62.** El coeficiente de flexión, *y*, para cualquier material dado, se obtiene experimentalmente, así: En el centro de cualquier viga rectangular, del material dado, colocada horizontalmente sobre dos apoyos, separados por cualquier distancia conveniente y conocida, colóquese cualquier peso que esté dentro del límite de elasticidad y mídase la flexión resultante, Y. Sea W = al peso central extraño $+ \,^5/_8\,w$, donde *w* = al peso de la luz de la viga. Entonces el coeficiente de flexión es

$$y = Y.\frac{b.d^3}{W.L^3};$$

---

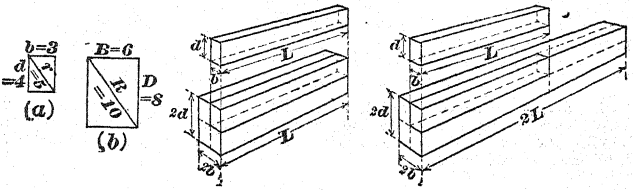
* Al calcular el ancho ó el alto, si es necesario contar con el peso de la viga misma, hacemos primero W = al peso extraño solamente, y luego procedemos por aproximaciones sucesivas, como en los §§ 45 y 46, recordando, sin embargo, que en el caso de flexiones 5/8 del peso de cada sección adicional ha de tomarse como peso central equivalente, y no 1/2 como en el caso de resistencias.

** *N. del T.*— Puede tomarse, para la luz, el decímetro como unidad; pero recuérdese entonces que, suponiendo iguales las otras condiciones, las flexiones son proporcionales á los cubos de las luces; ó bien las cargas que producen una misma flexión, están en razón inversa de los cubos de las luces. (Véase comienzo pág. 504.)

donde $b$ y $d$ = al ancho y el alto, en centímetros, y L = á la luz, en metros, de la viga con que se hace la experiencia.

**63.** La relación entre cualesquiera dos líneas homólogas, en cualesquiera dos figuras ó cuerpos semejantes, es constante. En consecuencia, al determinar ó usar coeficientes, ya para resistencia, ya para flexión, comparando vigas de secciones semejantes pero de tamaños diferentes, podemos usar cualesquiera dos líneas homólogas en lugar de los dos anchos, ó en lugar de los dos altos.

Así, en las figs. 11, B/$b$ = D/$d$ = R/$r$ = 2.




**Figs. 11.** **Fig. X.** **Fig. Y.**

Por tanto,

$$\frac{BD^2}{bd^2} = \frac{384}{48} = 8 = 2^3 = \frac{RR^2}{rr^2}; = \frac{1,000}{125} = 8 = 2^3;$$

También,

$$\frac{BR^2}{br^2} = \frac{6\times100}{3\times25} = \frac{600}{75} = 8 = 2^3.$$

También,

$$\frac{B^3D}{b^3d} = \frac{1,728}{108} = 16 = 2^4 = \frac{R^3R}{r^3r} = \frac{10,000}{625} = 16 = 2^4.$$

**63 a.** De lo precedente y del § 31, pág. 495, se sigue que, con *secciones transversales semejantes* pero *aberturas ó luces iguales*, fig. X, las resistencias son proporcionales á los *cubos* de cualesquiera líneas homólogas en las dos secciones; pero, con vigas semejantes *en todos respectos, inclusive la luz*, fig. Y, las resistencias están como los *cuadrados* de cualesquiera líneas homólogas en las dos secciones; esto es : como las *áreas* de las dos secciones.

**64. Coeficiente de flexión** para vigas de sección transversal rectangular. Véanse §§ 60, 61. Viga de 1 cm en cuadro y 1 metro = 100 cm de luz. W (= peso central + $^5/_8$ × peso de la luz) = 1 kg. Del cuadro pág. 503, tenemos :

$$\text{Coef de flex} = \text{flex, } y, \text{ en cm, en el centro} = \frac{L^3W}{48EI} = \frac{100^3\times12}{48E} = \frac{250,000*}{E}$$

**65. Advertencia.** Las flexiones de maderas de la misma clase varían mucho con el grado de sazón, la edad del árbol, la parte de donde se corta la viga, etc. En nuestros propios experimentos sobre buenas piezas, bien sazonadas, en las cuales se dejaron obrar los pesos durante meses, menos del 2 por ciento del peso de ruptura produjo deformaciones permanentes en pocos meses. Varios de los maderos soportaron sus pesos de ruptura durante meses antes de ceder efectivamente. Las

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* *N. del T.* — Ayudará al lector la siguiente ampliación : el caso tercero del cuadro de la pág. 503 (que es el actual) da para flexión la fórmula $\frac{1}{48} \times \frac{L^3W}{EI}$. El valor de I tomado de cuadro pág. 491, para una sección cuadrada es $\frac{bd^3}{12}$; pero como aquí $b = d = 1$ cm; el valor $\frac{bd^3}{12}$ queda reducido á $\frac{1}{12}$ y como L = en nuestro caso á 100 cm, la fórmula $\frac{1}{48}\frac{L^3W}{EI}$ se convertirá recordando que W = 1 kg en $\frac{1}{48} \times \frac{100^3\times1}{E\times1/12} = \frac{100^3\times12}{48E} = \frac{250,000}{E}$. (Véase además *N. del T.* **, pág. 506.)

vibraciones y trepidaciones á que están expuestas todas las construcciones aumentan las flexiones.

**66. Pesos excéntricos concentrados.** Sea Y, fig. 12 (*a*), la flexión, en el centro de la luz (esto es : en el punto de aplicación del peso) de una viga sostenida en cada extremo, y debida á un peso central W (dentro del límite de elasticidad). Entonces, si el mismo peso, W, se coloca excéntricamente en la misma viga, como en la fig. 12 (*b*), la flexión, Y′, en el punto, *c*, de aplicación del peso, y debida al peso, W, es

$$Y' = Y \frac{16\, m^2 n^2}{l^4} ; *$$

donde

$l$ =á la luz ó abertura;
$m$ y $n$ =los segmentos en que el peso divide la luz.

**67. Cargas uniformes.** Sea Y la flexión, debida á cualquier peso central extraño (dentro del límite de elasticidad), sobre una viga sostenida en cada extremo. Entonces, la flexión, Y′, de la misma viga, debida al mismo peso uniformemente distribuído sobre la luz, es :

$$Y' = \frac{5}{8} Y.$$

**68. Vigas inclinadas.** Si la viga es inclinada, úsese la proyección horizontal de su luz, en lugar de $l$, al determinar sus flexiones.

**69. Vigas cilíndricas.** Sea Y la flexión de una viga cuadrada bajo cualquier peso dado. Entonces, para una viga cilíndrica cuyo diámetro = al lado del cuadrado, la flexión, bajo el mismo peso es = 1.698 Y.

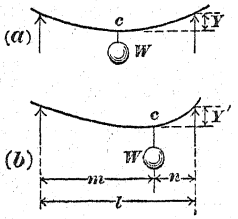
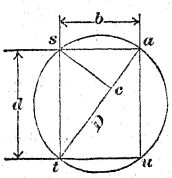
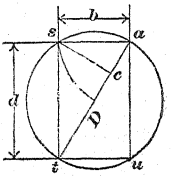



Fig. 12.




Fuerte. (*a*)




Rígida. (*b*)

Fig. 13.

**70.** Las figs. 13 (*a*) y (*b*) muestran, respectivamente, las más fuertes y más rígidas secciones rectangulares que pueden cortarse de un madero cilíndrico de diámetro D. En la sección más fuerte, fig. (*a*), $ac = \frac{D}{3}$, y $b = \sqrt{\frac{1}{3} D^2}$. En la sección más rígida, fig. (*b*), $ac = \frac{D}{4}$ y $d = \sqrt{\frac{3}{4} D^2}$.

**71. Flexión máxima permitida.** Bajo un peso aun perfectamente seguro, puede una viga tener demasiada flexión para ciertos fines. Así, para impedir que

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* *N. del T.*—Esta fórmula de la flexión para carga excéntrica en función de la flexión para carga central se deduce así: el tercer caso del cuadro, pág. 503 (que es este caso), da para la flexión con carga central, que la llamaremos Y como el autor: $Y = \frac{1}{48} \times \frac{l^3 W}{EI}$; y la fórmula que dimos al final de nuestra observación, pág. 502, para carga en cualquier punto, da, llamando Y′ la flexión excéntrica: $Y' = \frac{1}{3} \times \frac{W m^2 n^2}{EIl}$. Dividiendo ésta por aquélla, resulta $\frac{Y'}{Y} = \frac{\frac{1}{3} \times \frac{W m^2 n^2}{EIl}}{\frac{1}{48} \times \frac{l^3 W}{EI}} = \frac{16\, m^2 n^2}{l^4}$.

se agriete el enyesado de los cielos rasos, es usual limitar la flexión de las vigas á $\frac{\text{luz}}{360}$. En líneas largas de transmisiones, para maquinarias, la flexión se limita de ordinario á $\frac{\text{luz}}{1{,}200}$; en puentes de camino real á $\frac{\text{luz}}{640}$; en puentes de ferrocarril á $\frac{\text{luz}}{1{,}600}$.

**72.** Sea Y = á la flexión máxima permitida, en cm por metro de luz, en cualquier caso dado;
$y$ = al coeficiente de flexión, §§ 60, etc.;
L = la luz neta de la viga, en metros;
$w$ = al peso de la luz neta de la viga, en kg;
W = al peso central + $^5/_8$ $w$;
= $^5/_8$ (peso uniforme + $w$).

Entonces, YL = la flexión, en cm, para toda la luz, L, y tenemos, para el peso permitido, W, el ancho, $b$, y el alto, $d$, requeridos para una viga rectangular (véase 61)

$$\textbf{Peso} \quad = \mathrm{W} = \frac{\mathrm{YL}bd^3}{\mathrm{L}^3y} = \frac{\mathrm{Y}bd^3}{\mathrm{L}^2y};$$

$$\textbf{Ancho}^* \quad = b = \mathrm{W}.\frac{\mathrm{L}^2y}{d^3\mathrm{Y}};$$

$$\textbf{Alto}^* \quad = d = \sqrt[3]{\mathrm{W}.\frac{\mathrm{L}^2y}{b\mathrm{Y}}}.$$

## Cargas aplicadas repentinamente.

**73.** Supóngase una carga aplicada repentinamente á una viga flexible, sin golpe ni trepidación, como, por ejemplo, sostenida por una cuerda que la deja precisamente tocar la viga sin descansar sobre ella, y cortando luego la cuerda. La flexión de la viga, en tal caso, es teóricamente doble que cuando el mismo peso se aplica gradualmente, como, por ejemplo, aflojando la cuerda lentamente, ó dividiendo el peso en fragmentos pequeños y aplicándolos á intervalos, uno por uno. Véanse §§ 35, etc. (Resistencia de Materiales.) En consecuencia, la resistencia de la viga (dentro del límite de elasticidad) es mucho más intensamente atacada en el primer caso que en el último. Un tren pesado que pasa muy rápidamente por un puente es un caso intermedio entre los dos citados.

## « Cantilevers » y vigas de resistencia uniforme.

**74.** Para el equilibrio, el momento de resistencia, R, de cualquier sección, debe contrarrestar el momento de flexión, M, en esa sección. Es decir,

$$\frac{\mathrm{S}}{\mathrm{T}}.\mathrm{I} = \mathrm{M};\ \mathrm{S} = \mathrm{M}.\frac{\mathrm{T}}{\mathrm{I}};$$

donde S = esfuerzo por unidad de superficie en las fibras extremas;
T = distancia del eje neutro á las fibras extremas;
I = momento de inercia de la sección.

**75.** En una viga de sección transversal uniforme, puesto que T ó I son uniformes en toda la luz, la unidad de esfuerzo, S, en las fibras extremas, varía con el momento de flexión, M. Para resistencia uniforme contra los momentos de flexión, la sección transversal debe variar de modo que $\frac{\mathrm{T}}{\mathrm{I}}$ sea inversamente proporcional á M, á fin de que S permanezca constante.

**76.** El cuadro siguiente muestra, en elevación y en plano, las formas teóricas de *cantilevers* y vigas rectangulares, de resistencia uniforme contra los momentos de flexión, bajo pesos concentrados y uniformes. En la práctica, algunas de estas formas deberían hacerse más fuertes cerca de sus extremos, para que tengan una sección suficiente para resistir al esfuerzo cortante.

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* Véase nota al pie del § 61.

**77.** No obstante la reducción del material empleando vigas de resistencia uniforme, resulta su uso raras veces económico, excepto en el caso del hierro fundido. Con la madera, el material suprimido no se ahorra todo; y con el acero la economía del material queda con frecuencia compensada por el costo del trabajo adicional.

Además, las *flexiones* de vigas de resistencia uniforme, bajo un peso dado, son considerablemente mayores que las de vigas de sección transversal uniforme.

En el cuadro pág. 511.

W=carga concentrada;
$w$=carga uniforme por unidad de luz;
$l$=luz;
$x$=distancia desde un apoyo á cualquier sección dada;
$d$=alto de la viga en esa sección;
$b$=ancho de la viga en esa sección;
D=alto máximo de la viga;
B=ancho máximo de la viga;
S=esfuerzo por unidad de superficie en las fibras extremas
E=coeficiente de elasticidad;
Y'=flexión, debida al peso extraño, en vigas de resistencia uniforme;
Y=flexión, debida al peso extraño, en vigas de sección transversal uniforme= sección transversal máxima de vigas de resistencia uniforme.

**Símbolos** en el cuadro pág. 512:

W=carga concentrada; $w$=carga uniforme por unidad de luz;
$l$=luz; $x$=dist de un apoyo á una sec dada;
$d$=alto de la viga en esa sección; $b$=ancho de la viga en esa sec;
D=alto máximo de la viga; B=ancho máximo de la viga;
S=unidad del esfuerzo en las fibras extremas;

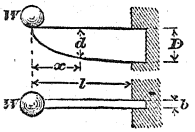
$$\text{E=coeficiente de elasticidad} = \frac{\text{tensión por unidad de superficie}}{\text{alargamiento por unidad de longitud}};$$

Y'=flexión, debida al peso extraño, en vigas de resistencia uniforme;
Y=flexión, debida al peso extraño, en vigas de *sección* transversal uniforme= sección transversal máxima de vigas de *resistencia* uniforme.

## « Cantilevers » de sección transversal rectangular y de resistencia uniforme. Perfiles, planos y flexiones.

**Para símbolos**, véase § 77.

### Carga concentrada en el extremo=W.

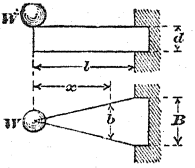



Ancho, $b$, constante.

Perfil, parábola, con vértice en la carga.

$$d = \sqrt{\frac{6Wx}{Sb}}; \quad Y' = \frac{8Wl^3}{EbD^3} = 2Y;$$

D = altura máxima.

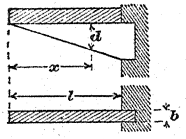



Alto $d$, constante.

Proyección, triángulo.

$$b = \frac{6Wx}{Sd^2}; \quad Y' = \frac{6Wl^3}{EBd^3} = \frac{3}{2}\,Y.$$

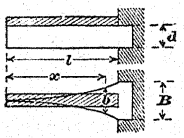
### Carga uniforme por unidad de luz=$w$.




Ancho, $b$, constante.

Perfil, triángulo.

$$d = x\sqrt{\frac{3w}{Sb}}.$$




Alto, $d$, constante.

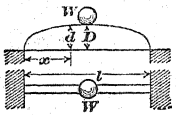
Proyección, dos curvas parabólicas con vértices en el extremo libre.

$$b = \frac{3wx^2}{Sd^2}, \quad Y' = \frac{3Wl^3}{EBd^3} = 2Y.$$

## Vigas de sección transversal rectangular, y resistencia uniforme
## Perfiles, planos y flexiones.

**Para símbolos**, véase § 77, al final.

### Carga concentrada en el centro=W.

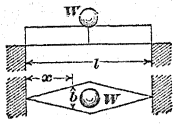



Ancho, $b$, constante.

Perfil, dos curvas parabólicas con vértice en los apoyos.

$$d = \sqrt{\frac{3Wx}{Sb}} \qquad D, \text{ en el centro de la luz.}$$

$$Y' = \frac{Wl^3}{2EbD^3} = 2Y.$$

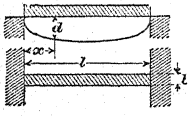



Alto, $d$, constante. B = ancho máximo.

Proyección, dos triángulos.

$$b = \frac{3Wx}{Sd^2}; \quad Y' = \frac{3Wl^3}{8EBd^3} = \frac{3}{2}\,Y.$$

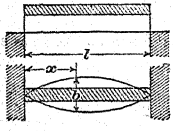
### Carga uniforme por unidad de luz=$w$.




Ancho, $b$, constante.

Perfil, elipse ó semielipse.

$$d = \sqrt{\frac{3w}{Sb}\,(lx - x^2)}.$$




Alto, $d$, constante.

Proyección, parábolas con vértices en el centro de la luz.

$$b = \frac{3w}{Sd^2}\,(lx - x^2).$$

**Vigas continuas.** Véanse también §§ 134-6.

**78.** Una viga continua es la que descansa sobre más de dos apoyos.

**79.** Las resistencias y flexiones de las vigas continuas, como las de vigas empotradas, se determinan por medio de la curva de elasticidad usando el cálculo. Los hechos más importantes, así deducidos, se indican en la fig. 14 y en el cuadro ilustrado, pág. 514.

**80.** La fig. 14 representa el carácter general de las flexiones y las variaciones de los momentos y de los esfuerzos cortantes en vigas continuas uniformemente cargadas.

**81. Momentos**, fig. 14 (*b*). Ordenadas trazadas *por encima* de la línea cero, *a'b'*, representan momentos *positivos*, ó aquellos en que el segmento de la viga, á la izquierda de cualquier sección, tiende á girar *como el minutero* *, y viceversa.

**82.** En cada extremo de la viga, en un punto, *i* (llamado punto de inflexión ó de flexión contraria), en cada tramo de los extremos, y en dos puntos semejantes en cada tramo ó luz restante, el momento es cero.

**83.** En otro punto, *m*, en cada tramo ó luz, el momento positivo llega á un máximo para ese tramo, mientras que los momentos negativos llegan á sus máximos en los apoyos. Tanto los momentos positivos como los negativos varían con aberturas ó luces diferentes; pero, si las luces son iguales, entonces los momentos, en cualesquiera dos puntos equidistantes del centro de la viga entera, son iguales.

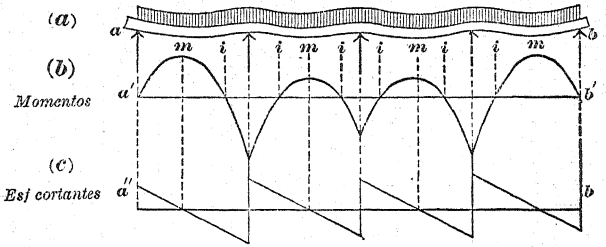



**Fig. 14.**

**84.** El diagrama de los momentos, entre cada apoyo y el punto, *m*, del momento positivo máximo, á uno ú otro lado de él, es una semiparábola, con su vértice en *m*.

**85. Esfuerzos cortantes**, fig. 14 (*c*). Las ordenadas trazadas *por encima* de la línea cero, *a''b''*, representan *esfuerzos cortantes positivos* ó aquellos en que el segmento *izquierdo*, en cualquier sección, tiende á resbalar *hacia arriba*, más allá del segmento *derecho*, y viceversa.

**86.** En el punto, *m*, del momento máximo, en cada luz, el esfuerzo cortante es cero. Entre cada uno de dichos puntos y el próximo apoyo á la izquierda, el esfuerzo cortante es positivo y viceversa.

**87.** En cada apoyo el esfuerzo cortante cambia de súbito con una intensidad=á la reacción del soporte.

**88.** El diagrama del esfuerzo cortante es una serie de líneas rectas.

**89. El cuadro ilustrado**, pág. 514, representa las condiciones teóricamente existentes en vigas continuas uniformemente cargadas desde dos hasta cinco luces iguales. Sólo se ve en la figura la mitad izquierda de cada una de tales vigas, siendo la mitad derecha simétrica con la izquierda.

**90.** Los números indican la intensidad del momento positivo máximo en cada luz, la del momento negativo en cada apoyo y el esfuerzo cortante en cada lado de cada apoyo.

**91.** Los números muestran también el coeficiente, *a*, para la distancia, *al*, desde el apoyo izquierdo de cada luz hasta el punto del momento máximo en esa luz; y el coeficiente, *x*, para la distancia ó distancias, *xl*, desde el mismo apoyo hasta el punto ó puntos de inflexión en esa luz. En cada luz central, la suma de los dos valores de $x=1$. En cada luz extrema, $x=2a$.

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* *N. del T.* — Veáse nuestra nota pág. 463.

$m\,w\,l^2$ = Momento.

$v\,w\,l$ = Esfuerzo cortante.

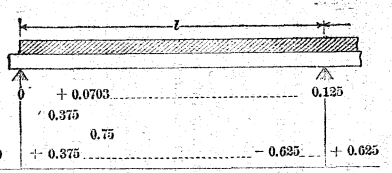
$al$ = Distancia del apoyo izquierdo de cualquier luz, al punto máximo de momento positivo de esa luz.

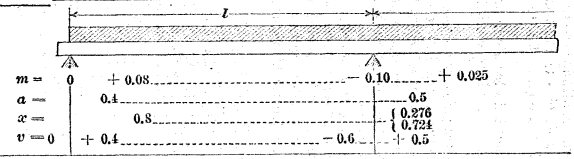
$xl$ = Distancia de cualquier apoyo izquierdo al punto de inflexión.

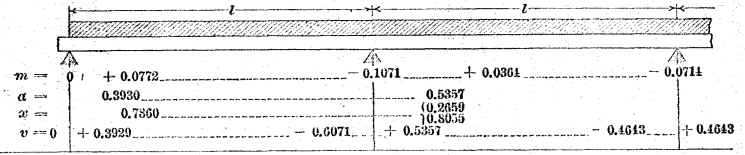
$w$ = Carga por unidad de luz.

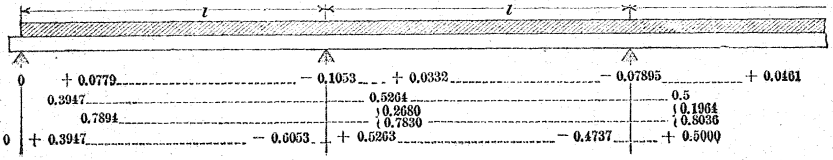
$l$ = Luz.

| Número de | |
|---|---|
| Luces. | Apoyos. |
| 2 | 3 |
| 3 | 4 |
| 4 | 5 |
| 5 | 6 |



**92.** En cada luz central, el punto del momento positivo máximo está en el centro de la luz. En otros términos, la flexión en esa luz es simétrica, ó bien $a = .5$.

**93.** La suma numérica de los dos esfuerzos cortantes, uno en cada lado de un apoyo, es=á la reacción de ese apoyo. En cada apoyo central, los esfuerzos cortantes, en sus dos lados, son iguales.

En el cuadro :

$w$=peso por unidad de luz;
$l$=luz;
$m$=el coeficiente para el momento;
$mwl^2$=momento;
$v$=el coeficiente para el esfuerzo cortante;
$vwl$=esfuerzo cortante;
$a$=el coeficiente para la distancia al punto del momento máximo;
$al$=distancia desde el apoyo izquierdo de cualquier luz al punto del momento positivo máximo en esa luz;
$x$=el coeficiente para la distancia al punto de inflexión;
$xl$=distancia del apoyo izquierdo de cualquier luz á uno ú otro punto de inflexión en esa luz.

**94.** La fig. 15 muestra los valores de $m$ y de $v$ en una viga no continua, uniformemente cargada. Comparando éstos con los correspondientes valores en vigas continuas, como se ven en el cuadro ilustrado último, se observa que la viga continua tiene considerable ventaja teórica. Pero véase § 95.

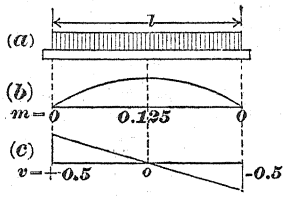



**Fig. 15.**

**95.** Ciertas consideraciones prácticas, empero, reducen considerablemente estas ventajas en muchos casos. Así, en un puente continuo de ferrocarril de 30 m, por ejemplo, de luz, de tal modo calculado que la flexión máxima no exceda de 2 cm, una bajada ó hundimiento de 2 cm en un pilar intermedio privaría al puente del apoyo de tal pilar y prácticamente pondría así dos luces adyacentes en una sola, haciendo obrar sobre sus miembros fuerzas muy superiores á aquellas para las cuales fueron calculadas y destinadas. Además, con cargas móviles, la ventaja teórica es á veces mucho menor que la debida á una carga fija.

## Vigas en forma de cruz *.

**96.** En una viga en forma de cruz, fig. 16, de material homogéneo, cargada en el centro, sean :

W = á la carga;

E=al coeficiente de elasticidad = $\frac{\text{esfuerzo por unidad de superficie}}{\text{alargamiento por unidad de longitud}}$;

Y=á la flexión en el centro;
L, $l$=á la luz ó abertura de
D, $d$=á los altos de
T, $t$=á las semi-alturas de
I, $i$=á los momentos de inercia de
S, $s$=esfuerzo por unidad de superficie en las fibras extremas de
P, $p$=las porciones de W sostenidos por

} los dos brazos respectivamente.

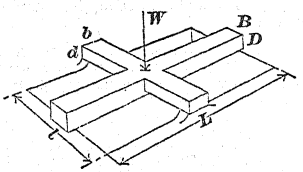



**Fig. 16.**

Entonces (véase cuadro ilustrado pág. 503), puesto que la flexión es necesariamente la misma para ambos brazos,

$$\frac{L^3P}{EI} = \frac{l^3p}{Ei}; \text{ ó}$$

$$\frac{P}{p} = \frac{I.l^3}{i.L^3}$$

y, puesto que $P = 4.\frac{S.I}{T.L}$, y $p=4.\frac{s.i}{t.l}$ (véase cuadro pág. 496), tenemos (despejando S y $s$, dividiendo sus valores, reemplazando $\frac{P}{p}$ por su valor y á $\frac{T}{t}$ por su equivalente $\frac{D}{d}$), $\frac{S}{s} = \frac{D}{d}\left(\frac{l}{L}\right)^2$.

**97.** En otros términos, á fin de que ambos brazos sean igualmente fuertes, sus altos (independientemente de sus anchos) deben estar en razón inversa de los cuadrados de sus luces, ó sus luces en razón inversa de las raíces cuadradas de sus altos.

## Resistencia transversal de planchas planas.

Para Planchas ondeadas, convexas, véase pág. 1212.

**98.** Las leyes que rigen la resistencia de las planchas, á presiones perpendiculares á sus superficies, no están bien comprendidas, y las fórmulas que las expresan deben usarse con cautela y sólo como aproximaciones probables.

**99.** En el siguiente cuadro, los momentos son los dados, como suficientemente aproximados, por Rankine, Ingeniería Civil, pág. 544, para planchas apoyadas en sus bordes. Los esfuerzos se deducen de estos momentos por medio de la fórmula, $S=M\frac{6}{Ld^2}$. Véanse Símbolos, § 101. Véanse las fórmulas para pesos, en las vigas rectangulares (cuadro pág. 496). Para planchas con los extremos *fijos, empotrados*, véase § 100.

Una plancha oblonga homogénea tiende á henderse á lo largo de su eje mayor.

* F. Reuleaux, « Der Konstrukteur ». Braunschweig, 1889. « The Constructor », traducido por H. H. Suplee, Filadelfia, 1893.

**100. Extremos fijos ó empotrados.** Si M es el momento, y S el esfuerzo de la fibra máxima, para una plancha apoyada en sus bordes, y si $Mf$ y $Sf$ son los valores correspondientes para el mismo peso en la misma plancha con los extremos *fijos* (empotrados), podemos suponer :

para carga central, $Mf = {}^1/_2 M$; $Sf = {}^1/_2 S$;
para carga uniforme, $Mf = {}^1/_3 M$; $Sf = {}^2/_3 S$.

Véanse fórmulas para vigas ordinarias, cuadro pág. 496.

**101.** En una plancha apoyada en sus bordes, sean :

$d$ = espesor de la plancha;
$w$ = peso por unidad de superficie;
W = peso total;
M = momento máx de flexión;
S = esfuerzo máximo de la fibra.

En una plancha oblonga, sean :
L = la luz más larga;
$b$ = la luz más corta;
en una plancha cuadrada ó circular, $L = b$;
en una plancha circular, $r$ = radio.

**Momento máximo de flexión**, M, y **esfuerzo máximo**, S, de la fibra en planchas apoyadas en los bordes. Para bordes empotrados, véase § 100.

### Peso ó carga central.

| Plancha. | Momento, M. | Esfuerzo, S. |
|---|---|---|
| Oblonga | $\frac{1}{4}\,W b$ | $\frac{3}{2}\,\frac{b}{L}\,\frac{W}{d^2}$ |
| $L < 1.19\,b$ | | |
| $L < 1.19\,b$ | $\frac{3}{8}\,W\,\frac{L^4 b}{L^4+b^4}$ | $\frac{9}{4}\cdot\frac{L^3 b}{L^4+b^4}\cdot\frac{W}{d^2}$ |
| Cuadrada | $\frac{3}{16}\,WL$ | $\frac{9}{8}\,\frac{W}{d^2}$ * |
| Circular | $W\,\frac{r}{\pi}$ | $\frac{3}{\pi}\,\frac{W}{d^2}$ * |

### Carga uniforme.

| Plancha. | Momento, M. | Esfuerzo, S. |
|---|---|---|
| Oblonga | $\frac{1}{8}\,W\,\frac{L^4 b}{L^4+b^4} = \frac{1}{8}\,w\,\frac{L^5 b^2}{L^4+b^4}$ | $\frac{3}{4}\cdot\frac{L^4 b}{L^4+b^4}\cdot\frac{W}{d^2} = \frac{3}{4}\cdot\frac{L^4 b^2}{L^4+b^4}\cdot\frac{w}{d^2}$ |
| Cuadrada | $\frac{1}{16}\,WL = \frac{1}{16}\,wL^2$ | $\frac{3}{8}\cdot\frac{W}{d^2}$ * $= \frac{3}{8}\cdot L^2\cdot\frac{w}{d^2}$ |
| Circular | $\frac{1}{3}\,W\,\frac{r}{\pi} = \frac{1}{3}\,wr^3$ | $\frac{1}{\pi}\cdot\frac{W}{d^2}$ * $= r^2\,\frac{w}{d^2}$ |

**102. En una plancha circular,** uniformemente cargada, la **flexión** en el centro $= c\,\frac{wr^4}{Ed^3}$ donde $c = \frac{2}{3}$ para planchas *apoyadas* en sus bordes ; $c = \frac{1}{6}$ para planchas *empotradas;* E = módulo de elasticidad.

---

* Puesto que el esfuerzo $\left(S = M\,\frac{6}{Ld^2}\right)$ es proporcional á $\frac{WL}{Ld^2}$, la tensión, S, bajo un peso *total* dado, W, en planchas *cuadradas* ó *circulares*, y en planchas rectangulares donde $L/b$ es constante, es independiente de las dimensiones de la superficie.

## ESFUERZOS TRANSVERSAL Y LONGITUDINAL COMBINADOS

**103.** Aunque es censurable la combinación de los esfuerzos longitudinal y transversal en la misma pieza, es á menudo inevitable. Así, en un techo de madera, los contrapares obran por lo general como columnas y como vigas.

En tales casos, el esfuerzo total por unidad de superficie, S, en las fibras extremas, es la suma del esfuerzo uniforme, $S_c$, debido á compresión ó tensión directa, y el esfuerzo de las fibras extremas, $S_b$, debido á los momentos de flexión solamente, bajo la acción de las cargas transversales y longitudinales combinadas. Es decir, $S = S_c + S_b$.

Sea $M_b$ = al momento de flexión debido al peso transversal; $M_c$ = al momento de flexión debido al peso longitudinal, P; y M = al momento de flexión resultante total, $= M_b - M_c$, cuando la carga longitudinal ejerce tensión; $= M_b + M_c$ cuando ejerce compresión.

Pero $M_c = Pd$, donde P = al peso ó carga longitudinal, y $d$ = su brazo palanca, = la flexión de la viga, debida á todas las causas; y (véase § 57) $d = \frac{l^2 S_b}{ETc}$; donde $l$ = luz, $S_b$ = esfuerzo por unidad de sup en las fibras extremas, debido á la flexión; E = módulo de elasticidad; T = distancia del eje neutro á las fibras extremas, y $c$ = un coeficiente, cuyos valores, para diferentes casos, se dan en el § 57.

En consecuencia, $M_c = P\frac{l^2 S_b}{ETc}$, y el momento resultante $M = M_b \pm P\frac{l^2 S_b}{ETc}$. El momento de resistencia, R (véase § 10), es $= S_b \frac{I}{T}$; y, para el equilibrio, R = M.

Por tanto, $S_b \cdot \frac{I}{T} = M_b \pm P\frac{l^2 S_b}{ETc}$; de donde deducimos, para el esfuerzo de la fibra extrema, S, debido á la flexión solamente, bajo la acción de las cargas ó peso transversal y longitudinal combinados,

$$S_b = \frac{M_b T}{I + \frac{Pl^2}{Ec}}$$ donde el esfuerzo longitudinal es de tensión

$$S_b = \frac{M_b T}{I - \frac{Pl^2}{Ec}}$$ donde el esfuerzo longitudinal es de compresión

Además de esto, tenemos el esfuerzo por unidad de sup (esfuerzo unitario), $S_c$, debido directamente al peso longitudinal, P, que es $= \frac{P}{A}$, donde A es el área de la sección transversal de la viga. Por consiguiente, para la unidad total del esfuerzo, S en las fibras extremas, tenemos

$$S = S_c + S_b = \frac{P}{A} + \frac{M_b T}{I \pm \frac{Pl^2}{Ec}}.$$

Cuando la flexión, $d$, es despreciable, $M_c = 0$; $M = M_b = S_b \frac{I}{T}$; y $S_b = \frac{M_b T}{I} = \frac{MT}{I}$ como en el § 10; y $S = \frac{P}{A} + \frac{MT}{I}$. En la práctica, $d$ es frecuentemente despreciable y se usa esta fórmula.

## ESFUERZOS DIAGONALES EN LAS VIGAS

### Unidad máxima del esfuerzo.

**104.** Cuando un cuerpo (como un perno) está **sometido á fuerza de tensión (ó compresión) solamente**, la tendencia del cuerpo, *considerando las secciones perpendiculares á los esfuerzos*, es á separarse, rajarse (ó aplastarse) en la dirección del esfuerzo, ó perpendicularmente á la sección, y todo el esfuerzo obra perpendicularmente sobre la sección; pero, en los *planos oblicuos al esfuerzo*, éste se resuelve en dos componentes, una (*n*) de tensión (ó comp) perpendicular al plano, y una (*t*) tangencial al plano (esfuerzo cortante).

**105. Bajo el esfuerzo cortante solamente**, el efecto, sobre un plano paralelo á las dos fuerzas, y entre ellas, resulta de exclusivo esfuerzo cortante; pero en planos *oblicuos* á las fuerzas que producen el esfuerzo cortante, aquéllas se resuelven en (*t*) fuerzas tangenciales ó de esfuerzo cortante, y (*n*) fuerzas perpendiculares (de tensión ó comp).

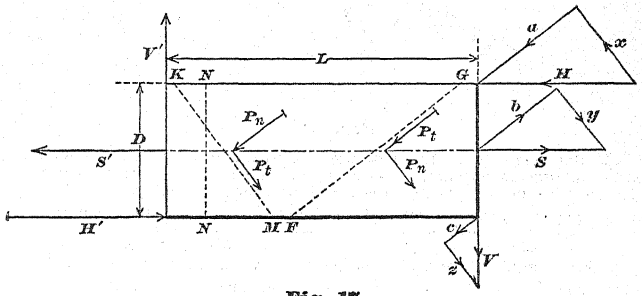



**Fig. 17.**

**106.** Así, fig. 17, sea una barra, de largo, L, y alto, D, sometida á una tensión S=S', en línea con su eje horizontal, y dos pares de fuerzas=V=V' y H=H', como se indican; constituyendo V y V' un esfuerzo cortante vertical *á la derecha*, mientras que H y H' forman un esfuerzo cortante horizontal *á la izquierda*.

Supóngase dividida la barra por una sección, como NN, FG ó KM, y considérese que las fuerzas obran, en uno ú otro caso, sobre el segmento á la derecha de la barra según se haya dividido.

Sobre la sección *perpendicular*, NN, la tensión, S, y el esfuerzo cortante horizontal, H, obran perpendicularmente (S como tensión, H como compresión), y el esfuerzo cortante vertical, V, tangencialmente (como esfuerzo cortante); pero, para una sección *oblicua*, FG ó KM, resolvemos primero cada fuerza, S, V y H, en dos componentes, *b* é *y*, *c* y *z*, *a* y *x*, respectivamente perpendiculares y paralelas á la sección, como se demuestra por los triángulos de las fuerzas á la derecha *. Entonces, sumando estas compresiones algebraicamente, obtenemos las fuerzas resultantes, $P_n$ (perpendicular) y $P_t$ (tangencial ó de esfuerzo cortante) que obran sobre la sección de que se trata. Con las fuerzas, S, V y H, como se indica en la fig. 17, tenemos:

En la sección FG, $P_n$, tensión, $=y+z-x$;

$P_t$, esfuerzo cortante *derecho*, $=a+c-b$;

En la sección KM, $P_n$, compresión, $=a+c-b$;

$P_t$, esfuerzo cortante *derecho*, $=y+z-x$.

**107.** Si ahora examinamos *todos los planos posibles* cortando el cuerpo en un punto dado, encontraremos (1) uno de tales planos en que la unidad de fuerza de *tensión* resultante llega á su máximum; (2) otro, perpendicular al (1), en el cual la unidad de fuerza *compresiva* resultante llega á su máximum, y (3) dos planos, perpendiculares entre sí y que dividen en dos partes iguales los ángulos rectos

* A fin de que, para una ú otra fuerza, S, V ó H, los dos triángulos de fuerzas (para las dos secciones, FG y KM) sean idénticos, simplificando así la figura, tomamos las dos secciones, FG y KM, perpendiculares entre sí.

entre los planos (1) y (2). Sobre los dos planos últimamente nombrados, (3), la unidad resultante de las fuerzas que producen el esfuerzo cortante llega á su máximo,

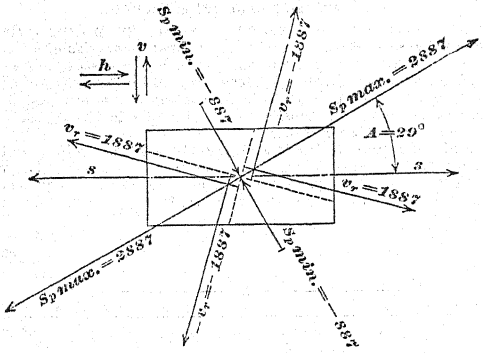



**Fig. 18.**

**108.** Represente la fig. 18 un **pequeño elemento** en una barra sometida á tensión y esfuerzo cortante; y trátese de determinar las **posiciones de estos planos y las correspondientes tensiones máximas.** Sean

$s$ = esfuerzo normal original por unidad de superf (de tens ó comp);
$v$ = — vertical — — — (esfuerzo cortante);
$h$ = — horizontal — — — (esfuerzo cortante);
$s_p$ = la resultante normal de la unidad máx ó mín del esfuerzo;
$v_r$ = — — — máxima del esf cortante;
A = ángulo formado por $s$ con $s_p$.

$$\text{Entonces } \tan 2A = \frac{v}{s/2} \quad \ldots\ldots (1)$$

$$v_r = \sqrt{(s/2)^2 + v^2} \quad \ldots\ldots (2)$$

$$s_p \text{ máx} = s/2 + v_r = s/2 + \sqrt{(s/2)^2 + v^2} \quad \ldots\ldots (3)$$

$$s_p \text{ mín} = s/2 - v_r = s/2 - \sqrt{(s/2)^2 + v^2} \quad \ldots\ldots (4)$$

Si $s$ es { tensión { signo + da tensión máx; — — — comp — = tensión mín. / comp { — + — comp —; — — — tensión — = comp mín.

**109. Ejemplo** : Sean

$s$ = 2,000 kg/cm cuad, tensión (no trazada en escala);
$v = h$ = 1,600 — — esf cortante ( — — ).

Aquí $v$ es izquierdo, $h$ derecho. Si esto se invierte, el ángulo, A, entre la tensión resultante, $s_p$, y la horizontal, estará *por debajo* del eje neutro.

**110.** Entonces $\tan 2A = \dfrac{v}{s/2} = \dfrac{1{,}600}{1{,}000} = 1.6$; $2A = 58°$; $A = 29°$;

$$v_r = \sqrt{(s/2)^2 + v^2} = \sqrt{1{,}000^2 + 1{,}600^2} = 1{,}887;$$
$$s_p \text{ máx} = s/2 + v_r = 1{,}000 + 1{,}887 = 2{,}887 \text{ (tensión)};$$
$$s_p \text{ mín} = s/2 - v_r = 1{,}000 - 1{,}887 = -887 \text{ (comp)}.$$

**111.** En otras palabras, **tenemos, como resultantes,** (1) una unidad máx de tensión, $s_p$ máx = 2,887 kg/cm cuad formando un ángulo, A = 29°, con el eje de la barra ó con la dirección de $s$; (2) una unidad mín de tensión ó máx de compresión, $s_p$ mín = — 887 kg/cm cuad, perpendicular á $s_p$ máx; (3) una unidad de esfuerzo cortante derecha, $v_r$ ‖ 1,887 kg/cm cuad; y una unidad de esfuerzo

cortante izquierdo $-v_r = -1{,}887$ kg/cm cuad. Las direcciones de los esfuerzos cortantes dividen en dos partes iguales los ángulos rectos entre los esfuerzos normales máx.

**112.** La tensión y compresión máx, en cualquier punto, se llaman los « **esfuerzos principales** » para ese punto.

## Esfuerzo cortante horizontal y vertical en las vigas.

Véanse también los §§ 301, etc., 325, etc., 348, etc., en Estática, y los §§ 51 á 56 en Resistencia de Materiales.

**113.** Supongamos que la fig. 19 representa la mitad izquierda de una **viga homogénea**, de sección rectangular; ancho, $b_1=1$ cm; alto, $d_1=10$ cm; luz, $L_1=100$ cm; con carga cent, W *, de 200 kg; reacción izquierda, $R=W/2=100$ kgs. Despréciase el peso de la viga. El mom de flexión, en el cent de la luz, es $M=RL/2=WL/4$ *$=5{,}000$ kg-cm; y el mom disminuye uniformemente *, desde su máx, en el cent de la luz, hasta cero en los apoyos. En las fibras extremas, superiores é inferiores, la unidad de fuerza longitudinal (§ 10), $s_1=MT/I$, donde $T=d/2$=dist del eje neut á las fibras extremas$=5$ cm; I=mom de inercia de la sección transversal$=bd^3/12=1{,}000/12$. En consecuencia, en la fig. 19, $s=12\times 5M/1{,}000=.06M$. Ahora $s$, siendo así proporcional á M, también disminuye uniformemente *, desde su máx, en el cent de la luz, hasta cero en los apoyos. Los valores de M y de $s$, para las secciones 0, $a$, $b$, $c$, $d$, $e$, están indicados con números en el diagrama.

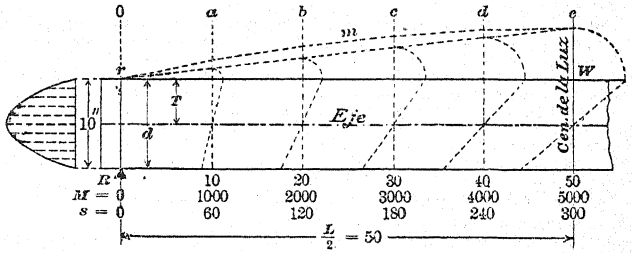



**Fig. 19.**

**114.** Las unidades de **fuerzas** horizontales de **tensión y compresión**, $s$, en los varios puntos en cualquier sección vert, son **proporcionales á las dists** de esos puntos **al eje neutro**, como lo indica el diagrama en cada sección vert, fig. 19.

**115.** En la fig. 20, sean $n$ y $g$ dos sécciones vert de esta viga, tales que, en $n$ y en $g$, las unidades de las fuerzas de las fibras extremas sean : $mn=15$, y $ug=25$, respectivamente. Entonces sobre la **parte rectangular**, $nf$, de la viga, entre las secciones $n$ y $g$, obran una serie de fuerzas que varían de compresión, $eg=ug-mn=-25-(-15)=-10$, en el borde superior, á tensión, $=+10$, en el inferior, como lo indica el diagrama en $ek$.

**116.** Supóngase la pieza $gf$ dividida en 10 fajas horiz de igual altura, $=1$ cm. Entonces las unidades de fuerza neta, $s$, que obran en estas fajas, respectivamente, son las $(-10, -8, -6, \ldots. 6, 8, 10)$ numeradas de $e$ á $k$, y la fuerza media, ó (puesto que el alto de cada faja$=b=1$) **la fuerza, que obra sobre cada faja**, es la $(-9, -7, -5, \ldots. 5, 7, 9)$ numeradas entre $g$ y $f$.

**117. Estas fuerzas son transmitidas**, de faja á faja, por sus superficies de contacto; y, al determinar el esfuerzo cortante, que obra en el plano horizontal

* Bajo un peso *uniformemente distribuido*, el mom de flex, en el cent de la luz, es $WL/8$; y los moms de flex, M, y la unidad del esfuerzo longitudinal resultante, $s$, varían como las ordenadas de una parábola, como lo indica la parábola de puntos, $rmc$, en la parte superior de la fig. 19, que corresponde á un peso uniforme $=400\,\text{kg}=2W$. La unidad de esfuerzos cortantes $v$, en una sección horiz dada, disminuye entonces uniformemente, desde un máx, en los apoyos, hasta cero en el cent de la luz. Compárense 3.ª y 4.ª figs, en el cuadro de la pág. 496.

entre cualesquiera dos fajas, consideramos la faja superior (ó inferior) como movida por su propio empuje ó impulso positivo *más* (suma algebraica) de los de todas las fajas por encima (ó por debajo si se tomó ésta) de ella.

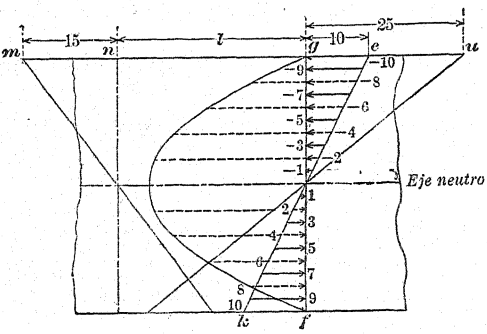



**Fig. 20.**

**118.** Así, la 3.ª faja, á partir de arriba, es empujada hacia la izquierda por una fuerza de — 9 — 7 — 5 = — 21, mientras la 4.ª faja, precisamente por debajo de ella, es impulsada hacia la derecha por una fuerza de 9+7+5+3+1 — 1 — 3=21. En consecuencia, la superficie entre la 3.ª y 4.ª fajas, sostiene un **esfuerzo cortante** « contra el del minutero »* de 21; que, dividido por el área, $bl=l$, de esa superficie, da el **esfuerzo cortante** por unidad de superficie en el plano entre la 3.ª y la 4.ª fajas. Con peso central **, esta unidad de esfuerzo cortante es uniforme desde cada apoyo al centro de la luz, donde cambia de sentido (de más á menos, ó vice versa), pero es de la misma intensidad en la otra media luz. Véase 3.ª figura del cuadro, pág. 496.

**119. En cualquier sección vertical de la viga**, sean

V = al esfuerzo cortante total
  = á la reacción de uno ú otro apoyo, menos la suma de todos los pesos entre ese apoyo y la sección;
I = al momento de inercia con respecto al eje neut;
$b$ = ancho; $d$ = alto;
$a$ = al área por encima (ó por debajo) de cualquier punto en la sección;
$c$ = la distancia del eje neut, al cent de grav de $a$;
$M_s = ac$ = al mom estático de $a$, con respecto al eje neut;
$v$ = al esfuerzo cortante por unidad de superf vert = esfuerzo cortante por unidad de superficie horizontal en un punto dado.

**120.** Entonces

$$v = \mathrm{V}\frac{\mathrm{M}_s}{\mathrm{I}} \div b = \mathrm{V}.\frac{ac}{\mathrm{I}b}$$

$$\text{En el eje neut, } \mathrm{M}_s\ (=ac) = \frac{db}{2}\cdot\frac{d}{4} = \frac{d^2b}{8}$$

En consecuencia, **en el eje neutro :**

$$v = \mathrm{V}\frac{d^2}{8\mathrm{I}} = \mathrm{V}\frac{12d^2}{8bd^3} = \frac{3}{2}\cdot\frac{\mathrm{V}}{bd}$$

$$= \frac{3}{2} \times \text{el esfuerzo cortante vert medio en la sección transversal.}$$

Véanse también §§ 51, etc.

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* Véase *N. del T.* pág. 463.
** Véase nota pág. 521.

En el § 115 hemos tomado dif en $s$, entre $n$ y $g$, fig. 20, $=10=V/bd$.

En consecuencia, $v = \frac{3}{2} \cdot \frac{V}{bd} = \frac{3}{2} \cdot \frac{100}{10} = 15 = \frac{3}{2} \times$ dif en $s$.

En el eje neut, fig. 20, tenemos esfuerzo cortante horiz total $=9+7+5+3+1=25$; y dist $ng=l=bl=$esfuerzo cortante horiz total $\div$ unidad del esfuerzo cortante $=25/15=1.666\ldots$; y $s$ máx $\times \frac{l}{L/2} = (300 \times 1.666\ldots)/50=10=$dif en los esfuerzos, $s$, de las fibras horiz, entre $n$ y $g$.

**121.** A la izquierda de la fig. 19 hay un **diagrama que indica las unidades de los esfuerzos cortantes** en las varias secciones horizontales.

**122.** Representemos por la fig. 21 un pequeño elemento de un cuerpo, de unidad de espesor, perpendicular al papel, sobre el que obra un esfuerzo cortante vert derecho, $V=vD$ (donde $v=$á la unidad del esfuerzo cortante vert, y $D=$la altura del elemento) y por un esfuerzo cortante horiz izquierdo, $H=hL$ (donde $h=$á la unidad del esfuerzo cortante horiz, y $L=$el largo del elemento). Para el equilibrio de los momentos debemos tener

$$VL = HD; \quad \text{ó } vDL = hLD; \quad \text{ó sea } v = h.$$

En otros términos,

**la unidad del esfuerzo cortante vert = la unidad del esfuerzo cortante horiz.**

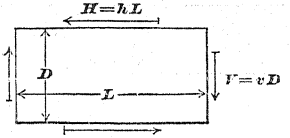



**Fig. 21.**

## Esfuerzos máximos por unidad de superficie en las vigas.

**123. La teoría común de las vigas** (§§ 1-103, págs. 488 á 518) considera solamente las **fuerzas longitudinales, tensión y compresión** y los **esfuerzos cortantes** vert y horiz, debidos directamente á la carga y á las reacciones ascendentes de los apoyos, y que obran, en cualquier punto, sobre *planos verts y horizs* que pasan por determinado punto; pero, excepto en ciertas porciones limitadas de la viga, estos esfuerzos no son **los esfuerzos máximos** que obran en dicho punto; pues ellos se combinan para formar esfuerzos **diagonales** resultantes, que obran sobre planos **diagonales** (que pasan por el mismo punto); y, sobre algunos de estos planos diag, los esfuerzos resultantes normales y tangenciales son mayores que los esfuerzos originales.

**124. La teoría común** se **adapta** bastante bien á vigas de muchas clases, y especialmente á **vigas de acero**, donde las alas de la viga resisten á las fuerzas longitudinales, y los esfuerzos cortantes los resiste el alma de la viga; pero, en ciertas porciones de vigas altas y fuertemente cargadas, especialmente las de **cemento armado**, las resultantes diagonales ó **esfuerzos máximos** son los esfuerzos *predominantes*, y no deben desatenderse.

**125.** En una viga, en la parte alta y baja, tenemos, respectivamente, sólo esfuerzos de tensión y compresión horiz, y, en el eje neut, esfuerzos cortantes (vert y horiz) solamente; pero en todos los otros puntos, tenemos esfuerzo cortante (vert y horiz), **obrando conjuntamente con esfuerzos horiz**, ya de tensión, ya compresivos. En todos los puntos, estos esfuerzos cortantes y las fuerzas longitudinales pueden **resolverse en componentes**, normales y tangenciales á cualquier plano, como en el caso de la barra ó del perno, fig. 17.

**126.** Así, sobre cada elemento de la viga, figs. 22, 23, 24, actúan fuerzas horiz y verts (unidades de esfuerzos), que, obrando sobre planos diagonales, se resuelven en componentes diagonales, y estas componentes algebraicamente **sumadas forman resultantes**; pero las fuerzas originales varían en intensidad, y las fuerzas resultantes tanto en intensidad como en dirección, de un punto á otro.

Para las **direcciones y valores** de estos esfuerzos resultantes en sus máximums, tenemos la ecuación (1), § 108, pág. 520.

$$\text{Tang } 2A = \frac{v}{s/2} \quad \ldots\ldots (1)$$

$$v_r = \sqrt{(s/2)^2 + v^2} \quad \ldots\ldots (2)$$

$$s_p = s/2 \pm v_r = s/2 \pm \sqrt{(s/2)^2 + v^2} \quad \ldots\ldots (3)\ (4)$$

donde

$s$ = al esfuerzo original por unidad de superf (tens ó comp) en el punto;
$v$ = esfuerzo cortante original (vert ú horiz) por unidad de superf, en el punto.

Los esfuerzos perpendiculares máx, $s_p$, se llaman **esfuerzos principales**.

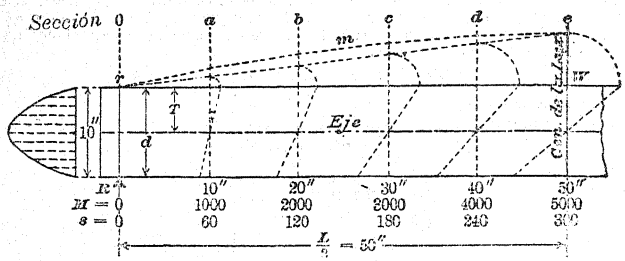
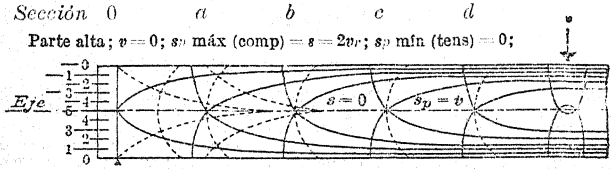



**Fig. 22.**




Parte alta; $v = 0$; $s_p$ máx (comp) $= s = 2v_r$; $s_p$ mín (tens) $= 0$;

Parte baja; $v = 0$; $s_p$ máx (tens) $= s = 2v_r$; $s_p$ mín (comp) $= 0$;

**Fig. 23.**

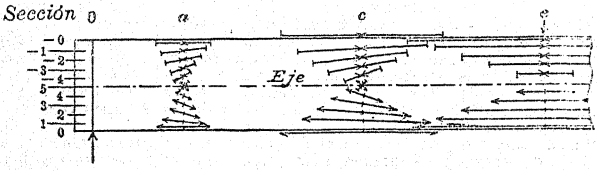



**Fig. 24.**

**127.** Aplicando estas fórmulas en numerosos puntos del perfil de la viga, fig. 22, se llega á construir **curvas**, fig. 23, que muestran las **direcciones** de los esfuerzos, y á trazar, como en la fig. 24, para puntos dados, las **direcciones é intensidades** de los esfuerzos que allí obran. En cualquier punto, fig. 24, tenemos esfuerzos resultantes normales y esfuerzos cortantes análogos á los de la fig. 18, pág. 520; pero, en esta fig. 24, debido á la falta de espacio, sólo se muestra para cada punto elegido el esfuerzo **principal máx**, $s_p$.

**128.** En la fig. 23, las direcciones de los **esfuerzos principales**, $s_p$, están representadas por las curvas continuas; las de los **esfuerzos cortantes resultantes**, $v_r$, por las de puntos.

| De las curvas continuas (esfuerzos principales) | cóncavas | horizontal en el cent de la luz | á 45° con | á 90° con |
|---|---|---|---|---|
| Las curvas de tensión son | hacia arriba | por debajo del eje neut | eje neut | borde sup de viga |
| Las curvas de compresión son | hacia abajo | por encima del eje neut | eje neut | borde inf de viga |

Las curvas de tens y comp son perpendiculares entre sí en sus intersecciones.

**129.** Siguiendo cualquier curva (cóncava hacia arriba) de tensión normal *, encontramos que :

(1) para su punto de **tangencia con la horiz** (esto es : en el cent de la luz) $s_p$ máx=tensión=$s$; $s_p$ mín=comp=0;

(2) para el punto **donde la curva cruza el eje neut** (á 45°) $s_p$ máx (tensión) $=s_p$ mín (comp)$=v_r=\pm v$ (esfuerzo cortante);

(3) **Por sobre el eje neut**, la tensión se hace $s_p$ mín, y continúa disminuyendo, á medida que la dirección se acerca á la vert, haciéndose cero en el borde sup, donde A=90°. Por encima del eje neut, para puntos de la misma curva, la *compresión* (*normal* á la curva) es ahora $s_p$ máx, y aumenta desde $s_p=v_r=\pm v$, en el eje neut, hasta $s_p$ máx (comp)$=s$, en el borde sup.

**130. Donde v=cero** (esto es : en cualquier punto de la sección transversal vert en el cent de la luz, y á lo largo de las fibras extremas superiores é inferiores) tenemos (§ 126) :

$$v_r=s/2$$
$$s_p \text{ máx}=s/2+v_r=s; \quad \text{tang } 2A=0;$$
$$s_p \text{ mín}=s/2-v_r=0; \quad \text{tang } 2A=0.$$

**131.** La ecuación, tang 2A=0, da ó 2A=0°, ó 2A=180°; esto es : A=0°, ó A=90°; pero sabemos que, en el cent de la luz y á lo largo de las fibras extremas (superiores é inferiores), $s_p$ máx es horiz, ó A=0°; y $s_p$ mín es vert, ó A=90°.

**132. Donde s=cero** (como en la superf neut y donde el momento de flexión= cero), tenemos (§ 126) : $v_r=\pm v$; $s_p$ máx$=s_p$ mín$=\sqrt{v^2}=\pm v$; tang 2A$=\infty$; 2A= 90°; y A=45°.

**133.** De las **curvas** (de puntos) **del esfuerzo cortante**, fig. 23, unas son tangenciales al eje neut y llegan al borde sup é inf de la viga en ángulos de 45°, tendiendo á separarse del cent de la luz; mientras que las otras son perpendiculares á éstas y al eje neut en sus intersecciones, llegando al borde sup é inf de la viga en áng de 45°, tendiendo hacia el cent de la luz.

* *Al contrario* para curvas (cóncavas hacia abajo) de compresión normal.

## MOMENTOS EN LAS VIGAS CONTINUAS

Véanse también §§ 78, etc.

**134.** Las figs. 25 y 26 muestran **momentos de flexión** positivos y negativos **en dos vigas continuas**; la fig. 25 de dos luces iguales, y la fig. 26 de tres luces iguales, descansan libremente sobre sus apoyos. Cada luz=1. La fig. 26 (tres luces) puede usarse, con bastante aproximación, para casos de mayor número de luces.

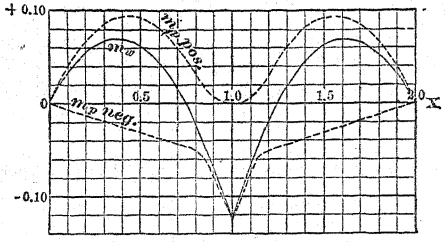



**Fig. 25.**

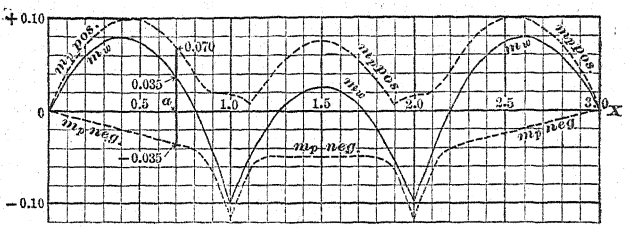



**Fig. 26.**

**135.** En cualquier sección transversal, la ordenada, entre el eje, 0X, de las abscisas, y la curva (1) $m_w$, (2) $m_p$ *pos*, ó (3) $m_p$ *neg*, representa, respectivamente, por la escala de las ordenadas, á la izquierda, (1) el momento de la carga muerta, $m_w$, (2) el mom máx positivo de la carga viva, $m_p$ *pos*, ó (3) el mom máx negativo de la carga viva, $m_p$ *neg*, en esa sección, estando la carga muerta (1 por unidad de luz) uniformemente distribuída sobre toda la longitud (dos ó tres luces, como se indica) de la viga, y estando el peso vivo (1 por unidad de luz) uniformemente distribuído alternativamente sobre dos porciones del largo de la viga, y siendo dichas porciones, para cada sección transversal, tales, que la carga viva uniformemente distribuída, puesta sobre aquéllas, produzca, alternativamente, el mom pos máx y el mom neg máx en dicha sección.

**136. En una viga real**, en cualquier punto, tenemos, para mom de flexión

$$M = m_w w L^2 + m_p p L^2;$$

donde

$m_w$=la ordenada, en el punto, desde 0X hasta la curva $m_w$;

$m_p$= — — — 0X — — $m_p$ *pos* ó $m_p$ *neg*;

$w$=carga muerta uniforme, por unidad de luz;

$p$= — viva — — — situada como se explica en el § 135.

L=la luz efectiva.

Así, en el punto, *a*, fig. 26 (distante .7 L de 0), tenemos, por la escala, $m_w$=.035; $m_p$ *pos*=.070 $m_p$ *neg*= — .035.

En consecuencia, en el punto *a*,

mom pos máx = $.035\, wL^2 + .070 pL^2$;

mom neg máx = $.035\, wL^2 - .035 pL^2$.

Si $p=w$, el mom neg máx, en $a$, es cero, y no hay momento neg resultante á la izquierda de $a$; pero, si $p=2w$, tendremos $w=p/2=(w+p)/3$; y, en $a$, con $p=2w$:

mom neg máx $= .035\,wL^2 - .035\times 2\,wL^2$;

$= .035\,wL^2 - .070\,wL^2 = - .035\,wL^2$

$= - .035\,(w+p)L^2/3$.

## COLUMNAS EN GENERAL

### Resistencia de las columnas, pilares, postes, de sección transversal uniforme.

Para Columnas de hierro y acero, véanse págs. 1236, etc., y para Columnas de madera, véanse págs. 1190, etc.)

**1. El eje** de una columna es una línea que pasa por los centros de gravedad de todas sus secciones transversales.

**2.** Si la línea de acción de la carga coincide con el eje de la columna en las secciones extremas, se dice que la columna está **cargada según el eje**; de otro modo, **excéntricamente cargada.**

**3.** Bajo una carga en el eje, si éste queda matemáticamente recto, coincide con la línea de presión en todo el largo de la columna, y el esfuerzo está uniformemente distribuído sobre cada sección transversal, como se indica por el paralelogramo, *ab*, fig. 2; pero la excentricidad de la carga, fig. 1 *a*, ó falta de rectitud en el eje, fig. 1 *b*, ó de homogeneidad en el material, hará que el eje diverja de la línea de presión.

**4.** Cuando el eje y la línea de presión dejan de coincidir, en cualquier sección, su divergencia establece, en esa sección, un « par » (véanse §§ 148, pág. 420, etc., y §§ 155, etc., pág. 423) representado por los dos triángulos, *bcd* y *ecf*, fig. 2, § 6 abajo.

**5.** En todos los casos, por tanto, **el diagrama** de los esfuerzos en la sección transversal consiste en el **paralelogramo**, *ab*, modificado ó no (según el caso) por los **dos triángulos**, *bcd* y *ecf*. En la fig. 2, los esfuerzos de las fibras extremas son : $s=af\ (=ae+ef)$ y $gd\ (=gd-bd)$.

**6. En columnas muy cortas**, cargadas en el eje, el paralelogramo está poco (si está algo) afectado por los triángulos; esto es : la presión está casi ó toda uniformemente distribuída sobre la sección transversal; mientras que, en **columnas muy delgadas**, por el contrario, la presión está desigualmente distribuída; y los efectos perturbadores, representados por los dos triángulos, pueden llegar á ser factores principales; de modo que la unidad máxima del esfuerzo del lado cóncavo, puede exceder considerablemente al esfuerzo medio, *ae* ó *gb*, fig. 2; mientras que la unidad del esfuerzo, *gd*, sobre el lado convexo, puede bajar á cero y aun llegar á ser tensión.

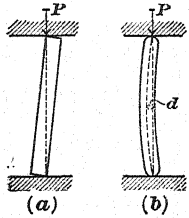



Fig. 1.

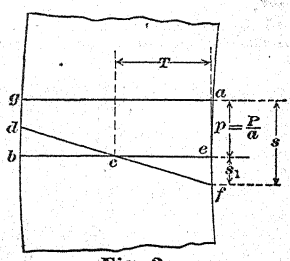



Fig. 2.

**7. En columnas muy delgadas, bajo flexiones moderadas**, los cambios de longitud en las fibras extremas, y por tanto los esfuerzos ejercidos por ellas, son relativamente pequeños. En consecuencia, la columna puede dejar de sostener su carga mucho tiempo antes de que estos esfuerzos lleguen á la última resistencia, ó al límite de elasticidad del material; como cuando una barba de ballena es comprimida por sus extremos entre el pulgar y otro dedo y salta lateralmente.

**8.** Pero, á medida que la flexión, *d*, fig. 1 *b*, aumenta, el momento, $Pd$, del peso

ó carga, P, aumenta también. Los cambios de longitud de las fibras, y por tanto las fuerzas ejercidas por ellas, aumentan también á causa del aumento de curvaturas de modo que es eventual poder impedir el movimiento ulterior de la carga, y si ésta es suficiente, puede la columna romperse ó quedar encorvada.

**9.** Una oblicuidad muy ligera, fig. 1 *a*, ó flexión, fig. 1 *b*, puede reducir su resistencia en un 50 por ciento; y pueden encontrarse diferencias de 10 por ciento ó más, en las cargas, entre dos pilares que, según todas las apariencias, y probados en las mismas condiciones, son perfectamente semejantes. En consecuencia, el modo de conducirse las columnas es mucho menos cierto que el de las piezas en tensión y aun que el de las vigas; y **hay que emplear factores muy liberales de seguridad** al usar fórmulas ó tablas para columnas.

**10.** Esta falta de certeza, con respecto á las resistencias de las columnas, es especialmente lamentable en el uso necesario, tan generalizado, **de las columnas como piezas de compresión en las armaduras de puentes y de techos.** (Véase Puente de Quebec en Columnas de hierro y de acero, pág. 1246.)

**11. El eje neutro** de cualquier sección es una línea, de esa sección, que pasa por su centro de gravedad, y perpendicular al plano en que se supone ó se efectúa la flexión. Una sección transversal puede en consecuencia tener uno ó varios ejes neuts, pues la col puede cimbrarse en uno ó varios planos. De ordinario, sin embargo, cuando una col de material homogéneo, se deja que cimbre libremente en cualquier plano, el plano de flexión se determina por la forma de su sección transversal y es el de su menor radio de giro. Así, si la sección es circular, sea sólida ó hueca, la flexión puede ocurrir igualmente en cualquier plano; pero, en una sec cuadrada, la flexión ocurre en un plano paralelo á uno ú otro lado; en un rectángulo, en un plano paralelo á los lados más cortos, etc., etc.

**12. Cuando el plano de flexión está predeterminado** por algún motivo, como por las espigas ó pernos en la fig. 3 *c*, **la economía de material** requiere que la sección esté de tal modo dispuesta, en lo que se refiere á la espiga, etc., que el **menor radio de giro** (véanse págs. 371 y 372) sea perpendicular al plano de flexión ya determinado. Cuando el plano de flexión no está determinado, la economía de material requiere que la sección esté trazada de tal modo, que los varios radios de giro sean tan próximamente iguales como sea posible.

**13.** Puesto que una construcción está en peligro cuando cualquier porción de ella sostiene una fuerza mayor que el **límite de elasticidad**, ese límite debe tomarse como valor del esfuerzo máximo.

**14. Fatiga.** En las consideraciones siguientes sobre este asunto, suponemos que los pilares sostienen una carga *constante;* y la carga última ó de ruptura á que se hace referencia es aquella que, durante su primera aplicación, dañaría ó rompería el pilar en corto tiempo. Pero los puntales en puentes, etc., tienen á menudo que soportar esfuerzos que varían considerablemente en intensidad de tiempo en tiempo. Su carga máxima ó de ruptura es entonces menor. Véanse págs. 486 y 487.

**15.** La resistencia de una columna está considerablemente afectada por la disposición que se dé á sus extremos; véase fig. 3. Así:

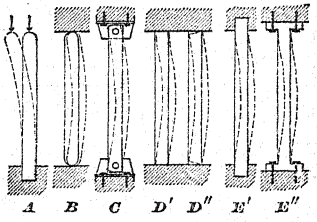



**Fig. 3.**

**A. Extremo libre.** Columna libre para flexarse en cualquier dirección, respectos al otro extremo (fijo).

En los siguientes casos, se supone que los extremos permanecen estacionarios aunque el eje esté más o menos libre para doblarse.

**B. Extremos redondeados.** Eje libre para doblarse en cualquier plano vertical.

**C. Extremos con pernos ó espigas.** Flexión limitada á un plano vertical.

**D. Extremos planos ó cuadrados.** Tangentes al eje de la columna, en los extremos, fijos en posición (véase D') hasta que haya rotación en los extremos, como en D".

**E. Extremos fijos.** Tangentes al eje de la columna, en los extremos, fijos en posición. Los dos extremos de una columna pueden estar diferentemente dispuestos. Así, puede uno estar fijo y el otro redondo, ó uno plano y el otro fijo con pernos ó espigas, etc.

## Cargadas según el eje (carga axil).

**16.** Se desprecia **el peso de la columna misma.**

**17.** Exprésense todas las dimensiones en una sola y misma unidad, como el centímetro, y todas las fuerzas en una sola y misma unidad, como el kg; y sean:

$a$ = área de la sección transversal;
$r$ = radio menor de giro;
$I = ar^2$ = su menor momento de inercia;
T = distancia desde el eje neutro de la sección á la fibra extrema en el lado cóncavo de la col flexada;
L = largo de la parte no apoyada de la col (véase § 41);
$K = \frac{L}{r}$ = coeficiente de delgadez de la col;
$b$ $m$, $c$ = coeficientes, como se explican abajo;
P* = carga de la columna;
$p^* = \frac{P}{a}$ = carga por unidad de superficie en la columna;
$s^*$ = unidad máxima del esfuerzo en la sección transversal;
$s_e$ = límite de elasticidad del material;
E = coeficiente de elasticidad del material;
$e$ = excentricidad de la carga, P; fig. 5;
$d$ = flexión bajo la carga, P; fig. 1$b$
= dist máx de la línea de pres al eje de la col.

**18. Fórmula de Euler.** Véase fig. 4 **. Supongamos que la columna sea ligeramente flexada por una fuerza lateral. Entonces el peso medio $p$, por unidad de superf, que bastará precisamente para tener en equilibrio la columna flexada, es:

$$p = \frac{P}{a} = b.\frac{\pi^2 E}{K^2}; \quad K = \pi\sqrt{\frac{bE}{p}} \ldots\ldots\ldots\ldots\ldots (1)$$

Para extremos redondos, $b=1$; para extremos fijos $=b=4$. Para un extremo redondo y un extremo fijo, $b$ se toma ordinariamente entre 2 y 2¼.

Bajo cualquier carga menor, ó K menor, volverá la columna á las mismas condiciones que cuando estaba derecha. Bajo cualquier peso mayor, la flexión aumentará. Véase § 8.

**19.** Para columnas muy largas, la **fórmula de Euler**, § 18, da resultados que concuerdan bien con la experiencia; pero, para longitudes usuales, da pesos excesivos. Véanse líneas *Ee*, *E'e'*, fig. 4. Para longitudes usuales (si la columna pudiera quedar derecha y cargada según el eje) y hasta para bloques cortos, recomendaríamos, para resistencia máxima, $p = P/a = s_e$ = límite de elasticidad, y el diagrama sería *Jbe* ó *Jb'e'*, fig. 4. Véase § 13.

**20.** Pero, por imperfecciones inevitables, las columnas de longitudes usuales, son susceptibles de flexión lateral, y el diagrama por tanto pasa más abajo de la línea *Jbb'*. Véase « Fórmula parabólica », § 28.

**21. Fórmula de Rankine** *** para columnas de longitudes usuales cargadas según el eje, fig. 2.

Como en el § 17, sean

---

* Bajo cualesquiera condiciones dadas, P y $p$ son la carga total y la carga media por unidad de superficie respectivamente, correspondientes al esfuerzo, $s$, de la fibra extrema bajo esas mismas condiciones. Así, P, $p$ y $s$ pueden corresponder á la carga de ruptura ó de seguridad, ó á cualquiera otra.

** *N. del T.* — Como el autor trae toda esta teoría con el diagrama (fig. 4) y las presiones, coeficientes, etc., en sistema inglés, hemos tenido que transformarlo todo al sistema métrico.

*** W. J. M. Rankine, « Ingeniería Civil », pág. 523. « La fórmula de Gordon », atribuída al prof. Lewis Gordon, de Glasgow, usa el *diámetro* menor, en vez del menor *radio de giro*, de la sección transversal.

**p**=carga media por unidad de superficie en la columna;
**s**=unidad máx del esfuerzo en la sección transversal;
**K**=L/*r*=longitud parte no apoyada ÷ radio menor de giro;
**m**=un coeficiente.

**22.** En una columna cargada según el eje (§ 2), mientras no está flexada, tenemos:

$$s=p=\mathrm{P}/a;$$

**23.** Pero, cuando la columna se *dobla*, obra por ese hecho un esfuerzo *adicional*, $s_1$, en su lado cóncavo (véase § 6), y tenemos, para el esfuerzo total por unidad de sup, *s*, en dicho lado :

$$s=p+s_1=(\mathrm{P}/a)+s_1$$

**24.** El momento de flexión, debido á la flexión, *d*, fig. 1 *b*, de la columna, $\mathrm{M}=\mathrm{P}d$.

**25.** Como en las vigas, págs. 489 y 490, tenemos, para equilibrio entre el momento de flexión, M, y el de resistencia, R,

$$\mathrm{M}=\mathrm{R};\quad \text{ó } \mathrm{P}d=\frac{s_1}{\mathrm{T}}\,\mathrm{I}.$$

En consecuencia,

$$s_1=\frac{\mathrm{MT}}{\mathrm{I}}=\frac{\mathrm{P}d\mathrm{T}}{ar^2}=p\,\frac{d\mathrm{T}}{r^2}$$

$$s=p+s_1=p+p\,\frac{d\mathrm{T}}{r^2}=p\left(1+\frac{d\mathrm{T}}{r^2}\right).$$

**26.** En una viga, pág. 504, para esfuerzo de una fibra dada, la flexión, *d*, es proporcional al cociente $\mathrm{L}^2/\mathrm{T}$. Suponiendo que esto es también verdad respecto á las columnas, tenemos :

$$d=m\ (\mathrm{L}^2/\mathrm{T}).$$

En consecuencia,

$$s_1=p\,.\,\frac{d\mathrm{T}}{r^2}=p\,.\,m\,\frac{\mathrm{L}^2}{\mathrm{T}}\,.\,\frac{\mathrm{T}}{r^2}=pm\mathrm{K}^2;\ \text{y}$$

$$s=p\ (1+m\mathrm{K}^2) \qquad (2)$$

Por tanto, la **fórmula de Rankine** ·

$$p=\frac{\mathrm{P}}{a}=\frac{s}{1+m\mathrm{K}^2}=s\,\frac{1}{1+m\mathrm{K}^2} \qquad (3)$$

$$\mathrm{K}=\sqrt{\frac{\frac{s}{p}-1}{m}} \qquad (3a)$$

**27.** Ritter * da $m=s_e/(b\pi^2\mathrm{E})$; y Crehore ** da $m=s/(b\pi^2\mathrm{E})$. Para valores de *b*, correspondientes á diferentes disposiciones de los extremos de la columna, véase § 18 §.

(*Obs. del T.* — Para valores de *s* y de $1/m$, usados en la práctica, en columnas de hierro y acero, véase pág. 1242. Allí están los valores de *s* en libras por pulg cuad, y debajo de cada uno, entre paréntesis, su equivalente en kg por cm cuad (véase *Obs. del T.*, pág. 1241, § 19). Para obtener el valor de $1/m$, divídase simplemente á la unidad por el correspondiente valor de B en la misma pág. 1241, etc. Para las columnas de madera, véanse fórmulas y tablas prácticas, págs. 1190, etc.)

**28. Fórmula parabólica. J. B. Johnson** ‖ .

$s_e$=límite de elasticidad del material; **q**=un coeficiente.

$$\text{Resistencia máx en kg por cm cuad}=p=\frac{\mathrm{P}}{a}=s_e-\frac{s_e^2}{q\pi^2\mathrm{E}}\,\mathrm{K}^2 \qquad (4)$$

$$\mathrm{K}=\sqrt{\frac{(s_e-p)\pi^2\mathrm{E}q}{s_e^2}} \qquad (4a)$$

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* Construcciones de Techos y Puentes, 1873.
** Van Nostrand's Magazine, 1879.
Véase *N. del T.*, § 18.
‖ Estructuras armadas modernas, Nueva York, John Wiley and Sons, 1873, pág. 150.

Para extremos de espiga ó articulados $q=6.4$; para extremos chatos $=10$. Para columnas de madera, véanse fórmulas y tablas prácticas, págs. 1190, etc.

**29 *. Fórmula (línea recta). Thos. H. Johnson**.** En vista de la sencillez, propone el Sr. Thos. H. Johnson, para columnas de longitudes prácticas, la fórmula siguiente :

$$\text{Resistencia máx en kg por cm cuad} = p = P/a = s_s - Kc \quad \ldots\ldots (5)$$

$$K = (s_s - p)/c \quad \ldots\ldots (5a)$$

**Valores de $s_s$ y de c, para columnas de hierro y acero. Cargas de ruptura** (kg/cm cuad).

| | | Extremos | | | | | |
|---|---|---|---|---|---|---|---|
| | | chatos. | | espigados ó articulados. | | redondeados. | |
| | $s_s$ | $c$ | K | $c$ | K | $c$ | K |
| Hierro forjado..... | 2,950 | 12.96 | 218 | 10.99 | 178 | 14.21 | 138 |
| Acero blando; carbón=.12%. | 3,700 | 12.43 | 195 | 15.40 | 159 | 19.88 | 123 |
| — duro; — =.36%.. | 5,600 | 23.59 | 158 | 28.98 | 129 | 37.38 | 100 |
| Hierro colado..... | 5,600 | 30.66 | 122 | 37.59 | 99 | 48.51 | 7 |

(Para columnas de madera, véanse fórmulas y tablas prácticas, págs. 1190, etc.)

El error, debido al uso de esta fórmula, es probablemente mucho menor que los inevitables provenientes de la naturaleza de las columnas.

**30.** La fig. 4 muestra una **comparación de los resultados de experimentos y fórmulas para columnas de acero soldable, de extremos redondos**, como sigue : Para la fórmula de Euler, $p$ es la carga, en kg/cm cuad, que sostendrá, ligeramente cimbrada, una columna de los E y K dados. **Para las otras fórmulas** y para los experimentos de Tetmajer, $p$ es el peso *flexor.* en kg/cm cuad.

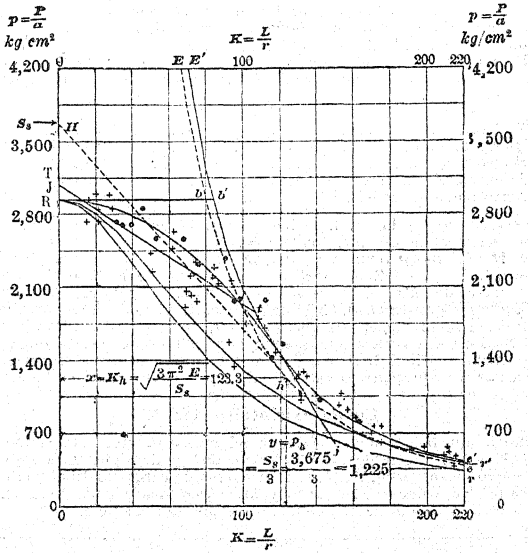


**Fig. 4.**

* Véase *N. del T.*, § 18.

** Trans. Am. Soc. Civ. Engrs, julio, 1886, vol. 15, pág. 547.

## Fórmulas.

(*Obs. del T.* — Al diagrama anterior y á las fórmulas les hemos hecho las modificaciones necesarias para adaptarlas al sistema métrico.)

| | |
|---|---|
| *Ee.* Euler, $p=\pi^2 E/K^2$; | $E=1{,}890{,}000$ kg por cm cuad; |
| *E'e'.* Euler, $p=\pi^2 E/K^2$; | $E=2{,}100{,}000$ kg por cm cuad; |
| *Hh.* T. H. Johnson, línea recta, | $p=3{,}690-19.88\,K$; |
| *Tt.* Tetmajer, línea recta, | $p=3{,}080-11.34\,K$ *; |
| *Jj.* J. B. Johnson, parabólica, | $p=2{,}953-.105\,K^2$ ** |
| *Rr.* Rankine, | $s=2{,}952;\ 1/m=6{,}000$; |
| *Rr'.* Rankine, | $s=2{,}953;\ 1/m=8{,}000$. |

## Ensayos.

(De Tetmajer *: dimensiones aproximadas.)

16 barras redonds, de $^3/_4$ á $1\ ^3/_4$ pulgadas (19 á 44 mm) de diámetro.

102 experimentos, que comprenden ángulos, $4\times4$ pulgs ($102\times102$ mm) formas en **T** $5\times2\frac{1}{2}$ pulgs ($127\times63$ mm), canales, $5\frac{1}{2}\times2\ ^1/_4$ pulgs ($140\times57$ mm), vigas I, $7\times3\ ^3/_4$ pulgs ($178\times82$ mm), columnas remachadas como sigue : — 2 ángulos, $3\ ^1/_4$ pulgs (82 mm); 4 ángulos, $2\frac{1}{2}$ pulgs (57 mm); 2 formas en **T**, $3\frac{1}{2}\times1\ ^3/_4$ de pulg ($89\times44$ mm); 2 canales, $3\ ^1/_4\times1\frac{1}{2}$ pulgs ($82\times38$ mm). Cada punto representa el término medio de dos experimentos.

**31. T. H. Johnson** encuentra que, **para todas las tangentes á la curva de Euler**, las coordenadas del punto (como *h*, fig. 4) de tangencia son :

$$\text{Abscisa, } x=K=\sqrt{3zE/s_s}; \qquad \text{Ordenada, } y=ps=s_s/3;$$

donde $z=\pi^2=9.87$; para extremos redondos; $z=(5/3)\pi^2=16.45$, para extremos con goznes, y $z=(5/2)\pi^2=24.67$, para extremos planos; y $s_s$=la ordenada de ese punto, H, donde la tangente encuentra el eje de las ordenadas.

El Sr. Johnson toma $E=1{,}890{,}000$ kg por cm cuad, y $s_s=3{,}690$.

En consecuencia, en la fig. 4, $x=123.3$; $y=1{,}230$.

**32. Tetmajer** da los siguientes valores, $x'$, para la abscisa del punto donde su línea recta encuentra la curva de Euler (E, está dado en kg/cm cuad) :

| | E | $x'$ |
|---|---|---|
| Madera | 98,420 | 100 |
| Hierro fundido | 984,200 | 80 |
| Hierro forjado | 1,968,400 | 112 |
| Acero blando de construcción | 2,100,000 | 105 |
| Acero medio de construcción | 2,249,000 | 105 |

En consecuencia, en la fig. 4, su línea recta, *Tt*, encuentra la curva de Euler, *E'e'* (no tangencialmente) en *t*, donde $K=105$.

**33. J. B. Johnson** toma $E=2{,}100{,}000$ kg por cm cuad; y su curva, *Jj*, es tangente á la curva de Euler, *E'e'*, en el punto donde $K=120$. Para columnas de acero suave con extremos de espigas, él limita el valor de K á un máx de 150.

**34. Fórmula gradual. J. R. Worcester.** En vista de la amplia divergencia encontrada en la resistencia de las columnas, por las varias fórmulas y coeficientes en uso, y de la variación igualmente amplia en los resultados experimentales, considera inútil el Sr. J. R. Worcester ‖ darle á *p* un valor diferente para cada valor de K; y propone un valor *fijo* de *p* para cierta *serie* de valores de K.

## Carga excéntrica. Fig. 5.

**35.** Hasta aquí hemos supuesto que la columna está axilmente cargada (según el eje) (véase § 2) y, considerando la desigualdad de distribución de los esfuerzos en una sección transversal, fig. 2, con su exceso de carga resultante en las fibras extremas del lado cóncavo, hemos tomado en cuenta solamente aquel exceso, $s_1$, que se debe á la *flexión* de una columna cargada según su eje.

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* Comunicaciones del Instituto de Examen de Materiales á la Politécnica Suiza en Zurich. Entrega VIII, 1896.

** $0.105=s^2/\pi^2bE=2{,}953^2/(9.87\times2{,}100{,}000\ b)$. Para extremos redondos, $b=4$.

‖ Trans. Am. Soc. Civ. Engrs., vol. 54, p. 417, junio 1905.

**36.** Pero muy comúnmente sucede que las columnas están (accidental ó intencionalmente) cargadas excéntricamente; y esta excentricidad, *e*, trae sobre las fibras extremas del lado más próximo á la línea de presión, un exceso *adicional* de la unidad del esfuerzo, fig. 5. Si la columna quedase perfectamente recta, tendríamos (véanse págs. 489-90) $s_2 = TM/I = TPe/ar^2 = (P/a)(Te/r^2) = pTe/r^2$; y en consecuencia, en una columna cimbrada (véase ecuación (2), donde $s_1 = pmK^2$),

$$s = p + s_1 + s_2 = p\left(1 + mK^2 + \frac{Te}{r^2}\right) \qquad (6)$$

$$p = \frac{P}{a} = \frac{s}{1 + mK^2 + \frac{Te}{r^2}} \qquad (7)$$

$$K = \sqrt{\frac{\frac{s}{p} - 1 - \frac{Te}{r^2}}{m}} \qquad (7a)$$

donde, como en el § 17 :

**p** = carga media de la columna;
**e** = excentricidad, fig. 5;
**s** = unidad máxima del esfuerzo en la sección transversal;
**K** = $L/r$ = longitud entre soportes ÷ radio menor de giro;
**r** = dist del eje neutro de la sección hasta la fibra extrema en el lado cóncavo,

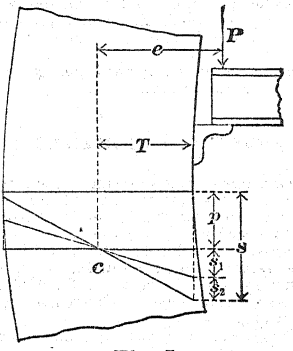


**Fig. 5.**

**37.** Como asunto de hecho, la flexión de la columna, establecida en el término, $mK^2$ (Ecuaciones 6 y 7), aumenta ligeramente la excentricidad, *e*, del peso (véase fig. 5) y esto se desprecia en el término $Te/r^2$. En consecuencia, el esfuerzo de la fibra, *s*, dada por la ecuación (6) es un poco pequeño, y el peso, *p*, dado por la ecuación (7), es un poco grande; pero la discrepancia de ordinario se desprecia en la práctica.

**38.** Para columnas de extremo redondo, da **J. M. Moncrieff** ★:

$$p = \frac{P}{a} = \frac{s}{1 + \frac{Te}{r^2}\left(\frac{48E + pK^2}{48E - 5pK^2}\right)} \qquad (8)$$

$$K = \sqrt{\frac{48E}{5s + p(Te/r^2 - 5)}\left(\frac{s}{p} - 1 - \frac{Te}{r^2}\right)} \qquad (8a)$$

★ Trans. Am. Soc. Civ. Engrs., vol. 45, pág. 349, junio 1901.

**39.** Sean

M = P*e*, fig. 5 = momento de un peso excéntrico, P, respecto de *c*;
I = momento de inercia de la sección transversal;
T = distancia del eje neutro á la fibra más distante;
X = I/T = módulo de sección transversal en la dirección de la flexión;
$s_2$ = esfuerzo máximo de la fibra debido solamente á la excentricidad.

Entonces, $s_2 = MT/I = M/X$ ..... (9)

**40.** Con columnas que se suponen axilmente cargadas, la carga es, en realidad y muy comúnmente, más ó menos excéntrica, y el tamaño de la excentricidad se desconoce; pero cuando una carga se aplica en un punto del borde de la columna ó más allá de él, como en la fig. 5, la excentricidad es tan grande, que ocurren incertidumbres, inevitables en cuanto á su exacto valor, y éste se hace relativamente despreciable; las Ecuaciones (7 y (7 *a*) pueden usarse con más ó menos confianza.

**41.** Si una columna muy larga fuere de tal modo **reforzada á intervalos** que se impida su flexión en esos puntos, entonces su longitud disminuye virtualmente, y se aumenta su fuerza. Así, si una columna de 30 m de largo fuere suficientemente reforzada á intervalos de 10 m, entonces puede sostener la carga de una columna de sólo 10 m de largo.

### Advertencias.

**42.** Las columnas de hierro fundido están sujetas á tener huecos y otros defectos ocultos y se rompen fácilmente por golpes laterales. Las columnas de todas clases están sujetas á trepidaciones y vibraciones provenientes de cargas movibles. Muy raras veces sucede que la presión esté igualmente distribuída sobre toda el área del pilar; ó que los extremos estén igualmente apoyados y sostenidos en todas sus partes.

**43.** En ensayos de columnas pueden lograrse por excepción buenos resultados cuando sus condiciones se acercan al ideal.

## ESFUERZO CORTANTE

**Esfuerzo cortante ocurre** cuando sobre un cuerpo obran dos fuerzas opuestas en planos paralelos y adyacentes, que tienden á hacer deslizar unas partículas sobre las otras. En la fig. 1, las dos fuerzas son (1) la presión descendente del peso, W, y (2) la reacción ascendente del soporte A.

En el **esfuerzo cortante simple**, fig. 1, el área del esfuerzo cortante, $a$ = á la sección *gg*. En el **esfuerzo cortante doble**, fig. 2, $a = gg + oo = 2 \times gg$. En la fig. 3, $a$ = 6 × sección transversal de la pieza. En la fig. 4 (esfuerzo cortante simple), $a$ = sección *cc*. Al horadar agujeros para remaches, $a$ = circunferencia del agujero × espesor de la plancha.

En cualquier caso, si S = al esfuerzo cortante por unidad de superficie, el esfuerzo cortante total será = S*a*.

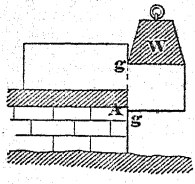

Fig. 1

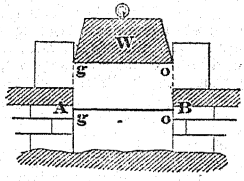

Fig. 2

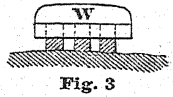

Fig. 3

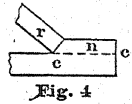

Fig. 4

**El esfuerzo cortante, S,** por unidad de superficie, en kg por cm cuadrado, en los metales y en las maderas, está indicado por las cifras siguientes:

Metales: Hierro forjado, 2,500 á 3,800; hierro fundido, 1,400 á 2,100; acero, 3,200 á 5,300; cobre, 2,300.

| | | Según la fibra, fig. 4. | A través de la fibra. |
|---|---|---|---|
| Maderas: Según nuestras experiencias: | Abeto | 17 á 35 | 228 |
| | Pino blanco | » | 175 |
| | Abeto del Canadá | » | » |
| | Pino amarillo | ..... | 300 á 400 |
| | Roble | 30 á 50 | ..... |
| | Roble blanco | ..... | 300 |

## TORSIÓN

**La torsión** ocurre cuando sobre un cuerpo obran dos pares ó momentos de sentido contrario y en planos diferentes. Por ejemplo: en el eje de un freno cuando lo hacemos girar mientras su extremo inferior está sujeto por la cadena del freno; y en los ejes transmisores de fuerza motriz cuando transmiten ésta de la máquina á los aparatos. Supóngase que el cuerpo sometido á torsión se divida en capas, por medio de secciones transversales. Entonces cada capa tiende á ejercer un esfuerzo cortante desde la próxima capa. En consecuencia, para mantener el equilibrio, cada dos capas adyacentes deben ejercer, en la sección transversal entre ellas, un momento de resistencia interno igual á uno de los dos momentos de torsión externos y contrarios.

**Momento de resistencia,** en una sección transversal circular de un árbol cilíndrico. Sean:

$P$ = la fuerza de torsión de uno de los dos momentos externos;
$l$ = su palanca = su distancia al eje del árbol;
$M = Pl$ = momento externo ó de torsión;
$T$ = distancia del eje á las fibras más distantes = radio del árbol;
$D$ = diámetro del árbol, $= 2T$;
$S$ = esfuerzo cortante por unidad de superficie en las fibras más remotas;
$t$ = distancia del eje á cualquier fibra dada;
$s$ = esfuerzo cortante por unidad de superficie en dicha fibra dada;
$a$ = área de dicha fibra dada;
$F$ = esfuerzo cort total en dicha fibra;
$r$ = momento de resistencia de dicha fibra respecto al eje;
$R$ = momento interno de resistencia de la sección transversal entera
$= \Sigma r$ = suma de los momentos de resistencia de todas las fibras;
$I_p$ = momento polar de inercia* de la sección transversal;
= momento de inercia de la sección transversal respecto del eje del árbol;
$= \Sigma t^2 a$, = la suma, para todas las fibras, de $t^2 a$.

(*Obs. del T.* — Hay que tener mucho cuidado en la elección de estas unidades para que resulte la homogeneidad de las fórmulas al aplicarlas. Por ejemplo, si P son kilogramos y $l$ y $D$ son centímetros, han de calcularse los esfuerzos S, $s$ en kg por cm cuad y las áreas, como $a$, en cm cuad. Es claro que M vendrá en kg-centímetro y el momento de inercia I será una unidad de cuatro dimensiones, cuya base es el centímetro. Estas unidades serán las que adoptaremos en los ejemplos que van después. Todos los que trae el autor en medidas inglesas los hemos transformado para adaptarlos al sistema métrico.)

Entonces el esfuerzo por unidad de superficie en cualquier fibra dada, es $s = St \div T$; su esfuerzo total, F, es $= as = Sat \div T$; y su momento de resistencia, $r$, es $= Ft = Sat^2 \div T$. Para el equilibrio, el momento interno de resistencia, R, de la sección entera, debe ser = al momento externo de torsión, M.

En consecuencia, para el **momento interno de resistencia,** R, tenemos

$$R = M = \Sigma r = \Sigma Sat^2 \div T = (S \div T)\, \Sigma t^2 a = (S \div T)\, I_p.$$

De donde, también, $S = MT \div I_p$; $M = SI_p \div T$; y $P = M \div l = SI_p \div (Tl)$.

**En un círculo sólido,** $I_p = \pi T^4 \div 2$. Por tanto, $S = 2MT \div (\pi T^4) = 2M \div (\pi T^3)$; $M = S\pi T^3 \div 2$; $P = S\pi T^3 \div (2l)$; y $T = \sqrt[3]{\frac{2lP}{\pi S}}$; de donde

---

* En cualquier figura, el momento polar de inercia, $I_p$, es = á la suma de los momentos de inercia más grandes y más pequeños de la misma figura, respecto á dos ejes que estén en la figura y que se intersecten en su centro. En un círculo sólido, cada uno de éstos es un momento de inercia respecto de un diámetro, y es $= \pi T^4 \div 4$. Por tanto, dicho círculo, $I_p = \pi T^4 \div 2$.

**Diámetro,** $D, = 2\,T = 2 \times \sqrt[3]{\frac{2Pl}{\pi S}} = \sqrt[3]{\frac{5.1\,M}{S}} = 1.72 \sqrt[3]{\frac{M}{S}}$.

(*Obs. del T.* — Pongamos un ejemplo: supongamos el momento M=86,400 kg-cm y $s$=422 kg por cm cuad; tendremos $1.72 \sqrt[3]{\frac{86,400}{422}} = 1.72 \times 5.85 = 10.06$, digamos 10 cm.)

**Para valores aproximados de S, para la torsión,** úsense los valores para el esfuerzo cortante, pág. 528*f*, con **factores de seguridad** de 5 á 10.

**Fuerza en caballos de un eje de transmisión.** En una revolución, la fuerza, P kg, que describe una circunferencia con radio=$l$ cm, efectúa un trabajo=$2\pi lP$ kg-cm, y, en $n$ revoluciones, trabajo=$2\pi lPn$ kg-cm. Si $n$ es el número de revoluciones *por minuto,* el caballo de fuerza es:

$H = 2\pi lPn \div (100 \times 75 \times 60)$ * $= 2\pi Mn \div (100 \times 75 \times 60) = \pi Mn \div (50 \times 75 \times 60)$ ó puesto que $Pl = M = R = SI_p \div T$, tenemos:

$H = S\pi nI_p \div (50 \times 75 \times 60\ T)$; de donde $S = 50 \times 75 \times 60\ TH \div (\pi nI_p)$.

**En un árbol cilíndrico sólido** (véase nota pág. 528*g*): $I_p = \pi T^4 \div 2$. Por tanto, $H = S\pi n\pi T^4 \div (100 \times 75 \times 60\ T) = S\pi^2 nT^3 \div (100 \times 75 \times 60)$; de donde

$S = H\,(100 \times 75 \times 60) \div \pi^2 nT^3 = 100 \times 456.4\ H \div nT^3$;

$n = 45,640H \div S\,\frac{D^3}{8} = 365,120\ H \div D^3S$; y

$$D = \sqrt[3]{\frac{365,120\ H}{Sn}} = 71 \sqrt[3]{\frac{H}{Sn}}.$$

**Mientras mayor es la velocidad, menor es la fuerza,** y por tanto menor es la fuerza del eje de transmisión para transmitir un *caballo;* pero si se aumenta la velocidad aumentando la fuerza de torsión, se aumenta así la fuerza transmitida.

**Ejemplo.** Dado un árbol de hierro forjado; sea S=422 kg por cm cuad; P=3,456 k; $l$=25 cm; M será=86,400 kg-cm. Tendremos para el diámetro (véase ejemplo del traductor al comienzo de esta página) D=10.06 cm.

Supongamos que la potencia en caballos sea H=25. Entonces $n = 365,120 \times 25 \div (D^3S) = 9,128,000 \div (10.06^3 \times 422) =$ digamos 21.

Comprobemos ahora: $D = 71 = \sqrt[3]{H \div Sn} = 71 \times \sqrt[3]{25 \div (422 \times 21)} = 71 \times .14 =$ digamos 10 cm.

**Secciones rectangulares.** Las ecuaciones que preceden se basan en el supuesto de que la fuerza aumenta uniformemente desde el eje del árbol hacia afuera. Se ha demostrado (especialmente por St. Venant) ** que esta hipótesis no es aplicable á secciones cuadradas y rectangulares. En un rectángulo, sea B=lado más largo, $b$=más corto, y $c = b \div B$. Entonces $S = M\,(3 + 1.8\ c) \div (Bb^2)$; y $P = SBb^2 \div [(3 + 1.8\ c)l]$. En un cuadrado, de lado=$b$, resulta $S = 4.8\ M \div b^3$; $M = Sb^3 \div 4.8$. $P = Sb^3 \div (4.8\ l)$.

**El ángulo de torsión** es el descrito por uno de los momentos externos de torsión respecto al otro. Dentro del límite de elasticidad este ángulo es proporcional al momento de torsión, M, y (suponiendo constante á $l$) proporcional á la fuerza, P. En igualdad de las otras circunstancias, el ángulo es proporcional á la distancia entre los planos de los dos momentos opuestos externos, y, en un árbol sólido cilíndrico, es inversamente proporcional á $D^4$. No es bueno que el ángulo de torsión exceda de 1° en una longitud=á 20 diámetros, en árboles que giran en una sola dirección. En árboles alternativos concédase aún menos. Véase Fatiga, pág. 486, etc.

**Consideraciones prácticas.** En muchos casos el diámetro del árbol debe hacerse más grande que el requerido por las fórmulas que preceden; como, por ejemplo, en un árbol largo, á fin de mantener el ángulo de torsión dentro de los límites permitidos, en las ruedas volantes y otros árboles que llevan considerables pesos de flexión además de las fuerzas de torsión; y, en los más de los casos, para prever los momentos adicionales debidos á alternativas aceleraciones y retardaciones.

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* *N. del T.* — *Como* $2\pi lPn$ son kg-cm *por minuto,* para tener kg-metro, se divide por 100; ahora, como un caballo = 75 kgm por seg, se divide aún por 75 y por 60.

** Véase Tratado de Física, por Sir William Thomson y Peter Guthrie, parte II, nueva edición, Cambridge, 1890, págs. 236, etc.

# HIDROSTÁTICA

**Art. 1. La hidrostática trata de** la presión del agua y de otros líquidos en reposo.

En cualquier punto dado dentro de un fluido **la presión es igual en todas direcciones**, y la presión contra cualquier punto de cualquier superficie, plana ó curva, es **perpendicular á la superficie comprimida** (ó á un plano tangente á esa superficie) en ese punto.

**La intensidad de la presión** es proporcional á la profundidad del punto bajo la superficie del agua.

**Presión contra cualquier superficie plana.**

Sean

$a$=el área de la superficie comprimida;
$h$=la profundidad vertical del centro de gravedad de la superficie comprimida por debajo de la superficie libre del fluido;
H=la profundidad total del fluido;
$w$=el peso de una unidad de volumen del fluido *;
$p$=la presión media por unidad de la superficie comprimida;
P=la presión total sobre la superficie comprimida.

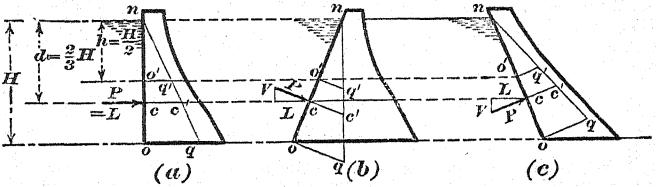


**Fig. 1.**

Entonces la **unidad media de presión**, $p$, es igual al peso de un prisma del fluido, cuya base es=1, y cuyo largo es=$h$; ó

$$p=hw;$$

y la **presión total**, P, es igual al peso de un prisma del fluido, cuya base es=$a$ y cuyo largo es=$h$. O bien

$$P=ahw=ap.$$

**En los diagramas** de las figs. 1 y 2, las *ordenadas* (que se suponen trazadas desde las superficies comprimidas y perpendicularmente á ellas, respectivamente) representan las *unidades* de presión (como en lbs por pulgada cuad, kilogramos por centímetro cuad, etc.) y el *área*, enfrente de cualquier superficie dada, representa la presión *total* sobre esa superficie. Así, en la fig. 1 (*a*), las unidades de presión en *n*, en *o'*, en *c* y en *o* están representadas por *n* (=0), por *o'q'*, por *cc'* y por *oq* respectivamente; y la presión total sobre *no* está representada por el área *no'q*, la sobre *no'* por el área *no'q'*, la sobre *o'o* por el área *oq'*, y la sobre *o'c* por el área *cq'*.

**El centro de presión** sobre cualquier superficie está enfrente del centro de gravedad del área que representa la presión total sobre esa superficie. Así, en las figs. 1, el centro de presión sobre *no*, está enfrente del centro de gravedad del triángulo, *noq*, ó á una profundidad, $d$, $=\frac{2}{3}$H, por debajo de la superficie de agua. Véase « El Centro de Presión », arts. 8, etc., y también §§ 133, etc., de « Estática ».

* Para agua, $w$ = cosa de 62.5 lbs por pie cúbico = 1 kg por decímetro cúbico.

**La paradoja hidrostática.** Para una profundidad dada, *h*, tanto la unidad de presión media, *p*, como (para un área dada *a*) la presión, P, son independientes de la cantidad de agua. Así, en las figs. 1, las paredes sostienen una presión tan grande, ya provenga de una lámina vertical de agua de sólo un cm de espesor ó de una masa de agua que se extendiese kilómetros detrás del muro.

En la fig. 2 (*b*), el exceso de peso de agua sobre el de la fig. 2 (*a*), es soportado por la pared inferior inclinada de la vasija. Las presiones totales, P, sobre las bases iguales, *ab* y *a'b'*, figs. 2 (*c*) y (*d*), son iguales. En la fig. 2 (*c*), la presión total sobre la base es mayor, y en la fig. 2 (*d*), menor que el peso del agua; pero, en uno ú otro caso, la suma algebraica de todas las presiones verticales en la vasija (tomadas como negativas las presiones ascendentes) es=al peso de todo el agua de la vasija, y la suma algebraica de todas las presiones horizontales es=0.

Así, supongamos que la parte inferior de la fig. 2 (*c*) representa una caja cúbica, de 2.40 m por lado, llena de agua. Ahora, llénese de agua el tubo *no*, de 11 m de alto y de 7 mm de diámetro. El agua en el tubo solo, aunque pesa solamente cosa de 450 gramos, causará una presión adicional (que tiende á reventar la vasija) de 1.1 kg por cm cuad, ó digamos 384 toneladas totales, que se ejercerá sobre la parte superior, el fondo y los lados de la caja.

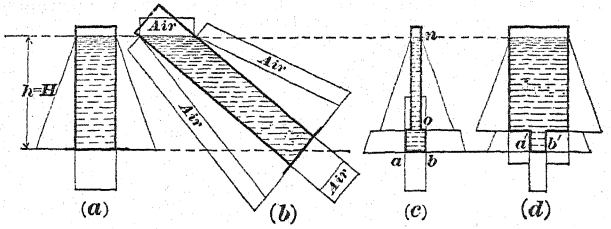


**Fig. 2.**

**Presión del aire en la superficie del agua.** Además de la presión del agua, la superficie libre de cualquier masa de agua sostiene también la presión del aire, =cosa de 1 kg/cm cuad. Esta presión (transmitida por el agua á las paredes de la vasija) está indicada por los paralelogramos marcados « aire » en la fig. 2 (*b*). En los más de los casos, la presión debida al aire contra una superficie de agua, es contrarrestada por una presión igual del aire en la dirección opuesta, como contra los lados externos de las paredes de las vasijas en las figs. 2, y contra los lados de las presas en las figs. 1.

Estrictamente hablando, la presión del aire, ejercida directamente contra las paredes, figs. 2, ó contra las presas, figs. 1, siendo ejercida prácticamente en los centros de gravedad de esas superficies, y por tanto á más profundidad que la presión opuesta debida al aire sobre la superficie libre del agua, es muy poco mayor que la última. En una presa de 30 m de profundidad, esta diferencia es más ó menos de .0018 kg **por** cm cuad=.0018 atmósfera.

**Refuerzos para presas.** Siendo la presión del agua perpendicular á la superficie comprimida, los postes, fig. 3, deben también ser perpendiculares á la super-

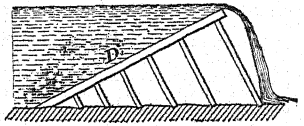


**Fig. 3.**

ficie D, para que reciban la presión longitudinalmente y evitar así momentos de flexión; pero otras consideraciones pueden prohibir que se coloquen de esa manera. Así, si la cara, D, de la presa, es casi vertical, los postes tendrían que hacerse

extraordinariamente largos; y por consiguiente muy débiles, como pilares, á no ser que se hagan exageradamente gruesos. Así puede quedar más que compensada la ventaja debida á la acción longitudinal de la presión. Además, los pies de los postes más largos se proyectarían más allá del borde de la presa y podrían dañarse con el hielo, troncos, etc., que cayeran por el vertedero de la presa. Como la presión aumenta uniformemente desde la superficie del agua hacia abajo, los postes se colocan de ordinario más próximos cerca del fondo que cerca del tope, aunque lo corto de los más bajos los hace más fuertes como pilares. De modo semejante, los aros de las tinas, si son de la misma fuerza, se colocan más próximos unos á otros cerca del fondo.

**Componentes horizontales y verticales.** En las figs. 1 (*b*) y (*c*) el triángulo de fuerza (Estática, §§ 46, etc.) nos da los componentes horizontales y verticales, L y V, de la presión perpendicular total, P. O bien, si *n o* se toma, en cada caso, para representar la presión perpendicular total, P, por escala, entonces H=la presión horizontal total. En la fig. 1 (*a*), tratándose de una presión contra una superficie vertical, L=P, y V=0.

En la fig. (*b*), la componente vertical, V, comprime la pared hacia abajo contra su base; pero en la fig. (*c*) tiende á levantarla y volcarla.

Siendo la profundidad, H, la misma en cada una de las tres figuras, figs. 1 *a*, *b* y *c*, las proyecciones *verticales* de las tres superficies sumergidas son iguales, y en consecuencia las presiones *horizontales* totales son iguales en los tres casos; pero la proyección *horizontal* y en consecuencia la presión *vertical* total, varían con la inclinación de la superficie. Así, en la fig. 1 (*a*), la proyección horizontal y la presión vertical son cada una=0.

**Presiones en vasijas cúbicas de otras formas, llenas de agua.** Sean F=el peso de agua contenida en una vasija prismática; y $f = \frac{F}{3}$ = el peso de la que contiene una vasija cónica ó piramidal de las mismas base y altura.

En una vasija cúbica, la presión sobre la base=F; presión sobre un lado $= \frac{F}{2}$;

presión sobre la base y cuatro lados juntos $= F + 4\,\frac{F}{2} = 3F$.

En una vasija cónica ó piramidal, presión sobre la base $= 3f = F$.

En una vasija esférica, presión total = 3 × peso del agua.

**Art. 2. Presión desigual en direcciones opuestas.** En las figs. 4, represente *n o* el lado de un paralelogramo, cuyo lado superior coincide con la superficie del agua, y cuyo fondo sea paralelo á esa superficie; sea *a*=el área de la porción,

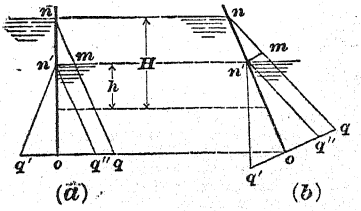


Fig. 4.

*n'o*, que está sometida á presión en ambos lados; y sean H y *h*=las profundidades verticales del centro de gravedad de *n'o* por debajo de las dos superficies de agua *n*, y *n'*, respectivamente. Entonces, en cada fig., el triángulo grande, *noq*, representa la suma de las presiones del agua más honda á la izquierda contra todo el muro, *no;* el trapezoide *mo*, representa la suma=*a*H*w* de las presiones del agua á la izquierda contra *n'o;* y el triángulo más pequeño, *n'oq'*=*n'oq''*, representa la suma, =*ahw*, de las presiones del agua más baja á la derecha contra la porción, *n'o*. Entonces el paralelogramo, *mq''*, representa el *exceso* de presión, del lado izquierdo, contra la porción, *n'o*. Este exceso, debido á la diferencia, H — *h*, entre los dos niveles, está uniformemente distribuído sobre *n'o*, estando representada la *unidad* de exceso de presión uniforme por la ordenada *n'm*=*q''q*. El exceso *total* de pre-

sión, representado por el paralelogramo, $mq'' = aHw - ahw = (H - h)\ aw$, es por tanto proporcional á $(H - h)$.

La presión procedente de la derecha contra la porción *n'o*, y representada por el triángulo *n oq'* está equilibrada por una porción igual (representada por el triángulo *n'oq''*) de la presión total, *mo*, desde la izquierda contra la porción *n'o;* y los centros de estas dos presiones, estando cada uno á una profundidad$=\frac{1}{3}$ *n'o* por debajo de *n* , son opuestos y estas dos presiones están en equilibrio. Pero el centro del exceso de presión, *n'q*, de la izquierda, está frente al centro de gravedad del paralelogramo, *n'q*, ó en el centro de gravedad de *n'o*. En consecuencia, la porción *n'o*, considerada independientemente de *n'n*, es impulsada por la fuerza no contrarrestada *n'q*, procedente de la izquierda, obrando por su centro de gravedad y tendiendo á moverla hacia la derecha, sin rotación.

Esto se comprenderá por medio de la fig. 5 *, que representa cinco tablones 1, 2, 3, 4 y 5, formando un dique visto de canto. Cada taldón tiene 1 pie de alto y 20 pies de largo, horizontalmente, formando, por tanto, una superficie comprimida de 20 pies cuadrados. La presión en libras contra cada 20 pies cuad de superficie separadamente, calculada por la regla del art. 1, se indica en la figura. Ahora bien, la presión hacia afuera contra los 20 pies cuad del primer tablón *sumergido* superior n.º 3, es de 3,125 lbs, mientras que la presión contraria que obra del otro lado es de 625 lbs, produciendo un *exceso* de presión hacia afuera de 3,125 — 625 = 2,500 lbs. Ahora bien, en el tablón más bajo n.º 5, la presión hacia afuera excede de la que obra hacia adentro en 5,625 — 3,125 = 2,500 lbs lo mismo que en el de arriba; y así en cualquiera superficie igual sumergida á cualquiera profundidad; dependiendo el exceso de la altura vertical *mn*, se distribuirá igualmente sobre *ab*. Falta sólo demostrar que el exceso total de la presión hacia afuera ejercida contra *ab* es igual en intensidad al peso de una columna de agua con una base igual en superficie á *ab* y una altura igual á *mn*. Así hemos visto que en el ejemplo presente el exceso asciende á 3 veces 2,500 lbs ó sean 7,500 lbs. Ahora el peso de la columna de agua es 60 (área de *ab*) × *mn* (ó 2 pies) × 62.5 lbs = 7,500 lbs, que es lo mismo que el *exceso* de presión sobre *ab*. El exceso de presión entre la cara entera *sb*, y la cara *n o*, es evidentemente la diferencia de las dos presiones calculadas respectivamente según la regla del art. 1.

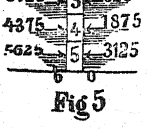

Fig 5

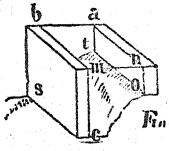

Fig 6

**Art. 3. Las superficies verticales como** *bmcs, anot*, **fig. 6, ó de otra manera, de iguales anchos** *bm, an*, **que comienzan al nivel** *banm* **del agua, pero que se prolongan á diferentes profundidades,** *mc, no*, **medidas verticalmente, con la misma inclinación hacia la superficie del agua, sufren presiones totales proporcionales á los cuadrados de aquellas profundidades.**

En la fig. 6, supongamos que las dos caras verticales *anot* y *bmcs* de un envase, tengan el mismo ancho *an* y *bm*. Si la profundidad *mc* es de 2, 3, 4, 5, etc., veces mayor que la profundidad *no*, la presión contra la superficie *bmcs* será 4, 9, 16, 25, etc., veces mayor que la que obra contra *anot*. Esto se verá observando las presiones indicadas del lado izquierdo de la fig. 5, donde, como se ha dicho en el art. 2, la superficie del tablón 1, expuesta á la presión que obra contra la cara izquierda, es de 20 pies cuad; la de los tablones 1 y 2, de 40 pies cuad; la de los 1, 2 y 3, de 60 pies cuad., etc. Todas estas superficies principian al nivel del agua, y siendo todas ellas verticales, tienen, por supuesto, la misma inclinación con la superficie del agua; pero sus profundidades son de 1, 2 y 3 pies respectivamente.

* *N. del T.* — Dejamos el ejemplo en medidas inglesas, porque para la teoría no tiene importancia la unidad elegida.

La presión contra la superficie 1 es de 625 lbs; la que se ejerce contra la superficie 1 y 2, es de 625+1,875=2,500; y la que obra contra las superficies 1, 2 y 3 es de 625+1,875+3,125=5,625. Pero 2,500 es *cuatro* veces 625, y 5,625 es *nueve* veces 625. Y la presión total contra la superficie entera *sb* (que tiene una profundidad 5 veces mayor que el tablón 1), es 25 veces mayor que la que actúa contra el tablón 1, ó 625×25=15,625 lbs, suma de todas las presiones marcadas en el lado izquierdo de la fig. 5. (*N. del T.* — Véase nuestra nota al pie de la página anterior.)

Se sigue de la regla del art. 1, que para una superficie de área doble, multiplicada por el doble de la altura vertical del centro de gravedad á la superficie del agua, se obtendrá una presión cuádrupla; para una área triple multiplicada por profundidad triple debe dar una presión nueve veces mayor, etc. Se sigue también que para cualquier *punto* ó contra cualquier área *dada* situados á diferentes profundidades, la presión aumentará simplemente como la profundidad vertical; de manera que si hay tres áreas, cada una de 1 p cuad colocadas en las mismas posiciones, pero con sus centros de gravedad respectivamente á 8, 16 y 24 p debajo de la superficie, la presión que obrará contra ellas será respectivamente *como* 8, 16 y 24, ó *como* 1, 2 y 3.

**Art. 4. La presión del agua en equilibrio en cualquiera dirección dada** contra cualquiera superficie dada, vertical, horizontal, inclinada, plana ó curva, es igual al peso de una columna de agua, con una base igual al área de la proyección *de la superficie comprimida tomada perpendicularmente á la dirección dada;* y una altura igual á la profundidad vertical del centro de gravedad de dicha superficie bajo el nivel superior del agua.

*Regla.* **Para encontrar la presión en** *kg*, multiplíquense entre sí el área en decímetros cuad de la proyección perpendicular á la dirección de la presión, por la profundidad vertical en decímetros del centro de gravedad de la superficie comprimida bajo el nivel del agua. Sea *mcsn*, fig. 7, una superficie inclinada que soporta la presión del agua, cuyo nivel superior está en *mc*. La presión total sufrida por *mcsn*, normal á ella, es por la regla del art. 1 un caso particular de la presente regla; porque la proyección de *mcsn*, tomada perpendicularmente á la dirección dada, ó paralela á *mcsn*, es de hecho igual á *mcsn*. Por lo tanto, la regla del art. 1 es meramente una simple modificación de la presente, aplicable al caso de la presión *total* contra cualquier superficie.

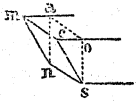

Fig 7

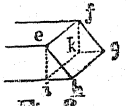

Fig 8

Fig. 9.

**Pero si se quiere solamente encontrar la presión vertical ó la presión hacia abajo contra** *mcsn* en kg, multiplíquense entre sí el área de la proyección horizontal *aocm*, en decímetros cuad, por la profundidad vertical en decímetros del centro de gravedad de *mcsn* á la superficie del agua. O si solamente se busca la componente horizontal de la presión sobre *mcsn*, multiplíquense entre sí el área de la proyección vertical *aosn*, la dist vertical del centro de gravedad de la superf *mcsn* á la superf del agua.

En la figura 8, la presión *total* contra *efgh* se halla también por la regla del art. 1, y las presiones horizontales y verticales se hallan como en la fig. 7, usando las proyecciones *efki* y *kygh*. En la fig. 7, la presión vertical es hacia abajo, mientras en la fig. 8, es hacia arriba; pero esta circunstancia no altera la regla por ningún respecto.

*Nota* 1. A cualquier profundidad dada, la presión perpendicular á una superficie cualquiera es la misma en todas direcciones; pero las figuras 7 y 8 demuestran que la presión total, *oblicua* á una superficie dada, es menor que la presión normal á la misma profundidad; porque la proyección oblicua de una superficie es siempre menor que la superficie misma, mientras que en el cálculo de la presión entra toda la superficie cuando la presión es perpendicular á ella. De modo que, en un estanque, la total presión perpendicular á una pared inclinada como *mnsc*, fig. 7, es mayor que la presión vertical ú horizontal ejercida sobre ella.

Sea la fig. 9, **un envase cónico lleno de agua,** cuya base *bc* es de 2 decímetros

de diámetro y su altura vertical *an* de 3 dcm. La circunferencia de la base será entonces de 6.2832 dcm; el área de la base, 3.1416 dcm cuad; el largo del lado oblicuo *ab* ó *ae*, 3.16 dcm; el área de la superficie curva lateral será $\frac{6.2832 \times 3.16}{2} = 9.93$ dcm cuad, y el centro de gravedad de los lados oblicuos estará á dos terceras partes de la altura vertical *an* á partir del vértice *a*, es decir, á 2 dcm de dicho vértice.

Para hallar aquí la presión total contra la base, tenemos, por la regla del art. 1, $3.1416 \times 3 = 9.425$ kg (porque un decím cúb de agua pesa un kilogramo). Para obtener la presión total contra la superficie oblicua lateral, la misma regla nos dará: $9.93 \times 2 = 19.86$ kg. Para la presión vertical hacia arriba ejercida contra el área total de la superficie oblicua lateral mencionada, tenemos el área de la base (que representa aquí la proyección horizontal de dicha superficie oblicua) = 3.1416, y la altura vertical 2 dcm del centro de gravedad de la superficie oblicua, *y*, por lo tanto, tenemos: $3.1416 \times 2 = 6.2832$ kg, presión vertical hacia arriba.

Finalmente, para la presión horizontal en cualquier dirección dada contra la superficie lateral oblicua de la *mitad* del cono, tenemos la proyección vertical de esta mitad, representada por el triángulo *abc;* su base de 2 dcm y su altura de 3; y, por consiguiente, con un área de 3 dcm cuad: Su centro de gravedad se encuentra á 2 dcm de *a*, de consiguiente tenemos: $3 \times 2 = 6$ kg, la presión horizontal buscada *.

En la fig. 10, que representa un envase lleno de agua, la presión *total* normal á la superficie semicilíndrica *avemdk*, debe ser horizontal, porque la superficie es vertical; pero puesto que la superficie es curva, la presión total, por la regla del art. 1, obra sobre ella en muchas direcciones, representadas por un número infinito de radios tirados desde *o* como centro. Pero si se solicita la presión horizontal en una dirección solamente, por ejemplo, paralela á *oe*, ó perpendicular á *ad*, que sería la fuerza que tendería á separar la superficie curva de las caras planas *abnv* y *desk*, produciendo fracturas á lo largo de *av* y *dk*, ó la tendente á reventar un tubo ú otro cilindro. En este caso, multipliquense entre sí el área de la proyección vertical *adkv* en decímetros cuad por la altura vertical desde la superficie libre del líquido al centro de gravedad de la superficie curva en decims (el cual, en un semicilindro, estará en la mitad de *em* ó de *oi*). Desde que á la resultante de las presiones se opone la resistencia del envase á lo largo de las líneas *av* y *dk*, es claro que es suficiente que el espesor del envase resista la mitad de la presión solamente, y así respecto de los tubos ú otros cilindros, tales como estanques, tinas, etc. (Véase art. 17.)

Para buscar la presión ejercida solamente contra la mitad de la superficie curva *edmk* y en una dirección paralela á *od*, tendiendo la presión á producir fracturas á lo largo de las líneas *em* y *dk*, se toma la proyección vertical *oemi*, la misma altura vertical y se multiplican entre sí como anteriormente.

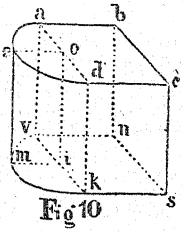

Fig 10

Fig. 10 1/2.

Se sigue de esto que si la base de un émbolo metálico es cóncava ó convexa, no se necesitaría más presión, para introducirlo á cualquiera distancia, que la que se necesitaría si fuera plana, porque la presión que obra contra la base del émbolo en la dirección en que éste se mueve, debe ser medida por el área de una proyección de aquella cara, tomada perpendicularmente á dicha dirección, y el área de la proyección será la misma en todos los casos.

* En una esfera llena de un fluido, la presión total interior es igual á 3 veces en peso del fluido.

*Nota* 2. **Si la pila de un puente ú otra construcción, fig. 10½, tiene sus cimientos sobre arena ó granzón,** ó sobre cualquier substancia, á través de la cual el agua puede filtrar por debajo, aunque sea en una capa muy delgada, entonces habrá una presión de abajo hacia arriba que tiende á levantar la pila con una fuerza igual al peso del agua desalojada por la pila. (Véanse arts. 18, 19.) En otras palabras, el peso real de la porción de pila *sumergida* será disminuído á razón de 1 kg por decímetro cúbico, que es próximamente la mitad del peso de la mampostería.

**Pero si los cimientos están sobre roca,** cubiertos con una capa de cemento para evitar la filtración del agua por debajo de la mampostería, no se producirá este efecto, sino que, al contrario, la presión vertical de arriba abajo, producida sobre las caras oblícuas de la pila y sobre el cimiento saliente, se agregará á su peso y aumentará su estabilidad, la cual, en agua tranquila, será mayor en este caso que en tierra.

**Art. 5. Para dividir una superficie rectangular, sea vertical como** *abcd*, **ó inclinada como** *mnop*, **fig. 11, cuya parte superior** *ab* **ó** *mn* **está**

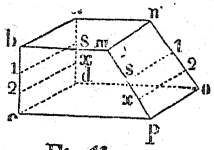


Fig 11

**al nivel de la superficie del agua, por medio de una línea horizontal** *x*2 **tal, que la presión total contra la parte superior de dicha línea horizontal sea igual á la de la parte que está debajo, se observará la siguiente regla:**

*Regla.* Multiplíquese la mitad de la longitud *bc* ó de *mp*, según el caso, por el número constante 1.4142, y el producto será *b*2 ó *mx*.

Ej. Sea *bc*=12 m; entonces tendremos 6×1.4142=8.4852 m ó *b*2.

Sea *mp*=16 m; entonces 8×1.4142=11.3136 m ó *mx*.

*Nota.* La línea *x*2, hallada así, no debe confundirse con el *centro de presión*, que es del todo diferente. (Véase art. 8.)

**Art. 6. En una superficie rectangular, sea vertical como** *abcd*, **ó inclinada como** *mnop*, **fig. 11, cuya parte superior** *ab* **ó** *mn* **coincide con la superficie del agua, encontrar cualquier número de puntos como 1, 2, etc., por los cuales, si se tiran líneas horizontales, 1*s*, 2*x*, etc., éstas dividan la superficie dada en rectángulos más pequeños, que sufran iguales presiones.**

*Regla.* Fíjese primero el número de rectángulos pequeños que se desea. Luego, para fijar el punto 1 de la parte superior, multiplíquese el número 1 por el número de los rectángulos; tómese la raíz cuadrada del producto; multiplíquese esta raíz cuadrada por el largo entero de *bc* ó *mp*, según el caso; divídase el producto por el número de rectángulos, y el cociente será la distancia *b*1 ó *n*1, según el caso.

Para la distancia *b*2 ó *n*2, procédase exactamente de la misma manera, solamente que en lugar del número 1, úsese el número 2 para multiplicarlo por el número de rectángulos, y así empléense sucesivamente los números 3, 4, 5, etc., si se necesitan estos números de puntos.

Ej. Sea *bc*=10 m, y que se requiera hallar 2 puntos, 1 y 2, para dividir la superficie rectángular *abcd* en tres partes rectangulares, que soporten iguales presiones. Tendremos para el punto 1:

$$1 \times 3 = 3.$$ La raíz cuad de 3=1.732; y $1.732 \times 10$; ó $(bc) = 17.32$

y $$\frac{17.32}{3 \text{ rectáng}} = 5.773 \text{ m} = b1.$$

Para el punto 2 tenemos:

$$2 \times 3 = 6.$$ La raíz cuad de 6 = 2.449; y $2.449 \times 10$; ó $(bc) = 24.49$

y $$\frac{24.49}{3 \text{ rectáng}} = 8.163 \text{ m} = \text{ó } b2,$$

Y así para cualquier número de puntos.

*Nota* 1. **Esta regla será útil en la determinación de los espacios á que deben colocarse los atravesaños de las puertas de canales, los aros de los envases cilíndricos y los puntales de una construcción como la fig. 3.**

*Nota* 2. **Para dividir de la misma manera cualquier superficie** *obed.* **fig. 12, que no sea rectangular,** con la exactitud suficiente para la mayor parte de los usos prácticos, quizás la siguiente regla sea la más conveniente.

*Regla.* Primeramente, divídase la superficie como en la fig. 12, en varias partes pequeñas horizontales, iguales ó no, como se quiera. Luego, por la regla del art. 1, búsquese la presión contra cada parte por separado, como se supone hecho, é indicado por los números situados del lado izquierdo de la figura. La suma de éstos (en este caso = 15,510) es la presión total ejercida contra la superficie entera *obed.* Ahora, suponiendo que deseamos dividir esta superficie en cuatro partes que soporten la misma presión; divídase primeramente á 15,510 por 4 = 3,878. Luego, principiando en la parte superior, súmese un número de presiones suficiente para producir la suma de 3,878; por este medio se busca el punto 1. Después procédase á la adición hasta que la suma ascienda al doble de 3,878 ó 7,756, que indicará el punto 2, y de esta misma manera búsquese el punto 3, sumando hasta obtener el triple de 3.878 ú 11,634, entonces las líneas horizontales de puntos, tiradas por 1, 2, 3, darán las divisiones buscadas aproximadamente. De este modo encontramos las separaciones que deben darse á los aros de los envases cónicos, ó de otras formas, con exactitud suficiente para las aplicaciones prácticas.

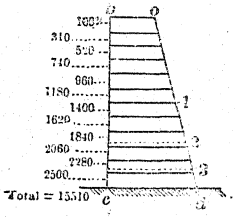


Fig. 12.

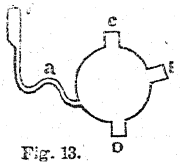


Fig. 13.

**Art. 7. Transmisión de presión por el agua. El agua y los otros fluidos poseen la propiedad importante de transmitir presiones iguales en todas direcciones.** Así, suponiendo que el envase, fig. 13, está enteramente cerrado y lleno de agua, y suponiendo las áreas transversales de T, *c*, D, E, iguales á un cm cuad cada una. Entonces, si por medio de un émbolo ó de otro modo se aplica la presión de un kg, de una tonelada ó de cualquier otro peso al área de un cm cuad de T, *c*, D ó E, cada cm cuad de la superficie interior del envase recibirá instantánea y normalmente una presión igual de 1 kg, 1 ton., etc., además de la presión producida por el agua misma; y esto sucede aunque el envase tenga varios km de largo, como por ejemplo si T distara varios km de E y estuvieran unidos por una larga tubería. Si este envase fuese una fuerte caldera de vapor llena de agua, una simple presión de pocos quintales aplicada en T, *c*, etc., podría hacerlo reventar. Véase también fig. 2 (*c*) y el párrafo anterior.

**La prensa hidrostática* está basada en este principio.** Cualquier cuerpo que se halle en el interior del envase recibirá también igual presión adicional en cada cm cuad de su superficie.

Si la parte superior de T está abierta, el aire ejercerá una presión como

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* *N. del T.* — El autor, como todos los que hablan inglés, llama esta prensa *hidrostatic press*, es decir, *prensa hidrostática*. En español se llama *prensa hidráulica*, pero nos parece más adecuado el nombre de *prensa hidrostática*, toda vez que esta máquina se funda en el principio de la transmisión de presiones en una masa líquida, que es principio exclusivamente *hidrostático*, y á la hidráulica sólo pertenecen las aplicaciones de la hidrodinámica que estudia exclusivamente el movimiento de los líquidos. Podría rebuscarse en defensa del nombre español, que el agua de la prensa se mueve alterando su nivel al transmitir la presión; pero éste es un movimiento insignificante y no esencial para la transmisión de la fuerza ni tan importante en el aparato, como la multiplicación de la fuerza transmitida por entre el líquido, que es *principio de hidrostática*.

de 1.033 kg sobre cada cm cuad de la superficie libre del agua y una presión de igual intensidad será transmitida también á cada cm cuad de la superficie interior del envase y de los tubos que los conectan; pero no hay peligro de que reviente á consecuencia de esta presión atmosférica, porque el aire también ejerce una presión igual sobre cada cm cuad en la superficie exterior del envase.

**El aire y otros fluidos gaseosos transmiten la presión igualmente en todas direcciones**, como los líquidos; pero no con tanta rapidez.

**Art. 8. Centro de presión.** Sea la fig. 14, un envase lleno de agua, y supongamos que la cara P está perfectamente floja, de manera que con la menor presión que se ejerza sobre el agua contenida, sea empujada ó botada hacia afuera. Ahora bien, sólo existe un punto único, P, en toda superficie comprimida de este modo, cualquiera que sea su forma, al cual, si se aplica una fuerza igual á la presión del agua, y en dirección opuesta á aquélla, impedirá que la cara P ceda al empuje antes citado. Este punto se llama *centro de presión*.

Pero no debe entenderse por esto que la intensidad total de la presión efectiva del agua contra la parte de superficie que está encima de la línea horizontal de puntos que pasa por P, sea igual á la del agua que está debajo de dicha línea; sino que la suma de los productos de las diversas presiones que se ejercen de dicha línea hacia arriba, multiplicada por sus diversos brazos de palanca respecto á P, es igual á la suma de los productos de las diferentes presiones que obran de la línea hacia abajo, multiplicada por sus brazos de palanca respecto al mismo punto; ó, en otras palabras, que la suma de los *momentos* referidos al punto P, de las presiones que obran de la línea hacia arriba, es igual á la suma de los *momentos* de las que actúan de ella hacia abajo; de manera que si pasamos una barra de hierro, *bb*, por toda la cara P al nivel de la línea de puntos para que sirva como de eje á cuyo alrededor pueda girar, esta cara se mantendrá en equilibrio.

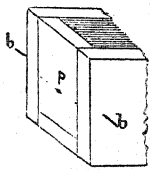


Fig. 14.

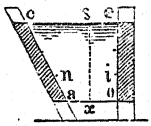


Fig. 15.

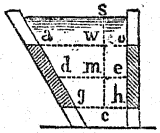


Fig. 16.

**Art. 9. Encontrar el centro de presión de un fluido en reposo contra una superficie plana**, fig. 15.

1. El centro de presión de un fluido en reposo contra cualquiera superficie plana, de anchura uniforme en toda su profundidad, sea que dicha superficie esté vertical, como *eo*, ó inclinada, como *ca* (ó inclinada en la dirección opuesta), y cuya parte superior *e* ó *ce* coincida con la superficie horizontal del agua, se halla á una distancia vertical bajo la superficie del agua, igual á dos terceras partes de la profundidad *sx* de dicha superficie del agua, como en *n* é *i*. Puesto que una línea horizontal situada á $^2/_3$ de la profundidad *sx*, corta á *ca* y á *eo* á $^2/_3$ de sus longitudes, respectivamente, podíamos decir de una vez, que el centro de presión contra una superficie plana de ancho uniforme se encuentra á dos tercios de su longitud á partir de la superficie del agua.

A todo lo aludido en el art. 9 puede aplicársele la **unidad de medida** que se desee.

2. Pero si el extremo horizontal *a* ú *o*, fig. 16, del plano rectangular *ag* ú *oh*, está cubierto de agua hasta alguna altura, entonces la dist vertical *sm* del centro de presión *d* ó *e* á la superficie del agua será igual á

$$\frac{2}{3} \text{ de } \frac{\text{cubo de } sc - \text{cubo de } sw}{\text{cuad de } sc - \text{cuad de } sw}.$$

en que *sc* es la profundidad vertical del fondo y *sw* la profundidad de la parte superior de la superficie comprimida de que se trata; ó, en otras palabras, del cubo de *sc* réstese el cubo de *sw* y llámese el residuo *a*. Luego, del cuadrado de *sc* réstese el cuad de *sw;* y llámese el residuo *b*. Divídese á *a* por *b* y tómese $^2/_3$ del cociente para obtener á *sm*.

3. Cuando una superficie plana de cualquier forma, ya sea rectangular, triangular ó circular, etc., sea vertical como *op*, fig. 17, ó inclinada como *nm*, se halla *enteramente sumergida*, de manera que la presión se ejerza sobre toda el área de *ambas caras*, pero por *diferentes alturas* de agua en dichas dos caras, el centro de presión coincidirá con *el centro de gravedad* de la superficie comprimida. En las tres figuras que anteceden, las superficies supuestas se ven de canto, de modo que no se ven sus anchos.

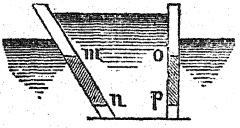


Fig. 17.

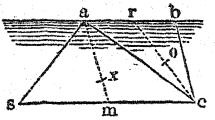


Fig. 18.

4. En cualquiera superficie plana triangular, en ángulo recto ó situada de otro modo, como *abc*, fig. 18, vertical ó inclinada, cuya *base ab* coincida con la superficie horizontal del agua, el centro de presión *o*, estará en el centro de la línea *cr*, que divide la base *ab* en dos partes iguales.

5. Pero si el triángulo *asc*, vertical ó inclinado, tiene su *vértice a* en la superficie del agua, y su base *sc* horizontal, entonces el centro de presión *x* estará también en la línea *am*, que divide la base en dos partes iguales; pero *ax* será $^3/_4$ de *am*.

6. Si cualquier triángulo plano, *abc*, fig. 19, con la base hacia arriba, y horizontal, tiene su base *ab*, á una profundidad *nd*, del líquido, el centro de presión *o*, estará en la línea *cs*, que divide la base en dos partes iguales, y *no* será igual á

$$\frac{mx^2 + (2mx \times ma) + 3ma^2}{(mx + 2ma) \times 2}.$$

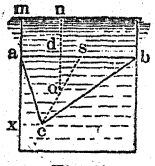


Fig. 19.

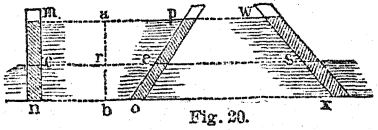


Fig. 20.

7. El centro de presión contra cualquier superficie plana *rectangular*, fig. 20, vertical como *mn*, ó inclinada como *op* ó *wx*, que tenga su parte superior coincidiendo con la superficie del agua comprimida por diferentes alturas de agua y sus caras opuestas como se ve en la fig. 20, estará, en una *vertical* debajo de la superficie *superior* del agua, á una distancia igual á

$$\frac{\left[\text{área de la superf } mn, \text{ ó } op, \text{ ó } wx \times \frac{\text{cuad de la prof vert } ab}{3}\right] - \left[\text{área de la superf } cn \text{ ó } eo, \text{ ó } sx \times \frac{\text{cuad de la prof vert } rb}{3}\right] - \left[\text{prof vert } ar \times \text{área de la superf } en \text{ ó } eo \text{ ó } sx \times \frac{\text{prof vert } rb}{2}\right]}{\left(\text{área de la superf } mn, \text{ ó } po, \text{ ó } wx \times \text{mitad de } ab\right) - \left(\text{área de la superf } cn, \text{ ó } eo, \text{ ó } sx \times \text{mitad de } rb\right)}.$$

8. Encontrar el centro de presión de una superficie circular, ó elíptica, comprimida por uno de los lados solamente; ya sea vertical ó inclinada, y cuya parte superior coincida con la superficie del agua ó esté debajo de ella.

Llámese *h* la altura ó dist vertical del centro de presión á la superficie del agua; *r*, el *semi*-diámetro vertical (ó inclinado, según el caso) de la superficie; *d*, la distancia *vertical* del *centro* de la superficie comprimida á la superficie del agua.

Entonces tenemos, $h = \frac{r^2}{4d} + d$. En un **círculo vertical** con su parte superior al nivel de la superficie, tenemos $h = 1\frac{1}{4}$ radio.

**Art. 10. Muros para resistir la presión de aguas tranquilas.** En relación con esta materia sería de utilidad hacer un estudio de lo que sabemos sobre los muros para el sostenimiento de tierras (pág. 647). Se supone, por supuesto, que el agua no encuentra entrada por debajo del muro y que éste no puede resbalar. Al hacer los cálculos de muros de sostenimiento, ya sea para contener tierras ó agua, es conveniente suponer que el muro no tiene sino un m de *largo* (no de altura, ni de espesor); porque entonces el número de ms cúb que contenga, será igual á los ms cuad del área de su sección transversal ó perfil; de manera que estos ms cuad multiplicados por el peso de un m cúb de mampostería darán el peso del muro. En los casos ordinarios sería bueno suponer, para mayor seguridad, que el agua llega hasta la línea de los mismos cimientos del muro.

Ahora bien, según el art. 1, la presión total de agua tranquila contra el paramento interior rectilíneo del muro, vertical ó inclinado, se halla en toneladas métricas (1,000 kg), multiplicando entre sí el área en m cuad de la parte realmente comprimida (ó en contacto con el agua) por *la mitad* de la profundidad vertical del agua en ms (que es la distancia *vertical* del centro de gravedad de su paramento interior rectilíneo á la superficie). Esta presión total es siempre *perpendicular al área de la superficie comprimida.*

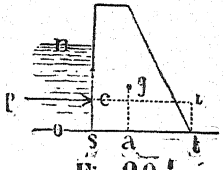


Fig 20 $\frac{t}{2}$

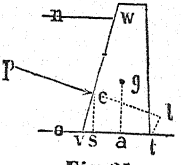


Fig 21.

Cuando el paramento interior del muro es vertical, como en la fig. 20½, esta presión, *p*, es naturalmente menor que cuando es inclinado, es también *horizontal* y tiende á volcar el muro haciéndolo girar alrededor de su arista exterior *t*. El centro de presión está en *c;* siendo $cs = {}^1/_3$ de la profundidad *vertical on*. En otras palabras, la presión total del agua, considerada como tendiendo á volcar el muro, y á éste como formado de un solo cuerpo, concentrado en el punto *c*, donde obra tendiendo á volcarle con el brazo de palanca *tl*.

La presión en kg multiplicada por este brazo de palanca en metros, da el *momento* en kilográmetros de la fuerza que tiende á volcar el muro. El muro, por otra parte, resiste en una dirección vertical *ga* con un momento igual á su peso (que se supone concentrado en su centro de gravedad *g*) multiplicado por la distancia horizontal *at*, que constituye el brazo de palanca del peso con respecto al punto *t* como punto de apoyo. Si el momento del agua es mayor que el del muro, este último será volcado; pero si es menor, resistirá. Aquí, fig. 21, el momento con que el agua tiende á volcar el muro es igual á su presión calculada *p* × su brazo de palanca *tl*, mientras que el momento de estabilidad del muro es igual á su peso × su brazo de palanca *at*. Con el auxilio de un trazado en escala podemos, basados en estos principios, saber si un muro dado podrá resistir; porque tenemos solamente que calcular la presión *p;* aplicarla entonces en *c*, perpendicular al paramento interior del muro, prolongarla hasta *l*, y medir á *tl*, por la misma escala. Luego calcúlese el peso del muro; búsquese su centro de gravedad *g;* trácese la línea *ga* vertical, y mídase el brazo de palanca *at*. Entonces tenemos los datos para calcular los dos momentos.

Si el agua en lugar de estar tranquila está expuesta á ser agitada formando olas, el muro debe hacerse de más espesor.

**Art. 11. Hallar el espesor de un muro en su base** (que esté á cubierto del *volcamiento*) bajo la presión del agua en reposo al nivel de su parte superior y ejercida contra todo el paramento interior vertical. **Advertencia**, véase art. 13.

(1) **Muro vertical**, fig. 22.

$$\text{Espesor} = \text{altura} \times \sqrt{\frac{\text{Coeficiente de seguridad*}}{3 \times \text{densidad del muro}}} = \text{Altura} \times \text{la decimal que le corresponde en la tabla que sigue.}$$

$$* \text{ Coef de seguridad} = \frac{\text{Momento de estabilidad requerida para el muro}}{\text{Momento de volcamiento del agua}}.$$

**(2) Muro triangular á ángulo recto,** fig. 23.

$$\text{Espesor en la base} = \text{Altura} \times \sqrt{\frac{\text{Coeficiente de seguridad*}}{2 \times \text{densidad del muro}}} = \text{Altura} \times \text{la decima}$$

que le corresponde en la tabla que sigue = espesor *mo* del muro vertical × 1.225.

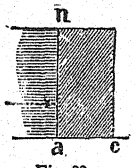


Fig. 22.

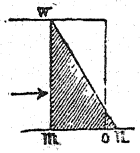


Fig. 23.

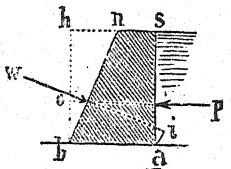


Fig. 24.

A pesar de su mayor espesor en la base, estos muros triangulares no contienen, como se ve por la figura, sino poco más de la mitad de la cantidad de mampostería requerida para muros rectangulares de igual estabilidad. Esto es debido á que su centro de gravedad está situado más adentro, aumentando de este modo el brazo de palanca, con que el peso del muro resiste al volcamiento.

**(3) Muro de paramento interior vertical y de paramento exterior inclinado,** fig. 24.

$$\text{Espesor de a base en m} = \sqrt{\frac{(\text{altura en m})\ (\text{Coef de seguridad*}) + (nh' \text{ en m} \times \text{densidad del muro})}{3 \times \text{densidad del muro}}}$$

= altura × la decimal correspondiente de la tabla siguiente.

| Fig. 22. | Densidad | Resist =1.5 presión. | Resist =2 presión. | Resist =3 presión. |
|---|---|---|---|---|
| Granito labrado | 2.5 | .447 | .516 | .633 |
| Piedra arenisca labrada | 2.2 | .447 | .550 | .674 |
| Mampostería de piedra bruta | 2 | .500 | .578 | .707 |
| Mampostería de ladrillo | 1.8 | .527 | .609 | .746 |
| **Fig. 23.** | | | | |
| Granito labrado | 2.5 | .548 | .633 | .775 |
| Piedra arenisca labrada | 2.2 | .584 | .675 | .826 |
| Mampostería de piedra bruta | 2 | .613 | .707 | .866 |
| Mampostería de ladrillo | 1.8 | .646 | .746 | .913 |

| Fig. 24. | Densidad. | Resist=1.5 presión. | | | | Resist = 2 presión. | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Incl 1/12 | Incl 1/6 | Incl 1/3 | Incl 1/2 | Incl 1/12 | Incl 1/6 | Incl 1/3 | Incl 1/2 |
| Granito labrado | 2.5 | .449 | .458 | .487 | .532 | .519 | .526 | .551 | .593 |
| Piedra arenisca labrada | 2.2 | .480 | .488 | .515 | .558 | .552 | .560 | .583 | .622 |
| Mampostería de piedra bruta | 2 | .502 | .510 | .536 | .578 | .571 | .586 | .609 | .646 |
| Mampostería de ladrillo | 1.8 | .530 | .539 | .562 | .602 | .610 | .618 | .640 | .674 |

**Art. 12. Tabla que demuestra cómo afecta la estabilidad de un muro de sostenimiento y contención de aguas, un cambio en la forma de dicho muro,** siendo la cantidad de mampostería la misma. *Nota.* Cuando la base de un muro triangular con una densidad de 2, es menor que la mitad de su altura, tiene la mayor estabilidad cuando el agua ejerce presión contra la cara vertical; pero si la base excede de la mitad de la altura, posee entonces la

* Véase nota * página anterior.

mayor estabilidad cuando el agua empuja del lado ó paramento inclinado. **Advertencia.** Véase art. 13.

| | **Todos estos muros contienen precisamente la misma cantidad de mampostería.** Se supone que la mampostería es de ripios ó piedra bruta y mortero, con peso de 125 lbs el pie cúb, ó 2,000 kilgms el m cúb, doble del peso del agua, ó más ó menos lo mismo que la mampostería ordinaria. Si la densidad de la mampostería es realmente mayor ó menor, la seguridad será también mayor ó menor, exactamente en las mismas proporciones. | Base en partes de la altura. | Resistencia aproximada del muro. |
|---|---|---|---|
| 1 | Muro vertical..... | .5 | 1.5 |
| 2 | Paramento ext vert; paramento int inclind $\frac{1}{10}$ de la altura. | .55 | 1.8 |
| 3 | — — — — — — $\frac{1}{5}$ — | .6 | 2.2 |
| 4 | — — — — — — $\frac{1}{4}$ — | .625 | 2.6 |
| 5 | — — — — — — $\frac{1}{3}$ — | .667 | 3.5 |
| 6 | — — — — — — $\frac{4}{10}$ — | .7 | 4.9 |
| 7 | — — — — — — $\frac{1}{2}$ — | .75 | 14.0 |
| 8 | Paramento int vert; paramento ext inclind $\frac{1}{10}$ de la altura. | .55 | 1.8 |
| 9 | — — — — — — $\frac{1}{5}$ — | .6 | 2.1 |
| 10 | — — — — — — $\frac{1}{4}$ — | .625 | 2.2 |
| 11 | — — — — — — $\frac{1}{3}$ — | .667 | 2.4 |
| 12 | — — — — — — $\frac{4}{10}$ — | .7 | 2.6 |
| 13 | — — — — — — $\frac{1}{2}$ — | .75 | 2.9 |
| 14 | Paramentos ext é int inclinados ambos, $\frac{1}{10}$ de la altura..... | .6 | 2.2 |
| 15 | — — — — — $\frac{1}{5}$ — ..... | .7 | 3.4 |
| 16 | — — — — — $\frac{1}{4}$ — ..... | .75 | 4.6 |
| 17 | — — — — — $\frac{1}{3}$ — ..... | .833 | 9.0 |
| 18 | — — — — — $\frac{4}{10}$ — ..... | .9 | 36.0 |

**Art. 13. Propensión de un muro ó fundación al desmoronamiento ó ruptura por compresión bajo presiones desiguales.** Los artículos 11 y 12 se aplican solamente á la *estabilidad* de un muro *rígido*, levantado sobre una base *rígida*, y por consiguiente incapaz de ceder sino al *volcamiento, considerando el cuerpo como un todo.* Ellos demuestran que la *estabilidad* es la mayor, cuando el agua ejerce su presión sobre el lado *inclinado.* Pero en la práctica, el punto donde la resultante de todas las presiones corta ó atraviesa la base, no debe estar tan cerca de la arista ó pie, que haya el riesgo del rompimiento ó *desmoronamiento* del muro ó los cimientos. Esta consideración hace á menudo preferible que la presión se ejerza contra el paramento interior *vertical*, no obstante la pérdida consiguiente de estabilidad.

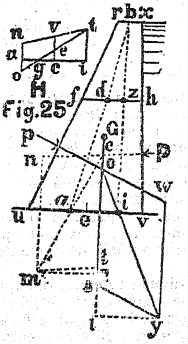
**Art. 14.** Así, la fig. 25 representa, en escala, un dique en Poona, India, proyecto del Sr. Fife, I. C. de Inglaterra. Está construído de mampostería de piedra bruta con mezcla, con peso de 2,400 kilg m cúb. Su altura vertical total es de 30.47 m; espesor *uv* en la base, de 18.50 m; en la parte superior, 4.19 m. La cara ó paramento exterior *ru* tiene una inclinación de 42 por 100 y el paramento interior *xv* de 5 por 100. Los cimientos tienen 2.13 m de profundidad; pero se supone aquí que el agua ejerce presión contra *todo* su paramento interior *xv*. Por el centro de gravedad G trácese la línea *Gs*, vertical. Desde *c*, donde la dirección de la presión P del agua corta á *Gs*, hágase la línea *cn*, por la escala, igual al número de toneladas de la presión del agua contra un m de longitud de *xv* y *ct* igual al peso de un m de longitud del muro. Complétese el paralelogramo *cnmt* de las fuerzas, cuya diagonal *cm* representa la resultante de todas las presiones ejercidas contra la base *uv* y corta la base en *a* á 6.09 m de la arista *u* hacia el interior. Hágase lo mismo con la presión *p* contra *ru* y obtendremos la resultante *oy*, que es mayor que *cm*, y corta la base (en *i*) solamente á distancia de 3.86 m de la arista *v*, 2.23 m menos de lo que dista *a* de *u*.

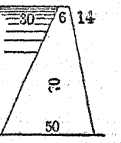
Por lo tanto, cuando el agua ejerce presión contra *xv*, el muro está menos expuesto á romperse, y en el terreno *uv* donde se apoya se distribuye más uniformemente el peso y, por consiguiente, está menos expuesto á descender desigualmente hasta producir grietas en el muro. Por esta razón *xv* se toma como paramento interior del muro, sin embargo de que entonces el momento de estabilidad es solamente 2.2 (llamando 1 el momento del agua tendente al volcamiento

del muro), mientras que si el agua comprimiese á *ru*, sería 3, 4 es decir, 36 por ciento mayor.

**Art. 15.** Los puntos *a* é *i*, fig. 25, se llaman **centros de presión** sobre la base ó **centros de resistencia** de la base. Si considerando porciones de muro (tales como *rxhf*) como si fueran muros enteros, encontramos puntos *d* y *z* semejantes á aquéllos, situados de la misma manera con respecto á *fh*. La línea ligeramente curva que los une se llama **curva de presión.** Así, *ba* es la curva de presión cuando el agua ejerce presión contra *xv*. Cada punto, como *d* en *ba*, indica en donde una junta cualquiera, como *fh*, trazada por este punto, es cortada por la resultante de todas las fuerzas que obran sobre dicha junta; *bi* es la curva de presión cuando el agua comprime á *ru*. Estas líneas no muestran las *direcciones* de las resultantes. Así es que, en *a*, esta dirección es *cm* y no *ba*. El ángulo comprendido entre la dirección de la resultante y una línea perpendicular al asiento ó junta del muro, debe ser menor que el ángulo de rozamiento de los materiales en contacto en la junta.

Si desde el extremo *m* ó *y* de la resultante de las presiones que se ejerzan sobre cualquiera junta se trazan *m*2 ó *yl*, horizontales, entonces *c*2 ú *ol* (según el caso) medirán toda la presión *vertical* que obra en esa junta y *m*2 ó *yl* medirán la presión *horizontal* que actúa contra el paramento interior del muro, que tiende á producir resbalamiento en la misma junta. Si la dirección de la resultante llega al límite mencionado en el párrafo precedente, *m*2 ó *yl* será menor que la resistencia de rozamiento que se opone al resbalamiento, la cual es = *c*2 (ú *ol*) × el coeficiente de rozamiento de las superficies que forman la junta. Por lo expuesto no hay resbalamiento. En la *mampostería* de los muros de formas ordinarias no ocurren jamás resbalamientos. La mezcla buena, bien seca, contribuye á evitarlos, pero es mejor no confiarse. Muros enteros se han deslizado sobre cimientos resbaladizos.




**Art. 16. En California se encuentra la represa,** fig. 14, de un estanque de una mina, hecha de piedra bruta sin mortero y cimentada en la roca. Altura, 21.33 m; base, 15.23 m; parte superior, 1.82 m; inclinación ó talud del lado del agua, 9.14 m; talud exterior, 4.26 m. Para evitar mermas por filtraciones, el talud interior está revestido con tablones de 76 mm, asegurados horizontalmente con pernos á unos zoquetes de 30×30 cm fijos al muro. Toda ella fué hecha á mano y con cuidado, con excepción de una parte interior como de un quinto de toda la masa que se llenó echando el material sin cuidado. El costo fué de 3,6 dólares el metro cúb más ó menos. Está en uso desde 1860.

**Observación. Si se refuerza ó rellena sólidamente un dique ó represa con tierra** que tenga su talud natural, en cantidad suficiente para evitar que el agua llegue hasta la represa, la presión contra ésta no aumentará por causa de la tierra.

**Art. 17. Encontrar el espesor de un cilindro,** que resista con seguridad la presión del agua, del vapor, etc., contra su interior. Si está remachado, véase un poco más abajo.

**Donde el espesor es menor que una treintava parte del radio,** como lo es en el mayor número de casos, se emplea la fórmula ordinaria :

$$(1) \qquad \text{Espesor en cm} = \frac{\text{presión}}{\text{resist de seguridad}} \times \text{radio} *.$$

Se considera el material como sometido solamente á un esfuerzo de tensión, directo, lo cual es bastante exacto en espesores tan delgados.

**Para espesores y presiones algo mayores,** el profesor F. Reuleaux-(Der Konstrukteur, pág. 52) da :

$$(2) \quad \frac{\text{Espesor}}{\text{en cm}} = \frac{\text{presión}}{\text{resist de seguridad}} \left(1 + \frac{\text{presión}}{2 \times \text{resist de seguridad}}\right) \times \text{radio} *.$$

**Para presiones y espesores muy grandes,** como en las prensas hidráulicas, cañones, etc., el profesor Reuleaux (Konstruckteur, pág. 53) da la fórmula de Lamé :

$$(3) \quad \frac{\text{Espesor}}{\text{en cm}} = \left(\sqrt{\frac{\text{resist de seguridad} + \text{presión}}{\text{resist de seguridad} - \text{presión}}} - 1\right) \times \text{radio} **.$$

Las tres fórmulas dan el siguiente resultado en pulgs, tomando las presiones y resistencias en lbs por pulg cuad y el radio en pulgs :

| Diám. | Radio. | Presión. | Resist de seguridad á la tensión. | Espesores en pulgs. | | |
|---|---|---|---|---|---|---|
| | | | | Fórmula (1). | Fórmula (2). | Fórmula (3). |
| 20 pulgs | 10 pulgs | 50 | 10000 | **.05** | .050125 | .05 |
| — | — | 500 | — | .50 | **.5125** | .513 |
| — | — | 5000 | — | 5.00 | 6.25 | **7.32** |

Los espesores dados por las fórmulas adaptables á las diversas presiones, se han impreso en **tipos gordos.** Se verá que en estos casos los resultados difieren muy poco, excepto en las presiones muy *grandes.*

*Observación.* **La falta de uniformidad en el enfriamiento** de las fundiciones de gran espesor las hace más débiles relativamente que las de poco espesor; de modo que, para reducir éste en casos importantes, debemos usar solamente el mejor hierro refundido tres ó cuatro veces, por cuyo medio se puede conseguir una cohesión máxima de 2109 kg por cm cuad más ó menos. Pero aun con esta precaución, **no hay regla que pueda aplicarse con seguridad** en la práctica á los cilindros de hierro fundido, cuyos espesores excedan de 8 á 10 pulgs (20 á 25 cm) próximamente.

Bajo una presión de (8,000 lbs por pulg cuad) 562.48 kg por cm cuad **se filtra el agua á través del hierro fundido de 8 á 10 pulgs (20 á 25 cm) de espesor,** y bajo una presión de sólo (250 lbs por pulg cuad) 17.58 kg por cm cuad se filtrará al través de (0.5 pulg) 12 ½ mm.

**Tabla de espesores de tubos de hierro forjado de roblonadura sencilla,** para estanques, tubos de alimentación, etc., calculados por la regla precedente, para resistir con un coeficiente 6 de seguridad, la presión de una carga de 304 m de agua en reposo ó sean 30.4 kg por cm cuad, tomando la cohesión máxima de las planchas de hierro de regular calidad en 3374 kg por cm cuad, ó sean 562 kg por cm cuad, para un coeficiente 6 de seguridad; aun reduzcámoslo á $562 \times .56 = 314.72$ kg por cm cuad por la pérdida de resistencia producida por los agujeros de los roblones; porque los cilindros de **roblonadura**

---

* En todas las tres fórmulas tómese el *radio en cm*, y la presión y la resistencia en *kg/cm cuad.*

** *N. del T.* — Tomando el radio en cm y la presión y resistencia en kg por cm cuad, obtendremos el espesor en cm. Pongamos un ejemplo para la primera fórmula : ¿Qué espesor nos dará para un radio = 25.4 cm (10 pulgs), una presión de 3.52 kg por cm cuad (50 lbs por pulg cuad) y una resistencia de 703.095 kg por cm cuad (10,000 lbs por pulg cuad) ?

(1) Espesor en cm $= \frac{3.52}{703.095} \times 25.4 = 0.127$ cm (.05 pulg). Que es igual al que da la tabla.

**sencilla** no tienen sino una resistencia de .56 más ó menos de la que tienen las planchas sólidas, y los de **roblonadura doble**, como .7.

*N. del T.* — La tabla que sigue es la **original** del autor, convertida al sistema métrico. Los diámetros y espesores están en centímetros. Hemos tomado sólo hasta el diámetro de 108 pulgs.

| Diám. | Esp. | Diám. | Esp. | Diám. | Esp. | Diám. | Esp. | Diám. | Esp. |
|---|---|---|---|---|---|---|---|---|---|
| 1.3 | .06 | 12.7 | .60 | 40 | 2.00 | 75 | 3.80 | 150 | 7.60 |
| 2.54 | .13 | 15. | .76 | 45 | 2.32 | 84 | 4.32 | 167 | 8.50 |
| 3.8 | .19 | 20. | 1.00 | 50 | 2.54 | 91 | 4.47 | 182 | 9.30 |
| 5.00 | .25 | 25. | 1.30 | 55 | 2.79 | 105 | 5.41 | 213 | 10.80 |
| 7.60 | .37 | 30. | 1.52 | 60 | 3.05 | 122 | 6.20 | 243 | 12.40 |
| 10.1 | .50 | 35. | 1.78 | 67 | 3.48 | 135 | 6.96 | 274 | 13.90 |

**Para una carga ó presión menor** ó para cualquier coeficiente de seguridad menor que 6, satisface, y es casi suficiente en la práctica, reducir los espesores de los cilindros de hierro forjado en la misma proporción en que dicha carga, presión ó coeficiente de seguridad sea menor que los de la tabla.

**Los cilindros de roblonadura doble** son, según Fairbairn, 1.25 veces más fuertes que los de roblonadura sencilla. Por tanto deben tener $^1/_5$ menos de espesor. **Las soldaduras recubiertas** son casi 1.8 veces más fuertes que las remachadas sencillamente, y por tanto no requieren sino .56 del espesor.

**En California** han estado en uso, durante muchos años, muchas millas de tubería de **roblonadura doble** con un coeficiente de seguridad de 2 á 2.6 solamente. En un caso tiene una carga de 524 m con una presión de 52.45 kg por cm cuad; diám, 292 mm; espesor, 8½ mm.

Los tubos **de hierro fundido para la distribución del agua en las poblaciones** deben tener más espesor que el dado por la fórmula (1) para que soporten el recio manejo, y el efecto de los choques producidos por el agua misma debido á la parada repentina de la corriente, véase observación, pág. 545, y para prevenirlos contra las irregularidades de la fundición, y de las burbujas de aire, ó vacíos á que están más ó menos expuestos todos los objetos fundidos.

En la tabla que sigue se ha tomado, como resistencia máxima á la tensión del hierro fundido, 1.265 kg por cm cuad. Estos espesores están calculados por la fórmula del Sr. J. T. Fanning (Hidráulica, pág. 454). Está de acuerdo con los términos medios de la práctica.

(*N. del T.* — Hemos convertido la tabla del autor al sistema métrico.)

| Diámetros en milímetros. | Carga en metros. | | | | | |
|---|---|---|---|---|---|---|
| | 15 | 30 | 60 | 90 | 150 | 300 |
| | Presiones en kg /cm cuad. | | | | | |
| | 1.5 | 3 | 6 | 9 | 15 | 30 |
| | Espesores en milímetros. | | | | | |
| 50 | 9.1 | 9.4 | 9.5 | 9.6 | 10.7 | 12.2 |
| 75 | 9.4 | 9.5 | 10.2 | 10.7 | 11.7 | 13.7 |
| 100 | 9.6 | 10.2 | 10.7 | 11.7 | 12.7 | 15.5 |
| 150 | 10.4 | 10.9 | 11.7 | 12.7 | 14.5 | 19.0 |
| 200 | 11.7 | 11.9 | 13.2 | 14.2 | 16.7 | 22.9 |
| 250 | 11.9 | 12.7 | 14.2 | 15.7 | 18.8 | 26.4 |
| 300 | 12.4 | 13.5 | 15.2 | 17.0 | 20.8 | 30.0 |
| 400 | 14.0 | 15.2 | 17.8 | 20.0 | 24.9 | 37.1 |
| 450 | 14.5 | 16.0 | 18.8 | 21.6 | 26.9 | 40.6 |
| 500 | 15.5 | 17.0 | 20.0 | 23.1 | 29.2 | 44.4 |
| 610 | 16.7 | 18.5 | 22.1 | 25.6 | 33.0 | 51.5 |
| 760 | 18.8 | 21.1 | 25.5 | 30.2 | 39.4 | 62.5 |
| 910 | 20.8 | 23.6 | 29.2 | 34.5 | 45.7 | 73.1 |
| 1220 | 24.9 | 28.7 | 36.1 | 43.2 | 57.9 | 94.7 |

**Tablas de espesores de tubos de plomo** para soportar presiones interiores, con un coeficiente de seguridad de 6, tomando la cohesión máxima del plomo por 98 kg por cm cuad.

*Observación.* Aunque estos espesores ofrecen seguridad para las presiones de líquidos en reposo, quizás no resistan los choques producidos por el agua corriente al cerrar de repente las llaves de retención.

*Obs. del T.* — Damos la siguiente tabla aplicable lo mismo que la trae el autor pero en sistema métrico: cohesión máxima del plomo=98 kg por cm cuad; coeficiente de seguridad=6.

| Diám en cm. | Carga en metros. 25 | 50 | 75 | 100 | 125 | 150 |
|---|---|---|---|---|---|---|
| | Presión en kg por cm cuad. 2.5 | 5 | 7.5 | 10 | 12.5 | 15 |
| | Espesores en centímetros. | | | | | |
| ½ | .04 | .10 | .16 | .22 | .26 | .39 |
| 1 | .08 | .18 | .28 | .40 | .52 | .67 |
| 1½ | .12 | .26 | .42 | .60 | .79 | 1.00 |
| 2 | .16 | .35 | .56 | .80 | 1.05 | 1.33 |
| 2½ | .20 | .44 | .70 | .99 | 1.32 | 1.67 |
| 3 | .25 | .52 | .84 | 1.20 | 1.58 | 2.00 |
| 3½ | .29 | .61 | .98 | 1.39 | 1.84 | 2.33 |
| 4 | .33 | .70 | 1.12 | 1.59 | 2.10 | 2.66 |
| 4½ | .37 | .79 | 1.26 | 1.79 | 2.37 | 3.00 |
| 5 | .41 | .88 | 1.40 | 1.99 | 2.63 | 3.33 |

*Nota.* **Las llaves de los tubos de agua deben cerrarse lentamente**, y la necesidad de esta precaución aumenta con el diámetro. De lo contrario la detención repentina, dada la velocidad adquirida, y conservada por el agua corriente en virtud de la inercia, causaría una gran presión contra los tubos en todos sentidos y en toda la porción de tubo situado detrás de la llave, aun cuando el tubo tenga muchos km de longitud, corriendo el peligro de romperse en cualquier punto. Por esta razón se cierran las llaves de retención por medio de tornillos que impidan cerrarlos de repente; pero en los tubos de gran diámetro, hasta los tornillos deben manejarse muy lentamente, para evitar que se revienten.

**Art. 18. Flotación en los líquidos.** Cuando se coloca un cuerpo en un líquido, sea que flote ó que se sumerja, desaloja un volumen de líquido igual al volumen de la parte sumergida del cuerpo. En ambos casos, y en cualquier profundidad y posición, el cuerpo sumergido recibe un empuje vertical de abajo hacia arriba igual al peso del líquido desalojado. De manera que si sumergimos enteramente en agua un pedazo de corcho, fig. 26, ó cualquier otro cuerpo cuyo peso específico sea menor que el del agua, el corcho tiende, por su peso, á descender á mayor profundidad; pero el empuje del agua hacia arriba siendo mayor que el peso del corcho, obligará á este último á subir con una fuerza igual á la diferencia de las dos fuerzas. En este caso el corcho recibe una presión total de arriba abajo igual al peso de la columna vertical de agua que tiene encima (indicada en el envase por líneas verticales) y una presión total de abajo hacia arriba igual al peso de la columna de agua indicada en el envase 2. *La diferencia* de estas dos columnas (véanse las figs.) es evidentemente igual al volumen del corcho mismo; por consiguiente, la diferencia de sus pesos ó presiones (ó, en otras palabras, el empuje del agua) es igual al peso ó presión del aguá que ocupa el lugar del corcho, ó, en otros términos, al peso del agua desalojada por el corcho.

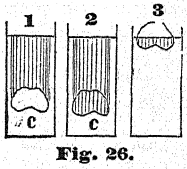


**Fig. 26.**

Esta *diferencia* ó empuje será evidentemente la misma, cualquiera que sea la profundidad á que esté sumergido. El corcho abandonado á sí mismo subirá hasta que una parte salga fuera de la superficie, como se ve en el envase 3; de modo

que la presión de la columna de arriba abajo deja de existir, y entonces el corcho está comprimido hacia abajo solamente por su propio peso. Pero como ahora permanece fijo, y sabemos que cuando dos fuerzas opuestas mantienen un cuerpo en equilibrio, estas dos fuerzas deben ser iguales, por tanto la presión del agua de abajo hacia arriba debe ser igual al peso del corcho. Pero esta presión del agua hacia arriba es producida por la columna sombreada que se muestra en el envase 3, y esta columna es igual *al volumen del agua desalojada.*

Por consiguiente, *en todos los casos* la fuerza de empuje es igual al peso del agua desalojada, y cuando el cuerpo *flota* en la superficie, el empuje ó peso del agua desalojada resulta igual al peso del cuerpo.

**Si el cuerpo sumergido, *c*, *c*, fuera de hierro** ó de cualquiera otra substancia de un peso específico mayor que el del agua, la diferencia de las presiones hacia arriba y hacia abajo sería siempre la misma : igual al peso del agua desalojada. Pero el peso del cuerpo es ahora mayor que el del agua desalojada, ó, en otras palabras, el empuje hacia abajo producido por el peso del cuerpo es mayor que el que, el agua desalojada ejerce hacia arriba, y por consiguiente el cuerpo desciende ó se sumerge con una fuerza igual á la diferencia de las dos. Por lo expuesto se ve que si el cuerpo fuera un sólido de hierro fundido de un decímetro cúb, como un decím cúb de agua dulce pesa 1 kg solamente, y el peso del hierro fundido es de 7.2 kg por litro, dicho cuerpo descenderá con una fuerza igual á 7.2 — 1=6.2 kg por cada litro de volumen que tenga. **Si el cuerpo sumergido tiene el mismo peso específico que el fluido,** no ascenderá ni descenderá, sino que permanecerá estacionario en donde se le ponga; porque entonces el peso del cuerpo y el empuje del agua son iguales.

**El aire también ejerce contra los cuerpos empujes hacia arriba iguales al peso del aire desalojado**; por consiguiente, aunque un kg de hierro y otro de plumas pesados en el aire se equilibran, sin embargo, colocados ambos en la redoma de una máquina neumática, el peso de las plumas excederá al peso del hierro, tanto más cuanto más exceda el volumen del aire desalojado por las plumas al volumen del aire desalojado por el hierro.

**Un globo sube en el aire por la misma causa que un corcho en el agua.** Su fuerza ascendente es igual á la diferencia entre su peso cuando está lleno de gas y el peso del volumen de aire que desaloja. El globo no tiende realmente á subir, sino á descender; pero siendo el aire, á volumen igual, más pesado que el globo, aquél empuja á éste hacia arriba con una fuerza mayor que con la que el peso del globo tiende á bajar. Lo mismo sucede con el humo caliente, que no tiende á subir por sí mismo, sino que es *empujado hacia arriba* por el aire frío que es más pesado. Ninguna materia tiende á subir, todas tienden á bajar hacia el centro de la tierra.

La acción de la gravedad hacia abajo puede considerarse como concentrada en el centro de gravedad G de un cuerpo flotante. La presión hacia arriba ó el empuje * del agua puede considerarse del mismo modo concentrado y obrando en el centro de gravedad W del agua desalojada †. W se llama también **centro de presión** del agua, y una línea vertical que pase por él se llama **eje de flotación.** Ordinariamente ‡ W varía de posición con cada cambio de posición del cuerpo. Así,

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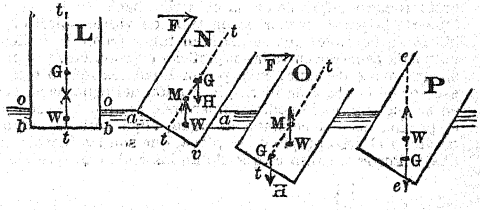
* **Este empuje se compone** de las presiones paralelas ejercidas de abajo hacia arriba por los innumerables filamentos verticales del agua desalojada, como se ve en la fig. 26, y **el eje de flotación** es la resultante de ellas, como en el caso de las fuerzas paralelas.

† **Sobre el cuerpo actúan de hecho otras fuerzas**, tales como las presiones horizontales del agua contra las partes sumergidas; pero como todas éstas están en una dirección dada, equilibradas por las de la dirección opuesta, no tienen influencia sobre las fuerzas G y W. También sufre el cuerpo la acción del aire, que lo comprime hacia abajo, con una fuerza de 1.033 kilg por cm cuad; pero ésta está equilibrada por la presión del aire que rodea la superficie del agua, la cual se transmite verticalmente hacia arriba á la parte inferior ó fondo del cuerpo sumergido.

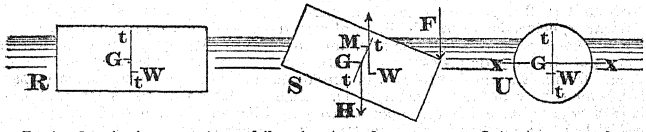
‡ La forma de un cuerpo (como la de una esfera ó un cilindro U) puede ser tal, que la posición de su centro de presión W, con relación á la de su centro de gravedad G, no cambie por la rotación del cuerpo alrededor de un eje dado (como cualquier eje de la esfera ó del eje *longitudinal* del cilindro), sino que permanezca siempre en la misma línea vertical que G, de manera que el cuerpo al rotar permanece en equilibrio. Un cuerpo así se dice que está en **equilibrio indiferente** alrededor de su eje. Pero si se hace rotar á un cilindro U en la dirección de su eje *transversal xx*, quedará comprendido en el caso de las observaciones hechas acerca de las figs. R y S, y puede hallarse (antes de rotar) en equilibrio estable ó inestable alrededor de aquel eje según el modo de distribución de su peso.

en L, está en el centro de gravedad del rectángulo *oobb;* y en N, está en el de gravedad del triángulo *aav.*

Cuando un cuerpo flotante, L, P ó R, está en reposo, sin que una tercera fuerza, F, lo perturbe, se dice que está en **equilibrio**, y G y W se hallan entonces en la misma línea vertical *tt*, figs. L y R, ó *ee*, fig. P. Esta línea se llama **eje ó vertical de equilibrio.**




Cuando una tercera fuerza, F, produce la inclinación del eje de equilibrio, como en las figs. N, O y S, entonces, se tira una línea vertical desde el centro de presión W hacia arriba, el punto M, donde dicha línea corta el eje mencionado, se llama **metacentro** del cuerpo*; G y W no continúan entonces en la misma línea vertical**, y las dos fuerzas opuestas y verticales de gravedad y de empuje, *obrando* sobre aquellos puntos respectivamente, forman un « par »; cuando se suprime la tercera fuerza F, ellas no mantienen por más tiempo el cuerpo en equilibrio, sino que le comunican un movimiento de rotación. Si las posiciones de G y W (como en las figs. O y S) son tales que el metacentro M se halle más *arriba del centro de gravedad* G, está rotación tenderá á *volver el cuerpo á su posición anterior*, y se dice que el cuerpo ha estado (antes de la aplicación de la tercera fuerza F) en **equilibrio estable** §. Pero si (como en N) M se halla *debajo* de G, la dirección de la rotación es tal que puede *volcar* el cuerpo, *alejándolo más de su posición anterior*. Se dice entonces que el cuerpo estaba en **equilibrio instable**. Véase nota ÷, pág. 546.




La tendencia ó momento en kilográmetros de un cuerpo flotante para volcarse ó para enderezarse es

= peso del cuerpo (ó su igual, es decir, la presión del agua hacia arriba) en kg × dist horizontal comprendida entre WM y GH, figs. N, O, y S, en m.

La tercera fuerza F puede ser tan grande que venza la tendencia del cuerpo á enderezarse. De esta manera, un barco puede ser volteado por un huracán á pesar de haberse cargado y lastrado bien para los vientos ordinarios. La sección horizontal de un cuerpo por la línea de flotación se llama **plano de flotación.**

**Art. 19. Un cuerpo menos denso que el agua colocado en el fondo de un envase que contenga aquel líquido, no subirá, á menos que el agua penetre por debajo de él y lo haga subir del mismo modo que el aire hace subir á un globo.** Así, si un bloque de madera liviana, perfectamente plano y liso, se coloca en el fondo de un envase igualmente plano y liso, y si se le sujeta allí hasta que se haya llenado de agua dicho envase, la presión hacia abajo le retendrá en su lugar hasta que el agua penetrando por los

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* Este metacentro cambia de posición en la línea *tt*, según la inclinación de esta última.

§ **Una carga desigual** puede hacer las veces de una tercera fuerza y lograr que un envase en reposo se incline como en P, y sin embargo el envase inclinado de este modo puede estar en equilibrio, porque el eje *ee* de equilibrio puede ser vertical aunque no coincida con el **eje de simetría** del envase, como pasa con *tt* en L.

** Este caso puede ocurrir algunas veces en los cuerpos *flotantes* (como en las figs, R y S), aun cuando el *centro de presión* W (*no el metacentro*) se halle *debajo* del centro de gravedad G, porque cuando el cuerpo está forzado á inclinarse, W cambia de lugar en el cuerpo y esta nueva posición puede ser tal, que M venga á quedar arriba de G. W está siempre debajo de G en los cuerpos de densidad uniforme (homogéneos) que flotan en reposo, si una parte cualquiera del cuerpo está fuera del agua. Si los cuerpos están enteramente sumergidos W y G coinciden.

poros de la madera se aloje debajo de él. Pero si se alisa y se le da barniz al pedazo de madera, á fin de que el agua no penetre en sus poros, aquél permanecerá en el fondo.

**Por otra parte, se puede evitar que una pieza de metal se vaya á fondo,** sometiéndola solamente á la acción de una presión suficiente, *de abajo hacia arriba* y suprimiendo la de arriba hacia abajo. Así pues, si se alisa suficientemente la parte inferior de un tubo de vidrio abierto *t*, fig. 27, y una placa de hierro *m*, de modo que se adapten tanto (como se indica en la fig.) que el agua no pueda penetrar entre las superficies en contacto, y si en esta posición se colocan en un envase de agua, cuya profundidad sea más ó menos mayor que 8 veces el espesor de la placa de hierro, la presión del agua hacia arriba mantendrá la placa en su lugar y le impedirá sumergirse, porque está comprimida hacia arriba por la presión de una columna de agua, cuyo peso es mayor que el de la columna de aire y el peso propio de la placa, que son las fuerzas que empujan á ésta hacia abajo. Por esta causa es que flotan las embarcaciones de hierro.

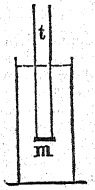

Fig 27

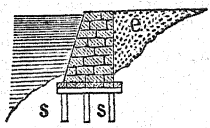

Fig 28

*Nota* 1.ª **Un muro de sostenimiento construído sobre estacas como en la fig. 28,** puede ser suficientemente fuerte para resistir el empuje de las tierras *e*, colocadas detrás de él, en el caso de que el agua no penetre por debajo, y sin embargo, puede volcarse si el agua penetra por debajo y aun si la tierra *ss*, situada alrededor de las estacas, se satura de agua y se convierte en barro ó pantano fluído. En uno ú otro caso la presión hacia arriba del agua contra la pared inferior ó base del muro, reducirá virtualmente el peso de todas las partes, *que están bajo la superficie del agua* á razón de 1000 kg por metro cúb ó casi en la mitad del peso ordinario de la mampostería de piedra bruta y mezcla.

*Nota* 2.ª Aunque las estacas situadas debajo de un muro, como en la fig. 28, sean más que suficientes para sostener el peso del muro, y éste *por sí mismo* sea igualmente fuerte para resistir la presión del relleno *e*, sin embargo, si la tierra *ss* del contorno de las estacas fuere blanda, tanto ellas como el muro pueden ser empujadas, y este último volcado por la presión del relleno *e*. Por esta razón, las alas de los puentes, *si están construídas sobre estacas* en un suelo muy blando, se abomban frecuentemente hacia afuera y se deforman. En estos casos, la estaca y la plataforma de madera que se coloca sobre ésta deben extenderse sobre todo el espacio comprendido entre los muros; ó aplicar cualquier otro preventivo.

**Art. 20. Calado de los barcos.** Como un cuerpo *flotante* desaloja un peso de líquido igual al peso del cuerpo, podemos calcular el peso de un barco y de su carga determinando el número de metros cúb de agua que desaloja. El número de m cúb multiplicado por 1,000 nos dará el peso en kilog. Suponiendo, por ejemplo, un barco plano con lados de 25 m de largo, 5 m de ancho y .15 m de calado; cuando está sin carga desalojará $125 \times .15 = 18.75$ m cúb de agua, que pesan $18.75 \times 1{,}000 = 18{,}750$ kilgs, que será también el peso del barco. Si dicho barco después de cargado cala un metro más, tendremos para peso del agua desalojada por la carga solamente $25 \times 5 \times 1{,}000 = 125{,}000$ kilgs, que es al mismo tiempo el peso de la carga. De igual modo, conociendo de antemano el peso del barco y de la carga y las dimensiones de aquél, podemos hallar el calado. Así, si el peso fuera como antes de $143{,}750 = (18{,}750 + 125{,}000)$ kilgms y las dimensiones del barco $25 \times 5$, tendremos $25 \times 5 \times 1{,}000 = 125{,}000$; y $\frac{143{,}750}{125{,}000} = 1.15$ m, cala buscada. En los barcos de formas más complejas, como los barcos de vela ordinarios, el cálculo del agua desalojada es más complicado; pero el principio es el mismo.

# HIDRÁULICA

## Movimiento del agua en las tuberías.

Muchas de las teorías hidráulicas son todavía materia de discusión. Esto, y las imperfecciones de los trabajos, hace que sea prudente usar liberales coeficientes de seguridad al emplear las fórmulas hidráulicas. Aun los tubos nuevos se oxidan y se forman escorias que disminuyen la corriente; y aquéllas se aumentan á veces considerablemente por las substancias químicas que contiene el agua. El aire en los tubos también disminuye la corriente.

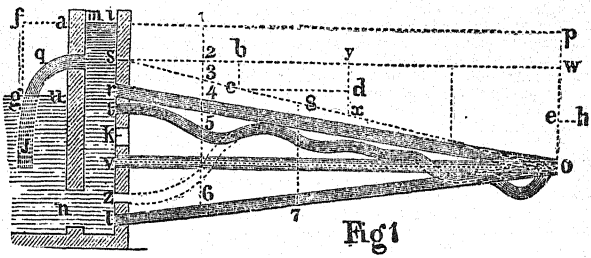

Fig 1

El término **carga ó carga total de agua** aplicado al derrame del agua por canales, tubos ú orificios en estanques, etc., quiere decir la distancia *vertical iv* ó bien *op*, fig. 1, de la superficie *m i* del agua en el estanque, ó fuente de alimentación al centro (ó más bien el centro de gravedad) *o*, del orificio (sea éste el extremo del tubo *ro*, *to*, *vo*, *zo*, *lo*), ó cualquiera otra clase de orificio por el cual tiene lugar la salida del agua libremente *al aire;* ó la distancia vertical *au*, ó *fg*, de la misma superficie *mi*, á la *superficie del nivel gu*, del agua en el estanque más bajo, cuando la salida se efectúa *por debajo del agua*. Así, en el caso de la salida del agua al aire, la distancia vertical *iv* ú *op* es la carga total para cualquiera de los tubos *ro*, *to*, *vo*, *zo* ó *lo;* siendo *ik* la carga para el orificio *k*, en la pared del estanque. Y para la salida por *debajo del agua*, *au*, ó *fg*, es la carga, tanto para el tubo *f*, como para la abertura *n*, independientemente de sus profundidades bajo la superficie del agua más baja, lo cual, según los autores más antiguos, no influye en nada en la salida del agua. Una parte del tubo puede tener una carga mayor que la carga correspondiente al tubo entero. Así, el punto 6 del tubo *ol* tiene la carga 6.1, mientras que el tubo entero no tiene sino una carga *op*.

**Tanto en la teoría como en la práctica, la velocidad y el gasto son independientes de la posición del tubo, ya esté inclinado hacia abajo, como *ro*, fig. 1, horizontal, como en *vo*, ó inclinado hacia arriba, como *lo*, con tal que la carga total del agua *op* y el largo del tubo sean los mismos.** Cuando un tubo es más largo que otro, sus paredes evidentemente producirán más rozamiento contra el agua y disminuirán de este modo la velocidad y el gasto. Los tubos inclinados *ro*, *lo*, siendo, por supuesto, un poco más largos que el horizontal *vo*, producirán por tanto, cada uno, un gasto un poco menor; pero si el tubo horizontal se prolongase un poco más allá de *o* á fin de darle la misma longitud de los otros, entonces cada uno de los tres producirá el mismo gasto.

**Art. 1 *a*. División de la carga total.** En cualquier tubo *so*, *ro*, *to*, *vo*, *zo*, ó *lo*, fig. 1, tiene la carga que realizar tres trabajos : 1.°, vencer la *resistencia* de la entrada en *s*, *r*, *t*, *v*, *z* ó *l;* 2.°, vencer la resistencia *dentro* del tubo; y 3.°, comunicar al agua que entre en el tubo, la *velocidad* con que realmente corre. Por conveniencia consideramos la carga total dividida en tres partes, correspondientes á estos tres trabajos; á saber: 1.°, la carga de *entrada;* 2.°, la carga de *resistencia* ó *rozamiento;* y 3.°, la carga *de velocidad*.

**Art. 1** *b*. **La carga de velocidad** es la altura de la cual un cuerpo tiene que caer en el vacío para adquirir la velocidad efectiva con que el agua corre dentro del tubo. Es por lo tanto $= \frac{v^2}{2g}$, en que $v$ es la velocidad en metros por segundo; y $g$ (9.81 m), la aceleración debida á la gravedad.

**Art. 1** *c*. Los experimentos hechos han demostrado que **la carga de entrada** en los orificios comúnmente usados, de arista viva, es, con una aproximación suficiente en la práctica, igual á la mitad de la carga de velocidad. Si la entrada ó boca del tubo tiene la forma de la fig. 7, casi no se necesita carga de entrada; pero en los tubos de una longitud mayor que 1,000 veces el diám, la carga de entrada es una fracción tan pequeña de la carga total, que la ventaja que ofrece aquella forma es de poca importancia; es de alguna consideración en tubos más cortos.

**Art. 1** *d*. Supongamos que, en la fig. 1, *is* represente la suma de las cargas de velocidad y de entrada para cualquiera de los tubos. Entonces lo restante *sv* ó *wo* de la carga total, es la **carga de rozamiento** ó la carga exactamente suficiente para hacer equilibrio al rozamiento y á las otras resistencias del interior del tubo, y como la carga de *entrada* equilibra la resistencia á la entrada del tubo, la carga de velocidad sólo tiene que comunicar velocidad al agua en el envase, haciéndola entrar en el tubo con la misma rapidez que corre en él, conservando así el tubo siempre lleno. Si recortando el tubo ó alisando su superficie interior, disminuímos el rozamiento total, se necesitará entonces menor carga de rozamiento; pero la velocidad aumentará al mismo tiempo y ésta necesitará una carga mayor de velocidad y de entrada, de modo que las tres unidas, juntas producen la misma carga total que antes. Como el *rozamiento* es igual á la presión ó carga que se necesite para vencerla, se representa también por *wo*.

**Art. 1** *e*. Toda la carga de rozamiento puede hallarse como en *vo*, *zo* y *lo*, fig. 1, por *encima* de la boca de entrada del tubo, y de consiguiente *por fuera* de él; ó como en un tubo colocado de *s á o*, toda ella *debajo* de la entrada, y por tanto en *el interior* del tubo, ó parte de ella por encima y parte por debajo de dicha entrada, como en *ro* y *to*, y de consiguiente parte en el exterior y parte en el interior del tubo. La velocidad y el gasto, *después de lleno el tubo*, no son afectadas por esta diferencia de la posición de la boca de entrada; pero sí lo son las presiones y las velocidades *en el tubo mientras se llena el tubo vacío* con el agua, como se explica después.

**Art. 1** *f*. **Pero es necesario que el extremo del tubo por donde entra el agua esté colocado á una distancia tal, debajo de la superficie** *mi* del agua, que deje por encima de su centro de gravedad por lo menos una carga *is* suficiente para efectuar los trabajos de la entrada y la velocidad. Si la boca de entrada de cualquiera de los tubos se eleva sobre *s*, una parte de la carga de *velocidad* estará por dentro del tubo. En otras palabras: la carga en el *interior* del tubo será más que suficiente para vencer la resistencia en dicho tubo y el sobrante obrará como carga de velocidad, comunicándole mayor velocidad al agua *dentro* del tubo. La carga *reducida*, que de este modo se deja por sobre la boca de entrada dicha, será claramente insuficiente para mantener la alimentación con esta velocidad *mayor*, y el tubo no se llenaría sino parcialmente.

En los casos ordinarios de tuberías de una longitud considerable, la suma de las cargas de entrada y de velocidad que se necesitan teóricamente, es tan sólo una pequeña fracción de la carga total, y raras veces pasa de 30 cm. En un tubo de considerable diám, la mitad superior de su sección transversal en la boca de entrada, es á menudo más que suficiente para proporcionar la carga de entrada y de velocidad con la distancia que existe entre el centro de gravedad de dicha sección transversal y el borde superior del tubo, de manera que la parte superior de la boca del tubo podría sobresalir de la superficie del agua en el estanque; pero en la práctica, la boca del tubo debe estar siempre enteramente debajo de la superficie del agua, pues de lo contrario el aire y algunas impurezas flotantes entrarían en el tubo y formarían obstrucciones. Además, la superficie del agua en los estanques siempre está expuesta á considerables cambios de altura, y el extremo ó boca de entrada del tubo debe colocarse á tal profundidad que el agua pueda entrar con suficiente velocidad cuando esté en *su nivel más bajo*. Como ya se ha dicho, esto no causará disminución ni aumento de gasto.

**Art. 1** *g*. **Hallar la carga de rozamiento requerida para una parte cualquiera de un tubo**, conociendo la carga de rozamiento de todo el tubo. Como el rozamiento en un tubo de diám uniforme (en igualdad de circunstancias)

es proporcional á su longitud, y como *wo*, fig. 1, representa la carga total de rozamiento, tenemos :

Longitud total de tubo : Longitud de la porción dada :: *wo* : Carga de rozamiento para esta porción.

O habiendo trazado *wo* por escala, *sw* horizontal, y á *so* tendremos :

Longitud total del tubo : Longitud de la porción dada :: *so* : Una dist como *sc* tomada desde *s* sobre *so*.

O :: *sw* : Una dist como *sb* tomada desde *s* sobre *sw*.

Entonces una línea vertical *bc*, tirada desde *b* ó *c*, que una á *sw* y á *so*, dará en partes de la escala la carga de rozamiento buscada.

**Art. 1** *h*. Si el tubo es recto como *ro*, *vo*, *lo*, el rozamiento en una parte cualquiera, *que empiece en el estanque*, por ejemplo, como la porción *l* 6 del tubo *lo*, puede encontrarse inmediatamente trazando la línea 6–1 verticalmente hacia arriba á partir del eje del tubo en el punto 6. La línea 2–3 dará entonces el rozamiento de *l*–6. También dará el de *r*–4 ó el de la porción de *vo* comprendida entre *v* y la línea de puntos 1–6. Debe tenerse presente que, en la fig. 1, todos los tubos se suponen de la misma long efectiva. En este caso terminarán todos en puntos distintos del punto *o*, y debe hacerse un diagrama separado para cada uno. En una porción de tubo que no principie en el estanque, como en los tubos *ro*, *vo*, *lo*, la parte comprendida entre las verticales de *c* y *x*, el rozamiento está dado, por ej., por la línea *d x*, porque es igual á *yx* — *bc*.

**Art. 1** *j*. **Si el tubo es vertical como** *vo*, fig. 1 A, sea *is* (sobre su eje *io*), como antes, la suma de las cargas de velocidad y de entrada. Desde *s*, *v* y *o* res-

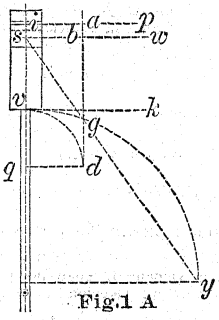

Fig. 1 A

pectivamente tírense las líneas horizontales *sw*, *vk*, *oy*, haciendo *oy*=*vo*. Tírese la oblicua *sy* para luego hallar el rozamiento en una porción cualquiera *vq* que empiece en el estanque; tírese desde *q* la línea *qd* horizontal é igual á *vq;* tírese la línea vertical *ad*, que corta á *sy* en *g*. Entonces *bg* dará el rozamiento en *vq*.

**Art. 1** *k*. **Si el tubo es curvo** y la curvatura está uniformemente distribuída en toda su longitud, ó es tan suave que puedan despreciarse las cargas de rozamiento requeridas para las diferentes partes del tubo, se pueden hallar de la misma manera que para tubos rectos, como en el art. 1 *h*. De lo contrario, deben buscarse por medio de proporciones como en el art. 1 *g*.

**Art. 1** *l*. **Durante el tiempo en que se llena de agua un tubo vacío,** el exceso de la carga total sobre la de rozamiento, etc., requerida, comunica al agua una **velocidad mayor** que la que tiene después de lleno el tubo; pero esta velocidad disminuye gradualmente, á medida que el agua que avanza va sufriendo el rozamiento de la porción creciente de tubo lleno, y llega á su mínimum cuando el agua llena todo el tubo y comienza á salir por la boca de descarga *o*. Pero si sólo queda por encima de la boca de entrada, la carga de velocidad y de entrada, como sucede en los tubos colocados en la posición *so*, es claro que no habrá tal exceso de la carga total, y en consecuencia ningún cambio de velocidad mientras se llena el tubo.

**Art. 1** *m-r**. **Relación entre el gasto, área, velocidad y presión.** En la fig. 1 B-D, el agua corre en el tubo, *b*F, á caño lleno y aquél se surte, en *b*, de un estanque ilimitado, R, y se descarga por un orificio, F; el volumen de agua que pasa por cualquier sección transversal del tubo, *b*F, es constante é igual á la que sale por F. Así, si la descarga en F es de Q litros por segundo, igual cantidad pasará por seg en cualquier sección de *b*F.

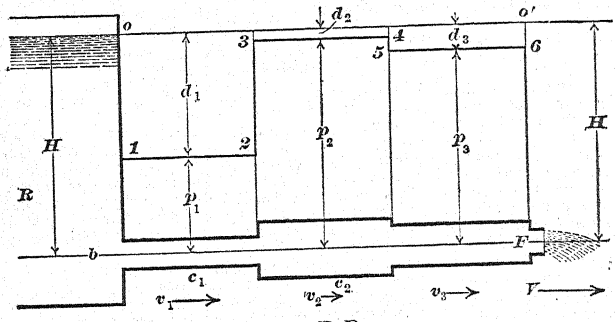


**Fig. 1 B-D.**

Sea $a$=área de la sección transversal, V=velocidad de la corriente en el tubo estrecho, después de F. V se llama la velocidad de salida. Sean $A_1$, $A_2$, etc., las diversas secciones del tubo *b*F, y $v_1$, $v_2$, etc., las respectivas velocidades del líquido en dichas secciones. Entonces $Q = aV = A_1v_1 = A_2v_2$, etc.; ó bien $V = \frac{Q}{a}$, $v_1 = \frac{Q}{A_1}$, $v_2 = \frac{Q}{A_2}$, etc. En otras palabras, la velocidad está en razón inversa del área de la sección transversal. También, $a = \frac{Q}{V}$, $A_1 = \frac{Q}{v_1}$, etc.

Las pérdidas de presión debidas á las respectivas velocidades, son $d_1 = \frac{v_1^2}{2g}$, $d_2 = \frac{v_2^2}{2g}$, etc.; como están representadas por las ordenadas entre la línea *oo'*, de la presión estática y la línea quebrada, *o* 1 2 3 4 5 6 F, de las presiones efectivas. La diferencia, debida á la velocidad, entre las presiones en dos puntos cualesquiera, como $c_1$ y $c_2$, donde las velocidades son, respectivamente, $v_1$ y $v_2$, es $p_2 - p_1 = d_1 - d_2 = \frac{v_1^2}{2g} - \frac{v_2^2}{2g} = \frac{v_1^2 - v_2^2}{2g}$ *. Lo que queda de la presión (parte de la carga) en cada punto, $p_1$, $p_2$, etc., es=á la carga total estática en el depósito — la carga de velocidad en dicho punto, $= H - d_1$, $H - d_2$, etc. La pérdida de carga en F, es (6F)=$p_3 = H - d_3$; y luego la presión baja hasta cero, es decir, á la presión atmosférica.

**Art. 1** *s*. **Piezómetros abiertos.** Si el extremo bajo de tubos verticales ó inclinados se inserta en un tubo, *b*F, fig. 1 B-D, digamos en $c_1$, ó $c_2$, etc., la superficie del líquido en estos tubos llegará á las alturas respectivas, $p_1$, $p_2$, que corresponden á las cargas (presiones) en los puntos donde se inserte el tubo. Estos tubos se llaman piezómetros abiertos. Para poder observar la altura á que llega el líquido, se hacen de cristal ó vidrio, por lo menos en la parte adonde se cree que llegará el nivel. Una obstrucción en el tubo entre $c_2$ y F, *levantará* el nivel del agua en el piezómetro en $c_2$; mientras que una obstrucción entre *b* y $c_2$ lo *bajaría*.

**Art. 1** *t*. Si imaginamos un tubo cualquiera lleno de agua provisto de cierto

* En el art. 1 *m-r*, se desprecian, para mayor sencillez, todas las resistencias, inclusives las debidas á los aumentos ó disminuciones bruscas del diámetro de los tubos.

número de piezómetros, la línea que junte los extremos superiores de las columnas de agua en los diferentes piezómetros se llama **línea de pendiente hidráulica.**

**Art. 1** *u.* En un tubo recto de diámetro uniforme en toda su extensión, como *ro*, *vo*, ó *lo*, fig. 1, lleno de agua corriente y con salida libre al aire, la línea de pendiente hidráulica es una línea recta trazada del punto de salida *o* á un punto *s* situado inmediatamente sobre la boca de entrada del tubo, y á una profundidad, bajo la superficie, igual *á la suma de las cargas de velocidad y entrada.*

**Si se reduce el orificio en** *o*, la línea de pendiente hidráulica debe tirarse de *s* á un punto *e* situado inmediatamente encima de *o*, cuya altura depende de

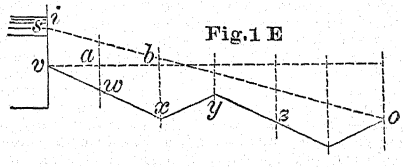

Fig.1 E

la reducción hecha en *o*. Pero en este caso el punto *s* estará también más alto que antes, porque la velocidad en el tubo está disminuída por la reducción, y la suma *is* de las cargas de velocidad y de entrada será menor.

**Si la salida ó descarga por** *o* **se efectúa debajo del agua,** el efecto que esto produce sobre la posición de la línea de pendientes, será análogo al que produce la contracción del orificio en *o*. El punto *e* estará en la superficie del agua más baja é inmediatamente encima de *o*.

**Art. 1** *v*. Si el tubo de diám uniforme (sea de salida libre ó á través de un orificio reducido en *o*, ya sea al aire libre ó bajo el agua) **está doblado ó encorvado,** la línea de pendiente hidráulica será sin embargo recta con tal que las resistencias sean iguales en cada una de las divisiones iguales en que consideremos partida la longitud horizontal del tubo, como en la fig. 1 E, en que á divisiones iguales *vw*, *wx*, etc., de la longitud total, corresponden divisiones iguales *va*, *ab*, etc., de la longitud horizontal. Pero en la fig. 1 F, la línea de pendiente hidráulica tomará la forma *sao;* porque si, de acuerdo con el art. 1 *g*, dividimos á *so* en dos partes guales *sm*, *mo* que correspondan á las partes iguales *vr*, *ro* de la longitud del tubo, obtendremos *me=ae* para la carga consumida en las resistencias de *vr*, dejando solamente *ra* para la carga de presión en *r*.

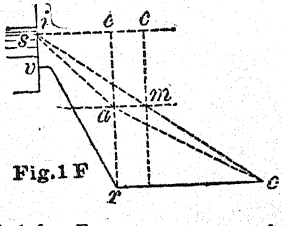

Fig.1 F

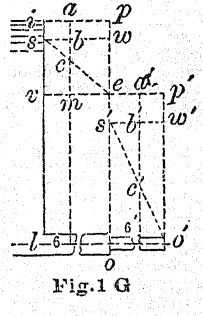

Fig.1 G

**Art. 1** *w*. En un envase muy grande, la carga total en cualquier punto al nivel de la entrada *l*, de un tubo *loo'*, fig. 1 G, está representada por *il* como ya se ha explicado; pero de esta carga total, una parte *is* obra como carga de velocidad y de entrada de la abertura *l*, quedando *sl* como carga de presión en un punto del *tubo*, situado inmediatamente á la derecha de *l*. Así, mientras que la presión en lbs por pulg cuad es en el *envase* en *l*, $p=il\times.434$; la presión en el *tubo* en *l*, es $p=sl\times.434$, ó empleando el sistema métrico tendremos, en el primer caso, $p=il$, (en m)$\times.1$, y en el segundo, $p=sl$ (en m)$\times.1$, = la presión en kg por cm cuad

Pero una parte *sv* de *sl* se emplea en *lo* en (vencer) las resistencias de esa parte del tubo; y al realizar este trabajo, disminuye gradualmente desde *sv* que es su valor (en *l*) hasta llegar á nada (en *o*), como lo indica la línea de puntos *se*. De manera que en el punto 6 una parte=*bc* se ha consumido ya en vencer las resistencias del tubo entre *l* y 6, quedando *c*6 como carga de presión en 6, de la cual *cm* se gastará aún en vencer las resistencias del tubo ancho entre 6 y *o*, quedando *m*6=*vl*=*eo* como carga de presión para un punto situado justamente al lado izquierdo de la contracción existente en *o*. La presión en *lo* ha disminuído gradualmente de *sl* que era (en *l*) á *eo* = *vl* (en *o*). Ahora se necesita que una parte *es'* de *eo* obre como carga de velocidad y de entrada en *o* en la parte más angosta *oo'* del tubo, porque necesitamos en *o* no solamente una carga de *entrada* adicional para vencer la resistencia que opone el recodo formado por el cambio de diámetro, sino también una carga de *velocidad* adicional para comunicar el aumento de velocidad que debe tener efecto cuando el agua pasa del tubo ancho *lo* al más angosto *oo'*, porque mientras que un tubo permanece *lleno* de agua corriente y la salida se conserva constante, la *velocidad* en cada parte del tubo debe estar *en razón inversa del área de la sección transversal* de aquella parte, puesto que en cada segundo pasa la misma cantidad de agua por cada uno de los puntos, y esta cantidad constante es igual al área×la velocidad. Por esto, como el área disminuye, la velocidad tiene que aumentar. Por tanto, *s'o* queda como presión sobre un punto de la porción estrecha situada justamente á la derecha de *o*, y ésta, á su vez, disminuye gradualmente hasta llegar á cero en el extremo *o'*, del tubo, como lo indica la línea de puntos *s'o'*, habiéndose gastado toda en vencer la resistencia de *oo'*. Tenemos, pues, como línea de pendiente hidráulica en la fig. 1 G, la línea quebrada *ises'o'*.

Cuando la presión disminuye de este modo con las resistencias ó por aumentos de velocidad, la disminución se llama **pérdida de carga**. Así decimos que *is* **se pierde** á la entrada *l*; *sv* como carga de rozamiento en la porción *lo*; *es'* por la contracción existente en *s* y *s'o* como carga de rozamiento del tubo *oo'*.

**Cuando el aumento de velocidad** en cualquier punto *o* **es muy grande**, entonces la carga de velocidad requerida para tal aumento (sumada con la carga de entrada) puede llegar á ser tan grande como la carga de presión total disponible en aquel punto. De consiguiente, en tal caso la presión cesa enteramente, y la línea de pendiente hidráulica desciende al nivel del eje *oo'* del tubo. Ciertamente, la carga de velocidad requerida puede *exceder* y á menudo *excede* á la carga de presión disponible, causando una presión *negativa* ó presión *hacia el interior*, es decir, una **succión** (tendencia al vacío); de manera que si, se coloca hacia abajo un piezómetro desde el tubo al interior de un envase que contenga agua, la presión del aire sobre la superficie de ésta sostendrá en el piezómetro una columna de agua que se levanta desde el envase hacia el tubo; y la línea de pendiente hidráulica, para el punto donde el piezómetro se une al tubo, se hallará *debajo del eje oo*, del tubo á una distancia vertical, igual á la altura de dicha columna. Véase medidor Venturi, págs. 570, etc.

**El sifón.** Si una rama *ab* de un tubo encorvado *abc*, fig. M, de cualquier diám, lleno de agua y tapado por ambos extremos, se coloca en un estanque con agua,

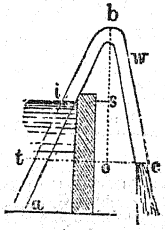


Fig. M.

como el de la fig., y si entonces se destapan ambos extremos, el agua del estanque principiará á salir en *c* y continuará saliendo hasta que su nivel llegue en *t* á la

misma altura del extremo más *alto c*, del tubo ó sifón. La salida del agua se detendrá entonces. Las partes *ab* y *bc* se llaman ramas del sifón, siendo *b* el vértice, y así se dice en lo que se refiere á dicho sifón considerado como un pedazo de tubo; pero si lo consideramos como una máquina hidráulica, la parte, *ta*, bajo el nivel del extremo más alto *c*, no tiene ninguna importancia, porque el agua en el estanque no descenderá bajo el nivel de la boca más alta, sea ésta la exterior ó la interior. Por esta razón, si el extremo de salida está más alto que la superficie del agua, por ejemplo en *w*, no habrá salida. La altura *vertical ob*, del vértice del sifón al nivel más bajo *t* (hasta donde descenderá el agua en el estanque), no debe teóricamente exceder de 10.33 m, que es la altura de una columna de agua igual á la presión atmosférica. En la práctica debe ser menor, teniendo en cuenta el rozamiento del agua corriente con el tubo y el aire que se introduce. Y aun menos todavía en los lugares situados sobre el nivel del mar, porque en estos lugares el peso reducido de la presión atmosférica no equilibrará una columna de agua de aquella altura. Para que se comprenda fácilmente, ó para que se recuerde en cualquier tiempo el principio en que se basa el sifón, imaginemos que podemos considerar teóricamente el extremo de la rama interior, como no sumergida *bajo* la superficie del agua, sino que se mantiene precisamente *en* ella, á medida que ésta va descendiendo por causa de la salida del agua en el otro extremo; pero si debemos considerar la altura *bo* como longitud de la rama exterior, el largo de la interior será una dist variable que al principio es *bs* y que finalmente llega á ser *bo* (cuando desciende el agua en el estanque). La corriente solamente continúa mientras *esta rama exterior permanezca más larga que la interior*. No hay razón en lo que dicen los libros que la rama exterior *bc debe* ser más larga que la interior *ba* para que el agua salga. El principio es simplemente el siguiente : que habiendo llenado primeramente de agua ambas ramas *bc*, *bi* (considerada primero la porción *ta* del *estanque* y no del sifón), cuando se destapan los extremos *c* y *a*, el aire ejerce la misma presión sobre éstos; pero la gran carga vertical *bo*, obrando en la rama exterior *bc*, ejerce presión contra el aire en *c* con una fuerza mayor que la que ejerce la pequeña carga *bs* en la rama interior *bi* contra el aire en *a* ó *i* *. Por consiguiente, el agua en *bc* tiende á salir más rápidamente que en *bi*, y al comenzar á salir se produciría un vacío en *b* si no fuera que la presión del aire contra el otro extremo *a* ó *i* empuja el agua de abajo arriba hacia *ib*, para ocupar el lugar del agua que sale por *c*.

De esta manera continúa la salida hasta que la superficie del agua en el estanque descienda hasta *t* al mismo nivel que *c*, y siendo entonces las presiones verticales iguales á *bo*, tanto en *bc* como en *bt*, cesa dicha salida.

El principio del sifón puede emplearse para desaguar estanques ó pozos, trasegando el agua á lugares más bajos situados á distancias considerables. En la práctica no debe *exceder* más ó menos de 8.53 m sobre el nivel á que llegará el estanque después de vaciado. En estos casos debe colocarse una **válvula** adecuada en el vértice ó vértices (si hubiere más de uno) de los codos, para dar salida al aire que inevitablemente penetra y pronto detendría el curso del agua si no se toma esta precaución. La válvula de aire no dará resultado aquí; porque tan pronto como se abre la válvula *v*, se convierte el sifón en dos tubos separados abiertos en la parte superior y el agua saldrá por ambos. Se necesita de un orificio en el escape para llenar el sifón al principio, y para evitar que el agua introducida se salga deben colocarse llaves de retención en los extremos, las que se conservarán cerradas hasta que se haya llenado el sifón completamente. Debe tenerse mucho cuidado en unir las juntas de los tubos herméticamente.

**La fuerza motriz ó carga** que produce la corriente en un sifón, es la dist vertical *so* de la superficie del estanque al extremo de salida *c*, ó, en otras palabras, la dif *so* entre las longitudes teóricas *bo* y *bs* de las dos ramas. Por consiguiente, mientras más baja esté la boca *c* respecto de *s*, más rápida será la corriente, y, por el contrario, á proporción que el nivel del estanque baje con respecto á *s*, más lenta va siendo la salida. Con esta carga, la longitud total *abc* del sifón y su diám, puede hallarse aproximadamente el gasto por cualquiera de las reglas dadas en el art. 2.º para tubos rectos. Estas reglas dan 252.16 litros por minuto en lugar de 197.64 litros que realmente salen por el sifón del coronel Crozet con una carga de 6.09 m. Véase más abajo.

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* Se entiende que dicha presión de aire no se ejerce directamente sobre *a* ó *i* sino que se transmite á través del agua del envase hacia *a* y luego hacia *i* á través del agua del sifón.

**En un sifón real**, *agnyo*, fig. 1½, sin aire en su interior, y lleno de agua corriente, **la carga total** *op* se mide verticalmente de la superficie, *mi*, en el estanque al centro de gravedad del orificio de salida *o*, como en la fig. 1; **la línea de pendiente hidráulica** (con la restricción mencionada en el art. 1 *v*), es como antes una línea recta *sro* trazada desde el pie *s* de las cargas de entrada y de velocidad combinadas, al extremo *o*, y el gasto y la velocidad son las mismas

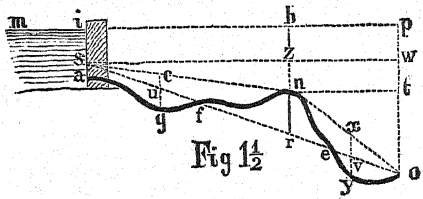


Fig 1½

que serían si todo el tubo estuviese debajo de *sro;* pero véanse las advertencias 1 y 2 que siguen.

**La presión en un punto cualquiera** *g*, *n* ó *y*, está representada por una línea vertical *gu*, *nr* ó *yv* trazada del punto en cuestión á la línea *sro;* pero para puntos como *n*, situados sobre *sro*, esta presión es *negativa* ó hacia *el interior;* mientras que en los puntos en que *sro* y el tubo están al mismo nivel, como en *f* y en *e* no hay ni presión ni vacío ó succión.

**Advertencia 1.** Pero si entra el agua á un tubo vacío por *a* estando el orificio de salida *o*, abierto, el tubo no será un sifón verdadero. En la parte *agn* correrá el agua llenando el tubo y *scn* será la línea de pendiente hidráulica de dicha parte; pero al llegar á *n* en la parte del tubo más inclinada *no*, el agua correrá con una velocidad mayor que aquella con que llegó de *an*.

Por lo tanto la vena líquida tendrá una sección transversal menor en *no* que en *an*, y no podrá llenar la parte *no* sino que correrá como en un canal abierto.

**Advertencia 2.** La tendencia al vacío en los puntos situados encima de *sro*, es causa de una acumulación de **aire** en *n* arrastrada por el agua, dentro del sifón, que penetra por las juntas no bien cerradas, etc., y esto produce un efecto que se aproxima á lo descripto en la advertencia 1.ª; porque su fuerza elástica reduce la presión negativa ó succión *nr* en *n*, disminuye la carga total *hr* de la parte *agn*, cuando al estrechar efectivamente el área transversal del sifón en *n* hace que se aplique una parte de la carga restante en *n* como carga de entrada para vencer la resistencia ocasionada por la contracción; y como carga de velocidad á dar el aumento de velocidad necesario para pasar por la sección angosta en *n*. Ahora, desde que la carga de rozamiento necesaria para la parte *agn* permanece la misma más ó menos, la carga de velocidad en el estanque está disminuída considerablemente, y el agua llega á *n* con tan poca velocidad que no puede llenar á *no*. La acumulación de aire en *n* retarda así la corriente, y perturba la distribución de las presiones de tal manera, que ya éstas no están bien representadas por las líneas verticales trazadas hacia *sro*.

**En el tunel de Blue Ridge** (Virginia), el coronel C. Crozet construyó un sifón de agua que tenía de largo 546.19 m, con tubos de hierro fundido de .076 m de diámetro interior y de 2.75 m de longitud.

El vértice estaba á 2.75 m sobre la superficie del agua que debía agotarse, y la boca de salida á 6.09 m bajo dicha superficie, teniendo así una carga de 6.09 m. En el vértice, situado á una dist de 173.73 m de la entrada, **había una válvula de aire** ordinaria, de hierro fundido, con una cámara de .91 m de altura y .38 cm de diám interior. En el mango de conexión con el sifón había una **llave de retención**, y en la parte superior un orificio de .15 cm de diám cerrado herméticamente con una tapa de tornillo. En cada extremo del sifón había una llave de retención. **Para lanzar la corriente** se cierran estas llaves terminales, y todo el sifón y recipiente de aire se llena de agua por el orificio de la parte superior del recipiente. Luego se cierra este orificio herméticamente, y se abren en seguida las llaves terminales. La llave de retención se deja abierta. Ahora principia la corriente, y aunque **teóricamente** deberá continuar sin disminución (excepto la que proviene del descenso de la carga por lo que baja el nivel de la superficie del agua en el estanque), **en la práctica** no sucede así en los sifones

muy largos, porque el aire principia á separarse del agua y sube en el sifón hasta la parte superior ó vértice donde entra al recipiente de aire, y elevándose gradualmente hasta la parte superior de la cámara, obliga al agua á salir. Si no se evita que continúe esto, el aire llenará primero toda la cámara y luego la parte superior del sifón mismo, donde obrará como un tapón deteniendo la corriente completamente. Puede saberse **el nivel á que está el agua en la cámara de aire** por el sonido producido por golpes suaves de martillo dados en el exterior de dicha válvula.

**Para evitar esta detención** se cierra la llave de retención del pie de la cámara antes que haya salido de ella toda el agua, y quitando luego la tapa de la parte superior, se llena nuevamente la cámara de agua, se coloca otra vez la tapa y se abre la llave de retención. La corriente del agua continúa mientras tanto sin interrupción, pero disminuyendo siempre gradualmente, á pesar de haber llenado de nuevo la cámara. Después de llenarla varias veces nuevamente, cesará la corriente por completo, debiendo repetirse toda la operación de llenar todo el sifón y cámara como al principio. Al principio, en el sifón del coronel Crozet, y debido á la porosidad del calafateo de las juntas que se cogían con estopa y brea solamente, el aire entraba con tanta rapidez, que desalojaba el agua de la cámara y era necesario llenarla nuevamente cada 5 ó 10 minutos; á pesar de esto el sifón se secaba á las dos horas. Entonces las juntas se calafatearon perfectamente con plomo, protegido por una capa preparada de minio y albayalde, mezclado con barniz del Japón y aceite de linaza cocido. Sin embargo, la cámara tenía que llenarse cada dos horas; y pasadas 6 horas, el sifón se secaba y había que llenarlo nuevamente. De esta manera continuaba trabajando. En opinión del autor, se ganaría mucho dándole á los tubos y al recipiente de aire una mano de barniz con alquitrán de carbón de piedra, en el interior y en el exterior.

**Art. 2. Fórmulas aproximadas de la velocidad** del agua en los tubos de hierro rectos, lisos y cilíndricos, como *ro*, *vo*, *lo*, fig. 1. Siendo conocida la carga total *op*, y el largo y diámetro del tubo.

$$\left.\begin{matrix}\textbf{Velocidad}\\ \textbf{media aprox}\\ \text{en m por segundo}\end{matrix}\right\} = \left.\begin{matrix}\text{Coeficiente } \mathbf{m},\\ \text{según la tabla}\\ \text{que sigue}\end{matrix}\right\} \times \sqrt{\frac{\text{diám en m} \times \text{carga total en m}}{\text{largo total en m} + 54\ \text{diám en m}}}$$

**Tabla de los coeficientes «m»*.**

| **Diámetro** del tubo. Metros. | **m.** | **Diámetro** del tubo. Metros. | **m.** |
|---|---|---|---|
| .03 | 12.69 | .45 | 29.25 |
| .06 | 16.56 | .60 | 31.46 |
| .09 | 18.77 | .75 | 33.12 |
| .12 | 20.42 | .90 | 34.22 |
| .15 | 21.53 | 1.06 | 35.33 |
| .18 | 23.18 | 1.22 | 36.43 |
| .21 | 24.29 | 1.52 | 37.53 |
| .24 | 25.38 | 1.83 | 38.64 |
| .27 | 25.94 | 2.13 | 39.74 |
| .30 | 26.50 | 3.05 | 42.50 |

(*N. del T.* — La fórmula y tabla del autor, las hemos transformado para usarlas en sistema métrico.)

Para cargas menores de .75 m por kilómetro, esta fórmula da resultados que prácticamente corresponden á los de la fórmula de Kutter, pág. 558, con el coeficiente *n* de asperezas=.012. Pero, pequeñas variaciones, como por ej. en la aspereza, etc., producirán considerables cambios en la velocidad, especialmente en los tubos pequeños, porque en éstos una pérdida de superficie en la sección por una

* Para diámetros intermedios, tómense coeficientes intermedios de la tabla por una simple proporción.

aspereza la disminuye en mayor proporción que cuando la sección es grande. En estas materias no se puede contar con mucha exactitud.

Como en un río la velocidad es generalmente mayor en el medio y en la superficie que en el fondo y en la orillas, asimismo la velocidad en un tubo es mayor en el centro de su sección transversal que en su circunferencia. La **velocidad media** á que se refieren nuestras reglas es una velocidad uniforme, que produciría el *mismo gasto* que la velocidad variable verdadera. Por consiguiente, tenemos :

$$\text{Gasto en m}^3 \text{ por segundo} = \text{Vel media en m por seg} \times \text{Area de la sección del tubo en m}^2.$$

En el caso de tubos largos y pequeñas cargas, la suma de las cargas de entrada y velocidad es frecuentemente tan pequeña, que puede despreciarse. Cuando esto acontece ó cuando puede saberse aproximadamente á cuánto asciende, puede usarse la **fórmula de Kutter,** también aplicable á canales abiertos. Esta fórmula es de los eminentes ingenieros suizos E. Ganguillet y W. R. **Kutter,** pero comúnmente se la cita con el nombre del último.

Es, propiamente hablando, una fórmula para hallar el coeficiente $c$ en la muy conocida fórmula :

$$\textbf{Velocidad media} = c\sqrt{\text{radio medio} \times \text{pendiente}}.$$

$$= c\sqrt{\frac{\text{diám}}{4} \times \text{pendiente}}.$$

según Kutter,

| **Para medidas inglesas.** | | **Para medidas métricas.** |
|---|---|---|
| $c = \dfrac{41.6 + \dfrac{.00281}{\text{pendiente}} + \dfrac{1.811}{n}}{1 + \dfrac{\left(41.6 + \dfrac{.00281}{\text{pendiente}}\right) n}{\sqrt{\text{radio medio en } pies}}}$ | $=$ | $\dfrac{23 + \dfrac{.00155}{\text{pendiente}} + \dfrac{1}{n}}{1 + \dfrac{\left(23 + \dfrac{.00155}{\text{pendiente}}\right) n}{\sqrt{\text{radio medio en } metros}}}$ |

Véanse también las **tablas de** $c$, en el **art. 22.**

**El radio medio** es el cociente, en metros, que se obtiene dividiendo el área de la sección transversal mojada en m cuad, por el perímetro mojado (véase abajo) en metros. En tubos llenos ó *exactamente medios llenos,* y en canales semicirculares llenos, el radio medio es igual á la cuarta parte de su diámetro interior.

**El perímetro mojado** es la suma *abco,* fig. C (pág. 561), de las porciones *ab, bc, co,* en pies ó metros hallados midiendo perpendicularmente al eje del canal la parte de sus caras laterales y del fondo, en contacto con el agua. En tubos llenos es, por su puesto, igual á su circunferencia interior.

$$\textbf{La pendiente es} = \frac{\text{Carga de } \textit{rozamiento wo}\text{, fig. 1.}}{\text{Largo del tubo medido en línea recta de extr á extr}}$$

$$= \text{seno del ángulo } \textit{wso} \text{ fig. 1.}$$

**En canales abiertos,** tenemos :

$$\textbf{pendiente} = \frac{\text{altura de la caída desde la sup del agua en una porción cualquiera de la longitud del canal}}{\text{largo de dicha porción.}}$$

$$= \text{caída desde la sup del agua, por unidad de longitud del canal.}$$

$$= \text{seno del ángulo formado por la inclinación de la sup del agua del canal y la horizontal.}$$

« **n** » es el « **coeficiente de asperezas** » del perímetro mojado, y por supuesto depende principalmente del estado de la superficie interior del tubo. Para tubos de hierro en buenas condiciones y de 2 á 120 cm de diám, puede tomarse de .010 á .012, tomando números más pequeños, cuando el tubo está en condiciones excepcionalmente buenas.

**Las curvas y codos** no afectan mucho la descarga, siempre que la carga total y la longitud de la tubería queden las mismas, y por supuesto que los vér-

tices de las curvas y codos estén bajo la línea de pendiente hidráulica y se tenga cuidado de hacer salir debidamente el aire que se acumula en la parte alta de las curvas y codos.

**Relación entre el área, la velocidad y la descarga.**

Sea $q$ = el gasto por seg,
$v$ = la velocidad media por seg,
$a$ = área de la sección transversal.

Entonces : $q = av$; $v = \frac{q}{a}$; $a = \frac{q}{v}$.

**Relación entre la descarga, el diámetro* y la pendiente.** Si suponemos la velocidad $= c\sqrt{\text{radio medio} \times \text{pendiente}}$, ó bien $v = c\sqrt{rs}$, véase pág. 558, y si el tubo es de sección circular, tenemos para el gasto (volumen de líquido por seg), Q, de un tubo de diámetro, $d$, y área, A, de la sección, corriendo lleno.

$$Q = Av = \frac{\pi d^2}{4} \cdot c\frac{d^{1/2}}{2} s^{1/2} = \frac{c\pi d^{5/2} s^{1/2}}{8};$$

donde se ve que Q es proporcional á la raíz cuad de la quinta potencia del diámetro y á la raíz cuadrada de la pendiente. Para tablas de quintas potencias y de raíces cuadradas de quintas potencias, véanse págs 69 a 71.

## Efecto de las resistencias.

**La presión** del agua que corre por un tubo, en cualquier punto de éste, entre el orificio de salida y el estanque, es :

$$= \left\{\begin{array}{c}\text{carga total}\\ \text{en}\\ \text{ese punto}\end{array}\right\} \text{menos} \left\{\begin{array}{c}\text{la carga}\\ \text{de velocidad}\\ \text{en}\\ \text{dicho punto}\end{array}\right\} + \left\{\begin{array}{c}\text{la}\\ \text{carga}\\ \text{de}\\ \text{entrada}\end{array}\right\} + \left\{\begin{array}{c}\text{la carga consumida}\\ \text{en vencer las resistencias}\\ \text{entre}\\ \text{el estanque y el punto.}\end{array}\right\}$$

Así, en el punto 6, del tubo *lo*, fig. 1, pág. 549, la presión es $h = (3\,6) = (1\,6) - [(1\,2) + (2\,3)]$; en donde $(1\,2) = is$ = suma de las cargas de velocidad y de entrada. En 4, en el tubo *ro*, $h = (3\,4) = (1\,4) - [(1\,2) + (2\,3)]$. Supongamos en la fig. 11 que la línea recta, *so*, representa la verdadera longitud del tubo, ya sea recto, con codos ó curvo, etc.; y *sv*, la suma de las resistencias (suponiéndolas uniformemente distribuídas) dentro del tubo. Entonces el ángulo, *sov*, se llama la pendiente hidráulica, y seno $sov = sv \div so$.

En el tubo vertical, *vo*, fig. 1A, la presión en $q = gd$.

### PESO DEL AGUA CONTENIDA EN UN METRO DE TUBO DE DIFERENTES DIAMETROS

(*N. del T.*—Hemos reemplazado con esta tabla la que el autor trae en medidas inglesas.)

| Diámetro en cm. | Peso agua en kg. | Diámetro en cm. | Peso agua en kg. | Diámetro en cm. | Peso agua en kg. | Diámetro en cm. | Peso agua en kg. |
|---|---|---|---|---|---|---|---|
| 1¼ | .123 | 10 | 7.854 | 30 | 70.686 | 60 | 282.744 |
| 2½ | .491 | 12½ | 12.272 | 35 | 96.212 | 65 | 331.832 |
| 3 | .707 | 15 | 17.672 | 40 | 125.664 | 70 | 384.846 |
| 4 | 1.257 | 17½ | 24.053 | 45 | 159.044 | 80 | 502.656 |
| 5 | 1.963 | 20 | 31.416 | 50 | 196.350 | 90 | 636.174 |
| 7½ | 4.418 | 25 | 49.088 | 55 | 237.584 | 100 | 785.400 |

**El peso del agua** en cualquier longitud dada (como un pie ó un metro) de un tubo cualquiera ú otro cilindro circular **es proporcional al cuadrado del diámetro interior.** De aquí que el peso del agua en un m de longitud, de cualquier cilindro de diámetro distinto á los que hemos dado en la tabla, puede hallarse

* Diámetro $= 4 \times$ radio medio, ó bien $d = 4r$, pág. 558

multiplicando el correspondiente á un tubo de un decímetro de diám por el cuadrado del diámetro interior del cilindro dado en dcm.

**Art. 3. Teoría de la corriente en tubos largos y canales.**

**Resistencia á la corriente.** Cuando un cuerpo sólido se mueve sobre otro, las más de sus partículas conservan prácticamente sus relativos puestos; pero cuando corre agua, como en un tubo ó canal, las asperezas y las irregularidades, que existen aun en las superficies más lisas, estorban la corriente, describiendo las varias partículas caminos espirales ó de otras formas no paralelos á la dirección general de la corriente. La resistencia á la corriente créese que se debe á tales estorbos antes que á la fricción propiamente dicha, como la entendemos en los sólidos. Véase pág. 425, y § 194, pág. 436.

A falta de un completo conocimiento respecto de la naturaleza de estos estorbos, estamos obligados á atenernos á los experimentos al determinar la manera y la extensión de su influencia en la corriente.

**Velocidad media. Gasto.** Figs. A, B. En un tubo ó canal, á causa del contacto con los lados y fondo, las partículas de agua se mueven en caminos tortuosos. La

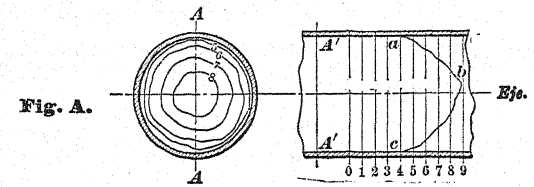
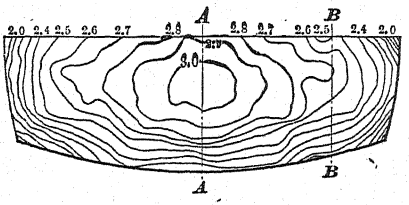
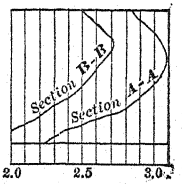



Fig. A.

Fig. B.




**velocidad media**, al través de la sección transversal entera en cualquiey punto, es el cociente obtenido dividiendo el gasto por el área de la sección transversal. **En cualquier punto dado** en una sección transversal, la **velocidad**, tal como la registra cualquier medidor corriente, es la componente, paralela al eje del tubo, de la velocidad efectiva de lás partículas que pasan por ese punto. Tratamos, actualmente, sólo de casos de « **corriente constante** », esto es : en que la velocidad, en cada punto, permanece la misma.

**Las velocidades, medidas en puntos diferentes en la sección transversal** de un tubo ó canal, son generalmente mínimas cerca del borde de los tubos y cerca de los lados y fondos de los canales.

En la fig. A, la sección longitudinal muestra una de muchas series de medidas de velocidad por los Sres. Williams, Hubbell y Fenkell* en un tubo de hierro fundido de 16 pulgadas de diámetro. Las medidas se hicieron, por medio del tubo de Pitot, en un diámetro vertical, tal como AA de la sección transversal. Las distancias horizontales de los varios puntos en la curva, *abc*, desde la línea vertical, 0, representan, por la escala que está debajo de la figura, las velocidades en los varios puntos en el diámetro. En la sección transversal, las varias líneas curvas son líneas de igual velocidad.

La fig. B muestra los resultados de medidas de velocidades en una sección transversal, del Sudbury Conduit, Mass. †, de 9 pies (2.74 m) de ancho, y 3 pies

* Trans. Am. Soc. Civ. Engrs, vol. XLVII, lámina LV, pág, 66.
† F. P. Stearns. Trans. Am. Soc. C. E., agosto 1883, vol. VII pág. 324.

(.91 m) de hondo. En la sección longitudinal están aproximadamente indicadas las velocidades, en diferentes profundidades, de las líneas AA y BB de la sección transversal, como se hizo en la fig. A. En la sección transversal, las varias líneas curvas son líneas de igual velocidad. La interior corresponde á una velocidad de 3 pies (.91 m) por segundo; la inmediata á 2.9 pies (0.884 m.) por segundo, y así de lo demás.

**Fórmulas para el gasto** $q$, **y la velocidad media,** $v$, en una longitud dada, L, de un **tubo ó canal recto** con vena líquida de sección transversal uniforme.

Refiriéndonos á la fig. C, sean

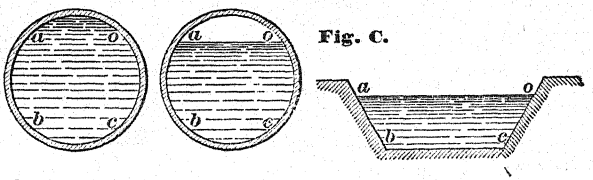



**Fig. C.**

L=longitud dada,

$p$=el perimetro mojado *abco* ;

A=L$p$ = el área de la superficie mojada;

$a$=el área de la sección transversal de la vena líquida ;

$r = \frac{a}{p}$ = el radio medio de su sección transversal;

$d$=diámetro del tubo*;

$q$=el gasto

=volumen que pasa por una sección transversal dada, $a$, en la unidad de tiempo;

$v = \frac{q}{a}$ = la velocidad media;

$h_v = \frac{v^2}{2g}$ = la carga=altura de caída;

$m$, $c$, $f$, F=factores de resistencia, como se explica abajo;

$h_f$= la resistencia (ó « fricción ») en la longitud L;

$s = \frac{h_f}{L}$ = la pendiente.

Créese generalmente que las resistencias á la corriente son directamente proporcionales á A y á alguna potencia (generalmente $v^2$)† de $v$, é inversamente proporcional á $a$.

Es decir :

**pérdida por fricción,** $h_f = m\frac{Av^2}{a} = m\frac{Lpv^2}{a} = m\frac{Lv^2}{r}$ ; y $m = h_f \frac{r}{Lv^2}$ $\frac{rs}{v^2}$

de donde

$v^2 = h_f \frac{a}{mLp} = \frac{rs}{m}$ ; y **velocidad,** $v = \sqrt{\frac{1}{m}}\,rs = \sqrt{\frac{1}{m}} \times \sqrt{r} \times \sqrt{s}$.

Sea $c = \sqrt{\frac{1}{m}}$ ; $m = \frac{1}{c^2}$. Entonces tenemos la *fórmula Chezy* : —

**velocidad,** $v = c\sqrt{rs} = c\sqrt{r} \times \sqrt{s} = cr^{0.5}s^{0.5}$.

Para la fórmula de « Kutter », que da valores de $c$, véanse arts. 2 y 20 á 23. Para tablas de $c$, por la fórmula de Kutter, véase art. 22.

**Los coeficientes de resistencia, ó fricción,** tales como $m$, $f$, F ó $c$, se

---

* En canales semicirculares llenos, y en tubos llenos ó medio llenos, $d = 4r$.

† Véanse « Fórmulas Exponenciales », art. 3 *a* que sigue.

deben elegir á nuestro juicio, ó determinarse, como por la fórmula de Kutter, arts. 2, 20, etc., según se conozca ó suponga la superficie mojada.

*N. del T.* — Reemplazando unas dos fórmulas y la tabla que da el autor en medidas inglesas, damos otra tabla, págs. 564 y 565, que suple á la del autor.

Para velocidades mayores de tres metros, que no figuran en la tabla, damos los siguientes:

| $v =$ | 4 m | 5 m | 6 m | 7 m | 8 m | 10 m |
|---|---|---|---|---|---|---|
| Coef de frotamiento = | .0191 | .0187 | .0183 | .0180 | .0177 | .0174 |

**Art. 3 a. Fórmulas « exponenciales ». El estudio** cuidadoso de los experimentos sobre tubos grandes y en canales, bajo condiciones variables, indica que la velocidad media depende, no de las raíces *cuadradas* (ó potencia ½), de *r* y de *s*, sino generalmente de potencias algo más altas; y que la inexactitud, implicada en tomar .5 como exponentes en la fórmula Chezy explica la notable variación de la *c* Chezy con *r* y con *s*, para una condición dada de la superficie interior del tubo ó del canal.

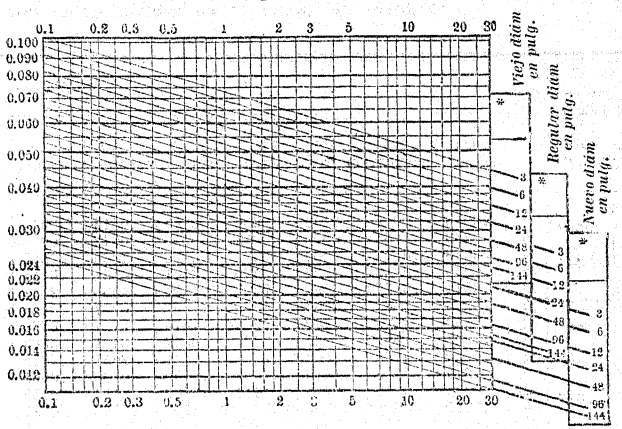
*N. del T.* — Damos á continuación algunas fórmulas para los coeficientes de frotamiento.

Según Weisbach llamándolo $\pi = .01439 + \dfrac{.0094711}{\sqrt{v}}$

— Zeuner — $= .014312 + \dfrac{.010327}{\sqrt{v}}$

Darcy cree que depende menos de la velocidad que del diámetro $= .0198 + \dfrac{.0005078}{d}$.

**Valores de la fricción, F, para tubos de hierro.** (Véase pág. 566.)




*N. del T.* — Por ser este diagrama muy práctico y de muy fácil manejo en el sistema métrico (con las pequeñas modificaciones que le hemos hecho) lo hemos dejado. En efecto: dado el diám del tubo en metros, basta multiplicarlo por 40 (suficiente exactitud en este caso) para tener su diám en pulgs, y entonces se procede así :

**Uso del diagrama.** Dado un tubo de 6 pulgadas, por ejemplo, en condición regular. En la columna, á la derecha, titulada « Regular », encuéntrese la línea, 6 pulgadas. Siguiendo á la izquierda la dirección de la línea corta inclinada, (con preferencia por medio de una regla ó una orilla derecha de papel) encontramos que ella coincide casi con una de las líneas inclinadas que cruzan el diagrama. Por medio de las intersecciones de esta línea con las otras, encontramos que, para el tubo en cuestión, una velocidad de 5 pies ó sea 1.50 m por segundo corresponde aproximadamente á F=.035; para una velocidad de .15 m ó .5 pies por segundo, F=.048, etc., etc.

| Diám en pulg..... | 3 | 6 | 12 | 24 | 48 | 96 | 144 |
|---|---|---|---|---|---|---|---|
| Diám en metros..... | .075 | .15 | .30 | .60 | 1.22 | 2.44 | 3.66 |

(*N. del T.* — Cuando se quiera entrar á la tabla con una velocidad intermedia de las marcadas con cifras, es muy fácil encontrar la línea que le corresponde por el valor de las velocidades extremas, entre las que viene quedando la velocidad dada, distribuyendo la diferencia entre aquéllas, para el número de líneas intermedias.

El factor fricción es poco sensible á las variaciones de velocidad y diámetros; de modo que este diagrama resulta suficientemente exacto para los cálculos ordinarios.)

**Valores del gasto $Q_1$ en metros cúbs; por**

| Velocidades. | Coeficiente de frotamiento | | Diámetros. .025 metros. | .050 metros. | .075 metros. | .100 metros. | .125 metros. | .150 metros. | .200 metros. |
|---|---|---|---|---|---|---|---|---|---|
| .05 | .0568 | $Q_1$ | .0015 | .0059 | .0133 | .0236 | .0368 | .0513 | .0942 |
| | | $h_1$ | .0289 | .0145 | .0096 | .0072 | .0058 | .0048 | .0036 |
| .10 | .0444 | $Q_1$ | .0029 | .0118 | .0265 | .0471 | .0736 | .1060 | .1885 |
| | | $h_1$ | .0905 | .0452 | .0302 | .0226 | .0181 | .0151 | .0113 |
| .15 | .0389 | $Q_1$ | .0044 | .0177 | .0398 | .0707 | .1104 | .1590 | .2827 |
| | | $h_1$ | .1783 | .0892 | .0594 | .0446 | .0357 | .0297 | .0223 |
| .20 | .0356 | $Q_1$ | .0059 | .0236 | .0530 | .0942 | .1473 | .2121 | .3770 |
| | | $h_1$ | .2902 | .1451 | .0967 | .0723 | .0540 | .0484 | .0363 |
| .25 | .0333 | $Q_1$ | .0074 | .0295 | .0663 | .1178 | .1841 | .2651 | .4712 |
| | | $h_1$ | .4249 | .2125 | .1416 | .1062 | .0850 | .0708 | .0531 |
| .30 | .0317 | $Q_1$ | .0088 | .0353 | .0795 | .1414 | .2209 | .3181 | .5655 |
| | | $h_1$ | .5817 | .2908 | .1939 | .1454 | .1163 | .0969 | .0727 |
| .40 | .0294 | $Q_1$ | .0118 | .0471 | .1060 | .1885 | .2045 | .4241 | .7540 |
| | | $h_1$ | .9584 | .4792 | .3195 | .2396 | .1917 | .1597 | .1198 |
| .50 | .0278 | $Q_1$ | .0147 | .0589 | .1325 | .2356 | .3682 | .5301 | .9425 |
| | | $h_1$ | 1.4166 | .7083 | .4722 | .3542 | .2833 | .2361 | .1771 |
| .60 | .0266 | $Q_1$ | .0177 | .0707 | .1590 | .2827 | .4418 | .6362 | 1.1310 |
| | | $h_1$ | 1.9545 | .9772 | .6515 | .4886 | .3909 | .3257 | .2443 |
| .70 | .0257 | $Q_1$ | .0206 | .0825 | .1856 | .3299 | .5154 | .7422 | 1.3195 |
| | | $h_1$ | 2.5694 | 1.2847 | .8565 | .6423 | .5139 | .4282 | .3212 |
| .80 | .0250 | $Q_1$ | .0236 | .0942 | .2121 | .3770 | .5890 | .8482 | 1.5080 |
| | | $h_1$ | 3.2607 | 1.6303 | 1.0869 | .8152 | .6521 | .5434 | .4075 |
| .90 | .0244 | $Q_1$ | .0265 | .1060 | .2386 | .4241 | .6627 | .9543 | 1.6965 |
| | | $h_1$ | 4.0261 | 2.1030 | 1.3420 | 1.0065 | .8052 | .6710 | .5033 |
| 1.00 | .0239 | $Q_1$ | .0295 | .1178 | .2651 | .4712 | .7363 | 1.0603 | 1.8850 |
| | | $h_1$ | 4.8665 | 2.4332 | 1.6222 | 1.2166 | .9733 | .8111 | .6083 |
| 1.25 | .0229 | $Q_1$ | .0368 | .1473 | .3313 | .5890 | .9204 | 1.3254 | 2.3562 |
| | | $h_1$ | 7.2852 | 3.6426 | 2.4284 | 1.8213 | 1.4570 | 1.2142 | .9107 |
| 1.50 | .0221 | $Q_1$ | .0442 | .1767 | .3976 | .7069 | 1.1045 | 1.5904 | 2.8274 |
| | | $h_1$ | 10.151 | 5.0757 | 3.3838 | 2.5378 | 2.0303 | 1.6919 | 1.2689 |
| 1.75 | .0216 | $Q_1$ | .0515 | .2062 | .4639 | .8247 | 1.2885 | 1.8555 | 3.2987 |
| | | $h_1$ | 13.461 | 6.7306 | 4.4871 | 3.3653 | 2.6923 | 2.2435 | 1.6825 |
| 2.00 | .0211 | $Q_1$ | .0589 | .2356 | .5301 | .9425 | 1.4726 | 2.1206 | 3.7699 |
| | | $h_1$ | 17.199 | 8.5994 | 5.7329 | 4.2997 | 3.4398 | 2.8665 | 2.1499 |
| 2.50 | .0204 | $Q_1$ | .0736 | .2945 | .6628 | 1.1781 | 1.8408 | 2.6507 | 4.7124 |
| | | $h_1$ | 25.981 | 12.991 | 8.6604 | 6.4953 | 5.1962 | 4.3302 | 3.2476 |
| 3.00 | .0198 | $Q_1$ | .0884 | .3534 | .7952 | 1.4137 | 2.2089 | 3.1809 | 5.6549 |
| | | $h_1$ | 36.357 | 18.178 | 12.119 | 9.0891 | 7.2713 | 6.0594 | 4.5446 |

*N. del T.* — Damos esta tabla para reemplazar la del autor en medidas inglesas. (Tomada del Manual del Constructor, por Soroa y Castro.)

**minuto y de la pérdida de carga $h_1$ para 100 metros.**

tros = $d$ =

| .250 metros. | .300 metros. | .350 metros. | .400 metros. | .500 metros. | .600 metros. | .750 metros. | 1.000 metros. | |
|---|---|---|---|---|---|---|---|---|
| .1473 | .2121 | .2618 | .3770 | .5891 | .8482 | 1.3254 | 2.3562 | $Q_1$ |
| .0029 | .0024 | .0022 | .0018 | .0014 | .0012 | .0010 | .0007 | $h_1$ |
| .2945 | .4241 | .5236 | .7540 | 1.1781 | 1.6965 | 2.6507 | 4.7124 | $Q_1$ |
| .0090 | .0075 | .0068 | .0057 | .0045 | .0038 | .0030 | .0023 | $h_1$ |
| .4418 | .6362 | .7854 | 1.1310 | 1.7672 | 2.5447 | 3.9761 | 7.0686 | $Q_1$ |
| .0178 | .0149 | .0134 | .0111 | .0089 | .0074 | .0059 | .0045 | $h_1$ |
| .5890 | .8482 | 1.0472 | 1.5080 | 2.3562 | 3.3929 | 5.3014 | 9.4248 | $Q_1$ |
| .0290 | .0242 | .0217 | .0181 | .0145 | .0121 | .0097 | .0072 | $h_1$ |
| .7373 | 1.0603 | 1.3090 | 1.8850 | 2.9452 | 4.2412 | 6.6268 | 11.781 | $Q_1$ |
| .0425 | .0354 | .0319 | .0266 | .0213 | .0177 | .0142 | .0106 | $h_1$ |
| .8836 | 1.2723 | 1.5708 | 2.2619 | 3.5343 | 5.0894 | 7.9521 | 14.137 | $Q_1$ |
| .0582 | .0485 | .0436 | .0364 | .0291 | .0242 | .0194 | .0145 | $h_1$ |
| 1.1781 | 1.6965 | 2.0944 | 3.0159 | 4.7124 | 6.7858 | 10.603 | 18.850 | $Q_1$ |
| .0985 | .0799 | .0719 | .0599 | .0479 | .0399 | .0319 | .0240 | $h_1$ |
| 1.4726 | 2.1206 | 2.6180 | 3.7699 | 5.8905 | 8.4823 | 13.254 | 23.562 | $Q_1$ |
| .1417 | .1181 | .1062 | .0885 | .0708 | .0590 | .0472 | .0354 | $h_1$ |
| 1.7671 | 2.5447 | 3.1416 | 4.5239 | 7.0686 | 10.179 | 15.904 | 28.274 | $Q_1$ |
| .1954 | .1629 | .1466 | .1222 | .0977 | .0814 | .0651 | .0489 | $h_1$ |
| 2.0617 | 2.9688 | 3.6652 | 5.2779 | 8.2467 | 11.875 | 18.555 | 32.987 | $Q_1$ |
| .2594 | .2141 | .1927 | .1606 | .1285 | .1071 | .0856 | .0642 | $h_1$ |
| 2.3562 | 3.3929 | 4.1888 | 6.0319 | 9.4248 | 13.572 | 21.206 | 37.699 | $Q_1$ |
| .3261 | .2717 | .2446 | .2038 | .1630 | .1359 | .1087 | .0815 | $h_1$ |
| 2.6507 | 3.8170 | 4.7124 | 6.7854 | 10.603 | 15.268 | 23.856 | 42.411 | $Q_1$ |
| .4026 | .3355 | .3020 | .2516 | .2013 | .1678 | .1342 | .1007 | $h_1$ |
| 2.9452 | 4.2412 | 5.2360 | 7.5398 | 11.781 | 16.965 | 26.507 | 47.124 | $Q_1$ |
| .4866 | .4055 | .3650 | .3042 | .2433 | .2028 | .1622 | .1217 | $h_1$ |
| 3.6816 | 5.3015 | 6.5450 | 9.4248 | 14.726 | 21.206 | 33.134 | 58.905 | $Q_1$ |
| .7285 | .6071 | .5464 | .4553 | .3643 | .3036 | .2428 | .1821 | $h_1$ |
| 4.4179 | 6.3617 | 7.8540 | 11.310 | 17.671 | 25.447 | 39.761 | 70.686 | $Q_1$ |
| 1.0151 | .8459 | .7613 | .6345 | .5076 | .4230 | .3384 | .2538 | $h_1$ |
| 5.1542 | 7.4220 | 9.1630 | 13.195 | 20.617 | 29.688 | 46.388 | 82.467 | $Q_1$ |
| 1.3461 | 1.1218 | 1.0096 | .8412 | .6731 | .5609 | .4487 | .3665 | $h_1$ |
| 5.8905 | 8.4823 | 10.472 | 15.080 | 23.562 | 33.929 | 53.014 | 94.248 | $Q_1$ |
| 1.7199 | 1.4332 | 1.2899 | 1.0749 | .8599 | .7166 | .5733 | .4300 | $h_1$ |
| 7.3631 | 10.603 | 13.090 | 18.850 | 29.452 | 42.411 | 66.268 | 117.81 | $Q_1$ |
| 2.5981 | 2.1651 | 1.9486 | 16.238 | 1.2991 | 1.0825 | .8660 | .6495 | $h_1$ |
| 8.8357 | 12.723 | 15.708 | 22.620 | 35.243 | 50.894 | 79.521 | 141.37 | $Q_1$ |
| 3.6357 | 3.0297 | 2.7267 | 2.2723 | 1.8178 | 1.5149 | 1.2119 | .9089 | $h_1$ |

Las velocidades, los coeficientes y los diám están dados en metros.

**Art. 4. Para encontrar el gasto, $q$, de una tubería de diámetro variable, fig. 1 H.**

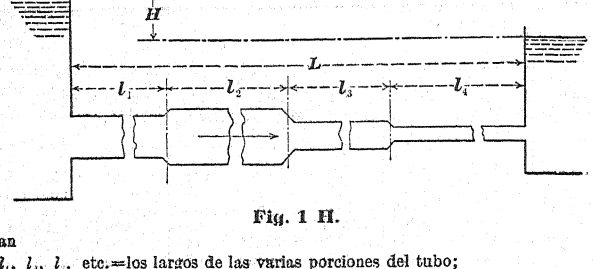



**Fig. 1 H.**

Sean

$l_1$, $l_2$, $l_3$, etc.=los largos de las varias porciones del tubo;
$d_1$, $d_2$, $d_3$, etc.=los correspondientes diámetros;
$v_1$, $v_2$, $v_3$, etc.=las correspondientes velocidades;
$F_1$, $F_2$, $F_3$, etc.=los correspondientes valores de la resistencia ó coeficiente de fricción. Véase pág. 563.
$L=l_1+l_2+l_3+$etc.=el largo total del tubo;
H=carga total (art. 1).
$q=\frac{1}{4}\pi d_1^2 v_1=\frac{1}{4}\pi d_2^2 v_2=$etc.=gasto.

En un tubo largo, las cargas de velocidad y de entrada son ordinariamente despreciables con relación á la carga de fricción. Despreciándolas, tenemos

H=carga total=carga de fricción.

En cada porción del tubo, la resistencia, y la correspondiente carga de « fricción » $hf$ *, se cree que son proporcionales directamente á la longitud $l$ y á la carga de velocidad, $\frac{v^2}{2g}$, é inversas al diámetro, $d$, ó

$$h_f = F \cdot \frac{l}{d} \cdot \frac{v^2}{2g}.$$

En consecuencia

$$H = F_1 \cdot \frac{l_1}{d_1} \cdot \frac{v_1^2}{2g} + F_2 \frac{l_2}{d_2} \cdot \frac{v_2^2}{2g} + F_3 \frac{l_3}{d_3} \cdot \frac{v_3^2}{2g} + \text{etc.};$$

y puesto que

$$v_1 = \frac{4q}{\pi d_1^2}, \; v_2 = \frac{4q}{\pi d_2^2}, \text{ etc.},$$

tenemos, también,

$$2gH = F_1 \cdot \frac{l_1}{d_1} \cdot \frac{16\,q^2}{\pi^2 d_1^4} + F_2 \frac{l_2}{d_2} \cdot \frac{16\,q^2}{\pi^2 d_2^4} + \text{etc.}$$

$$= \frac{16\,q^2}{\pi^2}\left(F_1 \cdot \frac{l_1}{d_1^5} + F_2 \frac{l_2}{d_2^5} + \text{etc.}\right)$$

de donde

$$q = \frac{\pi}{4}\sqrt{\frac{2g\,H}{F_1 \frac{l_1}{d_1^5} + F_2 \frac{l_2}{d_2^5} + \text{etc.}}}$$

**Art. 4 a †. El medidor Venturi** se destina á la medida de la corriente de líquidos en tubos de grandes dimensiones y llenos.

**El medidor** propiamente dicho, patentado por Clemens Herschel, consiste

---

* *N. del T.* — En español decimos « pérdida de carga por frotamiento ó fricción »; pero como es tan largo y hay que repetirlo tanto, abreviamos como está.

† Extracto de una descripción preparada por el autor como presidente de una Comisión del Instituto de Franklin. *Diario del Instituto de Franklin*, febrero de 1899.

esencialmente en una simple contracción del área de la sección transversal del tubo, con aberturas en sus diámetros reducidos para medir, por piezómetros ó manómetros, las presiones en esos puntos; mientras que **el registrador** patentado por los Sres. Frederick N. Connet y Wálter W. Jackson, es un complicado mecanismo, provisto de movimientos de relojería y cuadrantes.

**Teoría** *. Supongamos que las figs. 1 á 3 representen un tubo de medidor Venturi, con tres piezómetros en su lugar, esto es · n.º 1, sobre el tubo aguas arriba

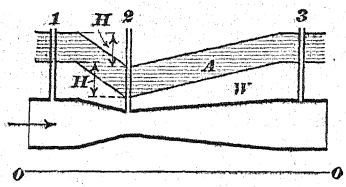



**Fig. 1.**

de la contracción; n.º 2, sobre la contracción misma; y n.º 3, sobre el tubo agua abajo de la contracción. Represente el área no sombreada W, en las figs. 1 á 3 las alturas á que llega el agua (por efecto de la presión en cada punto) respecto de cualquier plano horizontal dado O-O; y representemos por el área sombreada A la presión uniforme de la atmósfera, que, por conveniencia, podemos suponer representada en peso por una capa de algún líquido de la gravedad específica del agua, pero que se distingue del agua por su apariencia.

La distancia vertical, entre la línea superior de esta última área y cualquier punto dado en el tubo, representa la presión combinada de aire y del agua en tal punto.

Las velocidades en el tubo medidor, en cualquier instante, son inversamente proporcionales á las áreas de las secciones transversales respectivas, y, como las cargas correspondientes á las varias velocidades son proporcionales á los cuadrados de esas velocidades, las cargas ó presiones deben variar también, estando la más pequeña ó más baja carga ó presión por encima de la más estrecha sección, donde la velocidad es más grande.

El aumento de velocidad, adquirido por el fluido al pasar de la sección 1 á la sección 2, lo pierde al pasar de la sección 2 á la sección 3; y, en el caso de un

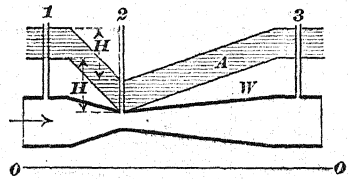



**Fig. 2.**

fluido perfecto, la presión perdida entre las secciones 1 y 2 la readquiriría exactamente al pasar de la sección 2 á la sección 3. En la práctica, hay siempre una pequeña pérdida total. Esta pérdida es mayor con grandes que con pequeñas velocidades.

Para una carga ó presión dada en el piezómetro n.º 1 y diámetro de tubo dado en la sección 1, la pérdida de carga para producir velocidad entre las secciones 1 y 2 aumenta á medida que disminuye el área del cuello ó sección y á medida que

* El medidor Venturi, aparte de sus méritos como aparato de medida, está basado en importantes principios hidráulicos. Por esto se expone aquí su teoría con bastante extensión.

la velocidad del cuello aumenta *. En la fig. 2 se muestra el caso en que toda la carga de agua que estaba sobre el cuello ó sección estrecha se ha empleado en mantener la velocidad por el cuello.

En las figs. 1 y 2, la carga, H, invertida en el aumento de velocidad entre las secciones 1 y 2, está representada por la diferencia de nivel de las dos columnas de agua 1 y 2, ó entre los límites superiores de las dos columnas de aire correspondientes. En la fig. 2, sección 2, esta diferencia es igual á la altura vertical total de la columna de agua en la sección 1.

Si ahora, fig. 3, se reduce aún más la sección del cuello (quedando como antes las demás circunstancias), la velocidad en el cuello aumentará más aún; pues la presión *total* aprovechable para aumentar la velocidad entre las secciones 1 y 2 está formada, no solamente de la presión del *agua*, *w*, sino también en la *presión atmosférica*, representada por el área sombreada A, por encima del agua W.

En la fig. 3, toda el agua ha desaparecido del piezómetro 2, y aun una porción del líquido que representa la presión del aire ha desaparecido también, dejando

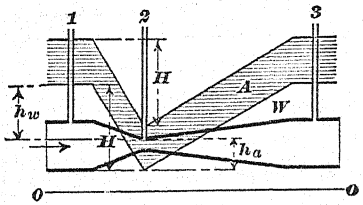



**Fig. 3.**

sólo una porción del último para representar la presión que queda en el cuello. En otras palabras, la presión dentro del cuello es ahora menor que la presión atmosférica.

En la fig. 3, la pérdida de carga, debida al aumento de velocidad entre las secciones 1 y 2, es $H=h_w+h_a$=á toda la carga de agua aprovechable, $h_w$, más una porción, $h_a$, de la presión atmosférica. La última porción, $h_a$, se llama frecuentemente « el vacío ».

Habiendo desaparecido ahora la columna de agua debajo del cuello, no se puede averiguar la pérdida de carga tomando la diferencia entre los niveles del agua en los piezómetros 1 y 2.

El grado de vacío puede encontrarse, como se indica en la fig. 4, usando, en lugar de los piezómetros, un tubo de cristal encorvado *hacia abajo*, y cuya boca

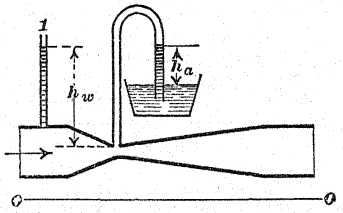



**Fig. 4.**

se introduce en una cubeta abierta que contenga agua ó mercurio. La altura á que el agua (ó el mercurio, convertido en metros de agua) sube en este tubo, da la medida del vacío, ó la porción, $h_a$, de la presión de aire que ha contribuído

* En un tubo de Venturi dado pueden también variarse la presión y la velocidad en el cuello modificando las mismas en las secciones 1 y 3; ó regulando las aberturas de las válvulas de aflujo al tubo medidor y de derrame; ó cambiando la carga total en el sistema, etc.

á producir la gran velocidad en el cuello. Añadiendo esto á $h_w$, obtenemos, como arriba, la pérdida total de carga H entre las secciones 1 y 2.

Cuando la reducción del área en el cuello ha llegado hasta el punto de que toda la presión de agua y aire disponible en la sección 1 se emplea en mantener la correspondiente velocidad en el cuello (esto es : cuando la línea superior de la superficie que representa el aire baja al nivel del borde superior del cuello), no puede conseguirse ningún otro aumento de velocidad en el cuello (con la carga total dada sobre la sección 1) estrechándolo más aún. Si el cuello se angosta más,

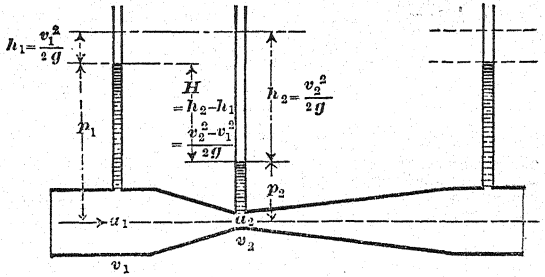



**Fig. 5.**

la velocidad en él seguirá siendo la misma, y, disminuído así el gasto, la velocidad en la sección 1 se reducirá necesariamente.

Sean $v_1$ la velocidad en la sección 1, aguas arriba del cuello, y $v_2$ la velocidad en el cuello ó sección 2.

Refiriéndonos á la fig. 5, la carga de velocidad en la sección 1, medida desde una altura ó presión supuesta representada por las líneas horizontales superiores de la figura, es :

$$h_1 = \frac{v_1^2}{2g},$$

y la de la sección 2 es

$$h_2 = \frac{v_2^2}{2g}.$$

Despreciando las resistencias á la corriente, la pérdida de carga, entre las secciones 1 y 2, es igual á la pérdida de presión, entre $a_1$, y $a_2$, ó

$$H = h_2 - h_1 = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} = \frac{v_2^2 - v_1^2}{2g} = p_1 - p_2 \text{ *}.$$

En consecuencia, $$h_2 = \frac{v_2^2}{2g} = H + h_1$$

y la **velocidad en el cuello** $= v_2 = \sqrt{2g\,(H+h_1)} = \sqrt{2g\left(H + \frac{v_1^2}{2g}\right)}$.

Dicho de otro modo, la velocidad en el cuello es la correspondiente á la carga H, más la carga correspondiente á la velocidad $v_1$ en la sección 1.

Pero, puesto que las velocidades están en razón inversa de las áreas de las secciones $a_1$ y $a_2$,

$$v_1 = \frac{a_2}{a_1} v_2, \quad \text{y } v_1^2 = \frac{a_2^2}{a_1^2} v_2^2,$$

$$H = \frac{v_2^2 - v_1^2}{2g} = \frac{v_2^2 - \frac{a_2^2}{a_1^2} v_2^2}{2g} = \frac{\left(1 - \frac{a_2^2}{a_1^2}\right) v_2^2}{2g} = \frac{\frac{a_1^2 - a_2^2}{a_1^2} v_2^2}{2g}$$

* Por el teorema de Bernouilly, $p_1 + h_1 = p_2 + h_2$.

$$v_2^2 = \frac{2g\,H a_1^2}{a_1^2 - a_2^2},$$

y la velocidad en el cuello $= v_2 = \frac{a_1}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gH}.$

La razón $\frac{a_2}{a_1}$,

entre el área $a_2$ de la sección transversal en el cuello, y la $a_1$, en el extremo superior del cono de aguas arriba, es llamada **razón del cuello**. Para una razón de 1 : 9 tenemos

$$\frac{a_1}{\sqrt{a_1^2 - a_2^2}} = \frac{9}{\sqrt{9^2 - 1^2}} = \frac{9}{\sqrt{80}} = \sqrt{\frac{81}{80}} = = 1.0062.$$

ó $$v_2 = 1.0062\sqrt{2gH}.$$

**El tubo de Venturi**, para tubos no mayores de 1.52 m (60 pulgs) de diámetro, se forma de varias secciones cortas de tubo de hierro fundido, que tienen la disminución requerida y están provistos de rebordes, por medio de las cuales se empatan las secciones para formar los dos conos truncados.

En los tamaños más pequeños, el cono más corto lo forma generalmente una sección y el cono más largo dos ó más secciones.

La sección del cuello se hace generalmente en una pieza separada, y ó se hace de bronce ó se forra con ese metal.

Los extremos del tubo Venturi están provistos de campana, de espita ó con extremos guarnecidos de rebordes, según el sistema de la tubería en que el tubo vaya á usarse.

**Para corrientes aún más grandes**, tales como las de conductos de mampostería ó de canales remachados, el tubo de Venturi puede hacerse de duelas de madera, láminas de acero, hormigón de cemento, ladrillos ú otro material conveniente, usándose metal para la pieza del cuello y donde lo exige la presión.

**La pieza del cuello** está rodeada por una cámara anular llamada la **cámara de presión**, que se comunica con el interior del cuello por medio de varios agujeros taladrados radialmente al través de las paredes del último á distancias iguales ó casi iguales en torno á la circunferencia.

Hay una cámara de presión semejante en el extremo más grande del cono corto para observar la presión en la sección normal aguas arriba del cuello; y, si se desea conocer la pérdida final de carga debida al paso del agua por el Venturi, debe proveerse una cámara semejante en el extremo más grande del cono más largo ó de aguas abajo.

**Al designar el tamaño** del medidor, úsase el diámetro del tubo de que él orma parte, y no el diámetro del cuello. Así, un medidor para usarlo en un tubo de 15 cm, se llama un medidor de 15 cm.

**El registro** da indicaciones periódicas, ordinariamente cada diez minutos, en las cuales la pérdida de carga $H = h_2 - h_1$, existente en el instante del registro, es anotada en pies cúbicos, litros, etc., de la descarga total desde el último registro y como un aumento en los volúmenes registrados. En otros términos, el registro supone que la velocidad media entre dos indicaciones, es igual á la velocidad en el instante de la indicación subsiguiente.

El registro puede colocarse á considerable distancia (que no exceda, digamos, de 150 m) del tubo de Venturi. Debe colocarse á tal profundidad por debajo de la línea de presión, que las presiones existentes en el tubo Venturi sean transmitidas al registro.

Las tuberías que unen el tubo de Venturi con el registro, deben estar cubiertas y el registro debe estar abrigado contra la intemperie y la helada.

El tamaño y costo del registro son independientes del tamaño del Venturi.

**Resultados.** De los experimentos hechos por el Sr. Herschel * †§, por la Oficina del Agua, Filadelfia †, y por otros †, resulta que puede contarse ordinariamente

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* Trans. Am. Soc. Civil Engrs, nov. 1887, vol. XVII, pág. 228.

† Diario del Instituto de Franklin, feb. 1899.

§ Diario de la Asociación de las Obras Hidráulicas de Nueva Inglaterra, vol. VIII, nº 11, sept 1893.

con que el medidor Venturi da resultados dentro del 3 por cierto del verdadero gasto.

Con un Venturi de 1.22 m (48 pulgs), encontró el Sr. Herschel una **pérdida total** de carga, debida al paso del agua por el tubo Venturi, de cerca de 10.6 por ciento de la carga H en el Venturi. Con dos Venturis de 1.37 m (54 pulgs), encontraron los profesores Marx Wing y Hoskins* una pérdida de 14.9 por ciento; parte de la cual se debía, sin duda, á la presencia de una llave de válvula de 1.07 m (42 pulgs) en el cono, aguas abajo. Este último resultado añadiría cosa de .34 m á la carga necesaria para bombear diariamente 75,600 m cúb por un conducto de 1.22 m y un Venturi que tuviese una estrechez de cuello de 1 : 9.

Hase encontrado que el medidor Venturi da resultados perfectamente satisfactorios midiendo una corriente de salmuera y agua muy caliente.

Los tubos de Venturi se hacen con cuellos (sección estrecha) que varían de 1 : 4½ (6.2 : 9) á 1 : 16. Los primeros se adaptan á grandes velocidades y los últimos á pequeñas; porque, cuando la velocidad en el tubo es pequeña, es necesario aceleraría considerablemente en el cuello, á fin de obtener suficiente pérdida de presión para conseguir indicaciones precisas en el registro. Éstas no pueden obtenerse cuando la velocidad en el cuello es menor m. ó m. de 1 metro por segundo. Con una estrechez del cuello de 1 : 16, daría en dicho caso una velocidad en el tubo de 57 mm (3/16 de pie) por segundo. Por otra parte, un medidor con una fuerte estrechez de cuello, adaptada á pequeñas velocidades, excedería, con las grandes, el límite superior del registro.

Debido á su conducto sin obstrucción ni partes movibles, con el medidor Venturi hay muchas menos probabilidades de estorbar el movimiento del líquido que con las formas de medidor en uso común.

Los **precios** de los tamaños principales del medidor Venturi son como siguepuestos en los carros en Providencia, R. I.:

| | | |
|---|---|---|
| .15 m, $600.00 | .60 m, $1,130.00 | 1.22 m, $3,060.00 |
| .30 m, $770.00 | .91 m, $1,680.00 | 1.52 m, $4.890 00 |

Estos precios incluyen el registro que, en los tamaños más pequeños, constituye a parte principal del costo. Descuento, 1901, 10 por ciento.

**Art. 4 b. El medidor Ferris-Pitot**, inventado y patentado por el Sr. Wálter Ferris, de Filadelfia, sirve para medir la corriente de líquidos en tubos á caño lleno. Consiste en la invención de un aparato registrador de los resultados obtenidos por el tubo Pitot, descrito más adelante, art. 19, y de otros inventos especiales para impedir la perturbación del movimiento en los tubos y permitir su examen estando en uso.

En la fig. 6, sea P el nivel del agua en el tubo recto *s* de Pitot. Entonces $h = kv^2$, ó la diferencia de nivel entre las columnas en los dos tubos, es la carga (teóricamente $= \frac{v^2}{2g}$) debida á la velocidad del agua en el tubo á medida que ella choca

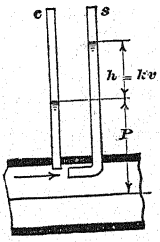


**Fig. 6.**

contra el extremo abierto aguas arriba del tubo encorvado, *s*. Para una velocidad dada, *v*, esta diferencia, *h*, es constante é independiente de la presión representada por P.

El registro Ferris, como el medidor Venturi, indica la velocidad (en el momento

* Trans. Am. Soc. Civil Engrs, vol. XL, dic. 1888, págs. 471, etc.

en que se registra) en partes del gasto total desde el último registro y como un aumento en los volúmenes registrados. El registro supone así que la velocidad media entre los valores registrados es igual á la velocidad al fin de ese período. En el medidor Ferris el registro se hace cada dos minutos.

Evidentemente el instrumento mide la velocidad en sólo un punto de la sección transversal del tubo, y así puede usarse para determinar sucesivamente las velocidades en cualquier número de puntos de dicha sección; pero la velocidad en tal punto puede ó no ser igual á la velocidad media de toda la sección transversal. El instrumento es por tanto calibrado de ordinario con referencia á alguna medida ó medidas aceptadas, y el coeficiente ó coeficientes obtenidos se usan en las observaciones subsiguientes.

El mecanismo del registro se mueve con un pequeño motor hidráulico, movido á su vez por el agua del tubo mismo. Para esto, un segundo par de tubos de Pitot se insertan en el tubo; y la corriente, fluyendo por estos tubos, pone en acción al motor sin pérdida de agua, pues ésta vuelve al tubo. Si la velocidad en el tubo es menor de 1 m por segundo, debe aumentarse por medio de un « reductor ».

Los experimentos hechos por el Sr. Ferris y por la Oficina del Agua, Filadelfia, indican que el medidor Ferris-Pitot es exacto hasta dentro de un 3 por ciento.

En general, el tamaño y costo del aparato registrador son independientes del tamaño del tubo.

**Art. 5. Resistencia de las curvas y codos en las tuberías de agua.** Mucha incertidumbre existe respecto de estos asuntos. La **fórmula * de Weisbach** para la resistencia debida á una curva circular, figs, 2 y 3, es de

$$h = C\,\frac{A}{180}\cdot\frac{v^2}{2g} = \left[.131 + 1.847\left(\frac{r}{R}\right)^7\right]\frac{A}{180}\cdot\frac{v^2}{2g}$$

$h$ = pérdida de carga para vencer la resistencia debida á la curva ó codo.
C = coeficiente experimental;
A = ángulo de flexión, en grados;
$v$ = velocidad media de la corriente en el tubo;
$g$ = aceleración de la gravedad = 9.81 m;
$\frac{v^2}{2g}$ = carga teórica para la velocidad $v$;
D = diámetro interior del tubo;
$r$ = radio interior del tubo;
R = radio del eje de la curva.

| Si $r \div R =$ | .1 | .2 | .3 | .4 | .5 | .6 | .7 | .8 | .9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| entonces C = | .131 | .138 | .158 | .206 | .294 | .440 | .661 | .977 | 1.408 | 1.978 |

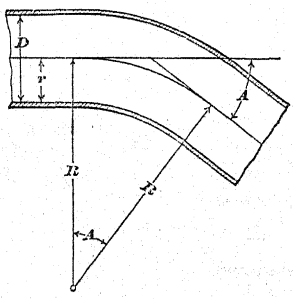


Fig. 2.

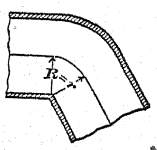


Fig. 3.

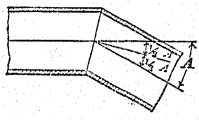


Fig. 4.

Conforme á esta fórmula, la resistencia debida á la curvatura disminuye tan rápidamente como R aumenta desde ½ D á 2 D; y sólo hay otra pequeña disminución más allá de R=5 D; pero, de experimentos muy cuidadosos en canales,

* El Ingeniero, págs. 444, 445.

desde .30 m á .75 m de diámetro, en Detroit, Mich. *, deducen los investigadores que una línea de tubos con una curva de *corto* radio R (hasta un límite de R = 2½ D) produce *menos* resistencia que una línea de igual largo é igual ángulo total A, con una curva de radio mayor que R. Sus resultados fueron aproximadamente los siguientes; donde

H = resistencia debida á una sección de 80 diámetros de largo, con una curva de A = 90° en el medio;

$h$ = resistencia en una tangente de largo = 80 diámetros.

| Si R ÷ D = | 1 | 2 | 2.5 | 3 | 4 | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|
| entonces H ÷ $h$ = | 1.35 | 1.14 | 1.13 | 1.14 | 1.18 | 1.24 | 1.50 | 1.66 | 1.80 | 1.93 |

Ellos encontraron también que **la pérdida de carga**, debida á una curva, ocurre **no sólo en la curva misma**, sino que se sigue perdiendo carga en la tangente que sigue por alguna distancia **aguas abajo** de la curva.

De sus experiencias se deduce que, aun muy **pequeñas flexiones**, A, en la línea, producen **importantes pérdidas de carga**, y que, por tanto, es muy recomendable tratar de colocar las tuberías rectas. Para **codos**, véase lo que sigue.

**Para ángulos**, fig. 4, da Weisbach : Pérdida de carga por resistencia =

$$h = c\frac{v^2}{2g} = (.95\ \text{sen}^2\ ½\ A + 2.05\ \text{sen}^4\ ½\ A)\frac{v^2}{2g}.$$

| Si ½ A = | 10° | 20° | 30° | 40° | 45° | 50° | 55° | 60° | 65° | 70° |
|---|---|---|---|---|---|---|---|---|---|---|
| entonces $c$ = | .03 | .14 | .36 | .74 | .98 | 1.26 | 1.56 | 1.86 | 2.16 | 2.43 |

**Además de la resistencia** opuesta á la corriente, las curvas y codos envuelven trabajos y gasto adicionales en la manufactura y en la colocación; y los codos y curvas verticales conducen á la formación de depósitos de sedimento á los pies de los declives y de cámaras de aire en sus cimas.

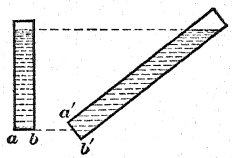


**Fig. 5.**

**Art. 6.** Aunque, en la fig. 5, las presiones estáticas sobre las bases iguales, *ab* y *a'b'*, de los dos tubos son iguales (véase Hidrostática, art. 1); con todo, para extraer agua por uno ú otro tubo, con una velocidad dada, necesítase una *fuerza adicional*, para vencer las resistencias de la corriente; y estas resistencias y la fuerza adicional requerida para vencerlas serán más grandes en el tubo más largo que en el más corto.

**Art. 7. Salida del agua por orificios.** *Teóricamente*, la velocidad con que el agua debe salir por tales aberturas es igual á la que adquiriría un cuerpo pesado que cayese libremente de una altura igual á la *carga h* ó profundidad del agua, medida *verticalmente* desde el nivel del líquido en el estanque al *centro de gravedad* de la abertura. Esta velocidad teórica se encuentra así :

$$v = \sqrt{2gh} = \sqrt{2 \times 9.81\ h}\ ;\quad h = \frac{v^2}{2 \times 9.81}\ ;$$

$g$ = á la intensidad de la pesantez = 9.81 m.

**Estas leyes se aplican igualmente á todos los fluidos.** Así, teóricamente, el mercurio, el agua, el aire, etc., todos salen con igual velocidad por un orificio dado, con una carga dada. **Para las diferencias** entre la experiencia y la teoría, véase art. 9, etc.

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* Gardner S. Williams, Clarence W. Husbell y George H. Fenkell, Transacciones Sociedad Americana de Ingenieros Civiles, vol. XLVII, abril de 1902.

**Art. 8. Salida del agua por aberturas verticales provistas de tubos cortos.** Cuando el agua sale de un estanque, fig. 6, á través de una pared vertical *mmaa*, cuyo espesor *am* es más ó menos de 2½ á 3 veces la menor dimensión transversal de la abertura (sea esta dimensión el alto ó el ancho de la abertura), ó si la pared *nn*, es muy delgada y el agua sale por un tubo *t*, cuya longitud es 2 ó 3 veces la menor dimensión de la abertura, entonces, el agua que afluye llenará toda la abertura, ó el tubo, como se indica en la fig. 6; es decir, *correrá á caño*

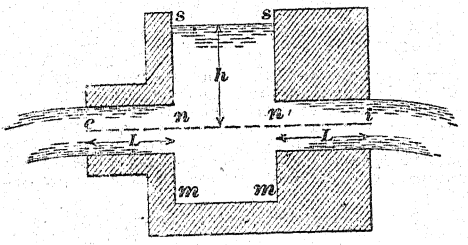


**Fig. 6.**

*lleno* y suministrará un gasto mayor; por tanto, saldrá más agua en un tiempo dado, que si fuera el tubo más largo ó más corto. Porque siendo su longitud más de tres veces la menor dimensión de la abertura, la corriente sufrirá mayor rozamiento contra las paredes del tubo; y si fuere menor que el doble de la menor dimensión transversal, más ó menos, el agua no saldrá á caño lleno, sino como vena contracta, como se ve en la fig. 11. Así sucederá ya sea la sección del tubo circular ó poligonal.

**Hallar aproximadamente la velocidad y el gasto real del agua que sale al aire libre por un tubo ó abertura, de sección circular ó poligonal, cuya longitud *n' i* ó *n e* en la dirección de la corriente sea más ó menos 2½ á 3 veces su menor dimensión transversal, cuando el nivel de la superficie *s*, fig. 6, es constante y de una altura mayor ó igual á la del borde superior del tubo ó abertura.**

*Regla* 1. Tómese la velocidad teórica por la fórmula del art. 7 correspondiente á la carga medida *verticalmente* desde el centro *e* (ó más exactamente desde el centro de gravedad) del orificio hasta el nivel de la superficie *s*. Multiplíquese por el coeficiente del gasto .81. El producto será la velocidad buscada por segundo. Multiplíquese esta velocidad real por la sección transversal del orificio. Si el orificio es circular y se conoce su diámetro $=d$, el área será $\frac{1}{4}\pi d^2 = .7854\ d^2$. El producto será el gasto ó cantidad de agua que sale por segundo con una aproximación de 3 á 4 por ciento.

*Regla* 2. Búsquese la raíz cuadrada de la carga (Tablas págs. 57 á 64) en metros y multiplíquese por 3.586. El producto es la velocidad real por segundo.

Ej.: Sea una abertura *n e* ó tubo en forma de cajón *ct*, fig. 6, de 1 metro de ancho por .25 m de alto y su longitud en la dirección de *n' i* ó *n e* de la corriente del agua de .62 m ó más ó menos 2½ veces su menor dimensión transversal ó su altura. La carga, desde el centro de gravedad *e* de la abertura al nivel constante *s*, de 2 metros. ¿Cuál será la velocidad del agua y cuál su gasto?

*Por la Regla* 1. La velocidad teórica que corresponde á una carga de 2 metros es de $\sqrt{2\times 9.81\times 2} = 6.26$ m por segundo, y $6.26\times .81 = 5.070$ m por segundo será la velocidad real buscada. El área transversal de la abertura ó del tubo, es $1\times .25 = .25$ m cuad y $.25\times 5.07 = 1.268$ metros cúbicos, el gasto ó cantidad que sale por segundo.

*Por la Regla* 2. La raíz cuadrada de 2 es 1.4142 y $1.4142\times 3.586 = 5.071$ por segundo, velocidad buscada como antes.

*Nota* 1. **Si el tubo corto, *n e*, sobresale en parte hacia el interior de la pared vertical, *s m*,** el gasto disminuirá más ó menos en $\frac{1}{8}$ parte. En este caso, úsese .71 ó .7 en lugar de .81 (*Regla* 1), y el coeficiente 3.15 en lugar de 3.586 de la *Regla* 2.

*Nota* 2. Cuando el espesor L de la pared vertical, ó el largo *n e* del tubo, fig. 6, llega á ser 4 veces más ó menos la menor dimensión transversal de la abertura, ó del diámetro, si fuere circular, entonces el rozamiento adicional contra sus paredes principia á disminuir apreciablemente la velocidad y el gasto. En este caso, y para longitudes mayores todavía, hasta 100 diámetros, puede encontrarse aproximadamente usando en lugar del coeficiente .81 de la *Regla* 1, los coeficientes que siguen, por los cuales deben multiplicarse las velocidades teóricas.

**TABLA 11.**

| Longitud del tubo en diám. | Coeficientes. | Longitud del tubo en diám. | Coeficientes. |
|---|---|---|---|
| 4 | .80 | 40 | .62 |
| 6 | .76 | 50 | .60 |
| 10 | .74 | 60 | .57 |
| 15 | .71 | 70 | .55 |
| 20 | .69 | 80 | .52 |
| 25 | .67 | 90 | .50 |
| 30 | .65 | 100 | .48 |

(*Obs. del T.* — Esta tabla se aplica cualquiera que sea la unidad que se use : el pie, el metro, etc.)

*Nota* 3. — Cuando el largo de la abertura ó del tubo, en la dirección de la corriente del agua, se hace, más ó menos, *menor* de dos veces su menor dimensión transversal, el gasto disminuye; de manera que para longitudes desde 1½ veces ó menores hasta aberturas en paredes muy delgadas, podemos usar .61 en lugar de .81 de la *Regla* 1. Para estas aberturas véanse los arts. 9 y 10.

*Nota* 4. Pero, por otra parte, el gasto de los tubos y aberturas tan cortas como la de la fig. 6, puede aumentarse hasta llegar casi á las teóricas $(\sqrt{2gh})$ con sólo redondear bien los bordes del orificio de entrada ó boca, como en la fig. 7, que es de la misma forma y tiene la mitad del tamaño real de una con que Weisbach obtuvo .975 de la velocidad y gasto teóricos, con una carga de 3.05 m, y .958 con una carga de .305 m; de modo que en casos semejantes puede usarse .975 y .958 en lugar del coeficiente .81.

Puede obtenerse de .92 á .94 ensanchado la abertura *nm* hacia la boca exterior *os*, fig. 8, dando al ángulo *a* más ó menos 5° ó haciendo el ensanche hacia su boca interior, como en *tc*, fig. 9, pero aumentando el ángulo de divergencia del *b* de 11° á 16°. En todos los casos consideramos el extremo pequeño como la abertura, cuya área debe multiplicarse por la velocidad para hallar el gasto.

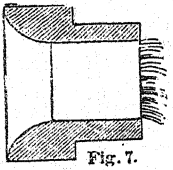

Fig. 7.

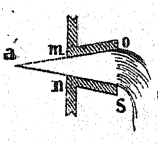


Fig. 8.

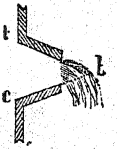


Fig. 9.

**En algunos experimentos hechos con canales piramidales de madera** de 2.90 m de largo con una boca interior de .97 m×.73 m., y una de salida de .19 m×.13 m con una carga de 2.90 m, se encontró un gasto de .98 m del gasto teórico, debido á que era más pequeña la boca; por lo tanto puede usarse .98 en semejantes casos en lugar de .81 de la *Regla* 1.

*Nota* 5. Usando un regulador de salida de la forma de la fig. 10, el gasto puede aumentarse y llegar á ser mayor que el correspondiente á la carga sobre el *centro de gravedad s* del orificio *mn*, porque en estos casos, como se dijo en el art. 1 *w* de Hidráulica, la carga *verdadera* en *s*, ó sea la carga que causa la salida rápide por la parte más angosta *mn*, puede ser mucho mayor que la carga sobre *s*. Véasa el medidor Venturi, art. 4 *a*.

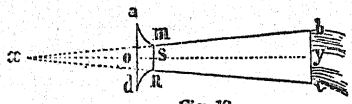


Fig. 10.

**Art. 9. Gasto por aberturas practicadas en paredes verticales delgadas de caras planas** *ee* ó *nn*, fig. 11 *. Si la cara *ee* ó *nn*, en lugar de ser plana y vertical, fuere curva ó inclinada en diferentes direcciones, hacia la abertura, el gasto se alterará. Cuando el agua sale de un estanque, fig. 11, por una pared vertical plana *nn*, cuyo espesor no sea mayor que la menor dimensión transversal de su abertura, más ó menos, ya sea esta dimensión el ancho de ella ó su altura *oo* †, ó cuando siendo la pared *ee* mucho más gruesa, damos á la abertura la forma que se ve en *b* (lo que viene á ser lo mismo); entonces la vena líquida afluente no saldrá cilíndrica ó á *caño lleno* como en la fig. 6, sino que tomará la forma de la fig. 11, formando muy cerca de la parte exterior de la abertura lo que se llama la *sección contracta*. Para que esta contracción sea *completa* ó llegue á su máximum, el borde interior del orificio no debe hallarse de la superficie del agua, del fondo ni de las paredes del estanque, á menos de 1½ veces la más pequeña dimensión transversal de la abertura. La vena se contrae á una dist del orificio igual á la mitad, más ó menos, de su menor dimensión. En un orificio circular, dicha contracción tiene efecto á una dist de medio diám más ó menos, y de ordinario, el área de la sección contracta es de .62 ó cerca de $^{2}/_{3}$ de la del orificio. En este punto la velocidad media *verdadera* de la vena líquida es muy aproximadamente .97 de la velocidad teórica dada por la fórmula $\sqrt{2gh}$. *El gasto* efectivo no es sino .62 ó casi $^{2}/_{3}$ del gasto teórico.

*Caso* 1. **Hallar el gasto efectivo cuando el agua sale al aire ‡ libre, por una abertura de sección circular ó poligonal § practicada en una pared delgada vertical, cuando la contracción es completa y cuando el nivel de la superficie *s* se conserva constantemente á la misma altura entrándole agua al estanque en igual cantidad á la que sale.**

*Regla* 1. Cuando la carga medida verticalmente desde el centro *c* (ó más bien desde el centro de gravedad) de la abertura, al nivel de la superficie *s*, no es menor de .30 m ni mayor de 3 m; y cuando la menor dimensión transversal de la abertura no es menor de .025 m, multiplíquese la velocidad teórica $\sqrt{2gh}$ por el coeficiente de gasto .62. El producto será la velocidad media verdadera del agua

* Creemos que estas Reglas son bastante aproximadas en casi todos los casos prácticos, para paredes delgadas, si la abertura está en el *fondo* del estanque, ó en una cara *inclinada* en lugar de una *vertical*.

† Cuando el lado del estanque, ó la arista del tablón, etc., por donde fluye el agua no tiene mayor espesor que ésta, se dice que el agua sale por *pared delgada*.

‡ Si la salida tiene lugar por *debajo del agua* como en la fig. 12, *permaneciendo constantes ambas superficies*, entonces la carga que debe usarse es la diferencia *vertical*

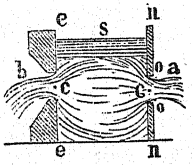


Fig. 11.

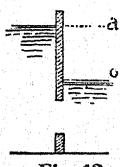


Fig. 12.

*ao* de los dos niveles. Después de hacer el cálculo con esta carga, debemos, según Weisbach, deducir 1/75; porque él dice que el gasto es menor en esta cantidad cuando la salida se efectúa por debajo del agua (á caño ahogado) que cuando tiene efecto al aire libre. Sin embargo, otros experimentadores afirman que el gasto es exactamente igual en ambos casos.

§ Si la forma del orificio es ovalada, triangular ó irregular, la carga debe medirse verticalmente sobre su centro de gravedad.

en la abertura. Multiplíquese esta velocidad por el área de la abertura, y el producto será el gasto por segundo aproximadamente.

Cuando la carga es mayor de 3 m, úsese .60 en lugar de .62.

*Regla* 2. Búsquese la raíz cuadrada de la carga en metros y multiplíquese por 2.75; el producto será la velocidad verdadera en metros por segundo, la cual multiplicada por el área como antes, dará el gasto.

(*Obs. del T.* — Comparados los resultados con estos diversos coeficientes, pueden resultar diferencias por razón de las decimales que se aprecien, pero serán insignificantes.)

Ej.: ¿Cuál será el gasto por una abertura con contracción completa, cuyas dimensiones sean .15 m verticalmente y 1.22 m horizontalmente, y la carga vertical sobre el centro de gravedad de la abertura de 1.83 m?

*Por la Regla* 1. Empleando el metro, tenemos:

$$\text{Velocidad verdadera} = .62\sqrt{2gh} = .62\sqrt{2\times 9.81\times 1.83} = .62\times 4.43\times 1.35 = 3.71 \text{ m.}$$

El área de la abertura es

$$= .15\times 1.22 = .183, \text{ y } 3.71\times .183 = .679 \text{ m}^3, \text{ ó sean 679 litros por segundo.}$$

*Por la Regla* 2. La raíz cuad de 1.83 es=1.35 y $1.35\times 2.75 = 3.71$ m velocidad verdadera como antes, la cual multiplicada por el área dará el gasto. Ambos muy aproximados aun cuando el orificio llegue hasta la superficie del agua que sale.

*Nota* 1. **El coeficiente .62 es el término medio** de los resultados de muchos experimentadores antiguos. En 1874, el general T. G. Ellis, de Massachusetts, dirigió una serie de experimentos (Trans. Am. Soc. C. E., Feb. 1876) en grande escala, cuyos resultados generales, con menos de 1% de diferencia, se dan en la tabla siguiente. Véase también la *Nota* 3. Los orificios de arista viva se hicieron en plancha de hierro de 6 á 12 milímetros de espesor.

| Orificios. | Cargas sobre el centro. | Coeficientes. |
|---|---|---|
| .609 × .609 m | .609 m á 1.067 m | de .60 á .61 |
| .609 × .305 » | .549 » 3.444 » | » .60 » .61 |
| .609 × .152 » | .427 » 5.182 » | » .61 » .60 |
| .609 m de diámetro. | .549 » 2.926 » | » .59 » .61 |

*Nota* 2. Se requiere **un cuidado extremo** para obtener resultados exactos; pero para muchas aplicaciones de ingeniería es de poca importancia un error de 5 á 10%.

Raras veces habrá necesidad de una exactitud mayor que la que se obtenga por las reglas precedentes; pero cuando esto ocurra, servirá de auxiliar la **tabla siguiente, deducida de los experimentos de Lesbros y Poncelet**, en orificios de 8 pulgadas (203 mm) de ancho, de diferentes alturas y con cargas diferentes. Usese el coeficiente de la tabla que venga al caso en lugar del .62 de la *Regla* 1. En algunos casos de esta tabla la arista superior del orificio dista menos de 1½ veces su menor dimensión transversal, del nivel de la superficie del estanque.

**TABLA 12. Coeficientes para aberturas rectangulares practicadas en paredes delgadas verticales con contracción completa.**

El ancho en todas las aberturas es=8 pulgs=.203 m.

| Carga sobre el centro de gravedad de la abertura en metros. | Carga sobre el centro de gravedad de la abertura en pies. | Altura de la abertura. | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | Metros .203 Pulg. 8 | Metros .152 Pulg. 6 | Metros .102 Pulg. 4 | Metros .076 Pulg. 3 | Metros .051 Pulg. 2 | Metros .025 Pulg. 1 | Metros .010 Pulg. .4 |
| .0102 | .0330 | ..... | ..... | ..... | ..... | ..... | ..... | .70 |
| .0203 | .0666 | ..... | ..... | ..... | ..... | ..... | .65 | .69 |
| .0254 | .0833 | ..... | ..... | ..... | ..... | ..... | .64 | .68 |
| .0381 | .125 | ..... | ..... | ..... | ..... | .61 | .64 | .68 |
| .0508 | .1666 | ..... | ..... | ..... | .60 | .62 | .64 | .68 |
| .0635 | .2083 | ..... | ..... | .59 | .61 | .62 | .64 | .67 |
| .0762 | .250 | ..... | ..... | .60 | .61 | .62 | .64 | .67 |
| .0889 | .2917 | ..... | .57 | .60 | .61 | .62 | .64 | .66 |
| .1016 | .3333 | ..... | .58 | .60 | .61 | .63 | .64 | .66 |
| .1143 | .3750 | .56 | .59 | .60 | .61 | .63 | .64 | .66 |
| .1270 | .4167 | .57 | .59 | .61 | .62 | .63 | .64 | .66 |
| .2032 | .6666 | .59 | .60 | .61 | .62 | .63 | .64 | .65 |
| .3048 | 1 | .60 | .60 | .61 | .62 | .63 | .63 | .64 |
| .9144 | 3 | .60 | .60 | .61 | .62 | .62 | .63 | .63 |
| 1.524 | 5 | .60 | .60 | .61 | .61 | .62 | .62 | .62 |
| 3.048 | 10 | .60 | .60 | .60 | .60 | .60 | .61 | .61 |

*Nota* 3. **Experimentos hechos cuidadosamente con aberturas de 1.30 m de ancho y 0.46 m de altura**, bajo cargas de 1.83 á 4.57 m muestran que el coeficiente .62 da resultados exactos con $^1/_{30}$ de aproximación para aberturas también del mismo tamaño, bajo grandes cargas; aunque el espesor de la pared varíe en sus diferentes lados de .30 m á .51 m. Debe tenerse, sin embargo, en cuenta que en esta materia sólo puede llegarse á resultados *muy aproximados*.

*Nota* 4. Algunos escritores aseguran que **cuando el agua sale por dos ó más aberturas** *contiguas* al mismo tiempo y del mismo estanque, sale en proporción menos agua por ellas que cuando el agua sale por una sola abertura de las existentes. Sin embargo, otros experimentos demuestran que esto no es así, y es muy probable que la diferencia, si existe, sea de poca importancia.

*Caso* 2. **Salida del agua á través de paredes verticales, delgadas, con contracción completa, cuando el nivel de la superficie, *m*, fig. 13, desciende á medida que el agua sale al aire.** En este caso, si el estanque es prismático, es decir, si sus secciones horizontales son iguales en toda su altura, y si no entra agua al estanque para reponer la que sale, entonces, para hallar el tiempo en que se vacía, se observará la siguiente

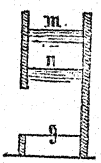


Fig. 13.

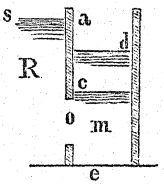


Fig. 14.

*Regla.*—Como el tiempo que se requiere para que este estanque se vacíe es doble del que se necesita para que salga igual cantidad de agua, bajo una carga constante (como en el caso 1), calcúlese el gasto por segundo, por la *Regla* 1, art. 9, divídase el número de metros cúbicos contenidos en el estanque desde el nivel *g*

de la parte inferior de la abertura hacia arriba, fig. 13, por este gasto; y el cociente dará el número de segundos en que saldría del estanque hasta la profundidad $g$, un volumen igual de agua al calculado en el caso 1, *con una carga constante;* y *el doble* de este número de segundos será el tiempo que se necesita para vaciar dicho estanque en el caso 2, con una *carga variable.*

*Nota.* Si fuere necesario buscar el tiempo durante el cual dicho estanque prismático se vacía *parcialmente*, como por ejemplo de $m$ á $n$, fig. 13, entonces calcúlese primeramente por la regla anterior, el número de segundos necesario para vaciarlo en el caso en que sólo estuviese lleno hasta $n$, y en seguida calcúlese como si lo estuviera hasta $m$. La diferencia entre los dos números de segundos es evidentemente el tiempo que se requiere para vaciarlo de $m$ á $n$. Si la abertura no produce una contracción completa, véase el art. 11, etc.

**Si el agua se vacía en un estanque más bajo, cuyo nivel sea constante,** procédase de la misma manera; sólo que se tomará como carga la diferencia de nivel de las dos superficies, y después (según Weisbach) se aumentará el tiempo en $^1/_{27}$.

**Art. 10. Salida del agua de un estanque, B, fig. 14, de nivel constante, $s$, á un estanque prismático, $m$, por una abertura, $o$, practicada en pared delgada, con contracción completa y situada enteramente debajo del agua.** (Medidas en m.)

**Número de segundos requeridos** para verter una cantidad = *cda* permaneciendo el nivel $c$ constante.

$$= \frac{\text{área horizontal del envase } m \times \sqrt{\text{altura } ac}}{\text{área de la abertura } o \times .62 \times 4.43.}$$

**Número de segundos requeridos para** elevar el nivel del envase $m$ de $c$ á $a$.

$$= \frac{\text{área horizontal del envase } m \times 2 \times \sqrt{\text{altura } ac}}{\text{área de la abertura } o \times .62 \times 4.43.}$$

**Número de segundos requeridos para** elevar el nivel del envase $m$ de $c$ á cualquier otro nivel $d$.

$$= \frac{\text{área horizontal del envase } m \times 2 \times (\sqrt{ac} - \sqrt{ad})}{\text{área de la abertura } o \times .62 \times 4.43.}$$

*Nota* 1. **Si se busca el tiempo necesario para llenar el envase $m$ desde su fondo $e$ hasta** $d$, podemos obtenerlo muy aproximadamente calculando, por la primera regla del art. 9, el tiempo gastado de $e$ hasta el centro del orificio $o$, como si la salida del agua en esta porción tuviera lugar al aire libre, y luego desde el centro del orificio hasta $d$, por la regla que acabamos de dar. Este caso es análogo al de llenar una esclusa desde la parte superior del canal en el que se considera constante el nivel del agua.

*Nota* 2. **Si la parte inferior de la abertura $o$ coincidiese con el fondo del estanque,** entonces el coeficiente se hará mayor que .62. Véase art. 11, que trata de coeficientes para contracciones imperfectas.

*Nota* 3. **Si la abertura en lugar de ser de contracción completa,** tiene una de las formas de las figuras 6 á 9, entonces el art. 8 indicará cuál coeficiente debe usarse en lugar de .62.

*Caso* 3. **Salida del agua de un receptáculo prismático, W, fig. 15, á otro X, de cualquier tamaño, cuando el agua asciende en X, al**

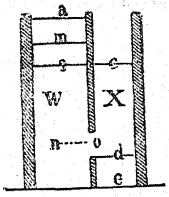


Fig. 15.

**bajar en W, efectuándose la salida por una abertura $o$ practicada en una pared plana, delgada y vertical y con contracción completa.**

*Hallar el tiempo durante el cual el agua, pasando de* W *á* X *por la abertura o desciende la distancia* a s, *hasta situarse al mismo nivel* s c, *en ambos estanques.* El agua requerida para llenar el envase X de *e á d* (siendo *d* el fondo de la abertura *o*) efectúa su salida al aire libre, y el tiempo que se requiere para ello debe calcularse aparte del que se necesita para llenar la porción del envase de *d* hacia arriba; pues en esta parte la salida se efectúa por debajo del agua.

*Regla.* Estudiemos primeramente de *e á d*: Búsquese el área horizontal de cada uno de los envases ó estanques en m cuad; multiplíquese el área horizontal de X por la altura vertical *d e* en m para obtener el número de m cúb contenidos en aquella porción; divídase este número de m cúb por el área ó sección horizontal de W. El cociente será la dist *am* en m que debe descender el agua en W para llenar á X hasta el punto *d*.

**Número de segundos que se requieren** para descender de *a á m* y por tanto para ascender de *e á d*.

$$= \frac{\text{Doble del área horizontal del envase W} \times \left(\sqrt{\text{carga } an} - \sqrt{\text{carga } mn}\right)}{\text{Area de la abertura} \times .62 \times \sqrt{2g}}$$

**Número de segundos que se requieren** para descender de *m á s* y para ascender de *d á c*. (Muy aproximadamente.)

$$= \frac{\text{Area horizontal del envase X} \times \text{doble del área horizontal del envase W} \times \sqrt{\text{carga } mn}}{\text{Area de la abertura } o \times \left(\begin{matrix}\text{área horizontal} \\ \text{de W}\end{matrix} + \begin{matrix}\text{área horizontal} \\ \text{de X}\end{matrix}\right) \times .62 \times \sqrt{2g}}$$

(*Obs. del T.* — Estas fórmulas se usan indiferentemente en pies ó en metros. Cuando se usan en pies, $g = 32.2$; y cuando se usan metros, $g = 9.81$.)

Ej.: Supongamos el área de W = 100 pies cuad = 9.29 m cuad; la de X = 60 pies cuad = 5.574 m cuad; *an* = 20 pies = 6.096 m; *mn* 16 pies = 4.88 m; y el área de la abertura, *o* = 3 pies cuad = .279 m cuad. ¿En cuánto tiempo descenderá el agua de *a á s* y ascenderá de *e á c*? Por cuanto el método para hallar el tiempo que se gasta en llenarse de *e á d* con el agua que cae de *a á m* no requiere más explicación, nos limitaremos al tiempo *adicional* que se necesita para llenar dicho envase de *d á c* con el agua que cae de *m á s*. Para lo cual tenemos: raíz cuad de la carga *mn* = 4 pies; y la suma de las dos áreas = 100 + 60 = 160. Además, con *g*, en pies, $\sqrt{2g} = 8.03$; y con *g*, en metros, $\sqrt{2g} = 4.43$. Usando la fórmula, nos da: $\frac{60 \times 200 \times 4}{3(100 + 60) \times .62 \times 8.03} = \frac{48000}{2389.73} = 20.1$ seg, tiempo adicional requerido, muy aproximadamente.

Aplicando el mismo ejemplo en sistema métrico, tenemos:

$$\sqrt{mn} = \sqrt{4.88} = 2.2091 \text{ m};$$

suma de las dos áreas = 9.29 + 5.574 = 14.864. Luego obtendremos

$$\frac{5.574 \times 2 \times 9.29 \times 2.2091}{.279 \times 14.864 \times .62 \times 4.43} = 20.09 \text{ segundos.}$$

Como antes, con una pequeña dif debida á las decimales que no se han apreciado.

*Advertencia* 1. **Cuando la abertura *d*, fig. 16, llega hasta el fondo del estanque,** podemos considerar toda el agua que pasa de R á T como efectuando su salida dentro del agua. Por lo tanto, al usar la carga *am*, calculamos desde luego el tiempo necesario para que el agua de los dos estanques llegue al

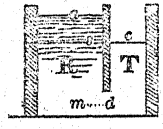


Fig. 16

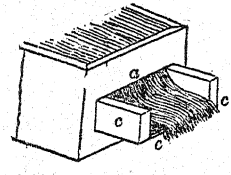


Fig. 17.

mismo nivel *sc* por el procedimiento seguido en el ejemplo precedente. Pero en este caso debe tenerse presente que la abertura *d* ya no dará contracción *completa*

puesto que está suprimida la contracción á lo largo de su borde inferior. El gasto aumentará algo y, de consiguiente, debe emplearse un coeficiente mayor de .62. El método para hallar éste se da en el siguiente caso 4. En el art. 8 se da el coeficiente en el caso de que la abertura tenga una de las formas de las figs. 6 á 9.

**Art. 11**, *Caso* 4. **Salida del agua por aberturas hechas en paredes delgadas, planas y verticales, pero con contracción incompleta.** La abertura puede ser tal, que la contracción tenga lugar en una porción de su perímetro ó en la parte superior de la abertura *a*, fig. 17, donde, por ejemplo, está suprimida en el fondo y las dos caras de dicha abertura *a*, por la adición de piezas *cortas* en el fondo y costados *c*, *c*, *c* del orificio. En estos casos el gasto será mayor que en los de contracción completa, pero menor que en los que el agua sale á caño lleno, puesto que la abertura se asemeja ahora en parte á los tubos *cortos* del art. 8; y el coeficiente subirá de .62 ó del que *generalmente* corresponda á las aberturas de contracción completa, acercándose á .8 ó al que corresponde á caño lleno, según sea la extensión del perímetro en que se ha suprimido la contracción; ó aun á .9 ó .98 usando las aberturas que se indican en las figs. 7, 8, 9.

**Hallar aproximadamente un coeficiente nuevo de gasto; y el gasto mismo en casos de contracción incompleta.**

*Regla.* Primero : búsquese por las reglas anteriores cuál sería el gasto en el caso particular que se considera, suponiendo que la contracción fuese completa. Entonces divídase la parte del perímetro de la abertura en que está suprimida la contracción por el perímetro total. Multipliquese el cociente por la decimal .152 si la abertura es rectangular, ó por .128 si es circular. Al producto agréguese la unidad. Llámese la suma *g*. Entonces dígase : la unidad : *g* :: el coeficiente para la contracción completa en los casos ordinarios (generalmente .62) : al coeficiente nuevo que se busca. Finalmente, repítase el cálculo primitivo, sustituyendo solamente este coeficiente nueve en lugar de .62.

Según esta regla, tenemos los coeficientes de gasto siguientes para aberturas rectangulares, con una aproximación de 3 ó 4 por ciento, cuando la contracción no está suprimida en más de $^{7}/_{8}$ del perímetro. El gasto teórico multiplicado por el coeficiente correspondiente, dará el gasto real. Cuando la contracción aumenta, el coeficiente se hace extremadamente irregular, y probablemente es indeterminable.

| | | | | |
|---|---|---|---|---|
| *Para contracciones completas (generalmente)* | | | | .62 |
| *Cuando la contracción está suprimida en un* | | | $^{1}/_{4}$ *del perímetro* | .64 |
| — | — | — | $^{1}/_{2}$ — | .67 |
| — | — | — | $^{3}/_{4}$ — | .69 |
| — | — | — | *enteramente alrededor de la abertura* | .80 |

Los coeficientes intermedios pueden calcularse con suficiente exactitud por una simple proporción.

*Nota* 1. **Cuando, en lugar de un vertedero corto como el de la fig. 17, la abertura está provista de un canal horizontal de un largo indefinido,** adherido de un modo semejante y abierto en su parte superior, no habrá prácticamente disminución sensible del gasto, comparado con el que suministra la abertura sencilla *a*, fig. 11, con tal que la carga medida por sobre el centro de gravedad de la abertura sea por lo menos de 2 á 2½ veces la altura de la abertura misma. Por lo tanto, en tales circunstancias debe calcularse el gasto por la regla del art. 9. Pero con cargas menores el gasto disminuye considerablemente; de tal manera que cuando la carga sobre el centro es sólo de la altura de la abertura, el gasto no será sino $^{1}/_{2}$ del calculado. Con carga más pequeñas aún, la salida se hace menor, mucho más rápidamente. Para este caso no se ha establecido todavía ninguna regla.

*Nota* 2. **Si en lugar de ser horizontal el canal, es inclinado como de 1 en 10,** el gasto *aumentará* muy poco (como 3 ó 4 por ciento) sobre el calculado por las reglas del art. 9, para una abertura ordinaria. Estos resultados se han obtenido por experimentos en muy pequeña escala y deben considerarse como meras aproximaciones.

**Art. 12. En casos semejantes al de la fig. 18, en el cual se supone suprimida la contracción en el fondo y en los lados verticales de la abertura** *o*, por su coincidencia con el fondo y los lados del depósito, pero en que el frente del depósito en lugar de ser vertical está inclinado como en *f*, y cuando el agua después de haber pasado la abertura fluye por sobre un canal *g*, ligeramente inclinado hacia abajo, entonces el gasto puede hallarse aproximadamente por la *Regla* 1, caso 1, art. 9, sustituyendo solamente .8 en lugar de .62

cuando *f* tenga una inclinación hacia atrás de 45° ó de 1 en 1; se tomará el coeficiente .74 si *f* está inclinado hacia atrás 63°, ó, lo que es lo mismo, para *uno* de base y *dos* de altura. En estos casos de frentes inclinados, la altura de la abertura debe medirse *verticalmente ó, más bien, en ángulo recto al piso del depósito*, y no en la línea inclinada del frente.

*Nota.* **Cuando el frente *f* del estanque es vertical, y se aplica una prolongación ó canal *g* inclinada**, con su cara superior al nivel del fondo de la abertura, el gasto no disminuye en cosa apreciable, comparado con el que se obtiene libremente al aire, con tal que la carga sobre el centro de gravedad de la abertura no sea menor

| | | | |
|---|---|---|---|
| de 46 á 60 cm | para una abertura de | 10 á 23 cm de altura; |
| de 30 á 40 | — | — | 10 cm de altura; |
| de 23 | — | — | 5 ó menos de altura. |

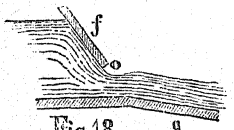

Fig 18

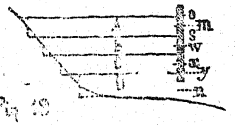

Fig 19

**Art. 13. Hallar aproximadamente el tiempo necesario para vaciar un pozo ó cualquier otro estanque como el de la fig. 19, cuya forma no sea prismática, por una abertura *n* practicada cerca del fondo.**

*Regla.* Fijemos primeramente la forma y dimensiones exactas del estanque. Si es grande y de forma irregular, debe medirse con cuidado, sondearse y hacerse un plano de él y secciones transversales exactas. Luego considérese dividido todo el volumen de agua en capas horizontales delgadas A, B, C, D, situando la parte superior de la capa más baja D, por lo menos algunos cm sobre la parte superior de la abertura *n*. No es necesario que estas capas sean del mismo espesor; sin embargo, mientras más delgadas sean, más exacto será el resultado.

El espesor de la capa más baja D, depende de la altura de la abertura; las de arriba próximas á ésta, no deben tener más de 30 cm de espesor hasta una altura de 1.80 m ó 2.40 m; entonces se puede aumentar convenientemente y con suficiente exactitud hasta 60 cm más ó menos durante otro espacio de 1.80 m ó 2.40 m; y así sucesivamente, dándoles á las capas más espesor á proporción que se aproximan más á la superficie del agua. Con la ayuda de los planos calcúlese el volumen de cada capa. Ahora bien: como las capas son delgadas, podemos, sin error sensible, suponer que todas ellas son prismáticas, como lo indican en las figuras las líneas de puntos; y podemos también suponer que la carga, bajo la cual cada una de las capas (con excepción de la última ó más baja) fluye por *n*, es igual á la altura vertical medida desde el centro de la abertura hasta el centro de la capa de agua. De este modo *mn* será la carga de A; *wn*, la de B; *xn*, la de C. Entonces, respecto de la capa A, por la *Regla* 1. art. 9 (tomando solamente á *mn* como carga en lugar de *on*), y en lugar del coeficiente .62 de aquella regla (que solamente puede usarse si *n* es de contracción completa), tómese .64 ó cualquier otro coeficiente que pueda aplicarse según el final del art. 11, y calcúlese el gasto por segundo. Divídase el volumen de la capa A por este gasto, y el cociente dará el número de segundos que se necesita para vaciar á A. Tomando la carga *wn*, procédase exactamente lo mismo con la capa B; y tomando la carga *xn*, hágase lo mismo con C. Finalmente, para la capa más baja D, búsquese por la *Regla* 1, del art. 9 (con la misma precaución que antes respecto al coeficiente adecuado), en cuánto tiempo se vaciaría bajo una carga *constante* igual á *yn* medida desde su *superficie* al centro de la abertura. El *doble* de este tiempo será el que se necesita para vaciarse en el caso presente, bajo su carga *variable*. Por último, súmense todos estos tiempos: y la suma total será el tiempo que se necesita para vaciar el estanque ó pozo con suficiente aproximación en la práctica.

**Art. 14.** (*a*) **Salida del agua por vertederos.** El vertedero proporciona un medio muy conveniente para el aforo de pequeños arroyos, como cuando se quiere calcular la cantidad de agua que se suministra á las ruedas hidráulicas, etc.

(*b*) Un dique de aforo ó vertedero está siempre dispuesto de manera que su cara interior *ab*, ó sea la situada aguas arriba, fig. 20, sea vertical, y tan próximamente como se pueda en ángulo recto á la dirección de la corriente. Las

caras *ah*, *ah*, figs. 21 y 22, son verticales, y el borde ó arista *aa*, horizontal. (Véase más adelante « Ranuras ó Muescas triangulares », art. 15, pág. 598.)

(*c*) **Contracciones en los extremos del vertedero.** Cuando el vertedero *aa* ocupa transversalmente todo el canal de entrada, como en la fig. 21, de manera que sus caras extremas *ah*, *ah*, coincidan con los costados *ss* del canal ó formen parte de él, la contracción, art. 9, tiene efecto solamente en la parte *superior* é *inferior* de la lámina de agua que pasa por el vertedero, como en *mc* y en *a*, fig. 20, y está del todo « suprimida » en los *extremos*, de manera que la salida del agua se hace como lo indica la fig. 21 *a*.

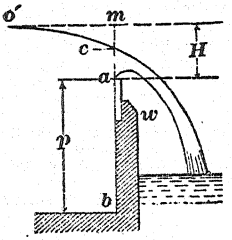


Fig. 20.

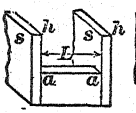


Fig. 21.

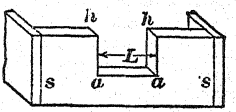


Fig. 22

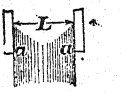


Fig. 21 *a*.

Fig. 22 *a*.

Esta clase de vertederos se llaman **vertederos sin contracciones extremas.** Pero como cuando en las figs. 22 y 22 *a*, las caras extremas *ah*, *ah* se hallan distantes de los costados *ss* del canal ó del estanque, también hay contracciones en los *extremos*, como se ve en *a* y *a*, así como sobre la arista. Tales contracciones disminuyen el gasto. Esta especie de vertederos se llaman **vertederos con contracciones extremas.** En igualdad de circunstancias, la magnitud de la contracción y su efecto sobre el gasto aumenta con la carga H.

Cuando el largo *aa* ó L del vertedero es mayor de 10 veces la carga H, el efecto de las contracciones en las extremidades, sobre el gasto, es casi imperceptible; pero á medida que el largo disminuye en relación á la carga, el efecto de la contracción aumenta rápidamente. Francis (art. 14 *m*) halló que cuando L era solamente $=4\times H$, el gasto se reducía un 6 por ciento, por la contracción completa de las extremidades. En vista de la incertidumbre de los efectos producidos por la contracción de las extremidades, es mejor evitarlas y usar vertederos como los de la fig. 21, donde la contracción está suprimida; pero si se admite la contracción en las extremidades, debe hacerse *completa* *, porque los coeficientes dados no sirven para **contracciones incompletas**, es decir, con contracciones sólo parcialmente suprimidas.

(*d*) **En un vertedero sin contracción en las extremidades**, debe tenerse cuidado de que *el aire tenga acceso libre* al espacio (*w*, fig. 20 ó 22 *b*) por

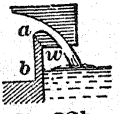


Fig.22b

detrás de la caída formada por la lámina de agua, de lo contrario se formaría allí un vacío parcial y la lámina de agua sería atraída hacia el vertedero, modificándose el gasto notablemente. Al mismo tiempo debe evitarse que la lámina de agua se *esparza lateralmente* cuando abandona el borde del vertedero. Ambos objetos pueden lograrse prolongándose un poco la *parte superior solamente* de los dos costados del canal aguas abajo, más allá del borde ó arista y de la parte

* La contracción se dice que es « completa » cuando es prácticamente tan grande como puede hacerse por un aumento cualquiera ulterior de la dist *as*, figs. 22 y 22 *a*, y se cree que éste se obtiene cuando se hace *as* igual á la carga H.

superior de la caída formada por la lámina de agua, como en la fig. 22 *b*. Francis encontró que estas prolongaciones, limitando lateralmente la lámina de agua, disminuían el gasto en .4 por ciento.

(*e*) Ordinariamente el **copete** del vertedero es en « pared delgada » (véase nota †, pág. 576), de modo que la lámina de agua que pasa por sobre el vertedero, lo toca solamente en la propia arista ó ángulo *a*, fig. 20. Un ángulo ó borde *redondeado* aumenta el gasto, del mismo modo que los bordes redondeados de un orificio (art. 8), y un copete ó borde suficientemente ancho para desviar la caída de la lámina de agua, disminuye el gasto (véanse los coeficientes para este caso en la tabla 15, pág. 592); pero ambas formas producen mucha incertidumbre y deben por lo tanto evitarse.

(*f*) **La longitud L del copete ó cresta**, figs. 21 y 22 *a*, debe ser por lo menos el triple de la carga H, con el fin de reducir los efectos del rozamiento de los costados *ss* y el de la contracción de las extremidades cuando ésta existe. **La altura** *p*, fig. 20, de la cara interior vertical *ab* en contacto con el agua, no debe ser menor del doble de la carga H; porque con el fin de reducir la velocidad al acercarse el agua á la compuerta del vertedero, ó sea la velocidad de acceso (véase art. 14 *u*), la sección transversal **del canal** que conduce al vertedero debe ser grande en proporción con la de la corriente *ac*. La sección transversal del canal de entrada ó acceso debe ser lo más *regular* posible.

(*g*) El vertedero debe ser de construcción fuerte, porque las **vibraciones** pueden modificar seriamente el gasto.

(*h*) *Teóricamente*, **la carga** es la dist vertical H', fig. 24, de la cresta *a*, á un punto *o' en donde el agua se encuentra perfectamente tranquila*, y por consiguiente la superficie es *horizontal*. Pero de hecho la carga se mide generalmente desde la cresta *a* á un punto *o*, á algunos metros detrás del vertedero, donde el agua está sólo *relativamente* en equilibrio, siendo perceptible la velocidad de acceso. (Véase art. 14 *u*.) La diferencia entre la carga H realmente medida y la carga H' en el agua *tranquila* es muy pequeña generalmente. En la figura está muy exagerada. **La medida correcta de la carga** es una operación delicada, aumentando ó disminuyendo el gasto más ó menos 1½ por ciento por un aumento ó disminución de 1 por ciento de la carga. Las agitaciones del agua y otras perturbaciones de la superficie, y la atracción capilar son las causas principales de error.

(*i*) Para evitar esta última dificultad se emplea la **sonda aforadora de gancho**, para medir, en casos importantes, la altura de la superficie del agua. Se compone ésta de una larga regla graduada, provista en su parte inferior de un gancho ó punta vuelto hacia arriba, el cual se desliza verticalmente (por medio de tornillos) en un soporte fijo provisto de un nonio que indica en la escala la altura del gancho ó punta. Se baja primeramente la regla corredera hasta que la punta se halle bien debajo de la superficie, y luego se levanta gradualmente por medio del tornillo hasta que la punta ó gancho llegue justamente á la superficie, lo que se conoce por la aparición de un « botoncito » en la superficie del agua inmediatamente sobre el gancho. Bajo circunstancias favorables, con un buen aforador de gancho se pueden hacer lecturas con aproximaciones hasta de 6 á 15 centésimos de milímetro.

(*j*) Para evitar inexactitudes, debidas á las **perturbaciones de la superficie** producidas por las corrientes, el viento, etc., se toma el nivel algunas veces (con la sonda aforadora de gancho ó de cualquiera otro modo) en un departamento lateral que comunique con el canal principal del acceso. La superficie del agua en el departamento conserva el mismo nivel que el del canal mismo, pero está relativamente libre de perturbaciones. También puede servir al efecto un balde ó tobo comunicado con el canal por medio de un tubo. Ambos deben estar, por supuesto, á cubierto del viento. **Advertencia.** Fteley y Stearns hallaron que, cuando el tobo ó departamento estaban comunicados con el agua *cerca del fondo y próximamente detrás del vertedero*, la carga así obtenida era generalmente algo mayor que la hallada por medidas hechas cerca de la superficie y á 1.80 m detrás del vertedero. Pero Francis encontró esta diferencia apenas perceptible.

(*k*) Debe tenerse mucho cuidado en **arreglar la sonda aforadora de gancho con respecto á la altura de la cresta ó copete**, porque un error en esto afecta todas las operaciones subsiguientes. El gancho se ajusta generalmente á la altura de la superficie cuando esta última llega exactamente al nivel de dicho borde ó cresta; pero este método se hace inexacto por la atracción capilar de aquélla. Más exacto es tener, además del aforador de gancho, un punzón fuerte, *fijo*, con la punta hacia arriba, cuyo nivel con relación al de la cresta pueda deter-

minarse por medio de un nivel de ingeniero colocando la regla graduada sobre dicho borde ó arista.

Entonces se deja descender la superficie del agua lentamente hasta que aparezca un *botoncito ó menisco* en la punta del gancho fijo. Luego se mantiene el agua á este nivel y, de acuerdo con esto, se ajusta el aforador de gancho. O, si el aforador es fuerte, puede colocarse desde luego la regla graduada sobre su extremidad sin tener que recurrir al gancho fijo. Lo mejor es arreglar el aforador de manera que el *cero* corresponda al nivel de la cresta del vertedero, que servirá entonces de origen ó punto de partida de la graduación, pues la lectura del aforador con respecto á la superficie del agua dará entonces desde luego la carga H sin tener que restar la altura de la cresta del vertedero.

**(1) Fórmulas del gasto de los vertederos.**

Sean

$\mathbf{Q}$ = gasto real del vertedero por segundo*;
$\mathbf{Q'}$ = gasto teórico del vertedero por segundo;
$\mathbf{H} = \mathbf{H'}$ + = dist vertical ó carga *am*, fig. 24 (art. 14 *u*), medida desde la arista *a* á la superficie horizontal *o'* del agua *tranquila*, aguas arriba del vertedero *:
$\mathbf{L}$ = largo *aa* del vertedero *, figs. 21 á 22 *a*;
$g$ = intensidad de la gravedad = 32.2 pies = 9.81 metros * por segundo.

$$\left.\begin{aligned} c &= \text{coeficiente de gasto} = \frac{\text{gasto real}}{\text{gasto teórico}} = \frac{Q}{Q'}; \\ m &= \frac{2}{3}c; \\ x &= \frac{2}{3}c\sqrt{2g} = m\sqrt{2g} \end{aligned}\right\} \div$$

Luego, para **el gasto teórico**, tenemos:

$$\mathbf{Q'} = \frac{2}{3}\,\mathrm{LH}\sqrt{2g}\,\mathrm{H},\ \S\ldots\ldots\ (1)$$

y para el **gasto real**

$$\left.\begin{aligned} \mathbf{Q} &= \mathbf{cQ'} \\ &= \frac{2}{3}\,\mathbf{cLH}\sqrt{2g\mathrm{H}} \ldots\ldots\ (2) \\ &= \mathbf{mLH}\sqrt{2g\mathrm{H}} \ldots\ldots\ (3) \\ &= \mathbf{xLH}\sqrt{\mathrm{H}} = \mathbf{xL}\sqrt{\mathrm{H}^3} = \mathbf{xLH}^{\frac{3}{2}} \ldots\ (4) \end{aligned}\right\} \text{Véanse notas al pie } *\dagger\ddagger.$$

**Para el valor del coeficiente** ($c$, $m$, ó $x$ ‡) hemos recurrido á experimentos, midiendo el gasto real y comparándolo con el gasto teórico como se indica en los artículos siguientes.

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* Estas fórmulas se aplican igualmente á cualquier sistema de medidas: inglés, métrico, etc.; sólo se requiere que la carga, el largo y la intensidad de la gravedad ($g$) se expresen en la *misma* unidad: todos en pies, todos en metros, etc., y el gasto en el cubo de esta unidad. En medida métrica $g$ es = 9.81 *m* por segundo.

† *Por ahora* suponemos que la carga se mide en el agua *tranquila*, de modo que H = H'; cuando no sea este el caso, véase « Velocidad de acceso », art. 44 (*u*) más adelante.

‡ Se notará que las fórmulas (2), (3) y (4) con sus correspondientes coeficientes, $c$, $m$ y $x$, son realmente idénticas, diferenciándose solamente en la forma. La última es la más conveniente en la práctica, pero todas ellas se usan en hidráulica.

§ Cuando el agua sale bajo una carga H, por un orificio *horizontal* practicado en el *fondo* de un envase, la velocidad teórica (art. 7) es = $\sqrt{2g\mathrm{H}}$; y ésta también se puede considerar como exacta para orificios *verticales* practicados en las *paredes* de los envases, con tal que la carga H sobre el centro de gravedad del orificio sea á lo menos dos ó tres veces la dimensión vertical del orificio; porque en ambos casos las velocidades teóricas en los diferentes puntos del orificio pueden considerarse *iguales*. Pero cuando un orificio vertical está más cerca de la superficie, ó *llega* hasta ella como en un vertedero, entonces debemos tomar en consideración las diferencias de velocidades de la salida del agua en los puntos de profundidades diferentes.

Teóricamente las moléculas atraviesan el plano oblicuo *ao'*, fig. 23, en líneas horizontales, con velocidades (= $\sqrt{2gh}$ = 4.43 $\sqrt{h}$) (hablando en metros) proporcionales á las raíces cuads de sus respectivas y diferentes profundidades verticales $h$ (que no se indi-

*(m)* **James B. Francis*** experimentó en « las esclusas bajas » en Mass, 1852, con vertederos de 3.05 m de largo, 1.52 m y .61 m de alto, bajo cargas de .18 m á .48 m. Para aplicar sus resultados, deben existir las condiciones siguientes : la carga H, fig. 20, debe estar comprendida entre .15 m y .61 m. La altura *p* de la cara vertical interior del vertedero sobre el fondo *b* del canal, fig. 20, debe ser por lo menos el doble de la carga H. El copete *a* debe ser de « pared delgada » (nota †, pág. 576) y su longitud L, figs. 21 á 22 *a*, debe ser por lo menos el triple de la carga H. Las extremidades *ah*, *ah*, deben ser verticales, y cuando haya contracción de extremidades, que sean de « pared delgada ».

Cuando hay contracción de extremidades, Francis rebaja primeramente de la longitud L del vertedero *una décima † parte de la carga* H, por *cada extremidad* donde haya contracción. Así : si *n* es=al número de contracciones de las extremidades (dos en la fig. 22),

$$Q = x\left(L - n\frac{H}{10}\right)H\sqrt{H} = x\left(L - n\frac{H}{10}\right)H^{\frac{3}{2}} \div \ldots. \quad (5)$$

$$\text{En la fig. 22, } Q = x\left(L - \frac{H}{5}\right)H\sqrt{H} \div = x\left(L - \frac{H}{5}\right)H^{\frac{3}{2}}$$

Pero entre los límites especificados arriba, la fórmula es muy aproximada *sin* correción por las contracciones de las extremidades, con tal que el largo L del vertedero sea por lo menos 10 veces la carga H; y con un 6 por ciento de aproximación cuando L=4 H. Cuando no existe contracción en las extremidades, tal corrección no es necesaria, y la fórmula permanece igual á $Q = xLH\sqrt{H} \div = xLH^{\frac{3}{2}}$.

---

can en la figura) bajo la superficie tranquila del agua en *o'*. Por lo tanto, si imaginamos trazadas desde *am* líneas horizontales *aa'*, *dd'*, *vv'*, *cc'*, etc., etc., que representen dichas velocidades en cualquiera escala, entonces los extremos exteriores *a'*, *d'*, *v'*, *c'*, etc., de estas líneas, formarán con *am* y *aa'* un segmento parabólico *amc'a'* cuya área será = 2/3 del área del rectángulo *an* (véase parábola, pág. 203)=2/3 *am* × *aa'*=2/3 H $\sqrt{2gH}$; y esta área multiplicada por el espesor de la lámina de agua que fluye por el largo L del vertedero dará el gasto teórico por segundo. O

$$Q' = L \times \text{área } amc'a' = L \times 3/2\ H\sqrt{2gH}.$$

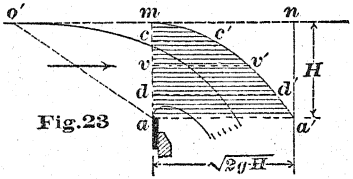

Fig.23

Por lo tanto, el área *amc'a'* en metros cuads. representará el gasto teórico en metros cúbs por segundo, suministrado por un metro de longitud del vertedero bajo la carga H. La *velocidad media* teórica en la sección *ao'* es

$$V = vv' = \frac{\text{gasto teórico } Q'}{\text{área } am} = 2/3\ LH\sqrt{2gH} \div LH = 2/3\sqrt{2gH},$$

ó dos tercios de la velocidad horizontal teórica *aa'* de las moléculas que pasan inmediatamente por sobre el vertedero. Como en el caso de los orificios (art. 9), la velocidad real *en la sección mas pequeña* de la lámina de agua después de haber pasado el vertedero (que corresponde á la « sección contracta » de la vena) es probablemente casi igual á esta velocidad teórica.

* « Lowell Hydraulic Experiments ». Van Nostrand, Nueva York, 1883

† En los experimentos de Fteley y Stearns, este número no permanecía constante en .10, sino que variaba entre .061 y .124, aumentando generalmente á medida que disminuía la carga.

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÷ Suponemos aquí que la carga se mide hasta la superficie del agua *tranquila*, de manera que H y H' (véase art. 14 *h*, pág. 584), son iguales. Véase « Velocidad de acceso », art. 14 *(n)*.

**Francis da x=1.838** empleando unidades métricas; ó

$$\textbf{Gasto} = 1.838 \times \left( \text{largo} - \frac{\text{número de contracciones de las extremidades}}{10} \times \text{carga H} \right) \times H^{3/2}, *$$

El término medio de 88 experiencias dió $x = 1.8394$. El mayor valor obtenido fué $^1/_{105}$ mayor; y el menor, resultó $^1/_{112}$ más pequeño. Usando, $x = 1.838$, tendremos para **c y m**, los valores siguientes : $c = .622$; $m = .415$.

La altura de la superficie se midió (1.83 m) detrás del vertedero, por medio de dos sondas aforadoras de gancho, situada una de cada lado del canal, y se tomó el término medio de sus lecturas para calcular el coeficiente $x$.

**Tabla 13 **. Gasto en pies cúbicos por segundo por cada pie de longitud del vertedero,** en pared delgada y sin contracción de las extremidades por la fórmula de Francis : ‖

$$\textbf{Gasto } Q = 3.33\ LH^{3/2} = 3.33\ LH\sqrt{H}.$$

Muy aproximada también, cuando haya contracción de las extremidades, con tal que L sea por lo menos=10 H; y con sólo 6 por ciento de exceso sobre el verdadero valor, si L=4 H. Francis limita la fórmula á cargas H desde .5 pies hasta 2 pies; pero no resultarán errores de importancia del uso de la tabla para cualquiera de las cargas dadas. **Para vertederos de otras longitudes** distintas á 1 pie, multiplíquese el gasto tabular por el largo real en pies. La primera carga de la tabla es .01 pie = .12 pulg = $^1/_8$ pulg escasa = .003 m = 3 milímetros.

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* Véase esta nota pág. 586.

** Esta tabla es una ampliación de la tabla « original » publicada en la primera edición de 1872. La mayor parte de los valores dados ahora se han tomado (con permiso) de una tabla publicada por los Srs. A. W. Hunking y Frank S. Hart, de Lowell, Mass, en mayo de 1884.

‖ Véase *N. del T.* después de la tabla que sigue.

| Carga H en pies. | Pies cúbicos por segundo. | Carga H en pies. | Pies cúbicos por segundo. | Carga H en pies. | Pies cúbicos por segundo. | Carga H en pies. | Pies cúbicos por segundo. | Carga H en pies. | Pies cúbicos por segundo. |
|---|---|---|---|---|---|---|---|---|---|
| .01 | 0.003 | .51 | 1.213 | 1.01 | 3.380 | 1.51 | 6.179 | 2.01 | 9.48 |
| .02 | 0.009 | .52 | 1.249 | 1.02 | 3.430 | 1.52 | 6.240 | 2.02 | 9.56 |
| .03 | 0.017 | .53 | 1.285 | 1.03 | 3.481 | 1.53 | 6.302 | 2.03 | 9.63 |
| .04 | 0.027 | .54 | 1.321 | 1.04 | 3.532 | 1.54 | 6.364 | 2.04 | 9.70 |
| .05 | 0.037 | .55 | 1.358 | 1.05 | 3.583 | 1.55 | 6.426 | 2.05 | 9.774 |
| .06 | 0.049 | .56 | 1.395 | 1.06 | 3.634 | 1.56 | 6.488 | 2.06 | 9.846 |
| .07 | 0.062 | .57 | 1.433 | 1.07 | 3.686 | 1.57 | 6.551 | 2.07 | 9.917 |
| .08 | 0.075 | .58 | 1.471 | 1.08 | 3.737 | 1.58 | 6.613 | 2.08 | 9.989 |
| .09 | 0.090 | .59 | 1.509 | 1.09 | 3.790 | 1.59 | 6.676 | 2.09 | 10.062 |
| .10 | 0.105 | .60 | 1.548 | 1.10 | 3.842 | 1.60 | 6.739 | 2.10 | 10.134 |
| .11 | 0.121 | .61 | 1.586 | 1.11 | 3.894 | 1.61 | 6.803 | 2.11 | 10.206 |
| .12 | 0.138 | .62 | 1.626 | 1.12 | 3.947 | 1.62 | 6.866 | 2.12 | 10.279 |
| .13 | 0.156 | .63 | 1.665 | 1.13 | 4.000 | 1.63 | 6.930 | 2.13 | 10.352 |
| .14 | 0.174 | .64 | 1.705 | 1.14 | 4.053 | 1.64 | 6.994 | 2.14 | 10.425 |
| .15 | 0.193 | .65 | 1.745 | 1.15 | 4.107 | 1.65 | 7.058 | 2.15 | 10.498 |
| .16 | 0.213 | .66 | 1.786 | 1.16 | 4.160 | 1.66 | 7.122 | 2.16 | 10.571 |
| .17 | 0.233 | .67 | 1.826 | 1.17 | 4.214 | 1.67 | 7.187 | 2.17 | 10.645 |
| .18 | 0.254 | .68 | 1.867 | 1.18 | 4.268 | 1.68 | 7.251 | 2.18 | 10.718 |
| .19 | 0.276 | .69 | 1.909 | 1.19 | 4.323 | 1.69 | 7.316 | 2.19 | 10.792 |
| .20 | 0.298 | .70 | 1.950 | 1.20 | 4.377 | 1.70 | 7.381 | 2.20 | 10.866 |
| .21 | 0.320 | .71 | 1.992 | 1.21 | 4.432 | 1.71 | 7.446 | 2.21 | 10.940 |
| .22 | 0.344 | .72 | 2.034 | 1.22 | 4.487 | 1.72 | 7.512 | 2.22 | 11.015 |
| .23 | 0.367 | .73 | 2.077 | 1.23 | 4.543 | 1.73 | 7.577 | 2.23 | 11.089 |
| .24 | 0.392 | .74 | 2.120 | 1.24 | 4.598 | 1.74 | 7.643 | 2.24 | 11.164 |
| .25 | 0.416 | .75 | 2.163 | 1.25 | 4.654 | 1.75 | 7.709 | 2.25 | 11.239 |
| .26 | 0.441 | .76 | 2.206 | 1.26 | 4.710 | 1.76 | 7.775 | 2.26 | 11.314 |
| .27 | 0.467 | .77 | 2.250 | 1.27 | 4.766 | 1.77 | 7.842 | 2.27 | 11.389 |
| .28 | 0.493 | .78 | 2.294 | 1.28 | 4.822 | 1.78 | 7.908 | 2.28 | 11.464 |
| .29 | 0.520 | .79 | 2.338 | 1.29 | 4.879 | 1.79 | 7.975 | 2.29 | 11.540 |
| .30 | 0.547 | .80 | 2.383 | 1.30 | 4.936 | 1.80 | 8.042 | 2.30 | 11.615 |
| .31 | 0.575 | .81 | 2.428 | 1.31 | 4.993 | 1.81 | 8.109 | 2.31 | 11.691 |
| .32 | 0.603 | .82 | 2.473 | 1.32 | 5.050 | 1.82 | 8.176 | 2.32 | 11.767 |
| .33 | 0.631 | .83 | 2.518 | 1.33 | 5.108 | 1.83 | 8.244 | 2.33 | 11.843 |
| .34 | 0.660 | .84 | 2.564 | 1.34 | 5.165 | 1.84 | 8.311 | 2.34 | 11.920 |
| .35 | 0.690 | .85 | 2.610 | 1.35 | 5.223 | 1.85 | 8.379 | 2.35 | 11.996 |
| .36 | 0.719 | .86 | 2.656 | 1.36 | 5.281 | 1.86 | 8.447 | 2.36 | 12.073 |
| .37 | 0.749 | .87 | 2.702 | 1.37 | 5.340 | 1.87 | 8.515 | 2.37 | 12.150 |
| .38 | 0.780 | .88 | 2.749 | 1.38 | 5.398 | 1.88 | 8.584 | 2.38 | 12.227 |
| .39 | 0.811 | .89 | 2.796 | 1.39 | 5.457 | 1.89 | 8.652 | 2.39 | 12.304 |
| .40 | 0.842 | .90 | 2.843 | 1.40 | 5.516 | 1.90 | 8.721 | 2.40 | 12.381 |
| .41 | 0.874 | .91 | 2.891 | 1.41 | 5.575 | 1.91 | 8.790 | 2.41 | 12.459 |
| .42 | 0.906 | .92 | 2.939 | 1.42 | 5.635 | 1.92 | 8.859 | 2.42 | 12.536 |
| .43 | 0.939 | .93 | 2.987 | 1.43 | 5.694 | 1.93 | 8.929 | 2.43 | 12.614 |
| .44 | 0.972 | .94 | 3.035 | 1.44 | 5.754 | 1.94 | 8.998 | 2.44 | 12.692 |
| .45 | 1.005 | .95 | 3.083 | 1.45 | 5.814 | 1.95 | 9.068 | 2.45 | 12.770 |
| .46 | 1.039 | .96 | 3.132 | 1.46 | 5.875 | 1.96 | 9.138 | 2.46 | 12.848 |
| .47 | 1.073 | .97 | 3.181 | 1.47 | 5.935 | 1.97 | 9.208 | 2.47 | 12.927 |
| .48 | 1.107 | .98 | 3.231 | 1.48 | 5.996 | 1.98 | 9.278 | 2.48 | 13.005 |
| .49 | 1.142 | .99 | 3.280 | 1.49 | 6.057 | 1.99 | 9.348 | 2.49 | 13.084 |
| .50 | 1.177 | 1.00 | 3.330 | 1.50 | 6.118 | 2.00 | 9.419 | 2.50 | 13.163 |

(*N. del T.* — Por ser la tabla anterior muy minuciosa y precisa y, además, por ser muy fácil utilizarla, empleando medidas métricas, hemos resuelto dejarla.

En efecto, cuando se tenga la longitud del vertedero y la carga en metros, se puede hacer uso de esta tabla, teniendo presente que para convertir metros en pies se multiplican los metros lineales × 3.2809 = pies lineales, y los pies cúbicos que nos da la tabla se convierten en metros cúbicos así : pies cúbicos × .0283 = metros cúbicos.

Ej. Tenemos un vertedero de 2 m de largo y una carga .50 m, es decir, L = 2 m; H = .50 m.

L = 2 × 3.2809 = 6.5618 pies; H = .5 × 3.2809 = 1.6404 pies;

y entrando á la tabla con esta carga de 1.64 pies nos da para un vertedero de 1 pie

de largo solamente 6.994 pies cúbicos por segundo. Pero como el vertedero tiene de largo 6.5618 pies, tendremos el gasto=6.5618×6.994=45.893 pies cúbicos y 45.893 pies cúb×.0283=1.299 metros cúbicos por segundo = 1299 litros por segundo.)

(*n*) **A. Fteley y F. P. Stearns** * hicieron experimentos en Boston, Mass, de 1877 á 79, con vertederos de 5 pies (1.52 m) y 19 pies (5.79 m) de largo, 3 pies 2 pulgs (.96 m) y 6 pies 6 ½ pulgs (1.99 m) de alto y bajo cargas de .8 pulg (.02 m) á 19 pulgs (.49 m). Para vertederos en pared delgada y sin contracción en las extremidades, con un canal de acceso rectangular y uniforme, y bajo cargas mayores que .07 pies (.02 m); (las otras condiciones iguales á las especificadas en (*b*) y (*d*)), la fórmula de ellos es como sigue:

$$\textbf{Gasto, } Q=3.31\,LH^{3}/_{2}+.007\,L \dagger \ldots\ldots\ldots \quad (6).$$

O la siguiente (por el Traductor) aplicable al sistema métrico, y deducida de la anterior:

$$Q=1.82605\times LH^{3}/_{2}+.0006503\,L$$

(*Obs. del Traductor.* — Para que estas fórmulas den resultados exactos deben apreciarse siempre suficiente número de decimales.)

En sus experimentos (los de A. Fteley y P. F. Stearns) las cargas se midieron á 6 pies (1.83 m) detrás del vertedero. La variación total en el valor de los coeficientes obtenidos fué más ó menos de 2½ por ciento.

(*Obs. del Traductor.* — Damos el siguiente ejemplo para que sirva de comparación de las fórmulas: Sea

| | | | |
|---|---|---|---|
| $H=.1$ | (en pies)..... | .0304794 | (en m) |
| $L=.4$ | — ..... | .1219176 (=4 H) | — |
| $H^{3}/_{2}=.03162$ | — ..... | .00532 | — |

tendremos:

En pies... $Q=3.31\times.4\times.03162+.007\times.4=.044664$ pies$^3$ = .001264 m$^3$
En m.... $Q=1.82605\times.1219176\times.00532+.0006503\times.1219176=.001263636$m$^3$)

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* *Transactions, American Society of Civil Engineers, Jan., Feb. and March,* 1883.
† Véase corrección de la velocidad de acceso, art. 14 (*n*).

(*o*) **M. Bazin** * hizo experimentos en Dijón, Francia, en 1886-88, en vertederos como de .45 m á 1.97 m de largo y de .23 m á 1.14 m de alto, bajo cargas

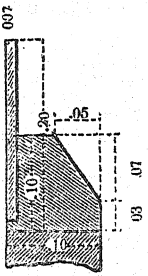


**Fig. 23 *a*.**

pe .063 m á .53 m. La parte superior del vertedero se muestra en la fig. 23 *a*. Los vertederos se pusieron en diferentes puntos de un canal rectangular y regular de 213.35 m de largo revestido con cemento y liso.

Mientras que Francis y Fteley y Stearns previenen el efecto de la velocidad de acceso (véase art. 14 *w* y *v*) modificando la *carga* medida H, Bazin la previene introduciendo el *coeficiente m* en la fórmula $Q = mLH\sqrt{2gH}$.

Sea M = el valor de *m* para el caso en que la velocidad de acceso = 0. Entonces, muy aproximadamente, $M = .405 + \frac{.003}{H\ (\text{metros})}$. Cuando se toma en cuenta la velocidad de acceso, tenemos: $$\mathbf{m} = M\left[1 + .55\left(\frac{H}{H+p}\right)^2\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (7)$$

en la cual H es la carga verdadera medida en el agua corriente, y *p* la altura *ab* del vertedero, fig. 20 H y *p* deben, por supuesto, medirse con la misma unidad, ambos en metros ó ambos en pies, etc.

M. Bazin cree que, excepto en el caso de vertederos muy bajos (lo que debe evitarse), los valores de *m* dados por la fórmula (7) y en la Tabla 14, calculados con ella, se hallarán con 1 por ciento de aproximación para vertederos en pared delgada, sin contracción en las extremidades, si se reproducen exactamente las condiciones de sus experimentos y si se tiene especial cuidado de que la lámina de agua no se esparza lateralmente, después de pasar el copete (art. 14 *d*) y que el aire tenga acceso libre al espacio *w*, fig. 20, por detrás de la caída formada por la lámina de agua al salir del vertedero.

Para cargas comprendidas entre 10 cm y 30 cm, M. Bazin da, como **suficientemente aproximado,**

cuando no hay velocidad de aceso, $M = .425$

y teniendo en cuenta la velocidad de acceso $\mathbf{m} = .425 + .21\left(\frac{H}{H+p}\right)^2$.

---

* *Expériences nouvelles sur l'écoulement en déversoir. Extrait des Annales des Ponts et Chaussées*, oct. 1888, *Paris, Vve Ch. Dunod*, 1888. Traducción de A. Marichal y John. C. Trautwine Jr., presentada al *Club de Ingenieros de Filadelfia* en 1889 para que la publicase en sus trabajos.

**Tabla 14. Valores de m en la fórmula :**

$$Q = mLH\sqrt{2gH} * \ldots\ldots\ldots\ldots 6$$

**Gasto en metros cúbicos** $= m \times$ largo en metros carga medida H, en metros $\times \sqrt{2gH}$, en metros.

*Nota.* El coeficiente $m$ para cualquier caso dado es el mismo, úsese medida inglesa, métrica ú otras; con tal que se tome la carga, el largo y $g$ en la misma unidad y el gasto en el cubo de aquella unidad, porque $m$ es simplemente $\frac{2}{3}ac = \frac{2}{3} \times \frac{\text{gasto real}}{\text{gasto teórico}}$.

Se notará que **bajo las líneas gruesas** de la tabla, la carga H es mayor que la mitad de la altura $p$, y esto excede al límite fijado en (*l*) y (**m**).

| Carga H, fig. 24 más adelante. | | | Altura $p$, fig. 20, de la cresta del vertedero sobre el fondo del canal aguas arriba. | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Metros. | aproximadamente. Pies. | Pulgadas. | Metros | .20 | .30 | .40 | .50 | .60 | .80 | 1.00 | 1.40 | 2.00 | Infinito |
| | | | | m | m | m | m | m | m | m | m | m | M† |
| .05 | .164 | 1.97 | ——— | .458 | .453 | .451 | .450 | .449 | .449 | .449 | .448 | .448 | .4481 |
| .06 | .197 | 2.36 | ——— | .456 | .450 | .447 | .445 | .445 | .444 | .443 | .443 | .443 | .4427 |
| .07 | .230 | 2.76 | ——— | .455 | .448 | .445 | .443 | .442 | .441 | .440 | .440 | .439 | .4391 |
| .08 | .262 | 3.15 | ——— | .456 | .447 | .443 | .441 | .440 | .438 | .438 | .437 | .437 | .4363 |
| .09 | .295 | 3.54 | ——— | .457 | 447 | .442 | .440 | .438 | .436 | .436 | .435 | .434 | .4340 |
| .10 | .328 | 3.94 | ——— | .459 | .447 | .442 | .439 | .437 | .435 | .434 | .433 | .433 | .4322 |
| .12 | .394 | 4.72 | ——— | .462 | .448 | .442 | .438 | .436 | .433 | .432 | .430 | .430 | .4291 |
| .14 | .459 | 5.51 | ——— | .466 | .450 | .443 | .438 | .435 | .432 | .430 | .428 | .428 | .4267 |
| .16 | .525 | 6.30 | ——— | 471 | .453 | .444 | .438 | .435 | .431 | .429 | .427 | .426 | .4246 |
| .18 | .591 | 7.09 | ——— | .475 | .456 | .445 | .439 | .435 | .431 | .428 | .426 | .425 | .4229 |
| .20 | .656 | 7 87 | ——— | .480 | .459 | .447 | .440 | .436 | .431 | .428 | .425 | .423 | .4215 |
| .22 | .722 | 8.66 | ——— | .484 | .462 | .449 | .442 | .437 | .431 | .428 | 424 | .423 | .4203 |
| .24 | .787 | 9.45 | ——— | .488 | .465 | .452 | .444 | .438 | .432 | .428 | .424 | .422 | .4194 |
| .26 | .853 | 10.24 | ——— | .492 | .468 | .455 | .446 | .440 | .432 | .429 | .424 | .422 | .4187 |
| .28 | .919 | 11.02 | ——— | .496 | .472 | .457 | .448 | .441 | .433 | .429 | .424 | .422 | .4181 |
| .30 | .984 | 11.81 | ——— | .500 | .475 | .460 | .450 | .443 | .434 | .430 | .424 | .421 | .4174 |
| .32 | 1.050 | 12.60 | ——— | —— | .478 | .462 | .452 | .444 | .436 | .430 | .424 | .421 | .4168 |
| .34 | 1.116 | 13.39 | ——— | —— | .481 | .464 | .454 | .446 | .437 | .431 | .424 | .421 | .4162 |
| .36 | 1.181 | 14.17 | ——— | —— | .483 | .467 | .456 | .448 | .438 | .432 | .424 | .421 | .4156 |
| .38 | 1.247 | 14.96 | ——— | —— | .486 | .469 | .458 | .449 | .439 | .432 | .424 | .421 | .4150 |
| .40 | 1.312 | 15.75 | ——— | —— | .489 | .472 | .459 | .451 | .440 | .433 | .424 | .421 | .4144 |
| .42 | 1.378 | 16.54 | ——— | —— | .491 | .474 | .461 | .452 | .441 | .434 | .425 | .421 | .4139 |
| .44 | 1.444 | 17.32 | ——— | —— | .494 | .476 | .463 | .454 | .442 | .435 | .425 | .421 | .4134 |
| .46 | 1.509 | 18.21 | ——— | —— | .496 | .478 | .465 | .456 | .443 | .435 | .425 | .421 | .4128 |
| .48 | 1.575 | 18.90 | ——— | ——— | ——— | .480 | .467 | .457 | .444 | .436 | 425 | .421 | .4122 |
| .50 | 1 640 | 19.69 | ——— | ——— | ——— | .482 | .468 | .459 | .445 | .437 | .426 | .421 | .4118 |
| .52 | 1.706 | 20.47 | ——— | ——— | ——— | .483 | .470 | .460 | .446 | .438 | .426 | .421 | .4112 |
| .54 | 1.772 | 21.26 | ——— | ——— | ——— | .485 | .472 | .461 | .447 | .438 | .426 | .421 | .4107 |
| .56 | 1.837 | 22 05 | ——— | ——— | ——— | .487 | .473 | .463 | .448 | .439 | .427 | .421 | .4101 |
| .58 | 1 903 | 22.83 | ——— | ——— | ——— | .488 | .475 | .464 | .449 | .440 | .427 | .421 | .4096 |
| .60 | 1.969 | 23.62 | ——— | ——— | ——— | .490 | .476 | .466 | .451 | .441 | .427 | .421 | .4092 |

Debido á que se ha tomado la carga H y la altura $p$ entre límites más extensos en estos experimentos, encontramos en ellas una **divergencia mayor** en los valores de los coeficientes que los que resultaron de las investigaciones anteriores. Así, el valor más pequeño de $m$ que está de la línea gruesa hacia arriba, es .4092 ó más ó menos $^1/_{10}$ menor que el término medio .4325; y el mayor es .459 ó como $^1/_{10}$ mayor que .4325.

* En estos experimentos la carga H se midió en un punto situado á 5 m detrás de vertedero. La corrección por la velocidad de acceso está incluída en el coeficiente $m$.

† M es el valor de $m$ cuando no hay velocidad de acceso; es decir, cuando la sección transversal del canal de acceso es demasiado grande comparada con la de la corriente del agua que pasa por sobre el vertedero.

(*p*) Comparando un número de datos experimentales, **el autor** ha deducido la siguiente:

**Tabla 15. Valores aproximados del coeficiente** $m$ en la fórmula

$$Q = mLH\sqrt{2gH}$$

para vertederos de diferentes formas y espesores. (Original.)

| **Carga H.** | Arista viva *. | Espesor .05 m. | Espesor .9144 m liso, con inclinación hacia afuera y hacia abajo desde 1 en 12 hasta 1 en 18. | Espesor .9144 m liso y horizontal. |
|---|---|---|---|---|
| **m** | **m** | **m** | **m** | **m** |
| .0254 | .41 | .37 | .32 | .27 |
| .0508 | .40 | .38 | .34 | .30 |
| .0762 | .40 | .39 | .34 | .31 |
| .1016 | .40 | .41 | .35 | .31 |
| .1270 | .40 | .41 | .35 | .32 |
| .1524 | .39 | .41 | .35 | .33 |
| .1778 | .39 | .41 | .35 | .32 |
| .2032 | .39 | .41 | .34 | .31 |
| .2540 | .38 | .40 | .34 | .31 |
| .3048 | .38 | .40 | .33 | .31 |
| .6096 | .37 | .39 | .32 | .30 |
| .9144 | .37 | .39 | .32 | .30 |

(*Obs. del T.* — A esta tabla, original del autor, le hemos agregado la carga y espesores en metros, para facilitar su uso en el sistema métrico, pues la fórmula $Q = mLH\sqrt{2gH}$, como ya se ha dicho, puede aplicarse en ambos sistemas, con el mismo coeficiente $m$ con tal que se sustituyan en ella los valores en metros de los otros factores.)

(*q*) **Hallar la carga H aproximadamente** teniendo el gasto Q. Según las fórmulas (3) y (4), art. 14 (*l*), tenemos :

$$Q = mLH\sqrt{2gH} = xLH\sqrt{H} = mL\sqrt{2g}\sqrt{H^3} = xL\sqrt{H^3}$$

$$H = \sqrt[3]{\frac{Q^2}{m^2L^2 2g}} = \sqrt[3]{\frac{Q^2}{x^2L^2}} \quad \ldots\ldots\ldots\ldots\ldots \quad (8)$$

ó bien (sistema métrico).

$$\text{Carga H en metros aproximadamente} = \sqrt{\frac{\text{Cuad del gasto de la corriente en metros cúbs por segundo}}{m^2 \times (\text{largo en metros})^2 \times 19.62}}$$

El coeficiente $m$ ó $x$ mismo varía algo con la carga.

(*r*) **Vertederos sumergidos ó ahogados.** fig. 23 *b*, son aquellos en los cuales la superficie del agua del lado *abajo* de la corriente en $h$, es, después de la contracción del vertedero, *más alta* que la arista $a$.

En un vertedero que efectúe la salida del agua libremente al aire como en la fig. 20, Francis halló que con una carga de 30 cm disminuyó el gasto solamente como en una milésima parte, poniendo un piso sólido horizontal como á 15 cm debajo y frente al borde ó arista del vertedero para que el agua cayese sobre dicho piso. También, cuando la carga era de 25 cm y el agua caía libremente á través del aire en aguas de profundidad considerable (como en la fig. 20), el

* Estos valores son menores que los dados en el art. 14 (*m*) y (*n*) y mucho menores que los dados en (*o*).

gasto era el mismo, aunque estuviese la superficie del agua del lado abajo de la corriente, como 7 cm ½ ó 33 cm bajo la cresta *a*.

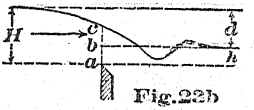


Fig. 23b

En los experimentos hechos por Francis y Fteley y Stearns, dejando entrar libremente el aire por debajo de la caída formada por la lámina de agua, justamente debajo de la cresta *a*, el gasto no se afectó sensiblemente por una sumersión de *h*=desde .017 de H á .023 de H. Cuando el aire sólo penetraba parcialmente, el gasto se afectaba (*aumentando*) en menos de 1 por ciento mientras *h* permanecía menor que .15 H.

**Fórmula de Dubuat para vertederos sumergidos.** Sea :

**H** y *h*=las cargas medidas verticalmente desde la cresta *a* del vertedero á la superficie del agua en reposo * del lado arriba y abajo de la corriente del vertedero respectivamente;

$d = H - h$ =su diferencia=diferencia de nivel de las dos superficies del agua en reposo, * (la sup de arriba y la de abajo del vertedero);

*c* =coeficiente del gasto $= \frac{\text{gasto teórico}}{\text{gasto real}}$.

Entonces tenemos :

$$Q = cL\ (h + {}^2/_3\ d)\ \sqrt{2gd}\ **. \ldots\ldots\ (9)\ \text{ó sea}$$

**Gasto** *real* en mets cúbs por segundo $= c \times$ largo del vertedero en mets $\times 4.43\ \sqrt{d \text{ en mets}} \times (h + {}^2/_3 d)$ en mets.

(*s*) **Fteley y Stearns** † hicieron experimentos en Boston, en 1877, con **vertederos sumergidos** bajo cargas H aguas arriba como de 10 á 25 cm, y **Francis** § en Lowel, 1883, bajo cargas como de 30 á 71 cm.

De estos experimentos deducimos la siguiente **tabla 16** de los valores aproximados del **coeficiente c** en la fórmula del gasto en **vertederos sumergidos.**

$$Q = cL\ (h + {}^2/_3 d)\ \sqrt{2gd}.$$

Deducidos de experimentos hechos por Fteley y Stearns y por J. B. Francis. En los experimentos de Francis, el valor de *c* para un valor dado de $h \div H$, aumentó generalmente en proporción de H.

---

* Para la velocidad de acceso, véase art. 14 (*u*), etc.

** Al deducir esta fórmula, el agua que pasa por el vertedero entre *c* y *b*, se supone que corre como por sobre un vertedero cuya arista es *b* y que efectúa la salida del agua libremente al aire, como por sobre la arista *a* en la fig. 20; y para esta parte, el gasto lo dará la fórmula (2), art. 14 (*l*) :

$$Q_b = cL\ \frac{2}{3}\ d\ \sqrt{2gd}$$

(para pies y metros introduciendo en cada caso los valores respectivos) mientras que el agua que pasa por la parte inferior entre *b* y *a* se considera como saliendo por un *orificio* vertical sumergido, cuya altura es $ba = h$, bajo una carga $= d$. Por consiguiente, para esta parte inferior el gasto sería :

$Q_a = cLh\ \sqrt{2gd}$. (La misma para metros y pies introduciendo para cada caso los valores respectivos.)

Se supone que el coeficiente de gasto *c* sea el mismo para la sección superior *cb* que para la inferior *ab*. Por tanto, sumando estos dos gastos, tendremos para todo el gasto :

$$Q = Q_b + Q_a = c\,L\,(h + \frac{2}{3}\ d)\ \sqrt{2gd}\ \text{(Para metros y pies.)}$$

† *Transactions, American Society of Civil Engs, March* 1883, págs. 101, etc.
*Transactions, American Society of Civil Engs, Sept.* 1884, págs. 295, etc.

| | Fteley y Stearns (H=.099 á .248 mets). | J. B. Francis (H=.3048 á .7071 mets). |
|---|---|---|
| $h \div H$ | c | c |
| .05 | ..... | .623 á .632 |
| .10 | .625 á .635 | .620 á .630 |
| .20 | .618 á .628 | .610 á .625 |
| .30 | .600 á .610 | .598 á .615 |
| .40 | .590 á .600 | .586 á .610 |
| .50 | .585 á .595 | .585 á .607 |
| .60 | .583 á .593 | .585 á .607 |
| .70 | .580 á .590 | .585 á .607 |
| .80 | .581 á .591 | .585 á .607 |
| .90 | .590 á .600 | ..... |
| .95 | .610 á .615 | ..... |

**(1) Clemens Herschel** *, comparando estos experimentos con algunos anteriores hechos por Francis, da lo siguiente :

Habiendo determinado las profundidades H y $h$ á que se halla la cresta del vertedero bajo el nivel del agua en reposo, aguas arriba y aguas abajo, respectivamente, divídase $h$ por H; búsquese el cociente tan aproximadamente como se pueda en la columna encabezada $h \div H$ de la Tabla 17. Tómese el coeficiente correspondiente $a$, y multiplíqueselo por la carga aguas arriba H**.

El producto $a$H es la carga que haría que un vertedero dado produjese el mismo gasto saliendo el agua al *aire libremente*, como en la fig. 20. Búsquese el gasto ó cantidad de agua que vierte al aire el mismo vertedero con la carga $a$H; y este gasto será aproximadamente el mismo que el del vertedero *sumergido* realmente bajo la carga aguas arriba H, y contra la carga aguas abajo $h$; ó (siendo H la carga efectiva aguas arriba del vertedero *sumergido*) el gasto es :

$$Q = mLa\text{H}\sqrt{2ga\text{H}} = xLa\text{H}\sqrt{a\text{H}} \ldots\ldots \quad (10)$$

(*Obs. del T.* — Tanto la fórmula como la tabla se aplican al sistema métrico, sustituyendo los valores de $g$, $h$ H y L en metros, y teniendo en cuenta que $x = m\sqrt{2g}$.)

**TABLA 17**

| $h \div H$ | a | $h \div H$ | a | $h \div H$ | a |
|---|---|---|---|---|---|
| .10 | 1.000 to 1.010 | .45 | 0.894 to 0.930 | .72 | 0.762 to 0.784 |
| .20 | 0.975 to 0.995 | .50 | 0.874 to 0.910 | .74 | 0.747 to 0.769 |
| .25 | 0.960 to 0.984 | .55 | 0.853 to 0.889 | .76 | 0.732 to 0.752 |
| .30 | 0.945 to 0.973 | .60 | 0.829 to 0.863 | .78 | 0.713 to 0.733 |
| .35 | 0.928 to 0.960 | .65 | 0.803 to 0.833 | .80 | 0.693 to 0.713 |
| .40 | 0.912 to 0.946 | .70 | 0.775 to 0.799 | | |

*(u)* **Velocidad de acceso.** Véase fig. 24. Generalmente es impracticable la medida de la carga H en el agua perfectamente en reposo, desde $o'$ aguas arriba. La carga se mide generalmente en un punto $o$ situado desde 60 cm ó 1 m hasta 1½ á 2½ m ó más distante del vertedero aguas arriba, según el tamaño de este último. En estos puntos la velocidad es generalmente apreciable, y la superficie por lo tanto un poco más baja que en $o'$. Por consiguiente, una fórmula en que se use la carga más pequeña H, medida de este modo en lugar de la H', y con

* *Transactions, American Society of Civil Engineers, May* 1885, págs. 189, etc.

** La tabla de Herschel, de la cual se ha extractado la nuestra, da el valor de $a$ para cada .01 pie de $h \div H$; pero los valores de $a$, intermedios á los que hemos elegido, pueden deducirse de nuestra tabla con bastante exactitud por una simple proporción. El orden de los coeficientes $a$ de la tabla, correspondientes á cada valor de $h \div H$, es el indicado por las experiencias que variaban de un modo semejante. No sabemos qué escoger entre estos extremos; pero en la mayor parte de los casos los valores *medios* son probablemente exactos.

los coeficientes basados en H', dará un gasto demasiado pequeño. Francis halló que una corriente de .305 m por segundo, ó casi 1,098 m por hora, en el punto, *o*, donde se midió la carga, aumentó el gasto como en un 2 por ciento solamente

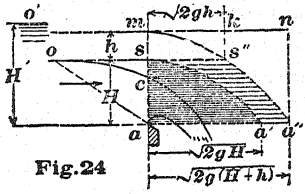

Fig. 24

cuando la carga era de 33 cm; y con una corriente de 15 cm aumentó el gasto en 1 por ciento más ó menos cuando la carga era de 20 cm.

Sin embargo, si la velocidad de acceso es tal que se haga necesario tenerla en cuenta, procédase como sigue : Para obtener la velocidad media de acceso, aproximadamente tenemos :

$$v \text{ en mets} = \frac{\text{gasto aproximado en mets cúbs}}{\text{área total de la sección transversal del arroyo en } o, \text{ en mets cuads}} = \frac{1.838\ \text{LH}^{3/2}}{\text{área en } o \text{ en mets cuads}}$$

y para la carga debida á esta velocdad, $h = \frac{v2}{2g}$.

Luego, para todas las aplicaciones prácticas, podemos decir : $H' = H + h$; ó

$$Q = mL\ (H+h)\ \sqrt{2g\ (H+h)} = xL\ (H+h)^{3/2} \ldots\ldots \quad (11)$$

(para ambos sistemas de medidas, como ya se ha dicho, y recordando que $x = m\sqrt{2g}$; $g = 9.81$ m $= 32.2$ pies).

Sin embargo, hablando estrictamente, la diferencia de nivel entre *o'* y *o* es realmente (como se indica en la fig. 24) algo *mayor* que en *h*, ó que $\frac{v2}{2g}$, porque se pierde alguna carga por el rozamiento entre *o'*, y *o*.

(*y*) **Fteley y Stearns** hacen á $H' = H + 1.5\ h$ para vertederos sin contracción en las extremidades y $H' = H + 2.05\ h$ como término medio, para vertederos en que es completa la contracción en las extremidades; y **Hamilton Smith, Jr.**, después de comparar los experimentos de ellos con otros hechos por Lesbros, Castel y Francis, da $H' = H + 1\frac{1}{3}\ h$, y $H = H' + 1.4\ h$ para los dos casos respectivamente.

(*w*) Por otra parte, la fórmula de Francis, modificada para la velocidad de acceso,

$$Q = xL * (\sqrt{(H+h)^3} - \sqrt{h^3}) = mL * \sqrt{2g} \times (\sqrt{(H+h)^3} - \sqrt{h^3}) ** \ldots \quad (12).$$

(*Obs. del T.* — Esta última fórmula sirve también para medidas métricas.)

---

* Si hay contracciones en el extremo, L se convierte en $\left(L - n\frac{H}{10}\right)$. Véase el art. 14 (*m*).

** Esta fórmula se deduce como sigue : Supongamos que el área del segmento parabólico *asa'* fig. 24, representa el gasto teórico del vertedero en una unidad de longitud (como se ha explicado en la nota § al pie del art. 14 (*l*), bajo la carga medida $H = a\ s$, como si no hubiera corriente alguna en *o*. Sea $ms = h = v^2 \div 2g$. Las velocidades teóricas de las moléculas que atraviesan el plano oblicuo, *oa*, bajo sus cargas efectivas, estarán ahora representadas por líneas horizontales *ss''*, *aa''*, etc., trazadas desde cada punto de *s a* á la curva *exterior* *s'' a''* ; la línea *ss''* representa el valor de $v$ = velocidad de acceso $= \sqrt{2gh}$, y *aa''* representa á $\sqrt{2g\ (H+h)}$.

(*x*) **A. W. Hunking y Frank S. Hart,** ingenieros civiles é hidráulicos, han sustituído por la expresión $(\sqrt{(H+h)^3} - \sqrt{h^3})$ en la fórmula (12), la expresión equivalente $K\sqrt{H^3}$, en la cual K es un coeficiente deducido de la expresión anterior, y por lo tanto dependiente de la relación existente entre H y *h*, ó de la relación entre la sección transversal *as*, fig. 24, en el vertedero y de toda la sección transversal de la corriente en *o*.

Después de hallar el área de la sección transversal en *o*, divídasela por $\left(L - n\frac{H}{10}\right)$, usando cualquiera unidad, que es el largo del vertedero corregido con respecto á la contracción. Véase art. 14 (*m*). Llámese el cociente D §. Divídase la carga medida H por D; búsquese este último cociente en la columna $\frac{H}{D}$ de la tabla. Multiplíquese el gasto aproximado, $Q_1 = 1.838\left(L - n\frac{H}{10}\right)H^{3/2}$, (en m) por el coeficiente correspondiente K; ó sea:

$$\textbf{Gasto real en } \textit{mets cúbicos} = 1.838\ K\left(L - n\frac{H}{10}\right)H^{3/2} \ldots\ldots\ldots (13)$$

(El coeficiente K es el msimo, úsense metros ó pies.)

**Tabla 18. Coeficiente K en la fórmula (13).**

| $\frac{H}{D}$ | K | $\frac{H}{D}$ | K | $\frac{H}{D}$ | K | $\frac{H}{D}$ | K | $\frac{H}{D}$ | K |
|---|---|---|---|---|---|---|---|---|---|
| .01 | 1.0000 | .09 | 1.0020 | .17 | 1.0072 | .24 | 1.0143 | .31 | 1.0239 |
| .02 | 1.0001 | .10 | 1.0025 | .18 | 1.0081 | .25 | 1.0155 | .32 | 1.0254 |
| .03 | 1.0002 | .11 | 1.0030 | .19 | 1.0090 | .26 | 1.0168 | .33 | 1.0271 |
| .04 | 1.0004 | .12 | 1.0036 | .20 | 1.0100 | .27 | 1.0181 | .34 | 1.0287 |
| .05 | 1.0006 | .13 | 1.0042 | .21 | 1.0110 | .28 | 1.0195 | .35 | 1.0305 |
| .06 | 1.0009 | .14 | 1.0049 | .22 | 1.0121 | .29 | 1.0209 | .36 | 1.0322 |
| .07 | 1.0012 | .15 | 1.0056 | .23 | 1.0132 | .30 | 1.0224 | .37 | 1.0341 |
| .08 | 1.0016 | .16 | 1.0064 | | | | | | |

K es muy aproximadamente $= 1 + \frac{1}{4}\left(\frac{H}{D}\right)^2$. Por lo tanto,

Q en pies cúbs, es $= 3.33\left[1 + \frac{1}{4}\left(\frac{H}{D}\right)^2\right]\left(L - n\frac{H}{10}\right)H^{3/2} =$

$$\left[3.33 + .83\left(\frac{H}{D}\right)^2\right]\left(L - n\frac{H}{10}\right)H^{3/2} \ldots\ldots\ldots (14)$$

(*Obs. del T.*: Q en mets cúbs $= \left[1.838 + .459\left(\frac{H}{D}\right)^2\right]\left(L - n\frac{H}{10}\right)H^{3/2}$. ....(14)

Véase *Journal of the Franklin Institute Philadelphia, August*, 1884, del cual hemos extractado la tabla arriba mencionada.

(*y*) **M. Bazin,** para la velocidad de acceso, modifica (véase art. 14 *o*) el coeficiente *m* en lugar de la carga H, haciendo $m = .425 + .21\left(\frac{H}{D}\right)^2$; (para pies y metros),

---

Entonces, el área $ss''a''a$ es = área $ma''a$ − área $ms''s = \frac{2}{3}$ área del rectángulo $ais$ $- \frac{2}{3}$ área del rectángulo $sk$

$$= \frac{2}{3}(H + h)\sqrt{2g(H + h)} - \frac{2}{3}h\sqrt{2gh}, = \frac{2}{3}\sqrt{2g}\left(\sqrt{(H + h)^3} - \sqrt{h^3}\right);$$

y el gasto efectivo ó real es:

$$Q = c \times \text{largo del vertedero} \times \text{área } ss''a''a = cL\frac{2}{3}\sqrt{2g}\left(\sqrt{(H + h)^3} - \sqrt{h^3}\right)$$

$= m\,L\sqrt{2g}\left(\sqrt{(H + h)^3} - \sqrt{h^3}\right) = x\,L\left(\sqrt{(H + h)^3} - \sqrt{h^3}\right)$; porque $x = m\sqrt{2g}$

En un vertedero sin contracción en las extremidades, $D = H + p$.

mientras que por el método de Hunking y Hart (basado en los experimentos de Francis), $m$ se hace $= .415 + .10 \left(\frac{H}{D}\right)^2$ (para pies y metros).

**Art. 15. Vertederos inclinados.** Si la cara de un vertedero situada del lado aguas arriba, en lugar de ser vertical, como en la fig. 25, está inclinada como en la fig. 25 *a*, ó hacia aguas abajo como en la fig. 25 *b*, se modifica la forma de la vena y el gasto. Con una inclinación aguas arriba, fig. 25 *a*, la cara inferior de la lámina

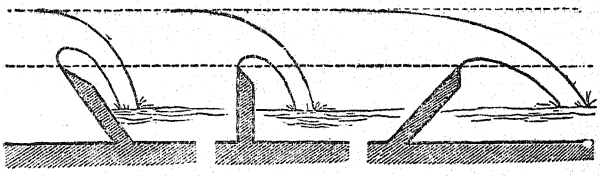

FIG. 25 *a*. Inclinado aguas arriba.

FIG. 25. Vertical.

FIG. 25 *b*. Inclinado aguas abajo.

de agua al pasar por sobre el vertedero salta más alto y su curva interior penetra más hacia aguas arriba cuanto más se aumente la inclinación.

Con una inclinación aguas abajo, fig. 25 *b*, es al contrario: cuando la inclinación aumenta, el salto de la lámina de agua disminuye, su perfil se hace más y más plano y la curva de la superficie superior, debida á la caída, se extiende más, aguas arriba, del borde del vertedero.

Una inclinación aguas arriba, fig. 25 *a*, disminuye el gasto, y una inclinación aguas abajo, fig. 25 *b*, lo aumenta, como lo indican los coeficientes siguientes obtenidos por M. Bazin *.

**Para obtener el gasto suministrado por un vertedero inclinado.** Conociendo el gasto de uno vertical de la misma altura, y bajo condiciones semejantes por otros respectos, multiplíquese el gasto suministrado por el vertical por los coeficientes aproximados que siguen:

| | Inclinación. | | Angulo. | | Coeficientes. |
|---|---|---|---|---|---|
| | Horizontal. | Vertical. | con la horizontal. | con la vertical. | |
| Vertederos inclinados aguas arriba, fig. 25 *a*..... | 1 | 1 | 45° | 45° | .93 |
| | ⅔ | 1 | 56°19′ | 33°41′ | .94 |
| | ½ | 1 | 71°34′ | 18°26′ | .96 |
| Vertederos verticales, fig. 25. | 0 | 1 | 90° | 0° | 1.00 |
| Vertederos inclinados aguas abajo, fig. 25 *b*..... | ¼ | 1 | 71°34′ | 18°26′ | 1.04 |
| | ⅔ | 1 | 56°19′ | 33°41′ | 11.70 |
| | 1 | 1 | 45° | 45° | 1.10 |
| | 2 | 1 | 26°34′ | 63°26′ | 1.12 |

El gasto aumentará también si se redondea la arista interior ó filo del copete de vertedero en lugar de dejarlo con aristas vivas; ó si las paredes del estanque convergen más ó menos á medida que se aproximan al vertedero, de manera que guíen el agua más directamente hacia él; ó si *ab*, fig. 20, es menor que el doble de *am*. Realmente son tantas las circunstancias susceptibles de modificaciones que existen para embarazar los experimentos en ésta, y en materias análogas, que algunos de los ya hechos con gran cuidado han resultado inaplicables, y también otros de aproximaciones tolerables, debido á que por negligencia ó ignorancia no se tomó en consideración alguna peculiaridad local, al tiempo de hacer los ensayos,

* *Expériences Nouvelles sur l'Ecoulement en Déversoir*, 2e article; *Annales des Ponts et Chaussées*, enero 1890, traducido en *Proceedings, Engineers Club of Philadelphia*, vol. IX, 1892.

por no creerse que ejercería un efecto apreciable. A menos que las circunstancias no nos permitan combinar todas las condiciones mencionadas en los arts. 14 (*d*), (*f*) y (*m*), consiguiendo con ello resultados *muy* aproximados, debemos ocurrir ó á una medida directa del gasto en un envase de capacidad conocida, ó conformarnos con reglas que puedan conducir á errores de 5, de 10 ó más por ciento, en proporción á lo que nos desviemos de aquellas condiciones. A menudo hasta un error de 10 por ciento se considera de poca importancia.

*Nota* 1. Cuando el agua después de pasar por un vertedero, fig. 26, en lugar de caer libremente al aire, corre por encima de un canal, T, de poca inclinación, cuyo fondo coincida con el borde ó arista, *a*, del vertedero, entonces el gasto no dismi-

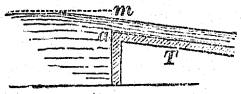


*Fig. 26.*

nuye por eso apreciablemente, cuando la carga *am* es de 38 cm ó más. Pero si la carga *am* es solamente de 30 cm, entonces el gasto calculado debe reducirse más ó menos en una décima parte; si es de 15 cm, dos décimas; si de 6 cm, tres décimas; y si es de 2½ cm, cinco décimas más ó menos.

*Nota* 2. El profesor Thomson, de Dublín, propuso el uso de vertederos triangulares para medir el gasto; porque entonces el perímetro siempre tiene la misma relación con el área del arroyo que fluye por él, lo que no sucede con cualquiera otra forma. Experimentando con una abertura triangular rectangular practicada en una plancha de hierro delgada, fig. 26 A, con cargas desde 5 á 18 cm, medidas

*Fig. 26 A.*

verticalmente del fondo de la abertura *al nivel de la superficie del agua en reposo*, encontró: *gasto* en pies cúbs por seg = 2.54 × $\sqrt{}$ quinta potencia de la carga en pies *.

***Del Traductor.*** — Gasto en mets cúbs = 1.40254 $\sqrt{}$ quinta potencia de la carga en mets ó en general si $m$ es = coeficiente de contracción (art. 9, pág. 576) T = tangente del semiángulo de la escotadura = ½ ancho de la superficie del agua ÷ la profundidad en la abertura; $g$ = la intensidad de la gravedad = digamos 9.81 m por segundo y $h$ = carga, medida como se dijo; y entonces tenemos:

$$\text{Gasto} = \frac{8m}{15}\,\text{T}\sqrt{2gh^5} * = 4.28\,m\text{T}\sqrt{h^5}\text{ en pies }*.$$

(***Obs. del T.*** — $m$ y T tienen el mismo valor en cualquiera unidad que se calcule, por tanto, cuando se de la altura $h$ en metros, la fórmula del gasto en mets cúbs será = 2.363 m T $\sqrt{h^5}$ en mets *.)

*Nota* 3. Al construir el canal de irrigación, Canale Villoresi, cerca de Milán, en 1881-4, el ingeniero italiano, Cesare Cippoletti **, adoptó **una abertura trapezoidal**, con su arista inferior horizontal y sus costados inclinados á razón de 4 vertical : 1 horizontal, para evitar la necesidad de suprimir ó de permitir las contracciones de las extremidades. (Véanse arts 14 *c* y 14 *m* de este mismo capítulo.) Se encontró que la contracción sólo afectaba los espacios triangulares situados encima de los extremos inclinados del vertedero, y el largo efectivo del vertedero permaneció así constante (é igual al largo de la arista del fondo) para todas las cargas. Al usar estos vertederos la contracción es completa tanto á lo largo del fondo como en sus extremidades.

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Para esta raíces, véanse págs. 69 á 71.

** Véase su obra *Canale Villoresi; Modulo per la Dispensa della Acqua, etc.* Milán, 1886; publicada por la *Società Italiana per Condotte d'Acqua*. Los resultados fueron recopilados por L. G. Carpenter, en el Bulletin n.º 13, *Agricultural Experiment Station, Fort Collins*, Colorado, octubre, 1890.

## DE LA CORRIENTE DEL AGUA POR CANALES DESCUBIERTOS

**Art. 16. La velocidad media de la corriente** es una velocidad imaginaria uniforme con que suponemos animada el agua en cada punto de su sección transversal y que da el *mismo gasto* que el producido por la velocidad real no uniforme. O de otro modo:

$$\text{Velocidad media} = \frac{\text{Gasto}}{\text{área de la sección transversal}}.$$

En los canales de sección transversal uniforme la **velocidad máxima** se halla más ó menos en el medio ó centro entre sus dos orillas, y generalmente á cierta distancia bajo la superficie; esta distancia varía en las diferentes corrientes; pero por término medio parece ser, más ó menos, la tercera parte de la profundidad total. Cuando la profundidad es grande en relación á su ancho (por ejemplo, la mitad del ancho ó más), se ha encontrado que la velocidad máxima se halla como á la mitad de la profundidad de la superficie al fondo, mientras que en arroyos pequeños de poca profundidad parece aproximarse á la superficie hasta una distancia de ella como de .1 á .2 de su profundidad total. Muchos experimentos hechos en ríos ó corrientes de poca profundidad han indicado ciertamente que la velocidad máxima está *en* la superficie.

**La relación entre las velocidades de los diversos puntos de la sección transversal** es sumamente variable en las diferentes corrientes, de modo que hay que fiarse poco en las reglas empleadas para deducir de la velocidad de un punto de la sección, la de otro punto de la misma sección. Con la misma velocidad, los ríos anchos y hondos tienen mayor velocidad media y en el fondo, que los ríos de poca profundidad. Para obtener **una grosera aproximación de la velocidad media** cuando se ha dado la máxima de la superficie, á menudo se supone que la primera es igual á [illegible] (ó .80) de la última. Pero Francis encontró en sus experimentos hechos en Lowell, que los flotadores *superficiales* de cera, de 5 cm de diámetro, que flotaban en el centro de un canal rectangular descubierto de 3.65 m de ancho y 2.44 de hondo, realmente se movían como 6 por ciento *más despacio* que un flotador compuesto por un tubo de hoja de lata de 5 cm de diámetro, que sobresalía pocas pulgadas de la superficie del agua y llegaba hasta 3 ó 4 cm del fondo del canal, provisto por supuesto de plomo en su parte inferior para que se mantuviese próximamente vertical. Mientras que el flotador superficial de cera se movía á razón de 1.14 m por segundo, el tubo se movía (evidentemente con una velocidad casi igual á la del hilo ó filamento líquido *vertical* del centro) á razón de 1.21 m por segundo. Igualmente encontró que en el mismo canal, con velocidades del tubo central que variaban desde .46 m, á 1.22 m, la velocidad del tubo era *menor* que la velocidad media de toda la sección transversal del agua en el canal como .96 á 1 para la velocidad menor y .93 á 1 para la mayor. Mientras que, en otro canal rectangular de 6.10 m de ancho y 2.43 m de profundidad, con velocidades que variaban de .35 m á .56 por seg, la velocidad de los tubos flotadores era *mayor* que la de la masa total del agua, más ó menos como de 1.04 á 1. En un canal de 8.84 de ancho por 2.47 m de hondo, con velocidades de .91 m por seg más ó menos, era como de 1 á .9, y en un canal de 11.12 m de ancho por 2.56 m de hondo con velocidades como de 1.07 m por segundo, era, la del flotador, de 1 á .97 de la de toda la masa.

**Charles Ellet hijo, ing. civil, halló que en el Misisipí**, en diferentes puntos del río á profundidades que variaban de 16 á 30 m, y en corrientes que variaban de 5 á 11 km por hora, más ó menos, la velocidad de un flotador provisto de una cuerda de 15 m de largo, es casi siempre mayor que la de un flotador superficial solo. Los mismos resultados se obtuvieron con cuerdas de 8 y 23 m de largo, siendo el exceso de velocidad de los flotadores de cuerda más ó menos de un 2% mayor que la de los flotadores solos; y M. Ellet dedujo de esto, que la velocidad media de la sección transversal total del Misisipí, en lugar de ser menor, es absolutamente 2% mayor que la *velocidad media de la superficie*. El, sin embargo, toma .8 de la velocidad máxima de la superficie para representar aproximadamente, en su opinión, la velocidad media de toda la sección transversal del agua. En ríos de *poca profundidad* encontró siempre que el flotador superficial corría más ligero que el de cuerda.

Ensayos hechos en Europa sobre la velocidad *media en diferentes verticales de una corriente*, en ríos de profundidad regular, han dado de .85 á .96 de la velo-

cidad de la superficie en cada vertical. Debe tomarse á .90 como término medio de todo.

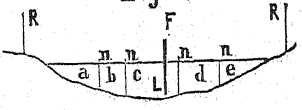
**Velocidad en el fondo.** En ríos ó arroyos de pendiente y sección transversal próximamente uniformes, hay una gran disminución de velocidad cerca del fondo. Como ligera aproximación, la velocidad mensurable más profunda, en ríos de pendiente uniforme, etc., parece ser de ½ á $^{3}/_{4}$ de la velocidad media.

**Art. 17. Para medir la velocidad en la superficie**, elíjase un lugar donde el río sea en alguna distancia (mientras más grande mejor) de una sección transversal aproximadamente uniforme y libre de contracorriente, remolinos, saltos de agua, etc. Obsérvese en un reloj de segundos cuánto tiempo emplea un flotador (como un pedazo pequeño de madera), colocado en el punto más *rápido* de la corriente, en atravesar una distancia determinada y medida previamente. Es suficiente distancia, en los ríos de poca corriente, 15 metros y hasta 50 metros en los rápidos. Esta dist dividida por el número total de segundos que el flotador emplea en recorrerla, dará en metros por segundo, la mayor velocidad de la superficie.

**La velocidad en la superficie debe medirse siempre en tiempo de perfecta calma** para que el viento no perturbe el flotador; y por la misma razón, el flotador no debe sobresalir mucho de la superficie del agua. La medida debe repetirse varias veces para obtener exactitud. En arroyos muy pequeños pueden arreglarse las orillas y el lecho en una distancia corta, de manera que presente un canal uniforme. El flotador debe ponerse en el agua un poco más arriba del punto en donde principia la observación, para que pueda adquirir la velocidad total del agua antes de llegar á dicho punto.

**Art. 18. Aforo de un arroyo por medio de su velocidad.** Elíjase un lugar en que la sección transversal permanezca, en una corta distancia, medianamente uniforme, y libre de contracorrientes, remolinos, aguas muertas ú otras irregularidades. Prepárese con cuidado una sección transversal, como en la fig. 27.

Fig 27




Por medio de jalones ó boyas, *nn*, divídase el arroyo en secciones, *a*, *b*, *c*, etc. Pónganse dos jalones R, R, en el extremo superior, y otros dos en el inferior de la distancia que han de recorrer los flotadores, para observar el tiempo, con un reloj ó péndulo, que emplean en recorrerla. Luego mídase la velocidad *media* de cada sección, *a*, *b*, *c*, etc.; separada y directamente, por medio de flotadores largos, como FL, que lleguen hasta cerca del fondo del río, y que sobresalgan un poco de la superficie. Los flotadores pueden ser tubos de hoja de lata ó reglas de madera, poniéndoles plomo en su extremo inferior hasta que floten casi verticalmente. Deben ser de diferentes longitudes, á fin de que se adapten á las profundidades de las distintas secciones. Por esta razón deben hacerse los flotadores en piezas, con juntas ó articulaciones provistas de tornillos. El área de cada sección separada del arroyo en metros cuadrados, multiplicada por la velocidad media observada del agua en metros, por segundo, dará el gasto de aquella sección en metros cúbicos por segundo. Y los gastos de todas las secciones separadas obtenidos así, darán, sumados, el gasto total del río, y este gasto total, dividido por el área total de la sección transversal del río, dará la velocidad media de *toda* el agua.

*Nota.* **Si el canal es de tierra común**, especialmente si es arenosa, las pérdidas por las infiltraciones y por la evaporación extraen á menudo tanta agua, que el gasto disminuye gradualmente más y más mientras más abajo se haga el aforo de la corriente. Por esta razón, los canales de alimentación muy largos no suministran al canal principal sino una parte pequeña del agua que les entra por su extremo superior.

**El flotador doble** se usa para determinar las velocidades á diferentes profundidades. Se compone de un flotador que descansa sobre la superficie del agua y de un cuerpo más pesado, ó « flotador inferior », que está suspendido del flotador superior por medio de una cuerda. La altura del flotador inferior depende, por supuesto, del largo de la cuerda de suspensión (que puede aumentarse ó dismi-

nuirse según se quiera hasta que se crea que el flotador inferior ha llegado á la profundidad, cuya velocidad se necesita) y de su posición vertical, que es más ó menos modificada por la corriente. Debida á esta última circunstancia es difícil saber si el flotador inferior está realmente á la profundidad deseada. Además, no se sabe hasta dónde llega la acción contraria que los flotadores ejercen en sus movimientos el uno sobre el otro. En aguas profundas la cuerda suele oponer á la corriente una área mayor que el flotador inferior. Por esta razón no se puede fijar con certeza hasta qué grado puede confiarse en la velocidad del flotador superior como indicador de la velocidad del agua á la profundidad donde se halla el flotador inferior.

**Art. 19. El cuadrante de Castelli, ó péndulo hidrométrico,** se compone de una bala metálica suspendida por un hilo del centro de un arco graduado. Se pone el instrumento en la corriente, con el arco paralelo á la dirección del agua, y la velocidad se calcula por el ángulo que forma el hilo con la vertical.

**La placa de presión de Gauthey** era una lámina de metal suspendida de uno de sus extremos, á cuyo alrededor podía girar libremente. Se sumergía la placa en la corriente, colocando su cara en ángulo recto á ella. Se estimaba la velocidad por el peso necesario para que la lámina tomase la posición vertical opuesta á la fuerza de la corriente.

**El tubo de Pitot** era en su origen un simple tubo de vidrio, fig. 27 A, abierto por ambas extremidades y doblado en forma de L. Una rama de la L estaba fija horizontalmente debajo del agua con su extremidad abierta dándole el frente á la corriente, y se medía la velocidad $v$ en el punto $o$ donde estaba colocado, por la altura vertical $h$ $\left(\text{teóricamente} = \frac{v^2}{2g}\right)$ á la cual el agua subía en la otra rama sobre la superficie de la corriente.

Fig. 27A

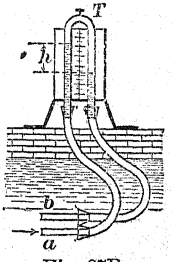


Fig. 27B

**Modificado por M. Darcy y por el profesor S. W. Robinson,** toscamente dibujado en la fig. 27 B, el tubo de Pitot se compone esencialmente de *dos* tubos horizontales de vidrio ó de metal $a$ y $b$ de un diámetro interior muy pequeño, colocados uno al lado del otro en la corriente y dirigidos en contra de ella. El tubo $a$ recibe ésta por su extremo abierto colocado aguas arriba, mientras que el tubo $b$ tiene cerrado el extremo que mira hacia aquel lado y tiene solamente pequeños orificios *laterales*. La otra punta de cada tubo está en comunicación por medio de un pequeño tubo de caucho ó de metal con la rama de un tubo en forma de U invertida, de vidrio y graduado, fijo en un bote ó en la orilla de una corriente. Por conveniencia, los dos tubos flexibles deben unirse entre sí, formando un tubo doble. Por succión hecha en la llave T que está en la parte superior, el agua sube á una altura conveniente en las dos ramas del medidor. Cuando no hay corriente las dos columnas de agua permanecen á la misma altura; pero en una corriente, la diferencia $h$ de sus alturas es tal, que $v = \sqrt{2gh}$ sin necesitarse de coeficiente de corrección. El instrumento es notablemente sencillo y exacto, y puede usarse en corrientes de agua ó gas angostos y poco profundos. Este instrumento aprecia velocidades hasta de .10 m por segundo.

En la práctica, $a$ y $b$ se fijan juntos en una sola pieza y se colocan, para usarlos, en un marco de metal que se desliza verticalmente, ó sobre un alambre que pasa por él provisto de una plomada que descansa en el fondo y mantiene tenso el alambre (ó en corrientes de menos de 6 m de profundidad), se ponen sobre un

bastón vertical de madera cuya punta inferior se fija en el lecho de dicha corriente. En el primer caso, el marco está provisto de una veleta larga, para mantener el instrumento aguas arriba. En uno ú otro caso, se proveen medios para indicar la profundidad á la cual está sumergido el instrumento.

Haciendo que la escala graduada se adapte verticalmente y colocándola (en cada cambio de profundidad del instrumento) con el cero opuesto á la parte superior de la columna, evitamos la necesidad de observar la altura de ambas columnas de agua en cada lectura; porque la lectura de la columna superior sola, da desde luego la carga $h$.

**Art. 20. El medidor ó contador de rueda**, es una rueda que la corriente hace girar y que comunica su movimiento por medio de su eje y engranajes á indicadores que registran el número de revoluciones. Este instrumento puede fijarse en un punto cualquera de una vara larga que llegue hasta el fondo del arroyo, y por lo tanto puede usarse para cualquiera profundidad. El observador, por medio de un alambre, varilla ó cuerda, que llegue hasta el instrumento, engrana primero el aparato de registro y después lo desengrana con la rueda (por medio de un freno que se aplica al primero en el momento de desengranarlo), anotando cuidadosamente la hora á que lo hace. Luego se levanta el instrumento y se lee en los índices el número de revoluciones hechas en el tiempo anotado y con esto se calcula la velocidad. Pero el contador se hace á menudo de **registro automático**; la rueda en cada revolución interrumpe y restablece automáticamente una corriente eléctrica engendrada por una batería. El alambre que conduce esta corriente se hace obrar sobre un aparato registrador eléctrico de Morse, colocado en un bote ó en la orilla. Algunos medidores ó contadores arreglados así, pueden fijarse en diferentes puntos de la vara al mismo tiempo, y por consiguiente registran **observaciones simultáneas de velocidades á diferentes profundidades.**

La mayor parte de sus contadores se hacen de manera que puedan libremente moverse horizontalmente alrededor de una vara larga, á la cual están fijos, y cada uno está provisto de una aspa ó veleta semejante á la de los molinos de viento, para sostener la rueda en la posición que se desee con respecto á la corriente. Generalmente se hacen las ruedas como las de los molinos de viento, es decir, con sus aletas colocadas formando un ángulo tal que presente á la corriente una superficie inclinada, y el eje de la rueda paralelo á la dirección de dicha corriente. El eje se mueve en chumaceras de ágata. Si se quiere se puede proveer á la llanta de la rueda de un receptáculo de aire para equilibrar su peso y suprimir de este modo el rozamiento continuo debido á dicho peso. Los medidores ó contadores provistos de aparatos de registro eléctricos, tienen algunas veces el engranaje y los índices, etc., encerrados en una caja de vidrio para evitar que se emboten con la introducción en ellos de hierbas, sedimentos, etc.

**El contador de ruedas se regula** moviéndolo en el agua tranquila con una velocidad conocida y anotando el efecto producido. Por este medio se obtiene un coeficiente para cada contador, el cual, multiplicado por el número de revoluciones registradas en cada caso, dará la velocidad.

*Nota* 1. **Debe tenerse cuidado de que la velocidad en el fondo no sea tan grande que lo desgaste**; si hay ese temor debe protegerse la superficie del canal artificialmente; ó puede reducirse la altura de caída y aumentar la sección transversal del canal, de manera que se logre el mismo gasto con menos velocidad. También debe tenerse en cuenta, en la construcción de tales canales, lo que sube la corriente al tropezar y levantarse sobre las plantas acuáticas que se forman por los depósitos de lodo traídos en las aguas de lluvia, y á veces por el soplo de vientos fuertes contra la corriente.

*Nota* 2. **El agua que corre en un canal de fondo horizontal no puede tener una velocidad, ni una profundidad uniforme, á lo largo de su curso**, porque se necesita la acción de la gravedad debida al plano inclinado, y el agua puede sólo correr formándose ella misma una *superficie* inclinada, lo que trae una disminución de la profundidad á distancias sucesivas del estanque.

**Para la teoría de la corriente** en tuberías largas y canales, velocidad media, distribución de la velocidad en la sección transversal, fórmula de Chezy, coeficiente de frotamiento, etc., véanse págs. 560, etc.

**Para la fórmula de Kutter**, véanse págs. 558, 603, etc.

**Fórmula de Bazin para la corriente en canales.** Anales de Puentes y Calzadas, 1897, 4.º trim., pág. 40.

En la fórmula de Chezy, $v = c\sqrt{rs}$, donde (pág. 561)

$v$ = velocidad media;

$r$ = radio medio = $\frac{\text{área sección transversal}}{\text{perímetro mojado}}$;

$s$ = pendiente = $\frac{\text{carga de frotamiento}}{\text{longitud}}$.

Bazin considera $c$, independiente de la pendiente, y da:

**Para medidas inglesas,**

$$c = \frac{87}{.552 + \frac{\gamma}{\sqrt{r}}}$$

**Para medidas métricas.**

$$c = \frac{87}{1 + \frac{\gamma}{\sqrt{r}}}$$

donde $\gamma$ tiene los siguientes valores:

| | |
|---|---|
| Superficies muy lisas (cemento, maderas cepilladas, etc.)..... | $\gamma$ = .06 |
| Superficies lisas (tablas, ladrillos, piedras planas, etc.)..... | $\gamma$ = .16 |
| Mampostería..... | $\gamma$ = .46 |
| Tierra muy lisa ó pavimentada con piedras picadas..... | $\gamma$ = .85 |
| — en condiciones ordinarias..... | $\gamma$ = 1.30 |
| — escabrosa..... | $\gamma$ = 1.75 |

« Midiendo la pendiente de un gran río, los errores ordinarios, en la más cuidadosa nivelación, son á veces parte importante de la caída total; la variación de nivel en una sección transversal de la superficie, es á veces tan grande como la pendiente en 10 kilómetros, ó más; el punto preciso, donde debería tomarse el nivel, es á menudo incierto; el flujo y reflujo del agua hace muy difícil decidir cuándo deben tomarse los niveles en los puntos altos y bajos; el oleaje de transporte puede afectar la inclinación en cantidad grande é incierta y hasta darle á la superficie una pendiente contraria á la que tiene. » Genl T. G. Ellis.

## Fórmula de Kutter.

Véase también pág. 558. **Para la teoría de la corriente** en tuberías largas y canales, véase pág. 561.

Ganguillet y Kutter han dado una fórmula para el valor de $c$ en la fórmula de Chezy: $v = c\sqrt{rs}$; donde $v$ = velocidad media; $r$ = radio medio = área ÷ por perímetro mojado; $s$ = pendiente = carga de frotamiento ÷ longitud.

Los hidráulicos antiguos dieron (cada uno según el resultado de sus investigaciones) **valores fijos para el coeficiente c** (generalmente más ó menos de 95 á 100 para canales en tierra ó granzón, como en nuestras primeras ediciones), haciéndolo, en otras palabras, constante é independiente de la forma, tamaño, inclinación y aspereza del canal; pero, según E. Ganguillet y Kutter, ingenieros suizos eminentes, el coeficiente $c$ está afectado por las diferencias en cualquiera de estas circunstancias. Según la fórmula de ellos (llamada por costumbre **Fórmula de Kutter.**)

**Para medidas inglesas.** **Para medidas métricas.**

$$c = \frac{41.6 + \frac{.00281}{\text{pendiente}} + \frac{1.811}{n}}{1 + \frac{\left(41.6 + \frac{.00281}{\text{pendiente}}\right) \times n}{\sqrt{\text{radio medio } \textit{en pies}}}} = \frac{23 + \frac{.00155}{\text{pendiente}} + \frac{1}{n}}{1 + \frac{\left(23 + \frac{.00155}{\text{pendiente}}\right) \times n}{\sqrt{\text{radio medio } \textit{en metros}}}}$$

**Las tablas que dan los valores de c** en sus diferentes pendientes, radio medio y grados de aspereza, están en las págs. 606, etc.

Aquí $n$ es un « **coeficiente de aspereza** » de los lados del canal, como se da más abajo.

Estos valores de $n$ fueron obtenidos después de experimentos en que se tomó el término medio de muchos de éstos, hechos bajo circunstancias muy diferentes. Por consiguiente, abarcan todas las causas perturbadoras debidas á las obstrucciones que existen en el fondo y lados del canal, en todos los casos de las experiencias. En pequeños canales artificiales de secciones transversales y pendientes uniformes, puede decirse que estas obstrucciones sólo consisten en las comparativamente pequeñas debidas á la naturaleza del material, de que está hecho el

fondo del canal. Pero en canales de ríos y tierra, aun en los casos donde la dirección general, inclinación y sección transversal son tolerablemente uniformes (como lo fueron en los casos en que se basa nuestra lista), existen otras muchas irregularidades en los lados y en el fondo, y éstas ejercen sobre la velocidad una acción retardatriz mucho mayor que las asperezas del terreno. Encontramos, por consiguiente, mayor valor dado á $n$ en tales casos, que en los de pequeños y regulares canales artificiales, aunque el material de los lados, etc., fué en muchos casos barro suave; y no debemos aplicar á estos canales comparativamente irregulares los pequeños valores de $n$ obtenidos en corrientes rectas y de sección y pendiente uniforme, aunque supongamos el fondo y lados de los primeros tan lisos como los de los últimos.

No puede aplicarse una fórmula general á los casos de **curvas fuertes** en el curso de una corriente natural, ó de **marcadas irregularidades en la sección transversal**. Tales casos requerirían aún coeficientes $n$ más grandes que los que aquí se dan para ríos y canales; deben ser determinados por experimentos hechos para cada caso y no servirían para otros. Para tales corrientes debemos, por consiguiente, basarnos sobre la medida real de la velocidad, ya directamente, ya por medio del gasto.

Hay mucha latitud para la **elección del adecuado coeficiente n** en un caso determinado, aun en los casos en que se conocen bien las condiciones del canal. Frecuentemente es necesario darle á $n$ un valor intermedio al de los establecidos; porque una mampostería de ladrillo mal hecha puede ser más áspera que una mampostería ordinaria bien acabada; pendientes laterales sobre « un granzón firme » pueden tener grados muy diferentes de asperezas, etc., etc. El ingeniero debe hacer listas de los valores de $n$ y resultados de su propia experiencia, apuntando principalmente las peculiaridades en cada caso.

Una diferencia dada en el grado de aspereza $n$ ejerce un efecto mayor sobre el coeficiente $c$, y por tanto sobre la velocidad, en los canales pequeños que en los más grandes. Por consiguiente es muy necesario que, en los casos de canales pequeños, se tenga mucho cuidado en encontrar (por experimentos, si es necesario) el adecuado valor de $n$; y cuando se desee alcanzar un gasto grande, deben hacerse especialmente muy lisos los lados en los pequeños canales.

### Cuadro del coeficiente, n, de aspereza.

El valor de **n es el mismo si el radio medio se da en medidas inglesas, ó métricas**, etc.

| | n = |
|---|---|
| **Canales artificiales de sección transversal uniforme.** | |
| Lados y fondo del canal construído de madera bien lisa | .009 |
| Cemento puro (se aplica también á tubos barnizados y tubos de hierro muy lisos) | .010 |
| Mezcla de 1 medida de arena y 3 de cemento (ó tubos de hierro liso) | .011 |
| Madera no acepillada (también se aplica á tubos de hierro ordinarios) | .012 |
| Mampostería de ladrillo | .013 |
| Mampostería ordinaria | .017 |
| **Canales sujetos á irregularidades en la sección transversal.** | |
| Canales en cascajo muy firme | .020 |
| Canales y ríos de sección transversal bastante uniforme, pendiente y dirección bastante regular y libres de piedras y hierbas | .025 |
| Canales que tienen ocasionalmente piedras y hierbas | .030 |
| Canales en mal estado y orden, llenos de vegetación y de piedras y detritus | .035 |

**Art. 22. Las siguientes tablas dan los valores del coeficiente c** obtenido por la fórmula de Kutter para diferentes pendientes, radio medio y grados de aspereza ($n$) *.

**1.º Teniendo la pendiente S, el radio medio R y el grado $n$ de aspereza, encontrar el coeficiente c.** Búsquese en la parte del cuadro que corresponde, la

* Es necesario á menudo interpolar valores de S, R, $n$ y $c$ intermedios de los de la tabla; esto puede hacerse mentalmente por una simple proporción.

pendiente dada S. Búsquese en la primera columna el radio medio dado R. En la misma línea de esta R, y bajo la *n* dada, se encuentra el valor de *c* *.

**2.º** Teniendo la pendiente S, el radio medio R y uno de los dos, ó el coeficiente *c* ó la velocidad real ó la requerida *v*, **encontrar el verdadero ó el más próximo valor n de la aspereza** del canal. Si se conoce la velocidad y no *c*, búsquese primero $c = \frac{\text{velocidad}}{\sqrt{\text{pendiente} \times \text{radio medio}}}$. Búsquese la parte de la tabla que corresponde á S dada, y en la primera columna encuéntrese la R dada. En la misma línea encuéntrese el valor dado, ó el obtenido para *c*; sobre e cual se encontrará el valor de *n* *.

**3.º** Teniendo la pendiente S, el grado *n* de aspereza y la velocidad real ó la requerida, **encontrar el radio medio R**. Supóngase un radio medio; y de la parte de la tabla correspondiente á la S dada, tómese el valor de *c* correspondiente á la *n* dada y al supuesto R. Entonces dígase :

$$v' = c \text{ así encontrado} \times \sqrt{\text{supuesto radio medio} \times \text{pendiente}}.$$

Si esta *v'* es la misma que la velocidad dada, ó bastante próxima á ella, tómese la R supuesta como verdadera. O repítase toda la operación, con un nuevo valor de R, *mayor* que el anterior si *v'* es *menor* que la velocidad dada y *viceversa* *.

**4.º** Teniendo las dimensiones de la parte mojada (*abco*, fig. C, pág. 561) del canal, el grado de aspereza *n*, y la velocidad real ó requerida, **hallar la pendiente verdadera ó necesaria, S :**

$$\text{Búsquese el radio medio } R = \frac{\text{Area de la sección transversal mojada}}{\text{Largo } abco, \text{ del perímetro mojado}}.$$

Supongamos que una de las cuatro pendientes de las tablas sea la pendiente propia. De la columna correspondiente de la tabla tómase el valor *c*, correspondiente á los valores de R y *n* dados.

Si R es 1 metro, el valor de *c* hallado así es el valor propio (porque entonces *c*, para cualquier valor dado de *n*, permanece el mismo para todas las pendientes); y la pendiente S, puede hallarse desde luego, así :

$$\text{Pendiente } S = \left( \frac{\text{Velocidad dada}}{c \times \sqrt{\text{radio medio}}} \right)^2$$

Pero si R es mayor ó menor de 1 metro, dígase :

$$v' = c \text{ (así encontrado)} \times \sqrt{\text{radio medio} \times \text{pendiente supuesta}}.$$

Si esta *v'* es bastante próxima á la velocidad dada, tómese la pendiente S supuesta, como la verdadera. Si no es así, elíjase otra S *mayor* que la última, si *v'* es *menor* que la velocidad dada, y *viceversa*, y repítase la operación *.

---

* Á menudo es necesario interpolar valores de S, R, *n* y *c* intermedios á los de la tabla. Esto puede hacerse mentalmente.

**Tabla del coeficiente c, para radios medios** *en metros.*

| | Radio medio R en metros. | Coeficientes n de aspereza. .009 | .010 | .011 | .012 | .013 | .015 | .017 | .020 | .025 | .030 | .035 | .040 | Radio medio R en metros. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Pendiente = .000025** por unidad de longitud, = 1 en 40,000. | | c | c | c | c | c | c | c | c | c | c | c | c | |
| | .025 | 34 | 29 | 25 | 22 | 20 | 17 | 14 | 11 | 9 | 7 | 6 | 5 | .025 |
| | .05 | 44 | 38 | 33 | 30 | 27 | 22 | 19 | 16 | 12 | 9 | 8 | 7 | .05 |
| | .1 | 58 | 50 | 44 | 40 | 36 | 30 | 26 | 21 | 16 | 13 | 11 | 9 | .1 |
| | .2 | 72 | 63 | 56 | 51 | 46 | 39 | 34 | 28 | 21 | 18 | 15 | 13 | .2 |
| | .3 | 82 | 72 | 64 | 58 | 53 | 45 | 39 | 33 | 25 | 21 | 17 | 15 | .3 |
| | .4 | 89 | 79 | 71 | 64 | 59 | 50 | 44 | 37 | 29 | 23 | 20 | 17 | .4 |
| | .6 | 99 | 88 | 80 | 72 | 67 | 57 | 50 | 42 | 33 | 28 | 23 | 20 | .6 |
| | 1. | 111 | 100 | 90 | 83 | 77 | 67 | 59 | 50 | 40 | 33 | 28 | 25 | 1. |
| | 1.50 | 121 | 109 | 100 | 92 | 85 | 74 | 66 | 57 | 46 | 38 | 33 | 29 | 1.50 |
| | 2 | 127 | 115 | 106 | 98 | 91 | 80 | 71 | 61 | 50 | 42 | 37 | 32 | 2 |
| | 3 | 136 | 124 | 114 | 106 | 99 | 87 | 78 | 68 | 56 | 48 | 42 | 37 | 3 |
| | 4 | 142 | 130 | 120 | 111 | 104 | 93 | 83 | 73 | 61 | 52 | 46 | 41 | 4 |
| | 6 | 149 | 137 | 127 | 119 | 111 | 100 | 90 | 80 | 67 | 58 | 51 | 46 | 6 |
| | 10 | 158 | 145 | 135 | 127 | 120 | 108 | 98 | 88 | 75 | 66 | 59 | 53 | 10 |
| | 15 | 164 | 151 | 141 | 133 | 126 | 114 | 104 | 94 | 81 | 72 | 64 | 59 | 15 |
| | 20 | 167 | 155 | 145 | 137 | 130 | 118 | 108 | 98 | 85 | 75 | 68 | 62 | 20 |
| | 30 | 172 | 160 | 150 | 142 | 135 | 123 | 113 | 103 | 90 | 81 | 74 | 68 | 30 |
| **Pendiente = .00005** por unidad de longitud, = 1 en 20,000. | .025 | 40 | 35 | 30 | 26 | 24 | 20 | 17 | 13 | 10 | 8 | 7 | 5 | .025 |
| | .05 | 52 | 44 | 39 | 34 | 31 | 26 | 22 | 18 | 13 | 11 | 9 | 7 | .05 |
| | .1 | 65 | 57 | 50 | 44 | 40 | 34 | 29 | 24 | 18 | 14 | 12 | 10 | .1 |
| | .2 | 79 | 69 | 62 | 55 | 51 | 43 | 37 | 30 | 23 | 19 | 16 | 13 | .2 |
| | .3 | 87 | 77 | 69 | 62 | 57 | 48 | 42 | 35 | 27 | 22 | 18 | 16 | .3 |
| | .4 | 93 | 83 | 74 | 67 | 62 | 53 | 46 | 38 | 30 | 25 | 21 | 18 | .4 |
| | .6 | 102 | 90 | 82 | 74 | 69 | 59 | 52 | 43 | 34 | 28 | 24 | 21 | .6 |
| | 1. | 111 | 100 | 90 | 83 | 77 | 67 | 59 | 50 | 40 | 33 | 28 | 25 | 1. |
| | 1.5 | 118 | 107 | 97 | 90 | 83 | 73 | 65 | 55 | 45 | 38 | 33 | 28 | 1.5 |
| | 2 | 123 | 111 | 102 | 94 | 87 | 77 | 68 | 59 | 48 | 41 | 35 | 31 | 2 |
| | 3 | 129 | 117 | 108 | 100 | 93 | 83 | 74 | 64 | 53 | 45 | 40 | 35 | 3 |
| | 4 | 133 | 121 | 112 | 104 | 97 | 86 | 77 | 68 | 56 | 49 | 43 | 38 | 4 |
| | 6 | 138 | 126 | 117 | 109 | 102 | 91 | 82 | 72 | 61 | 53 | 47 | 42 | 6 |
| | 10 | 143 | 131 | 122 | 114 | 107 | 96 | 87 | 78 | 66 | 58 | 52 | 47 | 10 |
| | 15 | 147 | 135 | 126 | 118 | 111 | 100 | 91 | 82 | 70 | 62 | 56 | 51 | 15 |
| | 20 | 150 | 137 | 128 | 120 | 113 | 103 | 94 | 84 | 72 | 64 | 58 | 53 | 20 |
| | 30 | 152 | 140 | 131 | 123 | 116 | 105 | 97 | 87 | 76 | 68 | 62 | 57 | 30 |
| **Pendiente = .0001** por unidad de longitud, = 1 en 10,000. | .025 | 47 | 40 | 35 | 31 | 28 | 22 | 19 | 15 | 11 | 9 | 7 | 6 | .025 |
| | .05 | 59 | 50 | 44 | 40 | 35 | 29 | 25 | 20 | 15 | 12 | 10 | 8 | .05 |
| | .1 | 72 | 62 | 55 | 50 | 45 | 37 | 32 | 26 | 19 | 16 | 13 | 11 | .1 |
| | .2 | 84 | 74 | 66 | 60 | 54 | 46 | 39 | 32 | 25 | 20 | 17 | 14 | .2 |
| | .3 | 91 | 81 | 73 | 66 | 60 | 51 | 44 | 37 | 28 | 23 | 19 | 17 | .3 |
| | .4 | 97 | 86 | 77 | 70 | 64 | 55 | 48 | 40 | 31 | 25 | 21 | 18 | .4 |
| | .6 | 104 | 92 | 83 | 76 | 70 | 60 | 53 | 45 | 35 | 29 | 25 | 21 | .6 |
| | 1. | 111 | 100 | 90 | 83 | 77 | 67 | 59 | 50 | 40 | 33 | 28 | 25 | 1. |
| | 1.5 | 117 | 105 | 96 | 88 | 82 | 72 | 64 | 54 | 44 | 37 | 32 | 28 | 1.5 |
| | 2 | 120 | 109 | 100 | 92 | 85 | 75 | 67 | 57 | 47 | 40 | 34 | 30 | 2 |
| | 4 | 128 | 116 | 107 | 99 | 92 | 82 | 73 | 64 | 53 | 46 | 40 | 36 | 4 |
| | 6 | 131 | 119 | 110 | 102 | 96 | 85 | 77 | 67 | 56 | 49 | 43 | 39 | 6 |
| | 10 | 135 | 123 | 114 | 106 | 100 | 89 | 81 | 71 | 60 | 53 | 47 | 43 | 10 |
| | 15 | 137 | 126 | 116 | 109 | 102 | 92 | 83 | 74 | 63 | 55 | 50 | 46 | 15 |
| | 30 | 141 | 129 | 120 | 112 | 106 | 95 | 87 | 78 | 67 | 59 | 54 | 50 | 30 |
| **Pendiente = .0020** por unidad de longitud = 1 en 5,000. | .025 | 52 | 45 | 40 | 35 | 31 | 25 | 21 | 17 | 12 | 9 | 8 | 6 | .025 |
| | .050 | 63 | 55 | 48 | 43 | 39 | 32 | 27 | 21 | 16 | 12 | 10 | 8 | .050 |
| | .1 | 75 | 66 | 59 | 53 | 48 | 40 | 34 | 27 | 21 | 16 | 13 | 11 | .1 |
| | .2 | 87 | 77 | 69 | 62 | 57 | 48 | 41 | 34 | 26 | 21 | 17 | 15 | .2 |
| | .4 | 99 | 88 | 80 | 72 | 66 | 57 | 49 | 41 | 32 | 26 | 22 | 19 | .4 |
| | .6 | 104 | 93 | 84 | 77 | 71 | 61 | 53 | 45 | 36 | 29 | 25 | 22 | .6 |
| | 1 | 111 | 100 | 90 | 83 | 77 | 67 | 59 | 50 | 40 | 33 | 28 | 25 | 1 |
| | 2 | 118 | 107 | 98 | 90 | 84 | 74 | 65 | 56 | 46 | 39 | 34 | 30 | 2 |
| | 4 | 124 | 113 | 104 | 97 | 90 | 79 | 71 | 62 | 51 | 44 | 39 | 35 | 4 |
| | 10 | 130 | 119 | 110 | 102 | 96 | 85 | 77 | 67 | 57 | 50 | 45 | 40 | 10 |
| | 30 | 135 | 124 | 114 | 107 | 100 | 90 | 82 | 73 | 62 | 55 | 50 | 46 | 30 |

**Tabla del coeficiente c, para radios medios** *en metros.* (Continuación.)

| | Radio medio R en metros. | Coeficiente n de aspereza. | | | | | | | | | | | | Radio medio R en metros. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | .009 | .010 | .011 | .012 | .013 | .015 | .017 | .020 | .025 | .030 | .035 | .040 | |
| | | c | c | c | c | c | c | c | c | c | c | c | c | |
| **Pendiente = .0004** por unidad de longitud, = 1 en 2,500 | .025 | 55 | 47 | 41 | 37 | 33 | 27 | 22 | 17 | 13 | 10 | 8 | 7 | .025 |
| | .050 | 66 | 58 | 51 | 45 | 40 | 33 | 28 | 23 | 17 | 13 | 11 | 9 | .050 |
| | .1 | 78 | 68 | 61 | 55 | 50 | 42 | 35 | 28 | 21 | 17 | 14 | 12 | .1 |
| | .2 | 90 | 80 | 70 | 64 | 59 | 49 | 42 | 35 | 27 | 22 | 18 | 15 | .2 |
| | .3 | 95 | 85 | 76 | 70 | 63 | 54 | 47 | 39 | 30 | 24 | 21 | 17 | .3 |
| | .4 | 99 | 89 | 80 | 73 | 67 | 57 | 50 | 42 | 32 | 27 | 22 | 20 | .4 |
| | .6 | 105 | 94 | 85 | 78 | 72 | 62 | 54 | 45 | 36 | 30 | 25 | 22 | .6 |
| | 1 | 111 | 100 | 90 | 83 | 77 | 67 | 59 | 50 | 40 | 33 | 28 | 25 | 1 |
| | 2 | 117 | 106 | 97 | 89 | 83 | 73 | 65 | 56 | 45 | 38 | 34 | 30 | 2 |
| | 4 | 123 | 111 | 102 | 95 | 88 | 78 | 70 | 61 | 50 | 43 | 38 | 34 | 4 |
| | 6 | 125 | 114 | 105 | 97 | 91 | 81 | 72 | 63 | 53 | 46 | 40 | 36 | 6 |
| | 10 | 128 | 117 | 108 | 100 | 93 | 83 | 75 | 66 | 55 | 48 | 43 | 39 | 10 |
| | 30 | 132 | 121 | 112 | 104 | 98 | 87 | 79 | 70 | 60 | 52 | 48 | 43 | 30 |
| **Pendiente = .0010** por unidad de longitud, = 1 en 1,000. | 025 | 57 | 50 | 43 | 38 | 34 | 28 | 23 | 18 | 13 | 11 | 9 | 7 | .025 |
| | .050 | 69 | 59 | 52 | 47 | 42 | 34 | 29 | 23 | 17 | 13 | 11 | 9 | .050 |
| | .1 | 80 | 70 | 63 | 56 | 50 | 42 | 36 | 30 | 22 | 17 | 14 | 12 | .1 |
| | .2 | 90 | 80 | 72 | 65 | 60 | 50 | 43 | 35 | 27 | 22 | 18 | 16 | .2 |
| | .3 | 96 | 86 | 77 | 70 | 64 | 54 | 47 | 39 | 30 | 25 | 21 | 18 | .3 |
| | .4 | 100 | 89 | 81 | 74 | 67 | 58 | 50 | 42 | 33 | 27 | 23 | 19 | .4 |
| | .6 | 104 | 94 | 85 | 78 | 72 | 62 | 54 | 46 | 36 | 30 | 25 | 22 | .6 |
| | 1 | 111 | 100 | 90 | 83 | 77 | 67 | 59 | 50 | 40 | 33 | 28 | 25 | 1 |
| | 2 | 116 | 106 | 97 | 90 | 83 | 72 | 64 | 55 | 45 | 38 | 33 | 29 | 2 |
| | 4 | 121 | 111 | 102 | 94 | 87 | 77 | 69 | 60 | 50 | 42 | 37 | 33 | 4 |
| | 6 | 124 | 113 | 104 | 97 | 90 | 80 | 71 | 62 | 52 | 45 | 40 | 36 | 6 |
| | 10 | 127 | 115 | 106 | 99 | 92 | 82 | 73 | 64 | 54 | 47 | 42 | 38 | 10 |
| | 30 | 130 | 119 | 110 | 102 | 96 | 86 | 77 | 68 | 58 | 51 | 46 | 42 | 30 |
| **Pendiente = .01** por unidad de longitud, = 1 en 100. | .025 | 59 | 50 | 44 | 39 | 35 | 28 | 24 | 19 | 14 | 10 | 9 | 7 | .025 |
| | .05 | 69 | 60 | 53 | 48 | 43 | 35 | 29 | 24 | 18 | 14 | 11 | 9 | .05 |
| | .1 | 81 | 71 | 63 | 57 | 51 | 43 | 36 | 30 | 22 | 18 | 15 | 12 | .1 |
| | .2 | 91 | 81 | 72 | 65 | 60 | 50 | 44 | 36 | 27 | 22 | 18 | 16 | .2 |
| | .3 | 97 | 86 | 77 | 71 | 65 | 55 | 48 | 40 | 31 | 25 | 21 | 18 | .3 |
| | .4 | 101 | 90 | 81 | 74 | 68 | 58 | 50 | 42 | 33 | 27 | 23 | 20 | .4 |
| | .6 | 106 | 95 | 86 | 78 | 72 | 62 | 54 | 46 | 36 | 30 | 25 | 22 | .6 |
| | 1. | 111 | 100 | 90 | 83 | 77 | 67 | 59 | 50 | 40 | 33 | 28 | 25 | 1. |
| | 1.5 | 115 | 104 | 94 | 87 | 80 | 70 | 62 | 53 | 43 | 36 | 31 | 27 | 1.5 |
| | 2 | 117 | 105 | 96 | 89 | 83 | 72 | 64 | 55 | 45 | 38 | 33 | 29 | 2 |
| | 4 | 121 | 110 | 101 | 94 | 87 | 76 | 68 | 59 | 49 | 42 | 37 | 33 | 4 |
| | 10 | 126 | 114 | 105 | 98 | 91 | 81 | 73 | 64 | 53 | 46 | 41 | 37 | 10 |
| | 30 | 129 | 118 | 108 | 101 | 95 | 84 | 77 | 67 | 57 | 50 | 45 | 41 | 30 |

**Para pendientes mayores de .01** por unidad de longitud, =1 en 100, el coeficiente *c* se hace prácticamente el mismo que en esta pendiente. La *velocidad*, sin embargo, como es $= c \times \sqrt{\text{radio medio} \times \textit{pendiente}}$, continúa aumentando con la pendiente.

(*N. del T.* — Es más claro el método gráfico que damos á continuación que el que trae el autor.)

Diseño gráfico para la fórmula de la velocidad dada para Ganguillet y Kutter. (Véase la hoja págs. 608 *a* y 608 *b*).

En la fórmula de la velocidad media, pág. 558, $v = c\sqrt{\text{radio medio} \times \text{pendiente}}$; llamando R el radio medio y J la pendiente y tomando el valor *c* del coeficiente en la misma página 558, tenemos

$$v = c\sqrt{RJ} = \left[\frac{23 + \frac{1}{n} + \frac{.00155}{J}}{1 + \left(23 + \frac{.00155}{J}\right)\frac{n}{\sqrt{R}}}\right]\sqrt{RJ}$$

Las rectas $n$ dan el coeficiente $c$, en función de la naturaleza del perímetro mojado. Ellas emanan todas del punto que tiene por abscisa, $\sqrt{R} = 1$. Mientras más áspero, rugoso, es el lecho, más se aproximan las rectas $n$ á la horizontal. Las curvas J (curvas de pendiente) muestran la variación del coeficiente $c$, con la variación de la pendiente.

**Siendo dadas R, J, n**, encontrar $c$ : El punto $\sqrt{R}$ tomado como abscisa es el punto de intersección de la curva J con la recta $n$, determinando una recta que corta sobre el eje de las ordenadas un segmento igual á $c$. Ej. : Si $R = 1.7$ (es decir, $\sqrt{R} = 1.3$), $J = .0002$, $n = .025$, se obtiene el punto, $d$, juntando $a$ con $b$, y se lee $c = 45.3$.

**Dadas c, J, n**, encontrar **R** : El punto de intersección de la curva J con la recta $n$ y el punto sobre el eje de las ordenadas determinan una recta que encuentra al eje de las abscisas en un punto que es precisamente $\sqrt{R}$.

**Dadas c, R, n**, encontrar **J** : El punto $\sqrt{R}$ sobre el eje de las abscisas, y el punto $c$ sobre el eje de las ordenadas, determinan una recta que corta á la recta dada $n$ en un punto que corresponde al valor de J que se busca.

**Dadas c, R, J**, encontrar **n** : La recta que une el punto $\sqrt{R}$ sobre el eje de las abscisas y $c$ sobre el de las ordenadas, corta á la curva J dada en un punto que corresponde al valor de $n$, que se busca. »

**Cuando el agua corre á medio caño solamente,** la velocidad será la misma, según la fórmula, que cuando corre á caño lleno; pero no sucede lo mismo en cualquiera otra cantidad mayor ó menor. A profundidades mayores, la velocidad aumenta hasta que aquélla llega á ser próximamente igual á .9 del diámetro, y es entonces como 10 por ciento mayor que la velocidad á caño lleno ó á medio caño. A partir de la profundidad de .9 del diámetro, la velocidad disminuye aunque la profundidad aumente ó disminuya. A una profundidad de .25 del diámetro la velocidad es más ó menos .78 de la que corresponde al caño lleno; y luego disminuye mucho más rápidamente para menores profundidades. Todo esto se aplica igualmente á los tubos.

**Una velocidad de** 1.50 m por segundo más ó menos es la mayor velocidad que puede adoptarse en la práctica para evitar que las partes bajas de la cloaca se gasten demasiado pronto con las horruras que arrastra el agua.

**Art. 23. La proporción en que el agua de lluvia llega á una cloaca** ó alcantarilla puede hallarse aproximadamente por la fórmula siguiente de Burkly-Ziegler, según el admirable informe « *Report on European Sewerage Systems* », *de R. Hering, Civ. and San. Engineers of Philadel.;* véase *Trans. Am. Soc. C. Eng.*, noviembre 1881. (*N. del T.* — La hemos transformado para medidas métricas.)

Metros cúbicos por segundo y *por hectárea* = que llegan á la cloaca

Un coeficiente determinado á juicio según las circunstancias

× Término medio de metros cúb. de lluvia caída **por segundo por hectárea**

× $\sqrt[4]{\dfrac{\text{Pendiente media del terreno en metros por 1,000 metros (ó sea el tanto por mil)}}{\text{Número de hectáreas desaguadas} \times 2.47104}}$

**Los coeficientes** para ambos sistemas de medidas para calles pavimentadas es .75; para casos ordinarios, .625; para los alrededores de poblaciones con jardines, prados y calles macadamizadas .31.

(*N. del T.* — El ejemplo que sigue es análogo al del autor, pero cambiando todas las unidades de acres, pulgs, etc., al sistema métrico.)

**La caída media mayor de lluvia** es de 44 á 70 mm por hora, suponiéndola de 76.2 mm = .0762 m sobre un área de 1254.48 hectáreas con una pendiente, de 5 por 1,000 y adoptando el coeficiente de .5; tendremos para calcular los $m^3$, que llegarán por segundo y por hectárea á la cloaca, en la parte baja de las 1254.48 hectáreas, lo siguiente :

Los 76.2 mm de lluvia por hora equivalen á .2099 metro cúbico por segundo y por hectárea, y usando la fórmula tendremos :

Metros cúbs por seg y por hect que llegan á la cloaca =

$$.5 \times .2099 \times \sqrt[4]{\frac{5}{1254.48 \times 2.471}} = .02130485 \text{ metro cúb.}$$

Para 1254.48 hect llegarán á la cloaca por seg un total de $m^3 = 1254.48 \times .02130485 = 26.726\ m^3$.

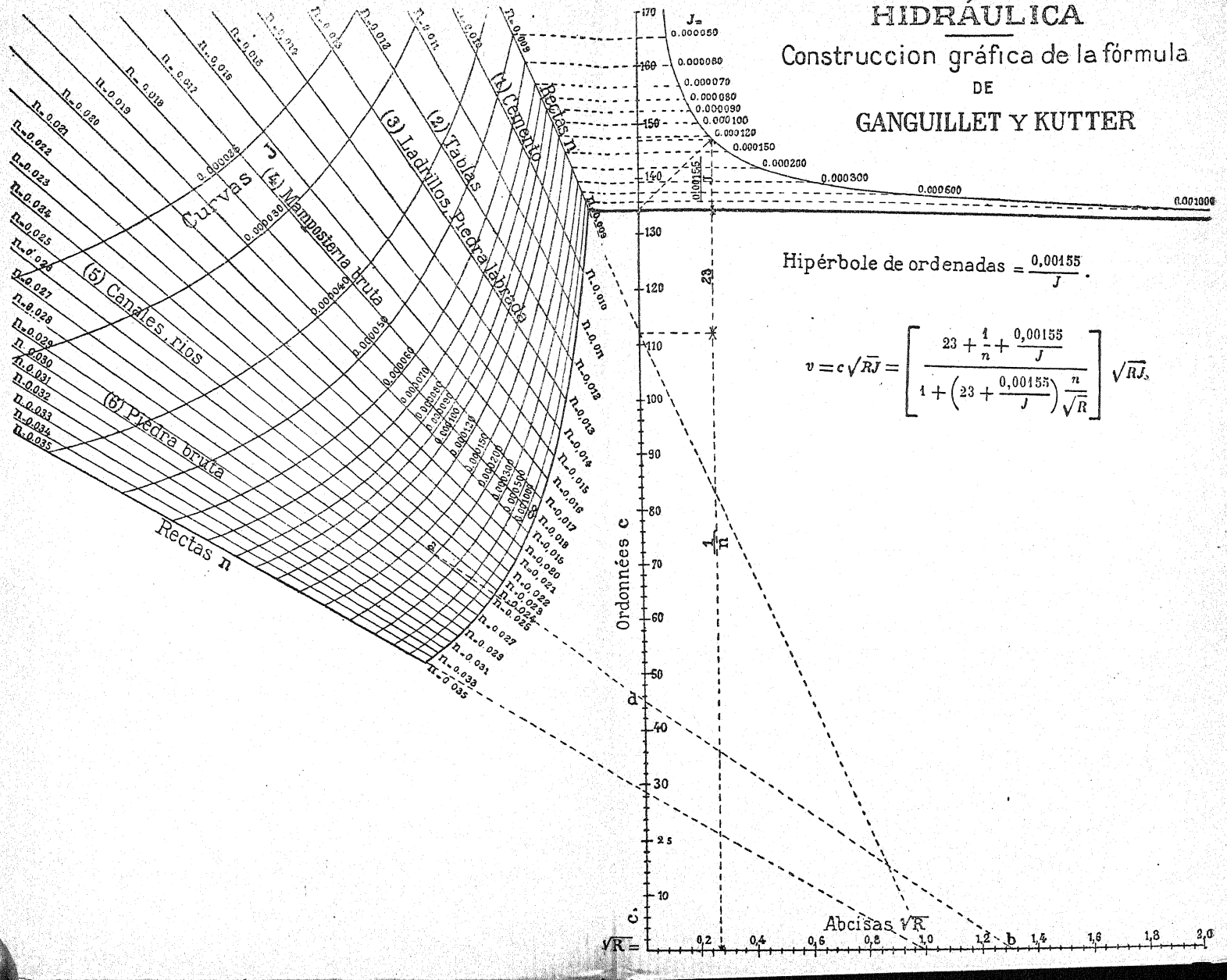
Cuando se tenga la pendiente J, la velocidad $v$ y el coeficiente de aspereza $n$, que corresponde á la especie de material con que se construye la superficie de la cloaca

# HIDRÁULICA

## Construccion gráfica de la fórmula

DE

## GANGUILLET Y KUTTER



en contacto con el agua, entonces, empleando la fórmula de Kutter ó su diagrama, se determina el radio medio R, y por ahí la sección de la cloaca, que dará pase al volumen calculado. »

La sección debe hacerse siempre un poco más amplia que la calculada.

Para una corriente de alguna constancia que arrastre basuras, la velocidad, para que el agua no gaste un fondo de ladrillos, no debe exceder de 1.50 m por segundo.

**Tabla. El Sr. Wicksteed, hidráulico inglés de experiencia, da la tabla siguiente de las velocidades, pendientes ó caídas menores, que deben darse á los tubos de desagües y á las cloacas en las poblaciones,** para que ellas se mantengan en las circunstancias ordinarias, limpias y libres de depósitos.

(*Obs. del T.* — La que sigue está deducida de la « que trae el autor en medidas inglesas »).

| Diám en metros. | Veloc en m por minuto. | Pendiente 1 en | Diám en metros. | Veloc en m por minuto. | Pendiente 1 en |
|---|---|---|---|---|---|
| .102 | 73.150 | 36 | .457 | 54.863 | 294 |
| .152 | 67.055 | 65 | .533 | 54.863 | 343 |
| .178 | 67.055 | 76 | .610 | 54.863 | 392 |
| .203 | 67.055 | 87 | .762 | 54.863 | 490 |
| .229 | 67.055 | 98 | .914 | 54.863 | 588 |
| .254 | 64.006 | 119 | 1.067 | 54.863 | 686 |
| .279 | 60.96 | 145 | 1.219 | 54.863 | 784 |
| .305 | 57.91 | 175 | 1.372 | 54.863 | 882 |
| .381 | 54.87 | 244 | 1.524 | 54.863 | 980 |

**Peso y precio** por pie corrido, **de tubos de tierra cocida vidriados,** para desagües, etc.; adoptados por los fabricantes unidos de tubos para cloacas de los Estados Unidos, marzo 1887.

(*N. del T.* — La siguiente tabla es equivalente á la que trae el autor en unidades inglesas. Para tener el precio **por metro,** multiplíquese, el que da la tabla, por pie lineal × 3.28.)

| Tubos de desagüe con juntas de campana. | | | | | | Tubos de cloacas con juntas en forma de mangas. | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diám | Peso. | Prec. | Diám | Peso. | Prec. | Diám | Peso. | Prec. | Diám | Peso. | Prec. |
| mets. | kilog. | dólar | mets. | kilog. | dólar | mets. | kilog. | dólar | mets. | kilog. | dólar |
| .050 | 1.81 | .14 | .152 | 8.16 | .30 | .381 | 20.41 | 1.25 | .762 | 68.04 | 5.50 |
| .076 | 3.17 | .16 | .203 | 9.98 | .45 | .457 | 29.48 | 1.70 | .914 | 88.45 | 7.00 |
| .102 | 4.54 | .20 | .254 | 13.61 | .65 | .533 | 40.37 | 2.50 | 1.067 | 92.08 | 8.50 |
| .127 | 5.44 | .25 | .305 | 14.97 | .85 | .609 | 45.36 | 3.25 | 1.219 | 104.33 | 10.50 |

Las juntas se cogen con mezcla de cemento, ó, cuando los tubos se usan para desagües solamente, con greda ó arcilla. Los tubos de desagüe de 76 á 300 mm tienen un espesor de 22 mm más ó menos. Un codo ó rama bifurcada cuesta tanto como 1 á 1½ m de tubo ordinario.

Los tubos de 1.22 m de diámetro tienen más ó menos 5 cm de espesor.

**Art. 24. Cuando el área de la sección transversal de un canal está reducida en cualquier punto,** sea por un dique, fig. 33, pág. 610, ó cuando se estrechan sus paredes, fig. 32, ó porque se coloca en él una pila, etc., fig. 34; una parte á lo menos de la gravedad (que de lo contrario comunicaría *velocidad* al agua, aguas arriba del punto en donde se halla el obstáculo) ejerce *presión* contra la represa ú obstáculo. Esta presión mantiene el agua del lado arriba á un nivel más alto del que tendría si esta obstrucción no existiese.

Dicha agua se halla entonces como si estuviera en un estanque, por tener menos velocidad y más presión que antes. Si el estanque no tiene salida, entonces *no* hay velocidad, y *toda* la carga, ó fuerza de la gravedad, actuando sobre el agua

se consúme ó emplea en ejercer *presión*. Pero si hay una salida, sea por encima de la represa ó por el espacio comprendido entre los pilares, etc., una parte *co*, figs. 31, 33 y 34, de esta presión ó carga se gasta en *darle velocidad* (ó aumentársela) al agua que se escapa por aquella salida; después de lo cual sólo se requiere la carga (producida por la pendiente) para vencer las resistencias de las obstrucciones del canal del lado abajo del obstáculo y *mantener así uniforme* la velocidad que le comunicó la carga *co*.

Cuando un gran canal está alimentado por un estanque, como los destinados á la navegación, la caída *co* en metros es aproximadamente

$$= (\text{velocidad media en mets})^2 \times .055777;$$

y en los canales más pequeños, como los que se usan en los molinos,

$$= (\text{velocidad media en mets})^2 \times .06562.$$

(*N. del T.* — Estas son las fórmulas equivalentes á las que trae el autor en pies.)

Lo brusco de la caída puede disminuirse redondeando ó inclinando las orillas de las pilas, ó los ángulos de los lados del canal, fig. 32, ó el acceso á la represa.

La fig. 33 es una sección transversal del **dique de Clegg**, que atraviesa el río Cape Fear, N. C. Está trazada con medidas hechas por Ellwood Morris, I. C., quien dió los datos al autor. El dique se hizo con maderos redondos de apuntalar y el borde superior ó copete horizontal, de 8 pies 5 pulgs (2.57 m) de ancho está cubierto de tablones, sobre los cuales el agua se desliza en una lámina plana de 6 pulgs (.15 m) de profundidad (cuando se midió). En la extremidad aguas arriba de esta lámina, y á una distancia de 2 pies (.61 m) más ó menos, se forma una carga *co* de 9 pulgs (.229 m) como lo indica la figura.

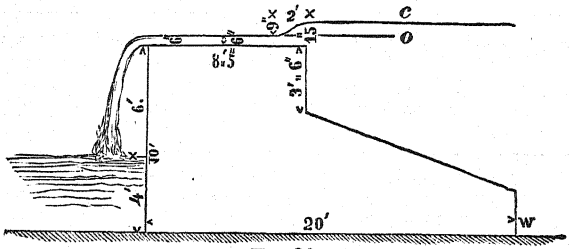



Fig 33

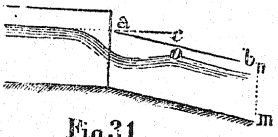



Fig 31

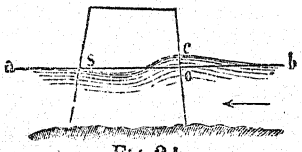



Fig 34

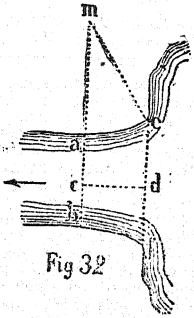



Fig 32

**Las dos tablas siguientes** fueron tomadas (con muchas correcciones) de la *Nicholson's Architecture*, y deben considerárselas meramente como aproximaciones probables. En ellas se supone que las pilas estén bien redondeadas ó que terminen por ángulos agudos en sus caras situadas hacia donde viene la corriente para dejar pasar fácilmente el agua. Nicholson dice que si dichas pilas son de caras cuadradas, la carga aumenta como 50 por ciento. Este asunto es complicado en extremo y no admite solución exacta. Si la velocidad aumentada socava ó degrada el fondo hasta el punto de que el área ó superficie del curso de agua se haga mayor de lo que era antes, la carga desaparece y la velocidad se reduce hasta su primitivo valor.

**Art. 25. Desgaste.** En un canal de pendiente y sección transversal uniforme y constante, la velocidad de las moléculas de agua contiguas al fondo y lados del canal es muy débil, y el desgaste es muy poco. Pero si hay irregularidad en la pendiente ó en la sección transversal, como se ha dicho en el último artículo, el desgaste aumenta en gran manera en sus inmediaciones.

La colocación de una ó dos pilas en una corriente de alguna magnitud produce frecuentemente un increíble deterioro ó erosión, si el fondo es de una substancia que cede á la acción de la corriente del agua. La mayor erosión tiene efecto, naturalmente, durante las crecientes y cerca de las obstrucciones.

**El desgaste se supone proporcional al cuadrado de la velocidad.**

Según Smeaton, una velocidad de 12.870 km por hora no mueve las piedras brutas de cantera que no excedan de 14 litros puestas alrededor de las pilas, etc., á menos que arrastre ó socave el terreno en que se apoyan.

*N. del T.* — Damos la tabla siguiente en sistema métrico deducida de la del autor en pulgs, pies y millas.

## Tabla de cargas producidas por obstrucciones de la corriente.

| Velocidad primitiva del curso de agua. ★ | | Naturaleza del fondo que comienza á desgastarse bajo la acción de velocidades de la corriente en el fondo iguales á las de las primeras tres columnas. | Parte del área primitiva del curso de agua ocupada por las obstrucciones. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 1/12 | 1/10 | 1/8 | 1/6 | 1/4 | 1/3 | 1/2 | 5/8 | 3/4 |
| Por seg. | Por hora | | Carga en metros producida en las obstrucciones. | | | | | | | | |
| Met. | Kil. | | | | | | | | | | |
| .0762 | .274 | Fango blando, fango..... | .00009 | .00012 | .00012 | .00018 | .00030 | .00053 | .00100 | .00204 | .00494 |
| .1524 | .549 | Greda ó arcilla..... | .00034 | .00043 | .00052 | .00070 | .00122 | .00177 | .00405 | .00814 | .01969 |
| .3048 | 1.096 | Arena..... | .00137 | .00171 | .00210 | .00277 | .00457 | .00704 | .01629 | .03278 | .07876 |
| .6096 | 2.189 | Granzón..... | .00555 | .00686 | .00841 | .01110 | .01829 | .02816 | .06486 | .13033 | .31576 |
| .9144 | 3.283 | Ripios menudos..... | .01247 | .01545 | .01893 | .02496 | .04114 | .06337 | .14593 | .29324 | .70895 |
| 1.2192 | 4.377 | Ripios grandes..... | .02219 | .02749 | .03365 | .04438 | .07315 | .11265 | .25 39 | .52120 | 1.26307 |
| 1.5240 | 5.488 | Esquisto blando..... | .03465 | .04298 | .05258 | .06934 | .11430 | .17602 | .40233 | .81441 | 1.9735 |
| 1.8288 | 6.582 | Roca estratificada..... | .04993 | .06187 | .07571 | .0985 | .16459 | .25347 | .58369 | 1.17285 | 2.8358 |
| 3.0479 | 9.960 | Roca dura.... | .13868 | .17190 | .2105 | .27736 | .45709 | .70407 | 1.60931 | 3.25824 | 7.89417 |

★ Esta es una expresión muy vaga. ¿A qué se refiere? ¿A la mayor velocidad de la superficie en medio del canal; ó á la velocidad media de la sección transversal total?

(*Obs. del T.* — Damos la siguiente en medidas métricas, deducida de la que trae el autor en medidas inglesas.)

**Tabla de los incrementos de velocidades producidos en, y por las obstrucciones redondeadas ó puntiagudas.**

| Velocidad primitiva del curso de agua *. | | Parte del área primitiva del curso de agua ocupada por las obstrucciones. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1/12 | 1/10 | 1/8 | 1/6 | 1/4 | 1/3 | 1/2 | 5/8 | 3/4 |
| Por seg. | Por hora. | Velocidad en mets por seg producida en las obstrucciones. | | | | | | | | |
| Metr. | Kil. | | | | | | | | | |
| .0762 | .274 | .083347 | .08839 | .09143 | .093334 | .106678 | .120089 | .153493 | .213356 | .320033 |
| .1524 | .549 | .17069 | .01768 | .18287 | .193068 | .213356 | .24018 | .320034 | .426712 | .640068 |
| .3048 | 1.096 | .34442 | .35356 | .36575 | .384041 | .426712 | .481373 | .640068 | .853425 | 1.28014 |
| .6096 | 2.189 | .69188 | .07101 | .73150 | .768082 | .853425 | .96315 | 1.28014 | 1.70685 | 2.56027 |
| .9144 | 3.283 | 1.03327 | 1.06068 | 1.09726 | 1.151123 | 1.28014 | 1.44473 | 1.92020 | 2.56027 | 3.840407 |
| 1.2192 | 4.377 | 1.38377 | 1.42034 | 1.46301 | 1.53616 | 1.70685 | 1.92630 | 2.56027 | 3.41370 | 5.120540 |
| 1.5240 | 5.488 | 1.70685 | 1.76781 | 1.82877 | 1.95068 | 2.13356 | 2.401779 | 3.200342 | 4.26712 | 6.40068 |
| 1.8288 | 6.582 | 2.06651 | 2.12137 | 2.19452 | 2.3042 | 2.56027 | 2.89045 | 3.840417 | 5.120546 | 7.68082 |
| 3.0479 | 9.960 | 3.44418 | 3.53592 | 3.657 3 | 3.84041 | 4.27712 | 4.815756 | 6.40068 | 8.53425 | 12.8014 |

Si son *cuadradas*, estas velocidades deben aumentarse en 1/8 según Nicholson.

**Art. 26. La resistencia que opone el agua al movimiento de una superficie plana colocada perpendicularmente á ella**, es casi como el cuad de su velocidad, y, según Hutton, su valor aproximado en libs por pie cuad es = cuad de la velocidad en pies por segundo.

(*Obs. del T.* — Este valor en kilog por metro cuad = (aproximadamente), al cuad de la velocidad en metros multiplicado por la constante 52.557.

Ej. : Supongamos $v$ = 1 pie = .304794; $v^2$ = 1 pie cuad = .0929 m cuad; luego tendremos :

$$\text{Resistencia en libs por pie cuad aprox} = v^2 = 1 \text{ libra.}$$

$$\text{Resistencia en kilog por m cuad.} = \begin{cases} v^2 \times 52.557 = .0929 \times 52.557 = 4.883 \text{ kilogramos} \\ (= 1 \text{ libra por pie cuad}), \end{cases}$$

**ó lo que es lo mismo, la presión que ejerce una corriente de agua** contra una superficie plana vertical fija, es = al peso de una columna de agua cuya base sea igual á la superficie comprimida y que tenga por altura la carga que corresponda á la velocidad $\sqrt{2gh}$. (En la unidad que se elija.)

La resistencia de una esfera es á la de su círculo máximo como 1 es á 2.9.

Cuando la superficie en movimiento en lugar de formar ángulo recto con la dirección en que se mueve, forma otro ángulo con ella, la resistencia se hace menor (más ó menos) en la relación que sigue. Por lo tanto, cuando la superficie es inclinada, calcúlese primero la resistencia como si estuviera en ángulo recto, y multiplíquese luego por las decimales que siguen situadas frente al ángulo de inclinación en este cuadro :

| | | | |
|---|---|---|---|
| 90° .... 1.00 | 60° .... .88 | 40° .... 58 | 20° .... .16 |
| 80 .... .98 | 55 .... .83 | 35 .... 46 | 15 .... .10 |
| 70 .... .95 | 50 .... .76 | 30 .... 34 | 10 .... .06 |
| 65 .... .92 | 45 .... .68 | 25 .... 24 | 5 .... .02 |

**La fuerza de desgaste del agua corriente** se supone proporcional al cuad de la velocidad.

**Art. 27**. Para calcular en caballos la fuerza de una caída de**

* Véase nota página anterior.

** *N. del T.* — Dejamos este ejemplo en ambos sistemas de medidas, por creer útil para los ingenieros, que respectivamente los usan, practicar esta conversión con unidades de tan frecuente empleo.

**agua.** Sabemos que por caballo de fuerza se entiende la fuerza necesaria para levantar 33,000 libs verticalmente á 1 pie de altura en un minuto. (La fuerza de un caballo ordinario es generalmente la que se necesita para levantar como las $^2/_3$ partes, ó sean 22,000 á un pie de altura en un minuto.) Multipliquense entre sí el número de pies cúbicos de agua que caen por minuto, la altura vertical de caída ó carga en pies y el número 62.3 (peso de un pie cúbico de agua), y divídase el producto por 33,000. O, traduciéndolo á una fórmula :

$$\text{Número de caballos de fuerza} = \frac{\text{Gasto en pies cúb por minuto} \times \text{altura vertical en pies} \times \text{libras que pesa un pie cúb de agua.}}{33{,}000}$$

*Obs. del T.* — Para mayor claridad en la apreciación de ambos sistemas de medidas (inglés y métrico), recordamos aquí que por *potencia de un motor*, se entiende el producto del esfuerzo (peso, presión, etc.) por el camino recorrido en un segundo; la unidad es **el kilográmetro**, por segundo, ó sea la potencia necesaria para elevar un kilogramo á 1 metro de altura en un segundo. **Setenta y cinco kilográmetros** (*potencia necesaria para elevar 75 kilogms á un metro de altura en un seg) es lo que* **se llama caballo de fuerza ó caballo de vapor** (Force de cheval), unidad adoptada en el sistema métrico para medir la potencia de una máquina, caída de agua, etc., etc. En consecuencia, la fórmula que en este sistema corresponde á la del autor es :

$$\text{Potencia en caballos, de una caída de agua} = \frac{\text{gasto en mets cúb por seg} \times \text{caída en metros} \times \text{kilogs que pesa un metro cúb de agua}}{75}.$$

**Ej.:** Sea una cascada de 16 pies (4.87671 m); gasto 800 pies cúbs=22.65225 mets cúbs por minuto=.377537 met cúb por seg). ¿Cuántos caballos de fuerza desarrollará esta caída?

(*Obs. del T.* — Para que sean comparables las fórmulas en medida inglesa con las métricas, tomemos el agua á 4° C; el pie cúbico á esta temperatura pesa 62.4245 libras y el met cúb 1,000 kilogs.)

Por la fórmula en medidas inglesas tenemos entonces :

$$\frac{800 \times 16 \times 62.4245}{33{,}000} = \frac{799{,}033.6}{33{,}000} = 24.21 \text{ caballos (Horse-power),}$$

y por la dada en sistema métrico :

$$\frac{.377537 \times 4.87671 \times 1{,}000}{75} = \frac{1840.86}{75} = 24.5484 \text{ (caballo métrico).}$$

Si multiplicamos este número de caballos obtenido por .9863, relación del caballo de fuerza, inglés al francés, obtendremos el mismo número suministrado por la primera fórmula. Ponemos también aquí la relación que hay entre uno y otro caballo de fuerza :

1 caballo de fuerza (inglés) =1.01385 caballos de fuerza (francés);
1 — — (francés)= .986337 — — (inglés).

**Las ruedas hidráulicas** no producen toda la fuerza que tiene el agua, calculada por esta última regla. Las ruedas que reciben el agua por debajo producen sólo de $^1/_4$ á $^1/_3$; las que les llega por un costado ½; y las que la reciben por arriba de $^2/_3$ á $^3/_4$; las turbinas $^3/_4$ á .85 de la fuerza total del agua; según la disposición de cada rueda, y la perfección de la obra de mano. Aunque giran dentro de una cabidad ó cubierta de fundición apropiada y bien ajustada, los cajones con codos ó en ángulo comunican más potencia que los cajones centrales ó en dirección del radio. De la fuerza real recibida por una rueda, una parte se consume en rozamientos, etc., mientras que la restante realiza el trabajo útil de elevar agua, moler granos, aserrar, etc.

**Las observaciones hechas por el general Haupt**, en 1886, dieron los resultados siguientes con un **ariete hidráulico** pequeño. Carga del ariete= 2.686 m; diámetro del tubo alimentador=38 mm; longitud=4.572 m; diámetro del tubo de salida=19 mm; longitud=60.959 m. Altura vertical á la cual se elevó el agua=19.324 m. Golpes del ariete por minuto 170. Cantidad de agua que actuaba sobre el ariete=.0125846 mets cúbs=12.58 kg.

Cantidad elevada á la altura de 19.324 m por minuto=.0007865 mets cúbs

por min = .78765 kg por min. Por lo tanto la fuerza gastada por minuto = $12.58 \times \frac{\text{kilog agua}}{\text{metros}}$ 2.686 = 33.79 kilográmetros; y efecto útil = .7865 × 19.324 = 15.199 kilogmtros

Por consiguiente la relación es $\frac{15.199}{33.79} = .45$. (*N. del T.* — El ejemplo anterior es el que corresponde al del autor, en medidas inglesas.) Sin embargo, la fuerza real desarrollada por el ariete es mayor que ésta porque aquel tiene que vencer el rozamiento del agua en todo el tubo de salida *.

**Hallar la fuerza (en caballos) de una corriente.** Las ruedas hidráulicas de paletas planas ** (sin cajones), son movidas algunas veces por la sola fuerza de una corriente natural sin caída apreciable. En estos casos debemos buscar la carga *virtual ó teórica* que es la carga capaz de producir la velocidad que la rueda tiene realmente. La carga teórica puede encontrarse de una vez por la fórmula $\sqrt{2gh}$. Así, si un río tiene una velocidad de 1.5 m por segundo, tendremos $h = \frac{v^2}{2g} = \frac{(1.5)^2}{19.62} = .114$. Hallada así la carga, debemos buscar ahora la cantidad de agua que pasa en un minuto por una sección transversa de la corriente. Si suponemos que la parte *sumergida* de una paleta, cuando está vertical, es de 1.524 m de largo, y de .305 m de ancho, entonces el área de esta parte que recibe el empuje de la corriente es 1.524 × .305 = .465 met cuad. Por lo tanto, .465 m² × 1.5 m = = .697 m³ por segundo. Teniendo ahora los metros cúbs por segundo y la altura vertical ó carga, se halla el número de caballos de fuerza de la corriente sobre el área dada por la siguiente fórmula.

$$\text{Fuerza en caballos} = \frac{\text{Gasto en mets cúbs por seg .697} \times \text{altura ó carga en mets .114} \times \text{peso del met cúb de agua 1,000}}{75} = \frac{79.458}{75} = 1.058.$$

Pero en la práctica las *ruedas* no producen sino, más ó menos, $^1/_{10}$ parte de la fuerza de *la corriente*. La velocidad de la corriente no debe medirse en la superficie sino en la mitad de la profundidad á que llegan las paletas. Esto es más necesario aún en los ríos de poca profundidad, donde la parte sumergida de la paleta es una tracción considerable de aquélla.

**La fuerza de una corriente (para cualquier sección transversal dada) aumenta como los cubos de las velocidades**; porque, como hemos visto, la fuerza se encuentra multiplicando entre sí el peso del agua que pasa por la sección transversal y la carga teórica; y como este peso aumenta como la velocidad, y esta carga como el cuadrado de la velocidad, el producto de los dos (ó la fuerza) está como el cubo de la velocidad.

---

* **Un comité del « Franklin Institute »**, en 1830, dió .71 como el coeficiente de un ariete en el Colegio Girard, en el cual el diámetro del tubo alimentador era de 63 $^1/_2$ mm; su largo, 48.77 m; su caída, 4.26 m. Tubo de salida, 25 mm diámetro: 688.84 m de largo; elevación vertical, á la cual se llevó el agua, 28.346 m. No hay detalles del experimento. En Francia algunos arietes grandes dan un *efecto útil* de .60 á .65 de la fuerza total empleada. El ariete *hidráulico* es una máquina excelente en muchos casos, y se usa muchas veces para llenar depósitos de agua en las estaciones de ferrocarriles.

** **Esta especie de ruedas, para molinos flotantes, en Europa**, raras veces tienen un diámetro mayor de 4.572 m. Cualquiera que sea el diámetro, ellas pueden tener de 18 á 20 paletas, las paletas son de 2.44 m á 4.88 m de largo; y profundizan más ó menos 1/8 á 1/10 del diámetro de la rueda. No deben sumergirse en toda su profundidad, y no deben estar en la prolongación de los radios de la rueda; sino inclinados 30° aguas arriba, para producir su efecto total. Todas estas observaciones se refieren á ruedas que se mueven libremente en un canal ó río indefinido; como en el caso de un molino flotante, construído sobre una lancha, y anclado en el río, pero no en ruedas, para las cuales se represa el agua y obra por caída. No puede esperarse gran exactitud de las reglas sobre esta materia. La mejor velocidad para una rueda es más ó menos .4 de la que tiene la corriente.

# DRAGADO

El dragado es hecho, generalmente, por contratistas diestros que poseen las máquinas necesarias, lanchas, etc., y que tienen este trabajo como una especialidad. Es necesario estipular si el material ha de medirse en el lugar donde esté antes de excavarlo ó después de excavado y depositado en la lancha; porque en este último caso hace más volumen. En las extensas excavaciones hechas para profundizar el río San Lorenzo, en el lago de San Pedro, se halló por término medio que, un metro cúbico de fango duro antes de excavarse, equivale á 1.4 m cúb puesto en la lancha, ó bien 1 en la lancha = .715 antes de excavar. También debe estipularse si la extracción de piedras grandes, árboles sumergidos, etc., constituye un trabajo extra ó no. Muchas veces es necesario aserrarlas ó romperlas debajo del agua.

El costo por metro cúbico de dragado varía mucho con la profundidad del agua, la cantidad y calidad del material, la distancia á la cual ha de llevarse éste; si la descarga se hace de una vez con la máquina por medio de canales colocados al lado de ella, ó si se hace en lanchas que puedan llevarse á una pequeña distancia por medio de remos, ó á una distancia mayor por remolcadores de vapor; si se le puede arrojar en aguas profundas por una puerta falsa practicada en el fondo de la lancha, ó si debe palearse de las lanchas para aguas **de poco fondo** (de 7 á 13 centavos por $m^3$), ó echarse á tierra (quizás de 10 á 17 ó 34 centavos por palear solamente, ó palear y transportar en carretillas de mano, según el caso) si hay que gastar mucho tiempo en mover á menudo la máquina hacia adelante como cuando la excavación es angosta y de poca profundidad, por ejem para ahondar un canal, etc.; si hay que levantar muchas piedras grandes ó árboles sumergidos ó que haya interrupciones por causa del oleaje en tiempo de tempestad; si se puede obtener fácilmente carbón ó leña para las máquinas, etc.

Estas consideraciones pueden hacer el costo por m cúb 2 ó 4 veces mayor. El costo real para profundizar el canal de navegación á través del lago San Pedro, á 5.5 m de su profundidad original de 3.3 m y por varias millas de fango moderadamente duro, fué de 18 centavos antes de excavar y 13 centavos en las lanchas; incluyendo la traslación del material á una distancia de 304 m más ó menos por remolcadores de vapor vaciándolo en agua profunda. Esto comprende reparación de instrumentos y material de toda clase; pero sin contar la utilidad. El caso era favorable. Cuando los tobos trabajan en aguas hondas no se llenan tan bien como en aguas de poca profundidad, porque tienen un movimiento más vertical y, por consiguiente, no se arrastran bien á una distancia grande del fondo. Esta es una de las causas por qué el dragado en aguas profundas es más costoso, y además el tener que levantarlo á una altura mayor.

Tal vez la tabla que sigue sea tolerablemente aproximada para grandes trabajos en fangos ordinarios, arena ó granzón, suponiendo los instrumentos pagados por la Compañía, y que el precio de jornal ordinario sea de un dólar.

**Tabla del costo real del dragado en grande escala, incluyendo la echada del material entre las lanchas al costado de la draga, ó en canales laterales á bordo. — Trabajo ordinario $ 1 por día. Se incluye la reparación de los útiles, pero no la utilidad del contratista.** (*N. del T.* — La tabla que sigue es equivalente á la del autor en medidas inglesas.)

| Profundidad en metros. | Centavos por metro cúbico antes de excavar. | Centavos por metro cúbico en la lancha. | Profundidad en metros. | Centavos por metro cúbico antes de excavar. | Centavos por metro cúbico en la lancha. |
|---|---|---|---|---|---|
| Menos de 3 m | 11 | 7.8 | 7.5 á 9 | 23.6 | 16.9 |
| — 3 á 4.5 | 12.8 | 9.1 | 9 á 10.5 | 32.7 | 28.4 |
| — 4.5 á 6 | 14.5 | 10.4 | 10.5 á 12 | 45.5 | 32.5 |
| — 6 á 7.5 | 18.2 | 13 | | | |

Para remolcar las lanchas con vapores á una distancia de 400 m y dejar caer el fango en aguas profundas, agréguese 5 centavos por metro cúb; á 800 m, como 8 centavos; por 1,600 m, como 13 centavos. Agréguese la utilidad del contratista. El trabajo en pequeña escala es menos ventajoso y por consiguiente debe hacerse un aumento en estos precios. También si el contratista mismo suministra la draga y accesorios, debe hacerse otra adición al precio. Es evidente que esta materia no admite gran exactitud. Los trabajos en pequeña escala, aunque el material sea favorable, pero en puntos no convenientes, pueden fácilmente costar dos ó tres veces más que el anterior, y en material muy duro, como granzón y greda compactados, puede costar su dragado cuatro ó cinco veces más. El costo de remolcar, sin embargo, es lo mismo siendo los salarios iguales en ambos casos.

El costo de las dragas, y remolcadores depende de su capacidad, solidez en la construcción, etc.; y además de la comodidad á bordo para vivir la tripulación. Cuando las excavaciones se hacen en agua salada, los fondos de las dragas, lo mismo que los de las lanchas, deben forrarse con cobre para protegerlas contra la acción corrosiva de aquélla; y si algunas veces están expuestas á oleajes, deben ser de construcción más fuerte. Las máquinas de más potencia usadas en el río San Lorenzo costaron más ó menos $45,000 cada una y mudaron en 10 horas de trabajo una cantidad media de 1376 $m^3$ medidos antes de excavar, ó 1927 $m^3$ en las lanchas. Sin embargo, las máquinas buenas, capaces de ejecutar igual trabajo, pueden construirse por $25,000 ó 30,000, más ó menos. Para trasladar esta cantitad de material excavado á una distancia de 800 á 1,600 m, se requieren dos remolcadores de vapor de un costo de $6,000 á $10,000, más ó menos cada uno; y de 4 á 6 lanchas (llenando unas mientras las otras están llevando el material) que carguen de 22 á 44 $m^3$ cada una, y que cuestan de $800 á 1,500 cada una en la fábrica. Las lanchas con dos tolvas son las mejores. Una draga como la descrita aquí, necesitaría, por lo menos, de ocho á diez hombres de tripulación, incluyendo capitán, ingeniero, fogonero y cocinero. Cada remolcador, cuatro ó cinco hombres, y cada lancha, dos. El ingeniero debería ser herrero al mismo tiempo, ó agregar un herrero. En ciertos casos pudiera necesitarse médico, contador, ingeniero auxiliar, etc.

Las dragas se construyen á menudo según el principio del « Excavador Yankee », con una sola tolva de .75 á 1.5 $m^3$. Casco de 7 á 18 m, más ó menos. Calado, 1 m. Cilindro de 18 á 20 cm de diámetro. Carrera de émbolo de 38 á 45 cm : presión de vapor ordinario de 3.5 á 6 kg por cm cuad, según la condición del material. Costo, $8,000 á 12,000. Esta draga extraerá por día de trabajo medio (10 horas) de 150 á 400 $m^3$ de material, medidos antes de la extracción, ó de 200 á 530 $m^3$ puesto en las lanchas, según la profundidad, clase de material, etc. Se requieren por todo de 5 á 7 hombres á bordo, incluyendo el cocinero. Gastos de carbón = ½, ó 1 tonelada por día. Las piedras grandes, de tamaño regular, y los árboles sumergidos pueden sacarse por medio del cucharón (*dipper*) *.

Cuando el material es duro y compacto, los tobos de las dragas deben estar provistos de dientes fuertes de acero que sobresalgan de su arista cortante. Al encontrar esta clase de material, algunas veces se aplican todos estos tobos alternativamente. Si se arreglan los tobos de tal manera que efectúen la excavación á pocos metros delante del casco de la draga, pueden excavarse tiras largas de tierra seca, haciendo la máquina de este modo su propio canal.

El trabajo de un día será en este caso, más ó menos, la mitad del trabajo en terreno húmedo.

En trabajos pequeños, las **dragas maniobradas por dos ó más caballos**, en vez de vapor, dan buen resultado si el material que se excava es blando, ó aun si el material es moderadamente duro, si se reduce el número y tamaño de los tobos. Una máquina de dos caballos puede levantar de 40 á 80 $m^3$ de fango ordinario, medido antes de excavar, ó 50 á 100 puesto en la lancha, por día, y de una profundidad de 3.5 á 4.5 m.

El material blando, á una profundidad moderada, puede sacarse en pequeñas cantidades por el método lento y costoso de **draga de mano**.

Ésta se compone simplemente de un saco B, hecho de lona ó cuero y provisto

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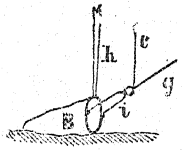
* El autor ha visto casos en que hubiera sido de gran utilidad la adición á la draga de una sierra circular para aserrar los trozos de madera sumergidos en aguas profundas, maniobrada al vapor y adaptable á diferentes profundidades; su precio sería $ 500 más ó menos.

en su boca de un anillo de hierro, el cual, en su parte inferior, tiene un filo cortante. Contiene un mango fijo *h* y uno giratorio *i*. El modo de usarlo es el siguiente : Un hombre introduce el saco dentro del fango por medio del mango *h*,




y otro hombre lo arrastra con la cuerda *g*, y cuando está lleno otro lo levanta con la cuerda *c* y lo vacía.

Si el saco es grande puede emplearse un cabrestante para izarlo. Los hombres pueden trabajar á bordo de una lancha ó de una balsa anclada convenientemente. Puede emplearse también una especie de cuchara de metal con un mango largo, de la forma de una azada honda y concava, la cual puede ser aplicada por un hombre solo, ó un hombre puede guiar una cuchara, y por medio de un caballo que marche á bordo, alrededor de un círculo, comunicarle movimiento hacia adelante y hacia atrás.

**El peso de un metro cúbico** de fango excavado mojado, arena pura ó granzón, es, por término medio, de 1 $^{3}/_{4}$ toneladas más ó menos; el granzón con lodo=2 ton; la arena pura y el granzón se dragan fácilmente; también las capas de conchas. La greda excavada mojada se desliza por un canal de tablas de una inclinación de 5 en 1, ó de 3 á 1, según se encuentre libre de arena, etc.; pero, la arena mojada ó granzón, no corre aun con una inclinación de 3 en 1, sin una corriente de agua que le ayude á deslizarse, pues de cualquier otro modo hay que empujarla mucho.

# CIMENTACIONES

Esta importante materia podría llenar un volumen. Sólo tenemos espacio para algunas apreciaciones generales, dejando al que estudia que determine hasta dónde pueden ser aplicables en un caso dado. En casos ordinarios, tales como para alcantarillas, muros, etc.; si las excavaciones, pozos, etc., hechos, en las inmediaciones, no han probado que el terreno permite una excavación profunda, será en general suficiente precaución, después de haber excavado y nivelado á una profundidad de 1 á 1.5 m, probar el terreno por medio de una vara de hierro ó un taladro, ó cavando agujeros en diversos puntos á la profundidad de 1 á 2 m (según el peso de la construcción que se pretenda hacer) para averiguar si el terreno continúa firme hasta aquella profundidad. Si así es, rara vez habrá peligro en proceder inmediatamente á construir la mampostería; pues una capa de terreno sólido de 1 á 1.5 m de espesor será firme para casi todos los edificios ordinarios, aun en el caso de que aquélla se apoyara en una capa mucho más blanda. Sin embargo, si la capa firme superior está expuesta á corrientes de agua, como en el caso del estribo de puente sobre un río, deben tomarse precauciones á fin de preservarlo del desgaste gradual por la corriente, ó de que se desmorone ó derrumbe por innundaciones fuertes, especialmente cuando arrastran grandes masas de hielo, árboles y otras materias flotantes. Éstas son detenidas á veces por las pilas y se acumulan de tal modo que forman diques que se extienden hasta el fondo de la corriente, creando con esto un aumento de velocidad y de acción destructora muy peligrosos, tanto para la estabilidad del cimiento como para la de la construcción. Cuando se necesita hacer la prueba á una profundidad considerable, será necesario encajar un tubo de hierro forjado ó fundido para impedir que la tierra caiga en el agujero sin terminar. Este tubo puede ser formado de trozos cortos unidos por empates de tornillo para manejarlos y dirigirlos mejor, y la tierra que se encuentra

en su interior puede removerse por medio de una cuchara pequeña con mango largo *.

**Taladrando en terrenos comunes** ó arcillosos pueden hacerse hasta 30 m de profundidad en un día ó dos por medio de un taladro de madera ordinario de 38 mm de diámetro, con palanca de un metro movida por 2 ó 4 hombres. Esto hará subir muestras del terreno.

Al empezar la mampostería deben, por supuesto, colocarse las piedras más grandes en la parte inferior de la fosa, de manera que se distribuya igualmente la presión lo más posible, y se debe tener cuidado de colocarlas sólidamente en el terreno á fin de que no haya tendencia á oscilar. Algunas de las capas subsiguientes por lo menos, deben ser también de piedras grandes colocadas de manera que sus junturas *no* se correspondan con las de abajo. Los huecos que quedan en las fosas deben rellenarse con tierra tan pronto como lo permita el trabajo de mampostería, á fin de evitar que la lluvia perjudique la argamasa y reblandezca los cimientos. Será conveniente apretar un poco la tierra á proporción que se deposita.

Si las pruebas demuestran que el terreno (no expuesto á corrientes de agua) es demasiado blando para soportar la mampostería, se harán los fosos considerablemente más anchos y profundos y se llenarán luego en toda su extensión (y á una profundidad como de 1 á 2 m ó más (según el peso que deban sostener) con capas pisadas, comprimidas de arena, cascajo ó piedras machacadas, ó con concreto en el cual haya una buena proporción de cemento. Sobre este depósito la mampostería se sostendrá bien. La práctica común, seguida á veces, de colocar planchas ó plataformas de madera en los cimientos para fabricar sobre ellos, es muy mala, porque si las planchas no se mantienen húmedas constantemente, se destruirán en pocos años y producirán grietas y hundimientos en la mampostería.

Algunas de las partes del acueducto de ladrillo ** que suple de agua á Boston presentaron muchos inconvenientes en los lugares en que los fosos pasaban por arena movediza y otros materiales traicioneros. Se empleó el concreto; pero la arena movediza mojada se mezclaba con él y lo destruía. Las armazones de madera, etc., tampoco dieron resultado, y finalmente se venció la dificultad depositando simplemente en los fosos como 60 cm de profundidad de cascajo grueso ***. La arena ó el cascajo, *cuando se evita que se esparza hacia los lados*, constituyen uno de los mejores cimientos. A fin de impedir este esparcimiento, el área donde se va á fabricar debe ser rodeada por un muro ó por estacas cuadradas colocadas, tan unidas, que se toquen unas con otras, y en casos de menos importancia sólo por tablas delgadas. Pero generalmente bastará hacer los fosos y rellenarlos con arena ó cascajo en capas (mejor si están húmedas), teniendo cuidado de compactarlas bien contra los lados del foso.

Los grandes pesos producen algún descenso (un pequeño hundimiento), como sucede en todas las fundaciones, con excepción de las que se hacen en roca. Si la construcción es muy pesada adóptese el sistema de estacas, etc. Véase « Empa-rrillado ».

**Cuando un terreno inseguro descansa sobre uno firme,** pero á tal profundidad que la excavación de los fosos (los cuales evidentemente deben ser más anchos y más profundos) se hace demasiado difícil y costosa, especialmente cuando (como generalmente sucede en estos casos) se filtra el agua rápidamente por las capas adyacentes, tenemos que recurrir á las estacas. Al hacer fosos para cimientos profundos en **arcilla húmeda**, debemos recordar que esta materia, siendo blanda, tiene tendencia á comprimirse hasta cierto punto en todas direcciones. Esto hace que haga barriga, es decir, se combe hacia dentro en los lados, y hacia arriba en el fondo. Las excavaciones para túneles ó para pozos verticales, á menudo se estrechan por todas partes, y se contraen mucho antes que puedan ser revestidas; por consiguiente, deben hacérselas más anchas de lo que sería necesario sin este motivo. La base de las excavaciones de canales y caminos de hierro sobre arcilla húmeda, se levanta generalmente hacia arriba por

* Las cavernas subterráneas en regiones de piedra caliza son fuentes de inconvenientes contra las cuales es difícil adoptar precauciones.

** El acueducto de Cochituate edificado de 1846-1848 : es ovalado de 1.93 m por 1.82 m en semicírculo invertido.

*** Smeaton menciona un puente de piedra construído sobre un lecho natural de arena de cerca de 60 cm de espesor solamente, colocado sobre un fango profundo tan blando, que una vara de hierro de 12 m de largo se hundió toda por su propio peso. Una de las pilas se hundió mientras echaban los arcos y fué repuesta por Smeaton. A pesar de este desgraciado precedente, este ejemplo demuestra el poder de resistencia d una capa gruesa de arena bien compacta.

el peso de los lados. La **arcilla seca** absorbe rápidamente la humedad y se hincha, produciendo efectos parecidos á los ya indicados. Su expansión produce grandes presiones, de tal modo que los muros que sostienen arcilla comprimida estarán en peligro de combarse si la arcilla se humedece. Es una substancia muy traicionera para trabajar en ella. Sobre cimientos de concreto, véase « Concreto ».

**Respecto al mayor peso** que se puede confiar con seguridad á fundaciones de tierra, Rankine aconseja no excederse de 10 á 16 toneladas por metro cuadrado; pero la experiencia prueba que en cascajo bueno y compacto, en arena ó marga, y á una profundidad fuera de la influencia atmosférica, puede llegarse sin riesgo, de 20 á 30 toneladas, y hasta de 40 á 60 si se conceden algunos centímetros de descenso para el asiento de la mampostería, como puede hacerse á menudo en edificios aislados, exentos de vibraciones. Pasarán años antes de que la construcción se asiente enteramente.

La arcilla pura, especialmente si es húmeda, es más susceptible de compresión y no se la debe cargar con más de 10 á 16 toneladas por $m^2$, según el caso. Todo cimiento en tierra cede algo. La **igualdad de presión** es un punto esencial y al cual debemos aspirar. Las vibraciones aumentan los hundimientos ó descensos y hacen que continúen por largo tiempo, especialmente en terrenos débiles, y se debe tener mucho cuidado en no exagerar la carga en dichos casos, aun cuando haya estacada. **Las cimentaciones en terrenos fangosos** descienden probablemente en años á razón de 7 á 30 cm por 10 toneladas (hasta 20 ton) por metro cuadrado, de carga *quieta*, si no está sobre estacas.

La fig. 2 enseña un modo fácil de obtener un cimiento en ciertos casos. Es el « **pierre perdue** » de los franceses, y en inglés el « **random stone** » ó « **riprap** ».

Es simplemente un depósito de piedras angulares de cantera que se arrojan al agua haciendo que queden las más grandes arriba á fin de que resistan el choque de las aguas, del hielo, y árboles flotantes, etc. Una parte del interior puede ser de piedras pequeñas, ripios de cantera, con algún cascajo, arena, arcilla, etc. Cuando la base es de roca imperfecta este procedimiento economiza el gasto de aplanarla para recibir la mampostería. Desde .60 á 1 m debajo de la superficie del agua las piedras deben ser colocadas á la mano de modo que queden juntas y firmes. Deben colocarse pequeños fragmentos de piedra entre las grandes para que el bloque sea más uniforme y menos propenso á ser removido. Se usan á veces lañas, grapas, ó cadenas de hierro para unir algunas de las grandes piedras y darles mayor estabilidad. **Este sistema, sin embargo, es propenso á hundimientos.**

**Si la base del cimiento es floja ó deleznable hasta el punto de que puede ser arrastrada ó socavada por inundaciones,** puede además protegérsela, como se ve en la fig. 2, por medio de una cubierta de la misma clase de piedras como en *c*, que se extienda completamente alrededor de la construcción. O bien el montón principal de piedras puede extenderse como lo indica la línea

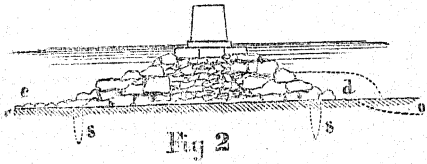


Fig 2

de puntos *d*, de manera que si la parte inferior fuese socavada como lo indica la línea de puntos *o*, las piedras en *d* caerán en la cavidad impidiendo así mayores daños. Se pueden colocar estacas delgadas como precaución adicional. Para mayor seguridad debe dragarse el lecho del río en todo el espacio que ha de ser cubierto por el depósito principal, como se ve en la fig. 3, y á tan gran profundidad que esté fuera del alcance de la acción corrosiva. Esta excavación debe ser rellenada también con piedras. Estas fundaciones, evidentemente, se adaptan más á aguas tranquilas. La mampostería debe descansar sobre una fuerte plataforma.

Grandes depósitos de piedra, como se ven en estas dos figuras, aumentan muchísimo la velocidad y la acción de la corriente alrededor de ellos, especialmente en las inundaciones, á menos que el fondo de cada lado del depósito sea excavado

de tal modo que el área primitiva del agua no se reduzca. Si el fondo es engañoso, esto debe hacerse antes de depositar las piedras de protección *c*, fig. 2.

Son muy necesarios en esta materia el buen juicio y la experiencia, como en toda otra que se relacione con la ingeniería. El mero estudio no nos preservará de constantes errores. La teoría y la práctica deben ayudarse mutuamente.

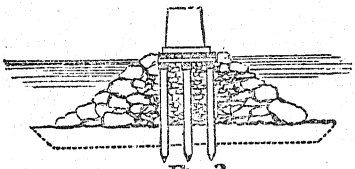


Fig 3

La fig. 3 indica otro método simple, muy eficaz, y cuando no se opone demasiado obstáculo al movimiento de la corriente ó al escape de las aguas en tiempos de inundaciones. En este caso, las estacas se clavan primero en el fondo del río para que sirvan de soporte á la pila; entonces se depositan las piedras para apoyar y proteger las estacas é impedir que se doblen bajo el peso que sostienen, y defenderlas contra los choques de los cuerpos flotantes. Luego que se hayan cortado las estacas á un mismo nivel, se colocará sobre ellas una fuerte plataforma de madera que servirá de base para la mampostería. La parte superior de la plataforma no debe tener menos de 30 á 40 cm debajo de las más bajas aguas para impedir que se deteriore. La estaca de tornillo de hierro de Mitchell ó las estacas huecas de hierro fundido se deben usar en lugar de las de madera.

La fig. 4 enseña un método muy conveniente para construir cimientos en el agua, por medio de una **caja de madera, AA, sin fondo**. Se construye con trozos de madera cuadrados, *escopleados en sus intersecciones* como en la fig. 5. Cada escopleadura debe ser ¼ del espesor de la madera. De este modo cada viga está sostenida en toda su extensión por la que está debajo, y resiste el empuje en ambas direcciones. Se le atraviesan también pernos en las intersecciones, por lo menos en los bordes de la caja, á fin de evitar que una parte sea arrastrada y separada de la otra. La caja se divide, pues, en celdas cuadradas ó rectangulares de .60 á 1.20, ó 1.5 m en cada lado, de acuerdo con las exigencias del caso. Las divisiones de las celdas se unen de la misma manera que las de los bordes de la caja, y por consiguiente forman, como las últimas, sólidos costados de madera.

La caja se armará á flote en el mismo sitio, en cualquier punto conveniente, y cuando esté terminada se llevará á su lugar, donde se anclará y hundirá cuidadosamente, depositando piedras en las celdas. Para esto se les pondrá fondo como se ve en *cc*. Estos fondos ó plataformas deben colocarse un poco más arriba del borde inferior de las celdas, á fin de no impedir que la caja penetre en el terreno, y de este modo llegue á fijarse perfectamente sobre el suelo.

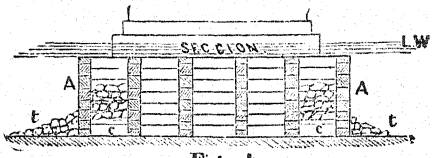
Después de hundir la caja se llenarán todas las celdas con piedras. Se agregará ó no, según el caso, una fuerte plataforma superior y también una defensa de piedras regadas para evitar el socavamiento que produce la corriente. Si los costados están expuestos á la abrasión ó desgaste producido por el hielo, etc., deben ser cubiertos todos ó en parte por tablones de madera ó por planchas de hierro y los ángulos deben ser guarnecidos por medio de láminas de hierro. En aguas profundas, el cimiento puede fabricarse en parte de piedras regadas, como en las figs. 2 y 3, y sobre éstas se hundirá una caja, cuya parte superior debe quedar como á .60 m bajo las aguas más bajas, sirviendo de base para la mampostería. Esto ofrece más seguridad que las piedras regadas solamente.

**En base ó fondo de roca desigual** es necesario ensamblar la parte inferior de la caja á fin de que calce en la roca, y la caja debe ser sumergida primero por medio de una plataforma cargada que se colocará sobre ella, ó llenando algunas de las celdas hasta que sus planchas inferiores se encuentren á una pequeña distancia del fondo. Conservándola así en una posición horizontal, se arrojarán piedras pequeñas en las celdas, las cuales, pasando á través de ellas, formarán una base igual donde podrá descansar la jaula. Entonces se llenarán

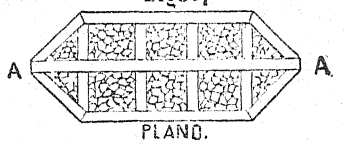
las celdas y se depositarán piedras grandes (*rip-rap*) alrededor de la caja para evitar que las pequeñas se salgan fuera de su sitio.

Se puede construir **una jaula que sólo tenga una hilera exterior de celdas**, y la cámara ó recinto interior puede llenarse de concreto debajo del agua. La mampostería puede descansar entonces sobre el concreto solamente. Si la jaula se apoya sobre cimiento de piedra picada, los intersticios de estas piedras deben rellenarse con piedras pequeñas ó cascajo para recibir entonces el concreto de la cámara.

**Ó se puede sumergir una jaula como la de la fig. 4**, clavar las estacas en las celdas y luego llenar éstas de piedra picada ó concreto. La mampostería descansará entonces solamente sobre las estacas, las cuales á su vez estarán pro-




Figs 4.



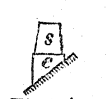
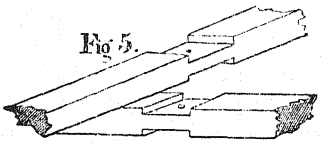




Fig. 5 *a*.

Fig 5.



tegidas por la jaula. Si el fondo ó suelo es susceptible de ser socavado, se colocarán estacas ó piedras sueltas alrededor de la base de la jaula.

**Se debe evitar por todos los medios** una jaula como la que representa *e* en la fig. 5½, con un lado mucho más alto que el otro, *si la construcción s* ha de descansar sobre el maderaje de la jaula ó cajón en lugar de apoyarse sobre las estacas, ó sobre concreto independiente del maderaje; pues que la parte alta de la jaula comprimiría más bajo su peso que la parte baja y esto haría que la construcción se inclinase y se desplomase.

Una jaula terminada por caras ya sea rectas ó circulares con sólo una hilera exterior de celdas para echar el concreto ó argamasa, puede usarse como un ataguía. (Véase « Ataguía », pág. 624.) Las ensambladuras exteriores de las vigas deben calafatearse bien y se debe tener cuidado de que el agua no se filtre por debajo, para lo cual se emplearán estacas, cascajo, etc.

**En el puente de hierro fundido que atraviesa el Schuylkill en la calle Chestnut, en Filadelfia**, el señor ingeniero Strickland Kneass nos presenta un ejemplo sorprendente de fundaciones con cajas. El estribo central descansa en una jaula de figura oblonga octógona de 9.45 × 26.53 m en la base, 7.32 × 24.40 m en la parte superior, y (con su plataforma) 9.15 m de alto. Las piezas de madera son de pino amarillo, de 30 cm en cuadro y construídas como en la fig. 5. Las maderas inferiores se labraron cuidadosamente á fin de adaptarlas á las irregularidades de la roca, medianamente á nivel, sobre la cual descansa. Ésta se encontró (después de haber dragado 2.4 m de profundidad de arena) de la manera acostumbrada. Luego se amarró sobre el lugar una plataforma de madera grande y flotante, compuesta de vigas que correspondían en posición con todas las de la jaula ó cajón que se ejecuta, tanto longitudinal como trans-

versalmente y se echaron las sondas á lo largo de esas líneas de vigas. Casi todas las celdas tienen como de 1 ¼ á 1.20 en sus lados, algunas de ellas tienen plataformas al nivel de la segunda serie para recibir piedras que hundan la jaula, y las otras están abiertas hacia abajo.

La jaula se construyó en el agua y se conservó flotante durante su construcción y, con la parte superior, no terminada, constantemente á flor de agua para llenarla gradualmente con piedras á proporción que se le agregaban vigas. La piedra que se empleó con este objeto solamente alcanzó á 300 toneladas. Al llevar la jaula á su posición y asegurarla, se le agregaron 150 toneladas para hundirla. Después se llenaron todas las celdas de piedra seca y escorias ordinarias de carbón, haciendo un total de 1,666 toneladas. Una plataforma de madera de vigas de 30 × 40 cm, cubría el todo y su parte superior estaba á 75 cm debajo del nivel más bajo de las aguas. El solo estribo que descansa sobre esta jaula pesa 3255 toneladas, y durante su construcción comprimió la caja 16 cm. El peso de la construcción que descansa sobre el estribo se puede calcular aproximadamente en 1000 toneladas más.

**Un cajón ordinario para construcciones bajo el agua es simplemente un lanchón fuerte, ó una caja sin tapa,** cuyas caras pueden abrirse ó separarse de su parte inferior, si se quiere. Se construye en tierra y luego se echa al agua. La mampostería puede ser construída en todo ó en parte, al estar á flote, y el todo, después de haber sido remolcado y fijado en su lugar, se sumerge hasta el fondo del río, y se apoya sobre un cimiento que ha sido preparado previamente por medio de estacas si es necesario, ó meramente rebajando y nivelando la superficie natural, etc.

El fondo del cajón constituye una fuerte plataforma de madera sobre la cual descansa la mampostería, y está preparada de tal modo, que después que ha sido sumergido se pueden desunir y quitarse sus lados para aplicarlos en la construcción de otra pila ó estribo. Esta separación puede efectuarse por medio del mecanismo indicado en la fig. 6, en que *PPw* es la base del cajón al cual están

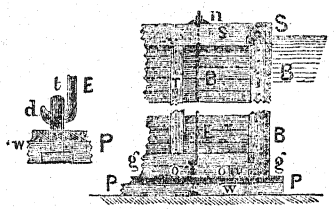



**Fig. 6.**

firmemente adheridas, á intervalos, fuertes argollas de hierro *t*, en las cuales se insertan los garfios de que están provistos los extremos inferiores de los pernos, E *n* que llegan hasta las vigas *s* superiores de la jaula, donde están fijos por medio de tuercas de tornillo *n*. Aflojando las tuercas, los garfios se separan de las argollas *t*, y de este modo puede quitárseles las piezas laterales del fondo. Los garfios y argollas se colocan generalmente en la parte exterior del cajón y las tuercas de los tornillos están sostenidas por los extremos prolongados de las piezas transversales *tt*, fig. 9. La impropia posición que les damos en nuestra figura fué debida simplemente á la conveniencia de ilustrar el principio. Sería necesario algunas veces tener un lado más sobresaliente que otros con el objeto de que el cajón flote libremente al separarse del estribo ya terminado, á menos que sea separado antes porque la mampostería se haya elevado ya á tal altura que haga innecesaria la precaución. La fig. 6 indica uno de los muchos modos de construir un cajón con caras formadas de postes verticales en las esquinas; piezas de tope y cabezales S en la parte superior y vigas de apoyo en la base, que descansan en la plataforma inferior *PPw;* postes intermedios T ensamblados á las vigas de tope y de apoyo, y todo cubierto exteriormente por uno ó dos espesores de tablón B, los cuales, así como la plataforma, deben ser calefateados para impedir que haga agua el cajón. Con el fin de contribuir á este objeto pueden clavarse también encurados por fuera. Donde más molestan las filtraciones es donde las caras del cajón se unen á la plataforma. En la parte superior de la plataforma está fuertemente fija una viga *oo* que se extiende alrededor de ella, justamente

del lado interior de la arista inferior de las caras del cajón. El objeto de esto es impedir que dichas caras sean comprimidas hacia adentro por la presión del agua. Los detalles de construcción variarán, naturalmente, según las exigencias del caso. A los cajones profundos es necesario aplicarles abrazaderas transversales interiores ó puntales *cc*, de una cara á otra, fig. 7, á fin de impedir que la presión del agua comprima las caras hacia adentro á medida que la balsa se sumerge gradualmente, á proporción que se construye la mampostería. A medida que la mampostería se eleva se quitan los puntales. Cuando el cajón es poco profundo sólo se necesitarán las abrazaderas superiores. Éstas sostendrán también una plataforma para los trabajadores y sus materiales. En cajones profundos, el entablado exterior puede, en parte, ser ejecutado gradualmente á proporción que la mampostería hace progresos, á fin de no estorbar á los albañiles.

Puede ser conveniente, algunas veces, hacer la mampostería hueca al principio, con tabiques interiores de poco espesor, para reforzarla, si es necesario, y rellenar el interior después de sumergir el cajón. En este caso, la mampostería descansa en la plataforma, formando así los lados del cajón; ó bien éstos se forman con una cubierta de hierro impermeable ó de madera de la forma del estribo ó pila que se ejecuta. Esta cubierta, limitada en la plataforma, viene á ser un molde en el cual se conforma la pila ó estribo, y se sumerge al mismo tiempo, llenándolo con concreto hidráulico. **Sobre cimentaciones de concreto,** véase « Concreto ».

**En suelos de roca** las vigas inferiores de la plataforma pueden labrarse de modo que se adapten á las irregularidades de la roca, como ya se ha dicho, ó puede emparejarse el fondo colocando primero grandes piedras alrededor del área sobre la cual debe descansar el cajón, y luego rellenar los intersticios con piedras pequeñas y cascajo, tanteando la profundidad por medio de la sonda; ó puede colocarse con cuidado en el agua una capa de concreto á nivel. Si la roca tiene grietas angostas y profundas, por medio de las cuales pueda escaparse el concreto, deben ser éstas cubiertas antes con tela encerada. Las campanas de buzo se usan á menudo con ventaja en dichas operaciones; pero cuando las rocas son muy irregulares será mejor recurrir á los cajones. Se aplican comúnmente válvulas de admisión de agua para sumergir el cajón. Si después de haberle sumergido fuese necesario elevarlo de nuevo, sólo se necesitará cerrar las válvulas y extraer el agua por medio de una bomba. Se pueden clavar estacas de guía y atarlas alrededor del cajón para efectuar su hundimiento verticalmente y en el lugar conveniente, ó puede hacerse descender por medio de tornillos sostenidos por un fuerte andamio provisional.

Suponiendo que los postes I, T, fig. 6, estén suficientemente sujetos como en *cc*, fig. 7, la siguiente tabla indicará el espesor de los tablones para diferentes distancias de un poste al otro para obtener un coeficiente de seguridad 6 contra la presión del agua á diferentes profundidades, y al mismo tiempo que no se encorve hacia adentro bajo dicha presión en una cantidad mayor de $^1/_{150}$ parte de la distancia á que se halla un poste de otro. Dicha tabla puede ser útil para cualquier otro problema análogo.

**Tabla de los espesores que han de tener los tablones de pino blanco para que no se encorven más de $^1/_{150}$ parte de su longitud horizontal bajo diferentes cargas de agua.** (Original.)

| Extensión en metros. | CARGA Ó ALTURA DEL AGUA EN METROS | | | | |
|---|---|---|---|---|---|
| | 12 | 9 | 6 | 3 | 1.5 |
| | Espesor en centímetros. | | | | |
| .90 | 9 | 7½ | 6½ | 5½ | 4½ |
| 1.20 | 11 | 10 | 8½ | 7 | 5½ |
| 1.80 | 17 | 15 | 13 | 10½ | 8 |
| 2.40 | 23 | 20 | 17½ | 13½ | 11 |
| 3.00 | 28½ | 25 | 22 | 18 | 14 |
| 3.60 | 34 | 31 | 27 | 21 | 17 |
| 4.50 | 42½ | 38 | 33 | 27 | 21 |
| 6.00 | 56 | 50 | 44 | 35 | 28 |

*N. del T.* — Esta es la tabla equivalente á la que trae el autor, pero apreciando sólo ½ cm en los espesores, pues eso es suficiente en la práctica.

**Las ataguías** (*coffer-dams*) son recintos de los cuales puede ser extraída el agua de manera que pueda hacerse el trabajo sin estorbo. En agua mansa y baja dará un resultado satisfactorio un simple terraplén bien construído con arcilla y cascajo, y cuando haya mucha corriente, sacos casi llenos de esas mismas materias, ó (según la profundidad) una hilera doble ó sencilla de estacas chatas ó de estacas cuadradas de grandes dimensiones clavadas de modo que se toquen una á otra, con sus extremos inferiores enterrados algunos pies dentro de la tierra y los superiores un poco más arriba del nivel de las altas aguas, y protegidas exteriormente por montones de tierra cascajosa ó lodazal (como en P, fig. 7) para impedir la filtración. Las estacas pueden ser de madera ó de hierro fundido de una forma fuerte.

**Está demostrada la suficiencia de un simple dique de tierra bien compactada** en aguas tranquilas, por los terraplenes y diques levantados en todos los países para impedir las inundaciones de los ríos en terrenos bajos. La altura media general de los diques que se extienden en un espacio de 1,026 kilómetros en el Misisipí, es de 1.83 m, sólo .90 m de ancho en la parte superior, é inclinados los lados de 1½ en 1. En las inundaciones el río sube cerca de .30 m ó menos en la parte superior, y frecuentemente rompe y se abre paso por en medio de ellos causando muchos perjuicios. Son demasiado débiles.

El sistema de una sola hilera de estacas cuadradas de 30 × 30 cm, clavadas juntas unas de otras, y resguardadas simplemente por un depósito exterior de terreno impenetrable, es muy eficaz, y con la adición de abrazaderas interiores transversales, semejantes á *cc*, fig. 7 (para impedir que se inclinen hacia adentro por la presión externa del agua y el relleno al extraer el agua con la bomba), se ha empleado con éxito en aguas de 6 á 7.5 m de profundidad, sin corriente suficiente para arrastrar el relleno. Las abrazaderas se intercalan sucesivamente á medida que se extrae el agua, comenzando, por supuesto, por las de arriba. Las puntas de estas abrazaderas deben rematar en las vigas longitudinales clavadas en las estacas con este objeto.

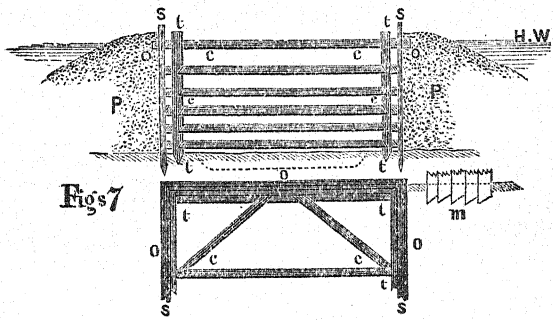
Hay otro método que consiste en una **caja fuerte** compuesta de postes fijos por sus extremos superior e inferior á piezas de madera que sirven de vigas de tope y de apoyo, forradas exteriormente con planchas de madera en contacto unas con otras, y bien calafateadas como en el cajón que indica la fig. 6, pero sin fondo. Entre los pares opuestos de postes, hay colocados fuertes puntales interiores como *cc*, fig. 7, que pasan de un lado al otro para impedir el hundimiento hacia adentro. Las series superiores de estos puntales generalmente sostienen una plataforma para los obreros, grúas, etc. Después de construída la jaula en tierra, se arrojará al agua, se colocará en su lugar definitivo, y se sumergirá amontonando piedras en una plataforma provisional que descansa sobre los puntales, habiendo preparado previamente para su recepción el fondo del río.

Para evitar la filtración por debajo de la caja ó jaula, se clavarán estacas de tablas á su alrededor, con las cabezas que sobresalgan del fondo, ó puede colocarse un depósito pequeño de escorias alrededor de ella, como se ve en los depósitos de piedras *tt*, fig. 4. O bien se clavará fuertemente alrededor, y un poquito hacia arriba de la arista inferior de la jaula, una tira ancha de encerado, arreglada de tal modo que pueda extenderse libremente en el fondo del río á una distancia de pocos metros alrededor de toda la parte exterior de la jaula; el relleno de escorias se pondrá sobre ella. Dichos encerados son también muy útiles en los casos en que el fondo del río es hasta cierto punto irregular, y no puede ser excavado sin grandes gastos, en cuyo caso la jaula no puede descansar perfectamente sobre él, y por consiguiente el agua filtraría ó correría libremente por debajo. Son adaptables especialmente á las rocas desiguales donde no es posible clavar estacas de madera. Sin embargo, puede depositarse sobre una roca lisa una capa impermeable, en cuyo caso el hundimiento de la jaula y las subsiguientes operaciones serán las mismas que sobre una capa natural. Estos medios son más ó menos aplicables en otros casos, los cuales no se mencionan especialmente para evitar repeticiones.

**La fig. 7 representa otra forma de ataguía** cuyas caras, en lugar de ser entabladas longitudinalmente como en el último ejemplo, están forradas con estacas de tablones, tablas verticales S, clavadas después de sumergir la jaula. Ésta es muy inferior á la última, debido á la facilidad con que puede filtrarse.

Se han usado con éxito en 5 m de agua con dimensiones de 10.27 × 24.40 m. En cada una de sus largas caras había 7 postes de 5.7 m de largo, 30 cm en cuadro,

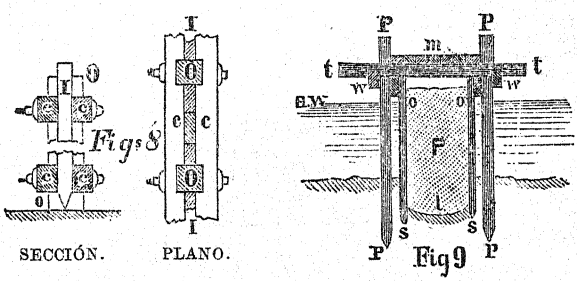
y á 3.88 m de distancia unos de otros. En cada par opuesto de éstos había escopleadas y sujetas por garfios de hierro abrazaderas de 30 cm en cuadro. La distancia entre las dos de encima era de .91 cm, disminuyendo gradualmente hasta .45 cm, que era la separación de las dos últimas. Así se hizo á causa del aumento de presión del agua al descender. En la parte exterior de los postes y opuestas á las abrazaderas extremas había vigas longitudinales atornilladas para soportar la presión exterior contra las estacas de los tablones *ss*. Otras piezas longitudinales *oo*




fijan las puntas de las estacas de tablas á la parte superior de la jaula, después que están clavadas. Las puntas inferiores se cortan en ángulos como en *m*, á fin de que al clavarse se unan y cierren bien el fondo.

Dichas estacas se clavarán de una manera mucho más regular y satisfactoria siguiendo la disposición adoptada en la fig. 8. En ella *oo* son los postes; *ee* son pares de piezas longitudinales escopleadas y ensambladas á los postes cerca de sus extremos superiores é inferiores, y en tantos puntos intermedios como se desee. Las estacas de tablas I están intercaladas entre éstas, y por tanto guiadas durante su descenso mucho más perfectamente que en la fig. 7.

Cuando la corriente es demasiado fuerte para permitir el uso del relleno exterior P, fig 7, se usa generalmente el tipo de caja de la fig. 9, en el cual ambos lados del relleno están protegidos contra el arrastre de las aguas. El espacio que debe




ocupar el dique está rodeado por dos hileras de estacas principales firmemente clavadas, *pp*, de las que depende principalmente la resistencia del aparato. Deben ser redondas. Al determinar el número de ellas debe recordarse que tienen que resistir la acción del hielo flotante ó choques accidentales de buques, etc.; contra esto, deben clavarse, además, estacas de defensa. Un poquito más abajo de las puntas superiores de las estacas principales están atornilladas dos piezas exteriores longitudinales *ww*, llamadas cintas, y opuestas á éstas dos interiores como se ve en la figura. Las exteriores sirven para sostener las vigas *tt* que unen cada par de estacas

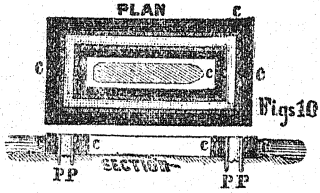
opuestas, haciéndolas inmovibles é impidiendo que se desunan con la presión del relleno P. Las interiores hacen el oficio de guías de las estacas *ss* mientras se encajan, después de lo cual las cabezas de las estacas de tablas se sujetan á ellas, En aguas profundas estas estacas deben ser muy gruesas, como de 30 cm en cuadro, para resistir la presión del relleno.

**Un pasamano, m**, se coloca á menudo sobre las piezas cruzadas *tt*, para el uso de los obreros al rodar el material, etc. El relleno se deposita en el agua en los espacios entre las estacas. Deben depositarse en capas compactadas, tan bien como se pueda, sin pandear las estacas, lo que ocasionaría la abertura de sus juntas.

El fondo del foso, donde se deposita el relleno, debe cavarse, como se ve en la figura, caso de que esté formado, como generalmente sucede, de materias sueltas, porosas, las cuales permitirían al agua filtrarse por debajo de él y de las estacas. Esta filtración por debajo del dique es á menudo fuente de muchos gastos y dificultades. El agua se abrirá paso prontamente á cualquier profundidad y á través de cualquier fondo pedragoso ó cascajoso que no esté mezclado con tierra. La arena es algo molesta también, y si se presenta una capa de cualquiera de estas dos especies que se extienda á gran profundidad, será un buen recurso apelar ó á una simple jaula, fig. 4, ó á cajones con estacas ó sin ellas, según las circunstancias. Pero si dicho cascajo, ó cualquiera otro material permeable ó movedizo, como barro blando, arena movediza, etc., se presenta en una capa de poco espesor, sobre arcilla resistente ú otra materia firme, se puede impedir la filtración, ó por lo menos reducirla mucho enterrando las estacas en este último á 60 ó 90 cm, y cavando el foso donde se va á echar el relleno á la misma profundidad. Es algunas veces mejor dragar toda la materia mala del espacio rodeado por el dique y á alguna distancia comenzar la construcción de éste.

Si la represa que indica la fig. 9 está provista (como debe estarlo) de abrazaderas transversales, como las *cc*, fig. 7, colocadas á través del recinto interior, el espesor ó ancho *oo* del relleno no es necesario que sea de más de 1.20 á 1.50 m para profundidades bajas, ó de 1.5 á 3 m para las grandes; porque su aplicación entonces es simplemente evitar la filtración. Pero si no hay abrazaderas debe hacerse más ancho, de modo que resista al volcamiento transversal, y entonces con un buen material de relleno puede adoptarse como regla empírica para dicho espesor $^3/_4$ partes de la profundidad vertical *o l* bajo la marea alta, excepto cuando ésta tenga menos de 1.20 m, en cuyo caso se hará dicho espesor de 1.20 m, á menos que se necesite más para el tráfico de los obreros, depósito de materiales, etc. Si la excavación para la mampostería penetra más profundamente que la masa del relleno, la represa debe ser más ancha, ó de otro modo caería dentro del pozo que ha sido excavado. **La tierra excavada** puede extraerse en baldes por medio de grúas, ó á mano por medio de andamios sucesivos. Las bombas pueden ser maniobradas á mano ó á vapor, según lo requiera el caso, lo mismo que las grúas que necesitan generalmente para descender el mortero, piedras, etc. Se debe contar siempre con que habrá más ó menos filtración, á pesar de todas las precauciones que se tomen.

Cuando una caja-dique ó ataguía está expuesta á corrientes violentas y á grandes peligros por el hielo, etc., se hace necesario adoptar el costoso sistema de la fig.



Figs 10.

Los dos rectángulos negros *cc* representan dos filas de jaulas ordinarias llenas de piedras y sumergidas, estando inscrita una en la otra con pocos metros de separación. Luego se clavarán estacas *pp* alrededor de las caras opuestas de las dos jaulas, y el relleno se depositará entre las dos, como se ve en la figura.

Donde la corriente no sea bastante fuerte para arrastrar el relleno, podemos, especialmente sobre roca, aislar el espacio donde se intenta construir, por simples cuadros formados con cajones, sumergidos por medio de piedras, y hacer el re-

lleno después de tomar precauciones, á fin de impedir que por la presión, el cascajo se introduzca por debajo de los cajones *.

Las figuras 10½ muestran el plano, visto lateralmente y la sección transversal en una escala de $^1/_{240}$, de un cajón colocado sobre roca á 2.4, ó 2.7 m de agua, empleado con éxito en el *Schuylkill Navigation*.

**Ataguías colocadas sobre roca.** Los postes *b*, como de 30 cm en cuadro, y á 3 m de distancia de centro á centro á lo largo de los costados del dique, y con 3 m en el espacio transversal de éste, sostienen dos cintas formadas de los largueros horizontales *ii*, en el interior de los cuales están las dos filas de estacas *ss*, conteniendo entre ellas una capa de relleno de cascajo con espesor de 2.1 m. Dos barras planas de hierro (*tt* de la sección transversal) unen los postes *bb* de

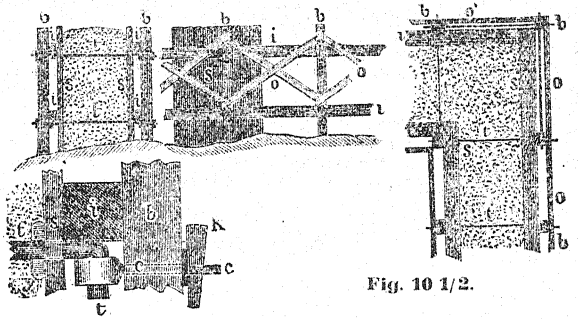


Fig. 10 1/2.

dos en dos. Estas barras son de 12 mm de grueso y 2.7 m de largo. Sus puntas encorvadas calzan en los huecos de los pernos *c* que atraviesan los postes *b*, y fijos en la parte exterior por clavijas (véase el diseño detallado). Entre las clavijas y *b* hay arandelas. En los ángulos del dique (véase el plano) hay tirantes adicionales. Una faja pequeña de paja como se ve en *y*, arrollada alrededor de las barras, justamente en la parte interior de las tablas, y mantenida en su lugar por el relleno, impide eficazmente la filtración, que es generalmente tan molesta. Los gruesos travesaños oblicuos *oo*, estaban simplemente sujetos á las caras exteriores de los postes *b*. No están indicados en la sección transversal. Este dique fué construído en tierra por secciones de 9 á 12 m de largo, las cuales fueron llevadas á flote á su lugar y rellenas con cascajo. El dique tenía compuertas, por medio de las cuales se introducía agua cuando era necesario, para impedir que la parte exterior excediese de 2.7 m. La longitud de los postes *oo* fué determinada de antemano por medio de cuidadosos sondeos.

**El anclaje de grandes cajones** es algunas veces difícil, particularmente en fuertes corrientes. A veces se necesita clavar grupos de estacas, ó sumergir temporalmente algunos cajones ordinarios, llenos de piedra, á los cuales se atan largas cuerdas de guía, por medio de las cuales se coloca en su posición. Frecuentemente se dejan sin sacar los diques después que el trabajo se ha hecho, si éste no queda en la vía de la navegación ó de modo que ofrezca obstáculos, pues que los materiales rara vez compensan el gasto de su remoción. Pero si se remueven, las estacas no deben arrancarse del suelo, sino recortarse cerca del fondo del río, porque si se arrancan, el agua que entra en los agujeros puede ablandar el terreno debajo de la mampostería. A menudo es conveniente clavar dos hileras de estacas desde la represa hasta la orilla para sostener un pasadizo para los obreros y aun para caballos y carros ó para rieles que faciliten la conducción de piedras grandes, etc.

**Estas ataguías pueden pasar á través de un suelo blando á uno firme**, gracias á su forma de cajón rectangular (y á veces circular) y sin fondo. Después de haberlos unido fuertemente y provisto de abrazaderas ó travesaños interiores temporales (para ser gradualmente removidos á proporción que se fabrica la mampostería) se dejan flotar en su lugar, y después de cargarlos de modo

* El cascajo ordinario, puro y limpio, es enteramente inútil para este objeto. — Se necesita una considerable proporción de tierra para impedir la filtración.

que descansen en el fondo blando, se sumergirán dragando el material blando del interior. Algunas veces es necesario agregar una carga adicional para vencer el frotamiento del terreno contra la parte exterior, ó puede hacerse necesario dragar también un poco el exterior. **Sobre roca** puede ser conveniente algunas veces taladrar hoyos bajo aguas profundas para meter las puntas de las estacas ó de barras de hierro, etc. Esto puede hacerse por medio de grandes barras de taladrar que trabajan dentro de un tubo de hierro ó conducto sumergido, como una guía para la barra, con su extremo inferior colocado en el punto donde se va á taladrar. Puede emplearse también una campana de buzo, ó **un cilindro** sin fondo de 10 á 30 cm de espesor suficientemente largo para que llegue á la superficie y provisto de una ancha orilla ó borde de tela alquitranada alrededor de su contorno inferior, que se cubrirá con cascajo para impedir la filtración; se podrá sumergir y extraer el agua con una bomba á fin de que pueda descender un obrero y trabajar al aire.

**Cuando las estacas** se clavan unidas como en la fig. 11, para impedir la filtración, para aislar la masa que constituye el relleno en una caja-dique, ó para cercar un pedazo de terreno blando ó arenoso, á fin de impedir que se esparza, ó si

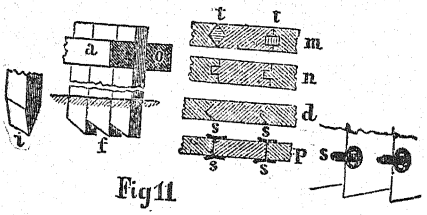


Fig 11

el suelo exterior que las rodea es socavado, etc., se llaman **estacas machihembras**. Son generalmente más delgadas que anchas, pero frecuentemente son cuadradas y tan largas como las estacas de sostén, y se llaman entonces estacas de cerca. A fin de poderlas clavar estrechamente juntas en el pie, se cortan oblicuamente como en *f*. Algunas veces, cuando se clavan en una roca por entre un terreno blando, sus puntas se labran además en filo como en *i*, de manera que estén un poco machacadas cuando lleguen á la roca, y de esta manera se adhieran mejor á su superficie. Sus cabezas se mantienen en fila mientras se clavan, por medio de una ó dos piezas longitudinales llamadas cintas. Estas cintas están sujetas por **estacas directrices** ó estacas de guía clavadas con anticipación en el alineamiento requerido y á alguna distancia con este objeto. Véase la figura 8.

**Un (dog iron) gancho, d,** de hierro redondo puede usarse también para mantener las puntas de las estacas unidas por las cabezas á las clavadas previamente, antes y después de clavarlas. Sus afiladas puntas *cc*, clavadas en las caras superiores de las cintas (como se ve en el plano), mantienen en su lugar la estaca descendente *o*. En *n*, *d*, *p*, fig. 11, se ven otros medios usados, algunas veces, para mantener en línea conveniente *las estacas*. En *p*, las letras *ss* indican pequeñas piezas de hierro bien atornilladas á las estacas un poquito más arriba de sus puntas inferiores, para que funcionen como guías. Se usan rara vez; *m* representa espigas

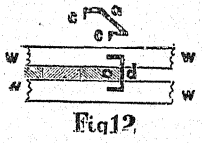
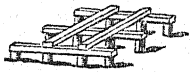


Fig 12

de madera *tt* que algunas veces se clavan entre las estacas después que éstas han sido clavadas, para ayudar ó impedir la filtración. En algunos casos se usan estacas de tablas ensambladas. Se cava un pozo primero á la profundidad á que han de ser enterradas, se colocan dentro las estacas, y entonces se rellena. Así se obtienen

juntas más cerradas que clavándolas. Se llaman estacas de cimiento cuando tienen que sostener pesos, sea enterrándolas completamente, ó en parte, como en la fig. 3. Son generalmente redondas, de 22 á 45 cm de diámetro en la parte superior, y deben ser derechas; pero no es necesario quitarles la corteza. El pino blanco, el abeto y hasta la cicuta son buenos para terrenos blandos, el pino amarillo, para terrenos más firmes y sólidos; la caoba, el olmo, etc., para los más compactos. Generalmente se entierran de .75 á 1.20 m de distancia unas de otras y de centro á centro; sin embargo su disposición depende de la clase de terreno y del peso que deben sostener. Es más económico, para suspender la maza ó martillo, emplear una rueda de escalones en vez de un torno, cuando el trabajo es hecho por hombres. Morin encontró que el trabajo de hombres trabajando 8 horas por día fué de .12 de caballo por hombre y por minuto con la rueda de escalones, y solamente de .08 por medio de cabria.

**Después que las estacas han sido enterradas** y sus cabezas cortadas á nivel cuidadosamente, si no están debajo del agua, se rellenan (en los casos de importancia, hasta el nivel de sus cabezas), los espacios que hay entre ellas, con cascajo bien apelmazado, fragmentos de piedra ó concreto, con el fin de comunicar



mayor resistencia al terreno que se encuentra entre las estacas. Serán atadas ó clavadas á las cabezas de éstas, como se ve en la figura, formando lo que se llama emparrillado, dos hileras de vigas gruesas de 20 á 30 cm en cuadro, según el peso que tengan que sostener. Sobre éste se clavará un tablado ó plataforma de tablas gruesas que sirva de soporte á la mampostería. Se pueden colocar juntas las vigas de la hilera superior del **emparrillado**. El espacio que queda debajo de la plataforma debe también ser macizado en casos de importancia con cascajo, fragmentos de piedra ó concreto. **Si es debajo del agua**, las estacas serán aserradas por un buzo ó por medio de una sierra circular movida por la máquina del martinete, y se omite el emparrillado. En lugar de dicho emparrillado puede fabricarse la mampostería al aire libre, en un cajón; y este se irá sumergiendo gradualmente, á proporción que se va llenando; o bien construir aquella en una fuerte plataforma que se hace descender sobre las estacas por medio de tornillos, á medida que progresa el trabajo. También se puede sumergir primero, enteramente debajo del agua, un cajón fuerte y llenarlo luego de concreto hasta cerca del nivel de las bajas aguas, dejando el cajón dentro del agua. Puede hacerse también del cajón una ataguía que se sumerja primero, y luego se extrae el agua con bombas. Si el terreno es susceptible de ser socavado por el agua alrededor de las estacas, como sucede en las pilas de los puentes, etc., se debe proteger por medio de estacas de tablas ensambladas, ó con piedras echadas, ó de ambos modos.

**El costo de los martinetes flotantes de vapor**, en Filadelfia, con lanchones de 7×15 m, de 45 cm de calado, con una máquina para clavar y otra (para economizar tiempo) para preparar la estaca que se va á clavar, con un martillo, de una tonelada, es como de $6,000, y con $500 más se obtendrá una sierra circular, para aserrar las estacas á cualquiera profundidad. Se necesita un maquinista, un fogonero y 4 ó 5 personas más; consume, más ó menos, media tonelada de carbón por día.

Por término medio se clavan de 15 á 20 estacas por día de 10 horas, enterrándolas á 6 m con cascajo, y aserrándolas luego. En fango, el doble; en tierra, la mitad.

**El martinete con motor de pólvora** inventado por el Sr. Thomas Shaw, el bien conocido ingeniero mecánico de Filadelfia, es máquina de mucho mérito. El martillo se mueve por medio de pequeños cartuchos de pólvora, colocados, uno á uno, en un receptáculo en la parte superior de la estaca, y que hace explosión por medio del martillo mismo. Puede fácilmente ejecutar de 30 á 40 golpes, de 1.5 á 3 m, por minuto, y como el martillo no se pone en contacto directo con las estacas, no maltrata sus cabezas, y no es necesario usar los aros de hierro, etc., para preservarlas. Cuando sólo se necesita un golpe suave se usará un cartucho más pequeño. Para clavar una estaca á 6 m dentro del fango se usa, poco más ó menos, ¼ libra de pólvora; dentro de cascajo, cuatro veces más. Esta máquina no ayuda á elevar la estaca y colocarla en su posición, como se hace por medio de los ordinarios martinetes comunes de vapor; éstos, sin embargo, sólo dan de 6 á 14 golpes por minuto.

**Se han clavado estacas por medio de explosiones** de pequeñas cargas de dinamita colocadas sobre sus cabezas, las cuales están protegidas por planchas de hierro.

**Los martinetes de vapor** que funcionaron bajo el principio indicado por **Nasmyth**, son económicos para clavar á grandes profundidades y en difíciles terrenos, donde haya como 200 ó más estacas en grupos ó hileras, de manera que la máquina pueda moverse fácilmente de estaca á estaca.

**El cilindro** de vapor es vertical y está encerrado entre las partes superiores de dos vigas verticales y paralelas en I, ó acanaladas, como de 1.8 á 3.6 m de largo, separadas 45 cm, cuyas extremidades inferiores sujetan un casquete hueco, cónico, de hierro fundido, el cual ajusta sobre la cabeza de la estaca. Este casquete está abierto en la parte superior, y á través de él el martillo, que está adherido á la varilla del émbolo, golpea la cabeza de la estaca. Cada una de las vigas verticales contiene una de las dos vigas de guía directrices del martinete, entre las cuales el aparato clavador descrito anteriormente puede deslizarse hacia arriba ó hacia abajo.

Cuando ha sido colocada una estaca en su posición y está lista para ser clavada, se le colocará sobre la cabeza el casquete de hierro haciendo actuar de este modo, el peso del volante, cilindro, martillo y casquete sobre la estaca. Este peso se apoya sobre la estaca á través del martinete, y el aparato se desliza hacia abajo por entre las guías ó directrices á proporción que desciende la estaca.

El vapor pasa de la caldera al cilindro por medio de un tubo flexible. Cuando entra en el cilindro, el martillo se levanta como .75 á 1 m, y al escaparse, cae golpeando la cabeza de la estaca. Se dan como 60 golpes por minuto. El martillo está provisto de una (*trip-piece*) pieza giratoria que permite automáticamente la entrada del vapor en el cilindro después de cada golpe y abre una válvula para su escape al fin del movimiento ascendente del émbolo. Alterando el ajuste de esta pieza (*trip-piece*), la longitud de caída para el golpe (y por consiguiente la fuerza de los golpes), puede aumentarse ó disminuirse. La entrada y escape del vapor en el cilindro puede también controlarse directamente por un empleado auxiliar. El número de golpes por minuto se aumenta ó disminuye regularizando el suministro de vapor.

Al hacer el movimiento ascendente del émbolo, el vapor comprimiendo la cabeza inferior del cilindro, comprime, por supuesto, la estaca y ayuda á su descenso.

**La ventaja principal de estas máquinas** consiste en la gran rapidez con la cual se suceden los golpes, no dando tiempo á la tierra movida á que se compacte de nuevo alrededor de los lados y bajo el pie de las estacas. Esto permite á la máquina hacer trabajos que no sería posible ejecutar con martinetes ordinarios. Se han clavado estacas de pino de Noruega á 12.60 m dentro de arena. Estas máquinas rajan ó desfloran menos las estacas, de manera que éstas pueden ser de madera más blanda y más barata. El casquete mantiene la cabeza de la estaca constantemente en su lugar, de manera que las estacas no se salen de línea. A veces se han incendiado las cabezas de las estacas á consecuencia de la rápida sucesión de golpes.

**Estas máquinas consumen** de 1 á 2 toneladas de **carbón** en 10 horas y **requieren una dotación** de 5 hombres. Trabajan con una presión de caldera de 3½ á 5¼ atmósferas.

## Reglas para calcular la resistencia de las estacas como soportes.

Hay grandes diferencias. No hay regla que pueda aplicarse bien en todas las circunstancias. El terreno mismo que está entre las estacas, sostiene, en la mayor parte de los casos, parte del peso, á pesar de que todo él está conflado á las estacas. Al mismo tiempo, en terrenos muy arcillosos hay mayor propensión á hundirse con el tiempo, á consecuencia de la introducción de agua entre las estacas y la arcilla, disminuyendo de este modo el frotamiento entre ellas. Mientras menos firme sea el terreno más daño hacen las trepidaciones, las que causan también hundimientos. En algunos casos este hundimento no será producido por el descenso de las estacas dentro del terreno que las rodea, sino por el de toda la masa compacta de estacas y tierra en las cuales fueron clavadas, que desciende á causa de alguna capa menos densa que se encuentre debajo de ellas; á veces se atribuye á las estacas el descenso que realmente se debe al aplastamiento de las vigas que descansan sobre sus cabezas.

En el hermoso **puente de Londres**, cada estaca debajo de algunos de los estribos sostiene el inmenso peso de 80 toneladas. Están enterradas 6 m solamente entre la fuerte arcilla azul de Londres y colocadas próximamente á 1.20 m de distancia de centro á centro, lo que es demasiado para tales estribos y arcos. A .90 m escasos de distancia hubieran tenido que sostener sólo 45 toneladas. Son de .30 m de diámetro en el medio de su longitud. Han ocurrido serios descensos, algunos de ellos de .30 m más ó menos, bajo estos estribos. El **puente de Blackfriars**, en aquellas mismas inmediaciones, muestra el mismo defecto. Algunas personas atribuyen esto en ambos casos á la penetración gradual del agua entre la arcilla y las estacas, quizás por la acción capilar de las mismas estacas ó por filtración directa. Puede ser debido, en parte al aplastamiento de las plataformas que están encima de las estacas ó á un descenso de la masa entera de arcilla y estaca entre la arcilla sin estaquear que está debajo, todo debido al inmenso peso que soportan. Este peso alcanza en este caso á 59 toneladas por m cuadrado de área cubierta por un estribo, y esto es quizás demasiado para arcilla húmeda cuando el más mínimo hundimiento es perjudicial.

**El mayor J. Sanders**, ingeniero de los Estados Unidos, hizo grandes experimentos en el fuerte Delaware con el fango del río y suministró lo siguiente al «Jour. Franklin Inst.» (nov. de 1851). Para obtener la carga de seguridad de una estaca común de madera enterrada hasta que se hunda sólo por cantidades pequeñas y casi iguales, bajo golpes sucesivos, divídase la altura de caída en metros por el pequeño hundimiento en metros producido por cada golpe; multipliquese el cociente por el peso del martillo, y divídase el producto por 8. El no establece un coeficiente especial de seguridad.

Ej.: En el puente de la calle Chesnut, en Filadelfia, el mayor peso sobre cualquiera de las estacas es de 18 toneladas; el Sr. Kneass hizo enterrar las estacas hasta que penetraron 19 mm (= .019 m) bajo cada golpe de un martillo de 547 kilogramos, enterrándolo hasta 6.10 m. ¿En esto procedía él sobre seguro? Aquí tenemos, aplicando la regla, $6.10 \div .019 = 321$ y $321 \times 547 = 175{,}587$ y $175{,}587 \div 8 = 21{,}948$ k, ó sean 21.95 toneladas como carga de seguridad según la regla del mayor Sanders. El terreno era fango de río. (*Obs. del T.* — Este ejemplo equivale al que trae el autor en medidas inglesas.)

**Nuestra regla** es la siguiente: multipliquense entre sí la raíz cúbica de la caída en pies, el peso del martillo en libras y la decimal .023. Divídase el producto por el último hundimiento de la estaca en pulgadas + 1. El cociente será la **carga extrema** que estará justamente á punto de causar más hundimiento. Para la carga de seguridad tómese de $^1/_{12}$ á $^1/_2$, de acuerdo con las circunstancias. Expresado en una fórmula métrica que equivalga á la regla del autor, resulta:

$$\text{Carga extrema en toneladas} = \frac{\text{Raíz cúb de la caída en metros} \times \text{peso del martillo en kg} \times 0.0019}{\text{Ultimo hundimiento en metros} + 0.0254}$$

Pongamos, como el autor, el mismo ejemplo anterior del puente, y tendremos:

$$\text{Carga extrema en toneladas} = \frac{\sqrt[3]{6.10} \times 547 \times 0.0019}{.019 + 0.0254} = \text{como } 42.8)$$

O digamos, la mitad de esto ó sea 21.4 toneladas, el peso, para un coeficiente 2 de seguridad. La regla del mayor Sanders da por carga de seguridad 21.95. La carga efectiva es 18 toneladas. Un coeficiente 2 de seguridad no es suficiente para fango de río. Pero no obstante que la regla del mayor Sanders y la nuestra están perfectamente de acuerdo en este caso, si se toma para cada uno un coeficiente 2 de seguridad, difieren muchísimo en otros. Así, en el puente Neuilly, Francia, el martillo más pesado tenía 912 kg, caída 1.52 m, hundimiento .006 m en los últimos 16 golpes, ó sean .00038 m por golpe. Las estacas sostienen 47 toneladas cada una. Nuestra regla da 40.6 toneladas para coeficiente 2 de seguridad, mientras que la regla de Sanders da 515 toneladas como carga de seguridad! Si como creemos probable no hubo hundimiento en el último golpe, entonces nuestra regla da 41.2 toneladas para una seguridad de 2, mientras que Sanders da el *infinito*. En los Hull Docks de Inglaterra, estacas de 25 cm en cuadro enterradas á 4.88 m en lecho de aluvión por un martillo de 675 kg á 7.32 m de caída con hundimiento, .05 m por golpe al fin de la clavada sostienen por lo menos 20 toneladas cada una ó, según algunos .25 toneladas. Nuestra regla da 33 toneladas como carga extrema, ó 16.

para un coeficiente 2 de seguridad. Sanders da como carga de seguridad 12.6 toneladas. Como hemos dicho antes, 2 no es coeficiente de seguridad suficiente para el fango. En el fango no son las estacas sino el terreno estacado el que desciende realmente con los años. En el puente Royal Border, Inglaterra, se clavaron firmemente estacas de 9.10 á 12.2 m en arena y cascajo húmedos en algunos casos. Se ensayó primero con pino, pero se resbalaban y se desfloraban tanto bajo el pesado martillo, que fué sustituído con éxito por el olmo americano. Fueron enterradas las estacas hasta que solamente se hundían .0012 m por golpe, bajo un martillo de 765 ks cayendo de 4.88 m de altura. Soportan 70 toneladas cada una. Nuestra regla da 47 toneladas para un coeficiente 2 de seguridad, mientras que Sanders da 364 de carga de seguridad. Sin embargo opinamos que no se hundieron realmente como supusieron los observadores, sino que fueron simplemente comprimidas ó parcialmente machacadas por clavarlas demasiado. La mayor parte de las estacas fueron clavadas hasta que se hundieron solamente 25 mm bajo 150 golpes. Pero dudamos si ofrecían seguridad ó si habían penetrado más en el terreno que recibiendo uno solo de los golpes. Consideramos peor que inútil esta precaución extrema. En algunos experimentos hechos (1873) en Filadelfia fué entetrada una estaca de prueba á 4.5 m en fango blando de río por un martillo de 720 kg, habiendo sido su último hundimiento .45 m. Bajo una caída 10.98 m. Solamente á las 5 horas después de haber sido clavada se cargó con un peso de 6.5 toneladas, que causó solamente un hundimiento de una fracción muy pequeña de pulgada. Nuestra regla da 6.1 toneladas como carga extrema. Bajo 9 toneladas se hundió 19 mm y bajo 15 toneladas 1.5 m. Según la regla del mayor Sanders, su carga de seguridad sería 2.14 notadas.

**Una estaca de prueba del Gobierno de los Estados Unidos** de cerca de .30 m en cuadro, enterrada á 8.84 m, á través de capas de fango, arena y arcilla, con un martillo de 409.5 kg, caída 1.5 m, último hundimiento .0094 m, soportó 26.6 toneladas; pero se hundió lentamente, bajo 27.9 toneladas. Nuestra regla da como 26 toneladas, carga extrema.

**Los ingenieros franceses** consideran que una estaca ofrece seguridad suficiente para un peso de 25 toneladas cuando rechaza el peso de 609 kg, cayendo de 1.22 m de altura. Nuestra regla da 24.2 toneladas para un coeficiente 2 de seguridad. Ellos estiman como peso rechazado el de un martillo que bajo 30 golpes no produce un hundimiento mayor de 1 cm. En muchos puentes importantes, etc., clavan las estacas hasta que cesa el hundimiento, bajo un martillo de 362 kg, cayendo de 1.5 m de altura. Nuestra regla da en este caso 31 toneladas por carga extrema, ó 15.5 para un coeficiente 2 de seguridad.

**Respecto á la carga propia de seguridad**, creemos que sólo debe tomarse la mitad del peso extremo que da nuestra regla para estacas enterradas perfectamente en *terrenos firmes*, no más de una *sexta* parte cuando sean clavadas en fango de río ó en pantano, suponiendo, como hemos hecho anteriormente, que sus puntas no descansan sobre roca.

**Si puede haber trepidaciones**, tómese solamente **la mitad** de estas cargas.

**Las estacas pueden hacerse de cualquier dimensión que se quiera,** sea respecto á longitud ó á sección transversal, atornillando ó empalmando latera, y longitudinalmente un número de vigas cuadradas.

**Estacas con puntas embotadas.** En el puente de la calle South Filadelfial fueron enterradas 1,200 estacas gruesas de abeto de Nueva Escocia, con las puntas embotadas, de 4.5 á 10.67 m, parte en fango sólido, por medio de un martinete común de vapor. Coste total (estaca y enterrada) de 7 á 8 dólares cada una. En el puente de Wilmington, California, el Sr. C. B. Scars, del ejército de los Estados Unidos (Diario Americano, Sociedad C. E., diciembre 1876), descubrió que en arena húmeda, firme y compacta, después de algunos primeros golpes, las estacas no penetran más de 12 á 37 mm á cada golpe, sea cual fuere la altura de la caída del martillo de 1087 kg. Las estacas sin puntas, de las cuales había muchos miles, se enterraron con la misma facilidad que las provistas de puntas, á una profundidad media de 4.5 m en esta arena, y con mucha menos tendencia á inclinarse. Como una gran caída no tenía otro efecto que desflorar las cabezas de las estacas, se redujo aquélla á 3 m, con lo cual se clavaban, por término medio, como 18 mm á cada golpe.

Para que las estacas puedan entrar rectamente deben hacerse ó cortarse sus puntas en ángulo recto á su longitud. En lugar de clavar las estacas, cuando la profundidad es mediana, es mejor, algunas veces, introducirlas simplemente en

hoyos practicados con taladros semejantes al taladro perforador de pozos.

El frotamiento máximo de estacas con su corteza, clavadas á .90 m, más ó menos, de distancia de centro á centro, probablemente nunca excederá mucho de 10.76 toneladas por m cuadrado en arena húmeda densa ó en cascajo arcilloso, ni más de 5 á 8 toneladas en terrenos y arcilla comunes, ó más de $1^1/_2$ á 2 toneladas en fango ó barro húmedo de río, dependiendo todo de la profundidad y densidad. **El frotamiento de los cilindros de hierro fundido** parece ser como de .3 del de las estacas.

**Hay gran diferencia en la penetrabilidad** de las diferentes arenas. Así, en el puente Lary no se encontró dificultad especial para enterrar estacas á 10.67 m en arena debajo del agua, mientras que en otras localidades, dentro del agua, estacas de muy buena madera, bien guarnecidas con hierro, no se pueden enterrar á 1.8 m de profundidad en arena sin que se rompan en pedazos. La misma dificultad se ha encontrado cuando las estacas son de tornillo. En el faro Brandywine no pudieron hacerse penetrar á más de 3 m en arena debajo del agua. La arcilla dura, entre el agua y el cascajo limpio, difieren también muchísimo en este respecto. Generalmente son penetrables á cualquiera profundidad que se desee con facilidad relativa; pero hemos visto estacas fuertes de cicuta hechas pedazos al tratar de enterrarlas á 1.80 m en cascajo entre el agua, y el Sr. Rendel en Plymouth « no pudo, con ninguna clase de fuerza, clavar estacas á más de 1.5 m en la arcilla, no siendo tan dura como la arcilla de Londres », sobre la cual los ya mencionados puentes nuevos de Londres y Blackfriars fueron cimentados y en cuya arcilla se enterraron estacas de madera á 6 m, sin ninguna dificultad.

La mezcla de barro y arena facilita muchísimo la enterrada de las estacas; pero antes de empezar un extenso sistema de estacado deben clavarse unas pocas por vía de experimento, á fin de conocer las dificultades y gastos que deban prevenirse. Simples perforaciones serán á menudo insuficientes.

Por regla general un martillo pesado á poca altura de caída, clava mucho mejor que uno liviano á gran altura. Cuando un martillo de $^2/_3$ de tonelada con caída de 7.5 m, en un terreno duro, destroza las estacas, uno de 2 toneladas con 2.10 m de caída, las clavará satisfactoriamente. Pueden darse mayor número de golpes en el mismo tiempo con poca caída, y de este modo se da menos tiempo á la tierra para compactarse alrededor de las estacas entre golpe y golpe. A veces una estaca puede resistirse al martillo después de haber penetrado algo, pero continuar penetrando después de un pequeño descanso; suelen rechazar un martillo pesado y funcionar bajo uno más liviano; ó penetrar lentamente al principio y más rápidamente después por causas difíciles de descubrir. Sucede muchas veces que la clavada de una estaca es causa de que algunas adyacentes, clavadas anteriormente, salten algunos decímetros hacia arriba. Una estaca se encuentra en la posición más favorable cuando su punta inferior se apoya sobre la roca después de haber atravesado por entre un terreno firme que la cubre y protege perfectamente, para que no se doble como una columna demasiado cargada, y al mismo tiempo desarrolla gran frotamiento contra sus caras, ayudándola así á sostener la carga, y por tanto relevándola de la presión sobre la extremidad inferior. Una estaca puede descansar sobre roca y ofrecer sin embargo muy poca resistencia, pues que si pasa á través de un terreno blando, la extremidad puntiaguda inferior soporta todo el peso, y viene á ser la estaca simplemente una columna, en peor condición que un pilar con una extremidad redondeada. En dichos terrenos las estacas requieren muy poca punta, y, en verdad, sería mejor clavarlas sin ninguna.

La clavada de una estaca en terreno blando á barro, generalmente hace que ana de las adyacentes que ha sido clavada anteriormente, se incline hacia fueru, á menos que se tomen precauciones para impedirlo.

Al estaquear una área de terreno firme, es mejor comenzar del centro hacia fuera, pues de otro modo el terreno se consolidará de tal manera en el centro, que no será posible le clavar las estacas.

Se ha visto que **la reacción elástica del terreno** ha levantado áreas enteras estacadas con las est casa, antes de que se hubiere fabricado sobre ellas.

En terrenos muy sólidos, especialmente si son de piedra, ó aun en terrenos blandos, si las estacas tienen punta y han de ser enterradas hasta la roca, deben protegerse sus puntas inferiores por zapatas de hierro forjado *a*, *s*, y *b*, fig. 13, fijas á la estaca por medio de piezas de hierro en ellas forjadas; ó de hierro fundid,o como en *c*, en que el casquillo es un cono sólido invertido, cuya ancha y chata base suministra un buen apoyo á la base plana del extremo de la estaca. La línea de puntos representa un perno fuerte de hierro, bien fijo al cono; éste sujeta el casquillo á la estaca. Los casquillos medianos de hierro forjadc generalmente pesan de

8 á 12 kg, pero se empleará hierro laminado cuando el terreno es sólo medianamente compacto; hierro batido cuando lo sea más, y hierro sólido ó puntas de acero de 5 á 10 cm en cuadro en su parte inferior cuando es muy compacto y pedregoso.

Fig. 13

Para impedir que las estacas se resbalen **se deben practicar agujeros en las rocas** é introducir sus puntas, lo cual se hará clavando primero tubos que sirvan de guía al barreno, después que se haya extraído la tierra del tubo.

**A fin de preservar las cabezas**, se rodean generalmente con un aro *h*, fig. *d*, de 1 á 2½ cm de espesor y de 4 á 8 cm de alto. Éstos son, á veces, ineficaces, pues cuando se clava fuertemente, la cabeza se machaca y se raja en todos los lados, frecuentemente varios decímetrosde bajo del aro, y por otra parte los aros se abren á menudo. Por lo tanto las cabezas tienen que ser recortadas repetidas veces antes de clavarlas del todo; al pedir estacas para cualquier trabajo debe tenerse en cuenta estas pérdidas, especialmente si el terreno es duro. El capitán Turnbull, ingeniero de los Estados Unidos, dice que en el acueducto del Potomac las cabezas de las estacas fueron preservadas dándole á la superficie cierta forma cóncava, ahondándola como 3 cm, y cubriéndolas con una lámina de palastro floja, como se ve en la sección *e*, fig. 13. Uñ ligero desflecamiento ó trituración de la cabeza disminuye sensiblemente la fuerza del empuje. Las estacas pueden ser clavadas á través de ripios sueltos sin mucho trabajo. La clavada con pisón de madera maltrata menos las cabezas de las estacas. Las que apoyan sus extremos inferiores sobre rocas inclinadas pueden resbalarse cuando estén cargadas.

**Para clavar una estaca debajo del agua**, se empleará un embutidor, *p*, fig. 13. El extremo de este embutidor calza en la parte superior de un molde *ff*, redondo ó cuadrado, según la forma de la estaca, que tiene una división *oo*. La parte inferior del molde se ajustará á la cabeza de la estaca, y el martillo caerá sobre la superior. Cuando se clavan estacas verticalmente en un suelo blando para soportar muros de sostenimiento ú otras construcciones expuestas á fuerzas horizontales ó inclinadas, se debe tener cuidado de que estas fuerzas no compriman las estacas mismas, pues que en dichos terrenos las estacas están dispuestas para resistir fuerzas verticales solamente, á menos que sean clavadas con una inclinación que corresponda á la fuerza oblicua.

**Una estaca rota puede arrancarse** ó, por lo menos, ser desencajada, si no está clavada muy firmemente, atándola á lanchones cuando la marea está baja, y dejar á cargo de la alta marea el aflojarla. O se puede usar una viga grande como palanca y tomar por punto de apoyo la cabeza de una estaca contigua, ó por medio de una grúa movida por la máquina del martinete. En casos muy difíciles se podrá usar el método aconsejado por el ingeniero civil Sr. J. Monroe. Un tubo de gas de 10 cm de diámetro y de 4.5 m de largo, calzado con una punta sólida de acero, y provisto de un reborde exterior para sostener un embutidor circular, fué clavado cerca y á 60 ó 90 cm más bajo que dos estacas que estaban enterradas hasta 3.60 m en 11 m de agua y rotas por el hielo. Luego se depositaron en el extremo nferior del tubo dos kg de pólvora, cuya explosión sacó las estacas completamente de su lugar. A menudo es mejor dejar en su sitio la estaca rota y clavar otra cerca de ella. Puede ser arrancada por presión hidráulica.

**El hielo se adhiere á las estacas** con una fuerza como de 2 á 3 kg por cm cuadrado, y al subir el agua puede arrancarlas si no han sido suficientemente clavadas.

Pilotes ó **estacas de hierro y cilindros**. En Europa se ha empleado el hierro fundido para estacas, y en diversas formas, particularmente cuando se dejan que permanezcan como una cubierta para proteger las obras de concreto, rellenas por detrás y apoyando el relleno *.

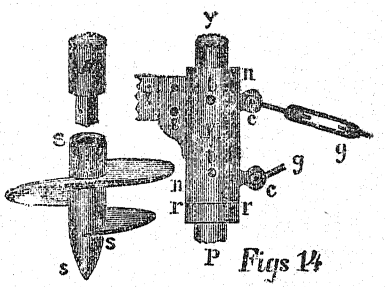
* El hierro fundido destinado á resistir el agua del mar debe ser de grano compacto,

Pueden emplearse como estacas de sostén cilindros de hierro fundido, abiertos en los dos extremos, y pueden limpiarse y llenarse de concreto si es preciso. El frotamiento que se produce al clavarlas es mayor que en las estacas sólidas, puesto que se desarrolla en las superficies interior y exterior. Puede disminuirse extrayendo gradualmente la tierra del interior á proporción que penetren. Se requiere mucho cuidado, debe emplearse un martillo más liviano ó menor caída que con las de madera para impedir que se rompan. Debería interponerse entre el martillo y la estaca una pieza de madera, ó el martillo puede ser de madera. Pero es mejor usarlos en la forma de cilindros de tornillo, los cuales tienen además la ventaja de una ancha base (véase fig.).

**Procedimiento de Brunel.** Experimentó con un cilindro abierto de hierro fundido de .90 cm de diámetro exterior, 37 mm de grueso en trozos de 3 m enchufados y con encajes internos y ensambladuras dentadas sujetas con tornillos y soldadas con plomo. Tenía un aro afilado ó cortador en el fondo, y un poquito más arriba de éste una vuelta de tornillo con un paso de 17½ cm y un vuelo de 30 cm en todo el contorno exterior del cilindro. Por medio de barras y de un cabrestante atornilló éste á través de greda dura y arena á 18 m hasta encontrar la roca. Para descender esta distancia el cilindro hizo 142 revoluciones, penetrando por término medio como 12½ cm en cada una. El tiempo verdaderamente empleado para atornillarlo fué de 48¼ horas ó como 36 cm por hora. Sin embargo hubo intervalos largos de reposo para sacar la tierra del interior. Después de descansar no hubo gran dificultad en comenzar de nuevo. Las próximas figuras darán una idea de la disposición dada al tornillo.

**El pilote de roca** de Alexandre Mitchel, de Belfast, consiste generalmente en una espiga de hierro laminado A, fig. 14, de 7½ á 20 cm de diámetro, provista por debajo de un tornillo de hierro fundido SSS con una espira cortante de .45 cm á 1.5 m de diámetro. Los tornillos que se usan para lugares expuestos á mares mansos son ordinariamente de .90 m de diámetro más ó menos y de 1½ vueltas ó pasos: pesan como 270 kg. Los mangos ó espigas redondos laminados tienen de 12 á 20 cm de diámetro. El tornillo penetra de 3 á 6 m en greda, arena ó coral por el impulso de 30 ó 40 hombres comunicado á 6 ú 8 barras del cabrestante, cuyos extremos describen un círculo como de 9 á 12 m de diámetro.

Con este objeto se prepara frecuentemente una plataforma sobre estacas. En agua mansa se sostiene sobre lanchones. Puede emplearse una balsa bien amarrada cuando la clavada es fácil; ó el puente de una balsa grande, con un hueco en el centro, para que pase el pilote ó estaca. Jaulas y cajones hechos ordinaria y temporalmente rellenos de piedras, y sumergidos, pueden sostener una plataforma en



Figs 14

ciertas posiciones. La plataforma debe de ser, evidentemente, capaz de resistir los movimientos de rotación horizontales comunicados por la gran fuerza de empuje de los hombres que maniobran en las barras del cabrestante; por este motivo no es

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blanco y duro. La pequeña cantidad de carbón contenida está químicamente combinada con el metal; pero en el más obscuro está mecánicamente combinado, y dicho hierro pronto se vuelve dúctil (más ó menos, como la plombagina) cuando se expone al agua del mar. El hierro blanco duro se ha probado que resiste por lo menos 40 años, sin deteriorarse, esté constantemente debajo del agua ó alternativamente mojado y seco.

Al cobre y al bronce los afecta sólo ligera y superficialmente; pero tiene efecto la acción destructiva galvánica si entran en contacto metales diferentes.

difícil clavar tornillos á una profundidad suficiente en arena compacta por medio de una plataforma flotante. Los extremos inferiores de las estacas deben estar sujetos á los tornillos para impedir que sean separados de ellos por la fuerza de las olas contra la parte superior de la construcción. En *y p*, fig. 14, se ve un modo de empalmar ó unir diferentes puntos ó secciones de una estaca. El punto de unión es *v; rr* es un anillo fuerte de hierro forjado en la estaca baja *p* como 30 ó 45 cm debajo de su parte superior. Un manguito *nn* fuerte, cilíndrico, de fundición, que envuelve los extremos de las secciones, descansa en este anillo y se une á los pilotes por medio de clavijas *tt*. En estos manguitos también hay piezas salientes de fundición *ccc*, para fijar barras *gg*, y vigas, etc., necesarios para ligar la construcción de estaca á estaca. El tiempo que realmente se necesita para enterrar un tornillo es de 2 á 10 horas, bajo favorables circunstancias.

En el faro Brandywine, en un banco de arena muy pura, cubierta de 1.8 á 2.4 m por la baja marea y de 3.30 á 3.90 m por la alta, no pudieron hundirse más de 3 m desde una plataforma fija. En otros lugares se llega á 6 m en arena, sin mucha dificultad, cuando la arena contiene una buena cantidad de barro; pero entonces su resistencia como soporte es menor. Esta (última) varía como de 10 á 60 toneladas por metro cuadrado, según la fuerza, profundidad, compactibilidad, etc., de la arena. En casos de importancia debe probarse su resistencia.

Los pilotes de Mitchell han sido atornillados cerca de 12 m en una mezcla de greda y arena, con tornillos de 1.20 m de diámetro. Atraviesan, sin mucha dificultad, por entre piedras picadas y roca de coral y echan á un lado las piedras de tamaño mediano. Ordinariamente la greda ó la arena no oponen mucha resistencia; pero accidentalmente cualquiera de las dos pueden oponerla. Como regla general la arena pura es la que presenta mayores dificultades. En el bajo de Brandywine la hinca fué ayudada con un espolón (*spur*) y un piñón colocados tan bajo como lo permitió el agua; manejaron las palancas 30 hombres. El peligro de que se fuerza el árbol es lo que limita la línea del tornillo. Son muy usadas para asegurar cadenas, amarrar boyas, etc. En tierra, pequeños tornillos huecos con ejes ó espigas, constituyen un soporte bueno y durable para pilares de almacenes de depósito, para postes de madera, para telégrafos, estaciones de señales en la marina, etc. Se podrán emplear caballos ó bueyes para clavar grandes tornillos. El faro Brandywine descansa sobre 9 pilotes, los cuales están rodeados por otros 30 de 12½ cm de diámetro que le sirven de defensa. Tienen que resistir no solamente mares moderados, sino también inmensas cantidades de hielo flotante que tienen millas de extensión. Un edificio no terminado fué destruído por el hielo, el cual perjudica de tiempo en tiempo al que hoy existe.

**Deben practicarse reconocimientos con los barrenos para asegurarse** de que los tornillos no se apoyan precisamente sobre una capa muy débil, que perjudique su resistencia. Esto para cualquiera estaca.

**Por medio de un chorro de agua** lanzado fuertemente por una bomba impelente, se afloja la más resistente arena (pero no la arcilla dura ni el cascajo compactado) y se facilitará muchísimo la hinca de los pilotes de rosca de madera, y hasta los cilindros más grandes. Pueden desatornillarse fácilmente cuando se quieran quitar. En un muelle del Gobierno, en el cabo Henlopen, en arena muy compacta, se rompieron 6 de 7, antes de haber penetrado 3 m y se descubrió que el uso del chorro les quitaba más de las tres cuartas partes de la resistencia *.

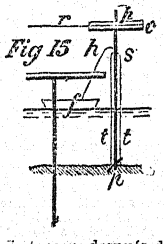

Fig 15

Para clavar la estaca *p*, después de haberla colocado en su puesto, como en la

* Informe de la Sección de guerra.

fig. 15, se apoyan los extremos inferiores abiertos *tt*, de un tubo de hierro *tst*, de 31 mm de diámetro, sostenido por la parte superior del disco del tornillo, y allí se mantenía firme por tres ó cuatro hombres, mientras la estaca se atornillaba por medio del cabrestante *c*, el cual era movido por una cuerda *r*. Desde la curvatura *s* del tubo, una manguera *h*, de 5 cm de diámetro conducía á la bomba impelente, cuyo cilindro tenía 12½ cm de diámetro, una carrera de émbolo de 23 cm, y daba como 80 golpes de émbolo completos por minuto, por medio de una rueda movida por una mula en una plataforma flotante *t*. Entonces no había trabajo en atornillar las estacas á cualquiera profundidad.

**En Mobile Bay** fueron clavadas de 3 á 6 m de profundidad, por medio de un chorro, algunos miles de estacas de madera desde .45 hasta 1.20 m de diámetro en arena con un hundimiento medio de .30 m por segundo. Aquel era impelido por una bomba de vapor, de incendio, colocada sobre un bote de vapor y usando la misma manguera de la bomba con una boca de 31 mm. Durante el descenso la boca *nn* de la manguera se mantuvo sujeta flojamente en su lugar cerca del pie de la estaca, por medio de dos grapas *ss* y por una cuerda *t* que llegaba á la superficie. Las estacas estaban suspendddas por sus cabezas por medio de tijeras con un aparejo para regular su descenso. La arena se asentó firmemente alrededor de las estacas pocos minutos después de haber sido enterradas.

En el río Tensas (Alabama), para cilindros de hierro de 1.80 m de diámetro en profunda y ligera arena movediza, el chorro fué arrojado por una bomba giratoria de 200 á 300 revoluciones por minuto, por medio de una manguera de 7½ cm de diámetro dentro de una cubeta cónica central de hierro fundido de 25 cm de diámetro, de la cual irradiaban 12 tubos de gas de 2½ cm de diámetro y como 75 cm de largo. En el extremo exterior de cada uno de estos radios había un codo, al cual estaba adherido un tubo largo vertical que penetraba dentro del cilindro, compuesto de trozos de 3 m de longitud y terminados en tornillos para prolongarlo á proporción que descendía el cilindro. Este aparato se levantaba y bajaba por medio de una pieza liviana de madera y un cordel: con esto sólo se hizo penetrar cada cilindro como 48 m, en pocas horas, dentro de una arena movediza *.

**En el viaducto Levan, el Sr. James Brunlee**, Inglaterra, en una marga ligera y arenosa de gran profundidad, enterró cilindros huecos de hierro fundido de 25 cm de diámetro exterior, á una profundidad de 6 m, por medio de un tubo surtidor de 5 cm de diámetro que pasaba por debajo, dentro del cilindro y á través de un hueco en su base, la cual era un disco de hierro fundido de 75 cm de diámetro y 2½ de espesor, reforzado por bordes exteriores. Los bordes de conexión de los trozos del cilindro eran exteriores, impidiendo así el descenso, como lo hacía también el disco ancho del fondo. Hasta 3 ó 4 horas bastaban, comúnmente, para sumergir cada uno á la profundidad de 6 m. Ensayos practicados prueban que su resistencia segura como soporte era de 50 toneladas por m cuadrado del disco de la base.

**En el viaducto de Lock Ken** cada estribo constaba de dos cilindros de hierro fundido abiertos en ambos extremos, de 240 cm de diámetro, y 28 mm de espesor, en trozos ó secciones de 1.80 m de longitud con peso de 4 toneladas cada uno, unidos por medio de bordes interiores con soldaduras de hierro entre ellos. Los cilindros tienen una separación de 2.40 m y se encuentran en 11 m de agua. Se erigió un fuerte andamio y fueron clavadas cuatro estacas de guía para cada cilindro. Habiendo sido previamente unidas las diferentes porciones de cilindro, fueron descendidos á sus puestos. Cada cilindro penetró por su propio peso de 30 á 60 cm á través del barro superficial y luego se asentó sobre la arena y el cascajo que forman el lecho, de gran espesor. En este último se hundieron después como 2.40 ó 2.70 m más, extrayendo la tierra interior debajo del agua por medio de una *vasija de tornillo*, cónica é invertida, ó draga, de planchas de palastro de 6 mm. Su diámetro mayor era de .60 m y .30 m de profundidad, y tenía adherido á su fondo un tornillo como de .30 m de largo para ayudar á atornillarla en la tierra. Estaba provista de aberturas en sus costados para que entrara la tierra, y de válvulas de cuero abiertas hacia adentro para impedir que se escape aquélla. A los lados opuestos de la vasija ó paila había tres barras de 19 mm de diám, prolongadas hacia arriba 1.20 m, y allí juntas forjadas y conectadas por medio de una ranura y un perno, unidos á una larga barra ó árbol, en cuyo extremo superior había una cruceta transversal de 4 brazos, por medio de la cual 4 hombres montados en un andamio atornillaban la vasija.

« Cuando una vasija estaba llena, se la subía por medio de una polea. Traía como 27 litros de un golpe, una menor de sólo 30 cm de diám por 30 cm de alto,

* Gabriel Jordan, I. C. Trans. Am. Soc. C. E., febrero 1874.

traía la cuarta parte, y se empleaba cuando el material era muy duro. Así se vaciaba el cilindro á razón de 5 á 45 cm por día. El trabajo lento de 5 cm por día, se debía á las piedras, algunas de 23 kg. Éstas se aflojaban con un pico en forma de barra de .90 de largo con brazos de 30 cm. Después de aflojarlas se subían con el aparato. El costo de estos aparatos era de poca importancia, y la excavación se hizo fácil y barata. Después que se terminó la excavación y se vació el cilindro, antes de bombear afuera el agua se echó concreto (2 volúmenes de granzón, 1 de cemento hidráulico) en una profundidad de 3.60 m, con baldes de fondo movible y quedó aquél 12 días para que fraguara. Entonces se bombeó el agua hacia afuera y se mamposteó al aire. Sin embargo, en algunos de los cilindros el agua ascendía tan pronto, á pesar de los 3.60 m de concreto, que las bombas no pudieron agotarla y hubo que agregar 1.80 m de concreto. Finalmente se echaron piedras sueltas y ripio alrededor de los cilindros para preservarlos de los golpes y evitar que se socaven.* » La mampostería llega á 6 m sobre los cilindros y sobre el agua.

**Procedimientos por el vacío y por aire comprimido.** Apenas podemos aludir á los principios generales de estos dos modos de sumergir grandes cilindros huecos. Por el método del vacío, del Dr. Holher Potts, de Londres, el cilindro *c*, fig. 16, durante la inmersión, está cerrado herméticamente en su parte superior por una puerta falsa, que se abre hacia arriba. Un tubo de caucho flexible *p*, suficientemente largo para adoptarse á la sumersión del cilindro, y provisto de una

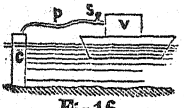

Fig 16

llave *s*, comunica el cilindro con el envase *v*, que puede colocarse sobre una balsa, ó lancha, ó estar en tierra, según convenga.

Después de haber colocado el cilindro en su posición, como en la figura, se saca por medio de una bomba el agua y el lodo ú otro material suelto que se encuentre en su interior si se ha sumergido alguna cantidad por su propio peso. Entonces se cierra la llave *s*, y se saca el aire del envase *v* por medio de una bomba de aire. Abriendo entonces la llave, la mayor parte del aire del cilindro se precipita al envase vacío *v*; dejando de este modo al cilindro comparativamente vacío, y por consiguiente menos capaz de resistir la presión hacia abajo del aire exterior sobre su parte superior. Esta presión, como es bien sabido, es cerca de 1 kg por cm cuadrado, ó diez toneladas por m. En consecuencia, el cilindro está impelido hacia el fondo del río, por esta presión, más su propio peso. Al mismo tiempo, la presión del aire sobre la superficie del agua, se transmite por ésta al terreno, alrededor de la parte inferior abierto del cilindro, de manera que si el terreno está blando ó en un estado semifluido, habrá una presión hacia arriba en el interior del cilindro, donde no existe presión hacia abajo que se oponga á ella. El descenso del cilindro varía desde algunos cm hasta 1.20 ó 1.50 m en cada vez. Luego se repite el procedimiento admitiendo nuevamente aire en el cilindro, abriendo la puerta falsa, sacando el agua y la tierra, etc., etc. Pueden agregarse otros cilindros sobre el primero, por medio de rebordes interiores y pernos.

**Este método se adapta sólo á terrenos blandos**, como también á terrenos arenosos mojados; pero no tiene suficiente fuerza para los muy compactos; ni se puede aplicar donde hay obstáculos como piedras grandes, trozos de madera, etc., cuya remoción requiere que los hombres desciendan dentro del cilindro, lo que no pueden hacer en el aire rarificado. El tubo *p* debe ser de un diámetro suficientemente grande para permitir la salida del aire con rapidez, de modo

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* **Los pilotes de hierro, huecos**, sean fundidos ó forjados, están provistos en su pie de puntas sólidas, para clavarlos por medio de un martinete que cae en su *interior* y golpea contra la parte superior del pie sólido; son de gran utilidad en muchos casos. Se fabrican en secciones ó partes que pueden unirse gradualmente para llegar á la profundidad necesaria. Así en secciones se evita el peligro de que se doblen, peligro que siempre existe cuando se golpea sobre la parte superior. La base sólida de hierro es un poco más grande que el diámetro del pilote, para disminuir el frotamiento del terreno contra dicho pilote en el resto de su parte superior.

que la presión exterior pueda actuar lo más rápidamente posible sobre la tapa del cilindro.

En el faro de los Goodwin Sands, Inglaterra, se han sumergido cilindros huecos de .75 m de diámetro por este método hasta la profundidad de 10.2 m en 6 horas más ó menos, en lugares donde una barra de acero no se podía hacer entrar á fuerza de martillo sino á 2.4 m de profundidad. Otros cilindros de 30 cm de diámetro se han sumergido 4.8 m en menos de una hora. En este último caso la bomba de aire tenía dos cuerpos de bomba de 11 cm de diámetro, 40 cm de carrera de émbolo, movida por 4 hombres. El tubo *p* era de plomo y solamente de 12 mm de diámetro.

**El procedimiento por el aire comprimido**, inventado por el Sr. Triger, de Francia, consiste en introducir aire en el cilindro CC, fig. 17, hasta una presión suficiente para hacer salir el agua obligándola á escapar por debajo del extremo abierto del cilindro, y á través de la que lo circunda. Estando de esta manera seco el fondo dentro del cilindro, pueden bajar los trabajadores y sacar el material en su base y debajo de ella. Hecho esto, salen, y se deja escapar el aire comprimido, y el cilindro, no sostenido ya por la presión del aire comprimido que se ejerce de abajo hacia arriba, se sumerge en la cavidad ó entre el material aflojado en su pie. La fig. 17 enseña una disposición sencilla para que los trabajadores puedan entrar ó salir del cilindro sin dejar escapar el aire comprimido y también demuestra el principio general del método. LL es una pequeña cámara separada, la **cámara de extracción** (**air-lock**), la cual se quita cuando se agrega otro pedazo de tubo; después se vuelve á poner y se asegura firmemente. Esta cámara tiene una pequeña puerta *d* que puede cerrarse herméticamente y por la cual puede entrarse. Tiene otra puerta *o* que se abre hacia el cilindro. Esta cámara tiene también dos llaves de retención, una, *a*, en un piso comunicando con el cilindro; y otra, *e*, arriba comunicando con el aire libre. En *s* hay un tubo doblado, también con su llave que pasa herméticamente por el lado y el fondo de la cámara de extracción. Por esta llave entra el aire comprimido al cilindro por medio de una bomba impelente y por esta misma llave se le deja después escapar. En *nnn* se ve un sifón. Se emplea un torno *w* para izar el material excavado del fondo del cilindro á la cámara de extracción; los ejes *ii* del torno pasan por cajas de estopas situadas en los costados de la cámara de extracción; el material se iza por trabajadores colocados del lado afuera de dicha cámara. Esta es la disposición generalmente adoptada por el Sr. W. J. Mc Alpine, I. C. de Nueva York, en el puente de Harlem. Nuestra descripción condensa la de este señor. Los cilindros eran de 1.8 m de diámetro, de 30 mm de grueso y en secciones de 2.70 m de largo, unidos por pernos en bordes interiores *f*, á medida que se iba efectuando la inmersión de ellos. La caja de extracción tiene 1.8 m de diámetro por casi 1.8 m de alto, con sus caras construídas con planchas de hierro de calderas y su parte superior y fondo de hierro fundido.

Ahora bien; suponiendo sumergido el cilindro CC y fijo en la posición de la figura, y la caja de aire LL ajustada en la parte superior del cilindro, lo que se hace en seguida es impeler el aire por el tubo doblado *s*, habiéndose cerrado previamente a válvula *t* de la puerta inferior *o* y la llave *a*. Cuando el aire comprimido se acumula

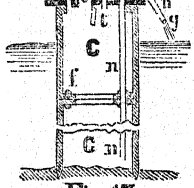

Fig 17

en el cilindro, hace salir el agua que en parte se escapa por debajo del fondo del cilindro y en parte sube por el sifón *nn* y sale por *g*. Como la puerta *o* está cerrada y

la puerta *d* abierta, el aire en la cámara de extracción se halla en la misma condición que el aire exterior, de modo que los trabajadores pueden entrar fácilmente en la caja. Ya adentro, ellos cierran la puerta *d* y la llave *e* y abren la llave *n*, por la cual se precipita el aire comprimido del cilindro hacia arriba, llenando prontamente la caja de extracción. Hecho esto, se abre la válvula *t*, y los hombres descienden al fondo por la puerta *o*, por medio de una escalera de mano ó en un tobo que se maneja con el torno *w*. Una vez en el fondo, aflojan y excavan el material hasta la profundidad que pueden; y llenando el tobo ó saco con el material, avisan por una señal á los trabajadores de afuera, que izan el tobo á la cámara de extracción. Hecho esto, ascienden á dicha cámara, cierran la puerta *o* y la llave *n*, y abren la llave *e*, por la cual se escapa el aire comprimido dentro de ella dejando el aire interior lo mismo que el exterior. Entonces se abre la puerta *d*, los tobos del material se sacan y los trabajadores salen. Finalmente, se abre la llave en *s*, el aire condensado del cilindro se escapa por ella al exterior y el cilindro se sumerge por su propio peso en la cavidad y terreno aflojado y preparado al efecto en su base para este fin. Dicho terreno es de nuevo impelido dentro del cilindro por el agua que se precipita otra vez en él. Luego se repite el mismo procedimiento.

La inmersión varía algunas veces de 0 á 3 m ó más en una sola operación. La mayor parte de los hombres pueden soportar el aire comprimido hasta una profundidad de 12 á 15 m; pero si aumenta ésta, se hace más difícil y hasta positivamente peligrosa. Se han sumergido de este modo cilindros de hierro fundido de 4.5 m de diámetro y grandes cajones, fig. 18; pero muchas veces con mucho costo y trabajo. El cilindro debería guiarse en su descenso por medio de un andamio fuerte que pueda apoyarse sobre estacas. De lo contrario está expuesto á inclinarse hacia un lado y es muy trabajoso hacerle llegar á su puesto otra vez. Estos cilindros han sido sumergidos en aguas profundas por buzos empleados en socavar interiormente el terreno.

**Procedimiento por aire comprimido.** El que se aplicó en el puente de South Street, en Filadelfia, por Mr. John W. Murphy, ingeniero contratista, difiere esencialmente del descrito arriba; y además merece atención por la gran sencillez y eficacia de su aparejo. Consiste éste, en parte, de dos lanchas, cada una de 30 m de largo, 5.2 m de ancho y 2.4 m de profundidad. Se anclaron paralelas una á otra á una distancia de 4.5 m. En dicha distancia ó espacio, y apoyada sobre ambas lanchas, estaba levantada una cabria fuerte de cuatro patas, de 15 m de altura más ó menos, y en cuya parte superior estaba sujeto un aparejo para manejar los cilindros de hierro fundido. En la bodega de una de las lanchas se colocó un **compresor de aire Burleigh** provisto de dos émbolos de 25 cm de diámetro y 23 cm de carrera, junto con su caldera de vapor. Sobre la cubierta de la misma lancha había un **depósito ó regulador de aire vertical** de 6.60 m de alto por .60 m diámetro, hecho de planchas de hierro de 6 mm de grueso. Este regulador servía para mantener un gran refuerzo de aire comprimido en el cilindro sumergido en el caso de una parada accidental del compresor de aire, que de otro modo hubiera quizás sido muy fatal para los trabajadores dentro del cilindro. El aire comprimido pasaba de este depósito á la cámara de extracción del cilindro por un tubo de 10 cm de diámetro, hecho de goma y lona, y colocado de tal manera que se extendía por sí mismo á medida que se sumergía el cilindro, manteniendo así la comunicación en todo tiempo.

A través de las dos lanchas se extienden dos durmientes ó travesaños de madera, de .90 m de ancho por .45 m de alto cada uno, compuestos de tres piezas de madera de .30×.45 m, unidas fuertemente por pernos. Estos travesaños tenían en el centro sus dos caras interiores verticales cóncavas hasta la profundidad de .30 m, correspondiendo estas concavidades al diámetro exterior de los cilindros. La distancia de los dos travesaños se fijaba por dos barras fuertes de hierro provistas con tal fin de tornillos y tuercas en sus extremidades.

De manera que cuando una sección del cilindro se izaba en su posición, por medio de la cabria situada entre las dos lanchas, las dos caras cóncavas se ponían en contacto con los lados del cilindro, y apretando luego los tornillos, el cilindro quedaba fijo en su lugar. Entonces se podía colocar otra sección de cilindro encima de la primera por medio de la cabria y unirlos con los pernos. Repitiendo esta maniobra, la altura del cilindro sería pronto demasiado grande é impediría poner otra sección encima por medio de la cabria, en cuyo caso, aflojando ligeramente las tuercas de los tornillos, se permitía al cilindro deslizarse suavemente en el agua hasta que su parte superior sobresaliera muy poco de la superficie del agua. Apretando nuevamente los tornillos, el cilindro estaba sujeto hasta que se unieran otras secciones encima de él, aseguradas por pernos. Cuando había temor

de que la presión hacia arriba del aire comprimido pudiera levantar un cilindro, entonces se levantaban los travesaños, por medio de la cabria, fuera de las lanchas y se colocaba sobre el cilindro una plataforma cargada de piedras.

**La cámara de extracción** estaba arreglada de tal manera que no era necesario quitarla cuando se ponía una sección nueva del cilindro. Esto se efectuaba del modo siguiente: Se unían las secciones del cilindro, como se ha descrito, hasta que el pie descansaba en el fondo del río, con su parte superior algunos decímetros sobre la superficie de la marea alta. Un diafragma pesado, de hierro fundido, de 30 mm de grueso, se ponía entonces sobre el cilindro para formar el piso de la cámara de extracción. Luego se arreglaba otra sección de 3 m del cilindro para formar la caja de la cámara de extracción. Éstas se unían por medio de pernos, y entonces se ponía otro diafragma en la parte superior para formar el techo de dicha cámara de extracción. Estos diafragmas estaban provistos de aperturas y puertas y válvulas correspondientes á las ya descritas en la fig. 17, y se dejaban en el cilindro cuando el trabajo estaba concluído. Si la profundidad del suelo que debía atravesarse para llegar á la roca firme, era tan grande que se hacía necesario añadir otras secciones de cilindro más arriba de la cámara de extracción, podía hacerse hasta donde fuera necesario, tanto más cuanto que no es esencial que la caja de extracción esté debajo del agua ó no. **Para impedir que ni el agua ni el aire puedan penetrar en el cilindro**, las caras de los rebordes se unen con una mezcla de minio y albayalde con fibras de algodón, antes de asegurarlas por medio de los pernos.

**En el puente de South Street**, los cilindros eran de 1.20, 1.80 y 2.40 m de diámetro, en secciones de 3 m de largo; de un espesor de 30 mm. Rebordes interiores de 70 mm de ancho y 30 mm de grueso, con pernos de 30 mm diámetro á distancia de 12½ cm entre sí de centro á centro. La arista de la sección inferior no tenía reborde. Una sección de cilindro de 3 m y de 2.4 de diámetro pesa 6,570 kg; de 1.8 m diám, 4,860 kg, y de 1.2 m, 3,060 kg. Un diafragma de 2.4 m, 1,260 kg; de 1.80 m, 720 kg; de 1.2 m, 352 kg. La roca debajo del suelo era desigual en algunos lugares; pero se nivelaba á medida que bajaban los cilindros, y éstos se aseguraban entonces á la roca firme por medio de abrazaderas y pernos.

**El trabajo se hizo día y noche**, verano é invierno, sin ninguna interrupción por las mareas, crecientes ó hielo flotante; las trece columnas fueron sumergidas rellenas de concreto, y el trabajo acabado en 11 meses; una gran parte del tiempo se gastó nivelando la roca y asegurando las abrazaderas. La falta de guías para la inmersión de los cilindros ocasionó mucho trabajo y dilación.

La diferencia entre la marea alta y baja era de 2.10 m más ó menos. La obra se hizo bajo la inspección del Sr. John Anderson, muy entendido y enérgico en esta clase de trabajos. **El costo total neto** de los cilindros colocados en su lugar, y rellenos de concreto hidráulico, fué aproximadamente de $302 por m de los de 2.4 m; $310 por los de 1.8 m, y $131 por los de 1.2 m de diámetro. Había tres cuadrillas de trabajadores, y cada cuadrilla trabajaba cuatro horas seguidas. La casa del Sr. Anderson (*Anderson and Barr; Tribune Building, N. Y.*), ha sumergido desde entonces con mucho éxito un número de estos cilindros, incluyendo (1884-5) cuatro de ellos de hierro forjado de 2.40 m de diámetro y 20 m de largo; inclinado 45° y colocados como puntales para impedir el movimiento de uno de los estribos del puente de la calle de Chesnut, Filadelfia.

**Se han reventado** varios cilindros de hierro fundido en varios puntos de su circunferencia en muchos puntos de los E. U. y en tiempo muy frío, debido á la diferencia de contracción del hierro y del relleno de concreto.

La ignorancia al usarlos puede traer grandes peligros.

La parte sombreada de la fig. 18 muestra una sección transversal de un **cajón de madera de pino amarillo** y cemento, para la torre del puente colgante sobre el **East River, en Brooklin** (N. Y.), de una abertura neta de 488 m. El cajón tiene un largo de 51.24 m en el fondo, y 31.11 m de ancho. La sección longitudinal se parece á la transversal con la diferencia de que es más larga y exhibe más tubos verticales J. De estos tubos hay 6, arreglados en pares para el trabajo rápido y como una precaución contra los accidentes. Es decir, 2 conductos de agua J, cada uno de $2.10 \times 1.95$ m, para sacar por medio de tinas y del aparato de suspensión el material excavado debajo del cajón, junto con el agua que pueda acumularse en *oo;* dos conductos de aire de 53 cm de diámetro, por los cuales se impele el aire de arriba hacia abajo para expulsar el agua de la parte CSSD debajo del cajón, y permitir que los trabajadores puedan minar por debajo, escapándose el agua debajo del pie CD del cajón al río; y dos conductos de suministro de 1.05 m de diámetro, para que puedan penetrar trabajadores, herramienta, etc. Todos estos

conductos tienen depósitos de aire en su parte superior, bajo el mismo principio que en la fig. 17, para evitar que el aire comprimido se escape por *ss*.

Los conductos son de planchas de hierro de calderas de 6 mm de grueso. El pie CD, de una altura igual á nueve espesores de trozos de madera, es continuo y se extiende alrededor del cajón; su parte inferior está calzada con hierro fundido; y

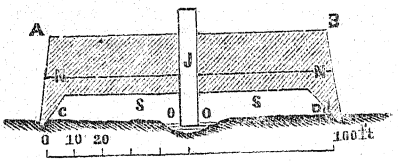

Fig 18

sus 4 esquinas reforzadas interiormente con ángulos de madera de 6 m de largo.

Desde el fondo hasta la línea NN, á 4.20 m, el cajón está construído de capas horizontales de maderas de 30 cm en cuadro cruzándose en ángulo recto, y las maderas de cada capa en contacto unas y otras bien unidas entre sí por pernos; todas las juntas cogidas con brea. Para impedir la filtración del agua, las tuercas y las cabezas de los tornillos tienen arandelas de goma elástica; también todas las juntas del lado exterior, lo mismo que las capas de madera NN, están bien calafateadas; y además un forro de hojalata, intercalado entre dos capas de fieltro, está colocado sobre cada junta del lado exterior, lo mismo que sobre la parte superior de la capa que está debajo de NN.

Cuando el cajón estaba concluído hasta NN, sobre la tierra, se echó á flote en su lugar y se fijó por medio de anclas; después de esto se agregaron, para sumergirlo quince capas de madera de 30×30 cm, colocadas á distancia de 30 cm entre sí, llenando los intersticios con concreto. La capa superior AB, es de madera sólida, para que sirva de sostén á las maquinarias, etc. Se sumergió pocos metros más bajo que el mismo fondo del río, para evitar el daño que podía producir la broma. Se sumergían á los lados cajas abiertas de madera para que sirvieran de muelles provisionales á las lanchas cargadoras del material excavado y á las embarcaciones que traían piedras, etc.

Después de sumergido el cajón, y expelida el agua del espacio CSSD, los trabajadores principiaron á excavar uniformemente el área cercada del fondo del río, para que de este manera el cajón pudiese descender hasta una capa inferior ó lecho firme. Después se construyó una ataguía sobre la parte superior del cajón, y en éste se principió la mampostería de la torre. La altura total de esta torre, incluyendo el cajón, es de 91.50 m más ó menos. Con respecto á los detalles, véase el informe de W. A. Roebling, ingeniero en jefe (1873).

**Los cilindros huecos, ú otras formas de obras de ladrillo ó mampostería, provistos en su parte inferior** de un brocal fuerte ó anillo de madera ó hierro, pueden sumergirse gradualmente, minando y excavando por debajo de ellos en su interior, y entonces forman cimientos muy estables. Debajo del agua puede hacerse esto en medio de excavadoras de forma adecuada, con ó sin el auxilio de campanas de buzos, según la profundidad, etc. En tierra este método es mucha veces el más económico y satisfactorio, especialmente en terrenos duros. Puede ayudarse su descenso colocando una carga sobre ellas, si el frotamiento de sus lados contra la tierra exterior impide que se sumerja por su peso solo, como sucede algunas veces. Un cilindro de mampostería de ladrillos de 14 m de diámetro exterior con sus muros de .9 m de espesor, se ha sumergido hasta una profundidad de 12 m en arena seca y granzón sin dificultad alguna. Se construyó de 4 m de alto (sobre un brocal de madera de 53 cm de grueso), y pesaba 300 toneladas cuando se principió la inmersión. La tierra interior se excavaba lentamente, de manera que el descenso era, más ó menos, de 30 cm por día, levantándose los muros á medida que descendía el cilindro. Conductos verticales para túneles se construyen algunas veces de esto modo.

**En las orillas del Rhin** para construir un pozo vertical de una mina de carbón, se sumergió primero de este modo y por su peso propio, á una profundidad de 23.18 m, un cilindro de mampostería de ladrillos de 7.5 m de diámetro por entre arena y granzón; luego un cilindro interior de 4.5 m de diámetro se sumergió del

mismo modo á una profundidad de 78 m debajo de la superficie, de cuya profundidad los primeros 55 m debajo del primer cilindro fueron de una arena movediza. A 78 m de profundidad el frotamiento era tan grande que el cilindro quedó inmóvil. La arena movediza se excavó por medio de taladros : no se extrajo el agua con la bomba sino que se dejó lleno el cilindro.

Los cimientos enteros de una pila grande de mampostería se han sumergido de este modo en un solo bloque, dejando un numero suficiente de aberturas verticales para el descenso de los trabajadores ó para la herramienta. Ésta es, generalmente, una operación muy lenta y tardía, especialmente debajo del agua. Muchas veces puede hacerse más expedita por medio de campanas de buzos. Es preferible, generalmente, hacer el bloque de mampostería más ancho en el pie que sobre él, para disminuir así el frotamiento contra el terreno exterior. Cuando se trabaja en tierra se emplea de vez en cuando agua para humedecer el fondo. Conservando seco el interior de esta mampostería hueca, se puede **construir de la superficie hacia abajo**, socavando solamente una parte de su circunferencia, llenando dicha parte de mampostería, y luego minando y llenando la otra porción, y así sucesivamente 60 ó 90 cm á la vez. Puede adoptarse este método cuando el frotamiento ha detenido el descenso de la masa de mampostería por su peso propio, estando socavada por debajo.

**La bomba de arena**, como se usó en el puente de San Luis, puede ser de utilidad algunas veces para sacar arena de los cilindros, mientras se están sumergiendo en agua. Con un tubo de bomba de 9 cm de diámetro, y un chorro de agua bajo una presión de 10 kg por cm cuad, se sacaron 15 metros cúbicos de arena por hora á una altura de 38 m. También se usó con éxito un chorro de aire como en el puente colgante de Brooklyn.

**Fajinas.** Sobre terrenos cenagosos ó de arena movediza mojada pueden hacerse los cimientos depositando primero capas de fajinas ó ramos pequeños ó varillas gruesas, amarradas firmemente en líos de 1.8 á 3.6 m de largo, y de 15 á 60 cm de diámetro. Las capas de los líos deben cruzarse. Se hace primeramente una especie de balsa flotante ó colchón, y ésta se sumerge cargándola con tierra, piedras, granzón, etc. De este modo se hicieron (1852) los cimientos para los estribos y pilares del gran puente colgante de Kieff, en Rusia, con aberturas de 134 m y sobre una arena movediza. En este caso las balsas de fajinas se extienden 30 m más allá de las bases de la mampostería que descansa sobre ellas.

Pueden usarse la fajinas del mismo modo para sostener terraplenes de ferrocarriles, etc., en terrenos cenagosos; pero bajan considerablemente.

**Pilotes de arena.** Hemos aludido ya al uso de la arena bien pisada en capas dentro de zanjas ó excavaciones de cimientos; pero también puede usarse en terrenos blandos en la forma de estacas. Primero se entierra una estaca corta y gruesa á la profundidad de 1.5 ó 3 m ó más según el caso; luego se saca ésta y se llena el agujero de arena bien mojada y bien pisada. Entonces se entierra la estaca en otro lugar y se repite la operación. Los espacios de una á otra pueden ser de 30 á 90 cm. Sobre estas estacas ó pilotes de arena pueden ponerse plataformas lo mismo que sobre estacas de madera. Si se echa arena seca y no mojada, en los pozos, hay peligro de que baje mucho después con el agua de lluvia ó de manantial. En este caso, lo mismo que en las fajinas, es bueno probar los cimientos por medio de una carga de prueba. Algún descenso debe efectuarse inevitablemente mientras todo se sitúa en su posición definitiva; pero será relativamente de poca importancia. Lo mismo ocurre hasta cierto grado en toda construcción de cierta magnitud : como en los techos ó arcos de gran abertura, sean de madera, hierro ó mampostería; lo mismo que en pilas, muros elevados, etc. Los cimientos en terrenos arenosos debajo del agua debían estar cerrados por pilotes de palastro para impedir que salga la arena en caso de que la exterior fuera arrastrada por el agua y también debían estar defendidos por un depósito de piedras colocadas alrededor. Sobre fondos malos debajo del agua se han depositado ó formado pequeñas islas artificiales de buen terreno para servir de cimientos á la mampostería. Pueden construirse algunas veces y ventajosamente por este sistema esclusas de canales y otras obras en tierras cenagosas. Si fuere necesario pueden dragarse pocos m de profundidad antes de depositar el material más firme, y este último puede cargarse primero para probar su estabilidad. Se ha empleado últimamente con buen éxito, en aguas profundas, el sistema de construir cimientos bajo el agua construyendo la mampostería sobre una plataforma de madera colocada sobre el agua, sostenida por tornillos y sumergida en el agua á medida que se termina al aire libre una hilada. Sin embargo esto no es nuevo. Fué deado por Belidor hace más de un siglo. Se entierran estacas ó pilotes á distancia de 1.8 á 3 m unas de otras alrededor del lugar que debe ocupar el pilar ó estribo,

unidas sus cabezas por fuertes piezas. Estas últimas sostienen los tornillos que trabajan á través de ellas. La construcción entera tiene ligazones contra el movimiento lateral. Un grupo de estacas bien enterradas y encerradas después por un cilindro de hierro sumergido hasta un fondo duro y lleno de concreto, constituye un cimiento excelente. Las estacas pueden llegar hasta la parte superior del cilindro y de este modo entrar en el concreto. Un sistema como éste ha sido patentado por S. B. Cushing, I. C. El cilindro y el concreto sirven para proteger las estacas de la broma é impedir que se pique la porción que queda fuera de la baja marea, pero no están puestos con la idea de soportar carga alguna.

# TRABAJO EN PIEDRA*

En los lugares donde se hacen obras en grande escala, la extracción de piedras por medio de explosivos puede hacerse algunas veces al precio de 13 á 27 centavos menos por m cúbico **taladrando á máquina y con dinamita** que usando taladros de mano y pólvora. Sin embargo, ordinariamente **el costo es más ó menos lo mismo** y la ventaja de los métodos más modernos consiste más bien en la economía del tiempo y en la conveniencia de tener el trabajo mejor inspeccionado. En trabajos ordinarios de ferrocarriles sobre roca, medianamente dura, y con jornal común de $1 por día de 10 horas, el costo por m cúbico de aflojar la piedra, es, generalmente, de 40 á 80 centavos, incluyendo herramienta, perforaciones, pólvora, etc.

**Las perforaciones para los barrenos, hechas á mano**, son generalmente de .75 á 1.20 de profundidad, y de 37 á 50 mm de diámetro. La perforación con chompa** grande es mucho más ligera y económica que la ejecutada con chompín, á martillo. La chompa es una barra redonda de hierro, generalmente de 1.8 á 2.40 m de largo, con un filo cortante de acero, que es un poco más ancho que el diámetro de la barra. Un hombre la levanta unos cm, ó, más bien, la agarra cuando rebota, la hace girar parcialmente, y la deja caer de nuevo. De este modo puede taladrar un agujero de 1.5 á 4.5 m de profundidad de cerca de 5 cm de diámetro en un día de 10 horas de trabajo, según la clase de la roca. La ejecución de 2.10 á 2.40 m de barreno de 37 mm de diámetro, es, más ó menos, la tarea regular de un día en *gneis* duro, granito ó piedra cálcarea compacta; de .9 á 1.5 en cuarzo sólido; de 2.4 á 2.70 m en mármol común ó piedra cálcarea, y de 2.70 á 3 m en piedra arenisca; pero todo esto puede variar dentro de los límites expresados. Cuando la perforación tiene más de 1.2 m, dos hombres trabajan al mismo tiempo con la misma barra.

La chompa *(jumper)*, como se usa ahora es mucho más corta ***. Un hombre (el *sostenedor*) sentado la levanta un poco, y la da vueltas parcialmente durante los intervalos entre cada dos golpes de un martillo de 3 á 5 kg de peso más ó menos, manejado por otro. Este barreno puede aplicarse á perforaciones de un diámetro menor que el que puede hacerse con una chompa. También es mejor en rocas conglomeradas, en las cuales los guijarros duros silíceos hacen salir el barreno grande de su dirección vertical, de modo que el agujero sale torcido y la barra se atraca dentro de él. Los conglomerados de carbón pueden perforarse fácilmente con la chompa. Anteriormente se empleaba la chompa también para perforaciones profundas, antes de que se reconociera la superioridad del taladro giratorio (*churn-drill*); es necesario sacarle nuevo filo cuando se ha perforado de 15 á 45 cm, y el desgaste del filo de acero obliga á renovarlo cada dos á cuatro días. En los *chompines* de hierro la parte superior se destruye prontamente. A medida que la perfo-

* *N. del T.* — Aunque en otros países los cálculos de valor del material y costo de las obras varían con los diversos jornales, cualidades del obrero, etc., dejamos esta clase de ejemplos porque siempre serán de utilidad para el cálculo de obras y presupuesto. Como el autor los trae todos en medidas inglesas: pies cuadrados, yardas cúbicas, etc., hemos reemplazado los ejemplos y presupuestos por los análogos y equivalentes, en sist metr con suficiente exactitud para cada caso.

** *N. del T.* — El término chompa, aunque no es castizo, es muy corriente en Venezuela.

*** *N. del T.* — Esta chompa más corta se llama en Venezuela *chompín*.

ración se hace más honda, se emplean frecuentemente barrenos más largos que al principio. Mientras más pequeño sea el diámetro de la perforación, tanto más profundo puede hacerse en un tiempo dado; están en razón inversa. Bajo circunstancias análogas, tres obreros, con una chompa, harán, más ó menos, la misma perforación que un hombre con un taladro giratorio.

**El taladro de mano**, en el cual el mismo hombre maneja el martillo y la barra, se usa principalmente para perforaciones de poca profundidad y de poco diámetro. Con él un obrero puede hacer tantos metros de perforación de 15 á 30 cm de hondo cada una y 18 mm de diámetro, como los que pueda hacer otro obrero con perforaciones de 90 cm de profundidad, más ó menos, y de 50 mm de diámetro, en el mismo tiempo, pero con un taladro giratorio (*churn-drill*).

Solamente la chompa y el taladro de mano son prácticos para hacer perforaciones horizontales, ó muy inclinadas.

**Costo de la piedra de cantera.** Después de los gastos preliminares para comprar el sitio de una buena cantera, limpiar la tierra de la superficie y despejar la roca superior; y después de proveer las herramientas, carros, grúas, etc., necesarios, los gastos netos totales para *extraer* la piedra bruta para albañilería, por metro cúbico, lista para entregarla, pueden calcularse aproximadamente así: piedras de tamaños tales que dos hombres puedan fácilmente levantarlas, medidas *en montones*, costarán, poco más ó menos, tanto como de $^1/_3$ á $^2/_3$ del jornal de un obrero de cantera. Grandes piedras, que varíen desde ½ á $^1/_4$ metro cúbico cada una, extraídas por explosión, de 1 $^1/_2$ á 2 $^1/_2$ jornales por metro cúbico. Grandes piedras, que varíen desde 1 á 1½ metros cúbicos cada una, en las cuales la mayor parte del trabajo se haya hecho con cuñas, á fin de que cada piedra salga en forma tolerablemente regular, y conforme á dimensiones estipuladas, desde 3 á 5 jornales por metro cúbico. Los precios más pequeños son bajos para el asperón, mientras que los más altos son altos para el granito. En circunstancias ordinarias, cosa de 1 $^1/_2$ de metro cúbico de buen asperón puede extraerse al mismo costo que 1 de granito; ó, en otros términos, fijando el costo del granito en 1, el del asperón será $^2/_3$; de modo que los promedios de los límites precedentes pueden considerarse más bien como precios completos para asperón; escasos para granito, y casi regulares para piedra caliza ó mármol.

**Costo de la labrada de piedra.** En primer lugar hay que hacer una concesión liberal por desperdicio. Aun cuando la piedra se saque bonita por todos lados de la cantera, en grandes bloques de casi la forma y tamaño requeridos, de $^1/_4$ á $^1/_4$ del bloque bruto, se gasta generalmente en desperdicios cuando está bien labrado. En bloques de tamaño moderado (digamos de un tamaño medio de ½ metro cúbico cada uno), y extraídos por explosión, una pérdida de $^1/_4$ á $^1/_3$ no será exagerada para piedras que tienen mediana facilidad para henderse derecho. Por este motivo hay también que calcular mucha pérdida para las piedras pequeñas bien desbastadas. Mientras más pequeñas sean las piedras, mayor debe ser la concesión por desperdicio en el labrado. En grandes trabajos, es bueno hacer labrar las piedras lo más posible en la cantera, á fin de disminuir el costo del transporte, pues cuando la distancia es grande, y especialmente por tierra, y por caminos comunes, el transporte constituye un gasto importante.

**Un cantero** labrará cómodamente con martillo adecuado cosa de $^1/_3$ á 1 metro cuadrado de cara lisa en granito duro en 8 horas de trabajo; ó el doble de un labrado inferior que ordinariamente se ejecuta en los lechos y cerca de las juntas; y también en las caras de piedras para puentes, etc., cuando no se requiere un acabado muy perfecto. En buen asperón, ó mármol, se puede hacer como $^1/_4$ más que en granito. Del **más perfecto** acabado á martillo, en **granito**, se hace ½ metro cuadrado, más ó menos.

**Costo de la mampostería.** Todos los elementos del costo total son susceptibles de mucha variación; en consecuencia, sólo podemos dar un ejemplo tratando el asunto desde un punto de vista muy general, para que sobre esto pueda hacerse un cálculo más aproximado. Supondremos que el salario de un obrero sea $2.00 por día de 8 horas de trabajo; y $3.50 para un albañil. El monopolio de las canteras afecta mucho * los precios.

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* Los bloques de granito para el **monumento de Bunker Hill**, por término medio de 1½ metros cúbicos cada uno, fueron trabajados á cuña y entregados por la Asociación del Monumento en el sitio mismo, á un costo líquido de $7.03 por metro cúbico, y de una cantera abierta con ese fin. La Asociación no obtuvo ninguna utilidad; sus servicios fueron gratuitos. Las ofertas medias de contrato para ellos fueron de $32 metro cúbico. El costo de los bloques brutos en la cantera fué de $3.50. La cargada en carros en la cantera, cosa de 20 centavos. Transporte de 13 kilómetros por ferrocarril y camino

**Costo de la mampostería de piedra labrada** *. Tamaño medio de las piedras, digamos 1.50 m de largo, .60 m de ancho y .42 m de alto; entran casi tres piedras en metro cúbico. Entonces, suponiendo que la piedra sea granito ó *gneis*, el costo por métro cúbico de mampostería **con tales salarios**, será :

| | |
|---|---|
| Obtención de la piedra de la cantera por explosión, concediendo $^1/_4$ por desperdicio en el labrado; $1^1/_2$ metros cúbicos, á $4.00 por metro | $5.33 |
| Labrado de 1½ m cuadrados de cara exterior á $3.5 | 5.25 |
| — de 6 m cuadrados de lechos y juntas á $1.8 | 10.80 |
| Costo neto de la piedra labrada en la cantera | 21.38 |
| Acarreo, digamos 1 milla; carga y descarga | 1.56 |
| Mortero, digamos | .52 |
| Colocación, inclusive andamios, máquina de elevar pesos, superintendencia, etc. | 2.60 |
| Costo neto | 26.06 |
| Utilidad del contratista, digamos 15 por ciento | 3.90 |
| Costo total | 29.96 |

El labrado costará más, si las caras han de ser redondeadas ó moldeadas. Si las piedras son más pequeñas que las que hemos supuesto, habrá que labrar mayor superficie por metro cúbico.

Si en el caso precedente, las piedras han de labrarse *perfectamente* bien en todos los lados, inclusive la parte posterior, el costo por metro cúbico aumentará cosa de $13; y, si algunos de los lados fueren encorvados, como en las piedras de arco, calcúlese $15 ó $18 de aumento; y si los bloques han de ser cuidadosamente trabajados con cuña y con dimensiones dadas, $21 ó $24; llegando así el costo neto de la piedra labrada *en la cantera* á $34, $36 ó $39 por metro cúbico.

El gasto de colocación se aumentará mucho, si la piedra ha de levantarse á grandes alturas, ó si ha de ser muy movida, como cuando se lleva en lanchones para ser depositada en muelles, etc. Casi todo gran trabajo presenta ciertas peculiaridades que alteran los cálculos y que deben dejarse al juicio del ingeniero y del contratista. El tanto por ciento de la utilidad del contratista es ordinariamente menos en grandes trabajos que en pequeños.

**Costo de la mampostería de piedra labrada.** Si la piedra es asperón con buenos lechos naturales, la obtención puede fijarse en $4.00 por metro cúbico. El labrado de las caras exteriores á $2.86 por metro cúbico. Lechos y juntas á $1.40 por metro cuadrado; digamos $8.40 por metro cúbico. El costo neto, colocado, como $22.

Y el **costo total de mampostería de grandes asperones bien desbastados con argamasa** puede apreciarse en cosa de $13 por metro cúbico.

**Costo de piedras gruesas de granito desbastadas**, como se usan de ordinario para rellenar el sillar precedente; piedras que tienen por término medio cosa de ½ metro cúbico cada una :

| **Trabajo á $1 por día.** | Costo por metro cúbico de albañilería. |
|---|---|
| Obtención de la piedra de la cantera por explosión, concediendo $^1/_5$ por desperdicio en el desbaste; $1^1/_7$ m cúbico á $4.00 | 4.57 |
| Acarreo 1600 m, carga y descarga | 1.56 |
| Mezcla (73 litros de cal viva, en terrón ó en polvo; y 365 litros de arena ó cascajo; y la mezclada) | 1.95 |
| Desbaste; colocación, inclusive andamios, máquinas para levantar, etc. | 3.25 |
| Costo neto | 11.33 |
| Utilidad del contratista, digamos 15 por ciento | 1.70 |
| Costo total | 13.03 |

**Las piedras pequeñas** **, que puedan manejar dos hombres, cuesta

ordinario, $3.32. Total, $7.02. En 1825 á 1845 el trabajo de un obrero inhábil, común, valía por término medio $1 por día.

* Véase *N. del T.* * pág. 644. Los $ son dólares.

** Véase *N. del T.* * pág. 644.

extraerlas de la cantera cosa de 1.04 por metro cúbico en montón; ó calculando desperdicio, digamos $1.30. Acarreo 1 milla, $1.30. Pueden ser desbastadas toscamente y colocadas por $1.56 más; mezcla como queda dicho, $1.95. Costo neto total, $6.11; y, con 15 por ciento de utilidad, $7.03 *con los salarios dichos.*

**Con piedras más pequeñas,** como las que puede manejar un hombre, podemos calcular: piedra 90 centavos; acarreo, $1.30; colocación, andamios, herramientas, etc., $1.30; mezcla, $1.95. Resultando el costo neto, $5.45; ó, con 15 por ciento de utilidad, $6.27. El trabajo de ordenación mediana nitidamente desbastaba, cuesta de $2.6 á $4 más por metro que con ripio ó piedras sueltas, según la naturaleza de la piedra, etc. El levantamiento de paredes delgadas cuesta relativamente más que el de paredes gruesas.

**El costo de revestimientos de sillería lisos de 20 cm de espesor** para habitaciones, etc., en Filadelfia, en 1888, es próximamente como sigue por metro cuadrado de paramento exterior, todo incluído. Asperón, $16 á $24.30; mármol de Pensilvania, $27; mármol de Nueva Inglaterra, $30 á $35; granito, $24 á $30. Si es de 15 cm de espesor, hay que deducir la octava parte. *Con piedra artificial de primera clase* podía hacerse por la tercera parte del precio. **Baldosas de piedra azul de North River,** de 7½ cm de espesor, para aceras, colocadas, inclusive arena gruesa, etc., $7.50 por metro cuadrado. **Pavimento belga para calles,** con arena gruesa, completo, $4.55 por metro cuadrado en las ciudades del Este.

Cuando el revestimiento de piedra labrada está respaldado por ripio, el gasto por metro cúbico de la masa entera variará por supuesto según las proporciones de los dos. Así, si la piedra labrada cuesta $15.5 por metro y está respaldada por igual espesor de ripio á $6.5, el costo medio será ($15.5+6.5) ÷ 2=$11; ó si el ripio es dos veces tan ancho como el sillar, entonces, (15.5+6.5+6.5) ÷ 3=9.5. **Tales paredes compuestas son débiles** y susceptibles de separarse con el tiempo, como también las paredes de piedra tallada respaldadas por concreto, ó por ladrillo, á causa de que no se asientan lo mismo las dos partes. A veces al contratista deben hacérsele concesiones extraordinarias cuando tiene que abrir nuevas canteras, ó construir caminos ó abrir cimientos, ó por tener que bombearlos, estaquearlos, etc., cuando inesperadamente se encuentran manantiales; para las cimbras, para arcos, etc., á menos que estas partidas estén incluídas en el contrato.

# MUROS DE SOSTENIMIENTO

**Art. 1. Muro de sostenimiento** es el destinado á soportar la presión de las tierras, arenas ú otros *rellenos* que se depositen detrás de él, después de construído, para distinguirlo de los **muros de revestimiento,** que es una construcción semejante, erigida para evitar la caída de las tierras, que permanecen inmóviles en posición natural, á las cuales se ha excavado una cara vertical ó inclinada. La tierra, en este caso, está tan consolidada que no ejerce ninguna, ó muy poca, presión lateral, y por lo tanto, el muro puede, generalmente, ser más delgado que el de sostenimiento.

Esto, sin embargo, depende de la naturaleza de la posición de las capas del terreno en el cual se ha preparado la cara. Si las capas son de roca, con camadas interpuestas de arcilla, de tierra ó arena, ó si están inclinadas hacia el muro, puede necesitar un espesor mayor que cualquier muro de sostenimiento ordinario, porque cuando las capas delgadas de tierra se ablandan por la filtración de la lluvia, actúan como un lubricante semejante al jabón ó al sebo, y facilita el deslizamiento de las capas de roca y de este modo producen una presión enorme contra el muro. O la roca puede entrar en movimiento por la acción de la escarcha sobre las capas de greda, ó, como suele suceder, por la trepidación producida por los trenes que pasan. Aun no existiendo roca, si las capas del suelo se inclinan hacia el muro, habrá siempre peligro de un resultado semejante, y deben tomarse precauciones adicionales, especialmente cuando las capas alcanzan á una altura mucho mayor que el muro. **En un muro vertical,** tanto *c o* como *d s* son verticales.

**La experiencia, más bien que la teoría,** debe guiarnos en la construcción de ambas clases de muros. Recomendamos que el espesor horizontal *ab*, fig. 1, en la base de un muro de sostenimiento vertical ó casi vertical *cdba*, que sostiene un relleno, sea de arena, granzón ó tierra, á nivel con su parte superior *cd*, como en la figura, no sea menor de lo que se indica en seguida en la práctica de ferrocarriles, cuando los cimientos no tengan sino una profundidad de .90 m más ó menos.

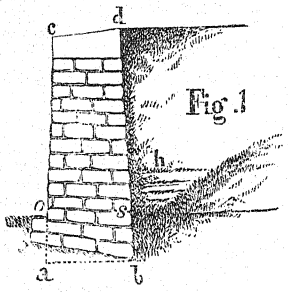


Fig. 1

**Cuando el relleno se ha depositado flojamente (es decir, cuando es de tierra echada) como de costumbre, tal** como cae de los carros, carretas, etc.

| | | |
|---|---|---|
| Muros de piedra tallada ó de piedras brutas dispuestas en hiladas grandes de primera clase, en mortero *ab* | .35 | de su altura total vert *db*. |
| Muro de buenas piedras brutas comunes desbastadas en mortero, ó de ladrillos | .4 | — — — |
| Muro de piedra en seco, bien desbastada | .5 | — — — |

Con buena mampostería, sin embargo, podemos tomar altura *ds* en lugar de *db* y entonces las fracciones de la altura arriba dadas darán un espesor suficiente en la línea de tierra *os*. Cuando el relleno está algo consolidado en capas horizontales, pueden reducirse cada uno de estos espesores, pero no se pueden dar reglas fijas para este caso. La parte saliente *oe* de la base del muro no está comprendida en estos espesores. Sin embargo, cuando el relleno es de arena limpia pura ó de granzón debemos darle las dimensiones enteras, por cuanto la trepidación producida por los pasos de los trenes perturbando la cohesión neutraliza cualquier ventaja supuesta al apisonar el material. Tales arenas pueden apisonarse con mucha ventaja con el propósito de comprimirlas en los cimientos; pero esto hace diferente el caso. Cuando se hace esto, aun en tierras de cohesión para economizar mampostería en los muros de sostenimiento, es probable que el costo resulte igual al de la mampostería economizada. La base, *ab*, fig. 1, es $^4/_{10}$ de su altura *bd*. En los espesores mencionados de la base la cara interior del muro *db* se supone vertical y la *ac* vertical ó inclinada hacia atrás, pero no más de 1 en 8, más ó menos. De este límite no debe pasarse en la práctica, debido á los malos efectos de la lluvia, etc., sobre el mortero, cuando la inclinación es mayor. La base de un muro vertical o requiere en efecto tanto espesor como uno inclinado; pero cuando la inclinación no pasa de 1 en 8, la diferencia es muy pequeña. Véase tabla, art. 7.

*Nota* 1. **Una mezcla de arena ó de tierra con una fuerte proporción** de guijarros redondos grandes, ó de piedras de las usadas para empedrar, etc., pesa considerablemente más que los materiales usados ordinariamente para rellenos y ejerce una presión mayor contra el muro, cuyo espesor debe aumentarse de un octavo á un sexto, cuando el relleno es de aquella naturaleza.

*Nota* 2. El muro será más fuerte, si todas las **hiladas de la mampostería se ejecutan con una inclinación hacia adentro,** como en, *oeb*, especialmente en la mampostería de piedra en seco, ó cuando no se le puedadar al mortero el tiempo suficiente para que tragüe bien, antes de colocar el rellen detrás de él. El objeto de esta inclinación de las hiladas de mampostería es colocar las juntas lo más próximamente en ángulo recto á la dirección de la presión *fP*, figs. 6, 7, 8,

contra el paramento interior del muro, para de este modo disminuir la tendencia de las piedras á deslizarse una sobre otra, y causar el desplomo del muro.

Cuando las hiladas son horizontales, no hay otra cosa que impida este deslizamiento, que el frotamiento de una piedra contra otra, si la mampostería es de piedra en seco; ó el frotamiento con el mortero, cuando se usa este último. Pero cuando, como ocurre á menudo (especialmente en muros gruesos construídos con mucho apuro), el mortero no tiene tiempo de fraguar, no opondrá sino muy poca resistencia al deslizamiento. Cuando las hiladas están inclinadas, no pueden *deslizarse*, sin que al mismo tiempo tengan que subir por el plano inclinado formado por ellas mismas. En los muros de sostenimiento, como en los estribos de arcos importantes, el ingeniero debe (en cuanto á la estabilidad) confiar lo menos posible en el mortero, y más bien en la posición de las juntas. Hay que hacer una objeción á esta inclinación de las juntas en los muros de piedra en seco, y es que el agua de lluvia, al caer sobre la cara inclinada de un muro, corre por las juntas hacia el interior del muro, en el relleno, el cual se ablanda y se asienta ó baja más. Pero esto puede obviarse en gran parte haciendo horizontales las hiladas exteriores ó de la cara, ó aplicando mortero en esta parte hasta una profundidad de .30 m, más ó menos. La parte superior del muro debe defenderse con un caballete *cd*, fig. 1, que sobresalga algunos centímetros del frente. Después de construída la mampostería hasta la superficie del suelo, deben rellenarse las excavaciones hechas para los cimientos, y es bueno consolidar este relleno apisonándolo, especialmente enfrente del muro.

**El paramento interior del muro debe dejarse áspero.** En los muros de mampostería de ladrillos sería bueno que cada tercera ó cuarta hilada sobresaliese de 3 á 5 cm. Esto aumenta el frotamiento de la tierra contra el paramento interior del muro, y de este modo la resultante de las fuerzas que actúan detrás del muro se acerca más á la vertical y cae más hacia dentro en la base, aumentando la estabilidad. También conduce á aumentar la resistencia el no hacer cada hilada de mampostería de una altura uniforme en todo el espesor del muro, sino colocar algunas piedras (especialmente cerca del paramento interior) de la altura de dos ó tres hiladas. De este modo toda la mampostería está más entrelazada, más unida, y por consiguiente más sólida. Se pueden hacer muros muy gruesos con el paramento exterior de mampostería y el interior de concreto.

*Nota* 3. La presión de la tierra misma contra la cara interior es la que produce el frotamiento, el cual á su turno modifica la acción de la presión; porque el peso ó la presión de un cuerpo sobre un plano inclinado produce frotamiento entre el cuerpo y el plano, suficiente á veces para impedir que el cuerpo resbale. Un muro de sostenimiento se *vuelca* al hacerle girar alrededor de su arista exterior *e*, fig. 1, como apoyo, pero para esto su paramento interior debe, evidentemente, elevarse primero; y al hacer esto tiene que rozar contra el relleno, y de consiguiente vencer este frotamiento. El frotamiento es el mismo esté el muro firme ó no; como en el caso de un cuerpo sobre un plano inclinado; la única diferencia es que en un caso *impide* el movimiento y en el otro sólo lo *retarda*.

**En el lugar donde pueda ocurrir una congelación fuerte,** al paramento interior del muro debe dársele una escarpa hacia adelante de 60 á 90 cm

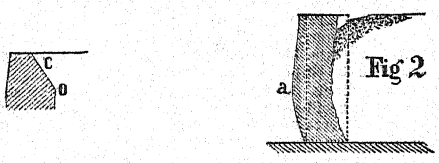


Fig 2

en su parte superior, *co*, la que debe ser enteramente lisa para disminuir la adhesión de la escarcha.

***Nota* 4.** Cuando el muro es muy delgado forma una *comba* ó *abombamiento* hacia fuera, próximamente á un tercio de su altura, sobre la superficie, y generalmente se cae, fig. 2.

Un ligero abombamiento en un muro nuevo no prueba *necesariamente* que esté inseguro. Esto se debe generalmente á lo nuevo del mortero, y á la presión mayor que ejerce el relleno nuevo; muchas veces deja de aumentar en algunos meses

No debe uno preocuparse si el abombamiento no excede de $^1/_{48}$ del espesor en *a*. Véase nota 3, art. 7.

**Art. 2.** El ingeniero joven no tiene necesidad de ocuparse mucho en la práctica **del peso específico exacto del relleno**, ni del ángulo de inclinación bajo el cual se sostiene éste, porque el material que se deposita detrás del muro hoy, puede estar seco é incoherente, como para que resbale por un talud de 1½ á 1; y mañana la lluvia puede convertirlo en un fango líquido, que busca su nivel como el agua; al día siguiente puede estar helado y capaz de sostener una carga considerable, como por ej. : un pilar vertical.

Además, no puede saberse de antemano, cual pueda ser la naturaleza del relleno; porque como regla general, éste debe ser de cualquier material que la excavación más cercana produzca de tiempo en tiempo; hoy arena, roca mañana, etc. Los muros de sostenimiento por lo regular se construyen antes de que el ingeniero conozca la condición del relleno; de modo que, en la práctica, estas consideraciones teóricas tienen comparativamente poca importancia. La teoría, sin estar sancionada por la observación y el sentido común, nos conducirá á cometer graves errores en todos los ramos de la ingeniería; pero, por otra parte, ningún grado de experiencia, por sí solo, compensa la ignorancia de la teoría. Las dos deben andar juntas.

Por otra parte, **el asiento del relleno por su propio peso**, ayudado por la trepidación que produce el pasaje rápido de trenes pesados, su expansión cuando se hiela, ó la filtración de la lluvia, la presión hidrostática producida por la penetración de ésta en las grietas que se forman en el relleno durante un prolongado tiempo seco, lo mismo que la acción lubricante que ejerce sobre él (que disminuye su frotamiento y le comunica una tendencia á deslizarse), etc., ejercen algunas veces una tendencia al volcamiento, tan fuerte que alcanza á lo indicado por la presión teórica. La acción de estos agentes es gradual. La observación atenta de los muros de sostenimiento ha demostrado, año tras año, que sus paramentos inclinados se vuelven verticales. Luego principian á inclinarse hacia afuera, y, por último, algunos se caen. La teoría hace omisión de las cargas que puedan sobrevenir y aumentar la presión del relleno.

Suponiendo correctas las ideas avanzadas por el profesor Moseley como teorías, los espesores que hemos recomendado en el art. 1 para muros de mampostería hechos con morteros, corresponden á presiones calculadas por él de 7 hasta 14 veces mayores, y en los muros de piedra en seco, más ó menos de 10 á 20 veces la presión señalada por él; y no obstante, no consideramos que nuestros espesores sean mayores de lo que la experiencia exige. Véase la tabla 3. Muros de sostenimiento calculados por buenos ingenieros, pero muy ajustados á la teoría (la cual supone suficiente una resistencia igual al doble de la presión teórica), han fallado; y es natural deducir que muchos de los existentes tienen un coeficiente de seguridad demasiado pequeño.

El hecho es (ó por lo menos así nos parece) que deben existir defectos en las suposiciones teóricas de algunos de los autores más prominentes que suministran reglas prácticas sobre esta materia. Así, Poncelet, que ciertamente está á la cabeza de ellos, dice que sus tablas de uso práctico dan espesores en la base para soportar 1.8 veces la presión teórica, y la considera suficientemente segura. Sin embargo, él trae para un muro vertical de granito tallado, construído para sostener arena seca hasta el nivel de su parte superior, como en la fig. 1, una base de .35 de su altura vertical, y para ladrillos .45; pero el autor halló que no habiendo trepidación, un modelo de madera de muro vertical, que pesaba sólo 441 kg por metro cúbico, con una base de .35 de altura, hizo equilibrio perfectamente á arena seca con talud de 1½ por 1, pesando 1402 kg por metro cúbico.

Ahora bien; la resistencia de muros semejantes de las mismas dimensiones, varía como sus pesos específicos; y como el granito pesa más ó menos 2,600 kg por m cúbico ó como 6 veces más que nuestro modelo, se sigue que un muro de dicho material con una base de .35 de su altura debe tener una resistencia 6 veces mayor, en lugar de 1.8 solamente, y un muro de ladrillos más ó menos 5 veces la resistencia teórica. Nuestros experimentos se hicieron en la habitación alta de un edificio sólidamente construído, y hallamos que la trepidación producida por los vehículos que pasaban por la calle, cerradas las puertas, ó simplemente por caminar en el cuarto, bastó para producir, gradualmente, desplomos en los muros, y creemos que los efectos perjudiciales de un tren pesado serían no menos grandes comparativamente sobre un muro de sostenimiento que soporte un material tan incoherente como la arena. Desde que el muro de Poncelet es en este ejemplo suficientemente estable en la práctica, nos parece que su teoría, que no considera los esfuerzos de la trepidación, debe ser defectuosa. Él también da $^1/_5$ de la altura como espesor de sufi-

ciente seguridad para un muro vertical de granito que sostenga una tierra dura; pero sospechamos que á pocos ingenieros les gustaría fiarse de esta proporción especialmente durante las lluvias y cuando, como generalmente se hace, la tierra está simplemente echada. Si la tierra se deposita, y se consolida en capas, la teoría nos dará poquísimo espesor para el muro, porque el relleno se vuelve, por decirlo así, una masa de ladrillos crudos, que no ejerce ninguna presión horizontal, y que sólo requiere la protección contra la influencia atmosférica, para obtener su estabilidad sin muro de sostenimiento. Es con mucha timidez y desconfianza en nuestras opiniones, que nos atrevemos á expresar nuestras dudas con respecto á las ideas de un investigador y escritor tan profundo como Poncelet; lo hacemos solamente con la esperanza de que personas más competentes que nosotros emitan sus opiniones. Las nuestras no tienen otra base que los experimentos hechos por nosotros mismos con modelos de madera y de ladrillos, en combinación con las observaciones realizadas sobre muros existentes.

**Art. 3.** Después de trazar un muro *abco*, fig. 3, de paramento interior vertical según las proporciones de nuestra regla del art. 1, **se puede convertir en uno de paramento interior en escalones como** *aino*. Este presentará mayor resistencia al volcamiento, y sin embargo no contiene más material. Por el centro *t* del paramento interior trácese una línea cualquiera, *in*, por *n* trácese *ns* vertical; divídase *is* en un número *par* de partes iguales (en la fig. hay 4), y divídase *sn* en el mismo número de partes iguales más una (en la fig. hay 5). Por los puntos de división trácense líneas horizontales y verticales para formar los escalones como en la figura.

En el muro de escalones, el centro de gravedad se encuentra más al interior de la arista *o* que en el otro muro, dando así un brazo de palanca y resistencia mayores; pero dentro de los límites ordinarios de la práctica, la diferencia es muy pequeña, y como el prisma triangular de las tierras sostenidas es más grande que cuando el paramento interior es vertical, la presión es mayor también; de modo que probablemente por este respecto no hay ventaja apreciable. **El aumento de espesor cerca de la base disminuye sin embargo, el brazo de palanca** *va*, **fig. 3, del empuje** *P*, **de la tierra** contra el paramento interior del muro. El *centro de presión* de este empuje está en ambos casos á la tercera parte de su altura vertical, medida desde su parte inferior, y por consiguiente, es claro que mientras más distante del paramento exterior se aplique, más corto será *va*. Además, en el paramento interior en escalones, la *dirección* de la presión se hace más aproximadamente vertical que cuando dicho paramento interior es vertical. Es debido á estas causas, más bien que á la transposición hacia atrás del centro de gravedad, á lo que el muro escalonado debe su aumento de estabilidad sobre los de paramento interior vertical.

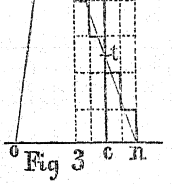


Fig 3

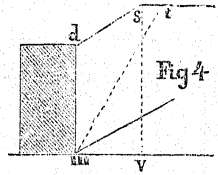


Fig 4

**Art. 4. Cuando, como en la fig. 4, el relleno tiene mayor altura que el muro y presenta una rampa hacia el interior de la arista** *d* **con una pendiente igual al talud natural,** *ds*, de 1½ á 1, creemos que los espesores de la base que siguen serán á lo menos *suficientes* para muros verticales con relleno de arena. Se han deducido de los experimentos aludidos arriba, y son solamente ligeras aproximaciones, sin ninguna base científica. No las daríamos á no ser por el hecho de que no conocemos ningunas otras que convengan á este caso.

La primera columna contiene la altura vertical *sv* de las tierras, comparada con la altura vertical del muro, el cual suponemos ser=1; de modo que la tabla prin-

cipia con el relleno de altura igual á la del muro mismo, como en la fig. 1. Estos muros verticales pueden cambiarse en otros, con caras inclinadas ú oblicuas, según el art. 8; ó sin este método, sus caras pueden hacerse oblicuas en cualquier grado que no pase de 1 en 8, sin afectar sensiblemente la estabilidad y sin aumentar su base.

**TABLA 1.** (Original.)

| Altura total de la tierra comparada con la altura del muro sobre la superficie del suelo. | Muro de piedra tallada con mortero. | Muro de piedra ó ladrillo con buen mortero. | Muro bueno de piedra en seco. | Altura total de la tierra comparada con la altura del muro sobre la superficie del suelo. | Muro de piedra tallada con mortero. | Muro de piedra ó ladrillo con buen mortero. | Muro bueno de piedra en seco. |
|---|---|---|---|---|---|---|---|
| | Espesor de la base, en partes de la altura. | | | | Espesor de la base, en partes de la altura. | | |
| 1. | .35 | .40 | .50 | 2. | .58 | .63 | .73 |
| 1.1 | .42 | .47 | .57 | 2.5 | .60 | .65 | .75 |
| 1.2 | .46 | .51 | .61 | 3. | .62 | .67 | .77 |
| 1.3 | .49 | .54 | .64 | 4. | .64 | .68 | .78 |
| 1.4 | .51 | .56 | .66 | 7. | .64 | .69 | .79 |
| 1.5 | .52 | .57 | .67 | 9. | .65 | .70 | .80 |
| 1.6 | .54 | .59 | .69 | 14. | .66 | .71 | .81 |
| 1.7 | .55 | .60 | .70 | 25. | | | |
| 1.8 | .56 | .62 | .71 | ó más | .68 | .73 | .83 |

**Art. 5.** Pero cuando la pendiente *nr*, fig. 5, de 1½ á 1, parte de la arista exterior del muro, se requiere mayor espesor. Poncelet da la tabla siguiente para este caso, y para *arena seca*.

**TABLA 2.**

| Altura total de la tierra comparada con la altura del muro. | Muro de piedra tallada con mortero. | Muro de mampostería de ladrillo. | Altura total de la tierra comparada con la altura del muro. | Muro de piedra tallada con mortero. | Muro de mampostería de ladrillo. |
|---|---|---|---|---|---|
| 1 | .35 | .432 | 2.4 | .762 | 1.02 |
| 1.1 | .393 | .498 | 3.0 | .814 | 1.11 |
| 1.2 | .439 | .548 | 4.0 | .852 | 1.18 |
| 1.3 | .485 | .604 | 6.0 | .883 | 1.25 |
| 1.4 | .532 | .665 | 11.0 | .909 | 1.28 |
| 1.5 | .579 | .726 | 21.0 | .922 | 1.31 |
| 1.6 | .617 | .778 | 31.0 | .926 | 1.32 |
| 1.7 | .645 | .824 | Infinito. | .934 | 1.34 |
| 1.8 | .668 | .847 | | | |
| 1.9 | .690 | .903 | | | |
| 2.0 | .707 | .930 | | | |

Cuando la tierra pasa más arriba de la parte superior del muro, como en las figs. 4 y 5, el muro está **sobrecargado**, y á la tierra que esté encima de la parte

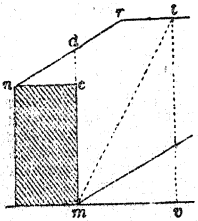



**Fig. 5**

superior se la llama **sobrecarga**; cuando se deposita con cuidado dicha sobrecarga, de modo que las tierras formen una escarpa de un ángulo mayor que 1½ á 1, como por ejemplo de 1 á 1, la teoría no exige que el muro sea tan grueso. Sin embargo de la gran reputación de Poncelet, el autor no puede imaginarse que la base de un muro de ladrillos debe ser de 1⅓ veces su altura para cualquiera altura de arena que pueda haber.

**Art. 6. Teoría de los muros de sostenimiento.** Sea *beam*, fig. 6, un muro tal, que sostenga un relleno *csmg*, cuya superficie superior *cs* sea horizontal, y á nivel con la parte superior *bc* del muro; y que *ms* represente el talud natural de las tierras de que se compone el relleno, siendo *mg* horizontal.

Muchas experiencias hechas en las obras públicas demuestran que este talud, sea arena, ó tierra, estando estas secas, puede prácticamente tomarse como de 1½ á 1; es decir, 1½ horizontal para 1 vertical; que corresponde á un ángulo *smg* de 33°41′ con el horizonte; que es también más ó menos el ángulo bajo el cual el ladrillo ó la piedra toscamente labrada, principian á resbalar una sobre otra.

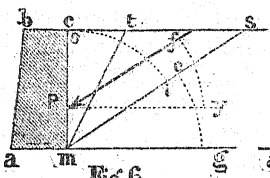


Fig. 6

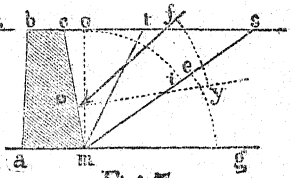


Fig. 7

Este ángulo, sin embargo, varía considerablemente, influyendo en gran manera el grado de sequedad ó humedad del material, de modo que tierra ó arena medianamente húmeda, se sostiene en un talud de 1 á 1, es decir, bajo un ángulo de 45°. Sea como fuere, se llama **ángulo del talud natural** del material. En los cálculos teóricos de los muros, lo más acertado es suponer (como lo hemos hecho continuamente) que el relleno está perfectamente seco, ya que entonces es que produce mayor presión, á menos que se le suponga tan mojado que adquiera algún grado de fluidez. El prisma triangular *oms* de las tierras que reposan sobre el talud natural *ms*, tiende á resbalar por dicho talud, pero el muro se lo impide.

Debe suponerse siempre que el muro no puede *resbalar* sobre su base, art. 9, que tiene un espesor suficiente para no combarse, y que solamente puede destruirse por el volcamiento girando alrededor de su arista exterior *a* como arista de apoyo. El espesor de seguridad requerido para que no suceda esto último, será también suficiente para impedir el desplome. Refiriéndonos ahora solamente á la fig. 6, de paramento interior vertical, si el ángulo *oms*, entre el talud natural *ms* y la línea vertical *mo*, tirada por la arista inferior del muro, se divide por una línea *mt*, en dos ángulos iguales, *omt*, *tms*, entonces el ángulo *omt* y la línea *mt* se llaman respectivamente **el ángulo** y **el talud de la presión máxima**. El prisma triangular de las tierras, del cual *omt* es una sección, se llama **el prisma de presión máxima**, porque considerándolo como una cuña que actúa contra el paramento interior del muro, produciría sobre éste una presión mayor que el triangulo entero de tierra *oms*, considerándolo como una cuña sola. Porque aunque este último es más pesado, está más sostenido por la tierra que está debajo. Los cálculos demuestran que si consideramos la tierra *oms* dividida en cuñas por *cualquiera* línea *mt*, la cuña que ejerce más presión contra el muro es la que se forma cuando *mt* divide el ángulo *oms*, ó el arco *oi* en dos partes iguales. Pero véase art. 11.

Como *mg* es horizontal, y *mo* vertical, las dos forman un ángulo de 90°; por consiguiente el ángulo de la presión máxima se encuentra restando de 90° el ángulo, *smg*, del talud natural y dividiendo el residuo por dos. De este modo un talud de 1½ en 1, ó de 33°41′ restado de 90°, da un residuo de 56°18′ y 56°18′ ÷ 2 = 28°9′ **ángulo, o m t, correspondiente á la presión máxima**. Para facilitar el cálculo se considera generalmente una unidad de longitud de muro y de relleno. El número de unidades cúbicas de muro ó de relleno es entonces igual al de las unidades cuadradas de sus respectivos perfiles ó secciones transversales. Ahora bien; según Moseley, si suponemos que las partículas de tierra que componen el relleno están perfectamente secas y sin cohesión (ó tendencia á pegarse unas con otras, lo

que casi sucede con la arena pura), y si suponemos que se quita el muro repentinamente, entonces el prisma triangular de las tierras, *cmt*, comprendido entre el talud de presión máxima, *mt*, y el paramento vertical, *cm*, del muro, fig. 6, se deslizaría hacia abajo, bajo la influencia de una fuerza (que puede representarse por *yP*) que actúa en una dirección perpendicular á la cara, *cm*, de dicho prisma (ó, en otras palabras, en ángulo recto al paramento interior del muro vertical), estando su centro de presión en P á una distancia de ⅓ entre *m* y *c* medida desde el suelo, y su intensidad igual á cualquiera de las siguientes:

**N.º 1.** Presión perpendicular $yP = \dfrac{\text{Peso del prisma triangular de las tierras } cmt \times ot}{\text{Altura vertical } om.}$

**N.º 2.** Presión perpendicular $yP = \dfrac{\text{Peso de un solo m cúbico de relleno} \times ot^2}{2}$

} Véase art. 11.

En vista de la gran incertidumbre que envuelve en la práctica el asunto relativo á la presión real de las tierras contra los muros de sostenimiento (véase art. 2), y para dar una regla sencilla, la cual, aunque no apoyada en la teoría, es, sin embargo (en opinión del autor), suficientemente aproximada para las operaciones comunes de la práctica, supondremos que el n.º 1 de las dos fórmulas arriba mencionadas, se adapta también con suficiente aproximación á los muros de **paramento interior inclinado** *cm*, figs. 7 y 8 (exactamente con las mismas letras), á lo menos **para inclinaciones del paramento interior hacia adelante como de 1 horizontal para 2 vertical**, ó de un ángulo *cmo* de 26°34'. **Lo que sigue sobre muros de sostenimiento está basado en esta suposición incorrecta y debe considerarse que sólo dan una seguridad aproximada.**

Parece que algunos suponen que esta presión perpendicular sea la única que obra contra el paramento interior del muro, y por consiguiente, llegan á conclusiones erróneas en la práctica. Cuando para impedir que esta fuerza cause el resbalamiento del prisma de las tierras, colocamos el muro de sostenimiento frente á él, entonces, en lugar de *movimiento*, la fuerza producirá presión de la tierra contra el muro, desarrollando frotamiento entre las superficies comprimidas de la tierra y el muro.

Es decir, si un muro estuviese á punto de principiarse á volcar alrededor de su arista *a* como apoyo, su paramento *cm* tendría, por supuesto, que levantarse, y al hacer esto debe rozar contra el relleno de tierra en contacto con él, y este rozamiento obraría evidentemente en el sentido de *impedir* que se volcase. Mientras el muro no se mueva el mismo frotamiento contribuye á *evitar* que se vuelque.

Para hallar la intensidad y efecto del frotamiento, representemos por *yP*, fig. 8, *en escala*, la fuerza perpendicular al paramento interior *cm*; y supondremos haberla calculado previamente por la fórmula n.º 1 que antecede. Hágase el ángulo *yPf* igual al ángulo de frotamiento del muro*; trácese *yf* en ángulo recto con *yP*, ó bien paralela á *mc*; hágase *Px* igual á *yf*, y complétese el paralelogramo *Pyfx*. Luego *xP* representará en la misma escala la **intensidad del frotamiento contra el paramento interior del muro.**

Tenemos por consiguiente dos fuerzas obrando en P, es decir, la fuerza perpendicular *yP*, y el frotamiento *xP*; por consiguiente, por la composición y descomposición de las fuerzas, la diagonal *fP* del paralelogramo *Pyfx*, medida en la misma escala, nos dará la intensidad de su resultante; **la cual es aproximadamente la única fuerza teórica, tanto en intensidad como en dirección, que el muro tiene que resistir**, incluyendo el frotamiento del muro.

Pero esta fuerza, *fP*, es también igual á la fuerza perpendicular *yP*, multiplicada por la secante del ángulo *yPf* de frotamiento del muro (ó dividida por su coseno), y puede por supuesto encontrarse de este modo:

$$\text{Presión teórica aproximada } fP = \frac{\text{Peso del prisma triangular } cmt}{\text{Altura vertical } om.} \times ot \times \frac{\text{Secante del ángulo } yPf \text{ de frotamiento del muro}}{} = \frac{\text{Peso de } cmt}{\text{Coseno } yPf} \times \frac{ot}{om}$$

O finalmente, si suponemos, como lo hemos hecho, que la tierra está perfectamente seca (su presión es entonces la mayor) y que los ángulos del talud natural

* Este ángulo de frotamiento es el ángulo de inclinación con la horizontal, que debe dársele á un plano de mampostería para que la arena ó tierra seca se deslicen sobre él. Es, más ó menos, lo mismo que el talud natural 33°41', ó 1.5 á 1. Su secante es 1.202, y su coseno = .832.

son entonces cada uno iguales á 33°41′ ó de 1.5 á 1, entonces en las figs. 6, 7 y 8, si el ángulo *cmo* que forma el paramento interior *cm* y la vertical *om* no excede de 26°34′, más ó menos, podemos suponer que la

**Presión aproximada** *fP* = **Peso del prisma triangular** *cmt* × .643

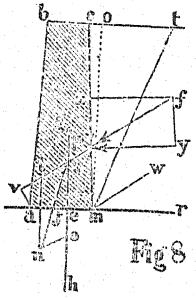


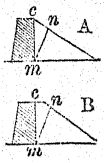
Fig 8

la cual comprende la acción del frotamiento de la tierra contra el paramento interior del muro.

*Nota* 1. **Cuando el paramento interior del muro está construído de escalones**, como en la fig. 3, en lugar de ser simplemente oblicuo, ó inclinado como en las figs 7 y 8, la dirección de la presión de la tierra será la misma que si el paramento interior tuviese la inclinación, *i n*.

*Nota* 2. **Para encontrar tanto la tendencia de las tierras á volcar el muro** como la resistencia de éste á ser volcado alrededor de *a* como apoyo, búsquese primero el centro de gravedad *g* del muro, y por él trácese una línea vertical *gh*. Prolónguese *fP* hacia *v*, y tírese *av* perpendicular á ella. Hágase en cualquiera escala *so* = peso del muro, y *si* = presión calculada *fP*. Complétese el paralelogramo *s i n o* y tírese su diagonal *sn*, que será la resultante de la presión *fP* y del peso del muro; y para que ofrezca seguridad, debe resultar tal que *aj* no sea *menos* de la quinta parte de *am*, *aun con la mejor mampostería y con el terreno más firme*. De lo contrario, la gran presión, tan próxima á la arista *a*, puede, ó fracturar el muro, ó comprimir el suelo cerca de aquel punto, de modo que el muro se incline hacia adelante. En los muros construídos según nuestra regla, art. 1, ó según la tabla, pág. 657, *aj*, es mayor que la quinta parte de *am*. Multiplicando la presión *fP* por su brazo de palanca, *av*, tenemos el momento de la presión en *a;* y el peso del muro multiplicado por su brazo de palanca, *ea*, nos da el del muro. *El muro ofrece seguridad contra el volcamiento* en la misma proporción en que está su momento con el del empuje. Se supone incapaz de resbalamiento, fractura ó hundimiento.

*Nota* 4. **Si la tierra se inclina** de C hacia abajo, como en la fig. A ó B, en lugar de estar horizontal como en las figs. 6, 7 y 8, úsese el peso de la tierra *cmn* en




lugar de, *cmt*, siendo, *mn*, el talud de la mayor presión. En A el punto de aplicación estará todavía en P (á una tercera parte de *mc*) como en las figs 6, 7, 8; pero en B estará un poco más alto como se indica abajo en la fig. 9.

**Muros sobrecargados** son aquellos en los cuales el relleno se prolonga por sobre la parte superior de los muros. Según la teoría, cuando, como en la fig. 9, hay una sobrecarga, *vck*, del relleno que se prolonga desde *c*, según el talud natural

*ev*; se produce la presión máxima contra el muro cuando la tierra llega al nivel *d*, donde la dirección inclinada de la presión máxima, *md*, intersecta la cara del talud natural, *ev*; de modo que si después llega la tierra hasta *v*, ó más arriba, no se ejerce por eso más presión contra el paramento interior del muro. Lo mismo deci-

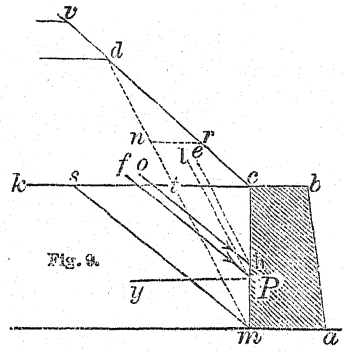


Fig. 9.

mos si el talud de las tierras parte de, *b*; ó de un punto entre *c* y *b*, con la diferencia de que entonces la dirección inclinada *md* de la presión máxima debe prolongarse hacia arriba para encontrar á este otro talud.

**La intensidad aproximada** de la presión oblicua, cuando el muro está sobrecargado (como en las figs. 4, 5 y 9), puede hallarse por el mismo principio que cuando la tierra está al nivel de su parte superior; es decir, en lugar del prisma *triangular cmt* de las tierras, figs. 6, 7, 8 y 9, búsquese el peso de todas las tierras, *dsmt*, en la fig. 4; *dmtr*, en la fig. 5; ó *cdm*, en la fig. 9 (si la sobrecarga llega á *d* ó *v* ó más arriba) entre la dirección inclinada *md*, fig. 9, *mt*, figs. 4 y 5 de la presión máxima, el paramento interior del muro y el talud, omitiendo á *den*, fig. 5, que descansa sobre la parte superior del muro (y de este modo ayuda á su estabilidad), cuando el talud principia enfrente de *c*. Después de haber hallado este peso, tenemos entonces para relleno seco que la

**Presión aproximadamente = Peso de la tierra × .643**

incluyendo la acción del frotamiento de las tierras contra el paramento interior del muro. Esto con suficiente aproximación (según la opinión del autor) para todas las operaciones prácticas en una materia tan incierta, pero esencialmente empírica.

**La dirección de la presión** hallada así es la misma que cuando la tierra está al nivel de la parte superior *bc;* es decir, como en las figs. 6 y 7, trácese primero una línea *y*P perpendicular al paramento interior *cm*, sea vertical ó inclinado. Luego tírese otra línea P*f* formando el ángulo de frotamiento del muro, el cual suponemos siempre de 33°41′, ó 1.5 á 1. Entonces P*f* dará la dirección de la presión. Pero su punto de aplicación no estará siempre en P (al tercio de la altura del muro sobre *m*) como hasta ahora, porque en todos los casos estará en un punto P, ó en alguno más arriba, *h*, tal que una línea *h*P ó *eh*, fig. 9, tirada por el centro de gravedad de las tierras sostenidas, corte el paramento interior (omitiendo las tierras que descansan inmediatamente sobre la parte superior *bc*) y paralela á la dirección *md* de la presión máxima. Una línea tal encuentra al muro á una tercera parte de su altura cuando la tierra sostenida *tcm* ó *dcm* **forme un triángulo completo**, uno de cuyos ángulos esté en el extremo superior *c* de la arista interior del muro. En todos los otros casos dicha línea, cuando haya sobrecarga, tocará más arriba de P.

**Art. 7.** En la pág. 648, fig. 1, recomendamos que la base *os* en la línea de tierra de muros **verticales** bien construídos no fuese menos de .35, .4 ó .5 de la altura *ds* sobre dicha línea de tierra, según la clase de mampostería. Pero un muro con paramento inclinado como el trazado en el art. 8 (por el cual se construyó la tabla siguiente), será tan resistente, conteniendo al mismo tiempo menos mampostería que un muro vertical, aunque el muro inclinado tenga la base *os* más ancha.

**Tabla 3, de los espesores *so* en la base, fig. 1, y en la parte superior *cd* de los muros de paramentos exteriores inclinados, y tan resistentes como los muros verticales que contienen más mampostería.**

| Todos los muros á continuación tienen la misma resistencia que un muro vertical cuya base *os*, fig. 1, sea igual á .35 de su altura *ds*. | | Todos los muros á continuación tienen la misma resistencia que un muro vertical cuya base *os*, fig. 1, sea igual á .4 de su altura *ds*. | | Todos los muros á continuación tienen la misma resistencia que un muro vertical cuya base *os*, fig. 1, sea igual á .5 de su altura *ds*. | |
|---|---|---|---|---|---|
| **Piedra tallada.** | | **Piedra bruta con mortero** | | **Piedra seca.** | |
| Base en partes de la altura. | Espesor de la parte superior en partes de la altura. | Base en partes de la altura. | Espesor de la parte superior en partes de la altura. | Base en partes de la altura. | Espesor de la parte superior en partes de la altura. |
| .350 | .350 | .400 | .400 | .500 | .500 |
| .352 | .310 | .404 | .359 | .501 | .459 |
| .355 | .270 | .403 | .320 | .503 | .420 |
| .359 | .234 | .408 | .283 | .506 | .384 |
| .364 | .197 | .413 | .246 | .510 | .343 |
| .371 | .163 | .419 | .210 | .516 | .308 |
| .379 | .129 | .425 | .175 | .522 | .272 |
| .389 | .096 | .435 | .143 | .528 | .236 |
| .400 | .066 | .445 | .110 | .547 | .204 |
| .425 | .007 | .468 | .051 | .555 | .138 |
| .429 | .000 | .490 | .000 | .612 | .009 |

Moseley y otros autores suponen un *talud de arena seca de* 21° Sería mejor no continuar la propagación de errores tan evidentes como éste. La arena seca no se mantiene sin resbalar á un ángulo menor para un sabio que para cualquiera otra persona. En las aplicaciones prácticas podemos decir que la arena, el granzón y las tierras secas toman un talud de 3:2041 ó 1½ á 1; como lo prueban las numerosas experiencias hechas en los rellenos de ferrocarriles. Poncelet da tablas para muros que sostienen tierra seca con un talud de 1 á 1, ó 45°; pero nosotros no creemos en la existencia de tal tierra, y omitimos estas tablas. La arena, el granzón y la tierra pueden humedecerse á grados diferentes, como para tomar un talud cualquiera entre la horizontal y la vertical; y humedeciéndolas y apisonándolas, pueden convertirse las tierras en masas compactas, que ejercen poca ó ninguna presión; y pueden hasta continuar así después de secas; siendo entonces, de hecho, una especie de ladrillo crudo. Algunas veces es difícil saber si la tierra ó la arena están perfectamente secas ó no, y un grado excesivamente pequeño de humedad les hace tomar un talud de 1 á 1, en pilas pequeñas, como las que probablemente han observado las autoridades en esta materia. El autor observó que la arena fina de playa bajo techo tomaba un talud de 1½ á 1 en tiempo seco y de 1 á 1 en tiempo húmedo; sin embargo, era imposible por el tacto descubrir diferencia alguna en el grado de humedad. Su sensibilidad á la humedad era debida de seguro á la sal. Algunos puñados de tierra seca pueden obligarse quizás á tomar un talud de 1 en 1 sobre una mesa; pero lo que podemos asegurar, como consecuencia de nuestras observaciones, es que cuando se la deja caer de carretas, ó carros de mano, su talud es más ó menos 1½ á 1, y creemos que éste debe ser el talud que debe usarse en la práctica, donde la seguridad es la condición de suprema importancia.

**Mientras menor sea el talud natural, mayor será la presión,** y como dicho talud menor tiene efecto cuando el relleno está perfectamente seco comitiendo por supuesto el caso en que esté tan mojado que se encuentre parcialmente líquido), hemos limitado para mayor seguridad nuestras tablas al relleno seco. Como dijimos en el art. 1, no podemos recomendar dimensiones menores de las que hemos dado allí, cuando consideramos el trato tan recio á que están expuestas las mamposterías en obras públicas.

**Al hacer un camino á lo largo de precipicios peligrosos** estuvimos tentados algunas veces á hacer muros de más espesor todavía. Creemos, por ejemplo, que la fuerza centrífuga de un tren pesado, que corre rápidamente por una curva de poco radio, convexa del lado peligroso, no debe descuidarse como elemento del cálculo para muros así situados. Esta fuerza es horizontal y se aplica cerca de la parte superior del muro; y, por consiguiente, su brazo de palanca puede considerarse igual á su altura; en tanto que la presión teórica de la tierra es oblicua, y

está aplicada á una tercera parte de altura sobre el suelo; de modo que su brazo de palanca respecto de su pie es muy corto. Además, el solo *peso* del tren produce presión contra el muro; lo mismo que el peso del relleno. Los teóricos omiten todas estas consideraciones. La presión perjudicial causada por la trepidación, etc., no se puede suponer aplicada, á la tercera parte de la altura, ni puede calcularse de manera alguna.

*Nota* 2. **Los muros de muelles** son ejemplo de muros donde el espesor debe aumentarse, á pesar de que la presión del agua frente á ellos ayuda á sostenerlos. La tierra detrás de estos muros, no solamente está expuesta á una carga pesada durante la descarga de barcos, sino que también es capaz de saturarse de agua, especialmente bajo el nivel de la marea baja; y esto la hará ejercer una presión muy grande contra los muros. Además, el agua penetra debajo del muro; y por su presión hacia arriba, reduce realmente su peso, y por consiguiente su estabilidad. La misma causa disminuye también de seguro el frotamiento del muro sobre su base. Estos muros son por lo tanto muy propensos á resbalar, si los cimientos son horizontales y lisos, y se han resbalado aun teniendo los cimientos una inclinación considerable hacia atrás, como en la fig. 1. Véase art. 5.

*Nota* 3. Un muro de sostenimiento está generalmente en mayor peligro durante pocos meses después de su construcción, que cuando el mortero ha tenido tiempo de fraguar perfectamente, y de asentarse el relleno. Si hay sospechas de que pueda peligrar la seguridad de un muro nuevo, es bueno reforzarlo colocando fuertes puntales contra el muro, á ¼ ó á la mitad, más ó menos, de su altura sobre el terreno. En algunos casos pueden construirse contrafuertes ó estribos permanentes con este objeto; éstos deben estar bien trabados penetrando en el muro.

*Nota* 4. La presión del relleno de tierra puede reducirse mucho, si se hacen las primeras capas delgadas, y que se vayan consolidando al usarlas como andamio de los albañiles, como se ve en *h*, fig. 1. A menudo puede hacerse esto sin inconveniente y á un costo muy pequeño.

**Art. 8. Cambiar un muro de sostenimiento vertical en uno de paramento exterior inclinado, que oponga igual resistencia al volcamiento y con menos mampostería.** Esto se llama **transformación del perfil.** (Original.)

Sea *aboi*, fig. 10, un muro vertical. Multiplíquese su base *oi* por 1.225 (1.22475 es más aproximado); y su producto será la base *oe*, de un muro triangular *boe*, que posee la misma estabilidad, y que sin embargo no requiere mucho más de la mitad de la mampostería del muro vertical. Véase nota 1. Hecho esto, supongamos que

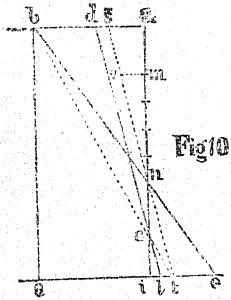


Fig 10

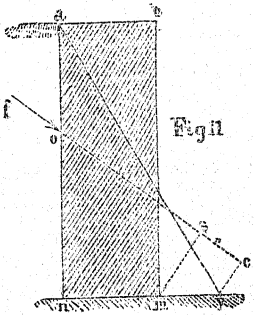


Fig 11

se desea un paramento inclinado, por ejemplo de 1 en 4. Del punto *n*, donde el paramento del muro triangular, intercepta el del vertical, márquense verticalmente hacia arriba 4 espacios pequeños iguales, y del superior de ellos *m* llévese á *v* un espacio horizontal; por *v* y *n* trácese la línea de puntos *st*, la cual evidentemente dará una inclinación de 1 en 4. Entonces, *bsto*, es aproximadamente el muro que se requiere, pero con un poco más de espesor del necesario. Para reducir éste, de *t* tírese la línea de puntos *tb*. Márquese el punto *c*, donde encuentra el paramento *ai*, del muro vertical, y por *c* trácese *ld*, paralela á *st*; entonces *bdlo* es el muro que se desea. Nuestra figura está trazada de un modo exagerado, para evitar confusión.

en las líneas. La base *oe* del muro triangular no sería en realidad tan grande como está representada.

Se observará que, á medida que la base aumenta, la cantidad de mampostería disminuye.

*Nota* 1. **El muro de paramento inclinado ofrece de hecho más seguridad que el vertical.** El muro de paramento inclinado tiene el mismo momento de estabilidad que el vertical; y la presión de la tierra contra él tampoco cambia, pero el *momento ó tendencia* de la presión á volcar el muro se hace menor. Sea *abmn*, fig. 11, un muro vertical, y *fo* la intensidad y dirección de la presión que obra detrás de él. (Para ilustrar con más facilidad el asunto hemos colocado á *o* **más arriba** del verdadero centro de presión del relleno, el cual debería estar más ó menos al tercio de *an* á contar de *n*.) Ahora bien, el brazo de palanca con el cual esta presión tiende á volcar el muro alrededor de su pie *m*, es la distancia *ms*, medida del pie ó punto de apoyo *m*, en ángulo recto á la dirección, *fose*, de la presión; y este brazo de palanca, multiplicado por la fuerza *fo*, da la tendencia al volcamiento ó el momento de dicha fuerza. Por otra parte, sea *a n y* un muro triangular de la misma estabilidad que el otro, como se halló por nuestra regla. Tendremos todavía la misma intensidad *fo*, la dirección, *fose*, de la presión contra el muro; pero obrando ahora para volcar el muro *any* alrededor de la arista *y*, y por tanto, con un brazo de palanca reducido, *ye*. Por consiguiente, su tendencia al volcamiento es menor que antes. Por eso, en lenguaje vulgar, podemos decir que el muro es más fuerte que antes, aunque su momento de estabilidad, ó tendencia á permanecer firme, no ha sufrido cambio. Si la presión *fo* contra el paramento vertical fuera horizontal, como sucede cuando es agua, entonces su brazo de palanca sería evidentemente el mismo en ambos muros; y la proporción entre el momento de volcamiento de la presión y los momentos de estabilidad de los dos muros serían constantes.

*Nota* 2. Al pretender reducir la mampostería adoptando un muro, *obe*, fig. 10, de sección triangular, ó de una forma casi aproximada á un triángulo, debe tenerse especial atención con la calidad de la mampostería que se coloque cerca de la arista delgada *e*, pues de lo contrario puede fallar bajo la presión.

*Nota* 3. **Por otra parte, cuando se usa mortero ordinario sin mezcla de cemento**, lo que jamás debería hacerse en muros de sostenimiento, donde la durabilidad es uno de los objetos, no se puede aconsejar una gran inclinación

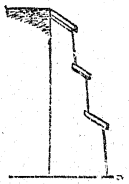


Fig 12

por cuanto la lluvia combinada con la escarcha, etc., destruye prontamente el mortero. En estos casos la inclinación debe, por consiguiente, no exceder de 1 en 12 ó 1 en 8, y aun entonces deben cogerse las juntas con cemento, como también deben hacerse con cemento ó con mortero de cemento las capas inferiores de la mampostería hasta la altura de algunos decímetros. Hemos observado una diferencia muy marcada en la corrosión del mortero cuando en muros iguales situados del mismo modo se construye una parte de paramento vertical y la otra con una inclinación de solamente 1 en 8. El mortero común expuesto á la humedad de la tierra no fragua jamás bien ni se conserva firme. Esto se puede observar muy bien cerca de las cabezas y pies de los estribos, muros de sostenimiento, etc., en cuyos puntos se verá que el mortero de cal se ha desvirtuado generalmente. Un perfil algo semejante á la fig. 12, puede algunas veces ser útil en lugar del perfil triangular. Este es el diseño de los estribos ó contrafuertes góticos que probablemente tuvieron su origen en la causa de que acabamos de hablar.

**Art. 9. Un muro de sostenimiento puede resbalarse sin perder su verticalidad**, y ciertamente sin peligro alguno de volcarse. Esto es fácil que

ocurra si está construído sobre una plataforma de madera ó sobre una superficie á nivel, de roca ó greda, sin otra resistencia que el frotamiento, para impedir el resbalamiento. Éste puede evitarse dándole á la base una inclinación como en la fig. 1; haciendo los cimientos del muro de una profundidad tal que le dé una resistencia adecuada al terreno situado enfrente del muro; ó en caso de una plataforma, fijar una ó más hileras de vigas fuertes á su parte superior transversalmente á la dirección en que el resbalamiento pueda tener efecto. Sobre la **greda mojada** el frotamiento puede ser tan pequeño como de .2 á $^1/_3$ del peso del muro; sobre tierra seca es, más ó menos, de $^1/_2$ á $^2/_3$; y sobre arena ó granzón, de $^2/_3$ á $^3/_4$, más ó menos. El frotamiento de la mampostería sobre una plataforma de madera es, más ó menos, de $^9/_{10}$ del peso si está seca, y $^7/_8$ mojada.

**Contrafuertes.** Como se muestran en el plano en *ccc*, fig. 13, se componen de un aumento de espesor en el paramento interior del muro, á intervalos regulares de su longitud. Nosotros no los consideramos sino como un desperdicio verdadero

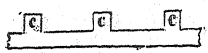


Fig 13

de mampostería. Cuando un muro de esta clase se cae, casi siempre se separa de sus contrafuertes, á los cuales está unido solamente por la adhesión del mortero, y muy poco, por la traba de la mampostería. La tabla del art. 7 demuestra que una muy pequeña adicion que se haga á la base de un muro causa un aumento considerable de su resistencia; y por consiguiente creemos que la mampostería de los contrafuertes podía emplearse mejor y más barato dando al muro un espesor adicional en toda su longitud, y hasta la tercera parte de su altura á partir del suelo. Los contrafuertes son usados generalmente en los muros de sostenimiento por los ingenieros europeos, pero raras veces por los americanos.

**Los estribos** (*buttresses*) son lo mismo que los contrafuertes, con la excepción de que se colocan *enfrente* del muro en lugar de colocarlos detrás, y que su perfil es generalmente triangular, ó casi triangular. Aumentan la resistencia de los muros considerablemente, pero son de mal efecto á la vista; se usan raras veces, exceptuando el caso en que haya que impedir la caída de un muro.

**Amarras de tierra** (*land-ties*), ó vientos, ó largas barras de hierro se han empleado algunas veces como medio provisional de sostener muros débiles. Atraviesan el muro y se fijan sus extremidades en tierra anclándolas en mampostería ó á postes de hierro ó de madera enterrados á alguna distancia bajo tierra.

**Los muros de sostenimiento de perfiles curvos** se mencionan aquí solamente para advertir á los jóvenes ingenieros que no los construyan. Aunque su uso está sancionado por la práctica de algunas altas autoridades, ellos realmente no poseen mérito suficiente para recompensar el costo adicional y las dificultades de su construcción.

(*Obs. del T.* — Aunque en español (por lo menos en Venezuela) no existen las diferencias que el autor señala en el próximo art. 10, hemos querido dejarlas.)

**Art. 10.** Entre los militares, un muro de sostenimiento es llamado de **revestimiento escarpado.** Cuando la tierra está al nivel de su parte superior, se llama **revestimiento escarpado;** cuando la tierra está sobre el muro, **revestimiento contraescarpado** ó **semirevestimiento.** Cuando el paramento exterior del muro está inclinado, se llama *rampa* (sloping), y cuando el fondo está inclinado, se llama *contra rampa*. La pendiente se llama **talud.**

**Art. 11.** La presión de la arena, etc., contra un muro, fig. 6, á nivel de su parte superior, no disminuye reduciendo la cantidad de arena, en tanto que su anchura superior *cs* no sea menor que la (*ct*) correspondiente al ángulo *cmt* de mayor presión. La presión principia entonces á disminuir; pero en la práctica la disminución no es apreciable en tanto que el ancho no se reduzca á una sexta parte, más ó menos, de la distancia (*cs*) correspondiente al ángulo *cms* del talud natural, ó más ó menos á la mitad de *ct*. Entonces la presión principia á disminuir rápidamente á medida que la anchura se reduce.

# PUENTES DE PIEDRA

**Art. 1.** En un arco *sis*, fig. 1, la distancia *eo* se llama **luz**; *ia*, **flecha**; la línea del límite inferior *eao*, **intradós**; la línea superior *rtr*, **trasdós**. Los términos **intradós** y **trasdós** se aplican también ó la *superficie* interna inferior y superior de la bóveda entera. Las extremidades del arco, ó las áreas visibles comprendidas entre el intradós y el trasdós, son sus **caras**; de modo que el área *stsa* es una cara. Las superficies inclinadas, ó juntas *re* y *ro*, sobre las cuales descansan las extremidades del arco, y de las cuales *arranca* el arco, se llaman **arranques**. Los bloques de que se compone el arco, son las **dovelas**; la dovela del centro *ta* es la **clave del arco**, y las inferiores *ss*, **las impostas**. Las partes *tr*, *tr*, son los **riñones del arco** **, y los espacios *trl*, *trb*, encima de ellos, **las enjutas**. El material depositado en estos espacios se llama **relleno de las enjutas** y se compone algunas veces de tierra ó de mampostería, ó de ambas como en la fig. 1.

En los arcos grandes, el relleno lo forman á menudo varios muros **paralelos** *ll*, fig. 2½, sobre la enjuta, que se prolongan á lo largo de la calzada, ó á través del arco. Se cubren en su parte superior ó por arcos pequeños de muro á muro, ó por piedras chatas, para sostener el material de la calzada. Algunas veces están conectados por

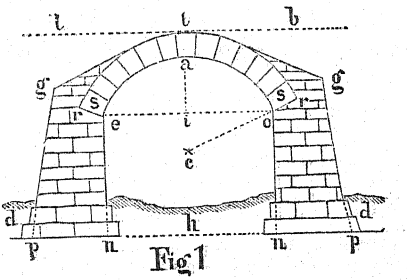


Fig 1

muros transversales verticales de distancia á distancia, para afianzarlos lateralmente, como en *tt*, fig. 2½. Las partes *gpen*, *gpon*, fig. 1, son los **estribos ó pies derechos del arco**; *en*, *on*, los **paramentos exteriores**; *gp*, *gp*, los **paramentos interiores**, y *pn*, *pn*, *las bases de los estribos*. Estas bases se ensanchan generalmente por *escalones dd*, para distribuir el peso del puente sobre una área mayor, disminuyendo así el peligro que hay al asentarse. La distancia *at* de cualquiera dovela es su espesor.

Los únicos arcos de uso común en los puentes son los circulares (á menudo escarzanos) y los elípticos.

**Art. 2. Para encontrar el espesor en la clave para arcos de piedra tallada de primera clase, sean circulares ó elípticos** *. Búsquese el radio *co*, fig. 1, que toca el arco en *a*, *o*, y *e*. Súmese este radio y la *mitad* de *la luz*, *oe*. Tómese la raíz cuadrada de esta suma. Divídase esta raíz por 4, al cociente agréguese $^2/_{10}$ de pie, ó empleando una fórmula,

$$\text{Espesor en pies} = \frac{\sqrt{\text{Radio} + \text{mitad de la luz}}}{4} + .2 \text{ pies.}$$

* Por cuanto las reglas que damos para arcos y estribos son enteramente originales y nuevas, no estará demas hacer constar que no son enteramente empíricas, si no que están basadas sobre diseños y cálculos exactos hechos por el autor, de líneas de presión, etc., en arcos desde 0.30 cm hasta 90 m. de luz y de todas flechas, desde = radio hasta 1/15 de la luz. De estos diseños él trató de encontrar proporciones, las cuales, aunque quizás no soporten la prueba de una crítica estricta, pueden sin embargo aplicarse en todos los casos con una exactitud suficiente para las operaciones ordinarias de la práctica.

** Haunches. — *Diccionarios de Ponce de León y de Cuyás.*

(*N. del T.* — La misma fórmula para el sistema métrico:

$$\text{Espesor de la clave en metros} = \frac{\sqrt{\text{Radio} + \text{mitad de la luz}}}{7.244} + .061 \text{ metros.})$$

**Para obras de segunda clase**, este espesor puede aumentarse $^1/_8$ parte más ó menos, y para obras de ladrillos ó piedras brutas $^1/_5$ más ó menos.

En los arcos grandes es prudente aumentar el espesor hacia los arranques; pero si la luz no es mayor de 18 ó 24 ó 30 m, no es necesario si la piedra es buena; aunque el arco será más fuerte dándole dicho espesor. En la práctica este aumento, aun en las aberturas más grandes, no excede de ¼ á ½ el espesor en la clave; aunque la teoría indique mucho más en arcos de una gran flecha.

*Nota.* **Para hallar el radio** *co*, sea el arco circular ó elíptico; tómese el cuadrado de la *mitad* de la luz *eo*. Tómese el cuadrado de la flecha entera *ia*. súmense estos dos cuadrados, y divídase la suma por el doble de la flecha *ia*. O puede hallarse con suficiente aproximación para el objeto por medio del compás, trazando un arco pequeño en escala.

**Intensidad de la presión soportada por las dovelas.** En puentes del mismo ancho que la calzada, si todas las otras partes soportan en la misma proporción que la luz, la presión total aumentaría como el cuadrado de ésta, mientras que la presión *por m²* aumentaría como la abertura. Pero en la práctica el espesor del arco aumenta con *mucho menos rapidez* que la luz; mientras que el espesor del material de la calzada, y la carga extraña por metro cuadrado, permanecen lo mismo para todas las aberturas. Por tanto la presión total, en la clave y los arranques, aumenta con *menos* rapidez que los *cuadrados* de *las luces;* pero más rápidamente que las luces simples, como sucede también con las presiones *por metro cuadrado*.

Con la misma luz, la presión en la clave se hace menor, mientras que la de los arranques aumenta á medida que aumenta la flecha. También cuando las dovelas tienen espesor uniforme, la presión en cada arranque de un arco semicircular es más ó menos 4 veces mayor que cuando la flecha es solamente de una sexta parte de la luz, la presión en los arranques es por término medio solamente una tercera parte mayor que la de la clave. Estas proporciones varían algo en diferentes luces.

La mayor presión por metro cuadrado en los arranques, puede reducirse aumentando el espesor de las dovelas hacia los arranques. Esto, sin embargo, no es *necesario* en luces medianas, por cuanto la piedra buena ofrecerá seguridad aun bajo estas presiones mayores.

**Por el uso de muros de enjuta paralelos**, fig. 2½, ó rellenando en parte con tierra en lugar de mampostería, la presión sobre las dovelas puede disminuirse, por término medio aproximado, en $^1/_5$ parte más ó menos.

**Tabla 1. De algunos arcos existentes**, con espesores en la clave, efectivos y calculados (según nuestra regla). Cuando se dan dos espesores en la columna de claves, el menor es para piedra tallada de primera clase, y el mayor para buena piedra bruta, ó para ladrillos. Los que no están especificados, son de piedra tallada de primera clase. C quiere decir circular; E, elíptico. Para obras de segunda clase, agréguese más ó menos ⅛ parte; y para ladrillos ó piedras brutas buenos, ¼ parte más ó menos.

| | | Luz. | Flecha. | Radio. | Espesor real en la clave. | Espesor en la clave según nuestra regla. | INGENIERO. |
|---|---|---|---|---|---|---|---|
| | | Metros. | Metros. | Metros. | Metros. | Metros. | |
| Gabin John, Acueducto de Washington | C | 67.10 | 17.46 | 40.04 | 1.37 | 1.25 | Meigs. |
| Puente Grosvenor, Chester, Inglaterra | C | 61.00 | 12.81 | 42.70 | 1.52 | 1.44 | Hartley. |
| Puente de Londres, Lugl | E | 49.75 | 8.53 | 54.33 | 1.45 | 1.10 | Rennie. |
| Gloucester, á través del Severn, Ingl | E | 45.75 | 10.67 | 25.89 | 1.37 | 1.06 | Telford. |
| Dora Riparia, Turín, Italia | C | 43.74 | 5.49 | 24.38 | 1.50 | 1.23 | Mosca. |
| Pont-y-Pryd, Gales, Piedra bruta | C | 42.70 | 10.67 | 25.68 | .45 | 1.60 \| 1.28 | Edwards. |
| Puente de l'Alma, París, Pequeña piedra bruta en cemento | E | 41.12 | 8.69 | 31.35 | 1.50 | 1.07 \| 1.35 | Darcel. |
| Souppes, Francia. Un arco experimental de piedra tallada en mortero de Portland* | C | 37.11 | 3.12 | 85.35 | .80† | 1.47 | Vandrey. |
| Waterloo, sobre el Támesis, Londres | E | 33.00 | 9.76 | 32.92 | 1.37 | .94 | Rennie. |
| Tongueland, Escocia, Carretera | C | 33.39 | 11.59 | 12.76 | 1.07 | .91 | Telford. |
| Puente Napoleón, Viaducto de ferrocarril, Piedra pequeña bruta en cemento | G | 33.37 | 4.50 | 76.69 | 1.32 | 1.08 \| 1.37 | Couche. |
| Holy Trinity, Filadelfia | C | 29.15 | 4.60 | 25.38 | .84 | .84 | Annimati. |
| Puente Dean Escoc. Camino carretero | C | 27.43 | 4.15 | 14.71 | .61 | .80 | Telford. |
| » Dunkeld » » | C | 27.45 | 4.15 | 14.94 | .85 | .90 | Telford. |
| Acueducto Licking, Canal de Chesapeake y Ohio | C | 27.40 | 4.57 | 22.87 | 1.22 | .83 \| .93 | Fink. |
| Viaducto de Posen, Alemania. Ladrillos en cemento | C | 24.38 | 4.38 | 11.60 | .76 | .87 | Stevenson. |
| Allanton, Escocia. Carretera | C | 23.87 | 3.30 | 20.45 | .91 | .75 | |
| Falls Bridge, Ferrocarril, Filadelfia | C | 23.79 | 7.62 | 15.11 | .91 | .91 | Rennie. |
| Puente de Staines, Inglaterra. Carretera | C | 22.57 | 2.83 | 23.79 | .91** | | |
| Viaducto del ferrocarril Brent, Ingl. Ladrillos en cemento | E | 21.33 | 3.35 | 13.20 | .91 | .71 \| .92 | Brunel. |
| Puente de Bow, Carretera, Ingl | E | 20.13 | 4.39 | 12.18 | .76 | .74 | Walker. |
| Viaducto Royal Border, Ingl. Ladrillos en cemento | C | 18.76 | 3.37 | 9.57 | .84 | .66 \| .88 | |
| Bewally, Carretera, Inglaterra | C | 18.30 | 6.10 | 10.06 | .67 | .66 | Telford. |
| Puente de Chesnut Street, Filadelfia. Ladrillos en cemento | G | 18.30 | 5.49 | 10.37 | .76 | .67 \| .85 | Kneass. |
| Acueducto Monocacy, Canal de Chesapeake y Ohio | E | 16.47 | 2.77 | 15.72 | .76 | .71 | Fink. |
| Llanrwast, Gales, Carretera | E | 11.09 | 3.18 | 10.06 | .76 | .66 | Inigo Jones. |
| Viaducto sobre el Avon, Inglaterra | C | 15.25 | 4.57 | 8.63 | .81 | .62 \| .81 | Edwardes. |
| Acueducto sobre el río James, Virginia | C | 15.25 | 2.13 | 14.33 | .81 | .70 | Elliet. |
| Alcantarilla de Tonoloway debajo del canal Chese y Ohio. Piedra bruta en cemento | C | 12.20 | 4.57 | 6.40 | .61 | .54 \| .70 | Fisk. |
| Ferrocarril de Filadelfia y Reading | C | 13.42 | 2.44 | 10.46 | .76 | .63 | Steele. |
| » » » | C | 9.51 | 1.52 | 8.17 | .51 | .56 | Steele. |
| Edimburgo y Dalkeith, Escocia | E | 7.32 | 1.22 | 6.10 | .25 | .49 | Jardine. |

* Véase arco de experiencia en Souppes, Francia.
** Puente de Staines. Algunos autores dan .71 metros para el espesor en la clave.

**Arco de prueba en Souppes, Francia.** Véase la tabla. Abertura, más ó menos, 18 × flecha.

| | Abertura. | Flecha. | Radio del intradós. | Espesor del arco. En los arranques. | Espesor del arco. En la clave. En las caras. | Espesor del arco. En la clave. Entre las caras. | Anchura. |
|---|---|---|---|---|---|---|---|
| Metros..... | 37.886 | 2.125 | 85.5 | 1.10 | 1.10 | .80 | 3.5 |
| Pies..... | 124.30 | 6.97 | 280.52 | 3.61 | 3.61 | 2.264 | 11.5 |

El arco es de granito; se dejaron las cimbras durante cuatro meses sobre arena en 16 cilindros de .30 m de diámetro, .30 m de alto, hechos de planchas de hierro de $^3/_4$ mm de espesor. El arco descargado descendió 15 milímetros al quitar las cimbras. El descenso adicional bajo cargas extrañas fué como sigue.

| Carga extraña. | Kilogramos. | Libras. | Aumento en el descenso. Milímetros. | Pulgadas. |
|---|---|---|---|---|
| Distribuída..... | 367,000 | 809,000 | 21 | .8 |
| En el centro..... | 4,975 | 11,000 | .3 | .012 |
| Distribuída..... | 132,600 | 292,000 | 1.2 | .047 |

Con la carga distribuída de 367,000 kg, una carga de 4,975 kg cayendo sobre la clave de una altura de .3 mets causó vibraciones de 2.88 mm. *Annales des Ponts et Chaussées*, 1866, *Part* 2. 1868, *Part* 2.

El arco del ferrocarril Bourbonnais, es probablemente el arco de piedra más atrevido*, y el de Cabin John, por el capitán, ahora general, M. C. Meigs, del ejército de los E. U., el arco de piedra más grande existente. Pont y Prydd, en Gales, es un puente en un camino ordinario, de una construcción muy ruda, con una calzada de una pendiente peligrosa. Fué construído enteramente de piedras brutas, con mortero, por un albañil ordinario del campo, en 1750, y está todavía en perfecta condición. Sólo las dovelas exteriores ó las que *están á la vista* son de .75 m de espesor, y este espesor se compone de dos piedras. Las dovelas interiores no tienen sino .45 m de espesor y de 15 á 23 cm de grueso. Las piedras son de cantera con caras medianamente buenas, y se labraron muy poco ó nada. Este puente es un buen ejemplo de esa ignorancia, que muchas veces pasa por audacia. El puente Napoleón soporta un ferrocarril que atraviesa el Sena en París. Los arcos son de un espesor uniforme, de 1.20 m de la clave á los arranques. Los arcos se componen principalmente **de pedazos de piedra pequeños, sobrantes de canteras**, bien lavados para librarlos del sucio y del polvo; y luego colocados en buen lecho de cemento, con las juntas del mismo material. Este arco es por tanto como un arco de concreto. El puente de l'**Alma** cerca de él, y **construído del mismo modo**, tiene arcos elípticos de 38.40 á 43 m de luz, con flechas de $^1/_9$ de la abertura y clave de 1.49 m. Estos dos puentes, considerada la escasez de precedentes en estas construcciones en grande escala, deben considerarse como muy atrevidos y dan una gran reputación de prácticos á sus ingenieros señores Darcel y Cauche. La contracción desigual de los diferentes espesores de cemento originó algunos trastornos. Estos puentes demuestran lo que fácilmente puede hacerse en arcos de luz mediana con piedras pequeñas y *cemento hidráulico bueno*, á falta de piedras grandes propias para arcos. En el **puente Napoleón** el espesor del arco es menor de lo que nuestra regla da para piedras talladas de segunda clase.

**Art. 3. Los espesores de las claves calculadas para arcos grandes elípticos** por los mejores ingenieros, tienen generalmente una tercera parte más del que da nuestra regla, ó del que se da á los arcos circulares de la misma

* Construído como el de Souppes.

luz y flecha, para conservar así la línea de presión dentro de las juntas; aunque el arco elíptico, con las enjutas rellenas, tiene un peso algo menor; este peso tiene un brazo de palanca ligeramente menor que en un arco circular, y por consiguiente ejerce menos presión, tanto sobre la clave como sobre los arranques. Véanse los puentes Londres, Gloucester y Waterloo en el cuadro precedente.

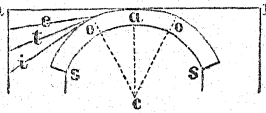


Fig. 1/2

*Nota.* Los ingenieros jóvenes son afectos á los arcos delgados, pero sería mucho mejor adoptar un sistema opuesto; porque no solamente los arcos gruesos hacen que la obra sea más estable, sino que un arco delgado es de un efecto tan desagradable á la vista, como una columna demasiado delgada. El arco es preferible hacerlo con 1/3 más de espesor del que da nuestra regla para las piedras talladas de primera clase, atendiendo á la apariencia. Especialmente cuando un arco es de piedra bruta, cuya mampostería cuesta más ó menos lo mismo en el arco que en el relleno de la enjuta, es una necedad construir los arcos de poco espesor. La estabilidad y la duración debe ser el objeto deseado, y cuando podemos lograrlo, aun con exceso, sin aumentar el costo, siempre es mejor hacerlo.

**Tabla 2. Espesor en las claves para arcos** de piedra tallada de primera clase, según el art. 2. Para los de segunda clase auméntese 1/8 parte; y para obras superiores de ladrillo, de 1/4 á 1/3, si el arco excede de 4.5 á 6 m más ó menos. (Original.)

| Luz en metros. | FLECHA, EN PARTES DE LA LUZ | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1/2 | 1/3 | 1/4 | 1/5 | 1/6 | 1/8 | 1/10 |
| | Clave en m. | Clave en m. | Clave en m. | Clave en m. | Clave en m. | Clave en m. | Clave en m |
| .610 | .1677 | .1708 | .1769 | .1830 | .1860 | .1952 | .2074 |
| 1.220 | .2135 | .2196 | .2257 | .2318 | .2409 | .2531 | .2684 |
| 1.830 | .2470 | .2531 | .2623 | .2714 | .2806 | .2958 | .3141 |
| 2.440 | .2775 | .2836 | .2928 | .3050 | .3141 | .3324 | .3538 |
| 3.050 | .3019 | .3080 | .3172 | .3263 | .3385 | .3599 | .3843 |
| 4.575 | .3568 | .3629 | .3721 | .3843 | .3965 | .4270 | .4575 |
| 6.100 | .4026 | .4117 | .4209 | .4361 | .4514 | .4849 | .5185 |
| 7.625 | .4422 | .4510 | .4666 | .4819 | .5002 | .5368 | .5734 |
| 9.150 | .4788 | .4880 | .5032 | .5215 | .5429 | .5825 | .6222 |
| 10.675 | .5124 | .5185 | .5378 | .5581 | .5795 | .6222 | .6679 |
| 12.200 | .5429 | .5520 | .5734 | .5947 | .6191 | .6649 | .7106 |
| 15.250 | .6008 | .6100 | .6344 | .6588 | .6862 | .7350 | .7869 |
| 18.300 | .6527 | .6649 | .6893 | .7167 | .7442 | .7991 | .8540 |
| 24.400 | .7442 | .7594 | .7869 | .8174 | .8479 | .9089 | .9699 |
| 30.500 | .8235 | .8387 | .8723 | .9058 | .9424 | 1.0126 | 1.0827 |
| 36.600 | .8967 | .9119 | .9455 | .9821 | 1.0217 | 1.1010 | 1.1834 |
| 42.700 | .9638 | .9790 | 1.0156 | 1.0553 | 1.098 | 1.1803 | 1.2657 |
| 44.800 | 1.0248 | 1.0492 | 1.0919 | 1.1346 | 1.1803 | 1.2718 | |
| 54.900 | 1.0858 | 1.1071 | 1.1437 | 1.1805 | 1.2383 | 1.3359 | |
| 61.000 | 1.1407 | 1.1620 | 1.2047 | 1.2566 | 1.3084 | | |
| 67.100 | 1.1925 | 1.2200 | 1.2596 | 1.3115 | 1.3664 | | |
| 73.200 | 1.2413 | 1.2657 | 1.3115 | 1.3664 | | | |
| 79.300 | 1.2901 | 1.3145 | 1.3633 | 1.4213 | | | |
| 85.400 | 1.3359 | 1.3603 | 1.4121 | | | | |
| 91.500 | 1.3816 | 1.4091 | 1.4640 | | | | |

**Art. 4. Encontrar las dimensiones de los estribos de un arco de piedra ó ladrillo, circular ó elíptico.** (Original.)

El autor se toma la libertad de ofrecer la regla siguiente en la creencia de que servirá para combinar las exigencias de la teoría con las de la economía y fácil eje-

cución, tanto como es posible, para reducir materia tan complicada **á una regla sencilla y segura que sirva para los trabajos de constructores prácticos de puentes.** Esto es todo lo que reclama. Sin embargo de su sencillez, es el resultado de mucha labor personal. Esta regla se aplica igualmente á la alcantarilla más pequeña como al puente más grande, cualesquiera que sean las proporciones de luz y flecha, y para cualquier altura de estribos. También se aplica á todos los rellenos sobre los arcos, generalmente en uso, háganse con mampostería sólida hasta el nivel *v f*, fig. 2, de la parte superior del arco, ó con tierra; ó con ambas; ó bien, con muros paralelos en las enjutas, prolongados hasta el paramento interior de los estribos, fig. 2½. Aunque la estabilidad de un estribo no puede permanecer exactamente la misma bajo todas estas condiciones, sin embargo la diferencia de espesor que resultaría de una investigación estricta en cada caso particular, no es suficiente fundamento para que compliquemos una regla destinada al uso práctico con una multitud de excepciones y modificaciones que destruirían el verdadero objeto para que fué creada.

**Damos espesores de estribos que sin relleno alguno detrás, ofrecen por sí solos seguridad; y también resisten las presiones en todos los casos, cuando el puente está sin carga.** Además, en los arcos muy grandes, en los cuales la carga máxima que probablemente tienen que soportar en la práctica es pequeña, comparadas con el peso del arco mismo y del relleno que hay encima, nuestros estribos ofrecen también seguridad para el puente *cargado*, independientemente de la tierra que está detrás de ellos; pero, cuando los arcos son más pequeños, y por consiguiente el peso de la carga es mayor, en proporción al peso del arco y del relleno, debemos contar más y más con la resistencia de la tierra colocada detrás de los estribos, para evitar la necesidad de dar á éstos un espesor extravagante. *Por lo tanto debe entenderse (excepto en el caso en que haya muros paralelos en las enjutas)* que nuestras reglas suponen que, como de costumbre, se depositen tierras detrás de los estribos, á la altura de la calzada, después de construído el puente.

En puentes pequeños ó alcantarillas grandes de ferrocarriles de primera clase, sujetos á la trepidación de los trenes pesados de gran velocidad, el poco costo, relativamente, con el cual se puede obtener un exceso de resistencia para construcciones importantes, ha inducido en muchos casos al uso de estribos de un espesor desde una cuarta parte, hasta la mitad mayores de los que da la regla siguiente.

**Si es de piedra bruta** agréguense 15 cm para obtener un espesor completamente seguro en todas partes.

$$\left.\begin{array}{l}\text{Espesor } on\text{, en pies, de los estribos en los arranques, cuando la elevación no excede de } 1\tfrac{1}{2} \text{ veces la base } sp.\end{array}\right\} = \frac{\text{Radio en pies}}{5} + \frac{\text{flecha en pies}}{10} + 2 \text{ pies.}$$

(*N. del T.* — La misma fórmula para el sistema métrico:

$$\left.\begin{array}{l}\text{Espesor } on\text{, en metros, de los estribos en los arranques, cuando la elevación no excede de } 1\tfrac{1}{2} \text{ veces la base } sp.\end{array}\right\} = \frac{\text{Radio en metros}}{5} + \frac{\text{flecha en m}}{10} + .61 \text{ m.)}$$

Marquénse los puntos *n*, *y*, así obtenidos. Después desde el centro *i*, de la luz ó cuerda *eo*, trácese *ih* igual á $^1/_{24}$ parte de dicha luz. Trácese *ha;* y por *n*, paralela á *ah*, trácese la línea indefinida *gnp* del estribo. Hágase lo mismo con el otro estribo. Hágase *ym* y *ng* ambas iguales á la mitad de la altura total *it* del arco; y desde *g* trácese una línea recta *gx*, tocando el trasdós del arco tan arriba como sea posible, ó, mejor todavía, como está representado en *tm*, con un radio *dt* ó *dm* (hallado por tanteo) descríbase un arco *tm; gx* ó *tm* será la parte superior del relleno de mampostería sobre el arco *, el cual debe terminarse antes de quitar las cimbras; antes de lo cual debe concluirse también el terraplén, por lo menos hasta *yn*.

---

***Excepto cuando la flecha es sólo 1/5 parte de la luz,** ó menos, en cuyo caso se lleva la mampostería sólida hasta el nivel *vtf*, de la parte superior del arco. O si el arco es grande, si excede, por ejemplo, de 18 m de abertura, más ó menos, y especialmente si su flecha es más ó menos mayor que $^1/_5$, entonces es mejor economizar la mampostería usando muros de enjuta interiores *ll*, fig. $2^1/_2$, los cuales se levantan hasta *vtf*. fig. 2. También se pueden introducir ventajosamente muros interiores como éstos en arcos mucho menores. Cuando son elevados pueden reforzarse con muros transversales *tt*. fig. $2^1/_2$. Sus bases debían terminar en escalones como se representa en *ooo*, de modo que descansen sobre toda la superficie del trasdós, igualando así la presión sobre

Búsquese luego por tanteos el punto *s*, fig. 2, donde el espesor *sp* es igual á dos tercios de la elevación vertical correspondiente *os*, y trácese *sp*. Entonces el *espesor on* ó *ey* será el de la línea de arranques del arco circular ó elíptico dado, de cualquier flecha ó luz; y la línea *gp* será el paramento interior del estribo, con tal que su elevación no exceda de 1½ veces *sp*; ó, en otras palabras, con tal que *sp* no sea menor de $^2/_3$ *os*. En la práctica *os* raras veces excede este límite; solamente en arcos de

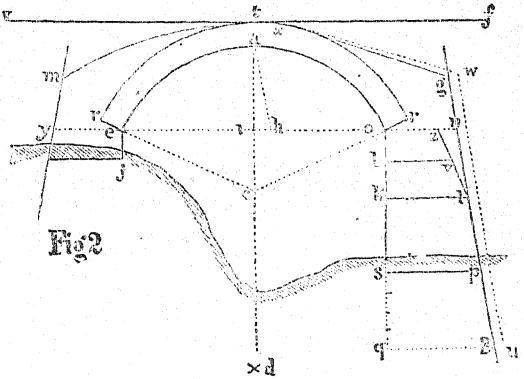


Fig 2

flecha considerable. Pero si sucede así, como por ejemplo en *oq*, entonces hágase la base *qu* igual á *sp*, más una cuarta parte de la altura *adicional sq*; y trácese el paramento interior *uw*, paralelo á *gp*, prolongado hasta la misma altura, etc., fig. 2. En el caso, sin embargo, de que esta adición de ¼ parte de *sq* dé en cualquier caso una base *qu*, menor de la *mitad* de la altura total *oq* (lo que en la práctica sucede raras veces), entonces hágase *qu* igual á la mitad de dicha altura total, trazando el paramento interior paralelo á *gp*, prolongado hasta la misma altura como se hizo antes. Los espesores adicionales hallados así debajo de *sp*, se refieren más bien á la presión de la tierra que está detrás de los estribos, que al empuje del arco. En un estribo muy elevado, la línea interior *gp* daría un espesor insuficiente para sostener la tierra con seguridad.

Cuando la altura *ob*, fig. 2, del estribo es menor que el espesor *on* en el arranque

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éste. En la parte superior de los muros pueden colocarse lajas, ó construirse pequeños arcos de un muro á otro, para sostener el balasto, etc., de la calzada. Los espacios que quedan debajo entre muro y muro se dejan vacíos. En la fig. $2^1/_2$ la parte más obscura,

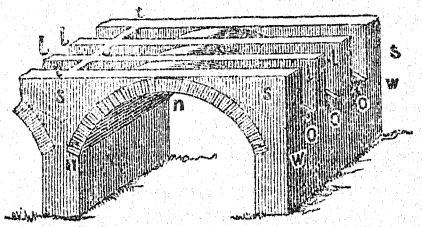

**Fig. 2 1/2.**

*ww*, se supone ser una sección transversal de un estribo, en la cual se ha omitido el segundo muro transversal semejante á *tt*. En puentes de ferrocarriles, colóquese un muro de enjuta *ll* debajo de cada riel.

puede economizarse un poco de mampostería (lo que no vale la pena, excepto en arcos muy grandes y rebajados ó en platabandas) reduciendo el espesor de todo el estribo de este modo: Hágase *ok* igual á *on* y trácese *kl*. Hágase *oz* igual á $^{4}/_{5}$ partes de *on* y trácese *lz*. Entonces, para cualquiera altura *ob* menor que *on*, trácese *bv*, terminando en *lz*. Esta *bv* será suficiente base si los cimientos son firmes. El paramento interior del estribo se trazará de *v* hacia arriba, paralelo á *gp*, y terminando á la misma altura que *g* ó *w*.

*Nota* 1. — Todos los estribos hallados así ofrecerán seguridad (con las precauciones del art. 6) independientemente de los muros en alas; no importando la altura á que llegue el terraplén sobre la parte superior del arco. Si el puente es angosto, y los paramentos de los muros en alas están, por consiguiente, tan próximos unos á otros que presten importante refuerzo á los estribos, estos últimos pueden tener menor espesor hasta un límite que fijará el ingeniero á su juicio.

Nosotros, sin embargo, advertimos á los jóvenes ingenieros que tengan cuidado al adoptar dimensiones más pequeñas que las dadas por nuestras reglas. Existen ciertas consideraciones prácticas, tales como descuidos en la ejecución de la obra; mortero nuevo ó reciente, el peligro de fuerzas indebidas al quitar las cimbras; la probabilidad de producir perturbación, descomposición, durante el proceso del relleno detrás de los estribos y sobre el arco, etc., á esto debe atenderse, aunque es imposible reducirlo al cálculo.

Siempre que sea posible, se deben dejar las cimbras colocadas hasta que esté concluído el relleno; y durante algún tiempo después para dar tiempo al mortero á que fragüe bien.

*Nota* 2. Algunos reducen con mucha libertad la cantidad de mampostería sobre la línea de los arranques de los arcos de flecha considerable y de luz mediana, teniendo cuidado de dejar las cimbras hasta que se concluye el relleno detrás de los estribos y sobre el arco, á fin de que no puedan producirse trastornos por algún accidente durante esta operación; esto se hace sobre todo cuando se usa buen cemento en lugar de mortero ordinario. Estos experimentos se pueden hacer con seguridad relativa, especialmente en arcos de alcantarillas, en los cuales el espesor es grande en proporción de la luz. Todo esto, sin embargo, debe dejarse al juicio del ingeniero, porque no se pueden dar reglas especiales para estos casos. Apenas pueden considerarse como fruto de la práctica y no podemos recomendarlo. Hemos visto puentes con arcos casi semicirculares, de 9 á 12 m de abertura, construídos así, y con éxito, teniendo apenas un poquito de mampostería sobre los arranques para reforzarlos. Estos arcos, sin embargo, están expuestos á caerse, al quitarles más tarde el relleno, si no se tiene la precaución de sostenerlos por medio de cimbras ó de otro modo. Aun cuando el terraplén pueda concluirse antes de quitar las cimbras, no podemos recomendar (y esto solamente en aberturas pequeñas) sino que se haga *ng*, fig. 2, *a lo menos* igual á $^{1}/_{5}$ parte de la elevación total *it* del arco; y desde *g* así encontrado, trazar una línea recta que toque el trasdós del arco tan arriba como se pueda.

*Nota* 3. Nada hemos dicho sobre la inclinación de los frentes de los estribos, porque, atravesando ríos, la inclinación ó disminuye el curso del agua, ó exige un arco mayor. Sin embargo una inclinación como de $^{1}/_{24}$ á $^{1}/_{8}$ es útil, como son los escalones, para distribuir el peso de la construcción y del terraplén sobre un área mayor de cimientos, especialmente cuando estos últimos no son muy firmes, ó cuando el terraplén llega á una altura considerable sobre el arco. En nuestra tabla n.º 3, de cantidades aproximadas de mampostería en puentes semicirculares de .61 á 15.25 m de luz, los paramentos exteriores de los estribos se suponen verticales.

**Art. 5. Pilas-estribo.** Cuando un puente consta de varios arcos sostenidos por pilas de un espesor ordinario solamente, si uno de los arcos fuere destruído por una creciente, ó de otro modo, los arcos adyacentes volcarían las pilas, y se caería un arco tras otro. Para impedir esto, se acostumbra, en puentes importantes, hacer algunas de las pilas de espesor suficiente parar resistir la presión de los arcos adyacentes, en el caso de un accidente semejante; y de este modo preservar una parte del puente de la ruina. Estas pilas se llaman pilas-estribo.

La fórmula (transformada por el traductor al sistema métrico) : $\frac{\text{Radio en m}}{5} + \frac{\text{flecha en m}}{10} + .61$ m para el espesor en los arranques; con la inclinación detrás como dijimos arriba, á razón de $^{1}/_{24}$ de la luz á la flecha, (*ih* para *ia*, (fig. 2) para paramentos exteriores verticales, (*sin modificación alguna para ele-*

*raciones grandes*) una pila-estribo perfectamente segura, para cualquier puente no cargado, y hasta cualquiera altura; teniéndose, sin embargo, debida consideración á las observaciones del artículo próximo. Así, para una pila-estribo tan elevada como *oq*, fig 2, ó de una elevación mayor aún, es solamente necesario hallar primero el espesor *on* en los arranques, como antes, y luego trazar el paramento interior inclinado *gnp*, extendiéndole hasta la base en B, sin agregar $\frac{1}{4}$ parte de la altura adicional *sq*. Esta adición se hace en el caso de estribos que deban ofrecer seguridad contra la presión de la tierra detrás de ellos, y contra la presión del arco; consideración que no se aplica á estas pilas, las que no deben resistir sino la presión de los arcos.

Pero aunque las pilas-estribos así calculadas por nuestra fórmula son suficientemente seguras, sin embargo su forma *abco*, fig. 3, es inadmisible. En la práctica se cambiaría por una forma más ó menos como la que indican las líneas de puntos, con una inclinación igual en ambas caras. Esto, por supuesto, requiere más mampostería, con poco aumento de estabilidad, pero no se puede evitar.

**Cuando se construye una pila-estribo en agua profunda** ó en corriente poco honda expuesta á crecientes, es necesario tener cuidado de que el agua no penetre por *debajo* de las pilas, y produzca una presión hacia arriba, la cual ó disminuye, ó enteramente destruye su eficacia como estribo. Véase nota 2, art. 4, de Hidrostática.

Fig 3

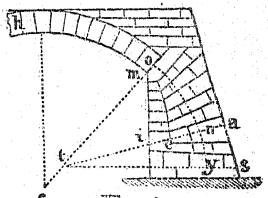

Fig. 4

**Art. 6. Inclinación de las hiladas de mampostería situadas debajo de los arranques de un arco.** Aunque nuestra regla precedente da el espesor que debe tener un estribo para no ser **volcado** por la presión del arco, sin embargo, si el arco es de una abertura grande y de pequeña flecha, su gran empuje horizontal puede producir un resbalamiento hacia afuera de la mampostería, cerca del nivel de los arranques, si la piedra se dispone en hiladas horizontales, especialmente si el mortero no ha fraguado bien.

Es cierto que este peligro podía evitarse ligando las hiladas de mampostería entre sí por medio de pernos ó lañas de hierro, ó aumentando considerablemente el espesor de los estribos; pero el costo necesario para esto nos induce á adoptar el método más económico de inclinar la mampostería como se indica entre *o* y *n*, fig. 4, dándoles á las hiladas cercanas á *o* más inclinación, y disminuyendo gradualmente la de las próximas á *n*. De este modo el arco se prolonga virtualmente dentro del cuerpo del estribo á tal distancia que, cuando la inclinación de las hiladas de mampostería inferiores cesa, como en *n*, la dirección de la línea de presiones, ó la presión del arco (representada por la línea curva de puntos *on*) esté casi en ángulo recto con las juntas de las hiladas horizontales situadas debajo de *n*; y, por consiguiente, dicha presión es incapaz de producir resbalamiento en este punto. Entre *o* y *n* la línea de presión está en todos sus puntos tan próximamente en ángulo recto con las diferentes hiladas inclinadas que llega á impedir también la posibilidad de un resbalamiento en dicho intervalo. Estando el estribo de este modo seguro contra el volcamiento y el resbalamiento, puede solamente fallar por causa de cimientos defectuosos ó por la inferioridad de las piedras con que esté construído, las cuales, si son blandas, pueden desmoronarse.

Esta inclinación de la mampostería es tan necesaria en un arco elíptico, fig. 4½, como en un arco circular. La forma elíptica, claramente, se presta poco para unir las dovelas con la mampostería inclinada cerca de los arranques, á fin de que resista

el empuje conveniente, ó más ó menos en ángulo recto á sus resultantes. En los casos ordinarios estas dificultades pueden vencerse haciendo solamente las juntas de las dovelas exteriores, ó de las que están á la vista, de conformidad con la curva elíptica, como entre *e* y *a*, mientras que las juntas interiores de atrás pueden tener la dirección indicada entre *g* y *u*, casi en ángulo recto á la línea de presiones. Rara vez

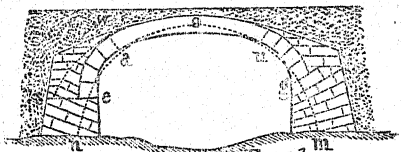

Fig. 4 1/2

sucederá, sin embargo, que un ingeniero joven tenga que construir arcos elípticos de suficiente magnitud para tener que emplear estos procedimientos. Para aberturas de menos de 15 m con flechas no menores de 1/4, más ó menos, de la luz no se requiere nada de esto, si el mortero es bueno y tiene tiempo de endurecerse *.

Para inclinar la mampostería de cualquier estribo con suficiente exactitud sería necesario trazar primero la curva de presión de dicho arco, para de este modo disponer las juntas de las hiladas de mampostería más ó menos en ángulo recto á dicha línea en cada punto; pero nosotros ofrecemos el método siguiente, que lo creemos suficiente para todos los casos prácticos, mientras que su sencillez lo coloca al alcance de un albañil ordinario. En puentes efectivos la dirección real del empuje varía con el paso de la carga; por consiguiente, en la práctica, ningún grado de inclinación dado de la mampostería de los estribos puede adaptarse exactamente á dicha dirección durante el paso de la carga. Por lo tanto, todo exceso de cálculo en este particular, es simplemente ridículo, especialmente en pequeñas luces.

**Reglas para fijar la inclinación de las hiladas de mampostería en los estribos.** Súmese el radio *cm*, fig. 4, con la luz del arco; divídase esta suma por 5 y al cociente agréguense 3 pies. (*N. del T.* — O, empleando el sistema métrico, súmese el radio *cm* y la luz del arco en metros, divídase la suma por 5 y al cociente agréguense .915 metros.) Hágase *ot* sobre el radio igual á esta última suma, y entonces *t* será el centro hacia el cual deben converger las direcciones de las juntas, como se indica en la figura. Trácese *ts* horizontal y desde *t* como centro descríbase el arco *oy;* siendo *o* el centro del arranque. Desde *y* tómese en el arco la dist *yn*, igual á una sexta parte de *ty*, trácese *tna*. Nunca *será necesario* inclinar la mampostería situada debajo de *tna*. Tampoco *es preciso* que la inclinación se prolongue enteramente hasta el paramento *mi* del estribo, sino que puede terminar en *e*, próximamente en medio de *i y n*. De, *e*, hacia arriba, la inclinación puede prolongarse hacia adelante hasta la línea *em*.

**El cemento** debe usarse sin restricción, no solamente en los arcos mismos, y en la mampostería de encima, para preservarlos de la filtración de la lluvia, sino también en los estribos, muros en alas y de sostenimiento, y en toda mampostería expuesta á la humedad. Todo el trasdós de los arcos importantes de ladrillos debía cubrirse con una capa de buen cemento, de 3 cm de espesor, más ó menos. La falta de esta precaución puede notarse en casi todas nuestras obras públicas. El mortero ordinario se pudre (descompone) y se cae del intradós de los arcos, y de las juntas de mampostería en general, próximamente á 1 ó 2 m de la superficie del suelo. La humedad sube por la acción capilar, hasta aquella distancia sobre la superficie del suelo; ó desciende á ella desde la superficie artificial del terraplén, etc.; por lo tanto, debería emplearse el mortero de cemento por lo menos en aquellas partes.

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*Los pies de los arcos, tanto elípticos como semicirculares, se hacen siempre horizontales; pero se ve claramente en la fig. 4 1/2, que esta práctica está en desacuerdo con los principios de estabilidad en el caso de la elipse. Lo mismo sucede en el semicírculo. En los puentes ordinarios de esta última forma, la presión vertical, ó peso que descansa sobre cada arranque (hablando en general), es comúnmente de 3½ á 4 veces la presión horizontal sobre la misma, y la presión total es, más ó menos, 4 veces la presión sobre la clave. Por consiguiente, teóricamente, el arranque debía tener un espesor 4 veces mayor que el de la clave, y su lecho, en lugar de ser horizontal, debía tener una inclinación á razón, más ó menos, de 1 vertical por 4 horizontal.

La lluvia y la intemperie perjudican más al mortero que se aplica en los paramentos exteriores de los muros inclinados (aun cuando la inclinación no sea sino de $^1/_8$ á $^1/_{12}$) que en los muros verticales, y por consiguiente aquél debe ser de mejor calidad. Véase Mortero, etc.

Hemos visto, no obstante, una regular filtración de aguas superficiales á través de arcos de ladrillos de casi 1 m de espesor, aun usando el cemento sin restricción. En puentes de acueductos, creemos que el cemento no basta á impedir las filtraciones, ya sean los arcos de ladrillo, ó de piedra tallada. ¿No sería esto efecto de grietas producidas por el asiento del arco, ó por la contracción y dilatación bajo las influencias atmosféricas? El cemento á lo menos impide que las juntas se desmoronen.

Cuando el arco es muy rebajado, esta inclinación puede llegar á ser tan fuerte, especialmente en sus partes superiores, que hay que emplear puntales de cualquier clase para impedir que la mampostería resbale, hasta que, completo el arco, lo impida. Las hiladas horizontales comprendidas entre el paramento *mi*, y la línea *oe*, ayudarán algo en este respecto.

Este método debía aplicarse *á todos los arcos muy grandes*, cuya flecha sea una tercera parte, ó menos de la luz. Como se observó anteriormente, no es muy necesario en los arcos que no exceden de 15 m de luz y no rebajados más de $^1/_3$ de dicha luz. Ciertamente, si se puede depositar el relleno de tierra antes de quitar las cimbras, estos límites pueden extenderse considerablemente sin peligro. Sin embargo, como cierto grado de inclinación puede hacerse con muy poco trabajo y costo, recomendamos, aun para tales arcos, un método algo semejante al que sigue: De la *mitad* de la abertura réstese la flecha; divídase el residuo por 3. (*N. del T.* — Lo mismo en pies que en metros.) Hágase *ot*, fig. 5, igual al cociente; trácese *tn*, y *om*, horizontales; divídase el ángulo *som* en dos partes iguales, por la línea *ao;* désele á la mampostería una inclinación tal que sea paralela á *ao*, y que llegue hasta *tn*. Las hiladas inclinadas pueden ó no prolongarse hasta el paramento *ot*, según se quiera.

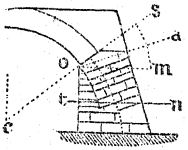


Fig 5

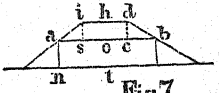


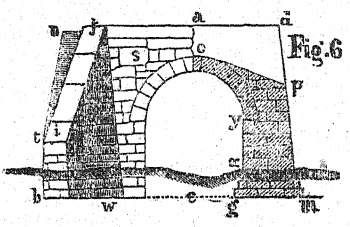
Fig 7

*Nota* 1. **Hallar la longitud** (*ab*, fig. 7) de boca á boca, **de una alcantarilla** que atraviesa un terraplén. De la altura *ht* del terraplén, tómese la altura, *na*, de la alcantarilla; el residuo será la altura, *ho*, del terraplén sobre la alcantarilla. Luego la longitud requerida *ab* es igual al ancho de la parte superior *id* del terraplén, más las dos distancias *as*, *cb*, que corresponden á los taludes laterales. Por tanto, si el talud lateral es, como de costumbre, de 1½ á 1, entonces *as* y *cb* serán cada una iguales á 1½ veces *oh;* ó las dos serán 3 veces *oh*. De modo que si el ancho *id* es = 6 m, y *ho* 1.80 m, la longitud *ab* será $6+(1.8\times3)=6+5.4=11.40$ m.

**Art. 7. Las tablas siguientes n.os 3 y 4**, servirán para facilitar los cálculos de presupuestos preliminares aproximados; por tanto, no se han hecho escrupulosamente exactas, sino más bien un poco exageradas. La primera columna de la tabla 3 contiene la altura vertical total *oc*, fig. 6, desde la clave *o* de un arco semicircular, hasta los cimientos ó base *gm* de sus estribos. Las otras columnas dan aproximadamente el número de *metros cúbicos* contenidos en cada metro corrido, ó metro de longitud de la alcantarilla ó puente, medido entre las caras del arco justamente, é incluyendo solamente el arco y sus estribos, como se indica en la fig. 1, ó en la semisección *opmgy*, fig. 6; incluyendo la base de los estribos, pero omitiendo los muros en alas (*wn*) y los muros en las enjutas (*s*), figs. 6 y 2½. Al pie de cada columna está el volumen aproximado en metros cúbicos de los dos muros de las enjutas por sí solos; calculando uno sobre cada frente de arco.

Estos últimos se calcularon en la suposición de que su espesor en la base, en su unión con los muros en alas, donde su altura es máxima, sea igual á $^1/_{10}$ de su altura

en dicho punto; exceptuando el caso en que esta proporción dé un espesor en la parte superior menor que .75 m y que tengan una altura de .60 m (*oa*) sobre la parte superior *o* del arco. En esta parte se suponen de .75 m de espesor, arriba, espesor que se juzga para estos casos, más ó menos, el mínimo admisible en muros de piedra bruta. Tanto el paramento interior como el exterior se suponen verticales. Los volúmenes ó cubos de estos muros de enjutas varían aún algo para la




misma luz, con la altura de los estribos y disposición de las alas. Sin embargo forman una parte tan pequeña del cubo total, que puede prescindirse de esta consideración en los presupuestos preliminares. Están unidos tan firmemente á la mampostería de los muros en alas en sus puntos más altos, y tan fuertemente conectados por mortero con el relleno del arco en su base, que no requieren un espesor mayor, cualquiera que sea la altura del terraplén.

**El volumen de los cuatro muros en alas**, de los cuales *njwb*, fig. 6, es uno, se encontrarán en una tabla (n.º 4) que sigue inmediatamente á las alcantarillas.

*Nota* 1. Aunque el espesor de los muros en alas aumenta en todas sus partes con su altura, no aparecen con un espesor mayor en *nj* que en *ti*, fig. 6; pero (como se ve en la figura) se construye un escalón en su paramento interior *tn*, un poco más abajo de la superficie superior inclinada *ij*, para dar de este modo una anchura uniforme á las lajas (ó piedras labradas, según el caso) con que están cubiertos. Para impedir que las baldosas resbalen en el plano inclinado *ji*, la piedra ó baldosa inferior *i* debe ser grande y gruesa, y colocada en un lecho horizontal. Algunas veces se fijan las lajas ó baldosas al muro con lañas de hierro y por medio de pernos. Haciendo escalones (lo que es preferible) no se requiere nada de esto, como se ve en *s*, fig. 11.

*Nota* 2. **Las tablas demuestran la inconveniencia de estrechar demasiado el cauce de las aguas**, por vía de economía, adoptando arcos de pequeña luz cuando pueda hacerse una alcantarilla de la misma altura *total* y de mayor luz.

Como las alas deben ser lo mismo, sea la abertura grande ó pequeña, con tal de que la altura total sea la misma en ambos casos, y como ellas constituyen una gran parte de la cantidad total de mampostería, en las alcantarillas de longitud ordinaria, la luz por sí, dentro de límites medianos, ejerce relativamente poca influencia sobre el volumen de aquélla. De este modo, la cantidad total de mampostería de una alcantarilla semicircular de .91 m de luz, de una altura total de 2.44 m, y de 18.30 m de longitud entre los frentes de los arcos, tiene la misma mampostería (dif uno ó dos m³) que una de 1.52 m de luz, de la misma altura y longitud.

*Nota* 3. En parte por esta razón, y en parte debido á que las **alcantarillas destinadas á una vía doble no tienen el doble de la longitud de las de una sola vía**, la cantidad de mampostería de alcantarillas de las primeras no es, por término medio, sino de $^{1}/_{4}$ á $^{1}/_{3}$ parte, más ó menos, mayor que la de las últimas; de manera que muchas veces es conveniente hacer las alcantarillas de una vez, con la longitud requerida para una vía doble, aunque los terraplenes se hagan al principio sólo para una vía sencilla, con la idea de hacerlos más tarde para vía doble.

**Tabla 3, del número aproximado de metros cúbicos de mampostería por metro corrido, contenidos en los arcos y estribos solamente,** como se muestra en la fig. 1 (omitiendo las alas y los muros de las enjutas colocados sobre los frentes de los arcos) de alcantarillas y puentes semicirculares, de .61 hasta 15.25 m de luz y de diferentes alturas totales *ht*, fig. 1, ó bien *oc*, fig. 6. Se verá que en muchos casos, un puente de una luz mayor contiene menos mampostería que otro, de luz más pequeña, *cuando sus alturas totales son las mismas* En dicha tabla se tienen en cuenta sin restricción alguna los zócalos ó escalones de las bases de los estribos.

**TABLA 3.** (Original.)

(*N. del T.* — Convertida al sistema métrico.)

| Altura total | Luz .61 m. | Luz .915 m. | Luz 1.22 m. | Luz 1.52 m. | Luz 1.83 m. | Luz 2.44 m. | Luz 3.05 m. | Luz 3.66 m. | Luz 4.57 m. |
|---|---|---|---|---|---|---|---|---|---|
| Metros. | M cúb. | M cúb. | M cúb. | M cúb. | M cúb. | M cúb. | M cúb. | M cúb. | M cúb. |
| .610 | 1.0525 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... |
| .915 | 1.5036 | 1.5787 | 1.679 | ..... | ..... | ..... | ..... | ..... | ..... |
| 1.220 | 1.9797 | 2.0790 | 2.1802 | 2.3055 | 2.4308 | ..... | ..... | ..... | ..... |
| 1.525 | 2.4809 | 2.6062 | 2.7064 | 2.8819 | 3.0322 | ..... | ..... | ..... | ..... |
| 1.830 | 3.2076 | 3.2076 | 3.2076 | 3.4332 | 3.6587 | 3.9594 | 4.2351 | ..... | ..... |
| 2.135 | 4.0597 | 3.9843 | 3.8843 | 4.1098 | 4.3103 | 4.6361 | 4.9368 | 5.3127 | ..... |
| 2.440 | 5.0670 | 4.9117 | 4.7864 | 4.8867 | 4.9869 | 5.3377 | 5.6635 | 5.9642 | ..... |
| 2.745 | 6.1397 | 5.9642 | 5.7888 | 5.7387 | 5.6886 | 6.0645 | 6.4153 | 6.6409 | 7.568 |
| 3.050 | 7.3676 | 7.1421 | 6.9165 | 6.8163 | 6.6910 | 6.9416 | 7.1922 | 7.3425 | 8.370 |
| 3.355 | ..... | 8.4702 | 8.1695 | 7.9941 | 7.8187 | 7.9189 | 7.9941 | 8.0943 | 9.197 |
| 3.660 | ..... | 9.9738 | 9.5729 | 9.3223 | 9.0717 | 8.9464 | 8.8211 | 8.8963 | 10.049 |
| 3.965 | ..... | ..... | 11.0765 | 10.7507 | 10.450 | 10.2746 | 10.0741 | 9.6731 | 10.926 |
| 4.270 | ..... | ..... | 12.7304 | 12.2794 | 11.9536 | 11.7030 | 11.4524 | 11.0514 | 11.828 |
| 4.575 | ..... | ..... | ..... | 13.9584 | 13.5825 | 13.2818 | 12.9560 | 12.5550 | 12.755 |
| 4.880 | ..... | ..... | ..... | 15.7878 | 15.3367 | 14.9608 | 14.5849 | 13.9333 | 14.259 |
| 5.185 | ..... | ..... | ..... | ..... | 17.2162 | 16.7902 | 16.3391 | 15.6875 | 15.888 |
| 5.490 | ..... | ..... | ..... | ..... | 19.2711 | 18.7448 | 18.2186 | 17.5670 | 17.642 |
| 5.795 | ..... | ..... | ..... | ..... | ..... | 20.8499 | 20.2234 | 19.3212 | 19.271 |
| 6.100 | ..... | ..... | ..... | ..... | ..... | 23.0552 | 22.3535 | 21.4513 | 21.275 |
| 6.405 | ..... | ..... | ..... | ..... | ..... | ..... | 24.6081 | 23.7067 | 23.406 |
| 6.710 | ..... | ..... | ..... | ..... | ..... | ..... | 27.0648 | 25.8118 | 25.561 |
| 7.015 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 28.3178 | 27.816 |
| 7.320 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 30.8238 | 30.322 |
| 7.625 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 33.079 |
| 7.930 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 35.585 |

Volumen de los dos muros de las enjutas; uno sobre cada frente del arco.

| 2.22 | 2.83 | 3.36 | 3.97 | 4.43 | 6.04 | 7.49 | 9.47 | 12.23 |
|---|---|---|---|---|---|---|---|---|

**TABLA 3.** (Continuación.)

| Altura total | Luz 6.4 m. | Luz 7.6 m. | Altura total | Luz 10.67 m. | Altura total | Luz 15.25 m. |
|---|---|---|---|---|---|---|
| Metros. | M. cúb. | M. cúb. | Metros. | M. cúb. | Metros. | M. cúb. |
| 3.660 | 11.527 | ..... | 6.100 | 26.313 | 8.235 | 45.108 |
| 3.965 | 12.480 | ..... | 6.405 | 27.566 | 8.540 | 46.862 |
| 4.270 | 13.457 | 15.287 | 6.710 | 29.070 | 8.845 | 48.616 |
| 4.575 | 14.460 | 16.063 | 7.015 | 30.573 | 9.150 | 50.371 |
| 4.880 | 15.487 | 16.941 | 7.320 | 31.826 | 9.455 | 52.375 |
| 5.185 | 16.540 | 17.943 | 7.625 | 33.330 | 9.760 | 54.130 |
| 5.490 | 17.617 | 19.071 | 7.930 | 34.583 | 10.065 | 56.134 |
| 5.795 | 18.720 | 20.299 | 8.235 | 36.337 | 10.370 | 57.889 |
| 6.100 | 20.349 | 21.552 | 8.540 | 37.841 | 10.675 | 59.893 |
| 6.405 | 22.103 | 22.604 | 8.845 | 39.344 | 10.980 | 61.898 |
| 6.710 | 23.982 | 24.358 | 9.150 | 40.848 | 11.285 | 63.903 |
| 7.015 | 26.062 | 26.062 | 9.455 | 42.602 | 11.590 | 65.908 |
| 7.320 | 28.318 | 28.067 | 9.760 | 45.359 | 11.895 | 67.913 |
| 7.625 | 30.573 | 30.323 | 10.065 | 48.115 | 12.200 | 70.168 |
| 7.930 | 32.829 | 32.578 | 10.370 | 51.122 | 12.505 | 72.173 |
| 8.235 | 35.335 | 35.084 | 10.675 | 54.380 | 12.810 | 75.180 |
| 8.540 | 38.091 | 37.590 | 10.980 | 57.638 | 13.115 | 78.939 |
| 8.845 | 40.848 | 40.347 | 11.285 | 60.896 | 13.420 | 82.698 |
| 9.150 | 43.604 | 43.103 | 11.590 | 64.404 | 13.725 | 86.708 |
| 9.455 | 46.612 | 46.110 | 11.895 | 68.163 | 14.030 | 90.968 |
| 9.760 | 49.869 | 49.118 | 12.200 | 71.922 | 14.335 | 95.479 |
| 10.065 | 53.127 | 52.375 | 12.505 | 75.681 | 14.640 | 99.739 |
| 10.370 | 56.637 | 55.633 | 12.810 | 79.671 | 14.945 | 104.250 |
| 10.675 | 60.144 | 59.142 | 13.115 | 83.451 | 15.250 | 109.262 |
| 10.980 | 63.652 | 62.650 | 13.420 | 88.241 | 15.555 | 114.023 |
| 11.285 | 67.414 | 66.409 | 13.725 | 92.471 | 15.860 | 118.754 |
| 11.590 | 71.421 | 70.168 | 14.030 | 96.982 | 16.165 | 123.796 |
| 11.895 | 75.431 | 73.927 | 14.335 | 101.744 | 16.470 | 129.310 |
| 12.200 | 79.440 | 78.187 | 14.640 | 106.505 | 16.775 | 134.572 |
| 12.505 | ..... | 82.497 | 14.945 | 111.266 | 17.080 | 140.085 |
| 12.810 | ..... | 86.457 | 15.250 | 116.278 | 17.385 | 145.599 |
| 13.115 | ..... | 90.168 | ..... | ..... | 17.690 | 151.362 |
| 13.420 | ..... | 95.479 | ..... | ..... | 17.995 | 157.126 |
| 13.725 | ..... | 100.240 | ..... | ..... | 18.300 | 163.141 |
| Volumen de los dos muros de las enjutas; uno sobre cada frente del arco. | | | | | | |
| | 21.4 | 32.1 | | 65.0 | | 149.0 |

**Art. 8. La tabla que sigue** da el volumen de mampostería de los muros en alas; es, como la anterior, de utilidad para la ejecución de presupuestos preliminares. Las alas *no*, *no*, representadas en proyección en la fig. 8, se suponen que forman un ángulo *aoc*, de 120° con la cara *oo* de la alcantarilla. Sus caras exteriores

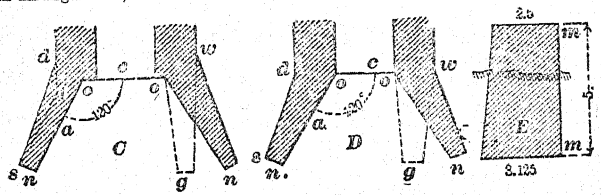



**Fig. 8**

extremas, *m*, se ven en una escala mayor en E. Espesor uniforme en toda la base igual á $^{4}/_{10}$ parte de la altura del muro en dicha parte; excepto el caso en que dicha cantidad resulte demasiado pequeña é impida darle á la parte superior un espesor de .75 m; en cuyo caso se ensancha aquélla lo suficiente. Véase nota 2. Esto sucede solamente cuando la altura *mm* del muro, fig. E, es menor de 2.70 m. Inclinación del frente 1 en 8. Paramento interior vertical, pero escalonado (si fuese necesario), á una pequeña distancia debajo de la parte superior, como para darle

á la parte superior visible un espesor uniforme de .75 m. La mampostería se supone de piedra bruta (bien picada) con mortero. Las alturas dadas de la primera columna son las mayores; es decir (*w y*, en la fig. 6), donde el ala se une con el frente de la alcantarilla. En la tabla no se tiene en cuenta el zócalo ó escalones de la base de los muros en alas; porque éstos se omiten frecuentemente cuando hay buenos cimientos. Al tomar las cantidades de la tabla, téngase en cuenta que la altura de las alas es generalmente un poco mayor que la de la alcantarilla misma.

El plano indicado en C es el que generalmente se usa; pero el plano D es preferible para alcantarillas; porque los recodos *oo* en la fig. C, además de que pueden sujetar los objetos arrastrados por la corriente, como ramas de árboles, etc., ofrecen por sí mismos una resistencia mayor á la entrada del agua de la que oponen las esquinas *oo*, fig. D.

**Tabla 4, del volumen aproximado, en metros cúbicos, de los cuatro muros en alas de un puente ó alcantarilla.**

| Altura de los muros en alas. | Largo de un ala. | M. cúb en las 4 alas. | Altura de los muros en alas. | Largo de un ala. | M. cúb en las 4 alas. |
|---|---|---|---|---|---|
| m. | m. | m cúb. | m. | m. | m cúb. |
| 1.830 | .5276 | 3.07 | 9.150 | 13.206 | 621 |
| 2.135 | 1.0553 | 6.73 | 9.760 | 14.274 | 757 |
| 2.440 | 1.586 | 11.10 | 10.370 | 15.341 | 906 |
| 2.745 | 2.1136 | 16.34 | 10.980 | 16.378 | 1,075 |
| 3.050 | 2.6413 | 22.95 | 11.590 | 17.446 | 1,262 |
| 3.355 | 3.172 | 31.08 | 12.200 | 18.513 | 1,465 |
| 3.660 | 3.691 | 40.81 | 12.810 | 19.581 | 1,687 |
| 4.270 | 4.758 | 64.75 | 13.420 | 20.618 | 1,939 |
| 4.880 | 5.825 | 97.28 | 14.030 | 21.685 | 2,213 |
| 5.490 | 6.862 | 139.08 | 14.640 | 22.753 | 2,512 |
| 6.100 | 7.930 | 187.72 | 15.250 | 23.790 | 2,843 |
| 6.710 | 8.997 | 250.04 | 16.775 | 26.443 | 3,756 |
| 7.320 | 10.034 | 323.76 | 18.300 | 29.066 | 4,867 |
| 7.930 | 11.102 | 411.16 | 19.825 | 31.720 | 6,179 |
| 8.540 | 12.139 | 510.72 | 21.350 | 34.465 | 7.717 |

(*N. del T.* — Convertida al sistema métrico.)

El volumen para alturas intermedias puede hallarse aproximadamente por una simple proporción.

*Nota* 1. No se recomienda prolongar todas las alas hasta hacer sus dimensiones tan pequeñas como las representadas en E, fig. 8. En los muros grandes en alas es generalmente más económico aumentar algo la altura de su extremidad *mm*. El cubo de la mampostería puede también hallarse fácilmente en este caso por la tabla. Así, suponiendo la altura de las alas de 9.15 m en una extremidad, y 2.44 m en la otra, tenemos solamente que restar el cubo dado por la tabla para 2.44 m de altura, del que da para 9.15 m. De modo que 621—11.10 = 609.90 es el volumen buscado.

*Nota* 2. Podría suponerse, que como los muros en alas tienen á menudo que sostener la presión de terraplenes que llegan hasta más arriba de su parte superior, debían hacerse en este caso de mucho más espesor, como muros de sostenimiento ordinarios. Pero el hecho de que dichos muros tengan mayor estabilidad, por estar unidos por sus extremidades elevadas con el cuerpo del puente ó alcantarilla, hace innecesario este aumento, cuando las alas tengan las dimensiones prescritas en nuestra tabla, y no importa hasta dónde se extienda la tierra sobre ellos; así está demostrado por numerosas experiencias.

Confiados en este refuerzo, podemos ciertamente, cuando la tierra no se prolongue sobre la parte superior, reducir la base en *o á'* un tercio de la altura, como se indica en *ot*, y por la línea de puntos *ts*. La experiencia demuestra que podemos hacer lo mismo, aun cuando la tierra llegue á una gran altura sobre la parte superior, con tal que las alas, en lugar de estar más separadas ó abiertas en sus extremidades como en *on, on*, sólo formen prolongaciones rectas de los estribos del arco, como se indica por las líneas de puntos *ogw*. En esto caso, la presión de la tierra contra las alas es menor que cuando están abiertas. Hemos sabido que el espesor en *o* se redujo en tales casos á menos de una tercera parte de su altura, cuando las alas tenían

4.5 m de alto, y la altura del terraplén *sobre sus partes superiores* era de 4.8 m en un caso, y de 11 m en otro. En otro puente, alas semejantes de 7.78 m de altura, y con un terraplén de 8.84 m sobre su parte superior, tenían en sus bases, *o*, más bien menos de $\frac{1}{4}$ parte de su altura. Siempre, el espesor uniforme en la parte superior era de .75 m, con los paramentos interiores verticales. Mencionamos estos casos, porque parece que en especial esta materia, no puede reducirse á una regla práctica. El último muro nos parece demasiado delgado, especialmente si la tierra no se deposita con cuidado y en capas, y después que el mortero esté endurecido. Sin embargo, el trabajo de depositar la tierra cuidadosamente en capas, puede costar más de lo que se economiza en la mampostería. El joven practicante debe tener esto en miente cuando desea economizar mampostería por estos medios; y también debe acordarse que un muro delgado puede desplomarse, ó caerse enteramente, si el relleno de tierra se deposita mientras el mortero no se ha endurecido perfectamente.

**Art. 9.** Debe tenerse cuidado especial en hacerles **cimientos firmes á las alcantarillas y desagües construídos debajo de terraplenes elevados**; porque de otro modo el peso, especialmente bajo la mitad del terraplén, puede hundirlos, si el suelo es blando ó cenagoso; y de este modo disminuir la sección de salida del agua, ó por lo menos causar una depresión de mal efecto en el centro de la alcantarilla. También, en un terreno blando, el relleno puede comprimir los muros laterales, y acercarlos, haciendo más angosto el canal. Esto puede impedirse por medio de un arco invertido ó por medio de un lecho de mampostería entre los muros. Una capa de granzón, arena, ó piedras trituradas de 1 á 2 m de espesor, será generalmente suficiente cimiento para alcantarillas en terrenos cenagosos y sospechosos; también puede usarse la arena suelta, pero en rellenos de altura mediana. Estos cimientos artificiales deben sobresalir algunos metros fuera de la mampostería, en todas direcciones, y apisonar si es posible cuando el granzón ó la arena están completamente mojados, para contribuir á la consolidación. Algunas veces será necesario clavar estacas. Si la mampostería se construye sobre una plataforma de madera, ó sobre superficie lisa de roca, debe tenerse cuidado de impedir que resbale por la presión de la tierra que está detrás. Esta misma puede volcar las estacas si no están bien sujetas.

**Art. 10. Desagües.** Los desagües de las dimensiones indicadas en la fig. 11 (cuyos números indican pies, 1 pie = .305 m. *N. del T.*) tienen 2.32 metros cúbicos de mampostería por metro lineal. Se construyen á menudo de piedra en seco

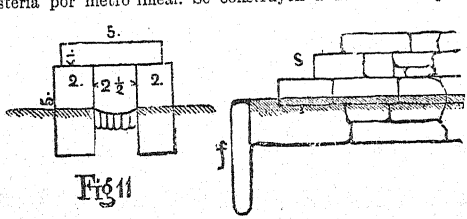


Fig. 11

desbastada haciéndoles el fondo ó piso con fragmentos de piedras. Cuando pasa una gran cantidad de agua por ellos y tienen una pendiente considerable, es mejor hacer el cimiento continuo á través de todo el desagüe. Muchas veces se hace esto aun sin los motivos indicados, porque la mampostería que se agrega es una cantidad insignificante, y la excavación de una sola fosa ancha para los cimientos, se hace más fácilmente que dos angostas. Puede enterrarse á la entrada y á cortas distancias en longitud lajas gruesas, *f*, fig. 11, tanto en las alcantarillas, como en los desagües para impedir que se socaven.

Estos desagues se prolongan debajo de todo el ancho del terraplén y pueden terminarse en escalones, como lo indica la vista lateral S. Por supuesto que es mucho mejor construirlos de mortero de cemento, para impedir cuando está lleno de agua que se filtre y ablande el terraplén.

Algunas veces se pueden colocar dos ó tres de estos desagües unos al lado de otros y paralelos, en lugar de una alcantarilla. Cuando se colocan dos de este modo, sólo contienen 1½ veces la mampostería de uno; sin embargo su uso no ofrece más economía que las alcantarillas. Un hombre puede penetrar en el interior para limpiarlos, fig. 11.

**Art. 11. Los desagües de las calzadas** en los puentes de mampostería, de varios arcos, se hacen generalmente por medio de canales abiertos en la clave de los arcos y en cada lado, hasta que llegan cerca de las extremidades de las luces respectivas. Aquí, descargan sobre tubos verticales de hierro introducidos en la mampostería. La extremidad superior de estos tubos debe cubrirse con un enrejado. Cuando el agua que sale de estos tubos causa un inconveniente, cayendo sobre la gente que pasa por debajo de los arcos, estos tubos deben prolongarse hasta el pie de dichos arcos; pero si no fuese así, pueden llevarse solamente hasta el intradós, dejando que el agua caiga libremente de esta altura.

**Art. 12. Arcos de ladrillo.** Como hasta los ladrillos buenos, propios para arcos grandes, tienen mucha menos resistencia á la ruptura por compresión que el buen granito ó piedra caliza buena, y es inferior aún á la buena piedra arenisca, mientras que su peso no difiere mucho del peso de la piedra, es claro que no pueden usarse en arcos de tan gran abertura como se usa la piedra. Algunos de ellos ya construídos, y que han durado muchos años, solamente tienen un coeficiente teórico 3 de seguridad, más ó menos; mientras que las autoridades nos enseñan á no confiar, aun á la piedra, en más de una vigésima parte de su carga de ruptura por compresión. Esto último, sin embargo, nos parece una de esas suposiciones ligeras, que, una vez admitidas en los libros profesionales, es difícil destruirlas. En nuestra opinión con buen cemento, y el cuidado necesario al quitar las cimbras, es suficiente la décima parte de la máxima resistencia aun contra los esfuerzos anormales producidos por el asiento de la clave y elevación de los riñones de la bóveda, cuando se quita la cimbra. Es inútil tratar de fijar los límites de seguridad de los materiales malos y peor dispuestos.

*Nota* 1. La práctica ordinaria de construir arcos de ladrillo en una serie **de anillos concéntricos** como en *acee*, fig. 12, sin otra trabazón entre ellos que la del mortero, la censuran las autoridades, porque la curva de presión al pasar del trasdós al intradós tiende á separar los anillos, y de este modo debilita el arco, rajándolo, por decir así, longitudinalmente. La razón de usar estos anillos, en lugar de hacer las juntas radiales continuas en todo el espesor del arco *mn*, como en *b*, es para evitar las juntas gruesas de mortero en el trasdós del arco, como lo indica la figura. Si en un arco, construído como se indica en *b*, se quita la cimbra demasiado pronto, el mortero blando de estas juntas gruesas, se comprimirá tanto, que causará un gran descenso de la corona, quitándole de este modo la forma al arco, y causando tal desigualdad de presión, que puede ocasionar hasta su caída, especialmente si el arco es rebajado. Como sistema intermedio entre el arco de anillos y el de juntas continuas, se emplean algunas veces los dos combinados, á fin de eliminar **algunas** de las juntas largas radiales, y al mismo tiempo alternar á **intervalos** la continuidad de los anillos. De modo que, en la fig. 12, que puede suponerse de un y medio ladrillo de espesor, principiando en el estribo *a*, podemos colocar anillos de medio ladrillo hasta *eoe;* y luego, cortando el ladrillo *o* hasta la línea *ee*, podemos colocar desde *ee* hasta *mn* una hilera de ladrillos con juntas radiales continuas, lo mismo que en *b*, y allí principiar nuevamente con los tres anillos; y así alternativamente. Otro método mejor todavía, pero más costoso, sería introducir en el espacio *eemn* una dovela de piedra tallada. Los intervalos propios para efectuar el cambio de los anillos por los bloques dependerán del número de los anillos y el espesor *ca* del arco; evitándose también lo más posible cortar los ladrillos.

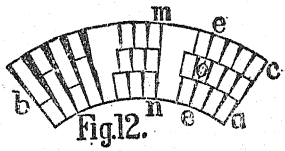


Fig.12.

Esto puede decidirse mejor en un plano de una parte del arco, en una escala de ¼ á ⅛. Generalmente los anillos se hacen de medio ladrillo solamente, ó de 10 á 12 c m de espesor, como en *ac*. En el viaducto de Maidenhead, los dos arcos elípticos de ladrillos de 39 m de luz y de 7.40 m de altura, son los más atrevidos construídos hasta ahora. Se les ha calculado que tienen un coeficiente 3 de seguridad solamente contra la ruptura por compresión en la corona.

Se han construído con éxito tantos otros arcos (de 21 á 30 m de abertura) entera

mente de anillos de ladrillos enteros ó de medio ladrillo, como para comprobarnos que no debemos prestarle poca atención á la objeción teórica arriba mencionada, con tal de que se use cemento de primera clase y se le dé el tiempo necesario para que se endurezca tanto ó casi como los ladrillos mismos, antes de quitar las cimbras. Bajo tales circunstancias no tendríamos objeción alguna que hacerle á una serie de anillos aun de 1.5 ladrillo de espesor, dispuestos alternativamente en hiladas como en *b*.

**Si los ladrillos tuviesen la forma de dovelas**, es decir, un poco más grueso en una punta que en la otra, podrían hacerse anillos del espesor de un ladrillo entero sin aumentar el espesor de las juntas del mortero en el trasdós de cada anillo. Sin embargo, con más de un anillo, las juntas radiales no serían continuas como en *b*, sino alternadas como en *ac*. Pero tales ladrillos serían de construcción más costosa, y además, para que convengan perfectamente al fin destinado, tendrían que hacerse muchos modelos diferentes para adaptarlos á los distintos radios del arco y aun para que se adapten á los radios de los diferentes anillos, cuando el espesor del arco requiere varios de ellos.

*Nota* 2. **Mójense los ladrillos** antes de asentarlos.

*Nota* 3. Cuando las caras ó frentes de un arco de ladrillo han de terminarse con dovelas de piedra tallada, es mejor no colocar estas últimas hasta algún tiempo después de la conclusión de la obra de ladrillos y del fraguado del mortero, y aun después de haber aflojado parcialmente las cimbras, para que no se rajen ó se agrieten al asentarse desigualmente con los ladrillos.

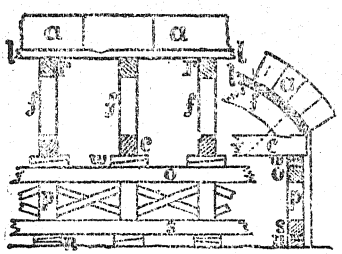
*Nota*. **Los arcos de ladrillo** á consecuencia del gran número de juntas, están expuestos á bajar más que los arcos de piedra tallada, y en consecuencia á cambiar de forma, y algunas veces, si no se tiene el cuidado debido, peligra su seguridad, especialmente cuando son grandes y rebajados. Cuando la abertura excede de 9 á 11 m más ó menos, y especialmente si es un arco rebajado, empléense solamente ladrillos de superior calidad con buen mortero de **cemento**. Aun con los mejores materiales y ejecución, aconsejamos á los jóvenes ingenieros que no construyan arcos de ladrillo para puentes de ferrocarriles con luces mayores que las siguientes. Se han construído algunos considerablemente más grandes, y han aguantado; pero su coeficiente de seguridad no es satisfactorio en todos los casos. En esta Tabla se da la flecha en partes de la luz.

| F. | L. | F. | L. | F. | L. | F. | L. | F. | L. |
|---|---|---|---|---|---|---|---|---|---|
| .5 | 30.5 | ⅜ | 26.8 | .225 | 20.7 | 1/10 | 15.2 | .134 | 10.7 |
| .4 | 29.6 | .29 | 25.0 | 1/5 | 18.3 | .155 | 13.7 | 1/8 | 9.1 |
| .36 | 28.4 | ¼ | 22.9 | .183 | 16.8 | 1/7 | 12.2 | | |

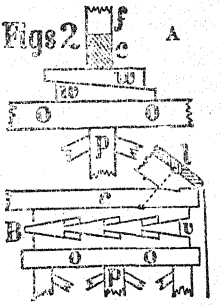
**En el ferrocarril de Pensilvania, en la calle Filbert, en Filadelfia**, hay 4 arcos de ladrillo de 15.27 m de abertura, con la muy pequeña flecha de 2.13 m. Tienen .76 m de espesor excepto en sus frentes visibles, donde sólo tienen 2.61m. Las juntas son de mortero ordinario, y de 6 mm de grueso, más ó menos. Estos 4 arcos, situados á una distancia de 182 m más ó menos unos de otros, y con un gran número de otros arcos de 7.93 m de abertura, forman un viaducto. Las pilas situadas entre los arcos cortos tienen 1.30 m de espesor, y las de las extremidades de los arcosgrandes, 5.64 m. Las líneas de arranques de todos estos arcos están más ó menos sè 1.80 á 2.4 m sobre el suelo. Uno de los arcos de 15.25 m de abertura bajó 7½ cm por haberse quitado la cimbra antes de tiempo; pero no se observó ningún descendo después, aunque el viaducto ha tenido desde su construcción (1880) un tráfico pesado de pasajeros y carga, á razón de 16 á 32 kilómetros por hora. Las calzadas son como de 30.5 m de ancho, con espacio para 9 ó 10 vías de ferrocarril.

# CIMBRAS PARA ARCOS

**Art. 1. Una cimbra** es una construcción de madera provisional (se la traza sobre una plataforma de tamaño natural, bajo techo ó no) para sostener un arco durante la construcción. La cimbra se compone de un número de cerchas ó **armaduras** *ff*, fig. 1, colocadas de .30 á 1.80 m de distancia de centro á centro y cubiertas con un piso *ll* de tablones ó tablas brutas, generalmente unidas, sobre el cual se colocan inmediatamente las dovelas. En la fig. 3 esta cubierta no está unida. No hay gran economía en colocar las cerchas á gran distancia unas de otras, porque así su espesor aumenta rápidamente. Para este espesor, véase nota 9, art. 8. Las cimbras descansan en las extremidades de sus cuerdas, *c*, sobre cuñas, *w*, fig. 1, que se apoyan en postes, *p*, con sus partes superiores unidas por **dinteles**, *o*, y cuyos pies descansan sobre durmientes, *s*, estando el conjunto ligado por piezas diagonales como lo indica la figura.



Figs 1.



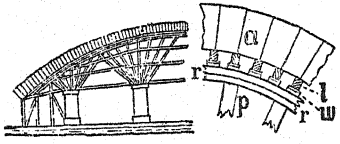
Figs 2

Cuando el terreno es muy firme y el arco liviano, los pedestales pueden descansar sobre él, con la interposición de bloques de ajuste, *n*, colocados debajo de los durmientes para adaptarlos á las irregularidades del terreno como en la figura. Estos bloques deben tener la forma de cuñas dobles, de modo que apretándolos á golpes de martillo, los pedestales pueden alzarse en caso de que desciendan ó penetren en el terreno. Pero para arcos pesados los pedestales deben descansar sobre cimientos mucho más firmes, como pilares de ladrillos enterrados un poco en el suelo, ó puede adoptarse cualquier otro medio. Frecuentemente se dejan en la mampostería de los estribos escalones ó huecos expresamente con este objeto, y en vista de esto es bueno hacer el proyecto de la cimbra al mismo tiempo que el del arco. Hasta aberturas de 15 á 18 m, una sola hilera de postes (uno debajo del extremo de cada armadura) es suficiente; pero para otras mucho más grandes pueden necesitarse 2 ó 3 hileras de .60 m ó más de separación unas de otras, como en la fig. 2. Las cuñas citadas arriba sirven para bajar ó quitar la cimbra después de concluído el arco. Se componen de piezas de madera dura en forma de cuñas *w w*, como en A, fig. 2, de .30 á .60 m de largo, 15 cm de ancho y 7 cm de grueso ó más (suficiente para bajar las cimbras de 5 á 15 cm, según la abertura ó luz y otras circunstancias) que descansan sobre el dintel, *o*, del pedestal mientras que la cuerda, *c*, de la armadura se apoya sobre ellos. Cuando la extremidad de una cercha está sostenida por dos ó más postes, *p*, como en B, fig. 2, en lugar de estarlo por uno solo, se hacen á veces las cuñas como se indica en dicha figura, y B*w* es una cuña larga en ángulo recto con el estribo y que obra como cuatro cuñas que pueden bajarse todas juntas golpeando en la extremidad B. En los arcos de 18 á 24 m de luz todas las cerchas pueden descansar sobre 2 cuñas semejantes á B*w* solamente; cada una de un largo tal que alcance **transversalmente** á todo el arco. Entonces todas las cerchas pueden bajarse en una sola operación, como se ha descrito al fin del art. 9.

Si tuviésemos que considerar solamente el frotamiento de la madera seca contra madera seca, el declive de las cuñas podría ser de 1 vertical para 3 horizontal, sin peligro alguno de resbalar una sobre otra; y entonces no necesitarían sino golpes suaves para moverlas; ó separarlas enteramente, cuando hay que quitar la cimbra. Pero es de la mayor importancia, especialmente en los arcos grandes, **bajar las cimbras muy lentamente**, porque de otro modo la fuerza adquirida por un cuerpo, tan pesado como el arco, al descender repentinamente aunque sean 5 ó 7 cm, es muy factible que dañe su forma y aun su estabilidad.

Por consiguiente, las cuñas no deben tener una inclinación mayor de 1 en 6 ú 8, más ó menos, en arcos de menos de 15 m de luz más ó menos; ó de 1 en 8 ó 10 para luces mayores. En los grandes lados de las cuñas deben trazarse líneas verticales equidistantes como guía para hacerlas descender la misma cantidad á un tiempo, y este descenso no debe exceder por todo de 12 mm por día más ó menos y á intervalos de 3 mm más ó menos á la vez, para arcos de 15 m; y de 3 á 6 mm por día por todo, en arcos mayores de 30 m. Se recomienda especialmente la lentitud del descenso en los **arcos de ladrillo**, no solamente porque el mayor número de juntas los expone á una deformación, sino también porque aun los ladrillos buenos tienen por término medio una resistencia á la ruptura por compresión mucho menor que el granito, la piedra caliza y la piedra arenisca buena, y por consiguiente están mucho más expuestos á rajarse, y aun á desmoronarse (como lo ha visto el autor) cuando las fuerzas todas, como se ha descrito en el art. 3, están obrando sobre sus aristas.

**En el puente de Gloucester**, Inglaterra, de piedra tallada de primera clase, con una luz de 45.7 m, una flecha de 10.67 m, las cimbras se quitaron del todo en el muy corto espacio de 3 horas; y la corona del arco descendió 25 cm! **En el puente de Grosvenor**, Inglaterra, de piedra tallada de primera clase, de 61 m de luz, y 12.81 m de flecha, se tuvo tanto cuidado al aflojar las cimbras, que la corona del arco no bajó sino 6.2 cm. Este caso sin embargo se distinguió por circunstancias que contribuyeron á tan favorable resultado. Es decir, las cimbras en lugar de ser una serie de cerchas ó armaduras sostenidas por sus extremidades como de costumbre, y de consiguiente permitiendo cierto grado, aunque pequeño, de descenso, se componía esencialmente de postes verticales é inclinados (véase fig. 3) que descan-




Figs 3,

saban sobre cuatro pilares provisionales de mampostería de 2.1 á 2.4 m de espesor, construídos dentro del río, paralelamente á los estribos, y tan largos como éstos. Estos pilares sostenían seis cerchas (ó más bien seis series) colocadas á 2.10 m más ó menos de distancia de centro á centro y descansando sobre zapatas de hierro fundido. La fig. 3 representa la mitad de una de estas series. Cada armadura ó serie consistía en cuatro juegos de postes en forma de abanico, todos colocados en el mismo plano vertical. Las piezas largas horizontales que se ven extend das de un lado al otro del arco se fijan á los postes por medio de pernos para aumentar la rigidez; y otras piezas, con el mismo fin, unían las seis series transversalmente. Cada poste ó puntal soportaba su correspondiente parte del peso de las dovelas transmitiéndolo á la cimentación inflexible del pilar, mientras que en las cimbras usuales de cerchas la carga entera descansa sobre las cerchas, y es transportada á los soportes relativamente instables de los puntales en sus extremidades.

Las partes superiores *p* de los puntales de una serie variaban de 1.5 á 2.4 m de dist de centro á centro y estaban unidas por una pieza curva *rr*, del espesor de dos tablones de 10 cm, adaptada aproximadamente al arco. Sobre esta pieza se colocaban pares de cuñas *w*, semejantes á las de la fig. 2, más ó menos de 40 cm de largo, 25 á 30 de ancho y con una disminución de 4 cm y próximas (de .75 á 1.07 m de centro á centro), de manera que había un par debajo de cada junta de las dovelas *aa*. Sobre estas cuñas, y prolongadas sobre todas las seis cerchas, se colocaron piezas de forro *l*, de 12 cm de grueso.

**Esta disposición especial de las cuñas** y de los forros, en luces grandes, tiene muchas ventajas sobre el sistema generalmente adoptado de colocar aquéllas solamente en las extremidades de las cerchas. En estas últimas, la cimbra y el arco enteros descienden juntos sin dar oportunidad para rectificar cualquiera irregularidad de forma ó desigualdad en los puntos de apoyo que puede haber ocurrido en el arco durante su construcción.

Esta cimbra, ideada por el Sr. Trubshaw, se presta para bajarla toda igualmente, ó para bajar una parte cualquiera de ella, un poco más ó un poco menos que las otras. Este señor tenía mucha experiencia respecto de los arcos grandes, y afirmaba que, durante el descimbramiento, había encontrado que se comportaba mejor el arco *o*, se asentaba mejor, bajándole un poco en los riñones, y suspendiendo la corona otro poco, hasta cerca del fin de la operación.

*Nota* 1. **En lugar de pilares de mampostería** para soportar los pies de los puntales, pueden emplearse á menudo cajones de madera, ó estacas, si el arco está sobre agua.

**El sistema de sostener las armaduras** por medio de puntales, situados en puntos de la cuerda, tan retirados de los estribos como las circunstancias lo permitan (á más de los situados en sus mismas extremidades), debía siempre aplicarse, cuando fuera posible, para reducir á un mínimo la tendencia á bajar. Pueden hacerse escalones en la mampostería de los estribos y pilas para apoyar los pies de los puntales, cuando éstos sean inclinados.

*Nota* 2. También *pueden usarse* **tornillos** en lugar de cuñas para bajar las cimbras. En el puente de *l'Alma*, en París, de arcos elípticos de 43 m de luz y de 8.6 m de flecha, las cerchas estaban sostenidas por columnas macizas de madera, cuyos pies descansan sobre **arena confinada en cilindros de palastro** de 30 cm de diámetro y de altura, que tenían cerca del fondo un tapón que podía sacarse ó ponerse á voluntad, y que regulaba de este modo la salida de la arena y el descenso de la cimbra. Este expediente tuvo éxito perfecto, y bien vale la pena de adoptarlo para arcos que excedan de 18 m de luz. Cuando son mucho más grandes, la encajada de las cuñas, golpeándolas, requiere golpes muy fuertes, y resulta una operación ruda, donde á veces no basta el simple martillo aun cuando las cuñas estén lubriadas. En las cortadas de ferrocarriles atravesadas por puentes, **la tierra situada debajo del arco** se ha aplicado como cimbra, dando á su superficie la curvatura adecuada, y colocando luego en ella piezas de madera curvas á conveniente distancia unas de otras, y de un estribo al otro, para sostener la cubierta de tablas.

*Nota* 3. **Toda cimbra debe ceder** ó asentarse más ó menos bajo el peso del arco, especialmente cuando está sostenido solamente cerca de sus extremidades; y como el arco mismo también se asienta algo, no solamente cuando se quitan las cimbras, sino durante algún tiempo después, es prudente hacerlos un poco más altos de como van á quedar. Este exceso de altura, cuando la cimbra está sostenida en sus extremidades, puede ser de 5 á 10 cm por 30 m de luz en los arcos de piedra tallada (según el tiempo en que se bajan, condición de la mampostería, ejecución), y del doble para los arcos de ladrillo.

*Nota* 4. **El tiempo adecuado para quitar las cimbras** es un punto discutido entre los ingenieros; algunos afirman que debe hacerse tan luego como esté concluído el arco y tenga el suficiente relleno, y otros dicen que debe dársele tiempo al mortero para que endurezca. Es opinión del autor, por cuanto en los arcos de piedra tallada las juntas de mortero son muy delgadas, y como en estos arcos el mortero sirve de poco, es de poca importancia quitar las cimbras, con tal que el relleno de mampostería y el terraplén estén concluídos hasta *yn*, fig. 2, pág. 667; pero en los arcos de ladrillo ó de piedra bruta, cuyas numerosas juntas requieren mucho mortero (el cual para que se endurezca debe contener bastante cemento), debe concedérseles 3 ó 4 meses, ó más todavía, si fuere posible, para que aquél se endurezca suficientemente, á fin de impedir una presión indebida, y el descenso consiguiente después de descimbrado. La permanencia de las cimbras no impide el tráfico sobre el puente.

**Art. 2.** La presión que ejercen las dovelas contra una cimbra es muy pequeña, mientras el arco no se ha construído á ambos lados hasta el punto en que las juntas formen ángulos de 25° ó 30° con la horizontal. Las discusiones teóricas sobre esta cuestión no acuerdan nada por los golpes accidentales durante la colocación de las dovelas, ó por la acumulación del material, tráfico de los albañiles que trabajan sobre él, etc. Sin ocuparnos de detalle alguno, aconsejamos solamente, para mayor seguridad, no apreciar esto en menos de lo que fijamos en las siguientes fracciones del peso del arco completo; á saber : en un arco semicircular, .47; para flecha de .35 de la luz, .61; para flecha de .25 ,.79; para flecha de .2 de la luz .86; para

flecha de .167 de la luz ó menos, 1, ó sea un peso igual al del arco. Esto da para la presión de un arco semicircular sobre sus cimbras, más bien menos de la mitad de su peso. **El peso de las cimbras mismas**, cuando están sostenidas solamente cerca de sus extremidades, debe considerarse como parte de la carga soportada por ellas.

**Art. 3.** Hemos visto que cuando se está construyendo un arco *aaa* gradualmente hacia arriba por ambos lados, después de pasar los puntos *ee*, fig. 4, donde sus juntas forman ángulos *ase* de 30° más ó menos con la horizontal *aa*, el arco principia á comprimir las cimbras más y más; tendiendo por esta razón á hundirlas en los riñones, como se indica en *h* con la línea de puntos, y por consiguiente, á alzarse en la corona como se ve en *c*. Pero como la conclusión continúa hacia arriba, las piedras ó dovelas que se agregan comprimen mucho más á la cimbra que las de más abajo, y por tanto tienden á deformarla exactamente á la inversa de como lo hacían las dovelas inferiores, es decir, á deprimirlas desde la corona *a* hasta *o;* y por consiguiente, á suspender los riñones como en *n;* y á éstos aún más, porque las piedras superiores realmente tienden á levantar ó á aflojar las piedras inferiores de la cubierta de la cimbra. En algunos casos cuando esta tendencia se aumenta *a*

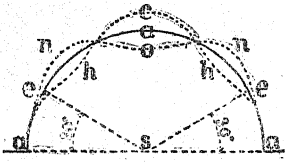


Fig. 4.

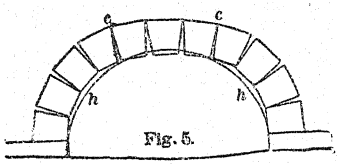


Fig. 5.

*forzar* la clave con golpes fuertes para que llegue á su lugar, podrían sacarse las piezas del forro que están bajo los riñones, sin trabajo alguno, antes de aflojar las cimbras. Al quitar éstas, dicha tendencia de los arcos á descender en la corona y á elevarse en los riñones, es capaz de ofrecer más ó menos peligro á las dovelas mismas, como se ve en la fig. 5, obligando á las situadas cerca de la corona á comprimirse muy fuertemente en el trasdós y á desunirse en el intradós, mientras que en los riñones resulta lo contrario. De aquí que los ángulos de las dovelas se astillan á menudo cerca de *c* y *h* por esta desigual presión.

Estas deformaciones son, por supuesto, mucho más importantes en arcos levantados que en los rebajados, especialmente si sus enjutas no están bastante adelantadas antes de quitar las cimbras.

En el puente Grosvenor arriba mencionado, de 61 m de luz, este exceso peligroso de presión cerca de *c* y *h* se evitó cubriendo las juntas, en la hilada de los arranques, en cada estribo con una cuña de plomo de 35 mm de grueso en el intradós, y reduciéndola hasta cero en el trasdós. Además de esto se colocó una tira de plomo de 22 cm de ancho en la arista del intradós de cada junta, hasta el punto donde se juzgó que la línea de presión pasaría del intradós al trasdós; en seguida se colocaron tiras iguales á lo largo de las aristas de las juntas del trasdós, hasta llegar á la corona. Por consiguiente, al quitar las cimbras, este exceso de presión no comprimió sino el plomo, y de este modo se distribuyó más igualmente sobre toda la extensión de las juntas. Véase « Trans. of the Ins. of Civ. Eng. », London, vol. 1.

**En el puente de Neuilly**, en Francia (de 5 arcos de 36.6 m de luz y 9 de flecha), se trazaron las cimbras de tal modo imperfectas desde su origen, que los arcos bajaron 33 cm en la corona, durante el tiempo de la construcción, y 26 cm inmediatamente después de quitar las cimbras, ó sea en todo 59 cm. Su construcción hizo el descimbramiento muy peligroso y fastidioso, poniendo en gran peligro la vida de los trabajadores y la existencia de los mismos arcos. Algunas de las juntas del trasdós en los riñones se abrieron 25 mm cada una, y las del intradós de la corona, 6 mm. Se redujeron estas aberturas debido al mucho cuidado, y por seguir el método más adecuado y propio, cuando se quitaron las cimbras.

*Nota* 1. **Biselando las aristas de las dovelas** disminuye mucho el peligro de que se astillen por la presión desigual; y también debe **rasparse el mortero de las juntas** hasta la profundidad de 2 á 4 cm antes de quitar las cimbras.

*Nota* 2. Es evidente que para evitar ó, á lo menos, disminuir la deformación alternada de la cimbra, las piezas que primeramente obraban como *puntales*, cerca de los riñones, fig. 4, para impedir que baje, como en *h*, deben después obrar como

cuerdas para impedir que se levante, como en *n;* mientras que, las que primeramente obraban como cuerdas, cerca de la corona, *a*, para evitar que se levante, como en *c*, deben después obrar como puntales para impedir que la corona descienda, como

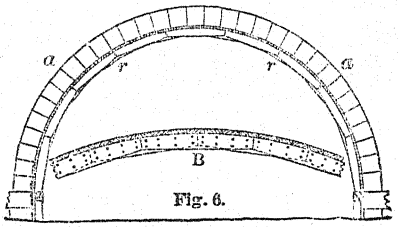


Fig. 6.

en *o*. En otras palabras, deben también usarse los contrapuntales tanto en la armadura de una cimbra como en la de un puente.

**Art. 4.** De lo que antecede resulta que **un simple arco de madera** es, á causa de su gran flexibilidad, poco adecuado para una cimbra, excepto para muy pequeñas luces; para éstas puede fácilmente obtenerse de un espesor adecuado. Sin embargo el autor las ha visto empleadas en un arco de 10.67 m de luz, con dovelas de piedra tallada de .61 m de espesor. La fig. 6 representa una de estas cimbras *rr* y el arco *aa*, trazados en escala. Cada elemento curvo de la cercha se componía de dos espesores de tablones de 5 cm de espesor y de 1.95 m de longitud más ó menos, fijos con clavijas ó tarugos de madera de modo de alternar las juntas, como en B. Cada pieza del tablón tenía 30 cm de ancho en el centro y 20 en las extremidades, con sus aristas superiores labradas para adoptarlas á la curva del arco. Las clavijas de madera eran de 3 cm de diámetro y había 12 de ellas en cada pieza. Estas cerchas se colocaron á una distancia de 43 cm de centro á centro y se afianzaron por medio de piezas de tablas de 2½ cm y de 32 cm de largo colocadas en cada junta de los tablones, es decir, á una distancia de un metro más ó menos. Como había necesidad de establecer una vía para el tráfico debajo del arco, no había cuerdas para unir los pies opuestos de los camones. Éstos estaban cubiertos por un forro unido, que también contribuía á afianzarlos transversalmente. Cuando el arco estaba construído más ó menos hasta las dos terceras partes de su altura en cada lado, las cerchas principiaron á descender en los riñones, como en *h*, fig. 4, y á elevarse en la corona, como en *c*. Esto se remedió cargando la clave con las piedras que debían usarse para terminar el arco, el que después se concluyó sin más dificultad.

**Un ejemplo más sorprendente todavía** del uso de una cercha simple sin trabazones, es el que ofrece el puente del antiguo camino nacional construído

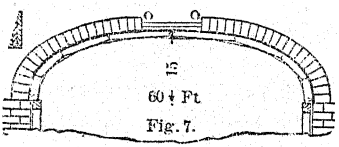


Fig. 7.

sobre *Wills Creek*, en Cumberland, Md. Este puente, del cual la fig. 7 representa en escala un arco con su cimbra, se componía de dos arcos elípticos de piedra tallada con una calzada de 8.08 m de ancho, 18 m de luz y 4.5 de flecha. Tenía .91 m de espesor en la corona y 1.20 en los arranques.

**Cada cimbra** se componía de cerchas simples de 15 cm de grueso, formadas de 3 espesores de tablones de roble de 5 cm en piezas de tamaños diferentes (2.10 á 4.50 m) para adaptarlos á la curva nueva, y al mismo tiempo para conservar un ancho de 40 cm más ó menos en el centro de cada pieza y de 30 cm en cada una de sus extremidades. Estos tablones estaban bien trabados por clavijas de madera, alternando las juntas, y había de 10 á 16 clavijas en la longitud de cada pieza.

Aquí, como en la fig. 6, no había cuerdas, debido á la violencia de las crecientes. Estas cerchas se colocaron á distancia de 45 cm de centro á centro, y ligadas unas con otras por medio de pedazos de tabla de 30 cm de largo colocados cada 1.5 m. A darles más firmeza contribuía, por supuesto, el revestimiento de tablas.

Cuando las dovelas se aproximaron como 3.66 m hacia el medio de la luz, el descenso de la corona, y la elevación de los riñones, se hizo tan alarmante, que se insertaron apuradamente, á ciertas distancias, piezas de madera de roble de 30×30 cm, *oo*, bien acuñadas contra las dovelas. Luego se concluyó el arco en secciones comprendidas entre estas piezas de madera que se removían una por una, según avanzaba la obra.

*Nota* 1. **Tales ejemplos de empujes parciales de un arco** son muy instructivos. Es cierto que más bien es debido á esto que á deducciones teóricas, que se ha llegado á conocer las dimensiones adecuadas en un gran número de casos pertenecientes á la ingeniería, maquinaria, etc. * De este modo podríamos emplear, con entera confianza cuando no hay serios peligros, cerchas de simples camones de las dimensiones expresadas en luces de la mitad de las arriba mencionadas.

*Nota* 2. Presumiendo que las planchas de los camones sean de 30 cm de ancho, sería mejor, como asunto de detalles, hacerlas de 25 cm más ó menos en sus extremidades en lugar de 20 cm, fig. 8, haciendo la curva superior de 5 cm.

Para conseguir esto, sus longitudes, que dependen del **radio** de las cerchas, no deben exceder de las **de la tabla siguiente** :

| Radio del arco. | Longitud mayor. | Radio del arco. | Longitud mayor. |
|---|---|---|---|
| Metros. | Metros. | Metros. | Metros. |
| 1.525 | .735 | 9.150 | 1.930 |
| 3.050 | 1.015 | 10.675 | 2.135 |
| 4.575 | 1.270 | 12.200 | 2.285 |
| 6.100 | 1.525 | 13.725 | 2.390 |
| 7.625 | 1.750 | 15.250 | 2.490 |

Si se cortan de 1½ veces, el tamaño dado por la tabla, tendrán muy aproximadamente 20 cm de ancho en sus extremidades, ó cada una tendrá en su parte superior una curva de 10 cm.

**Art. 5. En los casos donde sea necesario dejar cualquiera vía para el tráfico por debajo del arco**, durante la construcción de éste, como en los dos casos que anteceden, el autor indicaría un medio conveniente, ilustrado ligeramente en la fig. 8, que consiste en **colocar las cimbras encima del arco**, en lugar de **colocarlas debajo**, y después que el arco esté concluído en secciones ó por partes *aa*, desarmar las cimbras en lugar de bajarlas.

La fig. 8 es una sección transversal hecha por una parte de la cimbra y del arco *aa*. Aquí, *rc*, *rc*, *rc*, son cerchas de la cimbra colocadas como de 1.5 á 1.8 m de distancia entre sí, de un espesor cualquiera y de cualquier construcción que se necesite, para una absoluta seguridad; y *ll* es el forro ó revestimiento. Habiéndose construído el

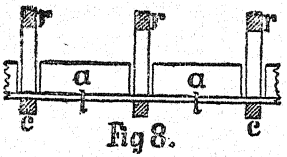


Fig 8.

arco de estribo á estribo en una serie de secciones *a*, *a*, *a*, necesariamente separadas como 30 cm ó más por el espesor de armaduras ó cerchas, podemos quitar las cimbras y llenar las secciones angostas intermedias sobre un forro suspendido por barras de hierro de las secciones ya concluídas. Podía emplearse un buen concreto en estas secciones angostas. En algunos casos puede ser conveniente emplear cambas **altas de hierro** en forma de I haciendo descansar el forro sobre el borde inferior.

* Los jóvenes ingenieros debían hacer y conservar notas ampliamente detalladas de todos los casos como éstos, que pueden llegar á su conocimiento.

Una parte de estas piezas podía quedar embutida en la mampostería, quitándose la parte superior después, cuando esté terminado el arco.

**Art. 6. Las cimbras con cuerdas horizontales** *cc*, fig. 9, son objetables (á pesar de su resistencia) en aberturas grandes de mucha flecha, como en el lado derecho de la figura, por la excesiva longitud requerida para las piezas; y en estos casos es conveniente adoptar algo análogo á lo que está representado en el lado izquierdo. Se ha sustituído aquí una armadura, *f*, más corta y menos elevada que la del lado derecho. En sus extremidades debe proveerse de apoyos no sólo para ella misma, sino también para las dovelas que están debajo de ella. Como la presión de estas dovelas inferiores es comparativamente pequeña, esto puede efectuarse generalmente dejando descansar la extremidad de la armadura *f* sobre otra armadura más baja *oa*. A ésta se puede reforzar en arcos grandes por medio de puntales verticales ó inclinados, solos ó ligados entre sí. Algunas veces una cercha baja como *f* está sostenida por otra en toda su longitud. Las cuñas pueden colocarse en las partes superiores ó en las inferiores, según sea más conveniente.

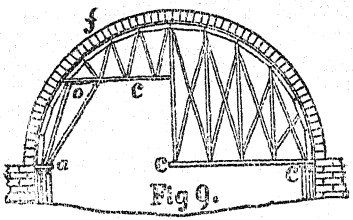


Fig 9.

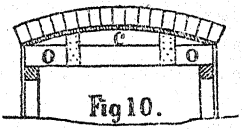


Fig 10.

**Art. 7. Para arcos rebajados de una luz de 3 m** bastará una simple tabla, *oo*, fig. 10, de 30 cm de ancho por 4 de grueso, con otra pieza *c* del mismo espesor, colocada sobre la primera, recortada según la curva, y sujeta en *oo* por dos listones de madera, á intervalos de 45 cm de centro á centro. Si la pieza superior también es de 30 cm de ancho en el centro, la luz puede extenderse hasta 4.5 m. Para luces de 4.5 á 6 m y de cualquiera flecha, servirán dos tablones de 3 á 5 cm de grueso, según la luz; de 20 á 30 cm de ancho en el centro de cada pieza, con longitudes dadas por la tabla, nota 2, art. 4, **bien clavadas** ó ensambladas, según el tamaño del arco, y alternando sus juntas como en la fig. 6, colocadas á distancias de .60 á .90 m de centro á centro. Para distancias mayores auméntese el espesor de los tablones.

**Cuando hayan que mudarse las cimbras de un lugar á otro** para utilizarlas, deben unirse sus pies para impedir que sufran, clavando en uno ó ambos lados de cada cercha una tabla de 2 ó 3 cm, á manera de cuerda, y una ó más piezas verticales del mismo tamaño, colocadas del centro de la cuerda á la parte superior de la cercha.

Aun cuando no hayan de moverse, dichas piezas aplicadas como cuerdas son convenientes hasta en arcos pequeños, porque facilitan el descimbramiento; estas cuerdas impiden que los pies se abran hacia afuera y ejerzan presión contra los estribos.

**Para luces de 4.5 á 9 m**, de flechas no menores de una sexta parte de la luz, pueden adoptarse las dimensiones siguientes que varían con la luz, para cerchas

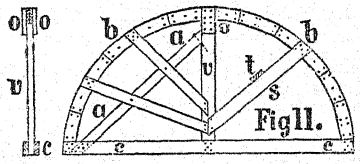


Fig 11.

distantes .90 m de centro á centro. Véase fig. 11. **Para el arco** *b*, dos tablones de 3 á 5 cm de grueso, de 20 á 30 cm de ancho en el centro, y de 17 á 25 cm en las extremidades, bien clavados como en B, fig. 6. **Para la cuerda** *c*,

dos espesores de tablas del mismo tamaño, fig. 11, que las del arco del centro, colocadas á los lados exteriores del arco, y bien clavadas en sus extremidades. **Un pendolón vertical, v,** en una sola pieza tan ancha como las del arco y de doble grueso. Su extremo superior está debajo del arco y fijado á él por dos piezas, *oo*, de una longitud doble del ancho del arco y clavadas al arco y al pendolón *v*. El pie de *v* pasa por entre la cuerda *c* y está clavado á ella. Dos puntales oblícuos, *s*, clavados en cada lado del arco y del pendolón *v*. Éstos, con *v*, dividen el arco en 4 partes.

*Nota* 1. Las dimensiones arriba establecidas son para flechas de $^1/_6$. Si la flecha es de $^1/_4$, los **espesores solos**, de las piezas, deben ser reducidos en una tercera parte, y para flechas de $^1/_3$ ó más, á la mitad.

*Nota* 2. Si los puntales, *s*, tienden á doblarse, clávense piezas, *t*, de cercha á cercha. En las más grandes, con flechas de más de $^1/_4$, pónganse 4 puntales dobles *s*, en lugar de dos, dividiendo así el arco en 6 partes. Para luces de 7 á 10 m, agréguense dos piezas como *aa*, de las mismas dimensiones de *v*.

**Art. 8. Para luces mayores de 9 m** más ó menos, el autor cree que, como regla general (sujeta á modificaciones según el juicio del ingeniero encargado), las ideas siguientes dan una seguridad práctica.

A saber: adoptar una armadura de arco y cuerda de piezas simples del sistema Warren ó de triángulos, como al lado izquierdo de la fig. 9. El arco ha de descansar sobre la cuerda, y cada uno debe ser de un solo espesor. Los miembros (especialmente en aberturas grandes) deben ser también de un solo espesor, colocados debajo del arco que descansa sobre la cuerda, y bien fijos á ambos por medio de abrazaderas, para que obren como puntales y como cuerdas. En aberturas más pequeñas, dichos miembros pueden ser cada uno de 2 espesores, uno de ellos fijado con clavijas á cada lado del arco y de la cuerda. Hay otros métodos; pero no tenemos espacio para estos pequeños detalles.

Pueden usarse también piezas del sistema Howe, ó Pratt, como en el lado derecho de la fig. 9. Pero con respecto á estas dos, debemos observar **que el uso de largas barras de hierro en las cimbras** de una abertura grande está sujeto á objeciones debido al diferente grado de dilatación entre el hierro y la madera. Por consiguiente, al usar estos sistemas, todos los miembros deben ser de madera. Puede adoptarse el sistema de **celosía**.

Aun cuando la flecha del arco exceda de .25 de la luz, es mejor no permitir que **las cimbras pasen** de este límite, sino adoptar el método demostrado al lado izquierdo de la fig. 9.

*Nota* 1. **Para fijar el número de triángulos de la** armadura Warren, para una cimbra, búsquese la raíz cuadrada de la abertura y agréguese á ella una décima parte de dicha abertura, divídase la suma por 2, y llámese el cociente *n*. Divídase la abertura por *n*. Si este cociente es en parte decimal, tómese el número entero más aproximado, como distancia en pies para marcarlos en la longitud de la cuerda, dividiendo así ésta en un número de partes iguales. Los puntos encontrados de este modo en la cuerda son los lugares donde han de situarse **los pies** de los triángulos *. Luego desde el centro de la distancia comprendida entre cada dos puntos trácense líneas verticales hacia el arco; donde éstas cortan el arco, es en donde deben terminar los vértices superiores de los triángulos.

En los arcos grandes **no debe desperdiciarse la madera** del arco de la cimbra, recortando sus aristas superiores conforme á la curva, sino que debe dejarse recta y colocarse con clavos en su parte superior piezas diferentes, *c*, fig. 10.

**El tamaño de la cuerda** debe ser el mismo que el del arco, y como él, uniforme de una punta á otra, teniendo sin embargo cuidado de no debilitarla demasiado para alojar los extremos del arco de la cercha en sus extremidades; y evitar por medio de las ensambladuras necesarias que sea alargada ó rota por el empuje del arco. Pero cuando la cuerda puede colocarse en los arranques, ó un poco debajo de ellos, puede evitarse todo peligro de esta especie acuñando bien sus extremidades contra los paramentos de los estribos.

**Con respecto al tamaño de las piezas,** cuando una armadura de arco y cuerda **está cargada completamente** en la parte superior del arco (como es aproximadamente el caso con la cimbra y las dovelas), los esfuerzos que obran

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* *N. del T.* — Empleando el sistema métrico y con suficiente aproximación: Multiplíquese la raíz cuadrada de la luz en metros por 1.81, agréguese la tercera parte de la luz, en metros, divídase la suma por 2; llámese el cociente *n*. Divídase la luz en metros por este cociente *n* y síganse todas las indicaciones (usando medidas métricas que el autor hace después para encontrar puntos de división de la cuerda.)

sobre sus miembros son enteramente insignificantes, y provienen principalmente del peso de la cimbra misma; **pero, durante el tiempo y á proporción que se carga**, los esfuerzos no son solamente mayores, sino que cambian constantemente de intensidad y de dirección, obrando como presiones en un período y como tensiones en otro.

Por consiguiente, sería muy fastidioso calcular los tamaños de las piezas. Afortunadamente la necesidad de hacerlo se ha evitado en muchos casos por el hecho de que una cimbra es solamente una construcción provisional, y que la madera de que se compone no se inutiliza y se puede emplear una cantidad mayor de la que se requiere estrictamente. Además, se facilita la obra de mano no teniendo que emplear maderas de muchos tamaños diferentes.

Por esto el autor se atreve á indicar como regla empírica, que se dé *á cada miembro la mitad del área de la parte curva*, teniendo cuidado de construir aquéllos de tal modo, que cada uno sirva tanto de puntal como de cuerda.

*Nota* 2. **Con respecto á los detalles de las juntas**, nos referimos á las figuras de las págs. 794-95; indicando aquí simplemente el uso de zapatas de hierro, largas y anchas, en el caso en que las maderas estén sometidas á gran presión lateral.

*Nota* 3. Para impedir que el empuje del arco, cuando su flecha es pequeña, raje las extremidades de la cuerda, los dos pueden unirse con muchos más tornillos de los que se emplean en las armaduras de techos, etc., donde generalmente no se pone sino uno cerca de cada extremidad de la cuerda. Los tornillos deben estar provistos de arandelas grandes y fuertes. Otro medio de lograr el mismo objeto en aberturas pequeñas, consiste en forrar completamente los dos lados del arco y de la cuerda por piezas cortas de tablones ó tablas, clavadas sobre ambos, á algunos decímetros de distancia de sus extremidades, con la misma inclinación más ó menos que la pieza más corta.

*Nota* 4. Constrúyase el arco por ambos lados al mismo tiempo, para forzar las cimbras lo menos posible.

*Nota* 5. Cuando un puente se compone de más de un arco, y hay que construir uno de ellos, entonces debe haber por lo menos dos cimbras; porque una cimbra no debe quitarse en tanto que los arcos contiguos á ambos lados no estén concluídos, por el peligro de que vuelquen la pila exterior que no está apoyada. Por consiguiente, si no hay más que dos arcos, deben construirse al mismo tiempo, y requieren dos cimbras.

*Nota* 6. **Colóquense siempre apoyos**, verticales ó inclinados, provistos con cuñas debajo de las armaduras y entre los apoyos extremos cuando sea posible; aun cuando no puedan sino colocarse á poca distancia de los estribos, como en el lado izquierdo de la fig. 9.

*Nota* 7. **El peso de las cimbras grandes** y de su revestimiento es mayor en los arcos rebajados que en los elevados de la misma luz.

*Nota* 8. **Espesor del revestimiento.** La siguiente tabla da espesores que no se doblan más de 3 mm (y generalmente no tanto) bajo el peso de cualquier dovela adecuada á sus respectivas luces.

## TABLA DE LISTONES. (Original.)

(*N. del T.* — Convertida en medidas metricas).

| Dist de las cerchas. | LUZ DE LA CIMBRA EN METROS | | | | | |
|---|---|---|---|---|---|---|
| | 3 | 6 | 15 | 30 | 45 | 60 |
| | Espesor de los listones para no flexarse más de 3 mm. | | | | | |
| metros. | m. | m. | m. | m. | m. | m. |
| 1.83 | .082 | .092 | .107 | .120 | .127 | .133 |
| 1.52 | .063 | .068 | .086 | .092 | .098 | .101 |
| 1.22 | .043 | .053 | .063 | .069 | .072 | .076 |
| .91 | .030 | .035 | .044 | .047 | .050 | .050 |
| .61 | .019 | .022 | .025 | .028 | .028 | .031 |

**Con espesores** iguales á las tres cuartas partes de éstos, la flexión puede llegar á 3 mm, lo que puede admitirse para separaciones de cerchas de 1 ó más metros entre sí.

*Nota* 9. **Las cimbras se arman** (lo mismo que los puentes de hierro) sobre una plataforma firme provisional, sobre la cual se ha trazado primero el diseño de un arco de tamaño natural. Cuando está lista cada cercha se lleva á su lugar y se coloca sobre las pilas ó estribos.

**Art. 9. El puente de Wissahickon**, del ferrocarril de Reading, en Filadelfia, consta de 5 arcos de 19.82 m de abertura, 7 m de flecha y 8.54 m de ancho. El espesor del arco es de .90 m (los lechos y juntas de mortero de cemento) y con cuatro pilas de piedra tallada de 2.90 m de espesor en la parte superior, y de 10.67 á 15.25 m de altura. El puente contiene 11704 metros cúbicos de mampostería*. **Cada cimbra** se compone de 7 armaduras ó cerchas de madera de abeto del Canadá; es del modelo de « arco y cuerda », el alma de celosía, y, lo más posible, de la misma abertura y flecha que los arcos. Se colocaron á una distancia de 1.35 m de centro á centro, y estaban sostenidos cerca de cada extremidad *f*, fig. 13 (una sec-

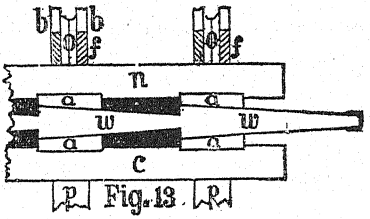


Fig. 13.

ción transversal en escala), por un poste *p* de abeto de 30 cm en cuadro. El arco de la cimbra consistía en dos tablones *bb* de abeto á una distancia de 15 cm entre sí, en trozos de 1.80 m de longitud con sus aristas superiores recortadas para adaptarlas á la curva del puente. Cada tablón era de 10 cm de espesor por 24 cm de ancho en su centro, y 30 cm en sus extremidades. Estas piezas no tenían las juntas alternadas, pero en cada junta había 4 pernos de $^3/_4$ de pulgada con tuercas y arandelas, que las unían á cuñas de madera.

El arco de la cimbra *bb* descansaba en la parte superior de las extremidades de las cuerdas *f*, y el ángulo formado por su intersección (representado aquí sólo en una vista lateral) se rellenaba (en un espacio de .75 m horiz y 1.65 vertical) con piezas de madera macizas y verticales, para formarle una base más firme en que apoyar en *n* la armadura, la cual se prolonga (en una vista lateral) 45 cm más allá.

**Las cuerdas** *f* se componían de dos tablones de abeto de 10×30 cm, colocados á una distancia de 15 cm entre sí, y la mayor parte de ellas constaban de dos ó tres piezas con juntas alternadas y fijas por medio de pernos de $^3/_4$ de pulgada de diámetro, provistos de tuercas y arandelas, y tacos de madera.

**Las almas** de cada armadura estaban formadas por un enrejado de 26 listones *o*, de madera de abeto de 7½×30 cm. que se cruzaban, más ó menos, en ángulo recto, á distancia de 1.05 m de centro á centro, más ó menos, y pasaban por entre los dos tablones, *bb*, del arco, *ff*, de las cuerdas. Algunos de estos listones que formaban la celosía se componían de dos piezas, pero sus juntas no estaban en los cruzamientos. Los listones estaban fijos en cada cruzamiento por 2 clavijas de madera dura de 23 cm de largo y de 5 de diámetro; y una de estas clavijas, de 46 cm de largo, pasaba por la intersección de cada extremidad del listón con el arco ó la cuerda. El primer listón se apoyaba como á 1.20 m del extremo de la cuerda. No sobresalían de la parte superior del arco de la cimbra. Todos los espacios comprendidos entre los dos tablones del arco de la cimbra y de las cuerdas, no ocupados por las extremidades de los listones, se rellenaron completamente con tacos, bien clavados.

**Cada armadura contenía** 10 metros cúb de madera, más ó menos, y pesaba como 5 toneladas. Eran muy flexibles, lateralmente, aun colocados en su lugar, y

* Este puente, concluído sin accidente en 1882, reflejó mucho crédito sobre el difunto Sr. William Loveng, I. C.; sobre Mr. Charles W. Buchholtz, ayudante encargado, y sobre los hábiles y enérgicos contratistas Srs. William y James Nolan, de « Reading, Pens ». Estos últimos ayudaron muy cordialmente al autor á hacer observaciones durante el progreso total de la obra.

estaban afianzadas por medio de 4 tablones horizontales transversales clavados sobre las cuerdas, y por 5 tablones más, clavados más arriba, en los listones.

Hasta la colocación de las claves, todas las juntas de las armaduras continuaron cerradas bajo la presión del arco del puente y del relleno no concluído hasta la altura de 4.20 m más ó menos sobre las líneas de los arranques; pero después de colocar las claves, todas las juntas de las cuerdas solamente, se abrieron de 6 á 18 mm; al mismo tiempo los listones de la armadura bajo los *riñones* del arco del puente se separaron ligeramente del intradós de la mampostería.

Cada cimbra bajó por todo 2½ cm hacia el medio, bajo la presión del arco de puente y de los 4.20 m de relleno.

La porción del puente sobre las pilas fué terminada como hasta las dos terceras partes antes de que las cimbras fuesen quitadas.

Debajo de cada extremidad de la cimbra **había una cuña de roble**, *ww* (de 9.90 m de largo, por .30 × .30 m). Se recortó de manera que formaran 7 cuñas más pequeñas, *ww*, cada una de 1.35 m de largo y disminuyendo hasta 17 cm; se colocó una debajo de la extremidad de cada armadura *f*. Se movían entre tarugos cónicos ó disminuídos *aa*, de roble de .61 de largo y .30 de ancho, embutidos unos 2½ cm en el dintel *c*, ó en la pieza *n*, sobre la cual descansaban las armaduras, *f*, *f*. Las superficies de resbalamiento se lubricaron bien con sebo al ponerlas en su lugar.

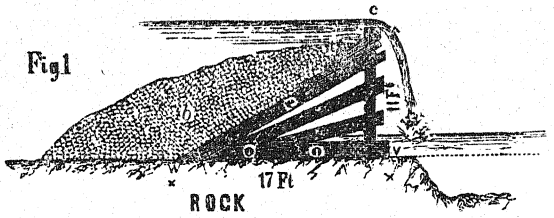
**Las cuñas se quitaron** fácilmente, en una extremidad de la cimbra á un tiempo, por medio de un mazo hecho de un trozo de roble, de 5.40 m de largo y de casi .30 m de diámetro, suspendido por cuerdas, y mecido y guiado por 4 hombres. Generalmente las cuñas cedían y se movían algunos centímetros al segundo golpe. Aunque cada una se aflojaba **enteramente** á los 2 ó 3 minutos, bajando así las cimbras muy repentinamente, sin embargo por la buena calidad de la mampostería no pudo notarse ni la más mínima rajadura en una junta de mortero ni en parte alguna de la obra. Después de tres días, el descenso medio de las claves fué sólo de 9 mm; el mínimo, de 6 mm, y el máximo, de 15 mm. Las cabezas y pies de los postes *p*, comprimieron los dinteles de abeto *c*, y los durmientes, como 9 mm cada uno, demostrando que para arcos de puentes de este tamaño los dinteles y durmientes deben ser de una madera más dura, como pino amarillo ó roble, aunque probablemente contribuyeron á la compresión las escopladuras grandes, de 7½ × 30 cm y de 15 cm de profundidad.

# REPRESAS DE MADERA

**Son requisitos importantes y elementales**, en la construcción de represas, unos cimientos suficientemente firmes para impedir que bajen y produzcan filtraciones, evitar también éstas en la cara atrás de la represa, ó bajo sus bases, é impedir la destrucción del fondo del río enfrente de la represa por la acción del agua que cae. Para lo primero, un lecho de roca dura nivelada es por supuesto lo mejor, y debe escogerse cuando sea posible. En este caso basta para evitar las filtraciones unos tablones gruesos, *tt*, fig. 6 (sencillos ó dobles, según el caso), bien unidos, y que lleguen desde el copete *c*, hasta la orilla inferior de la parte posterior *w*, donde deben estar clavados ó enterrados en la roca; y, además, un buen respaldo *b* de granzón. Granzón, ó más bien tierra con mucho granzón, es mejor que tierra sola, porque si el agua labrara por casualidad un hueco, el granzón de arriba lo llenaría otra vez. Para evitar que este montón de granzón sea movido cerca del copete de la represa, por cuerpos flotantes traídos por la corriente en tiempo de crecientes, se hace un empedrado en bruto, como lo indica la fig. 7, de más ó menos 40 á 45 cm de profundidad, en una extensión de 3 á 6 m ó hasta un punto situado de 1 á 2 metros más bajo que el copete de la represa, según las circunstancias.

En la fig. 1 (represa hecha en la navegación del Schuylkill) los maderos superiores *e*, están todos unidos en sus juntas y tocándose unos con otros, para evitar así la necesidad de cubrirlos con otros.

Fig 1

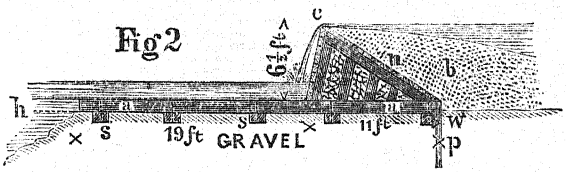



(*Obs. del T.* — Damos en seguida las medidas que trae el clisé *en pies* convertidas *en metros*, para más facilidad del lector; 11 pies = 3.35 m; 17 pies = 5.18 m.)

Pero si el lecho del río es de granzón ó de tierra, entonces deben usarse dos hileras de pilotes de palastro *p*, puestas una al lado de la otra, fig. 2, trabadas en sus juntas, y enterradas á la profundidad de algunos m, para evitar así las filtraciones por debajo de la base de la represa. Frecuentemente no se usa sino una sola hilera. Si el lecho ó fondo está blando ó poroso á la profundidad de pocos metros solamente, sería mejor quitarlo, y colocar la base de la represa más bajo sobre la capa más firme; usando sin embargo las estacas como se ha indicado arriba. Deben quitarse de la base las maderas podridas y otras basuras. En un suelo muy malo hasta una profundidad mayor, puede hacerse necesario sostener la represa enteramente con una plafatorma que descanse sobre estacas. Entonces son necesarias grandes precauciones para evitar las filtraciones; pero este caso ocurre tan rara vez, que no nos detendremos en considerarlo.

Con respecto á la destrucción del fondo producida por la corriente de agua que cae por el frente de la represa, deben tomarse precauciones en todos los casos, á menos que exista una roca muy dura, ó medianamente dura y protegida por una profundidad considerable de agua. La represa, fig. 1, se construyó sobre un lecho bastante firme de gneis micáceo en una capa casi vertical, cubierta, más ó menos, con .61 m de agua. En 39 años la roca se ha gastado enfrente de la represa, como se ve en la figura, hasta una profundidad de .91 m más ó menos, ó casi 2½ cm por año. La altura del agua sobre el copete era generalmente .15 m á .46 m, raras veces de 1.52 m á 1.83 m en las crecientes, y muy pocas veces durante todo el tiempo dicho, de 2.74 m de altura.

Fig 2

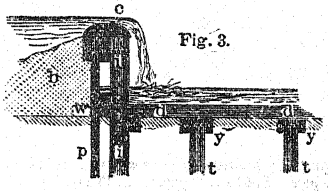



(*Obs. del T.* — Damos *en metros* las medidas que trae el clisé *en pies*, para mayor facilidad del lector; 6½ pies=1.98 m; 19 pies=5.79 m; 11 pies=3.35 m.)

**En la represa de Jones**, en el río Cape Fear, la altura era de 4.88 m y el frente vertical; la caída generalmente de 3.05 m en una profundidad de 1.83 m de agua; la roca blanda de pizarra, en capas verticales, se gastó en pocos años hasta la profundidad de 4.88 m, y la represa se minó por debajo al extremo de caer en la cavidad. En otro caso, en una represa de 10.97 m de altura, con el frente vertical, caía el agua sobre una roca de pizarra dura en capas verticales, cubierta generalmente con .61 m de agua; y en 20 años más ó menos se gastó hasta una profundidad irregular de 3.05 m á 6.10 m, extendiéndose la parte socavada de la cara 23.47 m á 26.52 m enfrente de ella.

En la fig. 2, en un río sometido á grandes y violentas crecientes, el granzón se lavó y desapareció en una profundidad y anchura considerable más allá de la plataforma en *h;* para evitar que se repitiera esto se llenó la cavidad con divisiones rectangulares hechas con maderos gruesos, y los cajones que quedaban en el centro se llenaron de piedras; toda esta construcción atravesaba el río de un lado al otro.

Un depósito de bloques de piedras sueltas, de una tonelada de peso y hasta más, no sirve de protección enfrente de una represa expuesta á grandes crecientes, pues sería arrastrada pronto por las aguas. Generalmente se toma una precaución contra




esta destrucción en las represas bajas; consiste en hacer una plataforma *aa*, fig. 2, ó *dd*, fig. 3, sea de trozas de árboles redondos, ó de madera labrada, colocadas muy juntas, extendiéndose bajo la base entera de la represa, á 5 ó 10 metros enfrente de ella. Estos maderos se aseguran algunas veces por medio de pernos á otras piezas *ss*, fig. 2, ó *yy*, fig. 3, colocadas debajo al través del río. En la fig. 3, estas piezas *yy* se suponen aseguradas por pernos á unas estacas cortas *tt*, enterradas con tal fin. Algunas veces se pone frente á la represa, para recibir la caída del agua, una especie de cajón ancho y bajo (hecho con maderos gruesos), lleno de piedra y cubierto en su parte superior con tablones. Esto es muy eficaz para proteger el fondo del cauce. También en algunos casos se ha construído una represa de menos altura, y de poco precio, á corta distancia aguas abajo de la represa principal, para formar en todo tiempo, enfrente de la última, un pozo hondo que destruya la fuerza del agua.

Otra precaución consiste en sustituir un frente inclinado como *cl*, fig. 4, ó como el que presentarían las figs. 1 y 2, invertidas, en lugar del frente casi vertical de las otras figuras, para disminuir de este modo la fuerza del agua. Este, sin embargo, es solamente un remedio parcial para fondos blandos, porque la lámina de agua que se desliza, siempre cae con una gran fuerza. La mejor forma para una represa en estos casos, es quizás la de la fig. 5, que consiste en una serie de escalones construídos en proporción de 1 de contrahuella para 3 ó 4 de huella. Éstos destruyen perfectamente la fuerza del agua; y agregándoles una plataforma *oo*, se logra un resultado satisfactorio. A esta forma y á las de las figuras 4 y 6 se ha objetado que los frentes están expuestos á ser rotos por árboles, hielo ú otros cuerpos, que des-

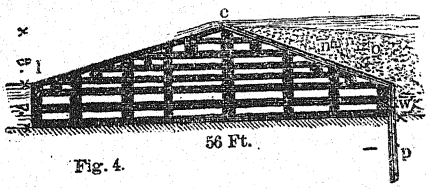


Fig. 4.

(*Obs. del T.* — Damos en seguida *en metros* las medidas que trae el clisé *en pies;* 8 pies=2.44 m; 7 pies=2.13 m; 56 pies=17.07 m.)

ciendan arrastrados por la creciente; pero la experiencia demuestra que esta objeción es de poco valor porque cuando estos cuerpos pasan por la represa, la lámina

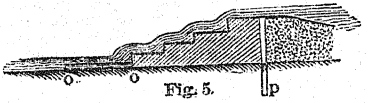


Fig. 5.

de agua es también de más espesor y protege las maderas del frente. En *Sch. Nav.* los maderos *cl*, fig. 6, apenas se gastan á razón de 25 mm cada 10 ó 15 años.

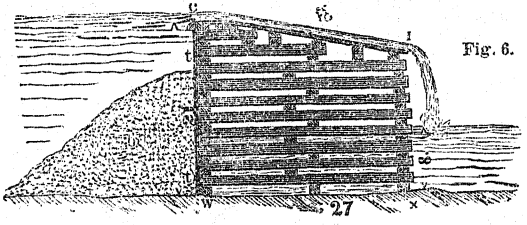


Fig. 6.

(*Obs. del T.*— Damos en seguida *en metros* las medidas que trae el clisé *en pies:* 8 pies=2.44 m; 21 pies=6.40 m; 27 pies=8.23 m.)

Las formas de represas de madera son muchas (véanse las figuras, que muestran las más usadas), variando con las circunstancias del caso y con el criterio del que proyecta. En los Estados Unidos se hacen generalmente de madera en bruto (es decir, de palos redondos) ó de madera labrada; pero en ambos casos de más ó menos de .30 m de grueso. Estos maderos están simplemente puestos uno sobre otro, formando en un plano una serie de rectángulos con lados de 2.13 m á 3.60 m. Están asegurados simplemente por pernos cuadrados (muchas veces arponados) de 60 á 75 cm más ó menos de largo, y colocados como para asegurar dos maderos en cada intersección. Estos pernos no se oxidan ni se gastan mucho, aunque están expuestos á la corriente. Los pernos cuadrados son los mejores. Lábrense chatos los maderos redondos en las partes donde descansan uno sobre otro. La experiencia ha demostrado que es innecesario ligarlos de un modo más firme y por tanto más costoso. Generalmente, pero no siempre, se llenan los cajones hechos con estos maderos con piedras brutas. En represas triangulares dispuestas como en las figs. 1, 2 y 7, estos rellenos de piedras no son tan necesarios como en otras formas, porque el peso del agua y del granzón sobrepuesto tienden á sujetar la represa en su base. Sin embargo puede hacerse necesario, aun en estos casos, cargarlos con unas piedras durante la construcción, si los maderos de la parte baja no están asegurados á la roca, para evitar que el agua se los lleve, antes de estar concluídos y antes que el granzón se haya depositado sobre ellos. Cuando el agua tiene una profundidad de algunos metros es necesario construir cajones con los maderos (ataguías) de pocos metros de alto solamente y llevarlos entonces flotando á un lugar y sumer-

girlos allí, cargándolos con piedras, para lo cual deben proveerse de una plataforma en bruto. Haciéndolo así no es necesario asegurarlos á la roca por medio de pernos. El agua pasa por las aberturas de este cajón ó especie de jaula mientras continúa la construcción de otro encima, teniendo cuidado de agregar siempre piedras suficientes para evitar que una creciente los arrastre. Estas jaulas *cc*, como se indica en proyección horizontal, fig. 8, pueden sumergirse, cargadas de piedras, dejando unos

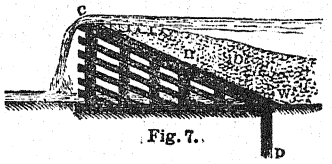


Fig. 7.

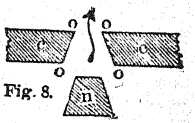


Fig. 8.

huecos *oooo*, entre ellas, para la salida libre del agua. Estas aberturas se cierran después con piezas *n*, que calcen bien.

La mano de obra en aguas profundas se ejecuta mejor construyendo ataguías (*cofferdam*) especiales para el caso, en lugar de sumergir simples jaulas de madera gruesa como hemos dicho. Las juntas pueden hacerse más cerradas, las piedras acomodarse mejor y colocar más juntos los pilotes.

Cuando la superficie del fondo es muy desigual y el agua es profunda, ó la introducción de compuertas en la represa, ó cualquiera otra circunstancia hacen prudente construir una ataguía (*cofferdam*), ambas deben construirse por *secciones* ó *partes*, para dejar de este modo una salida para el agua. Comenzando en una ó ambas orillas la primera sección del agua puede llegar á una cuarta parte del ancho del río ó más. En la sección de la represa construída dentro de esta ataguía, deben dejarse suficientes compuertas para darle salida al agua mientras se construye la última sección del dique. Cuando la represa está construída, se cierran dichas compuertas por medio de travesaños corredizos ó colgantes de madera gruesa *.

Antes de quitar una sección, la extremidad exterior de la represa misma debe concluirse de firme. Es imposible dar detalles para cada contingencia; el ingeniero debe con su propio ingenio hacer frente á las particularidades que se le presenten. En algunos casos, cuando el agua es poco profunda, bastan simples montones de tierra ó de piedras, con tierra y granzón sobrepuesto.

Después que el agua haya pasado por el copete *c*, como en las figuras, no hay necesidad de evitar que se filtre entre los maderos; al contrario, los forros de los tablones gruesos (ó de maderas cuadradas, según el caso) *cl*, figs. 4 y 6, que forman la inclinación por la cual el agua corre en algunas represas, no están generalmente colocados muy cerrados, sino con una separación de 12 milímetros, para permitir que parte del agua pase, con el fin de tener mojadas las maderas por debajo, lo que, en cierta cantidad, las hace más durables. En las figs. 1, 4, 6 y 7 el agua del pozo más bajo penetra libremente entre las maderas y entre las piedras brutas empleadas en llenar en parte ó totalmente los cajones. En las figs. 4 y 6 no se ven estas piedras. En la represa, fig. 1, no se usaron. En la fig. 2, estaban como se indican.

Una represa poco costosa es la de la forma de la fig. 7; puede construirse de piedras brutas con cemento. Deben ponerse horizontalmente algunas piezas de madera cuadrada y asegurarlas firmemente en la mampostería del plano *cnw*, á conveniente distancia entre sí, con su parte superior al nivel de la mampostería. Sobre estas maderas debe clavarse un forro de tablones *cnw*, para proteger la mampostería de la acción del agua y de otros cuerpos flotantes. El granzón *b*, que va detrás, puede omitirse; pero las estacas *p* y una plataforma enfrente de la represa son indispensables en los pisos blandos. Las figs. 1, 2, 4, 6 y 7 son secciones de represas existentes en Pensilvania, trazadas en escala, que han aguantado con éxito la fuerza de crecientes grandes por una larga serie de años**. Estas crecientes traen algunas veces moles grandes de hielo, árboles, casas, puentes, etc., y llegaron á subir

* Maderos preparados de antemano para cerrar una abertura por donde sale el agua y que se dejan caer de repente en su lugar por medio de guías. Varios de estos maderos pueden fijarse en un marco, para después dejarlos caer al mismo tiempo cerrando así la abertura ó compuerta de una sola vez; sobre todo si es pequeña.

** Las de *Schuylkill Navigation*.

hasta 3.35 m sobre los copetes. La de la fig. 1 se construyó en 1819 en la *Sch. Nav.*, y sirvió perfectamente durante 39 años, hasta que en 1858, á consecuencia de estar podridos muchos de los maderos, especialmente los de su parte superior *e*, se construyó una nueva, inmediata y enfrente de la otra. Se construyó muy sencilla y nunca se llenó de piedras. Los maderos inferiores *oo*, á 3.05 m de distancia entre sí, se sujetaron á la roca con pernos, é inmediatamente encima de cada uno de ellos había una serie de maderos inclinados, como se ve en la figura. Los superiores *e*, sin embargo, se colocaron tocándose unos con otros, para formar así el forro superior, en lugar de hacerlo con tablones más delgados. Las piezas cortas en *t* se colocaron del mismo modo. No se usaron ataguías, sino que las piezas del fondo se fijaron en la roca con pernos, y separadas 3.05 m; entonces se colocaron las piezas diagonales y verticales, y por último las piezas inclinadas. El forro *e* que tapaba se extendió desde las extremidades de la represa hasta dejar en el centro un espacio de 18 m más ó menos para la salida del agua. Estando listo el forro para cubrir este espacio que había quedado abierto, un número considerable de hombres principiaron con el trabajo de cubrir el espacio, lo que se ejecutó tan rápidamente, que el río no tuvo tiempo de subir á una altura suficiente para impedir la operación.

La fig. 2 representa una represa de alimentación de canal en el río Juniata; *ss* son maderos atravesados en la corriente (como 90 metros) que sostienen el entarimado *aa*, de fuertes maderos que se tocan. Esta represa se llenó con piedras, y para sostenerlas se agregó el forro de tablones al frente.

La fig. 6 es la del *Sch. Nav.*; fué construída en 1855. Esta estructura ha sido muy alabada para esta forma de represa; está situada sobre pura roca, con una considerable profundidad de agua por delante. La más alta represa (9.60 m) sobre el *Sch. Nav.* es muy semejante á ésta y fué construída en 1851. Todas estas represas son de maderos labrados, en su mayor parte de pino blanco y amarillo. El agua corre, á veces, hasta con 2.40 á 3.60 m de espesor sobre sus copetes, y luego se derrama y cubre muchos de los estribos. La parte vertical posterior permite al agua derramada colarse por entre *todos* los maderos inferiores de la represa contribuyendo á su conservación.

La fig. 4 representa las represas sobre el río Monongahela para la navegación en el repunte de la marea, construídas por el ingeniero W. Milnor. Son de maderos redondos con sus cortezas, aplanados en los cruzamientos. Los más largos que se ven en la figura están colocados á 3 m de separación á lo largo de la represa. La experiencia ha demostrado que estas represas tienen la necesaria resistencia para fuertes corrientes. En roca pura los maderos inferiores se aseguran con pernos.

La fig. 7 se ha empleado con éxito en alturas de 12 m *.

La fig. 3 da solamente una idea para represas bajas en fondos que ceden á la presión. Sus principales apoyos son los postes *ii*, separados de 1.2 á 2.4 m, según la altura de la represa y otras circunstancias, y *tt* son postes pequeños para sostener el entarimado *dd*. Se puede aumentar á mayores alturas con vigas, que se pueden cubrir con fuertes tablones, para formar un declive para el agua desbordante. Al ingeniero se le ocurrirán muchas combinaciones de maderos y declives para represas en terrenos blandos. Por ej., la colocación, á intervalos de algunos metros, de hileras de 3 ó más postes situados transversalmente á la represa, dejando el tope del pilote de cada hilera á la altura que se desee dar al copete, mientras que los de atrás se hincan más y más bajos sucesivamente, de manera que al ser unidos después por maderos transversales y longitudinales, y cubiertos con tablones fuertes y granzón, formarán una represa de forma triangular semejante á la fig. 7. Sería conveniente hincar los pilotes inclinando sus topes corriente arriba.

En la construcción de represas en diversas circunstancias hay mucho campo para lucir el ingenio y el dibujo; por ej. : en el cambio de curso de las aguas por medio de acequias, tubos ó sifones, etc., á diversas alturas, aprovechando á veces represas provisionales, montones de tierra ó tabiques de estacas, etc., ó por ataguías, para mantenerlas separadas de la parte en construcción. Cada localidad presenta sus peculiaridades y el ingeniero debe emplear su criterio para aprovecharlas.

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* **Costo de las represas de cajón.** Con jornales ordinarios á $1.50 por día; maderos á $20 por 4,000 pies (2.36 m³) de medida ancha, entregado en el lugar; piedra para llenar, $1.3 por metro cúbico; granzón, $0.70 por metro cúbico; hierro para pernos, etc., $0.14 por kg. Estas represas en aguas bajas cuestan, generalmente, de $3.2 á $4.2 por metro cúbico.

**Los estribos de las represas** no deben, por regla general, reducir el ancho natural de la corriente, y si es preciso, debe ser lo menos posible, pues la contracción aumenta la altura y la violencia del desbordamiento en tiempo de crecientes, cuando precisamente se desea gran amplitud para facilitarlo. Los estribos deben estar muy bien unidos á los extremos de las represas, y si el terreno lo permite, deben ser tan altos y penetrar tanto en el terreno, que las aguas de las crecientes no pasen por sobre ellos ó alrededor de sus extremos, amenazando socavarlos y destruirlos. En valles anchos y muy llanos, no se pueden extender sin mucho costo y la única alternativa es cimentarlos tan profunda y firmemente que puedan soportar aquella acción, dándoles suficiente altura para que los desbordamientos sean lo menos frecuente posible. Sus extremos adyacentes á las represas deben redondearse para facilitar el paso del agua por los copetes.

La mejor construcción es la de grandes piedras en cemento, pues aunque los maderos le darían bastante resistencia, éstos, así expuestos, se pudren rápidamente. Si se construyen de tierra solamente pueden ser arrastrados por una fuerte creciente que les pase por encima.

**Toda represa importante debe** *tener compuertas*, á fin de que se pueda extraer toda el agua si fuese necesario, en caso de reparaciones, ó para extraer el fango, ó para buscar artículos de valor perdidos, etc. Las compuertas pueden ser tan sólo fuertes cajas, con fondo, paredes y tapa de maderos cuadrados y colocados en el espesor de la represa sobre el fondo. Para impedir que árboles, etc., penetren y se claven en ellas, sería conveniente emplear fuertes traviesas. En casos ordinarios las compuertas no deben *exceder* de $1 \times 1\frac{1}{2}$ m en sus secciones transversales; de otra manera son difíciles de manejar. Cuando el agua que se va á sacar es mucha, se usan dos ó más de estas salidas. Deben estar situadas cerca de los estribos. Las puertas, ó válvulas, para abrirlas ó cerrarlas deben estar aguas arriba, pues situadas abajo, el fango acumulado, etc., llenaría las compuertas impidiéndoles trabajar. Se hacen generalmente de maderos; corren verticalmente en cajuelas y suben y bajan por medio de un engranaje de cremallera y piñón pero, en las represas de gran importancia se pueden hacer de hierro colado. Es conveniente establecer dos juegos de compuertas de manera que una esté siempre lista para usarse si la otra está en reparación.

La parte del entarimado frente á la compuerta debe ser especialmente firme, de manera que no sufra con la presión del agua cayendo de mucha altura.

En represas de mampostería ordinaria y de concreto, si las márgenes son de roca, se le da frecuentemente la forma de un arco rebajado convexo corriente arriba. Utilizando así las márgenes como estribos para el arco (horizontal), se puede lograr una importante reducción en el volumen del material de la represa; pero, á menos que las márgenes sean de roca, ellas constituyen muy imperfectos estribos para el arco, y están expuestas á ser destruídas por la corriente.

**Una represa construída oblicuamente á la corriente**, tendrá menos espesor de agua sobre su copete que otra normal y, por consiguiente, producirá menor derrame sobre el terreno adyacente. En igualdad de circunstancias, mientras menor sea el desnivel de la corriente, mayor será la altura de la creciente.

Las figs. 9 y 10 son adecuadas para pequeñas represas de aforo, en aguas de poca profundidad y de no más 30 m de anchura. La 9 para fondos de tierra ó granzón, y la 10 para roca.

En el primer caso, se ponen primero, atravesando el fondo del río, durmientes de cicuta $S_1$ y $S_2$ de $20 \times 25$ cm, haciendo en dicho fondo las zanjas necesarias y teniendo cuidado de colocar $S_1$ en línea recta. Los durmientes deben extenderse de 2 á 3 metros fuera del río y estar encajados en los bordes. Se entierran detrás é inmediatos al durmiente superior $S_1$, y con .60 á 1.20 m de profundidad, pilotes de palastro P de sección rectangular machihembrados y con una escuadría de $8 \times 25$ centímetros, y se clavan con clavos grandes en $S_1$. Un tercer durmiente, $S_3$, se pone entonces detrás de pilotes, y los dos durmientes $S_1$ y $S_2$ se unen por medio de pernos de 2 ó 3 cm de diámetro, colocados á cada 50 ó 60 cm de distancia más ó menos. Las partes superiores de la estacada P sobresalen más ó menos 30 cm á los durmientes y para hacerlos más consistentes se ligan con maderos *w* de 10 cm de escuadría, asegurados también con pernos. Este piso, lo mismo que los durmientes, sobresale pocos metros á las extremidades de la represa y están enterrados en los bordes del río y cargados de piedras pesadas. Cualquier hueco que resulte debajo de ellos por desigualdad del fondo, debe nivelarse también con piedras ó granzón.

Un poste M de pino amarillo de 25 cm de escuadría y un metro de largo se coloca por medio de una espiga entre los durmientes $S_1$ y $S_2$ en cada extremidad de la

vertiente y se asegura por un puntal N de pino amarillo de 20 × 25 cm, embutido en M y también en el durmiente $S_2$. Más allá de los postes M, la estacada de pilotes de palastro continúa á la altura de M hasta el borde del río. Las partes superiores

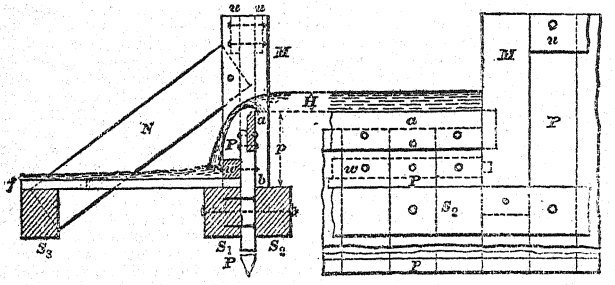


**Fig. 9.**

*N. del T.* — Esta figura está construída en la escala $\frac{1}{40}$

de las estacas se mantienen en una misma línea por unos listones *uu*, de 5 × 20 cent asegurados con pernos.

En la fig. 10, los durmientes $S_1$ de 25 × 25 cm y los durmientes $S_2$ de 15 × 20 cm, descansan sobre un muro de concreto, hecho con cemento Portland con alturas diferentes, para acomodarse á las desigualdades del fondo de roca, y asegurado por pernos de 25 mm de diámetro á 1.20 m de distancia uno del otro. Estos pernos pasan por la mampostería, como está indicado, y penetran á 30 cm ó más de profundidad en la roca de abajo. Entre los dos durmientes se colocan tablones, P, de 8 × 20 cm machihembrados y se aseguran con pernos. En cada extremidad de la represa, se coloca, entre los durmientes de espiga, un poste de pino amarillo de 25 × 25 cm, lo mismo que en la fig. 9, y asegurados en los extremos de la mampostería de la represa, que también debe penetrar en los bordes del río.

En ambas figuras la pieza del copete *a* es de roble de 5 × 20 cm, cortado al sesgo para dejar una cara superior horizontal de 12 mm. La pieza del copete está embutida en todo su grueso en los tablones verticales P, asegurándola con pernos, y también está embutida de 5 á 8 cm en los postes de las extremidades M. En tiempo de poca agua, la corriente puede hacerse pasar por sólo una parte del copete por medio de tablones ó tabiques colocados á lo largo de la represa.

Un copete hecho de una lámina de hierro de 20 cm por ½ cm es preferible á los de madera. Hay menos trabajo para embutirlo, y la cara superior no se destruye tan pronto. El filo superior y las puntas de tope de las diferentes piezas deben estar acepillados, lisos y cuadrados, para tener un borde interior afilado, *a*, por donde pase el agua, y para evitar la filtración.

Como otra precaución contra la filtración, puede embutirse *entre las estacas y el copete, enfrente* de cada junta de la pieza del copete, otra tira de hierro de 20 × ⅓ cm, de 30 cm de largo más ó menos, y cuyas puntas cubran las puntas de la pieza del copete. Si esta lámina de tope en la parte alta del copete se colocara del lado aguas arriba, destruiría la continuidad de la lámina de agua que pasa por la represa, y se opondría algo á la exactitud de los aforos. Estos hierros pueden obtenerse en cualquier centro comercial, en planchas de 5 metros más ó menos.

Todas las juntas deben calafatearse con estopa. Para aplicar la fórmula ordinaria de las represas (véase el art. 14 *f* de Hidráulica) el dorso de éstas debe ser vertical en una profundidad *p* debajo del copete igual lo menos al doble de la carga H sobre la represa. Es mejor proteger la cara, corriente arriba de la represa (excepto la porción cerca del fondo) con alquitrán, en lugar de greda pisada.

En una represa larga de poca caída de agua, es difícil asegurarle un acceso suficientemente libre al aire por detrás de la lámina de agua que cae, especialmente cuando la corriente tiene poca profundidad y la lámina tiende á acercarse mucho

á la represa. En estos casos se forma un vacío* parcial entre la lámina de agua y la cara de la represa, y aumenta la salida, impidiendo la exactitud de los resultados. Por consiguiente es muy importante, al hacer los planos para represas de aforos, disponerlos (hasta donde sea posible) de modo que la lámina de agua pueda caer

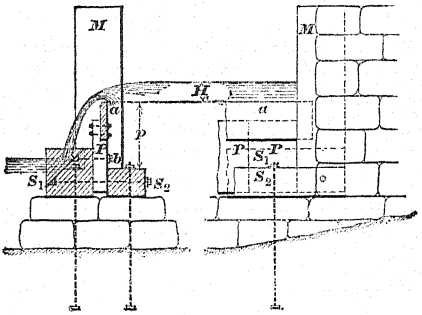


**Fig. 10.**

*N. del T.* — Esta figura está construída en la escala $\frac{1}{40}$

libremente sin adherirse en parte alguna á la represa misma, porque así disminuiría el espacio detrás de la lámina y hace más difícil evitar la formación del vacío.

**Vibraciones en las represas.** Las represas en que el agua cae formando una lámina larga lisa y sin interrupción y de una altura considerable, están más ó menos sometidas á vibraciones ú oscilaciones causadas por la rarefacción y compresión alternativa del aire por la lámina de agua que cae perpendicularmente en el espacio (W, fig. 20, art. 14 (*a*) de Hidráulica) detrás de la lámina, donde muchas veces se forma un vacío parcial, porque el aire se mezcla con el agua que cae y es lanzado aguas abajo en forma de espuma y burbujas.

Estas vibraciones producen algunas veces un ruido en las ventanas de las casas, situadas hasta mil metros ó más de distancia. Hemos sabido que se han suprimido estas vibraciones construyendo una vez (sin intención) un tabique ancho de troncos de madera de altura conveniente y colocado como antepecho frente á la represa para evitar que se le gaste el fondo. En otros casos ha sido suficiente para suprimir las vibraciones una hilera de maderos puestos oblicuamente contra el frente de la represa, y hasta una parte de su altura inclinados más ó menos de 1½ á 1 cubiertos con tablones. En fin, cualquier sistema que permita al aire entrar más libremente detrás de la lámina que cae, ó que destruya la continuidad de ésta, ó interrumpa su altura y su extensión, debe disminuir y hasta evitar este inconveniente.

**La buena época para la construcción de represas** es, naturalmente, el período más largo de las aguas bajas.

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* Este vacío hace que el agua río abajo cerca de *w* suba por detrás de la lámina, fig. 9. Cuando la rarefacción del aire ha llegado á cierto grado, el aire exterior la rompe, entra y llena el vacío. Entonces se vuelve á formar otro, que á su vez desaparece al penetrar el aire, siguiendo así, alternativamente. Durante estos movimientos se ha notado que los cuerpos livianos, como un pedacito de madera, etc., que flotan en el agua río abajo cerca de las extremidades de la represa, son atraídos hacia el espacio detrás de la lámina y llevados hasta la mitad del ancho de la represa, y de aquí otra vez al punto donde entraron para ser botados hacia afuera, después de vaivenes hacia adelante y hacia atrás en el espacio formado detrás de la lámina de agua.

**Tabla de los espesores de tablones de pino blanco, que no se doblan sino 1/480 de su extensión horizontal libre, bajo diferentes cargas de agua.** (Original.)

| Largo en metros. | Carga en metros. | | | | |
|---|---|---|---|---|---|
| | 12 | 9 | 6 | 3 | 1.50 |
| | Espesor en mm. | | | | |
| 0.91 | 89 | 76 | 70 | 57 | 44 |
| 1.22 | 114 | 100 | 89 | 70 | 57 |
| 1.83 | 171 | 150 | 133 | 108 | 82 |
| 2.44 | 228 | 203 | 178 | 140 | 120 |
| 3.05 | 286 | 254 | 222 | 178 | 140 |
| 3.66 | 340 | 310 | 273 | 216 | 171 |
| 4.57 | 422 | 381 | 330 | 266 | 216 |
| 6.10 | 565 | 508 | 444 | 356 | 279 |

# ABASTO DE AGUAS

**Consumo de agua.** Debido en gran parte á la adecuada extensión del uso del agua en las habitaciones, la cantidad requerida en las ciudades aumenta más rápidamente que la población. Dicho de otro modo, el consumo por cabeza aumenta.

**Uso.** Una larga experiencia demuestra que una provisión de 50 galones (189 litros) por cabeza y por día es abundante para todas las necesidades y gustos de familias acomodadas en las ciudades americanas. El consumo de los manufactureros, por supuesto, no lleva relación fija con la población. En las ciudades, es generalmente mucho menor que el consumo doméstico.

**Desperdicio.** En las ciudades americanas, el desperdicio sube á dos ó tres veces la cantidad realmente usada. De los 438 lit por cabeza y por día, suministrados en Nueva York en 1899, estima el Sr. Freeman* que de 117 á 208 lit se usaban : 38 inevitablemente desperdiciados; y de 189 á 283 desperdiciados, pero, pudiendo evitarse.

En Filadelfia, las investigaciones hechas por medio del indicador del desperdicio de agua de Deacon, en 142 habitaciones modernas, de siete piezas, de dos pisos, con baño, etc., en dos calles intermedias, resultó que de 839 lit por cabeza y por día, suministrados por 782 instalaciones, 726 lit, ó el 86.5 por ciento, se desperdiciaban, y sólo 13.5 por ciento eran usados. La ciudad ha construído vastas obras con el fin de bombear, filtrar, conducir, rebombear, acopiar y distribuir el agua desperdiciada, como también la cantidad más pequeña usada. Del costo total **, menos de la mitad habría bastado para el agua usada é inevitablemente desperdiciada.

**Fuentes de desperdicio.** El desperdicio es causado por descuido : por dejar correr el agua para que se bote á fin de impedir que se congele en el invierno y para obtener agua más fresca en verano; por instalaciones defectuosas, por filtraciones fijas no sospechadas en los canales y tubos de servicio, etc.

Como una « conjetura » clasifica el Sr. Freeman los 189 á 283 lit por cabeza y por día, desperdiciados en Nueva York, como sigue :

| | | | |
|---|---|---|---|
| Filtraciones en canales..... | 38 á 57 lits | por cab, | por día. |
| — en tubos de servicio..... | 38 á 57 — | — | — |
| — en tuberías defectuosas..... | 57 á 94 — | — | — |
| Desperdicio por descuido voluntario..... | 53 á 64 — | — | — |

El desperdicio evitable lo hace de ordinario una pequeña fracción (digamos de un quinto á un tercio) de la población, pues el resto usa el agua razonablemente. En el

* Informe sobre el Abasto de Agua de Nueva York, hecho á Bird S. Coler, por John R. Freeman, ingeniero civil, 1900.

** El costo total se aproxima á $30,000,000.

caso de Filadelfia, arriba citado, de las 782 instalaciones se halló que 22 dejaban perder el agua poco á poco y 32 lo hacían continuamente.

**Restricción del desperdicio. Medidores de agua.** El mejor modo de restringir el desperdicio consiste en interesar pecuniariamente al consumidor para que lo evite, y esto como mejor se realiza es usando el medidor de agua, á lo menos en todos los servicios (doméstico, industrial y público), en donde se encuentre desperdicio. Los medidores deben ser propiedad y conservados por la corporación que suministra el agua.

**Derecho mínimo.** A fin de estimular el *uso* liberal del agua, al mismo tiempo atenuando el deseo de *desperdiciarla*, y evitando una economía indebida (que conduce al desaseo), á cada consumidor debe cobrársele periódicamente un derecho mínimo, suficiente para cubrir ampliamente toda el agua de que él probablemente pueda usar y gozar.

El Sr. Freeman estima el costo medio de los medidores domésticos, para Nueva York y Brooklyn, por lo común de 5-8 pulgadas y 3–4 pulgadas, con unos pocos de tamaños más grandes, en $12.50 cada uno, y el costo de instalación, trabajando sistemáticamente y en grande escala, en $2.50, ó sean $15 cada uno. Él supone que la duración media del medidor doméstico ordinario, de un buen tipo, bien cuidado, y con reparaciones oportunas y renovación de las partes gastadas, es de « no mucho menos de 20 años »; y los gastos anuales como sigue :

| | Providencia, R. I. Aprox. | Nueva York. Se suponen. |
|---|---|---|
| Interés sobre el costo del medidor y colocación... | $.50 | $.45 |
| Depreciación y renovación del medidor (supuesta duración de 20 años)..... | .75 | .75 |
| Conservación y reparaciones, prueba y recomposición..... | .46 | .70 |
| Anotación de medidores y cómputo de cuentas, etc. | .42 | .60 |
| Costo anual total, por medidor..... | $2.13 | $2.50 |

**Agua gratis para defenderse del fuego.** En las ciudades, se da algunas veces á los manufactureros un suplemento de agua gratis, por especiales conexiones, para usarla sólo contra el fuego, garantizando no emplearla con otro objeto, y la ciudad pone un medidor en la conexión para conocer cualquier uso ilícito.

**El agua para el uso de una ciudad no debe tomarse del mismo fondo del estanque,** por que se llevaría los sedimentos, que no sólo dañan el agua, sino producen obstrucciones en la corriente. Debe esto tenerse en cuenta al fijar la capacidad necesaria del estanque, considerando el agua que se halla debajo del nivel de la toma como perdida. Cuando las circunstancias justifican el costo, sería bueno encorvar la parte del tubo matriz en el estanque, y proveerlo con válvulas á diferentes alturas, para tomar el agua de la capa de agua más pura en el estanque. En vista de esto, la torre de las válvulas generalmente tiene estas válvulas comunicando con el agua del estanque, y por este medio no se da entrada á la torre sino al agua limpia, y de ésta pasa á los tubos para la ciudad. Este esmerado sistema es, sin embargo, pocas veces practicable. Estas válvulas deben ser manejadas por empleados de guardia.

**Art. 1. Estanques.** En estanques de tierra, de alguna importancia, para almacenar agua á profundidades moderadas, para ciudades, parece que la experiencia no permite **dimensiones** menores de 3 metros de ancho en la parte superior; talud interior 2 en 1, talud exterior 1½ en 1. Se han adoptado en algunos casos de 4.50 á 6 m con taludes interiores de 3 á 1 y exteriores de 2 á 1. Algunas veces ambos taludes se hacen de 1½ en 1. La superficie del agua debe mantenerse por lo menos á 1 metro bajo la parte superior del terraplén, y más si está expuesto á oleajes. En un estanque grande, con una brisa moderada, se llegaron á formar olas hasta de 1 metro. Se usa algunas veces un muro pequeño, ó una palizada cerrada, *w*, fig. 37, que sirve como defensa contra las olas. La parte superior yel talud exterior deben estar protegidos, por lo menos, con césped. Para mantener la parte superior seca debe redondearse un poco, ó tener una inclinación hacia el exterior. Deben suprimirse y limpiarse cuidadosamente las partes blandas fangosas y toda materia vegetal en toda la base de los terraplenes; esta limpieza se hace hasta donde el suelo es impenetrable al agua, con el fin de que no se puedan formar filtraciones *debajo* de los terraplenes. Con este objeto y en malos terrenos se establece una hilera doble de pilotes de palastro ó un muro enterrado de mampostería de concreto hasta una profundidad adecuada y á lo largo del pie del talud

interior. Si hay manantiales de agua debajo de la base, deben taparse y desaguarlos con cuidado hacia otra parte por medio de tubos. El terraplén debe hacerse por capas, un poco cóncavas hacia el centro y que no pasen de 30 centímetros de grueso; y deben excluirse cuidadosamente todos los troncos de árboles, piedras ó cualquier otro material extraño, como granzón *puro*, arena, esquistos micáceos, etc., que puedan producir filtraciones. Estas capas de tierra deben ser pisadas, lo que es tanto más fácil de hacer cuanto más suaves sean los taludes. Debe procurarse que las capas no sean de distintas clases de tierra y unirlas muy bien entre sí. Esto se logra algunas veces haciendo traficar mucho y en todas direcciones sobre cada capa un rebaño de ganado y hasta de ovejas, á más del tráfico de las carretas. No recomendamos los rollos pesados porque tienden á producir grietas entre las capas.

La tierra arenisca es un material excelente, quizás el mejor. Los materiales más escogidos deben colocarse en el talud más cerca del agua: y comprimirlos con especial cuidado en esta parte, para evitar que el agua pueda filtrarse en la parte interior del terraplén y debilitarlo. No está demás cubrir la parte baja del terraplén exterior *b*, fig. 37, para disminuir el peligro de que el agua de lluvia lave el terraplén, y para disminuir también la rapidez de su descenso.

Si al fondo del estanque corresponde un suelo poroso, ó una roca rajada, por donde el agua pueda escaparse, debe cubrirse cuidadosamente con greda pisada en una capa de 40 á 80 centímetros, la cual á su vez se protege con una capa de granzón, ó de concreto, según las circunstancias.

Los estanques construídos según las dimensiones que acabamos de fijar, y con cuidado, *pueden* conservarse en buen estado durante un período de tiempo indefinido, pero deben tomarse las **precauciones** *adicionales* que siguen si resultaren daños serios por algún defecto. El talud interior hasta la parte superior debe cubrirse con una capa de piedras en seco de 40 á 45 cm de grueso y colocadas con sumo cuidado, como protección contra la acción del agua y de los animales, como ratas, etc., que pueden minar el terraplén. Estos animales, según creemos, principian á minar debajo del agua. Si los taludes están más escarpados que de 2 en 1, las capas de piedra en seco están expuestas á resbalar. Será mucho mejor, pero por supuesto más costoso, colocar las piedras en cemento hidráulico, ó mejor todavía, sobre una capa de concreto de algunos centímetros de espesor, especialmente si debajo de la capa de concreto hubiese una capa de greda pisada de 40 á 80 centímetros de grueso, extendida sobre la cara del talud; esto con el principal objeto de proteger el talud interior del contacto del agua. Si estas condiciones se realizan, basta para una completa seguridad que los taludes tengan una inclinación de 1½ á 1. Debe tenerse mucho cuidado en consolidar la parte inferior del talud interior, para evitar que el agua allí se filtre y ablande la tierra minando por la parte baja las capas de piedras superpuestas. Hacia la parte superior debe tenerse cuidado con los efectos de destrucción producidos por el frío, hielo, lluvias y olas. Los taludes interiores muy inclinados no sólo evitan que se disloquen ó desacomoden las piedras, sino que aumentan la estabilidad del terraplén, á causa de que la presión del agua (que siempre obra en ángulo recto contra el talud) se hace más vertical y comprime el terraplén sobre su base con más fuerza que cuando no hay agua sobre él. Algunas veces las patas de ambos taludes, interior y exterior, se protegen con un pequeño muro de sostenimiento hecho de concreto.

Muchos ingenieros, para evitar filtraciones, sea en el terraplén ó debajo de él, construyen un muro de mezclote amasado de arcilla y tierra bien pisados, *p*, fig. 37,

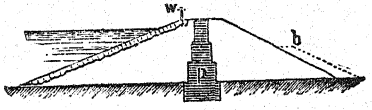


Fig. 37.

impenetrable al agua, que llega desde la parte superior hasta algunos decímetros bajo la superficie de la tierra. Este muro no debe tener menos de 1.50 á 2.50 metros en su parte superior, y debe ensancharse hacia abajo por medio de escalones (y no por taludes) en proporción de 1 de espesor por 3 ó 4 de profundidad. Otros ingenieros se oponen á esta clase de muros pisados, y afirman que la filtración debe evitarse haciendo los taludes interiores y el fondo del estanque impermeables, por medio de greda pisada, concreto, ó cubriéndolos con una capa de piedras colocadas

en cemento, como se ha dicho. Arguyen que si el terraplén está bien construído, por sí mismo constituye un muro pisado en toda su extensión.

**Cerca de San Francisco, Cal, hay dos estanques de terraplenes de tierra,** construídos en 1864, uno tiene 28.95 m de alto, 7.92 m en la parte superior; el talud interior 2.75 por 1; el exterior 2.5 por 1.

El otro tiene 28.°6 m de alto, 7.62 m en su parte superior; el talud interior 3.5 á 1; el exterior 3 á 1. En ambos el muro de tierra pisada llega á 14.32 m bajo su base. No hay revestimiento de piedras.

**Es difícil evitar bajo una fuerte presión que el agua penetre á distancias considerables por los intersticios** que resultan donde la tierra está en contacto con roca lisa, madera ó metal, como, por ejemplo, en los tubos de hierro colocados bajo los terraplenes del estanque; y lo mismo puede ocurrir bajo las bases de los terraplenes que descansan sobre roca lisa. Debe tenerse un cuidado especial en que la tierra usada en estos puntos no sea porosa, que esté muy compacta á lo largo de los filones ó intersticios, procurando que no queden en línea recta, sino que se alternen ó crucen. En el caso de roca ó mampostería, en los intersticios debe hacerse una especie de reborde con piedras y cemento.

Se acostumbra, y es prudente, dividir los estanques en dos partes, de modo que mientras se usa el agua de una división, el agua de la otra decanta todos sus sedimentos y se purifica. También sirve para usar una parte mientras se limpia la otra. Se necesitan algunas veces muchos días y hasta 2 ó 3 semanas, según la profundidad del estanque, para que se asiente completamente el agua cuando contiene partículas muy finas en suspensión. Deben colocarse una ó dos escaleras hasta el fondo del estanque.

**Del fango en los estanques.** Los estanques de la Comp. *New River, London, Ingl.*, se dejaron sin limpiar durante 100 años, y en este tiempo se depositó un fango de 2.40 m de alto, es decir, más ó menos, 2½ centímetros por año. En Filadelfia se depositan más ó menos 6 milímetros por año en las aguas del río Schuyklil y 25 en las del río Delaware. En San Luis, Misuri, más ó menos de .90 m á 1.20 m por año!

En los estanques de poca profundidad se desarrolla mucha vegetación, y en los de mucha profundidad, en las orillas, especialmente durante la estación cálida. Cuando estas plantas se pudren dañan el agua.

**Las aguas que corren por pantanos** no son á veces potables, como sucede, por ejemplo, en algunos parajes del río Concord, en Massachusetts, que han sido declaradas venenosas por el eminente ingeniero hidráulico, Loammi Baldwin, de Boston.

La construcción de un estanque grande y profundo es, no tan sólo muy costosa, sino muy arriesgada, pues á pesar de la mayor vigilancia y cuidado es casi imposible impedir las filtraciones, y éstas suelen no ser visibles durante meses y aun años. En los casos de rotura, sobre todo cerca de una ciudad, puede producir grandes pérdidas de vidas y de propiedades. Si el agua encuentra una vía y se forma una corriente, tendrá gran fuerza de destrucción.

**Art. 1a. Estanques de depósito.** El total rendimiento anual de una corriente puede ser más que suficiente para suministrar el agua á cierta población; pero puede suceder, sin embargo, que no se haga uso de ella con aquel objeto, porque en verano se seque casi por completo y, además, porque las lluvias y las nieves derretidas producen en otras estaciones crecientes que dan mucha más agua de la requerida, y que se desperdicia. Un estanque de depósito sirve para recoger y conservar este exceso de agua, y usarlo durante las sequías del verano, igualando así el abasto de agua en todo el año. Cuando la localidad lo permita, se logra esto construyendo una represa á través de la corriente, para formar un lado del estanque, mientras que las faldas de las colinas que rodean la corriente forman los otros lados. La corriente entra en este estanque por su extremo superior. Cuando la corriente pueda agotarse durante las largas sequías del verano, la experiencia demuestra que la *capacidad* del estanque debe hacerse igual á la provisión de 4 á 6 meses, según las circunstancias. Durante la construcción de la represa se debe hacer un ramal para que pase la corriente sin dañar la obra. Si la represa se construye exactamente como la fig. 37, toda de tierra, sería fácilmente destruída por la corriente en caso de que el estanque llegase á llenarse tanto que el agua pase sobre su borde. Para impedir esto se puede, por medio de una mampostería, ó por cajones llenos de piedras trituradas, ó de otra manera, hacer que toda, ó parte de la represa, sirva como *vertedero*. O por canales (abiertos ó por entubados) á cada extremo de la represa, y sobre el suelo natural, á un nivel que pueda dar salida al exceso de las crecientes antes de que lleguen éstas á pasar por encima

del borde de tierra de la represa. Además de éstos, y de los tubos que lleven el agua á la población, debe existir una salida, con válvula ó compuerta á nivel del fondo del estanque, para que se pueda sacar toda el agua en caso de reparaciones ó de limpieza. Las bocas de los tubos para la ciudad deben estar protegidas por rejas, para impedir la entrada de peces, etc.

**Para facilitar las reparaciones ó la renovación de todas las válvulas, etc., que están bajo el agua**, los extremos de los tubos ó alcantarillas encajados en el estanque pueden estar rodeados por cajas ó cámaras impermeables, que para el servicio se dejan abiertas hacia el estanque y se cierran en caso de reparaciones. Se puede llegar á ellas entrando por el otro extremo, después que haya salido el agua. En el caso de que la salida sea al través de una larga extensión de tubos y que no se pueda entrar en ella, se puede hacer una abertura especial con este objeto en el tubo mismo cerca de la orilla exterior del terraplén y mantenerla cerrada cuando no hay reparaciones. A veces, la mejor manera de llegar á estas válvulas es levantando sobre ellas una *torre de válvulas* hecha de mampostería. Ésta es una cámara hueca, vertical, impermeable, semejante á un pozo, pero situada cerca de la base del talud interior, con sus cimientos en el fondo del estanque, desde donde se levanta la torre á través del agua hasta su superficie. Esta cámara está provista de válvulas ó compuertas generalmente abiertas hacia el estanque, pero que se pueden cerrar en caso de reparaciones. Así construída, los obreros pueden bajar por la torre con escaleras, entrando por la parte superior.

A veces las salidas para el exceso de agua de las crecientes, así como las de limpieza, se colocan sobre el nivel del fondo del estanque. Para que éstas puedan funcionar en caso de creciente repentina nocturna, etc., deben estar provistas de válvulas automáticas, que se abren por sí mismas cuando la creciente sube demasiado. Esto se puede lograr asegurándolas á boyas, las cuales, al subir el nivel del agua, las abren tirando de ellas. Todas estas salidas deben ser suficientemente amplias para dar paso á los obreros. De ninguna manera se deben hacer en el cuerpo de tierra de la misma represa, sin que estén apoyadas en bases de mampostería que lleguen á cimientos sólidos; de otra manera estarían expuestas á romperse al asentarse el terraplén. Es, generalmente, más seguro hacerlas en firme, en el suelo natural, cerca de uno de los extremos de la represa. Sus válvulas, si son sencillas, deben estar situadas hacia la extremidad interior del estanque, de ese modo las salidas se mantienen generalmente libres facilitando su inspección; pero es mejor tener dos válvulas, para usar una mientras se repara la otra, y en este caso se puede colocar una á cada extremo. En los estanques que se llenan por medio de bombas, no se requieren precauciones contra las inundaciones; porque las bombas se pueden parar cuando se hayan llenado lo suficiente. Los estanques grandes de provisión inundan generalmente mayor ó menor cantidad de terrenos que deben comprarse. Estos estanques al interceptar el agua corriente evitan frecuentemente las crecientes de primavera que perjudican los terrenos más bajos. Si hay molinos río abajo de la represa, quedan privados del agua para moverlos, si no se les destina parte de la almacenada en el estanque. El agua que se aplica para **compensar** la pérdida de una corriente natural, se llama **agua de compensación**, y el estanque, **estanque de compensación**.

**Art. 1 b. Estanques de distribución.** Frecuentemente un valle á propósito para un estanque de provisión, no se halla sino á una distancia grande (de muchos kilómetros algunas veces) de la ciudad; entonces, es conveniente construir también otro estanque más pequeño que el de provisión cerca de la ciudad y á una altura tan grande como las circunstancias lo permitan; pero siempre más bajo que el estanque de provisión. Éste se llama, para distinguirlo, estanque de *distribución*, porque el agua, después de haber llegado á él por *tubos surtidores que la traen* del estanque de provisión, sale de aquél para ser distribuída, en diferentes direcciones por la ciudad, en la tubería de las calles. Este estanque pequeño debe contener agua suficiente para algunos días por lo menos, y mejor para algunas semanas. La extremidad de los tubos surtidores debe estar provista de una llave para suspender cuando se quiera la entrada del agua que viene del estanque de provisión. Estas precauciones permiten que se hagan reparaciones á lo largo de los tubos surtidores, sin privar á la ciudad del agua durante este tiempo. En vista de estas reparaciones, como también con el fin de poderlos limpiar, estos tubos deben estar provistos **de llaves de desagüe** en varios puntos bajos de su longitud entre los dos estanques, especialmente en aquellos puntos donde se pueden desaguar en vertientes naturales. Al abrir estas válvulas, la fuerza del agua se lleva el sedimento, y deja el tubo libre para inspeccionarlo.

**Al calcular los diámetros de los tubos** para surtir de agua una ciudad,

es necesario tener en cuenta, que la mayor parte del producto del agua durante las 24 horas, se gasta en realidad durante 8 á 12 horas; y por consiguiente, la capacidad del tubo debe ser tal, que supla la cantidad necesaria para un día en mucho menos de 24 horas. También, durante los meses cálidos de verano, se gasta mucha más agua que en los meses de invierno, y esto hace necesario un diámetro mayor aún.

(*N. del T.* — Esto sucede solamente en países donde las diferencias de las estaciones es muy notable. En climas análogos al nuestro (Venezuela) no pasa esto.)

**Art. 2. Sistemas de tubos de distribución en las ciudades.** El autor no conoce ninguna regla práctica para calcular el diámetro de los tubos en estos sistemas. Las varias complicaciones que envuelve hacen de poco ó ningún valor las investigaciones puramente científicas. Después de mucha vacilación nos atrevemos á dar las siguientes reglas que son puramente empíricas, basadas sobre las observaciones, que han llegado á nuestro conocimiento.

*Regla* 1. *Cuando en ningun punto del sistema de tubos de distribución en las calles, la carga ó la distancia vertical bajo la superficie del estanque, comparada con la distancia horizontal del estanque, es menor de* 9.47 *m por kilóm, entonces la población de una ciudad dada en la última columna de la Tabla A, puede surtirse abundantemente para todos los usos, sea por un solo tubo del diámetro interior de la* 1ª *columna, ó por* 2, 3, *etc., tubos de los diámetros que se indican en las otras columnas.* Estos diámetros se han dado con una aproximación de 3 milímetros más ó menos. El gasto por habitante se supone ser de 60 galones (227 litros) por día *.

(*Obs. del T.* — Suponemos que se trata de galones de los Estados Unidos, de 3.78 litros.)

**TABLA A.** (Original.)

(*N. del T.* — La hemos convertido al sistema métrico.)

| Número de tubos. | | | | | | | | Población. |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 6 | 8 | 12 | 24 | |
| Diám. mm. | Diám. mm. | Diám. mm. | Diám. mm. | Diám. mm. | Diám. mm. | Diám. mm. | Diám. mm. | |
| 152 | 117 | 98 | 88 | 76 | 66 | 60 | 47 | 1647 |
| 203 | 155 | 133 | 117 | 101 | 88 | 79 | 60 | 3465 |
| 254 | 193 | 165 | 149 | 127 | 111 | 98 | 76 | 5908 |
| 304 | 231 | 196 | 177 | 149 | 133 | 114 | 88 | 9324 |
| 355 | 269 | 231 | 206 | 177 | 155 | 136 | 104 | 13706 |
| 406 | 307 | 269 | 234 | 199 | 174 | 152 | 117 | 19141 |
| 457 | 349 | 295 | 266 | 225 | 199 | 171 | 130 | 25677 |
| 508 | 387 | 330 | 295 | 250 | 222 | 190 | 146 | 33426 |
| 558 | 425 | 361 | 323 | 276 | 241 | 209 | 158 | 42433 |
| 609 | 463 | 393 | 352 | 298 | 266 | 228 | 174 | 52671 |
| 660 | 501 | 425 | 381 | 323 | 285 | 247 | 190 | 64447 |
| 711 | 539 | 460 | 412 | 349 | 311 | 266 | 203 | 77565 |
| 762 | 569 | 492 | 447 | 374 | 333 | 285 | 219 | 91580 |
| 812 | 625 | 524 | 469 | 397 | 355 | 301 | 228 | 108160 |
| 863 | 657 | 558 | 498 | 425 | 381 | 323 | 247 | 125840 |
| 914 | 695 | 590 | 530 | 450 | 400 | 342 | 260 | 144480 |
| 1016 | 774 | 657 | 587 | 498 | 447 | 381 | 292 | 188320 |
| 1117 | 850 | 720 | 644 | 549 | 495 | 419 | 320 | 239600 |
| 1219 | 927 | 787 | 704 | 600 | 536 | 457 | 349 | 297600 |
| 1371 | 1041 | 885 | 793 | 673 | 606 | 511 | 390 | 391200 |
| 1524 | 1159 | 984 | 879 | 749 | 669 | 568 | 434 | 511200 |
| 1676 | 1273 | 1082 | 968 | 822 | 739 | 625 | 476 | 650400 |
| 1828 | 1390 | 1181 | 1057 | 898 | 803 | 682 | 520 | 800000 |
| 2032 | 1546 | 1311 | 1174 | 996 | 895 | 758 | 577 | 1064000 |

* Para apreciaciones rápidas y aproximadas, nada más útil ni más práctico que estas dos tablas del autor. Continúese leyendo hasta la aplicación de la Tabla B, y lo que falta del art. 2, porque lo que sigue está tan íntimamente ligado á la Tabla A, que la completa. *(N. del T.)*

(*Obs. del T.* — Cuando se quiera calcular la tubería para surtir de agua una población, pero dándole á cada habitante *más ó menos* de los (60 galones) 227 litros que supone la tabla, también es útil ésta, procediendo así:

Multiplíquese el número de habitantes por el número de litros que se quieran dar y divídase por 227; tómese el número que resulte como población para entrar en la Tabla A, y los tubos que ella indica servirán para surtir la población dada con los litros que se deseaban.

Ej.: Se quiere surtir con cuatro tubos una población como la de Caracas (Venezuela), que tiene 100,000 habitantes, con 160 litros por habitante: Tendremos: $100{,}000 \times 160 \div 227 = 71{,}748$. Búsquese en la tabla A, en la columna *Población*, el número más próximo á 71,748, que es 77,565, y, como se ve en la columna 4, con 4 tubos de 412 mm se surtiría la población. Es claro que se supone la carga de 9.47 m por kilómetro.

Más adelante la tabla B, también original del autor, da el modo de hacer el cálculo para otras cargas.)

Será bueno también aumentar además uno ó dos centímetros para cada diámetro (según la clase de agua) por obstrucciones y sedimentos.

El agua después de llegar á la ciudad por uno ó más tubos matrices, debe distribuirse por las calles, por medio de tubos pequeños que se ramifican de los tubos mayores. El diámetro de estos tubos más pequeños también puede hallarse con la Tabla A, así: Si una calle con sus callejones, etc., contiene 6,000 personas, más ó menos (y la proporción de la carga, como antes se dijo, no es menor de 9.47 m por kilóm en cualquier punto) vemos entonces por la tabla que un tubo de 254 mm es suficiente. Aconsejamos no usar tubos de un diámetro menor de 150 mm en cualquier calle de la ciudad.

**Los tubos matrices que se cruzan deben estar conexionados en algunas de sus intersecciones**, para dar al agua una circulación más libre por todo el sistema; si se quita el agua provisionalmente cerrando las llaves para alguna reparación ó se la disminuye por un gasto excesivo, el servicio se mantiene siempre con la corriente de agua de las otras tuberías.

**Evítense los extremos de tuberías cerrados**, porque el agua en ellos se vuelve sucia y malsana.

*Regla* 2. *Con los mismos diámetros, pero con cargas distintas á la de* 9.47 *m por kilóm, supuesta por el autor en la Tabla A, se surtirán poblaciones que estén con las poblaciones del cuadro A, en la proporción que indican los números que se hallan en la columna* 3 *de la Tabla B. O dicho de otro modo: para hallar los diámetros de los tubos que surtan las mismas poblaciones dadas en la última columna de la Tabla A,* **pero con diferentes cargas**; multiplíquese el diámetro dado en la Tabla A, por el número correspondiente en la columna 4 de la Tabla B, enfrente de la nueva carga en la columna 1.

**TABLA B.** (Original.)

| Columna 0. | Columna 1. | Columna 2. | Columna 3. | Columna 4. |
|---|---|---|---|---|
| Carga en metros por kilómetro. | Carga en pies por millas. | Relación de la carga comparada con la de la Tabla A. | Proporción en que están las poblaciones. | Diámetros proporcionales para suplir las poblaciones en la Tabla A. |
| .946 | 5 | .1 | .32 | 1.58 |
| 1.893 | 10 | .2 | .45 | 1.37 |
| 2.366 | 12½ | .25 | .50 | 1.32 |
| 2.839 | 15 | .3 | .55 | 1.27 |
| 3.786 | 20 | .4 | .64 | 1.20 |
| 4.732 | 25 | .5 | .71 | 1.14 |
| 5.679 | 30 | .6 | .78 | 1.11 |
| 6.626 | 35 | .7 | .84 | 1.07 |
| 7.099 | 37½ | .75 | .87 | 1.06 |
| 7.572 | 40 | .8 | .90 | 1.05 |
| 8.519 | 45 | .9 | .95 | 1.02 |
| 9.465 | 50 | 1.0 | 1.00 | 1.00 |
| 14.198 | 75 | 1.5 | 1.23 | .92 |
| 18.930 | 100 | 2.0 | 1.41 | .88 |
| 23.662 | 125 | 2.5 | 1.59 | .83 |
| 28.395 | 150 | 3.0 | 1.73 | .80 |
| 37.860 | 200 | 4.0 | 2.00 | .76 |
| 47.324 | 250 | 5.0 | 2.25 | .73 |
| 56.790 | 300 | 6.0 | 2.46 | .69 |
| 75.720 | 400 | 8.0 | 2.83 | .66 |
| 94.650 | 500 | 10.0 | 3.18 | .63 |

(*Obs. del T.* — Hemos hecho, como siempre, en los ejemplos y en esta tabla, las modificaciones necesarias para que puedan usarse en el sistema métrico.)

**Ejemplo.** Por la tabla A observamos, que con la carga de 9.47 m por kilómetro (en ella supuesta) un tubo de 762 milímetros) suplirá una población de 91,580; pero con tres veces esa carga ó 150 pies por milla (como 28.4 m por kilómetro), véase tabla B; encontramos por la columna 3 (tomando el número enfrente á la presión nueva 28.39) que el mismo tubo surtirá á 1.73 veces tantas personas ó 91,580 × 1.73 = 158,433 personas. Pero si con esta carga mayor deseamos surtir solamente 91,580 personas, entonces hallamos en la columna 4, Tabla B, enfrente á 1.73), que debemos disminuir el diámetro del tubo de 762 mm hasta 762 × 0.80 = 610 mm.

También después que el agua haya llegado á la ciudad por el tubo de 762 mm de la Tabla A, si queremos distribuirla por ocho ramales ó tubos más pequeños, vemos por la sexta columna, Tabla A, que cada uno de ellos debe tener á lo menos 333 mm de diám. De estos ocho tubos, otros más pequeños pueden ramificarse en las diferentes calles transversales, callejones, etc. Al apreciar el gasto necesario para cualquier tubo principal de una calle, debemos agregar evidentemente lo que se requiera para las calles transversales á la calle principal y aquéllas deben ser alimentadas por dicho tubo.

Si ciertas partes limitadas de un sistema de tubos de una ciudad tiene una carga considerablemente menor en relación al resto de los tubos, entonces puede ser conveniente proveerla con tubo especial, separado, y de mayor diámetro, que saldría directamente del estanque mismo ó de uno de los tubos matrices que llevan el agua á las partes bajas de la ciudad.

Debe tenerse presente que aumentando los diámetros se puede obtener un gasto tan abundante como se quiera, compensando así, *en cuanto al volumen*, la carga grande; pero el agua no sube entonces á tanta altura en los tubos secundarios como para abastecer los diferentes pisos de las casas, etc.

Con los diámetros y carga de la tabla A, el agua con un gasto ordinario no subirá á la altura de la superficie del agua en el estanque, y si hay un gasto extraordinario y al mismo tiempo en muchas partes del sistema como en el caso de un incendio

extenso ó durante los meses cálidos del verano, quizá no sube ni á la mitad de aquella altura.

**Art. 3. Las siguientes reglas ó condiciones han dado muy buenos resultados para evitar la formación de concreciones en los tubos de agua** y la consiguiente obstrucción. Antiguamente en Boston, los tubos de hierro fundido de 4 pulgadas (10 centímetros) de diámetro, se tapaban en 7 años, y á los de mayor diámetro se les redujo éste considerablemente en el mismo tiempo.

Pero más tarde, y durante ocho años en que se usó este charol, no se formaron concreciones *.

**Reglas para la aplicación del charol de brea mineral á tubos y fundiciones construídos para el Departamento de Aguas de Filadelfia, bajo las condiciones siguientes:**

*Primero.* Todo tubo debe estar enteramente arreglado y limpio, libre de tierra ó arena, que se adhiera al hierro en las partes mohosas y deben usarse cepillos duros para quitarles el último polvo.

*Segundo.* Ningún tubo debe tener ninguna parte oxidada cuando se aplica el charol. Si el tubo no puede sumergirse inmediatamente después de haberse limpiado, debe dársele una mano de aceite de linaza para preservarlo hasta que esté listo para el charol, y éste no debe aplicarse después que haya principiado la oxidación.

*Tercero.* La brea mineral se hace de alquitrán de hulla, por medio de la destilación, hasta que haya salido toda la nafta, y el material se haya hecho inodoro. La destilación se hace hasta que la brea tenga la consistencia de la cera. Se recomiende mezclarla con 5 ó 6 por ciento de aceite de linaza. La brea que al enfriarse se vuelva dura y quebradiza, no sirve para esto.

*Cuarto.* Después de haber obtenido la brea propia para el efecto, debe calentarse en un envase adecuado, á una temperatura de 149° C, y debe mantenerse á esta temperatura durante el tiempo de la inmersión. El material se pondrá más espeso y deteriorará después que se haya sumergido un número de tubos; entonces se agrega con frecuencia brea fresca; y algunas veces debe extraerse del envase su contenido viejo; y llenarse de nuevo con brea fresca; el sobrante será duro y quebradizo como la brea ordinaria.

*Quinto.* Ningún tubo debe sacarse del envase de brea caliente, hasta que no haya alcanzado 149° C. Entonces puede sacarse lentamente y ponerse sobre varas de madera para que gotee.

Todos los tubos de 50 cm ó más de diámetro, deben quedar á lo menos treinta minutos en el fluído caliente para alcanzar aquella temperatura, y probablemente por más tiempo cuando haga frío.

*Sexto.* La aplicación del charol debe hacerse á satisfacción del ingeniero en jefe del Departamento de Aguas, y el material estar sujeto en todo tiempo á su examen é inspección, para ser rechazado por él si no satisface las condiciones exigidas.

*Séptimo.* No se pagará lo que cuesta la aplicación del charol sino en los tubos que están en buen estado y satisfacen á todas las condiciones antes de aplicarles el charol.

*Octavo.* Ningún tubo debe sumergirse sin que el inspector autorizado haya examinado si está limpio y libre de oxidación, y lo haya sometido á la prueba del martillo. Entonces puede sumergirse y después se pasa á la prensa hidráulica, para hacer la prueba de la presión requerida.

*Noveno.* Si está bien aplicada, la capa de charol después que esté fría debe estar rígida y muy adherida al tubo, y no ser quebradiza ni tener tendencia á desconcharse. Cuando el charol no se haya aplicado bien á un tubo, sea por defecto del material, herramientas ó manipulación, no debe pagarse; si se desconcha, ó parece que puede desconcharse, debe limpiarse en su interior antes de charolarlo nuevamente.

**Art. 4.** Los tubos se colocan siguiendo la configuración de las calles en el sentido vertical. Las partes superiores de los tubos no deben estar nunca á menos de un metro bajo la superficie de la calle; y en tubos de 7 cm el agua se ha helado algunas veces aun á esta profundidad.

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* El Sr. Braccket, de Boston, nos informa (1892) que allá se forman tubérculos en los tubos no charolados hasta un espesor como de 18 milímetros, lo que hace poco útiles ó inútiles los tubos de 10 centímetros para los casos de incendio; pero que, no obstante, no ha observado ninguna interrupción total durante los veintitrés años que lleva en el Departamento del Ingeniero Municipal. También dice que hasta los tubos charolados después que están enterrados diez ó quince años, se corroen en la superficie interior.

(*Obs. del T.* — En nuestros climas templados basta para tubos de pequeño diámetro que su parte superior esté 60 centímetros bajo la superficie del suelo.)

**En Filadelfia, en 1885, había como 1,262 kilómetros de tubos en las calles**; ó 1,770 metros para cada 1,000 habitantes. La población era de 860,000 y vivían en 150,000 edificios más ó menos. **Berlín**, 1887-8, tenía 1,400,000 habitantes en 20,000 casas (70 personas por casa término medio). Consumo medio por persona=64 lit por día, máximo 90 lit; mínimo 47 lit; todo aproximado; había 25,000 medidores de agua en uso.

**Ninguna acción galvánica** ha sido observada donde los tubos de plomo ó cobre están unidos á los de hierro fundido. Ningún tubo de menos de 15 centímetros debe colocarse en una ciudad; y cuando se empleen que sea solamente para distancia de algunos centenares de metros *. La insuficiencia de ellos se nota principalmente en casos de incendio. Un mínimo de 20 centímetros es mejor. No hay más filtraciones en el invierno que en el verano; excepto cuando se revientan los *tubos de servicio* porque se hiela el agua en ellos.

Pisando bien la tierra alrededor de los tubos, se excluye el aire, y se evita mucho la oxidación.

Los tubos de agua se corroen por las filtraciones de gas.

## PESO DE TUBOS DE HIERRO COLADO

**Como se usan en Filadelfia** y probados con la prensa hidráulica á una presión interior de 21 kilogramos por centímetro cuadrado. En esta tabla está incluída la campana.

Los tubos deben hacerse de un hierro gris, duro y fuerte, fundido de manera que puedan taladrarse y cortarse fácilmente; y todos los que tienen más de 7 cm de diám deben fundirse verticalmente, con la campana hacia abajo. Se permite una diferencia de 5%, en más ó en menos del peso teórico, por las irregularidades imposibles de evitar en la fundición. Los tubos se hacen con 4 ó 5 cm de exceso en longitud sobre 3.66 m, para que colocados midan 3.66 m de la boca *l*, fig. 38, de una campana á la boca de la otra.

| Diámetro. | Espesor. | Peso por tubo. | Diámetro. | Espesor. | Peso por tubo. | Diámetro. | Espesor. | Peso por tubo. |
|---|---|---|---|---|---|---|---|---|
| mm. | mm. | kilog. | mm. | mm. | kilog. | mm. | mm. | kilog. |
| 76.2 | 7.94 | 71.668 | 406.4 | 16.0 | 599.632 | 914.4 | 23.81 | 1965.815 |
| 101.6 | 9.53 | 95.709 | 508.0 | » | 750.221 | 914.4 | 26.99 | 2205.305 |
| 152.4 | 11.11 | 174.635 | 508.0 | 17.46 | 815.536 | 914.4 | 30.16 | 2433.910 |
| 203.2 | » | 208.656 | 762.0 | 20.62 | 1502.710 | 1219.2 | 28.58 | 3302.969 |
| 254.0 | 12.7 | 302.551 | 762.0 | 22.82 | 1637.423 | 1219.2 | 34.93 | 3931.177 |
| 304.8 | 14.3 | 407.775 | 762.0 | 25.4 | 1798.091 | 1219.2 | 38.10 | 4253.677 |

*Obs. del T.* — En la tabla que agregamos á continuación, los espesores de los tubos son los siguientes: De 40 á 70 mm de diámetro, van aumentando desde 7 hasta 8.5 mm; de 75 y 80, son de 9 mm; de 90 á 162, son de 10 mm; de 200 y 250, son de 12 mm; de 300 á 400, son de 14 mm; de 400 á 500, son de 16 mm; de 600, son de 18 mm; de 700 á 1,000, son de 22 mm; y de 1,100 á 1,500, varían de 25 á 28 mm.

Siendo $e$ el espesor del tubo en metros; D, el diámetro del tubo, y $n$, la presión en atmósferas, á la cual se ensayan los tubos, la fórmula del espesor es:

Tubos colados horizontalmente, $e = 0.010 + 0.0020\ Dn$.
» » verticalmente, $e = 0.008 + 0.0016\ Dn$.

* *N. del T.* — Esto puede hacerse en las grandes ciudades, porque en ciudades de poca población y donde ésta está muy esparcida, hay que recurrir hasta á tubos de 5 centímetros de diámetro. Además, en nuestros climas no hay el temor de que, á causa del pequeño diámetro, el agua se congele.

| Diámetro interior en milímetros. | Longitud útil en metros. | Peso del tubo en kilog. | Peso del metro corriente útil. | Diámetro interior en milímetros. | Longitud útil en metros. | Peso del tubo en kilog. | Peso del metro corriente útil. |
|---|---|---|---|---|---|---|---|
| **40** | 2.00 | 19 | 9.5 | **220** | 3.00 | 210 | 70 |
| **50** | 2.50 | 30 | 12 | **250** | 3.00 | 240 | 80 |
| **54** | 2.50 | 33.7 | 13.5 | **300** | 4.00 | 388 | 97 |
| **60** | 2.50 | 37.5 | 15 | **350** | 4.00 | 472 | 118 |
| **70** | 2.50 | 42.5 | 17 | **400** | 4.00 | 560 | 140 |
| **75** | 3.00 | 57 | 19 | **450** | 4.00 | 680 | 170 |
| **80** | 3.00 | 60 | 20 | **500** | 4.00 | 780 | 195 |
| **90** | 3.00 | 66 | 22 | **600** | 4.00 | 1,000 | 260 |
| **100** | 3.00 | 75 | 25 | **650** | 4.00 | 1,120 | 280 |
| **110** | 3.00 | 81 | 27 | **700** | 4.00 | 1,280 | 330 |
| **120** | 3.00 | 90 | 30 | **750** | 4.00 | 1,460 | 365 |
| **125** | 3.00 | 99 | 33 | **800** | 4.00 | 1,600 | 400 |
| **135** | 3.00 | 105 | 35 | **900** | 4.00 | 1,870 | 467 |
| **150** | 3.00 | 120 | 40 | **1,000** | 4.00 | 2,254 | 560 |
| **162** | 3.00 | 138 | 46 | **1,100** | 4.00 | 2,698 | 674 |
| **175** | 3.00 | 156 | 52 | **1,250** | 4.00 | 3,420 | 855 |
| **200** | 3.00 | 180 | 60 | **1,500** | 4.00 | variable según la presión. | |

Los siguientes tamaños de **tubos para agua, de hierro forjado con soldaduras solapadas,** los hace la Compañía Nacional en McKeesport, Pa. Un extremo de cada tubo está provisto de su unión. El peso por metro incluye el peso del plomo en la unión. El peso del plomo, por unión es el que se requiere para echarlo derretido en aquélla al colocar los tubos, ó el plomo que se requiere por un solo lado.

| Diámetro interior en milímetros..... | 50 | 75 | 100 | 125 | 150 | 200 | 250 | 305 | 406 |
|---|---|---|---|---|---|---|---|---|---|
| Peso por metro en kilogramos..... | 2.88 | 5.61 | 7.82 | 10.91 | 13.02 | 19.63 | 25.40 | 37.37 | 70.92 |
| Plomo para cada junta en kilogramos.... | .28 | .68 | .96 | 1.47 | 1.59 | 2.60 | 2.72 | 3.97 | 7.25 |

Los tubos se someten á una prueba de presión de 35 kilogramos por centímetro cuadrado. Están provistos con una capa de estaño ó asfalto, ó, si se desea, primeramente estañados y después cubiertos con una capa de asfalto.

El proceso de estañar consiste en « incorporar, sobre y entre las partículas del hierro, una liga de metal no corrosivo compuesto en su mayor parte de estaño ». La superficie formada así no se cuartea con golpes ni doblando el tubo, sea frío ó caliente.

La junta ó unión, es de hierro fundido, y tiene concavidades internas que reciben y sujetan los bordes exteriores en cada extremo del tubo.

La junta se llena entonces de plomo derretido de la manera ordinaria (véase pág. 712) sea con collares de greda, ó por medio de una laña ó abrazadera especial hecha para el efecto.

Esta laña se parece á la pieza de unión, fig. 39, con la diferencia que aquélla consiste en dos piezas semicirculares unidas por bisagras, y provistas de dos mangos, semejantes á un exprimidor de limones con un agujero en un lado, para echar el plomo derretido. Esta unión forma una superficie interior lisa con el tubo en la junta, disminuyendo así en parte la resistencia á la corriente que presenta el hierro fundido.

En los casos donde se hace necesario los cambios frecuentes, la unión se compone de dos piezas sujetadas en sus pestañas con pernos.

**El hierro forjado para tubos** tiene la gran ventaja sobre el hierro colado de ser más liviano y más flexible. El poco peso facilita su manejo, y resulta más económico *el metro lineal* á pesar de que su costo por *tonelada* es, más ó menos, 25 por ciento mayor. No están expuestos á romperse durante el transporte por torpe manejo y pueden doblarse hasta un ángulo de 25°. No requieren ser fundidos expresamente para formar estos ángulos; hay máquinas para doblarlos, que pueden

manejar dos hombres. Una máquina cambiando los dados, puede usarse para todos los tamaños de tubos. Estos tubos se hacen en piezas de 4.57 m á 5.49 m en lugar de 3.66 m que tienen los de hierro fundido, así es que se emplean **menos juntas por kilómetro.**

La Compañía provee « lañas especiales » de servicio y taladros (aparatos con terraja, etc.) **para conectar los tubos de las casas á los tubos principales de las calles.** Esto puede hacerse, como se hace con el taladro de Payne, estando el tubo principal bajo la presión del agua. Esta laña es una pieza de hierro fundido en forma de silla, la cual se asegura al tubo antes de hacer el taladro, por medio de un perno en forma de U, y queda permanentemente fijo después de taladrarse el tubo. Entre la laña y el tubo se calza un anillo de plomo. Aquélla tiene un agujero cilíndrico provisto de roscas, en las cuales se atornilla el tubo, llaves, etc., suministrados por la Compañía para hacer *la toma*. El aparato de taladrar tiene un barreno que pasa por el orificio cilíndrico de la laña, y taladra el tubo.

La Compañía también suple máquinas para cortar tubos y piezas especiales, como para hacer reducciones y también cruces, etc., todo de acuerdo con las uniones y diámetros de los tubos que se van á conectar.

**Art. 5. Los tubos de hierro forjado se oxidan** mucho más pronto que los de hierro fundido.

**Un tubo de gutapercha**, de 3 mm de espesor y 18 milímetros de diámetro interior, **ha resistido** con seguridad á una presión interna de más de 17.58 kilogramos por centímetro cuadrado, igual á más ó menos 175 mets de carga. Dicho tubo tan sólo se hinchó un poco con la presión de más ó menos 24 kilogramos por centímetro cuadrado. En 1851, un tubo de esta materia, de 64 milímetros de diámetro interior y 12 milímetros de espesor y de 412 mets de largo, fué sumergido en el East River, N. Y., para llevar el agua de Craton á la isla de Blackwell. El tubo lo sostuvieron en el fondo con pesos, y no dió un resultado satisfactorio debido al rozamiento causado por las corrientes de la marea y al daño sufrido con las anclas arrastradas por las embarcaciones que pasaban. Con un forro de lona se evitó la primera causa de daño, pero no la última. Este tubo fué reemplazado por otro de hierro forjado en 1870.

**Tubos de hierro y cemento patentados** por la Compañía de Tubos de Agua y Gas de Jersey City, N. J. Estos tubos están formados de hierro en planchas remachadas y cada tubo se sumerge en una capa de una mezcla caliente de alquitrán mineral y asfalto. Después se revisten con una capa de cemento hidráulico. Esta capa alcanza un espesor de 15 mm en un tubo de 300 mm de diám y hasta 25 mm en los de 500 mm de diám. Se hacen estos tubos hasta de .91 m de diámetro. Se colocan en un lecho de mezcla de cemento y se cubren enteramente con éste. Se los provee de todo lo necesario para hacer las conexiones que después puedan necesitarse, y para las ramificaciones de tubos en la ciudad. Más de 2,000 kilómetros están en uso en varias ciudades, y algunos tienen ya 35 años; parece que han dado muy buen resultado. No se forman obstrucciones en estos tubos como en los de hierro. Hay muchas razones para suponer que son duraderos. Haciéndose las zanjas, la Compañía de Jersey City suple los tubos y los coloca (incluyendo el cemento).

A. Wychoff y Son, Elmira, N. Y., hace **tubos de madera para agua.** Para una presión de 1 kg por cm cuad, ellos suplen tubos simples de 9 á 18 cm en cuadro, en el exterior, y de 2½ á 10 cm de diámetro interior; y tubos redondos, de 2½ á 40 cm de diámetro interior, provistos de una capa exterior de cemento asfaltado. En sus extremidades, tanto los tubos cuadrados como los redondos están forrados con láminas de hierro. Para presiones de 2 á 11 kg por cm cuad, á los tubos redondos de madera, antes de darles la capa de cemento, se los envuelve con unos aros de hierro en espiral, aplicados por medio del vapor, pasando antes por una preparación de alquitrán mineral. Los aros están tan apretados, que penetran entre la madera hasta formar una sola superficie. Las extremidades están envueltas doblemente, y entonces se aplica la capa de cemento asfaltado. Estos tubos se han usado extensamente y con buen éxito tanto para agua como para gas. Se proveen con lo necesario para empatarlos y conectarlos.

**Los tubos para agua, de trozos de roble y de pino taladrados,** colocados en Filadelfia hace 50 á 60 años, están casi todos enteramente sanos, y todavía servibles, excepto en los casos en que se pudre la madera blanca del exterior. Quitando esta parte, muchos de ellos se han colocado otra vez. Comprimiendo *bien un poco de greda* alrededor de los tubos de madera, se evita el contacto del aire

y aumenta mucho su durabilidad. Es, al contrario, muy desfavorable un medio suelto y poroso como la arena ó tierra.

**Los tubos de papel betuminado** hechos bajo una gran presión, se han usado tanto para agua como para gas. No están tan expuestos á quebrarse como los de hierro fundido, y su peso no es sino la mitad más ó menos. Con un diámetro interior de 125 milímetros y 12 milímetros de espesor han resistido á una presión de 15 kilogramos por centímetro cuadrado; igual á una carga de agua de 150 metros más ó menos.

**Costo de la tubería de agua y de su colocación.** Las siguientes cifras se deducen de un cuadro bondadosamente suministrado por el Sr. Allen J. Fuller, superintendente general (Oficina de las Aguas, Filadelfia). Ellas representan condiciones medias para tubería recta, colocada en el suelo, en esa ciudad. El costo, en cualquier caso dado, puede diferir considerablemente de esas cifras, según las circunstancias. « La colocación » incluye todo movimiento de materiales, después de su entrega en el terreno, para colocarlos en la zanja, hacer conexiones, calafateo, etc. Los calafateadores principales reciben $2.50, los otros $2, y los peones $1.75 por día de 8 horas. (*N. del T.* — Hemos dejado estos cálculos y costos, porque dados estos valores de jornal y materias primas, es muy fácil deducir los que corresponderán á otros países conociendo los jornales y precios para estos últimos.)

El costo del material se calcula como sigue: Fundiciones de tubos, 1.2 centavos; plomo, 5 centavos; empaquetadura, 3½ centavos; combustible, .27 centavos por libra; acuñar, nivelar, 5½ centavos por metro. Agréguese para llaves de retención, bocas de ramales, de incendios, fundiciones especiales, reempedrado, perjuicios, sueldos de caporales, costo de herramientas, etc. Para presupuestos brutos, que cubran instalaciones, excavaciones en roca, y algunas adicionales requeridas por la zanja, deterioro de herramientas y reempedrado ordinario, pero sin incluir perjuicios, nuevo pavimento de asfalto ó caballetes, el costo en el cuadro puede aumentarse como sigue:

| | | | | | | |
|---|---|---|---|---|---|---|
| Diámetro del tubo... | 100 | 150 | 200 | 250 | 300 | 400 á 1,200 mm. |
| Agréguese..... | 80 | 70 | 65 | 60 | 50 | 40 por ciento. |

MATERIALES

| Tubo. | | Por cada tubo. — Longitud de 3.66 m. | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Diám. | Espesor. | Hierro. | | Plomo. | | Calzar (estopear). | | Cok. | | Madera. | Total. |
| mm. | mm. | kil. | $ | kil. | $ | kil. | $ | kil. | $ | $ | $ |
| 101.6 | 9.53 | 97.066 | 2.57 | 2.721 | 0.30 | 0.0907 | 0.01 | 1.8144 | 0.01 | 0.03 | 2.92 |
| 152.4 | 11.11 | 166.463 | 4.40 | 4.535 | 0.50 | 0.0907 | 0.01 | 1.8144 | 0.01 | 0.03 | 4.96 |
| 203.2 | 11.11 | 222.254 | 5.88 | 5.443 | 0.62 | 0.436 | 0.01 | 1.8144 | 0.01 | 0.03 | 6.56 |
| 304.8 | 14.29 | 446.386 | 11.02 | 8.164 | 0.90 | 0.2721 | 0.02 | 2.268 | 0.01 | 0.03 | 11.98 |
| 457.2 | 15.87 | 684.905 | 18.42 | 13.608 | 1.50 | 0.3628 | 0.03 | 3.1751 | 0.02 | 0.03 | 19.70 |
| 609.6 | 19.0 | 1114.899 | 29.50 | 18.144 | 2.00 | 0.4536 | 0.03 | 3.6287 | 0.02 | 0.05 | 31.60 |
| 762.0 | 20.62 | 1508.153 | 39.90 | 34.019 | 3.75 | 0.5896 | 0.05 | 4.0823 | 0.02 | 0.07 | 43.79 |
| ..... | 25.4 | 1848.402 | 48.11 | 34.019 | 3.75 | 0.5896 | 0.05 | 4.0823 | 0.02 | 0.07 | 52.00 |
| 914.4 | 23.84 | 2094.003 | 55.32 | 52.164 | 5.75 | 0.9072 | 0.07 | 4.5359 | 0.03 | 0.09 | 61.25 |
| ..... | 30.16 | 2576.334 | 68.16 | 52.164 | 5.75 | 0.9072 | 0.07 | 4.5359 | 0.03 | 0.09 | 74.09 |
| 1219.2 | 31.75 | 3645.876 | 96.46 | 68.038 | 7.50 | 1.088 | 0.08 | 5.4431 | 0.03 | 0.44 | 104.21 |
| ..... | 38.10 | 4288.598 | 113.46 | 68.038 | 7.50 | 1.088 | 0.08 | 5.4431 | 0.03 | 0.44 | 121.21 |

## MOVIMIENTO DE TIERRA

| Tubo. | | Zanja. | | | Movimiento de tierra por metro lineal. | | | |
|---|---|---|---|---|---|---|---|---|
| Diámetro. | Espesor. | Ancho en metros. | | Profundidad en metros. | Excavación. | | Rellenar y botar lo que sobra. | Total. |
| | | Arriba. | Abajo. | | Metro cúbico. | $ | $ | $ |
| mm. | mm. | | | | | | | |
| 101. | 9.5 | 0.75 | 0.67 | 1.40 | 1.00 | 0.29 | 0.06 | 0.35 |
| 152. | 11.1 | 0.75 | 0.67 | 1.40 | 1.00 | 0.29 | 0.06 | 0.35 |
| 203. | 11.1 | 0.75 | 0.67 | 1.40 | 1.00 | 0.29 | 0.06 | 0.35 |
| 304. | 14.2 | 0.80 | 0.75 | 1.40 | 1.17 | 0.36 | 0.20 | 0.56 |
| 457. | 15.8 | 0.80 | 0.75 | 1.40 | 1.20 | 0.36 | 0.29 | 0.65 |
| 609. | 19.0 | 0.80 | 0.90 | 1.45 | 1.60 | 0.46 | 0.49 | 0.94 |
| 762. | 20.6 | 1.30 | 0.97 | 1.52 | 1.90 | 0.56 | 0.74 | 1.30 |
| .... | 25.4 | 1.30 | 0.97 | 1.52 | 1.90 | 0.56 | 0.74 | 1.30 |
| 914. | 23.8 | 1.50 | 1.40 | 1.70 | 2.43 | 0.72 | 0.98 | 1.70 |
| .... | 30.4 | 1.50 | 1.40 | 1.70 | 2.40 | 0.72 | 0.98 | 1.70 |
| 1219. | 31.7 | 2.10 | 1.40 | 2.00 | 3.68 | 1.44 | 1.84 | 3.28 |
| .... | 38.4 | 2.10 | 1.40 | 2.00 | 3.68 | 1.44 | 1.84 | 3.28 |

## ACARREO, COLOCACIÓN, RESUMEN

| Tubo. | | Movidas (pequeños acarreos en el trabajo) a 75 centavos fuertes la tonelada. | | | Colocación por metro lineal. | Total del costo por metro lineal. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Diám. | Espesor. | Tubo. | Miscelánea. | Total. | | Materiales. | Movimiento de tierra. | Movidas (pequeños acarreos) | Colocación. | Total. |
| mm. | mm. | $ | $ | $ | $ | $ | $ | $ | $ | $ |
| 101.6 | 9.5 | 0.03 | 0.03 | 0.06 | 0.12 | 0.79 | 0.35 | 0.07 | 0.42 | 1.33 |
| 152.4 | 11.1 | 0.03 | 0.03 | 0.06 | 0.12 | 1.34 | 0.35 | 0.07 | 0.43 | 1.90 |
| 203.2 | 11.1 | 0.03 | 0.03 | 0.06 | 0.12 | 1.80 | 0.35 | 0.07 | 0.43 | 2.36 |
| 304.8 | 14.2 | 0.10 | 0.03 | 0.13 | 0.16 | 3.28 | 0.56 | 0.43 | 0.46 | 4.43 |
| 457.2 | 15.8 | 0.13 | 0.03 | 0.16 | 0.20 | 5.38 | 0.66 | 0.16 | 0.49 | 6.39 |
| 609.6 | 19.0 | 0.23 | 0.03 | 0.26 | 0.26 | 8.63 | 0.94 | 0.26 | 0.26 | 10.09 |
| 762. | 20.6 | 0.29 | 0.06 | 0.35 | 0.26 | 11.97 | 1.30 | 0.36 | 0.26 | 13.90 |
| ..... | 25.4 | 0.36 | 0.06 | 0.42 | 0.26 | 14.20 | 1.30 | 0.43 | 0.26 | 16.20 |
| 914.4 | 23.8 | 0.42 | 0.06 | 0.48 | 0.26 | 16.73 | 1.70 | 0.49 | 0.26 | 19.19 |
| ..... | 30.2 | 0.52 | 0.06 | 0.58 | 0.29 | 20.24 | 1.70 | 0.59 | 0.29 | 22.83 |
| 1219.2 | 31.7 | 0.72 | 0.10 | 0.82 | 0.39 | 28.48 | 3.28 | 0.82 | 0.39 | 32.97 |
| ..... | 38.1 | 0.85 | 0.10 | 0.95 | 0.39 | 33.13 | 3.28 | 0.95 | 0.39 | 37.75 |

*N. del Trad.* — Esta tabla y las dos anteriores han sido reconstruídas con las unidades métricas conservando los precios en pesos fuertes americanos (dólares).

**Art. 6. Empalmes de tubos de hierro fundido.** Modelo de Filadelfia. La distancia *d*, entre el tubo y la campana es casi uniforme para todos los tamaños de tubos, variando solamente de 3 mm para tubos de 100 mm, á 10 mm para tubos de 750 mm. La profundidad de la campana *mn* varía de 75 mm en un tubo de 100 mm, á 100 mm en un tubo de 750 mm. Las pequeñas pestañas circulares *s*, *m*, *s'*, *m'* en el macho del tubo, que entra en la campana del otro tienen como 6 mm

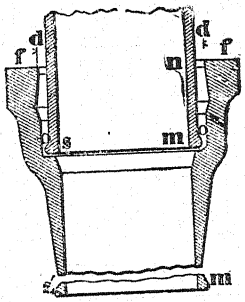


**Fig. 3b.**

Esta fig. está trazada en una escala de $\frac{1}{16}$.

de alto más ó menos, para evitar que el material con que se calafatea la junta, pueda penetrar al tubo. Se calafatea primero con estopa sin alquitrán bien apretada hasta 3 ó 5 centímetros y sobre ésta se echa el plomo derretido, encerrado para evitar que chorree, con una faja ó anillo de greda puesto alrededor de la campana. Después el plomo se compacta por medio de un martillo de calafatear.

El plomo derretido se echa por un orificio que se deja en la greda en el lado superior del tubo. En tubos gruesos se dejan dos orificios adicionales, uno en cada lado del tubo, y se echa el plomo primeramente por estos dos orificios empleando dos hombres, uno en cada lado y simultáneamente. Después se tapan estos orificios con greda; cuando el plomo ya vaciado se ha endurecido, los dos hombres echan el plomo por el orificio de la parte superior hasta llenar la junta. Es necesario proceder así, porque el peso grande del plomo derretido botaría el anillo de greda.

El agua que contiene la greda suele congelarse con el frío, y ponerla tan dura que no puede usarse. También esta agua que contiene la greda está expuesta, en todo tiempo, á convertirse en vapor por el calor del plomo, y á veces este vapor sale é « infla » la greda, saliéndose el plomo.

**Art. 7. Los « cinturones » * patentados**, de Watkins, para soldar tubos, evitan estas dificultades y hacen innecesario el anillo de greda. Consisten en un anillo R, figs. 39 y 40, de sección transversal cuadrada, y hecho de capas alternativas de lona y caucho. El anillo está forrado por una ó más tiras de acero delgado elástico que están remachadas al anillo de distancia á distancia, como se ve. EE son unos codos de hierro remachados á las tiras de acero. Se coloca el « cinturón » alrededor del tubo, cerca de la campana, en la posición de la fig. 40, y sujetada flojamente por el tornillo de presión, fig. 41, cuyas dos puntas entran cada una en una concavidad pequeña de los codos EE. Entonces se hace entrar el « cinturón » por medio de un martillo hasta que llegue á la punta *f*, de la campana, fig. 38, y se aprieta algo el tornillo para hacer que toque el « cinturón » al tubo; se coloca ahora un poco de greda enfrente de la abertura de los dos codos EE, y la junta está lista para vaciar el plomo en ella. Después de endurecido el plomo, se quita el « cinturón », y se puede usar entonces en otra soldadura. Uno de ellos sirve sin romperse para soldar hasta algunos centenares de juntas. Usando estos « cinturones » no se necesita, por su

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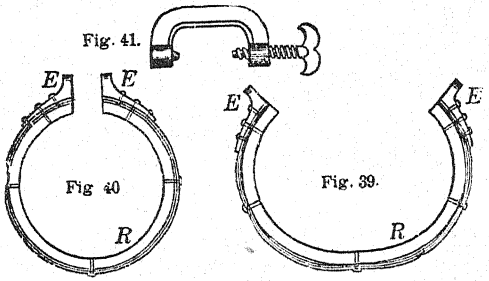
*N del T.* — No sabemos que en español tenga este aparato nombre especial; adoptamos el de « *cinturones* ».

puesto, el trabajo de los hombres que preparan los anillos de greda. Al quitar el




« cinturón », se encuentra el plomo liso, no siendo necesario cortarlo para emparejarlo, como muchas veces es preciso hacer cuando se trabaja con greda.

**Art. 8. Como otro preventivo para que ninguna parte de la soldadura penetre en los tubos**, se coloca un anillo de plomo en la junta antes de estopear el tubo. Este anillo de plomo es de un diámetro tal, que se le puede empujar exactamente por el espacio *d*, fig. 38, entre la boca y la campana, y de un tamaño suficiente para rodear exactamente el tubo de agua. Se le empuja lo más posible en el espacio angosto anular *oo*, fig. 38. Después se estopea y se vacía el plomo como de ostumbre.

**Art. 9. En la junta flexible de John F. Ward**, fig. 42, para tubos de hierro fundido que se colocan al través de los lechos irregulares de los ríos, etc., una parte, *ao*, de la campana B está torneada con exactitud para formar la zona media de una esfera cuyo centro es C; y el borde ó filo *mm* del lado exterior de la boca S se construye para formar una zona angosta, esférica, que corresponda con exactitud á la anterior. El plomo se echa cuando á cada dos tubos adyacentes se los hace descansar sobre una embarcación ó balsa para que estén en línea recta ó casi recta. El plomo ocupa el espacio *mn* pintado en negro y queda fijo en su lugar en la boca del tubo S dentro del espacio anular *dd*. Tan pronto como se llenan las juntas,

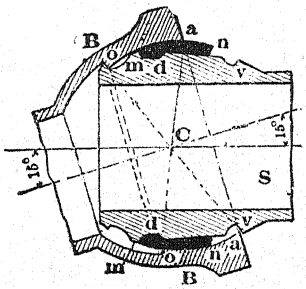
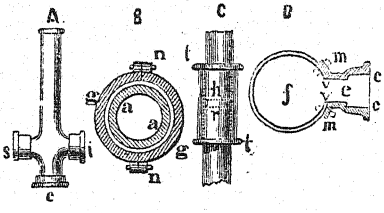



*Fig. 42*

las balsas se mueven hacia adelante, y los tubos que son de pequeñas dimensiones se sumergen sin más cuidado en el agua á poca profundidad. Un aparato apropiado se usa para sumergir los tubos grandes en aguas profundas sin que las juntas hagan gran esfuerzo. La junta permite una desviación de 15°; pero una desviación mayor expone á rajarse la campana y se la evita por los bordes *oo* de la campana y *vv* del macho. En algunos casos sería conveniente usar una draga para disminuir las irregularidades bruscas del suelo emparejando el fondo del río previamente. Más de

treinta tuberías provistas de estas juntas ó uniones, se han colocado con muy buen éxito, y con diámetros hasta de cerca de 1 metro.

**Art. 10.** En las figs. 43, A es un tubo en cruz; es decir, un tubo que tiene además de la campana *c*, en una punta, dos más *s*, *i*, en las cuales pueden conexionarse tuberías que conducen á direcciones opuestas, como sucede en las calles que se cruzan. Si se omite sea *s* ó *i*, el tubo se llama de **single brench** (adoptamos **ramal simple**). El tubo es más fuerte cuando las dos campanas *s*, *i*, se hallan cerca de sus extremos, que cuando están en el centro. En una tubería larga y para trabajar más ligero, se ponen frecuentemente á diferentes obreros á soldar tubos en diferentes partes de la línea, y cuando dos extremos de las tuberías que se vienen soldando se acercan á una distancia tan pequeña que no cabe un tubo completo para empatarlos, se unen como en *r* y *h*, fig. C, es decir, se usa un **sleeve** (adoptamos **tambor**) *tt* de hierro, el cual se pone primero sobre una parte del tubo, y (después de haber colocado el otro tubo) se corre hacia atrás hasta la posición que ocupa en la figura, dejando en su centro tapada la unión de los dos tubos. Los tambores se hacen generalmente de 30 cm de largo, del mismo espesor que el tubo; y su diámetro interior es suficiente para permitir la soldadura usual de estopa y plomo. La soldadura debe hacerse, por supuesto, en ambos extremos del tambor.




Figs. 43.

**Art. 11. Cuando ocurra una hendedura** en un tubo *aa*, fig. B, que ya está en uso, entonces se la compone por medio de *una brida* de hierro fundido *gg*, dividida en dos partes y asegurada con pernos en *mm*. Ésta se usa como el tambor que precede. El anillo blanco en la figura es la soldadura de plomo. Si la rajadura es demasiado grande ó en un estado que no puede remediarse con *una brida*, se rompe el tubo en pedazos y se derriten las juntas de plomo en las puntas, para que se puedan sacar. Entonces, como no se puede colocar un tubo nuevo entero, porque no cabe, se sustituyen con dos piezas cortas, una con un extremo macho y la otra con un extremo hembra. Una punta de cada pieza entra en los tubos ya colocados, mientras que las otras dos, que probablemente quedan apartadas algunos centímetros, se cubren con el tambor *tt*, fig. C.

Algunas veces se pueden componer provisionalmente las hendeduras envolviéndolas con lona enteramente saturada con pintura de albayalde; y bien apretadas al

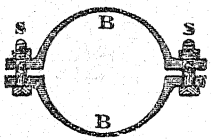



Fig. 44.

tubo por medio de flejes de hierro, delgados ó con alambre. O con una brida más angosta de hierro delgado, hecha de dos partes, BB, fig. 44, y unidas por pernos de tornillos, SS. Estos anillos son útiles también para reforzar tubos que se consideran en peligro de rajarse.

**Art. 12. Para conectar un tubo, e, fig. 43, á otro, f, que está ya en uso**, pero que no está dispuesto con tal objeto, se puede taladrar el tubo *f*, y fijar una pieza *e*, por medio de tornillos, en dos ó más taladros hechos en el tubo. Si el tubo nuevo es tan grande, que si se hace circular el taladro *vv*, resulte demasiado ancho, puede hacerse en *óvalo*, con su diámetro mayor en la dirección del *largo* del tubo *f*. En este caso la pieza *e* será oval en parte, pero redonda en *cc*.

**Art. 13. Válvulas de aire.** El aire es propenso á recogerse gradualmente en los puntos más altos de las curvas verticales de los tubos de alimentación; y si no tiene salida produce una obstrucción más ó menos fuerte en la corriente. Esto puede evitarse por medio de válvulas de aire, fig. 44 A. Este aparato consiste en una caja de hierro fundido *ccdd*, asegurada al tubo matriz *mm*, por medio de tornillos que pasan por las planchas *dd*. Tiene una tapa *gng*, asegurada por medio de tornillos *tt;* en la parte superior hay una abertura *n*, para la salida del aire de adentro. En esta caja hay un flotador *f*, que puede ser un envase cerrado de hojalata ó cobre, ó de

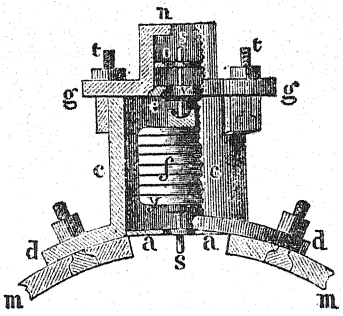



Fig. 44A.

capas de corcho, como se ve en la figura. Este flotador tiene un eje ó varilla *ss* que lo atraviesa y pasa por los orificios *aa* y *o* que permiten al flotador subir y bajar libremente; pero evitan que se mueva lateralmente. Cuando el tubo *mm* está vacío, el flotador está bajo y su base *y* descansa sobre *aa*. Una válvula *v* está fija al eje *ss*, y sube y baja con el flotador. Suponiendo que el tubo *mm* esté vacío, y por consiguiente el flotador y la válvula bajos, entonces, al entrar el agua en el tubo, ésta también sube á la caja y la llena hasta *e*, levantando el flotador y la válvula, obstruyendo la abertura *v* y evitando la salida al aire libre. Ahora, el aire llevado por el agua, por ser más liviano que ésta, sube naturalmente al punto más alto que encuentra. Por consiguiente, si este aire llega á la abertura *aa*, subirá por ella hasta llegar á *e*, donde la válvula cerrada le empide pasar más adelante. Así sucesivamente están subiendo partes de aire, que con el tiempo se acumulan en un grado tal, que comprime el agua hacia abajo y la hace salir de la caja. Sucediendo esto, el flotador que está sostenido por el agua solamente, también baja, y arrastra la válvula *v* hacia abajo. El aire acumulado se escapa instantáneamente al aire libre por las aberturas en *v* y *n*, y el agua en el tubo *mm*, sube instantáneamente otra vez á la caja, llevándose el flotador, y cerrando así nuevamente la válvula *v*. Esta válvula y su asiento *e*, están forrados de bronce, para evitar la oxidación y un mal ajuste. El todo está protegido por una tapa de hierro ó madera al nivel de la calle

**Ya en las poblaciones no se usan las válvulas de aire en los tubos**; las llaves de incendio á distancia de 150 m entre sí, más ó menos, las reemplazan. Estas llaves, colocadas, lo más posible, en las partes superiores de las curvas en la tubería, se usan para lavar las calles, y como se abren frecuentemente con tal fin, permiten también la salida del aire acumulado.

**La salida del aire comprimido por una válvula de aire, ú otro orificio, ha hecho reventar algunas veces los tubos matrices de las calles**; porque el escape es instantáneo, y las columnas de agua á ambos lados de la válvula se unen con gran fuerza chocando una contra otra y produciendo una reacción contra los tubos.

**Cámaras de aire.** Se da movimiento al agua en una tubería por medio de golpe, hacia adelante, del émbolo de una bomba de simple efecto; pero durante el movimiento del émbolo hacia atrás, el agua se detiene en su movimiento. Por lo tanto, el golpe próximo hacia adelante tiene que poner de nuevo en movimiento toda la columna de agua; y la fuerza que debe ejercerse por la bomba para producir este movimiento del agua, es mucho mayor que la que sería necesaria si el movimiento anteriormente producido se hubiese conservado durante el golpe de émbolo hacia atrás. La adición de una cámara ó depósito de aire asegura la conservación de este movimiento, y por lo tanto produce una gran economía de fuerza; además de que disminuye el peligro de que se revienten los tubos con tantos golpes de ariete. La cámara de aire consiste solamente en una caja de hierro alta y fuerte, herméticamente cerrada, asegurada fuertemente por medio de pernos á la parte superior de los tubos, colocada en la inmediación y más adelante de la bomba, y comunicándose libremente con los tubos por medio de una abertura en su base. Está llena de aire. El golpe hacia adelante de la bomba no solamente mueve el agua á lo largo de los tubos, sino también la hace entrar en la parte inferior de la cámara de aire por una abertura en la base de ésta, comprimiendo así el aire que contiene. Pero durante el golpe hacia atrás, este aire comprimido, libre ya de compresión por la bomba, se dilata y ejerce presión sobre el agua en los tubos, manteniéndola en movimiento hasta el próximo golpe hacia adelante; y así se continúa. Una cámara de aire también actúa como un *colchón de aire* (preferimos como un resorte), que permite al émbolo aplicar su fuerza al agua gradualmente, preservando así tanto á los tubos como á la bomba de choques violentos. Sin embargo, el aire en la cámara es absorbido y arrastrado gradualmente por el agua, y entonces cesa su acción como regulador. Para evitar esto, debe introducirse más aire en la cámara, de tiempo en tiempo, por medio de una bomba de aire impelente. En una bomba de *doble efecto* no es tan indispensable la cámara de aire como en una de simple efecto. No hay regla especial para el tamaño ó capacidad de las cámaras de aire. En la práctica parece variar de 5 á 50 veces el de la bomba; con una altura igual á 2 ó tres veces el diámetro.

Se usa algunas veces **un tubo vertical** en lugar de la cámara de aire (véase arriba). Es un tubo alto, abierto al aire en su parte superior, y en libre comunicación con los tubos de agua en su parte inferior, lo mismo que la cámara de aire. La parte superior ha de estar **más alta que el punto más alto** donde la bomba ha de llevar el agua por la tubería; porque de otro modo el agua se derramaría por la parte superior. El área de su sección transversal debe ser *á lo menos* igual al área del tubo, ó de los tubos que siguen y conducen el agua; pero es algunas veces mejor darle mucho mayor diámetro, porque un tubo vertical abierto, si la bomba deja de trabajar por algunas horas, puede servir entonces (y especialmente en una población pequeña) *como un estanque*. El tubo vertical debe ser cilíndrico, y no cónico; porque si se forma hielo grueso sobre el agua en uno cónico, una presión repentina hacia arriba producida por la bomba puede romperlo. Los tubos verticales en conexión con las tuberías de agua, en Filadelfia, son de una altura de 37 á 52 metros, y de 1.50 m de diámetro; están hechos de hierro de calderas remachado, de un espesor de 1 cm cerca de su base, y de ½ cm en su parte superior. No están protegidos contra la inclemencia del tiempo, ni están provistos de vientos, sino que se mantienen en su posición por su propio peso, aunque á veces están expuestos á vientos fuertes.

**Art. 14. Los tubos para surtir las habitaciones** se hacen de plomo y de un diámetro interior de 1 á 1½ centímetros. Están en conexión con los tubos

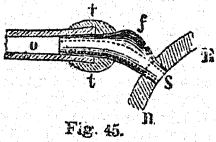


Fig. 45.

grandes de las calles *nn*, fig. 45, por medio de un **suncho ó tubo corto de latón** dibujado aquí como en ¼ parte de su tamaño natural. Las líneas de puntos muestran su diámetro, de 1 centímetro más ó menos. Este suncho cónico entra á martillo en un orificio cilíndrico del tubo principal en *s*. El tubo de plomo *o* está soldado en *t* á la otra punta del suncho y metido como 3 ó 4 centímetros. Con el

espesor mayor dado al suncho, en *f*, resulta de una forma especial y permite hacerlo entrar á martillo en el tubo mayor. El tubo y la soldadura se ven en sección longitudinal. Además de la llave de retención empatada con cada tubo de servicio, y á sus ramificaciones por toda la casa, hay otra llave subterránea, con la cual las autoridades municipales pueden quitar el agua en caso de que no paguen los derechos; y otra más, por la cual el latonero puede quitar el agua durante las reparaciones en los tubos del interior de la casa. **Los tubos de hierro galvanizados** se usan mucho para el servicio de las casas, especialmente para agua caliente, porque están menos sujetos á la dilatación y contracción que producen filtraciones.

**Art. 15. Las llaves de la Compañía de Aguas** se conectan en el tubo por una máquina especial, fig. 46. La gran ventaja de aquellas llaves sobre el suncho,

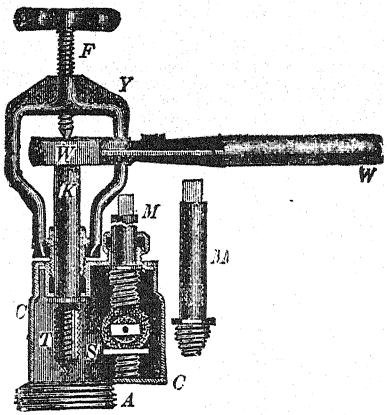



Fig. 46.

fig. 45, **consiste en que pueden conectarse en un tubo cuando éste. está lleno de agua y con presión.** Además, como ellas entran en el tubo por una rosca de tornillo, no tienen peligro de ser empujadas hacia afuera por la presión del agua. Están provistas de una llave de retención, la que se cierra mientras la llave de la toma se conecta en el tubo; y después se deja generalmente abierta.

**Las máquinas para taladrar y enroscar los taladros en los tubos,** y para insertar estas llaves, se hacen de varias formas. La de la fig. 46 es una fabricada por **Wálter S. Payne and Cy.**, Fostoria, Ohio. Cada una de estas máquinas trae un número de planchas de hierro fundido maleable que no se ven en la fig. y que sirven para los diferentes diámetros de los tubos con los cuales puede usarse. Se asegura esta plancha al tubo por medio de una cadena puesta alrededor de él, y se aprieta la cadena con perno y tuerca.

El cilindro de bronce ó latón grueso CC (en el cual se inserta primero el macho de la terraja del taladro T, y la llave S) se asegura entonces en la plancha por medio del tornillo A. La llave está atornillada provisionalmente á un mandril M. Este mandril y el eje del taladro K pasan por cajas de estopas (de hierro fundido). Por medio de una manigueta, no mostrada en la figura, la cabeza del cilindro da vuelta (mientras el cilindro mismo queda fijo) hasta que el taladro T y la llave S, están en sus puestos respectivos, como se ve en la figura. Cuando la cabeza del cilindro llega á su puesto, se para por medio de una uña en el interior del cilindro. El taladro se halla ahora inmediatamente sobre el centro de un gran orificio circular en la base del cilindro CC, y sobre otro orificio semejante que tiene la plancha que va sobre el tubo en el lugar donde se quiere taladrar. Se empuja hacia abajo hasta tocar dicho tubo. La *carraca* WW se pone ahora sobre la cabeza cuadrada del taladro K; y el marco Y con su tornillo F se ponen en la posición que indica la figura; moviendo ahora la *carraca* se taladra el tubo y se hace la rosca. Si el agua en el tubo tiene presión, sale del tubo por el taladro hecho y llena el cilindro.

Si ahora se cambia la posición del fiador de la carraca W y se vuelve á trabajar con ella, el macho del barreno sale ahora del orificio, pero se queda en el cilindro. Ahora se da vuelta á la cabeza del cilindro para cambiar las posiciones de S y T; una agarradera en el interior del cilindro impide á la cabeza dar más vueltas que las requeridas para poner la llave inmediatamente encima del taladro hecho. Por medio de la carraca, puesta ahora sobre la cabeza cuadrada del mandril M, la llave S (*cuya válvula debe estar cerrada*) se atornilla en el orificio del tubo, pero sólo lo suficiente para evitar que siga saliendo el agua del tubo, cuando se quita la máquina. Ahora la llave se atornilla firmemente por medio de una manigueta que encaja sobre la cabeza cuadrada de la misma llave.

El mandril M se hace de dos partes (que se atornillan una en la otra) para que su parte superior pueda quitarse y no estorbe á la carraca mientras se taladra. Tiene dos ó tres roscas diferentes, para servir á diferentes tamaños de llaves. Se suplen también diversas llaves para usar con la misma máquina.

La máquina puede trabajar en cualquiera dirección, y por consiguiente usarse para taladrar el tubo en cualquier punto de su circunferencia.

Después de conectada la llave, el tubo de servicio se asegura á ella por medio de una tuerca de empate con la rosca, como se ve en la figura.

**Art. 15a. La cúpula neumática**, figs. 46*a* y 46*b*, inventada por Mr. N. Monroe Hopkins, de Wáshington, son para evitar que se revienten los tubos al helarse el agua.

En tubos no protegidos, el agua al congelarse es expande transversalmente y revienta el tubo. La cúpula colocada en el tubo, como se ve en la fig., y á inter-

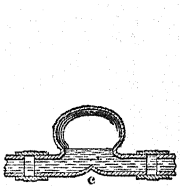



Fig. 46 *a*. Fig. 46 *b*.

valos de 3.60 m, permite la expansión del hielo hacia cada cúpula, donde comprime el aire allí encerrado. En la cúpula horizontal, los dos planos inclinados, *c*, en su parte inferior, compelen á las dos columnas de hielo hacia la cúpula, evitando que se compriman entre sí.

Con el objeto de asegurarse de que las cúpulas en un sistema de tubos (como los de una casa, industria ó puente) no se queden sin aire por la acción de la corriente, se coloca después de ellos un aspirador á la entrada del sistema; el aspirador consiste esencialmente en una contracción del tubo, que aumenta la velocidad del agua en ese punto y produce una succión del aire á través de una válvula colocada con ese objeto (véase medidor Venturi, pág. 566). El aire así introducido, es llevado á lo largo del tubo, en burbujas, entre la superficie del agua y la parte superior del tubo, las que son retenidas por las cúpulas. Cuando, al cerrar una llave, etc., se detiene la corriente, el exceso de presión cierra la válvula. Se han hecho ensayos muy estrictos en tubos anchos y estrechos (10 cm y 18 mm) protegidos por estas cúpulas, y siempre han resultado eficaces para evitar la ruptura.

**Art. 16. Las llaves de retención** que se abren verticalmente, figs. 47 y 48, se colocan en la tubería de la calle á distancias de 100 á 300 mets unas de otras. Se usan para quitar el agua de cualquiera sección durante las composiciones ú otros casos; y el agua de estas secciones se bota por tubos de desagüe. Hay mucha diferencia en los detalles de unas á otras según los diferentes fabricantes.

Las figs. 47 y 48 muestran una llave hecha por *Chapman Valve Mfg Co., Indian Orchard, Mass.* La válvula *v* está fundida en una sola pieza. Estando baja, como en las figs., cierra el tubo. Como en otros modelos, se abren verticalmente por medio de un tornillo D que hace penetrar la válvula á la caja de hierro fundido BB, dejando, cuando se la hace subir, una abertura de todo el diámetro del tubo. El tornillo se mueve por medio de una llave de mano que se ajusta á su cabeza cuadrada *h*. El tornillo D no puede moverse verticalmente porque se lo impide el reborde ó collar C.

Las dos fundiciones principales que componen la caja ó la cubierta están unidas

por pernos que atraviesan una plancha saliente, *g*. Las caras de unión de las fundiciones son muy lisas, y se inserta una tira delgada de plomo entre ellas para evitar

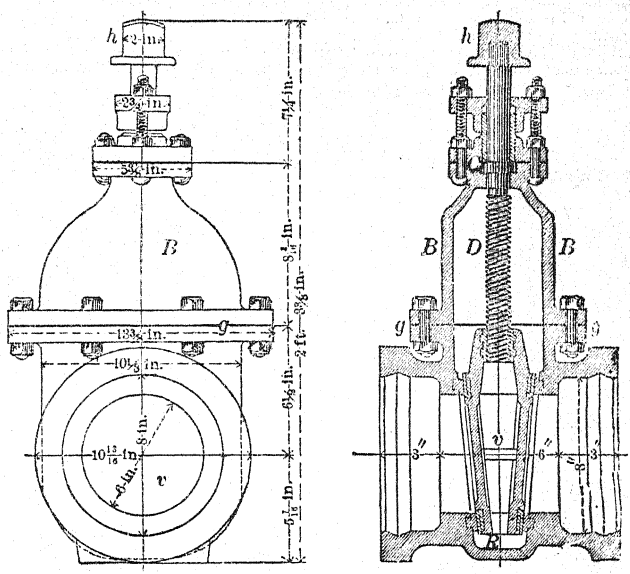



**Fig. 47.** Fig. 48.

(*N. del T.* — Para las medidas sobre la figura, recuérdese que una *in* (inch = pulgada) equivale á 25.4 mm y un *ft* (foot = pie) equivale á 305 mm.)

las filtraciones. La cavidad R admite pequeñas partículas de materias extrañas que de otro modo podían llegar á impedir que la válvula cerrara perfectamente. Los asientos de la válvula están revestidos con metal de *Babbitt* *. En la parte superior de la caja, el mango del tornillo pasa por una caja llena de estopa, que evita la filtración del agua en este punto.

(*N. del T.* — La tabla que sigue es la que trae el autor, convertida al sistema métrico.)

### Llaves « punta de campana » de Chapman.

| Milímetros. | Kilogr. | Milímetros. | Kilogr. | Milímetros. | Kilogr. | Milímetros. | Kilogr. |
|---|---|---|---|---|---|---|---|
| 50.80 | 14.515 | 152.4 | 88.449 | 304.8 | 272.148 | 508.0 | 771.086 |
| 76.20 | 24.928 | 177.8 | 111.727 | 355.6 | 382.367 | 600.6 | 1247.345 |
| 101.60 | 50.234 | 203.2 | 132.139 | 406.4 | 489.866 | 762.0 | 2902.912 |
| 127.0 | 61.234 | 254.0 | 199.124 | 457.2 | 669.030 | 914.4 | 3764.714 |

* *N. del T.* — Compuesto de 50 partes de estaño, 1 de cobre y 5 de antimonio.

**Art. 17.** En la fig. 49 se ve un aparato de estos **con el tornillo exterior** para subir y bajar la válvula. Aquí el tornillo D no da vuelta si no está asegurado á la válvula, y sube y baja con ella movido por la rueda W, en cuyo centro se halla una tuerca por la cual pasa aquél. La tuerca está fija en la rueda, y está dispuesta para que no pueda moverse verticalmente.

**Art. 18. Una llave de cuatro tubos** ó **cuatro vías**, figs. 50 y 51, se coloca en la intersección de dos tuberías matrices, cuyos cuatro extremos se aseguran

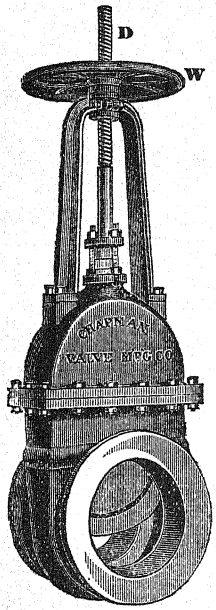



Fig. 49

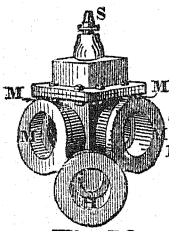



Fig. 50

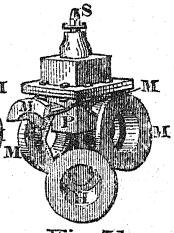



Fig. 51

respectivamente á las cuatro aberturas MMMM. En el fondo hay otra abertura que está en conexión, por medio de un codo H, con tubos que van á una boca de agua para incendios situada al lado de la acera. Véanse los arts. 20 y 21. Dos ó más aberturas pueden mandarse hacer, si se quiere; para suplir tantas bocas de agua como sean necesarias. Todas estas aberturas se abren y se cierran al mismo tiempo, subiendo ó bajando la válvula compuesta P, por medio de una llave aplicada á la cabeza cuadrada S del tornillo. Lo mismo que en las figs. 47 y 48, da vuelta el tornillo pero sin subir ni bajar, y la llave sube y baja en el tornillo mismo.

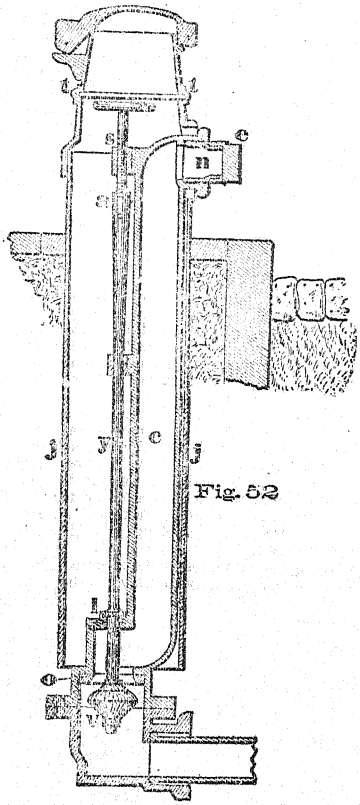
Como todo sedimento se escapa por la abertura en el fondo, que conduce á la boca de incendio, la llave no está expuesta á ser obstruída por esta causa. La boca para incendios, como está alimentada por ambos tubos matrices, obtiene una presión de agua mayor de la que se lograría, si estuviese alimentada por un tubo solamente.

**Art. 19.** Cualquiera que sea la forma ó clase de llave, está protegida, en su unión con el tubo, **por una caja que la contiene**, hecha generalmente de madera ó de hierro fundido, cuadrada, de sección horizontal menor arriba que abajo. La caja está abierta en el fondo, pero tiene una tapa movediza de hierro al nivel de la calle. Se quita la tapa cuando se quiere abrir ó cerrar la llave ó inspeccionarla. Dos de los lados opuestos de la caja tienen, por supuesto, aberturas para el pasaje y conexión de los tubos á la llave. Deben cerrarse muy lentamente, sobre todo las grandes,

de otro modo la violenta detención de la corriente en los tubos expondría á éstos á reventarse y á las llaves mismas á romperse. Como precaución contra esto, las cajas que cubren las válvulas se refuerzan con unas molduras salientes que se hacen en la fundición.

Ya no se usan las llaves de aire automáticas, fig. 44 A, en las partes altas de la tubería para dejar salir el aire. Las bocas de agua para incendios suplen su efecto. El radio para curvas horizontales en tubos matrices, no debe ser menor, si es posible, de 12 veces el diámetro del tubo; se les da el mayor radio que permite el ancho de la calle; generalmente de 15 m. Las bocas para incendios, figs 52, etc., se colocan, lo más posible, en las partes más altas de las calles, para que sirvan también para lavar las calles y para el escape del aire acumulado; el término medio son 8 para cada 2 kilómetros de tubo, ó una para cada cuadra edificada.

**Art. 20. La fig. 52 representa una boca de agua para incendios.**



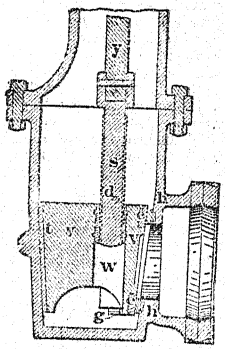



Fig. 53

La válvula *v* se hace de suela bien comprimida á martillo; y cuando se cierra, se ajusta contra un asiento de bronce ó latón en forma de anillo, que está asegurado en su puesto por una soldadura de plomo. Se abre la válvula haciéndola bajar por

el tornillo *s*, el cual, por medio de una articulación giratoria en *a*, puede dar vueltas sin que gire el eje de la válvula *y*. Si la válvula *v* está cerrada, después de haberse usado la llave de incendio, la cámara *c* queda llena de agua, expuesta á congelarse con el frío y romper el aparato. Pero, cerrando la válvula, levantamos el pestillo *l*, del eje, y de este modo se escapa el agua por la abertura en *l*, y se pierde corriendo por el suelo, por la parte inferior del cilindro vacío *jj* que envuelve el aparato.

La parte superior *t* de la caja de la boca de incendio está fundida en una sola pieza con la cámara *c*.

La tapa *e* se atornilla sobre la cajeta *n*.

**Art. 21. En las bocas de incendio de Chapman**, figs. 53, 54 y 55, hechas por *the Chapman valor Co*, **la válvula** *v es corrediza*. El vástago *y*, fig. 53, en que está fijo el tornillo *s*, no puede ni subir ni bajar á causa de un collar fijo en su parte

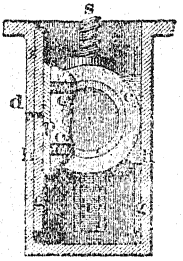



Fig. 54.

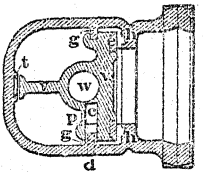



Fig. 55

superior, y confinado en una ranura circular. Cuando se da vuelta al vástago y al tornillo, la válvula se mueve hacia arriba y hacia abajo sobre el tornillo, admitiendo ó excluyendo, de este modo, el agua en la boca de incendio. La válvula corre sobre dos guías *gg*, fundidas en una sola pieza en los dos lados *verticales* del aparato. El borde circular, en el contacto con la caja del aparato, está cubierto con un anillo de bronce *ee*, que descansa contra un anillo igual, hecho de metal de *Babbitt*, embutido en la caja del aparato.

**El agua** que queda en la caja de la boca de incendio después de cerrarse la válvula, **se sale** por un agujero cilíndrico *d*, de 1 cm más ó menos, taladrado al través de la guía *g* y la caja *h*. Este agujero está á una altura tal, que quede exactamente sobre la parte superior de la plancha suelta *p*, cuando la válvula está cerrada como en la fig. 53. Esta plancha está en una ranura vertical, á un lado de la fundición de la válvula, y está comprimida contra el lado de la caja *h*, por dos resortes espirales, confinados en dos concavidades *cc*, de la fundición de la válvula. Cuando la válvula empieza á subir, el agujero, *d*, se cierra por la plancha *p*, y queda así hasta que la válvula se cierra de nuevo enteramente.

**Taladros de ensayo.** Las figs. 1 y 2 muestran un instrumento para **taladrar tierra, greda, arena y granzón**, aunque estén muy duros y aun helados. No

Fig. 1.

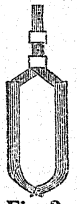


Fig. 2.

perfora roca dura ni piedras grandes sueltas. Se compone de dos segmentos cilíndricos de planchas de hierro SS llamadas « vainas », y cuyas aristas cortantes

inferiores están guarnecidas de acero. Estas aristas sobresalen (como se ve en la figura) á los lados del taladro, y hacen el pozo más ancho que el taladro mismo, de modo que éste no se atraque en él. Las dos aristas cortantes están equidistantes de la línea central del instrumento, y esto asegura una perforación recta y vertical. En *a* el taladro está sujeto á la extremidad inferior de una barra vertical de taladros compuesta de un número de barras de hierro de 37 mm en cuadro, ó tubos de 62 mm y de 3 á 4.5 m de longitud, unidos en sus extremidades por medio de un encaje cuadrado. En la parte superior de esta barra hay un gancho con movimiento giratorio, por el cual el aparato entero está colgado en un mecate, que pasa por encima de una polea en la parte superior de una grúa ó trípode y hacia abajo á un tambor movido por un torno y un engranaje. Por medio de este torno y del mecate se suspende el taladro y su barra (la cual al principio tiene una sola pieza) sobre la perforación que se hace. Luego se baja dicho taladro hasta el suelo; y por medio de una palanca fija á la barra, dos hombres ó un caballo le imprimen al taladro un movimiento de rotación horizontal. El gancho giratorio situado en la parte superior de la barra permite esta rotación sin que se tuerza el mecate. La forma del taladro es tal, que este movimiento rotatorio lo hace penetrar en la tierra á manera de tornillo, y el hombre que maneja el torno durante la perforación, no tiene sino que mantener el mecate tenso, para que de este modo impida que el taladro penetre demasiado ligero y se atraque. Con ocho revoluciones más ó menos se llena de tierra el taladro. Entonces se levanta por medio del torno á un metro más ó menos sobre la superficie, y soltando y quitando entonces el anillo *b*, el taladro se abre como unas tenazas, y cae la tierra extraída en un cajón de madera, que se ha colocado encima de la perforación. Después se sigue taladrando como antes. Cuando la perforación haya llegado á una profundidad de 3 m más ó menos, debe agregarse una segunda barra en la parte superior de la primera. Para hacer esto, el taladro, y la primera barra se levantan algunos centímetros, se coloca en el suelo un ligero andamio de tablas alrededor de la barra y luego se fija á ésta un reborde que la sostiene en el andamio, mientras se quite el gancho giratorio y se agrega la segunda barra, en cuya punta superior se fija el gancho giratorio, y clavando ésta hasta que su extremidad inferior puede asegurarse á la superior de la primera barra. Hecho esto, se estira el mecate, se baja el taladro al fondo de la perforación y se principia nuevamente. Del mismo modo se añaden las barras necesarias de tiempo en tiempo, según lo requiera el descenso del taladro.

Estos taladros se hacen de 15 á 20 cm de diám, ó más grandes por encargo especial. Si se quiere, puede hacerse la perforación de 60 á 90 cm de diámetro, fijando al taladro un ensanchador. Este taladro hará perforaciones hasta de 30 m ó más de profundidad, á razón de 1.20 á 6 m por hora. Saca piedras del tamaño de su semidiámetro. En terrenos secos se vacía un balde de agua en la perforación cada vez que se saca el taladro.

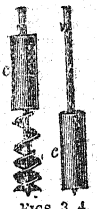
Puede emplearse ventajosamente en la perforación de los hoyos para **pilotes**, y algunas veces, **en lugar de hincar las estacas de madera** con el martinete, puede ser mejor enterrarlas (con la punta cuadrada hacia abajo, si se prefiere así) en perforaciones hechas con este instrumento, apretando la tierra alrededor de ellas después. Así se librará á las casas contiguas y cercanas, de la vibración y perjuicio producido por un martinete.

Si se encuentra **arena, fango, ó granzón suelto** al hacer la perforación, se ensancha aquélla 10 cm más y se colocan tablas forrando la excavación, las cuales se hacen penetrar á través de la arena ó granzón, y luego se extrae esta arena ó granzón por medio de una **bomba de arena**. Esta bomba se compone de un cilindro de hierro hueco de 12½ cm de diám por 75 cm de largo, más ó menos, con una válvula en su pie, que se abre hacia arriba. Este cilindro se baja hasta el fondo de la perforación; se cubre con agua hasta .60 á 1.20 m y se le da con la mano un movimiento rápido hacia arriba y hacia abajo, de 10 á 15 cm de amplitud y por 20 ó 30 veces, durante las cuales la bomba se llena de arena, y se saca entonces fuera De 3 á 6 m de altura de fango, ó arena, etc., pueden extraerse por hora de una perforación de 15 á 45 cm de diámetro. La bomba se usa también para extraer tierra, etc., de una perforación hecha en un suelo muy compacto. **El precio** de un taladro perforador con su cabrestante, barrenos, mecates, bomba de arena, etc., etc., completo, es más ó menos $175. El taladro pesa de 67 á 90 kg, según el tamaño. Los barrenos de 37 mm en cuadro, 5 kg por m. El cabrestante, 70 kg.

El perforador de arena, figs. 3 y 4, lo mismo que la bomba de arena descrita, se emplean entre tubos, con el mismo fin. El cilindro hueco de hierro C, de 25 cm de diámetro × 75 cm de largo, se desliza verticalmente en la barra. Al taladrar, la arena que está debajo y la que rodea al cilindro mantiene á éste en la posición

indicada en la fig. 3. Seis revoluciones de la barra y tornillo llenan el cilindro de arena. Luego se levanta la barra. Ésta hace entrar primero el tornillo dentro del cilindro como en la fig 4, y una válvula situada en el pie del tornillo cierra el fondo del cilindro, é impide que la arena se salga cuando se saca el taladro. La barra es un tubo abierto en ambas extremidades. Esto permite la introducción del aire libre evitando la resistencia producida por la sección al sacar el taladro. A éste se le comunica movimiento de rotación y se saca del mismo modo que el taladro de tierra descrito arriba. Precio, $30.

También se **usa un taladro** de acero de 5 á 10 cm de diám, para reconocimientos; tiene .60 m de longitud, sirve para perforaciones de 8 á 15 cm de diám y de 3 á 15 m de profundidad. Perfora greda, arena ó granzón fino, y saca muestras de todas estas substancias. Se le da vueltas por medio de llaves ó palancas, con fuerza de hombres ó de caballo. Véase también pág. 728.



FIGS 3, 4.

FIG 5.

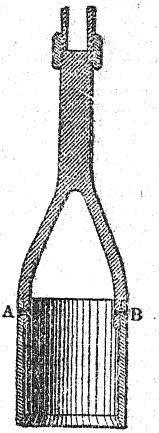


FIG 6.

**La perforadora cuya sección vertical se ve en la fig. 6,** y su sección transversal horizontal en la fig 5, es muy útil para **hacer á la mano perforaciones de poca profundidad á través** de greda, granzón, etc. El taladro mismo se compone de un cilindro de acero de resortes, de 7 á 10 cm de diámetro y de 10 á 13 cm de alto, con sus lados de 6 mm de espesor, con una abertura vertical (véase la fig. 5) en toda su altura, cortada al sesgo en el pie, para producir un filo cortante, como se ve en la fig. 6. En la parte superior está remachada ó soldada una pieza de hierro forjado en forma de V invertida, la cual, por medio de un encaje que tiene en su extremidad superior, se atornilla á un tubo de gas que le sirva de mango, y al cual se agregan otros, según vaya haciéndose la perforación.

Ésta se hace con dos hombres que agarran el mango, y teniendo verticalmente el instrumento, lo hacen entrar en el suelo levantándolo y dejándolo caer repetidas veces en el mismo lugar. A medida que el instrumento golpea el suelo, la forma sesgada de su filo cortante lo hace abrirse ligeramente, y cuando al levantarlo se suprime la presión hacia abajo, recupera su forma anterior cerrándose lo que se había abierto y recoge la tierra que entra en él. El instrumento se llena pronto, y los hombres, viendo que ya no entra con facilidad, lo sacan fuera para vaciarlo. La calidad de su contenido á diferentes profundidades (medida ésta á lo largo del mango), se anota.

En seis días de á ocho horas 3 hombres (uno de ellos descansando á intervalos) empleando uno de estos taladros, hicieron 20 perforaciones de 2.80 m de profundidad media, en marga, granzón greda y esquisto micáceo descompuesto, al costo de 72 centavos por metro. Salario ó jornal por día y por hombre, $2

Para trabajos en marga, greda ó arena no movediza, cualquier herrero bueno puede hacer un **taladro de tornillo que sirva** con sólo darle la forma de un

tirabuzón de .60 m de largo y con 6 vueltas completas de 15 cm diám á una barra de hierro ó acero de 2½ cm en cuadro, con su parte inferior formada de un filo cortante vertical, que sobresalga más ó menos 10 mm de la espiral del tornillo, para disminuir el frotamiento. Este instrumento sacará muestras completas, pero requiere un torno ú otro modo sencillo de levantarlo, cuando el tornillo está lleno.

## POZOS ARTESIANOS

Las perforaciones verticales profundas, de 15 á 20 cm diám, como se requieren para pozos artesianos ó para pozos de aceite, y exploraciones de minas, se hacen levantando y dejando caer repetidamente un barreno ó mecha pesada de hierro, fig. 1, pág. 727, con un filo cortante de acero. Después de cada golpe de la mecha se le da á ésta un pequeño movimiento horizontal de rotación, para hacer que la perforación sea redonda. El largo de la arista cortante de la mecha es algo mayor que el diámetro de ésta, y de este modo se hace la perforación suficientemente grande para impedir que la mecha se atraque. La mecha es la última pieza que figura en la serie de barras de hierro y acero, etc., unidas con tornillos por sus extremidades. Este conjunto de útiles varía en longitud de 7 á 18 m, según el tamaño y profundidad del pozo, la dureza de la roca; su diámetro general (más arriba del filo cortante) es de 2 á 5 cm menos que el del pozo. Su peso es de 360 á 1,800 kg. La pieza superior es siempre el portacuerda (véase la fig. 4), al cual se sujeta la punta inferior del **cable**. Este cable pasa al salir del orificio á una **palanca horizontal**, la que, por medio de una máquina de vapor ó fuerza animal, se tiene constantemente en movimiento alternativo de arriba abajo. Todo con el filo cortante de la mecha en la extremidad inferior, se levanta y se deja caer alternativamente de .60 á 1.20 m de alto y de 30 á 50 veces por minuto; de este modo se perfora la roca ó la tierra. Se conserva constantemente el hueco con 1 á 3 m de agua para facilitar la perforación y la extracción del material triturado. Después que la perforación llega donde hay agua, se puede continuar taladrando, aun cuando se llene de agua el pozo; pero una altura grande de agua disminuye la intensidad de la *fuerza de los golpes*. Debe darse una disposición conveniente para aflojar el mecate á medida, que el instrumento desciende. Una pinza horizontal está fija al mecate, y el hombre encargado de manejarla, dándole vuelta, tuerce el cable, y de este modo **da un movimiento de rotación horizontal** á la mecha, igual, más ó menos, á la quinta parte de una revolución entera, después de cada golpe, hasta que se hayan hecho de 6 á 8 revoluciones enteras en una sola dirección. Entonces invierte el movimiento, y hace un número igual de revoluciones, en el mismo tiempo, en dirección opuesta.

Después de haber barrenado á algunos metros de profundidad, se saca todo fuera del pozo, por medio del cable, para poder *extraer el material triturado* que se ha acumulado en la perforación. Esto se hace por medio de una **bomba de arena.**

**Los pozos se hacen generalmente de 15 á 20 cm de diámetro.** Para diámetros menores de 15 cm, los instrumentos son tan delgados, que están expuestos á romperse dentro de un pozo hondo.

El mismo aparato se usa **para perforar la tierra encima de la roca** antes de que se llega á esta última. Esto se llama « escardar » (*spudding*). En este caso debe impedirse que las paredes del pozo se derrumben. Al efecto se coloca un tubo de hierro forjado de un diámetro tal, que ajuste bien en la perforación y de 6 mm de espesor, y se empuja hacia abajo de tiempo en tiempo á medida que avance la excavación. El tubo se hace bajar por medio de un mazo pesado de roble, ó de otra madera dura, de 35 ó 45 cm en cuadro y de 3 á 4.5 m de largo. Este mazo se fija á una extremidad de la punta inferior del mismo cable que durante la perforación sostiene el instrumento. Varias veces se levanta y se deja caer sobre la cabeza del tubo, el cual está protegido por un **casquete** de hierro fundido. El pie del tubo está provisto de un anillo de acero con filo cortante. Cuando el tubo ha penetrado lo más que puede, se retira el mazo de la punta del cable, se sustituye por el instrumento y se principia nuevamente á perforar dentro del tubo.

Dicho tubo está formado de porciones de 2.40 á 5.40 m de longitud, y la perforación del pozo y colocación de aquél se efectúan alternativamente hasta que se llegue á la roca y el pie del tubo haya penetrado algunos centímetros dentro de ésta; lo suficiente para excluir la arena movediza ó el agua de la superficie.

**Si se encuentra arena movediza**, se saca el instrumento y se emplea la bomba de arena dentro del tubo.

**Para ensanchar las perforaciones ó para enderezar** las que estén torcidas, etc., se reemplaza la mecha por instrumentos especiales, tales como ensanchadores, etc.

Es necesario tener especial cuidado en aceitar bien **todas las superficies de rozamiento.** La polea situada en la cabeza de la grúa y las ruedas del piñón de la máquina deben aceitarse bien cada dos ó tres horas.

**En tiempo muy frío ó muy húmedo debe construirse un caney de tablas ordinarias,** ó un cobertizo de lona, de 2.4 m de altura más ó menos, para proteger los hombres contra la intemperie; y si se emplea máquina de vapor, debe cubrirse la correa con 2 ó 3 tablas que le sirvan de techo; pues cuando se moja se resbala.

La descripción que sigue está basada en las máquinas construídas por la Pierce Well Supply Co.

**Para perforaciones de 60 á 300 m de profundidad,** se usan **máquinas de taladrar portátiles,** que se pueden maniobrar con fuerza de vapor ó de sangre. En estas máquinas, el cable se extiende desde el instrumento y fuera del pozo, hasta pasar sobre una garrucha colocada en la cabeza del árbol de madera; desciende otra vez, y de allí, por intermedio de una polea fija al pie del árbol, va á un cilindro donde está enrollada. A este cilindro está fija una rueda dentada y una palanca para aflojar el cable á medida que desciende el aparato.

**El árbol tiene un gozne** á la altura de 1.8 m de su extremo inferior, de modo que su parte superior puede ponerse horizontal, cuando se traslada la máquina de un punto á otro. Cuando la máquina está trabajando, el árbol está sostenido en su posición por puntales de madera, sujetos con pernos en su parte superior, y sus extremidades inferiores fijas al burro de taladrar, que se compone de un andamio fuerte, de 2.70 m de largo, .90 de ancho y 1.2 de alto, colocado al pie del árbol, el cual contiene la palanca que suspende y deja caer el cable, el cilindro en que está enrollado el cable, el eje y la excéntrica que maneja la palanca, etc. El operador permanece parado al pie del árbol, y por medio de palancas de pie y de mano á su alcance, regula todos los movimientos de la máquina. Una de estas palancas gobierna la rueda de retención que regula el desarrollo del cable. Imprimiendo á la rueda de engranaje un movimiento de un diente á otro, baja el cable y la mecha 6 mm.

El operador, moviendo una corredera con su pie, aprieta hacia abajo la palanca principal, fuera del alcance de la excéntrica, y detiene de este modo el movimiento del cable y del perforador. Por medio de otra palanca puede engranar el cilindro del cable al eje motor principal, para que el cable se arrolle en el cilindro, y levante el perforador y lo saque del pozo. Otra palanca mueve un tambor pequeño, sobre el cual está enrollado un mecate liviano, que sujeta la bomba de arena. Todas estas operaciones se ejecutan por la misma fuerza (caballo ó vapor, según el caso) que trabaja sin interrupción; haciendo el hombre diferentes cambios para engranar y desengranar las diversas partes de la máquina con el eje motor principal, etc.

**Una de estas máquinas portátiles requiere** dos caballos ó un pequeño motor de vapor, un hombre para atenderla y otro para manejar la máquina, vaciar la bomba de arena, cambiar las piezas del taladro, etc. La máquina puede llevarse en una carreta de campo por cualquier camino ordinario. Dos hombres la pueden descargar, armar y principiar á perforar en dos horas; y á menos que se prefiera la fuerza de vapor, los dos caballos empleados en su transporte pueden suplir la fuerza motriz. La máquina puede desarmarse y meterse otra vez en el carro en el término de dos horas. Las figs. 1 á 4 muestran **los útiles empleados con estas máquinas.**

La fig. 1 es la **mecha de taladrar** (escariador) (*drilling bit*). El filo tiene 15 cm de largo y la mecha de 75 á 90 cm, y pesa 45 kg más ó menos. Su parte superior se atornilla en el pie **del mango de taladro,** fig. 2, que se hace de hierro redondo de 7½ cm diámetro, de 3.6 m de largo y pesa 150 kg. Su aplicación es la de un peso, que comunica una fuerza adicional á los golpes de la mecha.

Su parte superior se atornilla en el pie de la sonda de percusión (*drill-jars*), fig. 3, cuya parte superior se atornilla al cable. **Si la mecha se atraca por cualquiera causa dentro del pozo,** el operador detiene el movimiento del instrumento y afloja el cable, 30 cm más ó menos. Esto permite que el eslabón superior U de la sonda de percusión, fig. 3, se deslice hacia abajo en la ranura S inferior. Principiando entonces nuevamente con el movimiento de vaivén, el golpe adicional producido por la pieza U contra la parte superior del eslabón S, afloja el aparato.

Estas máquinas se hacen de diferentes tamaños para perforaciones hasta de 60 á 300 m de profundidad. El juego de instrumentos pesa de 360 á 1,000 kg, y la

máquina completa, incluyendo instrumentos, cable, árbol, etc., de 1,000 á 2,000 kg. Su precio es de $700 á $1,500, no incluyendo el de la fuerza motriz. Los tamaños más pequeños pueden manejarse con fuerza animal. Una máquina para fuerza animal de 1 caballo pesa 360 kg más ó menos, y su precio es de $75. Una máquina de vapor pesa 700 á 1,600 kg y vale de $150 á $300.

**Para pozos de 300 á 900 m de profundidad**, se usa una máquina fija con su balancín, semejante á las empleadas en las regiones de petróleo de Pensilva-

FIG 1.

FIG 2.

FIG 3.

FIG 4.

nia. Se levanta una grúa cuadrada piramidal, de 23 m de alto, 6 m en cuadro en su base y 1.2 m en cuadro en su parte superior. Cada una de las cuatro piernas de sus ángulos se construye de tablones de 5 × 20 cm y 5 × 25 cm, unidas de manera que formen una pieza angular de 25 × 25 cm y 2 pulgs de grueso. Estas piernas están unidas por piezas horizontales y verticales de madera. El balancín es de madera de 7.8 m de largo, de 30 cm de ancho, y 65 cm de grueso en su centro, donde gira por medio de muñones colocados en la parte superior de un poste de madera de 45 cm en cuadro y de 3.60 m de alto, llamado « Samson post ». La base de este poste está ensamblada en el quicio principal de la máquina, que tiene 45 cm de ancho por 60 cm de grueso.

La fuerza motriz es una máquina de vapor de 15 caballos, la cual por medio de una correa, polea, manivela y biela obrando sobre el balancín, le comunica el movimiento de vaivén. En la otra extremidad del balancín, é inmediatamente sobre el pozo, está suspendido, por medio de un gancho, un tornillo graduador. Este último se compone de dos barras de hierro de 15 × 50 mm más ó menos y 1.5 m de largo, colgando con 5 cm de separación y unidas por sus partes superiores, en cuyo punto

tienen un anillo suspendido del gancho del balacín. En las puntas inferiores de las barras y entre ellas hay un manguito de tuerca, por la que pasa un tornillo de 1.5 m de largo, el cual en su punta inferior tiene una cabeza que sostiene un eslabón giratorio. Éste agarra el cable de 5 cm de diámetro, de cuya extremidad inferior está suspendido el perforador. Éste, para una perforación de 600 m, está compuesto de una barrena de acero de .90 á 1.20 m de largo, con peso de 90 á 180 kg; de un mango de hierro redondo de 10 á 12 cm de diám y de 7 á 9 m de largo, con peso de 500 á 1,000 kg; de una sonda forrada de acero de 2.40 m de largo, 260 á 300 kg; una barra de inmersión de hierro redondo del mismo diámetro que la del mango de 3.60 á 4.50 m de largo, que pesa de 260 á 500 kg; y un encaje de cable de .75 m de largo, pesando 90 kg. Largo total del juego de piezas, de 15 á 18 m; peso total, 1,350 kg; y para una perforación de 20 cm en la roca más dura, 1,800 kg. **La barra de inmersión** se agrega como peso adicional, y de este modo ayuda á introducirse el cable entre el agua. Para extraer objetos que se hayan caído casualmente en el pozo, se usan **instrumentos especiales.**

**El cable del taladro está enrollado** á un tambor, y durante la perforación, pasa flojamente por sobre la polea situada en la parte superior de la grúa, y de allí á la parte inferior del tornillo de graduación en la extremidad del balancín. A medida que la perforación avanza, el hombre encargado de maniobrar le da vuelta al tornillo de graduación y al cable, para cambiar la posición del barreno después de cada golpe.

Cuando vaya á sacarse el instrumento de la excavación, se quita el cable de las lañas del tornillo de graduación y se arrolla en el tambor, el cual para este fin se engrana con la máquina de vapor, quitando al mismo tiempo la barra de conexión del manubrio, de modo que el balancín quede sin movimiento. Como en las máquinas portátiles, la bomba de arena se saca por medio de la misma fuerza que hace la perforación.

**6½ m³ de madera en bruto**, se llevan más ó menos para la grúa, balancín, quicios, etc., y como 2 m³ más para un caney que cubra la caldera, la máquina y la correa.

En roca calcárea de dureza ordinaria, **una máquina como la descrita** perfora más ó menos 45 cm por hora, en las circunstancias más favorables. Para el trabajo de la máquina **se requieren dos hombres**, el uno para atender á la caldera, afilar las barrenas, etc., y el otro para maniobrar la máquina.

**En la máquina Pierce**, para taladros de prueba, exploraciones mineras y pozos, se empujan los tubos con un martillo de hierro, como los de hincar estacas, pero cubierto con madera dura, en el extremo bajo con que golpea; el martillo se mueve con un *winch* de mano. Los tubos tienen de 1.5 á 3 m de largo; después que se ha enterrado cada tubo, se introduce agua á presión con una bomba de mano, hasta el fondo del agujero, por medio de un tubo que se agita de arriba abajo; el agua hace salir á la superficie el pantano, la arena y el granzón. Las máquinas pequeñas introducen tubos de 5 á 7½ cm de diám; las grandes, de 5 á 20. Las máquinas son desmontables, sus partes pesan de 5 á 30 kg. Cuatro tubos derechos, que sostienen una polea de hierro, obran como guías del martillo; sus extremos, forrados en hierro, entran en unas cavidades. Los martillos están compuestos por secciones que se atornillan juntas; en las máquinas pequeñas se les da así un peso de 45 á 90 kg, y en las grandes, de 45 á 900 kg. Los barrenos pueden hacerse á una profundidad de 30 á 120 m. Estas máquinas han sido muy usadas por la comisión del Canal Istmico de Nicaragua. Si se desea, las máquinas pueden suministrarse con herramientas especiales para taladrar roca, y con bombas de arena para sacar lodo, arena, pedazos de minerales, etc.

# MÁQUINAS PARA TALADRAR ROCAS

**Art. 1.** La roca se taladra mucho más rápida y económicamente con **máquinas de taladrar** que con taladros de mano, con tal que la obra sea de tal magnitud, que justifique los gastos de instalación. Perforan en cualquiera dirección, y pueden emplearse muchas veces en lugares donde es imposible barrenar á la mano. Son movidas directamente por el vapor; ó por aire comprido en un « recipiente », por medio del vapor ó fuerza hidráulica, y de allí llevado á los taladros por tubos de hierro.

El aire es la mejor fuerza motriz para taladrar en túneles y pozos, porque, escapándose después de haber funcionado en los taladros, contribuye á la ventilación

**Art. 2. Estos taladros son de dos especies : taladros de rotación y taladros de percusión.** En los primeros, la barra del taladro se compone de un tubo largo que gira alrededor de su eje. La extremidad de este tubo, templado de tal modo que forme una arista cortante anular, se mantiene en contacto con la roca, y al rotar practica una perforación cilíndrica, dejando generalmente un cilindro (alma) sólido en el centro. Este corazón ocupa el anillo central (*core-barrell*).

Art. 8. La barra del taladro avanza hacia adelante ó dentro del agujero, á medida que prosigue la perforación. La piedra triturada se extrae de la cavidad por medio de un chorro de agua continuo, que se dirige al fondo de ella por la cavidad central de la barra del taladro, y que saca fuera dichas trituraciones á través del angosto espacio comprendido entre la parte exterior de la barra del barreno y las caras de la perforación.

**En los taladros de percusión**, la barra del taladro es maciza, y su acción es como la del taladro giratorio.

**Art. 3. En el taladro rotatorio Brandt** (europeo) la arista cortante de la extremidad de la barra del taladro tubular está provista de dientes de acero templado. Está comprimido contra la roca por una presión hidráulica enorme, y hace sólo de 5 á 8 revoluciones por minuto.

**Art. 4. El taladro de diamante** es la única forma de taladros rotatorios para roca usados extensamente en América. La barra de este taladro se compone de un número de tubos unidos rígidamente en sus extremidades por medio de manguitos interiores hueces.

**Art. 5. La barrena ó mecha del taladro**, fig. 1, se llama mecha de anillo. En su arista cortante hay incrustados « **diamantes** » como se ve en la figura. Éstos están dispuestos de tal manera, que sobresalen ligeramente á ambas orillas, tanto interior como exteriormente. De este modo resultan espacios anulares entre el alma y el anillo central (*core-barrell*), y entre este último y las paredes de la perforación. Estos espacios permiten la entrada y salida del agua empleada en sacar la piedra triturada, y al mismo tiempo impide que el alma de piedra se atraque en el anillo, y éste en la perforación.

Fig. 1

*Mecha de anillo.*

Fig. 2

*Anillo elevador.*

Fig. 3

*Cabeza de taladro.*

**Art. 6.** Inmediatamente encima de la mecha de anillo se atornilla el anillo elevador (*core-lifter*), fig. 2, al anillo central (*corre-barrell*). Consiste el anillo elevador en un tubo de 20 cm de largo más ó menos y del mismo diámetro exterior que el anillo central. En el interior es ligeramente cónico, con la base del cono hacia arriba, y provisto de un anillo R, con dientes en el interior, é igualmente cónico. Cuando se está taladrando, el anillo se une al alma estrechamente y se separa del cilindro exterior; pero cuando el taladro cesa de funcionar, y la barra de taladro se principia á levantar, este anillo es cogido y levantado por el cilindro exterior; y

por causa de su forma en bisel, está comprimido fuertemente contra el alma de la roca, la cual se rompe en su parte inferior por la fuerza que levanta la barra del taladro.

**Art. 7.** Esta fuerza se suple por un **tambor de cable** (torno) fijo á la parte superior del andamio que sostiene el taladro, y maniobrado por la misma máquina que pone en movimiento rotatorio á la barra del taladro. El cable pasa del tambor hacia arriba á una polea situada en la parte superior de una grúa, y de allí hacia abajo á la extremidad superior de la barra del barreno. La altura considerable de la grúa permite que puedan sacarse unidos en una sola pieza, 12 ó 15 m de barra de taladro.

**Art. 8.** Encima del anillo elevador (*core-lifter*) se halla el **anillo central** (*core-barrell*). Éste está formado de un tubo de hierro forjado de 2.4 á 4.8 m de largo. En su lado exterior hay ranuras en espirales, que permiten que el agua y la piedra triturada puedan ascender y salir de la perforación; y algunas veces tiene diamantes incrustados en su exterior, para evitar que se gaste prontamente. La mecha del barreno, el elevador y el anillo central son de un diámetro exterior uniforme, un poco menor que el diámetro de la perforación. El diámetro exterior de la barra del barreno varía de 46 mm más ó menos para un anillo central de 50 mm hasta á 137 mm para uno de 300 mm.

**Art. 9.** Cuando no se desea conservar intacto el corazón ó alma del taladro, puede usarse una **cabeza de taladro**, fig. 3, en lugar de la mecha de anillo, fig. 1. Ésta consiste en una mecha maciza (excepto la que tiene perforaciones para que salga el agua) y está armada con diamantes, algunos de los cuales sobresalen á su circunferencia.

**Art. 10.** La barra de taladro da de **200 á 400 revoluciones** por minuto. **La máquina** que le comunica el movimiento de rotación se compone generalmente de dos cilindros, fijos ú oscilatorios, actuados por vapor ó aire comprimido, y trabajando en ángulo recto entre sí. Por medio de manubrios dan vuelta á un eje, que comunica su movimiento, por medio de un engranaje cónico, á la barra del barreno. Esta está dispuesta para que baje á medida que avanza la perforación, ya sea por otro **engranaje cónico** movido por la misma máquina, ó por estar fijo á una cruceta de cabeza que conecta los vástagos de émbolos de 2 **cilindros hidráulicos**, siendo los vástagos de los émbolos paralelos á las barras de los barrenos.

**Art. 11.** El barreno de diamante **hace perforaciones perfectamente circulares en líneas rectas y en cualquiera dirección, hasta grandes profundidades**, no siendo raro una de 90 á 450 m. Con la circunstancia de extraer el cilindro central (alma) de la roca **en pedazos de 3 á 5 m sin quebrarlas**; lo cual nos muestra las condiciones y estratificación de la roca perforada, tan útil en perforaciones de reconocimientos, de minas, etc., etc. Se consiguen también de tamaño suficiente para hacer perforaciones de 15 á 37 cm de diámetro, para **pozos artesianos**. La redondez de sus perforaciones permite el empleo de revestimientos de casi el mismo diámetro que el de la perforación; y su dirección recta es ventajosa cuando es necesario usar una bomba.

**Art. 12.** En roca blanda una mecha ó barrena puede perforar 60 ó más m sin amolarla; pero en roca muy dura, dichas mechas se gastan en 3 m ó menos. En 1883-4 un barreno de diamante hecho por la *Am'n Diamond Rock Boring Co* con peso por todo de 640 kg más ó menos, y de un costo de $2,800 próximamente, perforó en 1,428 horas de trabajo efectivo, 53 pozos de 2 pulgs de diámetro, haciendo un total de 2,788 m lineales. Profundidad media de las perforaciones, 52.6 m. **Término medio** de perforación, 1.95 m lineales por hora. Máximo por h, 3.8 m. **Costo medio total**, $1.95 por m lineal más ó menos. La roca era en su mayor parte calcárea, con algún cuarzo y piedra arenisca. Las perforaciones se hicieron en ángulos que variaban de 0° á 45° con la vertical.

Como un término medio aproximado podemos decir que **en la roca ordinaria**, tal como granito, piedra calcárea y arenisca dura, estos taladros pueden hacer perforaciones profundas de 5 á 7½ cm de diámetro, á razón **de 60 ó 75 cm por hora**, á un costo de **$3 á $6 por metro.**

**Art. 13.** Estos taladros se hacen de muchos tamaños **muy diferentes**, y montados de diverso modo, según la obra que se ejecuta. Los precios de catálogo para algunos tamaños de los más usados, son como sigue.

| Diámetro de la perforación. | Profundidad mayor de perforación. | H. P. de la caldera. | Precio. | |
|---|---|---|---|---|
| | | | Perforadora. | Bomba. |
| Milímetros. | Metros. | | $ | $ |
| 65 | 1,219 | 25 | 4,000 | ..... |
| 50 | 477 | 15 | 2,500 | 3,400 |
| » | 305 | 14 | 1,900 | 2,800 |
| 40 | 183 | 10 | 1,400 | 1,900 |
| » | 122 | á la mano. | 425 | ..... |

**Art. 14. En máquinas de taladrar de percusión** la barra del barreno está fuertemente impelida contra la roca **por la presión del vapor ó del aire comprimido**, que actúa sobre un émbolo, P, fig. 4, que se mueve en un cilindro CC, figs 4 y 5, y da 300 golpes de émbolo más ó menos por minuto. La rotación de la barra del barreno se hace automáticamente, como se explica en el art. 27.

**Art. 15. El cilindro** CC, se puede deslizar longitudinalmente en la **pieza** SS, fig. 5, á la cual está sujeto, la que está fija á su vez sobre una trípode, á otro sostén (véanse arts. 18 y 19) que soporta toda la máquina.

**Art. 16. La barra del barreno**, R, que corresponde á la del taladro giratorio está sujeta por medio del mandril K á la extremidad de la varilla del émbolo O. Se principia la perforación con una barra de barreno corta, y con el cilindro tan distante de la perforación como la extensión de la armadura S lo permita. A medida que la mecha penetra en la roca, el cilindro está impulsado hacia adelante, sea automáticamente ó á la mano (véase art. 28), hasta donde lo permita el largo de la armadura. Entonces se para el taladro, cerrando la válvula del vapor *, é invirtiendo el movimiento del aparato de alimentación, el cilindro corre hacia atrás. Se quita entonces la barra del barreno corta y se sustituye con una más larga, repitiendo el procedimiento.

**Art. 17**. Como á medida que se taladra se gasta el filo de la mecha reduciendo así su diámetro un poco, **la perforación se hará, por supuesto, cónica** ó de un diámetro ligeramente menor en el fondo que en su parte superior. La segunda mecha debe por consiguiente tener un diámetro ligeramente más pequeño, por ejemplo 1½ á 3 mm menos, la tercera debe ser menos que la segunda. Por otra parte, en perforaciones profundas, la barra del barreno estará raras veces en línea perfectamente recta, de modo que la mecha, en lugar de percutir siempre en el mismo lugar describe un círculo, y de este modo hace más ancha la perforación.

**Art. 18**. La armadura S, en que se desliza el cilindro, está dispuesta de manera tal que puede fijarse por medio de ganchos á una **trípode**, fig. 5, ó á una **barra ó columna**, á lo largo de la cual puede resbalar. La columna, cuando está horizontal, puede descansar sobre dos pares de patas, ó puede fijarse en cualquiera posición por medio de abrazaderas á los lados opuestos de una ranura angosta, ó contra el piso ó techo de una cuba, etc., en cuyo caso una de sus extremidades está provista de un tornillo que sobresale para hacer que las dos extremidades de la columna ejerzan una presión firme contra las paredes opuestas de la roca. En todo caso, los soportes del taladro están ajustados de manera que puedan perforar en cualquiera dirección.

**Art. 19**. Frecuentemente se fija el taladro á un **brazo corto por medio de ganchos**, cuyo brazo á su vez está sujeto á una columna, y sobresale de ella en ángulo recto. Al brazo puede hacérsele cambiar de posición vertical en la columna y también girar alrededor de ella tomando cualquiera posición. Esta disposición da al taladro un grado mayor de movilidad y le permite hacer perforaciones en una extensión mayor sin mover la columna.

**Art. 20**. En los túneles, uno ó más taladros pueden montarse sobre un **carro de taladros**, que corra sobre una línea de carriles de ferrocarril, colocados longitudinalmente dentro del túnel. Sobre estos carriles el carro se lleva donde se va á perforar. El ancho de la vía debe ser tal, que admita debajo del carro de taladrar una vía más angosta. Sobre esta vía angosta corren carros para sacar las piedras voladas en las explosiones. En este país se usan los carros de taladros menos que en Europa.

★ Para evitar repeticiones, usaremos la palabra **vapor** para significar tanto *vapor como aire comprimido.*

**Art. 21.** La presión aplicada á los cilindros de los taladros de percusión es generalmente de 4 á 5 kg por cm cuad más ó menos. **Un taladro hace en una hora** una perforación de 25 á 50 mm de diámetro y de 1 á 3 m de profundidad, según la calidad de la roca y el tamaño de la máquina, al precio **de 33 á 30 centavos por m lineal**, pagando los trabajadores á razón de $1 por día. **Es necesario afilar las mechas** á cada .60 ó 1.20 m que se perforen. Un herrero con un ayudante puede afilar y componer las mechas para 5 ó 6 máquinas.

**Art. 22.** Las mechas tienen muchas formas diferentes, variando con la naturaleza de la obra que se va á ejecutar. En roca dura uniforme se emplean mechas de dos filos cortantes, que forman una cruz de brazos iguales en ángulo recto. Para roca en capas ó filones, los brazos de la cruz son iguales, pero forman dos ángulos agudos y dos obtusos, como en la letra X. En roca blanda, se da algunas veces al filo cortante la forma de la letra Z.

**Art. 23. Cada taladro requiere** un hombre para su manejo. Dos ó tres hombres se requieren para mudar de un lugar á otro los tamaños más pesados. Un hombre puede atender á un compresor de aire pequeño y su caldera.

**Art. 24.** Las figs. 4 y 5 representan el taladro de percusión « **Eclipse** », de la *Ingersoll Rock Drill Co.*, N. Y. La fig. 5, muestra el taladro, montado (como casi siempre) sobre una trípode. La fig. 4 es una sección longitudinal del cilindro, de la cámara, de la válvula y del émbolo.

**Art. 25.** El cilindro C, está provisto en ambas puntas de una **almohadilla** de goma elástica N, para amortiguar los golpes del émbolo, el cual está expuesto, en todos los taladros de percusión, á golpear contra cualquiera de las cabezas del cilindro. El lado de la almohadilla más próximo al émbolo está protegido por una plancha delgada de hierro. Las almohadillas tienen que renovarse de tiempo en tiempo.

**Art. 26. La válvula** V tiene la forma de una devanadera. El perno B pasa flojamente por su centro y le sirve de guía. El vapor de la caldera entra en la caja de vapor, y ocupa todo el espacio comprendido entre los dos rebordes de las extremidades de la válvula, con excepción de *u*. El vapor empuja la válvula alternativamente de un extremo de la cámara al otro, según que uno ú otro punto esté libre de la presión por estar ó no, en comunicación con la cámara vacía E, por medio de los pasadizos DD' y FF'. D y D' están en comunicación con los extremos de la caja de vapor por pasadizos que no se ven en la figura; mientras F y F' comunican, por pasadizos semejantes, con la cámara vacía E. El émbolo tiene un canal anular, LL', en su alrededor. Cualquiera que sea la posición del émbolo, uno de los pasadizos D ó D' está siempre, por medio de este canal, en comunicación con su pasadizo correspondiente, F ó F', que comunican con la cámara vacía. De este modo uno ú otro extremo de la cámara de la válvula está siempre en comunicación con el aire libre; y la válvula está cerrada hacia ese extremo por la presión del vapor que la rodea, entrando aquél al cilindro, C, por el otro extremo.

**Art. 27.** La rotación del émbolo, y con ella la de la barra del barreno, se efectúan así : La barra cilíndrica de acero A, provista de ranuras en espirales, y llamada **barra estriada**, pasa á través de la **tuerca estriada** H y maniobra dentro de ella; dicha tuerca está fija firmemente á la extremidad del émbolo, y tiene ranuras espirales correspondientes á las de la barra estriada. Dicha barra está fija, en su extremidad superior, á la rueda dentada J dispuesta de tal manera, que cuando se efectúa el golpe de émbolo hacia abajo, la tuerca estriada H, actuando sobre las ranuras ó estrías de la barra estriada, comunica á esta barra, y por ella á la rueda dentada, un movimiento rotatorio alrededor de su eje común. El peso del émbolo y su fuerza al moverse son tales, que comunican fácilmente rotación á la rueda dentada sin girar el mismo. De este modo se impide que la mecha gire al golpear. Pero cuando el émbolo sube, la tendencia de la tuerca estriada es comunicar rotación á la barra estriada y á la rueda dentada en la dirección *opuesta:* pero este movimiento está impedido por los dientes de la rueda dentada, y la barra estriada permanece *estacionaria*, mientras que el *émbolo, el vástago del émbolo y la mecha giran* sobre su eje común.

**Art. 28. El tornillo de alimentación** M está provisto en su parte superior de un anillo ó collar, Q, en la armadura fija. De este modo se impide que se mueva longitudinalmente, cuando se le da vuelta por medio de un manubrio fijo en su parte superior. La parte inferior del tornillo gira en una tuerca T, fija al cilindro, que se mueve longitudinalmente al dar vueltas al manubrio. Los taladros grandes están frecuentemente provistos de un **sistema de alimentación automática**, además del manubrio de mano. En este sistema, cuando el cilindro se mueve hacia

adelante, y en consecuencia el émbolo avanza hasta cerca del límite de su carrera, el émbolo ejerce presión contra un tope inclinado dentro del cilindro cerca de su extremidad. El movimiento de este tope, por medio de una pieza exterior, que corre á lo largo del cilindro, está provisto en su parte superior de una pieza que comunica

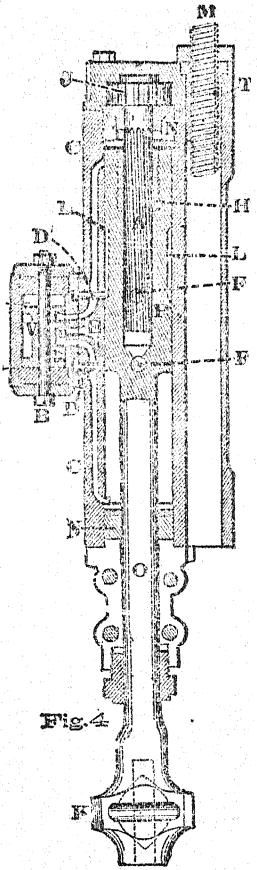



Fig. 4.

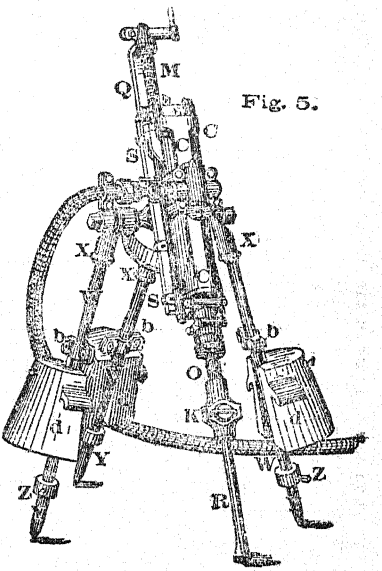



Fig. 5.

movimiento rotatorio á una rueda dentada fija al tornillo de alimentación. Cuando se desea, la alimentación automática puede desengranarse, y hacerla á la mano por medio del manubrio.

**Art. 29. Las piernas de la trípode** se componen de tubos de hierro forjado WW. Sus extremidades superiores están atornilladas en cajas XX. En sus partes inferiores reciben las barras cónicas puntiagudas de acero YY, de .60 á .90 m de largo más ó menos. Estas piernas pueden alargarse ó acortarse, por medio de los tornillos de presión ZZ, graduando así la distancia á que las barras YY pueden

entrar. Las abrazaderas *bb*, tienen ganchos en forma de L unidos á ellas. Sobre estos ganchos cuelgan **los pesos** *dd*, que sujetan la máquina contra la reacción de las percusiones hacia arriba.

**Art. 30.** La tabla siguiente da las dimensiones principales de estos taladros, con **los diámetros y profundidad de perforaciones**, á los cuales se adapta cada uno de ellos. El tamaño H se usa para trabajos submarinos, de túneles y cortadas grandes en roca. G y F se usan para túneles, nivelaciones de calles, pedreras y en trabajos de cloacas. E, D y C para trabajos ordinarios de minas. B sirve sólo para trabajos muy sencillos. Al solicitar datos acerca del valor de estos taladros y compresores, dese la descripción más amplia posible (acompañando un croquis) de la obra que va á hacerse, exponiendo su estado actual y lo que se quiere hacer. Dígase si el trabajo está en la superficie ó debajo de tierra; á qué distancia debe llevarse el vapor ó el aire comprimido; la profundidad de las perforaciones que van hacerse, y la clase de roca, etc. Los barrenos de percusión se venden sin restricciones con respecto á la naturaleza y extensión del trabajo, para el cual se van usar.

**Las letras indican las dimensiones de la máquina.**

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| Diám interior del cilindro. mm. | 44.4 | 63.5 | 69.8 | 76.2 | 79.3 | 88.9 | 107.9 | 127.0 |
| Carrera émbolo. mm. | 76.2 | 101.6 | 127.0 | 152.4 | 152.4 | 165.1 | 177.8 | 177.8 |
| Long alimentación. mm. | 304.8 | 508.0 | 609.6 | 609.6 | 609.6 | 660.4 | 863.6 | 863.6 |
| Long máquina. * mm. | 914.4 | | 914.4 | 1016.0 | 1066.8 | 1346.2 | 1524.0 | 1524.0 |
| Peso de la máquina sin montar. kilogr. | 36.287 | 70.304 | 88.448 | 104.332 | 113.395 | 156.485 | 274.415 | 303.898 |
| Peso de la trípode sin los pesos. kilogr. | *** | 56.698 | 56.698 | 56.698 | 56.698 | 68.038 | 124.735 | 124.735 |
| Peso de los 3 pesos de la trípode. kilogr. | | 113.395 | 113.395 | 113.395 | 113.395 | 158.753 | 181.432 | 181.432 |
| Peso de la columna, brazo y lanza. kilogr. | | 90.716 | 127.003 | 127.003 | 127.003 | 190.503 | 190.503 | 190.503 |
| Diám del agujero. milím. | 12.7 á 22.1 | 19.0 á 31.75 | 25.4 á 50.8 | 25.4 á 50.8 | 25.4 á 50.8 | 38.1 á 63.5 | 0.8 á 101.6 | 76.2 á 152.4 |
| Máxima profundidad ** del hoyo vertical. m. | *** | 1.220 | 2.440 | 3.050 | 3.660 | 4.880 | 9.150 | 12.200 |

* De la parte superior de la mangueta del manubrio alimentador, hasta el extremo inferior de la carrera del émbolo.

** Para tener el máximo de perforaciones horizontales, redúzcase una cuarta parte.

*** La máquina A está montada sobre una pequeña armadura.

**Art. 31. Las máquinas de taladrar de diferentes fabricantes varían principalmente** en el modo de actuar la válvula. Algunas de ellas son movidas por **vapor**, como la « Ingersoll Eclipse », art. 26. En otras la válvula se pone en movimiento por medio de una **palanca ó reborde** que penetra en el cilndro para así llegar al contacto del émbolo, que la mueve en cada carrera. Como estas carreras de émbolo se hacen con gran fuerza, algunas 300 veces por minuto, el juego de válvulas está sujeto á un deterioro muy grande.

**Art. 32.** En el barreno «**Pequeño gigante**», fabricado por la **Rand Drill Co.**, la válvula V, fig. 6, se desliza hacia adelante y hacia atrás en la misma dirección del movimiento del émbolo, por el reborde T.

**Art. 33. La misma fábrica** vende los taladros « **Economizer** » y « **Slugger** », en los cuales, como en el « Ingersoll Eclipse », la válvula se pone en movimiento por el vapor, pero bajo un principio enteramente diferente. En estos dos aparatos no existe ningún vapor comprimido (*steam cushion*) contra el cual pueda chocar el émbolo en su carrera hacia abajo, y de este modo, toda su fuerza se aplica y consume más completamente sobre la roca. El cojín de vapor comprimido que se acumula detrás ó encima del émbolo cuando éste hace su carrera hacia atrás, es formado por el vapor exhausto. Ambas máquinas suspenden la entrada del vapor antes del fin de la carrera del émbolo, usándolo por expansión. En la carrera de émbolo hacia abajo la « Economizer » corta el vapor primero que la « Slugger » y por esta razón se llama así.

**Art. 34.** En el taladro « Burleigh » mejorado, la válvula V, fig. 7, es movida por

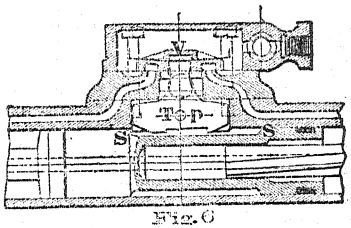



Fig. 6

medio de *dos* palanquitas TT' que están empujadas alternativamente por las extremidades del émbolo P.

**Art. 35.** En el taladro « Dynamic », inventado por el profesor De Volson Wood, la válvula, fig. 8, está adherida á un émbolo de válvula V, fig. 8, que se mueve hacia adelante y hacia atrás por el vapor, actuando alternativamente sobre sus dos

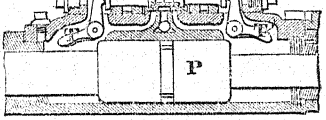



**Fig. 7.**

extremidades. La entrada de este vapor está regulada por una pequeña válvula auxiliar *a*, en cuya parte posterior se encuentra una proyección que ajusta en una ranura en espiral que se ve en el tarugo *n*. Este tarugo está continuamente comprimido hacia abajo (según la posición de la fig. 8) por el vapor que lo comprime en su parte superior, pero es levantado en cada carrera de émbolo por la superficie cónica del émbolo, P, que lo comprime en su pie. De este modo, el tarugo sube y baja continuamente, llevándose la válvula *a*. Dándole vuelta al tarugo, por medio del mango ajustador *s*, se hace que la proyección de la válvula ocupe un punto más alto ó más bajo en la ranura espiral, y de este modo puede variarse la carrera del émbolo, ó limitarla á una parte del cilindro.

En este taladro, diferente del Ingersoll, art. 27, el **émbolo tiene un movimiento rotatorio** en su carrera hacia abajo. **El vástago**, *o*, **del émbolo es**

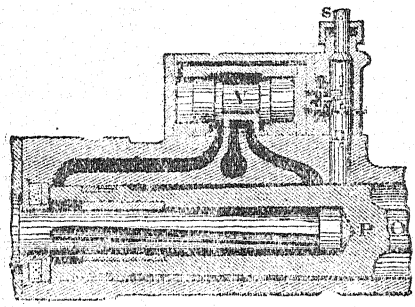



Fig. 8

**más liviano que en los otros taladros.** Éste presenta mayor superficie y por tanto tiene mayor fuerza.

Las piernas de la trípode son de barras de hierro y su longitud puede graduarse aumentándola ó recortándola.

**Art. 36.** El taladro de mano « **Pierce** » es un taladro de percusión. Se maneja por un manubrio, que hace girar un disco de 60 cm de diámetro más ó menos. Este disco tiene una hendedura ó muesca semicircular, en la cual actúa el brazo que levanta la barra del barreno. Este brazo, al levantarse, comprime un resorte enroscado, el cual, en la carrera hacia abajo del émbolo, empuja la mecha contra la roca. Una bola de hierro con peso de 13 kg ó más, se envía con cada máquina. Esta bola puede atornillarse sobre la parte superior de la barra del taladro, para dar mayor fuerza á los golpes de la mecha. Puede usarse la bola sin el resorte desengranando este último.

Este taladro da, más ó menos, 40 golpes de 25 á 30 cm por minuto, y hace las perforaciones de 18 á 62 mm de diámetro. Puede arreglarse de manera que perfore á una profundidad de 9 m ó más. Está provisto de una rueda ó mollejón de esmeril para afilar las mechas, la cual se mueve por medio del manubrio, desengranando al efecto el disco.

**El taladro está montado** sobre un andamio rectangular de dos patas de 1.5 m de alto por .60 m de ancho, hechas de tubos de hierro. En la parte superior de este andamio está sujeta una tercera pata, y al ajustarla puede variarse el ángulo que forma la barra del barreno con la vertical. Semejante á los otros taladros de mano de percusión, este taladro deja de trabajar con ventaja cuando dicho ángulo excede de 45° más ó menos.

**Art. 37. Acanaladura.** Consiste ésta en practicar cortes largos, profundos y angostos en la roca. De este modo pueden sacarse bloques grandes de piedras sin volarlas y sin temor de fracturarlas. Esto se hace ordinariamente ejecutando una hilera de perforaciones á unos 3 cm de distancia unas de otras, y luego rompiendo los espacios intermedios por medio de un cincel sin filo. Esto se llama acanaladura á cincel. Con este objeto se monta una máquina de taladro á vapor sobre una barra horizontal sostenida por dos pares de patas. La barra horizontal está colocada sobre la línea donde se va á perforar, y el barreno se desliza sobre ella de una perforación á otra. Al emplearlo así, se desengrana el aparato de rotación, de manera que el filo del cincel mantenga su posición en la línea de perforaciones.

**Art. 38. La máquina de acanalar de patente de Saunders,** de la Compañía Ingersoll, se compone de una máquina de taladrar roca, que tiene en lugar de la mecha ordinaria de taladrar, un juego de instrumentos compuestos de cinceles, uno al lado del otro, formando así un instrumento cortante de 17 cm de largo por 18 mm de ancho más ó menos. Este instrumento tiene tantos filos cortantes (cada uno tiene un largo igual al ancho del cincel) como cinceles hay en él. La máquina está sostenida por un carro, que se mueve sobre carriles paralelos

a canal que se va á cortar. Por supuesto, que el instrumento no tiene movimiento de rotación, sino que la barra estriada A, fig. 4, se emplea para mover el carro más ó menos una pulgada después de cada percusión. Durante la percusión no se mueve el carro. Bajo circunstancias favorables **esta máquina ha cortado de 7 á 9 m cuad** de canal en un día de 10 horas de trabajo. **Su peso,** incluyendo el carro, es de 2,250 kg.

El instrumento está provisto de una válvula, por la cual, cuando se desea, **puede cerrarse** el vapor del émbolo en su carrera hacia abajo; de modo que la percusión puede hacerse con el *peso* del émbolo, vástago y mecha solamente.

**Art. 39.** La Compañía Ingersoll tiene un método especial, ideado por el Sr. W. L. Saunders, I. C., **para barrenar y volar las rocas debajo del agua**, aunque estén cubiertas de fango á considerable altura.

**Art. 40. Los compresores de aire** para taladros de roca, como los hechos y usados en los E. U., son generalmente máquinas horizontales de acción directa. Es decir, que los ejes de los cilindros de vapor y aire son horizontales, y que el vástago del émbolo pasa directamente del cilindro de vapor al cilindro de aire. Un volante está fijo al vástago del émbolo por medio de un manubrio y barra de conexión. Algunas veces la máquina de vapor está separada del compresor, y se transmite la fuerza al último por medio de correas ó engranajes: puede emplearse de la misma manera fuerza hidráulica. El aire es impelido dentro de un receptáculo compuesto generalmente de un cilindro (de .90 á 1.20 m de diám) de planchas de hierro, y que tiene una longitud de 1.5 á 3.66 m.

Si el cilindro de aire está dispuesto de modo que deje entrar el aire en una sola carrera del émbolo, y lo impela al receptáculo en la carrera de retorno, se llama **máquina de simple acción**. Cuando en cada carrera deja entrar el aire y o lanza al receptáculo, se llama de **doble acción**. Cuando el compresor no tiene ri no un solo cilindro, se llama **sencillo**, y si tiene dos compresores sencillos, es **doble** (*duplex*). **Las válvulas** pueden ser **válvulas de tubo,** fijas por resortes y movidas por la misma presión del aire, ó **válvulas de corredera,** actuadas por medio de excéntricas y barras de conexión, como en las máquinas de vapor.

La compresión del aire desarrolla **calor,** y éste se hace desaparecer, haciendo circular agua fría por el émbolo de aire y por cubiertas que envuelven el cilindro de aire, ó inyectándola en dicho cilindro en forma de irrigación. O ambos métodos pueden usarse al mismo tiempo.

**Art. 41.** La siguiente lista parcial de los compresores de Clayton, formada con datos recibidos de los fabricantes, muestra las **dimensiones** y **trabajo** que efectúa cada uno de ellos. Damos también una lista de los **receptáculos** hechos por ellos.

## COMPRESORES DE AIRE DE DOBLE ACCIÓN DE CLAYTON.

Lista parcial.

| | Número de la máquina para diversos tamaños. | | | |
|---|---|---|---|---|
| | **1** | **2½** | **4** | **7** |
| Compresores de acción directa doble. | | | | |
| Diámetro de los cilindros de vapor.. mm. | 203.2 | 254.0 | 355.6 | 457.2 |
| » » » de aire.... mm. | 203.2 | 254.0 | 355.6 | 457.2 |
| Longitud de la carrera. ..... mm. | 304.8 | 330.2 | 381.0 | 609.6 |
| Número de revoluciones por minuto..... | 120 á 140 | 100 á 130 | 100 á 120 | 80 á 90 |
| Met cúbs de aire comprimido por minuto. Efectivo..... | 3.8 | 5.9 | 12.3 | 25.2 |
| Peso aproximado del compresor..... kg. | 1360. | 3175. | 6803. | 14339. |
| Compresores de acción directa simple. | | | | |
| Diámetro del cilindro de vapor..... mm. | 203.2 | 254.0 | 355.6 | 457.2 |
| » » de aire..... mm. | 203.2 | 254.0 | 355.6 | 457.2 |
| Longitud de la carrera..... mm. | 304.8 | 330.2 | 381.0 | 609.6 |
| Número de revoluciones por minuto ..... | 120 á 140 | 100 á 130 | 100 á 120 | 80 á 90 |
| Met cúbs de aire comprimido por minuto.. Efectivo..... | 1.9 | 2.9 | 6.1 | 12.6 |
| Peso aproximado del compresor..... kg. | 748.4 | 1746.28 | 3742·03 | 6236.7 |

* El precio de un compresor, para moverlo por medio de una máquina de vapor separada, ó por fuerza hidráulica, es, por supuesto, menor que el del compresor arriba descrito y máquina combinados.

**Recipientes de aire verticales y horizontales.**

| Diám. | Longitud. | Peso aproximado. | Diám. | Longitud. | Peso aproximado. |
|---|---|---|---|---|---|
| mm. | Metros. | Kilogr. | mm. | Metros. | Kilogr. |
| 838.2 | 1.52 | 317.5 | 1016 | 2.44 | 759 |
| 762.0 | 2.13 | 403.6 | 1016 | 3.05 | 861 |
| 914.4 | 2.44 | 707.5 | 1016 | 3.35 | 907 |
| 1016.0 | 1.83 | 725.7 | 1016 | 3.66 | 952 |

Los receptáculos están provistos de manómetros de bronce, de indicadores de cristal de nivel de agua, de válvula de seguridad, de válvulas de escape, llaves de prueba, rebordes y conexiones para alimentación automática.

# TRACCIÓN

**Tracción sobre caminos carreteros y canales,** ó la fuerza que se necesita para arrastrar por ellos vehículos y botes.

La siguiente tabla da (con aproximaciones tolerables) la fuerza en kg por toneladas que se necesita para arrastrar por caballos un coche ó diligencia con pasajeros, por la pendiente del camino (buen camino) Holyhead de Inglaterra, determinado por medio de un dinamómetro. El peso entero era de 1½ tonaladas; pero en la tabla se dan los resultados por toneladas. Por la naturaleza de tales experiencias no es posible alcanzar gran exactitud.

(*Obs. del T.* — Hemos convertido los ejemplos y la tabla del autor al sistema métrico.)

| Pendiente. | A 6,437 metros por hora. | A 9,656 metros por hora. | A 12,874 metros por hora. | A 16,093 metros por hora. |
|---|---|---|---|---|
| | kilogramos. | kilogramos. | kilogramos. | kilogramos. |
| 1 en 15½ | 95.25 | 97.97 | 102.06 | 108.86 |
| 1 » 20 | 88.91 | 91.62 | 96.15 | 103.87 |
| 1 » 26 | 70.31 | 72.57 | 75.29 | 79.37 |
| 1 » 30 | 62.14 | 64.41 | 66.68 | 69.83 |
| 1 » 40 | 51.71 | 54.43 | 56.24 | 58.97 |
| 1 » 64 | 49.44 | 52.17 | 54.43 | 57.15 |
| 1 » 118 | 46.26 | 48.53 | 51.26 | 54.43 |
| 1 » 138 | 44.91 | 46.72 | 49.44 | 53.06 |
| 1 » 156 | 44.45 | 45.81 | 48.08 | 50.79 |
| 1 » 245 | 42.18 | 43.54 | 45.81 | 48.53 |
| 1 » 600 | 36.74 | 38.56 | 41.27 | 43.54 |
| Horizontal. | 34.47 | 36.28 | 38.56 | 41.27 |

Los resultados siguientes apreciados en su mayor parte con el mismo instrumento son también en kilogramos por tonelada, con un carro de cuatro ruedas á paso, lento, en un plano á nivel y la vía en buena condición.

Sobre un pavimento de cubos de piedra, de 15 á 24 kilogs por tonelada métrica.

Sobre una vía macadanizada de pequeños pedazos de piedra, de 28 á 34 kilogramos por tonelada métrica.

Sobre una vía de cascajo, 64 kilogs por tonelada métrica.

Sobre una vía pavimentada con piedras pequeñas, 21 á 34 kilogs por tonelada métrica.

Sobre piedra picada en lecho de concreto, 21 á 34 kilogs por tonelada métrica.

Sobre vías de tierra común, 90 á 136 kilogs.
Sobre una vía entablada 14 á 23 kilogs.

**La fuerza de tracción de un caballo disminuye á proporción que aumenta su velocidad**, y quizá dentro de ciertos límites, de 1½ á 6 km por hora, esta disminución está en razón inversa con ella. De este modo el término medio de la tracción de un caballo, sobre una *superficie á nivel*, tirando por espacio de 10 horas al día, se puede suponer aproximadamente como sigue. (*Obs. del T.* — Al convertir esta tabla hemos puesto números sin fracciones y aproximados, porque bastan para la práctica y son más fáciles de retener en la memoria.)

| Metros por hora. | Kilogramos de tracción. | Metros por hora. | Kilogramos de tracción. |
|---|---|---|---|
| 1,200..... | 151 | 3,600..... | 50 |
| 1,600..... | 113 | 4,000..... | 45 |
| 2,000..... | 90 | 4,400..... | 41 |
| 2,400..... | 76 | 4,800..... | 37 |
| 2,800..... | 65 | 5,600..... | 32 |
| 3,200..... | 57 | 6,400..... | 28 |

Si trabaja * durante un número menor de horas, su tracción puede aumentar á proporción que disminuyen aquéllas; por ejemplo, para 5 horas por día, la tracción á 4,000 met por hora será de 90 kg.

Cuando sube una pendiente, su fuerza disminuye rápidamente porque tiene que suspender una parte de su propio peso (cuyo término medio es como de 450 á 500 kg), el cual puede apenas sobrellevarlo sin otra carga en una pendiente de 5 á 1.

En una pendiente semejante debe ejercer una fuerza igual á 199 kg por tonelada, es decir, de 90 kg por los 450 kg de su propio peso. Suponiendo que en un pedazo á nivel de buen camino tuviese al arrastrar un carro cargado que pese con carga y todo una tonelada, que ejercer una tracción de 27 kg; entonces al subir un pendiente de 4° (es decir, de 1 en 14.3), tendría una tracción de 70.72 kg para el peso de la tonelada, y de 30.39 kg para su propio peso, 101.11 kg por todo. Un caballo puede ejercer *por algunos minutos* hasta el doble de su tracción regular sin gran fatiga. Estos cálculos demuestran que en una pendiente de 4°, un caballo mediano se fatiga bastante cuando arrastra una carga total de una tonelada, y se le debe permitir por consiguiente andar al paso, y descansar á cortos intervalos. La carga justa para un caballo solo, con un carro, á un paso variable, en buen orden y trabajando 10 horas por día en un camino ordinario, onduloso, es como de media tonelada, además del carro, que tiene como media tonelada más. Con dos caballos y el mismo carro, el peso solo puede ser como de 1½ toneladas.

*Advertencia.* Como la acción de la gravedad es la misma en los caminos buenos que en los malos, se desprende que **se hacen más dificultosas las pendientes mientras mejor sea el camino**. Así, en una pendiente de 2°, es decir, de 3 1/2 por ciento, y teniendo en cuenta tan sólo el peso, se requiere una tracción de 35.38 kg por tonelada; que es como 10 veces la de un ferrocarril á nivel con velocidad de 9,655 metros por hora; pero es casi igual á la requerida en un camino común á nivel y á la misma velocidad. Por tanto (hablando así al poco más ó menos) se necesitarían 10 locomotoras en lugar de 1; pero sólo 2 caballos en lugar de 1.

Una pendiente de 1 en 35, ó 2.80 por ciento, ó de 1°38′ es la mayor pendiente en que pueden trabajar caballos bajando un camino sólido y plano á un trote ligero y sin peligro. De ella no debe, por tanto, pasarse sino cuando sea absolutamente necesario.

**En los canales y otras aguas**, el líquido como medio resistente hace las veces del rozamiento en los caminos á nivel; pero es distinto al rozamiento; su resistencia varía como los cuadrados de la velocidad, á lo menos desde la velocidad de 2 kilómetros por hora, hasta la de 13 kilómetros por hora. A proporción que la velocidad desciende, la resistencia disminuye más, y más rápidamente: se trata del caso en que el cuerpo en movimiento flota en parte sobre la superficie ó se sumerge enteramente. Al remolcar á lo largo de canales estancados, etc. la velocidad es generalmente de 1.5 á 4 km por hora, y para cargas en mas usado de

* *N. del T.* — Para nuestra clase de caballos carecemos de experiencias que nos den este dato; pero de seguro que es mucho menor, puesto que los caballos americanos son más grandes, dobles y mejor enrazados que los nuestros, y por tanto mucho más fuertes.

2.4 á 3.2 km. Se necesita menos fuerza para remolcar un bote con una velocidad de 2 kilómetros por hora donde no hay corriente, que á 1¼ por hora contra una corriente de ¼ de kilómetro porque en el último caso el bote tiene que *ser subido* por el plano inclinado que produce la corriente.

La fuerza que se requiere para remolcar un bote á lo largo de un canal, depende en *gran parte* de las áreas comparativas de las secciones transversales del canal y de la parte sumergida del bote. Cuando la anchura del canal en la línea de flotación es por lo menos 4 veces la del bote, y el área de su sección transversal por lo menos de 6¼ veces la de la sección transversal de la parte sumergida del bote, el remolque á velocidades usadas en los canales será casi tan fácil como en aguas más anchas y profundas. Con menos dimensiones se hace más difícil. (D'Aubuisson.) Depende mucho también de la forma de la proa y otras partes del bote, y de la proporción entre el largo, su anchura y profundidad. Por consiguiente, se ve que el solo peso de la carga no es un elemento de tanta importancia en estos casos, como lo es en tierra. El asunto es sin embargo muy complicado para tratar de él aquí. Morin dice que los constructores navales estiman la resistencia de los buques de vela y de los vapores en el mar sólo como de 2½ á 3½ kg por metro cuadrado de sección transversal sumergida, cuando la velocidad es de 3,200 metros por hora. Es mucho mayor en canales.

**En la navegación del Schuylkill de Pensilvania**, que es un canal mezclado con aguas de flujo y reflujo, para 173 kilómetros, la carga regular de 3 caballos ó mulas es un bote de muy buena construcción, sin quilla, como de 30 m de largo; con baos de 5.33 m y 2.43 m de profundidad de bodega, que cala 1.68 m cuando está cargado *. El peso del bote es como de 65 toneladas; carga 175 toneladas de carbón; peso total 240 toneladas, á 80 por caballo ó mula. En el viaje de descenso con los botes cargados, los animales trabajan durante cuatro días *remolcando* (excepto en las represas) por espacio de 18 horas de las 24 del día, excediendo en mucho á los límites concedidos generalmente para esfuerzos continuos.

En las secciones del canal (en que tiene 18.29 m la línea de flotación, y 1.83 m de profundidad, la velocidad es de 2,815 metros por hora, y en los estanques profundos y anchos, 3,200 metros más ó menos.

En el viaje de subida con los botes, de 65 toneladas, vacíos, la velocidad media es como de 4 km por hora. Los botes vacíos traen como 45 cm de agua, y frecuentemente continúan sin descansar día y noche en todo el trayecto de 173 km. Los animales tienen generalmente dos ó tres días de descanso á cada fin de viaje; pero quedan muy maltratados al fin de la estación.

Si nuestra suposición anterior, de 65 kg de tracción de un caballo á razón de 2.800 metros por hora, es correcta, la tracción de los botes cargados en las secciones del canal es $\frac{65}{80 \text{ toneladas}} = 0.81$ kg por tonelada.

El inteligente ingeniero y superintendente del *Sch. Nav., James F. Smith*, dió como resultado de sus propias y extensas observaciones, que uno de estos grandes botes cargados (240 toneladas por todo) sin fatigar los animales puede ser arrastrado por todo el canal durante 10 horas al día como sigue : por un caballo ó mula á razón de 1,600 metros; por 2 animales á razón de 2,400 metros, y por 3 á razón de 2,800 metros por hora. Cuando se emplean cuatro animales, la ganancia de tiempo es muy insignificante. En una época de rivalidad entre los boteros, uno de ellos empleó 8 caballos, pero con ellos no pudo pasar de 4 km por hora. Dos ó más caballos juntos no pueden, trabajando por horas, halar tanto como cuando trabajan separadamente.

Si nuestra pequeña tabla anterior de la tracción de un caballo á diferentes velocidades por 10 horas, es correcta, entonces la tracción de los anteriores botes cargados de carbón (240 toneladas) en las secciones del canal de navegación, es como sigue : La última columna da la tracción en kg por m cuadrado del área de la sección transversal sumergida donde es mayor; á saber : como de 8.83 metros cuadrados.

(*Obs. del T.* — Damos á continuación la tabla que trae el autor convertida al sistema métrico.)

| Caballos. | Mets por hora. | | Kg por tonel. | Kg por m². |
|---|---|---|---|---|
| 1..... | 1,600 | $\frac{113}{240}$ | .47 | 12.83 |

* **Valor de los botes**, en 1884 (Canal Schuylkill), como $1,800. Reparaciones anuales como $85. Los botes duraban de 16 á 20 años. Longitud como 31 m; bao de 5.33 m; calado de .45 m á 1.68 m; capacidad 180 toneladas; peso como 58 toneladas; velocidad con 3 mulas, 2,800 metros por hora.

| | | | | |
|---|---|---|---|---|
| 2..... | 2,400 | $\frac{152}{240}$ | .63 | 17.08 |
| 3..... | 2,800 | $\frac{195}{240}$ | .81 | 21.96 |
| 3 (*on pools*).. | 3,200 | $\frac{171}{240}$ | .71 | 19.28 |
| 8..... | 4,000 | $\frac{360}{240}$ | 1.50 | 41.10 |
| 3 (subiendo). | 4,000 | $\frac{135}{65}$ | 2.10 | 61.01 |

**En el canal Lachine**, Canadá, de 36.75 m de ancho en la línea de flotación, 24.38 m en el fondo y 2.74 m de profundidad en la compuerta, 6 caballos remolcaron con facilidad goletas cargadas.

Antes del ensanche del **canal Erie** * sus dimensiones eran 12.19 m en la línea de flotación, 8.53 m de fondo y 1.22 m de profundidad de agua. El término medio del peso de los botes era como de 30 toncladas. Con 75 toneladas de carga, ó un total de 105 toneladas, fueron remolcados por dos caballos á razón de 3,200 m por hora, lo cual da, según nuestra tabla (114 ÷ 105) una tracción de cerca de 1.09 kg por tonelada. Los botes eran como de 24.38 m de largo, con baos de 4.27 m, y calaban como .99 m cuando estaban cargados; por consiguiente, la tracción, según nuestra tabla, sería como de 27.82 kg por metro cuad de sección transversal sumergida. Mientras que para botes de 82 toneladas cargados en un canal más pequeño (casi tocando los botes el fondo), la tracción á 2,800 mets sería de 1.59 kg por tonelada. También da 27.83 kg por metro cuadrado de sección sumergida.

# FUERZA ANIMAL

**Art. 1.** En lo que se refiere á caballos, este asunto se ha tratado ya por separado bajo el encabezamiento precedente de « Tracción ». Todas las apreciaciones que se hagan á este respecto serán hasta cierto punto vagas, debido á las diferentes resistencias y velocidades entre animales de la misma especie, así como al grado de educación ó expedición que tengan para cualquier trabajo especial. Las autoridades en la materia difieren mucho en este punto, y algunas veces se expresan de una manera tan indefinida, que arrojan duda respecto á sus opiniones. Esperamos, sin embargo, que se encuentre el cálculo que sigue tan aproximado á los términos medios prácticos, como lo permite la naturaleza del caso con nuestros imperfectos conocimientos actuales. Supongamos un caballo medianamente educado, que no pese menos de media tonelada, bien tratado y bien mantenido. Dicho caballo trabajando 10 horas por día á razón de 4,000 m por hora, en un camino bueno y *á nivel*, tal como el andén de un canal, ó un andén circular de caballos **, **puede ejercer un tiro continuo de 45.35 kg (100 libras)**.

Ahora bien, 4,000 m por hora equivalen á 66.66 m por minuto, ó á $3^2/_3$ pies (1.11 m) por segundo ; y como 10 horas contienen 600 minutos, su trabajo de un día, halando constantemente nive con dicha velocidad, asciende á

$$600 \times 66.66 \times 45.35 = 1{,}813{,}818 \text{ kilográmetros por día;}$$

ó como 50 kilográmetros por segundo ***. Lo cual significa que ejerce fuerza suficiente durante el día para *suspender*, á 1 metro de altura, 1,813,818 kg, ó sean 181,381 á 10 m.

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* Extensión como 584 km ; precio $12.30 por metro lineal. El canal ensanchado tiene 21.33 m, 12.80 m ; y 2.13 de agua ; y cuesta el ensanche $35.40 por metro lineal.
El precio de los diferentes canales de Pensilvania varía de 14 á 31 dólares el metro lineal.

** Para hacer que un caballo trabaje con facilidad en **un andén circular**, el diámetro de éste no debe ser menor de 7 á 9 m, ó mejor aún, como de 11 m.

*** **Un caballo de fuerza (nominal)**, son 33,000 pies-libras (4,562.33 kilográmetros) por minuto; éste es el valor del caballo asignado por Boulton y Watt á las máquinas

Puede ejercer esta fuerza ya por *tracción* (*remolcando* o *halando*) ó ya *levantando* ó ***suspendiendo*** pesos. Si tiene que suspender una pequeña carga á una gran altura la maquinaria por medio de la cual lo hace debe estar dispuesta de tal modo que se pueda ganar velocidad con lo que se pierda en fuerza (así se dice vulgarmente aunque impropiamente). Sea que suspenda el gran peso á una pequeña altura ó un peso pequeño á grande altura, empleará siempre la misma cantidad de fuerza. La experiencia demuestra que entre los límites de 5 á 10 horas por día (permaneciendo la velocidad la misma), **puede aumentarse el tiro de un caballo casi en la misma proporción en que disminuye el tiempo**. De modo que cuando trabaja de 5 á 10 horas diarias, dicho aumento será como se muestra en la tabla que sigue. Por consiguiente, puede hacerse la cantidad total de 1,813,818 kilográmetros de trabajo por día, ya trabaje el caballo 5, 6 ú 8, etc., horas por día *. Esto, por supuesto, supone que realmente trabaje *todo el tiempo* sin pararse.

**Tabla del tiro á nivel de un caballo, á razón de 4,000 m por hora.**

| Horas por día. | Kilogs. | Horas por día. | Kilogs. |
|---|---|---|---|
| 10 | 45.36 | 7 | 64.80 |
| 9 | 50.40 | 6 | 75.60 |
| 8 | 56.70 | 5 | 90.72 |

**La experiencia nos demuestra también que, para velocidades comprendidas entre 1,200 m y 6,400 m por hora, su esfuerzo ó tiro estará en razón inversa de su velocidad.** Así á 3,200 m por hora, para las 10 horas del día, el tiro será

| metros | metros | | kilogs | kilogs |
|---|---|---|---|---|
| 3,200 | : 4,000 | = | 45.36 | : 56.70 |

A 2,400 m serían como 75 kg.

Por tanto, en este caso también la cantidad de trabajo diario permanece la misma §, y entre todos los límites anteriores de horas y velocidad, se puede tomar prácticamente como 13,200,000 pies-libras (1,824,930 kilográmetros) por día, ó 22,000 pies-libras (3,041.56 kilográmetros) por minuto de un día de 10 horas. Pero eso no significa que el caballo puede efectivamente en la práctica trabajar sobre pesos en esta proporción, porque generalmente una parte de su fuerza se emplea en vencer el *rozamiento* de la maquinaria que pone en movimiento, y, además, la naturaleza del trabajo puede requerir que el caballo se pare frecuentemente; de modo que en un *día de trabajo* de 8 á 10 horas, puede no haber *trabajado* efectivamente más de 5, 6 ó 7 horas.

Como cálculo aproximado, despreciando algo por la pérdida de fuerza en vencer el rozamiento de la maquinaria de suspensión, y el peso de las cadenas, cubos, etc., podemos decir que **el trabajo útil diario de un caballo levantando pesos**

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que ofrecen en venta, y en esta inteligencia los compradores que deseen sustituir vapor por caballo no sufrirán contratiempo alguno. Este valor supuesto puede llevarse á la práctica aplicando un caballo fuerte que trabaje diariamente á razón de 8 á 10 horas por día; pero como la máquina puede trabajar sin pararse durante meses, días y noches, lo cual no podría hacer un caballo, es claro que el caballo de vapor ó de una máquina puede efectuar mucho más trabajo que cualquier otro caballo. Por consiguiente, muchos objetan el término « caballo de fuerza » aplicado á las máquinas; pero desde luego que todo el mundo comprende su significado, y como dicho término es conveniente, no es en efecto objetable. Boulton y Watt quieren decir que una máquina de un caballo es la que efectúa en un momento cualquiera el trabajo de un caballo muy fuerte. Un caballo *mediano* no haría sino 22,000 pies-libras** (3,041.56 kilográmetros) por minuto.

* Es claro que aun cuando el *día de trabajo* sea el mismo, la hora ó el minuto de trabajo variarán con el número de horas asignadas al día de trabajo. Debe recordarse que *un día de trabajo* de un número de horas dado no implica absolutamente en todos los casos un número de horas de *trabajo efectivo*, sino que comprende las interrupciones y paradas.

§ **Esta advertencia sobre la velocidad no se refiere á las cargas remolcadas en el agua.** Así, si el tiro á razón de 2 millas por hora es de 125 libras (56.70 kg), y á 4 millas de 62 1/2 libras (28.29 kg), *en tierra* arrastrará pesos en esta proporción; pero al *remolcar* un bote *en el agua* á su mayor velocidad, tiene que vencer la resistencia aumentada del *agua misma*, la cual á 4 millas es mucho más del doble que á 2 millas; probablemente *cuatro* veces mayor. Por tanto, á 4 millas, sobre un canal, el tiro de 62 1/2 libras (28.39 kg) no sería suficiente para *una carga* mitad de la que podría arrastrar con el tiro de 125 libras (56.70 kg) á 2 millas.

** *N. del T.* — Para convertir pies-libra; en kilográmetros, multipliquese aquéllas por .138255.

**por medio de una grúa común**, es como de 10,000,000 de pies-libras (1,382,525 kilográmetros). Esto es, que él suspenderá un equivalente de 10,000,000 de libras netas de agua, mineral, etc., á un pie (ó sean 1,382,525 kilogramos á 1 metro de altura). La carga que él puede suspender de un golpe, incluyendo cadenas, cubos y las pérdidas por rozamiento será tanto mayor que su propia fuerza cuantas veces el diámetro del andén del caballo sea mayor que el del tambor de la grúa, y su velocidad estará en razón inversa. Su fuerza directa propia variará con el número de horas diarias que sea necesario que trabaje, según la tabla anterior. Con este dato se puede eligir el tamaño de los cubos, de los cuales debe haber por lo menos dos, de modo que el que está abajo, vacío puede llenarse mientras se vacía el de arriba, á fin de economizar tiempo. Lo mismo cuando el trabajo se hace por hombres.

**Art. 2. Un trabajador práctico tirando por medio de una cuerda sobre sus hombros en un camino á nivel**, ó empujando por delante de sí en un andén circular una barra horizontal, con una velocidad de 2,400 m á 4,800 m por hora, ejerce como la sexta parte de la fuerza de un caballo, ó sean 304,156 kilográmetros por día, ó 506.92 kilográmetros por minuto, de días de 10 horas de trabajo efectivo.

Pero los trabajadores tienen que trabajar frecuentemente bajo circunstancias menos ventajosas para la aplicación de sus fuerzas que cuando halan ó empujan de la manera indicada; y en dichos casos no pueden hacer tanto por día. Así, por ejemplo, al darle vuelta á un torno, la inclinación constante del cuerpo y el movimiento de los brazos es más fatigante. **El tamaño de un torno no debe exceder de 18 pulgadas (45 cm)** ó el radio de un círculo de 3 pies (91 cm) de diámetro; así un trabajador puede ejercer una fuerza como de 16 libras (7.25 kg) á una velocidad de 2½ pies (75 cm) por segundo, ó 150 pies (45.71 m) por minuto dando muy cerca de 16 vueltas por minuto, á 8 horas por día. Debe calcularse como una cuarta parte por los descansos cortos. Si se toman *los días de trabajo* á razón de 8 ó 10, etc., horas, se debe descontar generalmente de ellas una quinta parte para dichos descansos. Según los datos precedentes, una hora de *trabajo* de 60 minutos de *remolque efectivo* sería, empleando el sistema métrico :

kilogramos metros minutos
$$7.25749 \times 45.7192 \times 60 = 19{,}908.40 \text{ kilográmetros.}$$

y deduciendo la quinta parte dará como 16,000 kilográmetros por hora incluyendo descanso

En la práctica debe hacerse, sin embargo, otra deducción por el rozamiento de la máquina y por el peso de las cadenas elevadoras; y en caso de elevar agua, piedra, mineral, etc., también por el peso de los cubos. Como término medio aproximado, podemos suponer sólo 100,000 pies-libras (13,825 kilográmetros) de trabajo útil por hora; esto es, que **un hombre con una grúa suspenderá efectivamente un equivalente á 100,000 libras de agua, ó mineral, etc., á 1 pie de altura por hora, incluyendo descansos (13,825 kilogs á 1 metro de altura)**. Esto es, 230.42 kilográmetros por minuto en días de 10 horas, incluyendo los descansos. Por tanto, en un día de 10 horas de trabajo suspendería 138,250 kilográmetros á 1 metro, **ó justamente la décima parte de lo que haría un caballo con una grúa en el mismo tiempo**. Hemos visto anteriormente que cuando se remolca en un camino á nivel, se puede, á paso cortos ejecutar como la sexta parte del trabajo diario de un caballo. Puede también moverse la grúa con mayor fuerza hasta con 30 ó 40 libras (13.6 ó 18 kg); pero lo hará más despacio, efectuando el mismo trabajo diario. Con una grúa, como las destinadas á caballos, pero menos pesada con dos ó más tobos, un trabajador práctico suspenderá en un día de trabajo de 10 horas, de 165,903 y 193,554 kilográm respectivamente. En un aljibe ó foso de poco fondo se pierde más tiempo en vaciar los tobos (porque se suben mayor número de tobos) que en uno profundo; pero el profundo requiere mayor peso de cuerda; para economizar tiempo en todas estas operaciones deberá haber por lo menos dos tobos, para llenar el vacío mientras se vacía el lleno. También es mejor emplear dos ó más hombres para izar al mismo tiempo con palancas colocadas á los extremos del eje; y los hombres trabajarán con más facilidad si se colocan tornos en ángulos rectos. Cada palanca ó manubrio debe ser suficientemente largo para dos ó tres hombres. Se debe emplear un hombre más para vaciar los baldes. Éste debe alternar con los izadores. Las mismas observaciones son aplicables en algunos de los casos siguientes.

Con una rueda de pies ó de escalones un trabajador práctico hará como 40 por

ciento más de trabajo diario que con un torno; ó en un día de trabajo * de 10 horas, incluyendo descansos, hará como 1,400,000 pies-libras (193,554 kilográm). Y puede hacer esto, sea que trabaje en la circunferencia exterior de la rueda, subiendo sobre los escalones á nivel de sus ejes, ó caminando dentro de ella cerca de su parte inferior. En ambos casos obra con su peso, generalmente, 130 á 140 libras (58.96 á 63.50 kg), y no con la fuerza muscular de sus brazos. Cuando se encuentra á nivel de los ejes, su peso obra más directamente que cuando se halla en la parte inferior de la rueda; pero en el primer caso tiene que ejecutar un trabajo lento y fatigoso parecido al que se ejerce al subir una escalera continua; mientras que en el segundo es simplemente como si tuviera que ascender un plano inclinado muy poco sensible, lo que puede hacer más rápidamente, durante horas, con una fatiga relativamente pequeña, y esta rapidez compensa la menor acción directa de su peso. Por tanto, en cualquiera de los dos casos, como lo ha demostrado la experiencia, él ejecuta casi la misma cantidad de trabajo diario. Las ruedas de pie ó escalera deben ser de 5 á 25 pies (1.52 m á 7.61 m) de diámetro, según la naturaleza del trabajo. Son movidas generalmennte por varios hombres á la vez y pueden á veces emplearse para clavar estacas, como también para suspender agua, piedras, etc.

**Por medio de una buena bomba común**, adecuadamente proporcionada, un trabajador práctico elevará en un día de 10 horas de trabajo como 1,000,000 pies libras (138,252.52 kilográmetros) netos de agua **.

**Trabajando con un balde liviano ó con un achicador**, puede sacar como 200,000 pies-libras (27,650.5 kilográm) netos de agua. **Con balde y cigoñal** (*swape*) (una palanca larga oscilando verticalmente, y con un peso en un extremo que equilibre al balde colgado en el otro, usado á menudo en cisternas del campo), de 600,000 á 800,000 (de 82,951 á 110,602 kilográm).

En el último tiene solamente que tirar hacia abajo el balde vacío y suspender así el contrapeso. **Por medio de dos baldes suspendidos en los extremos de una cuerda sobre una polea**, hará de 500,000 á 600,000 (de 69,126 á 82,951 kilográm). En este caso trabajan los baldes tirando la cuerda á la mano.

**Por medio de un tímpano, tambor ó dado** ***, movido por una rueda de pie, hará como de 1,200,000 á 1,400,000 (de 165,903 á 193,554 kilográmetros).

**Con una rueda Persa §, una bomba de cadena, una cadena de cubos ‖, ó un tornillo de Arquímedes**, movidos por una rueda de pie, se elevan de 800,000 á 1,000,000 de pies-libras (de 110,602 á 138,252 kilográm). De estos cuatro los tres primeros pierden efecto útil por derramamiento, filtración ó por la necesidad de elevar el agua á un nivel más alto que aquel en el cual se bota.

Cuando cualquiera de las máquinas anteriores las mueven hombres por medio de tornos, el resultado es como la mitad menos que con la rueda de pie. Todas son frecuentemente movidas al vapor, agua, ó fuerza de un caballo.

**Caminando hacia adelante y hacia atrás sobre una palanca que oscile sobre su centro**, un hombre puede, según la *Filosofía Mecánica de Robinson*, ejecutar un trabajo mucho mayor que por medio de cualquiera de los métodos precedentes. Él dice que un joven con peso de 135 libras (61.23 kg), cargado con 30 litros (13.60 kg), trabajó de este modo 10 horas al día sin fatigarse y elevó 9¼ pies cúbicos (261 litros) de agua á 3.5 m de altura por minuto.

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* En el día de trabajo deben comprenderse los descansos necesarios, y además los intervalos que exija la naturaleza del trabajo; pero no el tiempo perdido en las comidas. *Un día de trabajo* de 10 horas no puede tener por tanto sino 8, 7 ó 6, etc., horas de *trabajo efectivo*. Así debe entenderse cuando en lo sucesivo hablemos de un día de trabajo, ó simplemente de un día.

** Los cálculos de *Desaguliers* respecto al trabajo diario de hombres y caballos exceden á los anteriores; son muy exagerados.

*** El tambor gira sobre un eje horizontal; y es una especie de rueda grande, cuyos rayos ó brazos son cajuelas que terminan en sus extremos exteriores en paletas que se hunden en el agua. A proporción que el agua se eleva gradualmente, se desliza á lo largo de los brazos de la rueda hasta sus ejes donde se descarga. La rueda de paleta es una modificación de ellas. Es una máquina admirable para elevar grandes cantidades de agua á una altura moderada. No podemos entrar en detalles respecto á esta y otras máquinas hidráulicas.

§ Una especie de rueda grande con cubos ó cajones en los extremos de sus radios. Gira sobre un eje horizontal; se descarga por encima. Los cubos están sujetos separadamente, de modo que cuelguen verticalmente, evitando así derramamientos hasta que llegan á su propio punto donde se ponen en contacto con una pieza adecuada para vaciarlos. La noria es parecida, excepto que los cubos están firmemente colocados en su lugar y derraman mucha agua. Es por tanto inferior á la rueda Persa.

‖ Una interminable cadena vertical giratoria de cajones. D'Aubuisson y algunos otros la llaman erróneamente *noria*. Es una máquina muy eficaz.

Esto es igual á 3,984,000 pies-libras (3,984,000 × .138253 = 550,792 kilográmts) por día de 10 horas ó 6,640 pies-libras (917.99 kilográmetros) por minuto, ó cerca de $^1/_{10}$ del trabajo diario, neto, de un caballo en una grúa.

**Un trabajador, de pie,** apenas puede sostener por algunos minutos una carga de 100 libras (45.36 kg) por medio de una cuerda que pasa por sus hombros y horizontalmente por una polea. Y más difícil aún de sostenerla si con las manos extendidas hacia adelante sujeta el extremo de una cuerda horizontal (frente á frente á la carga y la polea). No puede empujar horizontalmente con las manos á la altura de sus hombres más de 30 libras (13.60 kg).

Weisbach hace constar por su propia observación que cuatro hombres prácticos estuvieron suspendiendo un pisón de madera, con cuatro barras redondas horizontales y peso de 120 libras (54.43 kg) á 4 pies (1.20 m) de altura, á razón de 34 veces por minuto, por intervalos de 4½ minutos alternando con otros 4½ minutos de descanso, y así sucesivamente durante 10 horas del día de trabajo.

Por tanto, 5 de estas horas se perdieron en descansos, y el trabajo ejecutado por cada hombre durante las otras 5 horas ó 300 minutos, fué

$$\frac{120 \times 4 \times 34 \times 300}{4} = 1{,}224{,}000 \text{ pies-libras, ó sean}$$

$$\frac{54.43 \times 1.22 \times 34 \times 300}{4} = 169{,}331 \text{ kilográmetros.}$$

**Efectuando el tiro por medio de caballos.** (Véase Tracción.) Si se trabaja todo el día, digamos diez horas, el término medio de la velocidad con que camina un caballo yendo con el carro lleno y regresando con él vacío, es como de 2 á 2¼ millas (3,218 á 3,620 m) por hora; pero teniendo en cuenta las paradas y descanso, etc., es más aproximado calcularlo solamente como 1.8 millas (2,896 m) por hora es decir 48.3 por minuto.

El tiempo perdido en cada viaje, cargando y descargando, se puede generalmente calcular como en 15 minutos. Por tanto, para encontrar el número de cargas que pueden ser llevadas á cualquier distancia dada en un día, búsquese primero el tiempo que se necesita en minutos para llevar una carga y regresar vacío. Así: divídase el doble de la distancia á la cual debe ser llevada la carga, ó, en otras palabras, divídase la longitud del viaje redondo por 48.3 m. El cociente es el número de minutos que el caballo está en movimiento durante cada viaje redondo. A este cociente, agréguesele 15 minutos perdidos en cada viaje, mientras se carga y se descarga; la suma es el tiempo total en minutos empleado en cada viaje redondo. Divídase el número de minutos de un día de trabajo (600 minutos en un día de 10 horas de trabajo) por el número de minutos que se requiere para cada viaje; el cociente será el número de viajes ó de cargas tiradas por día.

Ejemplo. ¿Cuántas cargas conducirá un caballo en un día de 10 horas de trabajo, ó 600 minutos, á una distancia de 292.60 m? En este caso tendremos: $292.60 \times 2 = 585.2$ m. Y $\frac{585.2}{48.76} = 12$ m, y agregando 15 tendremos 27. Finalmente, $\frac{600}{27} = \frac{\text{minutos en 10 horas}}{\text{minutos por viaje}} = 22.2$, ó digamos 22 viajes ó cargas tiradas por día.

**Tabla del número de cargas tiradas por día de 10 horas de trabajo.** La primera columna es la distancia á la cual la carga es llevada efectivamente, ó la mitad de la extensión del viaje redondo. El costo por carga llevada se supone ser para un carro de caballo, cargando y descargando el carretero, calculando el gasto del caballo, el carro y el carretero á $2 por día.

| Distancia en metros. | Número de cargas. | Costo por carga en dólares. | Distancia en metros. | Número de cargas. | Costo por carga en dólares. | Distancia en metros. | Número de cargas. | Costo por carga en dólares. |
|---|---|---|---|---|---|---|---|---|
| 15 | 38 | .053 | 450 | 18 | .11 | 1,610 | 7 | .29 |
| 30 | 37 | .054 | 600 | 15 | .13 | 2,000 | 6 | .33 |
| 60 | 34 | .059 | 760 | 13 | .15 | 2,400 | 5 | .40 |
| 90 | 32 | .062 | 900 | 11 | .18 | 3,200 | 4 | .50 |
| 120 | 30 | .067 | 1,060 | 10 | .20 | 4,830 | 3 | .67 |
| 180 | 27 | .074 | 1,220 | 9 | .22 | 6,450 | 2 | 1.00 |
| 300 | 22 | .091 | 1,525 | 7 | .29 | 14,500 | 1 | 2.00 |

Si la carga y descarga es tal que no pueda hacerse por el carretero solo, sino que necesita de la ayuda de máquinas, puede hacerse necesario un aumento de 10 á 50 cts (de dólar) por carga. El tiro se puede hacer generalmente más barato usando 2 ó 3 caballos y un carretero para un vehículo. La carga neta por caballo, en adición al vehículo, será generalmente de 500 á 1,000 kg, dependiendo de la condición é inclinación del camino. De .368 á .425 m cúb de piedra sólida, ó de 23 á 27 pies cúb (.651 á .793 m cúb) de piedra suelta, hacen 1 tonelada (1,000 kg). **Al calcular la carretada de piedra ordinaria de cantera para muros, alcantarillas, etc.**, recuérdese que cada metro cúbico de mampostería común requiere la tracción como de 1.2 m cúb de piedra tal como se acostumbra amontonarla para la venta en la cantera, ó como $^{9}/_{4}$ de metro cúbico de la roca original antes de banquearla. **Cuando se parte en pedazos un metro cúbico de piedra sólida, ocupa generalmente como 1.9 m cúb, si están sueltas, en desorden las fracciones,** ó como $1\frac{3}{4}$ si están amontonadas. Un carro fuerte para cargar piedra pesa como $\frac{2}{3}$ de tonelada y contiene piedra suficiente para 700 litros de mampostería de piedra bruta. El peso medio de un buen caballo de trabajo es como de 500 kilogramos *.

**Morín da los resultados siguientes** tomados de sus experimentos hechos cuidadosamente para el Gobierno de Francia. El tiro del mismo vehículo de rueda en un camino se considera que cambia en la práctica :

**1.° En caminos empedrados y pavimentos,** en proporción á las cargas; en sentido inverso á los diámetros de las ruedas, y casi independiente de las dimensiones de la llanta. Aumenta hasta cierto grado con las desigualdades del camino, la dureza (falta de resorte) del vehículo, y la velocidad, pero en proporción considerablemente menor que la raíz cuadrada de la última.

**2.° En caminos blandos el tiro es menor con llantas anchas que con angostas;** y para aplicaciones agrícolas se recomienda una anchura de 10 cm.

Con velocidades desde el paso hasta un trote ligero, el tiro no varía sensiblemente.

# ARMADURAS

## INTRODUCCIÓN

### Principios generales.

**1. El diseño de las armaduras es una especialidad.** El diseño, construcción y erección de las armaduras ha llegado á ser una especialidad, á la cual algunas personas se dedican más ó menos exclusivamente, obteniendo de esta manera cierto grado de conocimiento superior al del ingeniero no especializado en esta materia. Este último, sin embargo, debe poseer un conocimiento del asunto suficiente á lo menos para hacerlo capaz de formar una opinión bien fundada de los méritos generales de un diseño y evitar la adopción del que contenga imperfecciones serias *. En un libro como éste podemo discutir solamente los principios generales.

**2. El principio de las armaduras.** Teóricamente una armadura se compone de cierto número de barras rectas, unidas, cerca de sus extremos, por uniones perfectamente flexibles, cargadas solamente en estas uniones, y arregladas de tal manera que sus piezas sostengan todas las presiones internas, y que sólo las presiones verticales **, debidas á los pesos de la armadura y su carga, se transmitan á los extremos.

**3. Diferencia entre las vigas y las armaduras.** Cuando una viga sólida (§ 7, fig. 2, « Resistencia transversal ») se encorva bajo su propio peso ó el de su carga, todas las fibras que están encima del eje neutro se comprimen, y las que se encuentran debajo de él, se alargan; y el cambio que resulta en el largo de cada fibra, es proporcinal á la distancia de la fibra al eje neutro; pero, en una armadura, las cargas (incluyendo el peso de la armadura), se consideran teóricamente divididas en partes que se encuentran en las uniones de las barras (nudos) y que obran por el centro de gravedad de sus secciones transversales. Colocadas así, las presiones por ellas ejercidas no podrían obrar transversalmente respecto á los miembros, como en una viga, produciendo lo que se llama presiones secundarias, pues tienen que obrar longitudinalmente por el eje de las barras y deben distribuirse uniformemente sobre las áreas enteras de sus secciones transversales. Este es el carácter distintivo de todas las armaduras.

**4.** En las armaduras se debe usar el material con mucha economía, y las presiones en cada pieza y en cada parte de dicha pieza pueden determinarse con prontitud y exactitud.

**5.** En una armadura de techo ó puente bien trazada, se llena esta condición ideal, usando como miembros principales, piezas rectas y más bien delgadas, y distribuyendo la carga exterior de tal manera que obre solamente en las uniones (nudos) de las barras, y disponiendo las fuerzas de modo que obren principalmente en sus extremos y á lo largo de las piezas de la armadura.

En las armaduras unidas con pasadores (véase § 175) en las uniones son prácticamente flexibles.

La mayor parte de las armaduras en uso común se componen de dos vigas largas, frecuentemente horizontales (pero véase § 49), llamadas **cordones**, que abarcan toda la luz del puente ó techo, y unidas por piezas ó barras, que están algunas veces todas inclinadas, y otras alternativamente verticales é inclinadas. Las piezas inclinadas se llaman diagonales.

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* Las Compañías de ferrocarriles y las Corporaciones municipales, casi siempre redactan las condiciones que deben satisfacer sus puentes; pero las proporciones generales, el número de tramos, etc., se dejan comúnmente á la elección de los constructores.

** Suponemos aquí que la armadura se va á cargar verticalmente. Si la carga obra de otra manera, como sucede con la presión del viento en una armadura reforzada horizontalmente, la presión en los apoyos puede ser horizontal, ó en cualquiera otra dirección, pero todas las fuerzas internas estarán aún soportadas por los miembros de la armadura.

**6. Tirantes y puntales.** El miembro sometido á tensión se llama tirante ó cuerda. El que sostiene presión se llama puntal ó poste. El que sostiene ambas se llama cuerda-puntal ó puntal-cuerda.

**7.** Las dimensiones de las armaduras se miden comúnmente en las líneas centrales de sus miembros; y en las armaduras unidas por pasadores, éstos se colocan en las intersecciones de estas líneas. De aquí que las medidas se hagan ordinariamente de « centro á centro de los pasadores ».

**8. En una viga llena** consideramos que las tablas ó alas ejercen la función de cordones de la armadura, y el alma de la viga representa la trabazón de la armadura.

## Cargas.

**9. Cargas muerta y viva.** En los puentes, hacemos distinción entre las cargas « muerta » y « viva »; la carga muerta comprende el peso permanente de la estructura; es decir, del mismo puente, con su pavimento, etc.; la carga viva comprende todas las cargas exteriores temporales, tales como máquinas, carros, caballos, vehículos, caminantes, etc., que trafiquen por el puente.

**10.** La carga muerta se distribuye siempre uniformemente á lo largo de la abertura; pero el cordón cargado (el que soporta la vía de tráfico) recibe, por supuesto, una parte mayor que el no cargado. La carga viva sólo obra sobre el cordón cargado. Para determinar los esfuerzos, se considera que el peso de la carga viva y del pavimento están sobre el cordon cargado, y el resto de la carga muerta dividido por igual entre los dos cordones. Algunas veces sucede, sin embargo, que ambos cordones, el superior y el inferior, soportan vías de tráfico. Es claro que se les debe considerar entonces como « cargados », aunque no por igual, porque quizás uno soporte un ferrocarril, y el otro una vía ordinaria solamente.

## Carga no simétrica. Contradiagonales.

**11. La carga no simétrica.** En las figs. 2 á 10, se supone que las cargas están colocadas simétricamente.

**12.** Si este fuese el caso en la práctica, no se someterían nunca las barras de compresión á resistir tensión, ni las de tensión á resistir compresión; y las armaduras de las figs. 2 á 10 serían suficientes (suponiendo que cada barra tenga la necesaria resistencia), aunque las barras de compresión no fueran capaces de resistir tensión y viceversa. Así pues, los tirantes ó barras de tensión podrían ser cadenas flexibles, y los de compresión, postes, solamente obrando contra los apoyos en sus extremos.

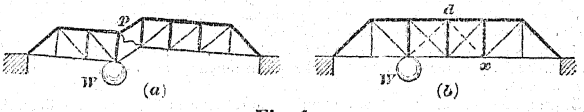



**Fig. 1.**

**13.** Pero en una armadura, fig. 1 (*a*) con un tirante flexible en el tramo, *p*, como está indicado, la carga W, colocada no simétricamente, causaría el efecto que indica el dibujo.

**14. Contradiagonales.** Para impedir esto, las barras que, bajo cargas movibles, puedan someterse alternativamente á tensiones y compresiones, pueden construirse de manera que resistan ambas especies de fuerzas. Es decir, las piezas de tensión pueden ser tan rígidas que puedan obrar como postes, y los extremos de las barras de compresión unidos de tal manera á las cuerdas que puedan obrar también como cuerdas. Este es el recurso que se emplea con frecuencia en las armaduras que no tienen barras verticales.

**15.** (*Obs. del T.* — Es oportuno hacer la siguiente explicación para lo que va á seguir. Marvá en su magnífica obra *Mecánica aplicada á las construcciones*, 4.ª edición, págs. 249 y 250, llama *brazos*, las barras inclinadas cuando trabajan por compresión, y *tirantes*, si están sometidas á extensión. Luego dice : « Para evitar que una barra sufra alternativamente esfuerzos de compresión y de extensión por la posición variable de las cargas móviles, se agrega en algunos sistemas otro grupo de barras inclinadas en sentido contrario, cuyo objeto es impedir el cambio de signo de las fuerzas en las barras primitivas, ó, lo que es lo mismo, hacer que cada barra trabaje

siempre por compresión ó por extensión, ó no sufra esfuerzo alguno. A este efecto, cuando una barra del primer sistema que está trabajando por compresión, por ejemplo, va á sufrir una extensión, hay una barra adicional del segundo sistema, que entrando á funcionar en aquel momento lo impide, sufriendo entonces una compresión y haciendo que la primera barra no experimente esfuerzo alguno. »

Llama *contradiagonales* á las del segundo sistema, ya sufran compresión ó extensión. *Contrabrazos* á las barras inclinadas en sentido contrario de las que trabajan por compresión, y *contratirantes* á las situadas en sentido contrario de las que trabajan por extensión.)

Así pues, la deformación, fig. (*a*), causada por una carga no simétrica, se evita introduciendo otros miembros que se llaman como hemos dicho, para distinguirlos de los miembros principales, cuya función es resistir las fuerzas normales debidas á cargas uniformes ó simétricamente distribuídas. Así, en la fig. 1 (*b*), la carga no simétrica W, tiende á convertir el rectángulo *p*, en un paralelogramo alargando su diagonal W*d*, lo cual puede evitarse introduciendo un miembro oblicuo de tensión en dicha diagonal, como lo indica la línea de puntos. Por la misma razón, se colocará otro igual en el tramo correspondiente, *xd*.

**16. Triángulos.** Se observará que la introducción de las contradiagonales hace que la armadura esté compuesta solamente de *triángulos*.

**17.** A primera vista aparecerá que las diversas partes de la armadura de un puente reciben la mayor presión cuando el máximum de la carga se extiende de extremo á extremo; pero esto es cierto solamente respecto al cordón y las principales partes, diagonales y verticales, próximas á los *extremos* de la armadura. Las otras barras podrán sufrir mayor esfuerzo de una parte de la carga, colocada no simétricamente en la armadura; de manera que, aunque de proporciones adecuadas para una carga total uniforme, puedan resultar débiles para una carga parcial. Si se construyen todas tan fuertes como la de los extremos, quedarán ciertamente seguras para una carga pasajera; pero esto requeriría un gasto de material, excusable sólo en casos en que la luz del puente sea moderada, y particularmente en los de madera, en que la construcción de algunos miembros usando pedazos de diferentes tamaños haría una economía en el material que quedaría más que anulada con la molestia de conseguir y unir adecuadamente piezas de tan diversas dimensiones.

**18.** En los puentes grandes en los que la carga viva es pequeña con relación á la muerta, se necesitan pocas contradiagonales, y solamente en el centro y cerca de él, mientras que en un puente muy ligero, las contradiagonales deben extenderse desde el centro, donde reciben más presión, hasta los extremos, donde la presión es la menor.

## Riostras.

**19. Ligazón entre las armaduras.** Se aprovecha la proximidad de dos ó más formas ó armaduras en un puente, uniéndolas con *riostras*, dando de esta manera á la estructura entera mayor estabilidad lateral que la que se obtiene en las armaduras sencillas.

**20.** De esta manera, las riostras de un lado al otro, fig. 39, forman verdaderas armaduras horizontales colocadas entre los dos cordones superiores de las armaduras principales, ó entre los dos inferiores, ó á la vez entre ambos; obrando los cordones de las armaduras principales como cordones de las armaduras horizontales laterales. Las **riostras laterales** impiden la desviación lateral de los cordones.

**21.** Las situadas como en la fig. 64 (*c*) (llamadas también laterales, de vibración contra el viento) se componen de armaduras cortas (ordinariamente verticales) que cruzan el puente transversalmente, uniendo de esta manera las dos armaduras mayores. Estas tienen su cordón propio, pero usan las partes de los postes de la armadura principal como sus postes extremos.

**22. Barras de portales**, fig. 54 (*a*), son los que (á veces en un plano inclinado) unen las partes superiores de los extremos de los postes, en las armaduras de altura suficiente para permitir su uso. Esta armadura de unión, con sus postes finales, forma un portal por el cual los trenes, etc., entran en el puente.

## Tipos de vigas armadas.

**23. La forma más simple de vigas armadas**, se compone de un triángulo simple, figs. 2 (*a*) y (*b*). En la fig. (*a*) la carga produce compresión en los pares

tensión en el cordón y compresión * (=tensión en el cordón) entre las cabezas de los pares; en la fig. (*b*), viceversa.

**24.** La viga representada en la fig. 2 (*a*) es de uso común para los techos de poca luz, en casas de habitación. En la práctica se carga, por supuesto, á lo largo de las vigas, y no solamente en el vértice como en la fig. (*a*); pero, calculando los esfuerzos de las barras de las armaduras, ordinariamente se supone que las cargas están concentradas en las intersecciones de las barras. El efecto de su distribución *á lo largo* de las barras se determina separadamente, considerando las piezas como vigas.

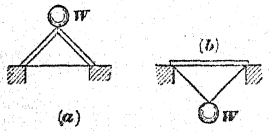



Fig. 2.

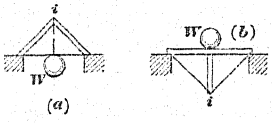



Fig. 3.

**25.** En la fig. 3 (*a*) la cuerda vertical llamada *pendolón* y en la fig. 3 (*b*) el poste vertical llamado mangueta, traen simplemente el peso de la carga al vértice, *i*, donde produce el mismo efecto que en las figs 2 (*a*) y (*b*).

**26.** De aquí que, despreciando los pesos de la cuerda vertical y de otras piezas, las presiones ejercidas por una carga dada, W, en las diagonales, y en la cuerda horizontal, fig. 3 (*a*), son las mismas (no solamente en su acción, sino en intensidad) que las producidas por una carga igual W, en la de la fig. 2 (*a*). Igualmente, las de la fig. 3 (*b*) corresponden á las de la fig. 2 (*b*).

**27.** Las figs. 4, 5 y 6, que indican modificaciones de las formas simples, dadas en las figs 2 y 3, muestran en principio, la mayor parte de las vigas armadas en uso común en los puentes, cuya luz es de 90, 120 y aun 150 m. Véanse las figs. 7 á 10, §§ 35, etc.

**28.** En las figs 4, 5, 6, hay un cordón superior en compresión y un cordón inferior en tensión: el cordón más corto sostiene la compresión entre los pares, figs. 2 (*a*) y 3 (*a*), ó la tensión horizontal entre los pies de las piezas diagonales, figs. 2 (*b*) y 3 (*b*). Las figs. 4 (*a*) y 5 (*a*) son modificaciones de las figs. 2 (*a*) y 3 (*a*); figs. 4 (*b*) y 5 (*b*) lo son de las figs. 2 (*b*) y 3 (*b*); la fig. 6 (*a*) de la fig. 2 (*a*), y la fig. 6 (*b*) de la fig. 2 (*b*).

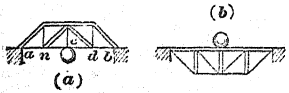



Fig. 4.

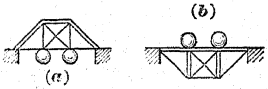



Fig. 5.

**29.** Las figs. 4 (*a*) y 4 (*b*) pueden considerarse como ilustraciones de las figs. 3 (*a*) y 3 (*b*) respectivamente, con la mangueta, así como la carga dividida en dos, y las dos partes separadas por un cordón horizontal. Si las cargas están colocadas simétricamente, de manera que las compresiones horizontales, fig. 4 (*a*), ó tensiones, fig. 4 (*b*), en los dos extremos del cordón más corto, sean iguales, las barras diagonales en el centro son innecesarias.

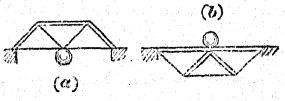



Fig. 6.

**30. Sistemas Howe y Pratt.** En la fig. 5 (*a*) las barras verticales de la viga están en tensión, y las diagonales en compresión, según el principio de « Howe », que se usa en los puentes de maderos, diagonales; mientras que, en la fig. 5 (*b*), las

* En las figs. 2 á 12, y 14 á 17, las líneas dobles ó gruesas indican postes ó puntal (sometidos á compresión), y las líneas delgadas indican cuerdas (sometidas á tensión

verticales sufren compresión, y las diagonales tensión, según el principio de «Pratt», usado en los puentes de diagonales metálicas. En tales puentes son objetables las barras sometidas á compresiones.

**31. Viga de armadura triangular ó de Warren.** En la fig. 6, que muestra la viga «triangular» ó de «Warren», las barras del tejido son todas diagonales y están alternativamente en tensión y en compresión. Dividen la armadura en triángulos *isóceles*.

**32. Armaduras de tablero superior, inferior, y armaduras pequeñas** (Pony). Las figs. 4 (*a*), 5 (*a*) y 6 (*a*), con la vía de tráfico entre los cordones inferiores, se llaman de «tablero inferior», y las figs. 4 (*b*), 5 (*b*) y 6 (*b*), con la vía de tráfico entre los cordones superiores, se llaman de «tablero superior». Éstas permiten el uso de barras diagonales en plano vertical contra la oscilación horizontal (véase § 21) entre, y por toda la altura de las dos ó más armaduras que forman el puente; la de tablero inferior, por supuesto que no lo permite. Pero el uso del tablero inferior es necesario á menudo, para dar amplia sección de paso (*head-room*) á los botes, las crecientes, los trenes en los caminos que se cruzan, etc., por debajo del puente. Una armadura, cargada en los cordones inferiores, pero demasiado baja para ligarla con barras laterales (véase § 20) entre los dos cordones superiores, se llama armadura (*pony*). (*N. del. T.* — No tiene que sepamos equivalente en español; la única palabra que puede aplicarse, es la muy familiar *rechoncha.*)

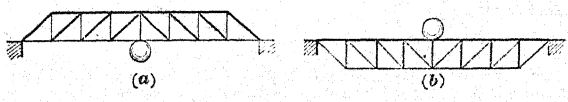
**33. Tramos.** Los puntos en que las barras verticales del tejido se encuentran con las cuerdas, en las figs. 4 y 5, se llaman puntos de división de los tramos, y los espacios rectangulares, *an*, *nc*, *cd*, etc., fig. 5 (*a*), entre las verticales, se llaman tramos.

**34. La viga Warren,** fig. 6, no tiene barras verticales, como partes esenciales de ella. Véanse los §§ 45 y 46 Sus subdivisiones se llaman triángulos simples, y un tramo es una longitud de la armadura igual al ancho de un triángulo. Una panela en cualquiera cuerda, es, sin embargo, la parte de ella comprendida entre dos puntos de división de los tramos.

**35.** Las modificaciones posteriores de estos diseños, con mayor número de tramos, se ven en las figs. 7 á 10. Las figs. 7 (*a*) y 8 (*a*), con barras verticales en tensión, representan la viga Howe de las figs. 4 (*a*) y 5 (*a*), mientras que las figs. 7 (*b*), 8 (*b*), 9 (*a*) y 9 (*b*), con las diagonales en tensión, representan la viga Pratt, de las figs. 4 (*b*) y 5 (*b*). Las figs. 10 representan la viga Warren de la fig. 6.

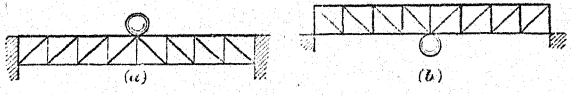
**36.** La fig. 8 (*a*) representa simplemente la fig. 7 (*a*), que ha descendido para convertirse en un puente de tablero superior en lugar de un puente de tablero inferior; y la fig. 8 (*b*) representa la fig. 7 (*b*) convertida de tablero superior en inferior, descansando en los extremos de postes verticales.

**37.** En las figs 8, los extremos de los postes verticales, y la pieza horizontal á cada extremo del cordón cargado, no forman parte de la armadura en sí. Estas últimas obran simplemente como vigas, soportando la carga durante su paso del estribo á la armadura y viceversa. El extremo del poste en la fig. (*a*) soporta solamente un extremo de esta viga, mientras que el de la fig. (*b*) soporta la mitad.




**Viga Howe, de tablero inferior.** **Viga Pratt, de tablero superior**

**Fig. 7.**




**Viga Howe, de tablero superior.** **Viga Pratt, de tablero inferior.**

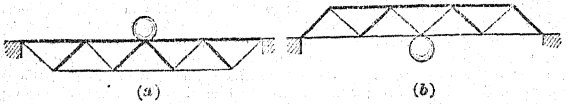
**Fig. 8.**

**38.** En las figs. 8 la vertical del medio no soporta ninguna parte de la carga. Teóricamente sirve sólo para impedir la flexión por su propio peso, de los dos tramos no cargados del medio del cordón.. Pero en la práctica tales miembros se

colocan con el objeto de obtener conexiones convenientes para las piezas laterales, tales como las vigas del piso.

**Viga Pratt, de tablero superior.** **Viga Pratt, de tablero inferior.**

**Fig. 9.**




**Viga Warren, de tablero superior.** **Viga Warren, de tablero inferior.**

**Fig. 10.**

**39.** En las figs. 9 (modificaciones de la fig. 7 *b*), y en la fig. 10, se muestran, en principio, las formas más comunes de las vigas de puentes metálicos, que se usan con tablero superior é inferior respectivamente.

**40.** En la fig. 9 (*a*) como en la fig. 8, los postes verticales de los extremos y las piezas horizontales á los extremos del cordón cargado no forman parte de la viga, y en la fig. 9 (*b*) como en la fig. 8, la vertical del medio soporta solamente los centros no cargados del cordón de los tramos.

**41. Intersecciones.** En las vigas altas se combinan algunas veces dos ó más cuerpos de trabazón en una sola viga, con un solo par de cordones. De esta manera las dos vigas simples de Pratt, de las figs. 11 (*a*) y (*b*), se combinan para formar la armadura de « Whipple », ó de « doble intersección de Pratt », fig. 11 (*c*), que está últimamente muy en uso.

(*Obs. del T.* — Estas son las vigas *compuestas* que se obtienen por la yuxtaposición de dos vigas simples del mismo sistema. Muévase paralelamente la fig. 11 (*a*) sobre a (*b*) hasta la completa coincidencia y resultará la viga, *c*.)

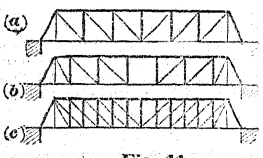



**Fig. 11.**

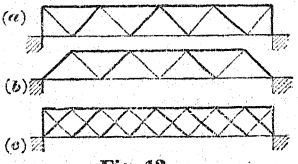



**Fig. 12.**

**42.** De igual manera las dos vigas simples de Warren, en las figs. 12 (*a*) y (*b*), se yuxtaponen para formar la de doble intersección (compuesta) de Warren de la fig. 12 (*c*).

**43.** La combinación de cuatro sistemas se llama viga de cuádruple intersección. Véase fig. 59 (*t*).

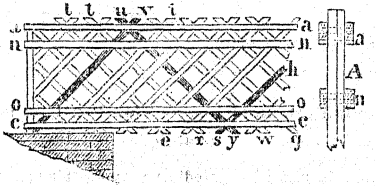


**Fig. 13.**

**44. La vieja viga de « celosía » «Towne »**, fig. 13, se compone de planchas que se entrecruzan (ordinariamente en ángulos rectos), y espigadas ó atornilladas en sus intersecciones, pueden considerarse como una combinación de varias vigas Warren.

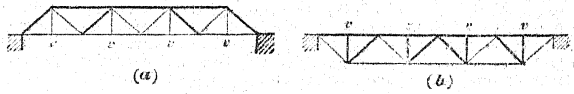


Fig. 14.

**45. Subverticales.** En las vigas altas, en que la extensión horizontal de los tramos es considerable, las subverticales, *v*, figs. 14 y 15, se usan á menudo, con especialidad en las armaduras Warren, para sostener los segmentos del cordón cargado. Véanse también las figs. 59 (*i*), (*r*), y (*s*).

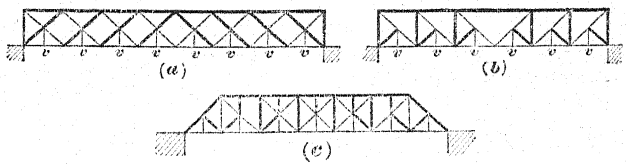


Fig. 15.

En la viga « **Baltimore** », fig. 15 (*b*), cada diagonal está apuntalada por un brazo diagonal corto inclinado en la dirección opuesta, y hay una subvertical suspendida en su unión. En los tramos muy largos, las subverticales algunas veces se usan también para los tramos del cordón no cargado. Véase fig. 15 (*c*).

**46. Puntales de colisión**, S, figs. 59 (*k*), (*m*), (*o*), y (*t*), y (73) *a*, se usan para apoyar largos postes extremos diagonales, contra el golpe de un tren descarrilado.

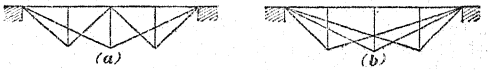


Fig. 16.

**47. Vigas Fink y Bollman.** Las figs. 16 muestran dos modificaciones anticuadas de la fig. 3 (*b*); es decir, la Fink, fig. 16 (*a*) y la Bollman, fig. 16 (*b*). El gran puente sobre el río Ohio en Louisville, Ky, terminado en 1870, es del tipo Fink. La Bollman se usó mucho en el ferrocarril de Baltimore y Ohio, hace años.

**48.** En las vigas Fink y Bollman había sólo un cordón como lo indicamos. A nivel de este cordón pasaba la vía ordinariamente. Cuando la vía se colocaba más baja, la viga parecía tener dos cordones. Bajo cargas distribuídas uniformemente, en la Fink, y en todas las circunstancias en la Bollman, la presión sobre este cordón era uniforme en todo él. En la Bollman (véase la fig.) los esfuerzos longitudinales en la viga se aplicaban todos en sus extremos. Cada tipo puede considerarse como una combinación de varias vigas de suspensión semejantes á la fig. 3 (*b*). En la Bollman, las vigas simples son todas de la misma luz y profundidad, y todas las manguetas, excepto la del centro, dividen desigualmente á las vigas simples. El sistema Fink se usa todavía mucho en las armaduras de metal para techos. Véanse las figs. 26.

Fig. 17.

**49. Vigas arqueadas.** Las vigas con cordones arqueados ó « quebrados », fig. 17, se usan ordinariamente en las grandes luces. Las barras colocadas entre los puntos de separación de los tramos son siempre rectas. En las armaduras de arco y cuerda (*bowstring*), fig. 17, los puntos de división de los tramos del cordón supe-

rior se encuentran en una línea curva, convexa hacia arriba. En la armadura de **media luna** (*crescent*), el cordón inferior es también convexo hacia arriba. La armadura de arco y cuerda tiene la ventaja, sobre las de cordón superior horizontal, de hacer que todos los esfuerzos en los cordones y barras sean más próximamente iguales, simplificando de esta manera la construcción y disminuyendo su peso. Tiene la desventaja de que no permite riostras transversales cerca de los extremos de la luz. Si la curva del cordón superior se hace parabólica, la presión de la carga muerta es uniforme en todo el cordón inferior, y en todas las barras verticales (ahora en tensión), el esfuerzo es igual á la carga muerta sobre la viga inferior. Las diagonales no trabajan con la carga muerta, sino con las cargas excéntricas.

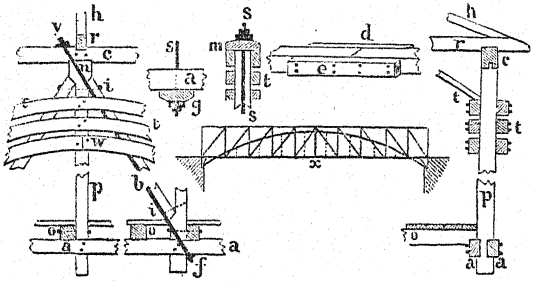



**Fig. 18.**

**50. La viga «Burr»**, fig. 18, en un tiempo muy usada en los puentes de madera, era una combinación de una armadura Howe y un arco.

### Ligera curvatura de los cordones.

**51.** En la práctica, los extremos de las barras del cordón superior é inferior no se colocan perfectamente en línea recta, sino de manera que los cordones se arqueen ligeramente, con la convexidad hacia arriba. Su objeto es impedir que la armadura descienda debajo de la horizontal, al estar cargada fuertemente. Cuando las vigas están combadas (véase *ys* y *cd*, fig. 19), se convierten aproximadamente en arcos

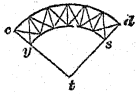



**Fig. 19.**

concéntricos de dos círculos de gran radio, cuyo centro está en *t;* el superior, por supuesto, más largo que el inferior. Las barras verticales, en lugar de permanecer verdaderamente verticales, se convierten en partes del radio de los arcos mencionados, y aunque sus largos permanezcan invariables, las partes superiores están, sin embargo, más separadas que los pies; haciendo necesario alargar las diagonales. Véanse §§ 211-214.

### Vigas consolas (*Cantilevers*).

**52. El principio de los cantilevers** se muestra en la fig. 20, en que A y B

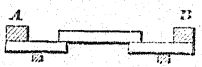



**Fig. 20.**

representan contrapesos. La fig. 21 hace ver el puente de (cantilevers) contrapesos del Niágara. Se compone de dos armaduras en cantilever, *ab*, *a'b'*, unidas por una

armadura ordinaria, que está suspendida por un eslabón, vertical en cada punta, de los extremos de las consolas. El peso de la armadura está contrabalanceado por anclajes A y B, ó por pesos, ó por ambos. La ventaja principal del cantilever, es que se puede construir hacia afuera de los estribos á través del canal; facilitando, de esta manera, grandemente, su erección en los casos en que no pueda usarse una construcción provisional.

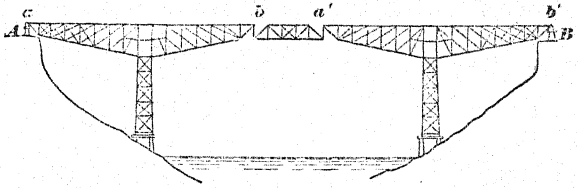



Fig. 21.

## Puentes movibles.

**53. Los puentes movibles**, incluyendo los levadizos, giratorios, y de alzamiento, son de tres clases generales, una en que la parte movible se desliza horizontalmente, otra en que gira horizontalmente, y otra en que gira verticalmente.

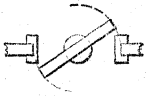


Fig. 22.

De ordinario la sección movible tiene su eje cerca del medio, y gira horizontalmente en el estribo central donde está el eje ó *pivote*, como en la fig. 22. Estos puentes se montan ordinariamente sobre un gran eje central, ó sobre un juego de cilindros, ó sobre ruedas que corren en una vía circular. Los puentes de esta clase deben construirse de manera que estando abiertos, ó sin haberse ajustado en los extremos al cerrarse, sostengan no solamente su propio peso, sino cualquiera otra carga que esté sobre ellos. Además, cada mitad debe resistir como un puente sostenido en ambos extremos, con todas las cargas vivas posibles, pues si no, al llegar una carga viva á cualquier extremo, dicho extremo no ajustaría. Los puentes bien construídos se disponen de manera que se puedan alzar los extremos de la parte movible, al cerrarse, haciéndolos ajustar firmemente, y quedando el pavimento á nivel con el del estribo fijo adyacente. Este alzamiento se hace ordinariamente lo suficiente para aliviar la pila del medio de una parte de la carga solamente. El puente obra entonces como una viga « continua » (véase Resistencia transversal, §§ 78, etc.), sostenido en tres ó cuatro puntos, que dependen de la disposición de la carga en la pila donde está el eje.

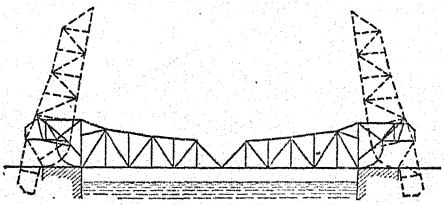


Fig. 23.

**54.** Los puentes levadizos en que la parte movible oscila verticalmente (teniendo el brazo corto un contrapeso), pueden girar en un *pivote* ó eje, ó rodar como en el

puente levadizo rodante Scherzer, fig. 23. En el puente Scherzer el centro de gravedad de cada uno de los dos brazos ó palancas movibles permanece á una elevación constante, de manera que no suba ni baje con ninguno de ellos, y el trabajo que se hace consiste sólo en vencer la fricción de rotación de la parte curva del brazo sobre su soporte.

**55. Los puentes oblicuos** se usan donde hay que cruzar un canal, un camino, etc., oblicuamente, y donde no es conveniente hacer los estribos perpendiculares á las vigas. Para la simplicidad al hacer las conexiones del pavimento, etc., la armadura se construye ordinariamente de manera que las divisiones de los tramos queden enfrente unas de otras, como en las figs. 24 y 25. Cuando la oblicuidad es ligera, se necesita una diferencia en la inclinación entre los dos postes de los

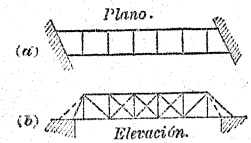



Fig. 24.

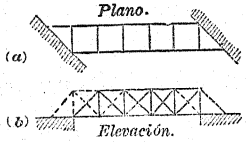



Fig. 25.

extremos, como en la fig. 24, complicando las conexiones de los cruceros en la entrada del puente. Pero cuando la oblicuidad es mayor, es posible hacerla justamente igual á uno ó más tramos completos, arreglando el largo de los tramos para que se correspondan, quedando las vigas simétricas, como en la fig. 25. En cada figura, la viga *más distante* (la superior en el plano) está indicada por líneas de puntos.

## Armaduras de techo.

**56. Las armaduras de techo** se construyen en una gran variedad de formas. Las que se ven en las figs. 26 son comunes. En la fig. 26 (*a*), parte de la carga, en *d*, comprime el par *ia* de *d* hasta *a*, mientras que el resto comprime el puntal, *dh*, y prensa la cuerda *hi* y la parte *ha* del tirante. Igualmente, una parte del peso de *e* pasa por *ca* hacia *a*, y el resto por *ckdhi* al vértice *i*. De esta manera cada carga es llevada por las piezas, parte al vértice y parte á lo largo de un par hasta un estribo. Se verá que los esfuerzos mayores en los pares y en los tirantes ocurren cerca de los extremos *. Algunas veces las piezas verticales de la fig. (*a*) se

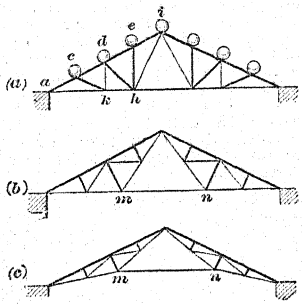



Fig. 26.

hacen inclinadas, ó el tirante se « quiebra » con la convexidad hacia arriba. Las armaduras de techo se componen, con frecuencia, como en las figs. (*b*) y (*c*), de dos vigas Fink, inclinadas, y apoyándose una contra la otra, ligadas las bases por una cuerda, *mn*, y formando los pares los cordones superiores de las armaduras Fink.

* Si las diagonales fueran paralelas, sus esfuerzos y los de las verticales serían mayores en el centro de la luz, y menores en los estribos.

## ESFUERZOS EN LAS PIEZAS DE LAS ARMADURAS

### Principios generales.

**57. Condiciones de equilibrio.** En las armaduras, como en las vigas, es necesario y suficiente, para el equilibrio, que los esfuerzos internos y sus momentos equilibren las fuerzas externas y sus momentos. Las fuerzas externas (por ej., las cargas y las reacciones en los extremos), los momentos resultantes y los esfuerzos cortantes, se estudian en Estática, § 285, etc. Aquí discutiremos la determinación de los esfuerzos internos. Para la distinción fundamental entre vigas y armaduras, véase Armaduras, § 3.

**58.** En general, los esfuerzos en las piezas se encuentran por la teoría de los momentos (Estática, §§ 301, etc.), y de los esfuerzos cortantes (Estática, §§ 325, etc.), usando el paralelogramo ó triángulo de las fuerzas (Estática, §§ 35, etc., y §§ 46, etc.) por el polígono de las fuerzas (Estática, §§ 72, etc., 86, etc.), y por el diagrama de influencia (Estática, §§ 339, etc.).

**59.** Un método muy conveniente y en uso común, se describe más ampliamente en los §§ 67, etc., más adelante, donde se consideran las armaduras en su sección transversal. Trataremos entonces de precisar qué fuerzas se requieren en las piezas así cortadas para mantener el equilibrio.

**60.** Antes de poder calcular los esfuerzos, y proporcionar la armadura á ellos, se debe conocer su peso, porque éste constituye una carga, y por consiguiente influye en los esfuerzos. Pero no podemos conocer su peso hasta que no conozcamos el tamaño de las diferentes piezas. En este dilema debemos atribuirle un peso aproximado, basados en el de algunas armaduras ya fabricadas. Éste es de la mayor influencia á proporción que la armadura aumenta de tamaño, de manera que su propio peso aumenta en mayor proporción que la carga.

**61. Para hacer distinción entre las cuerdas y los postes** del punto, en que está aplicada la fuerza, figs. 27, trácese *oc* representando la fuerza aplicada, en la dirección en que esa fuerza tiende á mover el punto *o*, y sobre *oc*, como diagonal, constrúyase el paralelogramo *ab* de las fuerzas. Por el punto *o* trácese *ii* paralela á la diagonal *ab*. Entonces, la pieza que se encuentre en el mismo lado de *ii* con *oc*, será un poste (sufriendo compresión), y si se encuentra en el lado opuesto, será una cuerda (sufriendo tensión).

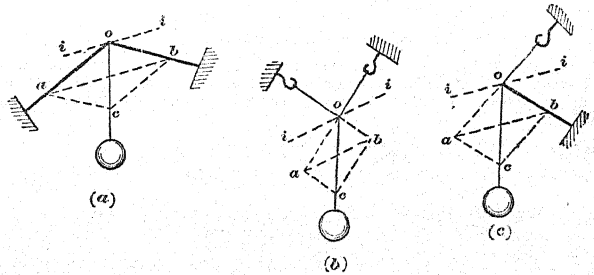



**Fig. 27.**

**62.** Las cuerdas y postes pueden distinguirse (como en la fig. 27) prontamente á la vista, imaginando que la pieza sea flexible, como un cabestro ó cadena. Si se juzga que resiste la fuerza que obra sobre ella, esa pieza es una cuerda, si no, es un poste. O bien, supongamos que la pieza no esté asegurada en sus extremos. Si vemos entonces que resiste la fuerza que obra sobre ella, esa pieza es un puntal, si no, será una cuerda.

**63.** O podemos proceder como sigue: En la fig. 28 (*a*), representando *a* un nudo, empezamos con la reacción vertical conocida R *; y encontramos las presiones

* Como cada una de las mitades de los tramos extremos descansa directamente en un apoyo y no agrega, de esta manera, nada á las fuerzas en las piezas, debemos, al determinarlas, usar solamente la **reacción neta** = *reacción — la mitad de la carga del tramo*.

desconocidas en la cuerda y en el extremo del poste por medio del triángulo de las fuerzas, haciendo que sus saetas sigan la dirección conocida de R. Transfiriendo estas saetas á las respectivas piezas de la armadura, fig: (*d*), encontramos que el cordón inferior tira de *a*, y es por consiguiente una cuerda; mientras que el extremo del poste empuja hacia *a*, y es por consiguiente un poste.

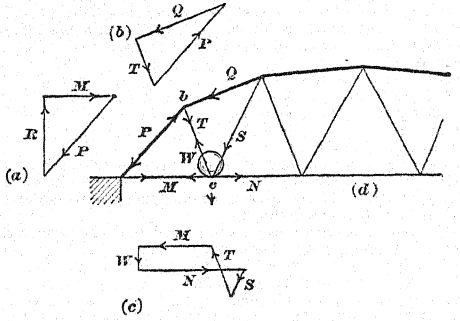



**Fig. 28.**

**64.** En la fig. (*b*), que representa un nudo (*b*), trazamos P hacia arriba para representar la presión del poste extremo hacia *b*, y los otros dos lados del triángulo de las fuerzas dan la presión en la pieza Q del cordón y la tensión en la cuerda T.

**65.** En la fig. (*c*), que representa un nudo *c*, conocemos á T, M, y la carga W, y obtenemos la tensión, N, y la presión, S, en las piezas correspondientes.

**66.** Las fuerzas *de tensión* (porque tienden á *alargar* la pieza), se consideran convencionalmente como *positivas*, y se designan por +; las compresivas se consideran como *negativas*, y se designan por —.

## Método por secciones.

**67.** Si la fig. 29 (*a*) representa una armadura de techo, con tres cargas iguales, W' de 2 toneladas cada una, aplicadas en *a*, *c* y *b* respectivamente, y si se desea encontrar las presiones producidas, por esas cargas solamente, en las piezas *ac* y *ad*;

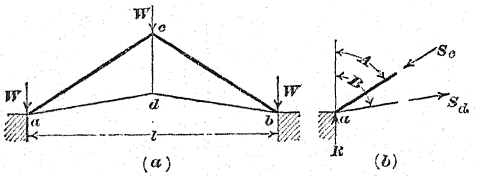



**Fig. 29.**

supongamos que la parte representada por la fig. 29 (*b*) sea separada del resto de la armadura, como se indica por un corte en las piezas *ac* y *ad*. Las partes inferiores de esas piezas, indicadas en (*b*), se supone, sin embargo, que están mantenidas en sus posiciones originales por los esfuerzos $S_c$ y $S_d$, ejercidos en dichas piezas. Tomando los momentos respecto al apoyo derecho, *b*, fig. 29 (*a*), tenemos, para la reacción hacia arriba del estribo izquierdo, *a*,

$$R = \frac{3W}{2}.$$

**68.** Tenemos, entonces, en *a*, fig. 29 (*b*), cuatro fuerzas, así: dos fuerzas conocidas, á saber: W, verticalmente hacia abajo, = 2 toneladas, y R, verticalmente hacia

arriba, $= \frac{3W}{2}$; y dos fuerzas desconocidas, $S_c$ y $S_d$. Ahora bien, $S_c$ y $S_d$ forman ángulos conocidos, A y B, con la vertical. Las fuerzas verticales, W y R, no tienen, por supuesto, componentes horizontales (véase Estática, §§ 54, etc.); y sus componentes verticales son las fuerzas mismas.

**69.** Las componentes horizontales de las fuerzas inclinadas $S_c$ y $S_d$ son, respectivamente : $S_c$ sen A y $S_d$ sen B, y las componentes verticales son : $S_c$ cos A y $S_d$ cos B.

**70.** Vemos por la simple inspección que la fuerza $S_c$ en el par *ac*, es una compresión, y en la viga inferior, $S_d$ es una tensión; pero por conveniencia podemos desde un principio suponer de antemano que todas las fuerzas cuya acción se desconoce son tensiones ó +. Entonces las que aparezcan al fin como + se sabe que son tensiones, y viceversa. Sus resultantes horizontales, en este caso, están ambas, por consiguiente, consideradas como si fueran de la derecha, ó positivas, y sus componentes verticales hacia arriba como positivas también. Se recordará (véase § 66) que consideramos las tensiones como positivas, y las compresiones como negativas.

**71.** Ahora bien, para que las cuatro fuerzas en *a*, á saber : W = 2 tons, hacia abajo; $R = \frac{3W}{2}$ hacia arriba, $S_c$ y $S_d$, puedan estar en equilibrio, es necesario :

(1) que la suma de sus componentes horizontales sea cero, ó que $S_c$ sen A + $S_d$ sen B = 0;

(2) que la suma de las componentes verticales sea cero, ó $R - W + S_c \cos A + S_d \cos B = 0$.

Así pues, sea A = 45°, sen A = .707; cos A = .707.
B = 75°, sen B = .966; cos B = .259.

Entonces $.707\, S_c + .966\, S_d = 0$;

$$R - W + .707\, S_c - .259\, S_d = 0;$$

$$S_c = \frac{-.966\, S_d}{.707} = \frac{-.259\, S_d - R + W}{.707}$$

$$.966\, S_d - .259\, S_d = .707\, S_d = R - W.$$

**72.** También, en la fig. 30, con la sección *u v*, el esfuerzo en $ed = W_1 - R = 6 - 15 = -9$. Con la sección *vy*, el esfuerzo en $ef = \frac{W_1 - R}{\cos \theta} = \frac{-9}{\cos \theta}$. Con la sección *ux*, el esfuerzo en $gd = \frac{W_1 + W_2 - R}{\cos \theta} = \frac{-3}{\cos \theta}$. Se verá que estas fuerzas, obrando todas *hacia abajo* en la parte de la armadura á la izquierda de la sección, dan tensión en *ed*, y *compresión* en *ef* y *gd*.

Con la sección *uz* cortamos dos miembros de la armadura *gd* y *gc*; pero la presión en *gd* ha sido encontrada ya $= \frac{3}{\cos \theta}$, cuya componente vertical es = 3. Por consiguiente, la presión en $gc = W_1 + W_2 + W_3 + 3 - R = 6$.

**73.** Es evidente, sin embargo, por una simple inspección que la vertical media soporta solamente la carga media $W_3 = 6$, porque, cortando la armadura por una sección curva, como en *c*, y examinando la parte pequeña separada de esta manera, veremos que sólo existen dos fuerzas verticales, á saber : la carga central, $W_3$ y la fuerza en la pieza vertical; y, para que haya equilibrio, estas dos deben ser iguales.

## Esfuerzos en los cordones. Momentos.

**74.** Para las tensiones en las cuerdas, fig. 30, sea P = largo del tramo = 10 pies *. Entonces el momento de flexión en el extremo, *d*, del tramo es

$$M = 2\, RP - W_1 P$$
$$= 15 \times 20 - 6 \times 10$$
$$= 300 - 60 = 240.$$

* *N. del T.* — Donde, como aquí, la unidad elegida puede ser cualquiera, sin que deje de comprenderse la teoría ó principio, dejamos las unidades inglesas.

Cortando la armadura por la sección *uv*, encontramos que, de las tres piezas cortadas, solamente el cordón, *eg*, tiene momento respecto al punto *d*. Llamemos su esfuerzo S. Su brazo de palanca es la altura, D, de la armadura = 12, y para que haya equilibrio, SD = M. Por consiguiente, $S = \frac{M}{D} = \frac{240}{12} = 20$.

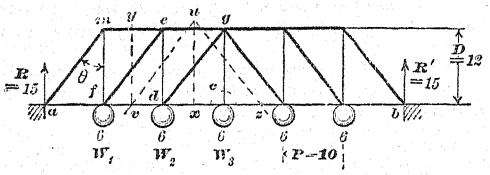



**Fig. 30.**

**75**. De igual manera, tomando momentos respecto al punto *e*, encontramos que el esfuerzo en el cordón inferior, *fd*, cortado por la sección *uv*, es $\frac{240}{12} = 20$, el mismo que el esfuerzo en la parte del cordón superior cortado por la misma sección. La sola inspección mostrará lo correcto de este resultado; porque el poste diagonal *ef* comunica evidentemente á la parte *eg* del cordón una compresión (véase § 77) igual á la tensión que le comunica á la parte *fd* del cordón inferior.

**76**. Si los cordones de la viga están inclinados, sus brazos de palanca deben, por supuesto, medirse *perpendicularmente á ellos*, y no podremos usar ya la altura vertical de la armadura como el brazo de palanca.

**77**. Cada diagonal comunica una compresión al cordón superior, y, en las armaduras de cordones paralelos, una tensión igual al cordón inferior. Encuéntrese el esfuerzo cortante, ó componente vertical, $V_1 V_2$, etc., de las fuerzas en cada diagonal, empezando por el poste del extremo. Entonces las tensiones ó compresiones $h_1, h_2$, etc. en las porciones de ellas, *af* y *me*, *fd* y *eg*, etc., debidas á las varias diagonales separadamente, son:

$$h_1 = V_1 \tan \theta$$
$$h_2 = V_2 \tan \theta$$
$$h_3 = V_3 \tan \theta$$

y para el esfuerzo total en cada cordón de la viga, tenemos: $H_1 = h_1$, $H_2 = h_1 + h_2$, $H_3 = h_1 + h_2 + h_3$, y así en adelante.

## Esfuerzo cortante.

**78**. Cualquier parte del diagrama de un esfuerzo cortante se aplica á todas las piezas atravesadas por él hasta llegar al extremo del tramo donde el esfuerzo cortante cambia. Así, en la fig. 31, el diagrama del esfuerzo cortante en la derecha de la figura incluye la vertical, *tu;* el de la izquierda incluye la diagonal *mo;* y el que está entre las cargas incluye la diagonal *tn*, y la vertical *mn*.

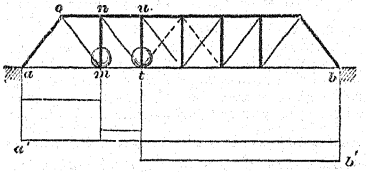



**Fig. 31.**

**79. Diagrama de influencia del esfuerzo cortante.** Véase Estática, §§ 325, etc. En una armadura, fig. 32, las ordenadas *c'g*, etc., á la línea *a"b'* (construída como en la fig. 156, Estática, § 349), dan las *reacciones* del extremo izquierdo, y las *c'h*, etc., hasta la línea *a'b"*, las *reacciones* del extremo de la derecha, para *cualquiera posición de la carga* y los esfuerzos cortantes resultantes de una carga W

(no indicada), *en cualquier división de tramo; pero los esfuerzos cortantes en un tramo, cd*, para una carga W *entre los extremos de los tramos*, se modifican por la acción de las vigas al distribuir la carga entre dichos extremos de tramo adyacentes, como lo indican las líneas de influencia, *qh*, etc., para los diversos tramos. Así, con W en *c* y en *d*, respectivamente, el esfuerzo cortante en el tramo *cd*, está representado, respectivamente, por *c'h* (negativa) y *d'q* (positiva), y al pasar la carga de *c* á *d*, el esfuerzo cortante en el tramo cambia de *c'h* á *d'q*.

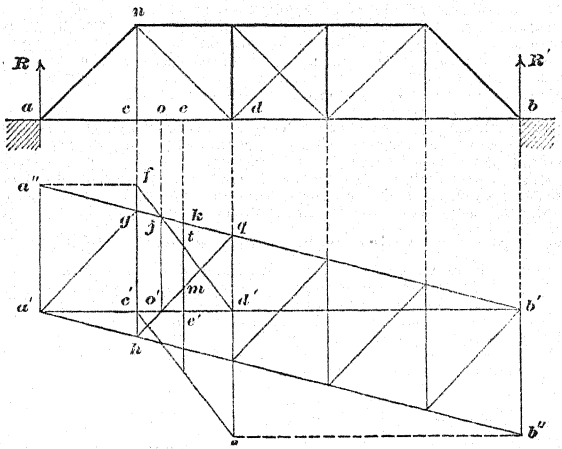



**Fig. 32.**

Pero cuando W se coloca en cualquier punto *e* entre *c* y *d*, su carga se distribuye entre *c* y *d*, por el cordón que obra como una viga.

**30.** De esta manera, trazando para esta viga *cd* (como para la viga entera en Estática, fig. 156), las líneas de influencia *jd'* y *c's* del *tramo*, vemos que, cuando W se mueve de *d* dentro del tramo *cd*, con respecto á *e*, la reacción de la armadura en *a* se aumenta ligeramente por esta razón de *d'q* hasta *e'k*; pero al mismo tiempo la parte de W, representada por *e't*, es transportada por el cordón á *c*, donde disminuye el esfuerzo cortante *e'k* (debido á la reacción de la armadura en *a*), dejando á *tk* como valor del esfuerzo cortante en el tramo. Al colocar el peso W en puntos, sucesivamente, más distantes de *d*, acercándose al punto *o*, la carga transportada por el cordón al punto *c*, y representada por las ordenadas de *c'd'* hasta *jd'*, continúa aumentando más rápidamente que la reacción R en el extremo izquierdo de la armadura, representada por las ordenadas de *a'b'* hacia *a''b'*, y los esfuerzos cortantes resultantes en el tramo están representados por las ordenadas entre *jd'* y *jq*. En *o*, la carga parcial, *o'j*, transportada al punto *c*, es=la reacción izquierda de la armadura, y el esfuerzo cortante en el tramo es cero. Con la carga entre *o* y *c*, las cargas parciales, transportadas á *c*, y representadas por las ordenadas de *o'c'* á *jf*, son *mayores* que las correspondientes reacciones del extremo izquierdo de la armadura, y el resultado es un esfuerzo cortante *negativo* en el tramo, indicado por las ordenadas de *jg* á *jf*. Se observará que los esfuerzos cortantes resultantes en todas partes, los positivos y los negativos, están indicados, por las ordenadas de *c'd'* hacia *hq* *.

Invirtiendo el proceso, se puede aplicar un argumento semejante á la línea de influencia del tramo *c's*, empezando con la carga en *c*, con esfuerzo cortante negativo en el tramo=*c'h*, y suponiendo que se mueve por el tramo hasta *d*, donde el esfuerzo cortante positivo en el tramo se hace=*d'q*.

**31.** En el caso de una **carga uniforme**, que se extienda por la luz desde

* Puesto que *fh*, *jo'* y *ke'* son paralelas, *me'* = *kt*, y *c'h* = *gf*.

el apoyo derecho, *b*, el punto *o* es la posición de la punta de la carga para el máximo esfuerzo cortante en el tramo *cd*, porque en el caso de una carga uniforme el esfuerzo cortante, con la cabeza de la carga en *e*, está representado por el *área* (suma de todas las ordenadas) *e'mqb'e'*, y evidentemente, esta área aumenta á proporción que la punta de la carga se acerca el punto *o;* pero, cuando llega al punto *o* el área sobre *a'b'*, no puede aumentar más, y cuando *pasa* de *o*, los esfuerzos cortantes *negativos*, representados por las ordenadas de *o'c'* á *o'h*, empiezan á *reducir* el esfuerzo cortante positivo que resulta.

**82.** Habiendo encontrado, por cualquier método, el esfuerzo cortante máximo, *d'q*, debido á una carga concentrada en *d*, para la diagonal *dn*, fig. 32, y el esfuerzo cortante máximo inverso, *c'h*, debido á la misma carga en *c*, podemos trazar una línea de influencia *hq*, que da como antes, el punto *o*, de la posición de la cabeza de la carga uniforme para el esfuerzo máximo en la diagonal, *dn*, por la cual (como anteriormente) encontramos la posición correspondiente de los extremos de una serie de cargas concentradas.

En la práctica, la línea de influencia del esfuerzo cortante es de importancia principalmente para encontrar la posición de la carga que produce esfuerzo máx, y los esfuerzos resultantes en las armaduras de cordones en curva como en la fig. 17. En tales armaduras, debido á la inclinación de las partes del cordón superior, estas partes reciben algo de los esfuerzos cortantes en sus respectivos tramos, y el esfuerzo en la diagonal es, por consiguiente, menor que el esfuerzo cortante en el tramo.

## Determinación gráfica de los esfuerzos de la carga muerta.

**83.** Constrúyase en primer lugar un diagrama de la armadura, como en la fig. 33 (*a*), poniendo letras en los espacios entre los miembros, y á los que están entre las saetas que representan las cargas muertas. Llámese el poste del extremo 1-3, entre A y B, « AB » el esfuerzo en él « *ab* », la carga en 2, « *cd* », etc., usando letras mayúsculas para los tramos y los miembros de la armadura, y letras minúsculas para las cargas y esfuerzos. Adóptese una escala de fuerzas adecuada, y constrúyase el diagrama, fig. 33 (*b*), como sigue :

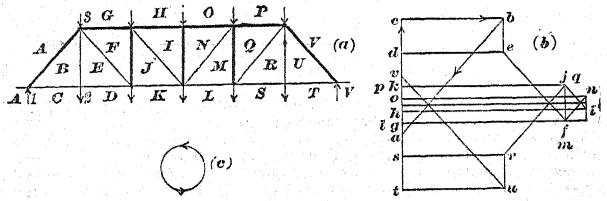



**Fig. 33.**

**84.** Consideremos en primer lugar el punto 1, fig. 33 (*a*). Aquí hay tres fuerzas en equilibrio, á saber : *ac*, *ab* y *bc*. Encuéntrese la reacción del extremo R = *ac*, trazándola hacia arriba (puesto que obra hacia arriba en 1), de un punto conveniente, *a*, hasta *c*, fig. 33 (*b*). Desde *a* trácese una línea indefinida *ab* paralela á AB, y desde *c* trácese *cb* paralela á BC, obteniendo el triángulo de fuerzas *acb* del punto 1. Las longitudes de *cb* y *ba* dan entonces los esfuerzos por la escala.

**85.** En la fig. 33 (*b*), la flecha en *ac* indica la dirección hacia arriba de esa fuerza. Siguiendo alrededor del triángulo, colocamos flechas (en la misma dirección), en *cb* y *ba*. Suponiendo que estas saetas se transfieran ahora á los miembros correspondientes en la fig. 33 (*a*), veremos que *bc* tira del punto 1, mostrando que *bc* es tensión ó +, y que *ba* empuja hacia 1, mostrando que *ba* es compresiva, ó —.

**86.** La naturaleza de las fuerzas puede encontrarse más rápidamente como sigue : trácese un círculo, fig. 33 (*c*), y pónganse saetas alrededor de él indicando la dirección (contraria á la del minutero en este caso) (véase *N. del T.* al pie, pág. 463) que se ha seguido alrededor de la armadura al construir la línea de la carga. Véase § 92, más abajo. Entonces consideremos un nudo cualquiera, fig. 33 (*a*), y sigamos las letras en los espacios alrededor de él en la dirección de las flechas del círculo. Obsérvese el orden de las letras y síguese el polígono de equilibrio, fig. 33 (*b*), en la misma dirección. Éste dará las direcciones según las cuales las fuerzas obran respectivamente en ese nudo.

**87.** Así, consideremos el nudo 2. Siguiendo alrededor de 2, en la dirección del círculo, leemos, B, C, D, E. Pasando ahora á la fig. 33 (*b*), y leyendo, *b*, *c*, *d*, *e*, encontramos que en *bc* vamos de derecha á izquierda (opuesto á la dirección indicada por la flecha trazada en el punto 1), por consiguiente *bc* obra hacia la izquierda en 2; BC está por consiguiente en tensión, y su fuerza *bc* es +.

**88.** Dada ya la fuerza *bc*, en BC, constrúyase sobre *bc* el polígono de fuerza *bcde*, de las cuatro fuerzas que obran en el punto 2. Desde *c*, trácese *cd*, hacia abajo, representando la carga muerta en el cordón inferior en 2. Puesto que *bc* obra como tensión hacia la izquierda en 2, y como las fuerzas deben seguirse una á otra alrededor del polígono, *cd* se deberá trazar evidentemente hacia abajo, desde *c*, y no desde *b*. Desde *d* trácese una línea indefinida paralela á la DE, y desde *b*, otra, paralela á BE. Éstas se interceptarán en algún punto, como *e*, y *eb* y *de* representarán entonces los esfuerzos en BE y DE.

**89.** Una inspección mostraría que $be = cd$, puesto que *cd* es la única fuerza que obra en 2, con una componente vertical, y que $bc = de$; pero la construcción del polígono de fuerzas *bcde* es necesaria para completar el diagrama.

**90.** Habiendo encontrado ya los esfuerzos en DE, BE y AB, y conociendo la carga del tramo ($= ga$) en el punto 3, constrúyase el polígono *gabefg*. Éste dará *ef* y *fg*, y desde éstas se podrá continuar el procedimiento y completar el diagrama.

**91.** Se observará que, en algunos casos, á un mismo punto del diagrama, fig. 33 (*b*), se le pone más de una letra. Ordinariamente, esto es una simple coincidencia, causada por la superposición de los polígonos de fuerza. En algunos casos, sin embargo, la coincidencia de las letras muestra que el esfuerzo en la pieza respectiva es cero.

**92.** En la práctica, se acostumbra construir primero la línea de carga entera de la manera siguiente: Trácese primero la reacción neta, *ac* hacia arriba, después siguiendo alrededor de la armadura, contra el movimiento del minutero (véase *N. del T.* al pie, pág. 463), trácense todas las otras fuerzas exteriores (carga muerta) en su orden respectivo, así: *cd*, *dk*, *kl*, *ls*, *st*, *tv*, *vp*, *po*, *oh*, *hg*, *ga*. El diagrama de los esfuerzos podrá entonces construirse como antes.

## Cargas vivas.

**93.** Podía suponerse, en primer lugar, que cada pieza de la armadura recibe el esfuerzo máximo cuando el tren cubre completamente el puente, pero esto es cierto solamente respecto á los cordones. En la armadura que muestra la fig. 33 (*a*) cada pieza entre cordones recibe el máximo de fuerza cuando el mayor esfuerzo cortante posible ocurre en una sección que corte esa pieza.

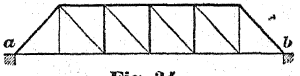



**Fig. 34.**

**94.** La fig. 34 indica las *principales* diagonales á la izquierda y las contradiagonales á la derecha del centro. Una cualquiera de estas piezas recibe el máximo de presión de una carga uniformemente distribuída cuando la carga se extiende desde él, hasta el soporte *derecho b*, con la cabeza de la carga en un punto, *o*, fig. 32 (*a*), que se encontrará como en el § 81; y viceversa para las diagonales inclinadas en dirección *opuesta*. Cada vertical recibe el máximo del esfuerzo cuando la carga se extiende desde el apoyo más distante hasta un punto *o* (véase § 81), en el tramo más allá de la vertical. Esta regla debe modificarse ligeramente cuando consideramos las cargas concentradas sobre ruedas. Véanse §§ 97, etc.

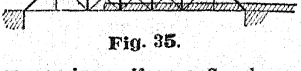


**Fig. 35.**

**95. Se supone la carga viva uniforme.** Con alguna aproximación la locomotora y el tren se consideran como una carga uniforme que cruza el puente, fig. 35; pero este método, donde no se toma en cuenta la gran concentración de peso en las locomotoras modernas, puede resultar, ó inseguro, ó produciendo desperdicio de material. Esta suposición es correcta al tratar de la presión del viento sobre el tren. Véase § 121.

**96. Exceso de cargas concentradas.** También para compensar las cargas de la locomotora se emplean algunas veces una ó más cargas de exceso concentradas. Los esfuerzos debidos á estas cargas pueden computarse separadamente, y agregarse á las presiones producidas por las cargas vivas uniformes. Para que produzcan el máximo de los esfuerzos en los cordones, las cargas de exceso deben estar en el

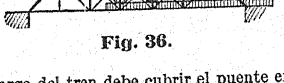


**Fig. 36.**

centro de la luz, y la carga del tren debe cubrir el puente entero. Este método es bastante aproximado, pero los ingenieros difieren respecto á si debe usarse este método ó el del peso real sobre las ruedas de la locomotora « típica » como se indica luego.

**97. Cargas sobre la rueda.** En el método de las cargas « típicas » sobre la rueda, se consideran los esfuerzos reales producidos por las máquinas más pesadas que puedan cruzar el puente. Aun en máquinas de casi el mismo peso, las cargas pueden estar diversamente separadas, y á intervalos de fracciones de centims, que hacen muy laborioso el cálculo. Por esta razón, y en previsón del uso de máquinas más pesadas en lo futuro, se acostumbra considerar una máquina imaginaria ó « típica », con cargas y espacios dados en números redondos, cuyas presiones sean por lo menos iguales á las que produzcan las máquinas más pesadas que puedan usarse en dicho puente. Las cargas vivas se consideran ordinariamente como compuestas de dos locomotoras normales con sus ténderes, seguidas de un tren de peso uniforme. Véase el Resumen de las especificaciones.

**98.** El siguiente, es un ejemplo del cálculo de las presiones de la carga viva por el método de las cargas sobre ruedas de locomotora. La fig. 37 (*b*) representa las cargas en un riel que corresponden al modelo de Cooper, clase normal E 40, que se compone de dos locomotoras de consolidación acopladas *, seguidas de un tren que se considera equivalente á una carga uniforme de 4,000 libras por pie lineal (5,960 kg por metro). En el diagrama, fig. 37 (*a*), todas las cargas están en miles de libra, los momentos en millones de pieslibras y las distancias en pies **.

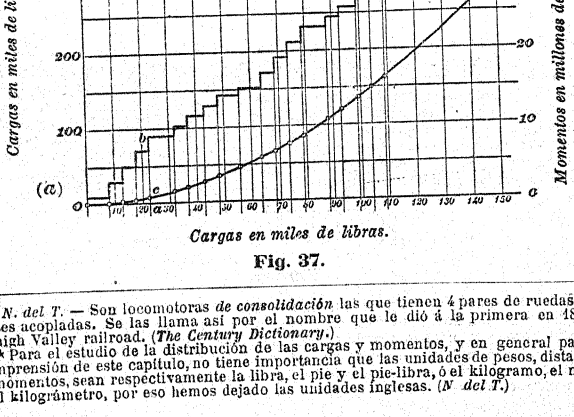



**Fig. 37.**

* *N. del T.* — Son locomotoras *de consolidación* las que tienen 4 pares de ruedas motrices acopladas. Se las llama así por el nombre que le dió á la primera en 1866 el Lehigh Valley railroad. (*The Century Dictionary.*)

** Para el estudio de la distribución de las cargas y momentos, y en general para la comprensión de este capítulo, no tiene importancia que las unidades de pesos, distancias y momentos, sean respectivamente la libra, el pie y el pie-libra, ó el kilogramo, el metro y el kilográmetro, por eso hemos dejado las unidades inglesas. (*N. del T.*)

**99. Esfuerzos producidos por la carga viva** sobre las piezas del tejido de la armadura. El máximo de estos esfuerzos ocurre en las barras ó piezas de cualquier tramo, figs. 34 ó 38, cuando la carga viva produce el máximo esfuerzo cortante en el tramo.

Puede demostrarse que esto ocurrirá cuando $P = \frac{W}{n}$, en que P=á la carga viva en el tramo cortado por la sección; W=la carga viva total en la armadura, y $n$=el número de tramos de la armadura. Esta ecuación se tiene como norma del esfuerzo cortante máximo.

**100.** La tabla siguiente se basa en esta relación. La segunda columna se obtiene agregando á P, las sucesivas cargas por rueda. En este caso W=6P, puesto que nuestra armadura tiene 6 tramos. Supongamos que una rueda esté en el punto de división de un tramo. Entonces, moviendo la rueda un poco á la izquierda ó á la derecha, estará incluída ó excluída de P. De aquí que P y W tengan cada uno un valor mínimo y máximo para cada rueda situada en un punto de división de un tramo.

| N.° de la rueda en cualquier punto de división de un tramo. | Valor, P, de la carga en el tramo á la izquierda del punto dado. | Valor correspondiente de W para el máximo del esfuerzo cortante en el tramo. |
|---|---|---|
| 1 | 0 á 10,000 | 0 á 60,000 |
| 2 | 10,000 á 30,000 | 60,000 á 180,000 |
| 3 | 30,000 á 50,000 | 180,000 á 300,000 |
| 4 | 50,000 á 70,000 | 300,000 á 420,000 |
| 5 | 70,000 á 90,000 | 420,000 á 540,000 |

**101.** La posición de la carga viva, para el esfuerzo cortante máximo, en cualquier tramo, se encuentra por ensayos sucesivos. Al encontrar la posición, se calcula el momento respecto al apoyo derecho, y de éste se obtendrá el esfuerzo cortante. Para ejemplos, véase más abajo.

**102.** Estas operaciones pueden ejecutarse por el cálculo con ó sin la ayuda de los métodos gráficos. Como el cálculo es un poco fastidioso, particularmente cuando la forma de la armadura es complicada por tener los cordones arqueados ó llevar subtramos, y como el método gráfico es bastante exacto para todos los usos prácticos, y tiene la ventaja de que habla á la vista directamente, damos este último solamente.

**103. El « diagrama de ruedas »**, fig. 37 (*a*) *, da (1) una « línea de carga » de escalones ó « diagrama del esfuerzo cortante », y (2) un diagrama en curva de los momentos ó « polígono de equilibrio ». Véase Estática, §§ 359, etc.

**104.** La línea de carga viva total á la izquierda de un punto cualquiera, incluyéndolo.

**105.** La línea de los momentos da, en cualquier punto, el momento de las cargas vivas (á la izquierda), con respecto á ese punto é incluyéndolo. Así, á la izquierda de la rueda n.° 5 é incluyéndola, tenemos :

| Ruedas. | Carga. | Distancia de la rueda 5. | Momento respecto á 5 en pies-lbs. |
|---|---|---|---|
| 1 | 10,000 | 23 | 230,000 |
| 2 | 20,000 | 15 | 300,000 |
| 3 | 20,000 | 10 | 200,000 |
| 4 | 20,000 | 5 | 100,000 |
| 5 | 20,000 | 0 | 0 |
| Total... | 90,000 | | 830,000 |

y las ordenadas, *ab*, á la curva de carga, y *ac*, á la curva de momentos, debajo de la rueda 5, miden 90.0 y .830 respectivamente.

**106.** La fig. 38 representa la armadura, en la misma escala que la fig. 37. Podemos llamar éste un « diagrama de armadura ».

*Método publicado por Ward Baldwin, « Engineering News », vol. XXII, sept. 28, 1889, pág. 295. Véase también « Eng. News », dic. 28, 1889, pag. 645.

**107.** Ejemplo. Para computar el esfuerzo cortante máximo en el tramo *bc*, fig. 38, encuéntrese primero aquella posición de la carga que producirá ese esfuerzo cortante máximo. Como tanteo, colóquese el diagrama de armadura, fig. 38 **, con el

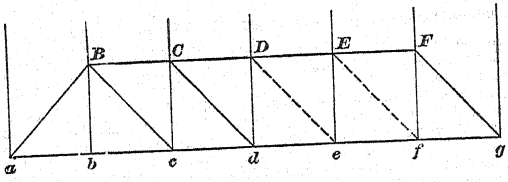



**Fig. 38.**

punto *c* debajo de la rueda 2, fig. 37. Examinando el diagrama de carga, en el extremo derecho, *g*, de la luz, vemos que ahora tenemos una carga total, W, de 284,000 libras en la luz, y que el diagrama de carga sobre la rueda 2 (colocada en el punto *c*) muestra (véase también la tabla § 100) que la carga P, en el tramo *bc*, está ahora entre 10,000 y 30,000 libras; pero para el esfuerzo cortante máximo en el tramo, *bc*, la carga, P, en ese tramo debe ser (véanse §§ 99 y 100° = W ÷ *n* = 284,000 ÷ 6 > 30,000 libras. De aquí que P deba aumentarse moviendo el diagrama de tren, fig. 37, hacia la izquierda (ó, lo que es lo mismo, moviendo el diagrama de armadura, fig. 38, hacia la derecha) hasta que la rueda 3 esté sobre *c*.

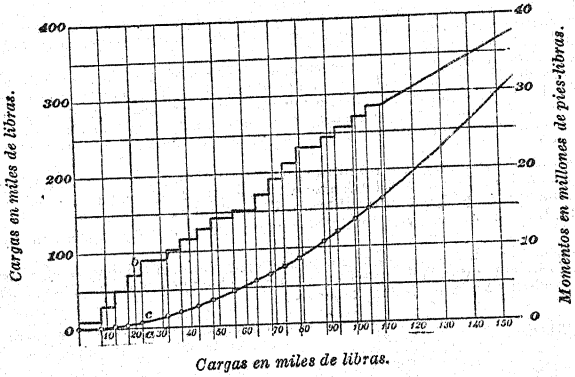



**Fig. 37 (repetida).**

Tenemos ahora, W = 292,000 libras, P = entre 30,000 y 50,000 y el valor requerido de P, para el esfuerzo cortante máximo = W ÷ *n* = 292,000 ÷ 6 = 48,667 libs. De aquí que las condiciones estén satisfechas, y el tramo *bc* reciba su esfuerzo cortante máximo cuando la rueda 3 está en *c*. El diagrama de los momentos indica verticalmente sobre *g* el momento de la carga viva, respecto al apoyo derecho = = 17,516,000 pieslibras, y el momento en *c* = 223,000 pieslibras.

** Para la discusión siguiente se encontrará cómodo hacer una copia de la fig. 38, ó simplemente del cordón inferior, en un pedazo de papel separado, que pueda aplicarse, en posiciones diferentes, á la fig. 37.

**108.** Sea M=el momento de la carga viva (de la izquierda) en el estribo de la derecha, debido á todas las cargas en la luz del puente.
L=la luz del puente.
$m$=el momento de la carga viva de la izquierda en la punta del tramo á la derecha del tramo en cuestión.
$l$=el largo del tramo.
V=el esfuerzo cortante en el tramo.

$$\text{Entonces } V = \frac{M}{L} - \frac{m}{l} = \frac{17{,}516{,}000}{150} - \frac{230{,}000}{25} = 107{,}600 \text{ lbs *.}$$

**109.** Los esfuerzos cortantes máximos de las cargas vivas en los otros tramos, computados de igual manera, son como sigue, estando la carga, en cada caso, colocada de manera que dé dicho esfuerzo cortante máximo.

| Tramo, n.º y posición de la rueda. | Mom M en el extremo derecho de la armadura. Pies-lbs. | Mom $m$ en el extremo derecho del tramo. Pies-lbs. | Esfuerzo cortante *. Libras. $V = \frac{M}{L} - \frac{m}{l}$ | Piezas ó barras. | Esfuerzo. Libras $S = \frac{V}{\cos \theta}$ **. |
|---|---|---|---|---|---|
| *ab* 4 á *b* | 27,176,000 | 480,000 | 162,000 | *a*B | — 217,200 |
| *bc* 3 á *c* | 17,516,000 | 230,000 | 107,600 | B*c* | + 144,200 |
| *cd* 3 á *d* | 10,816,000 | 230,000 | 63,000 | C*c* | — 63,000 |
| | | | | C*d* | + 84,500 |
| *de* 2 á *e* | 4,936,000 | 80,000 | 29,700 | D*d* | — 29,700 |
| | | | | D*e* | + 39,800 |
| *ef* 2 á *f* | 1,743,000 | 80,000 | 8,400 | E*f* | + 11,250 |

**110. El esfuerzo de la carga viva en el montante,** B*b*, es debida enteramente á las cargas sobre las dos porciones del cordón inferior, *ab* y *bc*. Así, pues, con la rueda cuatro en *b*, largo del tramo=*ab*=*bc*=25 pies, tenemos:

Sobre *ab*

| Rueda. | Carga, W. | Dist *d*, de *a* | Esfuerzo en B*d*= *wd* ÷ 25. |
|---|---|---|---|
| 1 | 10,000 | 7 | 2,800 |
| 2 | 20,000 | 15 | 12,000 |
| 3 | 20,000 | 20 | 16,000 |
| Total, | 50,000 | | 30,800 |

Sobre *bc*

| Rueda. | Carga, W. | Dist *d*, de *c*. | Esfuerzo en B*b*= *wd*÷25. |
|---|---|---|---|
| 5 | 20,000 | 20 | 16,000 |
| 6 | 13,000 | 11 | 5,720 |
| 7 | 13,000 | 6 | 3,120 |
| Total, | 46,000 | | 24,840 |

Carga total sobre *ab*= 50,000
— — — *bc*= 46,000
Rueda 4 = 20,000
Carga total sobre *ac*=116,000

Esfuerzo en B*b* de *ab* =30,800
— en B*b* de *bc* =24,840
— en B*b* de rueda 4=20,000
Esfuerzo en B*b*. Total, 75,640

**111.** Para cualquiera disposición de cargas sobre *ac*, el esfuerzo máximo tiene lugar cuando la carga sobre *ac* está igualmente dividida entre *ab* y *bc;* lo cual ocurre ordinariamente mientras alguna de las ruedas (encontrado por tanteo), esté pasando por *b*. Así pues, con la rueda 4 justamente á la derecha de *b*, tenemos, sobre *ab*, las ruedas 1, 2 y 3=50,000 lbs ; y, en *bc*, las ruedas 4, 5, 6 y 7 = 66,000 lbs ; pero con la rueda 4 justamente á la *izquierda* de *b* se tiene en *ab*, ruedas 1, 2, 3 y 4 = 70,000; y, sobre *bc* (despreciando la rueda 8 que entra ahora en *bc*), las ruedas 5, 6, 7=46,000 lbs. Por consiguiente, cuando a rueda 4 esté pasando por *b*, hay un instante en que las cargas sobre *ab* y sobre *bc* son iguales, y en ese momento la presión en B*b* llega á su máximo (75,640 lbs, véase § 110), para la serie de cargas dada.

**112. Esfuerzos de la carga viva en el cordón.** El criterio que guía para la posición de la carga, en el máximo momento flexor en una sección cualquiera, y

* A causa de ser $\frac{M}{L}$ = la reacción del soporte izquierdo, *a*, y $\frac{m}{l}$ = la porción de la carga del tramo que va al extremo izquierdo del tramo.
** $\theta$ = ángulo entre diagonal y vertical.

por consiguiente para el esfuerzo máximo en esa sección, es $\frac{W}{w} = \frac{L}{l}$ ó $l = L\frac{w}{W}$, en que W = la carga total en la armadura, $w$ = la carga á la izquierda de la sección, L = la luz del puente, y $l$ = el largo del segmento á la izquierda de la sección.

**113.** Para encontrar la posición de la carga para el momento máximo en cualquier tramo, por medio del diagrama de los momentos, fig. 37, colóquese una rueda, digamos la rueda 2, en el punto de división del tramo á la derecha del tramo dado. Desde la intersección de la línea de carga (que ordinariamente coincide con el eje de las $x$) verticalmente sobre el soporte izquierdo, colóquese una regla, ó extiéndase un hilo hasta la intersección de la línea de carga con una vertical sobre el soporte derecho. Si la línea construída de esta manera vuelve á cruzar la línea de carga en un punto situado sobre la sección en cuestión, la posición es correcta; si no, es incorrecta. Para facilitar este trabajo, es bueno usar un diagrama de armadura, fig. 38, dibujado en una hoja de papel de trazar, con las verticales prolongadas cuidadosamente desde los puntos de división de los tramos hacia arriba hasta donde las líneas de cargas ó de momentos puedan llegar.

**114.** Se encontrará con frecuencia que más de una posición satisface y que alguna de ellas puede dar momentos mayores que las otras. De aquí que sea conveniente buscar todas las posiciones posibles. Al encontrarlas, determinemos los momentos, como sigue: En la curva de momentos búsquense dos puntos que coincidan (verticalmente) con los apoyos izquierdo y derecho respectivamente, y únanse estos puntos con una línea recta. Cuando la cabeza del tren no ha llegado al apoyo izquierdo, el punto que corresponde á este apoyo, está en el eje de las $x$, prolongado.

**115.** El momento requerido se mide por la distancia vertical de la ordenada á lo largo de la sección, entre la curva de momentos y la línea recta que se acaba de trazar.

El esfuerzo en las partes del cordón es igual al momento dividido por la altura de la armadura. Empleando estos métodos se obtienen los siguientes resultados:

| Sección. | Rueda. | Momento en pies-lbs. | Presión lbs. | Piezas. |
|---|---|---|---|---|
| B$b$ | 4 | 4,049,333 | 144,600 | $ab = bc$ |
| C$c$ | 7 | 6,211,667 | 221,800 | $cd =$ — BC |
| C$c$ | 8 | 6,207,667 | No máx. | |
| D$d$ | 11 | 7,044,000 | No máx. | |
| D$d$ | 12 | 7,056,500 | 252,000 | — CD = — DE |

## Cargas debidas al viento.

**116.** Un puente está sujeto no solamente á las cargas *verticales*, debidas á la carga muerta, á la carga viva, y á los choques causados por desigualdades entre la vía y en el material rodante, sino también á las cargas horizontales. Estas cargas horizontales se deben á la acción transversal del viento, á las fuerzas centrífugas producidas

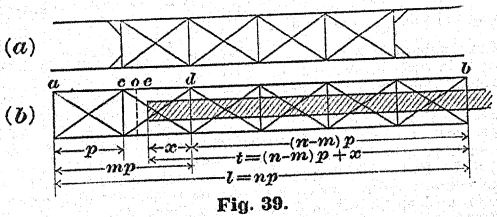



**Fig. 39.**

por el tren al pasar por una curva en el puente, á la tracción longitudinal ó «arrastre» causada por la parada de un tren en movimiento ó salida de un tren en reposo en el puente. Por consiguiente, es necesario construir barras horizontales que, con los dos cordones superiores ó los dos inferiores de las dos armaduras verticales, formen armaduras horizontales, como sistemas laterales superior é inferior, figs. 39 (*a*) y 39 (*b*), y también barras laterales, contra la vibración y el viento. Véanse §§ 21 y 22.

**117.** El viento se considera que sopla en ángulo recto con el puente.

**118.** El viento produce varios efectos y éstos deben considerarse separadamente, para determinar después sus efectos en conjunto. Entre estos efectos están:

(1) Los esfuerzos directos en ambos sistemas laterales, el superior y el inferior, causando presiones directas sobre los cordones, obrando horizontalmente sobre ellos como una carga uniformemente distribuída.

(2) Los esfuerzos directos adicionales en el sistema lateral del cordón cargado cuando un tren está en el puente, debidos á las presiones del viento contra el tren.

(3) Un momento de volcamiento sobre el puente como un todo, aumentando de esta manera las presiones de la carga viva y muerta en la armadura á sotavento y disminuyendo las de la armadura á barlovento.

(4) Una tendencia análoga á volcamiento sobre el tren y las ruedas, que modifica de igual manera sus presiones sobre las vigas del piso, y así, las presiones en las armaduras principales.

**119.** Supondremos que la carga que deriva del efecto del viento, obrando directamente, está igualmente dividida entre los cordones superior é inferior y entre la armadura á sotavento y la de barlovento.

**120.** (1) Las presiones directas del viento en las barras laterales, debidas á la presión del viento en la armadura, se encuentran como las presiones en las armaduras principales, debidas á la carga muerta; los puntales transversales horizontales de los tirantes laterales corresponden á los verticales de las armaduras principales.

**121.** (2) Presiones directas en el sistema lateral de cordones cargados, fig. 39 (*b*), debidas al viento contra el tren.

Examinando cualquier tramo, como *cd*, sea **

$w$ = la presión del viento, en lbs por pie lineal de tren.

$p$ = largo del tramo en pies.

$wp$ = presión del viento, en lbs, por tramo, todo ocupado por el tren.

$n$ = número de tramos en la luz (= 6 en este caso).

$l = np$ = la luz del puente en pies.

$m$ = número de tramos desde el apoyo izquierdo, $a$, hasta el tramo, $cd$, que consideramos é incluímos en el cálculo.

$mp$ = la distancia, $ad$, en pies.

$x$ = el largo, en pies, de la parte del tramo, $cd$, que está ocupado por el tren.

$t = (n - m)\,p + x$ = á la parte de la luz del puente que está ocupado por el tren.

= la presión del viento contra el tren para una presión de 1 lb por pie lineal.

R = reacción en $a$ (debida á la acción del viento en la armadura $= wt^2 \div 2l$*.

$r$ = reacción en $c$ debida á la acción del viento sobre el tramo *, $= wx^2 \div 2p$.

S = Esfuerzo cortante del viento * en el tramo, $cd$, $= \mathrm{R} - r = wt^2 \div 2l - wx^2 \div 2p$.

La reacción horizontal * de la *armadura*, en $a$, debida á la presión horizontal concentrada = 1, obrando á cualquiera distancia, $y$ (no indicada), de $d$, es $= \dfrac{(n-m)\,p+y}{np}$; y la reacción horizontal del tramo, en $c$, debida á la misma presión, es $= \dfrac{y}{p}$. El esfuerzo cortante máximo *, en el tramo $cd$, debido al viento contra el tren, ocurre (véanse §§ 79 á 81) cuando la cabeza del tren llega al punto, $o$, en el que, si las cargas concentradas estuvieran colocadas, estas dos reacciones serían iguales, es decir, $\dfrac{y}{p} = \dfrac{(n-m)\,p+y}{np}$. Con la cabeza del tren en $o$, tenemos $x = y = \dfrac{(n-m)\,p}{n-1}$.

Bajo cualesquiera condiciones, el esfuerzo cortante del viento *, S, en el tramo, es $= \mathrm{R} - r$, en que $\mathrm{R} = \dfrac{w\,|(n-m)\,p+x|^2}{2np}$ y, $r = \dfrac{x^2}{2p}$.

Sustituyendo aquí el valor de $x$ acabado de encontrar, para el esfuerzo cortante máximo, obtenemos, como valor máximo del esfuerzo cortante * del viento en el tramo,

$$\mathrm{S}_{max} = \frac{wp}{2\,(n-1)}\,(n-m)^2.$$

---

* Véase la nota ** al pie del § 2.

** *N. del T.* — Dejamos el pie y la libra, pues los ejemplos y la teoría son enteramente análogos usando el metro y el kilogramo.

**122.** (3) Esfuerzos en las piezas de las armaduras principales debidos a momento de volcamiento producido por el viento en la armadura: Momento de volcamiento = (carga debida al viento en el tramo en el cordón superior) × (número de puntos de división de tramos en la luz) × altura de la armadura.

$$\text{Reacción vertical en un apoyo} = \frac{1}{2}\,\frac{\text{momento de volcamiento}}{\text{ancho entre las armaduras}}.$$

Como el sistema lateral superior lleva todas las cargas debidas al viento á los extremos del puente, los postes de los extremos y los cordones (que reciben las componentes horizontales de los empujes del poste final) son las únicas piezas principales de la armadura, afectadas.

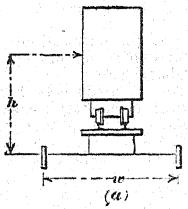



**Fig. 40.**

**123.** (4) Esfuerzos en los miembros principales de la armadura debidos al momento de volcamiento del viento contra el tren, fig. 40. Sea $h$ = altura del centro de gravedad del sistema lateral del cordón cargado, al centro de presión del viento contra el tren, $p$ = la presión del viento por pie lineal de tren, $w$ = la distancia entre los centros de gravedad de las armaduras, $m$ = momento de volcamiento por pie lineal de tren, $v$ = carga vertical añadida por pie lineal de la armadura de sotavento.

Entonces $m = hp$, y $v = \dfrac{m}{w}$.

## Percusión, choque, etc.

**124.** Los efectos del choque debidos á las desigualdades de la vía, los debidos á las paradas y salidas de los trenes, y los de la fuerza centrífuga de los trenes en las curvas, no son susceptibles de cálculo riguroso, y los ingenieros difieren en los medios de calcularlos y preverlos. Véase el Resumen de las especificaciones.

## Determinación de los esfuerzos máximos y mínimos.

**125.** Cuando las especificaciones permiten que la unidad de los esfuerzos dependa de la relación entre los máximos y mínimos esfuerzos en cualquier pieza, se deben calcular ambos.

Al calcular los esfuerzos máximos y mínimos en cualquier pieza, téngase presente que una condición que, por sí misma, tendría cierto efecto sobre un esfuerzo, puede traer consigo otras condiciones que produzcan un efecto mayor y contrario. Así, por ej.: aunque el efecto del viento en un tren reduce los esfuerzos en ciertas piezas, aquella acción puede tener lugar solamente con un tren en el puente, y la acción vertical de la carga del tren aumenta ordinariamente esos esfuerzos en más de lo que la acción del viento los disminuye.

Al calcular las presiones mínimas, aunque la carga viva se desprecia ordinariamente, no debemos despreciar la carga muerta que está siempre presente.

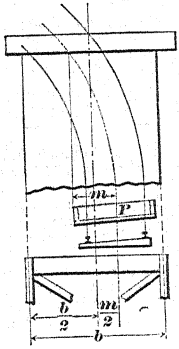
## Curvas en los puentes.

**126. Cuando la vía en un puente es curva,** se coloca ordinariamente de manera que la línea central del puente divida en dos la ordenada media, $m$, de la curva. Véase la fig. 40 (a). El centro de gravedad de una carga de tramo, P, en el centro de la luz del puente (suponiendo que esté en el centro de la vía), se proyecta en una distancia = ½ $m$ de la línea del centro del puente, ó una distancia = ½$b$ + ½$m$ de la armadura interior, en que $b$ = al ancho del puente entre los centros de las arma-

duras. Tomando los momentos respecto al centro de la armadura interior, tenemos por consiguiente, para la carga W, debida á P, en la armadura exterior,

$$W = P\,\frac{\frac{1}{2}b + \frac{1}{2}m}{b} = P\,\frac{b+m}{2b}.$$

Es costumbre (véase el Resumen de las especificaciones) proporcionar la armadura exterior suponiendo que su esfuerzo cortante debido á la carga viva, en cada punto de división del tramo, esté determinado por la fórmula que se acaba de dar, y trazando la armadura interior igual á la exterior.




Figs. 40 (a) y (b).

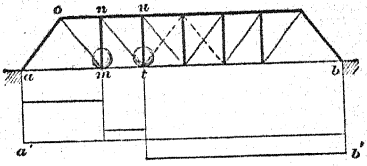



Fig. 31 (repetida).

## Contradiagonales.

**127.** En una armadura de forma ordinaria (como la de la fig. 31), bajo la acción de una carga viva ó muerta distribuída uniformemente ó de una carga viva distribuída simétricamente con respecto al centro de la luz, los esfuerzos cortantes en cada tramo á izquierda del centro de la luz son *positivos;* los de los tramos á la derecha son *negativos;* y los esfuerzos en toda la armadura son tales, que las cuerdas sufren tensión, y los postes compresión; siendo la tendencia en cada tramo *alargar* la diagonal ocupada por una *cuerda* y acortar la ocupada por un *poste.*

Pero, la tendencia de una carga *excéntrica,* igual á la que indica la fig. 31, es *invertir* los esfuerzos cortantes en los tramos entre ella y el centro, y si este esfuerzo, relativamente á las otras fuerzas, es de magnitud suficiente para invertir el esfuerzo cortante en cualquier tramo, la tendencia será acortar la diagonal *ocupada* por una *cuerda,* véase fig. 1 (*a*), y *alargar* cualquier diagonal ocupada por un *poste.*

Como se explica en los §§ 14 y 15, esta condición existe en la armadura triangular ó de Warren, haciendo que cada pieza ó barra pueda resistir ambas cosas: presión y compresión, y en las armaduras de barras verticales y diagonales por la adición de contradiagonales.

En un puente levadizo ó giratorio, no solamente los esfuerzos en las barras del entramado, sino también los de los cordones se invierten cuando la parte movediza se abre ó se cierra.

En previsión contra la posibilidad del aumento posterior en las cargas vivas, sobre las que se usan ahora, y cuando los esfuerzos de la carga viva y muerta sean contrarios, debe calcularse sólo el 70% de los esfuerzos de la carga muerta como cantidad efectiva para equilibrar los esfuerzos de la carga viva.

Para otros sistemas de calcular previsiones semejantes, véase Resumen de las especificaciones para puentes de acero, para ferrocarril.

## Armaduras de techo.

**128.** En las armaduras de techos, *la carga muerta,* es decir, el peso de la misma armadura, de sus accesorios, cubierta del techo, etc., y la carga de la nieve, se con-

sideran de ordinario distribuidas uniformemente. En muchos casos la suma de las cargas muertas y la de la nieve están divididas igualmente entre los dos apoyos. En otras palabras, las reacciones de los extremos son iguales.

**129. Los pesos de las armaduras de acero,** en kg por m cuadrado del espacio fabricado que cubren, puede tomarse, como apreciación preliminar, en (.80 á 1.28) × por la luz en m de acuerdo con el diseño y la carga. Los de las armaduras de madera, con tirantes de acero, hierro ó madera, pueden calcularse de un décimo á un quinto menos.

**130. El peso de los accesorios** de acero ó de madera puede calcularse en 9 á 15 kg por m cuadrado de espacio fabricado que cubra.

**131. El peso de la cubierta del techo** puede tomarse aproximadamente como sigue:

| | | | | | |
|---|---|---|---|---|---|
| Hierro corrugado..... | 9 á 15 kg por m cuadrado de superficie de techo. | | | | |
| Pizarra..... | 34 á 43 | — | — | — | — |
| Tejamaní sobre listoncillos. | 9 á 15 | — | — | — | — |
| Si están sobre tablas, agréguense..... | 13 | — | — | — | — |
| Si están con mezcla, agréguense..... | 29 | — | — | — | — |

**132. La carga debida á la nieve,** en los Estados al Norte de la latitud 35°, puede tomarse (variando principalmente con la latitud) de 48 á 146 kg por m cuadrado de proyección horizontal de superficie de techo.

**133.** Las piezas accesorias, las de refuerzo, etc., deben disponerse de manera de llevar el peso de la cubierta del techo y de la nieve directamente á los puntos de división de los tramos, evitando de este modo las presiones transversales en los cabios ó parecillos.

**134.** Cada armadura soporta, además de su propio peso, la mitad del peso del techo y de la nieve entre las dos armaduras (ó entre armadura y pared) adyacentes á ella, y cada punto de división de tramo soporta la mitad de la carga entre las dos duntas de tramo (ó punta de tramo y apoyo extremo) adyacentes á ella.

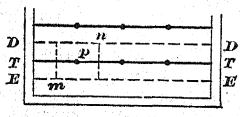



**Fig. 41.**

De esta manera, fig. 41, la armadura TT soporta un peso = al que está sobre la superficie entre las dos líneas de puntos, DD y EE, y el punto de división *p* del tramo R soporta un peso = al que se encuentra en el rectángulo *mn*.

**135.** El viento se considera que sopla horizontalmente sobre un lado del techo y que ejerce una presión normal distribuída uniformemente sobre ese lado. En la tabla siguiente de presiones normales supuestas, obrando contra las superficies inclinadas bajo una presión horizontal del viento de 195.2 kg por m cuadrado, los valores en la última columna están basados en los experimentos de Hutton. Aquí *a* es el ángulo entre la superficie inclinada del techo y el plano horizontal.

**Presión normal supuesta,** P, del viento, en kg por metro cuadrado. Presión horizontal = 195.2 kg por m cuad. *a* = ángulo entre la superficie del techo y un plano horizontal.

| | | P | | | | P | |
|---|---|---|---|---|---|---|---|
| *a.* | sen *a.* | 195.2 sen *a.* | Hutton. | *a.* | sen *a.* | 195.2 sen *a.* | Hutton. |
| 5° | .09 | 17.6 | 24.9 | 35° | .57 | 111.3 | 146.9 |
| 10° | .17 | 33.2 | 46.8 | 40° | .64 | 125.0 | 162.5 |
| 15° | .26 | 50.7 | 69.3 | 45° | .71 | 138.6 | 175.7 |
| 20° | .34 | 66.4 | 89.8 | 50° | .77 | 150.3 | 185.9 |
| 25° | .42 | 82.0 | 110.3 | 55° | .82 | 160.1 | 192.3 |
| 30° | .50 | 97.6 | 129.3 | 60° | .87 | 169.8 | 195.2 |

**136.** Las direcciones é intensidades de las reacciones de los extremos y de las presiones en las piezas, debidas al viento, dependen de que uno ó ambos apoyos estén

fijos. Si ambos están fijos, sus reacciones son paralelas á la presión normal del viento, es decir, están en ángulo recto con el lado del techo sobre el cual el viento está soplando; pero si uno de los extremos está libre para resbalar en el sentido de la armadura, su reacción se tomará como vertical y la del otro estará más cercana á la horizontal que la presión normal del viento. Cuando un extremo está libre, las presiones deben determinarse como si el viento soplara en el lado fijo (en cuyo caso tenderá á achatar el techo) y también como si el viento soplara en el lado libre, en cuyo caso su componente horizontal tiende á acortar el tirante y á levantar el vértice. Los esfuerzos sobre las piezas de las armaduras de techo se encuentran fácilmente por el método de secciones, §§ 67, etc., ó gráficamente como se indica más abajo.

**137.** La fig. 42 (*a*) ilustra el trazado gráfico de las presiones del viento en la fig. 42 (*b*), bajo las tres condiciones citadas, á saber: caso 1.°, con ambos extremos fijos; caso 2.°, con el viento soplando contra el lado fijo, y caso 3.°, con el viento soplando contra el lado libre *.

En la fig. 42 (*a*), los segmentos *ab*, *bc*, *cd* y *de* representan las presiones normales del viento en las puntas de los tramos AB, BC, CD y DE, respectivamente, y *de* por consiguiente representa la presión normal del viento total sobre el techo, estando todas ejercidas contra el lado izquierdo *.

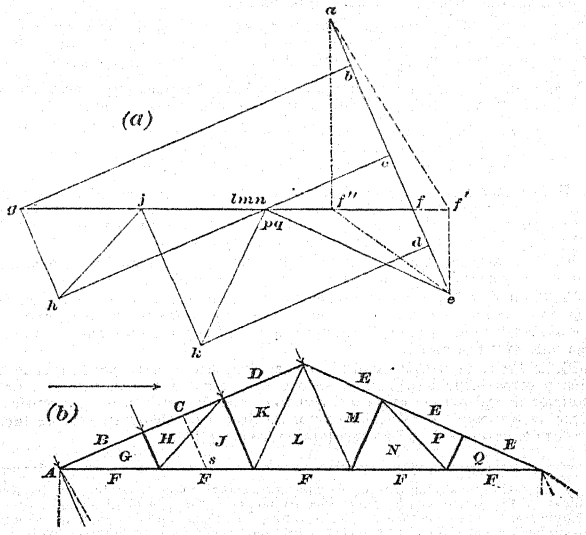



**Fig. 42.**

**138.** En el caso 1 (ambos extremos fijos) los segmentos *fa* y *ef* de la línea continua *ea*, representan las reacciones izquierdas y derechas respectivamente.

**139.** En el caso 2 (soplando contra el lado fijo) las reacciones están representadas por la línea de puntos *ef'a*, y en el caso 3 (soplando contra el lado libre) por la línea de puntos y rayas *ef''a*.

**140.** Los segmentos *f'f* y *ff''* representan respectivamente las componentes horizontales de las reacciones izquierda y derecha del viento en el caso 1, y *f'f''* la reacción del viento en el extremo fijo en el caso 2 ó 3, ó la de la reacción total del viento.

* Para evitar la necesidad de presentar dos figuras en esqueleto y dos diagramas,

**141.** Habiendo encontrado, por los momentos, las reacciones *ef* y *fa* de los extremos para el caso 1, en que, la fig. 42 (*b*),

$$ef = ae \frac{As}{\text{Luz de la armadura}};$$

las reacciones verticales, *ef'* y *af''*, para los casos 2 y 3 respectivamente, se encuentran bajando perpendiculares de *e* y de *a*, fig. 42 (*a*), sobre *gf* prolongada.

Las reacciones de los extremos fijos serán entonces dadas por la línea de unión *f'a*, en el caso 2, y por *ef''* en el caso 3.

**142.** Los esfuerzos en las barras del alma ó trabazón GH, MN, etc., fig. (*b*), y los de las varias partes de los pares BG, EM, etc., estarán dadas por las líneas correspondientes, *gh*, *mn*, *bg*, *em*, etc., en la fig. (*a*).

**143.** En el par á sotavento, el esfuerzo en los tres segmentos, ME, PE, QE, es uniforme en todas partes, y además es el mismo en cada uno de los tres casos, siendo $= me = pe = qe$.

En las cuatro piezas, LM, MN, NP, PQ, hacia sotavento, el esfuerzo en este caso es cero, representado como está por *lmnpq*, fig. (*a*).

**144.** Las presiones en las varias piezas, GF, JF, LF, NF y QF, del tirante horizontal, fig. (*b*), están representadas en la fig. (*a*).

en el caso 1 (ambos extremos fijos) por *gf*, *jf*, *lf*, *nf* y *qf*;
en el caso 2 (viento contra el lado fijo) por *gf'*, *jf'*, etc.;
en el caso 3 (viento contra el lado libre) por *gf''*, *jf''*, etc.

En cada uno de los tres casos hay una tensión uniforme en los tres segmentos de sotavento, LF, NF y QF, del tirante horizontal. Esta tensión uniforme es

en el caso 1 (ambos extremos fijos) $= lf = mf = nf$;
en el caso 2 (viento contra el lado fijo) $= lf' = mf' = nf'$;
en el caso 3 (viento contra el lado libre) $= lf'' = mf'' = nf''$.

**145.** Se verá así que, en nuestra figura, con tirante horizontal, la diferencia en el modo de fijar los extremos afecta sólo los esfuerzos horizontales en las partes de ese tirante, y, por medio, de ellas, la manera como la componente horizontal *f'f''* de la presión del viento se destribuye entre los dos apoyos.

**146.** Si la barra inferior ó tirante no fuera recta, los esfuerzos en las piezas interiores de la armadura y de los pares serían distintos en los tres casos.

**147.** El esfuerzo final ó resultante, en cualquier pieza, es la suma algebraica de las cargas muertas, de las debidas á la nieve y al viento en esa pieza. En algunos casos, la presión del viento puede disminuir y aun invertir los esfuerzos debidos á la carga muerta y á la nieve.

**148.** En las armaduras de madera de corta luz, figs. 43 á 47, para casas de habitación y otros edificios pequeños, se puede, con bastante exactitud, hacer una suposición liberal para la carga incluyendo la presión del viento. Al discutir estas figuras, investigaremos los esfuerzos por medio del paralelogramo de las fuerzas. Para las dimensiones de tales armaduras, véase el § 266.

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hemos supuesto que el viento sopla siempre en una dirección (contra el lado izquierdo), suponiendo fijo primero un lado de la armadura y después el otro. En la práctica, por supuesto, sucede lo contrario, es decir, uno ú otro extremo de la armadura está fijo (sino ambos), y permanece así, y el viento puede soplar contra uno ú otro lado. Las figuras, sin embargo, servirán también para esta última condición. Así pues, si el viento sopla contra el lado izquierdo, como está indicado, y si, como en el caso 2, ese lado está fijo, entonces el diagrama, usando las líneas quebradas *ef'a*, darán los esfuerzos en las piezas en el orden de las letras. Pero ahora (permaneciendo el extremo izquierdo fijo) supongamos que el viento sople contra el lado libre, es decir, de la derecha. Podemos sin embargo suponer que el extremo derecho esté fijo y que el viento sople contra el lado izquierdo, como en la fig. (*b*), y encontremos los esfuerzos en las piezas de la fig. (*a*) como se encuentra, usando el diagrama de puntos *ef''a*; pero debemos recordar entonces, que las presiones encontradas de esta manera para BG, GF, etc., á la izquierda de la armadura, fig (*b*), realmente se aplican á los miembros correspondientes, QE QF, etc., á la derecha, y viceversa.

**149.** En las armaduras de techo de madera, fig. 43, uniformemente cargadas á lo largo de los pares, sea HI=el peso de un par y su carga. Entonces EI=á la presión horizontal en la cabeza y en el pie de ese par (siendo esta última la tensión en el tirante), y HE=la presión inclinada en su pie.

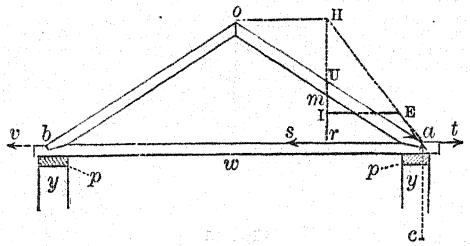


Fig. 43.

**150.** En la fig. 44, hágase GR=HI. Entonces GL será la presión transversal contra el par, como si fuera una viga, y LR será una presión longitudinal á lo largo del par, formando una *parte* de la presión longitudinal total.

**151.** Si GR estuviera concentrada en G, LR sería uniforme desde G hasta *a*, y no se ejercería por encima de G; pero como GR representa una carga distribuída uniformemente á lo largo del par desde el vértice hasta el pie, la presión representada por LR aumenta uniformemente, de cero, en el vértice, *o*, hasta ser LR, en el pie, *a*.

**152.** De la presión transversal, GL, una mitad=*op*, obra en el vértice, *o*, del par, y la otra mitad=*aq*, en el pie, *a*. En la parte superior, *op*, se descompone en una presión horizontal, *ob*=EI contra la cabeza del otro par, y en un empuje, *oz*, á lo largo de *oa*.

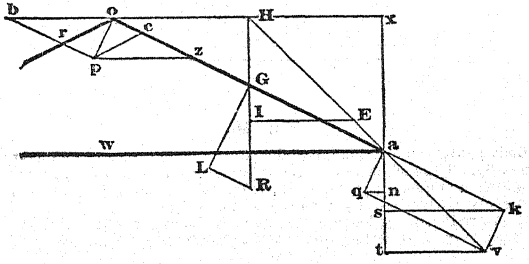


Fig. 44.

**153.** No importa que descompongamos á *op* directamente en *ob* y *oz* (como si la cabeza del par descansara contra una pared vertical en *o*), ó si la descomponemos primero entre los dos pares, en *oc* y *or*; porque en este último caso debemos añadir á *oc* un empuje (=*or*=*cz*) producido en *oa* por la presión transversal (semejante á *op*), de la cabeza del otro par, y la suma de estas dos (*oc* y *or*) es=*oz*.

**154.** El empuje longitudinal total en el par aumenta uniformemente de *oz*, en el vértice, á *oz* + LR=*ak*, en el pie, donde se combina con *aq* (=la mitad de la carga transversal) para formar á *av*=HE.

**155.** Tensión en el tirante=IE=*tv*=*sk* — *nq*. Presión vertical en el apoyo =HI=*at*=*as*+*an*=*as*+*st*.

**156.** En la fig. 45, habiendo encontrado, como en la fig. 43, los esfuerzos, etc., debidos á los pares y sus cargas, recordemos que el pendolón, *on*, soporta su propio peso más la parte *yy* del tirante = ½ del tirante. Haciendo *ot* = este peso combinado, tendremos *om* = *od* = una presión adicional uniforme en cada par, y *cm* = *cd* = una tensión adicional en el tirante.

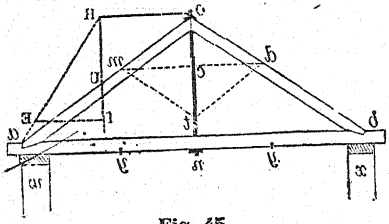

**Fig. 45.**

**157.** En la fig. 46, suponiendo para la seguridad, que el par *jb*, está dividido en su centro, U, hágase *eo* = el peso de *zr* y su carga (*zr* = la mitad del par).

**158.** Entonces *ei* = una presión adicional en U*b*, *ek* = presión en U*c*, *si* = *sk* = tensión adicional en la mitad del tirante, *cb* y *eo* = 2 *es* = carga de *zr* y en *zr*, = tensión adicional en el pendolón debida á ambos tornapuntas. Entonces hágase *ag* = *eo* + peso del pendolón + peso de los dos tornapuntas + peso de *yy* sobre *yy*, y procédase como en la fig. 45.

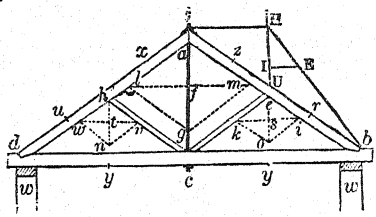

**Fig. 46.**

**159.** Cada puntal soportará la mitad del peso de *zr* y en *zr* ó *xu;* solamente cuando como en la fig. 46, la inclinación del puntal es la misma que la del par. Si el puntal está más inclinado que el par, soportará más de la mitad, si menos, menos de la mitad, y el resto lo soporta el par, en todo caso.

**160.** La introducción de los puntales convierte cada par, considerado como viga, en dos vigas de luz más corta y que soportan cargas menores.

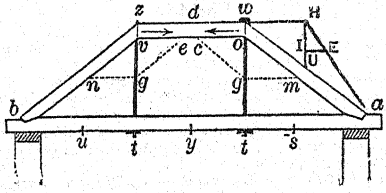

**Fig. 47.**

**161.** En la armadura de la forma, fig. 47, hágase *og* = la tensión total en la péndola + la mitad del peso de *zw*. La presión longitudinal sobre *zw* = la tensión en el tirante, *ba* = IE + *oc*.

## Deformaciones.

**162. La deformación total** de una armadura comprende (1) la deformación elástica ó temporal, debido al estiramiento * de sus varias piezas bajo la carga aplicada á la armadura, y (2) la deformación no elástica ó permanente, debida á la flojedad de sus uniones. En una buena construcción esta última es relativamente despreciable en las luces moderadas.

La deformación total elástica, D, de una armadura, en cualquier punto, *c*, se compone de las deformaciones elásticas parciales, *d*, *d*, etc., en *c*, cada una debida al alargamiento *k* *, en alguna pieza.

Supóngase que deseamos encontrar la deformación en un punto de división de tramo, *c* (ordinariamente, el centro de la luz ó el extremo del brazo de un puente levadizo ó el de un *cantilever*); y para cualquiera carga ó sistema de cargas, sea :

D = á la deformación total en *c*;
*d* = á la deformación elástica parcial en *c*; debida al alargamiento K * en cualquier pieza;
*p* = el esfuerzo en esa pieza por unidad de superficie;
P = el esfuerzo total en la pieza;
*l* = longitud de la pieza;
$k$ = alargamiento * de la pieza = $\frac{pl}{E}$;
W = la carga que, aplicada en *c*, produciría el esfuerzo P en la pieza;
$u = \frac{P}{W}$;
E = coeficiente de elasticidad del material = $p \div \frac{k}{l} = \frac{pl}{k}$.

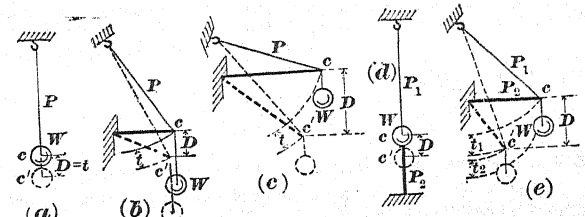


**Fig. 48.**

**163. Equivalencia de trabajo.** En la fig. 48, apliquese una carga, W, en cualquier punto, *c*, de una armadura ó barra. Entonces para una deformación pequeña ** como la que se permite en las armaduras, el trabajo exterior W*d* § de una deformación parcial, *d*, debida al estiramiento, *k*, en cualquier pieza, es prácticamente = al trabajo interno P*k*, en vencer la resistencia, P, en esa pieza, en la distancia *k*; ó bien

$$W d = P k.$$

Por consiguiente,

$$d = \frac{P}{W} \times k; \quad \text{ó bien} \quad \frac{k}{d} = \frac{W}{P}.$$

---

* Por brevedad usamos aquí la palabra alargamiento ó estiramiento para indicar cualquier cambio en la longitud refiriéndonos ya sea al acortamiento debido á la compresión, así como al alargamiento debido á la tensión.

** Para más claridad, los alargamientos y flexiones en nuestras figuras están exagerados mucho más allá del límite en que la razón $\frac{P}{W}$ permanece aproximadamente constante.

§ Estrictamente hablando, con una carga que aumente gradualmente de 0 á W, y con esfuerzos resultantes que aumenten gradualmente de 0 á P, deberíamos tomar la carga media = $^1/_2$ W, y la presión media = $^1/_2$ P, en cada pieza, pero se verá que esto no afecta las ecuaciones que se derivan.

Es decir: el estiramiento $k$, en cualquier pieza, está con la deformación parcial resultante, $d$, en $c$, en razon inversa del esfuerzo P, en esa pieza y del peso W, que aplicado en $c$, engendraría dicho esfuerzo P.

Así, en la fig. 48 $(a)$, en que $k$ está en la misma dirección que D*, P=W, y D=$k$.

En las figs. $(b)$ y $(c)$**, suponiendo el puntal incompresible; D se debe solamente al estiramiento de la cuerda, y $D = \frac{P}{W} \times k = uk$.

En la fig. $(c)$, $\frac{P}{W}$ es mayor, y (para un estiramiento dado, $k$, en la cuerda) D es por consiguiente mayor que en la fig. $(b)$.

**164. La deformación es independiente de la naturaleza de la fuerza.** Es evidente que la deformación $d$, en $c$, depende únicamente de la magnitud de $k$ en la pieza y es independiente de la naturaleza de la fuerza que produce el estiramiento. Es decir, cualquier cambio, $k$, en la longitud de la pieza, contribuye necesariamente con la cantidad fija, $d = \frac{P}{W} k$, á la deformación total, D en $c$. En otras palabras, puesto que $d$ y $k$ son meras distancias, y $u$ es simplemente un cociente, la relación entre $d$ y $k$ es puramente geométrica, y no está por consiguiente limitada á las deformaciones producidas por pesos, sino que es también aplicable á las producidas por cambios de temperatura, $o$, á los alargamientos y acortamientos intencionales de las piezas, ó á cualquiera otra causa.

Por consiguiente, si un miembro se alarga ó se acorta de cualquier modo, en una longitud $k$, ocurre un cambio correspondiente, $d, = \frac{P}{W} k = uk$, en la deformación en $c$.

Por ejemplo, si colocamos cualquier sistema de pesos sobre una armadura, y, por los principios de la estática, determinamos el esfuerzo total resultante, P, y la presión por unidad de superficie, $p$, en cualquier pieza, tendremos para la deformación parcial en $c$, debida al estiramiento, $k$, en esa pieza, bajo el sistema dado de cargas:

$$d = uk; \qquad \text{(Para } u\text{, véase § 165.}$$

y, puesto que $k = \frac{pl}{E}$,

$$d = \frac{pul}{E}.$$

**165. Para obtener la razón,** $u, = \frac{P}{W}$, para cada pieza, supondremos una carga concentrada aplicada en $c$, y por los principios de la estática, encontraremos los esfuerzos totales resultantes, P, P, etc., en los varios miembros. Si la carga que suponemos en $c$, se toma=la unidad, los esfuerzos, P, P, etc., así encontrados, serán los cocientes que deseábamos, $u$, $u$, etc.

**166. Suma de las deformaciones.** Siendo la deformación total D, en $c$, bajo el sistema dado de cargas=la suma de las deformaciones parciales, $d$, $d$, debidas (pero no necesariamente igual) respectivamente á los alargamientos $k$, $k$, en las varias piezas, tenemos

$$D = \Sigma d = \Sigma \frac{pul}{E}.$$

Así en las figs. $(d)$ y $(e)$ suponemos que la cuerda es extensible y el puntal compresible. En la fig. $(d)$, $W = P_1 + P_2$, y $WD = P_1 k + P_2 k = (P_1 + P_2)\, k$. Por consiguiente, $D = \Sigma uk = \frac{P_1}{W} \cdot k + \frac{P_2}{W} \cdot k = \frac{P_1 + P_2}{W} \cdot k = \frac{W}{W} \cdot k = k$. En la fig. $(e)$ $D = \Sigma uk = \frac{P_1}{W} \cdot k_1 + \frac{P_2}{W} \cdot k_2 = u_1 \cdot k_1 + u_2 \cdot k_2$.

**167. Alargamientos positivos y negativos.** En algunos casos puede suceder que el cambio de longitud de un miembro disminuya, en vez de aumentar, la deformación total en el punto $c$, en cuestión, y debe por consiguiente tomarse como

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* Cuando, como en las figs. $(a)$, $(b)$ y $(c)$ se supone que sólo una pieza cambia de longitud D = $d$.

** Véase la nota ** en la página anterior.

negativa al sumar los valores de $u\acute{c} = \frac{pul}{E}$; pero cuando $c$ es el punto medio de una luz, ó el extremo de *cantilever*, todos los cambios en la longitud de las piezas contribuyen ordinariamente al alargamiento y deben por consiguiente tomarse como positivos.

Teóricamente, la fórmula $D = \Sigma \frac{pul}{E}$ se aplica también á las deformaciones de los arcos, represas y otras estructuras compuestas de bloques; pero debido á la inseguridad de los valores de E, es de muy poca utilidad práctica en estos casos.

**168. Piezas estáticamente indeterminadas.** Las armaduras contienen ordinariamente piezas cuyos esfuerzos no pueden encontrarse por los principios de la estática. Así, en la fig. 11 (*c*), las dos piezas diagonales de tensión que se encuentren en la parte superior de uno ú otro poste extremo se dice que son estáticamente indeterminadas, porque dichos principios no nos permiten determinar qué proporción de la carga total va á los apoyos por cada uno de los dos sistemas, figs. 11 (*a*) y (*b*), que componen la fig. 11 (*c*). Pero la fórmula que se acaba de dar nos permite determinar los esfuerzos en tales piezas, porque, por medio de ella podemos encontrar, separadamente, la deformación en cada uno de los dos sistemas, figs. 11 (*a*) y (*b*), y la parte de la carga, transmitida á los apoyos por cada uno de estos dos sistemas, inversamente proporcional á sus deformaciones.

## DETALLES Y CONSTRUCCIONES DE LOS PUENTES

### Principios generales.

**169.** En general, un *puente* de armaduras se compone esencialmente de dos ó más *armaduras* verticales, AB, CD, fig. 49, colocadas una al lado de la otra, y unidas por el sistema de pavimento, que, á su vez, soportan, colocando donde sea practicable, barras transversales (que forman un « *sistema lateral* ») entre los dos cordones opuestos, para sostener las armaduras paralelas.

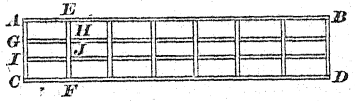


**Fig. 49.**

**170. El sistema de pavimento** se compone ordinariamente de *vigas de pavimento* y piezas de refuerzo. Las vigas del pavimento AC, EF, etc., fig. 49, se colocan transversalmente al puente, y están sujetas á las armaduras en los puntos opuestos de división de los tramos. Unidas á éstas y perpendiculares á ellas ó paralelas á las armaduras están las vigas longitudinales GH, IJ, etc. En los puentes para ferrocarriles hay ordinariamente dos ó más vigas longitudinales colocadas una al lado de la otra y paralelas á todo lo largo del puente, para sostener las traviesas. En los puentes de las vías de tráfico de las ciudades, estas vigas se colocan ordinariamente á pequeños intervalos y soportan placas unidas ú otra forma de apoyo sobre el cual se coloca el pavimento. En los puentes de los caminos reales en el campo, las vigas longitudinales son ordinariamente de madera, colocadas bastante juntas, y las planchas del piso se clavan directamente en ellas.

**Los pisos sólidos** (véase Pencoyd secciones de pisos) contribuyen á la rigidez y permanencia de un puente, y protegen mejor el tráfico debajo, contra la caída de cuerpos ó en caso de descarrilamiento. Su estructura plana es una ventaja, cuando se requiere espacio libre hacia arriba.

**171. Una carga** cualquiera es, pues, **transmitida** primero de los durmientes ó piso, etc., á las vigas longitudinales, después por éstas á las vigas del piso, y finalmente á los puntos de división de los tramos del puente, de donde se transmite por las armaduras á los apoyos.

**172. Los pedestales** ó placas de asiento, fig. 62, atornilladas á los estribos, soportan los extremos de las armaduras. Cuando el puente es de larga luz, de manera que la dilatación y contracción debidas el calor y al frío, sean considerables, deben colocarse placas ó rodillos, etc., **de expansión** en un extremo. Véanse §§ 205, etc. Para las riostras, barras laterales, etc., véanse §§ 19, etc.

## Naturaleza general del trazado.

**173. Piezas de tensión flexibles y rígidas. Ajustes. Contradiagonales.** Hasta hace poco, **las barras de ojo** se usaban como piezas de tensión de las armaduras. Éstas son barras largas chatas, que ceden bajo la compresión, y provistas en sus extremos con ojos ó aberturas, por los cuales entran los pasadores que las unen á las otras piezas del puente; pero hoy se usan como piezas de tensión, piezas rígidas, capaces de soportar alguna compresión, tan bien como la tensión. Las contradiagonales se hacían ordinariamente en dos largos y eran **ajustables**, uniendo las dos partes por tensores; pero éstos pueden causar esfuerzos indebidos y peligrosos en los tramos, y están reemplazados por contradiagonales hechas de una sola pieza.

**174. Las piezas de compresión** se hacen en piezas con ángulos y planchas, ó de canales y planchas, enrejadas, en forma hueca, que lleva la mayor parte del material tan lejos como sea posible del eje neutro de la sección transversal, aumentando así su momento de resistencia.

**175. Uniones de pasador y remachadas.** Las piezas del entramado se unen á los cordones por pasadores ó remaches. En el primer caso la armadura se llama de pasadores .y en el segundo, de remache. Hasta hace poco, las armaduras de pasadores han sido típicas en la práctica americana, pero ahora usan mucho las armaduras de remache con luces hasta de 45 á 50 m, mientras que los europeos usan en algunos casos pasadores. La ventaja principal que se les atribuye á los remaches, es que hacen el puente más rígido y no vibra, y que una armadura de remache comparada con la de pasadores, posee un margen adicional de seguridad á causa del aumento de rigidez. En los puentes de pasadores, por otro lado, se pueden determinar los esfuerzos con más exactitud, y las deformaciones pueden ocurrir sin producir esfuerzos de torsión ó de flexión en las mismas uniones.

**176. Tendencia á mayor rigidez.** Hay una tendencia creciente en usar barras más rígidas, por lo menos en todas las piezas cortas sometidas á compresión, y hacer aún las piezas más largas de tensión, de canales ó ángulos, formando una barra rígida. Las barras de ojos, unidas por pasadores, si no son de longitudes exactamente iguales, recibirán algunas, mayor esfuerzo del que les corresponde en el esfuerzo total.

**177. Uniones de las vigas del piso.** En los Estados Unidos, las conexiones de las vigas del piso se hacían antes suspendiendo las vigas de los pasadores por medio de soportes colgantes, pero ahora, donde es posible, sus extremos se remachan directamente á los lados interiores de los postes.

**178.** En las piezas de tensión, los remaches se arreglan de modo que reduzcan la sección neta efectiva lo menos posible.

**179.** Las piezas de compresión se disponen de manera que la mayor parte del material quede lo más lejos posible de sus ejes neutros, y son á veces reforzadas por cuerdas ó postes auxiliares que las apoyan en sus puntos medios, en los casos en que la economía de material hecha en la pieza sea considerablemente mayor que el que se gasta en el poste auxiliar.

**180.** Hasta donde sea posible, los miembros sometidos á compresión deben hacerse igualmente resistentes á la flexión en ambos ejes principales, AB y XY, fig. 52, de sus secciones transversales.

**181.** Cuando una pieza se repite muchas veces en un puente, y por consiguiente un exceso de material en su diseño causaría un gran desperdicio total, se repetirá el cálculo de la pieza hasta encontrar la sección más económica.

**182.** En las armaduras de metal, los miembros más cortos se hacen para resistir compresión, y los más largos, tensión, resultando así más economía de material. Por eso la armadura Pratt, con piezas de tensión diagonales, se usa en los puentes de acero, y la armadura Howe se fabrica ahora solamente con diagonales de madera.

## Piezas sometidas á tensión.

**183. En las barras de ojo, el área de la sección transversal** $= \frac{\text{tensión máxima}}{\text{unidad de tensión permitida}}$.

**184.** Las dimensiones de las cabezas de las barras de ojo las determinan ordinariamente los manufactureros, y las trazan dándoles un exceso de fuerza en los agujeros para los pasadores, de manera que, al someterlas á prueba, los dos tercios se quebrarán en el cuerpo de la barra; así se exige ordinariamente en las especificaciones. Es importante que las proporciones de las cabezas de las barras de ojo sean tales que tengan suficiente resistencia al recalcarlas.

**185.** La fig. 50 muestra, en dos escalas diferentes, el « acomodo » (arreglo de pasadores y barras de ojo) en la mitad izquierda del cordón inferior de un puente oblicuo, de 45 m de luz, fabricado por la « Phoenix Bridge Company », en 1900, para la Compañía del « Ferrocarril Philadelphia and Reading », cerca de Reynolds, Pa.

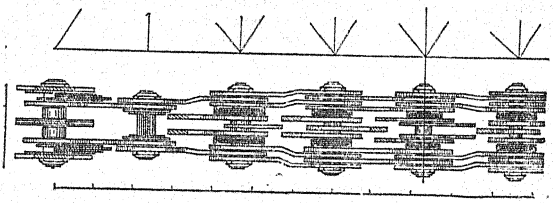

Fig. 50.

**186. Construcción de secciones.** Los montantes, las contradiagonales y sus partes principales se fabrican ordinariamente de acero laminado. Una sección de uso común es la de la fig. 51; se compone de cuatro ángulos unidos, á intervalos, por pequeñas barras chatas angostas remachadas á los ángulos en zigzag.

Cuando es sencillo como en la fig. 51, se llama « entrenzado » (*lacing*); cuando es doble como en la fig. 52 (*b*), « enrejado » (*latticing*). El área sombreada de los ángulos, fig. 51, menos la de los agujeros de los remaches, se toma como sección efectiva.

**187. Secciones mínimas.** Las especificaciones (véase Resumen) requieren á menudo el uso de alguna sección mínima. Así, en contradiagonales en que la presión es de 26,308 kg (58,000 libras), 22.57 cm cuad (3.5 pulgs cuad) de sección transversal serían suficientes; pero las especificaciones prohiben á menudo el uso en tales secciones, de un ángulo menor de $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{7}{16}$, que da 59.34 cm cuad (9.20 pulgs cuad) en bruto, ó, deduciendo un agujero de remache de cada ángulo, 49.5 cm cuad (7.68 pulgs cuad) de sección neta.

## Piezas sometidas á compresión.

**188. El cálculo** de una pieza sometida á compresión consiste en una serie de aproximaciones, porque la unidad de esfuerzo depende del radio de giro, del área de la sección y la disposición de material con respecto á los ejes, y ésta, á su vez, de la unidad de esfuerzo. Véase Pilares, en Resistencia de Materiales.

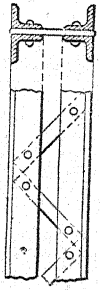

Fig. 51.

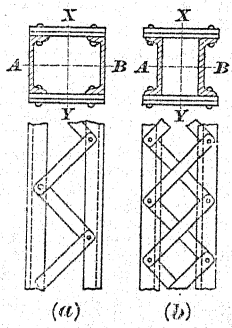


Fig. 52.

**189.** La fig. 52 (*a*), que se usa con frecuencia en los postes, compuesta de dos vigas en U, colocadas con la parte de atrás hacia fuera, ligadas entre sí con entrenzado (§ 186). En la fig. 52 (*b*) las vigas en U están colocadas con la parte posterior

hacia dentro. Por economía las vigas en U se deben separar de tal modo que el radio de giro sea el mismo respecto á uno ú otro eje, A-B ó X-Y. Este radio lo dan los manuales de los manufactureros de estas piezas de forma.

**190.** La sección del cordón superior se construye ordinariamente con dos vigas en U y una placa, ó en la forma que indica la fig. 53, que se compone de dos placas verticales ó almas, una placa horizontal superior ó « cubierta », cuatro « ángulos » y piezas chatas en cada lado de la parte inferior. El refuerzo de enrejado ó de entrenzado se coloca á lo largo de la parte inferior, excepto en los puntos de división de los tramos, donde se omite para que el poste y las cuerdas puedan entrar en la viga por debajo. En las armaduras de pasadores el eje del pasador se encuentra en la línea AB.

**191.** El ancho interior, *w*, depende principalmente del espacio que requieran el poste y las cuerdas que se encuentran en la división del tramo, y también de la altura de las cabezas de los remaches interiores. Ordinariamente, para comodidad en la construcción, el mayor ancho, *w*, que requieran se conserva constante hasta el cordón superior. La altura, H, depende principalmente del tamaño de la cabeza de la barra de ojo, y se conserva constante. El espesor de las placas del entramado ó alma, y algunas veces los de las barras y ángulos también, varían á lo largo del cordón, para tener, en cada punto, área suficiente para resistir los esfuerzos.

**192. El poste extremo** debe considerarse, no solamente como una columna, sino como una viga sujeta á esfuerzo constante, porque el viento sopla contra la parte superior del lado de la armadura. El trazado de esta construcción en piezas es muy parecido en principio al que se ha dado más arriba para los postes. Ciertas secciones se prueban, y después se cambian si es necesario.

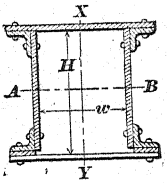


Fig. 53.

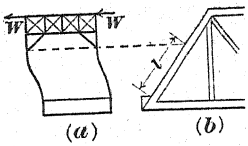


Fig. 54.

**193.** El poste extremo debe estar seguro, no solamente contra la flexión alrededor al eje AB, fig. 53, contra compresión, sino también contra flexión alrededor del otro eje X-Y, bajo el efecto combinado de la compresión y del momento de flexión, debido á que el viento sople contra la parte superior de la armadura. Véase fig. 54 (*a*).

**194.** Las riostras de la entrada ó portal del puente, arriba, y la viga del piso, abajo, se supone que impiden la flexión de la parte de los postes adyacentes á ellos, y podemos considerar cualquier mitad de uno ú otro poste ($=\frac{1}{2}l$) como un *cantilever* vertical, fijo en un extremo y cargado en otro extremo (que es el punto medio del poste) con una carga=la presión del viento en la mitad de la parte superior de una armadura.

**195.** Los máximos esfuerzos debidos á la compresión, obran, por supuesto, cerca del medio del poste, mientras que los debidos al viento obran cerca de los extremos. Por tanto sería irracional exigir que el poste resista ambos efectos simultáneamente en toda su longitud, y las especificaciones por consiguiente conceden que la unidad del esfuerzo debido á la carga muerta, viva, percusiones y fuerza del viento (combinadas), se aumente hasta 21,000 libras por pulg cuad (1,470 kg por cm cuad), debidamente reducida por la fórmula para compresión.

**196.** Para la resistencia de las piezas largas sometidas á compresión, véase más adelante Compresión en Resumen de especificaciones; § 19 en Columnas de hierro y acero; §§ 1 á 5 y 7 en Columnas de madera.

**197.** La fórmula para la presión extrema de las fibras debida á la compresión y flexión combinadas, es:

$$S = \frac{P}{A} + \frac{M_b T}{I - \frac{Pl^2}{Ec}}.$$

En que P=la fuerza compresiva longitudinal;
A=área de la sección;
$M_b$=momento de flexión debido á la carga transversal;
T=distancia del eje neutral á las fibras extremas;
I=momento de inercia;
$l$=largo de la viga;
E=módulo de elasticidad;
$c$=coeficiente. Véase Esfuerzo transversal, § 103.

## Uniones.

**198. Placas de pasadores.** Cuando un pasador pasa por una ó más partes de alguna pieza, sucede á menudo que las superficies combinadas de las piezas de la armadura, en contacto con el pasador, son insuficientes para transmitir todos los esfuerzos que deben transmitirse á esa pieza. Hay pues peligro de triturar el material que se comprime contra el pasador. Para evitar esto, otras partes, ordinariamente chatas y llamadas placas de pasadores ó placas de refuerzo, se remachan á la pieza, formando, suficiente superficie de apoyo para el pasador. Véase, fig. 55, en que las letras denotan:

AA, ángulos, W, alma, F, llenador,
C, cubierta, P, pasador, O, placa de pasador,
B, barra, J, telera, T, tabla.

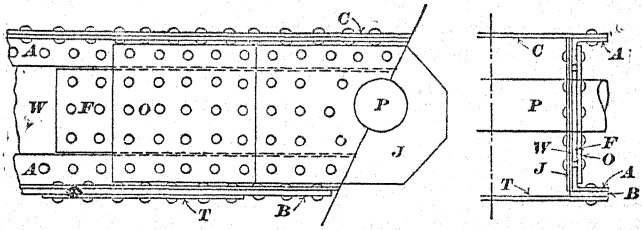


**Fig. 55.**

**199.** En la fig. 56, las dos vigas en U forman toda la pieza (excepto el enrejado que no puede incluirse para resistir compresión) y el pasador atraviesa por ambas vigas en U. En el caso de una sección del cordón ó de un poste extremo, fig. 53, las almas caladas forman solamente una parte de la sección, mientras que la cubierta, los ángulos y las piezas chatas no pueden recibir presión directamente del pasador, pero deben recibirla indirectamente del entramado del alma y de las placas de pasador, empatadas á él.

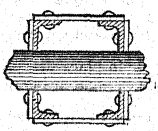

**Fig. 56.**

**200.** Cuando se coloca una placa de pasador en uno y otro lado del alma, la exterior debe, de acuerdo con la mayor parte de las especificaciones, cubrir los ángulos, y debe llevar además una placa de relleno entre ella y el alma.

**201.** Los ingenieros difieren respecto á la manera como se transmiten los esfuerzos por las diversas partes de una unión de pasador. Podemos suponer que los esfuerzos en las placas de pasadores se comunican casi directamente á las piezas. De este modo, la placa reforzadora exterior transmite probablemente la mayor parte ó todos sus esfuerzos á los ángulos, y poca ó ninguna al alma.

**202.** En cada ángulo, los remaches que pasan por la placa interior de pasadores,

deben transmitir, por medio de su apoyo contra el ángulo, la suma de los esfuerzos que reciben como esfuerzo cortante del interior y del exterior. En otras palabras, estos remaches están sometidos á doble esfuerzo cortante.

## Pasadores.

**203.** El pasador debe estar dispuesto de modo que resista los esfuerzos flexores de las piezas por las cuales pasa. Está también sujeto á esfuerzo cortante, pero esto es rara vez importante

**204.** El pasador que requiere la mayor sección transversal es ordinariamente el que está en el medio de la luz y en el cordón inferior, donde los esfuerzos del cordón son mayores, ó el que está en la unión del poste extremo y el cordón superior; pero como los pasadores son piezas pequeñas relativamente, todos los otros pasadores se hacen, por uniformidad, del mismo tamaño que él.

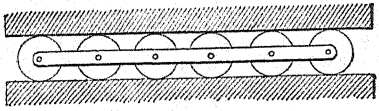

Fig. 57.

## Sistemas para la expansión en los apoyos.

**205. Las piezas de dilatación** se componen ordinariamente de un juego de cilindros cuidadosamente torneados que se colocan entre dos superficies planas, indicados en principio en la fig. 57.

**206. Los cilindros** son de acero, de 3 á 6 pulgs (7.5 á 15 cm) de diámetro, y de 1 á 4 pies (.30 á 1.22 m) de largo, con superficie lisa. De 4 á 8 de ellos ó más se unen por un marco, que se coloca debajo del extremo de una armadura por lo menos. Los cilindros descansan sobre una fuerte placa de hierro plana, fijada por debajo á la mampostería. Debajo del extremo de la armadura está una placa igual por la cual la armadura descansa sobre los cilindros. Como una armadura, aun de 60 m de luz, cambia apenas de longitud 7 cm por los extremos de temperatura, el juego de los cilindros es insignificante. Se les conserva en su lugar por medio de bordes alzados á su lado á lo largo de la placa. Los bordes deben también proyectarse de la placa superior hacia abajo para proteger los cilindros enteramente contra la lluvia, el polvo, etc.

**207.** El desplazamiento total en el extremo libre de una armadura se establece de antemano. (Véase Resumen de especificaciones.) Puede calcularse así, en metros:

$$D = 0.000011\ (T - t)\ \text{luz en m}$$

en que T y $t$ = las temperaturas máxima y mínima en grados C, respectivamente. La temperatura mínima que se puede esperar se obtendrá en los informes de la *Weather Bureau*, pero la máxima debe tomarse 10 ó 15° C más alta que la de la Oficina de Temperaturas, porque, con el sol fuerte, el puente se pone mucho más caliente que el aire.

**208.** Para disminuir el largo de la placa de apoyo, cuando el desplazamiento es moderado, se usan con frecuencia en lugar de los cilindros unas especies de balanceadores. Véase la fig. 62 *.

**209.** Para otros detalles respecto á la disposición de estas planchas y cilindros, véase el Resumen de las especificaciones, y las figs. 60 y 61.

## Cargas, etc.

**210. Cargas, sección libre, etc., para puentes de calzadas.** Véase también el Resumen de especificaciones para los puentes.

**Pesos de las multitudes.** En el puente Chelsea, Londres, los individuos agrupados y colocados sobre la plataforma de una romana, dieron una carga de

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* *N. del T.* — Estos no son cilindros de sección circular como se ven en la fig. 57. En la fig. 62 no se percibe la forma. Su sección es cercana á la elipse y á veces como la de

84 lbs solamente por pie cuad (como 410 kg por m cuad). Y en el Palacio de Buckingham individuos acuñados y tan apretados unos contra otros como era posible en un espacio de 20 pies (6.1 m) de diámetro, dieron 120 lbs por pie cuadrado (586 kg por m cuad). Pero con los experimentos modernos se han obtenido fácilmente cargas de 680 á 720 kg por m cuad. Con individuos, con peso medio de 73 kg cada uno, agrupados todos con la cara á un mismo lado, cuidadosamente colocados, y contenidos en un espacio de 1.83 m en cuadro, el profesor L. J. Johnson obtuvo, en la Universidad de Harvard, un máximo de 884 kg por m cuad. Véanse también las páginas. 817, etc.

Cuando la cerca del espacio es tal, que algunas de las personas que están paradas contra ella se inclinan hacia fuera, aumenta por supuesto la carga por unidad de superficie, y en las pequeñas, este aumento puede ser relativamente importante.

## Flecha de curvatura.

**211.** Dando al puente una flecha igual á ($^1/_{600}$) un seiscientosavo de la luz, tendrá una curvatura suficiente. La curvatura que debe usarse está ordinariamente estipulada en las especificaciones. Véase Resumen. Un puente bien trazado y bien construído bajo su carga mayor, no debe presentar una flexión mayor de 1 en 1200. Y en verdad, la flexión es siempre mucho menor que esto.

**212.** El exceso de longitud del cordón superior sobre el inferior, dadas la luz, la altura de la armadura y la curvatura, será =

$$\frac{8 \times \text{altura} \times \text{curvatura}}{\text{luz}}.$$

Esta regla se aplica rigurosamente con una curvatura que no exceda de .02 de la uz.

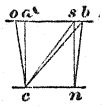


Fig. 58.

**213.** El largo de la diagonal, $cb$, fig. 58, $cb = \sqrt{ac^2 + ab^2}$ en que $ac$ = altura de la armadura y $ab = cn + \frac{ob - cn}{2}$.

**214.** Algunas veces, el alargamiento ó acortamiento, producido por la carga, se calcula para cada pieza y se cambia en proporción el largo de las piezas afectadas. Véanse §§ 162, etc.

## Ejemplos.

**215.** En las figs. 59 de ($a$) á ($u$) en una escala uniforme de 1 en 720 (una pulg por 60 pies), sirve para indicar la práctica corriente en la elección de los tipos de las diferentes luces, la relación, entre la luz, el largo del tramo y la altura, la disposición de los cordones y del entramado, el uso de las barras ó piezas rígidas y flexibles, las contradiagonales y tensores, en las armaduras, y, aproximadamente, las dimensiones de las piezas rígidas, escuadras ó placas de ángulos (*gusset plates*) y cubrejuntas, como se ven en la elevación.

**216.** En cada caso se muestra la mitad izquierda de la luz, estando la línea del centro de la luz indicada por una línea de puntos y rayas. Las armaduras de tablero inferior y superior se distinguen por la elevación de la vía, como se indica aproximadamente en el apoyo ó estribo izquierdo.

**217.** En las figs. de ($h$) á ($q$), que representan armaduras, las piezas rígidas están

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un abanico ó sector, con un movimiento de *balance*, siempre sobre la curva inferior, cuando el puente se alarga ó recoge. El eje mayor en el caso de la forma elipsoidal, es el vertical, y en el segundo se apoyan en la curva más amplia. Estos los llaman los ingleses *rockers*. No creemos que tienen nombre especial en español. Adoptamos balanceadores.

indicadas por líneas dobles; las flexibles en las verticales y diagonales principales, por líneas sencillas, y las contradiagonales, por líneas de puntos. En las vigas de

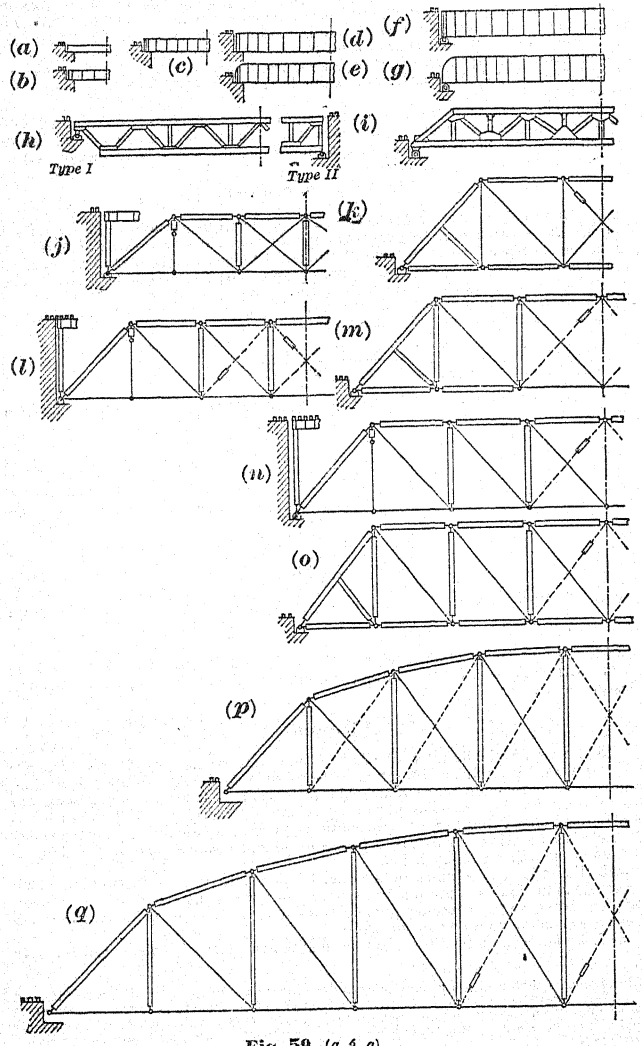



**Fig. 59.** (*a* á *q*).

pasadores, para evitar confusión, las piezas rígidas aparecen cortadas cerca de los pasadores.

**218.** Las figs. de (*a*) á (*o*) representan diseños modelos para luces de 7.5 á 61 m, por Mr. Ralph Modjeshy, I. C. de la Compañía de ferrocarril del Norte del Pacífico; la fig. (*p*), una luz de ferrocarril de 75 m trazada por la « Pencoyd Works » de la Compañía Americana de Puentes; la fig. (*q*), una luz de ferrocarril de 94 m por la Compañía de Puentes Phoenix; las figs. (*r*) á (*t*), diseños de armaduras roblonadas por la « Elmira Works of the American Bridge Co »; y la fig. (*u*), un puente de ferrocarril de 31.11 m de luz, trazado por la « Pencoyd Works ».

**219.** La fig. (*a*) representa una viga de remache; las figs. (*b*) á (*g*), vigas laminadas, y las figs. de (*h*) á (*g*), armaduras de remaches y de pasadores; las figs. (*h*) é (*i*) son de remaches; las figs. (*i*) á (*q*), de pasadores.

**220.** La fig. (*r*) representa una luz de 39 m para el Ferrocarril Central de Nueva York sobre el río Hudson, y las figs. (*s*) y (*t*), luces de 43.6 á 48.8 m respectivamente para el Ferrocarril Occidental de Delaware y Lackawanna. Las figs. (*r*) y (*s*) son modificaciones de la armadura Baltimore; la fig. 15 (*b*) y fig. (*t*) es una armadura

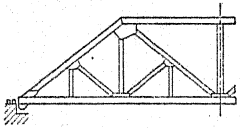


**Fig 59** (*r*).

*Vista del extremo.*

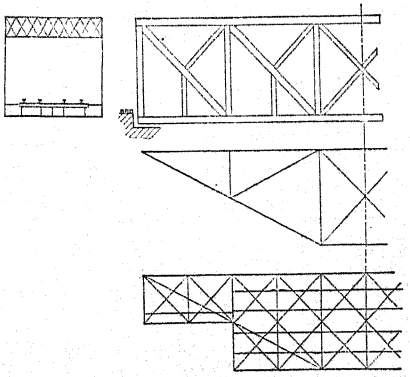


**Fig. 59** (*s*).

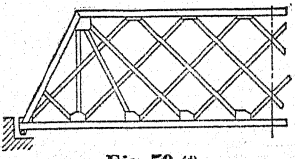
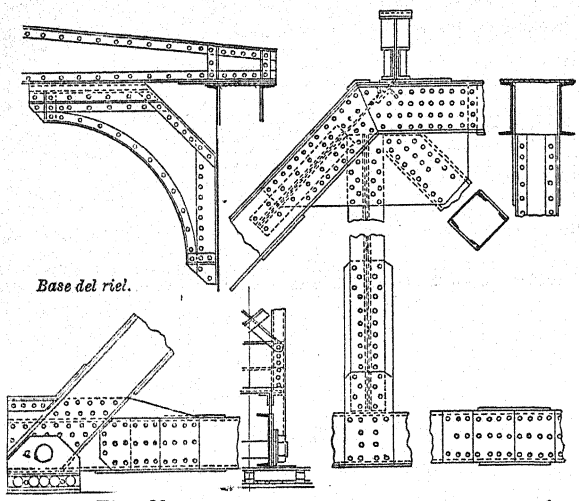


**Fig. 59** (*t*).

**Fig. 59** (*u*).

Warren cuádrupla. La fig. (*s*) es una luz de puente oblicuo. La fig. (*u*) es una luz (*pony*) *.

* *N. del T.* — « Pony » no puede, á nuestro juicio, traducirse por pequeño, pues indica algo más. Quizás sea para este caso : enana, chata, ó algo así el término conveniente en español y que no hemos podido encontrar.



*Base del riel.*

Fig. 60.

**221. Detalles.** En las figs. 60 á 65 se ven unos pocos detalles sobre las armaduras y las vigas remachadas.

**222.** Las figs. 60 y 61 indican las uniones del extremo izquierdo de dos armaduras de puentes de tablero inferior (con apoyos de cilindro para la dilatación) proyectadas por la « Pencoyd Works »; la fig. 61 representa una armadura de pasadores, de tablero inferior de 76 m de luz que corresponde á la fig. 59(*p*), y la fig. 60, una remachada, de tablero inferior de (124) 37.8 m, donde se ven las vigas del portal del puente.

(*N. del T.* — Para las figs. 60 y 61 la escala es $^1/_{40}$).

*Base del carril.*

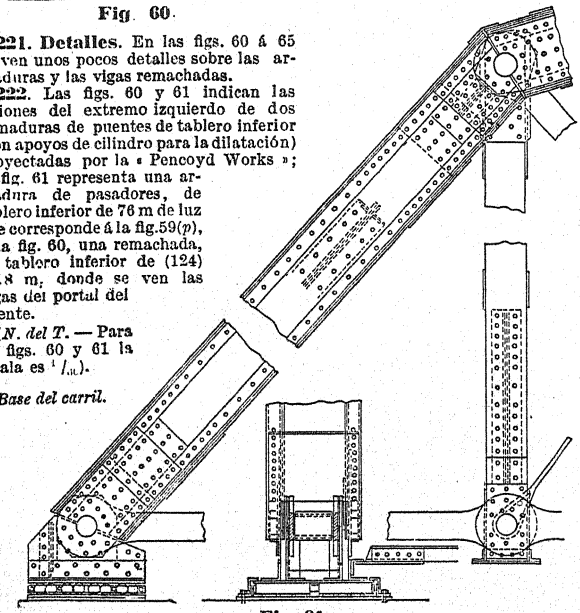


Fig. 61.

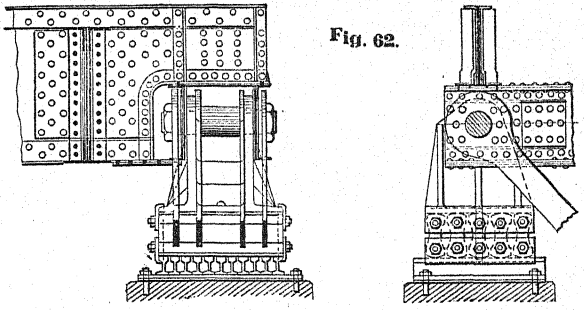


Fig. 62.

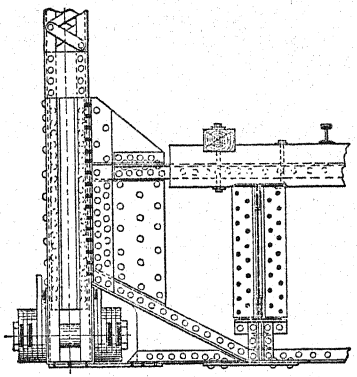


Fig. 63.

**223.** La fig. 62, muestra el apoyo del puente en el extremo izquierdo con balanceadores (véase *N. del T.* al § 208), y la unión de las vigas del piso de un puente de pasadores, de tablero superior de 61 m de luz, en el ferrocarril del Pacífico, fig. 59 (*n*); y la fig. 63 muestra la unión de las vigas del piso de un puente, de pasadores de tablero inferior de 48.8 m del mismo ferrocarril, fig. 59 (*m*).

(*N. del T.* — En las figs. 62, 63 y 64, la escala es $^1/_{50}$)

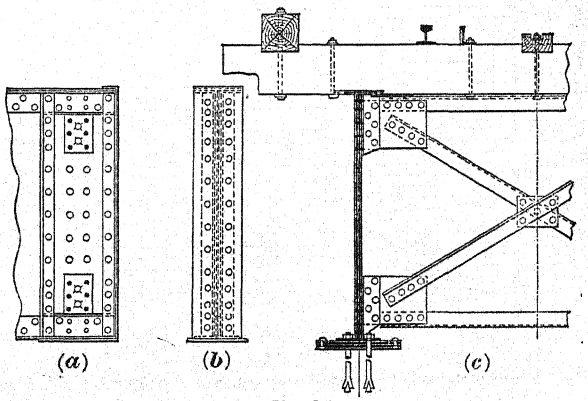



Fig. 64.

**224.** Las figs. 64 y 65 representan respectivamente una viga de 15 m, de tablero superior y otra de 25.9 m de tablero inferior del mismo ferrocarril.

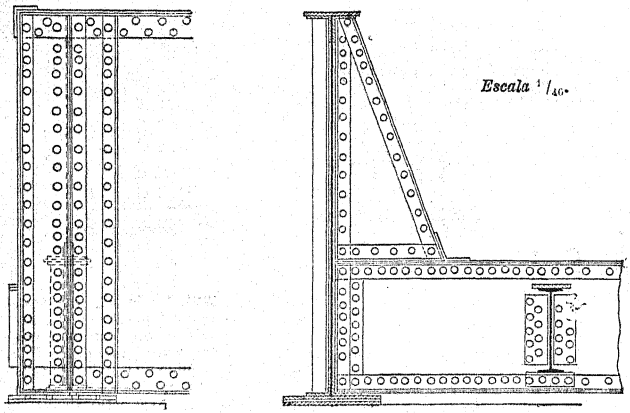



**Fig. 65.**

**225. Pesos de los puentes de acero para ferrocarril.** La fig. 66, fundada en la práctica del ferrocarril del « Northen Pacific Railway », 1902, muestra aproxi-

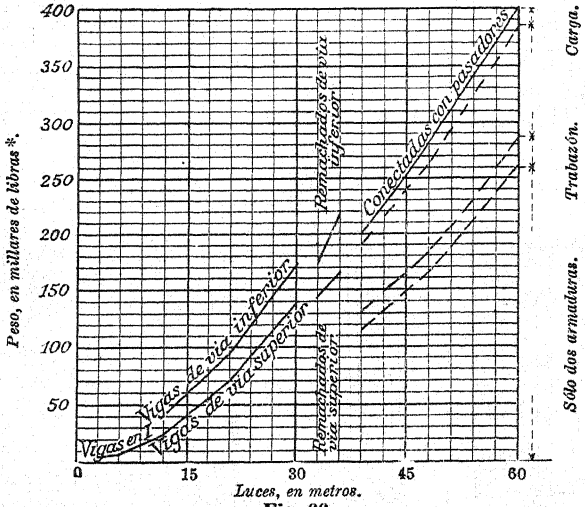



**Fig. 66.**

madamente los **pesos de los puentes de acero para ferrocarril,** proyec-

* *N. del T.* — 1,000 libras = 453.5 kg.

tados para dos locomotoras, de 146 toneladas cada una, y un tren cargado uniformemente con (4,000 libras por pie de vía) 5,960 kg por metro. Los pesos incluyen las dos vigas, las armaduras de una sola luz de una vía con sus riostras, refuerzos, sistema de piso metálico y apoyos extremos. Para los de sistema de piso de madera, agréguese 600 kg por m lineal. En los puentes de 40 á 60 m, unidos con pernos, los tres diagramas de rayas indican respectivamente los pesos de las dos armaduras solas, de las dos armaduras y trabazón, y de las dos armaduras, trabazón y piso de metal. La curva continua incluye los pesos de las piezas de apoyo extremo.

Para los pesos de los puentes mixtos (hierro y madera), de ferrocarril, véase § 249.

Los puentes para calzadas difieren tanto en la práctica y en los proyectos, que sería apenas posible dar aquí datos útiles sobre su peso.

### Lista de grandes puentes.

Todos los puentes que damos aquí creemos que son los más grandes de su tipo en el mundo.

| Tipo. | Luz sobre el | En | Luz, metros. | Edificado. |
|---|---|---|---|---|
| Armadura..... | Río Ohio | Louisville | 167.8 | 1893 |
| Levadizo..... | Río Missouri | Omaha | 158.6 | 1894 |
| De suspensión..... | Río del Este («Brooklyn») | Nueva York | 486.5 | 1882 |
| De suspensión..... | Río del Este («Nuevo») | Nueva York | 488.6 | |
| De arco (metal)..... | Río Niágara | Cataratas del Niágara | 256.2 | 1898* |
| De arco (piedra)..... | Petruff Valley | Luxembourg | 84.5 | |
| *Cantilever*..... | Firth of Forth | Queensferry | 518.5 | 1890* |

El viaducto más alto es el viaducto de Gokteik, en Birmania, con un alto máximo de 97.6 m y un largo total (compuesto de luces cortas) de 689.3 m, construído en 1901.

### Armaduras de madera.

**226.** La madera de construcción se ha puesto ahora tan costosa, excepto en las regiones no pobladas, y el trabajo de proyectos tan barato, que ya no se pone en práctica el uso de vigas innecesariamente pesadas, sólo con el objeto de asegurar la estabilidad y evitar cálculos. Por consiguiente, en los puentes importantes, cada parte de cada pieza que sufre un esfuerzo se calcula siempre. Además, la resistencia de las maderas es tan incierta que es mejor adoptar el cálculo que exige la mayor sección, debiéndose usar factores amplios de seguridad.

**227. Las piezas de compresión** se estudian como si fueran columnas (véase Pilares en Resistencia de Materiales), y si están sujetos á esfuerzos transversales también, deberán tomarse éstos cuidadosamente en cuenta. Todos los agujeros y otras reducciones de sección, deben por supuesto deducirse de la sección bruta.

**228.** En las piezas **sometidas á tensión**, también se deben considerar todas las reducciones de sección; pero ahora se usan generalmente barras de hierro ó de acero en lugar de madera, para dichas piezas.

**229.** Además, se debe tener cuidado de que las vigas de madera resistan cualquier esfuerzo transversal ó esfuerzo cortante que pueda presentarse ó estar obrando sobre ellas. Así pues, es necesario examinar los postes de los extremos para cerciorarse de que están asegurados contra cualquier esfuerzo transversal. Cuando un poste se encuentra con otra pieza bajo un ángulo y va á ser empotrada en él, es económico calcular la profundidad de la muesca que se requiere, pues mientras más honda es ésta, mayor será la sección requerida. Cuando los tornillos están sujetos á las vigas por tuercas, se deben poner siempre arandelas debajo de las tuercas, calculando el tamaño de las arandelas necesarias para impedir el hundimiento de la madera.

**230.** Cuando la madera está sometida á esfuerzo cortante, como con un tornillo que pasa por una viga, se transmite el esf por el apoyo contra el interior del taladro, ó cuando hay un escalón sobre el cual obre la presión de otra pieza, se debe siempre tener suficiente superficie á lo largo de la fibra de la madera, para que reciba el esfuerzo cortante, haciendo algún aumento por la posibilidad de que la

* En construcción, 1902.

fibra pueda deslizarse en la superficie ó hacia algún agujero antes de que todos los esfuerzos cortantes sean transmitidos.

**231. Sección transversal del cordón superior.** Como no sería conveniente, en la práctica, cambiar la sección de un cordón superior de madera en diferentes puntos, se le diseña todo para que resista el esfuerzo *máximo* que ocurra entre dos puntos de división de un tramo cualquiera. Supóngase el espesor del cordón. Encuéntrese á $r^2 = (\text{radio mínimo de giro})^2 = \frac{(espesor)^2}{12}$. Encuéntrese la unidad del esfuerzo permitido de acuerdo con la fórmula para columnas, dada en las especificaciones ó adoptada, usando la presión máxima dada. Encuéntrese el área que se requiere para esta unidad de esfuerzo. Encuéntrese la profundidad resultante, y, para una pieza horizontal ó inclinada, se prefiere un poco mayor que el ancho, teniendo en cuenta el momento de flexión debido á su propio peso. Si esto no da un buen tamaño comercial, revísese bien para obtener una sección mejor.

**232. Puntales.** Los puntales se hacen con preferencia del mismo ancho que la viga superior. Cada puntal debe diseñarse separadamente. Obténgase á $r^2$, la unidad de esfuerzo permitida, etc., como para el cordón superior. Por economía los puntales deben ser casi cuadrados por término medio, aunque fuera necesario alterar la sección del cordón superior para impedir que se separen mucho de la sección cuadrada.

**233. Los tirantes verticales** (de hierro) pueden estudiarse ahora. El área de la sección vertical $= \frac{\text{esfuerzo máximo}}{\text{unidad de presión permitida}}$. Pero véase Sección mínima, § 187. El tamaño de una tuerca se fija ordinariamente por el diámetro de la barra, pero las arandelas se usan de modo que no hundan la madera.

**234. Los apoyos ó cortes** que se requieren en el cordón superior é inferior para sostener las piezas inclinadas en su lugar, se calcularán ahora. Se calcula la componente (en el puntal) perpendicular á la cara ó caras que comprime, y se obtendrá la altura necesaria, suponiendo el ancho del cordón inferior igual al del superior y al de los puntales.

**235. La sección del cordón inferior** se podrá decidir ahora, puesto que la reducción de la sección, debida á las ensambladuras, se conoce.

$$\text{El área de la sección transversal} = \frac{\text{esfuerzo máximo}}{\text{unidad de esfuerzo permitido}}.$$

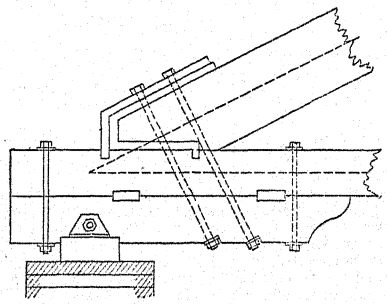


**Fig. 67.**

**236. Unión del extremo,** fig. 67. Muchos diseños diferentes se han hecho, propuesto y discutido para esta unión. Los extremos de las abrazaderas deben entrar en las muescas de la viga inferior, á tal profundidad que el esfuerzo total, recibido por el extremo de las fibras de los lados de las muescas, sea igual al esfuerzo que las abrazaderas de los extremos puedan resistir por flexión. Esta profundidad puede encontrarse por pruebas sucesivas, ó por medio de dos ecuaciones algebraicas en que la presión máxima permitida y la profundidad de la muesca son las dos cantidades desconocidas.

Determínese la superficie del esfuerzo cortante necesaria para transmitir este esfuerzo al cuerpo del cordón inferior. Esto determinará también el espacio entre las muescas y el extremo del cordón inferior.

Calcúlese el esfuerzo (si lo hay) que quede por transmitir, y trácense de acuerdo los tornillos largos inclinados y sus arandelas. Calcúlese también el área comprimida, y la profundidad de la cara vertical del extremo inferior del cordón superior que se necesita para transmitir la componente horizontal de su empuje. Véase también que el apoyo inferior presente bastante superficie para resistir la componente vertical. Los tacos entre la viga saliente del estribo ó pila y el cordón inferior deben soportar la componente horizontal del viento. Para más seguridad, la *fricción* entre las dos partes se desprecia.

**237.** La fig. 68 muestra las *uniones* ó ensambladuras que se usan más en las vigas de madera, etc. Necesitan muy poca explicación. La fig. (*a*) es un buen modo de ensamblar un poste, pero al hacerlo, la línea, *oo*, no debe quedar nunca inclinada, sino paralela al eje del poste, de otro modo, en caso de encogimiento, ó de gran presión, las partes en cada lado de ella tienden á resbalar una sobre otra, produciendo una gran presión sobre los tornillos.

Cuando se necesite mayor resistencia, se pueden usar abrazaderas de hierro, como en *b*, *h* y *j*, en lugar de tornillos. La fig. (*b*) muestra un poste de 4 piezas entablilladas, que pueden sujetarse por tornillos como en la parte superior, ó por aros como en la inferior. Los aros pueden apretarse por medio de rebordes y tornillos como en *s*, ó pueden introducirse cuñas delgadas de hierro entre ellas y la viga, si fuere necesario. La fig. C muestra un buen arreglo, fuerte, para unir una viga tirante *k*, ó un par, *l*, y una péndola *u*, haciendo que *k* y *l* empotren uno en otro dentro del pendolón ó péndola doble *tt*; *nn*, son dos bloques por los cuales pasan los tornillos. Un arreglo análogo es también bueno para unir un tirante *w* con el pie *v* de las péndolas, agregándoles una abrazadera como en la figura. La fig. (*e*) es un sistema de ensamblar una viga en otra en ángulo recto con ella. Por medio de una brida de hierro como en *f*, se logra el mismo objeto, y es más fuerte. Las figs. *g*, *h*, *i*, *j*, son vigas compuestas. Cuando se requiere una viga de gran espesor, si la construímos colocando simplemente una viga chata sobre otra, obtendremos sólo la resistencia que tendrían las vigas si estuvieran separadas. Pero si impedimos que una resbale sobre otra, insertando bloques transversales, como en *g*, ó ensamblando una entre otra, como en *i*, *j*, y luego atornillándolas ó con abrazaderas sujetándolas firmemente para crear la fricción, obtendremos próximamente la resistencia de una viga sólida de la altura total, resistencia que está como el *cuadrado* de la altura. Véase Esfuerzo cortante horizontal, § 51, en Resistencia Transversal.

**238.** La resistencia de una viga compuesta se aumenta, aumentando el espesor en su centro, donde trabaja más, como en los cordones superiores de un puente. Esto puede efectuarse agregando la pieza triangular, *yy*, entre las dos vigas.

**239.** Se puede colocar un pedazo de plancha de hierro en las uniones de las vigas de madera, cuando hay gran presión, para distribuir de esta manera el esfuerzo con más igualdad sobre todo el área de la unión.

**240.** Frecuentemente una simple abrazadera no basta cuando es necesario unir las dos vigas muy apretadas.

En tales casos un extremo de la abrazadera podrá, como en *x*, terminar en una especie de tornillo, y después de pasar por la barra transversal Z, se ajustará todo por medio de una tuerca en *x*. Puede emplearse el sistema que se indica en K. Algunas veces, como en A, se taladra primero el agujero para el tornillo, después se hace un agujero en un lado de la viga, que llegue hasta el agujero del tornillo, bastante grande para que la tuerca del tornillo pueda introducirse. Habiendo hecho esto, se rellenará el agujero con un tarugo de madera para sostener la tuerca en su lugar. Después se insertará el tornillo, de modo que entre en la tuerca. Haciendo girar el tornillo se ajustarán las vigas.

**241.** Cuando los extremos de la vigas, viguetas, etc., se introducen en la pared, como se usa, con sección cuadrada, hay peligro de que, en el caso de que se quemen y partan en dos, puedan, al caer, tumbar la pared. Esto se evita cortando los extremos en la forma que se indica en *m*.

**242.** Cuando una abrazadera, *o*, fig. R, tiene que soportar una presión tan grande que haya peligro de que se hunda en la viga, *p*, sobre la cual descansa, podrá usarse una pieza fundida, *v*, debajo de ella. La abrazadera pasará por la parte *r* de la pieza. Las pequeñas prolongaciones en la parte inferior impedirán que la pieza resbale bajo el esfuerzo oblicuo de la abrazadera. Esto mismo puede usarse con los tornillos oblicuos, y tanto sobre la viga como debajo de ella. Cuando se usa debajo, podrá ser necesario atornillar la pieza fundida en la parte baja de la viga. Cuando la tracción

en una abrazadera forma ángulo recto con la viga, si es muy fuerte aquélla, puede ponerse un pedazo de plancha de hierro, en lugar de la pieza dicha, entre la abra-

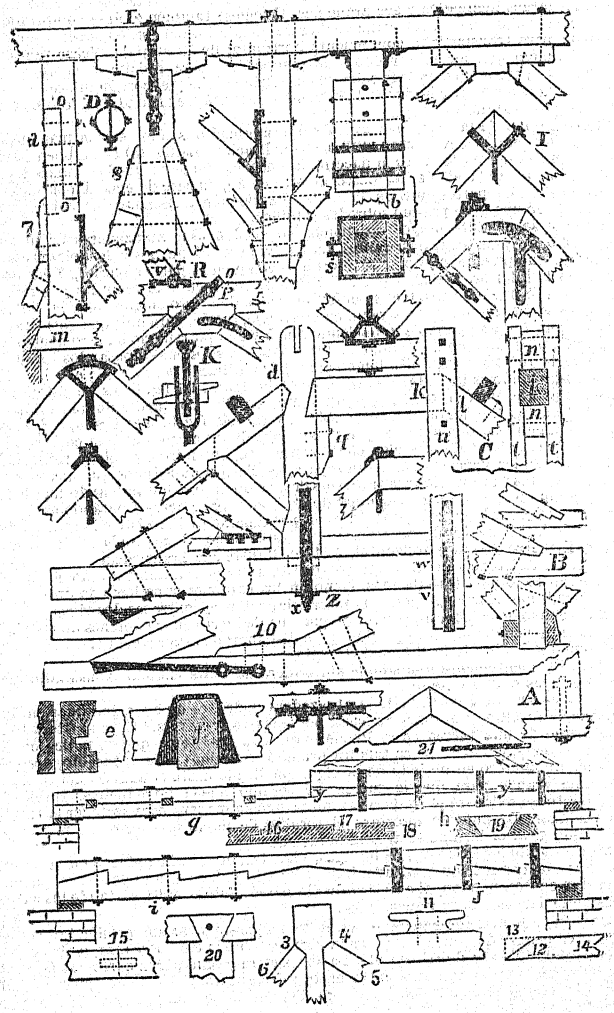



**Fig. 68.**

zadera y la viga, para impedir que esta última sea cortada en parte ó dañada; véase I é I.

**243. Ensambladura del cordón inferior.** Debido al largo del cordón inferior, puede ser necesario ensamblarlo (fig. 69). El número de planchas que se deben emplear es en gran parte un asunto de prueba. Si se ponen muchas, se necesitará mucho trabajo de carpintería, y como consecuencia incertidumbre en la distribución de las presiones, y si se ponen pocas, habrá que hacer muescas hondas, y esto puede reducir la sección en gran parte. Estas planchas deben trazarse para resistir empuje contra sus extremos y que no puedan ser arrancadas por el esfuerzo cortante. Se atravesarán con tornillos para impedir que se encorven hacia afuera, y las arandelas se dispondrán para que transmitan á la madera sin dañarla todas las presiones que obran en el tornillo.

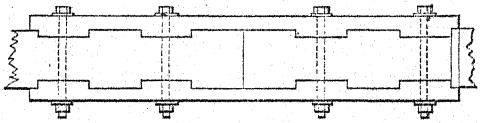


Fig. 69.

**244.** La fig. 70 muestra una forma de viga inferior usada en los puentes de armadura Howe combinada (madera y hierro), de los ferrocarriles de Chicago, Milwakee, y St. Paul. Véanse §§ 249-251, figs. 73. Cuatro de estas abrazaderas ó bridas se necesitan para cada unión, poniéndose un bloque en cada lado de cada trozo que se vaya á ensamblar. Los dos bloques opuestos que forman un par, se sostienen contra el trozo por medio de cuatro tornillos que lo atraviesan, y los tarugos cilíndricos fundidos (en el molde) en la superficie de cada bloque, entran en agujeros correspondientes taladrados en la cara del trozo. Los dos bloques de un mismo lado se sujetan por una barra de empalme y de gancho.

(*N. del T.* — Damos las equivalencias de las pulgs y pies del clisé en centímetros. 13 ½" = 34.3 cm; 15" = 38.1 cm; 8 ½" = 21.6 cm; 3" = 7.6 cm; 1" = 2.5 cm; 6 pies = 183 cm.)

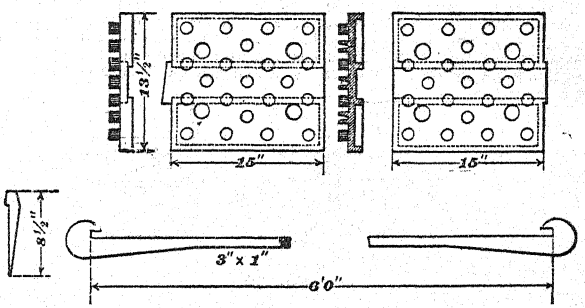



Fig. 70.

**245.** Las figs. 71 muestran un puente **pequeño de madera de armadura Howe.** Los cordones inferior y superior están formados de tres ó más maderos paralelos, *c, c, c,* colocados á poca distancia uno de otro para hacer que las barras verticales, *rr,* pasen entre ellos. Los brazos principales, *oo,* están en pares ó en tres. Las piezas que los componen se empotran en *bloques triangulares, s,* que si son de madera dura son sólidas, y si de hierro fundido son huecas, fig. (*d*) y reforzadas por bordes interiores. Estos bloques se extienden á través de tres ó más piezas del cordón. Contra su centro se apoyan también las contradiagonales *e.* Estas son piezas

sencillas en los puentes pequeños, ó dobles en los grandes. Los brazos de madera y las contradiagonales se sujetan en el punto donde se cruzan.

En la fig. (*d*) la línea de puntos muestra los bordes reforzadores, y el tarugo *x* retiene el bloque en su lugar. Las cuerdas verticales, *rr*, de hierro están en pares, en tres, ó en cuatro, etc., de acuerdo con el tamaño del puente, y con tornillo y tuerca en cada extremo. Las partes superiores é inferiores de los brazos y contradiagonales se apoyan con sección cuadrada contra los bloques de ángulo, y ordinariamente se fijan en su lugar, apretando simplemente los tornillos de las cuerdas verticales. Los postes *p* y *d* de los extremos, las cuerdas de los extremos, *ic* y *by*, y las porciones horizontales, *gi* y *wb*, del cordón superior, no forman propiamente parte de la armadura.

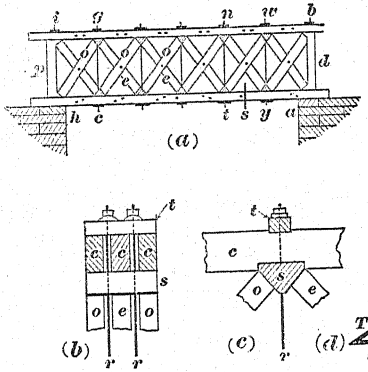



**Fig. 71.**

**246. En las luces grandes**, para impedir que la presión en los extremos de las diagonales aplasten los cordones, se funden los bloques de ángulo con bordes altos que se prolongan debajo de sus bases. Estos bordes al pasar entre las piezas que componen una viga, se prolongan hasta la cara opuesta de ella. Allí los bordes empujan sobre arandelas anchas en los extremos de los tirantes verticales.

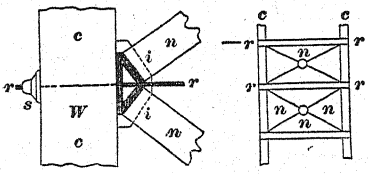


**Fig. 72.**

**247. Una forma común de refuerzo lateral** (riostras), fig. 72, se parece á la armadura Howe colocada á un costado. En ella las diagonales son puntales de madera y las piezas, *r*, *r*, barras redondas. Uno de los puntales está entero, á excepción de pequeñas escotaduras hechas para recibir la otra diagonal.

A los lados de los cordones, los extremos de las diagonales descansan sobre un saledizo (indicado por la línea de puntos *ii*), como de 4 cm de ancho en la parte inferior del bloque de ángulo de hierro fundido. La barra *rr*, pasando por los cordones de ambas armaduras, se aprieta por medio de la tuerca *s*, y sujeta las diagonales firmemente en su lugar; en caso de que se encojan mucho con el tiempo, podrán apretarse de nuevo del mismo modo.

El bloque de ángulo, de hierro fundido, es del mismo grueso que el brazo, su espesor no es necesario que pase de 13 mm en un puente grande. Tiene agujeros para el pase de la barra, *rr*.

**248. Dimensiones** para cada una de las dos armaduras de un **puente Howe**, de una sola vía para ferrocarril eléctrico en caminos urbanos ó rurales, con carros que pesen de 20 á 25 toneladas cada uno. La madera no debe forzarse más de 800 libras por pulgada cuadrada (56 kg por cm cuad), ni el hierro más de 5 toneladas por pulgada cuadrada (350 kg por cm cuad). Suponiendo que el hierro sea de calidad superior que requiera como 4,200 kg por cm cuad para quebrarlo. Las barras deben, cuando llevan tornillo, ser de rosca *recalcada*. A cada lado de cada uno de los cordones se supone que se ha añadido, uniéndola firmemente á ella en su centro, una pieza de un espesor igual por lo menos á la mitad del de una de las piezas que forman el cordón y de un largo igual al dé tres tramos.

(*N. del T.* — Hemos convertido la tabla del autor al sistema métrico.)

| Luz en metros. | Flecha en metros. | Número de tramos. | Un cordón superior. | | Un cordón inferior. | | Un brazo extremo. | | Un brazo del centro. | | Una contradiagonal. | | Una barra extrema. | | Una barra central. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | N.º de piezas. | Dimensiones en centímetros. | N.º de piezas. | Dimensiones en centímetros. | N.º de piezas. | Dimensiones en centímetros. | N.º de piezas. | Dimensiones en centímetros. | N.º de piezas. | Dimensiones en centímetros. | N.º de piezas. | Diámetro en cm. | N.º de barras. | Diámetro en cm. |
| 7.61 | 1.82 | 8 | 3 | 13.×15. | 3 | 13.×30. | 2 | 13.×20. | 2 | 13.×15. | 1 | 13.×15. | 2 | 3.8 | 2 | 2.5 |
| 15.24 | 2.74 | 9 | 3 | 15.×23. | 3 | 15.×35. | 2 | 15.×23. | 2 | 13.×20. | 1 | 13.×20. | 2 | 4.8 | 2 | 3.2 |
| 22.86 | 3.65 | 10 | 3 | 15.×30. | 3 | 15.×35 | 2 | 15.×28. | 2 | 15.×20. | 1 | 15.×20. | 2 | 5.7 | 2 | 3.5 |
| 30.48 | 4.77 | 11 | 3 | 15.×35. | 3 | 15.×41. | 2 | 20.×30. | 2 | 15.×15. | 1 | 15.×25. | 2 | 6.3 | 2 | 3.8 |
| 38.09 | 5.48 | 12 | 4 | 15.×35. | 4 | 15.×41. | 2 | 23.×35. | 2 | 15.×30. | 1 | 15.×30. | 2 | 7.6 | 2 | 4.8 |
| 45.72 | 6.40 | 13 | 4 | 20.×35. | 4 | 20.×46. | 3 | 20.×35. | 3 | 15.×25. | 2 | 15.×25. | 3 | 7.0 | 3 | 3.3 |
| 53.34 | 7.31 | 14 | 4 | 25.×41. | 4 | 25 ×51. | 3 | 20.×38. | 3 | 20.×25. | 2 | 20.×25. | 3 | 7.6 | 3 | 3.8 |
| 60.96 | 5.48 | 15 | 4 | 30.×41. | 4 | 30.×51. | 3 | 25.×41. | 3 | 20.×35. | 2 | 20 ×25 | 3 | 8.9 | 3 | 4.8 |

**249. Puentes típicos de ferrocarril de armadura Howe, mixtos (madera y hierro)**, ferrocarril de St. Paul, Milwakee, y Chicago, 1891-1892. Estos puentes están trazados para cargas de tren de (4,000 libras por pie lineal) 5,960 kg por m. Las presiones del viento se calculan para una presión de (500 libras) 745 kg por metro lineal de tren. Las presiones en los brazos: máx, 35 kg; mín, 6.44 kg por cm cuadrado. Tensiones: en las barras de la armadura principal, máx, 871; mín, 595 kg por cm cuadrado de área neta. Barras laterales: máx, 1,050 kg por cm cuadrado de sección neta. Maderas: cuerdas transversales y contracarriles, encina blanca; cordón superior, bloques ó tacos de pino blanco, el resto del maderaje, pino blanco ó de Noruega ó abeto Douglas.

**Puentes mixtos de combinación,** ferrocarril de St. Paul y Milwakee.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Largo total en metros*. | 24.993 | 28.346 | 31.394 | 34.747 | 38.0 9 | 41.452 | 44.805 |
| Luz en metros*..... | 23.469 | 26.822 | 30.175 | 33.528 | 36.880 | 40.233 | 43.586 |
| Tramos..... | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| Cordón superior. mm. | 4, 177.8×254 | 4, 177.8×254 | 4, 203.2×254 | 4, 203.2×254 | 4, 203.2×254 | 4, 203.2×254 | 4, 203.2×304.8 |
| Cordón inferior**. mm. | 4, 177.8×355.6 | 4, 177.8×355.6 | 4, 203.2×355.6 | 4, 203.2×355.6 | 4, 203.2×381 | 4, 203.2×381 | 4, 203.2×381 |
| Brazos principales M: | | | | | | | |
| En el centro***... cm. | 2, 25.4×25.4 | 2, 30.3×25.4 | 2, 25.4×25.4 | 2, 25.4×25.4 | 2, 25.4×25.4 | 2, 25.4×25.4 | 2, 25.4×25.4 |
| En los extremos.. cm. | 2, 25.4×30.4 | 2, 30.4×30.4 | 2, 30.4×30.4 | 2, 30.5×35.5 | 2, 30.5×35.5 | 2, 35.5×35.5 | 2, 35.5×35.5 |
| Contradiagonales C: | | | | | | | |
| En el centro***..... | 2, 20.3×25.4 | 1, 25.4×30.4 | 1, 20.3×25.4 | 1, 20.3×25.4 | 1, 20.3×25.4 | 1, 25.4×30.4 | 1, 20.3×25.4 |
| En los extremos..... | 1, 20.3×20.3 | 1, 20.3×20.3 | 1, 20.3×20.3 | 1, 20.3×20.3 | 1, 20.3×20.3 | 1, 20.3×20.3 | 1, 20.3×20.3 |
| Barras verticales: | | | | | | | |
| En el centro***..... | 3, 3.81 | 3, 3.81 | 3, 3.49 | 3, 3.81 | 3, 4.14 | 3, 3.81 | 3, 4.14 |
| En los extremos..... | 3, 5.39 | 3, 6.01 | 3, 6.35 | 3, 5.39 | 3, 5.71 | 3, 6.03 | 3, 6.35 |
| | | | | 2, 5.08 | 2, 5.08 | 2, 5.08 | 2, 5.08 |
| Peso en kilogramos §.. | 59.401 | 70.521 | 82.551 | 94.980 | 105.750 | 117.885 | 12..950 |

* El largo de los tramos es de 3.33 m. Cada luz es más larga, en un tramo, que la precedente. La altura, en todos los casos, es de 7.6 m libres entre cordones. Ancho, 4.32 m libres entre armaduras.

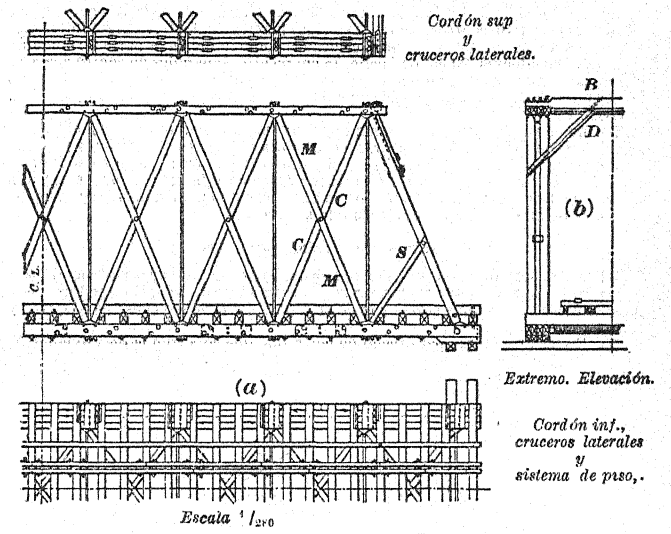
** Para las ensambladuras en el cordón inferior, véase fig. 70, § 244.

*** Debido á que las luces tienen alternativamente un número de tramos pares ó impares, hay alguna ambigüedad respecto á las dimensiones de las piezas del entramado en el centro de la luz.

§ El peso incluye las dos armaduras, riostras y pavimento para una vía. Del peso total del puente, las armaduras tienen como 53 0/0, los sistemas laterales 26, el sistema de pavimento 18, y las planchas sobre los muros 3 0/0.

(*N. del T.* — La tabla del autor la hemos convertido al sistema métrico. Desde la cuarta línea hasta la penúltima, la primera cifra separada de las demás indica el número de piezas elementales.)

**250.** Para todas las luces, traviesas laterales; 15 × 15 cm en el centro. 20 × 20 cm en los extremos de la luz; barras laterales, 28 mm en el centro, de 38 á 40 mm en los extremos; tornapuntas S (uno en cada extremo de cada armadura), 15 × 35 cm; tirante transversal, B, en el portal, entre los extremos de los cordones superiores



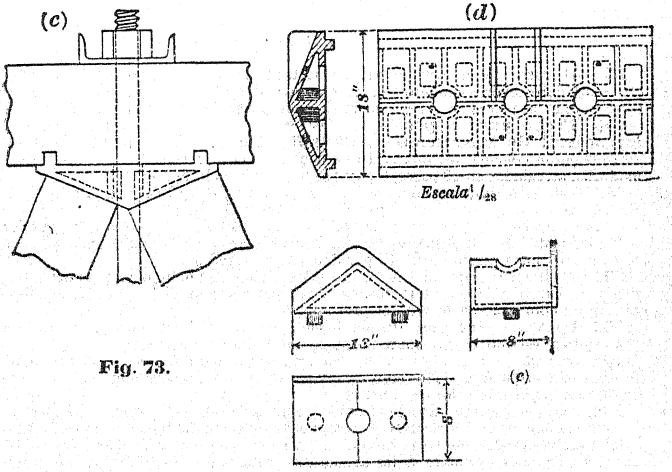




Fig. 73.

(*N. del T.* 18″=45 cm; 13″=33 cm; 8″=20.3 cm.)

(uno en cada extremo del puente), 25×30 cm; puntales angulares, D (dos en cada extremo del puente), 15×20 cm.

Las vigas del piso son de 25 × 40 cm, 6.1 m de largo, 4.27 m de luz neta, y con .76 de separación de centro á centro. Las vigas longitudinales ó refuerzos inferiores, 12½ cm de ancho por 30 cm de alto, colocadas como lo indica la parte terminal de la fig. (*b*). Durmientes, 20 cm de ancho, 15 cm de alto, separados de centro á centro 30 cm. Los guardarrieles, 20 cm de ancho, 12½ de alto. Debajo de cada extremo del cordón inferior hay dos planchas murales de madera, de 30×30 cm y 6.10 m de largo.

**251.** Las figs. 73 muestran una luz de 30.19 m. La fig. (*a*) es una elevación lateral de la mitad de la luz con tirantes laterales superiores é inferiores y el sistema de piso; la fig. (*b*) es la mitad del corte vertical normal al eje del puente; la fig. (*c*) una unión en los puntos de división de tramos (la misma para el cordón superior é inferior); la fig. (*d*) un bloque de hierro fundido para la misma; y la fig. (*e*) un bloque de hierro fundido para las traviesas laterales.

## Armaduras de metal para techos.

**252.** Entre los tipos que se usan ordinariamente en armaduras de metal para techos, está la armadura triangular, fig. 26, y la armadura de arco; la armadura triangular se usa para las luces cortas y la de arco para las *largas*. Véanse §§ 255, etc., y la fig. 75.

**253.** En las armaduras para techo de corta luz, que soportan cargas livianas, la sección mínima prescrita en las especificaciones de puentes, basta á menudo para todas las piezas. Las conexiones se hacen con remaches y planchas, más ó menos como se ve en las figs. 74.

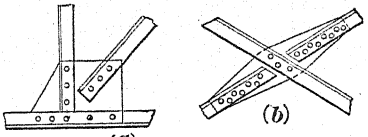



**Fig. 74.**

**254.** Al trazar dichas armaduras, no obstante lo pequeño que puedan ser los esfuerzos, debe tenerse cuidado de evitar las cargas excéntricas, que pueden presentarse cuando las piezas no se repiten simétricamente en ambos lados de las planchas.

**255. Techo para depósito de trenes. Estación de Broad Street, ferrocarril de Pensilvania, Filadelfia.** Figs. 75 de (*a*) á (*f*). Fabricado en 1893 por la Pencoyd Cº. Luz 91.64 m, flecha 33.14 m, largo 179.7 m. Veinte armaduras, arregladas en 10 pares, 3 pares se muestran en la fig. (*c*).

**256.** Cada armadura se compone de dos semiarcos, AB y BC, y un tirante horizontal, AC, con tres uniones de pasadores, A, B y C, fig. (*a*).

Cada semiarco está compuesto de dos cordones, 14 brazos ó montantes radiales y 26 diagonales. Por la buena apariencia, los cordones se prolongan á través de los tramos superiores, juntándose allí por una unión corrediza.

**257. El tirante horizontal** AC, fig. (*a*), se encuentra debajo del piso del depósito, y está suspendido, á intervalos, por vigas que soportan dicho piso, las que descansan en columnas de hierro del piso inferior.

**258. Apoyos extremos.** Un pie de cada armadura descansa sobre una pieza fija sujeta al estribo, el otro sobre un juego de 11 cilindros de acero.

**259.** En cada extremo del techo, una armadura horizontal contraviento WW, fig. (*a*), está suspendida de los cordones, y sus extremos descansan en barras remachadas en la parte inferior de aquéllos.

**260.** Entre estas armaduras horizontales contravientos y los semiarcos, y exactamente debajo de las vidrieras que cierran cada extremo del techo, están colocadas armaduras verticales, contravientos (no indicadas), con barras horizontales y diagonales en planos normales á las armaduras principales. Estas armaduras verticales están suspendidas de los puntos de división de tramos de los semiarcos, fig. (*a*). Ellas resisten la presión del viento en la vidriera, y la transmiten, por medio de las armaduras horizontales contravientos, á los cordones inferiores del arco.

**261.** Dos semiarcos AB y BC, fig. (*a*), de una armadura pesan juntos 69,916 kg. Un par de armaduras, 139,832. El techo completo 3,178,000 kg.

**262.** Las cargas extrañas supuestas en kg por metro cuadrado del área cubierta: pares, 46.4; correas, 37; ventilador y marco de la claraboya. 46.4; cubierta y claraboyas, 74; nieve, 83; viento, 171.

**263.** El andamio movible, figs. (*a*) y (*f*), era de madera y se diseñó así para usar el depósito viejo mientras se hacía el techo nuevo.

**264.** La sección superior del vértice á la junta 9, figs. (*a*) y (*c*), fué remachada

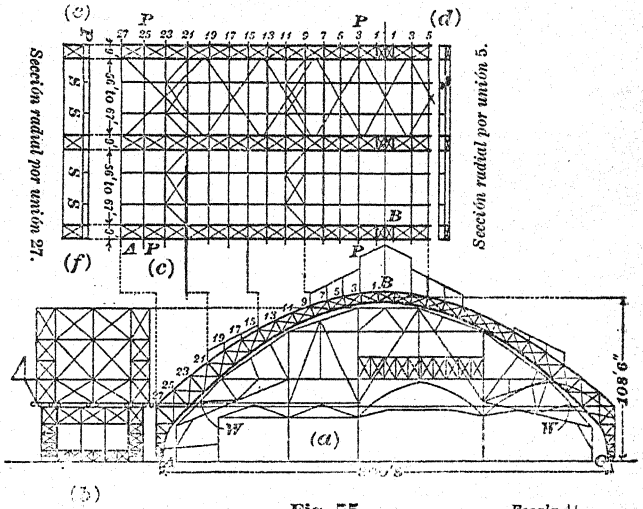



**Fig. 75.** *Escala* 1/150.

*(N. del T.).* — Para las cifras que trae la figura recuerdese que un pie (') = 30.5 cm y una pulgada (") = 2.5 cm.)

en la fábrica y colocada entera, como lo fué también la sección del pie desde la junta 27 hasta la parte superior del pie triangular. El resto fué remachado en el andamio ambulante. Para levantar una armadura con correas, viguetas, etc., uniéndola con las ya colocadas, se necesitaron cerca de 10 días. Para mudar el andamio ambulante á una distancia igual á su largo, para recibir el par de armaduras siguientes, se necesitaron como 3 minutos, y se hizo con las máquinas de suspender.

**265 Dimensiones de techos arqueados de luz grande.**
(*N. del T.* — Hemos convertido la tabla del autor al sistema métrico.)

| | Arcos. | | Techos. | |
|---|---|---|---|---|
| | Luz. | Flecha. | Largo. | Área cubierta. |
| | metros. | metros. | metros. | metros cuad. |
| **Sala de trenes :** | | | | |
| Ferrocarril de Pensilvania, Estación de « Broad Street »..... | 91.64 | 33.07 | 179.58 | 15939 |
| Ferrocarril de Pensilvania « Jersey City »..... | 77.01 | 27.43 | 168.88 | 14841 |
| Phila. and Reading Railroad, Phila. « Reading Terminal », Market Str..... | 78.94 | 26.90 | 154.43 | 11808 |
| New York Central and H. R. R. R., New York. Grand Central Station..... | 60.70 | 28.65 | 198.73 | 11682 |
| Londres, St. Pancras Station. Midland Railw..... | 73.15 | 32.61 | 215.49 | 15246 |
| Alemania, Colonia. Nave..... | 63.88 | 23.98 | 254.84 | 15768 |
| **Edificios de la Exposición :** | | | | |
| París, Salón de Máquinas, 1889..... | 110.56 | 45.41 | 420.62 | 45054 |
| Chicago 1893, Manufacturas y Artes Liberales ..... | 142.17 | 62.79 | 386.49 | 44994 |

## Armaduras de madera para techos.

**266. Dimensiones para armaduras de madera para techos pequeños.** Las figs. 43 á 47, §§ 148, etc., de pino blanco. Luz = 4 × flecha. Peso combinado de las armaduras, techo y carga, incluyendo nieve y una concesión para el viento, 195.2 kg por m cuadrado de superficie de techo. Armaduras separadas 3.66 m de centro á centro. Factor de seguridad = 3, $b$ = ancho, $d$ = la altura. En la fig. 46, los puntales de 11.4 × 11.4 cm, serían suficientes, pero por razones prácticas los puntales se hacen ordinariamente tan anchos como los cabios ó parecillos. En la fig. 47, el tirante compresor es de 30 × 30 cm. En las figs. 45, 46 y 47, se dan dos series de dimensiones, la primera para el tirante descargado, la segunda para la viga cargada con 488 kg por m cuadrado.

(*N. del T.* — Hemos convertido la tabla del autor al sistema métrico.)

| Fig. | Luz en metros. | Flecha en metros. | Pares centímetros. | | Tirante. Madera centímetros. | | Tirante. Hierro. Diámetro. | Pendolón y péndolas. Hierro. Diámetro. | Tirante. |
|---|---|---|---|---|---|---|---|---|---|
| | | | b. | d. | b. | d. | cm. | cm. | |
| 43 | 9.14 | | 13. | 25. | 13. | 25. | 2.54 | ... | sin cargar |
| | | | 16. | 23. | 16. | 23. | 2.54 | 2.5 | sin cargar |
| 45 | 9.14 | 2.26 | 21. | 28. | 21. | 28. | .... | 4.1 | cargado |
| 46 | 12.19 | 3.05 | 15. | 20. | 15. | 20. | 3.60 | 2.5 | sin cargar |
| | | | 20. | 25. | 20. | 38. | .... | 5.1 | cargado |
| 47 | 18.29 | 4.77 | 25. | 30. | 25. | 30. | 3.60 | 4.3 | sin cargar |
| | | | 30. | 35. | 30. | 30. | .... | 5.1 | cargado |

## TRANSPORTE Y ERECCIÓN

**267.** Las vigas deben **transportarse** en carros de plataforma, con el alma en posición vertical, y con apoyos en puntos situados á ¼ de su longitud, de los extremos hacia el centro. Si fuesen demasiado largas para dos carros, se colocan uno ó más carros sueltos, haciendo espacio, y se apuntalan los puntos de apoyo.

**268.** Las vigas se pueden **colocar en la obra** por medio de una cabria, malacate, grúa, etc., ó pueden ser sacadas de los carros rodándolas sobre rodillos y colocadas en sus puestos por medio de gatos y poleas. Las cabrias deben tener por o menos cuatro tirantes, para su maniobra y ajuste. La operación de levantarlas

se hace á menudo por medio de locomotoras moviéndolas sobre la parte de a vía ya construída. Las cuerdas se usan hasta un ¼ de su resistencia máxima.

**269. Los viaductos** se construyen generalmente desde sueparte superior, por medio de un andamio movible (torre movible). A veces por medio de una vía de cables, pero este método es más lento. En algunos casos la torre movible se coloca en el suelo y alcanza hasta la parte superior del viaducto. También se puede construir el viaducto por medio de andamios fijos ó de una estructura ya existente.

**270.** Los puentes de grandes luces se construyen generalmente en una plataforma de andamiaje sobre una hilera de caballetes, bien reforzados.

**271. Erección.** Primero se colocan los cordones inferiores sobre el andamio, tan á nivel como fuese posible. Luego se montan los cordones superiores sobre apoyos provisionales que descansan en los que sostienen el cordón inferior. Los cordones superiores se colocan primero á unas cuantas pulgadas más altos que su posición definitiva, á fin de que las piezas del entramado puedan entrar fácilmente en sus respectivos lugares. Cuando éstas se han colocado, se bajan gradualmente los cordones superiores hasta que todo descanse sobre los cordones inferiores. Luego se aprietan gradualmente los tornillos hasta llevar todas las superficies de las uniones á su posición de contacto, y por medio de esta operación (teniendo las piezas de los cordones superiores el necesario exceso de longitud) se forma la curvatura y se levantan los cordones superiores del andamio; las armaduras reposarán en sus apoyos permanentes.

**272. El andamiaje** se construye generalmente de pino ó pinabete, al costo de $20 por 1,000 pies (medida de tabla) = 2,360 metros cúbicos. Calcúlese un margen de unos $15 por 1,000 pies (medida de tabla) por armadura, etc. Se puede vender el material usado de $5 á $15 por los 1,000 pies (medida de tabla).

Las piezas principales son generalmente de 12 × 12 pulgadas (30 × 30 cm) y las diagonales 3 × 12 (7½ × 30 cm). Pernos, $^{7}/_{8}$ de pulgada (22 mm). Debido al carácter provisional de los andamios y al ahorro que se hace en su venta, si *no* está muy maltratado, se aconseja emplear bastante material de dimensiones corrientes, sobre todo en las piezas longitudinales, que se pueden colocar entre los pares alternados que forman las torres.

**273. En el caso de lechos de fondo blando**, el andamiaje puede descansar sobre pilotes á los cuales se fijan los maderos verticales del andamio por medios cortes y pernos, ó por ligaduras. No debe emplearse menos de cuatro pilotes por cada pata de caballete*. Se han empleado hasta 24. Los pilotes deben ir reforzados por debajo del nivel del agua. Los caballetes deben construirse en varios pisos, de 12 á 30 pies (3.66 á 9.1 m) cada uno. Deben conectarse por medio de piezas laterales ó placas de unión.

**274. En lechos de roca**, con fuerte corriente, sería conveniente sumergir cajones llenos de piedras, como cimiento del andamiaje.

**276. La renovación** de puentes se puede hacer por desalojamiento hacia adelante, rodando el nuevo tramo de luz á lo largo de la vía; por desalojamiento transversal por el cual los dos tramos de luz, el viejo y el nuevo, se colocan sobre vías que van en dirección normal al puente; por desalojamiento vertical : ya sea por levantamiento ó descenso; ó por desalojamiento por giración, haciendo girar el tramo de luz viejo sobre un eje y al nuevo sobre otro eje hasta colocarlo en su puesto.

**277. Precauciones.** Para la erección ó renovación, calcúlese el peso muerto del puente, los choques de la corriente del río, de los botes, del hielo, el arrastre de la corriente, etc., sobre todo cuando se pueden temer las crecientes, y el esfuerzo del aparejo. Para la rigidez debe emplearse un factor muy liberal de seguridad. Los arrastres de la corriente pueden amontonarse y formar una represa. Los caballetes*, en el agua aumentan la velocidad y fricción, pudiendo así producir socavaciones. El andamiaje se puede proteger con estacadas de defensa. Hay que preveer las excentricidades de la fuerza del viento. Numerosos accidentes han demostrado la necesidad de proteger las mismas armaduras (sin concluir) de los vientos fuertes. Todos los refuerzos y contravientos laterales deben estar colocados y asegurados antes de quitar el andamiaje y antes de que las armaduras descansen en sus apoyos definitivos.

Evítese la caída de la herramienta, etc. Aun las más pequeñas piezas, cayendo de una gran altura, son un peligro para las vidas y hasta para el mismo puente. Los garfios de los aparejos están expuestos á quebrarse ó estirarse. Los andamios deben estar bien sujetos por contravientos y asegurados por abrazaderas cuidadosamente arandeladas.

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* Véanse figs, pág. 879.

ESPECIFICACIONES PARA ARMADURAS, CONTENIDO PÁGINAS 805--831-

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(*Obs. del T.* — Estas páginas del Resumen que va á continuación, son las que más dificultades nos han presentado para su traducción. Son muchos los detalles de las innumerables piezas y elementos, y muy considerable la variedad de nombres para dichas piezas y para las operaciones de fundición, fraguado, etc., no bien determinados en español.

A lo que se agrega, la forma exageradamente sintética y los giros y modismos de los norteamericanos, tratándose de estas industrias y de las construcciones con ellas relacionadas. Pero hay tan copiosa enseñanza en esta condensada síntesis,

y como además es este uno de los varios capítulos de esta obra que no se encuentran en los Manuales análogos en español, no hemos omitido esfuerzo ni consulta para tratar de interpretar lo mejor posible sus dictámenes y fórmulas de irreemplazable valor en la práctica, sobre todo para los que han construído poco.

Es la ocasión de llamar la atención del lector sobre la manera de indicar la usanza de las diversas Compañías é industriales, á veces en parcial ó completa contradicción, y la importancia de conocer con tanta brevedad el desiderátum sancionado por serios estudios y larga experiencia.

Damos las gracias al notable ingeniero de puentes Sr. C. R. Grimm, por la eficaz ayuda que nos ha prestado en esta parte del trabajo.)

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# RESUMEN DE ESPECIFICACIONES PARA PUENTES Y EDIFICIOS

(1) Puentes de acero, para ferrocarriles, calzadas y ferrocarriles eléctricos.

(2) Puentes de hierro y madera (combinados) para ferrocarriles.

(3) Armaduras de acero para techos y edificios.

El siguiente Resumen de especificaciones para puentes y edificios tiene por objeto principal dar una idea general de los caracteres esenciales que presentan en la práctica aquellos materiales, y, á la vez, indicar el modo de construir de las diversas Compañías.

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## (1) RESUMEN DE ESPECIFICACIONES PARA PUENTES DE ACERO EN FERROCARRILES Y CALZADAS

### Abreviaciones usadas.

**A,** American Bridge Company,
Especificaciones generales para puentes de acero en ferrocarriles, 1900.

**Aa,** American Bridge Company,
Especificaciones generales para puentes de acero en calzadas, 1901.

**B,** Baltimore and Ohio Railroad Company,
Especificaciones generales para puentes en ferrocarriles y calzadas, techos y edificios de acero, 1901.

**C,** Cooper, Theodore,
Especificaciones generales para puentes de acero en ferrocarriles y viaductos, 1901.

**Cc,** Cooper Theodore,
Especificaciones generales para puentes de acero en calzadas y ferrocarriles eléctricos y viaductos, 1901.

**D,** Delaware, Lackawanna and Western R. R. Company,
Especificaciones para puentes de acero en ferrocarriles, octubre 1899: revisadas hasta julio 1900.

**E,** Erie Railroad Company,
Especificaciones generales para puentes, 1900.

**G,** Práctica ordinaria.

**Oo,** Osborn Engineering Company,
Especificaciones generales para las superestructuras en puentes de calzadas 1901.

**P,** Pennsylvania Railroad Company,
Especificaciones modelos para puentes de acero, enero 1 de 1901.

**R,** Philadelphia and Reading Railway Company,
Especificaciones para puentes de acero, 1898, revisadas febrero 1901.

**Y,** New York Central and Hudson River R. R. Leased and Operated Lines,
Especificaciones generales para puentes de acero, 1900.

(*N. del T.* — Hemos creído conveniente, por tratarse de las prácticas de Compañías americanas, dejar las medidas inglesas junto á las métricas.)

**A, Aa,** Am B Co; **B,** B y O; **C, Cc,** Cooper; **D,** D L y W; **E,** Erie;

## I. PROYECTO EN GENERAL

### Limitación de luces para diversos tipos.

| *Vigas.* | | **A** | **Aa** | **B** | **C** | **Cc** | **D** | **Y** |
|---|---|---|---|---|---|---|---|---|
| Vigas de acero laminado, pisos, etc. | pies. | hasta 20 | hasta 40 | hasta 20 | hasta 20 | hasta 40 | hasta 20 | hasta 25 |
| | metros. | (6.09) | (12.19) | ** | | | | (7.62) |
| Vigas de remache.. | pies. | 20 á 100 | 25 á 80 | 20 á 100 | 20 á 120 | 20 á 80 | 20 á 100 | 25 á 100 |
| | metros. | (6.09 á 30.48) | (7.62 á 24.38) | | (6.09 á 36.57) | (6.09 á 24.38) | | (7.62 á 30.48) |
| ***Armaduras.*** | | | | | | | | |
| Armaduras de remache..... | pies. | 100 á 140 | 40 y más | 100 á 120 | 75 á 150 | 40 y más | 90 á 160 | 100 á 200 |
| | metros. | (30.48 á 42.67) | (12.19) | (30.48 á 36.57) | (22.85 á 45.72) | | (27.43 á 48.77) | (30.48 á 60.96) |
| Armaduras de pasadores..... | pies. | más de 140 | más de 140 | más de 120 | más de 120 | más de 120 | más de 150 y más | más de 200 y más |
| | metros. | (42.67) | | (36.57) | | | (45.72) | (60.96) |

Armaduras de remache menos de 100 pies, armaduras de pasadores, más de 100 pies (30.48 m), **Oo.**

Altura de la armadura, mín, = un octavo de la luz, **Oo.**

### Clasificación para puentes en calzadas y ferrocarriles eléctricos.

**Aa Cc**

**A** A1* Puentes para ciudades con pisos de planchas cóncavas, pavimentados sobre base de concreto.

**A** A2* Puentes para ciudades con pisos de planchas de madera.

**B** B* Puentes suburbanos para tranvías eléctricos pesados.

**C** C Puentes urbanos y rurales para tranvías eléctricos livianos, ó grandes pesos.

**D** D Puentes rurales para tráfico común de calzadas.

**E1** E1 Puentes para tranvías eléctricos pesados ó automotores solamente.

**E2** E2 Puentes para los mismos, livianos.

### Curvatura.

Las secciones del cordón superior serán mayores que las del inferior : 1 en 960 **A, B, C, E, R, Y.**

En los puentes de calzadas tres dieciseisavos de pulg para cada 10 pies (=1 en 640), **Cc.**

Como $\frac{3}{4}$ de pulg (19 mm) en 100 pies (30.48 m) ó sea 1 en 1,600, **D.**

Lo suficiente para hacer que las uniones del cordón sometido á compresión trabajen en ángulo recto cuando la armadura tenga su carga completa. Cada pieza se construirá mayor ó menor en proporción al esfuerzo á que se someta con toda la carga muerta y viva, de manera que tengan su longitud normal, **Oo.**

### Sección transversal de puente.**

**Entrecarril,** usual. 4 pies 8½ pulgadas (1.43 m). **Distancia,** de centro á centro de las vías, 12 á 13 pies (3.67 á 3.96).

**Ancho entre las armaduras ó vigas con tablero superior.** Separación pasadores .05 de la luz. Armaduras ó vigas de remache (**D**), 10 pies (3.05 m). Vigas laminadas (**G**), 5 á 7 pies (1.52 á 2.13 m). Para luces no mayores de 60 pies (18.30 m), 7 pies (2.14 m); de 60 á 100 pies (18.3 á 30.48 m), 8 p (2.44 m); para más de 100 p (30.48 m). un duodécimo de la luz, **D.** Vigas laminadas (**Y**) de más de 60 pies (18.3 m), en proporción á la altura.

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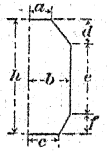
* Cc, las clases A y B se proyectan para llevar después doble vía de tranvía eléctrico

** *N. del T.*— Algunas veces suprimimos la conversión de algún número de pies, etc. en metros, porque ya se ha hecho esa misma no lejos de allí.

**G**, general; **Oo**, Osborn; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, Calzadas.

**Espacios varios á través de las luces, en tangentes, G.**

| | | | | |
|---|---|---|---|---|
| *a* | 3 á | 3½ | pies á | (.91 á 1.06 m) |
| *b* | | 7 | — | (2.14 m) |
| *c* | 5 á | 5½ | — | (1.52 á 1.67 m) |
| *d* | 4 á | 6 | — | (1.22 á 1.83 m) |
| *e* | 10 á | 14 | — | (3.05 á 4.27 m) |
| *f* | 1 á | 5 | — | (.30 á 1.52 m) |
| *h* | 20 á | 22 | — | (6.09 á 6.70 m) |




**El mínimo de espacios libres en las curvas.**

El mismo mínimo que en las tangentes, **A**. Lo mismo para vagón de 74 pies (22.56 m) de largo, 48 pies (14.63) de centro á centro de los bastidores de las ruedas de 10 pies (3.05 m) de ancho, **B**. Lo mismo para vagones de 75 pies (22.86 m) de largo, 54 pies (16.45 m) de centro á centro de bastidores. Espacio libre adicional = = 0.8 **d** pulgadas (una pulg = 25.4 mm) de cada lado; = 1.6 **d** pulgs entre las vías; en que **d** = al ángulo central subtendido por una cuerda de 100 pies (30.48 m), **C**; aumento lateral de espacio sobre el carro 2.5 pulg (63 mm) por cada pulg (cada 25.4 mm) de peraltación del riel exterior, **C**. El eje del puente es bisectriz de la ordenada media y paralela á la cuerda, **Y**.

**Puentes de calzada. Altura libre**, 14 pies (4.27 m), **Oo**. Para las clases **A**, **B**, **C** y **E**, mínimo = 15 pies (4.57 m), **Aa**, **Cc**; para ancho de 6 pies (1.83 m) sobre cada vía, **Aa**. Para clase **D**, 12.5 pies (3.81 m), **Aa**, **Cc**. **Espacio libre horizontal**. Mínimo 14 pulgadas (35.6 cm) más que el ancho entre los guardaruedas, **Aa**. Para tranvías eléctricos, 6.5 pies (1.98 m) del centro de la vía, **Aa**; 7 pies (2.13 m), **Cc**. En las curvas provéase espacio para vagones de 45 pies (13.71 m) de largo y 8 pies (2.44 m) de ancho en su máximo, 20 pies (6.09 m) entre los centros de los bastidores de las ruedas, **Aa**. Ancho entre los centros de las armaduras, mínimo = .05 de la luz, **G**.

## Piezas de tensión.

En general, los montantes así como uno ó dos tramos del cordón inferior, á cada extremo de la luz, deben ser de secciones rígidas para que soporten tanto la tensión como la compresión.

**Los ángulos**, empleados como piezas de tensión, deben asegurarse por ambas piernas, **C**, **D**; ó sólo se considera como efectiva la sección de una pierna, **C**.

**Tensores** (piezas de ajuste). Evítese el empleo de piezas de ajuste, **A**, **Aa**, **D**, (**P**, excepto en las contradiagonales). Evítense los tensores en las contradiagonales, **C**. Pueden ser ajustables las barras de las contradiagonales y tirantes en las armaduras de pasadores, **Y**; las puntas de las barras llevan roscas recalcadas, de mayor diámetro, **C**, **E**, **Y**; tornillos con rosca del patrón del Gobierno de los Estados Unidos, **C**, **Cc**, **D**, **E**; diámetro en la base de la rosca un dieciseisavo de pulgada (1.59 mm) mayor que el diámetro de la barra, **D**; un 10 por ciento, **Y**; 17 por ciento, **Oo**.

Las barras de cabeza remachada deben ser de hierro forjado, **Oo**. Los ganchos de unión deben desarrollar toda la fuerza de la barra, **Oo**.

## Piezas de compresión.

**Los postes extremos y los cordones superiores** tienen dos almas, una tabla de cubierta, una tabla inferior ó de tensión y aseguradas en el reborde inferior, **G**.

No más de una plancha, y ésta de no más de media pulgada (13 mm) de espesor

**A, Aa**, Am B Co; **B**, B y O; **C, Cc**, Cooper; **D**, D L y W; **E**, Erie;

(en puentes para calzadas de tres octavos de pulgada (9.5 mm), se empleará como lámina ó tabla de cubierta, **C, Cc**. Éstas no deben sobresalir más de 4 pulgadas (10 cm) fuera de la línea de remaches, **D**.

**Los empates entre las secciones** se empalman en todos sus lados con 2 hileras por lo menos de remaches, bien unidos á cada lado del empate, **C**. Colóquense de frente las superficies de los extremos **A** (**D, P** con excepción de los rebordes superiores de las vigas), **E, R, Y**. No se confíe en las superficies de los extremos, **E**.

**Barras de celosía.** Ancho de 1.5 á 2.5 pulgs (38 á 63 mm). Espesor de las celosías sencillas, un cuarentavo de la distancia entre los remaches; en las dobles, un dieciseisavo, **G**. Si son mayores de siete dieciseisavos de pulg (11 mm), empléense ángulos, **Oo**. El ángulo con el eje de la pieza, en celosías sencillas 60°; dobles 45°, **G**.

Distancia entre centros de remaches, ancho de la viga en U+9 pulgs (22.9 cm), **C, Cc**; 8×el menor ancho del segmento, **P, R**.

Las barras de celosías dobles remachadas en sus intersecciones, **C. D**.

Tablas inferiores de listón. (Planchas de tensión, planchas de sujeción.) Longitud mínima usual=.75 á 1.5×su propio ancho. Ancho mínimo, 9 pulgs (22.9 cm) ó .66×su propio largo, ó=al menor espesor de la pieza, **Oo**. Menor espesor=tres octavos de pulg (9.5 mm) ó de $^1/_{20}$ á $^1/_{40}$ de la distancia entre los centros de los remaches, **G**. Espacio entre los remaches (**D**), máximo 4 pulgs (10 cm) entre los centros.

## Uniones de pasadores.

**Barras de ojo.** Espesor mínimo, .625 pulgs (15.75 mm), o bien .2×el ancho de la barra, **Oo**. Cabezas recalcadas, laminadas ó á fragua **A, R**. No deben emplearse las soldaduras **A, D**, excepto (**R**) para formar argollas laterales, contradiagonales ó riostras, **R**. Recalcadas y moldeadas, **Y**. Las cabezas no deben tener más de $^1/_{16}$ de pulgada (1.5 mm) más gruesa que el cuerpo, **D, P**. Las barras templadas, **G**; antes de taladrar, **D**. No se debe hacer ningún trabajo de fragua después de taladrar, **R**. Las barras que han de colocarse unas al lado de otras deben ser taladradas á la misma temperatura, **A, Aa, Y**. Los pasadores deben penetrar sin esfuerzo. Las barras de ojo que trabajen juntas deben empalmarse y taladrarse en una misma operación, **Oo**.

La variación máxima de las distancias entre los taladros para los pasadores debe ser de un sesenticuatroavo á un trentidosavo de pulgada (.39 á .79 mm); ó un sesenticuatroavo de pulg (.39 mm) para distancia de 20 á 25 pies (6.10 á 7.60 m), **G**.

**Piezas de tensión compuestas.** Sección neta en el taladro del pasador=1.25 á 1.50×la sección neta á través del cuerpo de la pieza. Sección neta detrás del taladro=.75×la sección neta á través del taladro, ó=.80 á 1.00×la sección neta á través del cuerpo de la pieza, **G**; proporción para doble esfuerzo cortante en la sección situada detrás del pasador hasta el fin de la plancha, **Oo**; longitud de la plancha detrás del pasador, mínimo 2.5 pulgs (63 mm), **Oo**. Distancia detrás del ojo hasta el respaldo de la pieza, mayor que el radio del pasador, **Y**.

**Taladros para pasadores.** Espacio libre entre el pasador y el taladro, de un cincuentavo á un trentidosavo de pulgada, **G**.

**Planchas para pasadores ó de refuerzo.** Por lo menos una plancha á cada lado, que debe extenderse no menos de 6 pulgadas (15 cm) fuera de la lámina de tensión, **G**.

**Pasadores.** Hasta 7 pulgadas (17.8 cm) de diámetro, acero laminado, **P**; más de 7 pulgadas (17.8 cm) forjadas, **C**.

Diámetro mínimo de .66 á .85×la mayor dimensión de cualquiera de sus barras de ojo, **G**.

## Vigas de remache.

**Altura mínima :** de un noveno á un duodécimo de la luz, **G**.

**Proporción entre el alma y las alas ó tablas.** Los momentos flexores los resisten solamente las alas, y el esfuerzo cortante lo resisten enteramente las almas ó piezas del entramado, **C, Cc, R**; excepto cuando el alma está hecha de una sola pieza ó bien empalmada como para resistir á los esfuerzos de flexión, en cuyo caso $^1/_8$ del área de la sección transversal del alma puede considerarse como si fuese efectivamente área de las tablas, **Oo**.

**G**, general; **Oo**, Osborn; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa, Cc, Oo**, Calzadas.

Un octavo del área del alma, se considera como de la tabla, **A, Aa, B**; si su largo fuese=90 pies (27.43 m) ó más, **E**; si su largo fuese menor de 50 pies (15.24 m), sólo la plancha de cubierta y las piernas horizontales de los ángulos de la tabla deben incluirse en el área de las tablas, **E**; ninguna parte del alma debe incluirse en la tabla, **C, Cc, D, P, R, Y.**

**Alma.** Espesor mínimo, tres octavos de pulgada (9.5 mm), **G**; en puentes de calzadas (**Cc, Oo**), cinco dieciseisavos de pulgada (8 mm).

Todo el esfuerzo cortante, obrando hacia el lado del estribo, se considera como trasladado á los ángulos de la tabla dentro de una distancia=á la altura de la viga, **A, Aa, B, E, Oo, P.**

**Empates del alma.** Una lámina á cada lado del alma, **G**; por lo menos de tres octavos de pulgada (9.5 mm) de espesor, **A, B**; por lo menos cinco dieciseisavos de pulgada (8 mm) ó ¾ del espesor del alma, y suficientemente ancha para contener 2 hileras de remaches á cada lado del empate, **Oo.**

**Piezas de refuerzo** (que aumentan la rigidez). Se requieren generalmente á los extremos y en los puntos de concentración de las cargas. Estas piezas intermediarias se necesitan generalmente cuando las distancias sin apoyo entre los ángulos de las tablas de las vigas exceden de 50 á 60 veces el espesor del alma; cuando el esfuerzo cortante excede de

$$10.000 - 75 \times \frac{\text{altura}}{\text{espesor}}\text{; ó } \left[\left(703 - 5.27\ \frac{\text{altura}}{\text{espesor}}\right) \text{(en kg por cm cuad)}\right]\text{, C;}$$

en puentes para calzadas cuando el esfuerzo cortante excede de

$$12{,}500 - 90 \times \frac{\text{altura}}{\text{espesor}}\text{; ó } \left[\left(880 - 6.32\ \frac{\text{altura}}{\text{espesor}}\right) \text{(en kg por cm cuad)}\right]\text{; Cc,}$$

Dimensiones corrientes de los ángulos, $3\frac{1}{2} \times 3 \times {}^{5}/_{16}$ á $5 \times 3\frac{1}{2} \times {}^{5}/_{8}$ (8.8 cm × 7.6 cm × .8 cm) á (12.7 cm × 8.8 cm × .9 cm).

**Alas.** Longitud sin asegurar de las alas (alas de compresión **C, P**) máximo =12 × ancho, **B, P, R, Y**; 16 × ancho, **A, C, Cc**; 20 × ancho, **D**; en puentes para calzadas=20 × ancho, **Aa**=25 × ancho, **Oo.**

Las alas de compresión tienen la misma área que las de tensión **A, Aa, B, C, Cc, Oo, P.**

Las planchas de cubierta no deben extenderse más de 5 pulgs (12.7 cm), ú 8 × el espesor de la primera plancha fuera de la línea de remaches, **A, Aa, C, Cc.** Si fuesen de espesores diversos, las planchas más gruesas estarán sobre los ángulos y las más delgadas hacia fuera, **A, Aa, B, C, Cc, Y.** Una debe extenderse á todo el largo de la viga, **B, E.** Otras deben ser suficientemente largas para soportar dos hileras de remaches adicionales en cada extremo, **C, P.**

**Refuerzos, remache, resistencias.** Véase más abajo Refuerzos, Uniones de remache y resistencias.

## Vigas durmientes.

Vigas en grupos de 2, 3 ó 4 para cada carril, separadas por vigas en U de 10 pulgs (25 cm), y más ó menos á cada 3 pies (.91 cm) remachadas á las almas, **B.**

## Refuerzos.

Compuestos de piezas rígidas, remachadas, **A, Aa, C, Cc, Y**; las piezas deben cruzarse entre sí y con las otras á que están conectadas, sobre líneas centrales comunes, pasando por todos los centros de gravedad. Los accesorios de las piezas remachados simétricamente en todas direcciones, **Y.**

**Para vigas de durmiente.** Puntales de vigas en U de 10 pulgs (25.4 cm) á cada extremo; con 2 ó 3 vigas colocadas en ángulo reforzando las remachadas; con 4 vigas postes en ángulo, como á 6 pies (1.83 m) de separación. Las uniones deben llevar por lo menos 3 remaches, **B.**

**A, Aa,** Am B Co; **B,** B y O; **C, Cc,** Cooper; **D,** D L y W; **E,** Erie;

**Laterales.**

**B,** en armaduras de vía inferior. Refuerzos arriba; postes de portal (entradas de puente); puntales intermedios tan altos como los cordones; ángulos sencillos, $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ (8.9×8.9×1 cm), cortándose en cada tramo para vía simple; ángulos dobles de celosía para vías dobles.

**B,** para puentes de vía inferior, refuerzos interiores; puntal final al fondo y ángulos intermedios, remachados entre sí y á los refuerzos en cada intersección. No menos de 4 remaches en cada intersección y en cada conexión final.

**B,** para puentes de vía superior, sistemas completos superior é inferior en cada tramo.

**Oo,** puntales finales al fondo en todas las armaduras, ya sean de vía superior ó inferior.

**Y,** refuerzos laterales arriba y abajo en todos los puentes, de vía superior é inferior, que tengan suficiente altura libre de entrada. Refuerzo inferior lateral en todos los puentes de vía inferior. En el sistema superior todos los refuerzos deben estar trabados entre sí y remachados á las vigas del piso cuando el largo de las traviesas es de más de 15 veces el ancho de sus tablas.

**Y,** en puentes de vía superior sin piso metálico, el refuerzo superior es de puntales cruzados á cada punta de tramo, compuestos de cuatro ángulos enrejillados, con la misma altura que el cordón superior; las diagonales rígidas se cruzan en cada tramo y están remachadas entre sí en sus intersecciones.

**Para vigas laminadas.** Vía superior. Entre las tablas superiores, ángulos con cuatro remaches por lo menos, en las conexiones. Vía inferior; refuerzo lateral inferior de ángulos cruzados en cada tramo, remachados entre sí y á las traviesas en sus intersecciones, **B.**

Los ángulos de las riostras laterales son generalmente iguales á los de las de refuerzo, **R.**

**Cc,** en puentes para calzadas puede considerarse el piso de planchas encorvadas como suficiente refuerzo lateral á nivel del piso.

**Riostras** (diagonales, cruzadas, para vibración ó para el viento) y de **Portal.**

Proporcionadas para resistir pesos desiguales de armaduras en puentes de doble vía, **E, R;** riostras extremas para transmitir todas las fuerzas horizontales á los estribos, **E;** para soportar la mitad del aumento del esfuerzo máximo debido al viento y á la fuerza centrífuga, **A, Aa, B, P.**

En puentes de vía superior, en cada punto de división de los tramos, **A, Aa, D, E, P, R, Y.**

Refuerzos superiores en puentes de tablero inferior cuya altura exceda de 25 pies (7.5 metros), **C, D, P, Y;** en puentes para calzadas, 20 pies (6.1 m), **Cc;** 25 pies (7.6 m), **Oo.**

En armaduras enanas (*pony*) (véase *N. del T.*, § 220) y en vigas de tablero inferior, en los extremos y en cada viga del piso ó de los postes cruzados, **A, Aa, R;** en cada punto de división de los tramos, **D.**

En las vigas laminadas de vía inferior ó intermedia, en cada viga del piso y en los extremos, ó, si el piso fuese enterizo, á no más de 8 pies (2.44 m) de separación, **Y.**

En las vigas laminadas de tablero superior, se emplean cruceros rígidos en los extremos con separación máxima de 20 pies, **Y;** riostras en marcos con 4 ángulos, por lo menos, en los extremos y puntos de división de los tramos de 12 ó 14 pies 3.66 á 4.37 m) de separación, **B.**

## Uniones de remache.

**Taladros para remaches.** En vigas I deben ser taladrados, **B.**

Pueden hacerce por punción; en acero de no más de $\frac{5}{8}$ á $\frac{3}{4}$ de pulgada (16 á 19 mm) de espesor, **G.**

Subpunzados un octavo de pulgada más pequeños y escariados hasta un dieziseisavo de pulgada más grande que el remache, en acero de más de $\frac{5}{8}$ hasta $\frac{3}{4}$ de pulgada (16 á 19 mm) de espesor; en las uniones de las vigas del piso y las vigas que las cruzan con las armaduras maestras ó vigas de remache, **E.**

No se debe permitir (*drifting*)*, **A, B, C, D, E, R.**

* *N. del T.* — Sucede, á veces, que en dos ó más planchas perforadas, al yuxtaponerlas no coinciden los huecos y algún borde sobresaliente impide la entrada del perno—entonces, algunos, á golpes de martillo, fuerzan el perno, venciendo el obstáculo. Esta operación, no permitida, es el *drifting*. No encontramos palabra adecuada que la exprese en español.

**G**, general; **Oo**, Osborn; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, Calzadas.

No se deben cambiar las piezas después de escariadas, **D**, **P**.

Los huecos para remaches deben ser un dieciseisavo de pulgada (1½ mm) mayores que los remaches, **G**.

El taladro debe ser mayor que la perforación en un máximo de un dieciseisavo de pulgada (1½ mm), **G**.

Distancia entre la orilla de la plancha al centro del remache. Mínima, 1.25 á 1:5 pulgadas (32 á 38 mm), ó 1.5 á 2 diámetros del remache. Máxima, 4 á 5 pulgadas (10 á 12.7 cm), ú 8× el espesor de la plancha, **G**.

Distancia entre remaches. Mín=3×diámetro del remache, generalmente; con preferencia 4×diámetro, **Oo**.

Máxima distancia entre remaches en líneas de mayor resistencia, 5 á 6 pulgadas (12.7 á 15.2 cm), ó 16× espesor de la plancha exterior más delgada que se conecte; normales á los esfuerzos, 30 á 50× espesor de la plancha exterior más delgada que se conecte. En los extremos de las piezas de compresión (ó en piezas compuestas, de tensión, **B**); para una longitud de 1.5 á 2×el ancho ó la altura de la pieza, 3.5 á 4×diámetro del remache, **G**.

En vigas de remache, para los remaches del entramado á la tabla superior que soporta la vía, máx=3 pulgadas (7.5 cm), **R**.

**Remaches.** Diámetro generalmente de $^{3}/_{4}$ ó $^{7}/_{8}$ de pulg (19 á 22 mm). Cabeza hemisférica, **G**. Altura de la cabeza, mínimo=.6 del diámetro, **R**.

**La remachada.** Evítese remachar á mano. Usense máquinas de acción directa, á vapor, de presión hidráulica ó aire comprimido, capaces de mantener la presión aplicada después de ensanchar el extremo del remache, **G**.

## Piso.

**Vigas del piso.** Altura mínima=$^{1}/_{8}$×su longitud, **Y**. En puentes para ferrocarriles y puentes para calzadas importantes, remachadas á los postes de las armaduras ó al entramado de las vigas laminadas, **G**. También se les da un apoyo en la tabla inferior de la viga ó se apuntalan, **G**. A falta de estos apoyos auméntese en un 25 por ciento el número de los remaches, **R**.

Los suspensores, cuando se emplean, no deben ser ajustables, **C**, **Cc**. Éstos deben ser de lámina ó de forma, **Oo**.

**Vigas longitudinales.** Altura mín,=$^{1}/_{8}$×la longitud, **Y**. En puentes para ferrocarriles, las clases A1 y A2 de acero; las clases B, C y E, las vigas que van debajo del riel son de acero; clase D, de madera ó acero, **Cc**. En puentes de ferrocarril, y con preferencia en puentes para calzadas, remachadas al entramado de las vigas del piso, sostenidas por sus tablas ó por ángulos, **B**, **R**. No se tome en consideración la fuerza de estos apoyos al fijar el número de remaches necesarios, **R**.

Separación de centro á centro, 6 pies 6 pulgadas (1.98 m), **A**, **B**, **C**, **D**, **Y**; 5 pies (1.52 m), **E**; tablero inferior de doble vía, generalmente 6 pies (1.83 m), **R**. En vía simple, 8 pies (2.44 m), **R**.

**Pisos divididos en cajuelas.** Cajuelas rectangulares, construídas de láminas y ángulos, remachadas á las vigas principales ó armaduras, por ángulos, y, cuando fuese posible, por escuadras por debajo de las láminas horizontales inferiores. Placas angulares remachadas á las vigas y á las cajuelas á distancias no mayores de 8 pies (2.44 m), **Y**.

El fondo se llena con una pega compuesta de un pie cúbico de granzón limpio cernido en un tamiz de un cuarto de pulgada, con 1½ galones (5.67 lit) de asfalto n.º 4 para pisos, ó lo suficiente para llenar los vacíos. Se calienta primero el granzón á 300° F (149° C) y se hace toda la mezcla á esa temperatura, **Y**.

**Pisos de madera.** Se continúan por encima de los estribos, **A**, **B**, **C**, **B**.

**Durmientes ó vigas del piso.** Pino amarillo ó roble blanco, **G**.

Anchura, 8 pulgadas (20.3 cm), **A**, **B**; 9 pulgadas (22.8 cm), **R**.

Espesor, 8 pulgadas (20.3 cm) para 7 pies (2.13 m) de largo de luz del durmiente, á 14 pulgadas (35.6 cm) para 12 pies (3.66 m), **B**; 12 pulgadas (30.5 cm), **R**; 10 pulgadas (25.4 cm), **Y**.

Tájese media pulgada (12.7 mm); máx, 1½ pulgadas (38 mm), **R**.

Separación. Generalmente 6 pulgadas (15.2 cm) libres; 16 pulgadas (40.6 cm) de centro á centro, **R**. Cada 3.er, 4.º ó 5.º durmiente asegurado á las vigas con pernos de ¾ pulgadas (19 mm) ó con tornillos, **G**.

**Las vigas de madera en puentes para calzadas.** Ancho mínimo, 3 pulgadas (7.6 cm), ó .25×altura; separación máx, 2 á 2.5 pies (.61 á .76 m). Sus extremos descansan sucesivamente en sus puntos de encuentro sobre las vigas del

**A**, **Aa**, Am B Co; **B**, B y O; **C**, **Cc**, Cooper; **D**, D L y W; **E**, Erie;

suelo, con .5 pulgada (12.7 mm) de separación entre sí para la circulación del aire.

**Vigas de madera para pisos en puentes para ferrocarriles eléctricos.** Las clases E1 y E2. Mín, 6×6 pulgadas (15.2×15.2 cm), separación máx, 6 pulgadas, con tajaduras de media pulgada (12.7 mm) y aseguradas á las vigas por pernos de tres cuartos de pulgada (19 mm) separados por no más de 6 pies (1.83 m). Desde el centro de la luz hacia los extremos rebajadas á fin de reducir la flecha de curvatura de la viga.

**Guardarriel.** 6×8 pulgadas (15 × 20 cm) de pino amarillo ó blanco, **G**. Superficie interior á no menos de 3 pies 3 pulgadas (.99 m) del centro de la vía, **A**; 3 pies 7½ pulgadas (1.07 m), **B**; 5 pies 4 pulgadas (1.62 m), **Y**; 7 pies 1¼ pulgadas (2.16 m) de separación libre, **R**. Tajaduras de ½ á 1½ pulgadas (13 á 38 mm) sobre los durmientes, **G**. Aseguradas cada 3 ó 4 durmientes (á cada durmiente, **R**) y á los empates con pernos de tres cuartos de pulgada (19 mm) ó con tornillos, **G**. Los empates sobre los maderos del piso, con ensambladuras á medio corte de 6 pulgadas (15.2 cm) de largo, **G**.

**Guardarruedas y aceras en puentes para calzadas.** Guardarruedas 6×4 pulgs (15.2×10.1 cm) aseguradas á las planchas del piso por tacos de 2×6 pulgadas y (5×15.2 cm) y 12 pulgadas (30.5 cm) de largo, á no más de 5 pies (1.52 m) de separación, fijas por pernos á las vigas longitudinales á través de los tacos con pernos de tres cuartos de pulgada (19 mm), **G**. En puentes para ferrocarriles eléctricos (**Cc**, clase E) los maderos de las guardas, mín, 5×7 pulgadas (12.6×17.8 cm), con tajaduras de 1 pulgada (25.4 mm) sobre los maderos del piso y aseguradas por pernos de tres cuartos de pulgada á cada tercer madero y en cada empalme.

**Planchas convexas.** Mín $^{5}/_{16}$ de pulgada (8 mm) de espesor para calzada y un cuarto de pulgada (6.3 mm) para las aceras, flecha 2 pulgadas (5 cm) para anchos de 4 pies (1.22 m) bajo la calzada, 5 pies (1.52 m) bajo las aceras. Son preferibles en láminas continuas del tamaño de las secciones. Pueden conformarse sin calentar.

## Apoyos sobre los estribos y pilares.

**Peso permitido en cimientos de mampostería**, máx, libras por pulgada cuadrada, 400 (28 kg por cm cuad), **A**, **Aa**, **P**; 300 (21 kg por cm cuad), **B**; 250 (15.5), **C**, **Cc**, **D**, **E**, **R**; carga muerta, 500 (35); carga viva, 250 (15.5), **Y**.

**Planchas de apoyo.** De acero mediano, **C**, **Cc**. Espesor mín, tres cuartos á una pulgada (19 á 25 mm); en puentes para calzadas, media pulgada (12½ mm). Máx de la resistencia de las fibras, 12,000 libras por pulg cuad (840 kg por cm cuad), **E**.

Cuando los extremos de dos luces descansan en una pila, se unen los tramos ó se les pone planchas de apoyo enterizas, debajo de ambas, de tres cuartos á una pulgada (19 á 25 mm) de espesor, **G**.

Láminas de plomo de un octavo á un cuarto de pulgada (3 á 6 mm) de espesor, entre la plancha de apoyo y la mampostería, **G**.

**Pernos de anclaje**, 1 á 1.25 pulgadas (25 á 31.7 mm) de diámetro, 9 á 12 pulgadas (22.8 á 30.5 cm) dentro de la mampostería, **G**; asegurados con azufre, **R**; con cemento, **C**, **Cc**, **Y**.

**Pedestales.** De láminas y ángulos remachados, **C**; ó de acero fundido, **Y**. Planchas de apoyo y ángulos de conexión, mín, tres cuartos á siete octavos de pulgada (22 mm) de espesor, **B**, **C**, **Y**. 2 hileras de remaches en las lados verticales, **C**, **Y**.

**Apoyos para la expansión.** Dispuestos para cambios de temperatura de 150° F (65.6° C), **A**, **C**, **E**, **P**, **R**; para expansión de 1 pulgada (25 mm) en cada 100 pies (30.5 m), **D**, **Y**.

Un extremo libre para deslizarse, generalmente en tramos no menores de 60 á 90 pies (18.3 á 27.4 m), **G**.

Un extremo sobre rodillos de fricción, generalmente en tramos de mayores dimensiones, **G**; en todas las armaduras, **Y**.

Balanceadores (véase *N. del T.*, § 208) á cada extremo, en tramos de 80 á 100 pies (24.4 á 30.5 m), **G**.

Los rodillos descansan en barras de 3×1 pulgadas (7.5×2.5 cm) separados 2 pulgadas (5 cm) y remachados á las planchas de apoyo, **B**. Los extremos libres se anclan para que no se levanten ni muevan lateralmente, **C**, **Y**.

Rodillos, de acero de máquina, **C**, **Cc**. Diámetro mínimo, 3 á 4 pulgadas (7.5 á10 cm), **A**, **B**, **D**, **E**, **P**, **R**; 3 pulgadas (7.5 cm) para luces hasta de 100 pies

**G**, general; **Oo**, Osborn; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, Calzadas.

( 80.5 m), una pulg más para cada 100 pies adicionales, **C**, **Y**. Máximo de presión sobre los rodillos, en kg, por cm lineal, $80.5\sqrt{d}$, **R**; $d$=diámetro rodillos en cm. Largo, en cm= $1{,}444\sqrt{d}$, **Y**.

## II. MATERIAL

### Acero y hierro laminados y colados.

**Acero laminado** en superstructuras, por regla general.
**Acero colado** en planchas de apoyo, en casos especiales, en maquinaria de puentes movibles.
**Hierro laminado** en barras de argolla soldadas, **P**; en piezas laterales y sin importancia, **R**.
**Hierro colado** en planchas de asiento, en casos especiales y en la maquinaria de puentes movibles.

### Acero laminado, calidades.

**Blando**. En todas las piezas principales, por regla general.
**Mediano**. En pasadores, rodillos de fricción, pernos laterales, planchas de apoyo, barras de ojo, planchas corredizas y de asiento; se puede permitir (**C**) para compresiones, en los cordones, postes y pedestales.
**En remaches**.
**Mecanismos**. En rodillos para la expansión, **C**.

### Acero laminado. Manufactura.

Todas deben hacerse por horno de regeneración.
Las planchas de donde se sacan las láminas se hacen á martillo ó en laminadores de lingotes que tengan por lo menos el doble de la sección transversal de aquéllas, **A**, **B**.
Las láminas hasta 36 pulgadas (.91 m) de ancho se hacen en laminadores de rodillos horizontales y verticales (estos últimos aplanan el borde. *N. del T.*) (*universal-mill*), **D**, **R**; ó cizallando las orillas, **D**.

### Acero laminado. Manipulación.

Temple. Barras de ojo calentadas á un rojo obscuro uniforme, se dejan enfriar lentamente, **P**; las piezas llevadas á un rojo azul se calientan al rojo brillante uniforme (no expuestas directamente á las llamas) y se dejan enfriar lentamente, **B**.
El acero no se debe soldar, **B**. No se debe confiar en el acero soldado, **C**.

### Acero laminado. Su manejo en el taller.

Las aristas del acero de más de $^{5}/_{8}$ de pulgada (16 mm) cortado á cizallas deben acepillarse, **B**.
Todas las aristas cizalladas (de acero mediano, **D**) deben rebajarse .25 de pulgada (6 mm), **D**, **Y**; excepto en planchas para almas de vigas de más de 36 pulgadas (.91 m) de altura cuando van coronadas con tablas, y en las piezas para rellenar cuando no se les vean las aristas, **D**. No se debe aceptar que se limen las aristas en vez de acepillarlas, excepto en el caso de las barras para celosías, **Y**.
No se deben permitir rincones agudos ó sin llenar, **D**, **Y**. Cuando se ha cortado una lámina, un ángulo ó una pieza de forma, tanto el relleno como el corte deben igualarse con un instrumento muy cortante, ó con cincel y lima, de manera que no queden señales del corte á cincel, ó de las cizallas, **D**.
Los ángulos ó las piezas flexadas, empleadas como conexiones á los extremos de las vigas laminadas, ó de las vigas transversales y longitudinales, se deben ajustar muy bien, de manera que cuando la pieza se corte en su longitud no se les quite á estas conexiones más de un $^{1}/_{16}$ de pulgada (1.5 mm), **D**.
El material flexado por el taladro debe enderezarse antes de fijarse con los pernos, **R**. Cuando las planchas para los tramos estén deformadas, se deben repasar en frío para enderezarlas, **D**.
Las secciones empatadas de los cordones deben armarse en el taller, en trozos no

**A**, **Aa**, Am B Co; **B**, B y O; **C**, **Cc**, Cooper; **D**, D L y W; **E**, Erie;

menores de tres secciones, y después de ponerlas en contacto en sus empates y colocadas en sus respectivos lugares, se harán los agujeros para los remaches con exactitud antes de desarmarlas, y las partes así armadas, con sus respectivas planchas de empate, deben marcarse, **D**.

Todas las piezas armadas á remache deben asegurarse y juntarse entre sí antes de remacharlas, **D**.

En el caso de alguna obra oblicua, ó de conexiones complicadas, ó de un gran número de piezas de una misma clase, se armarán y se ajustarán en el taller, lo suficiente para evitar un mal ajuste.

Es necesario, generalmente, preparar las superficies que se tocan en los extremos de las secciones de las piezas de compresión, así como en las piezas que se han de armar.

## Acero laminado. (Requisitos.)

*Ensayos de tensión.*

Muestras de acero mediano, blando y de remaches. Para los ensayos de barras de ojo enterizas, véase á continuación.

Resistencia máxima *u* y límite de elasticidad, *el*, en millares de libras por pulg cuadrada *. Estiramiento, *s*, y reducción del área, *a*, en tanto por ciento de las dimensiones originales. El estiramiento medido en una longitud de 8 pulgadas (20.3 cm).

(*N. del T.* — Entre paréntesis van los kg por cm cuad equivalentes á las lbs por pulg cuad del número superior. La unidad, también el millar de kgs.)

| | Acero mediano ó de pasadores. | | | | Acero blando. | | | | Acero de remaches. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | u | el | s | a | u | el | s | a | u | el | s | a |
| **A**, **Aa**.... | 60 á 70 (4.2 á 4.9) | 0.5 u | 22 | .. | 52 á 62 (3.64 á 4.34) | 0.5 u | 25 | .. | 48 á 58 (3.36 á 4.06) | 0.5 u | 26 | .. |
| **B**..... | ..... | ..... | .. | .. | 58 á 63 (4.06 á 4.41) | 30 (2.1) | 25 | .. | 54 á 56 (3.57 á 3.92) | 27 (1.89) | 26 | .. |
| **Cc**.... | 60 á 68 (4.2 á 4.76) | 0.5 u | 22 | .. | 54 á 62 (3.78 á 4.34) | 0.5 u | 25 | .. | 50 á 58 (3.5 á 4.06) | 0.5 u | 26 | .. |
| **D**..... | 62 á 70 (4.34 á 4.9) | 0.5 u | 22 | .. | 54 á 62 (3.78 á 4.34) | 0.5 u | 26 | .. | 48 á 56 (3.36 á 3.92) | 0.5 u | 28 | .. |
| **E**..... | ..... | ..... | .. | .. | 56 á 64 (3.92 á 4.48) | 0.58 u | 27 | 45 | ..... | ..... | .. | .. |
| **Oo**.... | 60 á 70 (4.2 á 4.9) | 35 (2.45) | 22 | .. | 52 á 62 (3.64 á 4.34) | 32 (2.24) | 25 | .. | 50 á 60 (3.5 á 4.2) | 30 (2.40) | 26 | .. |
| **P**..... | 62 á 70 (4.34 á 4.9) | 33 (2.31) | 17 | 40 | 52 á 62 (3.64 á 4.34) | 28 (1.96) | 25 | 50 | 48 á 56 (3.36 á 3.92) | 28 (1.96) | 28 | 56 |
| **R**..... | 60 á 68 (4.2 á 4.7) | 0.5 u | 20 | .. | 52 á 60 (3.64 á 4.2) | 0.5 u 28 (1.96) | 25 | .. | 48 á 56 (3.36 á 3.92) | 28 (1.96) | 28 | .. |
| **Y**..... | 62 á 70 (4.34 á 4.9) | 0.6 u | 25 | 45 | 56 á 64 (3.92 á 4.48) | 0.6 u | 26 | 50 | 48 á 56 (3.36 á 3.92) | ..... | 28 | 55 |

Muestras del metal de más de $^5/_8$ pulg de espesor (16 mm), *el*=.56 *u*, **Y**.
— de las barras de ojo, los mismos requisitos que para acero mediano, **D**.
— — — *u*=63 (4.410 kg por cm cuad), **B**.
— — — de más de 1.5 pulgs de espesor (38 mm), dedúzcase de *el* 1 por cada $^1/_8$ de pulg (3 mm), *el* mín=20, **C**.
— — — y pasadores *u*=62 — 70 (4.340-4.900), *l*=.6 *u*, *s*=25, *a*=45, **Y**.
— os pasadores, *s*=15, **C**, **Cc**.
— — (mediano, suave ó de remaches) *s*, 5 por ciento menos, **A**.
— — (suave) *s*=20, **B**.
— — y rodillos, *s*=10, **D**.
— rodillos y planchas de apoyo, *u*=70 — 78 (4.900 á 5.460 kg por cm cuad), *s*=22, **Y**.

* *N. del T.* — Cada 1,000 lbs por pulgada cuadrada equivalen más ó menos á 70 kg por centímetro cuadrado.

**G**, general; **Oó**, Osborn; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, Calzadas.

*Pruebas de flexión.*

En acero mediano, la muestra debe doblarse en un ángulo de 180° alrededor de una barra de diámetro = 1 á 1½ × el espesor de la muestra, sin presentar fracturas del lado fuera de la flexión; en acero flojo y de remache, debe doblarse sobre sí mismo.

*Ensayo de hebra.*

Cuando se haya sacado el filamento alrededor de la barra y doblado ésta, el acero para remaches debe presentar una ruptura gradual fina, sedosa y homogénea, **D**.

*Prueba al punzón.*

El centro del agujero como en la práctica ordinaria, ó 1.5 á 1.87 pulgadas (37.5 á 47.5 mm) ó 2 diámetros del borde de la lámina; y ensánchese hasta 1.25 á 1.50 de diámetro, **G**.

*Prueba angular.*

Los ángulos de todo espesor deben poder abrirse planos (*flat.*). Los ángulos de no más de ½ pulgada (12.5 mm) de espesor deben cerrarse por completo, al frío, á golpes de martillo, sin dar señales de ruptura, **B**.

*Piezas de ensayo.*

Sección mínima, generalmente de ½ pulgada cuadrada (1.56 cm cuad). Longitud mínima, 8 á 12 pulgadas (20 á 30 cm). Generalmente deben hacerse pruebas de fundición ó golpe.

*Ensayos de las barras de ojo de grandes dimensiones.*

(*N. del T.* — Hemos convertido la tabla del autor al sistema métrico.)

| | Carga máxima kg por cm cuad (mín). | Límite de elasticidad en kg por cm cuad (mín). | Percentaje del alargamiento. |
|---|---|---|---|
| **A, Aa**.... | 350 menos que las piezas pequeñas | | 10 entre cuellos |
| **B**..... | 3,850 | .5 de la máx | 12 entre cuellos §<br>10 entre cuellos ‖ |
| **C, Cc**..... | 3,920 | | 10 entre cuellos |
| **D**..... | 4,060 | 2,100 | 12 en 3.05 m |
| **E**..... | | | 15 |
| **Oo**..... | 3,850 | | 12.5 en 4.5 m |
| **P**..... | 3,360 | 1,890 | 14 **** en 3.05 m |
| **R**..... | 4,060 *<br>3,920 **<br>3,360 *** | .5 de la máx<br><br>1,890 | 13 entre cuellos<br>10 entre cuellos<br>15 entre cuellos |
| **Y**..... | 4,060 | 2,310 | 10 en 6.1 m |

Por regla general no *más* del 4 por ciento del número total de las barras de un puente se deben someter á prueba, **R**; por lo menos 4 por ciento y no menos de 3 barras, **B**.

75 por ciento de las fracturas deben ser sedosas, el resto de grano fino, **R**.

Roturas en las cabezas puede no ser causa de rechazo.

(a) Si la barra desarrolla 10 por ciento de estiramiento (12.5 por ciento en 15 pies (4.57 m), **Oo**) y la resistencia máxima requerida (máxima 56,000, **C**, 55,000, **Oo**) (3,920 kg por cm cuad, **C**, 3,850, **Oo**), y si no, más de una tercera parte de todas las barras se rompen en la cabeza, **A, C, Oo**.

(b) si la barra se estira en 14 por ciento y si una segunda barra se rompe en el cuerpo y si el promedio del alargamiento de las dos barras no es menor de 16 por ciento, **P**.

La Compañía paga sólo por las barras que corresponden á los ensayos, **G**.

*** Acero mediano. * Barras de no más de 10 pulgs cuadradas (64.56 cm cuad). ** 20 pulgadas cuadradas (129.03 cm cuad). Valores proporcionales para las áreas intermedias. *** Acero blanco. § En barras de no más de 20 pies (6.1 m) entre cuellos. ‖ En barras de más de 20 pies (6.1 m) entre cuellos. **** Máx. 16.

**A, Aa,** Am B Co; **B,** B y O; **C, Cc,** Cooper; **D,** D L y W; **E,** Erie;

*Prueba sobre la construcción ya completa.*

La carga especificada, ó su equivalente, se pasa por encima de la construcción (en puentes de ferrocarril á una velocidad no mayor de 60 millas (96½ km) por hora, y detenida en cualquier punto por medio de frenos de aire comprimido ú otros) ó se coloca el máximo de la carga sobre la construcción por 12 horas. Después de la prueba, la obra debe volver á su posición original sin presentar señales de alteración permanente en ninguna parte, **C.**

*Composición.*

Fósforo, mayor percentaje:
En acero ácido, .06 á .08; en acero básico, .04 á .06; en piezas coladas, .08.
Azufre, mayor percentaje, .04 á .06.

*Máxima variación permitida en las secciones transversales y los pesos especificados.*

2.5 por ciento, **G**, excepto en planchas de ancho extraordinario, **D, Oo, P.**
En planchas de más de 40 pulgadas (1.02 m), en proporción del ancho, hasta el 5 por ciento en planchas de 90 pulgadas (2.29 m) ó más, **D.**
1.5 por ciento; cuando el 40 por ciento del total es de planchas de 36 pulgadas ó más de ancho, se aumenta el 2 por ciento, **Y.**
Planchas largas, de ½ pulgada (12½ mm) de desvío en 20 pies (6.1 m), ¾ pulgada (19 mm) en 40 pies (12.2 m), **R.**
Piezas de forma, 3 por ciento menos de espesor; planchas de 80 pulgadas (2.03 m) de ancho, 5 por ciento, **R.**

## Piezas de acero colado.

**Fabricación.** Acero Martín Siemens, **A, Aa, D, P, Y**; ácida, **Y**; templadas **P, R, Y.** Carbón, percentaje de .25 á .40, **G.**
Fósforo, percentaje, máx, .08, **B, Y.**

*Pruebas de tensión.*

(*N. del T.* — Para tener la tabla en ambas unidadas inglesas y métricas hemos puesto entre paréntesis las equivalencias en el sistema métrico.)

| | Tamaño de la pieza de la prueba en pulgs y cm. | Mayor resistencia en lbs por pulg cuad y kg por cm cuad mín. | Límite de elasticidad en libras por pulg cuad y kg por cm cuad mín. | Tanto por ciento de extensión en 2 pulgs (5 cm) mín. | Tanto por ciento de reducción del área. |
|---|---|---|---|---|---|
| **C, D, E, P, R.** | 1/2 cuadrada (1.56 cm cuad) ó 3/4 redonda (19 mm) | 65000 (4550 kilos) á 70000 (4900 kilos) | 33000 (2300 kilos) ó .5 últ. | 10 á 15 | 20, **P** |
| **Y.** | 3/4 redonda (19 mm) como 6 (15.2 cm) de largo | (a) 55000 (3850 kil) á 65000 (4550 kil) | ..... | 20 | |
| | | (b) 72000 (5040 kil) á 80000 (5600 kil) | 0.5 últ. | 15 | 25 |

*Prueba de flexión.*

**Y** (a), para usos generales como planchas de asiento, pedestales, etc., debe doblarse á 90°, para un radio=al diámetro de la pieza que se prueba.
**Y** (b), para rodillos de puentes levadizos ó giratorios, etc.

**G**, general; **Oo**, Osborn; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, Calzadas.

### Hierro laminado.

Condiciones de las especificaciones de Osborn para puentes de calzadas, **Oo**. Hecho con hierro pudelado ó laminado de atados ó montones de recortes de hierro forjado n.º 1, solo, ó con hierro de primera laminación. Tensión mín, 48,000 libras por pulg cuad (3,360 kg por cm cuad) (50,000) (3,500), **R**); punto *cedente* (véase *N. del T.*, § 31 de Resistencia de Materiales) (1,750 kg por cm cuad) (1,820 kg por cm cuad), **R**); estiramiento, 20 por ciento en 8 pulgadas (20.3 cm); en secciones de peso menor de .654 libra por pie lineal (.97 kg por m), 15 por ciento. Las piezas de prueba cortadas de la barra original deben doblarse en un ángulo de 180º con una serie de golpes ligeros, al ser tajada alrededor y doblada; la fractura debe ser generalmente fibrosa y libre de manchas gruesas cristalinas; no más del 10 por ciento de la superficie fracturada debe ser granular. Las muestras calentadas al rojo vivo deben doblarse completamente con golpes suaves sucesivos no dados directamente en el doblez. En barras planas y cuadradas, se pueden permitir variaciones en cualquiera de las dos dimensiones, de $^1/_{32}$ de pulgada (.8 mm), y en las redondas, .01 pulg (.25 mm), **Oo**.

### Hierro fundido.

Hierro gris duro de fundición, **A**, **D**, **E**, **R**; á menos que se especifique de otra manera, **A**, **R**.

Resistencia transversal. Barras de 1 pulg en cuadro (6.45 cm cuad), largas de 12 pulgadas (30.5 cm), soportarán 2,500 libras (1,134 kg) de carga central. Deben flexarse .15 pulgadas (3.8 mm) antes de romperse, **G**. Barras de 1 pulgada en cuadro (6.45 cm cuad) y de 4.5 pies (1.37 m) de largo, soportarán 500 libras (227 kg) de carga central, **E**, **R**.

### Bronce fosfórico.

Un cubo de una pulgada (16.38 cm cúb) bajo compresión, límite de elasticidad, 20,000 libras (9,076 kg). Bajo 100,000 libs (45,359 kg), deformación permanente, máxima $^1/_{16}$ de pulgada (1½ mm), **B**.

### Maderas.

No se debe emplear más del 10% de madera de savia en piezas de la misma calidad, y no se aceptarán piezas que presenten capas de savia de más de .25 × el ancho de la pieza en cualquier punto de sus caras, ó más de la mitad del espesor de cualquier tablón, **Oo**.

## III. CARGAS

### 1. Cargas verticales.

(Cargas muertas, vivas y choques.)

### Cargas muertas en puentes de ferrocarriles á vapor.

Carga muerta = peso del metal + $n$ kg, por m lineal de vía, **C**, **D**, **E**, **R**, **Y**. $n = 600$, **C**, **D**, **E**; $n = 750$, **R**; $n = 924$, **Y**.

La madera calculada á 4.5 libras por pie (6.7 kg por metro), **B**, **M**, **G**. Balasto 110 libras por pie cúbico (1,763 kg por m cúb), **C**.

Carriles, bridas y empates, calculados á 100 libras por pie lineal de vía (149 kg /m), **A**, **B**, **C**.

Carriles, bridas, guardacarriles, etc., á 160 libras por pie lineal de vía (238.4 kg por m), **P**.

Se supone que las dos terceras partes de la carga muerta la sostiene el cordón cargado, **Y**; en luces de menos de 300 pies (91.5 m), **B**; en luces mayores calcúlese la distribución, **B**.

### Cargas muertas en puentes de calzadas y de ferrocarriles eléctricos.

Hierro, 3.33 libras por pie lineal (4.96 kg por m) de barra de 1 pulgada cuadrada de área (6.45 cm cuad), **Oo**.

**A**, **Aa**, Am B Co; **B**, B y O; **C**, **Cc**, Cooper; **D**, D L y W; **E**, Erie;

Acero, 3.40 libras por pie lineal (5.06 kg por m) de barra de 1 pulgada cuadrada (6.45 cm cuad) de área, **Oo**.

Madera por pie (medida de tabla)*, 4, **Aa**; creosotada, 5, **Oo**; roble, 4.5, **Cc**, **Oo**; otras maderas duras, 4.5, **Cc**; pino amarillo, 4, **Oo**; abeto y pino blanco, 3.5, **Cc**; pino blanco y cedro, 3, **Oo**.

Concreto, etc., 130 libras por pie cúb (2,084 kg por m cúb), **Aa**; concreto de piedra 125 (2,003), **Oo**; concreto de escoria 100 (1,603), **Oo**. Piedra, 150 (2,404), **Oo**; granito, 160 (2,560), **Aa**.

Ladrillo, 150 (2,404), **Aa**; 125 (2,003), **Oo**; arena, 100 (1,603), **Oo**. Asfalto, 130 (2,084), **Aa**; 90 (1,440), **Oo**.

Carriles, ligazones, bridas y guardas de maderos, 100 libras por pie lineal (149 kg por m) de vía, **Aa**.

## Cargas vivas para puentes de ferrocarril á vapor.

### (CARGAS NORMALES DE TEODORO COOPER)

El número (27 á 50) que sigue á la letra E (Locomotora) en la clase que se expresa (véase la tabla abajo) da el peso $d$ en millares de libras sobre un par de ruedas motrices. En cada clase, $d = 2\,b = 40\,t \div 26 = 10$ U. Como son constantes estas proporciones para todas las clases, los esfuerzos, producidos por cualquier clase, son proporcionales al número de la clase. El peso del metal, en puentes, es, en cada clase, un 10 por ciento mayor que en la clase menor inmediata.

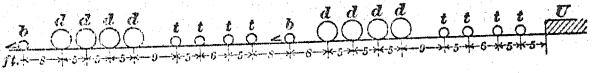



**Fig. 1.**

(*Obs. del T.* — Para tener en metros las distancias entre dos pares de ruedas (indicadas en pies, véase fig. 1) se multiplica la distancia expresada en la fig. por .305.)

*Dos locomotoras « Consolidation »** con sus tender y vagones.*

Carga en libras sobre un par de ruedas para cada vía.

(*Obs. del T.* — Hemos puesto entre paréntesis las equivalencias en sistema métrico.)

| Clase. | Carretilla delantera. | Motrices. | Ténder. | Tren cargado, lbs por pie lineal y kg por metro lineal. |
|---|---|---|---|---|
| | *b* | *d* | *t* | U |
| E 27 | 13,500 (6,124 kg) | 27,000 (12,247 kg) | 17,500 (7,938 kg) | 2,700 (4,023 kg) |
| E 30 | 15,000 (6,804 kg) | 30,000 (13,608 kg) | 19,500 (8,845 kg) | 3,000 (4,470 kg) |
| E 35 | 17,500 (7,938 kg) | 35,000 (15,876 kg) | 22,750 (10,319 kg) | 3,500 (5,215 kg) |
| E 40 | 20,000 (9,072 kg) | 40,000 (18,144 kg) | 26,000 (11,793 kg) | 4,000 (5,960 kg) |
| E 50 | 25,000 (11,340 kg) | 50,000 (22,680 kg) | 32,500 (14,742 kg) | 5,000 (7,450 kg) |

**A**, cargas de Cooper.
**B**, clase E, 50 de Cooper, á menos que se especifique de otra manera.

* *N. del T.* — Véase Tablas, págs. 273, etc. Un pie (medida de tablas) = á un pie cuad por una pulg de espesor. Mil pies = 2.360 m cúb. Entran 424 pies más ó menos (medida de tablas) en un metro cúbico.

** *N. del T.* — Llaman los americanos « *Consolidation* », las locomotoras que tienen cuatro pares de ruedas acopladas, como se ve en *dddd*, fig. 1. Este nombre proviene del que le dió la Compañía del ferrocarril Lehigh Valley á la primera de aquella clase fabricada en 1866. (*The Century Dictionary.*)

**G**, general; **Oo**, Osborn; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, Calzadas.

**C**, véase arriba.

**Y**, clase de Cooper E, 40.

**D**, **E**, **P**, **R**, las separaciones difieren poco de las de Cooper. Las cargas como aparecen abajo *.

| | *b* | *d* | *t* | U |
|---|---|---|---|---|
| **D** | 22,000 (9,979 kg) | 50,000 (22,680 kg) | 28,000 (12,700 kg) | 4,500 (6,705 kg) |
| **E** | 15,000 (6,804 kg) | 35,000 (15,876 kg) | 23,000 (10,433 kg) | 4,000 (5,960 kg) |
| **P** | 21,000 (9,526 kg) | 44,000 (19,958 kg) | 30,000 (13,608 kg) | 5,000 (7,450 kg) |
| **R** | 19,200 (8,709 kg) | 43,200 (19,595 kg) * 48,000 (21,773 kg) | 24,600 (11,158 kg) * 27,000 (12,247 kg) | 4,800 (7,152 kg) |

*Cargas alternativas.*

Empléese la fig. 1 ó las alternativas, fig. 2 ó 3, la que dé mayores esfuerzos.

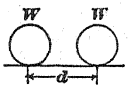



**Fig. 2.**

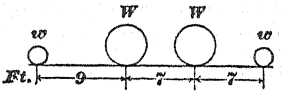



**Fig. 3.**

Carga sobre un par de ruedas.

Fig. 2. { *d*=6 pies (1.83 m); W=W=50,000 lbs (22,680 kg), sobre E 40, 60,000 lbs, 27,228 kg) **C**. *d*=7 pies (2.13 m); W=W=65,000 lbs (29,497 kg), **D**. *d*=7 pies (2.13 m); W=W=60,000 lbs (27,228 kg); U=4.500 lbs por pie lin (6,705 kg por m), **Y**.

Fig. 3. W=W=60.000 libras (29,951 kg); *w*=*w*=30,000 lbs (13,614 kg), **R**.

Agréguese 30 por ciento al calcular las vigas del piso, las riostras, soporte colgante, tirantes, y demás conexiones del piso. Agréguese de 0 á 30 por ciento por luces desde 100 pies (30.5 m) hasta 25 pies (7.62 m), **D**.

*En las curvas.*

Distribución de la carga viva entre las dos armaduras.

$W = P\,\frac{m+b}{2b}$; siendo W = la parte de la carga viva sobre a armadura de afuera; P=la carga viva en el tramo que se estudia; *m*=la ordenada media de toda la curva en la luz; *b*=la distancia entre los centros de las armaduras. Calcúlense ambas armaduras por igual, **B**, **Y**.

*Cargas especiales.*

Para los remaches que unen los ángulos superiores de las tablas con el alma de las vigas de tablero superior, que soportan el piso directamente sobre las tablas (ó alas) superiores, y en los tramos del tablero con las vigas de madera del piso, cuando la distancia entre las armaduras pasa de 6 pies (1.83 m), 50,000 lbs (22,690 kg) sobre un par de ruedas motrices, distribuídas por igual entre tres durmientes del piso, **P**.

Para pisos, la carga de un solo par de ruedas sencillas de locomotora distribuída entre 4 durmientes, **B**; entre 3 durmientes, **C**. Para piso de cajuelas, 60,000 libras (27,228 kg) en un par de ruedas distribuídas entre dos cajuelas, **Y**.

*Puentes de tres armaduras.*

En tramos de tablero superior de doble vía, las tres armaduras de la misma fuerza, **C**.

* Los ejes de la motriz y del tender con cargas desiguales, **R**.

**A, Aa**, Am B Co; **B**, B y O; **C, Cc**, Cooper; **D**, D L y W; **E**, Erie;

En puentes de vigas laminadas de más de una vía, la viga del centro se calcula en .75 × el peso de la carga viva, **E**.

*Aumento futuro de carga viva.*

Sólo el 70 por ciento (50 por ciento, **R**) de la carga muerta se debe considerar como efectiva al neutralizar la fuerza de la carga viva, **A, R**. Usese 1.5 × el peso de la carga viva, **E**.

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« No es improbable que la más pesada de estas locomotoras (véase arriba « **C** » en Cargas normales) esté muy próxima al máximo posible, considerando los límites permitidos para las secciones transversales en los ferrocarriles existentes y los detalles mecánicos de dibujo y proporciones. Es de esperarse que la tendencia económica hacia el constante aumento de peso de las locomotoras alcanzará pronto la clase más pesada, E 50, en los principales ferrocarriles. Los vagones también seguirán la misma tendencia para muchas clases de tráficos, pues la experiencia justifica este desarrollo. Existen hoy en uso vagones para carbón de descarga automática de capacidad nominal de 100,000 libras (45,380 kg), que tienen, en cuatro ejes, una carga total de 146,000 libras (66,138 kg) (10 por ciento de aumento sobre su capacidad nominal) en una base de rueda, para dos carros adyacentes, de 17 pies 2 pulgadas (5.23 m). Estos vagones en todos los puentes corrientes producen esfuerzos equivalentes á los de E 33. » — Theodore Cooper.

Las piezas sujetas al cambio de signo del esfuerzo deben disponerse de manera que una carga viva, *n* por ciento mayor que la especificada, no aumente sus unidades de esfuerzo en más de *n* por ciento, *n* = 25, **C**; 50, **B**; 100, **P**.

## Cargas vivas para puentes en calzadas y ferrocarriles eléctricos.

(*N. del T.* — Hemos puesto entre paréntesis los equivalentes en sistema métrico.)

| **Aa**, (Am. Bridge Co.) y **Cc**, (Theo. Cooper.) Clase ★. | Para pisos y sus apoyos. Concentrada. Vagones (a). tons. | Concentrada. Carros (b). en cada vía. tons. | Uniforme (c). Por pie cuad lbs | Para armaduras. Por pie lineal de vía simple. (Proporcional para luces intermedias.) Luz hasta 100 p (30.5 m) lbs. | Luz 200 p (61 m) y más lbs. | Por pie cuad del resto de piso. (Proporcional para luces intermedias.) Luz hasta 100 p (30.5 m) lbs. | Luz 200 p (61 m) y más lbs. |
|---|---|---|---|---|---|---|---|
| A..... | 24 | .. | 100 (488 kg por m cuad) | 1,800 (817 kg) | 1,200 (545 kg) | 100 (45.38 kg) | 80 (36.30 kg) |
| B..... | 12 ó 24 | | 100 (488 kg por m cuad) | 1,800 (817 kg) | 1,200 (545 kg) | 80 (36.30 kg) | 60 (27.23 kg) |
| C..... | 12 ó 18 | | 100 (488 kg por m cuad) | 1,200 (545 kg) | 1,000 (454 kg) | 80 (36.3 kg) | 60 (27.23 kg) |
| | | | | | | Hasta 75 p (22.87 m) | |
| D..... | 6 | .. | 80 (390 kg por m cuad) | ... | ... | 80 (36.30 kg) | 55 (24.96 kg) |
| E1..... | .. | 24 | ... | 1,800 (817 kg) | 1,200 (545 kg) | ... | ... |
| E2..... | .. | 18 | ... | 1,200 (545 kg) | 1,000 (454 kg) | ... | ... |

(a) En dos ejes 10 pies (3.05 m) entre centros (y **Aa**, 5 pies (1.52 m) entre carriles); en las clases **A, B** y **C**, que se suponen ocupan un ancho de 12 pies (3.66 m) en una línea (ó **Cc**, 22 pies (6.71 m) en línea doble) en cualquier parte de la vía.

(b) Sobre dos ejes 10 pies (3.05 m) entre centros.

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★ Clase A, puentes urbanos.
» B, puentes foráneos.
» C, calzadas foráneas de gran tráfico.
Clase D, calzada foránea corriente.
» E1, ferrocarril eléctrico de gran tráfico (sólo).
» E2, ferrocarril eléctrico de poco tráfico (sólo).

**G**, general; **Oo**, Osborn; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, Calzadas.

(c) En las clases A, B y C, en el resto del piso, incluso las aceras. En la clase D, en toda la superficie del piso.

**Oo** (Osborn Engineering Co). Calzada. Se puede especificar cualquier combinación de las siguientes cargas, según la condición del puente y de la carga.

Cargas uniformes, kg por metros cuadrados. Para luces hasta de 150 pies (45.75 m), 488 en la calzada y 390 en las aceras, ú 390 en ambas. Para luces de 150 pies (45.75 m) pies, 390 ó 290 en ambas.

Un aplanador de rodillos movido por vapor; ejes á 11 pies (3.36 m) de separación, rodillo delantero de 4 pies (1.22 m) de frente, dos rodillos posteriores de 5 pies (1.52 m) entre centros y cada uno de 20 pulgadas (50.8 cm) de frente, 15,000 (6,795 kg) ó 9,000 lbs (4,086 kg) en el rodillo delantero y 10,000 ó 6,000 libras (4,538 ó 2,723 kg) en cada uno de los rodillos posteriores.

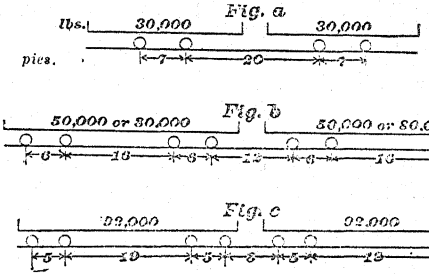
Un aplanador de rodillo tirado por caballo, 12,000 lbs (5,448 kg) en el rodillo, 5 pies (1.52 m) de frente.

Una carga de vagón, 10,000 libras (4,538 kg) en dos ejes, á 8 pies (2.44 m) de separación, 5 pies (1.52 m) entre carriles.

Dos vagones eléctricos en cada vía; fig. *a*.

Un tren de vagones (eléctrico) en cada vía; fig. *b*.

Un tren de vagones de carbón de 60,000 libras (27,228 kg) de capacidad; fig. *c*




(*N. del T.* — Las lbs se convierten en kgs multiplicándolas por 0.453 y los pies en metros por 0.305.)

*Aumento futuro de carga viva. Calzadas.*

En puentes para ferrocarriles eléctricos de la clase E, tan sólo se considerará como efectivo el 70 por ciento del esfuerzo de carga muerta al equilibrar el de la carga viva, **Aa**. En puentes por donde pasan tranvías eléctricos ó carros automotores, se emplean contradiagonales proporcionadas de manera que un aumento futuro de 25 por ciento de carga viva especificada no aumente la unidad del esfuerzo en más de 25 por ciento, **Cc**.

### Choques.

$I = S \frac{91.5}{l + 91.5}$; en que I=á la fuerza del choque que se debe agregar al esfuerzo de la carga viva; S=al máximo esfuerzo, calculado, de la carga viva; $l$=la longitud en m de la distancia cargada que produzca el mayor esfuerzo en la pieza, **A**.

En puentes para calzadas; I=25 por ciento de los esfuerzos de la carga viva, **Aa**.

### 2. Fuerzas horizontales.

(De arrastre Centrífuga y del viento.)

#### (a) Longitudinal.

**Arrastre.** En puentes para ferrocarriles de vapor y eléctricos, téngase en cuenta una fuerza longitudinal en los rieles= .2 del máximo de la carga viva. En vías dobles (**Y**) calcúlese para trenes en marcha en ambas direcciones.

**A, Aa,** Am B Co; **B,** B y O; **C, Cc,** Cooper; **D,** D L y W; **E,** Erie;

## (b) Transversales.

### (1) Fuerza centrífuga.

**F**=fuerza centrífuga; W=peso del tren sobre el puente; $d$=grado de curvatura=ángulo central subtendido por una cuerda de 100 pies (30.5 m); $v$=velocidad en millas por hora (1 milla = 1609 m); $c$=un coeficiente.

**A,** F=$c$, $d$, W. Para $d$ hasta 5°, $c$=.03. Dedúzcase de $c$ .001 por cada grado sobre los 5°. Tren en cada vía.

**B, R,** F=.02 de la carga viva para cada grado de curvatura. **B,** hasta 5°. Dedúzcase .001 para cada grado sobre los 5°.

**D,** $v$=60 = 96.54 kilómetros.

**E,** F=fuerza debida á la carga uniforme que produciría el momento flexor de máximo de la carga viva, especificada, sobre la luz; $v$=60 = 96.54 kilómetros.

### (2) Viento.

*(a) En puentes de ferrocarril.*

La presión del viento, en libras por pie cuadrado ***, = $w$; en libras por pie lineal = W.

$w$=30 libras por pie cuadrado (146.4 kg por m cuad) en las superficies expuestas de las armaduras y del piso y en la de un tren de un promedio de 10 pies (3.05 m) de altura, comenzando á 30 pulgadas (76 cm) sobre la base del riel; ó de 50 libras por pie cuadrado (244 kg por m cuad) en las superficies expuestas de las armaduras y del piso; cualquiera de las dos da el mayor esfuerzo, **A, P.**

En luces de armaduras de 200 pies (61 m) y en vigas laminadas, $w$=30 libras por pie cuadrado (146.4 kg por m cuad) de superficie expuesta de 1 viga y del piso, + W sobre el tren para el cordón inferior, como sigue, **B.**

W=L+U. L=presión en libras por pie lineal en el cordón cargado, U en el cordón sin carga. W comprende ambos vientos sobre el puente y sobre el tren ***.

Viento sobre el puente. L=U=150 (223.5 kg por metro lineal), **B, C** *, **D, R**; =200 (=298 kg por m lineal), **E.**

L=200 (298 kg por m lin), en doble vía 300 (447 kg por m lin) obrando á 8 pies (2.44 m) sobre el tope del riel;
U=150 (223.5 kg por m lin), en doble vía 225 (335.26 kg por m lin), obrando en el centro del cordón; } **Y.**

Viento sobre el tren. L=300 (447 kg por m lin), **B, D, R** **, **C,** §=400 (596 kg por m lin), **E.**

**A.** El esfuerzo del viento, S$w$ en cualquier pieza de la armadura, **C** (pieza de la armadura principal, **D**; cordón ó poste extremo, **R**), debe considerarse tan sólo (1) cuando S$w$ excede de 30 por ciento, **C** (25 por ciento, **D, R**), del esfuerzo máximo, S, producido por las cargas viva y muerta. Luego auméntese la sección para llevar á S$w$ á sus límites, **C, D, R.** (2) Cuando S$w$, sola ó combinada con la acción de la temperatura, puede equilibrar ó neutralizar á **S, C.**

Anclaje. Al determinar el anclaje necesario para la estructura cargada, se calcula que el tren pesa 800 libras por pie lineal (1,192 kg por m), **A, B, C, P**; 600 libras por pie lineal (894 kg por m), **R.**

*(b) En puentes para calzadas y para ferrocarriles eléctricos.*

30 libras por pie cuadrado (146.4 kg por m cuad) en las superficies expuestas de todas las armaduras y el piso, + 150 libras (223.5 kg por m) por pie lineal, (180 = 268.2 kg por m, **Oo**) de un tren que abarque toda la luz; ó 50 libras por pie cua-

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* En luces de más de 300 pies (91.5 m agréguese á U 10 libras (4.53 kg) por cada 30 pies (9.15 m) adicionales, **C.**

** Obrando á 75 pies (22.87 m) sobre el riel, **R.**

§ Obrando á 6 pies (1.83) sobre la base del riel. Incluye las vibraciones laterales de los trenes.

*** Las conclusiones son equivalentes, ya se usen las unidades inglesas (lbs por pie lineal, lbs por pie cuad) ó las métricas (kgs por metro lineal, kg por m cuad).

**G**, general; **Oo**, Osborn; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, Calzadas.

drado (244 kg por m cuad) en la superficie expuesta de todas las armaduras y del piso; cualquiera de las dos dará el esfuerzo máximo, **Aa**, **Oo**.

En cada cordón 150 libras por pie lineal (223 kg por m) de luz, para el puente, y en el cordón cargado 150 libras por pie lineal (223 kg por m) de luz adicional, para el tren. Para luces mayores de 300 pies (91.5 m), agréguese 10 libras (4.54 kg) en cada cordón por cada 30 pies (9.1 m) adicionales, **Cc**.

Se provee contra el esfuerzo del viento (en las piezas de las armaduras, **Cc**; en los cordones y postes extremos, **Oo**) tan sólo cuando el esfuerzo del viento exceda del 25 por ciento del esfuerzo máximo de las cargas muerta y viva (de la suma de todos los demás esfuerzos, **Oo**), ó cuando el esfuerzo del viento (solo ó en combinación con la acción de la temperatura, **Cc**) puede (neutralizar ó, **Cc**) invertir el esfuerzo sobre las piezas, **Cc**, **Oo**.

## IV. ESFUERZOS Y DIMENSIONES

### Luz y alturas verdaderas.

En las armaduras de pasadores la luz y la altura se miden entre los centros de los pasadores. En armaduras de remaches, se mide la luz entre los centros de los apoyos extremos, y su altura entre los centros de gravedad de las secciones del cordón. En las vigas laminadas se mide la luz entre los centros de los apoyos extremos, y la altura entre los centros de gravedad de las áreas de las tablas (de las vigas) ó detrás de los ángulos de las mismas, la que resulte menor. En las vigas del piso se mide la luz entre los centros de las armaduras, y en las longitudinales entre los centros de las vigas del piso, **G**.

### Limitación de la unidad de esfuerzos.

### Tensión.

**Sección neta.** La sección neta de cualquier pieza de tensión se determina por un plano que corte la pieza en ángulo recto en cualquier punto. El mayor número de perforaciones para remaches que pueda cortar el plano, ó que se encuentren á una pulgada (2.54 cm) del plano, se restan del total de la sección, **B**. La ruptura de una pieza de tensión remachada se considera tan sólo como igualmente probable, ya sea á través de una hilera transversal de perforaciones de remaches, ó á través de una línea diagonal de perforaciones donde la sección neta no excede en un 30 por ciento de la sección neta á lo largo de la línea transversal, **C**, **Cc**.

Al restar los agujeros de los remaches de la sección neta, se calculan sus diámetros en un octavo de pulgada (3 mm) más que el del remache frío, **G**; para remaches de cabeza embutida (**Oo**), un cuarto de pulgada más (6 mm).

**Máximas tensiones permitidas**, en libras por pulgada cuadrada.

(*N. del T.* — Los números que están entre paréntesis son los kg por cm cuad equivalentes á las libras por pulg cuad del número superior.)

| | Acero mediano. | Acero blando. |
|---|---|---|
| **A, Aa.** Bajo fuerzas verticales ú horizontales solamente.... | 17,000 (1,190) | 15,000 (1,050) |
| Bajo fuerzas verticales y horizontales combinadas.. | 21,000 (1,470) | 19,000 (1,330 |

**B.** Para acero blando (de puentes) y para remaches, como para el acero mediano rige **A**.

| **D.** Para acero blando : | Para carga muerta. | Para carga viva. |
|---|---|---|
| Barras de ojo..... | 14,000 (980) | 9,000 (630) |
| Secciones armadas..... | 12,500 (875) | 8,500 (595) |
| Contradiagonales..... | | 8,500 (595) |

**A, Aa**, Am B Co; **B**, B y O; **C, Ce**, Cooper; **D**, D L y W; **E**, Erie;

| | Para cargas viva y muerta. |
|---|---|
| Montantes, barras que suspenden las vigas del piso, piezas sometidas á cargas bruscas | 7,500 (525) |
| Tablas de tensión de las vigas laminadas y vigas laminadas | 9,000 (630) |
| Riostras y refuerzos | 12,000 (840) |

En las piezas principales de las armaduras, tablas y entramado de las vigas y las vigas del piso para vías dobles, tablas del piso y vigas con suelo para balasto, agréguese 10 por ciento.

Para acero mediano, agréguese 10 por ciento.

**Oo.** (Puentes para calzadas.) Acero mediano, 22,000 (1,540 kg por cm cuad); acero blando, 20,000 (1,400 kg por cm cuad); hierro forjado, 18,000 (1,260 kg por cm cuad).

**P.** M=esfuerzo máximo (calculado) en la pieza;
$m$=esfuerzo mínimo (calculado) en la pieza.

Hágase $r = \frac{m}{M}$; y $k = \frac{1-r}{1+r}$. Entonces M $(1 + k)$ no debe exceder de 15,000 (1,050 kg por cm cuad).

Los montantes verticales largos deben tener 25 por ciento de exceso de resistencia; las barras cortas que sostienen las vigas del piso, 50 por ciento de exceso, **P.**

**Y.** Acero blando. Los cordones y piezas del entramado de las armaduras y las tablas de las vigas laminadas y vigas de piso.

| | |
|---|---|
| Carga muerta y tracción | 16,000 (7,261 kg) |
| Carga viva y fuerza centrífuga | 8,000 (3,630 kg) |

*Resistencia máxima de las maderas, en libras por pulgada cuadrada.*

(*N. del T.* — Los números que están entre paréntesis, son los kg por cm cuad equivalentes á las lbs por pulg cuad del número superior.)

| Para puentes de calzada, **Oo.** | Carga transversal. | Apoyo extremo. | Columna corta *. | Carga á través de la fibra. | Esfuerzo cortante á lo largo de la fibra. |
|---|---|---|---|---|---|
| Roble blanco | 1,400 (98) | 1,300 (91) | 1,000 (70) | 550 (38.5) | 300 (21) |
| Pino de hoja larga | 1,600 (112) | 1,300 (91) | 1,000 (70) | 350 (24.5) | 200 (14) |
| Pino blanco | 1,100 (77) | 900 (63) | 700 (49) | 200 (14) | 150 (10.5) |
| Abeto | 950 (66.5) | 850 (59.5) | 650 (45.5) | 200 (14) | 100 (7) |

Resistencia máxima de la fibra, en vigas de piso, en pino amarillo y roble blanco, 1,200 libras por pulg cuad (84 kg por cm cuad); y pino blanco y abeto, 1,000 (70 kg por cm cuad), **Aa, Cc.**

* Longitud no mayor de 12 × lado menor.

**G**, general; **Oo**, Osborn; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, Calzadas

## Compresión **.

$p$ = el esfuerzo de trabajo permitido en las piezas de compresión, en kgs por cm cuadrado.
$f$ = esfuerzo generalmente permitido en las piezas de tensión, en kgs por cm cuadrado.
$a$ = un coeficiente.
L = longitud de la pieza, en cm, entre centros de conexiones.
$r$ = menor radio de giración de la sección transversal de la pieza, en cm.

$$p = \frac{f}{1 + \frac{L^2}{r^2 a}}$$

(*N. del T.* — Los números que van entre paréntesis son los kg por cm cuad equivalentes á las libras por pulg cuadrada del número superior ó anterior.)

| | | $f$ | $a$ |
|---|---|---|---|
| **Aa.** | En acero mediano | 17,000 (1,190) | 11,000 |
| | En acero blando | 15,000 (1,050) | 13,500 |
| **B.** | En acero blando | 17,000 (1,190) | 11,000 |

**C.** Véase más abajo.

| | Carga muerta. | | Carga viva. | |
|---|---|---|---|---|
| | $f$ | $a$ | $f$ | $a$ |
| **D.** | 12,000 (840) | 18,000 | 8,000 (560) | 18,000 |
| | á | á | á | á |
| | 12,500 (875) | 24,000 | 8,500 (595) | 24,000 |

**E.** $f = 560\left(1 + \frac{\text{resistencia mínima}}{\text{resistencia máxima}}\right)$; $a = 36{,}000$ con ambos extremos fijos $a = 24{,}000$ con un extremo fijo; $a = 18{,}000$ con ambos extremos articulados.

**Oo.** (Puentes para calzadas.) $f = 22{,}000$ (1,540) para acero mediano, 20,000 (1,400) para acero blando, 18,000 (1,260) para hierro forjado; $a$ como en **E**, arriba.

**P.** $f = 15{,}000$ (1,050); $a = 13{,}500$.

**R.** $f = 455\left(1 + \frac{\text{resistencia mínima}^*}{\text{resistencia máxima}}\right)$; $f$ máx = 8,000 (560); $a = 40{,}000$ con extremo chatos; $a = 20{,}000$ con extremos articulados.

Cuando un extremo está articulado, $p$ = promedio de valores deducidos arriba.

Para puntales angulares de hierro, véase más abajo.

**Y.** Acero blando en cordones y en piezas del entramado:

| | $f$ | $a$ |
|---|---|---|
| Para carga muerta y tracción | 16,000 (1,120) | 18,000 |
| Para carga viva y fuerza centrífuga | 8,000 (560) | 18,000 |

* Resistencia mínima = carga muerta — carga viva.

** *N. del T.* — Hemos modificado las fórmulas y coeficientes para hacerlas aplicables al sistema métrico.

**A, Aa,** Am B Co; **B,** B y O; **C, Cc,** Cooper; **D,** D L y W; **E,** Erie;

**C, Cc.** $p = M - c\frac{L}{r}$.

Para acero mediano en estructuras fijas.

| | Carga muerta. M | Carga muerta. c | Carga viva. M | Carga viva. c |
|---|---|---|---|---|
| Segmentos de cordón y piezas de refuerzo | 20,000 (1,400) | 6.33 | 10,000 (700) | 3.16 |
| Para puentes de calzada | 24,000 (1,680) | 7.73 | 12,000 | 3.87 |
| Postes finales y otros | 17,000 (1,190) á 18,000 (1,260) | 6.33 á 5.62 | 8,500 (595) á 9,000 (630) | 3.16 á 2.81 |
| Para puentes de calzada | 20,000 (1,400) á 22,000 (1,540) | 6.33 á 5.62 | 10,000 (700) á 11,000 (770) | 3.16 á 2.81 |
| Para puntales laterales, refuerzos rígidos para puentes de ferrocarril y de calzada | 13,000 (910) | 4.2 | 8,666 (607) | 2.81 |

Para acero blando dedúzcase 15 por ciento; para estructuras movibles dedúzcase 25 por ciento.

**R.** Puntales angulares de hierro.

Con extremos chatos, $p = 630 - 2.1\frac{L}{r}$; con extremos de pasador, $p = 630 - 2.4\frac{L}{r}$.

La puntales laterales y transversales agréguese 30 por ciento.

**Longitud** de las piezas de compresión, máxima = 40 á 45 diámetros, ó 100 *r* á 120 *r*. En puentes para calzadas, 120 *r* á 140 *r*, **Aa**; 100 *r* á 120 *r*, **Cc**; 125 *r* á 150 *r*, **Oo**, en que *r* = al menor radio de giro.

**Anchura sin apoyo** (distancia entre remaches) de las planchas sometidas á compresión, máx = 45 × espesor, **Oo**; 30 × espesor, **C, Cc, D**; en planchas de tapa de los cordones y postes extremos, 40 × espesor, **C, Cc, D**; ó si se emplea mayor anchura, se tomará la sección real como 40 × espesor, **C, Cc**. Distancia entre apoyos en la línea de resistencia, máximo = 16 × espesor, **Oo**.

Columnas de madera cuyo largo no exceda de 12 × sus lados menores, para puentes de calzada, **Oo**.

$$\text{Unidad de esfuerzo máximo} = \frac{C}{1 + \frac{L^2}{1{,}000\,d^2}}$$

en que C = 70 kg por cm cuad para roble blanco y pino de hoja larga, 49 para pino blanco, 45 para pinabete; L = largo de la columna, entre apoyos, en cm; *d* = lado menor, en cm, **Oo**.

### Esfuerzos alternativos.

El área total de la sección de la pieza se hará = la suma de las áreas necesarias para ambas resistencias, **A, B**.

Area suficiente para resistir cualquiera de los esfuerzos, .8 (.6, **R**; 1.0, **Y**) × la menor resistencia, **C, Cc, D, R, Y**.

Trabajo permitido, en kg por cm cuad:

$$a = 560\left(1 - \frac{\text{resistencia máxima de la menor clase}}{2 \times \text{resistencia máxima de la mayor clase}}\right)\ \textbf{E}.$$

M = resistencia máxima (calculada) de la clase mayor.

*m* = resistencia máxima (calculada) de la clase menor.

Sea $r = \frac{m}{M}$, $k = \frac{2 + r}{2 - r}$. Entonces M (1 + *k*) no debe exceder de 1,050 kgs por cm cuad. **P.**

**G**, general; **Oo**, Osborn; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, Calzadas.

*Puentes para calzadas y ferrocarriles eléctricos.*

En las clases A, B, C y D, las piezas proporcionadas para aquel esfuerzo que requiera la sección mayor. En las clases E1 y E2, hágase el área de la sección = á la suma de las áreas requeridas para las dos resistencias, **Aa**. Las piezas se hacen para resistir cualquiera de los dos esfuerzos y se les dan 25 por ciento de exceso de resistencia en sus uniones y conexiones, **Oo**.

### Esfuerzo cortante y fuerza de apoyo *.

**Esfuerzo cortante** en las almas, su máximo por pulgada cuadrada. 10,000 (700), **B**; 4,000 (280), **E**; 5,000 (350), **R**; 13,000 (910), **P**; en acero mediano, 10,000 (700), **A**, **Aa**; en acero blando, 9,000 (630), **A**, **Aa**; á través de la fibra, 6,000 (420), **D**; en la dirección de la fibra, 5,000 (350) (sección neta), **D**; carga muerta, 10,000 (700), **Y**; carga viva, 5,000 (350) (sección bruta), **Y**.

**Esfuerzo cortante y fuerza de apoyo sobre los remaches, pernos y pasadores.** Máximo en libras por pulg cuadrada.

| | Esfuerzo cortante. | | Resistencia (apoyo). | |
|---|---|---|---|---|
| | Acero mediano. | Acero blando. | Acero mediano. | Acero blando. |
| **A, Aa, B**..... | 12,000 (840) | 11,000 (770) | 24,000 (1,680) | 22,000 (1,540) |
| **C**..... | 9,000 (630) | 9,000 (630) | 15,000 (1,050) | 15,000 (1,050) |
| **Cc**..... | 10,000 (700) | 10,000 (700) | 18,000 (1,260) | 18,000 (1,260) |
| **Oo**..... | 10,000 (700) | 10,000 (700) | 22,000 (1,540) | 20,000 (1,400) |
| **D, R**..... | 7,500 (525) | 7,500 (525) | 12,000 (840) | 12,000 (840) |

**Y**, Esfuerzo cortante = .75 S; fuerza apoyo = 1.50 S.S = unidad de tensión permitida.

Al remachar (es la práctica), aumentar en 25 por ciento el número de los remaches, **A, Aa, B, Oo, P**; si son puestos á máquina 10 por ciento, **A, Aa, P**; en las vigas longitudinales y del piso, un tercio, **P**. Tómese .66 á .80 × esfuerzo como se dijo antes, **C, Cc, D, R, Y**.

En las conexiones del piso empléese .8 × los esfuerzos, como se dijo arriba, **C, Cc**; agréguese 20 por ciento al número de los remaches, **Y**.

En las riostras y contravientos empléese 1.25 á 1.5 × los esfuerzos, como se dijo antes, **C, Cr, D, R**.

Los remaches con cabezas embutidas se calculan en .75 × la resistencia de los de cabeza exterior, **P**.

**Apoyo**; en discos de bronce fosfórico, 5,000 libras por pulg cuad (350 kg por cm cuad), **B**.

### Resistencia á la flexión *.

Resistencia de las fibras extremas bajo los momentos flexores, máximo en libras por pulg cuad.

En pasadores y pernos, 25,000 (1,750), **B**; 18,000 (1,260), **C**; 20,000 (1,400), **Cc**; 15,000 (1,050), **D, R**; 16,000 (1,120), **Y**; en pasadores muy unidos, 25,000 (1,750), **Oo**; de acero mediano, 25,000 (1,750); de acero flojo, 22,000 (1,540), **A, Aa, P**. Los centros de apoyo de las piezas sometidas al esfuerzo se toman como puntos de aplicación de los esfuerzos, **A, Aa, R**. Las fuerzas aplicadas se consideran como uniformemente distribuídas sobre la línea media del apoyo de cada pieza, **C, Cc**. La flexión se calcula por las distancias entre los centros de apoyo, **Oo**.

En vigas laminadas ó vigas en U. 14.000. (980) **P**.

En vigas de madera para el piso, 1,000, (70) **A, B, C, P**.

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* *N. del T.* — Los números que van entre paréntesis, son los kg por cm. cuad equivalentes á las lbs por pulg cuad del número superior ó anterior.

**A, Aa,** Am B Co; **B,** B y C; **C, Cc,** Cooper; **D,** D L y W; **E,** Erie;

### Esfuerzos combinados **.

**Combinados** (axil y de flexión), máximo en libras por pulgada cuadrada.

En los postes de extremo en luces de vía inferior, carga muerta+carga viva+presión del viento+flexión máxima=15,000 (1,050), **R.**

Proporciónense las piezas para resistir la suma del esfuerzo directo más .75 de la flexión, **A, Aa, B, P, R.** Máximo $= \frac{560}{1 + \frac{L^2}{40{,}000\, r^2}}$, en que L = longitud en cm; $r$ = radio menor de giro en cm.

Si los pasadores están fuera del eje neutro de la sección, el máximo debe contener el esfuerzo adicional debido á la excentricidad, **R.**

El momento de flexión en los puntos de división de los tramos se supone igual y opuesto al del centro, **A, Aa.** Si el esfuerzo de la fibra producido por el peso de la pieza sola excede de 10 por ciento de la unidad del esf permitido en dicha pieza, se debe considerar el exceso en proporción á las áreas, **C, Cc, R.**

### Dimensiones mínimas.

Espesor mínimo de las láminas para puentes de ferrocarril, $^3/_8$ de pulgada (9.5 mm) para las piezas principales, $^5/_{16}$ de pulg (8 mm) para las laterales; en puentes para calzadas y ferrocarriles eléctricos, $^5/_{16}$ á $^1/_4$ de pulg (8 á 6.5 mm). Diámetro mínimo de las barras, $^3/_4$ de pulgada (19 mm), **Oo.** Mínima sección de las barras, 1 pulgada cuadrada (6.45 cm cuad), **D, R;** contradiagonales, 1.5 pulgada cuadrada (9.7 cm cuad), **D, P.** Postes en luces de pasadores, ancho mínimo 10 pulgadas (254 mm), **A.** En postes de luces de vía inferior y vigas en U, mínimo 10 pulgadas (254 mm), **B.** Angulos, mín $3.5 \times 3 \times {}^5/_{16}$ (8.9×7.6×.8 cm), **B.**

## V. PROTECCIÓN

**En el taller.** Al quitar las conchas sueltas y el orín; una mano de aceite de linaza puro, hervido, **A, Aa, B, D, E, P, R;** aceite de linaza crudo, **C, Cc;** con 10 por ciento en peso de negrohumo, **D;** pintura roja de plomo *, **Y.**

**Partes inaccessibles.** 2 manos de pintura de mineral de hierro en aceite puro de linaza, **A, Aa, B, C, Cc, E, R;** pintura de plomo roja *, **Y;** una mano, **D;** una mano espesa de pintura de plomo roja en aceite de linaza crudo, **P;** dos manos, á 18 lbs de pintura roja de plomo en 1 galón (3.78 lit) de aceite de linaza cocido, **Oo.**

**Mano final.** Capa de plomo blanco y sebo, **G.**

**Superficies en contacto.** Píntense antes de unirse, **A, Aa, B, C, Cc, R, Y;** con dos manos gruesas de rojo de plomo en aceite de linaza crudo en cada superficie, **Y.**

**Después de armar la obra.** 2 manos adicionales de pintura en aceite puro de linaza, **A, Aa, B, C, Cc;** dos manos de pintura de diversos colores, **R;** 2 manos gruesas de barniz de asfalto, **Y.**

Por lo menos se debe dejar secar cada mano durante 48 horas, **Y.**

**Columnas, etc.,** para 5 pies (1.52 m) sobre la superficie del suelo en calles, etc., dos manos espesas de charol de asfalto; **en los lados inferiores de los puentes,** resto de las columnas, etc., 2 manos espesas de pintura blanca *; lado del balasto en los pisos de cajuelas, 1 parte, por peso, de asfalto de Trinidad refinado y 3 partes de alquitrán fino á 300° F (149° C), **Y.**

En dondequiera que exista una tendencia á depositarse el agua, se deben llenar los espacios con un material impermeable, **C, Cc.**

La primera mano de pintura debe ser de grafito, **Oc.**

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* Pintura de rojo de plomo. 5 galones (19 lit) conteniendo 100 libras de rojo de plomo puro, 4 galones (15.12 lit) de aceite crudo de linaza puro, media pinta (1/4 lit más ó menos) de copal sin bencina, **Y.**

Pintura de blanco de plomo. 5 galones (19 lit) conteniendo 42 libras (19 kg) de blanco de plomo puro en aceite, 21 libras (9.5 kg) de blanco de cinc en aceite, 3 galones (11 1/3 lit) de aceite puro de linaza cruda, **Y.**

Por lo menos 48 horas entre las manos, y entre la última mano dada en el taller y la remoción de la pieza, **Y.**

** Véanse *N. del T.* al pie págs. 825 y 827.

**Gt** general; **Oo**, Osborn; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, Calzadas.

En puentes para calzadas, la superficie superior de las láminas metálicas de piso deben cubrirse enteramente con asfalto, **Oo**.

## VI. ERECCIÓN

El contratista está generalmente obligado :

(1) A descargar los materiales después de su entrega, á suministrar andamios y aparatos, á remover el puente viejo, á modificar los apoyos existentes del puente;

(2) A hacer taladros para los pernos de anclaje y poner éstos, á erigir y ajustar la superstructura, y á veces á suministrar y colocar las vigas de madera del suelo;

(3) A quitar los andamios y aparatos;

(4) A mantener libre la vía para el tráfico y no interceptar otras vías de comunicación por tierra ó por agua, y á no obstaculizar á otros contratistas; á suministrar y pagar vigilantes; á mantener el material limpio y en buen estado; y á asumir todos los riesgos de perjuicios personales ó de propiedad por causa de tempestades, inundaciones y otros accidentes (*Obs. del T.* — Creemos que estos últimos, son casos de fuerza mayor);

(5) A suministrar las piezas para la protección de las puntas de los pasadores al entrar en las vigas.

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## (2) RESUMEN DE ESPECIFICACIONES PARA PUENTES MIXTOS DE FERROCARRILES *

**(Por la Baltimore and Ohio Railroad Co., 1901.)**

## I. PROYECTO GENERAL

Tipo Howe.

Barras de acero, con extremos recalcados; tuercas normales y tuerca de cierre en cada extremo.

Fundiciones de hierro para uniones.

Láminas de acero para chavetas desde 1.25 pulgadas (32 mm) de espesor para barras de 1.25 pulgadas (32 mm) hasta 1.75 pulgadas (44 mm) de espesor para barras de 2.5 pulgadas (63 mm).

Las bridas (chapas de empalme) del cordón inferior son generalmente de acero.

## II. MATERIAL

Maderamen. Pino amarillo de Georgia, roble blanco ó pino blanco.

Acero laminado. Martín-Siemens. Resistencia máxima 60,000 lbs por pulg cuad (4,200 kg por cm cuad), variación permitida, 5,000 libras (350 kg); límite de elasticidad, 30,000 libras (2,100 kg); alargamiento 25 por ciento en 8 pulgadas (20.3 cm); flexión 180° sobre sí mismo.

## III. CARGAS

### Carga muerta.

Los maderos se calculan á razón de 4.5 libras (2 kg) por pie (en medida de tablas=2.36 litros). Enrielado 100 libras por pie lineal (149 kg por metro lineal).

### Carga viva.

Carga máxima calculada+25 por ciento para prever los aumentos y choques.

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* Para usarlos temporalmente.

**A, Aa**, Am B Co; **B**, B y O; **C, Cc**, Cooper; **D**, D D y W; **E**, Erie;

## IV. ESFUERZOS Y DIMENSIONES

### Límite de las unidades de esfuerzos.

(Véase *N. del T.*, pág. 827)

| Maderos, libras por pulg cuad. | Pino amarillo. | Pino blanco. | Roble blanco. |
|---|---|---|---|
| Flexión ó tensión directa..... | 1,200 (84) | 800 (56) | 1,000 (70) |
| Columnas de menos de 17 diámetros de largo..... | 900 (63) | 600 (42) | 750 (52½) |
| Columnas de más de 17 diámetros de largo..... | 1,200-15 *n* (84—1.26 *n*) | 800-12 *n* (56—0.84 *n*) | 1,000-15 *n* (70—1.05 *n*) |
| en que *n*=longitud ÷ menor grosor: *n* máx=40. | | | |
| Esfuerzo cortante á lo largo de la fibra ..... | 150 (10½) | 100 (7) | 200 (14) |
| Resistencia en dirección de la fibra. | 1,500 (105) | 1,000 (70) | 1,250 (87½) |
| Resistencia perpendicular á la fibra. | 350 (24½) | 200 (14) | 500 (35) |

En columnas hechas con varias piezas juntas y sujetas á intervalos por pernos cada pieza se tratará como una columna independiente.

Barras de acero, unidad máxima de esfuerzo=12,000 libras por pulgada cuadrada (840 kg por cm cuad).

Vigas del piso destinadas á soportar la carga muerta y las más pesadas locomotoras en uso, sin margen para choques. Refuércense para prever aumento futuro de cargas.

Para cargas mayores que las calculadas, redúzcase la velocidad de 60 á 15 millas (96.50 á 24.13 k) por hora, á medida que las cargas aumenten hasta el límite de 25 por ciento de los aumentos de pesos.

## V. PROTECCIÓN

Barras de acero, etc., 1 mano de pintura en el taller; 2 después de concluído.
Las maderas deben pintarse en los puntos de contacto.
Los agujeros de los pernos y barras deben llenarse con pintura.

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## (3) RESUMEN DE ESPECIFICACIONES PARA ARMADURAS DE TECHOS, ARMADURAS DE ACERO Y EDIFICIOS

### (Por el Baltimore and Ohio R. R. Co, 1901.)

## I. PROYECTO GENERAL

Se hace principalmente de piezas de forma. No deben emplearse piezas de ajuste, excepto en las trabazones laterales. Las riostras deben proporcionarse á la total presión del viento de 30 libras por pie cuadrado (146.4 kg por m cuad) de superficie expuesta, obrando en cualquier dirección. Las piezas de tensión en los cruceros deben siempre tirar directamente hacia un poste rígido. Si la construcción está encerrada y expuesta á la acción de gases, no se deben dejar espacios abiertos menores de 1 pulgada de ancho entre las piezas, para poderlos pintar bien.

## II. MATERIAL

Grosor mínimo, .25 pulgada ($6^1/_4$ mm). Cuando esté sujeta á la acción de gases, $^5/_{16}$ de pulgada (8 mm) si la construcción está al aire libre y .37 5pulg (9.3 mm) si está encerrada.

**G**, general; **Oo**, Osbora; **P**, Pa; **R**, R'd'g; **Y**, N Y C; **Aa**, **Cc**, **Oo**, Calzadas

## III. CARGAS

Nieve, 20 libras por pie cuadrado (97.6 kg por m cuad) de la proyección horizontal del techo. Viento, en cualquier dirección, 30 libras por pie cuadrado (146.4 kg por m cuad), horizontal. Total mínimo, 40 libras por pie cuadrado (195.2 kg por m cuad).

Cubiertas. Para techos y costados, á menos que se ordene otra cosa, hierro acanalado, n.º 22, de 26 pulgadas de ancho (66 cm); canales de 2.5 pulgadas (63 mm); 3 pulgadas (76 mm) de inclinación de 1 por 2; 6 pulgadas (152 mm) para inclinaciones menores. Clavados á no más de 4 pies (1.22 m) de distancia entre centros.

## IV. ESFUERZOS

Las columnas que sostienen el techo se consideran enterradas en la base, á menos que estén ancladas de manera absolutamente fijas.

La unidad de esfuerzo, si no está expuesta á ninguna otra carga movible que la del viento, véase **B**, en el Resumen de especificaciones para puentes de acero, y el Resumen (2) de especificaciones para puentes mixtos, de la B. and O. RR. Las resistencias dadas en el último se deben aumentar en un 25 por ciento.

## V. PROTECCIÓN

Tres manos de pintura. Sthay exposición á gases úsese pintura para puentes (véase **B** en las Especificaciones para puentes de acero); si no, úsense las pinturas corrientes para edificios.

# PUENTES COLGANTES

**Art. 1. Tabla de los datos que se requieren para calcular las cadenas ó cables principales de los puentes colgantes.** Original.

| Flexión * en partes del cordón. | Flexión en fracciones decimales del cordón. | Longitud de las cadenas principales entre las pilas de suspensión, en partes del cordón. | Tensión de todas las cadenas principales en una ú otra pila de suspensión, en partes del peso total del puente suspendido y de su carga. | Tensión en el centro de todas las cadenas principales, en partes del peso total del puente suspendido y de su carga. | Ángulo de la dirección de las cadenas en las pilas. | | Seno natural del ángulo de la dirección de las cadenas en las pilas. | Coseno natural del ángulo de la dirección de las cadenas en las pilas |
|---|---|---|---|---|---|---|---|---|
| | | | | | Gr. | Min. | | |
| 1-40 | .025 | 1.002 | 5.03 | 5.00 | 5 | 43 | .0995 | .9950 |
| 1-35 | .0286 | 1.002 | 4.40 | 4.37 | 6 | 31 | .1135 | .9935 |
| 1-30 | .0333 | 1.003 | 3.78 | 3.75 | 7 | 36 | .1322 | .9912 |
| 1-25 | .04 | 1.004 | 3.16 | 3.12 | 9 | 6 | .1580 | .9874 |
| 1-20 | .05 | 1.006 | 2.55 | 2.51 | 11 | 19 | .1961 | .9806 |
| 1-19 | .0526 | 1.007 | 2.43 | 2.38 | 11 | 53 | .2060 | .9786 |
| 1-18 | .0555 | 1.008 | 2.30 | 2.25 | 12 | 32 | .2169 | .9762 |
| 1-17 | .0588 | 1.009 | 2.18 | 2.12 | 13 | 14 | .2290 | .9734 |
| 1-16 | .0625 | 1.010 | 2.06 | 2.00 | 14 | 2 | .2425 | .9701 |
| 1-15 | .0667 | 1.012 | 1.94 | 1.87 | 14 | 55 | .2573 | .9663 |
| 1-14 | .0714 | 1.013 | 1.82 | 1.74 | 15 | 57 | .2747 | .9615 |
| 1-13 | .0769 | 1.016 | 1.70 | 1.62 | 17 | 6 | .2941 | .9558 |
| 1-12 | .0833 | 1.018 | 1.57 | 1.49 | 18 | 33 | .3180 | .9480 |
| 1-11 | .0919 | 1.022 | 1.46 | 1.37 | 19 | 59 | .3418 | .9398 |
| 1-10 | .1 | 1.026 | 1.35 | 1.25 | 21 | 48 | .3714 | .9285 |
| 1-9 | .1111 | 1.033 | 1.23 | 1.12 | 23 | 58 | .4062 | .9138 |
| 1/8 | .125 | 1.041 | 1.12 | 1.00 | 26 | 33 | .4471 | .8945 |
| 1-7 | .1429 | 1.053 | 1.01 | .881 | 29 | 45 | .4961 | .8726 |
| 3-20 | .15 | 1.058 | .972 | .833 | 30 | 58 | .5145 | .8574 |
| 1/6 | .1667 | 1.070 | .901 | .750 | 33 | 41 | .5547 | .8320 |
| 1-5 | .2 | 1.098 | .800 | .625 | 38 | 40 | .6247 | .7808 |
| | .225 | 1.122 | .747 | .555 | 42 | 0 | .6690 | .7433 |
| 1/4 | .25 | 1.149 | .707 | .500 | 45 | 00 | .7071 | .7071 |
| 3/5 | .3 | 1.205 | .651 | .417 | 50 | 12 | .7682 | .6401 |
| 1/3 | .3333 | 1.247 | .625 | .375 | 53 | 8 | .8000 | .6000 |
| 2/5 | .4 | 1.332 | .589 | .312 | 58 | 2 | .8483 | .5294 |
| 9-20 | .45 | 1.403 | .572 | .278 | 60 | 57 | .8742 | .4855 |
| 1/2 | .5 | 1.480 | .559 | .250 | 63 | 26 | .8944 | .4472 |

Estos cálculos están basados en la suposición de que la curva formada por las cadenas principales sea una parábola, lo cual no es estrictamente exacto. En un puente concluído, la curva está entre una parábola y una catenaria, y no es susceptible de una determinación rigurosa. **Puede evitarse algún trabajo al hacer los planos** de un puente colgante, acordándose que cuando la flexión no excede de $^1/_{10}$ de la luz más ó menos, puede usarse un segmento de un círculo en lugar de la curva verdadera, ya que las dos coinciden muy aproximadamente, y más todavía si la flexión es menor de $^1/_{10}$.

Las dimensiones tomadas en un segmento sirven para apreciar las cantidades del material necesario.

**La flexión* adoptada generalmente por los ingenieros** para grandes luces es de $^1/_{12}$ á $^1/_{15}$ de la luz. En luces pequeñas solamente se usa por lo general $^1/_{10}$. El puente será más fuerte, ó requiere menos área de cable, si la flexión es mayor; pero entonces son mayores sus undulaciones, y como éstas tienen tendencia á destruir el puente aflojando las juntas ó uniones y aumentando

* Relación de la flecha á la cuerda.

el « momento », debe tenerse especial cuidado de evitarlas cuanto sea posible. El modo más usado para conseguir esto es armando, entramando, las barandas ó parapetos del puente que para este fin pueden hacerse más altas, y de maderas más fuertes de lo que hubiese sido necesario. En luces grandes, puede sustituirse por vigas armadas ordinarias de puente, suficientemente altas para que puedan unirse con riostras por sus partes superiores, como en el puente del ferrocarril del Niágara, en el cual las vigas armadas tienen 5.49 m de alto y sostienen un ferrocarril de una sola vía en su parte superior, y una calzada ordinaria de 5.80 m de ancho en la parte inferior *.

Otro auxiliar muy importante se encuentra en los maderos longitudinales de piso de mucho espesor, unidos firmemente en sus extremidades. Ellos ayudan á distribuir el peso de las fuertes cargas que atraviesan el puente, entre varias barras de suspensión, y así sobre una extensión considerable de los cables principales, impidiendo la undulación que tendría lugar si la carga fuese concentrada solamente sobre dos barras de suspensión opuestas. En vista de esto, las vigas longitudinales de madera, debajo de los rieles del puente sobre el Niágara, son de 1.22 m de espesor. El mismo principio se aplica evidentemente también á los puentes de vigas armadas.

Otro modo de aliviar los cables principales es por medio de barras de hierro, ó cables metálicos, extendidos como *cy*, fig. 1, de los asientos en los puntos de suspensión *c*, *d*, oblicuamente al piso ó á alguna parte de la viga armada. En el puente del Niágara hay 64 de estos cables de 35 mm de diámetro; los más largos alcanzan á más de la cuarta parte de la abertura. Ellos transmiten gran parte de los esfuerzos debidos al peso del puente y de su carga directamente á las partes superiores de las torres, aliviando de este modo á los cables principales y disminuyendo la undulación. Las extremidades de ellos, *c* y *d*, no están sujetas á los cables, sino á las torres mismas.

**El mayor peligro proviene de la acción de vientos fuertes contra la parte inferior del piso,** sea suspendiendo la plataforma entera y dejándola caer repentinamente, ó sea comunicándole violentas undulaciones semejantes al movimiento de las olas. El puente en Wheeling, Ohio, de 308 m de abertura, construído por C. Ellet, Jr., fué destruído de este modo. Se dice que antes de caer hizo undulaciones verticales hasta de 6.10 m. No se tomaron precauciones contra las undulaciones, pues aunque tenía barandas ó parapetos armados, eran demasiado bajos y livianos para que sirvieran de mucho en una luz tan grande. Muchos otros puentes han sido destruídos ó deteriorados de igual manera. Si la altura de la calzada sobre el agua lo permite, puede adoptarse como medio de precaución, la colocación de tirantes ó vientos, que fijos en diferentes puntos, de la parte inferior de piso á lo largo de la abertura, vayan inclinados, á terminar en los estribos, donde pueden fijarse fuertemente. En el puente del ferrocarril del Niágara existen 56 de estos tirantes, hechos de cuerdas metálicas de 31 mm de diámetro, que se extienden diagonalmente desde el fondo del puente hasta las rocas que están debajo. Pero afean mucho el aspecto de una construcción.

El Sr. Brunel empleó también en algunos casos, para contrarrestar las undulaciones producidas por los fuertes vientos que chocan por debajo de la plataforma, *cables invertidos* con la convexidad *hacia arriba*, colocados por debajo del piso, con sus extremidades fijas fuertemente en los estribos á algunos metros más abajo de la plataforma, y conectados á ésta, tirando de ella hacia abajo.

**Art. 2.** El **ángulo** *adg* ó *aci*, fig. 1, que una tangente *dg* ó *ci* á la curva en ambos puntos de suspensión *c* ó *d*, forma con la línea *horizontal cd*, se llama **ángulo de suspensión del cable** ** en estos puntos; las extremidades *ch* y *dr* de los cables, llamadas **fiadores**, se colocan frecuentemente en línea recta desde los pilares ó

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* El autor cree haber sido el primero que sugirió la idea de la adición de vigas armadas muy altas y ligadas transversalmente, para puentes colgantes largos. Proyectamos un puente tal, de cuatro tramos de 305 m cada uno, y dos de 152.5 m, con cables de alambre, y vigas armadas de 6.1 m de alto. Este puente fué propuesto para cruzar el río Delaware en la calle « Market », en Filadelfia. El plano se expuso públicamente durante varios meses en el Instituto « Franklin » y en la Bolsa; y fué finalmente robado del salón de este edificio. El puente del Niágara, por el Sr. Roebling, con luz de 244 m y vigas armadas de 5.49 m de alto, no se principió sino en los últimos meses del 1852, ó más ó menos 18 meses después de haberse expuesto públicamente nuestro proyecto.

** *N. del T.*— Parece más sencilla esta definición que la del autor, y es la adoptada por E. Andrés en su *Manual del Const.de Puentes y Calzadas*, pág. 541, tabla de materias.

torres de suspensión hasta las placas de amarra, en cuyo caso los ángulos *ldr*, *ech*, formados por la línea horizontal *el*, y el cable mismo, se convierten en ángulos de dirección de los tiradores.

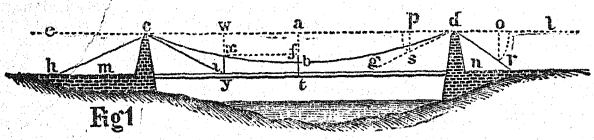



Fig 1

**Seno del ángulo de suspensión** *adg* $= \dfrac{\text{Doble de la flecha } ab.}{\sqrt{(\text{doble de la flecha})^2 + (\text{mitad de la cuerda})^2}}$

*Observación* 1.ª La dirección de la tangente *dg* ó *ci*, puede trazarse de esta manera: Prolónguese la línea *ab*, una cantidad igual al doble de su longitud, luego, las líneas trazadas por *d* y *c* hacia su extremidad inferior, serán las tangentes de la curva parabólica en los puntos de suspensión.

*Observación* 2.ª **Si la cuerda** *cd* **no es horizontal**, como sucede muchas veces, debe medirse el ángulo formado con una línea horizontal, trazada por cada punto de suspensión; los ángulos en este caso son desiguales, porque la altura de los pilares es desigual.

**Tensión de los cables principales juntos, en cualquiera de los pilares** *c* ó *d*, **fig. 1** $= \dfrac{\text{Mitad del peso suspendido en la parte libre de la luz y su carga}}{\text{Seno del ángulo de suspensión } adg.}$

ó $= \dfrac{\sqrt{(\frac{1}{2}\text{ abertura})^2 + (2\text{ flechas})^2}}{2\text{ flechas}} \times$ La mitad del peso total suspendido en la abertura y su carga.

**Tensión de todos los cables juntos en el medio** *b*, **de la luz, fig. 1.** $= \dfrac{\text{Mitad del peso total suspendido en la luz y de su carga} \times \text{Coseno del ángulo } adg}{\text{Seno del ángulo } adg.}$

**ó bien** $= \dfrac{\text{Mitad del peso total suspendido en la luz y su carga} \times \text{La mitad de la luz.}}{\text{Doble de la flecha}}$

La diferencia entre las tensiones del centro y las de los puntos de suspensión, es tan insignificante con la relación de la cuerda á la flecha generalmente adoptada en la práctica, de $^1/_{10}$ á $^1/_{15}$, más ó menos, que por lo común no se hace caso de ella; en tanto que lo que se gana en el peso del metal estaría más que compensado por el aumento de trabajo en la fábrica para reducir gradualmente las dimensiones de los cables de los extremos de suspensión hacia el centro, y en la preparación de los ajustes de las partes de diferentes tamaños. Esta reducción, sin embargo, se ha hecho en algunos puentes grandes en los cables de hierro forjado, pero nunca en los de alambre.

**Art. 2 A.** Como algunas veces es conveniente formarse una idea ligera en un momento dado, de las dimensiones de los cables de un puente, damos la regla siguiente para hallar aproximadamente el área metálica neta (*solid iron*) de alambre que se requiere en centímetros cuad para sostener, bajo un coeficiente 3 * de seguridad, el peso del puente mismo, junto con una carga extraña de 3.34 toneladas por m corrido de abertura, lo que corresponde á 488 kg por m cuadrado de plataforma de un ancho útil de 8.23 m. Esto basta para una calzada doble de

* No debe creerse que estamos por un coeficiente 3 de seguridad para 488 kg por m cuad además del peso del puente para todos los casos. Si creemos que aquel límite sea casi suficiente para un puente colgante bien calculado de alambres, para tráfico ordinario, pero para un puente de ferrocarril importante, preferiríamos adoptar (según la posición, exposición, etc.) un coeficiente de seguridad por lo menos de 4 á 6 para la carga máxima posible, más el peso del puente. Un tren de carros opone una gran superficie á la acción de los vientos laterales; y los trenes tienen que circular durante tempestades fuertes lo mismo que en tiempo de calma; pero en un puente grande descubierto de tráfico común, no se aglomera la gente durante una fuerte tempestad.

vía carretera con dos aceras. Se supone una flecha igual á $^1/_{12}$ de la luz; y el alambre de una resistencia máxima de 5.58 toneladas por cm cuadrado neto.

**Para luces de 100 pies (30.5 m) ó más**, tenemos la siguiente regla: Multiplíquese la luz en pies por su raíz cuadrada; divídase el producto por 100, y al cociente agréguese la raíz cuadrada de la luz. O expresando esto en una fórmula, tenemos:

$$\text{Area metálica neta de todos los cables, en pulgadas cuadradas, para luces de más de 100 pies} = \frac{\text{luz} \times \text{raíz cuad de la luz}}{100} + \text{raíz cuad de la luz.}$$

(*N. del T.* — Damos la fórmula que sigue como equivalente á la anterior, pero en sistema métrico. Llamando L la luz en *metros*, se tendrá:

$$\text{Area metálica neta de todos los cables en cm cuadrados} = 11{,}674\ \sqrt{L}\ (1+.0328\ L).$$

Para luces menores de 100 pies (30.5 m) tómese el área en proporción á la de 100 pies (30.5 m).

Adoptando una flecha de $^1/_{10}$, en lugar de $^1/_{12}$, el área de los cables puede reducirse casi $^1/_7$ parte.

**La tabla siguiente se ha hecho de acuerdo con esta regla.** La tercera columna da el área de todos los cables de alambre juntos, incluyendo los espacios vacíos.

(*N. del T.* — Hemos convertido la del autor al sistema métrico.) (Original.)

| Luz en metros. | Área metálica neta de todos los cables. | Area total de los cables terminados. | Luz en metros. | Área metálica neta de todos los cables. | Área total de los cables terminados. | Luz en metros. | Área metálica neta de todos los cables. | Área total de los cables terminados. |
|---|---|---|---|---|---|---|---|---|
| | cm cuad. | cm cuad. | | cm cuad. | cm cuad. | | cm cuad. | cm cuad. |
| 304.800 | 2245.156 | 2877.409 | 121.920 | 645.159 | 825.803 | 45.720 | 197.418 | 252.902 |
| 274.320 | 1935.480 | 2483.862 | 106.680 | 541.933 | 696.772 | 38.100 | 162.580 | 206.486 |
| 243.840 | 1638.704 | 2103.218 | 91.440 | 445.159 | 574.491 | 30.480 | 129.031 | 165.160 |
| 213.360 | 1367.738 | 1754.832 | 76.200 | 354.837 | 458.063 | 22.860 | 96.774 | 123.870 |
| 182.880 | 1103.222 | 1412.898 | 60.960 | 270.967 | 348.386 | 15.240 | 64.516 | 82.580 |
| 152.400 | 864.513 | 1109.673 | 53.340 | 234.838 | 301.289 | 7.620 | 32.258 | 41.290 |

Conociendo el área de todos los cables podemos fácilmente hallar el diámetro de cada uno. Así, supongamos que para una luz de 152.4 m, queremos aplicar 4 cables, el área de cada uno será 1,109.6÷4=277,4 cm cuad, y en la tabla de círculos, pág. 181, vemos que el diámetro correspondiente á un área de 277.59 es 18.8 (centímetros en este caso).

Las áreas arriba dadas se suponen calculadas tomando en cuenta el aumento en peso debido á la altura de la viga armada, y otras adiciones necesarias para defender el puente contra la acción de los vientos fuertes y contra las vibraciones indebidas ocasionadas por el paso de las cargas.

Cuando no se tengan en cuenta estas consideraciones, y se suponga una carga máxima más pequeña, las descripciones siguientes de los puentes de Wheeling y Freyburg indicarán las reducciones que pueden hacerse.

Es de gran utilidad para reducir las undulaciones dotar al puente de suficiente peso.

Al tablero del puente debe dársele una curvatura hacia arriba, digamos de 1/200 de la luz.

**Art. 3. Tensión de los fiadores *ch* y *dr*, fig. 1, y esfuerzos que obran sobre los pilares** ó torres. Si el ángulo de suspensión *adg* y el ángulo *ldr*, formado por los fiadores y la horizontal, son iguales, la tensión de los fiadores será igual á la de los cables principales en la parte superior de los pilares, y la presión que obre sobre éstos será vertical; pero si los dos ángulos son desiguales, estas tensiones y presiones dependerán, en gran parte, del modo como estén sujetos los cables á los pilares, ó del modo como estén colocados sobre la parte superior de ellos.

**Art. 4. En las figs 2, 3 y 4**, los pilares ó torres *dnm* se suponen inmóviles, y los cables *kdu*, que pasan por encima de ellos, descansan inmediatamente sobre rodillos horizontales, que *no tienen otro movimiento que el de girar alrededor de sus*

*ejes horizontales; y el marco, al cual están sujetos, está firmemente atornillado en la parte superior del pilar.* Los cables se deslizan sobre estos rodillos cuando las variaciones de carga ó de temperatura producen cambios en sus direcciones.

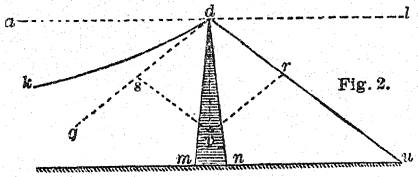


Fig. 2.

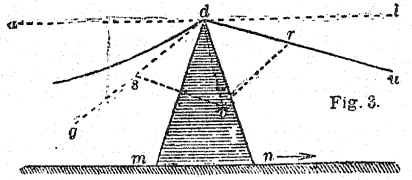


Fig. 3.

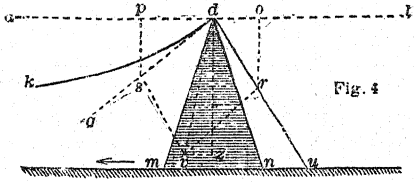


Fig. 4

En este caso **la tensión del fiador** es igual á la del cable. Véase Máquina funicular.

Para hallar la dirección é intensidad **de la presión que obra en el pilar**; por *d*, figs. 2, 3 y 4, trácese *ds* y *dr*, ambas iguales, por escala, á la tensión del cable que actúa en *d* (en toneladas), y por *s* y *r* fórmese el paralelogramo *dsvr* y su diagonal *dv*. Esta diagonal *dv* da la *dirección* é intensidad de la presión que obra en el pilar.

Cuando, como en la fig. 2, los ángulos *adg* y *ldu* son iguales, la presión *dr* será vertical, é igual al peso total del puente en la luz y su carga.

Si, como en las figs. 3 y 4, dichos ángulos *adg* y *ldu* son desiguales, la presión *dv* no será vertical, sino inclinada desde *d* hacia el ángulo más pequeño.

Si *adg* excede á *ldu*, fig. 3, la presión *dv* es *menor* que el peso total en la abertura y su carga.

Cuando, como en la fig. 4, *ldu* excede á *agd*, la presión *dv* es *mayor* que el peso total de la abertura y su carga.

Si suponemos que se usan en cada caso pilares simétricos *dnm*, la base *mn* del de la fig. 2 puede ser más angosta que la de las otras dos figuras, porque, siendo la dirección de *dv* vertical, la presión no tiene tendencia á volcar el pilar. En la fig. 2, la mampostería debe ponerse en hiladas horizontales como de costumbre, para que sus juntas estén en ángulo recto con la presión que obra sobre ellas.

Pero, en las figs. 3 y 4, si las bases se hicieran tan angostas como en la fig. 2, las líneas *dv*, dirección de la presión, caería, afuera de las bases, y por consiguiente los pilares estarían expuestos á volcarse. También las piedras de la mampostería, colocadas en hiladas horizontales, tendrían una tendencia á deslizarse unas sobre otras. Para evitar esto, las hiladas de piedra deben colocarse en ángulo recto á *dv*.

En la fig. 3, la oblicuidad de la presión tiende á que resbale la base del pilar *hacia afuera*, como se indica con la flecha; pero en la fig. 4, *hacia adentro*. Esta tendencia está producida por la componente horizontal de la fuerza *dv*; y su intensidad puede

hallarse en cualquiera de las figuras, así: Trácese una línea vertical de *d* hacia abajo como en la fig. 4, y desde *v* una horizontal, que la encuentre en *z*, luego *vz*, medida en la misma escala, dará esta fuerza horizontal, y *dz* será la componente vertical de la presión *dv*. El efecto de la presión *dr* sobre el pilar, es exactamente el mismo que produciría sobre él una fuerza vertical igual á *dz*, y otra horizontal igual á *vz*, actuando á un mismo tiempo, como se explica en la Composición y Descomposición de las Fuerzas.

Si en cualquiera de las figuras trazamos líneas verticales *sp* y *ro*, fig. 4, entonces *do*, medida por la escala anterior, dará las toneladas de tensión horizontal, y *ro* la presión vertical, producida en el pilar por el fiador, y *dp* y *ps*, darán, del mismo modo, las fuerzas correspondientes producidas por el cable. Si sumamos á *ro* y *ps*, se encontrará dicha suma igual á *dz*, y si restamos *do* de *pd*, la diferencia será igual á *vz*. Es esta diferencia solamente la que tiende á hacer resbalar ó á volcar el pilar; las otras partes de *do* y *pd* se neutralizan á este respecto mutuamente.

Los esfuerzos antedichos pueden calcularse así:

**Esfuerzo de tracción** (*pull*) **horizontal hacia adentro ejercido por el cable principal**=Tensión × Coseno de *adg*.

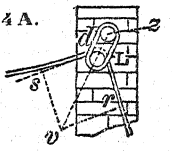
**Esfuerzo de tracción horizontal hacia afuera ejercido por el fiador** =Tensión × Coseno de *ldu*.

**Presión vertical ejercida por el cable**=Tensión × Seno de *adg*.

**Presión vertical ejercida por el fiador**=Tensión × Seno de *ldu*.

**Art. 5.** Si los cables pasan libremente por encima de un pasador suelto *d*, fig. 4 A, sostenido por el eslabón L, que cuelga del pasador fijo *z*, y capaz de moverse libremente en ambos apoyos, la tensión del fiador será, como antes, igual á la del cable, y la dirección é intensidad del esfuerzo que obra en el pilar se halla del mismo modo que en las figs. 2, 3 y 4; es decir, trácese *ds* y *dr*, ambas iguales á la tensión, y constrúyase el paralelogramo *dsvr*. Luego *dv* dará la intensidad y dirección del esfuerzo que obra en los pilares. Esta última será por supuesto transmitida por los pasadores y el elsabón. La intensidad de la tensión sobre el eslabón la dará la longitud de *dv*, y el eslabón (libre para moverse) estará en la misma línea de la tensión. El esfuerzo cortante que se ejerce sobre cada pasador está dado también por *dv*.

Fig. 4 A.



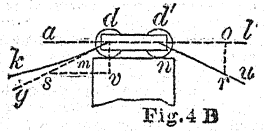




Fig. 4 B

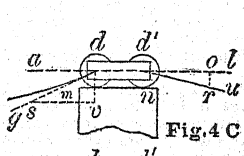



Fig. 4 C

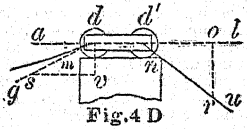



Fig. 4 D

**Art. 6.** Pero si las extremidades del cable y del fiador, figs. 4 B, 4 C y 4 D, en la parte superior del pilar, se fijan á un carrito apoyado sobre rodillos en una plataforma lisa colocada en la parte superior del pilar, y estando los ejes de los rodillos fijos á la carretilla, entonces el esfuerzo del fiador no será el mismo que el del cable, á menos que los ángulos *adg* y *ldu* sean iguales, como en la fig. 4 B.

Si *adg* excede á *ld'u*, como en la fig. 4 C, el esfuerzo que obra sobre el fiador será menor que el que actúa sobre el cable, y viceversa, fig. 4 D.

Pero, en uno ú otro caso, si la parte superior es horizontal, como lo es generalmente, las componentes horizontales de los esfuerzos de los cables y los fiadores serán iguales y contrarias, y de consiguiente se destruyen sus efectos, y por tanto no obrará ningún esfuerzo de tensión horizontal ú oblicuo sobre el pilar; es decir, que el esfuerzo que obra en él será vertical.

**Para encontrar la intensidad de la tensión del** fiador, y de la **presión que obra en el pilar**, tómese sobre *dg*, en cualquiera de las figs. 4 B, 4 C, ó 4 D, en escala, *ds* igual á la tensión del cable en *d*. Trácese *dv* perpendicular á la super-

ficie *mn* sobre la cual descansan los rodillos. Suponemos que *mn* es horizontal como lo es generalmente; *pero no necesariamente*, y por tanto que *dv* es vertical. Hágase *sv* horizontal ó paralela á *mn* *.

Entonces *sv* dará el esfuerzo de tensión horizontal del cable sobre el carrito, y *dv* dará la presión vertical del rodillo *d* sobre el pilar (á cuya presión tiene que agregarse todavía la del rodillo *d'*). En *d'*, trácese *d'o* horizontal é igual á *sv*, y trácese *ro* vertical. Luego *d'r* dará la intensidad del esfuerzo de tensión del fiador y *ro* la presión vertical del rodillo *d'* que actúa sobre el pilar, que debe agregarse á *dv* para obtener la presión vertical *total*.

También pueden calcularse los diversos esfuerzos así:

**Esfuerzo de tensión horizontal *sv* ó *d'o* en la parte superior del pilar** = Esfuerzo de tensión horizontal en el centro de la abertura = Tensión *ds* en el cable en *d* × Coseno de *adg*

**Esfuerzo de tensión *d'r* en el fiador** = esfuerzo de tensión horizontal *sv* ó *d'o* en la parte superior del pilar ó en el centro de la abertura ÷ coseno de *ld'u*.

**Presión sobre el pilar**, perpendicular á la superficie en que descansan los rodillos $= dv + ro = \left(\begin{matrix}\text{Tensión } ds \text{ sobre el cable en } d\end{matrix} \times \text{seno de } adg\right) + \left(\begin{matrix}\text{Tensión } d'r \text{ sobre el fiador}\end{matrix} \times \text{seno de } ld'u\right)$

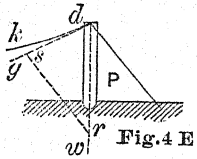


Fig. 4 E

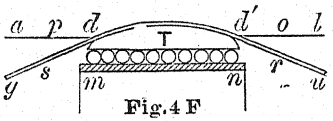


Fig. 4 F

**Art. 7.** Cuando, como alguna vez sucede en los puentes livianos, los pilares son postes, P, fig. 4 E, de madera ó de hierro, con los cables y fiadores sujetos firmemente en sus partes superiores; desde *d* trácese *ds*, igual por escala, á la tensión del cable principal en *d*, y *dw* hacia el pie del poste. Desde *s* trácese *sr* paralela al fiador, y que encuentre á *dw* en *r*. Entonces *sr* dará el esfuerzo en el fiador, y *dr* dará la intensidad y dirección de la presión sobre el poste.

**Art. 8.** Del mismo modo que en el puente del Niágara, los cables descansan á menudo simplemente sobre carritos movibles T, fig. 4 F, curvos aquéllos en su parte superior para evitar dobleces repentinos en los cables y apoyados sobre rodillos sueltos, colocados en una plancha gruesa de hierro, horizontal, remachada á la parte superior del pilar; los rodillos pueden moverse con libertad horizontalmente. En estos casos, los ángulos *adg* y *ld'u* permanecen siempre iguales; de este modo los esfuerzos de tensión *ds* y *d'r* serán iguales, como lo son también sus componentes horizontales *pd* y *d'o*; y las presiones que obran sobre el pilar son verticales; y si variaciones de temperatura o de cargas producen pequeñas variaciones en los ángulos *adg* y *ld'u*, el carrito de rodillos se moverá (por causa de la desigualdad producida en las componentes horizontales) lo suficiente para restablecer la igualdad de los ángulos y las componentes horizontales, y en consecuencia la presión sobre el pilar será siempre vertical.

**Art. 9. Para hallar, aproximadamente, la longitud de un cable** *cbd*, fig. 1, conociendo la abertura *cd*, y la flecha central *ab*. Véase la tabla que precede, art. 1.

La mitad de la longitud del cable $= \sqrt{1\,^1/_3\ (\text{flecha}^2) + (\tfrac{1}{2}\ \text{cordón})^2}$.

En el puente de Menai, la cuerda *cd* tiene 176.86 m (579.874 pies), y la flecha (43 pies) 13.11 m.

Según la fórmula anterior, la extensión total es de 179.4 m (588.3 pies). Según la medida efectiva, la cadena tiene exactamente 180 m (590 pies).

La regla aproximada que se da más abajo da 179.75 m.

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* Las líneas *al* y *sv* deben trazarse *paralelas á la superficie*, mn, *sobre la cual descansa el carrito con sus rodillos, aunque dicha superficie sea horizontal, ó inclinada.*

*Nota.* Las longitudes obtenidas por esta regla son aproximadas solamente; porque el cálculo está basado sobre la suposición de que la curva de las cadenas sea parabólica, mientras que en realidad la curva de un puente concluído, no es precisamente una parábola, ni una catenaria, sino una curva intermedia entre las dos.

La regla sencilla que sigue, del autor, es tan aproximada como la molesta regla que antecede, cuando la flecha no es mayor de $^1/_{12}$ parte de la cuerda, ó luz, como sucede generalmente.

Longitud del cable ó cadena cuando la flecha no excede de $^1/_{12}$ de la abertura = cuerda + .23 de la flecha.

**Art. 10. Encontrar, aproximadamente, la longitud de las péndolas *x*, *y*, etc., fig. 1, suponiendo la curva una parábola.**

Sea *x*, fig. 1, un punto cualquiera de la curva; y trácese *xw* perpendicular á la cuerda *cd*; y *xf* perpendicular á *ab*; tenemos en cualquira parábola, que $ac^2 : aw^2 = ab : bf$. Y *bf* hallado de este modo sumado con *bt* (que se supone ya conocida, por el pendolón) da á *xy*, longitud de la péndola que se requiere en el punto *x*, y así en cualquier otro punto.

**Si se resta á *bf*, así encontrada, de la flecha del centro *ab*, el residuo será *wx*.** Asimismo puede hallarse cualquiera otra.

En la regla que antecede, se supone que el tablero del puente es recto; pero generalmente está algo elevado en el centro, y en este caso las péndolas deben calcularse primeramente como si el tablero fuese recto, y después deben hacerse las deducciones necesarias. Cuando se eleva según dos líneas rectas que se encuentran en el centro, el modo de hacerlo salta á la vista. Cuando se emplea un arco de círculo, sus ordenadas deben calcularse y deducirse de las longitudes obtenidas por esta regla. O habiendo trazado la curva por el método usado para trazar una parábola, las dimensiones pueden tomarse aproximadamente por la escala. Los ajustes, para obtener las longitudes exactas, deben hacerse durante la construcción del puente aplicando tuercas en los tornillos de los extremos inferiores. Por consiguiente basta sólo, al principio, hacer las barras de longitud suficiente.

Las torres, ó pilares, que sostienen las cadenas ó cables, admiten variaciones infinitas. Según las circunstancias, pueden componerse de una simple pieza de madera vertical; ó de un pilar de hierro forjado ó fundido, ó de dos ó más de éstos, puestos oblicuamente con piezas de conexión ó sin ellas, como los inclinados de una pila armada en puentes de hierro; ó de planchas de hierro fundido (con cualquiera ornamentación), como los frentes de las casas de hierro; ó de mampostería de piedra, de ladrillo, de concreto ó de cualquiera de estos materiales combinados.

**Cada una de las péndolas** que mantienen suspendido de los cables, al piso ó tablero del puente, necesita sólo la resistencia suficiente para sostener, con seguridad, la carga máxima que pueda actuar en el intervalo formado por los dos espacios comprendidos entre cada una de ellas y el centro de la distancia á la péndola más próxima incluyendo el peso de la plataforma, etc., en el intervalo expresado.

Al fijar los fiadores en tierra es necesario dotar á las planchas de amarra, de una resistencia suficiente para que soporten con seguridad el esfuerzo de tensión igual al que obra en el fiador.

Con respecto á la fijación de los cables bajo la superficie del suelo, las rocas naturales de carácter firme constituyen el material más á propósito que pueda presentarse. Donde no exista dicho material, hay que hacer serios gastos de mampostería en aberturas grandes para obtener el peso capaz de resistir la tensión de los cables. Las figs. 4½ dan una idea de los métodos adoptados frecuentemente. Para un puente muy pequeño destinado al tráfico á pie, por ejemplo, los fiadores pueden fijarse simplemente á piedras grandes *t*, fig. A, enterradas á una profundidad suficiente. Si la tensión es demasiado grande para tan sencilla precaución, pueden agregarse bloques de mampostería *mm* que cubran el fiador. Una buena cubierta con el mortero ó cemento de la mampostería constituye una capa protectora del hierro.

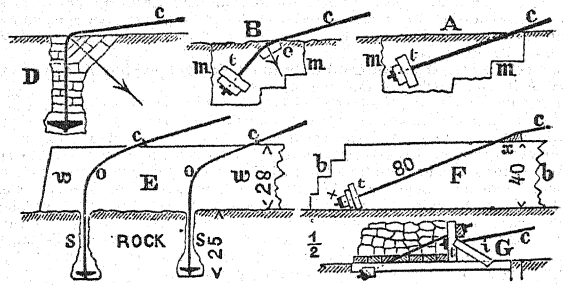
Para evitar la necesidad de prolongar los fiadores á gran distancia debajo de la tierra, se los encorva generalmente cerca del punto en donde penetran bajo la superficie del suelo, como se indica en B, D y E, con el fin de llegar así más pronto á la profundidad necesaria.

Esta curva, sin embargo, produce un nuevo esfuerzo en la dirección indicada por las flechas en las figs. B y D. La naturaleza de este esfuerzo, y el modo de hallar su intensidad (conociendo la tensión del fiador), es muy sencillo, y está claramente explicado en el capítulo Máquina funicular. La mampostería debe estar dispuesta como para resistir este esfuerzo así como el de tensión directo del fiador. En vista de

esto, los bloques de piedra, sobre los cuales descansa la extremidad doblada del fiador, debían estar colocados como se ven en la fig. D, ó como el bloque solo en B. Algunas veces se hace el doblez sobre un asiento ó pedestal de hierro fundido *x*, fig. F, fijo firmemente á la mampostería por medio de pernos.

La fig. E muestra la disposición adoptada en el puente de ferrocarril del Niágara, de 250.5 m de luz. Los fiadores de alambre terminan en *cc*, y de este punto hacia abajo hasta el anclaje, están formados de cadenas pesadas, cuyos eslabones están compuestos (alternativamente) de 7 ú 8 barras de ojo paralelas, de hierro chato, por las cuales pasan pernos.

Cada una de estas 7 barras es de 36 mm de grueso, por 178 mm de ancho cerca de las partes más bajas de la cadena; pero aumentan gradualmente de allí hacia arriba, hasta que en *cc*, donde se unen con el alambre, el área de la sección de cada eslabón sea de 600 cm cuad Estos fiadores de cadena atraviesan en curva los muros macizos




de acceso (de 8.54 m de alto) y descienden verticalmente por una cavidad *ss* á 7.6 m debajo de la roca sólida. Aquí pasan por las planchas de amarra de hierro fundido, á las cuales están fijas por un perno de 9 cm de diámetro. Las planchas de amarra son de 2 metros en cuadro, y 63 mm de grueso, con excepción de un espacio de 50 por 65 cm más ó menos, en el centro, por donde pasan las cadenas y donde tienen un espesor de 305 mm.

Esta parte gruesa está provista de una abertura para cada barra de que se compone el último eslabón. Las cavidades *ss*, tienen los lados ásperos como si fueran hechos con barrenos de pólvora y tienen de .91 por 2.10 m de sección transversal, excepto en el fondo, donde tienen 2.44 m en cuadro.

Están llenos completamente de mampostería de cemento, en capas bien dispuestas y bien en contacto con las caras de la cavidad; todas las juntas bien cogidas y macizadas, y cubriendo perfectamente, de este modo, las cadenas por todas partes; la mampostería de los muros *ww*, se eleva 8.54 m sobre la superficie del terreno con 1.83 de espesor en la parte superior y 3.05 m en su base sobre la roca natural.

En D, fig. 4½, está indicado el método que se usa en la mayor parte de los casos, para puentes de cualquiera abertura. La profundidad del pozo y el área de la sección transversal, y en consecuencia, la cantidad de mampostería, depende principalmente de si el pozo se hace en roca ó en tierra.

Si se hace en roca firme y las caras de la excavación son irregulares y se hace penetrar bien la mampostería en las irregularidades, puede confiarse en que contribuirá á que el peso de aquélla resista á la tensión de los fiadores.

La tierra también contribuye bastante á este respecto.

F es la disposición adoptada en el puente de Chelsea, de 101.5 m de luz, sobre el Támesis, en Londres. El espacio comprendido entre un muro *bb* y el muro opuesto es de 13.72 m y está construído sólidamente de ladrillo y concreto, excepto un pasaje interior de 1.22 m de ancho y 1.52 m de alto, á lo largo del fiador, y una cámara pequeña detrás de las planchas de anclaje. La mampostería descansa principalmente sobre estacas.

La disposición adoptada por el Sr. Brunel, en el puente de Charing Cross, Londres, es muy semejante. También aquí descansa todo el estribo sobre estacas; tiene 12.2 m de alto y 9.1 m de grueso, es macizo con excepción de un pasadizo angosto á lo largo de las cadenas. Los fiadores se prolongan 18 m en la mampostería. Luz 206.18 m, flecha 15.2 m.

G es solamente una idea general, la cual, con modificaciones variadas, puede aplicarse á puentes pequeños ó provisionales, y aun á puentes permanentes; con respecto al número de piezas, *i*, *t*, etc., pueden aumentarse hasta donde fuera necesario, y hacerse de hierro ó piedra en lugar de madera.

**Á fin de que los fiadores puedan ser accesibles**, se pasan frecuentemente por aberturas dejadas al efecto en la mampostería. De esta manera las masas *mm*, de mampostería en A y B, fig. 4½, en vez de ser sólidas pueden consistir en dos muros paralelos entre los cuales puede pasar el fiador; y las piedras ó planchas de anclaje, se extenderán á través de los espacios vacíos entre los muros, con sus apoyos sobre los extremos de los muros. Se puede suponer que en D, E y F, el cable esté sólidamente rodeado de la mampostería y encajado en ella, ó rodeado de un pasaje cilíndrico semejante á una cañería, para que sea siempre accesible.

Se debe excluir cuidadosamente toda piedra blanda desmenuzable en aquellas partes del anclaje que se encuentren más directamente opuestas al tiro del fiador.

Si no se pueden conseguir bloques de piedra suficientemente grandes para obtener una buena trabazón, se pueden emplear ventajosamente con este objeto vigas pesadas de hierro en T, en I, ó simples carriles ó barras de hierro.

Las masas de mampostería deben cimentarse á tal profundidad que no puedan resbalar porque ceda ó se derrumbe el terreno.

Para mayor seguridad sería conveniente descartar el efecto de la fricción sin disminuir la tensión del fiador y considerar aquella tensión como si continuara uniforme por todo el fiador hasta su extremo, aun cuando el fiador fuese curvo y estuviese empotrado en la mampostería, como en E, fig. 4½.

**Los parapetos laterales** del puente deben ser altos y fuertes, para que obren como armaduras de refuerzo, y no se debe limitar su empleo á meras barandas ó protección. Como regla aproximada, su altura en m debe ser $= 0.276 \sqrt{\text{de la luz}}$, con tal que la altura no sea menor que la requerida para un pasamano. Los parapetos deben construirse bien fuertes, teniendo especial esmero en reforzar sus empates, pues éstos están expuestos, por las undulaciones y movimientos laterales del puente, á violentos esfuerzos en todas direcciones.

# ROBLONES Y ROBLONADURA

Los **pesos** que se indican en la **tabla que sigue** comprenden, por supuesto, la cabeza del remache; pero la **longitud**, como de costumbre, se toma «por debajo de ella», ó mejor dicho, es la longitud del fuste solamente. En la práctica pueden resultar discrepancias, en el peso, de 5 ó 6 por ciento.

(*N. del T.* — 1.ª Para convertir pulgs en mm multipliquese el número de aquellas por 25,399 ó conviértanse por medio de la tabla que va al pie.

Para convertir libs en kg multipliquese por .453.)

| Long del fuste en pulgs. | Diámetro del remache ó roblón en pulgs. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 3/8 | 1/2 | 5/8 | 3/4 | 7/8 | 1 | 1 1/8 | 1 1/4 |
| | Peso de 100 remaches, en lbs. | | | | | | | |
| 1/2 | 3.0 | 8.7 | ..... | ..... | ..... | ..... | ..... | ... |
| 3/4 | 3.8 | 9.9 | 17.3 | ..... | ..... | ..... | ..... | ... |
| 1 | 4.6 | 11.2 | 19.4 | 25.6 | 38.9 | ..... | ..... | ... |
| 1 1/4 | 5.4 | 12.6 | 21.5 | 28.7 | 43.1 | 65.3 | 91.5 | 123 |
| 1 1/2 | 6.2 | 13.9 | 23.7 | 31.8 | 47.3 | 70.7 | 98.4 | 133 |
| 1 3/4 | 6.9 | 15.3 | 25.8 | 34.9 | 51.4 | 76 2 | 105 | 142 |
| 2 | 7.7 | 16.6 | 27.9 | 37.9 | 55.6 | 81.6 | 112 | 150 |
| 2 1/4 | 8.5 | 18.0 | 30.0 | 41.0 | 59.8 | 87.1 | 119 | 159 |
| 2 1/2 | 9.2 | 19.4 | 32.2 | 44.1 | 64.0 | 92.5 | 126 | 167 |
| 2 3/4 | 10.0 | 20.7 | 34.3 | 47.1 | 68.1 | 98.0 | 133 | 176 |
| 3 | 10.8 | 22.1 | 36.4 | 50.2 | 71.3 | 103 | 140 | 184 |
| 3 1/4 | 11.5 | 23.5 | 38.6 | 53.3 | 76.5 | 109 | 147 | 193 |
| 3 1/2 | 12.3 | 24.8 | 40.7 | 56.4 | 80.7 | 114 | 154 | 201 |
| 3 3/4 | 13.1 | 26.2 | 42.8 | 59.4 | 84.8 | 120 | 161 | 210 |
| 4 | 13.8 | 27.5 | 45.0 | 62.5 | 89.0 | 125 | 167 | 218 |
| 4 1/4 | 14.6 | 28.9 | 47.1 | 65.6 | 93.2 | 131 | 174 | 227 |
| 4 1/2 | 15.4 | 30.3 | 49.2 | 68.6 | 97.4 | 136 | 181 | 236 |
| 4 3/4 | 16.2 | 31.6 | 51.4 | 71.7 | 102 | 142 | 188 | 244 |
| 5 | 16.9 | 33.0 | 53.5 | 74.8 | 106 | 147 | 195 | 253 |
| 5 1/4 | 17.7 | 34.4 | 55.6 | 77.8 | 110 | 153 | 202 | 261 |
| 5 1/2 | 18.4 | 35.7 | 57.7 | 80.9 | 114 | 158 | 209 | 270 |
| 5 3/4 | 19.2 | 37.1 | 59.9 | 84.0 | 118 | 163 | 216 | 278 |
| 6 | 20.0 | 38.5 | 62.0 | 87.0 | 122 | 169 | 223 | 287 |
| 6 1/2 | 21.5 | 41.2 | 66.3 | 93.2 | 131 | 180 | 236 | 304 |
| 7 | 23.0 | 43.9 | 70.5 | 99.3 | 139 | 191 | 250 | 321 |
| 7 1/2 | 24.6 | 46.6 | 74.8 | 106 | 147 | 202 | 264 | 338 |
| 8 | 26.1 | 49.4 | 79.0 | 112 | 156 | 213 | 278 | 355 |
| 9 | 29.2 | 54.8 | 87.6 | 124 | 173 | 234 | 306 | 389 |
| 10 | 32.2 | 60.3 | 96.1 | 136 | 189 | 256 | 333 | 423 |
| 11 | 35.3 | 65.7 | 105 | 148 | 206 | 278 | 361 | 457 |
| 12 | 38.4 | 71.2 | 113 | 161 | 223 | 300 | 388 | 491 |

(*N. del T.* — 2.ª A mayor abundamiento, damos los equivalentes de los largos de los fustes en cm. Para los diámetros de los remaches, se usa mucho la pulg como unidad, por eso creemos inútil agregar dichas equivalencias.)

| Pulg. | 1/2 | 3/4 | 1 | 1 1/4 | 1 1/2 | 1 3/4 | 2 | 2 1/4 | 2 1/2 | 2 3/4 | 3 | 3 1/4 | 3 1/2 | 3 3/4 | 4 | 4 1/4 | 4 1/2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| mm. | 12.7 | 19 | 25.4 | 31.75 | 38.1 | 44.4 | 50.8 | 57.15 | 63.5 | 69.8 | 76.2 | 82.55 | 88.9 | 95.2 | 101.6 | 107.95 | 114.3 |

| Pulg. | 4 3/4 | 5 | 5 1/4 | 5 1/2 | 5 3/4 | 6 | 6 1/2 | 7 | 7 1/2 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| mm. | 120.6 | 127.0 | 133.35 | 139.7 | 146.0 | 152.4 | 158.75 | 177.8 | 184.15 | 203.2 | 228.6 | 254.0 | 279.4 | 304.8 |

**El diám de los roblones** para trabajos de puentes es de ½ á 1 pulg (de 12½ á 25 mm); comúnmente de $^{5}/_{8}$ á $^{3}/_{4}$ (de 15.87 á 19.05 mm); y para planchas de más de .5 de pulg (12½ mm) de grueso, el diám es como 1.5 veces el espesor; para más delgadas como el doble; pero estas proporciones no son muy exactas. **La forma común de los remaches** que se ofrecen en venta se indica en R, fig. 3, una **cabeza** y el **fuste** en una sola pieza; y S muestra el mismo remache cuando después de habérselo calentado hasta el blanco se ha insertado en el hueco formándole una nueva cabeza (cónica) por el remachamiento rápido á medida que enfría. **Cuando tienen más de 6 pulgs de largo**, se los enfría cerca del medio antes de introducirlos, si no, su contracción al enfriarse rompería las cabezas. Las cabezas hemisféricas que á menudo se ven, llamadas **cabezas redondas**, se hacen en máquina. **Las dos cabezas solas requieren** una cantidad de hierro como de 3 diámetros en longitud de la espiga. **Longitud de la cabeza**=como 1 diámetro de espiga; y su **ancho** como 2 diámetros de espiga.

**Roblonadura ó remache de juntas impermeables al vapor y al agua.**

**Las juntas ó empates de calderas** y cisternas impermeables se hacen generalmente como por la tabla siguiente de Fairbairn; y se construyen como lo indica la fig. 1 ó la fig. 2. La fig. 1 se denomina **junta ó empalme de roblonadura sencilla**, y la fig. 2, de **recubrimiento y roblonadura doble**. La dist *aa* ó *cc*, es la superficie recubierta.

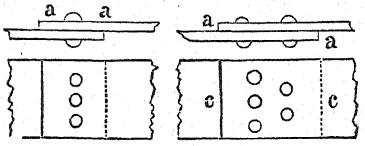



**Fig. 1.** **Fig. 2.**

Fairbairn considera la resistencia de la junta de roblonadura sencilla como de .56, y la de doble roblonadura, como .7 de la que corresponde á una placa sin agujerear, cuando ambas juntas están hechas con las proporciones indicadas en las tablas siguientes, de dicho autor.

Pero algunos experimentadores últimos consideran á .5 y .6 como término medio más próximamente exacto. Los experimentos hechos sobre esta materia son enteramente contradictorios, y es claro que ninguna serie de proporciones pueden precisamente convenir á todas las diferentes clases de placas de remaches. Con buenas clases de ambos hay razón para confiar en que .5 y .6 (ó como una séptima parte menos de lo que supone Fairbairn) sea una resistencia segura en la práctica. Esta proporción comprende el **rozamiento**, art. 4, sin el cual sería *como* .4 y .5.

**Tabla de Fairbairn que suministra las dimensiones proporcionadas de las roblonaduras de juntas solapadas impermeables al vapor y al agua.**

(*N. del T.* — Hemos convertido la tabla del autor al sistema métrico.)

| Espesor de cada placa. | Diámetro del roblón. | Long del fuste antes de insertarlo. | De centro á centro de roblón. | Recubrimiento con roblonadura sencilla. | Recubrimiento con roblonadura doble. |
|---|---|---|---|---|---|
| mm. | mm. | mm. | mm. | mm. | mm. |
| 4.76 | 9.52 | 22.22 | 31.75 | 31.75 | 52.39 |
| 6.35 | 12.70 | 28.57 | 38.10 | 38.10 | 63.50 |
| 7.94 | 15.87 | 34.92 | 41.27 | 47.62 | 77.79 |
| 9.52 | 19.05 | 41.27 | 44.45 | 50.80 | 85.72 |
| 12.70 | 20.63 | 57.15 | 50.80 | 57.15 | 95.25 |
| 15.87 | 23.81 | 69.85 | 63.50 | 69.85 | 117.47 |
| 19.05 | 28.57 | 82.55 | 76.20 | 82.55 | 139.70 |

## Roblonadura de vigas, puentes, etc.

**Art. 1. La cuestión roblonadura es asunto obscuro, de difícil comprensión**; está envuelto en dudas, y los resultados experimentales discrepan mucho. Nos proponemos aquí ceñirnos meramente á la que se considera como la mejor

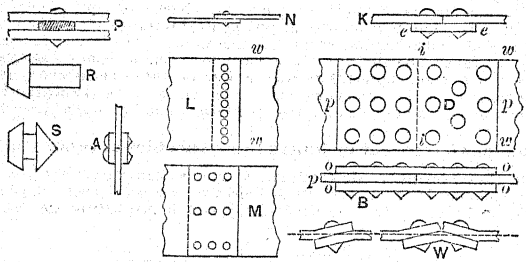



**Fig. 3.**

unión; y para mayor seguridad no contamos con el rozamiento; véase art. 4. En las vigas y trabajos de puentes, las juntas recubiertas arriba descritas se usan rara vez. En lugar de ellas, para unir las placas *p*, fig. 3, se empalman de tope una contra otra, formando de este modo **un empalme plano** *ii*, fig. D, y se unen por una **cubrejunta** *ee*, fig. K, ó mejor, por dos de ellas, como en A ó en *oo, oo*, fig. B. En lo que sigue, el término *plancha* ó *palastro*, según sea, jamás comprende las cubrejuntas. La cubrejunta sencilla semejante al recubrimiento permite tanto á las planchas como á las cubrejuntas doblarse bajo una fuerte tensión, como se indica más ó menos en W, debilitándolas, en tanto que la doble cubierta *oo, oo*, fig. B, mantiene la tensión directamente en el eje de las placas evitando la tendencia á doblarse. También somete á dos secciones de los remaches al esfuerzo cortante, doblando así su resistencia. Cuando no hay más que una cubierta, debe ser por lo menos del grueso de una placa, y cuando hay dos, la experiencia enseña que es mejor que cada una sea *dos tercios* del espesor de la placa, aunque la teoría exige que cada una tenga sólo la *mitad* de dicho grueso.

**La longitud** *ww* **de las cubrejuntas**, transversalmente á la junta, es igual á la de la junta. **La unión de los extremos, es decir el empalme plano requiere doble número de remaches** que los recubiertos, porque en éstos cada remache atraviesa las dos placas unidas, y en el empalme de tope ó plano, solamente una placa.

**Los remaches y placas de un solo lado** (á la derecha ó á la izquierda) de la línea *ii* de unión de cualquier **empalme plano ó de tope** D, de dimensiones adecuadas, representan la resistencia total de la junta, porque los de un lado ejercen tensión en una dirección contraria á los que se hallan del otro lado. Por esto al calcular tales juntas debemos tener presente únicamente los remaches y placas de un solo lado como se hace en lo que sigue: Así, **un empalme plano ó de tope** de roblonadura sencilla, doble ó triple, implica una, dos ó tres hileras de remaches **de cada lado** de la junta *ii*, paralelamente á ella. En una recubierta, de dimensiones adecuadas, el esfuerzo es como **todos** los remaches, porque la mitad de ellos no ejerce tensión contra la otra mitad, sino que un extremo de **cada** remache la ejerce en una dirección y el otro en la dirección opuesta.

**El hierro neto, la placa neta ó la junta neta** es aquella que queda comprendida **entre** los agujeros de los remaches y **del lado afuera** de los dos exteriores, todo en una línea recta trazada por los *centros* de los agujeros de *una* hilera. Su ancho y su área se denominan ancho neto y área neta de la junta. Los que se encuentran entre *otras* hileras no aumentan la resistencia.

En las figs. 3, N y K, están los remaches expuestos á esfuerzo cortante **simple**, es decir, que las tensiones opuestas de las dos planchas tienden á cortar á cada

remache transversalmente según una sola sección circular, mientras que por el contrario, en la fig. B, ó en la fig. A, cada remache está expuesto á un **doble esfuerzo cortante.**

**Art. 2. Las uniones ó juntas de los puentes no se requiere que sean impermeables al agua ó al vapor,** como las de las calderas ó depósitos, y por lo tanto, aumentando el ancho del recubrimiento superior ó la longitud de las cubrejuntas, pueden colocarse los remaches en varias hileras unas detrás de otras, como las tres hileras de á 3 remaches en M y D, en lugar de una sola hilera de 9 remaches como en L. Por este medio, sin que se pierda resistencia alguna de los 9 remaches ó del hierro **neto,** podemos disminuir el ancho de la placa hasta que se igualen los diámetros combinados (6 en este caso) de los agujeros omitidos en el caso de una hilera. Además, usando más de una hilera disminuímos el efecto debilitante que se ve en W. Este modo de colocar los remaches directamente unos detrás de otros en varias hileras, como se muestra en M y á la mitad izquierda de la fig. D, constituye el **roblonamiento de cadena** de Fairbairn; pero la junta será algo más fuerte si se colocan los remaches en **zizás,** como se ve en la mitad de la derecha de la fig. D.

**La separación de las hileras de centro á centro** no debe ser menor de 2 diáms. Es discutible hasta qué punto pueda llevarse este aumento en el número de hileras sin ocasionar una pérdida apreciable de resistencia en los remaches, proveniente de la imposibilidad de igualar los esfuerzos en las hileras separadas. Pero es probable que si no nos excedemos de dos ó tres hileras en los de recubrimiento, ó el mismo número *en cada lado* de la línea de juntura en los empalmes planos, podremos en la práctica suponer que cada hilera y cada remache están casi sometidos á iguales esfuerzos.

**Los agujeros** de los remaches son comúnmente como de un dieciseisavo de pulgada (1½ milímetro), mayor en diámetro que los remaches originales, de manera que permitan la introducción del remache caliente con facilidad.

Los golpes de martillo al recalcarlos aumentan el diámetro del remache hasta que éste llena el agujero. Podemos tomar este aumento del diámetro de los remaches en consideración, como lo hemos hecho al calcular su resistencia al esfuerzo cortante, como se explica adelante, ú omitirlo para aumentar la seguridad.

**Se cree que los agujeros de remaches, taladrados** son mejores que los hechos con punzón, porque el taladrado no maltrata al hierro que está á su alrededor, pero por otra parte se cree que sus aristas vivas cortan los remaches más fácilmente. Por eso, las aristas algunas veces se le desgastan ó suprimen. Estos dos puntos son discutidos y ambos modos son de uso común.

**La distancia del borde de un agujero** al extremo de una plancha ó cubrejunta no debe ser *menor* de 1.20 de diámetro, para evitar que los remaches rompan el extremo de la palanca, ni más cerca del borde del lado de una plancha que la mitad de la distancia exacta entre dos agujeros, como lo indica la Regla del art. 5. La primera es más bien más de lo que indica Fairbairn.

**Los agujeros de los remaches debilitan el hierro neto** que se deja entre ellos, no sólo por la pérdida de la parte de material extraído para hacer el hueco, sino también porque altera el hierro que los rodea, ó acaso porque cambia la forma de la línea neta de fractura, la que entonces no puede resistir las tensiones tan bien como cuando era una línea recta continua. Algunos niegan enteramente la causa y el efecto, basando cada cual su opinión en los experimentos. Pero por muchas razones cree el autor que el hierro pierde, por término medio, como un séptimo de la resistencia que tiene cuando es continuo. Por vía de seguridad, estableceremos en lo que sigue (mientras se resuelve la cuestión definitivamente) que **sí existe** tal pérdida de resistencia en el hierro.

**Las puntas remachadas para resistir compresión** deben depender, no, como podría suponerse, de los extremos que se junten, sino de su resistencia al cizallamiento ó á la distorción de los remaches, pues la contracción ó un trabajo mal ejecutado pueden traer la presión á obrar sobre los remaches. **El roblonamiento por medio de máquinas** es algo más fuerte que el hecho (como se supone en nuestros ejemplos) á mano. **El espesor de las planchas** que se usan en las vigas, puentes tubulares, etc., es comúnmente de .25 á .50 de pulgada (6¼ á 12½ mm); con las de mayor espesor se llega rara vez á 1 pulgada (25 mm) en los puentes largos. **Una pieza de guarnición ó cuña,** como la pieza sombreada en P, es la que se inserta entre dos placas para evitar que éstas se doblen ó se aproximen por causa de los remaches.

**Art. 3. Una junta remachada puede ceder de tres modos** (suponiendo que tiene las dimensiones convenientes), á saber : por el corte ó cizallamiento de los remaches; por la fractura de la placa continua comprendida entre los agujeros de los remaches, ó por el *desgarramiento* (especie de compresión, despachurramiento, ó distorción de la materia, etc.), de las placas por los remaches cuando se fuerzan las dos unas contra otras.

También se comprimen los remaches mismos transversalmente, **con un esfuerzo menor que el cortante**, y al ceder parcialmente las placas y los remaches, permiten á la junta **estirarse**, y puede de este modo originar fuerzas perjudiciales imprevistas en otras partes de la construcción, mucho antes de que haya verdadero peligro de fractura. También en las juntas impermeables al agua y al vapor, puede ocasionar filtraciones, sin posteriores inconvenientes ó peligros. Durante mucho tiempo, este efecto de la compresión se había escapado enteramente á la observación, y se suponía que lo único importante al hacer el cálculo de una junta remachada, era que la resistencia á la tensión de la placa neta, y la resistencia al esfuerzo cortante ó cizallamiento de los remaches debían ser iguales.

**La resistencia al desgarramiento * de una junta** es proporcional a número de remaches, en una junta de recubrimiento, ó al número de los que haya en un lado de la línea de unión, en las juntas de tope ó empalmes planos × diámetro × espesor de las placas unidas. Este producto da el *área de desgarramiento* (*crippled area*) de la junta ó empate. Podemos llamar aquí al producto diámetro × espesor de la placa, **área de desgarramiento del remache**. Si hay 2 ó más placas (no cubiertas) encima unas de otras, en una junta, su espesor total ó unido se emplea para hallar el **área de desgarramiento. La unidad del desgarramiento máximo** por la cual el producto de arriba debe multiplicarse para obtener la resistencia al desgarramiento máximo verdadero de la junta, puede tomarse con seguridad como en 60,000 lbs ó sea 26.8 tons por pulg cuad (como 4,200 kg por cm cuad).

**El diám en pulgs de un remache que resista con seguridad** un esfuerzo cortante dado, se encuentra así : Multiplíquese el esfuerzo cortante por el coeficiente de seguridad, esto es, por el número 3, 4 ó 6, etc. Llámese *g* el producto. Multiplíquese la resistencia máxima al esfuerzo cortante por pulg cuad del hierro del remache por la fracción decimal .7854. Llámese *b* el producto. Divídase *g* por *b*. Tómese la raíz cuad del cociente. La fuerza y la resistencia al esf cort deben *ambas* expresarse en libs ó en toneladas.

O empleando el sistema métrico (*N. del T.*), tendremos :

$$\text{Diám en cm} = \sqrt{\frac{\text{Esfuerzo cortante en kg} \times \text{coeficiente de seguridad} \times 8.214}{\text{Resistencia al esfuerzo cortante máximo en kg por cm cuad.}}}$$

**Si el remache está destinado á sufrir un doble esfuerzo cortante,** multiplíquese primeramente sólo la mitad del esfuerzo cortante por el coeficiente de seguridad, y después procédase como antes.

O lo que es suficiente en la práctica, multiplíquese el diám correspondiente al esfuerzo cortante sencillo por la fracción decimal .7.

**La unidad del esf cort máximo** del hierro de remaches, uno con otro, puede tomarse como en 20.1 tons por pulg cuad (3,163 kg por cm cuad), de sección circular cortada

### Tabla de la resistencia máxima (al esfuerzo cortante sencillo) de los remaches.

(dimensiones del mercado), expuestos al esf cort sencillo, á razón de 45,000 libs ó 20.1 tons por pulg cuad (3,163 kg por cm cuad).

**Esta tabla no debe usarse** cuando, como en nuestro « Ejemplo », art. 5, la resistencia al desgarramiento (*crippling*) del remache domine, sea mayor que la resistencia de la junta.

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* *N. del T.* — La expresión es *crippling strength*, que el autor usa mucho en estos párrafos.

**Si el remache está expuesto al esfuerzo cortante doble,** tendrá el doble de la resistencia de la tabla.

**Para calcular el diám que corresponde á un remache sometido al esfuerzo cortante doble** para iguales resistencias de la tabla, multiplíquese el diám de la tabla por .7, lo que es suficiente en la práctica; lo estricto es .707.

| Diámetro en milímetros. | Kilogramos. | Toneladas. | Diámetro en milímetros. | Kilogramos. | Toneladas. | Diámetro en milímetros. | Kilogramos. | Toneladas. |
|---|---|---|---|---|---|---|---|---|
| 3.175 | 250.4 | .246 | 14.2748 | 5,072.5 | 4.990 | 25.400 | 16,031.2 | 15.800 |
| 4.749 | 563.3 | .554 | 15.8750 | 6,262.3 | 6.160 | 26.974 | 18,097.8 | 17.800 |
| 6.350 | 1,002.0 | .986 | 17.4498 | 7,577.3 | 7.460 | 28.575 | 20,289.6 | 20.000 |
| 7.924 | 1,565.8 | 1.540 | 19.0500 | 9,017.4 | 8.880 | 30.149 | 22,606.0 | 22.200 |
| 9.525 | 2,254.4 | 2.220 | 20.6248 | 10,583.2 | 10.400 | 31.750 | 25,049.0 | 24.600 |
| 11.099 | 3,068.6 | 3.020 | 22.2250 | 12,274.2 | 12.100 | 33.324 | 27,616.9 | 27.200 |
| 12.700 | 4,008.0 | 3.940 | 23.7998 | 14,090.3 | 13.900 | 34.925 | 30,308.9 | 29.800 |

**La resistencia á la tensión de una junta ó conexión de dimensiones bien calculadas** es igualmente proporcional al área neta de la sección de la plancha (no cubierta) hecha á través del centro de **una** hilera solamente de remaches ó al área del esfuerzo cortante ó de (*crippling*) desgarramiento (según el caso) de **todos** los remaches contenidos en un recubrimiento; ó como todos los remaches **situados de un solo lado** de la línea de unión en una junta de tope ó plana. La resistencia á la tensión de hierro en planchas de regular calidad, antes de que se le hagan los agujeros de los remaches, es por término medio como de 45,000 libs ó 20.1 tons por pulg cuad (como 3,163.93 kg por cm cuad); pero para mayor seguridad supondremos, como se dijo en el art. 2, que la hechura de los agujeros disminuye la resistencia del hierro continuo, **neto**, que queda como en una séptima parte, ó á 38,500 libs, 17.2 tons por pulg cuad (2,706.91 kg por cm cuad).

*Observación.* **Esto es todavía considerablemente grande** para las juntas de recubrimiento ó para las juntas de tope de una sola cubrejunta, debido al debilitamiento del hierro en tales juntas ó conexiones por el doblez que se indica en W, fig. 3. Pero no estamos tratando de eso.

**El rozamiento entre las planchas** en los recubrimientos ó entre las planchas y las cubrejuntas en una junta plana ó de tope, producido por la presión que las une estrechamente en la contracción de los remaches al enfriarse, añade mucho á la resistencia de una junta cuando está nueva, quizás 1.5 ó 3 tons por pulg cuad (236.24 ó 472.48 kg por cm cuad) de sección circular de todos los remaches en una junta recubierta, ó de todos los situados á un lado de una junta plana de cubrejunta sencilla, ó de 3 á 6 tons (472 á 944 kg por cm cuad) de todas las situadas á un lado de una junta plana de doble cubierta. En construcciones perfectas, este rozamiento podría continuar existiendo, totalmente ó en parte, por un período indefinido; pero en los puentes, etc., sujetos á choques y vibraciones incesantes y violentos, es probable que disminuya pronto ó desaparezca del todo. De aquí que autoridades competentes en la materia recomiendan no confiar en él, y se omite por tanto en lo que sigue.

**Art. 5.** Damos fórmulas para encontrar el número de remaches que se necesitan en **una junta plana de doble cubrejunta** (de cuya clase sólo trataremos) y la dist neta que los separa. Esta dist + un diám es la dist de centro á centro de los remaches. El principio en que se funda la regla se explicará más adelante en el art. 7.

**Primero.** Elíjase un diám de remache igual ó mayor que .85 veces el espesor de la plancha. En la práctica es generalmente 1.5 veces para planchas de ½ pulg (12½ mm) ó más gruesas; ó 2 para planchas de un grueso menor de ½ pulg.

**Segundo.** Multiplíquese la tensión total más grande en libs (ó en kg si se usa el sistema métrico) que pueda obrar sobre toda la junta, por el coeficiente (3, 4 ó 6, etc.) de seguridad, y llámese *p* el producto.

**Tercero.** Multiplíquese el área de (*crippling*) desgarramiento del remache (esto es, su diám × el espesor (en pulgs ó en cm, según el sistema empleado) de la plancha por 60,000 (libs por pulg cuad) (4,218.573 kg por cm cuad). El producto es la resistencia máxima al desgarramiento de un remache. Llámeselo *m*.

**Cuarto**. Divídase á $p$ por $m$. El cociente será el número de remaches necesarios para soportar el esfuerzo de tensión dado con el grado de seguridad que se requiere. Entonces **la dist neta de los remaches será :**

$$\textbf{Dist en pulgs} = \frac{\text{Número de hileras} \times \text{diám en pulgs} \times 60{,}000}{38{,}500}$$

$$\textbf{Dist en cm} = \frac{\text{Número de hileras} \times \text{diám en cm} \times 4{,}218.573}{2{,}707}$$

**Quinto**. La dist neta de uno y otro agujero extremo de una hilera al borde lateral de la plancha no debe ser menor que la mitad de la dist neta comprendida entre dos remaches en una hilera.

(*Obs. del T.* — Pondremos un ejemplo en sistema métrico análogo al que trae el autor en medidas inglesas.)

Ejemplo : Una junta plana ó de tope de doble cubrejunta para una plancha de 1.27 cm de espesor, tiene que soportar una tensión efectiva de 15,308.794 kg con una seguridad igual á 4, es decir, que no se rompe con menos de 15,308.794 × 4 = 61,235.176 kg. ¿Cuántos remaches debe llevar esta junta y á qué dist se deben colocar?

**Primero**. Tendremos que .85 veces el espesor de la plancha es 1.27 × .85 = 1.08 cm, por tanto estos remaches no deben tener un diám inferior á 1.08 cm, pero tomaremos 2 cm.

**Segundo**. La tensión más grande × coeficiente de seguridad = 15,308.794 kg × 4 = 61,235.176 kg = $p$.

**Tercero**. El área de desgarramiento de un remache × 4,218.573 (= 60,000 libs por pulg cuad) = 2 × 1.27 × 4,218.573 = 10,715.175 = $m$.

**Cuarto**. $\frac{P}{m} = \frac{61{,}235.176}{10{,}206.205}$ = más de 5.5 (digamos 6 remaches) que se requieren de cada lado de la junta.

**El espacio neto** ó **ancho neto** entre ellos será, **si los 6 remaches están en una hilera :**

$$\frac{\text{Diám} \times 4{,}218.573}{2{,}707} = \frac{2 \times 4{,}218.573}{2{,}707} = 3.117 \text{ cm.}$$

**La dist de centro á centro** = espacio neto + diám = 3.117 + 2 = 5.117 = 2.558 diáms. En la práctica, para evitar molestas decimales, podemos hacer el espacio neto = 3 cm, y la dist entre los centros 5 cm, pero para ilustrar mejor el manejo de la regla tomamos un número de decimales más exacto.

**Quinto**. La dist neta de cada agujero extremo al borde lateral de la plancha es la mitad de 3.117 = 1.558 cm.

**El ancho total de hierro neto** es igual á un espacio neto × número de remaches = 3.117 × 6 = 18.702 cm; y el ancho total de plancha es igual á una dist de centro á centro × número de remaches = 5.177 × 6 = 31.062 cm.

El área de la sección transversal de plancha no agujereada es 31.062 × 1.27 = 39.448 cm cuad; su resistencia á la tensión **antes de hacerle los agujeros** es 39.448 × 3,163.93 = 124,810.7 kg.

La resistencia de nuestra junta, omitiendo el rozamiento, es por tanto :

$\frac{61{,}235.18}{124{,}810.70}$ = .52 de la resistencia de la plancha primitiva no agujereada.

**Si los 6 remaches están en 2 hileras** de á 3 remaches cada una, **la dist neta entre dos remaches** de una hilera será el doble de antes, es decir (siguiendo el ejemplo en sistema métrico. *N. del T.*), el doble de 3.117 = 6.234 cm. **Dist de centro á centro** = 6.234 + 2 = 8.234 cm = 8.234 ÷ 2 = 4.117 diáms. **Dist neta del agujero** extremo al borde lateral de la plancha = mitad de 6.234 = 3.117 cm. **Ancho total de hierro neto** = 6.234 × 3 = 18.702 cm. **Ancho total de plancha** = 8.234 × 3 = 24.702 cm. **Área de la sección transversal de la plancha no agujereada** = 24.702 × 1.27 = 31.371 cm cuad. **Resistencia máxima á la tensión** de la plancha no agujereada = 31.371 × 3,163.93 = 99,255.64 kg. **Resistencia máxima de la junta remachada**, omitiendo el rozamiento = $\frac{61{,}235.18}{99{,}253.64}$ = .62 de la de la plancha no agujereada.

De esta manera vemos que la disposición adoptada de dos hileras presenta la misma resistencia que una hilera con menor ancho y menor área de plancha. De seguro que requiere *cubiertas más grandes*.

**Si los 6 remaches están en 3 hileras** de á 2 remaches cada una, el **área de la sección transversal de la plancha no agujereada** es 4.2565 pulgs cuad. **Su resistencia á la tensión** 191,542 libs. Resist de la junta remachada $\frac{135.000}{191,542}$ = .7 de la de la plancha sin agujerear.

El ancho total de hierro neto (7.0128 pulgs); su área (7.0128 × .5=3.5064 pulgs cuads); y su resistencia máxima á la tensión (3.5064 × 38,500=135,000 libs), son los mismos en cada caso. El último valor es la resistencia requerida de la junta á la ruptura, como al comienzo de nuestro ejemplo, y es igual á la resistencia al aplanamiento combinada de los seis remaches.

Efectuando los cálculos en sistema métrico obtendremos resultados análogos, para el caso de que los **6 remaches estén dispuestos en tres hileras de á dos remaches**, á saber:

**Área de la sección transversal de la plancha no agujereada** es =27.46019 cm cuad (4.2565 pulgs cuad). **Su resistencia á la tensión** =86,882.132 kg (191,542 libs). Resistencia de la junta remachada = $\frac{61,235.176}{86,882.132}$ = .7 de la de la plancha no remachada.

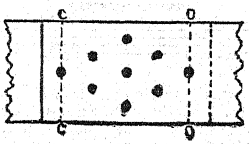
El ancho entero de hierro neto=17.8122 cm (7.0128 pulgs); su área=17.8122 × 1.27=22.6215 cm cuad (3.5064 pulgs cuad); y su resistencia máxima á la tensión 22.6215 × 2707=61,236.4 (135,003 libs), son los mismos en ambos casos. Este último valor es la resistencia á la ruptura de la junta requerida, como al comienzo de nuestra ejemplo; y es igual á la resistencia combinada de desgarramiento de los 6 remaches.

**Art. 6. La dist que separa las hileras de remaches,** de centro á centro de remahe, no debe ser menor de dos diámetros del agujero del remache.

*Nota* 1. Con nuestras constantes para tensión, esfuerzo cortante y compresión, **los remaches no ceden primero por el esfuerzo cortante** en una junta plana ó de tope de doble cubrejunta (y por supuesto sometida al doble esf cort), excepto cuando el diám es igual ó **menor** de .85 del espesor de la plancha, lo cual acontce rara vez. Cuando es igual á .85, la resistencia al desgarramiento y al esfcorte te un remache son iguales cuando se usan los coeficientes supuestos de desga ameto y esf cort.

*Nota* 2. Nuestro ejemplo fué escogido para ilustrar la regla. Rara vez ocurrirá en la práctica que la regla dé un número de remaches sin una fracción, ó que podamos dividirlo por 2 ó por 3 sin que quede un residuo. En el caso de una fracción, es claro, que será mejor tomar á ésta por un remache entero, aunque por ello sea el empate un poquito más resistente de lo necesario. Si el número de remaches resulta ser, por ejemplo, 7 ó 9, podemos formar dos hileras: una de 3 y otra de 4 ó una de 4 y otra de 5, etc. Además, el ancho de la plancha se fija frecuentemente de antemano por alguna necesidad de la construcción, y debemos disponer los remaches teniendo cuidado en todos los casos de conservar el área calculada de hierro neto en una hilera, etc.

*Nota* 3. Nos hemos limitado (como dijimos al principio que lo haríamos) á la junta plana ó de tope sencilla de dos cubrejuntas con los remaches colocados en 1, 2 ó más hileras paralelas á cada lado de la línea de unión, que es lo más resistente y lo que más comúnmente se usa en construcciones de ingeniería. La necesidad á veces




impone disposiciones más simples, para las cuales no podemos dedicar espacio en esta obra, y cuya resistencia no se calcula tan rápidamente. Éstas algunas veces dan resultados que parecen extraños á los poco conocedores; así esta junta recubierta se rompe transversalmente al hierro neto de una plancha por *cc* ó por *oo*, donde hay más hierro, y en donde, por lo tanto, podría creerse más resistente,

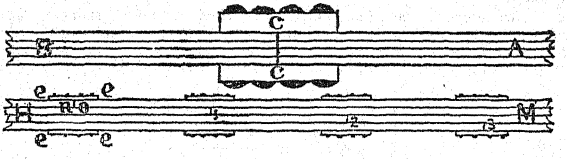
*Nota* 4. **La tabla siguiente** indica aproximadamente las resistencias relativas de las formas comunes de juntas de dimensiones adecuadas, variando con la calidad de las láminas y de los remaches

| | Con rozamiento. | Sin rozamiento. |
|---|---|---|
| La plancha primitiva sin agujeros | 1.00 | 1.00 |
| Junta plana ó de tope con roblonadura doble con dos cubrejuntas | .80 | .64 |
| Junta plana ó de tope con roblonadura doble con una cubrejunta | .65 | .52 |
| Junta plana ó de tope con roblonadura sencilla con una cubrejunta | .50 | .40 |
| Junta de recubrimiento con roblonadura doble | .65 | .52 |
| Junta de recubrimiento con roblonadura sencilla | .50 | .40 |

*Nota* 5. Las resistencias tubulares mencionadas para las **juntas de recubrimiento**, pueden obtenerse aproximadamente adoptando las dimensiones siguientes, según que la junta sea de roblonadura doble ó sencilla.

| | **Roblonadura doble en zizás.** | | **Roblonadura sencilla.** | |
|---|---|---|---|---|
| | En espesores. | En diáms. | En espesores. | En diáms. |
| Llamando los espesores de las planchas | 1 | .6 | 1 | .6 |
| Hágase entonces el diám del remache | 1.67 | 1.0 | 1.67 | 1.0 |
| — — ancho de la recubierta | 9.0 | 5.4 | 5.67 | 3.4 |
| — — dist de centro á centro | 7.0 | 4.2 | 4.5 | 2.7 |
| — — la dist del *extremo* de las planchas al *borde* de los agujeros | 2.0 | 1.2 | 2.0 | 1.2 |
| — — la dist de las hileras de remaches, de centro á centro | 3.33 | 2.0 | | |

*Nota* 6. **Si dos ó más planchas colocadas unas encima de las otras,** como las cuatro en AB ó MH, tienen que unirse de manera que actúen como una plancha del espesor *cc*, los diáms de los remaches y el espesor de las cubietras *cc*, *ee* dependerán de que las juntas de las planchas estén todas en una misma línea como *cc* en AB, ó de que se hallen formando una línea quebrada como en 0, 1, 2, 3 en MH;




Es claro que las cubrejuntas *cc* transmiten de A á B, por medio de los remaches ó pernos que las unen, á través de la junta *cc*, toda la resistencia que por partes compensa la que se pierde por la separación de las cuatro planchas en esa junta: mientras que las dos cubiertas *ee*, *ee* y sus remaches ó pernos, de un modo semejante transmiten de *n*, en una sola plancha, á *o*, en la plancha adyacente, á través de, la junta comprendida entre estas dos letras, solamente la resistencia que por partes compensa la que se pierde por la separación de esa sola plancha, y así respecto de las juntas 1, 2 y 3. Por tanto, las cubiertas *cc* y sus remaches, deben ser cuatro veces más resistentes que las que se hallan en una cualquiera de las cuatro juntas 0, 1, 1, 3. La primerá *cc* debe considerarse como uniendo dos planchas sólidas A y B, cada una de un espesor cuádruplo *cc*, y las otras como uniendo dos planchas de un espesor

sencillo: las cubrejuntas *cc* serán, por tanto, cada una como de dos tercios del espesor *cc*, y las otras tendrán cada una un espesor como de dos tercios del de una plancha sencilla. Así: supongamos que cada una de las cuatro planchas en AB ó en MH tienen un espesor de $^3/_4$ de pulgada (19 mm) siendo *cc* de 3 pulgs (76 mm). Cada cubierta *c* tendrá, en consecuencia, $^2/_3$ de 3 pulgs, es decir, 2 pulgs de espesor (ó $^2/_3$ de 76.2 mm=50.8 mm) ó las dos cubiertas *cc* juntas, cuatro pulgs (101.6 mm) que es el espesor real de la junta *cc*. Pero cada cubierta *ee* es sólo de $^2/_3$ de $^3/_4$ pulg ó de ½ pulg de espesor (es decir, $^2/_3$ de 19 mm=12.7 mm), y el espesor real de la junta en 0, 1, 2 ó 3, es el de las tres planchas enterizas (no desunidas) más el de las dos cubiertas, ó $(3\times\frac{3}{4})+(2\times\frac{1}{2})=3\frac{1}{4}$ pulg ó $(3\times19)+(2\times12.7)=82\frac{1}{2}$ mm).

**Art. 7. Principio en que se basa la Regla del art. 5.** Con nuestra constante del esfuerzo cortante (45,000 libs por pulg cuad=3,163.93 kg por cm cuad) y el de desgarramiento (60,000 libs por pulg cuad=4,218.573 kg por cm cuad), y con diáms de remaches iguales ó mayores que .85 del espesor de la plancha, según nuestra regla, la resistencia al desgarramiento de una junta plana ó de tope de doble cubrejunta será igual ó menor que su resistencia al esf cort. Por esto, para evitar gasto inútil de material, bien en las planchas ó en los remaches, debemos hacer.

Resistencia á la tensión de plancha neta transversalmente á una hilera de remaches = Resistencia al desgarramiento de *todos* los remaches. O, ancho total neto de plancha × espesor de plancha × unidad de tensión = área de desgarramiento (*crippling*) de un remache × unidad de desgarramiento × número total de remaches.

Ahora bien, por el art. 3, el área de (*crippling*) desgarramiento de un remache es = diám del remache × espesor de la plancha. Tomamos por unidad del esf cort 60,000 libs por pulg cuad = 4,218.573 kg por cm cuad, y por unidad de tensión 38,500 libs por pulg cuad = 2,707 kg por cm cuad y, deducido de lo anterior, tendremos (*N. del T.* — Damos las fórmulas métricas equivalentes á las del autor):

$$\text{Ancho total neto de la plancha en cm} = \frac{\text{Diám de remaches} \times \text{espesor de plancha} \times 4{,}216.573 \times \text{número total de remaches}}{\text{Espesor de plancha} \times 2{,}707.}$$

Haciendo la dist neta comprendida entre cada remache extremo de una hilera y el borde lateral de la plancha = la mitad de la dist neta comprendida entre dos remaches de una hilera; y llamando la suma de las dos dists extremas un espacio, tendremos, para ambos sistemas de medida:

$$\text{Número de } \textit{espacios} \text{ de una hilera de remaches} = \text{Número de } \textit{remaches} \text{ de una hilera}$$

De manera que

**La dist neta comprendida entre dos remaches de una hilera,** que es

$$= \frac{\text{Ancho neto total de plancha}}{\text{Número de } \textit{espacios} \text{ de una hilera}} = \frac{\text{Ancho neto total de plancha}}{\text{Número de } \textit{remaches} \text{ de una hilera}}$$

$$= \frac{\text{Diám de remaches} \times \text{Espesor de plancha} \times 4{,}218.573 \times \text{Número total de remaches}}{\text{Espesor de plancha} \times 2{,}707 \times \text{Número de remaches de una hilera}}$$

Pero

$$\frac{\text{Número total de remaches}}{\text{Número de remaches de una hilera}} = \text{Número de hileras.}$$

Por consiguiente, omitiendo el «espesor de plancha», común al numerador y al denominador, tenemos, como la regla del art. 5.

$$\textbf{Dist neta de los remaches en cm} = \frac{\text{Diám de remaches} \times 4{.}218{.}573 \times \text{Número de hileras}}{2{,}707}$$

**Pero si el diám de los remaches es menor que .85 veces el espesor de la plancha,** la resistencia *al esfuerzo cortante* de una junta plana ó de tope de doble cubrejunta (con nuestros coeficientes para esf cort y desgarramiento) es menor que su resistencia al desgarramiento. En tales casos, para la dist neta comprendida entre dos remaches de una hilera, tendremos:

$$\textbf{Dist neta} = \frac{\text{Area circular del remache} \times \text{unidad del esf cort}}{\text{Espesor de plancha} \times \text{unidad de tensión}} \times 2$$

*Nota* 1. **Por ser las juntas de dos cubrejuntas, expuestas al esf cort doble**, las únicas que aquí se consideran, y como los remaches pueden siempre tomarse con un diám mayor que .85 del espesor de la plancha, podemos en la práctica usar siempre la regla del art. 5 para tales juntas, y por eso dimos esa sola regla.

*Nota* 2. **Cuando se use esta regla para otra clase de juntas**, tales como de recubrimiento ó juntas planas ó de tope de cubrejunta sencilla, recuérdese que los remaches en dichas juntas están expuestos al esf cort **sencillo**, y por consiguiente podemos hacer uso de la regla del art. 5 (para el desgarramiento (*crippling*) solamente cuando el diám es 1.7 ó más veces el espesor de la plancha. **Si es menor**, úsese la regla que precede para el esf cort, todo en el supuesto de que se usen nuestros coeficientes precedentes de desgarramiento y esfuerzo cortante.

**Pero el coeficiente de tensión debe variarse** para cada clase de estas otras juntas, para tomar en cuenta los efectos de debilitamiento ocasionados por el doblez mostrado en W, figs. 3, como se ha deducido aproximadamente de los experimentos. El autor cree que las siguientes unidades de tensión darán resultados seguros aproximados, sin rozamiento. **Para juntas planas ó de tope**, de roblonadura doble, 38,500 libs por pulg cuad (2,707 kg por cm cuad) como se adoptó anteriormente. **Para juntas de recubrimiento de roblonadura doble**, ó juntas planas ó de tope de una cubrejunta, 28,000 libs por pulg cuad (1,968.67 kg por cm cuad). Para juntas de recubrimiento **de roblonadura sencilla** ó juntas planas de una cubrejunta, 24,000 por pulg cuad (1,687.43 kg por cm cuad). Pero como se dijo anteriormente, en materia de roblonadura no se llega á obtener gran certeza.

*Nota* 3. **Un empate ó unión puede ceder por desgarramiento** sin que se sepa y ni siquiera se haya sospechado, pues el hecho de que ceda no implica que alguna cosa se **rompa**, sino simplemente que la junta se ha **alargado**, y esto puede no descubrirse aun cuando se la observe *ligeramente*. Todavía puede suceder, y probablemente ha sido á menudo causa suficiente para dañar y aun destruir puentes y techos, el que se hayan desarrollado esfuerzos en donde no se habían tomado las precauciones necesarias.

# FERROCARRILES
## VÍA

### GENERALIDADES

**1.** La vía de ferrocarril como ordinariamente se construye en los Estados Unidos, se compone de las siguientes partes :

**2. Carriles,** ¶ 83. Las ruedas del material rodante ruedan sobre la cabeza del carril, el desgaste de la llanta de la rueda se verifica sobre la cabeza del carril y el de las pestañas sobre el costado de la cabeza del carril. Los carriles tambien sirven o hacen el oficio de vigas cortas cubriendo los espacios entre las traviesas, teóricamente la cabeza del carril trabaja por compresion, el alma por cizallamiento, y la base en tension; pero desde el momento que los carriles deben actuar como vigas *continuas*, los esfuerzos en las cabezas y bases necesariamente se reflejan sobre cada traviesa. La base del carril sirve tambien para distribuir la carga sobre la traviesa, y permitir que agarren los clavos; y ofrece la principal resistencia contra la flexion lateral.

**3. Uniones,** ¶¶ 145, etc. Los extremos del carril se unen entre sí y en línea por medio de bridas, las que se aseguran a los costados de los carriles, cerca de sus extremos por medio de tornillos que atraviesan las bridas y el alma del carril.

**4. Clavos,** ¶¶ 96, etc. Estos se introducen en las traviesas al lado de los carriles, asegurándolos en firme a las traviesas, y por medio de la cabeza que está ensanchada muerden los cantos de la base del carril; y la pata del clavo resiste el deslizamiento lateral de los carriles sobre las traviesas.

**5. Platinas,** ¶¶ 135, etc., se usan generalmente para distribuír las presiones del carril sobre una área mayor en la superficie de la traviesa. De este modo se reduce la unidad de presion y la tendencia a aplastar la traviesa por efecto de las cargas.

**6. Traviesas,** ¶¶ 31, etc., sirven para distribuir en, y al traves del balasto, las cargas verticales, y los empujes horizontales producidos por el material rodante.

**7. Balasto,** ¶¶ 19, etc., No solamente soporta las cargas y las trasmite al terreno sino que tiene la funcion adicional de prolongar la duracion de las traviesas, permitiendo que el agua fluya a traves de él.

**8. Lecho de la via.** La via propiamente, incluyendo el balasto, descansa sobre el lecho o apisonado, que no solamente debe ser de material firme para retener el balasto y la via a nivel, sino que debe también estar preparado para desalojar el agua que pueda recogerse. Con este objeto a menudo se provee de zanjas o cunetas a lo largo del apisonado para conducir el agua que fluye de la vía, excepto cuando la vía está en terraplen.

### Especificaciones.

**9.** En el curso de este artículo sobre Via, se ha hecho un uso liberal de especificaciones y otros testimonios de la práctica establecida; y cuando se hace mención de tales datos ó sintesis, el hecho se indica con notas al pie, la propia marca de llamada se coloca al comienzo y fin de cada párrafo o mencion, acompañado en algunos casos por números que denotan el año de su publicación.

Las referencias usadas con mas frecuencia son :

* nota al pie, se refiere a Asociacion Americana de Ingeniería de Ferrocarril; Manual 1915.

† nota al pie, se refiere a Especificaciones del Ferrocarril de Pennsylvania para la construcción y mantenimiento de Ferrocarril. Tipo 1909, y otras especificaciones del mismo ferrocarril, 1908-1914.

‡ nota al pie, se refiere al ferrocarril Unión Pacific. Reglas y Reglamentación para el mantenimiento de Via y Estructuras, 1909.

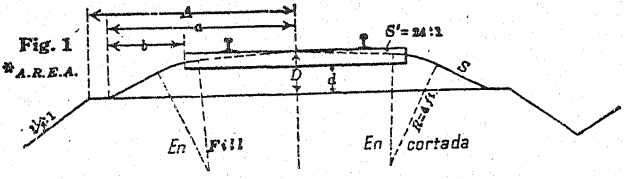
* Ascn Assion. Ing F. C. † F. C. Pen. ‡ F. C. U. Pacific.

## Clasificación de las Vías.

Ascn Am de Ing de F. C. Manual 1915, p. 15.

**10.** Los ferrocarriles, y partes (región o zona) de ferrocarriles, estan clasificados por la Ascn Am Ing F. C. según el volumen y carácter de su tráfico como sigue: (*N. del T.* — Una milla son 1.609 metros.)

| Clase. | Clase A, B ó C incluye todas las regiones de un ferrocarril. | Millaje de vagon de carga por año por milla. | Millaje de vagon de pasajeros por año por milla. | Velocidad máxima tren de pasajeros millas/hor. |
|---|---|---|---|---|
| A | Teniendo (1) mas de una via principal, ó (2) vias únicas principales con tráfico < ..... | 150.000 | 10.000 | 50 |
| B. | Vía única, con tráfico < A, y < .. | 50.000 | 5.000 | 40 |
| C. | No llenando los requisitos de tráfico de las clases A y B ..... | | | |




**11. Vía única.**

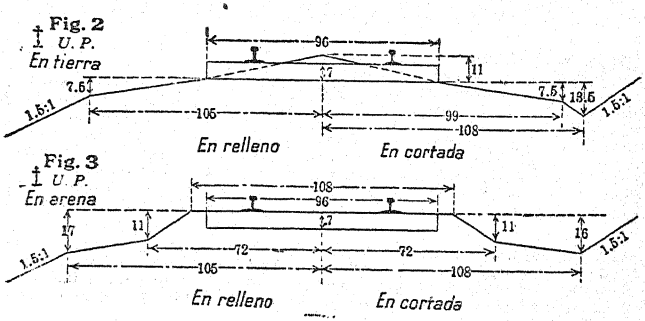
*Ascn Am Ing F. C. Fig. 1.

Se recomienda sembrar yerba o raices en la parte saliente del apisonado próximo a la zanja o cuneta y en el talud de la zanja.

| | Ancho, cm. | | | Hondo cm. | | Talud. |
|---|---|---|---|---|---|---|
| | *A* | *a* | *b* | *D* | *d* | *S* |
| Piedra triturada y escoria. | | | | | | |
| Clase A §..... | 300 | ... | 100 | 48 | 30 | 2 : 1 |
| Clase B..... | 240 | ... | 75 | 40 | 22 | 2 : 1 |
| Granzon, Cisco, etc. | | | | | | |
| Clase A §..... | 300 | ... | 125 | 48 | 30 | 3 : 1 |
| Clase B..... | 240 | ... | 100 | 40 | 22 | 3 : 1 |
| Granzon compacto y feldespato | | | | | | |
| Clase C*..... | 210 | ... | 45 | 32 | 15 | 3 : 1 |
| ∓ Unión Pacific, 1909 | | | | | | |
| Piedra picada o escoria. Fig. 1..... | 240 | 45 | 75 | 37 | 20 | ... |
| Granzon, arcilla quemada o cisco. Fig. 1... | 255 | 210 | 90 | 37 | 20 | ... |
| Tierra. | Vease Fig. 2. | | | | | |
| Arena. | Vease Fig. 3 ∓. | | | | | |

* Ascn Am Ing F. C. † F. C. Pen. ∓ F. C. U. Pacif. Vea tambien especificaciones, p. 853.

§ Las dimensiones de la clase A dan el hondo mínimo, *d* bajo las traviesas. Se recomiendan solamente en los afirmados más consistentes y provistos del mejor desague




‡ **Doble Vía.** 13 pies (3.965 m.) entre los centros de vía. * ‡

## LECHO DE LA VIA

**Especificaciones.** (Véanse los números y la tabla arriba.)

**12. Zanjas.** Las de amplias dimensiones y talud para buen desagüe, no son solamente esenciales para la mayor economía en la conservación, sino que, mejorando su corriente, pudieran también permitir el uso de balasto barato que de otro modo no se pudiera aceptar. En excavaciones largas de pequeña pendiente pudiera ser necesario darle á las zanjas un declive mayor que el del apisonado, necesitando un aumento gradual en su hondo de la cima hacia abajo, con el correspondiente aumento en su ancho. Las pendientes de las zanjas (comunmente de $\frac{1}{3}$ á $\frac{1}{2}$ por ciento) deben regirse por las condiciones en cada caso : tales como largo y hondo de la excavación; naturaleza del material (con materiales impermeables se necesitan zanjas más anchas y empinadas); costo del ancho del apisonado, área y condiciones de la vertiente, si el agua sale de los taludes de la excavación, etc. La sección transversal que facilita más el desagüe es la que da mayor área relativamente al perimetro mojado, véase páginas 558, 561, 603, etc.

**13. Para corrientes fuertes,** especialmente en material flojo, la zanja debe ser pavimentada ó cubierta con piedra.

**14.** El talud de la excavación y del terraplén se protegen con frecuencia con **zanjas de superficie** para desecar y que corren paralelas á la vía; hechas á unos 3 á 4, 5 m ó más del borde superior de la excavación.

**15. Desagüe inferior.** Algunas especificaciones exigen el uso de tubos de barro de 6" (15 cm) colocados 0,60 m por debajo de las zanjas ó cunetas.

**Miscelanea.**

**16.** Mr. Don J. Whittemore recomienda *redondear los angulos* que se ven con frecuencia en las secciones transversales del afirmado. Memorias de la Asociación de Ingenieros Civiles Americanos, Sept. 1894.

**17. Cima.** Para facilitar la desecación del balasto se acostumbra darle al apisonado una cima haciendo su elevación 4 ó 5 cm más alto en el centro que en los costados. El perfil de la sección transversal puede ser una curva, ó puede consistir en líneas rectas, descendiendo diagonalmente desde el centro.

**18. Línea de siembra.** En los terraplenes una tira de yerba de un pie (0,305 m) de ancho aproximadamente, á lo largo del afirmado próximo á cada borde protege el talud contra los arrastres que produce el agua y disminuye la pérdida del balasto.

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* Asen Am Ingo F. C. † F. C. Pen. ‡ F. C. U. Pacific. Véase tambien especificaciones pag. 853.

## BALASTO

**19. Piedra.** * Piedra picada en pequeños fragmentos por medios artificiales. La piedra picada es la preferida universalmente como balasto. Las mejores piedras en orden de preferencia aproximadamente son : roca verdosa, granito (ó roca granitica conteniendo hornablenda en lugar de mica), piedra calcárea y piedra cuarzosa. La piedra cribada del triturador se usa comunmente para andenes, y como balasto para vías laterales, pero se revuelve mucho con el agua debajo de las traviesas. *Requisitos;* — † " Roca verdosa ó una piedra ignea aceptable ó igualmente dura y apropiada..... es considerada como material típico. Fuerza de compresión $<$ 844 kg por cm cuad; piedra calcárea $<$ 703 kg por cm cuad. Debe estar rota en forma cúbica. Debe pasar por un aro de 7,5 cm, pero no por un aro de 3 cm. †

**20. Cascajo.** Es barato cuando se encuentra cerca del trabajo y se coloca fácilmente. La arena y la greda en el cascajo impide el desecado; mientras que la arena y el polvo causan el desgaste de llantas y muñones. *Requisitos.* * Guijarros que pasen un aro de 6 cm y que no pasen la criba nº 10 (1915) * † $<$ 6 cm (1908 †).

**21. Escoria dura.** La escoria de los Altos Hornos, depositada y enfriada al aire. Si contiene poca cal libre, es dura y vidriosa y cuando se tritura rivaliza con la piedra picada como material para balasto; pero á veces afecta las traviesas químicamente. Un exceso de cal libre es propenso á desintegrarlo y algunas veces fragua como el mortero.

**22. Escoria granulada.** Escoria á la cual se le haya inyectado agua cuando esté derretida. Se compone de particulas de tamaño muy uniforme como los granos de arena gruesa. Se pone fácilmente y se usa en patios y vías laterales. Es propenso á desintegrarse, solidificarse, é impedir el desecado si hay presente mucha cal libre.

**23. Cisco. Cenizas.** Sacado de las fosas de las locomotoras, Balasto de ceniza deseca bien, pero cede con el tráfico pesado, á veces daña las traviesas químicamente en especial cuando se mojan *. Recomendado para ramales con tráfico ligero, en desvíos y vías de patio cerca de los puntos de origen; como sub-balasto en lugares mojados y esponjosos; como sub-balasto en trabajo nuevo donde el relleno está cediendo; y en lugares donde la vía se levanta por las heladas. Dispongase lo conducente para mojar las cenizas tan pronto se saquen 1915 *.

**24. Greda quemada.** Como balasto deseca bien, es fácil de trabajar y soporta bien el tráfico ligero. *Requisitos;* — * Use stratum negro ú otra greda cualquiera libre de arena ó cieno. Antes de establecer la estufa ú horno para este proposito pruebese el material en una estufa pequeña. Quemese bien todo. El combustible debe ser fresco y bastante limpio para que queme con fuego limpio. Tengase a mano suficiente combustible para evitar interrupciones mientras dure la operación. Enfrie el balasto antes de cargarlo fuera de la fosa. Absorción de agua $>$ 15 % en peso 1911 *.

**25. Arena cuarzosa *.** Particulas de cuarzo, aproximadamente tan grandes como granos de trigo. Es el residuo de los molinos en el que los minerales de zinc y plomo se separan de la roca que los contienen *. Este balasto se trabaja fácilmente. No deja crecer la yerba, soporta el tiempo de agua y retiene su superficie bien; pero debido á la falta de cohesión se rueda fácilmente de costado. A grandes velocidades levanta polvo.

**25a. Arena.** Se maneja fácilmente como balasto. No es plástico cuando se moja; pero se levanta con las heladas, y cuando se seca es muy polvoriento y por consiguiente molesto á los pasajeros, además dañino al material rodante y es causa de que se calienten las cajas de las chumaceras.

**25b. Lodo.** Como balasto, en tiempo de agua, se convierte en fango. Mientras más arena contenga mejor.

**25c. Estrata.** Una clase de arcilla bastante tenaz que no contiene arena.

**25d. Horsteno *.** Un pedernal impuro, que se produce en depósitos naturales *.

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* Asch Am Ingo F. C. † F. C. Pen ‡ F. C. U. Pacific.

### Sección Trasversal del Balasto.

Vease figs 1-3.

**26. Profundidad.** El balasto debe ser de tal profundidad que la carga, distribuida sobre el afirmado no aplaste el balasto o rompa las traviesas.

**27. Ancho.** La sección del balasto debe ser bastante ancha de modo que retenga firmemente la via en su linea. El ancho necesario para esto depende del peso y angulosidades del material. El talud lateral del balasto debe ser de tal modo que la sección retenga su forma bajo la trepidación de los trenes.

**28.** La economia en el primer costo efectuada por la adopción de un afirmado *muy estrecho* puede ser contraproducente por el aumento en el costo de mantenimiento, debido a la dificultad en darle adecuado desagüe y en mantener el balasto en su lugar. En excavaciones, un apisonado estrecho aumenta las dificultades para reponer las traviesas.

**29. Traviesas en el balasto.** Con piedra picada, escorias y otros balastos favorables al desagüe, la superficie superior del balasto se hace pareja con la parte superior de las traviesas; y en materiales muy ligeros y que no apisonan el balasto, se extiende un pie o dos (30 a 60 cm) mas allá de los extremos de las traviesas a cada lado de la via, pero en materiales que no facilitan el desagüe, el extremo de la traviesa se deja descubierto a fin de facilitar la evaporación y evitar que se revuelva el balasto en tiempo de agua. En lugares húmedos el balasto debe ser mas hondo que lo acostumbrado.

*En vias de la clase C, úsese cualquier material que haga mejor via que la que haga el afirmado natural 1915*.

**Manejo, colocación, apisonado, limpieza y costo del balasto;** véase el epigrafe « Colocación y mantenimiento » ¶¶ 183, etc.

**30. Metros cúbicos por kilómetro de via.**

| Hondo total. | Via simple. Ancho superior en metros | | | | Via doble. Vias 3,96 m Aparte centro á centro. Ancho superior en ms. | | | | Hondo total. |
|---|---|---|---|---|---|---|---|---|---|
| | 2.74 | 3.05 | 3.35 | 3.66 | 6.40 | 6.70 | 7.01 | 7.31 | |
| cm | m c | m c | m c | m c | m c | m c | m c | m c | cm |
| 30 | 981 | 1066 | 1164 | 1236 | 2086 | 2180 | 2271 | 2364 | 30 |
| 35 | 1170 | 1270 | 1379 | 1488 | 2459 | 2568 | 2681 | 2784 | 35 |
| 40 | 1360 | 1483 | 1605 | 1741 | 2654 | 2996 | 3089 | 3212 | 40 |
| 45 | 1564 | 1706 | 1842 | 1987 | 3232 | 3371 | 3512 | 3650 | 45 |
| 48 | 1670 | 1815 | 1963 | 2116 | 3430 | 3577 | 3724 | 3871 | 48 |
| 50 | 1777 | 1931 | 2086 | 2240 | 3630 | 3785 | 3939 | 4093 | 50 |
| 52 | 1891 | 2048 | 2210 | 2372 | 3832 | 3994 | 4157 | 4325 | 52 |

## TRAVIESAS

### * Definiciones.

**31. Grietas.** Pequeñas hendiduras en la madera debido a su curación.
Traviesa entresacada. Una traviesa que no se ajusta a la especificación.
Traviesa tildada. Afectada por enfermedad del moho.

**Cara.** La superficie superior ó inferior de una traviesa.

**Traviesa de mediacaña.** La que tiene mas ancho en la cara inferior que la superior **Cabezal.** Un miembro o miembros de un juego usado para soportar el mecanismo para operar las agujas de un chucho.

**Traviesa de corazon.** Cuando tiene savia en una o dos esquinas solamente; la albura que mida $\geq$ 3 cm en cualquier esquina en lineas trazadas diagonalmente a traves del extremo de la traviesa. Vease tambien Traviesa toda de corazón.

**Traviesas intermedias.** Cualquier traviesa, usada entre traviesas de junta.

**Traviesa intermedia.** Una traviesa que se usa en una unión de carriles.

**Traviesa picada.** Hecha del arbol ciprés afectada de una enfermedad que forma excrecencia parecida al hongo, conocida localmente como *picada*.

* Asen Am Ingo F. C. † F.C. Pen ‡ F.C.U. Pacific. Vea tambien especificaciones, p. 853.

**Traviesa de poste.** Hecha de un arbol de tamaño tal que no puede dar mas de una traviesa de una sección. Labrada o aserrada en 2 caras paralelas.

**Traviesas en cuartos.** Hechas de un arbol de tamaño tal que puede dar solo cuatro traviesas en su seccion transversal.

**Traviesa de savia.** La que muestra mas cantidad de savia que la prescrita.

**Venteada.** Separacion de las fibras de la madera debido a la accion del viento.

**Traviesa de plancha.** Una traviesa hecha del corte exterior de un tronco.

**Traviesa costera.** Aserrada en la cara de arriba y de abajo.

**Traviesa hendida.** Hecha al hendir un arbol de tamaño tal que pueden hacerse de una seccion dos ó más traviesas.

**Traviesa de corazón.** Que no tenga savia. Vease tambien traviesa de corazón.

**Traviesa de reemplazo.** Cualquier traviesa que no sea de madera.

**Traviesa de chucho (1).** Traviesa de una pieza usada para soportar un desvio.

**Traviesa sangrada.** Hecha de un árbol al cual se le ha extraído la resina o trementina antes de tumbarlo.

**Traviesa tratada.** Una traviesa que ha estado sujeta a un procedimiento con objeto de que se conserve.

**Traviesa escasa.** Es cuadrada y mostrando parte de la superficie original del arbol en una o mas esquinas *.

## Generalidades.

### Dimensiónes, Colocación, etc.

**32.** Una traviesa actuando como una viga contínua, sirve para distribuir la presión del carril sobre el balasto que esta debajo de él, y sus dimensiones deben ser tales que permitan efectuar esta distribución con uniformidad tolerable. Cuando haya una traviesa mas apoyada en el medio de su longitud que cerca de los carriles, se dice que está *calzada al centro*. Una traviesa de suficiente largo trasmite una buena proporción de la carga al balasto distante de los carriles. El grueso es usualmente 6 ó 7 pulgadas (15 a 18 cm). Una traviesa de 6 pulgadas no es lo bastante rígida para tráfico pesado, especialmente cuando haya posibilidad de calzarla al centro; y propensa a rajarse con los clavos, teniendo en cuenta que éstos llegan hasta muy cerca de la cara inferior. Como la rigidez de una viga varia como el cubo de su altura, la resistencia a la flexión en una traviesa de 7 pulgs es á la de 6 pulgs como 343 es á 216, o como 100 es a 63. Rara vez se excede el peralto de 7 pulgadas (18 cm) excepto en maderas flojas.

**33.** Si mas de un 40 % del largo de un carril descansa en las traviesas, la proximidad de las traviesas estorba el apisonado. La luz mínima entre las traviesas es de 30 cm aproximadamente. Con la separación usual de traviesas un peralto de mas de 7 pulgs estorba el trabajo de apisonar. Con traviesas extremadamente anchas, no siempre se utiliza, este exceso como apoyo porque raravez tales traviesas son apisonadas del todo como las estrechas.

**34.** Cuando el clavo entra en la madera tangencialmente á las vetas es fácil rajar la traviesa.

**35.** Las traviesas deben ponerse con la cara del corazón abajo, a fin de que las vetas queden en tal posicion que dejen correr el agua.

**36.** Las traviesas de poste dan al carril un asiento de corazón que resiste mejor al rozamiento que la savia y los lados combados dan una superficie adicional de apoyo.

**37. Cortaduras por el carril.** La humedad que se aloja entre el carril y la traviesa tiende a desmejorarla, y la carga ayudada por el movimiento ondulatorio del carril rompe las fibras. Esta « cortadura » es la causa principal de que fallen las traviesas.

**38.** Las traviesas deben *protegerse* del desgaste mecánico por medio de *placas* y tornillos « tirafond ». Ascn. Am. Ingo F. C. Memoria de 1915. Volumen 16, p. 522.

* « El uso de traviesas tratadas se recomienda donde quiera que sea viable »*.

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* Ascn. Am. Igo F. C. † F. C. Pen. ‡ F. C. U. Pacific. Vease tambien especificaciones, p. 853.

(1) (*N. del T.*) Hemos adoptado el nombre de Chucho (usado en Cuba) para el conjunto de las agujas, etc., que se mueven al hacer un cambio.

## Maderas.

**39. Cedro.** Es liviano y resiste al deterioro bien; pero es aplastado fácilmente por el carril. En vias que tengan tráfico ligero, o donde esté protegido por sillas o platinas, las traviesas de cedro pueden durar de 15 a 20 años. Cedro sano viejo en traviesas dura tanto como el cedro fresco.

**Roble Blanco** es preferido para traviesas. Cuando está en sazón retiene los clavos bien; y resiste la acción cortante del carril mientras no se deteriora. Dura de 5 a 10 años; el promedio en muchas lineas, 8 años y ¼. Una traviesa de roble blanco en sazón de 7 pulgs × 9 pulgs, 8 pies largo (17,5 × 22,5 × 244 cm) pesa cerca de 185 lbs (85 kg).

**Roble roca** es menos sólido que el roble blanco. Su duracion es la misma poco mas ó menos.

**Pino amarillo del sur** en traviesas dura de 4 a 6 años en el sur; y de 8 a 12 años en el norte.

**Castaño** en traviesas dura de 7 á 9 años. Al sol se agrieta mucho; pero retiene los clavos bien y es de dureza mediana.

**Pino rojo**, es blando pero durable. Traviesas de pino rojo con platinas, duran de 10 á 14 años.

**Cerezo, Acacia y Nogal** en traviesas duran cerca de 8 años.

**40. Especificaciones.** * Maderas usables para traviesas sin *tratamiento preservativo*. La variedad de roble blanco, pino amarillo, todo de corazon de hoja larga, ciprés (que no sea blanco), pino rojo, cedro blanco, castaño, catalpa, acacia, nogal, cerezo negro *.

* Maderas preferibles que requieren *un tratamiento preservativo* aprobado por el comprador. La variedad del roble rojo, aya, olmo, meple, goma, *lodge pole*, pino amarillo del oeste, pino de Noruega, Carolina del Norte y otros pinos blancos, abeto rojo, pinabete, cicuta, alerce *.

**Traviesas para chuchos** (1). † Usables *sin tratamiento preservativo;* robles blancos, acacia negra, nogal negro, cerezo negro, pinos de hoja larga. Usables *unicamente despues del tratamiento preservativo;* robles rojos, aya, nogales americanos, meples duros, abedul 1913 †. Vease también bajo el epígrafe Desvios, Parte I, ¶ ¶ 128, etc.

## Dimensiones.

**41. Clasificación.** Dimensiones de **Asen Am Ingo F. C.** Manual, 1915, p. 59.

| * Clase. | Grueso en cm. | Ancho en cm. | Volumen de una traviesa, en decm cubs. Largo, 2.44 ms decm cubs. | Largo, 2.60 ms decm cubs. | Largo, 2.74 ms decm cubs. |
|---|---|---|---|---|---|
| * A | 17.50 | 25.40 | 113.00 | 117.00 | 124 00 |
| B | 17.50 | 22.86 | 99.00 | 105.30 | 111.30 |
| C | 17.50 | 20.32 | 88.00 | 93.40 | 99.00 |
| D | 15.00 | 22.86 | 85.00 | 90.00 | 95.40 |
| E | 15.00 | 20.32 | 75.50 | 80.10 | 85.00 |

***Exceso permitido:** — en grueso ½ pulg (1 ¼ cm); en ancho, 2 pulgs (5 cm); en largo, 1 pulg (2.5 cm). En traviesas de poste con costados redondos, y en traviesas de mediacaña, la cara puede ser < que lo arriba expuesto (< 15 cm), siempre que el área de la sección transversal < que la correspondiente a las dimensiones de la tabla, grueso < 15 cm *.

## Separación.

**42. Numero de traviesas** necesarias bajo un largo de carril: —

| | En vias de recorrido principal. | En vias laterales y ramales. | En patios y en desviaderos. |
|---|---|---|---|
| *Carril de* 33 *pies* (10,06). | 18 ≒ | 16 ≒ | 16 ≒ |
| ≒ Si las traviesas son < el modelo, | 20 ≒ | | |
| † Según el peso del tráfico, | 16 á 20 ≒ | 16 ≒ | 14 ≒ |
| *Carril de* 30 *pies* (9,15 m.). | 16 ≒ | 14 ≒ | 14 ≒ |
| Si las traviesas son < el modelo. | 18 ≒ | | |

* Asen Am Ingo F.C. † F.C. Pen. ≒ F. C. U. Pacific. Vea también Especificaciones, p. 853.

(1) Véase *N. del T.*, pag 858.

**Para carriles mas cortos** use un número proporcional de traviesas †.

**Distancia máxima permitida entre superficies de apoyo** †. Entre traviesas bajo uniones, 11 pulgs (28 cm); entre traviesas *intermedias* en vias de recorrido principal (18 pulgs) † 45 cm.

## Conservación de madera para traviesas.

**43.** Debido á la crecida escasez y costo de la madera para traviesas, se recurre (1) al tratamiento químico, ¶ ¶ 44, etc. (2), a la selvicultura, ¶ ¶ 54, etc., a la siembra de arboles para traviesas, (3) a la reducción del desgaste de las traviesas por medio de platinas, ¶ ¶ 135, etc., y (4) por el uso de traviesas de otros materiales, como acero ó cemento armado, ¶ ¶ 61, etc.

## Preservativos.

**44. La economia** producida por las traviesas tratadas no solamente consiste en su aumento de duración, sino en la reducción de jornales para renovarlas y la consiguiente alteración del lecho.

**45. Madera inferior.** El tratamiento químico y el uso de platinas, permite el uso de maderas que de otro modo no serian apropiadas y tambien el uso de la madera de savia muerta.

**46. Vapor.** Para traviesas del todo curadas al aire, un vacío preliminar $<$ 24 pulgs (60 cm) de mercurio mantenido $<$ 10 minutos *.

* Para traviesas no del todo curadas al aire una presión de 1,33 kg por cm cuad debe sostenerse durante 30 á 50 minutos y mantenerse (sin excederse) de 1 a 5 horas según la condición y estado de la madera. Un respiradero debe mantenerse abierto en el fondo del cilindro para desagüe y escape del aire y vapor condensado. Después de dar vapor, un vacío $<$ (24 pulgs) de 60 cm de mercurio al nivel del mar (y los grados correspondientes de vacío para otras alturas) mantenido por media hora. Entonces se admite el preservativo sin romper el vacío *.

* A las traviesas sin curar, que tienen que ser creosotadas, puede dárseles mas vapor o curarse en aceite de creosota caliente *.

**47. * Tratamiento al cloruro de zinc (Burnettizing).** Véase tambien *Conservación de la madera*, **mas adelante.** La solución se calienta á $<$ (140° Fahr) 60° centigrados, se admite sin reducir el vacío. La madera debe absorber 0.5 lb de cloruro seco soluble por (pie cub) (28 litros). La solucion debe ser lo suficientemente fuerte para que dé esta absorción y $>$ 5 %. La presion de vapor debe mantenerse en los serpentines de vapor de la máquina durante este tratamiento.

* El cloruro debe ser ligeramente básico, sin ácido libre; hierro $>$ 0,25% *.

* Muestras sacadas con barrenos deben observarse de vez en cuando, $<$ 6 traviesas tratadas en la misma remesa. Los agujeros deben taparse herméticamente con tarugos creosotados *.

* Las traviesas deben dejarse secar (a fin de que se endurezca su superficie) antes de colocarse en la via *.

**48. * Tratamiento al cloruro de zinc, tanino y cola.** Vease pag. 1180. Cloruro de zinc como se explica arriba. Se extrae la solución ó se desagua. Las traviesas se escurren 15 min. Solucion de 2 % de ácido tánico (6.67 lbs del 30 % del extracto de tanino en 100 lbs de agua) aplicado media hora bajo presión de 6.66 kg por cm². La solución de tanino se desagua y una solucion de 1 % de goma (2.1 lbs de goma que contenga un 50 % de gelatina en 100 lbs de agua) aplicada por media hora bajo la misma presion *.

**49. * Creosota.** (Vease pag. 1180). Alquitran aceite de creosota; a 38° centig, completamente liquido, y $<$ 1.03 de densidad; $>$ 3 % de agua; calentado a 170° centig. Presión, 6.33 kg por cm² *.

**50. * Emulsión de zinc creosotado.** La emulsión que contenga $<$ 10 % de creosota. Calentada á $<$ 60° centig, presion 6.33 kg cm². La madera que retenga un promedio de 0.4 lbs de cloruro seco soluble y de 1.25 a 1.5 lbs de creosota por 28 litros. La solución de cloruro de zinc a no mas de 35 %; que contenga $>$ 0.25 % de hierro.

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* Asen Am Ingo F. C, † F. C. Pen ‡ F. C. U. Pacific. Vea también Especificaciones, p. 853.

**51. * Doble injección de creosota y zinc.** La solución de zinc de no mas de 5 %; calentada á $\lessdot$ 60° centig. La madera que reciba 0.3 lbs de cloruro seco soluble por 28 litros. La solución se desagua inmediatamente, se admite aceite de creosota a 60° centig y se le aplica presion; 3 lbs de aceite por 28 lits. El agua que exceda del 6 % debe extraerse *.

**52. Duración.** Las traviesas de pino creosotadas duran cerca de 15 años, roble creosotado 18 años y de haya creosotada 20 años en línea principal; y pueden dar un tercio mas de servicio utilizándolos después en vías laterales.

**53. Curación, etc.** Las traviesas sin curar se *amontonan para curarlas;* se separan las traviesas verdes de las que están curadas en parte; todas descansando en durmientes tratados, con $\lessdot$ 15 cm de espacio, de aire, bajo la hilera mas baja; la hilera de arriba puesta en declive de modo que escurra el agua. Entre las pilas dejense pasillos de 1.20 m de espacio en una dirección y 30 cm en la otra. El grado requerido de curación se determinará por pruebas, buscando el peso al cual cada madera recibirá mejor el tratamiento; las traviesas que esten amenazadas de agrietarse deben protegerse por hierros dispuestos en forma de S, por tornillos ú otros medios. El labrado de azuela ó barrenado para poner platinas ó tirafondos debe hacerse antes del tratamiento.

**54. Selvicultura.** El cultivo de la madera debe emprenderse solamente por selvicultores expertos. Aquí solamente esbozamos los fundamentos de interés general para los ingenieros.

**55. Tiempo de tumba.** Cualquiera que sea la estación para la tumba de los árboles, las traviesas hechas de estos deben tener cuando menos seis meses para curarse antes de ponerse en la vía. No obstante, las estaciones para cortar y renovar traviesas caen tan cerca una de otra, que es difícil observar las traviesas sin guardarlas por más de un año.

* La madera es preferible cortarla de octubre á marzo *†. Septiembre a marzo †.

**56. Corteza :** Debe quitarse pronto después de la tumba para activar la evaporación y que se pudran (*souring*). Poniendo traviesas en la via con la corteza no solamente se activa el deterioro, sino que hace las traviesas más inflamables. * La corteza debe quitarse de todas las traviesas antes de su entrega á la Compañía. 1915 *.

**57. Los métodos de cortar** árboles y de cortar traviesas de los árboles son generalmente obra de derroche. Frecuentemente se pierde madera dejando tocones grandes innecesarios, y con esto se daña el árbol; hay árboles grandes que de sus ramas se puede sacar traviesas. Frecuentemente se pierde mucho cortando árboles que son suficientemente grandes para dar una traviesa de poste (una de cada sección); mientras que si se esperara 5 o 10 años, dos traviesas pudieran obtenerse de una sección.

**58. Propagación de la selvicultura.** La práctica antigua era plantar solamente árboles que crecieran rapidamente; pero se ha encontrado que otras consideraciones importantes (tales como la posibilidad de ser atacados por parásitos, baja resistencia o poca duración, etc.) pudieran aconsejar la selección de árboles de crecimiento lento. En cada region de terreno disponible, debe estudiarse qué especies pueden producirse mejor. Tambien es frecuente fomentar las tierras que tienen maderas imperfectas, sin tratar de hacer la completa selvicultura desde la semilla.

**59. Para evitar el fuego** y sus pérdidas considerables, se establecen rondas de vigilancia especialmente en tiempo seco.

**60. * Clavos fechados.** De hierro o acero, de galvanizado uniforme de ¼ pulg (6 mm) diám; 2 ½ pulg largo (63 mm) cabeza de $^{3}/_{8}$ (15 mm) diám. Con 2 números estampados de $^{1}/_{10}$ de pulg (1.5 mm) hondo y $^{5}/_{8}$ (9 mm) largo, designando el año. El clavo se introduce en la cara superior de cada traviesa tratada, á 25 cm de la parte de adentro del carril el dia que se coloque en la via. Cada traviesa tratada debe llevar estampado el año en ambas cabezas; esto se hará en la planta del tratamiento antes de tratar la madera *.

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* Ascn Am Ingo F. C. † F. C. Pen. ‡ F. C. U Pacific. Vea también Especificaciones, p. 780.

## Traviesas de reemplazo.

**61. Generalidades.** Una traviesa de reemplazo es « cualquiera otra traviesa que no sea de madera. » Memorias de la Asen Am Ingo F C 1915. Vol 16, pag. 522. Informe del Comité de Traviesas.

**62.** Debido al aumento rápido de la escasez de madera apropiada, las traviesas de reemplazo se están usando grandemente en Europa; y su uso se ha propagado en muchos ferrocarriles americanos, incluyendo algunos sistemas importantes.

El uso de la traviesa de reemplazo trae un cambio tan radical en la práctica de ferrocarril que se requiere una considerable experiencia en su implantación para determinar satisfactoriamente sus ventajas y desventajas relativas.

**63.** Las traviesas de reemplazo, debido a su sección uniforme en resistencia y en superficie de asiento (y especialmente las de acero), mantienen la vía en mejor alineación y superficie y el desgaste del carril es más uniforme. Las traviesas de madera, aun aquellas de la misma clase de madera, varían en su calidad cuando se ponen y se rinden desigualmente bajo una misma carga, y se deterioran desigual y rápidamente expuestas a la intemperie. Las traviesas de acero cuando están completamente balastradas se ruedan mas suavemente y la vía queda mas estable.

Las traviesas de reemplazo son mas pesadas que las de madera y mas costosas en principio. Se han experimentado dificultades en proveer medios satisfactorios para asegurar los carriles. Vease tirafondos, ¶ ¶ 107, etc.

Con el creciente uso de circuitos eléctricos de señal es importante producir el necesario aislamiento y en conjunto las traviesas de reemplazo ofrecen dificultades á este respecto.

Las traviesas de reemplazo alternadas con traviesas de madera precipitan el deterioro de estas últimas.

En igualdad de condiciones, el mayor peso de las traviesas de reemplazo tienden á dar mayor estabilidad a la vía.

Las traviesas de reemplazo son de acero o de concreto (usualmente armado) o compuesto (compuesto esencialmente de dos o mas materiales). Memorias de las Asen Am Ingo F. C., 1915. Vol. 6, pág. 522. Informe de la Comision de traviesas.

## Traviesas de acero.

**64. Tipos.** En Europa se han usado mucho las « **longitudinales** » o **durmientes** de acero, colocados debajo y paralelos al carril y de « tazas » invertidas de hierro fundido, colocadas opuestas unas á otras bajo los carriles á intervalos y conectados por traviesas extendidas al traves de la vía; pero en América la práctica y experiencia con traviesas de reemplazo se ha practicamente limitado a *traviesas cruzadas* con funciones similares a las que tienen las traviesas de madera.

**65.** Los primeros ensayos fueron con *traviesas de canal invertido* y todavía se estan usando en vias de tráfico ligero; pero la de *tipo de T invertida* llamada traviesa de acero de Carnegie, ¶ ¶ 70, etc., es la que más se usa.

**66.** Un fondo plano que se distingue de los que tienen proyecciones hacia abajo, facilita el apisonado.

**67. Las sujeciones** consisten generalmente en tornillos y presillas o en cuñas. Deben permitir el ajuste para la diferencia de ancho de vía y sección del carril.

**68. El costo** de las traviesas de acero es generalmente de 4 a 6 veces el de las traviesas de buen roble; pero las de acero deben tener mayor duración que la traviesa de roble y mucho más valor como desecho cuando no pueda utilizarse mas como traviesa.

**69. Ventajas y desventajas.** Las traviesas de acero *mantienen* la vía completamente á *cartabón.* Debido a su uniformidad en sección transversal, en resistencia y en superficie de asiento *mantienen la vía* en mejor *alineación y superficie*, y el desgaste del carril es mas uniforme que con traviesas de madera.

En la ausencia de datos suficientes de la experiencia su *duración* es dos o tres veces mayor que la de las traviesas de madera.

Debido á la pequeña sección transversal en metal las traviesas de acero necesitan *mas balasto* que las traviesas de madera de las mismas dimensiones exteriores.

Cuando las traviesas de acero han sido punzonadas con agujeros cuadrados, se encuentra que estos agujeros se empiezan á *rajar* por las esquinas de dichos agujeros.

Las traviesas de acero estan expuestas á *deteriorarse por el óxido*, especialmente en lugares húmedos, como túneles y bajo el goteo de la solucion de sal de los carros refrigeradores. Se han usado *cubiertas de proteccion*, pero son difíciles de aplicar a las traviesas que están en servicio.

La traviesa toda de acero es un substituto satisfactorio con tráfico pesado a media velocidad. Es durable. La alineación y superficie puede mantenerse. (Véase costos de mantenimiento, ¶ 73.) Tiene suficiente elasticidad y puede aislarse. La ligazón es generalmente inadecuada. Memorias de la Asen Am Ingo F. C. 1912, Vol 13, Comisión de Vía.

**70. Carnegie.** La fig. 4 representa la traviesa de acero de la Cía de Acero Carnegie. *A*, plano de toda la traviesa; *B*, elevación lateral; *C* y *D*, secciones transversales o mayor escala en *mn* y *op*, fig. *A*, respectivamente; *E* y *F* muestran la planta y elevación del carril y fijación tipica cuando el balasto es roca ó cascajo gordo ó en pendientes que tengan un tráfico pesado; la pestaña inferior de la traviesa puede rizarse como se muestra en *B* y *D*; pero este rizado obstrucciona el apisonado o calzada del balasto. La traviesa puede agujerearse para recibir dos pesos distintos de carril.

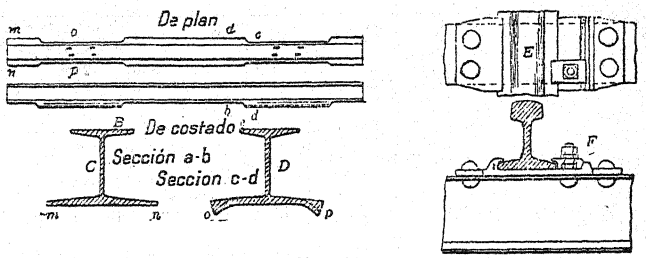


**Fig. 4.**

Dimensiones en pulgs., etc.

(*N. del T.* — Damos entre paréntesis las medidas métricas equivalentes en : Kgs por m; Kgs; cm y mm).

| Sección. | Lbs por pie. | Traviesa 8.5" largo, peso lbs. | Peralto. (21,60 cm). | Ancho pestaña inferior. | Ancho pestaña superior. | Grueso alma. |
|---|---|---|---|---|---|---|
| M 24 | 9.5 (14.15 kgs/m) | 81 (36.74 kg) | 3 (7.62 cm) | 5 (12.70 cm) | 3 (7.62 cm) | 1/4 (5.16 mm) |
| M 25 | 14.5 (21.60 kg/m) | 123 (60.32 kg) | 4 1/4 (10.66 cm) | 6 (15.24 cm) | 4 (10.16 cm) | 1/4 (6.35 mm) |
| M 21 | 20 (29.80 kg/m) | 170 (77.11 kg) | 5 1/2 (14.00 cm) | 8 (20.32 cm) | 4 1/2 (11.43 cm) | 1/4 (6.35 mm) |
| M 28 | 27.8 (41.42 kg/m) | 236 (107.40 kg) | 6 1/2 (16.50 cm) | 10 (25.40 cm) | 5 (12.70 cm) | 5/16 (7.94 mm) |

La *fijación del carril* deja 3 mm de juego vertical y horizontal en el carril.

**71. Aislamiento.** Este consiste en 1/8 pulg. (3 mm) de fibra del cual hay (1) una placa entre las traviesas de acero mostrada en la fig 4 F y la parte superior, de la traviesa, (2) una arandela bajo la arandela del remache, y (3) un buje alrededor de la espiga del remache.

**72. La primera** traviesa de acero Carnegie fué puesta en el Ferrocarril Bessemer y Lake Erie (que controlaba Carnegie) en 1904. Estas traviesas han sostenido bien el tráfico pesado de carga a velocidades moderadas. Sin embargo a velocidades altas su rigidez no ha sido satisfactoria.

Debido a lo restringido del movimiento entre el carril y la traviesa, se ha encontrado que *reduce la trepidación*.

*Cuando se renuevan*, se ha visto que una cuadrilla de peones puede remover, cambiar mas de estas traviesas que de las de madera en un tiempo dado.

Como *protección contra el óxido,* se han sumergido en alquitrán caliente, a un costo de 5 cts una.

*El aislamiento* (Véase ¶ 71). Es bueno mientras está en buen estado; pero a los 3 ó 4 años, las placas de fibra se gastan y los tornillos se aflojan en consecuencia. Véase ¶ 73.

Muchas se han quitado al ponerse instalaciones automáticas de señales.

*El costo* puede calcularse como de $2.00 a $2.60 por traviesa, con 20 cts adicionales por traviesa para la fijación. Esto casi es más que el doble del costo de la traviesa de madera.

**73.** *En el ferrocarril de Pittsburgh y Lake Erie,* las traviesas de acero Carnegie, con fijación de tornillo y grapa, fueron puestas en agosto de 1907 sobre un balasto de piedra calcárea en una extension de 1,342 mts de línea principal de carga y con velocidades de 49 km por hora. El costo de conservación fue como sigue :

| | 1908 | 1909 | 1910 | 1911 | 1912 | Total. | $ por mill. por año. | Por km por año. |
|---|---|---|---|---|---|---|---|---|
| Roble blanco..... | $ 417 | 95 | 128 | 116 | 94 | 850 | 204 | 128 |
| Acero Carnegie... | $ 280 | 153 | 428 | 184 | 348 | 1.393 | 334 | 209 |

En 1911 la via con traviesa de roble blanco fue renovada una vez; la via con traviesa de acero tres veces. Las 17 primeras placas de fibra se quitaron por estar cortadas por la base del carril; 20 tornillos se aflojaron en las presillas de las traviesas. En 1912 el costo comprende $102 por renovación de 1.000 placas de fibra. « Prácticamente no señalamos dificultad de ninguna clase. »

*Otras líneas* han dado informes de menos dificultades en mantenimiento que con traviesas de madera.

**74.** *La pérdida de peso* anual debido al óxido, frotación, etc., ha sido variablemente informado por diferentes ferrocarriles variando de (0.7 lbs), 0.317 kg por traviesa 0.40 % en escorias o cascajo, a (4.5 lbs) (2.041 kg) por traviesa, 0.2.55% en cenizas. Después de siete años de servicio han mostrado poca corrosión externa. De las traviesas puestas en el ferrocarril Pittsburg Shawmut Northern en 1907 se informó que daban « notable satisfacción » en 1913 y aparece que en 1915 estaban « fallando rápidamente. »

Traviesas de acero Carnegie, puestas en el F. C. Erie en abril 1909, se estaban oxidando en 1913 y « *deteriorándose en aire que contenía sal* ».

**75.** En el F. C. Duluth Missabe & Northern dieron gran resultado como reemplazantes de traviesas de madera que dieron mucho que hacer en *terreno pantanoso.*

**76.** Donde se ha usado la *fijación por cuñas* se han experimentado dificultades con materiales compactos, densos.

**77.** En el ferrocarril Lake Shore & Mich Southern; cerca de Sandusky, con *tráfico pesado a gran velocidad* en 1905-6 se pusieron traviesas de acero Carnegie con un bloque de madera asegurado en la parte de arriba de la pestaña superior debajo de cada carril en cada traviesa con un costo completo de $2.25 á $2.50 por traviesa. Los bloques quedaron asegurados con tornillos que pasaban al traves de abrazaderas de metal con forma de U bajo la traviesa. De este modo se aprisiona la fibra de la madera, aumentando la resistencia al tiro de los clavos. Hasta 1908 estas traviesas no dieron mas trabajo con respecto al aislamiento que si fueran de madera, pero mas tarde hubo dificultades con el aislamiento y todas las traviesas se cambiaron en 1915 debido á que se ablandó la madera.

**78.** El ferrocarril L. S. & M. S., cerca de Toledo, en 1907 puso traviesas de acero a las cuales se les quitaron las pestañas ó bordes superiores y se les atornillaron dos bloques de madera, uno á cada lado al traves del alma de la traviesa debajo de cada carril y descansando en las pestañas de la traviesa : los *carriles estaban clavados a los bloques de madera* como si fuera a una traviesa de madera. En 1910 se informó que estaban « en buen estado » y que « sostenían buena alineación y superficie como cualquier via en el ferrocarril L. S. & M. S. ».

**79.** *La traviesa de sección de canal ó artesa invertida,* debido a sus projecciones hacia abajo es desfavorable para el apisonado. De aquí el que se haga de poco peralto y así queda la traviesa débil verticalmente. Todavía se usa en tráfico ligero, como en vias industriales de construcción y vías de minas, etc.

**80.** La fig. 5 muestra dos formas de la traviesa de acero acanalada de Carnegie con sus fijadores.

Se hacen de las siguientes secciones, medidas en pulgadas.

(*N. del T.* — Damos entre parentesis las medidas métricas equivalentes en : kgs por metro; kgs, cm y mm.)

| Sección. | Lbs por pie. | Peralto. | Ancho mayor. | Grueso de l'Alma. | Fig. |
|---|---|---|---|---|---|
| M 19 | 2.5 | 5/8 | 4 | 9/64 | 5 *a* |
| | (3.72 kgs/m) | (15.87 mm) | (102 mm) | 3.57 mm) | |
| M 26 | 3.2 | 13/16 | 4 15/16 | 1/8 | 5 *a* |
| | (4.77 kgs/m) | (20.63 mm) | (126 mm) | (3.17 mm) | |
| M 20 | 6 | 2 | 6 | 3/16 | 5 *b* |
| | (8.94 kgs/m) | (51.6 mm) | (153 mm) | (4.76 mm) | |
| M 27 | 9 | 2 1/4 | 7 | 1/4 | 5 *b* |
| | (13.41 kgs/m) | (58 mm) | (178.60 mm) | (6.35 mm) | |

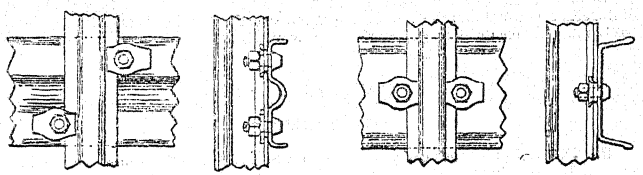

Fig. 5.

*Para trabajos de mina* las grampas son prensadas ó remachadas a la porción horizontal de la misma traviesa.

En 1900-1901 las traviesas de acero Carnegie *mas pesadas*, pesando (199 lbs) 91,5 kg cada una fueron instaladas en el ferrocarril *de Bessemer & Lake Erie*. Después de 8 años de servicio habían perdido (2.5 lbs) 1.15 kg por año y por traviesa = a 1.25 % por año y no mostraban bajo los carriles un desgaste apreciable.

## Traviesas de concreto y compuestas.

**81. Ventajas y desventajas.** Las traviesas de concreto y compuestas son generalmente mas pesadas que las de acero para el mismo servicio, pero mas costosas. Se necesitan meses para el preciso fraguado del concreto. Las superficies cóncavas, si no están protegidas por metal son vulnerables a los golpes como ocurre en los descarrilamientos. Se dificulta proveerla de fijadores satisfactorios. Su peso y lo basto de su superficie le favorecen para la estabilidad lateral, y la ausencia de proyecciones hacia abajo favorecen el apisonado y su cambio. El mismo concreto facilita un aislamiento tolerable para circuitos de señal; pero hay peligro de contactos al traves del metal de refuerzo.

**Generalmente han fallado** por razón de su fragilidad, peso excesivo (que hace su manejo dificultoso) y deterioro del relleno cuando éste es de masilla de asfalto.

Memorias de la Asen Am Ingº F. C., 1912, vol 13. Informe del Comité de Traviesas.

**82.** Las siguientes **relaciones de experiencias** con traviesas compuestas pueden ser de valor, pues indican lo que se ha hecho y lo que debe evitarse al hacer estas traviesas.

**82 a.** Concreto de 1 : 2 : 3 reforzado con tubos longitudinales de hierro dulce de 2 1/4 pulg (57 mm) y una lámina de tela alambrada pesada de 5 × 8,5 pulgs (13 × 21 cm) bajo cada carril. Peso 147 kg; 16,1 kg por fijadores (adicional) $1.50 a $1.75 cada uno.

**82 b.** Trapezoidal con extremos en talud. Reforzado con espirales de malla de alambre nº 16, ½ pulg (12 mm) de paso y con cabillas de ½". Costo $1,50 cada una En el patio de Scully Lineas del Oeste de Pennsylvania se instalaron en 1906 con

balasto de cenizas con tráfico pesado y lento. Los carriles clavados al traves de las placas de traviesas y bloques de madera; 16 traviesas por cada (30 pies) 9.15 m de carril de (85 lbs) 39 kg se desmenuzaron bajo los carriles, se aflojaron los tornillos. La trepidación hizo girar las presillas de fijación aflojando el carril.

**82 c.** Un carril desechado invertido de (65 lbs) 30 kg embebido en concreto. Extremos en talud. Estrechado al centro de la via, á un ancho de la base del carril. Esta sección estrecha evita el movimiento lateral, $0.95 + los fijadores y (180lbs) 82,8 kg, de carril de desecho. L S & M S. Todavía en la via en buen estado. Véase la gaceta de F. C., pag. 594, mayo 1.º 1908.

El ferrocarril Pg Ft W & C. Colocados en1903. Se cambió todo en 1906. El concreto se rompió y separó del acero.
Colocados en 1903. Tráfico ligero de media velocidad.
Se rompió. Se quitaron en 1903.

« La traviesa de concreto que ha tenido más éxito de las que he visto, y en condiciones favorables hace una via aceptable. » Sostiene los descarrilamientos bien. El balasto de roca puede ser demasiado rígido. Cascajo puede ser mejor. Véase la Memoria de 1907. Vol 8, p 465 de la Asen Am Ingo F. C. y mantenimiento de Via.

El Boletín 108, de Feb 1909, p 174 de la A. A. I. F. C. y M. de V. da unos informes favorables del F. C. de L. S. & M. S. y desfavorables de las Líneas de Pennsylvania, Enlace de Chicago, y Lake Erie & Western.

## CARRILES, etc.

### Generalidades.

**83. Peso.** Los carriles que pesan de 75 á 85 lbs por yarda (de 36,75 a 41,65 kg por metro) son de uso común en líneas de tráfico pesado; y carriles de 90 y 100 lbs (de 44 y 49 kg por metro) se usan donde el tráfico es extremadamente pesado. Carriles de más peso todavía se han laminado, pero algunas dificultades en su manufactura impiden que su uso se extienda.

**84. Largo** (standard) modelo —, * 33 pies á 60° Fahr (10,065 m á 15,6° C) 1915 *. † 33 pies á 60° Fahr el 10 % de la orden se acepta en largos de 30, 27.5 y 25 pies (9,15; 8,39 y 7,62 m). Se permite una variación de $\frac{1}{4}$ (6 mm) en los largos especificados, 1912. † Carriles con un largo de 60 pies (18,30 m) se usan algunas veces en cruceros de carreteras, etc., para evitar que hayan juntas bajo los tablones y pavimento donde serian inaccesibles al apisonado. También en general se usan para reducir el número de empates.

### Cómo se conserva el material.

**85. Desgaste.** Este es más rápido en las *curvas* cerradas donde las pestañas de las ruedas frotan contra el costado de la cabeza del carril. Disminuye á medida que disminuye lo cerrado de la curva. En curvas el desgaste de la parte superior del carril es muy pequeño comparado con el del costado de la cabeza del carril.

**86. En las tangentes.** El desgaste ocurre principalmente en la parte superior de la cabeza del carril, y es debido al esfuerzo de tracción de las ruedas motrices de las locomotoras y al resbalamiento de las ruedas de distintos diámetros en un mismo eje.

**87.** *En pendientes y en estaciones,* debido al necesario aumento de fuerza de tracción que se desarrolla para arrancar y parar, el desgaste del carril es mayor que en líneas a nivel o entre estaciones.

**88.** El *límite permitido* de desgaste en la parte superior de la cabeza del carril es generalmente de $\frac{3}{8}$ pulg (9 mm), pero, antes de llegar á este límite, el carril en muchos casos debe cambiarse debido a las asperezas de su superficie de contacto con las ruedas, causado (1) por el desgarramiento del metal, o (2) por deformación y excesivo desgaste de los extremos del carril, especialmente en el extremo que recibe el tráfico en doble vía.

**89. Duración de los carriles.** Está estimada mas o menos en 100 a 250 millones de toneladas en tráfico ,despendiendo en parte de su alineación pendiente y condiciones de la vía.

**90.** *Bajo tráfico pesado o en curvas cerradas*, los carriles deben volverse de un extremo al otro o renovarse una vez en dos años. En líneas de tráfico rápido, la renovacion de los carriles en las curvas debe en algunos casos hacerse en períodos menores de un año.

**91.** « **Corrugacion** ». En las cabezas de los carriles es asunto de mucha molestia y costoso, especialmente en curvas y en vias cruzadas por material rodante eléctrico. Consiste en una serie de desgastes parciales en la cabeza del carril, estos como lunares estan separados usualmente a una distancia de centímetros. No se ha llegado a un acuerdo en cuanto a la causa de esto a pesar de numerosas teorias planteadas. Una de las teorias mas aceptadas es la de que, al pasar una curva una rueda ó la otra de un par debe resbalar longitudinalmente; este resbalamiento puede ocurrir por tirones, y el marco de la carretilla donde van estas ruedas y el eje se deforman alternativamente para volver á tomar su forma.

El bajo coeficiente de fricción por resbalamiento daria lugar a permitir un deslizamiento apreciable (y por consiguiente excesivo desgaste) a lo largo de la parte superior del carril antes de que una rueda agarre junto con la otra. Entonces cuando el rodaje de ambas ruedas se restablece, el gran coeficiente de fricción estática evitaria el resbalamiento hasta que lo recorrido sobre la curva sea bastante para obligar a resbalar una vez más. Las ruedas cuando pasan una curva, deben también resbalar lateralmente, y este, lo mismo que el resbalamiento longitudinal, puede ocurrir por tirones simultaneos de ambos resbalamientos, y de estos combinados probablemente resulta un resbalamiento diagonal. Hay otra teoría basada sobre todo en la observación de que las corrugaciones varian con la reparación de las traviesas, etc., atribuyéndose estas corrugaciones a las vibraciones de la vía y del terreno.

## Composiciones, Requisitos, etc.

### Orden de Especificaciones.

**R**, Asociación Americana de Ferrocarriles. « Práctica Recomendada » propuesta por la Comisión en Tipo de Carriles y Ruedas, marzo 28, 1908.

**W**, Asociacion Americana de Ingenieria de Ferrocarril y Mantenimiento de Vía. « Manual de Práctica Recomendada », 1907.

**M**, Sociedad Americana para Prueba de Materiales. Especificaciones Tipo adoptado sept. 1, 1907, Memoria de 1907, vol. VII, p. 44.

**C**, Sociedad Americana de Ingenieros civiles, Especificaciones recomendadas por la Comision Especial en Secciones de Carril, julio 9, 1907, enmendada Enero 1908, Memorias, agosto 1907, vol. XXXIII, nº 6, p. 290; feb. 1908, vol. XXXIV, n.º 2, p. 85.

**A**, Todas = **R, W, M, C.**

### Composición.

**Composición, R.** Asociación Americana de Ferrocarriles.

| **Lbs por yarda.** | **60** | **70** | **80** | **90** | **100** |
|---|---|---|---|---|---|
| Kg por metro.... | 29.4 | 34.3 | 39.2 | 44.1 | 49 |

### Bessemer.

| | | | | | |
|---|---|---|---|---|---|
| Carbon % *..... | 0.37–0.47 | 0.40–0.50 | 0.43–0.53 | 0.45–0.55 | 0.46–0.56 |
| Manganeso %... | 0.80–1.10 | 0.80–1.10 | 0.80–1.10 | 0.85–1.15 | 0.90–1.20 |

Fósforo % * $\geq$ 0.10; sulfuro % $\geq$ 0.075; silice % 0.10 a 0.20.

### Martin Siemens.

| **Lbs por yarda.** | **60** | **70** | **80** | **90** | **100** |
|---|---|---|---|---|---|
| Kg. por metro... | 29.4 | 34.3 | 39.2 | 44.1 | 49 |
| Carbon % †..... | 0.50–0.60 | 0.55–0.65 | 0.60–0.70 | 0.65–0.75 | 0.70–0.80 |

Manganeso % 0,75 a 1.00 para todos los pesos.

Fosforo % † $\geq$ 0.04; sulfuro % $\geq$ 0.06; silice % 0.10 a 0.20.

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* Con menos fósforo, el carbon debe aumentarse en proporción, **R.**

† Con mas fósforo, el carbón debe de reducirse en proporción, **R.**

**Composición, M.** Sociedad Americana para Prueba de Materiales.

| **Lbs por yarda.** | **50 a 59** | **60 a 69** | **70 a 79** | **80 a 89** | **90 a 100** |
|---|---|---|---|---|---|
| Kgm por metro.. | 24.8 a 29.3 | 29.7 a 34.2 | 34.7 a 39.2 | 39.7 a 44.1 | 44.6 a 49.6 |

**Bessemer y Martin Siemens.**

| | | | | | |
|---|---|---|---|---|---|
| Carbon % ‡..... | 0.35–0.45 | 0.38–0.48 | 0.40–0.50 | 0.43–0.53 | 0.45–0.55 |
| Manganeso %... | 0.70–1.00 | 0.70–1.00 | 0.75–1.05 | 0.80–1.10 | 0.80–1.10 |

Fosforo % $\geq$ 0.10 †; silice % $\geq$ 0.20.

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**Composición, W, C.** Asen Am Ingo F. C. y Mant de Vía, Soc Am de Ingo Civ.

| **Lbs por yarda**..... | **70 a 79** | **80 a 89** | **90 a 100** |
|---|---|---|---|
| Kg por metro..... | 34.7 a 39.2 | 39.7 a 44.1 | 44.6 a 49.6 |

**Bessemer**

| | | | |
|---|---|---|---|
| Carbon %..... | 0.50–0.60 | 0.53–0.63 | 0.55–0.65 |
| Manganeso %..... | 0.75–1.00 | 0.80–1.05 | 0.80–1.05 |

Fosforo % $\geq$ 0.085; sulfuro % $\geq$ 0.075; silice % $\geq$ 0.20.

**Martin Siemens Básico.**

(Determinación Química completa para cada hornada.)

| **Lbs por yarda**..... | **70 a 79** | **80 a 89** | **90 a 100** |
|---|---|---|---|
| Kg por metro..... | 34.7 a 39.2 | 39.7 a 44.1 | 44.6 a 49.6 |
| Carbon %..... | 0.53–0.63 | 0.58–0.68 | 0.65–0.75 |

Manganeso % $\geq$ 0.90; fósforo % $\geq$ 0.05; sulfuro % $\geq$ 0.06; silice % $\geq$ 0.20.

**Manufactura.**

**Lingotes** se mantienen verticalmente (en los hornos con hornallas de calentar **W, M, C**) hasta que estén listos para laminarse, o hasta que el metal interior haya tenido tiempo de solidificarse, **A**; el uso de lingotes de « sangria » (bled) § esta prohibido, **A.**

« **Descarte** », ¶ serán cizallados del extremo de la masa formada arriba del lingote lo suficiente para garantizar carriles sanos, **R**; sujeto a convenio, **M**; $\leq$ 25 %, mas si es necesario hasta que el acero aparezca sólido, **W, C.**

**Contracción.** El número de pasadas y la velocidad del tren de laminar ha de ser tal, que el carril al dejar los laminadores en el pase final, su temperatura sera $\geq$ que la requerida; en las sierras calientes, un margen de contraccion para carriles de 33 pies (10.06 m) 100 lbs (49 kg por m) de 6.5 pulgs (16.25 cm) **R**; 7 5/16 pulgs (18.6 cm) **M**; 6 7/16 pulgs (18.80 cm) **W, C**; 1/8 pulgs (3mm) menos por cada 10 lbs de menos en sección, **R**; 1/16 pulg (1.5 mm) menos por cada 5 lbs, **W, M, C**; margen disminuido 0.01 pulg (0.25 mm) **M** (1/90 pulg **W, C**) por cada segundo de tiempo trascurrido entre el momento de dejar el laminador que lo termina y el aserrado, **W, M, C.**

**Enfriado.** Los carriles no deben enfriarse artificialmente entre el laminador de « entrada » y el de « terminacion » **R**; o después de haber dejado el laminador que lo termina, **R, C**; entre el pase de término y las sierras calientes, **W, M**; o aguantado antes de aserrarse para reducir su temperatura, **R, C.**

**Marcado.** El nombre del fabricante, peso del carril, mes y año de manufactura laminado en letras de relieve debe aparecer en el alma; el número de la fundicion estampado en el alma donde no sea cubierto por las bridas, **A**; tambien « **A** » en carriles de la parte de arriba del lingote; entonces « B » « C », etc., consecutivamente; « **A** » se omite cuando se descarta la parte de arriba del lingote $\leq$ 20 %; « **A** » carriles embarcados en carros separados, **R**; carriles obtenidos por sistema Martin Siemens deben marcarse « M S » **R.**

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‡ El carbón puede reducirse ajustándose á las condiciones locales, **W.**

† Con más fósforo, el carbón debe de reducirse en proporción, **R.**

§ Lingotes del interior del cual se ha escapado el acero líquido, **R.**

¶ Metal de la parte de arriba del lingote, y si es cortado del bloque ó del carril, **R.**

**Enderezado.** Los carriles en camadas calientes deben protegerse del contacto del agua o nieve, **R**; carriles que varian > 5 pulgs (12.5 cm) **M, C** ( > 3 pulgs) (7.5 cm) (**W, R**) de una linea recta en cualquier dirección, al llegar a la máquina de enderezar en frio o que tengan « retorcidos » cortos se clasificarán como de 2ª calidad, **A**; y asi marcados, **R**; y asi estampados **W, M, C**; soportes de carriles en las prensas de calibrar < 42 pulgs (1.05 m) aparte **R, W , M**; los soportes que tengan superficies planas, **R**; los carriles terminados deben ser rectos en linea y en superficie y suaves en las cabezas, el enderezado final se hará en frío, los extremos serán aserrados a escuadra, variaciones > 1/32 pulg (0.75 mm); las rebabas de la sierra se quitarán y los extremos se limpiarán antes de embarcarlos, **A.**

### Variaciones permitidas.

**En sección.** En altura 1/32 pulg (0.75 mm) **R**; 1/64 pulg (0.4 mm) menos, 1/32 pulgs (0.75 mm) mayor, **W, M, C**; en ancho de base 1/16 pulg (1.5 mm) **A**; el carril debe ajustarse a las barras de la brida, **R, W, C.**

**En peso.** 0.5 % de toda la orden; los carriles se aceptan y pagan por el peso actual, **A.**

**En largo.** 0.25 pulg (6 mm). Largo tipo, 33 pies (10.06 m) **A**; el 10 % de la orden aceptada en largos de 30, 28, 26 y 24 pies (9.15; 8.54; 7.93; 7.32 m) **R**; en largos variando por pies pares, hasta 27 (8.23 m) **W, M, C**; todos los carriles de 1ª < 33 pies (10.06 m) deben pintarse de verde en un extremo « **W, M**; en ambos extremos, **R**, C.

### Pruebas.

**Prueba de caída** de cabeza. (« Tup ») 2,000 lbs (920 kg), **A**; radio de la cara de golpeo, 5 pulgs (12.5 cm) **R**; > 5 pulgs **W, M, C**; bloque del yunque, 20.000 lbs (9200 kg) **R**; < 20.000 lbs, **W, M, C**; los soportes que formen parte, o que estén bien asegurados al yunque, **A**; pieza de prueba, largo, < 4 pies (1.22 m) > 6 pies 11.83 m) **A**; pieza de prueba se tomará de la parte de arriba del lingote, **A**; colocado con la cabeza hacia arriba, en soportes (5 pulgs (12.5 cm) radio superior, **R**), 3 pies (91 cm) aparte, **A**; una prueba de caída de cada fundicion, **R, W,** (para Bessemer, **C**); cada quinta fundicion, **M**; dos de cada fundicion para Martin Siemens básico, **C**; altura de caida :

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| lbs por yarda. | 60-80 | 90-100 | 45-55 | 55-65 | 65-75 | 75-85 | 35-100 |
| Kg por metro. | 29,7-39,7 | 44,6-49,6 | 22,3-27,3 | 27,3-32,2 | 32,2-37,2 | 37,2-42,2 | 17,4-49,6 |
| Caída en ms. | 4,88 | 5,20-5,50 **R** | 4,50 | 4,88 | 5,20 | 5,50 | 5,80 **M** |

**Temperatura** de las piezas de prueba entre 32° y 100° F, (0° y 38° C.), **R.** El informe que especifique la temperatura atmosférica, **W, M, C.**

### Aceptación y rechazo.

Si las piezas al romperse no muestran picaduras, cavidades, (pipe) o defectos físicos, todos los carriles procedentes de esa fundicion son rechazados, **R.**

Si la pieza rota muestra cavidad o defecto físico, el carril de arriba de cada lingote de esa hornada es rechazado, y el inspector selecciona un pedazo de un carril que no sea de la parte de arriba del lingote. Si esta pieza rompe, el resto de los carriles de la hornada son rechazados; si no son aceptados, **R.**

Si la primera pieza de prueba no rompe, se prueba hasta su destruccion. Si entonces muestra cavidades (*pipe*) o defecto físico, el carril de arriba de cada lingote es rechazado, el resto aceptado. Si no, todos los carriles de la hornada son aceptados, **R.**

Si la pieza de prueba rompe, se hacen dos pruebas adicionales de otros carriles (tomados del lingote de arriba, **W, M, C**) de la misma hornada. Si alguna de las pruebas adicionales falla, todos los carriles de la hornada son rechazados. Si no, todos son aceptados, **W, M, C.**

**92.** Los carriles **de acero manganeso** fundidos y laminados, y carriles de otros aceros especiales, se usan en curvas de lineas de tráfico rápido en las ciudades. El acero manganeso se usa grandemente en ranas de cruceros y chuchos. Algunos de los primeros carriles de acero manganeso fundido usados en las curvas del subterraneo de Boston, aunque diez veces mas costosos que los carriles ordinarios, duraron veinte veces y mas tiempo.

Donde se usa un carril durable, la renovación de carriles son menos frecuentes. En las curvas de Boston mencionadas, los carriles ordinarios tenían que renovarse dos veces en tres meses aproximadamente.

## Sección de carriles.
## Secciones tipo de la Sociedad Americana de Ings Civiles.

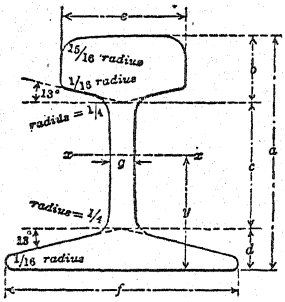


**Fig. 6.**

(*Nota del T.* — Para la concepción de la figura en sistema métrico : 15/16 = 23.8 mm; 1/16 = 1.5 mm; 1/4 = 6.25 mm.)

**93.** Fig. 6. Transacciones de junio 1893, vol. 28, No 6, pags. 425, etc. Informe final de la comision sobre sección de carriles.

En todos los tamaños.

Radio de la parte superior de la cabeza = al radio del lado del alma = 12 pulgs (30 cm).

Otros radios (en pulgs) y angulos, como se muestran en la fig.;

Ancho de la base *f* = altura del carril, *a;*

Distribución del area de la sección transversal; en la cabeza, 42 %; en el alma, 21 % en la base, 37 %.

Las siguientes propiedades de los carriles de la Asen de Ing Civiles americanos están tomadas del Libro de Bolsillo de Carnegie : —

*A* = area de la sección transversal;
*y* = altura del centro de gravedad sobre la base;
*I* = momento de inercia, pag. 490;
*X* = modulo de la seccion, pg. 489; } aprox. en el eje *x*.....*x*.
*r* = radio de giro = *I*/*A*.

Para secciones de carril tipo de la Sociedad Americana de F. C., veanse las dos pags siguientes.

Para los de la Asen Am de Ingo de F. C., vease ¶ 94.

Para los requisitos químicos y físicos de los carriles, véanse páginas

### Los números en tipo grueso dan las Dimensiones en 64 avos $^1/_{64}$ de pulg.

(*A* en pulgs cuad; *y* y *r* en pulgs. También véase ¶ y ||, y **nota** pag. 871).

| Peso del carril lbs por yarda | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 | 85 | 90 | 95 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *a* = *f*..... | **224** | **236** | **248** | **260** | **272** | **284** | **296** | **308** | **320** | **332** | **344** | **356** | **368** |
| *b*..... | **65** | **68** | **72** | **75** | **78** | **82** | **86** | **91** | **96** | **99** | **102** | **105** | **109** |
| *c*..... | **119** | **126** | **132** | **139** | **145** | **152** | **158** | **163** | **168** | **176** | **183** | **191** | **197** |
| *d*..... | **40** | **42** | **44** | **46** | **49** | **50** | **52** | **54** | **56** | **57** | **59** | **60** | **62** |
| *e*..... | **120** | **128** | **136** | **144** | **152** | **154** | **156** | **158** | **160** | **164** | **168** | **172** | **176** |
| *g*..... | **25** | **27** | **28** | **30** | **31** | **32** | **33** | **34** | **35** | **36** | **36** | **36** | **36** |
| *A*, pulg cuad... | 3.9 | 4.4 | 4.9 | 5.4 | 5.9 | 6.4 | 6.9 | 7.4 | 7.8 | 8.3 | 8.8 | 9.3 | 9.8 |
| *y*, pulgs..... | 1.7 | 1.8 | 1.9 | 2.0 | 2.1 | 2.2 | 2.2 | 2.4 | 2.4 | 2.5 | 2.5 | 2.7 | 2.8 |
| *I* ¶..... | 6.6 | 8.0 | 9.8 | 11.9 | 14.5 | 16.9 | 19.6 | 22.9 | 26.2 | 30.0 | 34.0 | 38.6 | 43.8 |
| *X*=..... | 3.6 | 4.2 | 4.9 | 5.8 | 6.7 | 7.4 | 8.2 | 9.3 | 10.0 | 11.0 | 12.0 | 13.3 | 14.6 |
| *r*, pulgs..... | 1.30 | 1.35 | 1.42 | 1.49 | 1.58 | 1.63 | 1.70 | 1.78 | 1.83 | 1.90 | 1.97 | 2.06 | 2.1 |

(*N. del T.* — Por tratarse de unas tablas de fabricantes americanos de carriles, útiles para pedidos de los mismos tal como están, creemos más conducente dejarlas así. Los fabricantes que usan el sistema métrico usan también otros tipos. Para tener una idea de sus dimensiones en metros diremos que para tener los kg de carril por metro, se multiplican las lbs por yarda por 0.496. Para obtener centímetros cuad, se multiplican las pulgs cuadradas por 6.451; las dimensiones en $^1/_{64}$ de pulgada, se multiplican por 0.397 y dan milímetros; las pulgs lineales, por 25.4 y dan milímetros).

**94. Sección de carriles pesados, Manual de 1915 de la Ascn Am de Ing de F. C.**, pags 77-83. Vease fig, pag. siguiente.

**Los números en tipo grueso dan dimensiones lineales en 64 avos (1/64) de pulg.**

(Las areas en pulgs cuadradas. Vease tambien ¶ y ||. Vease *(N. del T.)* anterior.)

| | **Adoptado** Memorias, Vol 16, 1915, pags 397, 1117. | | | | **Propuesto** por la Comisión | |
|---|---|---|---|---|---|---|
| Lbs por yarda Nominal / Actual. | 90 § 89.96 | 100 101.49 | 110 110.36 | 120 120.87 | 130 ... | 140 ... |
| *a*..... | **360** | **384** | **400** | **416** | **432** | **448** |
| *b*..... | **94** | **106** | **110** | **114** | **118** | **122** |
| *c*..... | **202** | **210** | **218** | **226** | **236** | **246** |
| *d*..... | **64** | **68** | **72** | **76** | **78** | **80** |
| *e*..... | **164** | **172** | **178** | **184** | ... | ... |
| *f*..... | **328** | **344** | **352** | **368** | **384** | **400** |
| *g*..... | **36** | **36** | **38** | **40** | **42** | **44** |
| Cot..... *h*..... | 4 | 4 | 4 | 4 | 4 | 4 |
| *j*..... | **163** | **176** | **181** | **187** | **194** | **201** |
| *m*..... | **896** | **896** | **896** | **896** | **896** | **896** |
| *n*..... | **24** | **24** | **24** | **24** | **24** | **24** |
| Area, pulgs cuad | | | | | | |
| cabeza..... | 3.20 | 3.80 | 4.04 | 4.40 | 4.63 | 4.93 |
| alma..... | 2.12 | 2.25 | 2.49 | 2.69 | 3.02 | 3.28 |
| base..... | 3.50 | 3.90 | 4.29 | 4.76 | 5.06 | 5.37 |
| Total..... | 8.82 | 9.95 | 10.82 | 11.85 | 12.71 | 13.58 |
| Mom de Inercia *I* ¶..... | 38.7 | 49.0 | 57.0 | 67.6 | 77.4 | 89.2 |
| Mod de seccion *X* ‖ | | | | | | |
| cabeza..... | 12.56 | 15.1 | 16.7 | 18.9 | 20.8 | 23.1 |
| base..... | 15.23 | 17.8 | 20.1 | 23.1 | 2.56 | 28.4 |

« No se han propuesto diseños nuevos para secciones de menos de 100 lbs. »

¶ *I* = momento de inercia, en pulgadas bicuadradas. Vea pág 490.
‖ *X* = módulo de la sección en pulgadas cuadradas. Vea pág 495.
§ Idéntico con el carril de 90 lbs, tipo A, p. 799 de la Ascn de Ferrocarriles.

**95. Secciones de carril, bridas y juntas.** Tabla, p. 873.

**Memorias de la Asociación Americana de Ferrocarriles.** Feb7, 1908, abril 22, 1908.

Las secciones de carriles propuestas por la Comisión de Secciones, Tipo de Carriles y Ruedas, « adoptadas como práctica recomendada ». Las bridas y juntas recomendadas por la Comision de Manufactureros representando las Cias de Acero de Maryland, Lackawanna, Illinois y Carnegie y el F. C. de Pennsylvania acordaron considerar las especificaciones y diseño de las bridas, y notificado por la Comisión de secciones, encuentran que los tipos de Carriles y Ruedas de la Asen Am de F. C. A y B presentan vistas divergentes.

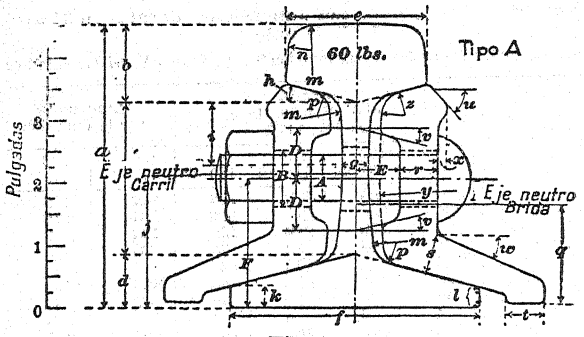


**Fig. 7a.**

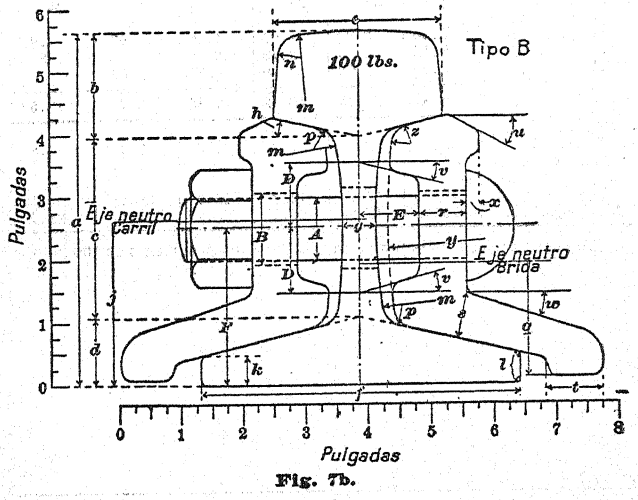


**Fig. 7b.**

*(N. del T.* : 60 lbs por yarda = 29,76 kg por m
100 — = 49.6 —

**Nota. Para evitar la impresión de fracciones,** las dimensiones lineales (excepto para *R*; véase nota al pie ††, abajo), están dadas en **tipo grueso,** y son en *sesenta y cuatro avos* ($^1/_{64}$) *de una pulg.* Las areas en pulgadas cuadradas.

Las letras *a, b, c,* etc. del caso abajo, refierense a dimensiones de *carril y brida.* Las dimensiones de bridas están anotadas en la brida del *lado derecho.* Las MAYUSCULAS se refieren a dimensiones de la **junta,** ó á la combinación del carril y la brida.

**Juntas de carril. Los números de tipo grueso dan la dimensión lineal en 64 avos de pulgada.**

(Véase para la conversión de estas unidades al sistema métrico la *N. del T.*: al principio pag 871.

| TIPO Carril lbs por yarda. | A 60 | 70 | 80 | 90 § | 100 | B 60 | 70 | 80 | 90 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Carril** | | | | | | | | | | |
| *a*..... | **288** | **304** | **328** | **360** | **384** | **268** | **291** | **316** | **337** | **361** |
| *b*..... | **79** | **86** | **92** | **94** | **100** | **80** | **87** | **94** | **103** | **109** |
| *c*..... | **157** | **160** | **174** | **202** | **216** | **132** | **145** | **153** | **168** | **183** |
| *d*..... | **52** | **53** | **62** | **64** | **68** | **56** | **59** | **64** | **66** | **69** |
| *e*..... | **144** | **152** | **160** | **164** | **176** | **136** | **152** | **156** | **164** | **170** |
| *f*..... | **256** | **272** | **296** | **328** | **352** | **236** | **259** | **284** | **305** | **329** |
| *g*..... | **30** | **32** | **33** | **36** | **36** | **31** | **33** | **35** | **36** | **36** |
| Cot . *h*..... | 4.0 | 4.0 | 4.0 | 4.0 | 4.0 | | angle | *h* = | 13° | |
| *i*..... | **64** | **64** | **72** | **80** | **96** | ... | ... | ... | ... | ... |
| *j*..... | **136.3** | **140.8** | **147.8** | **162.5** | **176.0** | **125.0** | **130.0** | **145.5** | **155.5** | **168.0** |
| *k*..... | **20** | **24** | **24** | **24** | **24** | **29** | **29** | **31** | **31** | **31** |
| *l*..... | **4** | **4** | **4** | **4** | **4** | **4** | **4** | **4** | **4** | **4** |
| *m*..... | **896** | **896** | **896** | **896** | **896** | **768** | **768** | **768** | **768** | **768** |
| *n*..... | **24** | **24** | **24** | **24** | **24** | **24** | **24** | **24** | **24** | **24** |
| *p*..... | **24** | **24** | **24** | **24** | **24** | **20** | **20** | **20** | **20** | **20** |

Para areas, etc., véase la siguiente página.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Brida** | | | | | | | | | | |
| *q*..... | **99.2** | **103.7** | **109.4** | **124.2** | **133.1** | **86.4** | **94.7** | **105.6** | **110.7** | **117.1** |
| *r*..... | **40** | **40** | **42** | **46** | **48** | **44** | **48** | **48** | **48** | **48** |
| *s*..... | **42** | **44** | **48** | **52** | **56** | **35** | **39** | **40** | **41** | **46** |
| *t*..... | **40** | **44** | **48** | **52** | **56** | **46** | **48** | **52** | **54** | **56** |
| *u*..... | 50° | 50° | 50° | 50° | 50° | 45° | 45° | 28° | 28° | 28° |
| Cot . *v*..... | **4** | **4** | **4** | **4** | **4** | **4** | **4** | **4** | **4** | **4** |
| *w*..... | 23° | 23° | 23° | 23° | 23° | 17° | 17° | 17° | 17° | 17° |
| *x*..... | **6** | **6** | **8** | **8** | **12** | **0** | **0** | **10** | **10** | **12** |
| *y*..... | **896** | **896** | **896** | **896** | **896** | **768** | **768** | **768** | **768** | **768** |
| *z*..... | **24** | **24** | **24** | **24** | **24** | **20** | **20** | **20** | **20** | **20** |

Para areas, etc., véase la siguiente página.

| **Junta** | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *A*..... | **48** | **48** | **53** | **64** | **64** | **48** | **48** | **53** | **64** | **64** |
| *B*..... | **53** | **53** | **64** | **72** | **72** | **53** | **53** | **64** | **72** | **72** |
| *D*..... | **53** | **55** | **64** | **65** | **66** | **55** | **55** | **64.5** | **66** | **66** |
| *E*..... | **45** | **51** | **56** | **60** | **62** | **52** | **52** | **53** | **62** | **62** |
| *F*..... | **130.5** | **138.0** | **149.0** | **165.0** | **176.0** | **122.0** | **131.5** | **143.0** | **159.0** | **160.5** |
| *S***..... | 43.41 | 37.67 | 36.59 | 43.64 | 44.34 | 32.33 | 34.51 | 34.62 | 33.68 | 34.86 |

| Carril lbs por yarda TIPO | 60 | 70 | 80 | 90 § | 100 | 60 | 70 | 80 | 90 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|
| | A | | | | | B | | | | |

Continua en la página siguiente.

¶ *I* = momento de inercia en pulgadas bicuadradas. Vea pág. 490.
‖ *X* = módulo de la sección en pulgadas cuadradas. Vea pag. 495.
†† *R* = radio medio *en pulgadas*
= al área en pulgs cuad ÷ Por la periferia en pulgs.
** *S* = 100 (*I* para 2 bridas) ÷ (*I* para el carril).
§ Adoptado por la Asen Am de Ings de F. C., Manual, 1915, pág. 78, Véase ¶94.

## Carriles y Juntas de la Asen Am de F. C. (concluye).

(Areas en pulgs cuad. ¶ † †.)

(Vease para la conversion de estas unidades al sistema metrico la *N. del T.* al principio pag 871.)

| TIPO Carril lbs por yarda. | A 60 | 70 | 80 | 90 § | 100 | B 60 | 70 | 80 | 90 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Carril** | | | | | | | | | | |
| Area | | | | | | | | | | |
| cabeza..... | 2.21 | 2.68 | 3.05 | 3.20 | 3.64 | 2.28 | 2.76 | 3.07 | 3.56 | 3.95 |
| alma..... | 1.41 | 1.49 | 1.65 | 2.12 | 2.29 | 1.14 | 1.34 | 1.54 | 1.70 | 1.89 |
| base..... | 2.24 | 2.65 | 3.16 | 3.50 | 3.91 | 2.45 | 2.79 | 3.30 | 3.61 | 4.01 |
| total..... | 5.86 | 6.82 | 7.86 | 8.82 | 9.84 | 5.87 | 6.89 | 7.91 | 8.87 | 9.85 |
| % | | | | | | | | | | |
| cabeza..... | 37.7 | 39.3 | 38.8 | 36.2 | 36.9 | 38.8 | 40.1 | 38.8 | 40.1 | 40.2 |
| alma..... | 24.1 | 21.8 | 21.0 | 24.0 | 23.4 | 19.4 | 19.5 | 19.5 | 19.2 | 19.2 |
| base..... | 38.2 | 38.9 | 40.2 | 39.8 | 39.7 | 41.8 | 40.4 | 41.7 | 40.7 | 40.6 |
| *I* ¶..... | 15.41 | 21.05 | 28.8 | 38.7 | 48.94 | 13.3 | 18.6 | 25.1 | 32.3 | 41.3 |
| *X* ‖ | | | | | | | | | | |
| cabeza ..... | 6.50 | 8.21 | 10.24 | 12.56 | 15.04 | 5.90 | 7.79 | 9.38 | 11.45 | 13.70 |
| base..... | 7.24 | 9.51 | 12.46 | 15.23 | 17.78 | 6.80 | 8.62 | 11.08 | 13.21 | 15.74 |
| *R* † † | | | | | | | | | | |
| cabeza..... | 2.35 | 2.12 | 1.93 | 1.90 | 1.80 | 2.10 | 1.99 | 1.79 | 1.68 | 1.64 |
| alma..... | 3.12 | 3.07 | 3.57 | 3.30 | 3.21 | 4.38 | 4.10 | 3.57 | 3.65 | 3.60 |
| base..... | 3.48 | 3.20 | 2.52 | 2.63 | 3.29 | 2.94 | 2.76 | 2.72 | 2.58 | 2.49 |
| total..... | 3.12 | 3.00 | 2.50 | 2.52 | 2.29 | 2.90 | 2.72 | 2.53 | 2.42 | 2.37 |
| **Brida** | | | | | | | | | | |
| 2 bridas | | | | | | | | | | |
| lbs por pie... | 21.76 | 23.94 | 27.14 | 33.64 | 38.28 | 19.44 | 23.60 | 26.24 | 29.04 | 34.48 |
| area..... | 6.40 | 7.04 | 7.98 | 10.26 | 11.26 | 5.72 | 6.94 | 7.72 | 8.54 | 10.14 |
| *I* ¶..... | 6.69 | 7.93 | 10.54 | 17.47 | 21.70 | 4.30 | 6.42 | 8.69 | 10.88 | 14.40 |
| *X* ‖..... | 3.45 | 3.93 | 4.72 | 6.87 | 8.28 | 2.53 | 3.45 | 4.41 | 5.18 | 6.26 |

### Clavos de ferrocarril.

**96.** Los clavos con cabeza de gancho *t*, comunmente usados para asegurar los carriles a las traviesas, varian dentro de los limites de la siguiente tabla; los mas ligeros, para carriles ligeros en ramales cortos locales; y los mas pesados para carril pesado en linea de primera clase. Los clavos se venden en cuñetes generalmente de 150 lbs (69 kg). Para saber el peso de clavos de dimensiones mayores podemos aproximarnos tomando el peso de una barra cuadrada del mismo largo. Lo que se economiza en la punta es suficiente para lo que se adiciona en la cabeza.

**Dimensiones**, etc.

| Tamaño en pulg. Largo. | Lado. | N.° por cuñete de 150 lbs. | N.° por 100 lbs. | Tamaño en pulgs. Largo. | Lado. | N.° por cuñete de 150 lbs. | N.° por 100 lbs. |
|---|---|---|---|---|---|---|---|
| 4 ½ | × 7/16 | 526 | 350 | 5 ½ | × ½ | 350 | 233 |
| 4 ½ | × ½ | 400 | 266 | 5 ½ | × 9/16 | 289 | 193 |
| 5 | × 5/8 | 705 | 470 | 5 ½ | × 5/8 | 218 | 146 |
| 5 | × 7/16 | 488 | 325 | 6 | × ½ | 310 | 207 |
| 5 | × ½ | 390 | 260 | 6 | × 9/16 | 262 | 175 |
| 5 | × 9/16 | 295 | 197 | 6 | × 5/8 | 196 | 130 |
| 5 | × 5/8 | 257 | 171 | | | | |

(*N. del T.* — La misma tabla en sistema métrico.)

¶*I* = momento de incercia en pulgadas bicuadradas. Véase pag. 490.
‖*X* = módulo de la seccion en pulgadas cuadradas. Véase pag. 493.
††*R* = radio medio *en pulgadas.*
= area en pulg cuad ÷ periferia en pulgadas.

| Tamaño en mm. Longitud. | Grueso. | Número por barril de 68 kg. | Número por 45 kg. | Tamaño en mm. Longitud. | Grueso. | Número por barril de 68 kg. | Número por barril de 45 kg. |
|---|---|---|---|---|---|---|---|
| 114 | 11 | 526 | 350 | 139 1/2 | 12 1/2 | 350 | 233 |
| 114 | 12 1/2 | 400 | 266 | 139 1/2 | 14 | 289 | 193 |
| 127 | 10 | 705 | 470 | 139 1/2 | 16 | 218 | 146 |
| 127 | 11 | 488 | 325 | 152 | 12 1/2 | 310 | 207 |
| 127 | 12 1/2 | 390 | 260 | 152 | 14 | 262 | 175 |
| 127 | 14 | 295 | 197 | 152 | 16 | 196 | 130 |
| 127 | 16 | 257 | 171 | | | | |

**97. Cantidad por milla y por kilometro.** Una milla de linea de una sola vía con carriles de 33 pies (10,06 m) y 18 traviesas por largo de carril, con 4 clavos en cada traviesa; tendra 160 tramos de carril, 2.880 traviesas, y 11.520 clavos, o cerca de 39 cuñetes de clavos de 5 ½ × 9/10" (14 × 1.4 cm) que pesan un poquito más de ½ lb por clavo.

(*N. del T.* — En un kilometro entran 99.4 tramos de carriles; 1.786 traviesas; 7.143 clavos o 24 cuñetes de clavos mas o menos.)

**98. Pero se debe dejar un margen** para guardacarriles en los cruceros, que podemos suponer sean de 33 pies (10,06 m) de ancho, o el largo de un carril. Estos se forman generalmente de 4 carriles extras para proteger la vía, y clavados a las 18 traviesas a las cuales estan sujetos los carriles de la vía. Por consiguiente, tal crucero requiere 18 × 8 = 144 clavos adicionales. Para desvíos, vías laterales, pérdidas, etc., podemos apreciar un promedio (dejando 1 milla de vía extra aproximadamente en la forma de desviaderos y vía lateral por cada 15 millas de camino) de 900 clavos mas por milla; arrojando por todo (asumiendo un crucero de carretera por milla) 11,520 + 144 + 900 = 12,564 clavos por milla o mas o menos 43 cuñetes de 150 lbs cada uno.

(*N. del T.* — Dejando 1 km por cada 15 resultan : 7.143 + 87 + 600 = 7.831 o 26 cuñetes de clavos.)

**99. Adhesion de los clavos.** El profesor **W.** R. Johnson encontró que con un clavo sencillo de 0.75, o de 3/4 de pulgs cuad (2.42 cm²) metido 3 3/4 de pulgada (8.5 cm) en pino de Jersey sazonado o castaño sin sazonar, se necesitó una fuerza como de 2.000 lbs (920 kg) para extraerlo; en roble blanco sazonado, cerca de 4.000 (1.840 kg); y con algarrobo bien sazonado cerca de 6.000 lbs (2.760 kg). Bevan encontró que una puntilla de 6 *penny nail* (medida inglesa de clavos) **6-penny**, metida una pulgada (25 mm) necesitaba para extraerlo : 667 lbs (307 kg) en haya sazonada, 557 (256 kg) en roble; 327 (150 kg) en olmo, 187 (86 kg) en pino.

**100.** En experimentos cuidadosos verificados en Hanover (Alemania), por el Ingeniero Funk, dió de 2.465 a 3.940 lbs (promedio de muchos experimentos 3.000 lbs) (1.380 kg), como la fuerza necesaria para extraer un clavo sencillo de hierro de ½ pulg cuadrado (12 mm) 6 pulgs largo (15 cm) y con la punta en forma de cuña de 1 pulgada largo (el doble del grueso del clavo), y metido 4 ½ pulgs (11.4 cm) en pino blanco o amarillo. Cuando se metió a 5 pulgs (12.7 cm) la fuerza necesaria fué cerca de 1/10 parte mayor. Clavos por el estilo de 9/10 en cuadro (14.3 mm) 7 pulgs largo (17.8 cm) metidos a 6 pulgs (15.2 cm) necesitaron de 3.700 a 6.745 lbs para extraerlos del pino, el promedio del resultado fué de 4.873 lbs (2.242 kg). En todos los casos *cerca del doble de esta fuerza fue necesaria para extraerlos del roble.* Todos los clavos fueron metidos al través del grano de la madera. La experiencia demuestra que cuando son metidos en **sentido** del grano, clavos o puntillas no aguantan ni la mitad de la fuerza.

**101.** Los clavos arponados o torcidos (como una barrena), o a los que se les haya aumentado o disminuido su sección cerca del medio de su longitud han resultado inferiores a los clavos cuadrados sencillos. Cuando el largo de la cuña de la punta se aumento a 4 veces el grueso del clavo, la resistencia para sacarlo resulta un poquito menor.

**102.** Cuando el largo del clavo es apropiado, probablemente no hay mejor forma que la de sección cuadrada sencilla, con una punta de cuña de largo doble al ancho del clavo.

**103. Resultado.** El clavo con cabeza de gancho con el tiempo se levanta por el movimiento ondulatorio del carril. Al meterse tritura la fibra de la madera; así

que (especialmente en maderas blandas) fallan al sostener la presión lateral del clavo. El agujero del clavo se agranda de este modo haciéndole perder al clavo su resistencia y el agua al penetrar en el hueco precipita su destrucción.

**104. Acero** *. El clavo no debe mostrar señales de fractura (a) cuando se dobla sobre si mismo a 180° y cerrado a martillazos, (b) cuando la cabeza se dobla hacia atras en frío, (c) cuando el cuerpo del clavo se tuerce en frio 1.5 vueltas *.

* Las variaciones máximas permitidas de las dimensiones expresadas, son : grueso $^1/_{32}$ de pulg (0,8 mm); largo bajo la cabeza o pulg menos, $^1/_4$ de pulg (6 mm) más; grueso de la cabeza, $^1/_{16}$ de pulg (1.5 mm); angulo del gancho, 1.° *.

**105. Agujeros.** Se barrenan a veces con $^1/_{16}$ de pulg (1.5 mm) menos en diámetro que el grueso del clavo, para recibir los clavos. De este modo se evita la trituracion de las fibras de la madera por el clavo, y asi aumenta la resistencia de clavo al tiro vertical y la de la madera al empuje lateral del clavo.

**106. Tarugos de traviesas.** Los tarugos de madera para traviesas, formados como el cuerpo del clavo, y metidos en los agujeros de los clavos cuando estos se aflojan, evitan la destrucción porque excluyen el agua. Si se mete un clavo después que se haya metido el tarugo en el mismo agujero, el tarugo aumenta el aguante en la traviesa. † La madera secada al aire; cortada longitudinalmente al grano; de lados opuestos paralelos; cortada a escuadra en el extremo que se mete; 4.5 pulgs largo (12 cm); $^{11}/_{16}$ pulg (17 mm) en cuadro; punta de cincel en 0.5 pulg (12 mm) en una punta. 1910 †.

## Tornillos tirafondos.

**107.** Los tirafondos son costosos y consumen tiempo (véase : Costos, ¶ ¶ 118, etc.) en su instalación, en renovaciones subsecuentes y otros cambios; pero prolongan la duracion útil de la traviesa quizás dos o tres veces más, y mantienen la vía en mejor estado y por mas tiempo, haciendo de este modo que las renovaciones sean menos frecuentes. Su uso por consiguiente reduce los gastos de instalacion **total** y mantenimiento **total.**

En el F. C. Lackwanna durante 5 años de uso no hubo necesidad de aumentar el número de hombres por milla.

**Resistencia.** En conjunto los tirafondos ofrecen de dos a tres veces la resistencia de los clavos ordinarios y mayor resistencia al empuje lateral, y mantienen estas resistencias mucho más tiempo. De este modo reducen la posibilidad del descarrilamiento y daños consiguientes.

**108. Corrosion.** Esta puede reducir el tamaño de la cabeza hasta hacer que estas no encaje bien en la llave de cubo y hacer difícil su cambio. Una cabeza cónica (fig. 8) evita esta dificultad hasta cierto punto; pero el atornillado a máquina (véase ¶ ¶ 130, etc.) produce agarraduras alternadas con resbalamientos de la llave que redondean las esquinas. Durante 5 años de uso en el F. C. Delaware Lackawanna & W'n no se encontró en las traviesas un tirafondo oxidado.

Cuando la cabeza se rompe se hace generalmente impracticable el cambio.

**109. Una letra en relieve** o marca en la parte de arriba de la cabeza excluye la posibilidad de usar mandarria al meterlos.

**110. Las variaciones en la sección del carril** aumentan la dificultad para usar los tirafondos.

**111. Resultados.** En general, el tirafondo al tener el carril y la traviesa en contacto, los obliga a descansar o moverse (segun sea el caso) juntos. Cuando se mueven juntos verticalmente, la traviesa puede, dañando el balasto, revolverlo o bien mejorándolo pisonarlo, segun las circunstancias. Algunos ferrocarriles prefieren aguantar en firme la plancha o la traviesa y dejar algun juego entre el carril y la plancha.

**112.** Como el agujero debe siempre barrenarse con anticipación, el tirafondo es menos propenso a rajar la traviesa que el clavo cortado metido en un agujero sin barrenar.

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* Asen am Ings de F. C. † F. C. Pen ‡ F. C. U. Pacific. Véase también Especificaciones, pág 853

**113.** Debido a la compresión de la traviesa, lo junto que están y el aumento de contacto entre la traviesa, la plancha y el carril en servicio, los tirafondos metidos nuevos, *deben generalmente apretarse* una o dos veces a intervalos de algunos meses; pero desde ese momento sostienen su aguante con muy poca o ninguna atención.

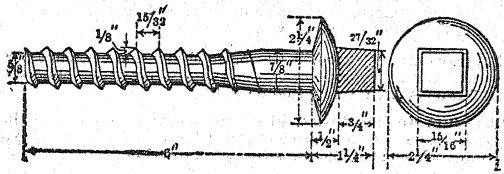



**Fig. 8.**

(*N. del T.* — 1/8 pulg = 3.16 mm; 15/32 pulg = 11.9 mm; 7/8 pulg = 22.12 mm; 2 1/4 pulg = 57 mm; 27/32 pulg = 22.8 mm; 15/16 pulg = 23.8 mm; 1 1/4 pulg = 32 mm; 3/4 pulg = 19 mm; 1/2 pulg = 12.7 mm; 5/8 = 16 mm.

**114. Dimensiones.** La fig. 8 representa el tirafondo tipo del F. C. D. L.[r] & —, 1915. Dimensiones en pulgadas. Otros diseños muestran la cabeza plana en el lado de abajo y sin cono. Véase ¶ 115.

**114a. El diseño,** una vez adoptado, no debe cambiarse, porque cualquier cambio aumentaría grandemente la dificultad de usar los tirafondos.

**115.** A fin de *proteger el tirafondo para que no se doble* bajo presiones laterales su cabeza, a cada lado debe estar bien soportada por medio de presillas adecuadas, o de tacones en la plancha de la traviesa, o ajustando la cabeza del clarvo a la pestaña de la base del carril.

**116. Espigas.** Se ponen tarugos o espigas de madera dura ó metal metidas ó enroscadas en agujeros preparados para ellos y con rosca interior para la rosca del tirafondo, y resultan ventajosos, especialmente en traviesas de madera blanda, cuando los tirafondos se meten nuevos o cuando el tirafondo ha empezado a aflojarse. Si son de madera, se tratan con preservativos antes de insertarse. El agujero que necesita la espiga es claro que debilita algo la traviesa. La espiga actúa como una columna y de este modo evita daños a la traviesa. Si es de madera, la fibra va vertical, y despues de metida, la parte de arriba de la espiga se corta parejo con la superficie superior de la traviesa. Las espigas se renuevan indefinidamente mientras dure la traviesa.

**117.** El muelle espiral de acero **The Thiollier** se ha usado como espiga en varios F. C. Americanos y en Francia. Resultan dificultosos para colocarlos y para quitarlos.

**117a.** En Francia se ha usado el buje o casquillo abierto de acero fundido **The Lakhovsky,** enroscado en la traviesa y que se expande a medida que el tirafondo se mete.

**118. Costos.** Los tirafondos cuestan de 2 a 3 cts cada uno, o, 2 o 3 veces más que los clavos cortados. Las espigas de madera dura, enroscadas por fuera y dentro, cuestan 1.5 ctvs cada una.

**119.** El costo de las varias **operaciones necesarias** varia mucho segun las condiciones. Ejemplo : *colocar* 2 traviesas, barrenar 4 agujeros y meter 4 tirafondos, costo 13,74 cts por traviesa cuando el trabajo se hizo *a mano*; 4,89 ctvs en *trabajo regado hecho con máquina* que tenia dos barrenas y herramienta para meter las piezas; y 2,9 ctvs en *trabajo continuo con máquina* que tiene dos barrenas y dos herramientas de meter las piezas.

**120. Barrenado.** Mr. J. W. Kendrick, en la Memoria de 1910 de la Asen de Ings Am de F. C. y mantenimiento de via, dice en la pag. 625, Part I, vol. 11 : Una máquina apropiada en la planta de tratar traviesas debe barrenar y enta-

rugar 600 traviesas con 8 tarugos cada una por dia de 10 horas a un costo de 3,5 ctvs por traviesa. » = 0,4375 cent por tarugo, pero al estimar el costo por milla, Mr Kendrick calcula para trabajo en el campo « barrenar traviesas y meter 24.000 espigas » (8 por traviesa) a 1 cent cada una.

**121. Uso.** Los tirafondos se han usado por mucho tiempo en Europa generalmente en lineas importantes, y se usan ahora en pocas lineas de los E. U. y notablemente en los F. C. Santa Fé y D. L. & W.

**122.** « Las traviesas deben protegerse contra el desgaste mecánico por medio de planchas y tirafondos. « Informe de la Comision de Traviesas. Memoria de 1915, vol. 16, pag. 522 de la Asen Am de Ings de F. C.

**123. Planchas de traviesas.** El uso de planchas de traviesas sin pestañas abajo (vease ¶ ¶ 135, etc.) se considera generalmente como esencial para dar un servicio satisfactorio con tirafondos. Vease ¶ ¶ 115 y 111.

**124.** La plancha de la traviesa se atornilla algunas veces a la traviesa y el carriles asegura por medio de clavos metidos al traves de ranuras en la plancha de la traviesa.

**125. Uniones o juntas.** Durante los 2 primeros años (1910-11) de uso de los tirafondos el F. C. D. L. & W. usó juntas de planchas de hierro maleable, y las barras angulares fueron ranuradas para clavos. Se usó un tirafondo en el exterior de la parte de fuera de la barra angular. Mas tarde, todas las planchas de union se hacen laminadas, y los tirafondos en barras angulares laminadas, con un tirafondo extra en el exterior de la barra angular.

**126. Metido.** Cualquiera *instrumento de meter* tirafondos debe estar dispuesto de modo que suelte su *agarre* sobre el tirafondo cuando este llegue a su lugar y asi se evita el forzar el tirafondo. Esto se arregla generalmente disponiendo un movimiento de friccion en la herramienta o instrumento.

**127. Metida a mano** : Aunque inseguro, y mucho mas costoso y gastándose mas tiempo que metido a máquina (véase ¶ ¶ 119, *etc.*), debe hacerse asi cuando son unos pocos los tirafondos que hay que meter, y en lineas de mucho tráfico (en las cuales no se pueda permitir un carro con maquinaria) a menos que no haya fuerza eléctrica o de aire comprimido para usar la herramienta.

**128.** Para el metido a mano, la herramienta *mas simple* es un husillo vertical con una mazorca abajo para sostener la llave, y un brazo horizontal, cruzado, o una chicharra en la cabeza.

**129.** Está descrita *una máquina de cigueña movida á mano para barrenar y meter material* montada sobre una trípode ajustable, y con un cigüeñal horizontal con engranes de angulo recto al husillo vertical, inventada por el Prof. A. L. Smith, del Instituto Politécnico de Worcester, Worcester, Mass., en el Boletin No. 50 del Negociado de Montes, pag. 53. Sometida a prueba esta máquina, metió dos tirafondos mientras se metieron 3 clavos. Puede ser movida por fuerza de vapor, etc.

**130.** *Máquinas movidas por vapor, etc.* Varian desde el pequeño aparato movido por electricidad o aire; a mano, ó por carros de gasolina que la cuadrilla puede levantar de la via, hasta la colección formidable de taladros y llaves, instalados en la planta de tratamientos o montados sobre carros para economizar la manipulacion de las traviesas y hacer mas fácil el cambiarlas de la via. Están generalmente provistas con sierras de carril, ruedas de esmeril, etc. Las máquinas grandes están provistas de contadores que llevan nota de las traviesas manipuladas y con un sistema de escape para sacar las virutas.

**131.** El carro motor Snow de gasolina tiene un motor que mueve un generador de corriente que por medio de un cable lleva a las herramientas de barrenar y llaves la corriente motriz; asi pueden estar aquellas a 300 m de distancia del carro. De este modo el carro puede quedarse en un desvio, fuera del tráfico de trenes. Pesa completo, 1.560 kgs, y lleva ademas del operador 10 hombres; generalmente mete de 2.000 a 2.500 tirafondos por dia. Puede ir a 50 millas por hora. Los fusibles se funden cuando se encuentra resistencia excesiva, de este modo se evita meter los tirafondos demasiado. Véase ¶ 126.

**132.** El « au-tra-kar » es un pequeño carro de gasolina; puede barrenar un agujero de tirafondo en roble en 5 a 10 segundos y meter un tirafondo en 20 segundos aproximadamente.

**133.** *Velocidad.* Un hombre con máquina puede barrenar 9 agujeros mientras se barrena 1 a mano, y puede meter 5 tirafondos mientras 2 hombres meten 1 con llave.

**134.** Una *máquina de hacer espigas*, usada por el F. C. de Santa Fé, tenia 4 herramientas; (1) para *barrenar* agujeros de espiga, (2) para hacer *rosca* a estos agujeros, (3) para insertar estas espigas, y (4) para recortar y enfrentar los extremos de las espigas que se proyectaban fuera de la traviesa. Podia entarugar traviesas para 1300 ms de via por dia, mientras que otra máquina metia los tirafondos necesarios para la misma distancia.

Una *barrena sola* barrenará 11000 agujeros, como 1500 agujeros entre cada dos afiladas.

El agujero debe de barrenarse *mas hondo que el largo del tirafondo.*

Si se barrena pasando la traviesa, *facilitará la salida de la viruta.*

## Planchas de traviesas, ó de asiento.

**135. Necesidad.** Donde los carriles descansan directamente sobre las traviesas la gran presion sobre la base estrecha del carril, las sacudidas del carril al pasar las ruedas y la friccion de las fibras de la madera, causarán el desgaste rápido de la traviesa debajo del carril.

**136. Economía.** Las planchas de traviesas aumentan grandemente su duración, en las curvas y puentes, la economia en gran número de casos, se ha estimado como en un 50 % del costo del material y de 60 a 75 % en jornales. Las planchas de traviesas han hecho a menudo innecesario el uso de pequeñas cuadrillas de hombres cuya mision era solamente reponer traviesas.

**137.** La plancha de traviesa se coloca en la traviesa debajo del carril. Aguantando la plancha y el carril en su lugar, se meten los clavos en la traviesa al traves de agujeros, en la plancha. Algunas formas de planchas tienen dos o mas filetes en la cara inferior. Estos filetes le dan rigidez a la plancha; y, al cortar estos filetes la superficie superior de la traviesa, resisten el resbalamiento de la plancha sobre ella. Los filetes algunas veces están al traves de las fibras de la traviesa y algunas veces paralelos. Algunas formas tienen un borde en la cara superior, para ayudar a los clavos a resistir y que se separen los carriles, actuando como mordaza del carril.

**138. Asiento.** Un asiento desigual bajo la plancha de la traviesa da lugar a que se abolle la plancha y la vía no quede a cartabón. Las traviesas deben estar provistas de sus planchas antes de ponerse en la vía. En traviesas labradas, los asientos para las planchas deben labrarse con hacha para tener los asientos del carril en el mismo plano.

**139. Costo.** El de las planchas de traviesas es de 5 a 15 cts cada una; y colocarlas cuesta de ½ a 1 ½ cts cada una.

**140. Materiales y pruebas.** * *Hierro dulce;* $<$ 3.150 kg por cm$^2$ de resistencia a la tensión; debe doblarse en frío a 90° al través de la fibra sin señal de fractura.

*Hierro maleable de horno.* La pieza de prueba debe doblarse y mostrar tenacidad. La fractura debe mostrar una franja de metal blanco en la superficie. La porción central obscura y sin fibra. 1915 *.

*Acero Bessemer o de Martin Siemens.* † El acero de Mart. Siemens se prefiere al Bessemer. Cortado y punzonado en frío. Piezas de prueba de 36 cm de largo de sección uniforme, area del extremo $<$ 3 cm$^2$ cortada del material terminado. Se requieren pruebas para cada orden y por cada fundición. Las planchas de traviesas de sección completa deben doblarse en frío en una dirección a angulo recto de la fibra, dobladas en plano sin señal de fractura. 1914 †. $\doteqdot$ (Bessemer o M. S.); carbón 0.2 % (0.05 % de variación en cualquier sentido), fósforo $>$ 0.1 %. 1905. $\doteqdot$ *Resistencia a la tensión,* kg por cm$^3$; 3867 *, 3796 a 4499 †. *Límite de elasticidad;* $<$ 0.5 ult * †. *Alargamiento;* en 5 cm $<$ 20 % * †. *Reducción de área;* $<$ 40 % * †.

**141. Dimensiones.** * *Ancho* $<$ 6 pulgs (15 cm).

$$Largo < \frac{\text{área del asiento de seguridad de la traviesa}}{\text{ancho de la plancha}}. \quad Borde < 12 \text{ mm alto.}$$

Distancia (uniforme) desde el canto de la base del carril al extremo de la plancha en el lado de afuera. $>$ proyección interior de la base del carril. *Filetes* (pocos en número) en la base, $>$ 6 mm de hondo. Con traviesas tratadas o con tirafondos, se prefieren planchas con *base plana*. 1915 *.

Sección trasversal. Espesor exagerado.

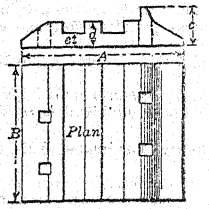



**Fig. 9.**

‡ Dimensiones, en pulgs, 1909.

Véase fig. 9.

Agujeros para clavos cuad de $^{11}/_{16}$ de pulg (17,46 mm).

| Carril, lbs por yarda. | *A* | *B* | *c* | *d* | *e* |
|---|---|---|---|---|---|
| 65 | 8.0 | 8 | $^{5}/_{8}$ | $^{5}/_{8}$ | $^{1}/_{4}$ |
| 75–80 | 8.5 | 8 | $^{11}/_{16}$ | $^{7}/_{16}$ | $^{1}/_{4}$ |
| 90 | 8.75 | 8 | $^{11}/_{16}$ | $^{7}/_{16}$ | $^{1}/_{4}$ ‡ |

(*N. del T.* — La misma tabla en sistema métrico).

Dimensiones en milímetros.

| Carriles kg por metro. | *A* | *B* | *c* | *d* | *e* |
|---|---|---|---|---|---|
| 32.24 | 200 | 200 | 15 | 9 | 6 |
| 37.2–39.7 | 213 | 200 | 17.4 | 11.11 | 6 |
| 44.6 | 222.5 | 200 | 17.4 | 11.11 | 6 ‡ |

***Para embarques** : Las planchas van amarradas juntas con alambre en mazos y en numero fijo, pesando $>$ 100 lbs (46 kg). 1915. *

**Uso.**

**142.** † « Las planchas de traviesas deben usarse en todas las traviesas en vias de alta velocidad, en las curvas de 2º ó más; en todas las traviesas de via sujetas a servicio pesado; en todas las traviesas de chucho, y en traviesas de mesas giratorias, fosas de cenizas, puentes y pilas; en aguadas y vias con canal de agua y en todos los cruces de via y andenes. Las planchas para traviesas deben usarse en traviesas de madera blanda y en todas las traviesas que han sido tratadas. » †.

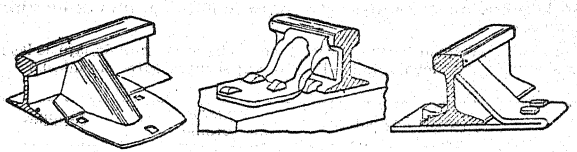
‡ A menos que no se indique otra cosa, las planchas para traviesa se aplicarán como sigue, siempre que se renuevan carriles o traviesas. (En « maderas blandas » incluyendo todas : las tratadas y no tratadas, excepto el roble);

* Asoc Am Ings F. C. † F. C. Pen. ‡ F. C. U. Pacific. Véase tambien Especificaciones, pág. 853.

En lineas nuevas : en todas las traviesas de madera blanda de la linea principal;
En lineas principales, en todas las traviesas de madera blanda;
En ramales; en todas las traviesas de madera dura en curvas de 3° y mas cerradas, y en todas las traviesas tratadas ya esten en curva ó en tangentes;
En los desvios; en todas las traviesas de desvios; en todas las curvas de 3° y mas fuertes y en todas las traviesas tratadas esten en curva ó en tangente ÷.

## Tacones o abrazaderas entre carriles.

**143.** Los tacones o abrazaderas de carriles se usan para evitar que se abra la carrilera, especialmente en las curvas. Para curvas de 5°, cinco abrazaderas por carril de (33) pies (10,06 m) es suficiconte Para curvas de 10° se usa una abrazadera en cada traviesa alternada.



Tacon *Alkins* de acero forjado | Tacon *Ajax* de acero. | Tacon y plancha *Edwards*, de traviesa.

**Fig. 10.**

**144.** La fig. 10 muestra tres tipos de abrazadera o tacones de carriles de uso corriente.

## Uniones (Bridas).

**145.** La tendencia de una via a ceder o rendirse en las bridas (mal llamadas eclisas. *N. del T.*) va en detrimento de la via y del material rodante.
El extremo de un carril sobre el cual se mueve una rueda cargada, se dobla mas que el extremo adyacente sin carga del proximo carril, y de este modo recibe un golpe fuerte de la rueda.

**146.** Cuando las traviesas están colocadas de modo inseguro, no se puede esperar que ninguna junta (brida) de carril haga buen servicio.

**147. Las bridas o uniones suspendidas** (aquellas donde los extremos del carril se encuentran entre dos traviesas) son generalmente preferidas a las bridas soportadas (que se encuentran directamente sobre una traviesa).

**148. El coeficiente de dilatacion**, en carriles de acero puede tomarse como (0.000 006 5 de pie) por grado Fahr. De aqui un carril de (33) pies (396 pulgs) bajo un aumento de 60° Fahr (33° C) en temperatura, se alargará en $(396 \times 60 \times 0.000\ 006\ 5) = (0.154)$ pulgs (ó sean 3,9 mm por 33° Cent). Los carriles se alargan ligeramente en sus extremos por el tráfico.

**149. Deslizamiento del carril** (Vease ¶ ¶ 163, etc.) Este ocasiona esfuerzo adicionales sobre las juntas. El deslizamiento ocurre en la direccion del tráfico mas pesado, y donde el tráfico es igual en ambas direcciones, el deslizamiento se hace en el sentido de la pendiente.

**150.** Si en las dos lineas de carril que forma la via, las juntas estan opuestas una a la otra « son juntas ó bridas parejas » y se llaman juntas « salteadas » cuando cada junta en una de las lineas de carril ocurre hacia el medio del carril de la otra linea. En este ultimo caso, el mas comun, la trepidacion al pasar de un carril a otro es menos fuerte que cuando las juntas son parejas, pero por supuesto de doble frecuencia.

**151.** Para disminuir esta trepidación, los carriles se cortan con extremos **biselados** de modo que el plano vertical formado por los extremos del carril haga un ángulo de 45° a 60° (y no angulo recto) con el plano vertical longitudinal del alma del carril.

**152. Las bridas angulares**, fig. 7, han reemplazado practicamente toda otra forma de brida. Sus pestañas horizontales le dan estabilidad lateral a la unión y llevan parte de la carga directamente a las traviesas, de este modo alivian a los extremos del carril, *de esa carga.*

**153. Las ranuras** en las pestañas del carril y de la barra o brida angular deben estar separadas de tal modo que los dos clavos, metidos en la traviesa, no deben quedar directamente opuestos unos a otros sino alternados o en « zig-zag », con el fin de evitar que se raje la traviesa.

**154. Dimensiones usuales, etc., para bridas angulares de uniones de carril.**

(*N. del T.* — Debajo de las libs por yard van los kg por metro; debajo de las pulgs van los milimetres y debajo de las lbs van kg; siempre entre paréntesis).

| Carril lbs/yarda. | Bridas angulares. Largo pulgs. | Bridas angulares. Lbs por par. | Agujeros de tornillo. No. | Agujeros de tornillo. Diam. pulgs. | Tornillos. Largo pulgs. | Tornillos. Diam. pulgs. |
|---|---|---|---|---|---|---|
| 70 | 40 | 70 | 6 | $^{13}/_{16}$ | 4 | $^{3}/_{4}$ |
| (34.3 kg/m) | (102 cm) | (31.75 kg) | | (20.6 mm) | (10 cm) | (19 mm) |
| 75 | 40 | 76 | 6 | $^{7}/_{8}$ | 4 $^{1}/_{8}$ | » |
| (36.75 kg/m) | (102 cm) | (34.47 kg) | | (22 mm) | (105 mm) | » |
| 85 | 40 | 80 | 6 | 1 $^{1}/_{16}$ | 4 $^{1}/_{2}$ | $^{7}/_{8}$ |
| (42.02 kg/m) | (102 cm) | (36.28 kg) | | (27 mm) | | |
| 90 | 27 | | 4 | 1 | 4 $^{1}/_{4}$ | » |
| (44.1 kg/m) | (68.6 cm) | | | (25.4 mm) | (10.7 cm) | |

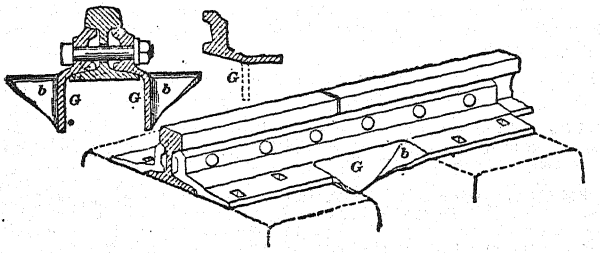



Fig. 11.

**155. Juntas en los puentes.** Fig. 11. En las juntas ó bridas en los puentes, la pestaña horizontal de la brida angular se lamina mas ancha que de ordinario y su porcion media se prensa hacia abajo con dados, formando una viga, G, que se extiende hacia abajo entre las dos traviesas en que cae la brida. Esto aumenta la resistencia vertical de la brida, y las pestañas extendidas aumentan la superficie de asiento de la carga y aumenta la resistencia lateral de la unión; pero las pestañas que se proyectan hacia abajo (que deben caer entre dos traviesas), restringen la libertad de colocar las bridas donde se quiera con respecto a las traviesas. Véase el final de ¶ 168. La fig. 11 representa las bridas del puente Bonzano.

**156. La fig. 12 representa otros tipos importantes de bridas angulares.**

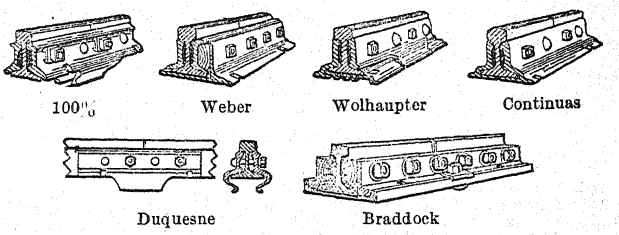



**Fig. 12.**

**157. Abbott.** En la brida Abbott de carril inventada por F. E. Abbott, Ingeniero Inspector de la Cia de Acero Lackawanna, el canto superior de cada barra angular está ligeramente hundido en el centro á fin de mantenerla separada de la parte baja de las cabezas del carril en la unión, evitandose con esto el efecto cortante y el desgaste de la parte superior de la barra angular por las cabezas del carril.

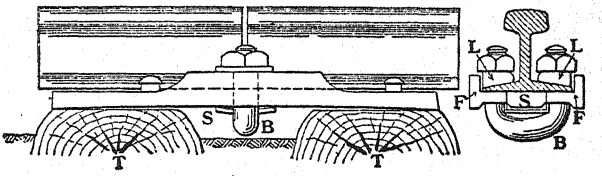



**Fig. 13.**

**158. Fisher.** Fig. 13. En la brida Fisher de carril (de la Fisher Rail Joint Works Trenton, N. J.) las *bases* de carril, en sus extremos, están soportadas por una placa *F* con pestañas, colocada bajo la base del carril y atornillada a él por medio de un tornillo *B*, en U, y grapas *L* como indica el diseño, en lugar de tener el soporte debajo de las *cabezas* del carril como sucede en las bridas de barras angulares, una muesca o entalladura y una pieza combada *S* de acero de resorte, colocada entre el tornillo *B*, y la placa con pestaña, *F*, y sostenida en su lugar por el tornillo en U que pasa al través de las muescas, le dan á la brida elasticidad, recibe y recoge los movimientos debidos á los desgastes, sirve de cojín á los esfuerzos en el tornillo, y mantiene la presión entre las roscas del tornillo y las tuercas, obrando como una tuerca de seguridad.

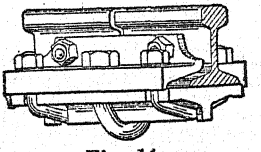


**Fig. 14.**

**159.** La junta « **triple fish** » (Fisher), fig. 14, tiene tambien una placa soporte bajo la base del carril. Es una junta corta con tres tornillos en U, como se indica en la fig. y formada de barras angulares especiales (« keepers ») « sostenedoras » atornilladas al través del alma del carril.

## Tornillos de Vía.

**160.** * El tornillo sin rosca debe doblarse en frío á 180° y aplastarse sin fractura afuera *.

*Rosca; Standard de los E. U. y de mayor diámetro, la parte de la rosca que el resto del tornillo, a menos que no se especifique de otro modo; cortada o laminada. $\lessdot 2 \gtrdot 5$ hilos en *dedo.* * (*N. del T.* — Un de do tiene $^5/_4$ pulg (19 mm). El pie tiene 16 dedos.)

* Para embarques, los tornillos deben aceitarse; y las tuercas en los tornillos metidas $\lessdot 2$ espiras. 1915. *

## Muelle espiral de tuercas de seguridad.

(Véase tambien pag. 1211.)

**161.** * Acero; fósforo $\gtrdot$ 0.05 5 %; sulfuro $\gtrdot$ 0.05 %.

La tuerca terminada (de un diámetro interno de ($^{13}/_{16}$ a 1 $^{5}/_{16}$ de pulg (20.64 a 33.34 mm) aguantada a plano una hora, debe recuperar $\lessdot$ $^2/_3$ de su altura o grueso si el grueso $<$ ancho; $\lessdot$ 0.5 del grueso si es cuadrada; $\lessdot$ ancho si la altura o grueso $>$ ancho.

No debe haber señales de fractura cuando se sujeta un extremo en un tornillo de banco, y el extremo opuesto se tuerce a 45°. 1915 *

## Partes de metal. Especificaciones.

### Acero. Requisitos mínimos.

(Para **carriles**, vease pag. 868 y 1106.)

**162.** Resistencia y límite de elasticidad en lbs por pulg cuadrada. Alargamiento en (2 pulgs) (50 mm).

(*N. del T.* — Los numeros de kg entre parentesis son : kg por cm cuadrado.)

Manual de la Ascn de Ings Am de F. C. 1915.

| | **Planchas de traviesa.** | **Clavos terminados.** Metidos. | Enroscados. |
|---|---|---|---|
| Ultima resistencia a la tension, *u*..... | 55,000 lbs (3,867 kg) | 55,000 lbs (3,867 kg) | 60,000 (4,218 kg) |
| Limite de elasticidad..... | 0.5 *u* | 0.5 *u* | 0.5 *u* |
| Alargamiento, en (2 pulgs) (50 mm)..... | 20 % | 20 % | 22 % |
| Reduccion del area..... | 40 % | 40 % | 40 % |

Manual de la Ascn de Ings Am de F. C. 1915.

| | **Tornillos de via.** Acero carbon. | Nikel u otra aleación sin tratar. | tratada. |
|---|---|---|---|
| Ultima resistencia a la tension, *u*..... | ..... | ..... | ..... |
| Limite de elasticidad..... | 35,000 (2.461 kg) | 45,000 (3.164 kg) | 75,000 (5.273 kg) |
| | 0.5 *u* | 0.5 *u* | 0.5 *u* |
| Alargamiento, en (2 pulgs)..... | 25 % | 20 % | 15 % |
| Reduccion de area..... | 50 % | 40 % | 40 % |

## Deslizamientos.

**163.** El deslizamiento del carril es debido al movimento ondulatorio del carril producido por el movimiento de los trenes, y es mas marcado en los lechos de via faltos de firmeza, y en pendientes pronunciadas. Los carriles ondulan y se deslizan en la direccion en que se mueven los trenes; por consiguiente el deslizamiento da mas que hacer en lineas de via doble. Aumenta tambien con la dilatacion del carril en tiempo de calor, y disminuye en las heladas.

**164.** « **Amarre cruzado** », fig. 15. Se emplea con éxito para evitar el deslizamiento. Consiste simplemente en meter los clavos de afuera, como se ve en la fig. mas adelante que los clavos de adentro en la direccion en la cual el carril tiende a

* Ascn Am Ings F. C. † F. C. Pen. ‡ F. C. U. Pacific. Véase tambien Especificaciones, pág. 853.

deslizarse. Entonces si, por ejemplo, el carril *A* se desliza en la direccion indicada por la flecha, llevandose consigo el extremo de la traviesa, la traviesa es llevada hacia adelante en la posicion indicada por la linea de puntos; (1) aumentando las

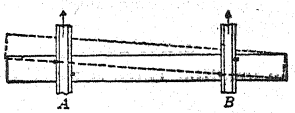



**Fig. 15.**

presiones laterales de sus dos clavos contra la pestaña; (2) aumentando el engrampe de la traviesa en el carril, y con esto (3) la resistencia al deslizamiento del carril en la traviesa. Y reciprocamente para el carril *B*.

**165. Las traviesas muy largas** (algunas veces hasta de 12 pies) (3,66 m) se han usado también para disminuir o evitar el deslizamiento.

**166. Ranuras para clavos** en las barras de brida son eficaces contra el deslizamiento, siempre que las traviesas de las juntas agarren en el balasto. Si no es asi, los carriles deben anclar en las traviesas intermedias con abrazaderas antideslizables, o las anclas pueden hacerse de barras de bridas desechadas cortadas al través en secciones, dejando un agujero de tornillo en cada sección, y haciendo una ranura de clavo en su pata horizontal. Entonces se barrena un agujero en el alma del carril, sobre la traviesa, y el ancla se atornilla al carril, clavado a la traviesa al través de la ranura. Se puede aumentar mas la resistencia al deslizamiento, ajustando bloques entre las traviesas delante de las traviesas ancladas.

**Otro** accesorio contra deslizamientos consiste en una grampa que muerda la base del carril sin atornillar, y lleva un tocón que descansa contra el costado de una traviesa.

Aunque a fin de evitar el deslizamiento del carril, las bridas algunas veces son ranuradas para clavos en las traviesas, ¶ 166, parece preferible emplear las anclas de carril o los otros medios; vease los anteriores ¶ ¶.

**167.** En el puente de St. Louis (arcos de acero) y en sus inmediaciones (vigas de planchas en columnas de hierro), los carriles se deslizan en la direccion del tráfico (un pie) 30 cm por dia aprox, en una pendiente de 1.5 % y con tal fuerza que ninguna de las varias sujeciones probadas han sido suficientes para evitar el deslizamiento del carril, y la vía ha sido ajustada diariamente. Vease el folleto del Prof. J. B. Johnson, Auxiliar de la Revista de la Sociedad de Ingenieria. Vol. IV, n.° 1, nov. 1884.

**168.** *Al reponer carriles*, cuando se hace necesario cambiar la posición de las bridas ha sido costumbre cambiar la separación de las traviesas para conformarlas con las nuevas posiciones; pero esta práctica se ha abandonado; el sitio de las traviesas y el balasto se dejan sin cambiar y parece que el resultado es satisfactorio.

Donde se usan bridas de barras angulares (tales como la junta Bonzano, ¶ 155) con pestañas verticales que se proyectan hacia abajo entre las traviesas, por supuesto que es inevitable mover las traviesas.

## Carriles continuos.

**169. Dilatación y contracción.** En los trabajos de lineas de ferrocarril en las calles, los carriles por lo regular van **soldados, fundidos** o remachados en sus extremos, formando practicamente carriles continuos; y una amplia experiencia ha demostrado que una linea larga de carril continuo no se dilata ni contrae considerablemente, considerada en conjunto, con los cambios de temperatura; la tendencia a expansionarse y contraerse es resistida con éxito por el terreno que actúa por medio de las traviesas y las sujeciones del carril. Por supuesto que el resultado es un aumento de esfuerzos longitudinales en los carriles, con el correspondiente cambio microscopico en sus areas de sección transversal.

**170.** Tomando el coeficiente de expansion lineal, $a$, del acero (o sea 0.000 006 5) o estiramiento por unidad de longitud $e$ (pp. 478-9) a 1° Fahr cambio de temperatura, y su módulo de elasticidad, E, como 29,000,000 lbs por pulg cuad (2.038.910 kg por cm cuad) tenemos la unidad de esfuerzo $= Ee =$ 188.5 lbs/pulg

cuad/grado Fahr (23,9 kg/cm cuad/grado C.). Tomando la variación de temperatura a 140° Fahr (60° C) y suponiendo que los carriles están puestos a temperatura media, estos estarian sujetos a un cambio máximo de temperatura de 70° (39° C) y una unidad máxima de esfuerzo de $70 \times 188.5 = 6$ sea 13,200 lbs/pulg cuad (928 kg por cm cuad), quedando comprendido dentro de la unidad del esfuerzo, permitido al carril de acero.

## Miscelánea.

**171. Diferencia en longitud entre el carril exterior y el interior** Fig. 16.

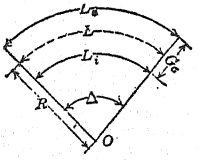



**Fig. 16.**

Sea

$R$ = radio de la linea del centro de la via;
$\Delta$ = abertura de la curva;
$G_c$ = ancho de via, medido entre los *centros* de carriles;
$L, L_o, L_i$ = largo del arco, en los arcos del centro de la curva, del carril exterior y del interior respectivamente.

En un circulo completo (360°) :

$$L_s = 2\pi(R + G_c/2) = 2\pi R + 2\pi G_c/2; = 2\pi R + \pi G_c;$$
$$L_i = 2\pi(R - G_c/2) = 2\pi R - 2\pi G_c/2; = 2\pi R - \pi G_c;$$

y, restando, $L_o - L_i = 2\pi G_c$.

**Por consiguiente, en un arco de** $\Delta^o$, tenemos :

$$L_o - L_i = 2\pi G_c \frac{\Delta^o}{360_i} = \frac{\pi G_c}{180}\Delta^o.$$

Con un ancho de via $G.$ = 4 pies 8.5 pulgs (1,435 m) tenemos $G_c$ = 4 pies 11 pulgs (1.499 m) aprox = 4.917 pies; y $L_o - L_i = 0.085818\ \Delta^o$.

(*N. del T.* — Empleando la medida métrica 1.499, resulta 0,02614.)

**En cualquier largo dado,** $L$, en la linea de centro, tenemos :

$$\Delta^o = 360^o \frac{L}{2\pi R} = \frac{180\,L}{\pi R}.$$

De donde $L_o - L_i = \frac{\pi G_c \Delta^o}{180} = \frac{\pi G_c}{180} \cdot \frac{180\,L}{\pi R} = \frac{G_c L}{R}$.

(*N. del T.* — Las mismas formulas se emplean para el sistema métrico.)

De otro modo : *para una abertura dada,* $\Delta$, la diferencia, $L_o - L_i$, es *independiente del radio ;* mientras que, *para un arco de longitud dada,* $L$, la diferencia *varía inversamente con el radio,* y es *independiente de la abertura.*

## Abertura de cartabón en curvas.

**172.** La necesidad y extensión de la abertura entre carriles, en las curvas, depende de muchos factores variables, tales como la longitud de la base rigida de la rueda (vease fig. 2 en Material rodante) el juego, S, o diferencia entre el ancho de via, $G$ (en tangentes), y el mismo entre *ruedas,* W, en ambos en ruedas nuevas y gastadas, etc. De aquí el que las opiniones y la práctica varien mucho.

**173. Práctica.** De 104 ferrocarriles preguntados por la Ascn de Ings Am de F. C., en 1897, 25 informaron que no debe haber aumento del ancho de via en las curvas. Los otros 79 ferrocarriles informaron tener prácticas variadas que en conjunto resumimos como sigue :

(*N. del T.* — Entre parentesis los ms equivalentes a los pies y los mm equivalentes a los $^1/_{16}$ de pulgs.)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Aumento de ancho de via empezando con D = | 1° | 2° | 3° | 4° | 5° | 6° | 8° |
| Radio en pies | 5730 | 2865 | 1910 | 1433 | 1146 | 955 | 717 |
| — en metros | 1746. | 873 | 582 | 437 | 349 | 291 | 219 |
| Aumento máximo, en $^1/_{16}$ de pulg | 4 | 4 | 6 | 8 | 8 | 8 | 12 |
| Lo mismo en mm | 6 | 6 | 9 | 13 | 13 | 13 | 19 |
| Numero de respuestas | 18 | 5 | 15 | 10 | 16 | 7 | 3 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Aumento de ancho de via empezando con D = | 9° | 10° | 12° | 13° | 20° | 21° |
| Radio en pies | 637 | 574 | 478 | 442 | 288 | 274 |
| — en metros | 194 | 175 | 146 | 135 | 88 | 84 |
| Aumento maximo en $^1/_{16}$ de pulg | 12 | 16 | 16 | 16 | 16 | 19 |
| Lo mismo en mm | 19 | 25 | 25 | 25 | 25 | 30 |
| Numero de respuestas | 1 | 2 | 0 | 1 | 0 | 1 |

**174. La Asoen Am de Ings de F. C. no recomienda** (Manual, 1915, p. 117) aumento en curvas de 8° ó más abiertas. En curvas cerradas, recomienda un aumento de ($^1/_8$) de pulg (3 mm) por cada 2° adicionales (ó fracción) de cierre, hasta un ancho de via máximo de (4 pies 9.25 pulgs) (1.454 m) para via ancha de (4 pies 8.5 pulgs) (1·435 m). El ancho de via incluyendo desgaste, en ningun caso excederá de (4 pies 9.5 pulgs) o (1.460 m).

**175. Los patrones de los maestros constructores de carros** (véase fig. 2 en « Material rodante ») se hacen :

$S\ (= G - W)$ = (4 pies 8.5 pulgs menos 4 pies 7 $^{11}/_{16}$ pulgs = $^{13}/_{16}$ pulgs) igual (20.637 mm).

Este juego es suficiente para evitar que los troles de 4 ruedas de los carros se tranquen en las lineas de tracción por vapor. Pero el caso es distinto con las bases rígidas largas en las voladoras (motrices) de las locomotoras.

**176. El F. C. Pennsylvania** empuja carros de carga en desviaderos de almacenes con 60 pies de radio (D = 113° aprox) : cartabón de via en tangentes, 4 pies 8.5 pulgs (1.435 m); en curvas donde sea necesario (4 pies 9 pulgs) o sean 1 m 448.

**177. Relaciones geométricas entre el sobreancho y la curvatura.**

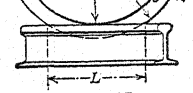



**Fig. 17**

Fig 17. Sea

$r$ = radio de la pestaña de la rueda;
$f$ = altura de la pestaña de la rueda;
$L$ = largo del contacto de la pestaña con el carril.

Tenemos

$$L = 2\sqrt{(r+f)^2 - r^2}.$$

**178.** Figs 18, 19, sea

$D$ = abertura de la curva;
$G$ = ancho de via en las tangentes;
$B$ = base rígida de la rueda;
$L$ = largo del contacto de la pestaña con el carril;
$R_1$ $R_2$ = radio de la curva;
$G_1$ $G_2$ = ancho de la via en las curvas que da un juego, $S$, igual al juego $G$ en las tangentes;
$S_1 = G_1 - G$ ; $S_2\ (= G_2 - G)$ = juego = aumento del ancho de via debido a las curvas.

Entonces, fig. 18, **para un trole de cuatro ruedas** :

**Juego**, $S_4 = G_4 - G = m\ n = (\text{aprox})\ m'\text{-}n'$

$$= L \operatorname{sen} n'\ E\ m'$$

$$= (\text{aprox})\ L \operatorname{sen} A\ O\ E = L \frac{B + L}{2\ (R_4 + G/2)\ *}$$

$$= \frac{L\ (B + L)}{2\ R_4 + G}$$

Bastante aproximado, $S_4 = \dfrac{L\ B}{2\ R_4} = \dfrac{L\ B\ D}{11{,}460}$.

(*N. del T.* — Aunque el autor nada explica para pasar de la fórmula $\dfrac{L\ B}{2\ R_4}$ a la $\dfrac{L\ B\ D}{11{,}460}$ se multiplican los dos términos del quebrado primero por $D$ y viene $\dfrac{L\ B\ D}{2\ R_4\ D}$ y en el párrafo 173 se toma como promedio la abertura de 5°, el Radio 1.146 y da para el denominador $2 \times 1.146 \times 5 = 11.460$. — Para usar el sistema métrico empléese la fórmula asi : $\dfrac{L\ B\ D.}{3{,}495}$ Entrando con $L$, $B$, $D$ en metros, el cuociente es metros.

Nos parece mucho mas práctico el procedimiento siguiente :

En vias anchas para radios de 1.000 ms o mas no se da sobreancho; para :

| Radios de..... | 850 | 600 | 400 | 250 | 150 metros. |
|---|---|---|---|---|---|
| Sobre anchos..... | 5 | 10 | 15 | 20 | 25 milimetros. |

En vias estrechas :

De 1 metro con radios de 80 a 250 metros. $S$ en mm $= 240 : \sqrt{R}$

De 0,75 — 50 a 150 — — $= 140 : \sqrt{R}$

De 0,60 — 30 a 100 — — $= 100 : \sqrt{R}$

(Tratado de ferrocarriles por S. Rahola, Madrid.)

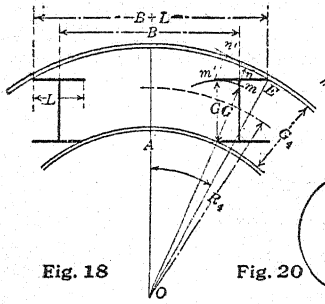



Fig. 18

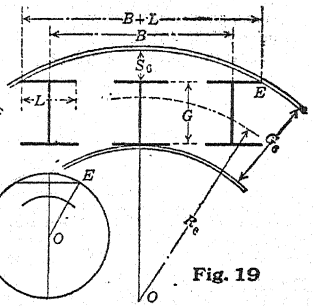



Fig. 19

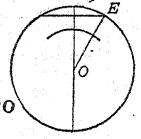



Fig. 20

Fig. 19. **Para un trole de seis ruedas** ; ejes igualmente separados, tenemos aprox (véase la pequeña escala fig. 20) :

$$S_6 = G_6 - G = \left(\frac{B + L}{2}\right)^2 \times \frac{1}{2\ R_6 + G} = \frac{(B + L)^2}{8\ R_6 + 4\ G\ *}$$

Bastante aproximado : $S_6 = \dfrac{B^2}{8\ R_6} = \dfrac{B^2\ D}{45.840}$.

(*N. del T.* — Poniendo la $B$ en metros úsese la fórmula $\dfrac{B^2\ D}{13{,}980}$ y el cuociente viene en metros.)

* Como una aproximación, $G$ se usa aqui para $G_4$ o $G_6$ (todavia sin conocerse). En la práctica, se puede prescindir de $G$.

Si, en un trole de seis ruedas, los ejes están **desigualmente** separados, o si hay **mas de tres ejes**, la ecuacion da para $S_0$ mas de lo necesario, pero aumentando la seguridad.

### Desgaste de los carriles en las curvas.

**179.** Fig. 21. Se han usado **carriles especiales** y otros accesorios para resistir o evitar las fuerzas laterales que se desarrollan en las curvas y el desgaste adicional debido a ellas.

**180.** Fig. 21a. En curvas estrechas, el F. C. de Lehigh Valley usa, en lugar de su carril normal de 100 lbs (49,6 kg p m) un carril de 110 lbs (54,57 kg por m) de la misma altura y ancho del patin, pero con un alma o nervio de $^1/_{32}$ de pulg (0,75 mm) mas grueso, y una cabeza de $^5/_{16}$ de pulg (7.5 mm) mas profunda, con una barra angular afuera de brida especial, que en el lado exterior, se proyecta hacia arriba, a lo largo de la cabeza para soportar esta. Se usa una junta escalonada o de « trancision » para efectuar el paso entre el carril normal y el especial.

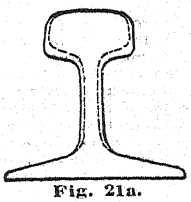


Fig. 21a.

**181.** Fig 21b. En el carril **Manning** usado en el F. C. Baltimore y Ohio, el grueso de la cabeza no es simétrico, el mayor grueso está en el lado sujeto al desgaste. Cuando este se ha gastado suficientemente, el carril se corre lateralmente hacia el centro de la curva a fin de cobrar el desgaste y de este modo restablecer el ancho de via.

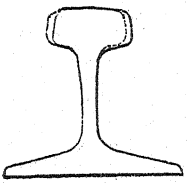


Fig. 21b.

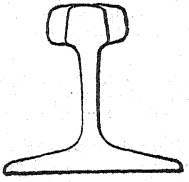


Fig. 21c.

**182.** Fig 21c. A fin de reducir el resbalamiento de las ruedas interiores cuando se pasan curvas, el F. C. Southern Pacific usa, en el lado interior de las curvas un carril con una cabeza 25 % mas estrecha que el del carril tipo. El carril especial es cerca de 8 % mas alto que el carril normal. Se usa un carril especial adelgazado para facilitar el pase del carril normal al especial.

**Peralte.** Véase ¶ ¶ 172 a 193, en « Curvas ».

### Colocacion y Mantenimiento.

**183.** La operacion completa de colocar la via se ejecuta generalmente en dos etapas (1). Colocando la via sobre el terreno lo bastante bien para permitir al tren de construccion pasar por ella, y (2). Balastrado y terminacion general del trabajo.

**184. Generalidades.** Una parte importante del trabajo es el de trasportar las traviesas, carriles y accesorios desde el lugar de abastecimiento hasta los lugares donde se van a colocar. Las traviesas algunas veces se mandan adelante en carretones, especialmente en las praderas, o donde haya un camino cerca. Los carriles suelen llevarse en *carros planchas* en el tren de construccion, que va empujándose adelante sobre la linea nueva a medida que va haciendose. Los carriles, y las traviesas, cuando se llevan en el tren, deben todavia llevarse desde los carros a puntos mas allá de donde llega la via, donde se echan, clavan y atornillan, lo suficiente para permitir que pase el tren de construccion.

**185. Tren de construccion.** La formación de un tren de construccion dependerá del adelanto de la via, distancia de la base de aprovisionamiento, pendiente y fuerza propulsora, y facilidades para la situacion de los desvios. Si los carros que van a utilizarse como vivienda se colocan a la cabeza del tren, los materiales

se deben trasportar por delante de ellos; si se dejan en un desvio mientras progresa el trabajo puede ocasionar atrazos serios en llevarlos y traerlos a la cuadrilla de construccion para las comidas y para dormir. Frecuentemente se colocan al frente del tren y se dejan ahi. El orden de colocacion de los carros en el tren es generalmente como sigue, empezando con el carro que esta mas adelante.

Carro « Avanzada »; con oficina y posiblemente taller y herramientas;
Carro almacen, con provisiones para vivir en él, etc.;
Carro o carros dormitorios y comedores, combinados o separados;
Carro cocina;
Carros cama y de comer adicionales, si es necesario;
Carro de herramientas, o de viveres y agua;
Carros de carriles; los necesarios para el dia, o que se puedan manejar sobre las cuestas y curvas de acuerdo con la fuerza de traccion o propulsora disponible;
Carro con accesorios de via; barras, bridas, tornillos, tuercas, etc;
Carros de traviesas; como se necesiten;
Carros con accesorios de linea telegráfica (si hace falta);
Carro con combustible (si el ténder no es de suficiente capacidad para un dia de servicio o si no hay puntos de aprovisionamiento disponible);
Locomotora; algunas veces varias.

**186. Entrega de Materiales.** De noche o cuando sea necesario volver por mas material de via, los carros-camas y de cocina se dejan en la parte mas avanzada de la via en construccion ó en un desvio, y la locomotora regresa con los carros de carriles y traviesas (y el carro de combustible si lo hay) a la base de aprovisionamiento, probablemente un patio. Los carriles y otros accesorios se cargan en el tren de construccion. Los carriles rectos y curvos de diversos largos, y las traviesas duras y blandas deben cargarse por separado. De otro modo se puede perder mucho tiempo en su manipulacion y en su busca cuando se necesitan. Por la mañana, esta parte del tren regresa al sitio de operaciones y conecta con el resto.

**187. El afirmado de la via** debe nivelarse bien, antes de recibir las traviesas y carriles; porque una superficie irregular puede forzar y doblar los carriles en mala forma cuando pase el tren de construccion y aumentar los descarrilamientos.

**188. Entongue de traviesas.** Cuando las traviesas se depositan cerca de la via, se requieren las siguiente condiciones. (Vease tambien « Sazonado » en « Traviesas », ¶ 53.) * Entongado $\lessdot$ (4 m) del carril mas cerca (0,90 m) de espacio entre las tongas *. † En terreno no mas bajo que el declive del F. C. pónganse sobre una fundacion de piedra o de traviesas de desecho. † * Tongas de 25 o 50 traviesas *. † Tongas $\gtrdot$ 12 camadas de alto. † * Cada tonga marcada con el nombre del propietario y fecha de entongue * † y el número de traviesas de cada clase de madera en la tonga. † * Las traviesas aserradas y labradas entongadas separadamente *. † Las de castaño entongadas separadamente. † * Las traviesas tratadas con cloruro de zinc u otra solución deben apilarse en tongas juntas en terrenos bien desaguados *.

**189. Reparto de traviesas.** Las traviesas son llevadas por parejas de animales ó á mano, ó con la « maquina de colocar via» (vease ¶ 199). Hasta donde sea posible, las traviesas de madera dura se reservan para las curvas. Las traviesas pueden tirarse, y colocarse en su lugar, poniendose en linea y graduando su separacion por una cuerda o liston graduado o por otros medios.

**190. Reparto de carriles.** Los carriles pueden correrse desde los carros hacia el suelo lateralmente sobre un par de carriles inclinados; pero en general no debe dejarse que se golpeen unos con otros, o dejarlos caer sobre un terreno irregular. † Deben distribuirse con la base hacia abajo, en una superficie uniforme de asiento sobre el afirmado de la via †. Deben llevarse hacia adelante con hombres, o arrastrados por caballos, o entregados por la « maquina de colocar via « (vease ¶ 199), como parezca mas expedito; y entonces se ponen sobre las traviesas.

† Carriles puestos uno a un tiempo. Los extremos puestos juntos a escuadra teniendo en cuenta la expansion †.

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* Ascn Am Ingo F. C. † F. C. Pen. ‡ F. C. U. Pacific. Vèase tambien Especificaciones, p. 853.

**191. Doblado de carriles en las curvas.** La necesidad de doblar los carriles de modo permanente, antes de ponerlos en las curvas, aumenta con el peso de los carriles y con la estrechez de la curva; y disminuye a medida que el largo de carril aumenta y segun se aguante el carril en la curva con el tráfico.

**192.** *En curvas medianas se colocan generalmente* los carriles sin doblarlos previamente, se obligan a encurvarse y se aguantan con los clavos, etc. Para curvas mas cerradas que 10°, y para ramales; los carriles se doblan en su sitio, por medio de un dobla-carril de mano, a un costo de $28 á $36 o mas, por kilometro (Camp). Para trazados especiales, algunas veces se doblan (en frio) en la fábrica, con máquinas adecuadas.

**193.** *En los ramales*, los « carriles de arranque deben encurvarse antes de ponerlos; de otro modo es difícil evitar que se doblen las abrazaderas cuando se trate de forzar una curva » (Camp).

## Ordenada media *m* ó flecha para doblar los carriles en las curvas.

C = longitud de la cuerda tomada como unidad;

D = grado de curvatura = ángulo central subtendido por la cuerda, C;

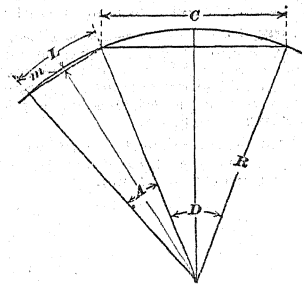
$R = \frac{C}{2 \operatorname{sen} \frac{1}{2} D}$ = radio de la curva, en metros;

L = longitud del carril;

$A = \frac{L}{R}$ = ángulo central subtendido por el carril, en radiales (véase *N. del T.* pág. 103, § 2);

A° = 57.2958 A;
log 57.2958 = 1.758 1226;
(1 radial = 57°.2958);

$m = R\left(1 - \cos\frac{A}{2}\right)$, en metros, tomando el R en metros.




(*N. del T.*— La que sigue es la tabla del autor convertida al sistema métrico. Para cualquier radio que no esté en la tabla, se encuentra la flecha por una simple proporción.)

| R metros | Largo L del carril en metros. | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 9.14 | 8.53 | 7.92 | 7.32 | 6.70 | 6.40 | 5.49 | 4.88 | 4.27 | 3.66 | 3.05 | 2.44 | 1.83 |
| | Flecha m en milímetros. | | | | | | | | | | | | |
| 3493.01 | 3.05 | 2.75 | 2.44 | 2.44 | 1.53 | 1.53 | — | — | — | — | — | — | — |
| 1746.50 | 6.10 | 5.49 | 4.58 | 3.97 | 3.36 | 2.75 | 2.14 | 1.83 | — | — | — | — | — |
| 1164.34 | 9.15 | 7.93 | 7.01 | 5.80 | 4.88 | 3.97 | 3.36 | 2.75 | 2.14 | 1.53 | — | — | — |
| 873.25 | 12.20 | 10.67 | 8.15 | 7.93 | 6.41 | 5.49 | 4.27 | 3.36 | 2.75 | 1.83 | — | — | — |
| 698.60 | 15.25 | 13.11 | 11.28 | 9.76 | 8.24 | 6.71 | 5.49 | 4.27 | 3.36 | 2.44 | 1.53 | — | — |
| 582.17 | 18.00 | 15.86 | 13.42 | 11.59 | 9.76 | 7.93 | 6.41 | 5.18 | 3.97 | 3.05 | 2.14 | 1.53 | — |
| 498.96 | 21.04 | 18.30 | 15.86 | 13.42 | 11.29 | 9.46 | 7.63 | 6.10 | 4.58 | 3.36 | 2.44 | 1.53 | — |
| 436.78 | 24.09 | 21.04 | 18.60 | 15.25 | 12.81 | 10.68 | 8.54 | 6.71 | 5.19 | 3.97 | 2.75 | 1.83 | — |
| 388.32 | 27.14 | 23.48 | 20.84 | 17.39 | 14.64 | 11.90 | 9.76 | 7.63 | 5.80 | 4.27 | 3.05 | 2.14 | — |
| 349.30 | 29.89 | 26.23 | 22.57 | 19.22 | 16.17 | 13.42 | 10.98 | 8.54 | 6.71 | 4.88 | 3.36 | 2.14 | — |
| 317.60 | 32.94 | 28.67 | 24.71 | 21.05 | 17.69 | 14.64 | 11.90 | 9.46 | 7.32 | 5.49 | 3.66 | 2.44 | — |
| 291.20 | 35.91 | 31.41 | 27.15 | 22.88 | 19.52 | 16.17 | 12.81 | 10.37 | 7.93 | 5.79 | 3.96 | 2.75 | 1.53 |
| 268.75 | 39.01 | 34.16 | 29.28 | 25.01 | 21.05 | 17.38 | 14.03 | 11.28 | 8.54 | 6.40 | 4.27 | 3.05 | 1.53 |
| 249.63 | 41.78 | 36.60 | 31.42 | 26.84 | 22.57 | 18.60 | 15.25 | 11.8 | 9.15 | 6.71 | 4.58 | 3.05 | 1.83 |
| 233.00 | 44.83 | 39.34 | 33.86 | 28.87 | 24.26 | 20.13 | 16.47 | 12.81 | 9.76 | 7.32 | 5.18 | 3.35 | 1.83 |
| 218.44 | 47.88 | 41.78 | 35.99 | 30.81 | 25.92 | 21.35 | 17.38 | 13.72 | 10.37 | 7.62 | 5.49 | 3.66 | 1.83 |
| 205.59 | 50.93 | 44.53 | 38.43 | 32.64 | 27.45 | 22.57 | 18.30 | 14.64 | 11.28 | 8.23 | 5.79 | 3.66 | 2.14 |
| 194.22 | 53.98 | 46.97 | 40.56 | 34.46 | 28.98 | 24.10 | 19.52 | 15.25 | 11.89 | 8.85 | 6.40 | 3.96 | 2.14 |
| 184.06 | 57.03 | 49.41 | 42.70 | 36.29 | 30.80 | 25.34 | 20.43 | 16.16 | 12.70 | 9.45 | 6.40 | 4.27 | 2.44 |
| 174.83 | 60.08 | 52.05 | 44.83 | 38.43 | 32.33 | 26.54 | 21.65 | 17.08 | 13.41 | 9.76 | 6.71 | 4.27 | 2.44 |
| 158.98 | 65.88 | 57.34 | 49.41 | 42.09 | 35.58 | 29.28 | 23.79 | 18.60 | 14.33 | 10.67 | 7.32 | 4.88 | 2.75 |
| 145.77 | 71.67 | 62.52 | 53.98 | 46.05 | 38.73 | 32.02 | 25.92 | 20.43 | 15.86 | 11.3 | 7.93 | 5.18 | 3.05 |
| 134.59 | 77.77 | 67.71 | 58.56 | 49.72 | 41.78 | 34.46 | 28.06 | 22.26 | 17.08 | 12.50 | 8.84 | 5.79 | 3.05 |
| 125.04 | 83.57 | 72.89 | 62.83 | 53.68 | 45.11 | 37.21 | 30.49 | 3.79 | 18.3 | 13.42 | 9.45 | 6.10 | 3.36 |
| 116.76 | 89.67 | 78.08 | 67.40 | 57.34 | 48.19 | 39.95 | 32.53 | 25.62 | 19.52 | 14.33 | 10.06 | 6.40 | 3.66 |
| 109.49 | 95.45 | 83.26 | 71.67 | 61.00 | 51.54 | 42.39 | 34.46 | 27.44 | 20.74 | 15.35 | 10.67 | 6.71 | 3.96 |
| 103.09 | 101.26 | 88.44 | 76.25 | 64.96 | 54.59 | 45.14 | 36.90 | 28.97 | 22.26 | 16.47 | 11.28 | 7.32 | 4.27 |
| 97.38 | 107.36 | 93.33 | 80.52 | 68.62 | 57.64 | 47.88 | 38.74 | 30.50 | 23.48 | 17.38 | 12.20 | 7.62 | 4.27 |
| 92.28 | 113.15 | 98.51 | 85.09 | 72.28 | 61.00 | 50.32 | 40.87 | 32.33 | 24.70 | 18.30 | 12.81 | 8.23 | 4.57 |
| 87.70 | 118.95 | 103.70 | 89.36 | 75.94 | 64.05 | 53.07 | 42.70 | 33.86 | 25.92 | 19.21 | 13.42 | 8.54 | 4.88 |
| 83.68 | 125.05 | 108.58 | 93.63 | 79.91 | 67.40 | 55.51 | 44.83 | 35.68 | 27.44 | 20.13 | 14.03 | 8.84 | 5.18 |
| 79.92 | 130.84 | 113.76 | 98.21 | 83.57 | 70.45 | 58.26 | 47.27 | 37.21 | 28.67 | 21.04 | 14.64 | 9.45 | 5.18 |
| 76.49 | 136.94 | 118.95 | 102.48 | 87.53 | 73.50 | 60.69 | 49.10 | 39.04 | 29.88 | 21.96 | 15.25 | 9.76 | 5.49 |
| 73.35 | 142.74 | 123.83 | 107.05 | 91.19 | 76.86 | 63.44 | 51.24 | 40.56 | 31.41 | 22.87 | 15.86 | 10.06 | 5.79 |
| 70.46 | 148.53 | 129.01 | 111.32 | 94.85 | 79.91 | 65.88 | 53.37 | 42.39 | 32.33 | 23.79 | 16.47 | 10.67 | 6.10 |
| 67.79 | 154.33 | 134.20 | 115.90 | 98.82 | 82.96 | 68.62 | 55.51 | 43.92 | 33.55 | 24.70 | 17.38 | 10.98 | 6.10 |
| 65.32 | 160.43 | 139.38 | 120.77 | 102.48 | 86.31 | 71.37 | 57.64 | 45.44 | 35.07 | 25.62 | 18.00 | 11.28 | 6.40 |
| 63.03 | 166.22 | 144.57 | 124.75 | 106.14 | 89.36 | 73.81 | 59.78 | 47.27 | 36.2 | 26.53 | 18.60 | 11.89 | 6.71 |
| 60.91 | 172.02 | 149.75 | 129.01 | 109.80 | 92.41 | 76.25 | 61.1 | 48.80 | 37.54 | 27.45 | 19.21 | 12.20 | 7.01 |
| 58.92 | 177.51 | 154.63 | 133.28 | 113.76 | 95.46 | 79.00 | 64.05 | 50.63 | 38.75 | 28.36 | 19.82 | 12.51 | 7.01 |

**194. Expansión y contracción. Suplementos de expansión.** Los hay de muchos tipos, desde cuñas graduadas especiales hasta clavos de alambre; pero deben ser de hierro o acero, no de madera, y deben tener estampada las temperaturas a que deben usarse; se colocan provisionalmente entre los extremos del carril y así sirven para separar las juntas del carril, dejando el juego para la expansión y contracción. Despues se quitan.

**195. Espacios para la expansión.** (Carriles de 33 pies, 10,065 m.) Temperaturas. Fahr, tomadas en el carril al tiempo de ponerse.
(*N. del T.* — Al lado de las pulgs van los milimetros y debajo de los grados Fahr van los Centig).

| | | | |
|---|---|---|---|
| Espacio, en pulgs, entre los ext del carril..... | $^{5}/_{16}$ (8 mm) | $^{1}/_{4}$ (6 mm) | $^{3}/_{16}$ (4.5 mm) |
| Ascn Am Ingo F. C..... | — 20 a 0° | 0° a + 25° | 25° a 50° |
| | ( — 30 a — 19 C) | (19 a — 4 C) | ( — 4 a + 10 C) |
| F. C. Pen..... | — 10 a + 14° | 14° a 38° | 38° a 62° |
| | ( — 24 a — 10 C) | ( — 10 a + 3.3 C) | (3.3 a + 17 C) |
| Espacio, en pulgs, entre los extr del carril..... | $^{1}/_{8}$ (3 mm) | $^{1}/_{16}$ (1.5 mm) | 0 § |
| Ascn Am Ingo F. C.... | 50° a 75° | 75° a 100° | sobre 100° |
| | (10 a 24 C) | (24 a 38 C) | (sobre 38 C) |
| F. C. Pen..... | 62° a 86° | 86° a 110° | sobre 110° |
| | (17 a 30 C) | (30 a 43 C) | (sobre 43 C) |
| En tuneles F. C. Pen... | 22° ( — 6 C) | 46° ( — 8 C) | sobre 70° (sobre 21 C) |

† En juntas aisladas, ½ pulg (12 mm) sin considerar la temperatura.†

**196. Las juntas** son las que siguen. Las barras de brida se ponen en su lugar, algunos si no todos los tornillos se pasan por los agujeros, y las tuercas se aprietan. Entonces los suplementos de expansión pueden quitarse. † *En las tangentes* cada junta debe caer frente al medio del carril opuesto de la misma via. *En las curvas*, una variacion de ⩾ 18 pulgs. † (45 cm).

**197. El clavado** de los carriles a las traviesas sigue ahora. La traviesa se sujeta contra el carril empleando un hombre con una « pata de cabra », (una palanca que se para por arriba del carril y agarra debajo de la traviesa), y dos hombres meten los clavos, uno a cada lado del carril, golpeando alternativamente. Cada clavo debe meterse verticalmente y debe empezarse de modo que el lado de la punta toque el canto de la base del carril, y que fuerce el carril. Cuando se usan *tirafondos* la operacion es por supuesto diferente. Véase en « tirafondos », ¶ ¶ 107, etc. Por supuesto que se tiene cuidado de que el cartabón esté bien puesto cuando los clavos se esten poniendo en el otro extremo de la traviesa.

**198. Especificaciones.** † Cuando no se usen planchas de traviesas, los clavos interiores deben meterse cerca del canto este ó sur de la traviesa; los clavos exteriores deben meterse cerca del lado oeste ó norte de la traviesa; ⩽ 2″ (5 cm) del canto de la traviesa. 1909. †

† Clavos por traviesa en cada carril; —

| | |
|---|---|
| En tangentes sin plancha de asiento..... | 1 dentro, 1 afuera |
| En tangentes con planchas de asiento planas o con nervios..... | 2 dentro, 1 afuera |
| En curvas con planchas de asiento planas o con nervios..... | 2 dentro, 2 afuera.† |

* Los clavos no deben enderezarse mientras se meten. Los clavos exteriores de ambos carriles, deben estar en un lado de la traviesa; ambos clavos interiores en el lado opuesto. Los clavos ordinariamente a (2.5 pulgs) (7 cm) de la parte exterior de la traviesa. Los agujeros de clavos viejos deben entarugarse. *

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* Ascn Am Ingo F. C. † F. C. Pen. ‡ F. C. U. Pacific. Véase tambien Especificaciones, p. 853.
§ Puesto junto, sin golpear.

**199.** «**Maquinas de colocar vias**». Asi se las llama, pero no colocan en verdad la linea. Simplemente facilitan la entrega de los carriles y las traviesas de los carros a los puntos avanzados donde se necesitan, consisten en cierta forma de rolletes ó via montada en el tren de construcción, a lo largo de uno ó de ambos lados, ó sobre la parte superior. En algunas máquinas la corredera está colocada en pendiente y el material rueda por gravedad; mientras que en otras es llevada en plataformas ó correas conductoras a otros accesorios movidos por una máquina especial fija. Estas máquinas rara vez aumentan la cantidad de trabajo en un tiempo dado, pero lo abaratan reduciendo el número de trabajadores necesarios.

**200. Velocidad** de colocación, segun Camp, en condiciones medias; cerca de una milla (1 609 m) en 10 horas sin apurarse, empleando 64 trabajadores, 3 capataces y 2 parejas de caballos.

**Balastado.**

**201. Entrega del balasto.** En carros o planchas corrientes, sin tablas de costado, cargados con paleadoras de vapor, con capacidad de (7.6 $m^3$) cada uno. Con costados, (2 pies 8 pulgs (80 cm) alto), un carro de 12 m. de largo llevará cerca de (32 yardas cubicas) 25 $m^3$. Las tablas de costado estan dispuestas como una serie de puertas, embisagradas arriba.

**202.** Los carros planchas se descargan economicamente con una pala que se arrastra por arriba de los carros planchas, por medio de un cable de alambre conectado la locomotora o al tambor de una máquina situada en un carro al extremo del tren. Con una locomotora el tren se estaciona, y cada carro deposita su carga completa en una distancia igual a su propia longitud; pero, con máquina y tambor, la distribución del material se puede hacer teniendo el tren en movimiento.

**203.** *Pala central*, esta distribuye el balasto a ambos lados de la via. Y se guia por estacas provisionalmente colocadas en las estaqueras laterales para este efecto, o por rolletes que descansan en el costado del carro. La punta de la pala central es movible para un lado ú otro : asi que, si se desea, se puede descargar mas de un lado que de otro en la via.

**204.** *Palas laterales.* Estas descargan a un lado solamente, están guiadas por estaqueras laterales. La descarga lateral con pala central o lateral significa el gasto de volver a tirar el balasto sobre la via. Vease tambien carros de volteo, (mas adelante) en Material Rodante (¶ 80).

**205.** Para evitar que el balasto caiga en la via entre los extremos de los carros se embisagra una plancha horizontal de caldera en una cabeza de cada carro; de modo que cubra el espacio donde se enganchan entre carro y carro.

**206. Nivelación.** Cuando el balasto se descarga (entre los carriles) por carros tolva, se nivela ó empareja por medio de un carro pala que lo extiende; este carro va conectado a la cola del tren, o por medio de nua traviesa atravesada y asegurada delante de las ruedas del tren, atrás del carro que descarga. Se necesitan algunos hombres con barretas ó palas en el carro, para empujar el material hacia abajo en las tolvas.

**207. Apisonado.** *Balasto de tierra o arcilla.* Equipo de pala con mango de hierro para apisonar : barretas de punta ancha para apisonar. * Apisónese cada traviesa la una distancia de (18 pulgs) (45 cm); (15 pulgs (38 cm) con arcilla quemada) desde el interior del carril al extremo de la traviesa con el mango de la pala ó con la barreta de apisonar. Si es posible apisónese primero el extremo de la traviesa y déjese pasar el tren antes de apisonar el interior del carril; désele especial atención al apisonado debajo del carril : apisónese el centro de las traviesas flojamente con la punta de la pala. La tierra ó arcilla entre las traviesas debe colocarse en camadas y bien prensada con los pies ó de otro modo, de manera que deje correr el agua facilmente; la tierra no se debe dejar amontonada mas arriba del fondo de las cabezas de las traviesas; el relleno entre las traviesas no debe tocar al carril y debe estar tan alto ó mas que la parte superior de las traviesas en el medio de la via. Con **piedra picada** ó **escorias de horno**, no se apisona el centro de las traviesas; arrímese balasto cerca del extremo de las traviesas a nivel con la parte superior de ellas. *

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* Asen Am Ingo F. C. † F. C. Pen. ÷ F. C. U. Pacific. Véase tambien Especificaciones, p. 853.

**208. Costos**; Balasto, ctvs por metro cúb.

| | |
|---|---|
| Piedra triturada a máquina en la cantera | 58 a 97 |
| Colocarla en la via abajo y apisonarla | 20 a 33 |
| En la via, completamente balastrada, alineada y emparejada | 97 a 162 |
| Cascajo, en via terminada | 26 a 39 |
| Incluyendo costo de jornal para colcarlo en la via apisonado y emparejado | 13 a 20 |
| Descargado, con pala y cable, incluyendo jornales para manejar el cable y el uso del equipo | 0.7 |

El aceite para el afirmado de la via de 2 a 3 1/2 cts por galón.

**209. Aceitado del Balasto.** *En afirmados de vias polvorientas* se puede evitar la mortificacion al pasaje, y el daño a las traviesas y muñones, rociando el balasto con aceite que penetre a una profundidad de varias pulgadas, disminuyendo no solo el polvo actual sino el que pueda asentarse mas tarde. Se evapora tan lentamente que una aplicacion por cada estacion del año es bastante.

**210.** « El aceite para el afirmado de la via « es un producto de la destilacion del petroleo con una densidad de cerca de 0.89. Es practicamente incombustible en las condiciones en que se usa, y se dice que se queman menos traviesas en via aceitadas que en las que no lo estan.

**211.** La capa de aceite retarda el crecimiento de la yerba en el balasto; y como lo hace no absorbente reduce el levantamiento de la via en las heladas. Se cree que conserva las traviesas porque excluye la humedad.

**212.** Los aparatos de regar o rociar se instalan por lo regular en un carro plancha, con enchufes para conectarse al carro-tanque de aceite. En tiempo de calor el aceite corre libremente por gravedad. En tiempo frio se puede ayudar a correr por medio de vapor, o con presion de aire tomado de la locomotora. El aceite se aplica al borde del afirmado de la via, y si es necesario al talud en excavaciones y terraplenes.

**213.** El tren de riego puede cubrir cerca de (4 millas) (6.400 m) por hora gastando de 1.200 a 1.500 gals de aceite por kilometro de via sencilla sin aceitar. En los riegos subsecuentes se necesita menos aceite. La penetracion aumenta en cada regada hasta que llega a cerca de (8 pulgs) 20 cm que es suficiente para evitar que el balasto seco y polvoriento se levante durante las renovaciones y el apisonado.

## Conservación de la Via.

**214. Emparejado.** Consiste en levantar las depresiones en la via (sobre todo en las juntas) a un nivel parejo. No es necesario mantener la via en las marcas que se pusieron cuando se hizo nueva mientras se mantenga bastante pareja.

**215. El emparejado completo** (consiste en emparejar la via en conjunto); es necesario cada cierto numero de años.

**216.** El *costo* de emparejar varia desde $60 a $120 kilometro por año, dependiendo del tráfico, y de los materiales de que esta construida la via.

**217. Balasto.** *Renovación.* Antes de distribuir el balasto nuevo sobre la via, el balasto viejo debe quitarse de entre las traviesas hasta el fondo de ellas para anchar el borde del afirmado de la via. Véase tambien « Embalastado ». ¶ ¶ 201, etc.

**218.** *Limpia* del balasto. * Intervalos de años entre las limpiezas; en las estaciones terminales, 1 a 3; en lineas de tráfico pesado, de carbon y coke, 3 a 5; en tráfico ligero, 5 a 8 *.

* En condiciones usuales, limpiese solamente la piedra y escoria dura. Úsese pala de balasto. Limpiese (a) el borde, hacia abajo ;(b) entre traviesas hasta el fondo de las traviesas; (c) centro de la zanja de doble via hacia abajo. Vuélvase a poner el balasto limpio *.

* Cascajo de orilla (Los promedios se refieren al volumen *original*).

| Clase de via. | Para lavarse o cernirse cuando contiene >. | El cascajo lavado ó cernido debe contener: |
|---|---|---|
| A. | 2 % de polvo ó 40 % arena | < 25 % > 35 % arena. |
| B. | 3 % — ó 60 % — | < 25 % > 50 % — * |

* Ascn Am Ingo F. C. † F. C. Pen. ÷ F. C. U. Pacific. Véase tambien Especificaciones, p. 853.

**219. Traviesas.** La persona que inspeccione las traviesas, con el propósito de renovarlas, las puede marcar con pintura blanca. El método usual de poner traviesas nuevas es excavar una zanja al lado de la traviesa vieja un poco mas honda que la traviesa; entonces se empuja de costado en la zanja, y se saca. Si la traviesa nueva es mas gruesa, o si la traviesa vieja estaba cortada por la base del carril, es necesario recorrer el asiento de la traviesa.

En balasto de roca, las traviesas se pueden renovar a razón de 8 o 10/dia/hombre; en cascajo, de 14 a 18.

**220.** *Gasto para Renovar traviesas.*

| | |
|---|---|
| En balasto de piedra..... | cerca de 20 ctos cada una. |
| En cascajo flojo..... | 6 a 10 ctos cada una. |
| En cascajo que fragua..... | 10 a 20 ctos cada una. |
| En escoria dura..... | 9 a 15 ctos cada una. |

**221. Carriles.** Para ponerse los carriles nuevos se colocan sobre las traviesas en la parte de afuera de los carriles viejos, se atornillan unos a otros en tramos que puedan manejarse. Los clavos en el interior del carril se sacan, el carril se levanta hacia afuera sin aflojar las juntas (excepto en el extremo de las secciones o tramo), y el carril nuevo se coloca en su lugar y se clava.

**222.** *En curvas*, se trasponen a menudo los carriles, el carril exterior gastado se pone al interior de la via, y el menos gastado interiormente va al exterior; los clavos interiores se sacan, los carriles se trasponen, y los clavos se reponen.

**223.** La via mas difícil de conservar es la de curvas estrechas para servicio rápido de pasajeros, y por la que corren trenes de carga cuya velocidad sea limitada, por las condiciones de las cuestas y declives.

**224. Carros registradores de via.** Los hay de muchos tipos, no hay aparentemente ninguno que prive. Casi todos llevan una larga tira de papel, movido por un mecanismo de relojería o por engranes de las ruedas, de modo, que por medio de lápices o plumas, se hacen trazados en los cuales la abcisa puede representar tiempo o distancia, como se desee. A fin de identificar, diferentes partes del registro, es costumbre dedicar una o dos plumas a registrar el tiempo por medio de un reloj impulsado por un electro-magneto que obra sobre una pluma; o registrar la distancia, automáticamente, por engranes desde las ruedas, ó por impulsos dados con un boton de contacto por un observador que vigile los postes kilométricos u otras marcas. Estas marcas son identificadas, haciendolas con cierta caracteristicas como dandoles, por ejemplo, doble ó triple longitud ó por otro operador registrando el nombre a la mano y frente a la marca.

**225.** Varios son los elementos que se registran, y de diferentes modos. Las desigualdades laterales o verticales de la via son registradas por pendulos (libres, ó contrapesados por « cojinetes »), por el movimiento relativo del trole (carretilla) y cuerpo del carro, o por los de una rueda loca ó un par de ruedas. Algunos carros estan dispuestos para manchar automaticamente con pintura el costado del carril, donde quiera que haya desigualdades en la via.

**Carros dinamómetros.** Véase Resistencia de trenes, ¶ ¶ 56, etc. (masadelante).

# DESVIOS Y CRUCEROS

## PARTE I. PRÁCTICA

### DESVÍOS

Para el estudio geométrico de los desvios, véase Part II, Pag 923, etc.

#### Generalidades.

**1. Elementos.** Figs 1. Un desvio se compone esencialmente de un chucho (1), dos carriles de « arranque », $L$ y $L_t$ *, una rana, y dos guarda carriles, $g$ y $g_t$.

**2. Enfrentado y Arrastre.** Figs 1. Cuando un tren entra en un desvio en la direccion de la flecha (pasando el chucho antes de llegar á la rana), se dice que « enfrenta » el chucho. Cuando se aproxima (viniendo de la linea principal, $M'$, ó del desvio $T$) en la direccion opuesta (pasando la rana antes de llegar al chucho) se dice que « arrastra » (trail) el chucho.

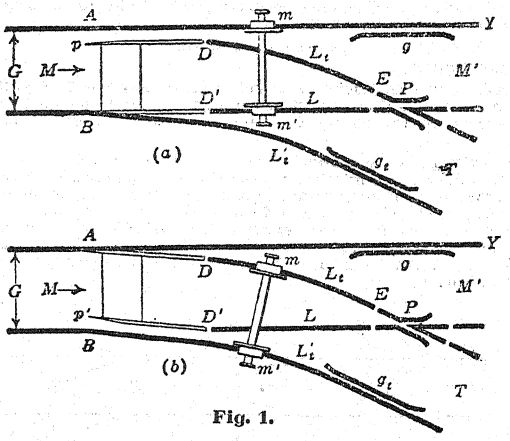


**Fig. 1.**

**3. Via doble;** Fig. 2; Los trenes se mantienen a la derecha, como se indica por las flechas. Aqui, para el tráfico normal, $X$ é $Y$ estan enfrentando los chuchos; mientras que $V$ y $W$ estan arrastrando los chuchos. Cuando se deja la via principal por arrastre de un chucho (por ej. : *enfrentando* un *chucho* de *arrastre*) un tren debe moverse en la direccion, contraria a la propia, a dicha via. No obstante la superior seguridad de los chuchos de arrastre justifica su uso.

**4.** Fig. 2. Un desvio es **derecho** o **izquierdo**, segun mande un tren que *enfrente* a la derecha o a la izquierda de la linea principal. Asi, $V$ y $W$ son desvios *derecho*, mientras $X$ e $Y$ son *izquierdo*.

**5. La posicion del chucho** (Figs 1), determina cual de las dos vias, $M'$ o $T$, seguira un tren que los enfrente. El carril principal de arranque o carril principal de cierre, $L$, y el carril de desvio de arranque o carril de desvio de cierre, $L_t$ *, conduce desde el chucho a la rana, y las ranuras de la rana para las pestañas permite a dichas pestañas de las ruedas en cualquiera de los dos carriles, $L$ y $L_t$, * pasar al otro (vease párrafo 68).

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* Algunos escritores llaman a los dos carriles encorvados, $Lt$ y $Lt'$, carriles de arranque. Vease Webb, construction de Ferrocarriles, pp. 265 y 272.

(1) (*N. del T.* — Asi se llama en Cuba el conjunto *movible* de las agujas. A veces comprende tambien todo el desvio. Lo hemos adoptado.)

Los dos guarda carriles, $g$ y $g_1$, mantienen dichas ruedas en sus propias vias dirigiendo las pestañas de sus compañeras mientras están pasando la rana.

**6. El avance** (algunas veces llamado la **distancia de la rana**), es la distancia $B\,P$, Figs 1, donde $P$, es el vértice teórico de la rana (¶42) y $B$ es la posicion del punto teórico del chucho (¶ 79) hacia el lado del desvio de la linea principal, $M\,M'$, cuando el chucho (1) esta puesto para la linea principal, como en la **Fig 1 a.**

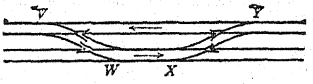



**Fig. 2.**

**7. La curva de arranque, o curva de desvio**, $D\,E$, Figs 1, es generalmente (por lo menos en teoría) una curva simple o circular. Fig 13. Las lineas de cartabon, $a\,y$ y $b\,z$, de la rana son generalmente rectas en todo su curso, o desde el principio hasta el fin de la rana; como tambien lo son los carriles del chucho en toda su longitud. Pero véase ¶ 77.

## Tipos de Desvios.

### Desvios dobles.

**8.** Figs 3. En un desvio doble (a menudo llamado desvio de **tres vias**) las dos vias laterales dejan la linea principal en el mismo punto. Comparándolos con dos desvios vecinos economiza, espacio, material, trabajo y tiempo, y facilitan su manejo.

**9. Curvatura.** La *linea principal* puede ser *recta o en curva* y las *vias laterales* pueden dejar la via principal *en la misma o en opuestas direcciones.* Cuando dejan o arrancan de lados opuestos, su agudeza (curvas ¶ 11) puede ser igual o diferente. Aunque las vias laterales se distinguen por su objeto ó funciones.

Es conveniente con frecuencia descartar esta distinción, y considerar las tres vias con referencia solamente a sus *posiciones* relativas, o como via *exterior* o del *medio.* Vease ¶ 11. Asi, $T$ o $T'$ puede ser una via principal en curva; en cuyo caso, $M'$ es un desvio recto.

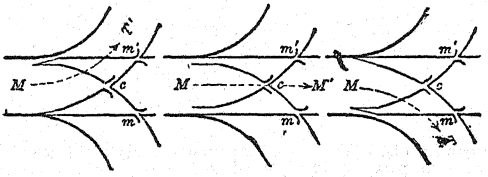



**Fig. 3.**

**10. Elementos.** Figs 3. Un desvio doble requiere *un chucho de 3 tiros* o tendidos, dos ranas principales, $m$, $m'$, una rana central $c$, y seis guarda carriles (dos para cada una de las tres ranas), a menos, que como en la Figs 3 los dos ángulos de la rana principal (vease Part II, ¶ 32) sean iguales, y las dos ranas principales sean por consiguiente opuestas : en este caso una pata de carril (vease ¶ 41) de cada rana principal actúa como guarda carril para la otra rana principal, y solamente se necesitan cuatro guarda carriles (véase figura) propiamente dichos. Cuando las dos ranas principales, $m$ y $m'$, son de poca diferencia en sus angulos, y por consiguiente casi opuestas, es impracticable colocar y mantener los dos guarda carriles adicionales de suficiente largo. En la Figs 3, los dos carriles de cada chucho de tres-tiros estan indicados por líneas finas, que muestran el chucho puesto para las vias $T'$, $M'$ y $T$, respectivamente, como se indica por las flechas. Cuando la curvatura del desvio es fuerte, las patas de la rana central estan encorvadas para que se adapten a la curvatura.

(1) Véase *N. del T.* ¶ 1.

**Vias principales, apartaderos y enlaces.**

**11.** De las dos vias, conectadas por un desvio, es generalmente fácil distinguir una, como la via *principal*, y la otra como el desvio. La linea o via entre la rana de la via principal y el desviadero se llama enlace : y la expresion « enlace « se aplica tambien a una via de alguna longitud que une dos vias, ninguna de las cuales es un desvio de la otra.

**12. Un chucho** (1) **muerto** es una via (generalmente corta), hecha con algun propósito especial, como para una cantera vecina, apartadero, foso o almacen, que no vuelve a enlazar con la linea principal.

**12 a. Una transferencia** conecta dos vias no-adyacentes, y el termino se usa especialmente cuando estas se encuentran a diferente nivel.

**12 b. Una maestra** es una via que cruza diagonalmente, y conecta una serie de vias paralelas o concéntricas.

**12 c. Un entronque** (vease parte II, ¶ 57) conecta una via principal con vias de patio adyacentes.

**Fig. 4 a.**

**Fig. 4 b.**

**Enlaces (Gauntlet) adheridos y (Intervolvd) envueltos.**

**13.** Fig. 4. En ciertos casos especiales, como en el caso de dos vias paralelas o concéntricas que deban pasar por un espacio demasiado estrecho para doble via como en un puente de via sencilla o túnel, puede usarse la disposicion que se indica. Donde el tráfico, en las dos vias, sea en direcciones opuestas se llama enlace *adherido*; cuando es en la misma direccion, se dice que las vias estan **envueltas**. Se necesitan dos ranas sin chuchos. Las dos vias se tiran sobre las mismas traviesas.

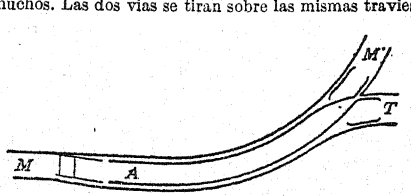



**Fig. 5.**

**14. Mejora en el Replanteo de Chucho.** Fig. 5. Para evitar la colocacion de un chucho en el exterior de una curva puede colocarse en la tangente, detras del punto *A* de la curva, y llevar el tráfico por una via envuelta concentrica, a la rana donde el desvio, *T*, deja la curva de la via principal, *A M'*. Esta disposicion requiere el uso de un chucho y una rana.

**15.** Un sistema de vias envueltas se usa algunas veces en las básculas o romanas **de via** para el material de tráfico que no se pesa. La via que lleve dicho trafico descansa en estribos independientes de la plataforma de la báscula. Esta disposicion exige el uso de dos chuchos, sin rana. Véase Patios y Estaciones, ¶ 60.

(1) Véase *N. del T.* ¶ 1.

**Desvio diamante o equilátero.**

**16.** Donde una via lateral paralela o concéntrica deje una via principal recta o curva, la linea principal ordinariamente continua su curso sin afectarse por el desvio; la deflexion de la via se concreta enteramente a el desvio.

**17.** No obstante, si la desviacion se divide igualmente entre la linea principal y la lateral, tenemos un desvio « diamante » o « equilátero ». Esto permite el uso de una rana de mayor angulo (numero bajo) sin estrechar la curvatura. Véase Part II, ¶ 52.

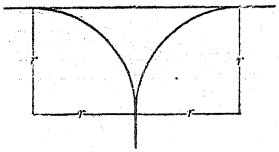



Fig. 6.

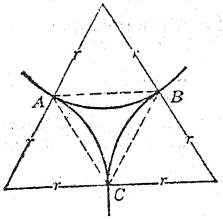



Fig. 7.

**Triángulos (desvios en Y).**

**18.** El triángulo, Figs 6 y 7, se usa para virar locomotoras (cambio de cabeza) de este modo sustituye a la mesa giratoria. Tambien se viran trenes enteros, si no son demasiado largos para el triángulo.

El sostenimiento del triángulo debe costar menos que el de la mesa giratoria : y puede costar menos su instalacion, segun sea el costo del terreno.

**19.** Fig. 7. En igualdad de condiciones, el triangulo *ocupa el mínimo de terreno* cuando las tres vias o « patas » son curvas de radios iguales, y de igual largo. Los tres chuchos caen entonces en los vértices de un triangulo equilátero, *A B C*, y cada lado es igual al radio comun, *r*.

**Descarrilamientos.**

**20. Generalidades.** Los carros dejados en los desvios, pueden moverse, por gravedad por el viento, descuidos, etc., a posiciones lo bastante cerca de la linea principal para hacer peligrar el tráfico por un choque. Los carros pueden arrancar solos por gravedad, sobre una pendiente de cerca de 0.4 por ciento (21 pies por milla); o (por la accion del viento) en pendientes mas planas o en vias a nivel. Es inseguro fiarse de los frenos.

**21.** Generalmente se considera segura una distancia de (12 pies) 3.66 m entre centros de via o sea un espacio de (6.5 a 7 pies) 2 a 2.15 m de espacio entre las cabezas de los carriles. El ferrocarril de Pennsylvania exige ∢ (16 pies) 4.90 m desde el centro de la via lateral al centro de la via principal (18 pies (5.50 m) donde sea viable); excepto en apartaderos paralelos a la via principal se encurvarán hacía afuera.

**22.** El punto, donde las dos vias se acercan mas de lo permitido, se indica con un **poste cartabón** de madera o de metal. Si se pone entre vias, tales postes son peligrosos de noche para los vigilantes. Por consiguiente, se colocan con preferencia 1,50 m a un lado de cualquier via. Algunas veces el punto de cartabón está indicado por una traviesa en forma de mediacaña pintada de blanco, puesta entre las dos vias.

**23.** Se usa en la via principal **un aparato de descarrilar** o apartaderos donde el descarrilamiento sea preferible antes que el carro llegue al punto de peligro, como un chucho (1), o un crucero, etc.

**24.** Figs 8 *a*, 8 *b*. *El extremo de empalme* de las agujas de chucho están indicados por tildes pequeños en el interior de la via.

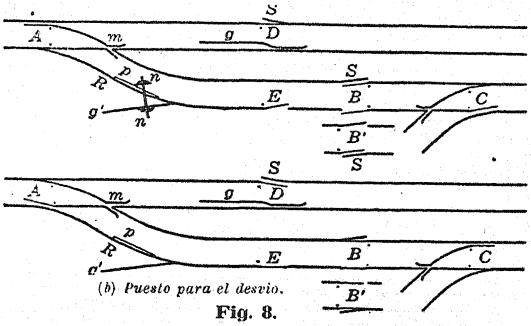
**25.** *Un carril solo de chucho de descarrilar*, como se muestra en *D* ó *E*, es suficiente para un descarrilo; pero un par de carriles de chucho, como se muestra en *B*, facilita el volver a la via los carros descarrilados. En posicion para descarrilar, el carril de chucho esta generalmente soportado por dos o tres mordazas de via. Véase Via, ¶ 143.

**26.** Es preferible que los chuchos de descarrilar, vayan *conectados con el chucho de la via principal A*, de modo que el aparato de descarrilar esté siempre en posicion adecuada con el chucho principal (compárense Figs 8 *a* y 8 *b*). De otro modo debe ponerse una señal o cartel, para recordarle al chuchero que los disponga para descarrilar.

**27. En las disposiciones para descarrilar de via principal**, como se muestra en *D*, el carril principal se dobla hacia afuera : y un carril de chucho de descarrilar simple se usa comunmente, con el empalme hacia la rana de la via principal *m*, y que abra hacia adentro, Fig. 8 *b*.

**28. En apartaderos**, basta un simple carril de chucho de descarrilar, *E*, fig. 8*a*, colocado en el carril exterior, con el empalme hacia la rana *m* de la via principal, y que obra hacia adentro.

*(a) Puesto para la Via principal.*




*(b) Puesto para el desvio.*

**Fig. 8.**

**29.** Fig. 8 *a*. Un carro, que descarrile en un desviadero, como en *E*, puede no obstante obstruir la via principal (como cuando el *descarrilamiento sea cerca de chucho de la via principal*, y la pendiente del apartadero sea considerable), y puede usarse un guardacarril largo *g'*, para mantener las ruedas *n*, mas lejos, fuera de la linea principal, y un tablon, *p*, para llevar las ruedas mas cercanas *n'*, sobre el carril del apartadero adyacente, *R*. Tambien puede usarse un *desviadero* de descarrilar, *C*.

**30.** Fig. 8 *b*. Descarrilado un carro en la linea principal, en *D*, se mantiene cerca de su propio curso por un largo guardacarril, *g*, colocado como a (20 cm) del carril que está opuesto al chucho de descarrilar. Este guardacarril puede continuarse hacia atrás, y doblarse, como se muestra, estrechando la abertura de la pestaña opuesta al aparato de descarrilar, de este modo se protege la punta o aguja del aparato de descarrilar cuando este esté puesto para la linea principal, como se muestra en la Fig. 8 *a*.

**31.** Figs. 8 *a*, 8 *b*. **En via doble**, si se usa un chucho de carril de descarrilar simple, *D* o *E*, se coloca generalmente en un carril *exterior*, con el objeto de evitar obstruir la otra via con los carros descarrilados.

**32. Direccion de Salida.** Figs. 8 *a*, 8 *b*. Donde se use un par de chuchos de descarrilar, *E* o *B'* en un apartadero, pueden como se indica en *B* huir de la rana, *m*, de la via principal, *separándose de* la via principal; o, como se muestra en *B'*, pueden huir *hacia* la rana de la via principal acercándose a la via principal; en cualquiera de os dos casos guiando el carro descarrilado *fuera* de la via principal.

(1) Véase *N. del T.*, ¶ 1.

**33. Desvio Wharton.** Se usa una forma especial de desvio corto Warton (vease ¶ ¶ 119, etc) para descarrilamientos en linea principal. Los carriles del desvio de descarrilar se dirigen *contra* los carriles de la linea principal para *descarrilar*, y *fuera* de ellos para el tráfico de la *linea principal*. Esta disposicion deja la linea principal entera sin interrupcion, y su ancho curso evita el peligro de un indebido e imperfecto descarrilamiento cuando esté puesto para la via principal.

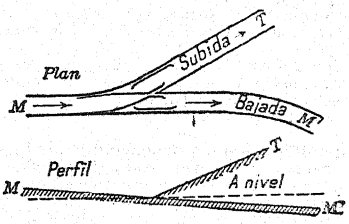



**Fig. 9.**

**Desvios de ataje.**

**34.** Fig. 9. A menudo, con el objeto de detener y desviar carros corridos en las pendientes, especialmente cuando se acercan a las curvas se usan los desvios de ataje. Estos desvios se extienden por una corta distancia hacia arriba al lado de la pendiente; y el chucho* se sujeta por medio de un muelle en posición para el desvio. De modo que el carro corrido se desvia de la principal al desviadero, donde se para por su propio peso. Entonces retrocede por gravedad al chucho pasando a la via principal y subiendo la pendiente. De este modo oscila hasta detenerse finalmente en el chucho. En operacion normal, el chuchero pone el chucho para la via principal, contra el muelle, el cual lo retiene asi hasta que el carro o tren haya pasado el chucho.

**Para Topes de via**, vease Señales, ¶ 47, pag. 1066.

**Enlaces.**

**35.** Figs 10 *a*, 10 *b*. Un enlace conecta dos vias paralelas o concéntricas. Consiste en dos desvios (enfrentados en direcciones opuestas) y una via que conecta, que puede ser una tangente o una curva invertida.

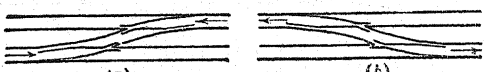



**Fig. 10.**

**36. Derecha e Izquierda.** Con tráfico que lleve su direccion a la derecha como se indica en las Figs. 10 *a* y 10 *b*, los dos desvios son ambos del lado *izquierdo*, si van de *frente*, como en la Fig. 10 *a*; y ambos del lado *derecho*, si *arrastran* o *siguen*, como en la Fig. 10 *b*.

**37. Desvios Doble.** Figs. 11 *a*, 11 *b*. Se usa un doble desvio en las lineas de via sencilla, con el objeto de que dos trenes que se encuentran, esperando para dar

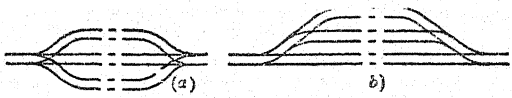



**Fig. 11.**

paso a un tercer tren puedan seguir entonces simultaneamente y sin retroceder, o estorbarse. Las vias laterales pueden ser una a cada lado de la via principal, Fig. 11 *a*, o ambas al mismo lado, Fig. 11 *b*.

* Véase *N. del T.*, ¶ 1.

**38. Desvios de doble enlace** (lap siding). Como se indica en la Fig. 12, tienen la ventaja de que la oficina de telégrafo puede colocarse en los enlaces del medio, y por consiguiente de modo conveniente para las máquinas de los dos trenes parados facilitando así las operaciones. Como se ve, el desvío de doble enlace, es en realidad una doble vía, de largo *A B*, con enlaces a la mitad de su tramo o largo.

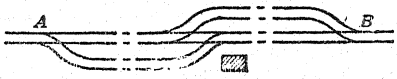



**Fig. 12.**

## Entrevía.

**39.** Algunos ferrocarriles mantienen la entrevía tipo en los desvíos. Otros la aumentan de (6 a 12 mm) entre la aguja del desvío y el extremo de la rana, fuera de cuyos puntos aquella se estrecha gradualmente hasta el ancho tipo en una distancia como de (9 m) a cada lado; mientras que en los ferrocarriles Europeos, con frecuencia se estrecha la vía en el desvío (6 mm) a fin de evitar el juego lateral de las ruedas, y dar estabilidad a los vehículos mientras están en el desvío. El desvío Wharton, vease ¶ ¶ 119, etc., requiere en el desvío una vía (12 mm) mas ancha que el tipo establecido.

## Ranas.

### Ranas rígidas.

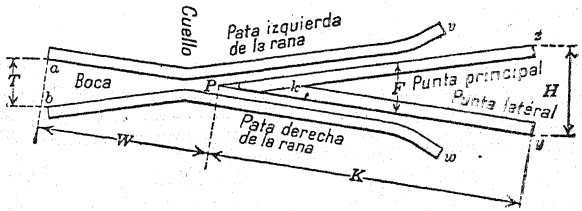



**Fig. 13.**

**40. Canales.** Fig. 13. Para la descripcion geométrica de la rana, véase Part II, ¶ 2, etc. A fin de que las pestañas de las ruedas, en el carril *a y*, puedan pasar en cualquiera direccion, del carril *b z* (o *vice* versa), la rana esta provista de « canales » al traves de los carriles *b z* y *a y*, respectivamente.

**41. Elementos.** Fig. 13. La rana se compone esencialmente de cuatro piezas de carril, así :

*a v*, la pata del carril lado izquierdo (pata izquierda de rana);
*b w*, la pata del carril lado derecho (pata derecha de rana);
*P z*, la punta principal o lengueta larga;
*k y*, la punta lateral o lengueta corta.

**42. Las lineas de la via**, a lo largo de la rana, son *a y* y *b z*. Fig. 13. A las *partes restantes* y *dimensiones* de la rana se le dan nombres como sigue : —

Abertura de la cabeza, $T$, $= a\ b$, medida entre las patas de la rana en las cabezas del carril;

Abertura de las patas de la rana, $H$, $= z\ y$; medida entre las cabezas de los carriles;

Cuello de la rana al punto de menor distancia entre las patas de rana;

Bocaa l espacio trapezoidal entre las patas, el cuello y la cabeza $T$;

Vertice de la rana el punto $P$; de la interseccion de las lineas de via $a\ y\ y\ b\ z$;
Lengueta, el triángulo entre $P$ y $k$;
Canales, los dos canales entre la punta y las patas;
Angulo de la rana, $F, = z\ P\ y = a\ P\ b$.

**43. El número de la rana**, $N$, es el cociente, $l/i$, donde $l$ (no se indica) = al largo de cualquier porcion de la rana, y $i$ (no se indica) = al aumento del ancho de la rana (medido entre los carriles de via y perpendicular al eje de la rana) en esa porcion. Algunos ingenieros miden $l$ a lo largo de la linea central de la rana; otros, a lo largo de los carriles de via; y el número dado de una misma rana varia segun el caso. Vease Parte II, ¶ 6, etc.

**44. Ranas derechas o izquierdas :** Fig 13. Una rana se llama *derecha* o *izquierda* si una persona dandole el frente a la rana, ve el *lado* de la punta principal ó lengueta larga $P\ z$ al lado derecho ó izquierdo.

**45. Bloques o Entre-dos.** Las dos puntas de carril van remachadas juntas y un bloque o entredos de acero va insertado entre la punta y cada pata de la rana. Los dos bloques algunas veces se llevan mas alla de la punta, y unidos alli forman un bloque.

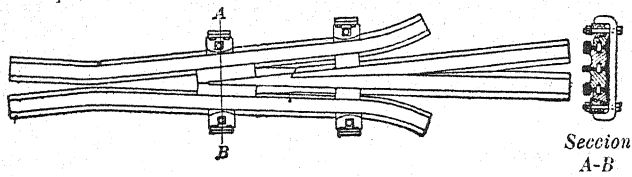



**Fig. 14a.**

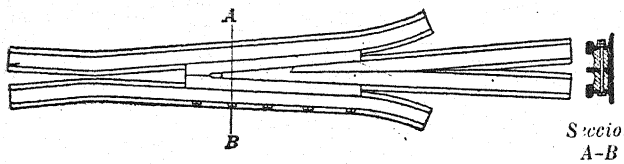



**Fig. 14b.**

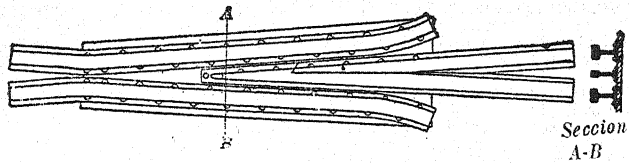



**Fig. 14c.**

**46. Amarres.** Las patas de los carriles y la punta, se aseguran, en sus posiciones relativas generalmente ó
(1) por grampas y cuñas, Fig. 14*a*;
(2) por tornillos, Fig. 14*b*;
(3) por remaches a una placa, Fig. 14*c*;
(4) por tornillos y placa [combinacion de (2) y (3)] ; o
(5) por remaches a las placas de asiento de las traviesas.

**47.** La práctica, respecto a la elección entre estos métodos, esta indicada por el resultado de dos investigaciones sobre los ferrocarriles que usan cada método.

| | (1) Grampa. | (2) Tornillo. | (3) Platina. | (4) Tornillo y Platina. | Total. |
|---|---|---|---|---|---|
| Asociación de Maestros de via, 1897..... | 7 | 29 | 9 | ..... | 45 |
| Revista Engineering News. Jun 4 1908..... | 7 | 32 | 15 | 10 | 64 |

Las ranas van conectadas con los carriles ordinarios de la via por los métodos usuales. Véase Via, ¶ ¶ 145, etc.

**48.** Aunque un lado de la rana esté en la curva del desvio, y el otro lado en la curva de la linea principal por lo corto de los lados deben hacerce rectos, excepto en desvios muy agudos y en casos especiales.

**49. Largo.** El largo de una rana debe ser tal, que los extremos del carril en la cabeza y las patas, esten separados lo necesario para dar lugar a las juntas del carril sin estorbarse. Aumentando el largo se hace mas fuerte la rana; pero tambien aumenta el desperdicio cuando hay que renovar las ranas, porque los carriles próximos duran mas que las ranas. La adopción de una rana, tipo fijo de ranas, reduce el corte de carriles cuando hay que cambiarlas, vease ¶ 70.

**50. Detalles.** El carril pata de rana en el lado del desvio se hace algunas veces ligeramente mas larga en la cabeza, que en la otra parte, con el objeto de que el carril principal y el del arranque del desvio (¶ 5) puedan ser del mismo largo, y dejar opuestos los extremos de las agujas o carriles del desvio.

## Desgaste, Refuerzo, etc.

**51. Acero Especial.** Aquellas partes de las ranas que están sujetas a un desgaste fuerte se hacen frecuentemente de manganeso especial u otro acero de alta resistencia para el desgaste, y construido en el cuerpo de la rana. Las ranas hechas de este modo cuestan cerca del doble de las ordinarias; pero su uso, es no obstante, económico en tráficos pesados.

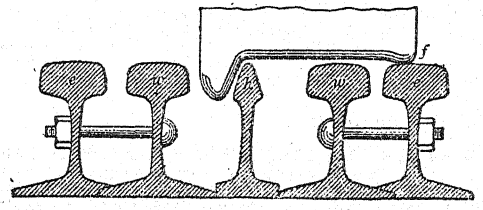



**Fig. 15.**

**52. Carriles de alivio ó de defensa.** Fig. 15. Se forman pestañas exteriores o falsas, *f*, por el desgaste de la llanta de la rueda. Un carril de alivio o de defensa corto, *e*, atornillado afuera del carril de las ancas de la rana (*w*), y formando, con *w*, un pata de dos cabezas, levanta la rueda suficientemente para mantener la llanta gastada fuera de contacto con la lengueta o punta de carril, *h*.

**53. Bloque en las patas de la rana.** Fig. 16. La pestaña falsa, *f*, de una rueda que arrastre, al caer entre las lenguetas de carril, *M* y *S*, tiende a separarlas. Para evitar esto se atornilla un bloque de patas *R*, entre las dos lenguetas de carril. El bloque tiene pendiente hacia la culata de la rana, como se muestra en la seccion con objeto de levantar ó bajar las ruedas gradualmente. Puede ser una pieza sólida de acero fundido, o un pedazo de carril invertido. Tambien sirve como guardapatas. Vease ¶ 75. Si no está bien asegurado, la pestaña puede forzarlo hacia adelante y actuar entonces como una cuña destructiva.

**54.** En la rana rígida (hasta ahora descrita) todas las partes son inmovibles y ambas canales están siempre abiertas. Las patas, Fig. 13, soportando la parte exterior de la llanta de la rueda, protege la relativa delgadez de la punta de la rana, que de otro modo se maltrataria con el paso de las ruedas; y las canales por donde pasan las pestañas de las ruedas, se hacen lo mas estrecha posibles, con el objeto de disminuir la intensidad de los golpes que dan las ruedas al pasar. No obstante, estos golpes son de consideración dado el volumen del tráfico.

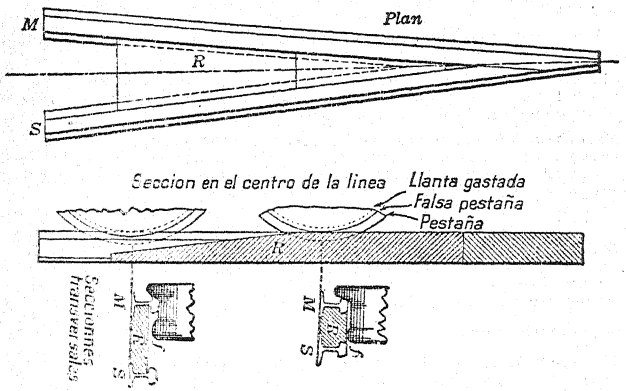



Fig. 16.

**55. Canales de Pestañas altas.** Al principio, con objeto de eliminar esta dificultad, las canales eran poco profundas de modo que las ruedas al atravesarlas, corrian sobre las pestañas, de este modo se levantaba la llanta de la rueda y no tenia contacto con la cabeza del carril; pero las pestañas pronto formaban ranuras, que hacian bajar las ruedas hasta que las llantas de las ruedas volvian a descansar sobre las cabezas del carril, dando golpes como antes.

## Ranas con Carril de muelles.

**56. Funcionamiento.** Figs 17 y 18. En la rana con carril de muelle la pata de la rana del desvio, *b s w* (que es el carril de muelle), aunque empalmado al carril de la via en su extremo, *b*, está clavado en su longitud solamente del lado de la via, y por consiguiente puede separarse del contacto con la punta, *P y*, contra la cual se apoya por muelles, como el de *h*. Cualquiera rueda de la via principal, *m'* (ó su compañera, *m*) tiene asi siempre un asiento completo en toda la rana. El carril con muelle se desliza sobre grandes placas fijas, que se extienden bajo todos los carriles de la rana.

De 59 ferrocarriles interrogados 58 usaron ranas con carril de muelle. Véase la revista Engineering News, de junio 4, 1908.

**57.** Las ruedas, *n'*, *o'*, yendo para o viniendo del *desvio*, al atravesar la rana, deben cruzar el canal de la pestaña entre la punta del carril, *P z*, y la pata de la rana *P v*. Para tener canal para la pestaña, entre el carril de resorte, *s w*, y *P y*, la pestaña de una rueda de *arrastre*, *o'*, empuja simplemente al entrar entre los dos; mientras que una rueda de *frente*, *n* (al estar separada de la rana por el guardacarril, *gt*), necesariamente guia su rueda compañera, *n'*, tambien en esa dirección y asi fuerza el carril de muelle, *s w*, fuera de, *P y*, y abre un canal para pestaña para la rueda *n'*. En cualquier caso, el muelle, *h*, vuelve al carril de muelle, *s w*, a su posicion normal, en contacto con *P y*, después que pasa cada rueda.

**58. Caja de guia.** Figs 17, 18. Para evitar que el extremo libre *w*, del carril de muelle, *b w*, se levante, cuando una carga se concentra en, o cerca de su otro extremo, *b*, se asegura al alma del carril de muelle, cerca de *w*, una pieza forjada, cuya lengua horizontal se desliza en una caja de guia, *c*, remachada a una plancha de corredera fija, extendida bajo todos los carriles de la rana en ese punto. La caja-guia obra tambien como retención para que el extremo libre, *w*, del carril de muelle no se salga demasiado.

**59.** Para evitar que la cabeza del carril de muelle pueda quedarse pegado al carril de lengueta de la punta de rana, su *patin o aleta debe cortarse* del lado próximo a la punta. El carril de muelle asi debilitado, se *refuerza* generalmente con una tira remachada o atornillada a su alma.

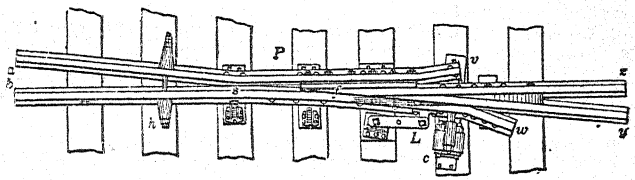



Fig. 17.

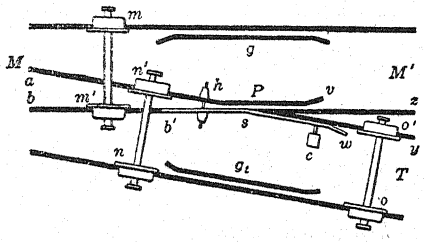



Fig. 18.

**60. Prevision contra las pestañas falsas.** Fig 18. Si el asiento entre la cabeza del carril de muelle, *s w*, y el de la punta del carril *P y*, es demasiado corto, una pestaña falsa (¶ 52) del canto exterior de la llanta de una rueda de arrastre de *m'* de la linea principal, pasando de *M'* hacia *M*, pudiera golpear la abertura, *w*, del carril de muelle, y de este modo acuñar el carril de muelle separándolo del punto, *P*. Para evitar esto al carril de muelle, *s w*, se le da un asiento o descanso mas amplio contra el carril de punta, *P y*, de lo que, sin esto seria necesario. Por una razon análoga, la cabeza del carril de muelle, donde se hace el contacto con la punta de la rana, se cepilla hasta formar una ranura paralela con la orilla de los carriles de la linea principal, como se indica por la sombra en la fig. 17 dando asi un paso a las pestañas falsas.

**61. Para Via Principal solamente.** Como las ranas corrientes con carril de muelle presentan un asiento completo a la llanta de las ruedas, *m'*, en la via *principal* solamente, fig. 18, y no a las ruedas *n'* y *o'*, que vienen por el desvio, son por consiguiente mas útiles donde la mayor parte del tráfico usa la linea principal y son pocos los trenes que usan el desvio. Vease ¶ ¶ 66, 67.

**62. Lado Derecho e Izquierdo.** Las ranas rígidas (exceptuando aquellas con patas de carril desiguales, ¶ 50) pueden usarse para desvíos del lado derecho o izquierdo; pero, las ranas con carril de muelle deben hacerse expresamente para el lado derecho o izquierdo.

**63. Resbalamiento.** Fig. 17. Para evitar que el carril de muelle se deslize o resbale; sin restringir su movimiento lateral, se usa el eslabón, *L*, que está puesto de tal modo que la tendencia a deslizarse forzaria el muelle de carril *contra* la lengueta, sin dejarlo *separarse* de ella.

**64.** *En la rana con carril de muelle embisagrado* (trazada para evitar la necesidad de emplear medios contra el deslizamiento del carril de muelle) este último se divide en *s*, fig. 17; y la parte *b s* se fija en su posición, mientras que la parte, *s a*, va embisagrada en *w*, en lugar de estarlo en *s*, donde se deja libre para girar hacia afuera contra un muelle, que lo hace volver a su puesto en contacto con la punta, despues que pasa cada rueda.

**65. En curvas.** Figs. 17, 18. Cuando una rana de carril de muelle se usa en la parte exterior de una curva en la via principal, la fuerza centrífuga de un tren en la via principal, empujando contra el carril de muelle, entre *b* y *s*, tiende a abrirlo. Esta tendencia se resiste solamente con el guarda carril, *g*, fig. 18, opuesto, pero se alivia abriendo ligeramente el carril de muelle fuera de contacto con la lengueta cerca de la punta.

**66. Rana con carril de Muelle Doble.** En la rana de carril de muelle doble, las dos patas de carril están equipadas como carril de muelle, y cada una se mueve independientemente de la otra. Ambas canales de las pestañas permanecen cerradas cuando no pasan ruedas. Estas ranas se adaptan en los desvíos donde el tráfico en la linea principal y en el desvío, es casi igual. Véase ¶ 61.

**67. Rana de carril de patas Corredizo.** En estas, los dos carriles de patas están conectadas rigidamente, a una distancia fija; pero libres para deslizarse juntos, de modo que cualquiera de las dos pueda descansar contra la lengueta fija, dejando una canal para las pestañas de las ruedas entre el otro y la lengueta. Las ruedas que entren de frente o arrastrando, abren la canal de pestaña (si no está ya abierta) por haberse deslizado los carriles (las ruedas que entran de frente lo efectuan por medio del guarda carril opuesto). Los carriles dichos permanecen entonces en la nueva posición para las ruedas que siguen. Estos carriles deben deslizarse, bajo alta presión, mientras uno de ellos soporta la carga de la primera rueda; una obstrucción alojada entre una pata del carril y la lengueta, puede trancar las patas del carril, y ocasionar un descarrilamiento. Como la rana de carril de doble muelle, ¶ 66, la rana de carriles de patas, corredizo esta dibujada, trazada, para casos donde el tráfico en la linea principal y en el desvío, es casi igual. Vease ¶ 61.

### Ranas de Carril Principal contínuo.

**68.** Para dejar siempre el carril de la via principal sin interrupción, enterizo, algunas ranas tienen el carril del desvío elevado de tal modo, que las ruedas en el desvío pasan por *encima* (en lugar de al través) del carril principal, las llantas quedan sostenidas por carriles cortos de apoyo que giran provisionalmente con este objeto, y que despues vuelven a girar hacia un lado, en direcciones opuestas, del carril principal, al despejar la linea principal. El chucho (1) se pone simultaneamente con la rana.

### Especificaciones, Uso, etc.

**69. Requisitos.** Al pedirse ranas, especifíquese tipo y detalles de construcción; sección y peso del carril; número de la rana o ángulo; largo total; largo desde la punta de la lengueta a la pata; reparto o espacio de los agujeros de las juntas de carril; grueso y dimensiones de la platina o placa usada; y, en el caso de una rana de carril de muelle, u otra rana con patas desiguales, decir si es lado derecho o izquierdo. A fin de que las partes sean cambiables, especifíquese tambien que todas las ranas de una clase deben hacerse iguales con respecto al barrenado de agujeros, de tornillos y remaches. Los carriles de las patas (que se gastan mas rapidamente que las lengüetas) pueden entonces reponerse con facilidad.

(1) Véase *N. del T.*, ¶ 1.

**70. Dimensiones. Assen Am Ings F. C.** Manual, 1915, pags 168-170. **Tamaños o números.** (Vease tambien el final de este parrafo para otros que no son de la Assen Am Ings F. C.) « Las ranas nos. 8, 11 y 16 se recomiendan como las que reunen todos los requisitos para patios, desvios de via principal y desviaderos o enlaces. Los trabajos nuevos deben hacerse hasta donde sea practicable, con estas ranas, con objeto de efectuar una eliminacion gradual de otros números, disminuyendo el costo de manufactura, y la cantidad que hay que tener en existencia. »

(*N. del T.* — Entre parentesis las medidas métricas correspondientes a pies y pulg y tambien los kg por metro de los carriles.)

*Ranas rigidas fig.* 13.

| No. de la rana. | *W* | *K* | *W* + *K* | *T* | *H* |
|---|---|---|---|---|---|
| 8 | 4′ 9″ (1,447 m) | 8′ 9″ (2,666 m) | 13′ 6″ (4,114 m) | 7 $^{1}/_{16}$″ (0,179 m) | 13 $^{1}/_{8}$″ (0,333 m) |
| 11 | 6′ (1,829 m) | 11′6″ (3,505 m) | 17′ 6″ (5,334 m) | 6 $^{9}/_{16}$″ (0,166 m) | 12 $^{9}/_{16}$″ (0,319 m) |
| 16 | 8′ (2,438 m) | 16′ (4,877 m) | 24′ (7,315 m) | 6″ (0,152 m) | 12″ (0,304 m) |

Véase tambien ¶ 71.

*Para la rana de muelle ó resorte* n.º 11, Fig. 17, las mismas dimensiones que para la n.º 11 rígida, indicadas arriba.

**Diámetros de los tornillos** para todas las ranas anteriores rígidas y de resorte o muelle.

| | | | | | |
|---|---|---|---|---|---|
| *Carriles*..... | 100 lb (49,60 kg/m) | 90 lb (44,64 kg/m) | 80 lb (39,68 kg/m) | 70 lb (34,72 kg/m) | 60 lb (29,76 kg/m |
| *Diam pulg*.... | 1 $^{1}/_{8}$ (28,57 mm) | 1 $^{1}/_{8}$ (28,57 mm) | 1 $^{1}/_{8}$ (28,57 mm) | 1 (25,4 mm) | $^{7}/_{8}$ (22,2 mm) |

*Para Ranas de Carril de Muelle,* Fig. 17.

*Los extremos de los carriles de las patas achaflanados* a 45° con la vertical.

*La rana de muelle n.º* 11 *tiene* cinco bloques de tope de $^{3}/_{8}$ (9 mm) con extremo abierto de acero prensado.

**Tamaños o Nos.** *Para linea principal* con desvios para *poca velocidad* se usan Ranas del n.º 8, 9 a 12; *para alta velocidad,* como en los extremos de via doble, se usan los n.os 15 o 16 á 20. *En patios,* se usan los n.os 6 ó 7 á 9; en *patios industriales* congestionados para locomotoras de cortas bases de ruedas, se usan los n.os 4 a 6.

**71. Canales para Pestañas.** Una investigacion en 59 ferrocarriles demostró que 38 usaban en las ranas canales de 1 $^{3}/_{4}$ (44 mm) de ancho; 18 usaban de 1 $^{7}/_{8}$ (47 mm) y 3 usaban de 2 pulgs (51 mm).

**Guarda-Carriles.**

**72.** Fig. 1. Cada rueda al pasar por la rana, se mantiene en su propio curso por medio del guarda-carril, *g, g1,* que sostiene sobre su rueda compañera.

**73. Detalles.** El patín del guarda carril que va pegado al carril de la via se corta (o el guarda carril se fabrica con el alma inclinada hacia el carril de la via) con el objeto de dejar espacio, para poder clavar en la canal estrecha de la pestaña. Se han usado como guarda-carriles en lugar de carriles arreglados de este modo ó deformados, angulares de acero pesados. El ancho tipo para canales de pestaña se lleva uno o dos pies (30 ó 60 cm) desde un punto opuesto a la punta de la rana. Los extremos del guarda carril se cepillan formando un bisel largo, para evitar que objetos arrastrados accidentalmente se engarcen : o tambien el alma, próxima al extremo se corta, y la cabeza del carril se dobla hacia abajo con la pendiente necesaria. En la via principal, los guarda carriles son de 15 a 18 pies (4,5 a 5,50 m) de largo.

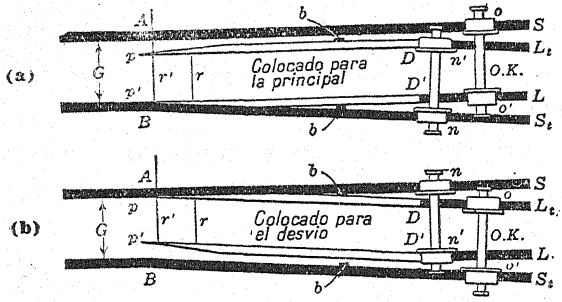
**74. Amarres.** Los guarda carriles (algunas veces debilitados por habérsele quitado el patín de un lado, ¶ 73) tienen que soportar presiones laterales fuertes; y por consiguiente están soportados por abrazaderas de carril aseguradas a las traviesas en el lado opuesto al carril de la via. Tambien planchas de asiento extendidas bajo el guarda carril y carril principal suelen usarse; o el guarda carril puede atornillarse al carril de la via, y quedar separado de él por bloques, que, cerca de os extremos actúan como guarda-pies. Véase ¶ 75.

**75. Guarda Pies.** Cuando el espacio libre, entre carriles adyacentes tiene entre 2 ½ y 5 pulgs (63 y 126 mm), hay peligro de que las personas se cojan los pies en el momento que avance un tren. Tales puntos se encuentran en los extremos de los guarda carriles, en la garganta y patas de las ranas, en la abertura de las patas de rana, y en los chuchos. Los guarda-pies se ponen para evitar tales accidentes. En ciertos estados los exige la ley. Están formados de piezas de tablón cortados para ajustarse a los espacios y clavados a las traviesas; o de bloques de madera o hierro con la forma necesaria o de tiras de acero, atornilladas al alma del carril; o si estan hechos de barras de acero, van doblados en una serie de angulos, y colocadas de canto.

## Chuchos (1).

**La Aguja de Chucho.**

Para el chucho de aguja de tope, véase ¶ ¶ 96-99.
Para el chucho Wharton, vease ¶ ¶ 119-126.




**Figs. 19.**

**76. Generalidades.** Figs 19 *a* y 19 *b*. Estas figs representan en lineas generales lo esencial del chucho de aguja, practicamente de uso universal. Para mas sencillez, los patines o bases del carril se han omitido en las figs, y solamente se muestran las cabezas del carril. En cada fig., se muestra el chucho puesto para las ruedas marcadas « O. K. ». Está mal colocado para las otras. Los carriles de arranque, $L_t$ y $L$ (vease ¶ 1, Fig. 1), y los carriles maestros, $S$ y $S_t$, están clavados de fijo a las traviesas. $S$, es el carril principal maestro (o carril continuo), y $S_t$ es el carril maestro del desvio (o carril articulado). En $B$, el carril articulado, $S_t$, está doblado formando un ángulo agudo = al angulo de chucho, $s$ (Fig. 21), y algunas veces doblado en este punto para recibir o alojar la punta de la aguja del carril, $p'$ $D'$, cuando esta puesto para la linea principal, como se indica en la fig. 19 *a*.

**Fig. 20.**

**77. Las agujas de chucho**, $p$ $D$, $p'$ $D'$, Figs 19, están formadas de carriles de via corrientes, asi cepillados, Fig. 20 (para afinar la punta), de modo que deje la via principal recta; estos carriles afilados se dejan rectos (excepto en los desvios muy agudos, donde algunas veces se encurvan). Permanecen rectos mientras se ponen para el desvio; girando en las juntas, $D$ $D'$, Figs 19.

(1) Véase *N. del T.*, ¶ 1.

**78. Largo de la Aguja.** En desvios corrientes, en lineas donde se usan carriles de (30 o 33 pies) 9,15 a 10,06 m en la via principal, el largo del carril de chucho (1) (por economia al cortar el carril) es usualmente (15′, ó 16′ − 6″), es decir la mitad del largo del carril; para altas velocidades, no obstante, como al final de una doble via, se usan largos de chucho de (20 y 30 pies), 6,10 y 9,15 ms; para patios y ramales, (10 o 11 pies) 3,05 ó 3,35 m. En cualquier caso, el largo del chucho debe ser cuando menos tal, que las juntas de carril en sus extremos, *D* y *D′*, Figs 19, no queden muy pegadas a los carriles maestros.

Véase la tabla, Part 2, ¶ 47.

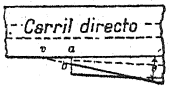



**Fig. 21.**

**79. El ángulo del chucho** (1) es el ángulo, *s*, Figs 20, 21, entre los lados de la via del chucho y el carril maestro (via principal). Debido al grueso, *a b*, Fig. 21, de la punta real, el punto teórico, o el vertice, *v*, de este ángulo está a una distancia, = *v a*, de dicha punta real. (Vease tabla de chuchos de aguja, ¶ 94.) El grueso de la punta de la aguja del chucho, *a b*, = *v a* tang *s*.

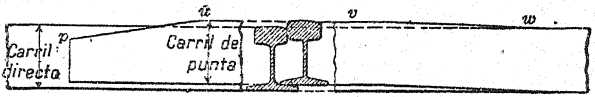



**Fig. 22.**

**80. Detalles.** Fig. 22. Para proteger la porcion delgada de cada aguja de carril, cerca de la punta, la parte superior de esa porción se rebaja cerca de (12 mm) la parte superior del carril maestro adyacente, en *p;* pero, desde *p*, la parte superior de la aguja del carril se levanta uniformemente en una distancia variable, *p u* (vease la tabla ¶ 94), hasta, digamos en *u*, donde la parte superior de la aguja del carril es cerca de (6 mm) *mas alta* que la del carril maestro adyacente, esto es con el objeto de que la falsa pestaña de una rueda que este acanalada, Fig. 23, no pueda chocar con el carril maestro, o separar los dos carriles cayendo en el espacio estrecho que hay entre ellos. Desde *u*, Fig. 22, la parte superior de la aguja del chucho mantiene esta sobreelevacion por algunos metros como se ve en *v*, y entonces declina hasta que las dos partes superiores de los dos carriles quedan a un mismo nivel, como en *w*. Algunas veces (con el mismo objeto) en lugar de *levantar la aguja del carril*, en *u*, el carril *maestro* se cepilla *rebajándolo* para dejar su parte superior por debajo del de la aguja de chucho.

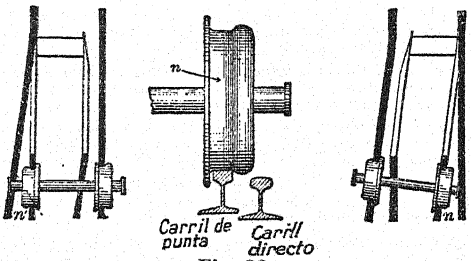



**Fig. 23.**

**81. Proteccion y Refuerzo.** Para evitar el riesgo de chocar o golpear la puntas abiertas, el empalme, en *p*, Fig. 19*a*; *p′* Fig. 19*b*, se hace mayor de lo que

(1) Véase *N del T.*, ¶ 1.

es necesario para que pasen las pestañas de las ruedas. La Asen Am de Ingos de F. C., (vease ¶ 94, abajo) especifica (12,5 cm).

**82.** Excepto en patios y para tráfico lento, las « agujas » son reforzadas, cuando menos en todo el largo de la parte cepillada, por un par de piezas de planchuela, de (6 a 15 mm) de grueso, y suficientemente anchas para llenar el espacio entre la cabeza y la pestaña, remachando o atornillando una a cada lado del alma del carril. Una barra angular o de canal sustituye a veces a una de las planchuelas.

**83.** En el *desvio de « canal »*, se atornilla una pieza ligera de carril en forma de T, de largo apropiado, a lo largo del carril de punta, del cual se mantiene como (15 cm) distante, por dos o tres separadores colocados a intervalos. La rigidez que resulta tiende a evitar que cierren el chucho (1) por equivocación cuando una obstrucción accidental separa los dos carriles, que deben traerse juntos. Vease ¶ 48, en señales.

**84.** Figs 19 *Bloques de tope, b.* Son bloques cortos de metal atornillados al alma del carril de aguja (Fig. 19*a*) o al alma del carril maestro (Fig. 19*b*) a intervalos convenientes, en puntos donde los dos carriles mismos nunca vienen en contacto. Su grueso es tal que presenta un asiento o sostén entre cada aguja de carril y el carril maestro adyacente, cuando estos están lo mas cerca. De este modo ellos apoyan la aguja del carril contra el carril maestro, permitiendole resistir mejor la presion lateral de las ruedas al pasar, que es fuerte sobre todo en las curvas debido a la fuerza centrifuga. Generalmente uno, dos o tres de estos bloques se colocan a cada lado de cada chucho (1).

**85. Platinas de corredera de acero.** Son de (12 a 15 cm) de ancho, y clavadas a las traviesas, extendidas bajo la aguja del carril y el carril maestro, entre la junta del chucho y el punto donde las pestañas de la aguja del carril y el carril maestro se separan. Hay generalmente de 6 a 8 platinas en cada lado del chucho. Sirven al doble propósito de guiar la aguja del carril a su asiento en la base del carril maestro, y (por medio de un asiento levantado en el cual la aguja del carril descansa y se desliza) de elevar la aguja del carril sobre el carril maestro, como se indica en el ¶ 80. Generalmente están hechas para recibir abrazaderas de carril para reforzar el carril maestro en el exterior.

**86. Platina de entrevia.** Se usa a veces una platina larga sencilla extendida enteramente al traves de la via, llamada platina de entrevia, en lugar de las dos platinas de correderas que se ponen en la junta del chucho. No solamente actúa como platina de corredera, sino tambien para retener los carriles maestros a cartabón, por medio de orejas o tacones adheridos a esta platina.

**87. Barras Tirante, Barras de desvio, Planchuela de Tirante Barras de Brida.** Figs 19. Las dos agujas de carril van conectadas, como se indica, por medio de barras tirante (barras de chucho, planchuelas de tirante, barras de brida), *r, r'* con preferencia sujetas al alma de las agujas de carril generalmente por conecciones embisagradas, para facilitar el cambio de angulo necesario al mover el chucho (1).

**88.** Las barras tirantes se hacen algunas veces amplias *en su largo* con objeto de rectificar el desgaste y para poder cambiarlas.

**89.** *Cantidad usada.* En linea principal, las barras tirantes son generalmente de 2 a 5. Su presencia estorba el apisonado; pero cuando se usan pocas barras, es necesario reforzar adicionalmente las agujas de carril.

**90.** *Situacion.* Las barras tirantes se colocan bajo el nivel de la parte superior de las traviesas para evitar que sean golpeadas por las ruedas de los carros, o que las arrastre alguna parte del material rodante que por un accidente se desprenda.

**91.** *Sección transversal.* En los chuchos de aguja, las barras tirantes son generalmente planas, y se colocan ó verticalmente de canto (en cuya posicion resisten el ladeo de las puntas del carril), u horizontalmente.

**92.** Figs 19. La barra tirante, *r'*, mas cerca de la junta del carril se llama *barra de cabeza* o barra *N.*° 1. A cada extremo se extiende bajo el carril maestro, con el propósito de evitar que el extremo de la junta del carril se levante fuera de su lugar. En uno de sus extremos esta unida a la barra de conección generalmente de 3 a 4 cm redonda o cuadrada.

(1) Véase *N. del T.*, ¶ 1.

**93. Requisitos en las Especificaciones.** Al pedirse chuchos (1) de aguja, especifíquese el ancho de la via; largo de las agujas de carril; carrera del chucho medido en la barra de cabeza; seccion del carril; barrenado para las juntas de carril, ángulo del chucho; y abertura.

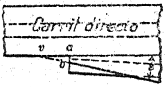



**Fig. 21 (Repetida.)**

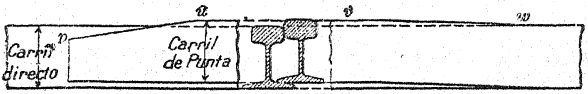



**Fig. 22 (Repetida.)**

**94. Especificación de Chuchos, Asen Am Ings F. C.,** Manual de 1915, paginas 178–9.

*Carrera del chucho* (5 pulgs), 12 $^1/_2$ cm, en la linea de centro de la barra n.° 1, *r'*, Fig. 19.

*Abertura,* en *D* y en *D'*, Figs 19 (6.25 pulgs) (16.5 cm) entre el carril maestro y el carril de chucho.

(*N. del T.* Las medidas en metros debajo de cada numero y entre paréntesis.)

| Número de la rana, *N* | | Largo del chucho $l = p\ D$ | $p\ u$ | $v\ a$ |
|---|---|---|---|---|
| Limitaciones. | Recomendadas. | Figs 19. | Fig. 22. | Fig. 21. |
| — | — | — | — | — |
| $\gtrless$ 6 | ... | 11 pies 0 pulg (3.353 m) | 5 pies (1.524) | 5.50 pulgs (0.139 m) |
| $>$ 6 $\gtrless$ 10 | 8 | 16 pies 6 pulgs (4.927 m) | 7 pies (2.134 m) | 8.25 pulgs (0.209 m) |
| $>$ 10 $\gtrless$ 14 | 11 | 22 pies 0 pulg (6.706 m) | 9 pies (2.743 m) | 11 pulgs (0.279 m) |
| $>$ 14 | 16 | 33 pies 0 pulg (10.058 m) | 12 pies (3.658 m) | 16.50 pulgs (0.406 m) |

*Barras de refuerzo,* $^3/_8$ (10 mm) grueso; altura para llenar el espacio entre la cabeza y la pestaña; largo lo mas que puedan permitir las conecciones en la junta.

Dos barras de chucho no ajustables, $^3/_4 \times 2\ ^1/_2$ (19 × 62 mm); colocadas horizontalmente; separadas (20″) 50 cm de centro a centro. El centro de la barra de cabeza a (12″) 30 cm mas atras de la punta de la aguja.

En cada traviesa, *dos placas de corredera* ($^7/_8$″ × 7″) 22 × 178 mm cepilladas para recibir el carril maestro y las abrazaderas.

**95. Un chucho de via triple,** Fig. 3 consiste virtualmente en dos chucho, de aguja separados; y asi estan dispuestos algunas veces, las puntas de dos pares de agujas de carril se colocan a veces separados cerca de un largo de carril de agujas o unos detras de los otros, en « *tandem* ».

**Chuchos de Aguja de Tope.**

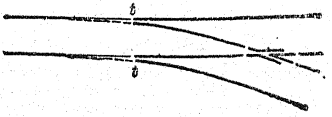



**Fig. 24.**

**96.** Fig. 24. El chucho de aguja de tope (todavia usado en localidades inferiores, como en desvios de lineas de ramales) es simple en construcción, y barato en su

(1) Véase *N. del T.*, ¶91.

primer costo, pero caro en su operación y mantenimiento. Para evitar la posibilidad de que se tranque con el calor, se debe dejar un gran espacio entre los topes o talones *tt*, y los extremos de los carriles adyacentes de la vía, y esto produce serios golpes con las ruedas al pasar por los extremos de los carriles. Tales golpes son perjudiciales a ambos : a los extremos de los carriles y al material rodante.

**97. Peligro.** La corrida de la punta del carril del chucho (1) o del extremo del carril adyacente causa un reborde, llamado « bemba », que puede hacerse peligroso. Pero la objeción mas seria del chucho de aguja de tope sin proteccion, es el inevitable descarrilamiento de los carros de arrastre cuando el chucho esta mal colocado.

**98. Largo.** Los carriles del chucho de aguja de tope son generalmente del largo completo de los carriles de vía, (30 o 33 pies) (9.15 a 10.06 mm) de largo. De este largo, (1.20 ó 1.50 m), próximo a la junta estan clavados a las traviesas, dejando el resto libre para encurvarlos cuando se ponen para el desvío.

**99. El curso** o distancia a que se mueve la punta del carril del chucho, debe ser cuando menos igual al ancho de la cabeza del carril (generalmente de 5 a 7 cm) mas un ancho (4 a 6 cm), lo suficiente para que pasen las pestañas de las ruedas. Generalmente la carrera ó juego, se hace de 12 a 14 cm.

**Pedestales de chucho.**

**100.** La palanca, por medio de la cual (ayudado por la barra de conexión) el chucho es puesto, se aloja en el « pedestal «. La forma o disposición varia mucho. La palanca puede girar en un plano vertical u horizontal. Si en plano vertical, este plano puede ser perpendicular, o paralelo a la vía; el movimiento de la palanca en este ultimo caso, se trasmite a la barra de coneccion por medio de un engrane o cigueña.

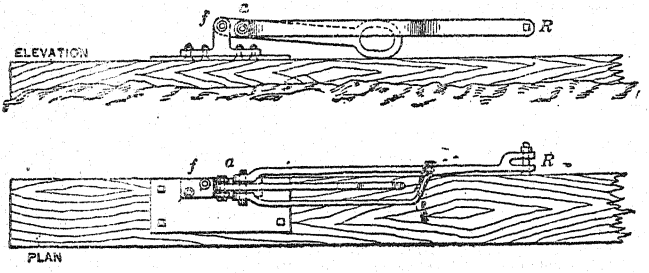



Fig 25.

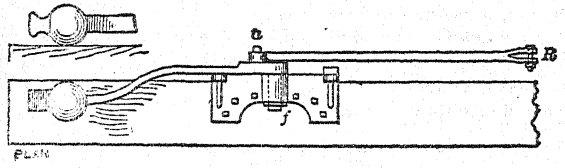



Fig. 26.

**101. Chucho de Palanca de Tumba.** Figs 25 y 26 muestran el pedestal de « palanca de tumba ». En la Fig. 25, la palanca se mueve en un arco de mas de 180°, en un plano vertical, perpendicular, o paralelo a la vía; de modo que cuando la palanca esta descansando, su unión, *a*, con la barra de conexión, está un poco mas abajo del fulcro, *f*, sobre el cual gira la palanca, o por debajo de su centro muerto; de modo que el chucho no puede ponerse o cambiarse porque lo empuje la presion lateral de las ruedas. En la Fig. 26, el extremo libre de la palanca

(1) Véase *N. del T.*, ¶ 1.

está contrapesado, y la junta, *a*, cuando la palanca, está descansada, queda por *arriba* del centro muerto. Esto permite reponer automáticamente un chucho (1) mal colocado por las ruedas de arrastre. Vease ¶ 57.

**102. Pedestal Giratorio.** La fig. 27 enseña una de las muchas formas de pedestal giratorio. La palanca generalmente embisagrada, de modo que cuelgue verticalmente, y fuera del paso (como se ve), cuando no está en uso para poner el chucho. Las escotaduras en *a*, *b* y *d*, lo retienen en sus tres posiciones, respectivamente. La cigueña, *c*, está generalmente construida como se indica, doblando el eje, *s*; y el pasador, *p*, al cual está unida la barra de conección, está formado doblando (para arriba o para abajo) el extremo de la cigueña.

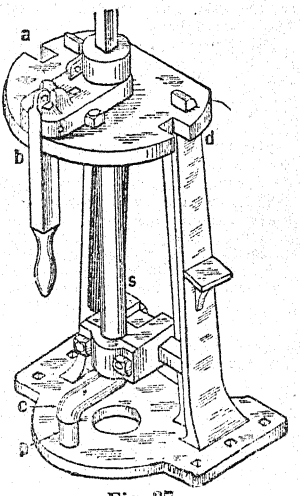


Fig. 27.

**103.** Fig. 28 *a*. Si en un pedestal giratorio el pasador de la cigueña se mueve 90° de *a* a *b* o *vice versa*, el movimiento lateral ( $= a'\,b' = b'\,a'$) del carril del chucho, es mayor que el que ( $= b'\,c'$) ocurre cuando el pasador de la cigueña se mueve 90° de *b* a *c* o viceversa. Esto puede rectificarse colocando el pedestal ligeramente sesgado o corrido, como se indica por las lineas de punto; porque esto corre materialmente $b'$ sobre $a'$, mientras que $a'$ y $c'$ cambian muy poco. O, como en la fig. 28 *b* el pedestal puede estar puesto de tal modo que la linea *o d*, normal a la barra del chucho, divida el curso de 90° de la cigueña, pero esta disposición no pone a la cigüeña en centro muerto en cualquiera posición. Para un juego dado de carril de chucho, y un ángulo dado de juego de cigüeña, fig. 28 *a* se requiere una cigueña mayor que la que se necesita en la Fig. 28 *b*.

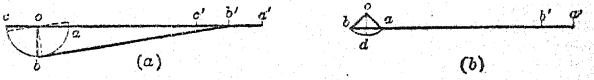


Fig. 28.

**104. Cerraderos.** Para evitar que se ande con el chucho, la palanca esta provista de un candado y argollas; pero se han usado pedestales con cerraderos complicados que forman cuerpo con aquellos. Véase tambien el final de ¶ 58, en señales.

**105. Una placa de señal,** en la parte superior de un eje vertical, que opera simultaneamente con el chucho, indica como está puesto el chucho. La placa de señal puede ser un disco sencillo, mostrando por su posición como está puesto el chucho (de plano o de canto a un tren que llega); o puede tener dos discos colocados a angulo recto, y diferenciándose en su forma, en su color o en ambos. Véase tambien señales, ¶ 14.

**106. Pedestales, Bajos, Intermedios,** o **Altos, son aquellos** en que la parte superior de la placa de señal está $>$ 60 cm, de (60 a 120 cm) de (180 a 240 cm o cerca de 540 cm) sobre el terreno respectivamente. Los pedestales intermedios son de uso corriente; los bajos se usan entre carrileras vecinas, y los altos donde la placa de señal de un pedestal bajo queda escondida por objetos que se atraviesan.

**107. Faroles.** Para chuchos que se tienen que usar de noche, el pedestal lleva tambien un farol, mostrando colores correspondientes a los que se usen con la placa de señal. Vease señales. Los faroles que usan *kerosene* están mas generalizados; pero, en patios grandes, a menudo se usan luces eléctricas incandescentes.

(1) Véase *N. del T.*, ¶ 1.

## Chuchos Automáticos.

**108.** Los chuchos (1) automáticos son aquellos en que las conexiones con el pedestal son tales que permiten el pasaje de un carro de arrastre cuando el chucho está mal puesto.

**109.** Figs 19 *a*, 19 *b*. La pestaña de una rueda de arrastre, *n*, en el carril maestro, se aprieta en el espacio estrecho entre él y el carril de chucho adyacente, y empujará a esta última (si no se evita) fuera del carril maestro, proporcionando así una canal para la pestaña, entre esos dos carriles; el otro carril de chucho simultaneamente se arroja contra el carril maestro opuesto, o en una posición apropiada para la rueda compañera, *n'*.

**110. El chucho automático « set-over »** es aquel en que las puntas del carril cuando se mueven automáticamente, como en ¶ 109, se llevan la palanca de chucho, y quedan en la nueva posición.

**111. En el chucho automático « fly-back »**, la palanca de chucho no se mueve cuando un carro de arrastre pasa un chucho mal puesto. Los carriles del chucho son separados temporalmente a un lado, y, despues del paso de cada par de ruedas, vuelven a sus posiciones anteriores por medio de un muelle en la barra de conexión, en la barra del chucho, o en el pedestal del chucho. El curso o carrera del pedestal se hace un poquito mayor que el curso del chucho, con el propósito de recoger el movimiento perdido.

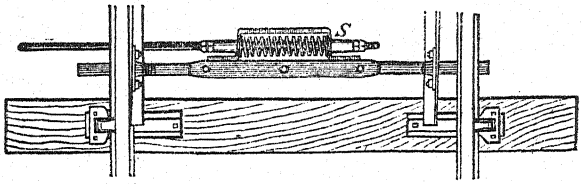



**Fig. 29.**

**112. Objeciones.** Ambas formas se prestan a la objeción de que al moverse el chucho parte de la carga que se mueve descansa sobre uno de los carriles de chucho relativamente delgados, que deben por consiguiente deslizarse mientras estan cargados.

**113. Muelle Lorenz.** La Fig. 29 enseña en principio, el muelle Lorenz, muy usado en los chuchos automáticos « fly back ». El muelle *S*, se coloca algunas veces fuera de la via.

**114.** Los *golpes repetidos* de las agujas de chucho contra el carril maestro en los chuchos automáticos es perjudicial, especialmente para las agujas delgadas; y la compresion permanente del muelle, como resultado del largo servicio, pudiera dejar una aguja de chucho (despues que la palanca haya sido cambiada) indebidamente fuera de contacto con su carril maestro, peligrando las ruedas que vengan de frente.

**115.** *El muelle* tambien pudiera permitir que la palanca sea forzada en su lugar, con la nieve o una piedra pequeña, u otra obstruccion alojada entre el carril maestro y la aguja impidiendo cambiar completamente el chucho.

**116. Automático y No-Automático.** Frecuentemente la barra pasando al traves del muelle espiral, está provista de dos bujes ajustables, por cuyo medio el chucho puede arreglarse como se quiera, de varios modos. Segun esta disposición (*a*, *b*, *c*, etc.), un carro de arrastre, entrando en un chucho mal puesto pudiera pasar libremente si viene.

(*a*) de la linea principal;
(*b*) del desvio;
(*c*) de la linea principal o del desvio;

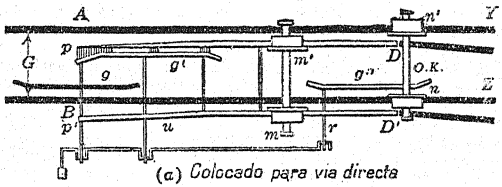
ó los bujes pueden estar ajustado de tal modo que eviten que el muelle actúe; de este modo el chucho se habilita no-automático, de manera que un carro de arrastre viniendo de la linea principal o del desvio, no puede pasar libremente el chucho si este último esta mal cambiado.

(1) Véase *N. del T.*, ¶ 1.

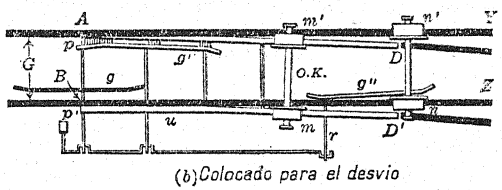
**117.** El *uso de* pedestales *automáticos* y *no-automáticos* («rigidos») de chucho (1), se compara como sigue : —

En 1900, 24 ferrocarriles usaban automáticos, y 23 usaban pedestales rigidos; en 1908, 23 ferrocarriles usaban automático y 29 usaban pedestales rigidos. Véase la Revista Eng News de junio 4, 1908.

**118. Especificaciones requeridas.** Al pedir pedestales de chucho, especifíquese el curso o carrera en el punto de conexion, tamaño de la barra de cabeza y si es vertical u horizontal, diámetro del agujero en la barra de cabeza largo y diámetro de la barra de conexion, estilo de la placa de señal y su altura sobre las traviesas, y dimensión del encaje de la linterna en la punta del eje.




(a) Colocado para via directa




(b) Colocado para el desvio

**Figs. 30.**

**Chucho** (1) **Wharton.**

**119.** Figs 30*a* y *b*. Omitimos las pestañas de carril, mostrando solamente las cabezas del carril. Los carriles *movibles* se indican en *blanco*. Las ruedas, para las cuales el chucho está propiamente puesto, están marcadas « O. K. ». La Fig. 30*a* muestra el chucho puesto para la linea principal. La Fig. 30*b* el chucho puesto para el desvio.

**120.** *A Y* y *B Z* son los carriles de la principal. Son continuos (excepto, por supuesto, que *B Z* está cortado en la *rana*, mas allá de la Fig.), y están clavados a las traviesas a todo lo largo. Estos no se mueven ni se interrumpen al operar el chucho. *p D* y *p' D'* son las agujas .

**121. La aguja de chucho,** *p' D'* (el « carril elevado »), termina en punta roma. En su extremo, *p'*, su parte superior está a nivel con el de *B Z;* pero, en *u*, 1,20 ó 1,50 m mas atrás de su extremo *p'*, su parte superior está unos 5 cm mas alto que en *B Z*. Esto permite que la pestaña *m*, al pasar viniendo o entrando en el desvio. Fig. 30*b*, deje libre el carril principal, *B Z*, *p' D'* conserva esta elevación desde *u* hasta su extremo, *D'*. Mas alla de *D'*, el carril del desvio a continuación de *p' D'*, desciende hasta que su parte superior queda al nivel normal. La « *aguja* » *p D*, se alza tambien, para evitar el balanceo lateral del material rodante que de otro modo resultaria de la elevacion de *p' D'*.

**122. El refuerzo de guardacarril,** *g'*, está atornillado y separado de la aguja del carril, *p D*, y se mueve con él. Se le deja canal para las pestañas de las ruedas entre sus cabezas. En la Fig. 30*b*, confinando la pestaña de la rueda *m'*, *g'* sostiene la pestaña de la rueda *m* libre de la cabeza del carril *B Z*, al bajar la inclinacion, *u p'*.

(1) Véase *N. del T.*, ¶ 1.

**123.** En la Fig. 30*b* el punto, *p*, se pega contra la cabeza del carril principal, *A Y*; de modo que la pestaña de la rueda *m'*, moviendose en cualquier dirección, deja libre el punto, *p*; y el guardacarril fijo, *g*, encerrando la pestaña de *m*, sostiene a *m'* fuera de *A Y*, dando de este modo protección adicional al punto, *p*.

**124. En los primitivos modelos** se ideaban accesorios especiales proyectados para recibir, del desvio un carro de arrastre cuando el chucho estaba puesto para la linea principal, y guiarlo con seguridad a la linea principal; pero estos accesorios se han abandonado por su costo y complicación.

**125. Arrastre.** Fig. 30*b*. Si un carro de arrastre se mueve a lo largo de la linea principal mientras el chucho (1) está puesto para el desvio, la pestaña de la primera rueda, *n*, empuja a *g''* a un lado (compárese la Fig. 30*a*) y esta empujando la barra, *r*, cambia el chucho a la posicion adecuada para la linea principal, como en la Fig. 30*a*.

**126. Entrevia.** Para la adecuada manipulación del chucho Wharton, al ancho de la via principal en el chucho, se le da media pulgada mas que al de la via normal.

### Cálculos.

**127.** Muchas de las aproximaciones permitidas al calcular desvios de pequeño ángulo, que son usuales en lineas de tracción por vapor, son inadmisibles en desvios y cruceros en lineas de tranvias, donde los trazados son frecuentemente muy complejos. De aqui que el diseño de tal trabajo se confia con preferencia a manufactureros especializados en ellos.

### Traviesas.

**128. Las traviesas de Desvios**, para via ancha tipo, varia en largo, entre la de una traviesa ordinaria, digamos de 2,60 y 7,4 m. Una o dos traviesas, de 3,6 a 4,80 m de largo (segun el estilo del pedestal del chucho), se usan en los extremos de los chuchos de aguja o de tope, como asiento del pedestal, y se llaman **cabezales.** El juego de traviesas debe extenderse mas allá de la rana, en un punto donde los extremos de las traviesas de las dos vias no se estorben. Las traviesas de (7 × 9 pulgs) 18 × 23 cm son las que se usan comunmente, excepto bajo la rana donde se usan traviesas de (7 × 10 pulgs) 18 × 25 cm para tener mayor area de asiento, y de resistencia a los choques de las ruedas que pasan sobre la rana. Generalmente se piden en largos que varian en (6 pulgs) 15 cm. Para traviesas en general, véase Via, pag 857,

**Traviesas de chuchos** (1).

**129. Recopilación de las Especificaciones de la Asen Am de Ing de F. C.** a continuación de la p 176. Manual de 1915.

Todos de 7″ × 9″ (18 × 23 cm).

Largos para traviesas de via de 8′ — 6″ (2.60 m). Para traviesas de 8′ — 0″ (2,44 m) los largos son en general 6 pulgs (15 cm) menos que los dados aqui.

| Long en m.... | 4.57 | 2.74 | 2.90 | 3.05 | 3.20 | 3.36 | 3.51 | 3.66 | 3.81 | 3.96 |
|---|---|---|---|---|---|---|---|---|---|---|
| | Número de traviesas del chucho empezando en la punta del chucho (1). | | | | | | | | | |
| Rana n.º 8..... | 2 | 8 | 7 | 5 | 4 | 3 | 3 | 3 | 3 | 3 |
| — n.º 11..... | 2 | 12 | 10 | 8 | 5 | 5 | 5 | 3 | 3 | 4 |
| — n.º 16..... | 2 | 20 | 14 | 10 | 9 | 6 | 6 | 5 | 5 | 5 |

(Continua la tabla.)

| Long en m.... | 4.11 | 4.26 | 4.41 | 4.57 | 4.72 | 4.87 | 5.02 | Total. | |
|---|---|---|---|---|---|---|---|---|---|
| | Número de traviesas del chucho. | | | | | | | trav. | M³ |
| Rana n.º 8..... | 2 | 3 | 2 | 3 | 2 | 3 | 2 | 58 | 8.762 |
| — n.º 11.... | 4 | 3 | 3 | 2 | 3 | 3 | 3 | 78 | 11.554 |
| — n.º 16.... | 5 | 5 | 4 | 5 | 5 | 4 | 5 | 115 | 17.066 |

Cada desvio tiene dos cabezales de 7″ × 9″ por 15 pies largo ó sean 18 × 23 cm por 4, 57 m largo.

(*N. del T.* — Vease pag 273 para conversión de pies de tabla en m³.)

(1) Véase *N. del T.*, ¶ 1.

**130. Separación de traviesas**, para desvíos. Empezando en la punta del chucho (1). Centro a centro. Para usarse con traviesas de vía de (8′ – 6″) 2.60 m. Traviesas de (7″ × 9″) 18 × 23 cm.

| | | | | | | | | | | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| Rana n.º 8. | Número de traviesas. | 5 | 4 | 18 | 3 | 7 | 9 | 6 | 5 | 57 |
| | Separación, ms..... | 0,50 | 0,53 | 0,50 | 0,48 | 0,50 | 0,45 | 0,48 | 0,50 | |
| | Total, ms..... | 2,50 | 2,13 | 9,00 | 1,45 | 3,50 | 4,05 | 2,89 | 2,50 | 28,02 |
| Rana n.º 11. | Número de traviesas. | 10 | 21 | 9 | 13 | 5 | 10 | 4 | 5 | 77 |
| | Separación, ms..... | 0,48 | 0,50 | 0,48 | 0,50 | 0,45 | 0,50 | 0,53 | 0,50 | |
| | Total, ms..... | 4,83 | 10,50 | 4,32 | 6,50 | 2,25 | 5,00 | 2,13 | 2,50 | 38,03 |
| Rana n.º 16. | Número de traviesas. | 18 | 19 | 4 | 16 | 4 | 20 | 15 | 18 | 114 |
| | Separación, ms..... | 0,48 | 0,50 | 0,48 | 0,50 | 0,48 | 0,50 | 0,48 | 0,50 | |
| | Total, ms..... | 8,64 | 9,50 | 1,93 | 8,00 | 1,93 | 10,00 | 7,20 | 9,0 | 56,20 |

**Notas de maderas.**

A continuación se veran listas tipicas de maderas para desvios (tres vias) dobles y para cruzamientos.

**131. Para Desvios Dobles.** Dos cabezales (7″ × 10″, de 16 pies) (18 × 25 cm × 4,80 m) de largo. Las traviesas bajo las ranas separadas a (23″) (58 cm). Las otras traviesas separadas a (24″) 60 cm.

| Largo, en ms.... | 2,64 | 2,79 | 3,05 | 3,20 | 3,35 | 3,50 | 3,65 | 3,81 | 3,96 | 4,11 |
|---|---|---|---|---|---|---|---|---|---|---|
| | Número de traviesas empezando en la punta del chucho (1). | | | | | | | | | |
| Rana n.º 6..... | 5 | 4 | 3 | 2 | 2 | 1 | 1 | 1 | 2 * | 1 * |
| — n.º 8..... | 5 | 4 | 4 | 2 | 3 | 2 | 1 | 2 | 1 * | 1 * |
| — n.º 10..... | 5 | 4 | 4 | 3 | 3 | 2 | 2 | 2 | 2 * | 1 * |

(Continua la tabla.)

| Largo, en ms.... | 4,26 | 4,41 | 4,57 | 4,72 | 4,87 | 5,03 | 5,18 | 5,48 | 5,63 | 5,78 |
|---|---|---|---|---|---|---|---|---|---|---|
| | Número de traviesas empezando en la punta del chucho (1). | | | | | | | | | |
| Rana n.º 6..... | 1 * | 0 | 1 | 1 | 1 | 1 | 1 * | 1 * | 1 * | 1 * |
| — n.º 8..... | 2 * | 1 | 1 | 1 | 2 | 1 | 1 * | 2 * | 0 | 2 * |
| — n.º 10..... | 1 * | 2 | 2 | 2 | 2 | 0 | 2 * | 2 * | 0 | 2 * |

(Concluye la tabla.)

| Largo, en ms.... | 6,09 | 6,39 | 6,70 | 6,86 | 7,00 | 7,30 | Total | Total m³ madera. |
|---|---|---|---|---|---|---|---|---|
| | Número de traviesas empezando en la punta del chucho (1). | | | | | | | |
| Rana n.º 6..... | 1 | 2 | 1 | 0 | 2 | 1 | 38 | 7.365 |
| — n.º 8..... | 2 * | 2 | 2 | 0 | 1 | 2 | 47 | 9.187 |
| — n.º 10..... | 3 | 2 | 2 | 2 | 2 | 3 | 57 | 11.410 |

**132. Para Cruzamientos.** (Cens 13 pies). Cuatro cabezales 7″ × 10″ (18 × 25 cm) de 16 pies (4,80 cm) largo. Las traviesas bajo las ranas separadas a (23″) 58 cm. Las otras traviesas separadas a (24″) 60 cm.

| Largo, en ms.... | 2,59 | 2,74 | 2,89 | 3,04 | 3,19 | 3,34 | 3,49 |
|---|---|---|---|---|---|---|---|
| | Numero de traviesas empezando en la punta del chucho (1). | | | | | | |
| Rana n.º 6..... | 8 | 8 | 6 | 6 | 6 | 4 | 4 |
| — n.º 8..... | 10 | 8 | 8 | 6 | 6 | 6 | 6 |
| — n.º 10..... | 10 | 10 | 10 | 8 | 8 | 8 | 8 |

(Concluye la tabla.)

| Largo, en ms.... | 3,66 | 3,81 | 3,96 | 6,55 | 6,55 | Total | Total m³ madera. |
|---|---|---|---|---|---|---|---|
| | Numero de traviesas empezando en la punta del chucho (1). | | | | | | |
| Rana n.º 6..... | 4 | 2 | 4 * | 4 * | 6 | 62 | 10.541 |
| — n.º 8..... | 6 | 6 | 4 * | 4 * | 9 | 79 | 13.306 |
| — n.º 10..... | 6 | 6 | 6 * | 6 * | 11 | 97 | 16.361 |

* Traviesas de rana (7″ × 10″). 18 × 25 cm. Otras traviesas (7″ × 9″) 18 × 23 cm.)

(1) Véase *N. del T.*, ¶ 1.

### Tendido.

**133. Tiempo necesario.** Cuando no hay muchas interrupciones por el paso de trenes, « un buen capataz y seis peones de vias regulares pueden poner un chucho de tope o de aguja para desviadero en un dia de diez horas, incluyendo quitar las traviesas viejas y poner las traviesas de chucho ». Camp, en Notas de Via, p. 298.

### Miscelánea.

**134. Para evitar el deslizamiento** de las puntas de los carriles se clavan en ranuras las cuatro juntas de carril en los extremos de la rana, las del amarre de las agujas, y las juntas intermedias de los carriles de arranque de la rana. El anclado o afirmado de las agujas de carril en sus extremos a los carriles maestros próximos, ha sido motivo de dificultades por apiñarse las juntas fuera de linea.

### Remoción del Hielo y la Nieve.

**135. Calefacción de vapor.** Cuando muchos chuchos están próximos pueden tenerse económicamente libres del hielo y nieve con tubos de vapor, alimentados de alguna planta central, y tendidos entre varios pares de traviesas, cerca del punto. Se quita primero el balasto. Los tubos se proveen con trampas automáticas que permiten al vapor condensado salir, y asi evitar su congelacion en los tubos. Véase la Revista « Railway Age Gazette », p. 1199, mayo 13, 1910.

**136. Fluido abrasador.** Los desviaderos se han mantenido con facilidad y sin gran costo libres (y despejados) de hielo y nieve, por medio de un fluido hidrocarbónico (un derivado de las plantas que producen el gaz Pintsch), aplicado donde se necesita por medio de una lata distribuidora de seguridad, que se enciende. El fluido arde a pesar del viento y de la nieve, derritiendo y finalmente evaporando la nieve. La temperatura de la llama no es lo bastante para afectar los carriles. El fluido cuesta de 3 a 5 ctvs por galon. Donde haya que atender varios chuchos vecinos, el liquido se almacena en estanques, y se distribuye por gravedad, o por aire comprimido, al traves de una tuberia.

Vease la Revista « Engineering-Contracting », p. 151, sep. 2, 1903.

**137. Protección de las barras de chucho** (1). Para evitar que se doblen o se rompan en caso de descarrilamientos se puede colocar una traviesa aserrada a cada lado de cada barra, dejando (2,5") (8 cm) de espacio libre, entre las traviesas para la barra. En tiempo frio las barras dobladas deben calentarse antes de tratar de enderezarlas, el balasto debe mantenerse separado de las barras, y bien desaguadas alrededor, especialmente en tiempo frio.

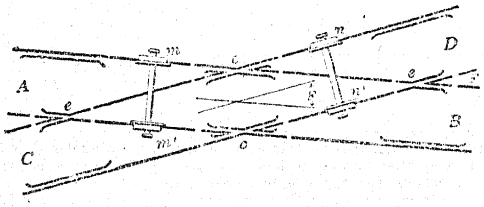



Fig. 31.

## CRUCEROS

**138.** Fig. 31. Un crucero consiste esencialmente en cuatro ranas, *e*, *e*, *c* y *c*, y lo guardarranas necesarios.

**139. El ángulo del crucero,** es el ángulo, *F*, entre las lineas de centro de as dos vias rectas que se cruzan.

**140. Cuando F = 90°,** las cuatro ranas son todas iguales. En la Fig. 31 las dos ranas de cabeza aguda, *e*, son iguales y parecidas a las ranas de desvio. Las dos ranas agudas del « medio », *c*, son iguales también. La presencia de guardacarriles, en las ranas del medio, da la apariencia de un segundo punto, opuesto al punto, *c*, propio; por consiguiente las ranas del medio se llaman con frecuencia de « punta doble », aunque por las « puntas » interiores formadas por los ángulos en los guardacarriles, no pasen ruedas.

(1) Véase *N. del T.*, ¶ 1.

**141. Donde una via o ambas estén en curva** todas las ranas tienen ángulos diferentes; y están encurvadas. Es mejor evitar cruceros de vias en curvas. Los problemas relacionados con ellas se resuelven mas facilmente por medio de trazados en escala mayor.

**142. Crucero de Doble-carril.** Fig. 31. Si los guardacarriles del medio estan extendidos, a ambos lados, hasta que encuentren los extremos opuestos de las ranas, y estan propiamente rellenados, atornillados y conectados, la resistencia y durabilidad del crucero aumenta grandemente, y se llama crucero de « doble-carril ». En este caso, las ranas de cabeza, *e*, (como tambien las ranas del medio, *c*) son de « doble punta » (¶ 140).

**143. Un carril de descanso** (¶ 52). Para que pasen « falsas pestañas » se atornilla algunas veces al exterior del carril que carga las ruedas.

**144. Cuando F excede de 35°**, las almas de ambos : del carril principal y de descanso de una de las dos vias, pasan algunas veces al traves del crucero sin cortarse; las canales para las ruedas en la otra via se cortan solamente al traves de las cabezas de estos carriles.

**145. Cuando F está entre 15° y 40°**, las cuatro ranas generalmente se encuentran directamente : pero en cruces, de pequeño ángulo, para evitar el manejo de ranas largas, se tienden pedazos de carril entre los extremos de los carriles de las ranas, como en la Fig. 31. Por conveniencia en los embarques y en su manejo, los cruceros se hacen con el menor número de uniones posible.

**146. Cuando F es menor de 10°**, Fig. 31, las puntas de las ranas del medio estan casi opuestas una a la otra en cualquiera de las dos vias, y no pueden protegerse bien. De este modo las ruedas *m* y *m'* pudieran salirse de su via propia, *A B*, en las ranas del medio, y tomar la via *C D* ; o las ruedas *n* y *n'* pudieran salirse de la via *D C*, y tomar la via *B A*.

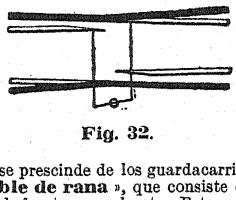


**Fig. 32.**

En tales casos (Fig. 32) se prescinde de los guardacarriles del medio, y se usa una « **punta central movible de rana** », que consiste en dos pares de agujas de carril de chucho (1) cortas, de frente una a la otra. Estos chuchos como requisito van asentados contra uno de los dos carriles maestros doblados, como se indica. Pueden operarse por un pedestal de chucho automático, de modo que si un tren se acerca a un crucero mal puesto, las primeras ruedas obligan a abrir las agujas de carril de arrastre del chucho, y automáticamente ponen las agujas de carril de frente en la buena posición.

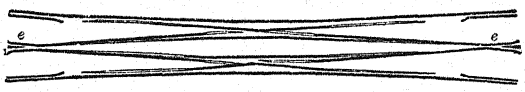



**Fig. 33.**

**147. Chucho-Corredizo o Crucero de combinación.** Fig. 33. En un crucero de pequeño ángulo, el tráfico puede intencionalmente llevarse de una carrilera a la otra, por medio de un chucho-corredizo o « crucero de combinación », que consiste en una disposicion de chucho y carriles de conección ademas de un crucero corriente, todo colocado entre las dos ranas de cabeza, *e*. La Fig. 33 muestra un chucho-corredizo doble. En el chucho-corredizo sencillo, los chuchos y carriles de conexión están puestos en un lado solamente.

(1) Véase *N. del T.*, ¶ 1.

**148.** El chucho (1) corredizo es especialmente útil para conectar un *enlace* (véase ¶ 12 *b*) con las vías paralelas con que se cruza. Tal combinación economiza mucho espacio, a lo largo, y da una línea recta si se compara con una serie de cruces separados.

**149.** *La punta central movible de rana* puede usarse, como en la Fig. 33, en chuchos-corredizos; pero en su lugar se usan *ranas corrientes rígidas en el medio*, ambas con chuchos-corredizos dobles y sencillos. Excepto donde el ángulo es tan pequeño que las patas de las ranas estorben a los carriles encorvados que conectan con las agujas del chucho.

**150.** *Palancas de chucho.* Estos cruceros pueden operarse por un pedestal de chucho corriente para cada par de agujas de carril y uno para la punta central movible de la rana; o todos los chuchos pueden conectarse por un sistema de barras y cigueñas, y operarse desde un pedestal sencillo, mientras que las puntas movibles de rana se manejan desde otro pedestal. O las agujas de chucho y los puntos movibles de rana se disponen de tal modo que puedan operarse juntas.

**151. Cruceros de Carril continuo.** En cruceros de ángulo muy grande, las ruedas al cruzar las canales, caen en ellas dando golpes fuertes al material rodante y a la vía; y se han construido sistemas que permiten carriles continuos en tales cruceros, por medio de un tramo corto de carril, o un equivalente, que por medio de un sistema entrelazado, puede colocarse provisionalmente en línea con el carril que se va a usar.

**152. Traviesas.** Las traviesas de cruceros, extendidos bajo ambas vías, son tendidas generalmente a ángulo recto, con la diagonal larga del rombo formado por los carriles que se cruzan; algunas veces se tienden a ángulo recto con la vía que tiene mas tráfico pesado; y algunas veces se usan traviesas de largo corriente, tendidos a ángulo recto con cada vía. Para traviesas, vease ¶ 128.

**153. Especificaciones requeridas.** Al pedir cruceros, especifíquese tipo de construcción; angulo del crucero, *F*, Fig. 31; ancho de vía; distancia de centro a centro, entre vías paralelas si es mas de una; sección del carril; largos de los brazos de las ranas; y separación de los agujeros de tornillo que se necesitan en los extremos para que ajusten con las juntas de carril en uso.

---

(1) Véase *N. del* T., ¶ 1.

# DESVÍOS Y CRUCEROS

## PARTE II. TEORÍA

### Geometria de los Desvíos.

Para la Construccion de los Desvios, véase Parte I, pag 897.

**1. Exactitud.** No es necesaria una exactitud escrupulosa en las medidas de los desvios. Una variación de unos pocos tantos por ciento en el largo del arranque o en el radio de un desvio, del valor teórico será rara vez notado. El ojo educado de un experto de via o un dibujo en escala mayor, puede dar tan buenos resultados como el cálculo mas cuidadoso. No obstante el ingeniero debe conocer bien el trazado de los desvios. Por consiguiente damos a continuación, ecuaciones (exactas y aproximadas) que tratan los casos mas simples.

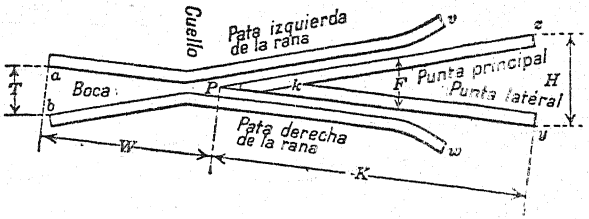



**Fig. 1.**

### Ángulo y número de la rana

Para descripcion de la rana, véase Parte I, ¶ 40, etc.

**2. Angulo de la Rana.** Figs 1 y 2. El punto teórico de la rana, $P$, es la intersección de las lineas de via que determinan la punta o lengueta de la rana. El ángulo, $F$, de la rana, es el ángulo, en el punto teórico, de la rana entre las dos lineas de via.

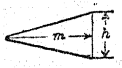
**3.** *Como se determina.* Figs 1 y 2. El ángulo, $F$, de la rana depende de la diferencia de agudeza (¶ 29) entre la curva del desvio y la curva de la linea principal (si la hay) y del ancho de la via.

**4.** *Como se expresa.* Fig. 2. El ángulo de la rana se expresa generalmente por medio de la relacion, $k/i$, entre el largo, $k$ (no se indica), de cualquier parte la rana, y el aumento, $i$ (no se indica), del ancho de la rana comprendida en dicha parte. Este es también la relación entre cualquier parte dada ($m$ ó $s$, empezando en $P$, vease abajo) del largo de la rana, y la correspondiente abertura, $h$. Esta relación es lo que llaman número de la rana, $N$.

**5. Número de la Rana.** *N se determina* de dos modos distintos, a saber:

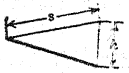
$$N = \frac{m}{h} = \frac{\cot (F/2)}{2}$$

$$2\,N = \cot (F/2)$$




$$N = \frac{s}{h} = \frac{1}{2 \operatorname{sen} (F/2)}$$

$$2\,N = \operatorname{cosec} (F/2)$$



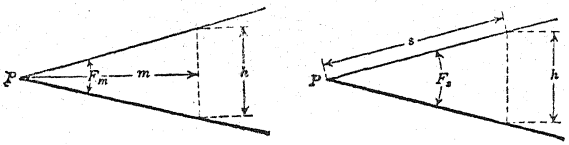

Ec (1)

**6.** La Asen Am de Ingo de F. C. hace $N = m/h = \frac{1}{2} \cot (F/2)$, como en la Fig. 2 *a* (véanse los valores de $F$, en la tabla, Manual de 1915, p 184); pero las

compañías de ferrocarril y los manufactureros están divididos en cuanto a sus prácticas a este respecto; muchos prefieren la segunda formula, $N = s/h$, Fig. 2 *b*, porque el largo de la rana se mide generalmente a lo largo de la via, *s*, mejor que a lo largo de la linea central *m*. Por otra parte, el largo asi medido, se da generalmente en numeros redondos de pies y pulgadas; y de aqui el que se encuentre facilmente la boca o abertura dividiendo mentalmente dicho largo por el numero de la rana.

**7. Comparación de los ángulos de rana, $F\,m$ y $F\,s$.**

| $N$ | $F\,m$ cot $(F\,m/2) = 2\,N$ Fig. 2 *a*. | $F\,s$ cosec $(F\,s/2) = 2\,N$ Fig. 2 *b*. | $F\,s - F\,m$ segundos. | $\frac{F\,m}{F\,s - F\,m}$ |
|---|---|---|---|---|
| 4 | 14° 15′ 00″ | 14° 21′ 41″ | 401 | 128 |
| 8 | 7° 9′ 10″ | 7° 10′ 00″ | 50 | 515 |
| 11 | 5° 12′ 18″ | 5° 12′ 38″ | 20 | 937 |
| 16 | 3° 34′ 47″ | 3° 34′ 54″ | 7 | 1841 |
| 24 | 2° 23′ 13″ | 2° 23′ 15″ | 1.87 | 4606 |

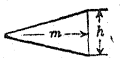
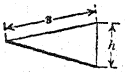



*Para* $h = 1$, $N = m$ (*a*)

*Para* $h = 1$, $N = s$ (*b*)

**Fig. 2.**

**8. Funciones del ángulo de la rana**, $F$, entrando el número $N$ de la rana. Vease p. 106, ¶ 16, y p. 107, ¶ 19.

| Fig. 2 *a*. | Fig. 2 *b*. |
|---|---|
|   |   |
| Tomando $N = \frac{m}{h} = \frac{1}{2}$ cot $(F/2)$ | Tomando $N = \frac{s}{h} = \frac{1}{2 \text{ sen } (F/2)}$ |
| tenemos tang $(F/2) = \frac{1}{2\,N}$, y | tenemos $\text{sen}^2\,(F/2) = \frac{1}{4\,N^2}$, y |
| tang $F = \frac{N}{N^2 - \frac{1}{4}}$ | sen $F = \frac{\sqrt{4\,N^2 - 1}}{2\,N^2}$ |
| sen $F = \frac{N}{N^2 + \frac{1}{4}}$ | tang $F = \frac{\sqrt{4\,N^2 - 1}}{2\,N^2 - 1}$ |
| cos $F = \frac{N^2 - \frac{1}{4}}{N^2 + \frac{1}{4}}$ | cos $F = \frac{2\,N^2 - 1}{2\,N^2}$ |

Ec (2)

**9. Para determinar el número** de una rana dada, Fig. 3; búsquese cualquier parte de la rana, donde la distancia, $h$ o $t$ entre *líneas de la vía* (¶ 2) sea una unidad (esto es 1 pie ó metro, o el largo de un lapiz o cuchilla de bolsillo). Entonces $N$ = distancia, $h$ $P$ o $t$ $P$ de esa parte al vértice teórico de la rana, $P$ (vease ¶ 2) medido, con la misma unidad a lo largo de la linea central $M$, o a lo largo de la vía, $S$, segun la definicion de $N$ adoptada (vease ¶ 6). O (si el vértice teórico, $P$, de la rana no se determina facilmente) mídase *ambas* aberturas, $h$ y $t$ (entre las *líneas de la vía*), en *dos* puntos convenientes en direcciones opuestas del vertice de la rana y la distancia, $M$ ó $S$, entre dichos puntos. Entonces,

$$N = \frac{M}{h+t} \text{ o } \frac{S}{h+t}.$$ Vease ¶ 5.

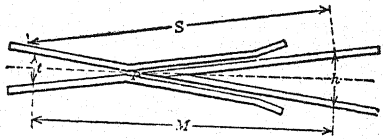



**Fig. 3.**

**10. Vértice de una rana dada.** Fig. 1. En la práctica, la lengüeta o vértice de ella termina en el « vértice real de la rana » (o « punto de media pulgada de ancho ») donde la lengueta tiene cerca de ½" de ancho; dejando un pequeño triángulo isoceles entre el punto real y el teórico. (Vease ¶ 2). El largo de este triángulo, o la distancia entre el punto teórico y el real de la rana, es el producto del ancho del punto real de la rana por el número de la rana, $N$.

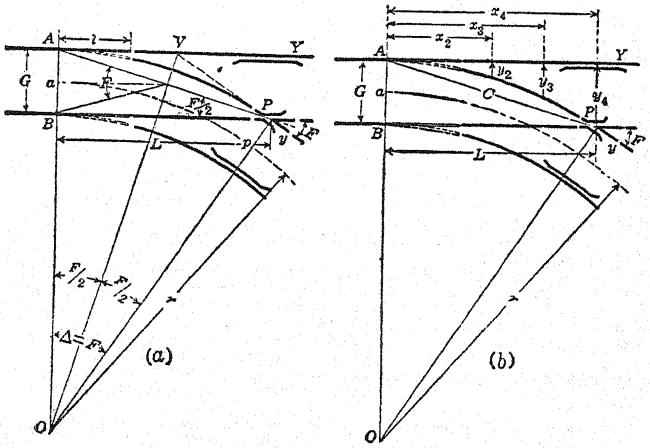



**Fig. 4.**

**11. Cuando el ángulo es pequeño.** Asi como un ángulo, $A$, se acerca a cero como límite, los valores de $A \times \cot A$ ( $= A/\text{tang } A$) y de $A \times \text{cosec } A$, ( $= A/\text{sen } A$) se acercan a la unidad, $A$ va expresado en radiantes o radiales de 57.295 779°. (¶ 2, pag. 103). Por tanto, los mismos valores expresados en

grados, se acercan, como límite, 57.295 779* (log = 1.758 1226); y para pequeños ángulos, tenemos approx.:

Para $A$ en grados, $A^o = 57.3^o/\cot A = 57.3 \text{ sen } A$;
$F = 2 \times 57.3/\cot (F/2) = 2 \times 57.3 \text{ sen } (F/2) = 57.3/N$.
Para $A$ en minutos, $A' = 3438'/\cot A = 3438 \text{ sen } A$.. (3)

## CHUCHOS (1) DE TOPE

(*N. del T.* — En las fórmulas que vienen inmediatamente y en las sucesivas, así como en muchas de las tablas, el Autor supone (como es costumbre entre los ingenieros ingleses y americanos) que los trazados de líneas, de desvíos, de chuchos, etc. se hacen teniendo como base, como unidad, la cuerda de 100 pies. Los ingenieros que usamos el sistema métrico, empleamos siempre la cuerda de 20 metros. Están tan mezcladas en este capítulo y en varios mas adelante así como en muchas tablas las fórmulas de estructura puramente trigonométrica, aplicables al Sistema Métrico y al Sistema Ingles indiferentemente, con las fórmulas y con muchos coeficientes y números que, presuponen el uso *de medidas en pies y el empleo de cuerdas de 100 pies*, que, despues de haber comenzado, a convertir muchas de estas al sistema métrico, nos resulta un embrollo que compromete la claridad y los datos tan prácticos y utiles, en esta parte de la obra y hemos resuelto dejarla como está recomendando como mas fácil y eficaz que los ingenieros usen la cuerda fija de 30,48 m que equivale a cien pies y las medidas que tomen en metros sobre el terreno, para entrar a las fórmulas y tablas las multipliquem por 3,28 para tenerlas en pies ó bien usar una cinta en pies y pulgs ingleses).

### Curva circular de desvío en una tangente.

**Generalidades.**

**12.** Fig. 4. Supongamos que la curva de arranque $A\,P$, del desvío (carril exterior) sea una curva circular simple desde su comienzo $A$ hasta el vertice teórico $P$ de la rana; dicha curva es tangente a $A\,Y$ y a $P\,y$ en $A$ y en $P$ respectivamente. Esta condicion rara vez se verifica con toda exactitud en la práctica; pero aproximadamente aun en chuchos de aguja ¶ 35, y da ecuaciones que, debido a su sencillez no solamente se usan algunas veces en chuchos de tope de desvío, sino tambien (por ser suficientemente aproximadas) en chuchos de aguja para desvío. Por consiguiente estas ecuaciones se dan a continuacion, aunque el chucho de tope se usa ahora en lugares de secundaria importancia.

**13. Ecuaciones.** Fig. 4. Chucho (1) de tope de desvío tangencial. Tenemos:

$G$ = $A\,B$..... = ancho de vía;
$r$ = $O\,a = O\,A - G/2$.
= $O\,B + G/2$..... = radio de la línea de centro del desvío;
$D$ ..... = agudeza de la curva del desvío;
$\Delta$ ..... = extencion del desvío, desde la union del chucho, $A$, al vértice $P$ de la rana;

$F$ = $\Delta = B\,P\,V$.
= $Y\,V\,P = A\,O\,P$..... = ángulo de la rana;
$N$ = $m/h$, Fig. 2.
= ½ cot $(F/2)$ (Vease ¶ 5)..... = número de la rana;
$L$ = $B\,P$..... = arranque = distancia de la rana;
$C$ = $A\,P$..... = cuerda del carril exterior del desvío;
$M$ ..... = ordenada media para $C$;
$Q$ ..... = ordenada de cuarto punto (¼ de la cuerda) para $C$;
$y_1$ (no se muestra), $y_2$, $y_3$, $y_4$..... = ordenadas perpendiculares de la tangente a los cuartos puntos del arco, $A\,P$;
$x_1$ (no se muestra), $x_2$, $x_3$, $x_4$..... = abscisa para las ordenadas, $y$;
$s$ (Fig. 10)..... = ángulo del chucho (1);
$l$ ..... = largo del chucho;
$t$ ..... = curso ó juego del chucho;

---

* Este es también el número de grados en el ángulo (= 1 radial) cuyo arco = el radio y es tambien el largo del radio de una curva en que un ángulo de 1° esté cubierta por un arco cuyo largo es la unidad.

(1) En Cuba se llama *chucho*, al conjunto movible de las agujas del desvío, y, por extensión a veces al desvío. — Lo hemos adoptado. (*N. del T.*).

**14. Ángulo de la Rana, F,** = extensión del desvío, $\Delta$.
Funciones de $F$ y de $(F/2)$.

$$\text{Sen } F = \frac{L}{r + (G/2)}\ ;\ \tan F = \frac{L}{r - (G/2)}\ ;$$

$$\text{Cos } F = \frac{\text{sen } F}{\tan F} = \frac{r - (G/2)}{r + (G/2)}$$

$$\text{Senoverso } F = \frac{G}{r + (G/2)}\ .\ F = \text{aprox } D\ L/100 \ldots \quad (4)$$

(*Vease N. del T.* — ¶ 11 pag 926.)
Para los valores de $F$ en función de $N$, vease ¶ 8.
Cot $(F/2) = L/G = 2\ N = r/G\ N$.
Tan $(F/2) = G/L = 1/(2\ N) = G\ N/r$.

**15. Número N de la rana.**

$$N = \frac{\cot (F/2)}{2} = \frac{L}{2\ G} = \frac{r}{L} = \sqrt{\frac{r}{2\ G}} = \frac{r \tan (F/2).}{G}$$

$$= \text{aprox } l/(2\sqrt{G\ t}) = \text{aprox } 5730^{o}/D^{o}L \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

**16. Radio de la linea central del desvio de radio, r.**

$r = L/\text{sen } F - G/2 = L/\text{tang } F + G/2 = G/\text{seno vers } F - G/2.$

$$= G\ \frac{\cot^2 (F/2)}{2} = G\ N \cot (F/2) = \frac{G\ N}{\text{tang } (F/2)}$$

$$= 2\ N^2\ G = L^2/2\ G = N\ L = \text{aprox } l^2/2\ t \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

**17. Agudeza D de la curva central del desvio.**

$$\text{sen } (D/2) = 50/r = 25/N^2\ G.$$

$$D = \frac{100\ F}{\text{arc; } a\ p,\ \text{en pies}} = \text{aprox } \frac{100\ F}{L,\ (\text{pies})} = \text{aprox } \frac{57.3^{o}}{N} \cdot \frac{100}{2\ N\ G.}$$

$$= \text{aprox } \frac{2865^{o}}{N^2\ G} = \text{aprox } \frac{608^{o}}{N^2} \quad \text{para } G = 4 \text{ pies } 8.5 \text{ pulgs } (1.435 \text{ ms}).$$

$$= \text{aprox } \frac{5730^{o}}{r,\ (\text{pies})} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

**18. Ancho de via, G.**

$$G = \frac{L}{\cot (F/2)} = \frac{L}{2\ N} = \frac{r}{2\ N^2} = \frac{L^2}{2\ r.}$$

$$= 2\ \left(\frac{L}{\text{sen } F} - r\right) = 2\left(r - \frac{L}{\text{tang } F}\right)$$

$$= \text{aprox } \frac{l^2}{4\ N^2\ t} = \text{aprox } \frac{L^2\ t}{l^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

**19. Arrangue o distancia de là rana, L** = $B\ P$, Fig. 4.

$L = [r + (G/2)]$ sen $F = [r - (G/2)]$ tan $F = G$ cot $(F/2)$.

$$= 2\ G\ N = r/N = \sqrt{2\ r\ G} = 2\ r \tan (F/2).$$

$$= (\text{aprox, en pies})\ 100\ F/D \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

(1) Véase *N. del T.* (1) pag. 926.

**20. Cuerda larga, C** ( $= A\,P$), del carril exterior de arranque. $C = 2\,[r + (G/2)]$ sen $(F/2) = \sqrt{G^2 + L^2}$.

$$= G\sqrt{1 + 4\,N^2} = \text{aprox } 2\,r \text{ sen } (F/2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

**21. Ordenadas, y**, desde el carril exterior de la linea principal hasta el carril exterior del arranque del desvio.

| *A*. Valores exactos de $y$. | *B*. Valores aproximados de $y$. | *A*/*B*. Cuando $F = 12^o$ $(N = 4.76)$ * |
|---|---|---|
| $y_1 = [r + (G/2)]$ senoverso $(F/4)$ = | $G/16$ ..... | 1.0034 |
| $y_2 = [r + (G/2)]$ » $(F/2)$ = | $G/4$ ..... | 1.0027 |
| $y_3 = [r + (G/2)]$ » $(3\,F/4)$ = | $9\,G/16$ ..... | 1.0016 |
| $y_4 = [r + (G/2)]$ » $F = G$ ..... | | (12) |

**22. Ordenadas** de la cuerda larga, $C$ ( $= A\,P$) hacia el carril exterior de la via.

**22 a.** *Ordenada Media*, M.

$M = [r + (G/2)]$ senover $(F/2)$

= aprox $G/4$ = aprox 14 $^1/_8$ pulgs (35,9 cm) cuando $G = 4$ pies 8.5 pulgs (1,435 m).

| Cuando $N =$ | 4 | 8 | 16 | |
|---|---|---|---|---|
| encontramos $M/(G/4) =$ | 1.0039 | 1.0010 | 1.000244 ..... | (13) |

**22 b.** Ordenada de cuarto-punto, $Q$.

$$Q = \sqrt{[r + (G/2)]^2 - (C/4)^2} + M - (G/2) - r \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

$$\text{Donde } C/4 = \frac{[r + (G/2)] \text{ sen } (F/2)}{2}.$$

$Q$ = aprox $3\,G/16$ = aprox $0.1875\,G$.

= aprox 10.6 pulgs (26,6 cm) cuando $G = 4$ pies 8.5 pulgs (1.435 m)... (15

Cuando $F = 12^o$, $(N = 4.76)$,

$$\frac{Q}{3\,G/16} = 1.0035.$$

Con pequeños valores de $F$ (mayores valores de $N$) la aproximación es mayor.

**23.** En desvios de tope, las ordenadas medias y de cuarto-punto son casi independientes del ángulo de la rana, $F$. Cuando $N = 4$. $N = 16$, y el ancho de via = 4 pies 8,5 pulgs (1,435) las ordenadas medias, al carril exterior de la via se comparan como sigue :

$$\frac{\text{Ord med, } N = 4}{\text{Ord med, } N = 16} = \frac{1.18165 \text{ pies}}{1.17737 \text{ pies}} = 1.00363.$$

Pero vease ¶ 38.

**24. Ángulo del desvio, s, largo, l, y curso, t.** Para la relación entre estos elementos podemos tomar :

$$t = l \text{ sen } s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

---

* Con pequeños valores de $F$ (mayores valores de $N$) la aproximación para $y_1$, $y_2$ y $y_3$, es mayor.

**25. Dimensiones.** Desvio circular de la tangente. Fig. 4.

Cartabón 4 pies 8.5 pulgs (1,435 m) = 4.70833..... pies.
Curso del chucho (1) = 5.5 pulgs (0,14 m) = 0.45833..... —

$N$ = numero de la rana; $C = A\ P$ = cuerda;
$F$ = ángulo de la rana; $L = B\ P$ = arranque;
$r$ = radio de la linea central del desvio; $l$ = largo del carril del chucho;
$D$ = agudeza del desvio; $t$ = juego de la aguja.

*N. del T.* Cuando se quiera usar el sistema métrico, recuérdese que los metros se convierten en pies, mult por 3,28 y los piés en ms mult. por 0,305).

| $N$ | $F$ | $r$ | log $r$ | $D$ | $C$ | $L$ | $l = 2N\sqrt{Gt}$ |
|---|---|---|---|---|---|---|---|
| 4 | 14°15′00″ | 150.67 | 2.17802 | 38°46′ | 37.38 | 37.67 | 11.75 |
| 5 | 11°25′16″ | 235.42 | 2.37184 | 24°32′ | 46.83 | 47.08 | 14.69 |
| 6 | 9°31′38″ | 339.00 | 2.53020 | 16°58′ | 56.30 | 56.50 | 17.63 |
| 7 | 8°10′16″ | 461.42 | 2.66409 | 12°27′ | 65.75 | 65.92 | 20.57 |
| 8 | 7°09′10″ | 602.67 | 2.78008 | 9°31 | 75.19 | 75.33 | 23.50 |
| 9 | 6°21′35″ | 762.75 | 2.88238 | 7°31′ | 84.62 | 84.75 | 26.44 |
| 10 | 5°43′29″ | 941.67 | 2.97390 | 6°05′ | 94.05 | 94.17 | 29.38 |
| 11 | 5°12′18″ | 1139.42 | 3.05668 | 5°02′ | 103.48 | 103.58 | 32.32 |
| 12 | 4°46′19″ | 1356.00 | 3.13226 | 4°14′ | 112.90 | 113.00 | 35.26 |
| 13 | 4°24′19″ | 1591.42 | 3.20178 | 3°36′ | 122.33 | 122.42 | 38.19 |
| 14 | 4°05′27″ | 1845.67 | 3.26615 | 3°06′ | 131.75 | 131.83 | 41.13 |
| 15 | 3°49′06″ | 2118.75 | 3.32608 | 2°42′ | 141.17 | 141.25 | 44.07 |
| 16 | 3 34′47″ | 2410.67 | 3.38214 | 2°23′ | 150.59 | 150.67 | 47.01 |

(1) Véase *N. del T*, ¶ 11, pág. 926.

## Desvios que arrancan de una curva.

**26 a.** Para desvios de tope en tangentes las fig. 5 *b*, y el ¶ 25 dan la agudeza $D$ de la curva, el radio, $r$, y el arranque, $L$, = $B\,P$, para numeros de ranas del 4 al 16. La ordenada media,

$M = \left(r + \frac{G}{2}\right) \text{senover} \frac{F}{2}$, de la cuerda $A\,P$ al carril de la rana del lado del carril exterior es practicamente el mismo para todos los números de las ranas. Véase ¶ 23.

**26 b.** Pero, figs 5 *a* y 5 *c*, cuando un número dado de rana se usa en un desvio de una linea o via principal *en curva*, todos estos valores (incluyendo las ordenadas) se afectan; y sus valores dependen no solamente de la agudeza, $Dm$, de la curva de la linea o via principal, sino tambien de si la curva del desvio va en la misma dirección de la curva de la via principal, fig. 5 *a*, o en dirección opuesta. Fig. 5 *c*.

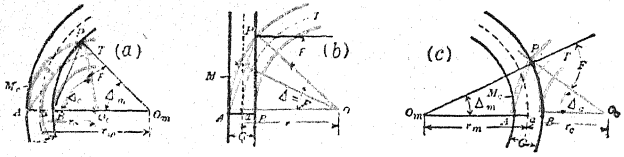



Fig. 5.

**27. Símbolos.** Figs 5. Supongamos :

$G$ = ancho de via; $F$ = ángulo de la rana; $N = \frac{\cot (F/2)}{2}$ = número de la rana y que otros símbolos denoten como sigue :

| | Para via principal en curva. Figs 5 *a*, *c*. | Para desvios. | |
|---|---|---|---|
| | | De tangentes Fig. 5 *b*. | De curvas. Figs 5 *a*, *c*. |
| Centro | $O^m$ | $O$ | $O_c$ |
| Radio de linea de centro | $r^m$ | $r$ | $r_c$ |
| Agudeza | $D^m$ | $D$ | $D_c$ |
| Longitud desde el comienzo, $a$, del desvio, al punto de la rana $P$ | $\Delta^m$ | $\Delta$ | $\Delta_c$ |
| Ordenadas, desde la cuerda larga, $A\,P$, hasta el carril de la rana del lado del carril exterior de la via. | | | |
| Ordenada media | ... | $M$ | $M_c$ |
| Ordenada de cuarto punto | ... | $Q$ | $Q_c$ |
| Arranque $B\,P$ | ... | $L$ | $L_c$ |

**28. Relaciones.** Entonces, para un ángulo dado $F$ de la rana :

Fig. 5 *a*, cuando las dos curvas están en la *misma* dirección :

$$D_c > D; \quad r_c < r; \quad L_c < L; \quad M_c > M.$$

Fig. 5 *c*, cuando las dos curvas están en direcciones *opuestas* :

$$D_c < D; \quad r_c > r; \quad L_c > L; \quad M_c < M.$$

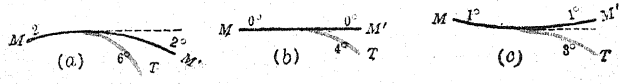



Fig. 6.

**29. Diferencia de Agudeza.** Fig. 6. Supongamos que la agudeza de la curva de la via principal, $M\,M'$, sea $D\,m$, y que la curva del desvío, $T$, sea $D_c$. (En fig. 6 *b*, $D\,m = 0$.)

Entonces para la diferencia de su agudeza, $D_d$ tenemos : —

$D_d = D_c \pm D^m$; mas para la Fig. 6 *c* ; menos para la Fig. 6 *a* ;

ó bien :

$D_c = D_d \pm D^m$ ; más para la Fig. 6 *d* ; menos para la Fig. 6 *c*. (17).

Si (Fig. 7*c*) la via principal tiene la curva *mas aguda*, o $D_m > D_c$, tenemos; $D_d$ $(= D_c - D_m)$ negativo, y $D_c = D_m - D_d$.

Las ecuaciones (17) de arriba, son exactas, cuando $D$ = extensión por unidad de *arco*. Vease curvas pag 954, ¶ 20.

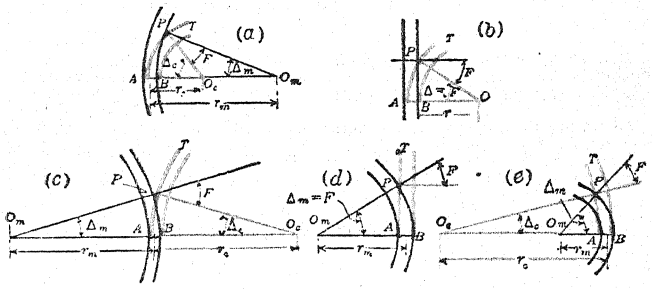



**Fig. 7.**

**30 a. Disposiciones.** Dejemos que las Figs 7 representen una serie de desvios, $T$ (que estan en rojo) cada uno dejando la via principal (esta, en negro) a la derecha con ángulo de la rana $F$ constante; y, comenzando con Fig. 7 *a*, dejemos que la via principal se separe progresivamente hacia la izquierda. Entonces tenemos : —

| | Fig. | | | | |
|---|---|---|---|---|---|
| | 7 *a*. | 7 *b*. $r_m = \infty$; $\Delta = F$ | 7 *c*. $r^m = r_c$. | 7 *d*. $r_c = \infty$; $\Delta_m = F$ | 7 *e*. |
| Centro (y separación de la tangente en nuestras Figs)..... | | | | | |
| Principal..., $O_m$ | Derecha. | 0 | Izquierda. | | |
| Desvio ...., $O_c$ | Derecha. | | | 0 | Izquierda. |
| El desvio deja la via principal en. | Interior. | | Exterior. | | |
| Carril de rana.. | | | | | |
| Principal ..., $B\,P$ | Dentro. | | Fuera. | | |
| Desvio ...., $A\,P$. | Fuera. | | | | Dentro. |
| Radio..... | | | | | |
| Mayor..... | Principal. | | | Desvio. | |
| Principal..., $r_m$.. | Aumento. | $\infty$ | Disminución. | | |
| Desvio ..., $r_c$.... | Aumento. | | | $\infty$ | Disminución. |
| Extension..... | | | | | |
| Principal....., $\Delta_m$ | Disminución. | 0 | Aumento. | | |
| Desvio....., $\Delta_c$... | Disminucion. | | | 0 | Aumento. |

**30 b.** Vemos, por consiguiente, que un desvio puede ser una tangente, Fig. 7 *d*, o una curva en cualquiera dirección; y que, una curva, puede dejar:

Fig. 7 *b*, una tangente de la linea principal, como en nuestros articulos anteriores;

Fig. 7 *c*, una linea principal encurvandose en la dirección *opuesta* (con centros en lados *opuestos* de la via);

Fig. 7 *a*, *e*, una linea principal encurvándose en la *misma* dirección (con centros hacia el *mismo* lado de la via).

**30 c. Cuando el desvio es una tangente** (Fig. 7 *d*), necesariamente deja el *exterior* de la curva de la via principal; y el ángulo de la rana, $F$, entonces el mismo (= extension del desvio, $\Delta_m$) que para un desvio (Fig. 7 *b*) de radio $r = r_m$, Fig. 7 *d*, dejando una tangente de la via principal.

**30 d. Cuando las dos curvas están en direcciónes opuestas** (Fig. 7 *c*), (centros, $O_m$ y $O_c$, en lados opuestos de la via), la curva del desvio necesariamente tambien deja el exterior de la curva de la linea principal.

**30 e. Cuando las dos curvas están en la misma direcciún** (Figs 7 *a*, *e*) (centros, $O_m$ y $O_c$, en el mismo lado de la via) el desvio puede dejar el interior o el exterior de la curva de la via principal, pero suponemos (Fig. 7 *a*) que, en este caso, a menos que no se especifique de otro modo, la curva del desvio tiene el radio mas corto, de modo que deja el *interior* de la curva de la via principal.

**31 a. Ecuaciones** para curvas de desvios. Para símbolos, vease ¶ 27.

Para ecuaciones aproximadas, vease ¶ 31 *b*.

$$\pm \Delta_m = F \pm \Delta_c\dagger; \qquad \pm \Delta_c = F \pm \Delta_m\dagger; \ldots\ldots\ldots\

| | En desvio de tangente. | En desvio de la curva. Exacto. | Aprox. |
|---|---|---|---|
| Radio de la linea de centro de la linea principal..... | $r_m$ ( $= \infty$ ) | $r_m$ | ... |
| del desvio..... | $r$ | $r_c$ | $r_{ca}$ |
| Agudeza de la linea principal..... | $D_m$ ( $= 0$) | $D_m$ | ... |
| del desvio..... | $D$ | $D_c$ | $D_{ca}$ |
| Diferencia de agudeza..... | $D_d$ ( $= D$) | $D_d$ ( $= D_c \pm D_m$)* | $D_{da}$ |
| Arranque..... | $L$ | $L_c$ | $L_{ca}$ |
| Ordenada..... | | | |
| media.—..... | $M$ | $M_c$ | $M_{ca}$ |
| cuarto-punto—. | $Q$ | $Q_c$ | $Q_{ca}$ |

Entonces, para un ancho de via dado, $G$, y un número dado de rana, $N$, tenemos: — Aprox; vease ¶ 31c.

$$D_{da} = D; \qquad D_{ca} = D_{da} \pm D_m \dagger; \qquad r_{ca} = 50/\text{sen } \tfrac{1}{2} D_{ca}$$

$$L_{ca} = L; \qquad M_{ca} = M\,(1 + D_m/D)\,\dagger; \qquad O_{ca} = Q\,(1 \pm D_m/D)\,\dagger \quad (26)$$

**31 c.** La siguiente tabla muestra los *grados de aproximación* en las ecuaciones aproximadas (26), ¶ 31 *b*.

| | | | | | | |
|---|---|---|---|---|---|---|
| | $D_m = 2°$ | $D_m = 4°$ | $D_m = 8°$ | $D_m = 2°$ | $D_m = 4°$ | $D_m = 8°$ |
| $N$ | $D_d/D$ | $D_d/D$ | $D_d/D$ | $D_d/D$ | $D_d/D$ | $D_d/D$ |
| 4 | 1.0042 | 1.0088 | 1.0196 | 0 99627 | 0 99299 | 0 98773 |
| 8 | 1 0019 | 1 0044 | 1 0115 | 0.99878 | 0.99821 | 0 99902 |
| 16 | 1.0018 | 1.0054 | 1.0176 | 0 99984 | 1.0014 | 1.0095 |
| | $r_c/r_{ca}$ | $r_c/r_{ca}$ | $r_c/r_{ca}$ | $r_c/r_{ca}$ | $r_c/r_{ca}$ | $r_c r_{ca}$ |
| 4 | 0.99621 | 0.99243 | 0.98490 | 1.0038 | 1.0076 | 1 0153 |
| 8 | 0 99845 | 0.99691 | 0.99383 | 1.0015 | 1.0031 | 1.0062 |
| 16 | 0.99900 | 0.99800 | 0.99599 | 1.0010 | 1 0020 | 1 0040 |
| | $L_c/L_{ca} = L_c/L$ | $L_c/L_{ca} = L_c/L$ | $L_c/L_{ca} = L_c/L$ | $L_c/L_{ca} = L_c/L$ | $L_c/L_{ca} = L_c/L$ | $L_c/L_{ca} = L_c/L$ |
| 4 | 0 99916 | 0.99827 | 0.99637 | 1.0008 | 1.0016 | 1.0029 |
| 8 | 0.99909 | 0 99801 | 0.99534 | 1.0007 | 1 0013 | 1.0019 |
| 16 | 0.99883 | 0.99698 | 0.99126 | 1.0005 | 1.0003 | 0 99779‡ |
| | $M_c/M_{ca}$ | | $M_c/M_{ca}$ | $M_c/M_{ca}$ | | $M_c/M_{ca}$ |
| 4 | 1 0013 | .... | 1.0043 | 0.99851 | .... | 0.99305 |
| 16 | 0.99990 | .... | 0.99285 | 0.99896 | .... | 0.99368 |

* Más para curvas en direcciones opuestas; menos para curvas en la misma direccion.

† Más para curvas en la misma direccion; menos para curvas en direcciones opuestas.

‡ Con $Dm = 8°$, $N$ 16, el desvio, del exterior de la curva de la linea principal; curvas en la misma direccion, vease fig, 7*e*; *i. e.*, los dos centros están en el mismo lado de la via. Vease ¶ 30 *e*.

## Doble Desvio de la Tangente.

### De lados opuestos.

**32.** Fig. 8. En un desvio doble de tope (vease Parte I, ¶ 10) de lados opuestos de una tangente, supongamos que las dos curvas del desvio sean de igual agudeza, y que

$G = A B$ = ancho de via;
$F = F_a = F_b$ = ángulo principal de la rana;
$F_c$ = ángulo de la bifurcacion de la rana;
$N = N_a = N_b$ = número de la rana principal;
$N_c$ = número de la bifurcacion de la rana;
$L = A P_a = B P_b$ = arranque de la rana principal;
$L_c = a P c$ = arranque de la bifurcacion de la rana;
$r = O_a a = O_b a$ = linea de centro del radio de cualquiera de las dos curvas del desvio;
$R = O_a E = O_b A = O_a P_a = O_b P_b = O_a P_c = O_b P_c = r + G/2$ = radio del carril exterior de cualquiera de las dos curvas del desvio.

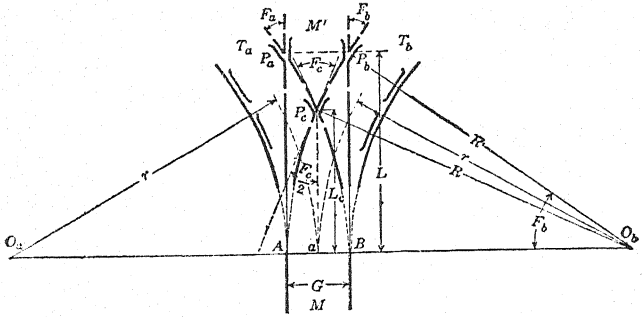



**Fig. 8.**

Entonces,

$$F_c = 2\ P_c\ O_b\ a;$$

$$L = 2\ G\ N \text{ (vease ¶ 19)}; \qquad L_c = 2\left[G + \frac{(G/2)^2}{r}\right] N_c$$

$$= \frac{G + \frac{(G/2)^2}{r}}{\tan\ (F_c/2)} \quad \ldots\ldots\ldots \quad (27)$$

O, para $L_c$, tenemos : —

$$\frac{L_c}{L} = \sqrt{\frac{R^2 - (R - G/2)^2}{R^2 - (R - G)^2}} = \sqrt{\frac{R - G/4}{2\ R - G}} \quad \ldots\ldots\ldots\ldots\ldots \quad (28)$$

De aqui, prescindiendo de $G$, tenemos, aprox : —

$L_c = \sqrt{\frac{1}{2}}\ L = 0.707\ L = \sqrt{2\ G\ N}$ ;

$L - L_c =$ aprox $(1 - \sqrt{1/2})\ L =$ digamos $0.293\ L$ ;

O, practicamente, $L_c = 0.7\ L$ ; y $L - L_c = 0.3\ L$ ..... (29)

$$N_c = \frac{\cot\ (F_c/2)}{2} = \frac{L_c}{2\ [G + (G/2)^2/r]}$$

$$= \text{aprox}\ \frac{L_c}{2\ G} = \frac{\sqrt{2\ G\ N}}{2\ G} = \frac{N}{\sqrt{2}} = 0.707\ N. \quad \ldots\ldots\ldots \quad (30)$$

**Del mismo lado.**

**33.** Fig. 9. Dos chuchos de tope de desvio, $T_a$ y $T_b$, con radio de linea de centro $r_a$ $r_b$, respectivamente, del mismo lado de la tangente, $M$ $M'$.

Sea $R_a$ ( $= O_a A = r_a + G/2) = 2 R_b = 2 (O_b A) = 2 (r_b + G/2)$.

Entonces, siendo $O_b P_b = O_b O_a$, tenemos :

$$F_a = F_b = F = \text{ángulo principal de la rana;}$$
$$N_a = N_b = N = \text{número de la rana principal} \ldots\ldots\ldots \quad (31)$$

y las dos puntas principales de la rana, $P_a$ y $P_b$, son opuestas, en la via, $T_a$.

Sea $D_a$ y $D_b$ la agudeza de la linea de centro de las curvas, $T_a$ y $T_b$, respectivamente.

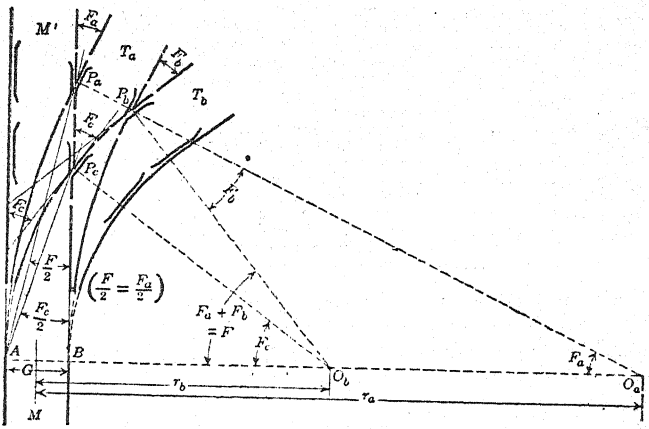



**Fig. 9.**

Entonces, aprox :

$$D_b = 2\ D_a\ ;$$

Senoverso $F_c = G/R_b = 2\ G/R_a$ ;

$L = B\ P_a$ = arranque $= R_a$ sen $F_a = 2\ G\ N$ ;

$L_c = B\ P_c$ = arranque de la bifurcación $= R_b$ sen $F_c = 2\ G\ N_c$

$$= \sqrt{0.5\ L} = \sqrt{2\ G\ N};$$

$$N_c = \frac{L_c}{2\ G} = \frac{\cot\,(F_c/2)}{2} = 0.707\quad N \ldots\ldots\ldots\ldots\ldots \quad (32)$$

## Desvios Dobles, de una Curva.

**34.** En desvios dobles del mismo lado o de lados opuestos de una curva de via principal de agudeza moderada, las dimensiones son practicamente las mismas que para desvios similares de via recta, usando los mismos ángulos de rana. Si la curvatura de la via principal es aguda, el problema puede resolverse gráficamente.

## CHUCHOS (1) DE AGUJA

### Desvios de Tangentes.

**35. Nomenclatura.** Fig. 10. Desvio de una tang con chucho de aguja. Compárese el Manual de la Ascn Am Ingo F. C. 1915, pp 182-3.

Sea

$G$ ..... = ancho de via;
$R = O\ D$..... = radio del lado de la curva del desvio, $D\ E$;
$r = O\ D - G/2$... = radio de la linea de centro de la curva de desvio, $D\ E$;
$D$ ..... = agudeza de la curva del desvio, $D\ E$;
$\Delta = D\ O\ E$..... = extension de la curva de desvio, $D\ E$, entre el extremo de la aguja del chucho y el extremo de la rana;
$M$ ..... = media ordenada de la cuerda, $D\ E$, al carril exterior;
$F = B\ P\ E = t\ O\ E$ = ángulo de la rana;
$N = \frac{1}{2}$ cot $(F/2)$.. = numero de la rana;
$C = D\ E$..... = cuerda larga del carril exterior;
$L = B\ P$..... = arranque del punto real de la punta de la aguja al punto teorico de la punta de la rana;
$T = D\ V = V\ E$.. = semitangente de la curva del desvio;
$s = Y\ A\ D$..... = angulo del chucho;
$l = A\ D$..... = largo del chucho;
$S$ ..... = distancia del extremo de la aguja, o extension, entre los anchos de via en $D$;
$W = P\ E$..... = distancia de la punta de la rana a la cabeza de la rana.
$e$ ..... = $G - S - W$ sen $F$;
$p$ ..... = grueso actual de la punta del chucho;
$t$ ..... = grueso actual de la punta de la rana;
$x, y, = t\ i, t\ k$..... = co-ordenadas de cualquier punto dado, como $n$, en el lado del carril exterior;
$a = D\ O\ n$..... = ángulo del arco, $D\ n$.

**36. Ecuaciones** para desvios de la tangente, de chuchos de aguja.

Fig. 10. Angulo de la rana, $F$; extensión del desvio, $\Delta$, y ángulo del chucho, $s$.

$F = \Delta + s$; $\Delta = F - s$;

$s = F - \Delta$ Sen $s = (S - P)/l$..... (33)

Arranque, $L$.

$$L = (l - W)\frac{\text{sen } \frac{1}{2}(F - s)}{\text{sen } \frac{1}{2}(F + s)} + (G - p)\ \text{cot}\ \frac{F + s}{2}$$

$$= (T + l)\ \text{cos}\ s + (T + W)\ \text{cos}\ F \quad \ldots\ldots (34)$$

Para *el arranque desde el vertice al punto teórico del chucho, agreguese* $p$ cot $s$.

Para el *arranque del punto real de la rana*, agreguese $N\ t$, $t$ = grueso real de la punta de la rana.

Como $T$ sen $s + T$ sen $F = T$ (sen $s$ + sen $F$) $= e$, tenemos;

$$T = R\ \text{tan}\ (\Delta/2) = \frac{e}{\text{sen}\ s + \text{sen}\ F} \quad \ldots\ldots (35)$$

*Radio r de la linea de centro del desvio.*

Como $e = D\ E$ sen $\frac{1}{2}(F + s)$

$= 2\ R$ sen $(\Delta/2)$ sen $\frac{1}{2}(F + s)$, tenemos;

$R = r + G/2 = T$ cot $(\Delta/2)$

$$= \frac{e}{2\ \text{sen}\ \frac{1}{2}(F + s)\ \text{sen}\ (\Delta/2)} = \frac{e}{\text{cos}\ s - \text{cos}\ F}\ ;\ \text{y}$$

$$r = R - G/2 \quad \ldots\ldots (36)$$

(1) Véase *N. del T.* (1), pag. 926.

Cuerda, $D\,E = 2\,R$ sen $(\Delta/2)$ ..... (37)

Agudeza $D$, de la curva del desvio.

Señ $(D/2) = 50/r$ ..... (38)

Arco $D\,E$.

$D\,E = 2\,\pi\,R\,\Delta^{\circ}/360 = 0.017\,453\,R\,\Delta.^{\circ}$ ..... (39)

*Distancia*, $D'\,E'$, desde el extremo $D'$ del chucho al extremo $E'$ de la rana.

$D'E' = L - (W + l)$ ..... (40)

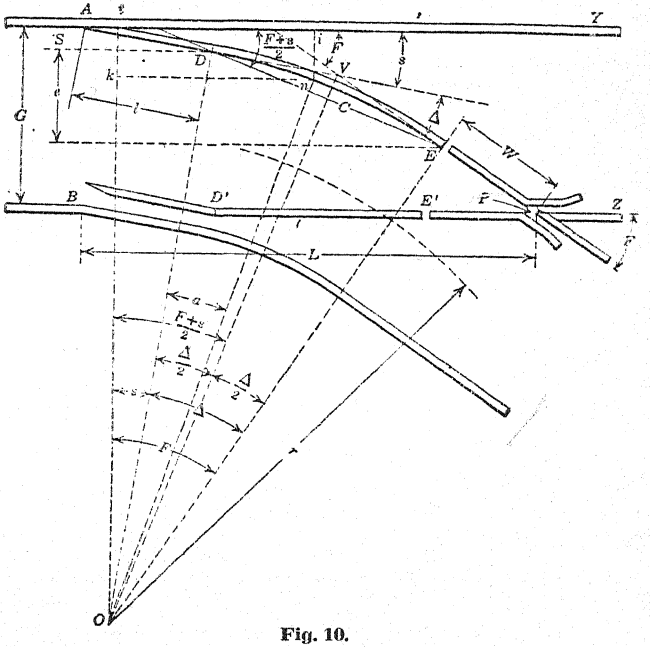



**Fig. 10.**

Co-ordenadas, $x = k\,n$; $y = i\,n$.

$x\ (= k\,n) = [l \cos s - R \text{ sen } s] + R \text{ sen } (a + s)$;

$y\ (= i\,n) = [S + R \cos s] - R \cos (a + s)$ ..... (41)

Las cantidades entre paréntesis son constantes para cantidades $N$, $l$, $s$, dadas en una rana.

La Asen Am de Ingo de F. C. Manual 1915, pagina 184, especifica: —

Grueso, $p$, de la punta del chucho.. = 0.25 pulgs ( 6 mm) = 0.020 833... pies;
Grueso, $t$, de la punta de la rana... = 0.50 — ( 12 mm) = 0.041 666... —
Juego o distancia del extremo de la aguja, o
extension, $S$..... = 6.25 pulgs (159 mm) = 0.520 833... pies;

Media ordenada, $M$ al carril exterior.

$M = R$ senoverso $(\Delta/2)$ ..... (42)

**37. Ecuaciones simples aproximadas,** usando « numero del chucho », (1) $n$.

Fig. 11. (Comparese Wellington B. Lee, Engineering News, p. 252, abril 21, 1898.

Siendo :

$G$ = ancho de via;
$W$ = distancia desde la punta a la cabeza de la rana;
$F$ = angulo de la rana;
$l$ = largo del chucho;(1)
$S$ = distancia del extremo de la aguja;
$N$ = numero de la rana;
$n = l/S$ = « numero del chucho »; (1)
$e = G - S - W \operatorname{sen} F$;
$C$ = cuerda, $D\,E$, desde el extremo de la aguja a la cabeza de la rana.

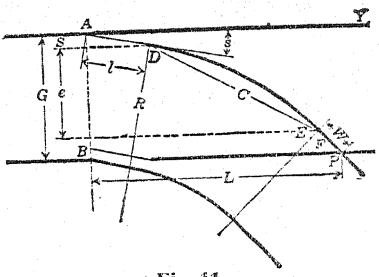



Fig. 11.

Entonces : —

$$C \text{ aprox} = 2\,e\,\frac{n\,N}{n+N}\,; \qquad (43)$$

$$R = r + \frac{G}{2}\,;\; R \text{ aprox} = C\,\frac{n\,N}{n-N}$$

= radio del lado del carril exterior ..... (44)

Con ancho de via, $G$, = 4'8.5'' (1.435 m) y $S$ = 6.25'' (158 mm) los cálculos muestran :

| $N$ = | $l$ = | $W$ = | $F$ = | $s$ = | $R - R_{app}$ | $R/R_{app}$ |
|---|---|---|---|---|---|---|
| 4 | 11 pies (3.35 m) | 3 pies 2'' (0.96 m) | 14°15'0'' | 2°36'19'' | 3.25 pulgs (82 mm) | 1.0024 |
| 16 | 33 pies (10.06 m) | 8 pies 0'' (2.44 m) | 3°34'47'' | 0°52'5'' | 8.53 pies (2.60 m) | 0.9958 |

## Ordenadas a la cuerda, C.

**38.** Donde, como se admitió, para el chucho (1) de *tope*, en ¶ 12, Figs 4, el carril exterior del desvio forma una curva continua circular, desde el comienzo $A$, del desvio al punto teórico de la rana, $P$, sus ordenadas medias y las de los cuartos puntos son aproximadamente independientes del numero de la rana, $N$ (y con ello independientes del radio); pero, donde, como en los chuchos de aguja, Fig. 10, la curva del desvio es tangencial al carril del chucho y a la pata de la rana, en $D$ y en $E$, respectivamente, las ordenadas disminuyen, como disminuye el angulo $F$ de la rana; y se convertiria en cero si el angulo de la rana, $F$, se convirtiera = al angulo del chucho, $s$; porque la curva del desvio se convertiria en una linea recta.

$$\text{Media ord} = R \operatorname{senoverso} \frac{\Delta}{2} = \left(r + \frac{G}{2}\right) \operatorname{senoverso} \frac{\Delta}{2}\,.$$ Véase Ec (42).

(1) Véase *N. del T.* (1) pag. 926.

**Dimensiones « Teóricas » y « Prácticas ».**

**39. Para disminuir y reducir el corte de carriles y el desperdicio de los mismos,** es deseable separarse de las dimensiones « teóricas » dadas por las ecuaciones 34-41 y representadas por las lineas *negras* en las figs 12, y hacer el carril de arranque recto, $D'E'$, de tal largo, $D_p'E'$, que permita la utilizacion eventual de las dos piezas en que queda cortado el carril; substituyendo asi las dimensiones « teóricas » por las « practicas ».

**40. El carril curvo de arranque,** ($D_p E_p$ fig. 12 *a*; $D_p'' E$, fig. 12*b*) puede entonces hacerse del mismo largo y numero de piezas con $D_p' E'$, poniendo opuestas las juntas de estos dos carriles, si el largo excede del carril encurvado entonces pudiera compensarse poniendolo con juntas separadas mas anchas que las $D_p' E'$. Pero, comparense las tablas de dimensiones teóricas y prácticas, ¶ ¶ 47-50.

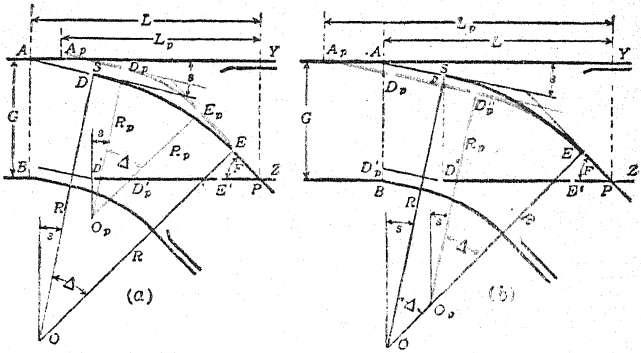



**Fig. 12.**

**41. Diagramas.** En figs 12 *a*, 12 *b*, las dimensiones « teóricas » se indican en lineas y letras *negras ;* y las dimensiones nuevas necesarias para el carril exterior del desvio, con objeto de dar dimensiones « *practicas* », en lineas y letras *rojas.* La fig. 12 *a* representa el caso donde el arranque « teórico », *L*, *excede* al arranque « practico » $L_p$ ; y *vice versa* en la fig. 12 *b*.

**42. Cambio en el largo del arranque.** Figs 12 *a*, 12 *b*. Con un largo dado de chucho, (1) $l = A\,D$, dando el angulo del chucho, $s = Y\,A\,D$, y dando el ángulo de la rana, *F*, hay solamente un largo de arranque, *L*, y solamente un radio del lado del carril exterior, $R = O\,D$, para un arco circular, $D\,E$, conectando tangencialmente el extremo del chucho, en *D*, y la cabeza de la rana, en *E*.

**43.** Si el arranque se *acorta* (como $L_p$, fig. 12*a*) debemos introducir una « tangente de rana « adicional, $E\,E_p$ ; ya que, si el arranque se alarga (como $L_p$, fig. 12 *b*) debemos introducir una « tangente de chucho » $D_p\,D_p''$ adicional. En cualquiera de los dos casos el *radio* es *acortado* como se indica.

**44. Radio Nuevo.** Para el nuevo (disminuido) radio del lado del carril exterior. $R_p$, y para las tangentes adicionales, tenemos. (Fórmulas de Via y Tablas de S. S. Roberts, pp. 12-13.)

Fig. 12 *a*.

$$R - R_p = (L - L_p)\,\frac{\text{sen } F \times \cot \tfrac{1}{2}\,(F - s)}{\text{sen } (F - s)}\,;$$

$$E\,E_p = (L - L_p)\,\frac{\text{sen } \tfrac{1}{2}\,(F + s)}{\text{sen } \tfrac{1}{2}\,(F - s)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (45)$$

Fig. 12 *b*.

$$R - R_p = (L_p - L)\,\frac{\text{sen } s \times \cot \tfrac{1}{2}\,(F - s)}{\text{sen } (F - s)}\,;$$

(1) Véase *N. del T.* (4) pag. 926.

$$D_p\ D_p'' = (L_p - L)\frac{\text{sen } \frac{1}{2}\ (F + s)}{\text{sen } \frac{1}{2}\ (F - s)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (46)$$

**45. Coordenadas** Para un punto dado en la curva de arranque,

Sea $x$ = la abscisa (coordenada *paralela* a la via principal), medida desde la punta del chucho, (1) $A_p$;

$y$ = la ordenada (coordenada *normal* a la via principal) medida normalmente de la tangente.

En la fig. 12 *a*, donde el arranque teórico excede al práctico, las coordenadas, $x$ e $y$, a cualquier punto en la curva de arranque, $D_p\ E_p$, son como las da la ecuacion 41, sustituyendo $R_p$ por $R$; pero, en la fig. 12 *b*, donde el arranque práctico excede al teórico, estas coordenadas se modifican, como se expresa abajo, por la introduccion de la tangente del chucho, $D_p\ D_p''$. (Fórmulas de Via y Tablas; S. S. Roberts, p. 13.) Aqui, $l$ = largo del chucho, $A\ D$; $S$ = extension del extremo de la aguja; $s$ = angulo del chucho, (1) $Y\ A\ D$; $\Delta_p$ = extension del desvio del extremo de la aguja al punto dado. Las cantidades en parentesis [ ] son constantes para $N$, $l$ y $s$ dadas.

$$x = [(l + D_p\ D_p'')\cos s - R_p \text{ sen } s] + R_p \text{ sen } (\Delta_p + s);$$
$$= (\text{aprox})\ [(l + D_p\ D_p'') - R_p \text{ sen } s] + R_p \text{ sen } (\Delta_p + s);$$
$$y = [S + D_p\ D_p'' \text{ sen } s + R_p \cos s] - R_p \cos (\Delta_p + s) \quad \ldots\ldots\ldots\ldots \quad (47)$$

**46. Ranas y Chuchos** (1) **Asen Am Ingo F. C.** Manual, 1915, pp. 184-5, Nuestras. Figs 1, 10 y 13. Para arranques teóricos y prácticos, vease ¶ ¶ 48-50.

$S$ = juego del extremo de la aguja = 6,25 pulgs (159 mm);

$N$ = numero de la rana; $F$ = ángulo de la rana;

$W$ = distancia de la punta a la cabeza de la rana, medida con un cartabón de linea.

$K$ = distancia de la punta al extremo de atras de la rana; $T$ = abertura de la cabeza de la rana;

$H$ = abertura de la parte de atras del extremo de la rana;

$l$ = largo del carril de chucho;

$s$ = angulo de chucho.

**47.** *Dimensiones de Rana y Chucho.* (1)

| $N$ | $F$ ° ′ ″ | $W$ pies pulgs | $K$ pies pulgs | $W + K$ pies pulgs | $T$ pies | $H$ pies | $l$ pies pulgs | $s$ ° ′ ″ |
|---|---|---|---|---|---|---|---|---|
| 4 | 14 15 00 | 3 2 | 5 4 | 8 6 | 0.79 | 1.32 | 11 0 | 2 36 19 |
| 5 | 11 25 16 | 3 7 | 6 5 | 10 0 | 0.71 | 1.28 | 11 0 | 2 36 19 |
| 6 | 9 31 38 | 4 0 | 7 0 | 11 0 | 0.66 | 1 16 | 11 0 | 2 36 19 |
| 7 | 8 10 16 | 4 5 | 8 1 | 12 6 | 0.63 | 1.15 | 16 6 | 1 44 11 |
| 8 | 7 09 10 | 4 9 | 8 9 | 13 6 | 0.59 | 1.09 | 16 6 | 1 44 11 |
| 9 | 6 21 35 | 6 0 | 10 0 | 16 0 | 0.67 | 1.11 | 16 6 | 1 44 11 |
| 9½ | 6 01 32 | 6 0 | 10 0 | 16 0 | 0.63 | 1 05 | 16 6 | 1 44 11 |
| 10 | 5 43 29 | 6 0 | 10 6 | 16 6 | 0.60 | 1.05 | 16 6 | 1 44 11 |
| 11 | 5 12 18 | 6 0 | 11 6 | 17 6 | 0.54 | 1.05 | 22 0 | 1 18 08 |
| 12 | 4 46 19 | 6 5 | 12 1 | 18 6 | 0.53 | 1.01 | 22 0 | 1 18 08 |
| 15 | 4 49 06 | 7 8 | 14 10 | 22 6 | 0.51 | 0 99 | 33 0 | 0 52 05 |
| 16 | 3 34 47 | 8 0 | 16 0 | 24 0 | 0.50 | 1.00 | 33 0 | 0 52 05 |
| 18 | 3 10 56 | 8 10 | 17 8 | 26 6 | 0.59 | 0 98 | 33 0 | 0 52 05 |
| 20 | 2 51 51 | 9 8 | 19 4 | 29 0 | 0.48 | 0.97 | 33 0 | 0 52 05 |
| 24 | 2 23 13 | 11 4 | 23 2 | 34 6 | 0.47 | 0.97 | 33 0 | 0 52 05 |

(*N. del T.* Se convierten pies en metros mult por 0,305; puls en m., mults por 0,0254 y ms en pies mult por 3.281.)

« Las ranas n.os 8, 11 y 16 se recomiendan por llenar todos los requisitos generales para patios, chuchos de via principal y empalmes ». Asen Am Ingo F. C. Manual, 1915, p. 168.

**48. Arranques Teóricos.** Para dimensiones de rana y chucho, (1) vease ¶¶ 46, 47. Para arranques « practicos » vease ¶ ¶ 49-50.

Asen Am Ingo F. C. Manual, 1915, p. 184.

$N$ = numero de la rana; $r$ = radio de la linea de centro; $D$ = agudeza;

$L$ = arranque = distancia del punto real del chucho (1) al punto teórico de la rana;

$L_s$ = cierre del carril recto, desde el extremo de la aguja a la cabeza de la rana;

(1) Vease *N. del T.* (1) pag. 926.

$L_c$ = cierre del carril encurvado, desde el extremo de la aguja a la cabeza de la rana.

(*Vease N. del T.* — ¶ 11, pag. 926.)

| $N$ | $r$ pies | $D$ ° ′ ″ | $L$ pies | $L_s$ pies | $L_c$ pies |
|---|---|---|---|---|---|
| 4 | 112.26 | 52 53 56 | 37.05 | 22.88 | 23 29 |
| 5 | 183.22 | 31 40 24 | 42.77 | 28.19 | 28.55 |
| 6 | 273.95 | 21 01 58 | 48.11 | 33.11 | 33.38 |
| 7 | 364.88 | 15 47 19 | 61 94 | 41.02 | 41.24 |
| 8 | 488.71 | 11 44 40 | 67.47 | 46.22 | 46.42 |
| 9 | 616.27 | 9 18 27 | 72 24 | 49.74 | 49.92 |
| 9½ | 699.27 | 8 11 33 | 74.90 | 52.40 | 52 58 |
| 10 | 790.25 | 7 15 18 | 77.51 | 55.01 | 55.17 |
| 11 | 940.21 | 6 05 48 | 92 06 | 64.06 | 64.20 |
| 12 | 1136.34 | 5 02 38 | 97 25 | 68.83 | 68.96 |
| 15 | 1744 45 | 3 17 06 | 130.50 | 89.83 | 89.94 |
| 16 | 2005.98 | 2 51 24 | 135.95 | 94.95 | 95.05 |
| 18 | 2587.66 | 2 12 52 | 146.38 | 104 54 | 104.61 |
| 20 | 3262.98 | 1 45 22 | 156.35 | 113 68 | 113 76 |
| 24 | 4932.77 | 1 09 42 | 175.09 | 130.66 | 130 77 |

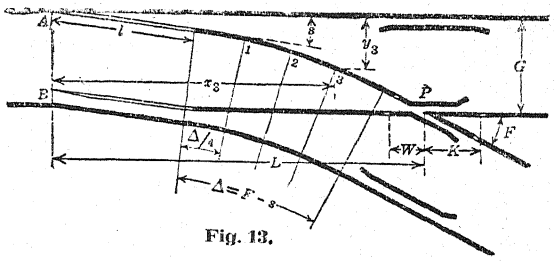



**Fig. 13.**

**49. Arranques prácticos.** Fig. 13. (Asen Am Ings F. C. Manual 1915, p 185 ccordenadas de la curva de arrastre.) Arranques teóricos, ¶ 48. Dimensiones de rana y chucho, ¶ ¶ 46-7. Largos de arranque y cierre, ¶ 50.

$N$ = numero de la rana; $r$ = radio de la linea de centro; $D$ = agudeza;

$x$ e $y$ = coordenadas desde el punto del chucho, a los cuartos puntos y al punto de centro en el lado del carril exterior.

(*Vease N. del T.* — ¶ 11, p. 926.)

| $N$ | $r$ pies | $D$ ° ′ ″ | $x_1$ pies | $y_1$ pies | $x_2$ pies | $y_2$ pies | $x_3$ pies | $y_3$ pies |
|---|---|---|---|---|---|---|---|---|
| 4 | 110.69 | 53 42 24 | 17.74 | 0.97 | 23.44 | 1.67 | 29.75 | 2 79 |
| 5 | 174.34 | 33 19 57 | 17.78 | 0.95 | 24.54 | 1.61 | 31.27 | 2.62 |
| 6 | 265.39 | 21 43 04 | 19.07 | 1.01 | 27 13 | 1.74 | 35 15 | 2.72 |
| 7 | 362.08 | 15 52 29 | 26.72 | 0.97 | 36.93 | 1.71 | 47.11 | 2.74 |
| 8 | 487.48 | 11 46 27 | 28.37 | 1.02 | 39.91 | 1.78 | 51.45 | 2 91 |
| 9 | 605.18 | 9 28 42 | 28.75 | 1.02 | 40.98 | 1.76 | 53.19 | 2 75 |
| 9½ | 695.45 | 8 14 45 | 30.31 | 1.06 | 43.35 | 1.82 | 56.37 | 2.83 |
| 10 | 790.25 | 7 15 18 | 30 28 | 1.06 | 44.05 | 1.84 | 57.81 | 2 85 |
| 11 | 922 65 | 6 12 47 | 40.74 | 1.08 | 56 47 | 1.84 | 72.19 | 2.87 |
| 12 | 1098.73 | 5 12 59 | 43.99 | 1.15 | 60.65 | 1.90 | 77.28 | 2.91 |
| 15 | 1744 38 | 3 17 01 | 55.49 | 1.01 | 77.95 | 1.78 | 100.41 | 2,85 |
| 16 | 1993.24 | 2 52 59 | 58.16 | 1.04 | 81.76 | 1.82 | 105.35 | 2.87 |
| 18 | 2546.31 | 2 14 31 | 58.73 | 1.04 | 84 46 | 1 82 | 110.10 | 2.86 |
| 20 | 3257.26 | 1 45 32 | 61.84 | 1.08 | 90.21 | 1 88 | 118.59 | 2.93 |
| 24 | 4886.16 | 1 10 21 | 68 72 | 1.27 | 100.21 | 1.97 | 132 59 | 3.00 |

(Vease tambien ¶ 50.)

(1) Vease *N. del T.* (1) p. 926.

**50.** Figs 12. Largos de arranques y cierres, etc.
(Ascn Am Ingo F. C. Manual, 1915, p. 183.)

Para « arranques teóricos » vease ¶ 48.
— dimensiones de rana y chucho, (1) vease ¶ ¶ 46-7.
— coordenadas practicas de curva de arranque ¶ 49.
$N$ = numero de la rana; $T_s$ = tangente del chucho, $D_p\ D_p''$, Fig. 12*b*;
— — $Tf$ = tangente de la rana, $E\ E_p$, Fig. 12*a*;
(Vease ¶ 43.)
$L$ = arranque, $B\ P$, del punto actual del chucho al punto actual de la rana;
$L_c$ = arranque para el carril recto;
$L_c$ = — — el encorvado;
( = largos de carril requeridos entre el extremo del chucho y la cabeza de la rana.)*

| $N$. | $L$. Pies. | $L_s$. Pies. | $L_c$. Pies. | $T_s$ | $Tf$. |
|---|---|---|---|---|---|
| 4 | 37.94 | uno 23.60..... | uno 24..... | 1.03 | 0.00 |
| 5 | 42.47 | — 27.68..... | — 28..... | 0.00 | 0.82 |
| 6 | 47.98 | — 32.73..... | — 33..... | 0.00 | 0.66 |
| 7 | 62.10 | — 13.89 uno 27..... | — 14.11 uno 27. | 0.00 | 0.19 |
| 8 | 67.98 | — 16.40 — 30..... | — 16.60 — 30. | 0.30 | 0.00 |
| 9 | 72.28 | — 16.41 — 33..... | — 16.59 — 33. | 0.00 | 0.57 |
| 9 ½ | 75.71 | — 25.82 — 27..... | — 26 — 27. | 0.76 | 0.00 |
| 10 | 77.93 | — 27 — 28..... | — 27.17 — 28. | 0.00 | 0.00 |
| 11 | 94.21 | — 32.85 — 33..... | dos 33..... | 2.99 | 0.00 |
| 12 | 100.80 | — 22.88 dos 24..... | tres 24..... | 5.33 | 0.00 |
| 15 | 133.28 | dos 33 uno 25.9..... | dos 33 uno 26. | 0.00 | 0.00 |
| 16 | 137.57 | uno 29.90 dos 33..... | uno 30 dos 33. | 1.56 | 0.00 |
| 18 | 146.51 | — 25.93 tres 26..... | cuat. 26..... | 0.00 | 1.08 |
| 20 | 157.42 | — 26.92 dos 27 uno 33 | tres 27 uno 33. | 0.44 | 0.00 |
| 24 | 117.22 | — 32.89 tres 33..... | cuat. 33..... | 2.43 | 0.00 |

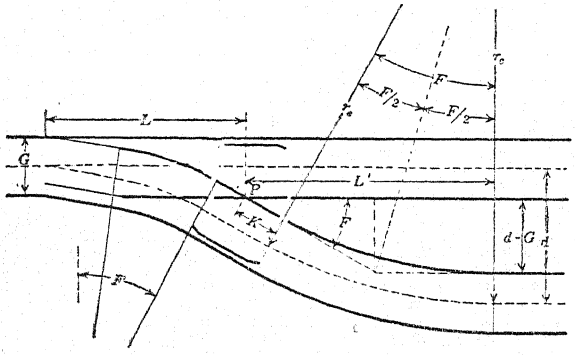



**Fig. 14.**

**Curva de Enlace.**

**51. Desvio Lateral.** Fig. 14. En el desvio « lateral » sea $d$ = distancia entre los centros de via; $d$—$G$ = dist. entre los carriles interiores de las vias; $K$ = distancia del punto teorico de la rana a la cabeza de la misma; $r_c$ = radio de la linea de centro de la curva de enlace; $L'$ = arranque del punto teórico de la rana al extremo de la curva de enlace. Entonces, recordando que

$$2\ N = \cot\ (F/2) = 1/\tan(F/2).$$

(1) Véase *N. del T.* (1) pag. 926.
* Véase *N. del T.* al pie tabla ¶ 47 y tambien ¶ 11 (*N. del T.*) pag. 926.

Tenemos :

$K + (r_c - G/2) \tan (F/2) = (d - G)/\text{sen } F$ ; de aqui :

$r_c = 2\,N\,(d - G)/\text{sen } F - 2\,N\,K + G/2$;

$L = (d - G) \cot F + (r_c - G/2)/2\,N$ ..... (48)

**52. Desvio Diamante o Equilateral.** Fig. 15. En el desvio « diamante o « equilateral », tenemos :

(Comparese el desvio Lateral, ¶ 51.)

$r$ = radio $O_t\,e$ de la linea de centro de la curva del desvio.

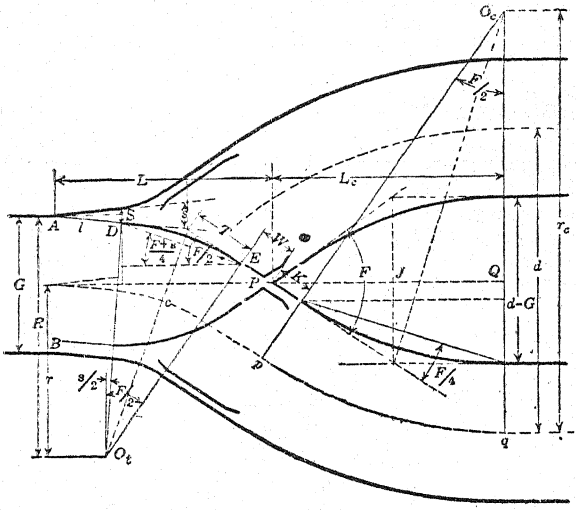



**Fig. 15.**

$$R\,(= r + G/2) = O_t\,D = T^* \cot \frac{F - s}{4} = \frac{D\,E^*}{2 \text{ sen } {}^1/_4\,(F - s)} \quad \ldots\ldots\ldots \quad (49)$$

$$L = (T + l) \times \cos\,(s/2) + (T + W) \times \cos\,(F/2) \quad \ldots\ldots\ldots\ldots\ldots \quad (50$$

*Comparacion de Dimensiones* entre un desvio diamante y un desvio de la via recta, dandose los angulos de la rana y del desvio.

Con $F = 8°$ $(N = 7{,}15)$; $s = 2°$; $G = 4'\ 8.5''$ (1,435 m). = 4.70833 pies (1.43608 m); $l = 16$ pies (4.877 m); $S = 0.5$ pies (0.152 m); $W = 5$ pies (1.524 m).

(*Vease N. del T.*, ¶ 11, pag. 926.)

| | Diamante. | Recto. | Diamante Recto. |
|---|---|---|---|
| $T$..... | 20.1285 | 20.1782 | 0.9975 |
| $R$..... | 768.6760 | 385.0226 | 1.9964 |
| $L$..... | 61.1903 | 61.0893 | 1.0017 |

$$^* T = \frac{G/2 - S/2 - W \text{ sen } (F/2)}{\text{sen } (s/2) + \text{sen } (F/2)};$$

$$D\,E = \frac{G/2 - S/2 - W \text{ sen } (F/2)}{\text{sen } {}^1/_4\,(F + s)} = 2\,R \text{ sen } {}^1/_4\,(F - s).$$

**53.** Radio y Arranque. Fig. 15. Radio, $r_c$, y arranque, $L_c$ de una curva de enlace, $p\ q$, para desvio diamante.

$d - G$ = distancia entre carriles interiores de las vias;

$r_c = O_c\ p$ = radio de la linea de centro de la curva de enlace, $p\ q$;

$K$ = distancia de la cabeza de la rana al punto teórico $P$ de la misma;

$L_c = P\ Q$ = arranque, del punto teórico de la rana al extremo de la curva de enlace;

$F$ = ángulo de la rana; $N$ = número de la rana $= \frac{1}{2}\cot(F/2)$.

Entonces

$(r_c - G/2)$ senoverso $(F/2) = (d - G)/2 - K$ sen $(F/2)$. De aqui,

$$r_c = \frac{d - G}{2 \text{ senoverso } (F/2)} - \frac{K}{\tan (F/4)} + G/2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (51)$$

$$L_c = P\ J + J\ Q = N\ (d - G) + (r_c - G/2) \times \tan (F/4) \ldots\ldots\ldots\ldots \quad (52)$$

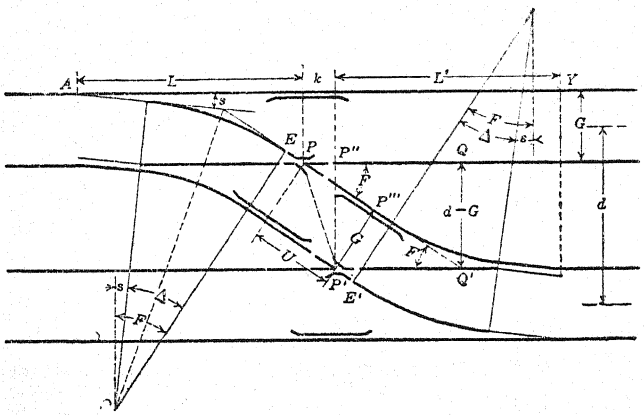



**Fig. 16.**

## Enlaces.

**54.** Figs 16, 17. Cuando la via de enlace es una tangente (fig. 16), los dos angulos de la rana son necesariamente iguales. Cuando es una curva invertida (fig. 17) se prefiere igualmente que sean iguales.

**Con via recta de enlace**, $E\ E'$. Fig. 16. Sea :

$R = O\ E$ = radio de la curva del desvio en el carril exterior;

$T$ = semitangente de la curva del desvio;

$G$ = ancho de via;

$d - G$ = distancia entre los carriles interiores de las vias.

$k = P\ P''$ = distancia teórica entre los puntos de la rana, $P$ y $P'$, || con las vias principales;

$U = P\ P'''$ = distancia teórica entre los puntos de la rana, $P$ y $P'$, || con la via de enlace;

$W$ = distancia del punto teórico de la rana, $P$, a la cabeza de la rana, $E$;

$l$ = largo del chucho. (1)

Entonces :

$$T = R \tan (\Delta/2); \qquad L = L' = (T + l) \cos s + (T + W) \cos F;$$

$$k = P\ Q - P''\ Q = P\ Q - P'\ Q' = \frac{d - G}{\tan F} - \frac{G}{\operatorname{sen} F};$$

$$A\ Y = L + k + L' = k + 2\ L;$$

$$U = P'\ Q' - P''\ Q' = \frac{d - G}{\operatorname{sen} F} - \frac{G}{\tan F} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (53)$$

Para el cambio de unidad en el valor de $d$, los cambios correspondientes en $k$ y en $U$, son, respectivamente; — en $k$, cambio = cot $F$ ; en $U$, cambio = cosec $F$.

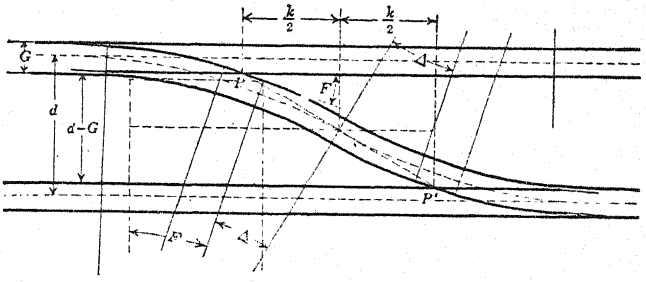



**Fig. 17.**

**55. Con curva de enlace invertida.** Fig. 17.

El uso de una curva de enlace invertida economiza espacio, especialmente donde las dos vias estan separadas ampliamente y donde el ángulo de la rana sea pequeño. El radio es generalmente el de una curva de desvio.

Sea :

$F$ = ángulo de la rana;

$\Delta$ = extension de cualquier ramal de la curva de enlace invertida;

$r$ = radio de la linea de centro de la curva de enlace invertida;

$R = r + G/2$;

= radio del carril exterior de la curva de enlace invertida;

$K$ = dist. del punto, $P$, teorico de la rana, a la cabeza de la misma;

$k$ = dist. paralela a la via, entre puntos de la rana, $P$ y $P'$.

Entonces :

$$r \cos (F + \Delta) = R \cos F + K \operatorname{sen} F - (d - G)/2;$$

$$k/2 = r \operatorname{sen} (F + \Delta) - R \operatorname{sen} F + K \cos F;$$

$$P\ P' = \sqrt{k^2 + (d - G)^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (54)$$

(1) Véase *N. del T.* (1) pág. 926.

## Desvio Doble.

**56.** Fig. 18. Desvio con chucho (1) de doble punta de lados opuestos a la tangente. Sean las dos ranas principales, $P$, de un mismo ángulo, y por consiguiente opuestos, y sea :

$r$ = radio de la linea de centro, del desvio;
$R = O\,E = r + G/2$ = radio del lado del carril exterior;
$\Delta_c$ = desarrollo de $D\,O\,P_c$ ;
$T_c$ = semitangente de la curva $D\,P_c$ ;
$W = E\,P$ = dist de la cabeza de la rana principal, $E$, al punto, $P$, teórico de la rana;
$S$ = juego del extremo de la aguja;
$s$ = angulo del chucho; (1)
$l = A\,D$ = largo del chucho;
$N$, $F$ = numero de la rana principal y ángulo;
$N_c$, $F_c$ = numero de la rana bifurcada y ángulo;
$L = B\,P$ = arranque a la rana principal, $P$ ;
$L_c = a\,P_c$ = arranque a la rana bifurcada, $P_c$.

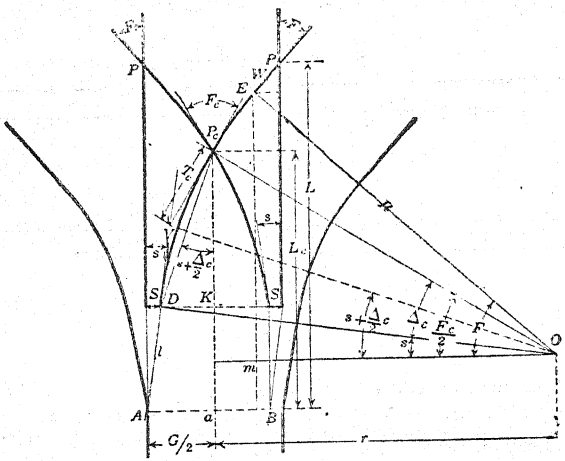



**Fig. 18.**

Entonces: —

$$\frac{F_c}{2} = \Delta_c + s\,; \qquad D\,P_c\,a = V\,O\,m = s + \frac{\Delta_c}{2} = \frac{F_c - \Delta_c}{2};$$

$$\text{Cos}\,\frac{F_c}{2} = \frac{r}{R} = \cos F + \frac{G/2 - W \operatorname{sen} F}{R}\,; \qquad \cos F = \frac{O\,m}{R};$$

$$N_c = \frac{\cot\,(F_c/2)}{2}\,;$$

$$T_c = \frac{G/2 - S}{\operatorname{sen} s + \operatorname{sen}\,(F_c/2)}$$

$$DP_c = \frac{G/2 - S}{\operatorname{sen}\,(s + \Delta_c/2)} = 2\,R \operatorname{sen} \frac{\Delta_c}{2}\,;$$

(1) Véase *N. del T.* (1) pág. 926.

$$R = \frac{D\ P_c}{2\ \text{sen}(\Delta_c/2)} = T_c \cot(\Delta_c/2) = \frac{G/2 - S}{2\ \text{sen}(\Delta_c/2)\ \text{sen}(s + \Delta_c/2)}$$

$$L_c = L - W \cos F - R\ [(\text{sen}\ F - \text{sen}\ (F_c/2)]$$

$$= a\ K + K\ P_c$$

$$= l \cos s + (G/2 - S) \cot \frac{F_c - \Delta_c}{2}$$

$$= (T_c + l) \cos s + T_c \cos (F_c/2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (55)$$

**Desvíos en escala.**

**57. Trazado.** Fig. 19. Ordinariamente, el patio o « conjunto » de vías 2, 3, 4, etc., conectados entre sí; son paralelas a la vía principal, 1; la escala es recta, de $P_1$ a $P_1$, etc.; las vías de patio, 2, 3, 4, etc. son rectas desde la rana de la escala, $P_2$, $P_3$, etc., y la dist, $d$, entre centros de vías adyacentes paralelas es constante.

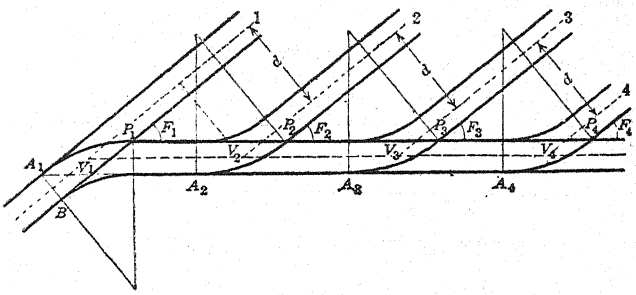



**Fig. 19.**

**58. Distancias.** Los ángulos de rana, $F_1$ $F_2$, etc., entre la vía principal y la escala, y entre la escala y las vías de patio, son entonces iguales; y las dist $A_1$ $A_2$, $A_2$ $A_3$, entre chuchos (1) consecutivos; y las dist. $P_1 P_2$, $P_2 P_3$, etc., entre ranas consecutivas; y las dist, $V_1$ $V_2$, $V_2$ $V_3$, etc., entre intersecciones consecutivas, medidas || a la escala, son todas iguales, y $V_1$ $V_2 = d/\text{sen}\ F$. (Pero véase ¶ 60). Para el cambio de unidad en el valor de $d$, el cambio correspondiente, en $d/\text{sen}\ F$, es $1/\text{sen}\ F = \text{cosec}\ F$.

**59.** La *mínima dist permitida*, a lo largo de la escala, entre un punto de la rana como $P_1$ y un punto opuesto al extremo del chucho, (1) $A_2$ del próximo desvío, es como de 10 pies (3.05 m).

**60.** « En condiciones ordinarias, *las vías deben separarse* de 13 a 14 pies (4 a 4.30 m) entre centros, y, donde sean paralelas a la vía principal o a otra vía de tráfico importante, el primer cuerpo de vía debe separarse. $\lessdot$ 15 pies (4.5 m) entre centros de la dicha principal o de otra vía importante. Las vías en escala deben espaciarse $\lessdot$ 15 pies (4.50 m) entre centros de cualquier vía paralela ». Am Ingo F. C. Manual, 1915, p. 469.

**61. Número de las Ranas.** Comunmente los patios de ferrocarril con líneas de vapor usan ranas N.os 7 y 8; pocos usan la n.º 6. Para uso general en vías en escala, la Asen Am Ingo F. C. (Manual, 1915, p. 469) recomienda ranas de no menor N.º que el n.º 8.

(1) Véase *N. del T.* (1) pág. 926.

**62. Para economizar espacio**, la escala puede sacarse de la rana de la via principal por una curva, *A B*, Figs 20, 21; de modo que forme con la via principal, un ángulo, $\theta$, mayor que el angulo, *F*, de la rana.

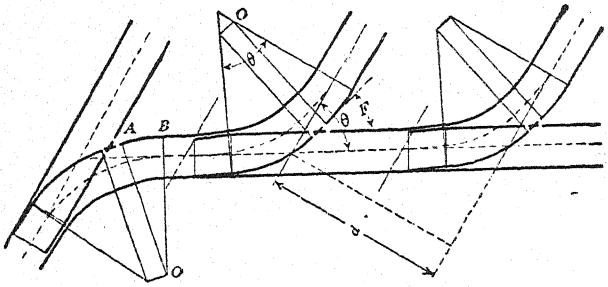



**Fig. 20.**

**63. Vias paralelas.** Fig. 20. Las vias de patio entonces dejan la escala por desvíos similares e iguales y encurvadas en direcciones opuestas, quedando asi paralelas con la via principal.

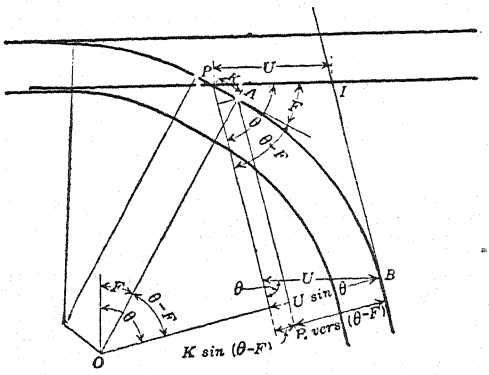



**Fig. 21.**

**64. Nomenclatura y fórmulas.** Fig. 21. Sea :
*U* = dist, *P I*, entre cualquier punto teórico de la rana, *P*, y la intersección *I*;
*K* = dist entre la cabeza de la rana y el punto teórico de la rana;
*R* = radio *O A* del carril exterior = *O B* (igual en todas las curvas).
Entonces :

$$U \text{ sen } \theta = R \text{ senoverso } (\theta - F) + K \text{ sen } (\theta - F) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (56)$$

**65.** Fig. 20. Dist entre chuchos,(1) entre ranas y entre intersecciones (vease ¶ 58) = $d$/sen $\theta$; y cambio de unidad, en $d$, cambios $d$/sen $\theta$ por 1/sen $\theta$.

(1) Véase *N. del T.* (1) pág. 926.

**66. Desvios (de tres cursos) dobles** (vease ¶ ¶ 34-4) se usan también en conexión con las escalas.

En tales casos, puede recurrirse a métodos gráficos.

Vease tambien ¶ 73 del capitulo; Patios y Estaciones, mas adelante.

**Desvio de una curva (chucho (1) de aguja).**

**67. El cálculo dificulta el trazado.** Las ecuaciones exactas, para las dimensiones de desvios de curva de chuchos (1) de aguja, serian muy complicadas; y los problemas, que envuelven tales dimensiones se resuelven mejor por medio de dibujos hechos en una escala liberal. El asunto se discute matematicamente, en « Formulas y Tablas de via », por S. S. Roberts; New York, John Wiley and Sons, publicistas.

**68. Diferencia de agudeza.** En general y para propósitos prácticos (aun con carriles de chucho y rana rectos), la *diferencia* de agudeza entre las curvas principales y de desvio, para un numero dado de rana, puede tomarse como igual al de un desvio de una via recta, con el mismo numero de la rana, como en el ¶ 26.

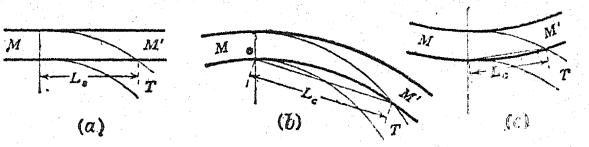



**Fig. 22.**

**69. Arranque.** Figs 22. Para un número dado de rana, el arranque puede tomarse aproximadamente como igual al calculado para un desvio de una tangente, por la ecuacion 10; pero, Mr. W. M. Camp en sus notas sobre via, p. 373 da lo siguiente : —

Sea $L_s$ = arranque para el desvio de la tangente, Fig. (*a*) ;

$L_c$ = arranque para el desvio de via encurvada con la misma rana, Figs (*b*), (*c*).

$D$ = agudeza de la curva de la via principal; $N$ = número de la rana.

Entonces :

$$L_c = L_s \pm D\ (N/12)^2 \ldots\ldots\ldots\

# CURVAS
# CURVAS SIMPLES

## GENERALIDADES

### Definiciones.

(Para ecuaciones, vease ¶ ¶ 25, etc).

**1. Fig. 4 a. Una linea de ferrocarril**, *a b c.....j*, se compone generalmente de una serie de curvas alternadas *b c, d e*, etc; y de lineas rectas, *a b, c d*, etc, que se llaman comunmente « tangentes » porque son necesariamente tangencial es a las curvas.

**Fig. 1. En una curva de ferrocarril circular o simple**, la linea de centro *A P B* de la via (y si no está a nivel, su proyeccion en un plano horizontal) es un arco circular.

**Fig. 1.** Supónganse las tangentes, *X A*, y *B Z*, en los extremos de la curva *A B*, prolongadas hasta su interseccion, *V*. Las distancias iguales, *A V*, *V B*, se llaman as *semitangentes*.

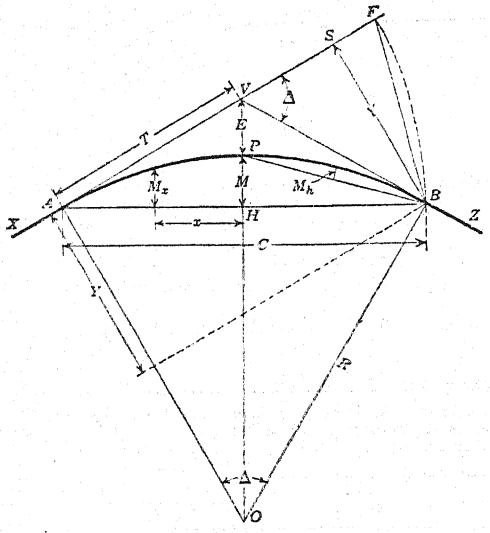



Fig. 1.

**2. Para los puntos de cambio** de la tangente a la curva, etc, etc., usamos los símbolos adoptados por la **Asen Am Ingo F. C.**, Manual de Práctica Reco mendada, 1915, p 135, como sigue : —

**Para curvas circulares**

| | | |
|---|---|---|
| **T. C.** | De tangente a curva; | Hasta aqui llamadas : |
| **C. T.** | De curva a tangente; | Punto de la curva P. C. |
| **C. C.** | — a curva; | — de tangencia P. T. |
| | | Punto de curva compuesta o invertida P. C. C. o P. R. C. |

**Para curvas espirales.** Vease p. ..., ¶ 201.

**El punto de la curva, Fig. 1. T. C.**, es el comienzo, $A$, de la curva, o el extremo a que llega el trazado en su progreso.

**El punto de tangente, C. T.**, es el otro extremo, $B$, de la curva.

**El vertice, o punto de interseccion, P, I.**, es el punto, $V$, donde las dos tangentes, $A\ V$ y $V\ B$, se encuentran.

## TRAZADO

### Funciones de la curva y sus símbolos.

**3. Fig. 1. Lista de las funciones** de una curva entera. Para una parte de curva subtendida por una unidad de cadena, vease ¶ 9.

| Funciones. | Símbolos. |
|---|---|
| Extension, desarrollo ó angulo central, (vease ¶ 10).. | $\Delta = A\ O\ B = F\ V\ B$ |
| Radio..... | $R = O\ A \quad = O\ B$ |
| Cuerda larga..... | $C = A\ B$ |
| Semitangente..... | $T = A\ V \quad = V\ B$ |
| Distancia exterior..... | $E = P\ V$ |
| Ordenada a la tangente (perpendicular a la tangente $A\ V$ o $B\ V$)..... | $Y = B\ S$ |
| Distancia tangencial (haciendo $A\ F = A\ B$)..... | $\ldots = B\ F$ |
| Ordenada media..... | $M = H\ P$ |
| Ordenada lateral, distante $x$ de $M$..... | $Mx$ |
| Media ordenada para $\Delta/2$..... | $Mh$ |

**4. Determinación de puntos en las curvas. Fig. 2.** Donde el radio $R$, es muy corto, es algunas veces practicable determinar el centro, $O$, de la curva, y trazar la curva en el terreno, usando una lienza de acero para el radio; pero ordinariamente, el centro no se determina, y los puntos, $a\ b$, etc., en la curva, se fijan poniendo el tránsito en un punto conocido de la curva (como el T. C., en $A$), trazando los ángulos periféricos conocidos, $v\ A\ a$, $v\ A\ b$, etc., de la tangente, $A\ v$ y midiendo las cuerdas, $A\ a$, $a\ b$, $b\ c$, etc., de longitud calculada.

**5. Cuerdas y Cadenas** *. En la práctica, estas cuerdas (excepto la primera y ultima, vease ¶ 24) son de igual longitud, igual (o casi igual, vease ¶ 20) a la de la lienza (generalmente 100 pies (30,48 m) o 20 metros de largo) usada en la determinacion de la linea, y se llaman « cadenas » *, con objeto de distinguirlas de otras cuerdas, como $A\ b$, $b\ d$, etc., que puedan adaptarse a la curva.

**6. Los ángulos periféricos iguales**, $a\ A\ b$, $b\ A\ c$, etc., subtendidos por estas cadenas iguales, se llaman **angulos de deflexion**, cada uno es igual a $D/2$ = mitad del **angulo central** correspondiente, $a\ O\ b$, $b\ O\ c$, etc.

**7. Angulos de Deflexion. Tangenciales y Centrales.** Samuel W. Mifflin en « Metodos de trazado para Ingenieros de Ferrocarriles » 1837, llamaba el ángulo, $e\ c\ d$ ( $= c\ O\ g = D/2$), el « *angulo tangencial* » por ser el angulo entre una cadena *, $c\ d$, y la tangente, $c\ e$, en cualquiera de sus extremos. En esto, Mifflin siguió al Coronel Stephen H. Long, del Ejercito Americano (« Manual de Ferrocarril », 1829) quien dio el nombre « *angulo de deflexion* » al angulo $f\ c\ d$, entre una cuerda $c\ d$, y la prolongacion $c\ f$, de la cuerda anterior $b\ c$; siendo igual al ángulo, $e\ h\ d$ (entre las tangentes, $c\ e$ y $h\ d$, en los dos extremos de una cuerda $c\ d$), desviación de *la linea* en la distancia subtendida por la cuerda. Es igual al ángulo *central*, $D = c\ O\ d$, subtendido por la cuerda $c\ d$.

**8.** Nosotros, « en la Practica en el Campo para trazar Curvas Circulares para Ferrocarriles » (primera edicion, 1851) y M. William Findlay Shunk en « El Ingeniero de Campo », 1880, seguimos esta nomenclatura. Ahora, no obstante el uso llama a $e\ c\ d$ el « **angulo de deflexion** » o simplemente la « deflexion » (como indicando una deflexion de la linea visual) y llama $D$ ( $= f\ c\ d$) el « **grado de la curva** ». Vease Agudeza ¶ 11.

---

* El uso de la palabra « cadena », para designar tal cuerda, viene de la época, cuando las *lienzas* de agrimensor eran desconocidas, y las medidas se tomaban por medio de cadenas hechas con eslabones de alambre. El *largo* completo tipo de tales cadenas se llamaba « una cadena ».

**9.** Figs 1 y 2. En una parte de la curva, como *a b*, fig. 2, subtendida por *una cadena* *, *c*, tenemos, extension o desarrollo $\Delta = D$, y designamos las otras varias funciones por minúsculas en lugar de mayúsculas. Así :

Las letras..... $C\ T\ E\ Y\ M\ Mx$ y $Mh$, fig. 1, ¶ 3,
se convierten, respectivamente en... $c\ t\ e\ y\ m\ mx$ y $mh$, en el arco *a b*, fig. 2.

**10. Fig. 1. La extension,** desarrollo, ó angulo del centro Δ, de la curva, es el angulo, *A O B*, de radio, *R*, y subtendido por la curva. Igual tambien al angulo, *F V B*, barrido por la tangente al girar de *V F* a *V B*. Es por consiguiente la desviacion total de la linea, debida a la curva; o es el ángulo de interseccion entre las dos tangentes. Cada *porción* de la curva tiene por consiguiente su propia extension, ó desarrollo.

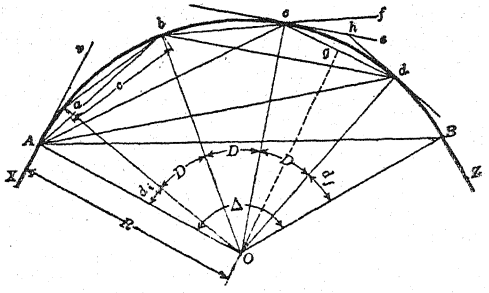



**Fig. 2.**

**11. Fig. 2. La agudeza** (comunmente llamado el « **grado de curvatura** ») se expresa generalmente por el angulo *central*, *D*, ( $= a\ O\ b = b\ O\ c$, etc.) subtendido por una *cadena* *, *a b o b c ; i. e.*, agudeza = la desviacion, *e h d* de la linea, en un largo de cadena. Una curva de « un grado », « dos grados », « tres grados treinta minutos », etc., es una curva cuya agudeza es 1º, 2º, 3º30′, etc. Vease ¶ 14.

**12.** Fig. 2. Pero algunas veces (especialmente cuando se usa el sistema metrico) la curva se nombra por el angulo de cadena *periferico*, $c\ b\ d = c\ A\ d$, etc, o angulo subtendido en la *circunferencia*, como en *b* ó *A*. Una curva dada, llamada 1º00′, 2º, 3º30 , etc., por la medida de su angulo *central* opuesto a una cadena, se llamaria (la mitad) 0º30′, 1º, 1º45′, etc., por la medida de su ángulo periférico respectivo.

Nosotros seguimos la definicion del ¶ 11.

**13.** Fig. 3. Cuando las curvas estan determinadas por medio de cadenas cortas * (vease ¶ 19), la agudeza se toma algunas veces por el desarrollo del arco subtendido por una *serie* de cadenas tales, cuya suma es igual al largo de la cadena unidad o tipo. Asi, una curva, *a p b*, en la que una serie de dos cadenas, *a p*, *p b*, de 50 pies cada una, subtende un angulo de *D*º, se llama entonces curva de *D*º, aunque sea mas aguda (tiene un radio mas corto, $R_2 = O\ b$) que la curva de *D*º, *A P B* ($R_1 = O\ B$) en el que el mismo ángulo de *D*º está subtendido por una cadena sencilla, *A B*, de 100 pies.

**14.** Fig. 1. En otros paises, fuera de los Estados Unidos, la agudeza de las curvas se determina a menudo por el largo de sus *radios*, $R = O\ A = O\ B$, etc. A mayor agudeza, menor radio.

**15. Representacion gráfica de la curvatura.** En la fig. 4, la linea de centro, *a f* (figs 4 *a* y 4 *b*) y los rectángulos (fig. 4 *b*) hacia arriba y hacia abajo representan el caso supuesto por la tabla a continuación. Como las abscisas, *a b*, *a c*, etc.

* Vease la llamada igual a esta * en el ¶ 7.

Fig 4 *b* representan distancias a lo largo de la linea, mientras que las ordenadas representan la agudeza de la curva, el área de los rectángulos representa *el desarrollo* de las curvas, o las desviaciones de las tangentes entre si. En la misma proporción las *porciones* de los rectángulos; por ej. : el rectángulo en *b' c* representa la extension o desarrollo de la porción, *b' c*, de la curva simple, *b c*.

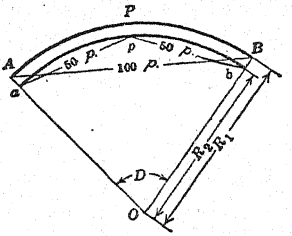



**Fig. 3.**

**16.** Las lineas inclinadas (Fig. 4*b*) representan la suavidad en los extremos de las curvas por medio de espirales (vease ¶ 194 y siguientes).

Como las posiciones de las tangentes no cambian por la introducción de las espirales, los desarrollos de las curvas, y las areas que las representan, quedan tambien sin cambiarse; asi el área del trapezoide en *B C* = área del rectángulo en *b c*.

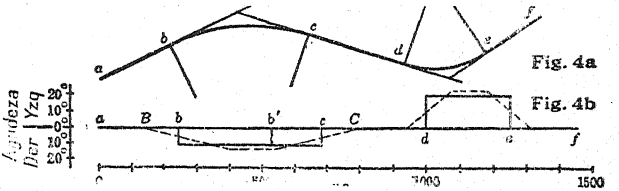



**Fig. 4.** Representaciones gráficas de las curvaturas.

| Punto..... | *a* | *b* | *c* | *d* | *e* | *f* |
|---|---|---|---|---|---|---|
| Estacion..... | 456 + 00 | 458 + 49 | 462 + 78 | 465 + 89 | 465 + 50 | 470 + 41 |
| Tramo..... | *a-b* | *b-c* | *c-d* | *d-e* | *e-f* | |
| Largo piés..... | 249 | 429 | 311 | 261 | 191 | |
| Agudeza, *D*..... | | | | | | |
| Izquierda..... | 0° | 0° | 0° | 20° | 0° | |
| Derecha..... | 0° | 10° | 0° | 0° | 0° | |
| Desarrollo, Δ..... | | | | | | |
| Tramo..... | | | | | | |
| Izquierda..... | 0° | 0° | 0° | 52.2° | 0° | |
| Derecha..... | 0° | 42.9° | 0° | 0° | 0° | |
| Sumas..... | | | | | | |
| Izquierda..... | 0° | 0° | 0° | 52.2° | 52.2° | |
| Derecha..... | 0° | 42.9° | 42.9° | 42.9° | 42.9° | |
| Total..... | 0° | 42.9° | 42.9° | 95.1° | 95.1° | |
| Diferencia..... | | | | | | |
| Izquierda..... | 0° | 0° | 0° | 9.3° | 9.3° | |
| Derecha..... | 0° | 42.9° | 42-9° | 0° | 0° | |

## Cadenas *. Longitud de las curvas.

**17. La unidad en los ferrocarriles y para la medida de curvas** es usualmente 100 pies (30.48 m) o 20 metros (= 65.617 pies).

**18. Cadena completa** *. Es costumbre dar (100 pies, o 20 metros) al largo de la *cadena;* y entonces el **largo de la curva** se expresa (aproximadamente) como igual a la suma de los largos de *cadenas* usados en determinarla. Así, Fig. 6, en la curva, $A\ a\ b\ c\ B$, el largo se toma como $= A\ a + a\ b + b\ c + c\ B$, medido a lo largo de estas *cuerdas*, $= 43 + 200 + 36.7 = 279.7$ pies. El largo *verdadero* de esta curva es por supuesto ligeramente *mayor* que el expresado.

**19. Cadena corta** *. Las curvas agudas se determinan usualmente por medio de una cadena corta cuyo largo es la mitad o la cuarta parte de una cadena tipo o menos; estas cadenas cortas subtenden ángulos correspondientes centrales mas chicos.

Así donde la cadena tipo es de 100 pies (30.48 m) (con largo de lienza) se usan en tales curvas cadenas de 50, 25 pies o menos. En general, el largo de cadena usado no es mayor de un duodécimo a un octavo del radio de la curva. La suma de una serie de largos de cadenas cortas subtendiendo un arco dado, es evidentemente mayor (casi igual al largo del arco mismo) que la suma de los largos de cadena tipo que subtende el mismo arco. Vease ¶ 13.

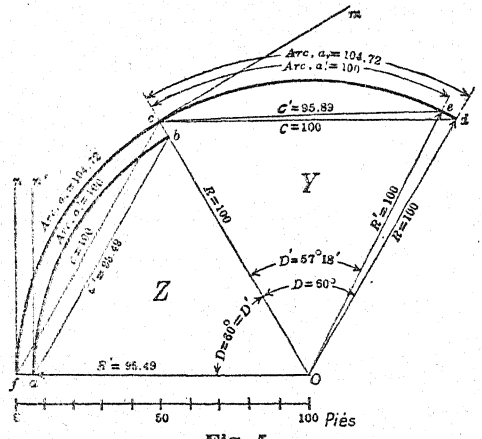



**Fig. 5.**

**20. Cadenas disminuidas** *. Si, en las curvas, el cadenero de atrás aguanta la lienza en la estaca con un pequeño margen de medida bien calculado, en lugar de ponerlo en la marca cero, disminuyendo así el largo de la cadena tomada como unidad el *arco* subtendido queda por consiguiente reducido y esta longitud así corregida corresponde entonces con su largo real, y ciertas fórmulas (¶ 42) para encontrar funciones de curvas (que de otro modo son solamente aproximadas) se hacen exactas.

**21. Agudeza. Esta se afecta por el uso de las cadenas disminuidas** *. Fig. 5 *Y*. En la curva, $c\ e\ d$, supongamos que la cadena completa, $c = c\ d = 100$ pies, subtende un ángulo central, $D = c\ O\ d = 60°$. Entonces $c\ e\ d$ es una curva de 60°, y su ángulo de deflexion, $D/2 = m\ c\ d = 30°$. Para recorrer la **misma**

* Vease la nota igual * en el ¶ 7 atrás. Y la *N. del T.* intercalada en el ¶ 11, pag. 926

**curva** con una cadena *disminuida*, $c' = c\,e$ ( = 95.89 pies, ¶ 22) debemos usar un ángulo de deflexion $m\,c\,e$, de solo 28° 39′, y $D'$ ( $= c\,O\,e$) = 57° 18′. Así, el uso de cadenas disminuidas produce una curva *nominalmente mas süave.*

**22.** Fig. 5 *Z*. Si, con cadenas disminuidas, $c' = a\,b$, usamos **el mismo ángulo de deflexion** ($n'\,a\,b = n\,f\,c$), como con la cadena completa ($c = f\,c$), debemos obtener una curva mas aguda, $a\,b$ (radio = $O\,a$) que con la cadena completa, quedaría a la curva $f\,c$ el (radio = $O\,f$). Así, el uso de cadenas disminuidas hace *realmente mas aguda*, la curva.

En las Figs 5 *Y* y 5 *Z*, las funciones se comparan como sigue :

($Q = a/c$ o $a'/c'$).

| Fig | $R$, piés | $R'$, piés | $D$ | $D'$ | $a$, piés | $a'$, piés | $c$ piés | $c'$ piés | $Q$ |
|---|---|---|---|---|---|---|---|---|---|
| 5 *Y*..... | 100.00 | ..... | 60°00′ | ..... | 104.72 | ..... | 100.00 | ..... | 1.047 |
| | ..... | 100.00 | ..... | 57°18′ | ..... | 100.00 | ..... | 95.89 | 1.043 |
| 5 *Z*..... | 100.00 | ..... | 60°00′ | ..... | 104.72 | ..... | 100.00 | ..... | 1.047 |
| | ..... | 95.49 | ..... | 60°00′ | ..... | 100.00 | ..... | 95.49 | 1.047 |

**23. Estaciones.** Como en las tangentes, los extremos de las cadenas en las curvas, se llaman « estaciones »; pero el nombre « estación » se aplica a menudo (vagamente) a *la distancia* entre dos de dichos puntos.

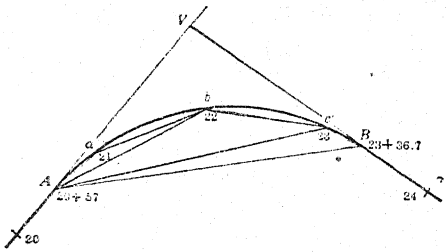



**Fig. 6.**

**24. Estacion « adicional »; sub-cadenas.** Fig. 6. Solo accidentalmente puede ocurrir que una curva empiece o termine exactamente en una estacion. En otras palabras, el T. C. y C. T. casi siempre cae *entre* dos estaciones por ej. : como en estacion $A = 20 + 57$, o en estacion $B = 23 + 36.7$; y esto requiere el uso de una sub-cadena, $A\,a$ o $c\,B$, y un sub-ángulo, $V\,A\,a$ o $V\,B\,c$ ($V\,B\,c = c\,A\,B$), en uno o en ambos extremos de la curva. Vease ¶ ¶ 44, etc.

## GEOMETRÍA

### Principios de frecuente aplicación.

**25.** Fig. 7. **Triángulos semejantes, y proporción entre cuerdas de segmentos.** Sea $O\ V$ un diámetro del círculo, $A\ V\ B\ O$; sea $A\ B\ (=C)$ una cuerda perpendicular a $O\ V$, dividida en $H$ en dos partes iguales; sea $O\ A$, $A\ V$ cuerdas en el semicírculo, $O\ A\ V$; y $O\ B$, $B\ V$, cuerdas en el semicírculo, $O\ B\ V$; sea $A\ V\ (=V\ B)$ = semitangente, $T$; y $O\ A\ (=O\ B)$ = radio, $R$, de la curva, $A\ P\ B$.

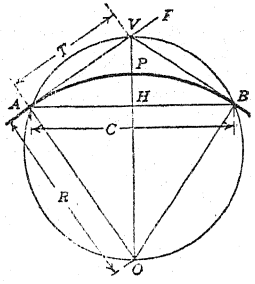



**Fig. 7.**

Entonces los 3 triángulos rectángulos, $O\ H\ A$, $A\ H\ V$ y $O\ A\ V$ son semejantes; como son tambien $O\ H\ B$, $B\ H\ V$ y $O\ B\ V$; ángulo, $O\ A\ V\ (=O\ B\ V)$, entre $R$ y $T$, $= 90°$; $A\ H = H\ B = C/2$; y

De los triángulos semejantes. Tenemos.

$O\ H\ B$, $B\ H\ V$; $O\ H : H\ B = H\ B : H\ V$;

$O\ H \times H\ V = (C/2)^2$; o bien $H\ V = (C/2)^2 \div O\ H$..... (1)

$O\ B\ V$, $B\ H\ V$; $O\ V : V\ B = V\ B : H\ V$;

$O\ V \times H\ V = T^2$; o bien $H\ V = T^2/O\ V$..... (2)

$O\ B\ V$, $O\ H\ B$; $O\ V : O\ B = O\ B : O\ H$;

$O\ V \times O\ H = R^2$; o bien $O\ H = R^2/O\ V$..... (3)

**26.** Fig. 8. **Ángulos entre dos cuerdas, entre dos tangentes, y entre una cuerda y una tangente. Valores de $\Delta/2$.**

Sea $A\ P\ B$ el arco, y $A\ B$ la cuerda que subtende el ángulo central, $A\ O\ B = \Delta = F\ V\ B = A\ O\ d + d\ O\ B = A\ O\ e + e\ O\ B$.

Entonces:

$\Delta/2$ = ángulo entre la tang, $A\ V$, en $A$ (o tang $V\ B$, en $B$) y la cuerda $A\ B$.

= ángulo externo entre dos cuerdas cualquiera, como $A\ d$ y $d\ B$, o $A\ e$ y $e\ B$, trazadas de $A$ y de $B$ respectivamente.

$= (A\ O\ d + d\ O\ B)/2 = (A\ O\ e + e\ O\ B)/2$.

= el ángulo periférico, $A\ E\ B = V\ A\ B = V\ B\ A$, subtendido por la cuerda, $A\ B$..... (4)

**27. Los ángulos pequeños** (tales como las deflexiones usuales en trabajos de ferrocarriles de vapor), son **aproximadamente proporcionales a sus senos**, y un poco menos, proporcionales a sus tangentes. Esto puede aprovecharse para facilitar muchos cálculos.

Asi, para cualquier ángulo pequeño, $A$, tenemos,
aprox : sen 0° 1′ : sen $A$ = 1 minuto : $A$, en minutos; o

$$A \text{ minutos} = \text{sen } A \div \text{sen } 0^\circ\, 1' = 3437 \text{ sen } A\,;$$
$$\text{sen } A = A \text{ minuto} \times \text{sen } 0^\circ\, 1' = 0.0002909\ A \text{ minutos} \ldots\ldots\ldots\ldots \quad (5)$$

De modo análogo (aprox) tan $A$ = $A$ minutos × tang 0° 1′ = 0.0002909 $A$ minutos.

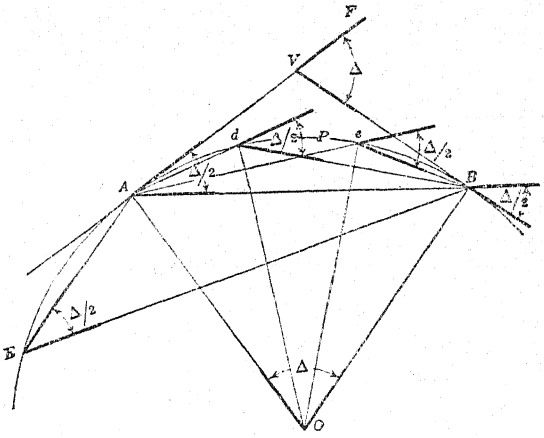



**Fig. 8.**

**Ejemplo.** Con cadena de 100 piés; donde el radio, $R$, = 1300 pies; averiguar la agudeza $D$ de la curva. Aqui, sen $(D/2)$ = 50/1300; y $D$ = 4° 24′ 30.44″.

Aprox, $D/2$ en minutos

$$= \text{sen } (D/2) \div \text{sen } 0^\circ\, 1'.$$

$$= \frac{50}{1300} \times \frac{1}{\text{sen } 0^\circ\, 1'} = 132.221.$$

Como, $D$ aprox = 264.442 minutos = $D$ − 0.065 minutos = 0.99975 $D$.

Reciprocamente; dando $D$ = 4° 24′, averiguar el radio, $R$. Aqui

$$R = \frac{50}{\text{sen } (D/2)} = 1302.497 \text{ piés}$$

$$R \text{ aprox} = \frac{50}{(D/2) \text{ en minutos} \times \text{sen } 0^\circ\, 1'}$$

$$= 1302.177 \text{ piés} = 0.99975\ R$$

Estas operaciones son convenientemente ejecutadas con la **regla de calcular**, pag. 78, etc.

**Errores de Aproximación.**

Sea $A$ en minutos; y sea

$$S = \frac{A \text{ sen } 0^\circ\,1'}{\text{sen } A}; \qquad T = \frac{A \tan 0^\circ\,1'.}{\tan A} \text{ Entonces : —}$$

| para $A$ = | $S$ = | $T$ = | para $A$ = | $S$ = | $T$ = |
|---|---|---|---|---|---|
| 0° 30′ | 1.000013 | 0.999995 | 6° 0′ | 1.001830 | 0.996342 |
| 1° 0′ | 1.000051 | 0.999898 | 7° 0′ | 1.002492 | 0.995020 |
| 2° 0′ | 1.000203 | 0.999594 | 8° 0′ | 1.003257 | 0.993493 |
| 3° 0′ | 1.000457 | 0.999086 | 10° 0′ | 1.005095 | 0.989825 |
| 4° 0′ | 1.000813 | 0.998375 | 20° 0′ | 1.020600 | 0.959050 |
| 5° 0′ | 1.001270 | 0.997460 | | | |

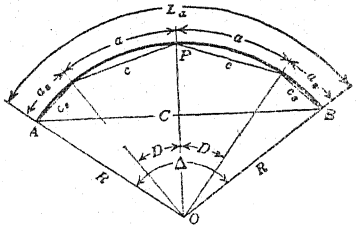



Fig. 9.

**28. Relaciones entre Δ, D, n, R, $L_a$ y $L_c$.** Fig. 9.

En cualquier curva circular, sea

$R$ = radio; $\pi = \frac{\text{circunf}}{\text{diam}}$; Δ = desarrollo; $C$ = cuerda larga;

$L_a$ = curva larga $A\ P\ B$, medida en el arco. Entonces : —

$\pi R$ = semi-circunf (subtendiendo 180°);

$\frac{\pi R}{180}$ = arco de 1°; y (Δ en *grados*)

$$L_a = \Delta \frac{\pi R}{180}; \Delta = \frac{180\ L_a}{\pi R}; \text{ y } R = \frac{180\ L_a}{\pi\ \Delta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (6)$$

Sea $c$ = largo cadena, en piés; $a$ = arco unidad, en piés; $n$ = número de cadenas, $c$, o de arcos de unidad, $a$, en la curva*; $D$ = agudeza; $L_c$ = largo de curva, $A\ P\ B$, medido en cadenas, = suma de largos de cadenas, $L_a$ = largo verdadero de la curva. Entonces : —

* Usualmente $n$ es un número mixto. Así con dos cadenas de 100 piés cada una y subcadenas de 33,6 piés 21,8 piés respectivamente, tenemos $n = 2 + 0{,}336 + 0{,}218 = 2{,}554$.

| Con cadena *completa*, ¶ 18 | Con cadena *disminuida*, ¶ 20 |
|---|---|
| $\text{sen}\,\frac{D}{2} = \frac{c/2}{R}$ ..... (7) | $D^\circ = \frac{180\,a}{\pi\,R}$ ..... (11) |
| $\Delta = n\,D$ ..... (8) | $\Delta = n\,D$..... (12) |
| $R = \frac{c/2}{\text{sen}\,(D/2)}$ | $R = \frac{180\,L_l}{\pi\,\Delta^\circ} = \frac{180\,c}{\pi\,D^\circ}$ ..... (13) |
| $= \frac{c}{2\,\text{sen}\,(D/2)}$ ..... (9) | |
| $L_c = n\,c$; | $L_a = n\,a$; |
| $n = L_c/c = \Delta/D$..... (10) | $n = L_a/a = \Delta/D$..... (14) |

**29. Por lo tanto, para el radio,** $R_f$, en piés, o $R_m$, en metros, tenemos: —

| | En cualquier curva | | En una curva de 1° | |
|---|---|---|---|---|
| | **Curva medida por** | | | |
| ¶ 18 | cadena de 100 piés | cadena de 20 metros | cadena de 100 piés | cadena de 20 metros |
| | **Radios** | | | |
| | $\mathbf{R}_f =$ | $\mathbf{R}_m =$ | $\mathbf{R}_f =$ | $\mathbf{R}_m =$ |
| Con cadena completa... | $\frac{50\text{ piés}}{\text{sen}\,(D/2)}$ | $\frac{10\text{ m}}{\text{sen}\,(D/2)}$ | $\frac{50\text{ piés}}{\text{sen } 0^\circ 30'}$ = 5729.650 piés log = 3.758 1281 | $\frac{10\text{ m}}{\text{sen } 0^\circ 30'}$ = 1145.930 m log = 3.059 1581 |
| | **Curva medida por** | | | |
| ¶ 20 | arco de 100 piés | arco de 20 metros | arco de 100 piés | arco de 20 metros |
| | **Radios** | | | |
| | $\mathbf{R}_f =$ | $\mathbf{R}_m =$ | $\mathbf{R}_f =$ | $\mathbf{R}_m =$ |
| Con cadena disminuida. | $\frac{18.000\text{ piés}}{\pi\,D}$ | $\frac{3.600\text{ m}}{\pi\,D}$ | $\frac{18.000\text{ piés}}{\pi}$ = 5729.578 piés log = 3.758 1226 | $\frac{3.600\text{ m}}{\pi}$ = 1145.916 m log = 3.059 1526 |
| | Ecuaciones (15) | | Ecuaciones (16) | |

| D | R | log R | D | R | log R | D | R | log R |
|---|---|---|---|---|---|---|---|---|
| 0° | | | 1° | | | 2° | | |
| 0′ | Infinito | Infinito | 0′ | 5729.65 | 3.758128 | 0′ | 2864.93 | 3.457115 |
| 1 | 343775. | 5.536274 | 1 | 5635.72 | 3.750950 | 1 | 2841.26 | 3.453511 |
| 2 | 171887. | 5.235244 | 2 | 5544.83 | 3.743888 | 2 | 2817.97 | 3.449937 |
| 3 | 114592. | 5.059153 | 3 | 5456.82 | 3.736939 | 3 | 2795.06 | 3.446392 |
| 4 | 85943.7 | 4.934214 | 4 | 5371.56 | 3.730100 | 4 | 2772.53 | 3.442876 |
| 5 | 68754.9 | 4.837304 | 5 | 5288.92 | 3.723367 | 5 | 2750.35 | 3.439388 |
| 6 | 57295.8 | 4.758123 | 6 | 5208.79 | 3.716737 | 6 | 2728.52 | 3.435928 |
| 7 | 49110.7 | 4.691176 | 7 | 5131.05 | 3.710206 | 7 | 2707.04 | 3.432495 |
| 8 | 42971.8 | 4.633184 | 8 | 5055.59 | 3.703772 | 8 | 2685.89 | 3.429089 |
| 9 | 38197.2 | 4.582031 | 9 | 4982.33 | 3.697432 | 9 | 2665.08 | 3.425710 |
| 10 | 34377.5 | 4.536274 | 10 | 4911.15 | 3.691183 | 10 | 2644.58 | 3.422356 |
| 11 | 31252.3 | 4.494881 | 11 | 4841.98 | 3.685023 | 11 | 2624.39 | 3.419029 |
| 12 | 28647.8 | 4.457093 | 12 | 4774.74 | 3.678949 | 12 | 2604.51 | 3.415727 |
| 13 | 26444.2 | 4.422331 | 13 | 4709.33 | 3.672959 | 13 | 2584.93 | 3.412449 |
| 14 | 24555.4 | 4.390146 | 14 | 4645.69 | 3.667051 | 14 | 2565.65 | 3.409197 |
| 15 | 22918.3 | 4.360183 | 15 | 4583.75 | 3.661221 | 15 | 2546.64 | 3.405968 |
| 16 | 21485.9 | 4.332154 | 16 | 4523.44 | 3.655469 | 16 | 2527.92 | 3.402763 |
| 17 | 20222.1 | 4.305825 | 17 | 4464.70 | 3.649792 | 17 | 2509.47 | 3.399582 |
| 18 | 19098.6 | 4.281002 | 18 | 4407.46 | 3.644189 | 18 | 2491.29 | 3.396424 |
| 19 | 18093.4 | 4.257521 | 19 | 4351.67 | 3.638656 | 19 | 2473.37 | 3.393289 |
| 20 | 17188.8 | 4.235244 | 20 | 4297.28 | 3.633194 | 20 | 2455.70 | 3.390176 |
| 21 | 16370.2 | 4.214055 | 21 | 4244.23 | 3.627799 | 21 | 2438.29 | 3.387085 |
| 22 | 15626.1 | 4.193852 | 22 | 4192.47 | 3.622470 | 22 | 2421.12 | 3.384016 |
| 23 | 14946.7 | 4.174547 | 23 | 4141.96 | 3.617206 | 23 | 2404.19 | 3.380969 |
| 24 | 14324.0 | 4.156064 | 24 | 4092.66 | 3.612005 | 24 | 2387.50 | 3.377943 |
| 25 | 13751.0 | 4.138335 | 25 | 4044.51 | 3.606866 | 25 | 2371.04 | 3.374938 |
| 26 | 13222.1 | 4.121302 | 26 | 3997.49 | 3.601787 | 26 | 2354.80 | 3.371954 |
| 27 | 12732.4 | 4.104911 | 27 | 3951.54 | 3.596766 | 27 | 2338.78 | 3.368990 |
| 28 | 12277.7 | 4.089117 | 28 | 3906.64 | 3.591803 | 28 | 2322.98 | 3.366046 |
| 29 | 11854.3 | 4.073877 | 29 | 3862.74 | 3.586896 | 29 | 2307.39 | 3.363122 |
| 30 | 11459.2 | 4.059154 | 30 | 3819.83 | 3.582044 | 30 | 2292.01 | 3.360217 |
| 31 | 11089.6 | 4.044914 | 31 | 3777.85 | 3.577245 | 31 | 2276.84 | 3.357332 |
| 32 | 10743.0 | 4.031125 | 32 | 3736.79 | 3.572499 | 32 | 2261.86 | 3.354466 |
| 33 | 10417.5 | 4.017762 | 33 | 3696.61 | 3.567804 | 33 | 2247.08 | 3.351618 |
| 34 | 10111.1 | 4.004797 | 34 | 3657.29 | 3.563160 | 34 | 2232.49 | 3.348789 |
| 35 | 9822.18 | 3.992208 | 35 | 3618.80 | 3.558564 | 35 | 2218.09 | 3.345979 |
| 36 | 9549.34 | 3.979973 | 36 | 3581.10 | 3.554017 | 36 | 2203.87 | 3.343187 |
| 37 | 9291.25 | 3.968074 | 37 | 3544.19 | 3.549517 | 37 | 2189.84 | 3.340412 |
| 38 | 9046.75 | 3.956493 | 38 | 3508.02 | 3.545063 | 38 | 2175.98 | 3.337655 |
| 39 | 8814.78 | 3.945212 | 39 | 3472.59 | 3.540654 | 39 | 2162.30 | 3.334916 |
| 40 | 8594.42 | 3.934216 | 40 | 3437.87 | 3.536289 | 40 | 2148.79 | 3.332193 |
| 41 | 8384.80 | 3.923493 | 41 | 3403.83 | 3.531968 | 41 | 2135.44 | 3.329488 |
| 42 | 8185.16 | 3.913027 | 42 | 3370.46 | 3.527690 | 42 | 2122.26 | 3.326799 |
| 43 | 7994.81 | 3.902808 | 43 | 3337.74 | 3.523453 | 43 | 2109.24 | 3.324127 |
| 44 | 7813.11 | 3.892824 | 44 | 3305.65 | 3.519257 | 44 | 2096.39 | 3.321471 |
| 45 | 7639.49 | 3.883065 | 45 | 3274.17 | 3.515101 | 45 | 2083.68 | 3.318832 |
| 46 | 7473.42 | 3.873519 | 46 | 3243.29 | 3.510985 | 46 | 2071.13 | 3.316208 |
| 47 | 7314.41 | 3.864179 | 47 | 3212.98 | 3.506908 | 47 | 2058.73 | 3.313600 |
| 48 | 7162.03 | 3.855036 | 48 | 3183.23 | 3.502868 | 48 | 2046.48 | 3.311008 |
| 49 | 7015.87 | 3.846082 | 49 | 3154.03 | 3.498866 | 49 | 2034.37 | 3.308431 |
| 50 | 6875.55 | 3.837308 | 50 | 3125.36 | 3.494900 | 50 | 2022.41 | 3.305869 |
| 51 | 6740.74 | 3.828708 | 51 | 3097.20 | 3.490970 | 51 | 2010.59 | 3.303323 |
| 52 | 6611.12 | 3.820275 | 52 | 3069.55 | 3.487075 | 52 | 1998.90 | 3.300791 |
| 53 | 6486.38 | 3.812002 | 53 | 3042.39 | 3.483215 | 53 | 1987.35 | 3.298274 |
| 54 | 6366.26 | 3.803885 | 54 | 3015.71 | 3.479389 | 54 | 1975.93 | 3.295771 |
| 55 | 6250.51 | 3.795916 | 55 | 2989.48 | 3.475596 | 55 | 1964.64 | 3.293283 |
| 56 | 6138.90 | 3.788091 | 56 | 2963.72 | 3.471836 | 56 | 1953.48 | 3.290809 |
| 57 | 6031.20 | 3.780404 | 57 | 2938.39 | 3.468109 | 57 | 1942.44 | 3.288349 |
| 58 | 5927.22 | 3.772851 | 58 | 2913.49 | 3.464413 | 58 | 1931.53 | 3.285902 |
| 59 | 5826.76 | 3.765427 | 59 | 2889.01 | 3.460749 | 59 | 1920.75 | 3.283470 |
| 60 | 5729.65 | 3.758128 | 60 | 2864.93 | 3.457115 | 60 | 1910.08 | 3.281051 |

| D | R | log R | D | R | log R | D | R | log R |
|---|---|---|---|---|---|---|---|---|
| 3° | | | 4° | | | 5° | | |
| 0′ | 1910.08 | 3.281051 | 0′ | 1432.69 | 3.156151 | 0′ | 1146.28 | 3.059290 |
| 1 | 1899.53 | 3.278646 | 1 | 1426.74 | 3.154346 | 1 | 1142.47 | 3.057846 |
| 2 | 1889.09 | 3.276253 | 2 | 1420.85 | 3.152548 | 2 | 1138.69 | 3.056407 |
| 3 | 1878.77 | 3.273874 | 3 | 1415.01 | 3.150758 | 3 | 1134.94 | 3.054972 |
| 4 | 1868.56 | 3.271508 | 4 | 1409.21 | 3.148975 | 4 | 1131.21 | 3.053542 |
| 5 | 1858.47 | 3.269155 | 5 | 1403.46 | 3.147200 | 5 | 1127.50 | 3.052116 |
| 6 | 1848.48 | 3.266814 | 6 | 1397.76 | 3.145431 | 6 | 1123.82 | 3.050696 |
| 7 | 1838.59 | 3.264486 | 7 | 1392.10 | 3.143670 | 7 | 1120.16 | 3.049280 |
| 8 | 1828.82 | 3.262170 | 8 | 1386.49 | 3.141916 | 8 | 1116.52 | 3.047868 |
| 9 | 1819.14 | 3.259867 | 9 | 1380.92 | 3.140170 | 9 | 1112.91 | 3.046462 |
| 10 | 1809.57 | 3.257576 | 10 | 1375.40 | 3.138430 | 10 | 1109.33 | 3.045059 |
| 11 | 1800.10 | 3.255296 | 11 | 1369.92 | 3.136697 | 11 | 1105.76 | 3.043662 |
| 12 | 1790.73 | 3.253029 | 12 | 1364.49 | 3.134971 | 12 | 1102.22 | 3.042268 |
| 13 | 1781.45 | 3.250774 | 13 | 1359.10 | 3.133251 | 13 | 1098.70 | 3.040880 |
| 14 | 1772.27 | 3.248530 | 14 | 1353.75 | 3.131539 | 14 | 1095.20 | 3.039495 |
| 15 | 1763.18 | 3.246297 | 15 | 1348.45 | 3.129833 | 15 | 1091.73 | 3.038115 |
| 16 | 1754.19 | 3.244077 | 16 | 1343.18 | 3.128134 | 16 | 1088.28 | 3.036740 |
| 17 | 1745.29 | 3.241867 | 17 | 1337.96 | 3.126442 | 17 | 1084.85 | 3.035368 |
| 18 | 1736.48 | 3.239669 | 18 | 1332.77 | 3.124756 | 18 | 1081.44 | 3.034002 |
| 19 | 1727.75 | 3.237481 | 19 | 1327.63 | 3.123077 | 19 | 1078.05 | 3.032639 |
| 20 | 1719.12 | 3.235305 | 20 | 1322.53 | 3.121404 | 20 | 1074.68 | 3.031281 |
| 21 | 1710.57 | 3.233140 | 21 | 1317.46 | 3.119738 | 21 | 1071.34 | 3.029927 |
| 22 | 1702.10 | 3.230985 | 22 | 1312.43 | 3.118078 | 22 | 1068.01 | 3.028577 |
| 23 | 1693.72 | 3.228841 | 23 | 1307.45 | 3.116424 | 23 | 1064.71 | 3.027231 |
| 24 | 1685.42 | 3.226707 | 24 | 1302.50 | 3.114777 | 24 | 1061.43 | 3.025890 |
| 25 | 1677.20 | 3.224584 | 25 | 1297.58 | 3.113136 | 25 | 1058.16 | 3.024552 |
| 26 | 1669.06 | 3.222472 | 26 | 1292.71 | 3.111501 | 26 | 1054.92 | 3.023219 |
| 27 | 1661.00 | 3.220369 | 27 | 1287.87 | 3.109872 | 27 | 1051.70 | 3.021890 |
| 28 | 1653.01 | 3.218277 | 28 | 1283.07 | 3.108249 | 28 | 1048.49 | 3.020565 |
| 29 | 1645.11 | 3.216195 | 29 | 1278.30 | 3.106632 | 29 | 1045.31 | 3.019244 |
| 30 | 1637.28 | 3.214122 | 30 | 1273.57 | 3.105022 | 30 | 1042.14 | 3.017927 |
| 31 | 1629.52 | 3.212060 | 31 | 1268.87 | 3.103417 | 31 | 1039.00 | 3.016614 |
| 32 | 1621.84 | 3.210007 | 32 | 1264.21 | 3.101818 | 32 | 1035.87 | 3.015305 |
| 33 | 1614.22 | 3.207964 | 33 | 1259.58 | 3.100225 | 33 | 1032.76 | 3.013999 |
| 34 | 1606.68 | 3.205930 | 34 | 1254.98 | 3.098638 | 34 | 1029.67 | 3.012698 |
| 35 | 1599.21 | 3.203906 | 35 | 1250.42 | 3.097057 | 35 | 1026.60 | 3.011401 |
| 36 | 1591.81 | 3.201892 | 36 | 1245.89 | 3.095481 | 36 | 1023.55 | 3.010107 |
| 37 | 1584.48 | 3.199886 | 37 | 1241.40 | 3.093912 | 37 | 1020.51 | 3.008818 |
| 38 | 1577.21 | 3.197890 | 38 | 1236.94 | 3.092347 | 38 | 1017.49 | 3.007532 |
| 39 | 1570.01 | 3.195903 | 39 | 1232.51 | 3.090789 | 39 | 1014.50 | 3.006250 |
| 40 | 1562.88 | 3.193925 | 40 | 1228.11 | 3.089236 | 40 | 1011.51 | 3.004972 |
| 41 | 1555.81 | 3.191956 | 41 | 1223.74 | 3.087689 | 41 | 1008.55 | 3.003698 |
| 42 | 1548.80 | 3.189996 | 42 | 1219.40 | 3.086147 | 42 | 1005.60 | 3.002427 |
| 43 | 1541.86 | 3.188045 | 43 | 1215.09 | 3.084610 | 43 | 1002.67 | 3.001160 |
| 44 | 1534.98 | 3.186103 | 44 | 1210.82 | 3.083079 | 44 | 999.762 | 2.999897 |
| 45 | 1528.16 | 3.184169 | 45 | 1206.57 | 3.081553 | 45 | 996.867 | 2.998637 |
| 46 | 1521.40 | 3.182244 | 46 | 1202.36 | 3.080033 | 46 | 993.988 | 2.997381 |
| 47 | 1514.70 | 3.180327 | 47 | 1198.17 | 3.078518 | 47 | 991.126 | 2.996129 |
| 48 | 1508.06 | 3.178419 | 48 | 1194.01 | 3.077008 | 48 | 988.280 | 2.994880 |
| 49 | 1501.48 | 3.176519 | 49 | 1189.88 | 3.075504 | 49 | 985.451 | 2.993635 |
| 50 | 1494.95 | 3.174627 | 50 | 1185.78 | 3.074005 | 50 | 982.638 | 2.992393 |
| 51 | 1488.48 | 3.172744 | 51 | 1181.71 | 3.072511 | 51 | 979.840 | 2.991155 |
| 52 | 1482.07 | 3.170868 | 52 | 1177.66 | 3.071022 | 52 | 977.060 | 2.989921 |
| 53 | 1475.71 | 3.169001 | 53 | 1173.65 | 3.069538 | 53 | 974.294 | 2.988690 |
| 54 | 1469.41 | 3.167142 | 54 | 1169.66 | 3.068059 | 54 | 971.544 | 2.987463 |
| 55 | 1463.16 | 3.165291 | 55 | 1165.70 | 3.066585 | 55 | 968.810 | 2.986238 |
| 56 | 1456.96 | 3.163447 | 56 | 1161.76 | 3.065116 | 56 | 966.091 | 2.985018 |
| 57 | 1450.81 | 3.161612 | 57 | 1157.85 | 3.063653 | 57 | 963.387 | 2.983801 |
| 58 | 1444.72 | 3.159784 | 58 | 1153.97 | 3.062194 | 58 | 960.698 | 2.982587 |
| 59 | 1438.68 | 3.157963 | 59 | 1150.11 | 3.060740 | 59 | 958.025 | 2.981377 |
| 60 | 1432.69 | 3.156151 | 60 | 1146.28 | 3.059290 | 60 | 955.366 | 2.980170 |

| D | R | log R |
|---|---|---|
| 6° | | |
| 0′ | 955.366 | 2.980170 |
| 1 | 952.722 | 2.978966 |
| 2 | 950.093 | 2.977766 |
| 3 | 947.478 | 2.976569 |
| 4 | 944.877 | 2.975375 |
| 5 | 942.291 | 2.974185 |
| 6 | 939.719 | 2.972998 |
| 7 | 937.161 | 2.971814 |
| 8 | 934.616 | 2.970633 |
| 9 | 932.086 | 2.969456 |
| 10 | 929.569 | 2.968282 |
| 11 | 927.066 | 2.967111 |
| 12 | 924.576 | 2.965943 |
| 13 | 922.100 | 2.964778 |
| 14 | 919.637 | 2.963616 |
| 15 | 917.187 | 2.962458 |
| 16 | 914.750 | 2.961303 |
| 17 | 912.326 | 2.960150 |
| 18 | 909.915 | 2.959001 |
| 19 | 907.517 | 2.957855 |
| 20 | 905.131 | 2.956711 |
| 21 | 902.758 | 2.955571 |
| 22 | 900.397 | 2.954434 |
| 23 | 898.048 | 2.953300 |
| 24 | 895.712 | 2.952168 |
| 25 | 893.388 | 2.951040 |
| 26 | 891.076 | 2.949915 |
| 27 | 888.776 | 2.948792 |
| 28 | 886.488 | 2.947673 |
| 29 | 884.211 | 2.946556 |
| 30 | 881.946 | 2.945442 |
| 31 | 879.693 | 2.944331 |
| 32 | 877.451 | 2.943223 |
| 33 | 875.221 | 2.942118 |
| 34 | 873.002 | 2.941015 |
| 35 | 870.795 | 2.939916 |
| 36 | 868.598 | 2.938819 |
| 37 | 866.412 | 2.937725 |
| 38 | 864.238 | 2.936633 |
| 39 | 862.075 | 2.935545 |
| 40 | 859.922 | 2.934459 |
| 41 | 857.780 | 2.933376 |
| 42 | 855.648 | 2.932295 |
| 43 | 853.527 | 2.931218 |
| 44 | 851.417 | 2.930142 |
| 45 | 849.317 | 2.929070 |
| 46 | 847.228 | 2.928000 |
| 47 | 845.148 | 2.926933 |
| 48 | 843.080 | 2.925869 |
| 49 | 841.021 | 2.924807 |
| 50 | 838.972 | 2.923747 |
| 51 | 836.933 | 2.922691 |
| 52 | 834.904 | 2.921637 |
| 53 | 832.885 | 2.920585 |
| 54 | 830.876 | 2.919536 |
| 55 | 828.876 | 2.918489 |
| 56 | 826.886 | 2.917446 |
| 57 | 824.905 | 2.916404 |
| 58 | 822.934 | 2.915365 |
| 59 | 820.973 | 2.914329 |
| 60 | 819.020 | 2.913295 |

| D | R | log R |
|---|---|---|
| 7° | | |
| 0′ | 819.020 | 2.913295 |
| 1 | 817.077 | 2.912263 |
| 2 | 815.144 | 2.911234 |
| 3 | 813.219 | 2.910208 |
| 4 | 811.303 | 2.909183 |
| 5 | 809.397 | 2.908162 |
| 6 | 807.499 | 2.907142 |
| 7 | 805.611 | 2.906125 |
| 8 | 803.731 | 2.905111 |
| 9 | 801.860 | 2.904098 |
| 10 | 799.997 | 2.903089 |
| 11 | 798.144 | 2.902081 |
| 12 | 796.299 | 2.901076 |
| 13 | 794.462 | 2.900073 |
| 14 | 792.634 | 2.899073 |
| 15 | 790.814 | 2.898074 |
| 16 | 789.003 | 2.897078 |
| 17 | 787.200 | 2.896085 |
| 18 | 785.405 | 2.895094 |
| 19 | 783.618 | 2.894105 |
| 20 | 781.840 | 2.893118 |
| 21 | 780.069 | 2.892133 |
| 22 | 778.307 | 2.891151 |
| 23 | 776.552 | 2.890171 |
| 24 | 774.806 | 2.889193 |
| 25 | 773.067 | 2.888217 |
| 26 | 771.336 | 2.887244 |
| 27 | 769.613 | 2.886272 |
| 28 | 767.897 | 2.885303 |
| 29 | 766.190 | 2.884336 |
| 30 | 764.489 | 2.883371 |
| 31 | 762.797 | 2.882409 |
| 32 | 761.112 | 2.881448 |
| 33 | 759.434 | 2.880490 |
| 34 | 757.764 | 2.879534 |
| 35 | 756.101 | 2.878580 |
| 36 | 754.445 | 2.877627 |
| 37 | 752.796 | 2.876678 |
| 38 | 751.155 | 2.875730 |
| 39 | 749.521 | 2.874784 |
| 40 | 747.894 | 2.873840 |
| 41 | 746.274 | 2.872898 |
| 42 | 744.661 | 2.871959 |
| 43 | 743.055 | 2.871021 |
| 44 | 741.456 | 2.870086 |
| 45 | 739.864 | 2.869152 |
| 46 | 738.279 | 2.868221 |
| 47 | 736.701 | 2.867291 |
| 48 | 735.129 | 2.866363 |
| 49 | 733.564 | 2.865438 |
| 50 | 732.005 | 2.864514 |
| 51 | 730.454 | 2.863593 |
| 52 | 728.909 | 2.862673 |
| 53 | 727.370 | 2.861755 |
| 54 | 725.838 | 2.860840 |
| 55 | 724.312 | 2.859926 |
| 56 | 722.793 | 2.859014 |
| 57 | 721.280 | 2.858104 |
| 58 | 719.774 | 2.857196 |
| 59 | 718.273 | 2.856290 |
| 60 | 716.779 | 2.855385 |

| D | R | log R |
|---|---|---|
| 8° | | |
| 0′ | 716.779 | 2.855385 |
| 1 | 715.291 | 2.854483 |
| 2 | 713.810 | 2.853583 |
| 3 | 712.335 | 2.852684 |
| 4 | 710.865 | 2.851787 |
| 5 | 709.402 | 2.850892 |
| 6 | 707.945 | 2.849999 |
| 7 | 706.493 | 2.849108 |
| 8 | 705.048 | 2.848219 |
| 9 | 703.609 | 2.847331 |
| 10 | 702.175 | 2.846445 |
| 11 | 700.748 | 2.845562 |
| 12 | 699.326 | 2.844679 |
| 13 | 697.910 | 2.843799 |
| 14 | 696.499 | 2.842921 |
| 15 | 695.095 | 2.842044 |
| 16 | 693.696 | 2.841169 |
| 17 | 692.302 | 2.840296 |
| 18 | 690.914 | 2.839424 |
| 19 | 689.532 | 2.838555 |
| 20 | 688.156 | 2.837687 |
| 21 | 686.785 | 2.836821 |
| 22 | 685.419 | 2.835956 |
| 23 | 684.059 | 2.835093 |
| 24 | 682.704 | 2.834232 |
| 25 | 681.354 | 2.833373 |
| 26 | 680.010 | 2.832515 |
| 27 | 678.671 | 2.831660 |
| 28 | 677.338 | 2.830805 |
| 29 | 676.008 | 2.829953 |
| 30 | 674.686 | 2.829102 |
| 31 | 673.369 | 2.828253 |
| 32 | 672.056 | 2.827405 |
| 33 | 670.748 | 2.826560 |
| 34 | 669.446 | 2.825715 |
| 35 | 668.148 | 2.824873 |
| 36 | 666.856 | 2.824032 |
| 37 | 665.568 | 2.823193 |
| 38 | 664.286 | 2.822355 |
| 39 | 663.008 | 2.821519 |
| 40 | 661.736 | 2.820685 |
| 41 | 660.468 | 2.819852 |
| 42 | 659.205 | 2.819021 |
| 43 | 657.947 | 2.818191 |
| 44 | 656.694 | 2.817363 |
| 45 | 655.446 | 2.816537 |
| 46 | 654.202 | 2.815712 |
| 47 | 652.963 | 2.814889 |
| 48 | 651.729 | 2.814067 |
| 49 | 650.499 | 2.813247 |
| 50 | 649.274 | 2.812428 |
| 51 | 648.054 | 2.811611 |
| 52 | 646.838 | 2.810796 |
| 53 | 645.627 | 2.809982 |
| 54 | 644.420 | 2.809169 |
| 55 | 643.218 | 2.808358 |
| 56 | 642.021 | 2.807549 |
| 57 | 640.828 | 2.806741 |
| 58 | 639.639 | 2.805935 |
| 59 | 638.455 | 2.805130 |
| 60 | 637.275 | 2.804327 |

| D | R | log R | D | R | log R | D | R | log R |
|---|---|---|---|---|---|---|---|---|
| 9° | | | 10° | | | 12° | | |
| 0′ | 637.275 | 2.804327 | 0′ | 573.686 | 2.758674 | 0′ | 478.339 | 2.679735 |
| 1 | 636.099 | 2.803525 | 2 | 571.784 | 2.757232 | 2 | 477.018 | 2.678535 |
| 2 | 634.928 | 2.802724 | 4 | 569.896 | 2.755796 | 4 | 475.705 | 2.677338 |
| 3 | 633.761 | 2.801926 | 6 | 568.020 | 2.754364 | 6 | 474.400 | 2.676145 |
| 4 | 632.599 | 2.801128 | 8 | 566.156 | 2.752937 | 8 | 473.102 | 2.674954 |
| 5 | 631.440 | 2.800332 | 10 | 564.305 | 2.751514 | 10 | 471.810 | 2.673767 |
| 6 | 630.286 | 2.799538 | 12 | 562.466 | 2.750096 | 12 | 470.526 | 2.672584 |
| 7 | 629.136 | 2.798745 | 14 | 560.638 | 2.748683 | 14 | 469.249 | 2.671403 |
| 8 | 627.991 | 2.797953 | 16 | 558.823 | 2.747274 | 16 | 467.978 | 2.670226 |
| 9 | 626.849 | 2.797163 | 18 | 557.019 | 2.745870 | 18 | 466.715 | 2.669052 |
| 10 | 625.712 | 2.796374 | 20 | 555.227 | 2.744471 | 20 | 465.459 | 2.667881 |
| 11 | 624.579 | 2.795587 | 22 | 553.447 | 2.743076 | 22 | 464.209 | 2.666713 |
| 12 | 623.450 | 2.794801 | 24 | 551.678 | 2.741686 | 24 | 462.966 | 2.665549 |
| 13 | 622.325 | 2.794017 | 26 | 549.920 | 2.740300 | 26 | 461.729 | 2.664387 |
| 14 | 621.203 | 2.793234 | 28 | 548.174 | 2.738918 | 28 | 460.500 | 2.663229 |
| 15 | 620.087 | 2.792453 | 30 | 546.438 | 2.737541 | 30 | 459.276 | 2.662074 |
| 16 | 618.974 | 2.791673 | 32 | 544.714 | 2.736169 | 32 | 458.060 | 2.660922 |
| 17 | 617.865 | 2.790894 | 34 | 543.001 | 2.734800 | 34 | 456.850 | 2.659773 |
| 18 | 616.760 | 2.790117 | 36 | 541.298 | 2.733436 | 36 | 455.646 | 2.658628 |
| 19 | 615.660 | 2.789341 | 38 | 539.606 | 2.732077 | 38 | 454.449 | 2.657485 |
| 20 | 614.563 | 2.788566 | 40 | 537.924 | 2.730721 | 40 | 453.259 | 2.656345 |
| 21 | 613.470 | 2.787793 | 42 | 536.253 | 2.729370 | 42 | 452.073 | 2.655208 |
| 22 | 612.380 | 2.787021 | 44 | 534.593 | 2.728023 | 44 | 450.894 | 2.654075 |
| 23 | 611.295 | 2.786251 | 46 | 532.943 | 2.726681 | 46 | 449.722 | 2.652944 |
| 24 | 610.214 | 2.785482 | 48 | 531.303 | 2.725342 | 48 | 448.556 | 2.651816 |
| 25 | 609.136 | 2.784714 | 50 | 529.673 | 2.724008 | 50 | 447.395 | 2.650691 |
| 26 | 608.062 | 2.783948 | 52 | 528.053 | 2.722677 | 52 | 446.241 | 2.649570 |
| 27 | 606.992 | 2.783183 | 54 | 526.443 | 2.721351 | 54 | 445.093 | 2.648451 |
| 28 | 605.926 | 2.782420 | 56 | 524.843 | 2.720029 | 56 | 443.951 | 2.647335 |
| 29 | 604.864 | 2.781657 | 58 | 523.252 | 2.718711 | 58 | 442.814 | 2.646221 |
| | | | 11° | | | 13° | | |
| 30 | 603.805 | 2.780897 | 0′ | 521.671 | 2.717397 | 0′ | 441.684 | 2.645111 |
| 31 | 602.750 | 2.780137 | 2 | 520.100 | 2.716087 | 2 | 440.559 | 2.644004 |
| 32 | 601.698 | 2.779379 | 4 | 518.539 | 2.714781 | 4 | 439.440 | 2.642899 |
| 33 | 600.651 | 2.778622 | 6 | 516.986 | 2.713479 | 6 | 438.326 | 2.641798 |
| 34 | 599.607 | 2.777867 | 8 | 515.443 | 2.712181 | 8 | 437.219 | 2.640699 |
| 35 | 598.567 | 2.777112 | 10 | 513.909 | 2.710887 | 10 | 436.117 | 2.639603 |
| 36 | 597.530 | 2.776360 | 12 | 512.385 | 2.709596 | 12 | 435.020 | 2.638510 |
| 37 | 596.497 | 2.775608 | 14 | 510.869 | 2.708310 | 14 | 433.929 | 2.637419 |
| 38 | 595.467 | 2.774858 | 16 | 509.363 | 2.707027 | 16 | 432.844 | 2.636331 |
| 39 | 594.441 | 2.774109 | 18 | 507.865 | 2.705748 | 18 | 431.764 | 2.635246 |
| 40 | 593.419 | 2.773361 | 20 | 506.376 | 2.704473 | 20 | 430.690 | 2.634164 |
| 41 | 592.400 | 2.772615 | 22 | 504.896 | 2.703202 | 22 | 429.620 | 2.633085 |
| 42 | 591.384 | 2.771870 | 24 | 503.425 | 2.701934 | 24 | 428.557 | 2.632008 |
| 43 | 590.372 | 2.771126 | 26 | 501.962 | 2.700671 | 26 | 427.498 | 2.630934 |
| 44 | 589.364 | 2.770383 | 28 | 500.507 | 2.699410 | 28 | 426.445 | 2.629863 |
| 45 | 588.359 | 2.769642 | 30 | 499.061 | 2.698154 | 30 | 425.396 | 2.628794 |
| 46 | 587.357 | 2.768902 | 32 | 497.624 | 2.696901 | 32 | 424.354 | 2.627728 |
| 47 | 586.359 | 2.768164 | 34 | 496.195 | 2.695652 | 34 | 423.316 | 2.626665 |
| 48 | 585.364 | 2.767426 | 36 | 494.774 | 2.694407 | 36 | 422.283 | 2.625604 |
| 49 | 584.373 | 2.766690 | 38 | 493.361 | 2.693165 | 38 | 421.256 | 2.624546 |
| 50 | 583.385 | 2.765955 | 40 | 491.956 | 2.691926 | 40 | 420.233 | 2.623490 |
| 51 | 582.400 | 2.765221 | 42 | 490.559 | 2.690692 | 42 | 419.215 | 2.622437 |
| 52 | 581.419 | 2.764489 | 44 | 489.171 | 2.689460 | 44 | 418.203 | 2.621387 |
| 53 | 580.441 | 2.763758 | 46 | 487.790 | 2.688233 | 46 | 417.195 | 2.620339 |
| 54 | 579.466 | 2.763028 | 48 | 486.417 | 2.687008 | 48 | 416.192 | 2.619294 |
| 55 | 578.494 | 2.762299 | 50 | 485.051 | 2.685788 | 50 | 415.194 | 2.618251 |
| 56 | 577.526 | 2.761572 | 52 | 483.694 | 2.684570 | 52 | 414.201 | 2.617211 |
| 57 | 576.561 | 2.760845 | 54 | 482.344 | 2.683357 | 54 | 413.212 | 2.616173 |
| 58 | 575.599 | 2.760120 | 56 | 481.001 | 2.682146 | 56 | 412.229 | 2.615138 |
| 59 | 574.641 | 2.759397 | 58 | 479.666 | 2.680939 | 58 | 411.250 | 2.614106 |
| 60 | 573.686 | 2.758674 | 60 | 478.339 | 2.679735 | 60 | 410.275 | 2.613075 |

| D | R | log R |
|---|---|---|
| 14° | | |
| 0′ | 410.275 | 2.613075 |
| 2 | 409.306 | 2.612048 |
| 4 | 408.341 | 2.611023 |
| 6 | 407.380 | 2.610000 |
| 8 | 406.424 | 2.608980 |
| 10 | 405.473 | 2.607962 |
| 12 | 404.526 | 2.606946 |
| 14 | 403.583 | 2.605933 |
| 16 | 402.645 | 2.604923 |
| 18 | 401.712 | 2.603914 |
| 20 | 400.782 | 2.602908 |
| 22 | 399.857 | 2.601905 |
| 24 | 398.937 | 2.600904 |
| 26 | 398.020 | 2.599905 |
| 28 | 397.108 | 2.598908 |
| 30 | 396.200 | 2.597914 |
| 32 | 395.296 | 2.596922 |
| 34 | 394.396 | 2.595933 |
| 36 | 393.501 | 2.594945 |
| 38 | 392.609 | 2.593960 |
| 40 | 391.722 | 2.592978 |
| 42 | 390.838 | 2.591997 |
| 44 | 389.959 | 2.591019 |
| 46 | 389.084 | 2.590043 |
| 48 | 388.212 | 2.589069 |
| 50 | 387.345 | 2.588097 |
| 52 | 386.481 | 2.587128 |
| 54 | 385.621 | 2.586161 |
| 56 | 384.765 | 2.585196 |
| 58 | 383.913 | 2.584233 |
| 15° | | |
| 0′ | 383.065 | 2.583272 |
| 2 | 382.220 | 2.582314 |
| 4 | 381.380 | 2.581358 |
| 6 | 380.543 | 2.580403 |
| 8 | 379.709 | 2.579451 |
| 10 | 378.880 | 2.578501 |
| 12 | 378.054 | 2.577553 |
| 14 | 377.231 | 2.576608 |
| 16 | 376.412 | 2.575664 |
| 18 | 375.597 | 2.574722 |
| 20 | 374.786 | 2.573783 |
| 22 | 373.977 | 2.572845 |
| 24 | 373.173 | 2.571910 |
| 26 | 372.372 | 2.570977 |
| 28 | 371.574 | 2.570045 |
| 30 | 370.780 | 2.569116 |
| 32 | 369.989 | 2.568189 |
| 34 | 369.202 | 2.567264 |
| 36 | 368.418 | 2.566340 |
| 38 | 367.637 | 2.565419 |
| 40 | 366.859 | 2.564500 |
| 42 | 366.085 | 2.563582 |
| 44 | 365.315 | 2.562667 |
| 46 | 364.547 | 2.561754 |
| 48 | 363.783 | 2.560843 |
| 50 | 363.022 | 2.559933 |
| 52 | 362.264 | 2.559026 |
| 54 | 361.510 | 2.558120 |
| 56 | 360.758 | 2.557216 |
| 58 | 360.010 | 2.556315 |
| 60 | 359.265 | 2.555415 |

| D | R | log R |
|---|---|---|
| 16° | | |
| 0′ | 359.265 | 2.555415 |
| 2 | 358.523 | 2.554517 |
| 4 | 357.784 | 2.553621 |
| 6 | 357.048 | 2.552727 |
| 8 | 356.315 | 2.551834 |
| 10 | 355.585 | 2.550944 |
| 12 | 354.859 | 2.550055 |
| 14 | 354.135 | 2.549169 |
| 16 | 353.414 | 2.548284 |
| 18 | 352.696 | 2.547401 |
| 20 | 351.981 | 2.546519 |
| 22 | 351.269 | 2.545640 |
| 24 | 350.560 | 2.544762 |
| 26 | 349.854 | 2.543887 |
| 28 | 349.150 | 2.543013 |
| 30 | 348.450 | 2.542140 |
| 32 | 347.752 | 2.541270 |
| 34 | 347.057 | 2.540401 |
| 36 | 346.365 | 2.539535 |
| 38 | 345.676 | 2.538670 |
| 40 | 344.990 | 2.537806 |
| 42 | 344.306 | 2.536945 |
| 44 | 343.625 | 2.536085 |
| 46 | 342.947 | 2.535227 |
| 48 | 342.271 | 2.534370 |
| 50 | 341.598 | 2.533516 |
| 52 | 340.928 | 2.532663 |
| 54 | 340.260 | 2.531811 |
| 56 | 339.595 | 2.530962 |
| 58 | 338.933 | 2.530114 |
| 17° | | |
| 0′ | 338.273 | 2.529268 |
| 2 | 337.616 | 2.528424 |
| 4 | 336.962 | 2.527581 |
| 6 | 336.310 | 2.526740 |
| 8 | 335.660 | 2.525900 |
| 10 | 335.013 | 2.525062 |
| 12 | 334.369 | 2.524226 |
| 14 | 333.727 | 2.523392 |
| 16 | 333.088 | 2.522559 |
| 18 | 332.451 | 2.521728 |
| 20 | 331.816 | 2.520898 |
| 22 | 331.184 | 2.520070 |
| 24 | 330.555 | 2.519244 |
| 26 | 329.928 | 2.518419 |
| 28 | 329.303 | 2.517596 |
| 30 | 328.681 | 2.516774 |
| 32 | 328.061 | 2.515954 |
| 34 | 327.443 | 2.515136 |
| 36 | 326.828 | 2.514319 |
| 38 | 326.215 | 2.513504 |
| 40 | 325.604 | 2.512690 |
| 42 | 324.996 | 2.511878 |
| 44 | 324.390 | 2.511067 |
| 46 | 323.786 | 2.510258 |
| 48 | 323.184 | 2.509451 |
| 50 | 322.585 | 2.508645 |
| 52 | 321.989 | 2.507840 |
| 54 | 321.394 | 2.507037 |
| 56 | 320.801 | 2.506236 |
| 58 | 320.211 | 2.505436 |
| 60 | 319.623 | 2.504638 |

| D | R | log R |
|---|---|---|
| 18° | | |
| 0′ | 319.623 | 2.504638 |
| 2 | 319.037 | 2.503841 |
| 4 | 318.453 | 2.503045 |
| 6 | 317.871 | 2.502251 |
| 8 | 317.292 | 2.501459 |
| 10 | 316.715 | 2.500668 |
| 12 | 316.139 | 2.499879 |
| 14 | 315.566 | 2.499091 |
| 16 | 314.993 | 2.498304 |
| 18 | 314.426 | 2.497519 |
| 20 | 313.860 | 2.496736 |
| 22 | 313.295 | 2.495953 |
| 24 | 312.732 | 2.495173 |
| 26 | 312.172 | 2.494393 |
| 28 | 311.613 | 2.493616 |
| 30 | 311.056 | 2.492839 |
| 32 | 310.502 | 2.492064 |
| 34 | 309.949 | 2.491291 |
| 36 | 309.399 | 2.490518 |
| 38 | 308.850 | 2.489748 |
| 40 | 308.303 | 2.488978 |
| 42 | 307.759 | 2.488210 |
| 44 | 307.216 | 2.487444 |
| 46 | 306.675 | 2.486679 |
| 48 | 306.136 | 2.485915 |
| 50 | 305.599 | 2.485152 |
| 52 | 305.064 | 2.484391 |
| 54 | 304.531 | 2.483632 |
| 56 | 304.000 | 2.482873 |
| 58 | 303.470 | 2.482116 |
| 19° | | |
| 0′ | 302.943 | 2.481361 |
| 2 | 302.417 | 2.480607 |
| 4 | 301.893 | 2.479854 |
| 6 | 301.371 | 2.479102 |
| 8 | 300.851 | 2.478352 |
| 10 | 300.333 | 2.477603 |
| 12 | 299.816 | 2.476855 |
| 14 | 299.302 | 2.476109 |
| 16 | 298.789 | 2.475364 |
| 18 | 298.278 | 2.474621 |
| 20 | 297.768 | 2.473878 |
| 22 | 297.260 | 2.473137 |
| 24 | 296.755 | 2.472398 |
| 26 | 296.250 | 2.471659 |
| 28 | 295.748 | 2.470922 |
| 30 | 295.247 | 2.470186 |
| 32 | 294.748 | 2.469452 |
| 34 | 294.251 | 2.468718 |
| 36 | 293.756 | 2.467986 |
| 38 | 293.262 | 2.467256 |
| 40 | 292.770 | 2.466526 |
| 42 | 292.279 | 2.465798 |
| 44 | 291.790 | 2.465071 |
| 46 | 291.303 | 2.464345 |
| 48 | 290.818 | 2.463621 |
| 50 | 290.334 | 2.462897 |
| 52 | 289.851 | 2.462175 |
| 54 | 289.371 | 2.461455 |
| 56 | 288.892 | 2.460735 |
| 58 | 288.414 | 2.460017 |
| 60 | 287.939 | 2.459300 |

| D | R | log R | D | R | log R |
|---|---|---|---|---|---|
| 20° 0′ | 287.939 | 2.459300 | 30° 0′ | 193.185 | 2.285974 |
| 10 | 285.583 | 2.455733 | 20 | 191.111 | 2.281286 |
| 20 | 283.267 | 2.452195 | 40 | 189.083 | 2.276652 |
| 30 | 280.988 | 2.448688 | 31° 0′ | 187.099 | 2.272071 |
| 40 | 278.746 | 2.445209 | 20 | 185.158 | 2.267541 |
| 50 | 276.541 | 2.441759 | 40 | 183.258 | 2.263062 |
| 21° 0′ | 274.370 | 2.438337 | 32° 0′ | 181.398 | 2.258632 |
| 10 | 272.234 | 2.434943 | 20 | 179.577 | 2.254250 |
| 20 | 270.132 | 2.431576 | 40 | 177.794 | 2.249916 |
| 30 | 268.062 | 2.428235 | 33° 0′ | 176.047 | 2.245628 |
| 40 | 266.024 | 2.424921 | 20 | 174.336 | 2.241386 |
| 50 | 264.018 | 2.421633 | 40 | 172.659 | 2.237188 |
| 22° 0′ | 262.042 | 2.418371 | 34° 0′ | 171.015 | 2.233035 |
| 10 | 260.098 | 2.415134 | 20 | 169.404 | 2.228924 |
| 20 | 258.180 | 2.411922 | 40 | 167.825 | 2.224855 |
| 30 | 256.292 | 2.408734 | 35° 0′ | 166.275 | 2.220828 |
| 40 | 254.431 | 2.405571 | 20 | 164.756 | 2.216842 |
| 50 | 252.599 | 2.402431 | 40 | 163.266 | 2.212895 |
| 23° 0′ | 250.793 | 2.399315 | 36° 0′ | 161.803 | 2.208988 |
| 10 | 249.013 | 2.396222 | 20 | 160.368 | 2.205119 |
| 20 | 247.258 | 2.393151 | 40 | 158.960 | 2.201288 |
| 30 | 245.529 | 2.390103 | 37° 0′ | 157.577 | 2.197494 |
| 40 | 243.825 | 2.387077 | 20 | 156.220 | 2.193736 |
| 50 | 242.144 | 2.384074 | 40 | 154.887 | 2.190014 |
| 24° 0′ | 240.487 | 2.381091 | 38° 0′ | 153.578 | 2.186328 |
| 10 | 238.853 | 2.378130 | 30 | 151.657 | 2.180863 |
| 20 | 237.241 | 2.375190 | 39° 0′ | 149.787 | 2.175475 |
| 30 | 235.652 | 2.372270 | 30 | 147.965 | 2.170160 |
| 40 | 234.084 | 2.369371 | 40° 0′ | 146.190 | 2.164918 |
| 50 | 232.537 | 2.366492 | 30 | 144.460 | 2.159747 |
| 25° 0′ | 231.011 | 2.363633 | 41° 0′ | 142.773 | 2.154645 |
| 10 | 229.506 | 2.360794 | 30 | 141.127 | 2.149610 |
| 20 | 228.020 | 2.357974 | 42° 0′ | 139.521 | 2.144641 |
| 30 | 226.555 | 2.355173 | 30 | 137.955 | 2.139736 |
| 40 | 225.108 | 2.352391 | 43° 0′ | 136.425 | 2.134895 |
| 50 | 223.680 | 2.349627 | 30 | 134.932 | 2.130114 |
| 26° 0′ | 222.271 | 2.346882 | 44° 0′ | 133.473 | 2.125395 |
| 10 | 220.879 | 2.344155 | 30 | 132.049 | 2.120734 |
| 20 | 219.506 | 2.341446 | 45° 0′ | 130.656 | 2.116130 |
| 30 | 218.150 | 2.338755 | 30 | 129.296 | 2.111584 |
| 40 | 216.811 | 2.336081 | 46° 0′ | 127.965 | 2.107092 |
| 50 | 215.489 | 2.333424 | 30 | 126.664 | 2.102655 |
| 27° 0′ | 214.183 | 2.330785 | 47° 0′ | 125.392 | 2.098270 |
| 10 | 212.893 | 2.328162 | 30 | 124.148 | 2.093938 |
| 20 | 211.620 | 2.325556 | 48° 0′ | 122.930 | 2.089657 |
| 30 | 210.362 | 2.322967 | 30 | 121.738 | 2.085425 |
| 40 | 209.119 | 2.320393 | 49° 0′ | 120.571 | 2.081243 |
| 50 | 207.891 | 2.317836 | 30 | 119.429 | 2.077109 |
| 28° 0′ | 206.678 | 2.315295 | 50° 0′ | 118.310 | 2.073022 |
| 10 | 205.480 | 2.312769 | | | |
| 20 | 204.296 | 2.310259 | | | |
| 30 | 203.125 | 2.307764 | | | |
| 40 | 201.969 | 2.305285 | | | |
| 50 | 200.826 | 2.302820 | | | |
| 29° 0′ | 199.696 | 2.300370 | | | |
| 10 | 198.580 | 2.297935 | | | |
| 20 | 197.476 | 2.295515 | | | |
| 30 | 196.385 | 2.293108 | | | |
| 40 | 195.306 | 2.290716 | | | |
| 50 | 194.240 | 2.288338 | | | |

## Ecuaciones generales de Curvas.

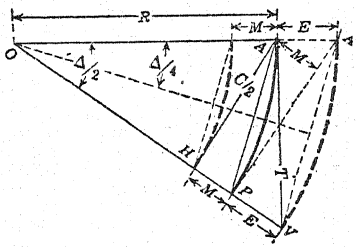
Para símbolos, vea ¶ ¶ 3 y 9, pags 951, 952.

**30.** Fig. 10. Para evitar repeticiones indebidas, toda ecuacion aqui se considera frecuentemente que sirve por trasposición y substitución, para todos los elementos que entran en ella y, aun para otros. Asi, de

$M = R$ senoverso $(\Delta/2)$; tenemos, tambien: —

$R = M/\text{senoverso}\ (\Delta/2)$; senoverso $\frac{\Delta}{2} = \frac{M}{R} = \frac{E \cos (\Delta/2)}{T \cot (\Delta/2)}$;

etc., etc. Vease ¶ 69.




$\frac{\Delta}{2} = H\ A\ V$; $\frac{\Delta}{4} = H\ A\ P = P\ A\ V = A\ V\ A'$.

**Fig. 10.**

## Funciones Angulares.

**31.** Fig. 10. **Funciones de** $\Delta/2$.

$$\text{Sen}\ \frac{\Delta}{2} = \frac{C}{2\,R} = \frac{T}{R+E} = \frac{E+M}{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

$$\text{Cos}\ \frac{\Delta}{2} = \frac{R-M}{R} = \frac{M}{E} = \frac{R}{R+E} = \frac{T^2-E^2}{T^2+E^2} = \frac{C^2-4\,M^2}{C^2+4\,M^2} \quad \ldots\ldots \quad (18)$$

$$\text{Tan}\ \frac{\Delta}{2} = \frac{T}{R} = \frac{E+M}{C/2} = \frac{C/2}{R-M} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

$$\text{Senoverso}\ \frac{\Delta}{2} = 1 - \cos\frac{\Delta}{2} = \frac{M}{R} = \frac{E}{R+E}$$

$$= \text{sen}\ \frac{\Delta}{2} \times \tan\frac{\Delta}{4} = 2\,\text{sen}^2\,\frac{\Delta}{4} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

$$\text{Exsec}\ \frac{\Delta}{2} = \frac{E}{R} = \frac{E-M}{M}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

**32.** Fig. 10. **Funciones de $\Delta/4$.**

$$\text{Sen } \frac{\Delta}{4} = \frac{P\,H}{A\,P} = \frac{M}{\sqrt{M^2 + (C/2)^2}} = \frac{\sqrt{M^2 + (C/2)^2}}{2\,R};$$

$$\text{Sen}^2 \frac{\Delta}{4} = \frac{A\,P}{2\,R} \cdot \frac{P\,H}{A\,P} = \frac{M}{2\,R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

$$\text{Tan } \frac{\Delta}{4} = \frac{E}{T} = \frac{2\,M}{C} = \sqrt{\frac{2\,T - C}{2\,T + C}} = \sqrt{\frac{E - M}{E + M}} = \frac{\text{senoverso } (\Delta/2)}{\text{sen } (\Delta/2)}$$

que puede demostrarse ser

$$= \frac{T}{2\,R + E} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$

**Funciones lineales.**

**33. Fig. 10. Relaciones entre R, E y M.**

$$R + E = \sqrt{T^2 + R^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (24)$$

$$R - M = \sqrt{R^2 - (C/2)^2} = \sqrt{(R + C/2)\,(R - C/2)} \quad \ldots\ldots\ldots\ldots \quad (25)$$

$$E + M = \sqrt{T^2 - (C/2)^2} = \sqrt{(T + C/2)\,(T - C/2)} \quad \ldots\ldots\ldots\ldots \quad (26)$$

**34. Fig. 11. Ecuaciones para R, C, T, E, Y, B F, M y M*h*.**

**D = agudeza,** *en grados* a menos que no se indique otra cosa.

$$R = \frac{T}{\tan (\Delta/2)} = \frac{C}{2 \text{ sen } (\Delta/2)} = \frac{M}{\text{senoverso } (\Delta/2)} = \frac{E}{\text{exsec } (\Delta/2)}$$

$$= \frac{T^2 - E^2}{2\,E} = \frac{(C/2)^2 + M^2}{2\,M} = \frac{E\,M}{E - M};$$

$$= \text{aprox } R_1/D = (R \text{ para una curva de } 1^\circ)/D = \text{aprox } 5729{,}65 \text{ piés}/D$$

$$= \text{aprox } C^2/8\,M \text{ (vease tabla, p. 969)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (27)$$

$$C = 2\,R \text{ sen } (\Delta/2) = 2\,T \cos (\Delta/2)$$

$$= 2\,M \cot (\Delta/4) = 2\sqrt{M\,(2\,R - M)}$$

$$= 100\,\frac{\text{sen } (n\,D/2)\,*}{\text{sen } (D/2)}$$

$$= 2E\,\frac{\text{sen } (\Delta/2)}{\text{exsec } (\Delta/2)} = \text{aprox } \sqrt{8\,R\,M}. \text{ (Véase tabla, pag. 969.)} \quad \ldots\ldots \quad (28)$$

$$T = R \tan (\Delta/2) = \frac{C}{2 \cos (\Delta/2)}$$

$$= M\,\frac{\tan (\Delta/2)}{\text{senoverso } (\Delta/2)} = E \cot (\Delta/4)$$

$$= (R + E) \text{ sen } (\Delta/2) = \sqrt{E\,(2\,R + E)}$$

$$= \sqrt{(C/2)^2 + (M + E)^2} = \frac{M + E}{\text{sen } (\Delta/2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (29)$$

* $n$ = número de cadenas en la curva.

$$E = R \text{ exsec } (\Delta/2) = T \tan (\Delta/4) = M \sec (\Delta/2)$$

$$= R \tan (\Delta/2) \tan (\Delta/4) = C \frac{\text{exsec } (\Delta/2)}{2 \text{ sen } (\Delta/2)} = \frac{R\ M}{R - M}$$

= aprox $M$ (vease tabla, p. 969) ..... (30)

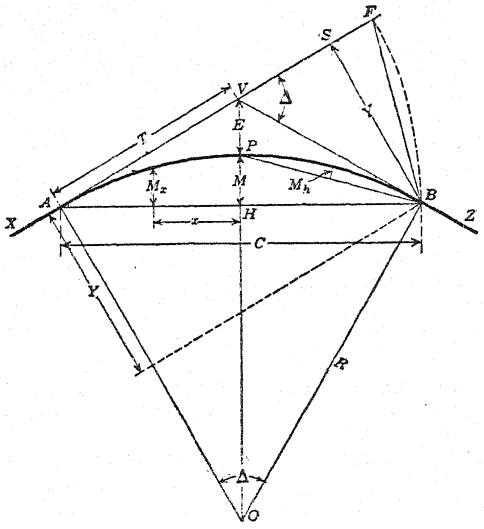



**Fig. 11.**

$$Y = R \text{ senoverso } \Delta = C \text{ sen } (\Delta/2)$$

$$= 2\ R \text{ sen}^2 (\Delta/2) = C^2/2\ R = T \text{ sen } \Delta$$

$$= E \frac{\text{senoverso } \Delta}{\text{exsec } (\Delta/2)} = M \frac{\text{senoverso } \Delta}{\text{senoverso } (\Delta/2)}$$

$$= \text{aprox (vease tabla, siguiente) } 4\ M = \frac{7}{8} n^2 D^{o}{}^{*} \text{ ..... (31)}$$

$$B\ F = 2\ C \text{ sen } (\Delta/4) \text{ ..... (31 } a\text{)}$$

$$M = R \text{ senoverso } (\Delta/2) = 2\ R \text{ sen}^2 (\Delta/4) = \frac{P}{2} \frac{B^2}{R} = T \frac{\text{senoverso } (\Delta/2)}{\tan (\Delta/2)}$$

$$= (C/2) \tan (\Delta/4) = E \cos (\Delta/2) = R\ E/(R + E)$$

$$= R - \sqrt{R^2 - (C/2)^2} = R - \sqrt{(R + C/2)\ (R - C/2)}$$

$$= R \text{ sen } (\Delta/2) \times \tan (\Delta/4);$$

$$= \text{aprox (vease tabla, p. 969) } 4\ M_h = Y/4 = B\ F/4$$

$$= \frac{M + E}{2} = E = \frac{C^2}{8\ R} = \frac{7}{32} n^2 D^{o}{}^{*} \text{ ..... (32)}$$

$$M_h = R \text{ senoverso } (\Delta/4) = M \frac{1 - \cos(\Delta/4)}{1 - \cos(\Delta/2)} = M \frac{\text{sen}^2(\Delta/8)}{\text{sen}^2(\Delta/4)}$$

$$= \text{aprox } \frac{M}{4} \text{ (vease la tabla abajo)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (33)$$

**Para la ordenada lateral,** $M_x$, vease ¶ 35.

**Multiplicadores para convertir** valores aproximados de las Ecuaciones arriba expresadas en **Valores Correctos.** La siguiente tabla de multiplicadores indican los grados de aproximacion de varias ecuaciones nuestras aproximadas en el ¶ 34, y nos permite obtener valores correctos de ellas.

**Ejemplo.** Dando $\Delta = 30°$; encontrar la ordenada media, $M$, para toda la curva. Aqui la ecuacion (32) da, aprox $M = B\ F/4$; pero, en la siguiente tabla, en la linea que empieza «(32) $M = B\ F/4$», y en la columna encabezada «$\Delta = 30°$», encontramos el multiplicador, 1.0088, que da el valor verdadero $M = 1.0088 \times B\ F/4 = 0.2522\ B\ F$.

(Vease *N. del T.* ¶ 11, pag. 926.)

| Ec. N.° | Ecuacion. | $\Delta =$ 1° | 10° | 20° | 30° | 60° | 90° |
|---|---|---|---|---|---|---|---|
| | | Para obtener valores correctos de las ecuaciones, multiplíquese el valor aproximado por el propio coeficiente que se da abajo. | | | | | |
| (32) | $M = 4\ M_h$ ..... | 1.0000 | 0.9995 | 0.9981 | 0.9957 | 0.9830 | 0.9619 |
| (33) | $M_h = M/4$..... | 1.0000 | 1.0005 | 1.0020 | 1.0043 | 1.0173 | 1.0396 |
| (31) | $Y = 4\ M$..... | 1.0000 | 0.9980 | 0.9925 | 0.9831 | 0.9328 | 0.8535 |
| (32) | $M = Y/4$..... | 1.0000 | 1.0020 | 1.0076 | 1.0172 | 1.0720 | 1.1716 |
| (32) | $M = B\ F/4$..... | 1.0000 | 1.0008 | 1.0040 | 1.0088 | 1.0352 | 1.0824 |
| (32) | $M = (M + E)/2$. | 1.0000 | 0.9980 | 0.9924 | 0.9826 | 0.9282 | 0.8284 |
| (30) | $E = M$..... | 1.0000 | 1.0038 | 1.0154 | 1.0353 | 1.1547 | 1.4142 |
| (32) | $M = E$..... | 1.0000 | 0.9962 | 0.9848 | 0.9659 | 0.8660 | 0.7071 |
| (27) | $R = C^2/8\ M$..... | 1.0000 | 1.0019 | 1.0076 | 1.0173 | 1.0718 | 1.1716 |
| (28) | $C = \sqrt{8\ R\ M}$.. | 1.0000 | 0.9981 | 0.9925 | 0.9830 | 0.9330 | 0.8535 |
| (32) | $M = C^2/8\ R$..... | 1.0000 | 1.0019 | 1.0076 | 1.0173 | 1.0718 | 1.1716 |
| (32) | $M = \frac{7}{32} n^2\ D^*$ | | | | | | |
| | Para $D = 1°$..... | 0.9975 | 0.9966 | 0.9947 | 0.9915 | 0.9746 | 0.9472 |
| | — $D = 10°$..... | 0.9984 | 0.9979 | 0.9961 | 0.9929 | 0.9760 | 0.9481 |
| | — $D = 20°$..... | 1.0025 | 1.0016 | 0.9998 | 0.9966 | 0.9797 | 0.9518 |
| (31) | $Y = \frac{7}{8} n^2\ D^*$ | | | | | | |
| | Para $D = 1°$..... | 0.9974 | 0.9949 | 0.9873 | 0.9747 | 0.9095 | 0.8085 |
| | — $D = 10°$..... | 0.9986 | 0.9961 | 0.9885 | 0.9760 | 0.9106 | 0.8095 |
| | — $D = 20°$..... | 1.0024 | 0.9999 | 0.9922 | 0.9798 | 0.9141 | 0.8126 |
| (27) | $R = R_1/D°$ | | | | | | |
| Cuando..... | | $D = 5°$ | $D = 10°$ | $D = 15°$ | $D = 20°$ | $D = 30°$ | $D = 40°$ |
| Mult $R_1/D$ por..... | | 1.0003 | 1.0013 | 1.0028 | 1.0051 | 1.0115 | 1.0206 |

$n$ = numero de unidades de cadena en la curva.

**Cuerdas Largas, C,** en pies, requeridas para subtender desde dos a ocho cadenas de 100 pies, para diferentes agudezas, *D*.

$$C = 2\ R \text{ sen } (\Delta/2).$$

Para la tabla de cuerdas largas para una curva de 1°, para distintos desarrollos $\Delta$, vease ¶ 40 *a*.

| D ° ′ | 2 estac. C ps | 3 estac C ps | 4 estac C ps | 5 estac C ps | 6 estac C ps | 7 estac C ps | 8 estac C ft |
|---|---|---|---|---|---|---|---|
| 0 10 | 200.00 | 300.00 | 400.00 | 500.00 | 599.99 | 699.99 | 799.98 |
| 20 | 200.00 | 300.00 | 399.99 | 499.98 | 599.97 | 699.95 | 799.93 |
| 30 | 200.00 | 299.99 | 399.98 | 499.96 | 599.93 | 699.89 | 799.84 |
| 40 | 200.00 | 299.99 | 399.97 | 499.93 | 599.88 | 699.81 | 799.72 |
| 50 | 200.00 | 299.98 | 399.95 | 499.89 | 599.82 | 699.70 | 799.56 |
| 1 0 | 199.99 | 299.97 | 399.92 | 499.85 | 599.73 | 699.57 | 799.36 |
| 10 | 199.99 | 299.96 | 399.90 | 499.79 | 599.64 | 699.43 | 799.13 |
| 20 | 199.99 | 299.95 | 399.87 | 499.73 | 599.53 | 699.24 | 798.86 |
| 30 | 199.98 | 299.93 | 399.83 | 499.66 | 599.40 | 699.04 | 798.56 |
| 40 | 199.98 | 299.92 | 399.79 | 499.58 | 599.26 | 698.82 | 798.22 |
| 50 | 199.97 | 299.90 | 399.74 | 499.49 | 599.11 | 698.57 | 797.85 |
| 2 0 | 199.97 | 299.88 | 399.70 | 499.39 | 598.93 | 698.30 | 797.44 |
| 10 | 199.96 | 299.86 | 399.64 | 499.29 | 598.75 | 698.00 | 797.00 |
| 20 | 199.96 | 299.83 | 399.59 | 499.17 | 598.55 | 697.68 | 796.52 |
| 30 | 199.95 | 299.81 | 399.52 | 499.05 | 598.34 | 697.34 | 796.01 |
| 40 | 199.95 | 299.78 | 399.46 | 498.92 | 598.11 | 696.97 | 795.46 |
| 50 | 199.94 | 299.76 | 399.39 | 498.78 | 597.86 | 696.58 | 794.87 |
| 3 0 | 199.93 | 299.73 | 399.32 | 498.63 | 597.60 | 696.17 | 794.26 |
| 10 | 199.92 | 299.70 | 399.24 | 498.47 | 597.33 | 695.73 | 793.60 |
| 20 | 199.92 | 299.66 | 399.15 | 498.31 | 597.04 | 695.27 | 792.91 |
| 30 | 199.91 | 299.63 | 399.07 | 498.14 | 596.74 | 694.79 | 792.19 |
| 40 | 199.90 | 299.59 | 398.98 | 497.96 | 596.42 | 694.28 | 791.43 |
| 50 | 199.89 | 299.55 | 398.88 | 497.77 | 596.09 | 693.75 | 790.63 |
| 4 0 | 199.88 | 299.51 | 398.78 | 497.57 | 595.74 | 693.20 | 789.80 |
| 10 | 199.87 | 299.47 | 398.68 | 497.36 | 595.38 | 692.62 | 788.94 |
| 20 | 199.86 | 299.43 | 398.57 | 497.15 | 595.01 | 692.02 | 788.04 |
| 30 | 199.85 | 299.38 | 398.46 | 496.92 | 594.62 | 691.40 | 787.11 |
| 40 | 199.83 | 299.34 | 398.34 | 496.69 | 594.21 | 690.75 | 786.14 |
| 50 | 199.82 | 299.29 | 398.22 | 496.45 | 593.79 | 690.08 | 785.14 |
| 5 0 | 199.81 | 299.24 | 398.10 | 496.20 | 593.36 | 689.39 | 784.10 |
| 10 | 199.80 | 299.19 | 397.97 | 495.94 | 592.91 | 688.67 | 783.03 |
| 20 | 199.78 | 299.13 | 397.84 | 495.68 | 592.45 | 687.93 | 781.93 |
| 30 | 199.77 | 299.08 | 397.70 | 495.41 | 591.97 | 687.17 | 780.79 |
| 40 | 199.76 | 299.02 | 397.56 | 495.12 | 591.48 | 686.38 | 779.61 |
| 50 | 199.74 | 298.96 | 397.41 | 494.83 | 590.97 | 685.58 | 778.41 |
| 6 0 | 199.73 | 298.90 | 397.26 | 494.53 | 590.45 | 684.75 | 777.17 |
| 10 | 199.71 | 298.84 | 397.11 | 494.23 | 589.91 | 683.89 | 775.89 |
| 20 | 199.70 | 298.78 | 396.95 | 493.91 | 589.36 | 683.02 | 774.58 |
| 30 | 199.68 | 298.71 | 396.79 | 493.59 | 588.80 | 682.12 | 773.24 |
| 40 | 199.66 | 298.65 | 396.62 | 493.26 | 588.22 | 681.20 | 771.86 |
| 50 | 199.64 | 298.58 | 396.45 | 492.92 | 587.63 | 680.25 | 770.46 |
| 7 0 | 199.63 | 298.51 | 396.28 | 492.57 | 587.02 | 679.29 | 769.01 |
| 10 | 199.61 | 298.44 | 396.10 | 492.21 | 586.40 | 678.30 | 767.54 |
| 20 | 199.59 | 298.36 | 395.92 | 491.85 | 585.77 | 677.28 | 766.03 |
| 30 | 199.57 | 298.29 | 395.73 | 491.47 | 585.12 | 676.25 | 764.49 |
| 40 | 199.55 | 298.21 | 395.54 | 491.09 | 584.45 | 675.19 | 762.92 |
| 50 | 199.53 | 298.13 | 395.34 | 490.70 | 583.77 | 674.12 | 761.31 |

**Cuerdas Largas, C,** en pies, requeridas para subtender desde dos a ocho cadenas de 100 pies, para diferentes agudezas, *D*.

$$C = 2\ R\ \text{sen}\ (\Delta/2).$$

Para la tabla de cuerdas largas para una curva de 1°, para distintos desarrollos Δ, vease ¶ 40 *a*.

(Continuación).

| | 2 estac | 3 estac | 4 estac | 5 estac | 6 estac | 7 estac | 8 estac |
|---|---|---|---|---|---|---|---|
| D ° ′ | C ps | C ps | C ps | C ps | C ps | C ps | C ps |
| 8 0 | 199.51 | 298.05 | 395.14 | 490.31 | 583.08 | 673.02 | 759.67 |
| 10 | 199.49 | 297.97 | 394.94 | 489.90 | 582.38 | 671.89 | 758.00 |
| 20 | 199.47 | 297.89 | 394.73 | 489.49 | 581.65 | 670.75 | 756.30 |
| 30 | 199.45 | 297.80 | 394.52 | 489.06 | 580.92 | 669.58 | 754.56 |
| 40 | 199.43 | 297.72 | 394.30 | 488.63 | 580.17 | 668.39 | 752.79 |
| 50 | 199.41 | 297.63 | 394.08 | 488.20 | 579.41 | 667.18 | 750.99 |
| 9 0 | 199.38 | 297.54 | 393.86 | 487.75 | 578.63 | 665.95 | 749.16 |
| 10 | 199.36 | 297.45 | 393.63 | 487.29 | 577.84 | 664.70 | 747.30 |
| 20 | 199.34 | 297.35 | 393.40 | 486.83 | 577.04 | 663.42 | 745.40 |
| 30 | 199.31 | 297.26 | 393.16 | 486.36 | 576.22 | 662.12 | 743.48 |
| 40 | 199.29 | 297.16 | 392.92 | 485.88 | 575.39 | 660.81 | 741.52 |
| 50 | 199.26 | 297.06 | 392.67 | 485.40 | 574.55 | 659.47 | 739.54 |
| 10 0 | 199.24 | 296.96 | 392.42 | 484.90 | 573.69 | 658.11 | 737.52 |
| 10 | 199.21 | 296.86 | 392.17 | 484.40 | 572.81 | 656.72 | 735.47 |
| 20 | 199.19 | 296.76 | 391.91 | 483.89 | 571.93 | 655.32 | 733.39 |
| 30 | 199.16 | 296.65 | 391.65 | 483.37 | 571.03 | 653.90 | 731.28 |
| 40 | 199.13 | 296.54 | 391.39 | 482.84 | 570.11 | 652.45 | 729.14 |
| 50 | 199.11 | 296.44 | 391.12 | 482.31 | 569.19 | 650.98 | 726.97 |
| 11 0 | 199.08 | 296.33 | 390.84 | 481.76 | 568.25 | 649.50 | 724.77 |
| 10 | 199.05 | 296.21 | 390.57 | 481.21 | 567.29 | 647.99 | 722.54 |
| 20 | 199.02 | 296.10 | 390.28 | 480.65 | 566.32 | 646.46 | 720.28 |
| 30 | 198.99 | 295.99 | 390.00 | 480.09 | 565.34 | 644.91 | 717.99 |
| 40 | 198.96 | 295.87 | 389.71 | 479.51 | 564.35 | 643.34 | 715.67 |
| 50 | 198.94 | 295.75 | 389.41 | 478.93 | 563.34 | 641.75 | 713.33 |
| 12 0 | 198.90 | 295.63 | 389.12 | 478.34 | 562.32 | 640.14 | 710.95 |
| 10 | 198.87 | 295.51 | 388.81 | 477.74 | 561.29 | 638.51 | 708.55 |
| 20 | 198.84 | 295.38 | 388.51 | 477.14 | 560.24 | 636.86 | 706.11 |
| 30 | 198.81 | 295.26 | 388.20 | 476.52 | 559.18 | 635.19 | 703.65 |
| 40 | 198.78 | 295.13 | 387.88 | 475.90 | 558.11 | 633.50 | 701.16 |
| 50 | 198.75 | 295.00 | 387.57 | 475.27 | 557.02 | 631.79 | 698.65 |
| 13 0 | 198.71 | 294.87 | 387.24 | 474.63 | 555.92 | 630.06 | 696.10 |
| 10 | 198.68 | 294.74 | 386.92 | 473.99 | 554.81 | 628.31 | 693.53 |
| 20 | 198.65 | 294.61 | 386.59 | 473.34 | 553.68 | 626.54 | 690.93 |
| 30 | 198.61 | 294.47 | 386.25 | 472.68 | 552.55 | 624.76 | 688.31 |
| 40 | 198.58 | 294.34 | 385.91 | 472.01 | 551.40 | 622.95 | 685.65 |
| 50 | 198.54 | 294.20 | 385.57 | 471.33 | 550.23 | 621.12 | 682.97 |
| 14 0 | 198.51 | 294.06 | 385.23 | 470.65 | 549.06 | 619.28 | 680.27 |
| 10 | 198.47 | 293.92 | 384.88 | 469.96 | 547.87 | 617.41 | 677.54 |
| 20 | 198.44 | 293.77 | 384.52 | 469.26 | 546.67 | 615.53 | 674.78 |
| 30 | 198.40 | 293.63 | 384.16 | 468.55 | 545.45 | 613.63 | 671.99 |
| 40 | 198.36 | 293.48 | 383.80 | 467.84 | 544.23 | 611.71 | 669.18 |
| 50 | 198.33 | 293.34 | 383.44 | 467.12 | 542.99 | 609.77 | 666.35 |
| 15 0 | 198.29 | 293.19 | 383.07 | 466.39 | 541.74 | 607.81 | 663.49 |
| 10 | 198.25 | 293.03 | 382.69 | 465.65 | 540.47 | 605.84 | 660.60 |
| 20 | 198.21 | 292.88 | 382.31 | 464.91 | 539.20 | 603.84 | 657.69 |
| 30 | 198.17 | 292.73 | 381.93 | 464.16 | 537.91 | 601.83 | 654.76 |
| 40 | 198.13 | 292.57 | 381.55 | 463.40 | 536.61 | 599.80 | 651.80 |
| 50 | 198.09 | 292.41 | 381.16 | 462.64 | 535.30 | 597.75 | 648.82 |

**Cuerdas Largas, C**, en pies, requeridas para subtender desde dos a ocho cadenas de 100 pies, para diferentes agudezas, *D*.

$$C = 2\ R\ \text{sen}\ (\Delta/2).$$

Para la tabla de cuerdas largas para una curva de 1°, para distintos desarrollos Δ, vease ¶ 40 *a*.

(Concluye).

| D ° ' | 2 estac. C ps | 3 estac C ps | 4 estac C ps | 5 estac C ps | 6 estac C ps | 7 estac C ps | 8 estac C ps |
|---|---|---|---|---|---|---|---|
| 16 0 | 198.05 | 292.25 | 380.76 | 461.86 | 533.97 | 595.69 | 645.81 |
| 10 | 198.01 | 292.09 | 380.37 | 461.08 | 532.64 | 593.61 | 642.78 |
| 20 | 197.97 | 291.93 | 379.96 | 460.29 | 531.29 | 591.51 | 639.73 |
| 30 | 197.93 | 291.76 | 379.56 | 459.50 | 529.93 | 589.39 | 636.65 |
| 40 | 197.89 | 291.60 | 379.15 | 458.70 | 528.56 | 587.25 | 633.55 |
| 50 | 197.85 | 291.43 | 378.74 | 457.89 | 527.17 | 585.10 | 630.43 |
| 17 0 | 197.80 | 291.26 | 378.32 | 457.07 | 525.78 | 582.93 | 627.28 |
| 10 | 197.76 | 291.09 | 377.90 | 456.24 | 524.37 | 580.75 | 624.12 |
| 20 | 197.72 | 290.92 | 377.48 | 455.41 | 522.95 | 578.55 | 620.93 |
| 30 | 197.67 | 290.74 | 377.05 | 454.57 | 521.52 | 576.33 | 617.72 |
| 40 | 197.63 | 290.57 | 376.62 | 453.73 | 520.08 | 574.09 | 614.49 |
| 50 | 197.58 | 290.39 | 376.18 | 452.88 | 518.63 | 571.84 | 611.23 |
| 18 0 | 197.54 | 290.21 | 375.74 | 452.02 | 517.16 | 569.57 | 607.96 |
| 10 | 197.49 | 290.03 | 375.30 | 451.15 | 515.69 | 567.29 | 604.66 |
| 20 | 197.45 | 289.85 | 374.85 | 450.37 | 514.20 | 564.99 | 601.35 |
| 30 | 197.40 | 289.67 | 374.40 | 449.39 | 512.70 | 562.67 | 598.01 |
| 40 | 197.35 | 289.48 | 373.94 | 448.50 | 511.19 | 560.34 | 594.66 |
| 50 | 197.31 | 289.29 | 373.48 | 447.61 | 509.67 | 558.00 | 591.28 |
| 19 0 | 197.26 | 289.10 | 373.02 | 446.71 | 508.14 | 555.63 | 587.89 |
| 10 | 197.21 | 288.91 | 372.55 | 445.80 | 506.60 | 553.26 | 584.48 |
| 20 | 197.16 | 288.72 | 372.08 | 444.88 | 505.04 | 550.86 | 581.04 |
| 30 | 197.11 | 288.53 | 371.61 | 443.96 | 503.48 | 548.46 | 577.59 |
| 40 | 197.06 | 288.33 | 371.13 | 443.03 | 501.91 | 546.04 | 574.12 |
| 50 | 197.01 | 288.14 | 370.65 | 442.09 | 500.32 | 543.60 | 570.63 |
| 20 0 | 196.96 | 287.94 | 370.17 | 441.15 | 498.72 | 541.15 | 567.13 |

**35. Ordenada lateral, $M_x$. Fig. 12.**

$$M_x = \sqrt{R^2 - x^2} + M - R = \sqrt{R^2 - x^2} - R\ \cos\ (\Delta/2) \quad \ldots\ldots\ldots \quad (34)$$

$$= \text{aprox}\ a\ b/2\ R\ \text{(vease la tabla, a continuacion)} \quad \ldots\ldots\ldots \quad (35)$$

donde *a* y *b* son los dos segmentos de la cuerda *C*,

$$= \text{aprox}\ 4\ M\ a\ b/C^2\ \text{(vease la tabla a continuacion)} \quad \ldots\ldots\ldots \quad (36)$$

Cuando $C = 100$ pies ( $= c$ ; $\Delta = D$), ec (36) se convierte :

$$m_x = \text{aprox}\ \frac{m\ a\ b}{2500} = \text{aprox}\ \frac{m}{0{,}25} \cdot \frac{a}{100} \cdot \frac{b}{100} \quad \ldots\ldots\ldots \quad (37)$$

o, como en el caso, $B\ F$ ( = « dist tangencial ») = 200 sen $(D/4)$ = aprox 4 *m*, tenemos, para cualquier curva y para cualquier valor de *C* :

$$M_x = \text{aprox}\ B\ F \cdot \frac{a}{100} \cdot \frac{b}{100} = \frac{B\ F \cdot a \cdot b}{10{,}000} \quad \ldots\ldots\ldots \quad (38)$$

Vease la tabla pag 969, usando coeficientes para $R = C^2/8\ M$. Asi, para $\Delta = 60°$,

$$M_x = 1{,}0718\ B\ F\ .\ a\ .\ b/10{,}000.$$

$$M_x = \text{aprox}\ D \times M_1\ \text{(vease la tabla que sigue)} \quad \ldots\ldots\ldots \quad (39)$$

donde *D* = la agudeza de la curva dada, y $M_1$ = el lado de la cuerda para una curva de 1°, a la dist dada, *x*, desde su media ordenada, *M*, en una cuerda igual, *C*.

**Coeficientes para las ecuaciones** aproximadas **anteriores** para ordenadas laterales, $M_x$.

La siguiente tabla de multiplicadores indica el grado de aproximación de varias de nuestras ecuaciones en el ¶ 35, y nos permite obtener valores correctos de ellas.

**Ejemplo.** Dando $\Delta = 20°$, y $x/C = {}^1/_4$; encontrar la ordenada, $M_x$, dist $x$ de la ordenada, media, $M$. Aquí la ecuacion (35) da, aprox, $M_x = a\ b/2\ R$; pero en la tabla siguiente, en la linea que empieza « (35) $M_x = a\ b/2\ R$ », y en la columna encabezada « $\Delta = 20°$, $x/C = {}^1/_4$ » encontramos el múltiplo correcto, 1.0096, que da el verdadero valor, $M_x = 1.0096\ a\ b/2\ R = 0.5048\ a\ b/R$.

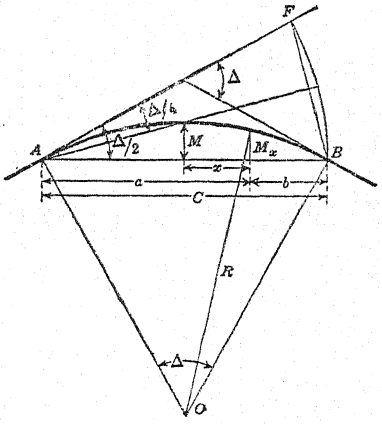



**Fig. 12.**

| Ec. No. | Ecuacion. | $\Delta = 10°$ $x/C =$ | | | $\Delta = 20°$ $x/C =$ | | |
|---|---|---|---|---|---|---|---|
| | | 1/4 | 3/8 | 19/40 | 1/4 | 3/8 | 19/40 |
| (35) | $M_x = ab/2R$ | 1.0024 | 1.0028 | 1.0024 | 1.0096 | 1.0128 | 1.0094 |
| (36) | $M_x = 4Mab/C^2$ | | | | | | |
| (37) | $m^x = mab/2500$ | 1.0005 | 1.0011 | 1.0012 | 1.0019 | 1.0051 | 1.0076 |
| (39) | $M_x = D \times M_1$ | 0.9995 | 1.0003 | 0.9965 | 1.0036 | 1.0064 | 1.0053 |

**36.** Para esa porción de una curva que está subtendida por una **cadena, c, o unidad de arco, a** (extensión, $\Delta$, = agudeza, $D$), úsense las ecuaciones del ¶ ¶ 34, 35.

Substituyendo..... $D$, $t$, $c$, $m$, $m_x$, $e$, y $y$ $m_h$.

Para..... $\Delta$, $T$, $C$, $M$, $M_x$, $E$, $Y$ y $M_h$ respectivamente.

(Vease *N. del T.* ¶ 11, pag 926.)

**Ordenadas Medias, m, en pies,** para cadenas de 100 pies, para diferentes agudezas, *D*.

| D Mins | 0° | 1° | 2° | 3° | 4° | 5° | 6° | 7° | 8° | 9° | D Mins |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.000 | 0.218 | 0.436 | 0.654 | 0.872 | 1.091 | 1.309 | 1.528 | 1.746 | 1.965 | 0 |
| 1 | 0.004 | 0.222 | 0.440 | 0.658 | 0.876 | 1.094 | 1.313 | 1.531 | 1.749 | 1.968 | 1 |
| 2 | 0.007 | 0.225 | 0.444 | 0.662 | 0.880 | 1.098 | 1.317 | 1.535 | 1.753 | 1.972 | 2 |
| 3 | 0.011 | 0.229 | 0.447 | 0.665 | 0.883 | 1.102 | 1.320 | 1.539 | 1.756 | 1.975 | 3 |
| 4 | 0.015 | 0.233 | 0.451 | 0.669 | 0.887 | 1.105 | 1.324 | 1.543 | 1.761 | 1.979 | 4 |
| 5 | 0.018 | 0.236 | 0.454 | 0.673 | 0.891 | 1.109 | 1.327 | 1.546 | 1.764 | 1.983 | 5 |
| 6 | 0.022 | 0.240 | 0.458 | 0.676 | 0.894 | 1.112 | 1.331 | 1.550 | 1.768 | 1.987 | 6 |
| 7 | 0.025 | 0.244 | 0.462 | 0.680 | 0.898 | 1.116 | 1.335 | 1.553 | 1.771 | 1.990 | 7 |
| 8 | 0.029 | 0.247 | 0.465 | 0.684 | 0.902 | 1.120 | 1.338 | 1.557 | 1.775 | 1.994 | 8 |
| 9 | 0.033 | 0.251 | 0.469 | 0.687 | 0.905 | 1.123 | 1.342 | 1.561 | 1.778 | 1.998 | 9 |
| 10 | 0.036 | 0.255 | 0.473 | 0.691 | 0.909 | 1.127 | 1.346 | 1.564 | 1.782 | 2.001 | 10 |
| 11 | 0.040 | 0.258 | 0.476 | 0.694 | 0.912 | 1.131 | 1.349 | 1.568 | 1.786 | 2.005 | 11 |
| 12 | 0.044 | 0.262 | 0.480 | 0.698 | 0.916 | 1.134 | 1.353 | 1.572 | 1.790 | 2.008 | 12 |
| 13 | 0.047 | 0.265 | 0.484 | 0.702 | 0.920 | 1.138 | 1.356 | 1.575 | 1.793 | 2.012 | 13 |
| 14 | 0.051 | 0.269 | 0.487 | 0.705 | 0.923 | 1.142 | 1.360 | 1.579 | 1.797 | 2.016 | 14 |
| 15 | 0.055 | 0.273 | 0.491 | 0.709 | 0.927 | 1.146 | 1.364 | 1.582 | 1.801 | 2.019 | 15 |
| 16 | 0.058 | 0.276 | 0.494 | 0.713 | 0.931 | 1.149 | 1.368 | 1.586 | 1.804 | 2.023 | 16 |
| 17 | 0.062 | 0.280 | 0.498 | 0.716 | 0.934 | 1.153 | 1.371 | 1.590 | 1.807 | 2.026 | 17 |
| 18 | 0.065 | 0.284 | 0.502 | 0.720 | 0.938 | 1.157 | 1.375 | 1.593 | 1.811 | 2.030 | 18 |
| 19 | 0.069 | 0.287 | 0.505 | 0.723 | 0.942 | 1.160 | 1.378 | 1.597 | 1.815 | 2.034 | 19 |
| 20 | 0.073 | 0.291 | 0.509 | 0.727 | 0.945 | 1.164 | 1.382 | 1.600 | 1.819 | 2.037 | 20 |
| 21 | 0.076 | 0.295 | 0.513 | 0.731 | 0.949 | 1.168 | 1.386 | 1.604 | 1.822 | 2.041 | 21 |
| 22 | 0.080 | 0.298 | 0.516 | 0.734 | 0.952 | 1.171 | 1.389 | 1.608 | 1.826 | 2.045 | 22 |
| 23 | 0.084 | 0.302 | 0.520 | 0.738 | 0.956 | 1.175 | 1.393 | 1.611 | 1.829 | 2.048 | 23 |
| 24 | 0.087 | 0.305 | 0.524 | 0.742 | 0.960 | 1.179 | 1.397 | 1.615 | 1.833 | 2.052 | 24 |
| 25 | 0.091 | 0.309 | 0.527 | 0.745 | 0.963 | 1.182 | 1.400 | 1.619 | 1.837 | 2.056 | 25 |
| 26 | 0.095 | 0.313 | 0.531 | 0.749 | 0.967 | 1.186 | 1.404 | 1.623 | 1.840 | 2.060 | 26 |
| 27 | 0.098 | 0.316 | 0.534 | 0.753 | 0.971 | 1.190 | 1.407 | 1.626 | 1.844 | 2.063 | 27 |
| 28 | 0.102 | 0.320 | 0.538 | 0.756 | 0.974 | 1.193 | 1.411 | 1.630 | 1.848 | 2.066 | 28 |
| 29 | 0.105 | 0.324 | 0.542 | 0.760 | 0.978 | 1.197 | 1.415 | 1.633 | 1.851 | 2.070 | 29 |
| 30 | 0.109 | 0.327 | 0.545 | 0.763 | 0.982 | 1.200 | 1.418 | 1.637 | 1.855 | 2.074 | 30 |
| 31 | 0.113 | 0.331 | 0.549 | 0.767 | 0.985 | 1.204 | 1.422 | 1.641 | 1.858 | 2.077 | 31 |
| 32 | 0.116 | 0.335 | 0.553 | 0.771 | 0.989 | 1.208 | 1.426 | 1.644 | 1.862 | 2.081 | 32 |
| 33 | 0.120 | 0.338 | 0.556 | 0.774 | 0.993 | 1.211 | 1.429 | 1.648 | 1.866 | 2.084 | 33 |
| 34 | 0.124 | 0.342 | 0.560 | 0.778 | 0.996 | 1.215 | 1.433 | 1.651 | 1.869 | 2.088 | 34 |
| 35 | 0.127 | 0.345 | 0.564 | 0.782 | 1.000 | 1.218 | 1.437 | 1.655 | 1.873 | 2.092 | 35 |
| 36 | 0.131 | 0.349 | 0.567 | 0.785 | 1.003 | 1.222 | 1.440 | 1.659 | 1.877 | 2.096 | 36 |
| 37 | 0.135 | 0.353 | 0.571 | 0.789 | 1.007 | 1.226 | 1.444 | 1.662 | 1.880 | 2.099 | 37 |
| 38 | 0.138 | 0.356 | 0.574 | 0.793 | 1.011 | 1.229 | 1.447 | 1.666 | 1.884 | 2.103 | 38 |
| 39 | 0.142 | 0.360 | 0.578 | 0.796 | 1.014 | 1.233 | 1.451 | 1.670 | 1.887 | 2.106 | 39 |
| 40 | 0.145 | 0.364 | 0.582 | 0.800 | 1.018 | 1.237 | 1.455 | 1.673 | 1.892 | 2.110 | 40 |
| 41 | 0.149 | 0.367 | 0.585 | 0.803 | 1.022 | 1.240 | 1.458 | 1.677 | 1.895 | 2.113 | 41 |
| 42 | 0.153 | 0.371 | 0.589 | 0.807 | 1.025 | 1.244 | 1.462 | 1.680 | 1.899 | 2.117 | 42 |
| 43 | 0.156 | 0.375 | 0.593 | 0.811 | 1.029 | 1.247 | 1.466 | 1.684 | 1.903 | 2.121 | 43 |
| 44 | 0.160 | 0.378 | 0.596 | 0.814 | 1.032 | 1.251 | 1.469 | 1.688 | 1.906 | 2.125 | 44 |
| 45 | 0.164 | 0.382 | 0.600 | 0.818 | 1.036 | 1.255 | 1.473 | 1.691 | 1.910 | 2.128 | 45 |
| 46 | 0.167 | 0.385 | 0.604 | 0.822 | 1.040 | 1.258 | 1.476 | 1.695 | 1.914 | 2.132 | 46 |
| 47 | 0.171 | 0.389 | 0.607 | 0.825 | 1.043 | 1.262 | 1.480 | 1.699 | 1.918 | 2.135 | 47 |
| 48 | 0.174 | 0.393 | 0.611 | 0.829 | 1.047 | 1.266 | 1.484 | 1.702 | 1.921 | 2.139 | 48 |
| 49 | 0.178 | 0.396 | 0.614 | 0.832 | 1.051 | 1.269 | 1.487 | 1.706 | 1.924 | 2.142 | 49 |
| 50 | 0.182 | 0.400 | 0.618 | 0.836 | 1.054 | 1.273 | 1.491 | 1.710 | 1.928 | 2.147 | 50 |
| 51 | 0.185 | 0.404 | 0.622 | 0.840 | 1.058 | 1.277 | 1.495 | 1.713 | 1.932 | 2.150 | 51 |
| 52 | 0.189 | 0.407 | 0.625 | 0.843 | 1.062 | 1.280 | 1.498 | 1.717 | 1.935 | 2.154 | 52 |
| 53 | 0.193 | 0.411 | 0.629 | 0.847 | 1.065 | 1.284 | 1.502 | 1.720 | 1.939 | 2.158 | 53 |
| 54 | 0.196 | 0.414 | 0.633 | 0.851 | 1.069 | 1.288 | 1.505 | 1.724 | 1.943 | 2.161 | 54 |
| 55 | 0.200 | 0.418 | 0.636 | 0.854 | 1.073 | 1.291 | 1.510 | 1.728 | 1.946 | 2.165 | 55 |
| 56 | 0.204 | 0.422 | 0.640 | 0.858 | 1.076 | 1.295 | 1.513 | 1.731 | 1.950 | 2.168 | 56 |
| 57 | 0.207 | 0.425 | 0.644 | 0.862 | 1.080 | 1.298 | 1.517 | 1.735 | 1.953 | 2.172 | 57 |
| 58 | 0.211 | 0.429 | 0.647 | 0.865 | 1.083 | 1.302 | 1.520 | 1.739 | 1.957 | 2.175 | 58 |
| 59 | 0.215 | 0.433 | 0.651 | 0.869 | 1.088 | 1.306 | 1.524 | 1.742 | 1.961 | 2.179 | 59 |

**Ordenadas Medias, m, en pies,** para cadenas de 100 pies para diferentes agudezas, *D*.

**(Concluye).**

| D Mins | 10° | 11° | 12° | 13° | 14° | 15° | 16° | 17° | 18° | 19° | D Mins |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 2.183 | 2.402 | 2.620 | 2.839 | 3.058 | 3.277 | 3.496 | 3.716 | 3.935 | 4.155 | 0 |
| 2 | 2.190 | 2.409 | 2.628 | 2.846 | 3.065 | 3.284 | 3.504 | 3.723 | 3.942 | 4.162 | 2 |
| 4 | 2.198 | 2.416 | 2.635 | 2.854 | 3.073 | 3.292 | 3.511 | 3.730 | 3.950 | 4.169 | 4 |
| 6 | 2.205 | 2.423 | 2.642 | 2.861 | 3.080 | 3.299 | 3.518 | 3.738 | 3.957 | 4.177 | 6 |
| 8 | 2.212 | 2.431 | 2.650 | 2.868 | 3.087 | 3.306 | 3.526 | 3.745 | 3.964 | 4.184 | 8 |
| 10 | 2.219 | 2.438 | 2.657 | 2.876 | 3.095 | 3.314 | 3.533 | 3.752 | 3.972 | 4.191 | 10 |
| 12 | 2.227 | 2.445 | 2.664 | 2.883 | 3.102 | 3.321 | 3.540 | 3.760 | 3.979 | 4.199 | 12 |
| 14 | 2.234 | 2.453 | 2.671 | 2.890 | 3.109 | 3.328 | 3.547 | 3.767 | 3.986 | 4.206 | 14 |
| 16 | 2.241 | 2.460 | 2.679 | 2.898 | 3.117 | 3.336 | 3.555 | 3.774 | 3.994 | 4.213 | 16 |
| 18 | 2.249 | 2.467 | 2.686 | 2.905 | 3.124 | 3.343 | 3.562 | 3.781 | 4.001 | 4.221 | 18 |
| 20 | 2.256 | 2.475 | 2.693 | 2.912 | 3.131 | 3.350 | 3.569 | 3.789 | 4.008 | 4.228 | 20 |
| 22 | 2.263 | 2.482 | 2.701 | 2.919 | 3.138 | 3.358 | 3.577 | 3.796 | 4.016 | 4.235 | 22 |
| 24 | 2.270 | 2.489 | 2.708 | 2.927 | 3.146 | 3.365 | 3.584 | 3.803 | 4.023 | 4.243 | 24 |
| 26 | 2.278 | 2.496 | 2.715 | 2.934 | 3.153 | 3.372 | 3.591 | 3.811 | 4.030 | 4.250 | 26 |
| 28 | 2.285 | 2.504 | 2.722 | 2.941 | 3.160 | 3.379 | 3.599 | 3.818 | 4.038 | 4.257 | 28 |
| 30 | 2.293 | 2.511 | 2.730 | 2.949 | 3.168 | 3.387 | 3.606 | 3.825 | 4.045 | 4.265 | 30 |
| 32 | 2.300 | 2.518 | 2.737 | 2.956 | 3.175 | 3.394 | 3.613 | 3.833 | 4.052 | 4.272 | 32 |
| 34 | 2.307 | 2.526 | 2.744 | 2.963 | 3.182 | 3.401 | 3.621 | 3.840 | 4.060 | 4.279 | 34 |
| 36 | 2.314 | 2.533 | 2.752 | 2.971 | 3.190 | 3.409 | 3.628 | 3.847 | 4.067 | 4.287 | 36 |
| 38 | 2.321 | 2.540 | 2.759 | 2.978 | 3.197 | 3.416 | 3.635 | 3.855 | 4.074 | 4.294 | 38 |
| 40 | 2.329 | 2.547 | 2.766 | 2.985 | 3.204 | 3.423 | 3.643 | 3.862 | 4.081 | 4.301 | 40 |
| 42 | 2.336 | 2.555 | 2.774 | 2.992 | 3.211 | 3.431 | 3.650 | 3.869 | 4.089 | 4.308 | 42 |
| 44 | 2.343 | 2.562 | 2.781 | 3.000 | 3.219 | 3.438 | 3.657 | 3.877 | 4.096 | 4.316 | 44 |
| 46 | 2.351 | 2.569 | 2.788 | 3.007 | 3.226 | 3.445 | 3.664 | 3.884 | 4.103 | 4.323 | 46 |
| 48 | 2.358 | 2.577 | 2.795 | 3.014 | 3.233 | 3.452 | 3.672 | 3.891 | 4.111 | 4.330 | 48 |
| 50 | 2.365 | 2.584 | 2.803 | 3.022 | 3.241 | 3.460 | 3.679 | 3.899 | 4.118 | 4.338 | 50 |
| 52 | 2.372 | 2.591 | 2.810 | 3.029 | 3.248 | 3.467 | 3.686 | 3.906 | 4.125 | 4.345 | 52 |
| 54 | 2.380 | 2.598 | 2.817 | 3.036 | 3.255 | 3.474 | 3.694 | 3.913 | 4.133 | 4.352 | 54 |
| 56 | 2.387 | 2.606 | 2.825 | 3.044 | 3.263 | 3.482 | 3.701 | 3.920 | 4.140 | 4.360 | 56 |
| 58 | 2.394 | 2.613 | 2.832 | 3.051 | 3.270 | 3.489 | 3.708 | 3.928 | 4.147 | 4.367 | 58 |
| 60 | 2.402 | 2.620 | 2.839 | 3.058 | 3.277 | 3.496 | 3.716 | 3.935 | 4.155 | 4.374 | 60 |

| Mins | 20° | 21° | 22° | 23° | 24° | 25° | 26° | 27° | 28° | 29° | Mins |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 4.374 | 4.594 | 4.814 | 5.035 | 5.255 | 5.476 | 5.697 | 5.918 | 6.139 | 6.360 | 0 |
| 10 | 4.411 | 4.631 | 4.851 | 5.071 | 5.292 | 5.513 | 5.734 | 5.955 | 6.176 | 6.398 | 10 |
| 20 | 4.448 | 4.668 | 4.888 | 5.108 | 5.329 | 5.549 | 5.770 | 5.992 | 6.213 | 6.435 | 20 |
| 30 | 4.484 | 4.704 | 4.925 | 5.145 | 5.366 | 5.586 | 5.807 | 6.029 | 6.250 | 6.472 | 30 |
| 40 | 4.521 | 4.741 | 4.961 | 5.182 | 5.402 | 5.623 | 5.844 | 6.065 | 6.287 | 6.509 | 40 |
| 50 | 4.558 | 4.778 | 4.998 | 5.218 | 5.439 | 5.660 | 5.881 | 6.102 | 6.324 | 6.545 | 50 |
| 60 | 4.594 | 4.814 | 5.035 | 5.255 | 5.476 | 5.697 | 5.918 | 6.139 | 6.360 | 6.583 | 60 |

| Mins | 30° | 31° | 32° | 33° | 34° | 35° | 36° | 37° | 38° | 39° | Mins |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 6.583 | 6.805 | 7.027 | 7.250 | 7.473 | 7.696 | 7.919 | 8.143 | 8.367 | 8.592 | 0 |
| 20 | 6.657 | 6.879 | 7.101 | 7.324 | 7.547 | 7.770 | 7.994 | 8.218 | 8.442 | 8.667 | 20 |
| 40 | 6.731 | 6.958 | 7.175 | 7.398 | 7.621 | 7.845 | 8.068 | 8.292 | 8.517 | 8.741 | 40 |
| 60 | 6.805 | 7.027 | 7.250 | 7.473 | 7.696 | 7.919 | 8.143 | 8.367 | 8.592 | 8.816 | 60 |

| Mins | 40° | 41° | 42° | 43° | 44° | 45° | 46° | 47° | 48° | 49° | Mins |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 8.816 | 9.041 | 9.267 | 9.493 | 9.719 | 9.946 | 10.173 | 10.400 | 10.628 | 10.856 | 0 |
| 30 | 8.929 | 9.154 | 9.380 | 9.606 | 9.832 | 10.059 | 10.286 | 10.516 | 10.742 | 10.970 | 30 |
| 60 | 9.041 | 9.267 | 9.493 | 9.719 | 9.946 | 10.173 | 10.400 | 10.628 | 10.856 | 11.085 | 60 |

**Ordenadas laterales, $m_x$, en pies separadas 5 pies**, para cadena de 100 pies.

$m_x = \sqrt{I.^2 - x^2} - R \cos (D/2)$;

$x$ = d'st, en pies, de la orde. ada lateral a la ordenada media.

(Vease *N. del T.* ¶ 11, pag 926.)

| D | x, ps. 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
|---|---|---|---|---|---|---|---|---|---|
| 0° 4′ | .014 | .014 | .013 | .012 | .010 | .008 | .008 | .005 | .003 |
| 8 | .029 | .028 | .026 | .024 | .022 | .018 | .015 | .010 | .005 |
| 12 | .043 | .041 | .038 | .037 | .033 | .028 | .022 | .015 | .008 |
| 16 | .058 | .056 | .052 | .049 | .044 | .037 | .030 | .020 | .011 |
| 20 | .072 | .070 | .066 | .061 | .055 | .047 | .037 | .026 | .014 |
| 24 | .086 | .083 | .077 | .074 | .066 | .056 | .045 | .031 | .017 |
| 28 | .101 | .098 | .092 | .086 | .077 | .065 | .052 | .036 | .019 |
| 32 | .115 | .112 | .106 | .098 | .088 | .075 | .058 | .042 | .022 |
| 36 | .130 | .126 | .119 | .110 | .099 | .084 | .066 | .047 | .024 |
| 40 | .144 | .140 | .133 | .123 | .110 | .093 | .074 | .052 | .027 |
| 44 | .158 | .153 | .145 | .135 | .121 | .103 | .081 | .057 | .030 |
| 48 | .172 | .167 | .158 | .147 | .132 | .112 | .088 | .062 | .033 |
| 52 | .187 | .181 | .171 | .159 | .143 | .122 | .095 | .068 | .035 |
| 56 | .202 | .195 | .185 | .171 | .154 | .131 | .103 | .073 | .038 |
| 1 | .216 | .209 | .198 | .183 | .164 | .140 | .111 | .078 | .041 |
| 4 | .231 | .223 | .211 | .196 | .175 | .150 | .118 | .083 | .043 |
| 8 | .245 | .237 | .224 | .208 | .186 | .159 | .125 | .088 | .046 |
| 12 | .260 | .252 | .237 | .220 | .196 | .168 | .133 | .094 | .049 |
| 16 | .274 | .265 | .251 | .232 | .207 | .177 | .140 | .099 | .052 |
| 20 | .288 | .279 | .264 | .244 | .218 | .187 | .148 | .104 | .055 |
| 24 | .303 | .293 | .277 | .256 | .229 | .197 | .155 | .109 | .057 |
| 28 | .317 | .307 | .291 | .269 | .240 | .206 | .163 | .114 | .060 |
| 32 | .331 | .321 | .304 | .281 | .251 | .215 | .171 | .120 | .063 |
| 36 | .345 | .335 | .317 | .293 | .262 | .224 | .178 | .125 | .066 |
| 40 | .360 | .349 | .330 | .305 | .273 | .233 | .185 | .130 | .069 |
| 44 | .374 | .363 | .343 | .318 | .284 | .242 | .192 | .135 | .072 |
| 48 | .389 | .377 | .356 | .330 | .295 | .251 | .200 | .141 | .075 |
| 52 | .403 | .391 | .370 | .342 | .305 | .261 | .208 | .147 | .077 |
| 56 | .418 | .405 | .383 | .354 | .316 | .270 | .215 | .152 | .080 |
| 2 | .432 | .419 | .397 | .366 | .327 | .280 | .222 | .157 | .083 |
| 4 | .446 | .433 | .409 | .379 | .338 | .289 | .230 | .162 | .086 |
| 8 | .461 | .447 | .425 | .391 | .349 | .298 | .237 | .167 | .088 |
| 12 | .475 | .461 | .437 | .403 | .360 | .308 | .245 | .173 | .090 |
| 16 | .490 | .475 | .450 | .415 | .371 | .317 | .252 | .178 | .093 |
| 20 | .504 | .489 | .463 | .428 | .382 | .326 | .260 | .183 | .096 |
| 24 | .518 | .503 | .476 | .440 | .393 | .334 | .267 | .188 | .099 |
| 28 | .533 | .517 | .489 | .452 | .404 | .346 | .275 | .194 | .102 |
| 32 | .547 | .531 | .503 | .465 | .415 | .355 | .282 | .199 | .104 |
| 36 | .562 | .545 | .516 | .477 | .425 | .364 | .289 | .204 | .107 |
| 40 | .576 | .559 | .529 | .489 | .436 | .373 | .297 | .209 | .110 |
| 44 | .590 | .573 | .542 | .501 | .447 | .382 | .304 | .214 | .113 |
| 48 | .605 | .587 | .555 | .513 | .458 | .391 | .312 | .219 | .116 |
| 52 | .619 | .601 | .569 | .526 | .469 | .401 | .319 | .225 | .118 |
| 56 | .634 | .615 | .582 | .538 | .480 | .410 | .326 | .230 | .121 |
| 3 | .648 | .629 | .595 | .550 | .491 | .419 | .334 | .235 | .124 |
| 4 | .662 | .643 | .608 | .562 | .502 | .428 | .341 | .240 | .127 |
| 8 | .677 | .657 | .621 | .574 | .512 | .438 | .349 | .246 | .130 |
| 12 | .691 | .671 | .635 | .587 | .523 | .448 | .357 | .251 | .132 |
| 16 | .705 | .685 | .649 | .599 | .534 | .457 | .364 | .257 | .135 |
| 20 | .720 | .699 | .662 | .611 | .545 | .466 | .371 | .262 | .138 |
| 24 | .734 | .713 | .675 | .623 | .556 | .475 | .378 | .267 | .141 |
| 28 | .749 | .727 | .688 | .635 | .567 | .485 | .386 | .272 | .144 |
| 32 | .763 | .741 | .702 | .648 | .578 | .494 | .394 | .278 | .146 |
| 36 | .777 | .755 | .715 | .660 | .589 | .503 | .401 | .283 | .149 |
| 40 | .792 | .769 | .728 | .673 | .600 | .512 | .408 | .288 | .152 |
| 44 | .806 | .783 | .741 | .685 | .611 | .521 | .415 | .293 | .155 |
| 48 | .821 | .797 | .754 | .697 | .621 | .531 | .423 | .298 | .158 |
| 52 | .835 | .811 | .768 | .709 | .632 | .541 | .431 | .304 | .160 |
| 56 | .850 | .825 | .781 | .721 | .643 | .550 | .438 | .309 | .163 |

**Ordenadas laterales, $m_x$, en pies, separadas 5 pies,** para cadena de 100 pies. (Vease *N. del T.* ¶ 11, pag 926.)

**(Concluye).**

$m_x = \sqrt{R^2 - x^2} - R \cos (D/2)$;

$x$ = dist, en pies, de la ordenada lateral a la ordenada media.

| D \ x, pies | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
|---|---|---|---|---|---|---|---|---|---|
| 4° 0′ | .864 | .839 | .794 | .734 | .655 | .559 | .445 | .314 | .166 |
| 10 | .900 | .874 | .827 | .764 | .682 | .582 | .464 | .327 | .173 |
| 20 | .936 | .909 | .860 | .795 | .709 | .606 | .482 | .340 | .179 |
| 30 | .972 | .944 | .893 | .825 | .736 | .629 | .501 | .354 | .186 |
| 40 | 1.008 | .979 | .926 | .855 | .764 | .652 | .519 | .367 | .193 |
| 50 | 1.044 | 1.014 | .959 | .886 | .791 | .676 | .538 | .380 | .199 |
| 5 | 1.080 | 1.048 | .993 | .917 | .818 | .699 | .557 | .393 | .207 |
| 10 | 1.116 | 1.083 | 1.026 | .947 | .845 | .722 | .576 | .406 | .214 |
| 20 | 1.152 | 1.118 | 1.058 | .978 | .872 | .746 | .594 | .419 | .220 |
| 30 | 1.188 | 1.153 | 1.092 | 1.009 | .900 | .769 | .613 | .432 | .228 |
| 40 | 1.224 | 1.188 | 1.124 | 1.039 | .927 | .792 | .631 | .445 | .235 |
| 50 | 1.260 | 1.223 | 1.157 | 1.070 | .954 | .816 | .649 | .458 | .241 |
| 6 | 1.296 | 1.258 | 1.191 | 1.100 | .982 | .839 | .668 | .472 | .248 |
| 10 | 1.332 | 1.293 | 1.224 | 1.130 | 1.009 | .862 | .686 | .485 | .255 |
| 20 | 1.368 | 1.328 | 1.256 | 1.161 | 1.036 | .886 | .705 | .498 | .262 |
| 30 | 1.404 | 1.362 | 1.290 | 1.192 | 1.064 | .909 | .724 | .511 | .269 |
| 40 | 1.440 | 1.397 | 1.323 | 1.222 | 1.091 | .932 | .742 | .524 | .276 |
| 50 | 1.476 | 1.432 | 1.355 | 1.253 | 1.118 | .956 | .761 | .537 | .283 |
| 7 | 1.512 | 1.467 | 1.389 | 1.284 | 1.146 | .979 | .779 | .551 | .290 |
| 10 | 1.548 | 1.502 | 1.422 | 1.314 | 1.173 | 1.002 | .798 | .564 | .297 |
| 20 | 1.584 | 1.537 | 1.454 | 1.345 | 1.200 | 1.026 | .816 | .576 | .304 |
| 30 | 1.620 | 1.572 | 1.488 | 1.375 | 1.228 | 1.048 | .835 | .590 | .311 |
| 40 | 1.656 | 1.607 | 1.521 | 1.405 | 1.255 | 1.071 | .854 | .603 | .318 |
| 50 | 1.692 | 1.641 | 1.553 | 1.436 | 1.282 | 1.095 | .872 | .616 | .324 |
| 8 | 1.728 | 1.677 | 1.587 | 1.467 | 1.310 | 1.118 | .891 | .629 | .332 |
| 30 | 1.836 | 1.782 | 1.687 | 1.559 | 1.392 | 1.188 | .946 | .669 | .353 |
| 9 | 1.944 | 1.886 | 1.787 | 1.651 | 1.474 | 1.258 | 1.002 | .708 | .373 |
| 30 | 2.052 | 1.991 | 1.887 | 1.742 | 1.556 | 1.328 | 1.057 | .748 | .394 |
| 10 | 2.161 | 2.096 | 1.987 | 1.834 | 1.637 | 1.398 | 1.114 | .787 | .415 |
| 30 | 2.269 | 2.201 | 2.087 | 1.926 | 1.719 | 1.468 | 1.170 | .827 | .436 |
| 11 | 2.377 | 2.305 | 2.186 | 2.018 | 1.802 | 1.538 | 1.226 | .866 | .457 |
| 30 | 2.486 | 2.411 | 2.286 | 2.110 | 1.884 | 1.609 | 1.282 | .906 | .478 |
| 12 | 2.594 | 2.516 | 2.386 | 2.203 | 1.967 | 1.680 | 1.339 | .946 | .499 |
| 30 | 2.703 | 2.621 | 2.485 | 2.295 | 2.049 | 1.750 | 1.395 | .985 | .520 |
| 13 | 2.811 | 2.726 | 2.585 | 2.387 | 2.132 | 1.820 | 1.451 | 1.025 | .541 |
| 30 | 2.920 | 2.832 | 2.685 | 2.479 | 2.214 | 1.891 | 1.507 | 1.065 | .562 |
| 14 | 3.028 | 2.937 | 2.785 | 2.571 | 2.297 | 1.961 | 1.564 | 1.105 | .583 |
| 30 | 3.136 | 3.042 | 2.884 | 2.664 | 2.379 | 2.031 | 1.620 | 1.144 | .604 |
| 15 | 3.245 | 3.147 | 2.984 | 2.756 | 2.462 | 2.102 | 1.676 | 1.184 | .625 |
| 30 | 3.354 | 3.252 | 3.084 | 2.848 | 2.544 | 2.172 | 1.732 | 1.224 | .646 |
| 16 | 3.462 | 3.358 | 3.184 | 2.941 | 2.627 | 2.243 | 1.789 | 1.264 | .667 |
| 17 | 3.680 | 3.569 | 3.384 | 3.125 | 2.792 | 2.384 | 1.902 | 1.344 | .709 |
| 18 | 3.897 | 3.779 | 3.584 | 3.310 | 2.958 | 2.525 | 2.014 | 1.424 | .751 |
| 19 | 4.115 | 3.990 | 3.784 | 3.495 | 3.123 | 2.666 | 2.127 | 1.504 | .793 |
| 20 | 4.332 | 4.201 | 3.984 | 3.680 | 3.288 | 2.808 | 2.240 | 1.583 | .836 |
| 22 | 4.768 | 4.624 | 4.386 | 4.050 | 3.620 | 3.093 | 2.467 | 1.744 | .922 |
| 24 | 5.204 | 5.048 | 4.789 | 4.423 | 3.952 | 3.379 | 2.695 | 1.905 | 1.008 |
| 26 | 5.642 | 5.473 | 5.192 | 4.798 | 4.286 | 3.665 | 2.924 | 2.068 | 1.094 |
| 28 | 6.079 | 5.898 | 5.595 | 5.171 | 4.622 | 3.952 | 3.154 | 2.232 | 1.181 |
| 30 | 6.517 | 6.323 | 5.999 | 5.544 | 4.958 | 4.239 | 3.385 | 2.396 | 1.268 |
| 32 | 6.957 | 6.751 | 6.406 | 5.922 | 5.297 | 4.530 | 3.619 | 2.565 | 1.356 |
| 34 | 7.398 | 7.179 | 6.813 | 6.300 | 5.637 | 4.822 | 3.854 | 2.733 | 1.445 |
| 36 | 7.841 | 7.609 | 7.222 | 6.679 | 5.978 | 5.115 | 4.090 | 2.901 | 1.535 |
| 38 | 8.286 | 8.041 | 7.633 | 7.060 | 6.320 | 5.410 | 4.327 | 3.069 | 1.626 |
| 40 | 8.731 | 8.474 | 8.044 | 7.442 | 6.663 | 5.705 | 4.565 | 3.238 | 1.718 |

**37. Relacion, Q, del arco a la cuerda.** Fig. 9. En cualquiera curva circular de radio $R$, y desarrollo $\Delta$, tenemos (vease ¶ 28) : —

$$\text{Ec (6) longitud, } L_l, \text{ del arco} = \frac{\pi R}{180} \cdot \Delta^{\circ}; \text{ y}$$

$$\text{Ec (28) cuerda larga, } C, = 2\ R \text{ sen } (\Delta/2).$$

Por lo tanto, para la relacion, $Q$, entre el arco, $L_l$, y la cuerda, $C$, tenemos : —

$$Q = \frac{L_l}{C} = \frac{\pi}{360} \cdot \frac{\Delta^{\circ}}{\text{sen } (\Delta/2)}; \quad \ldots\ldots\ldots\

**39.** Aproximadamente : —

$$C = L_1\left(1 - \frac{L_1^2}{24\,R^2}\right);\ \text{y}\ c = a\left(1 - \frac{a^2}{24\,R^2}\right) \ldots\ (42)$$

(Vease Rankine-Civil, Engng, p. 104.) Cuando $\Delta = 50°$ da para $C$, solo 0.031 por ciento muy corto.

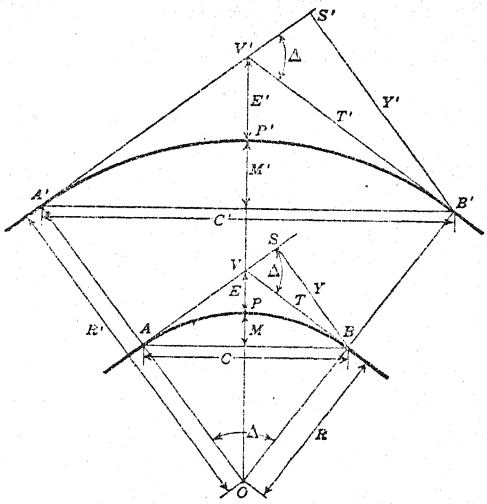



Fig. 13.

## Comparaciones de curvas.

**40.** Fig. 13. Comparación entre dos curvas de un desarrollo **dado**, $\Delta$. Relación entre las **funciones lineales** correspondientes. Sea $F$ una funcion lineal (como el radio, $R$, el arco, $A\ B$ ($A$), la semitangente, $T$, etc.) de una de las dos curvas dadas de igual desarrollo, $\Delta$, pero de diferentes agudezas, $D$, y sea $F'$ la correspondiente función, (como $R'$, $A'\ B'$ ($A'$), $T'$ etc.) de la otra curva. Entónces, de la semejanza de los triangulos, tenemos : —

$$F/F' = R/R' = C/C' = A/A' = T/T',\ \text{etc.};$$

ó bien $F' = FR'/R = FC'/C$, etc.;

$$F = F'R/R' = F'C/C',\ \text{etc} \ldots\ (43)$$

Asi, sea $C$ una unidad de cadena de 100 pies, y sea $C' = 200$ pies. Entonces, por ejemplo, $M' = MC'/C = 2\,M$; $R' = 2\,R$; etc.;

$$\frac{\text{sen}\,(D'/2)}{\text{sen}\,(D/2)} = \frac{50}{R'}\,.\,\frac{R}{50} = \frac{R}{R'}\,.$$

Nótese que, con un desarrollo dado, $\Delta$, la curva mas aguda tiene las funciones lineales mas cortas. Compárese ¶ 43.

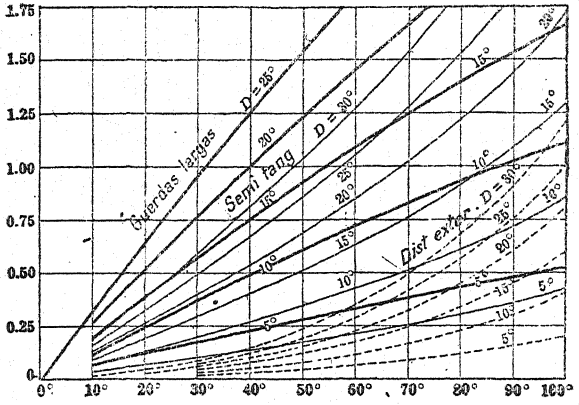
(*Vease N. del T.* — ¶ 11. — pag 926.)

## Funciones de una Curva de 1°.

**40a.** Figs 1 y 11. Tabla de Funciones, $F_1$ (semi tangentes, $T_1$; Distancias Externas $E_1$; y Cuerdas Largas, $C_1$); de una curva de 1° para diferentes desarrollos $\Delta$.
$T_1 = R_1 \tan (\Delta/2)$; $E_1 = R_1 \operatorname{exsec} (\Delta/2)$; $C_1 = 2\, R_1 \operatorname{sen} (\Delta/2)$.

**Para la funcion correspondiente**, $F$ ($T$, $E$ ó $C$) de una curva de **cualquiera otra agudeza**, $D°$, tenemos aproximadamente.

$$F = F_1/D°.$$




### Diagrama de Correcciones.

**Correcciones**, en pies, para ser sumadas a funciones de curvas aproximadas, $F$ (semitangente, $T$; Distancia Externa, $E$; y cuerda Larga, $C$) para una curva de $D°$; como se encuentra dividiendo la correspondiente función de la curva de 1°, $F_1$, de la Tabla, pags 981, etc, por $D$ en grados.

Las curvas continuas finas dan valores para sumarse a las semitangentes, $T$;
Las curvas de puntos finas dan valores para sumarse a las dist. externas, $E$;
Las curvas llenas gruesas dan valores para añadirse a las cuerdas largas $C$.

| Δ | $T_1, p$ | $E_1, p$ | $C_1, p$ | Δ | $T_1, p$ | $E_1, p$ | $C_1, p$ |
|---|---|---|---|---|---|---|---|
| 1° | 50.00 | 0.218 | 100.00 | 11° | 551.70 | 26.500 | 1098.3 |
| 10′ | 58.34 | 0.297 | 116.67 | 10′ | 560.11 | 27.313 | 1114.9 |
| 20 | 66.67 | 0.388 | 133.33 | 20 | 568.53 | 28.137 | 1131.5 |
| 30 | 75.01 | 0.491 | 150.00 | 30 | 576.95 | 28.974 | 1148.1 |
| 40 | 83.34 | 0.606 | 166.66 | 40 | 585.36 | 29.824 | 1164.7 |
| 50 | 91.68 | 0.733 | 183.33 | 50 | 593.79 | 30.686 | 1181.2 |
| 2° | 100.01 | 0.873 | 199.99 | 12° | 602.21 | 31.561 | 1197.8 |
| 10′ | 108.35 | 1.024 | 216.66 | 10′ | 610.64 | 32.447 | 1214.4 |
| 20 | 116.68 | 1.188 | 233.32 | 20 | 619.07 | 33.347 | 1231.0 |
| 30 | 125.02 | 1.364 | 249.98 | 30 | 627.50 | 34.259 | 1247.5 |
| 40 | 133.36 | 1.552 | 266.65 | 40 | 635.93 | 35.183 | 1264.1 |
| 50 | 141.70 | 1.752 | 283.31 | 50 | 644.37 | 36.120 | 1280.7 |
| 3° | 150.04 | 1.964 | 299.97 | 13° | 652.81 | 37.069 | 1297.2 |
| 10′ | 158.38 | 2.188 | 316.63 | 10′ | 661.25 | 38.031 | 1313.8 |
| 20 | 166.72 | 2.425 | 333.29 | 20 | 669.70 | 39.006 | 1330.3 |
| 30 | 175.06 | 2.674 | 349.95 | 30 | 678.15 | 39.993 | 1346.9 |
| 40 | 183.40 | 2.934 | 366.61 | 40 | 686.60 | 40.992 | 1363.4 |
| 50 | 191.74 | 3.207 | 383.27 | 50 | 695.06 | 42.004 | 1380.0 |
| 4° | 200.08 | 3.492 | 399.92 | 14° | 703.51 | 43.029 | 1396.5 |
| 10′ | 208.43 | 3.790 | 416.58 | 10′ | 711.97 | 44.066 | 1413.1 |
| 20 | 216.77 | 4.099 | 433.24 | 20 | 720.44 | 45.116 | 1429.6 |
| 30 | 225.12 | 4.421 | 449.89 | 30 | 728.90 | 46.178 | 1446.2 |
| 40 | 233.47 | 4.755 | 466.54 | 40 | 737.37 | 47.253 | 1462.7 |
| 50 | 241.81 | 5.100 | 483.20 | 50 | 745.85 | 48.341 | 1479.2 |
| 5° | 250.16 | 5.459 | 499.85 | 15° | 754.32 | 49.441 | 1495.7 |
| 10′ | 258.51 | 5.829 | 516.50 | 10′ | 762.80 | 50.554 | 1512.3 |
| 20 | 266.86 | 6.211 | 533.15 | 20 | 771.29 | 51.679 | 1528.8 |
| 30 | 275.21 | 6.606 | 549.80 | 30 | 779.77 | 52.818 | 1545.3 |
| 40 | 283.57 | 7.013 | 566.44 | 40 | 788.26 | 53.969 | 1561.8 |
| 50 | 291.92 | 7.432 | 583.09 | 50 | 796.75 | 55.132 | 1578.3 |
| 6° | 300.28 | 7.863 | 599.73 | 16° | 805.25 | 56.309 | 1594.8 |
| 10′ | 308.64 | 8.307 | 616.38 | 10′ | 813.75 | 57.498 | 1611.3 |
| 20 | 316.99 | 8.762 | 633.02 | 20 | 822.25 | 58.699 | 1627.8 |
| 30 | 325.35 | 9.230 | 649.66 | 30 | 830.76 | 59.914 | 1644.3 |
| 40 | 333.71 | 9.710 | 666.30 | 40 | 839.27 | 61.141 | 1660.8 |
| 50 | 342.08 | 10.202 | 682.94 | 50 | 847.78 | 62.381 | 1677.3 |
| 7° | 350.44 | 10.707 | 699.57 | 17° | 856.30 | 63.634 | 1693.8 |
| 10′ | 358.81 | 11.224 | 716.21 | 10′ | 864.82 | 64.900 | 1710.3 |
| 20 | 367.17 | 11.755 | 732.84 | 20 | 873.35 | 66.178 | 1726.8 |
| 30 | 375.54 | 12.294 | 749.47 | 30 | 881.88 | 67.470 | 1743.2 |
| 40 | 383.91 | 12.847 | 766.10 | 40 | 890.41 | 68.774 | 1759.7 |
| 50 | 392.28 | 13.413 | 782.73 | 50 | 898.95 | 70.091 | 1776.2 |
| 8° | 400.66 | 13.991 | 799.36 | 18° | 907.49 | 71.421 | 1792.6 |
| 10′ | 409.03 | 14.582 | 815.99 | 10′ | 916.03 | 72.764 | 1809.1 |
| 20 | 417.41 | 15.184 | 832.61 | 20 | 924.58 | 74.119 | 1825.5 |
| 30 | 425.79 | 15.799 | 849.23 | 30 | 933.13 | 75.488 | 1842.0 |
| 40 | 434.17 | 16.426 | 865.85 | 40 | 941.69 | 76.869 | 1858.4 |
| 50 | 442.55 | 17.066 | 882.47 | 50 | 950.25 | 78.264 | 1874.9 |
| 9° | 450.93 | 17.717 | 899.09 | 19° | 958.81 | 79.671 | 1891.3 |
| 10′ | 459.32 | 18.381 | 915.70 | 10′ | 967.38 | 81.092 | 1907.8 |
| 20 | 467.71 | 19.058 | 932.31 | 20 | 975.96 | 82.525 | 1924.2 |
| 30 | 476.10 | 19.746 | 948.92 | 30 | 984.53 | 83.972 | 1940.6 |
| 40 | 484.49 | 20.447 | 965.53 | 40 | 993.12 | 85.431 | 1957.1 |
| 50 | 492.88 | 21.161 | 982.14 | 50 | 1001.70 | 86.904 | 1973.5 |
| 10° | 501.28 | 21.886 | 998.74 | 20° | 1010.29 | 88.389 | 1989.9 |
| 10′ | 509.68 | 22.624 | 1015.35 | 10′ | 1018.89 | 89.888 | 2006.3 |
| 20 | 518.08 | 23.375 | 1031.95 | 20 | 1027.49 | 91.399 | 2022.7 |
| 30 | 526.48 | 24.138 | 1048.54 | 30 | 1036.09 | 92.924 | 2039.1 |
| 40 | 534.89 | 24.913 | 1065.14 | 40 | 1044.70 | 94.462 | 2055.5 |
| 50 | 543.29 | 25.700 | 1081.73 | 50 | 1053.31 | 96.013 | 2071.9 |
| 11° | 551.70 | 26.500 | 1098.33 | 21° | 1061.93 | 97.577 | 2088.3 |

| Δ | $T_1, p$ | $E_1, p$ | $C_1, p$ | Δ | $T_1, p$ | $E_1, p$ | $C_1, p$ |
|---|---|---|---|---|---|---|---|
| **21°** | 1061.9 | 97.58 | 2088.3 | **31°** | 1589.0 | 216.25 | 3062.4 |
| 10′ | 1070.6 | 99.15 | 2104.7 | 10′ | 1598.0 | 218.66 | 3078.4 |
| 20 | 1079.2 | 100.75 | 2121.1 | 20 | 1606.9 | 221.08 | 3094.5 |
| 30 | 1087.8 | 102.35 | 2137.4 | 30 | 1615.9 | 223.51 | 3110.5 |
| 40 | 1096.4 | 103.97 | 2153.8 | 40 | 1624.9 | 225.96 | 3126.6 |
| 50 | 1105.1 | 105.60 | 2170.2 | 50 | 1633.9 | 228.42 | 3142.6 |
| **22°** | 1113.7 | 107.24 | 2186.5 | **32°** | 1643.0 | 230.90 | 3158.6 |
| 10′ | 1122.4 | 108.90 | 2202.9 | 10′ | 1652.0 | 233.39 | 3174.6 |
| 20 | 1131.0 | 110.57 | 2219.2 | 20 | 1661.0 | 235.90 | 3190.6 |
| 30 | 1139.7 | 112.25 | 2235.6 | 30 | 1670.0 | 238.43 | 3206.6 |
| 40 | 1148.4 | 113.95 | 2251.9 | 40 | 1679.1 | 240.96 | 3222.6 |
| 50 | 1157.0 | 115.66 | 2268.3 | 50 | 1688.1 | 243.52 | 3238.6 |
| **23°** | 1165.7 | 117.38 | 2284.6 | **33°** | 1697.2 | 246.08 | 3254.6 |
| 10′ | 1174.4 | 119.12 | 2301.0 | 10′ | 1706.3 | 248.66 | 3270.6 |
| 20 | 1183.1 | 120.87 | 2317.3 | 20 | 1715.3 | 251.26 | 3286.6 |
| 30 | 1191.8 | 122.63 | 2333.6 | 30 | 1724.4 | 253.87 | 3302.5 |
| 40 | 1200.5 | 124.41 | 2349.9 | 40 | 1733.5 | 256.50 | 3318.5 |
| 50 | 1209.2 | 126.20 | 2366.2 | 50 | 1742.6 | 259.14 | 3334.4 |
| **24°** | 1217.9 | 128.00 | 2382.5 | **34°** | 1751.7 | 261.80 | 3350.4 |
| 10′ | 1226.6 | 129.82 | 2398.8 | 10′ | 1760.8 | 264.47 | 3366.3 |
| 20 | 1235.3 | 131.65 | 2415.1 | 20 | 1770.0 | 267.16 | 3382.2 |
| 30 | 1244.0 | 133.50 | 2431.4 | 30 | 1779.1 | 269.86 | 3398.2 |
| 40 | 1252.8 | 135.36 | 2447.7 | 40 | 1788.2 | 272.58 | 3414.1 |
| 50 | 1261.5 | 137.23 | 2464.0 | 50 | 1797.4 | 275.31 | 3430.0 |
| **25°** | 1270.2 | 139.11 | 2480.2 | **35°** | 1806.6 | 278.05 | 3445.9 |
| 10′ | 1279.0 | 141.01 | 2496.5 | 10′ | 1815.7 | 280.82 | 3461.8 |
| 20 | 1287.7 | 142.93 | 2512.8 | 20 | 1824.9 | 283.60 | 3477.7 |
| 30 | 1296.5 | 144.85 | 2529.0 | 30 | 1834.1 | 286.39 | 3493.5 |
| 40 | 1305.3 | 146.79 | 2545.3 | 40 | 1843.3 | 289.20 | 3509.4 |
| 50 | 1314.0 | 148.75 | 2561.5 | 50 | 1852.5 | 292.02 | 3525.3 |
| **26°** | 1322.8 | 150.71 | 2577.8 | **36°** | 1861.7 | 294.86 | 3541.1 |
| 10′ | 1331.6 | 152.69 | 2594.0 | 10′ | 1870.9 | 297.72 | 3557.0 |
| 20 | 1340.4 | 154.69 | 2610.3 | 20 | 1880.1 | 300.59 | 3572.8 |
| 30 | 1349.2 | 156.70 | 2626.5 | 30 | 1889.4 | 303.47 | 3588.6 |
| 40 | 1358.0 | 158.72 | 2642.7 | 40 | 1898.6 | 306.37 | 3604.5 |
| 50 | 1366.8 | 160.76 | 2658.9 | 50 | 1907.9 | 309.29 | 3620.3 |
| **27°** | 1375.6 | 162.81 | 2675.1 | **37°** | 1917.1 | 312.22 | 3636.1 |
| 10′ | 1384.4 | 164.87 | 2691.3 | 10′ | 1926.4 | 315.17 | 3651.9 |
| 20 | 1393.2 | 166.95 | 2707.5 | 20 | 1935.7 | 318.13 | 3667.7 |
| 30 | 1402.0 | 169.04 | 2723.7 | 30 | 1945.0 | 321.11 | 3683.5 |
| 40 | 1410.9 | 171.15 | 2739.9 | 40 | 1954.3 | 324.11 | 3699.3 |
| 50 | 1419.7 | 173.27 | 2756.1 | 50 | 1963.6 | 327.12 | 3715.0 |
| **28°** | 1428.6 | 175.41 | 2772.3 | **38°** | 1972.9 | 330.15 | 3730.8 |
| 10′ | 1437.4 | 177.55 | 2788.4 | 10′ | 1982.2 | 333.19 | 3746.5 |
| 20 | 1446.3 | 179.72 | 2804.6 | 20 | 1991.5 | 336.25 | 3762.3 |
| 30 | 1455.1 | 181.89 | 2820.7 | 30 | 2000.9 | 339.32 | 3778.0 |
| 40 | 1464.0 | 184.08 | 2836.9 | 40 | 2010.2 | 342.41 | 3793.8 |
| 50 | 1472.9 | 186.29 | 2853.0 | 50 | 2019.6 | 345.52 | 3809.5 |
| **29°** | 1481.8 | 188.51 | 2869.2 | **39°** | 2029.0 | 348.64 | 3825.2 |
| 10′ | 1490.7 | 190.74 | 2885.3 | 10′ | 2038.4 | 351.78 | 3840.9 |
| 20 | 1499.6 | 192.99 | 2901.4 | 20 | 2047.8 | 354.94 | 3856.6 |
| 30 | 1508.5 | 195.25 | 2917.6 | 30 | 2057.2 | 358.11 | 3872.3 |
| 40 | 1517.4 | 197.53 | 2933.7 | 40 | 2066.6 | 361.29 | 3888.0 |
| 50 | 1526.3 | 199.82 | 2949.8 | 50 | 2076.0 | 364.50 | 3903.6 |
| **30°** | 1535.3 | 202.12 | 2965.9 | **40°** | 2085.4 | 367.72 | 3919.3 |
| 10′ | 1544.2 | 204.44 | 2982.0 | 10′ | 2094.9 | 370.95 | 3935.0 |
| 20 | 1553.1 | 206.77 | 2998.1 | 20 | 2104.3 | 374.20 | 3950.6 |
| 30 | 1562.1 | 209.12 | 3014.2 | 30 | 2113.8 | 377.47 | 3966.3 |
| 40 | 1571.0 | 211.48 | 3030.2 | 40 | 2123.3 | 380.76 | 3981.9 |
| 50 | 1580.0 | 213.86 | 3046.3 | 50 | 2132.7 | 384.06 | 3997.5 |
| **31°** | 1589.0 | 216.25 | 3062.4 | **41°** | 2142.2 | 387.38 | 4013.1 |

| Δ | $T_1$, $p$ | $E_1$, $p$ | $C_1$, $p$ | Δ | $T_1$, $p$ | $E_1$, $p$ | $C_1$, $p$ |
|---|---|---|---|---|---|---|---|
| **41°** | 2142.2 | 387.38 | 4013.1 | **51°** | 2732.9 | 618.39 | 4933.4 |
| 10′ | 2151.7 | 390.71 | 4028.7 | 10′ | 2743.1 | 622.81 | 4948.4 |
| 20 | 2161.2 | 394.06 | 4044.3 | 20 | 2753.4 | 627.24 | 4963.4 |
| 30 | 2170.8 | 397.43 | 4059.9 | 30 | 2763.7 | 631.69 | 4978.4 |
| 40 | 2180.3 | 400.82 | 4075.5 | 40 | 2773.9 | 636.16 | 4993.4 |
| 50 | 2189.9 | 404.22 | 4091.1 | 50 | 2784.2 | 640.66 | 5008.4 |
| **42°** | 2199.4 | 407.64 | 4106.6 | **52°** | 2794.5 | 645.17 | 5023.4 |
| 10′ | 2209.0 | 411.07 | 4122.2 | 10′ | 2804.9 | 649.70 | 5038.4 |
| 20 | 2218.6 | 414.52 | 4137.7 | 20 | 2815.2 | 654.25 | 5053.4 |
| 30 | 2228.1 | 417.99 | 4153.3 | 30 | 2825.6 | 658.83 | 5068.3 |
| 40 | 2237.7 | 421.48 | 4168.8 | 40 | 2835.9 | 663.42 | 5083.3 |
| 50 | 2247.3 | 424.98 | 4184.3 | 50 | 2846.3 | 668.03 | 5098.2 |
| **43°** | 2257.0 | 428.50 | 4199.8 | **53°** | 2856.7 | 672.66 | 5113.1 |
| 10′ | 2266.6 | 432.04 | 4215.3 | 10′ | 2867.1 | 677.32 | 5128.0 |
| 20 | 2276.2 | 435.59 | 4230.8 | 20 | 2877.5 | 681.99 | 5142.9 |
| 30 | 2285.9 | 439.16 | 4246.3 | 30 | 2888.0 | 686.68 | 5157.8 |
| 40 | 2295.6 | 442.75 | 4261.8 | 40 | 2898.4 | 691.40 | 5172.7 |
| 50 | 2305.2 | 446.35 | 4277.3 | 50 | 2908.9 | 696.13 | 5187.6 |
| **44°** | 2314.9 | 449.98 | 4292.7 | **54°** | 2919.4 | 700.89 | 5202.4 |
| 10′ | 2324.6 | 453.62 | 4308.2 | 10′ | 2929.9 | 705.66 | 5217.3 |
| 20 | 2334.3 | 457.27 | 4323.6 | 20 | 2940.4 | 710.46 | 5232.1 |
| 30 | 2344.1 | 460.95 | 4339.0 | 30 | 2951.0 | 715.28 | 5246.9 |
| 40 | 2353.8 | 464.64 | 4354.5 | 40 | 2961.5 | 720.11 | 5261.7 |
| 50 | 2363.5 | 468.35 | 4369.9 | 50 | 2972.1 | 724.97 | 5276.5 |
| **45°** | 2373.3 | 472.08 | 4385.3 | **55°** | 2982.7 | 729.85 | 5291.3 |
| 10′ | 2383.1 | 475.82 | 4400.7 | 10′ | 2993.3 | 734.76 | 5306.1 |
| 20 | 2392.8 | 479.59 | 4416.1 | 20 | 3003.9 | 739.68 | 5320.9 |
| 30 | 2402.6 | 483.37 | 4431.4 | 30 | 3014.5 | 744.62 | 5335.6 |
| 40 | 2412.4 | 487.16 | 4446.8 | 40 | 3025.2 | 749.59 | 5350.4 |
| 50 | 2422.3 | 490.98 | 4462.2 | 50 | 3035.8 | 754.57 | 5365.1 |
| **46°** | 2432.1 | 494.82 | 4477.5 | **56°** | 3046.5 | 759.58 | 5379.8 |
| 10′ | 2441.9 | 498.67 | 4492.8 | 10′ | 3057.2 | 764.61 | 5394.5 |
| 20 | 2451.8 | 502.54 | 4508.2 | 20 | 3067.9 | 769.66 | 5409.2 |
| 30 | 2461.7 | 506.42 | 4523.5 | 30 | 3078.7 | 774.73 | 5423.9 |
| 40 | 2471.5 | 510.33 | 4538.8 | 40 | 3089.4 | 779.83 | 5438.6 |
| 50 | 2481.4 | 514.25 | 4554.1 | 50 | 3100.2 | 784.94 | 5453.3 |
| **47°** | 2491.3 | 518.20 | 4569.4 | **57°** | 3110.9 | 790.08 | 5467.9 |
| 10′ | 2501.2 | 522.16 | 4584.7 | 10′ | 3121.7 | 795.24 | 5482.5 |
| 20 | 2511.2 | 526.13 | 4599.9 | 20 | 3132.6 | 800.42 | 5497.2 |
| 30 | 2521.1 | 530.13 | 4615.2 | 30 | 3143.4 | 805.62 | 5511.8 |
| 40 | 2531.1 | 534.15 | 4630.4 | 40 | 3154.2 | 810.85 | 5526.4 |
| 50 | 2541.0 | 538.18 | 4645.7 | 50 | 3165.1 | 816.10 | 5541.0 |
| **48°** | 2551.0 | 542.23 | 4660.9 | **58°** | 3176.0 | 821.37 | 5555.6 |
| 10′ | 2561.0 | 546.30 | 4676.1 | 10′ | 3186.9 | 826.66 | 5570.2 |
| 20 | 2571.0 | 550.39 | 4691.3 | 20 | 3197.8 | 831.98 | 5584.7 |
| 30 | 2581.0 | 554.50 | 4706.5 | 30 | 3208.8 | 837.31 | 5599.3 |
| 40 | 2591.1 | 558.63 | 4721.7 | 40 | 3219.7 | 842.67 | 5613.8 |
| 50 | 2601.1 | 562.77 | 4736.9 | 50 | 3230.7 | 848.06 | 5628.3 |
| **49°** | 2611.2 | 566.94 | 4752.1 | **59°** | 3241.7 | 853.46 | 5642.8 |
| 10′ | 2621.2 | 571.12 | 4767.3 | 10′ | 3252.7 | 858.89 | 5657.3 |
| 20 | 2631.3 | 575.32 | 4782.4 | 20 | 3263.7 | 864.34 | 5671.8 |
| 30 | 2641.4 | 579.54 | 4797.5 | 30 | 3274.8 | 869.82 | 5686.3 |
| 40 | 2651.5 | 583.78 | 4812.7 | 40 | 3285.8 | 875.32 | 5700.8 |
| 50 | 2661.6 | 588.04 | 4827.8 | 50 | 3296.9 | 880.84 | 5715.2 |
| **50°** | 2671.8 | 592.32 | 4842.9 | **60°** | 3308.0 | 886.38 | 5729.7 |
| 10′ | 2681.9 | 596.62 | 4858.0 | 10′ | 3319.1 | 891.95 | 5744.1 |
| 20 | 2692.1 | 600.93 | 4873.1 | 20 | 3330.3 | 897.54 | 5758.5 |
| 30 | 2702.3 | 605.27 | 4888.2 | 30 | 3341.4 | 903.15 | 5772.9 |
| 40 | 2712.5 | 609.62 | 4903.2 | 40 | 3352.6 | 908.79 | 5787.3 |
| 50 | 2722.7 | 614.00 | 4918.3 | 50 | 3363.8 | 914.45 | 5801.7 |
| **51°** | 2732.9 | 618.39 | 4933.4 | **61°** | 3375.0 | 920.14 | 5816.0 |

| Δ | $T_1, \rho$. | $E_1, \rho$. | $C_1, \rho$. | Δ | $T_1, \rho$ | $E_1, \rho$ | $C_1, \rho$ |
|---|---|---|---|---|---|---|---|
| **61°** | 3375.0 | 920.14 | 5816.0 | **71°** | 4086.9 | 1308.2 | 6654.4 |
| 10′ | 3386.3 | 925.85 | 5830.4 | 10′ | 4099.5 | 1315.5 | 6668.0 |
| 20 | 3397.5 | 931.58 | 5844.7 | 20 | 4112.1 | 1322.9 | 6681.6 |
| 30 | 3408.8 | 937.34 | 5859.1 | 30 | 4124.8 | 1330.3 | 6695.1 |
| 40 | 3420.1 | 943.12 | 5873.4 | 40 | 4137.4 | 1337.7 | 6708.6 |
| 50 | 3431.4 | 948.92 | 5887.7 | 50 | 4150.1 | 1345.1 | 6722.1 |
| **62°** | 3442.7 | 954.75 | 5902.0 | **72°** | 4162.8 | 1352.6 | 6735.6 |
| 10′ | 3454.1 | 960.60 | 5916.3 | 10′ | 4175.6 | 1360.1 | 6749.1 |
| 20 | 3465.4 | 966.48 | 5930.5 | 20 | 4188.4 | 1367.6 | 6762.5 |
| 30 | 3476.8 | 972.39 | 5944.8 | 30 | 4201.2 | 1375.2 | 6776.0 |
| 40 | 3488.2 | 978.31 | 5959.0 | 40 | 4214.0 | 1382.8 | 6789.4 |
| 50 | 3499.7 | 984.27 | 5973.3 | 50 | 4226.8 | 1390.4 | 6802.8 |
| **63°** | 3511.1 | 990.24 | 5987.5 | **73°** | 4239.7 | 1398.0 | 6816.3 |
| 10′ | 3522.6 | 996.24 | 6001.7 | 10′ | 4252.6 | 1405.7 | 6829.6 |
| 20 | 3534.1 | 1002.3 | 6015.9 | 20 | 4265.6 | 1413.5 | 6843.0 |
| 30 | 3545.6 | 1008.3 | 6030.0 | 30 | 4278.5 | 1421.2 | 6856.4 |
| 40 | 3557.2 | 1014.4 | 6044.2 | 40 | 4291.5 | 1429.0 | 6869.7 |
| 50 | 3568.7 | 1020.5 | 6058.4 | 50 | 4304.6 | 1436.8 | 6883.1 |
| **64°** | 3580.3 | 1026.6 | 6072.5 | **74°** | 4317.6 | 1444.6 | 6896.4 |
| 10′ | 3591.9 | 1032.8 | 6086.6 | 10′ | 4330.7 | 1452.5 | 6909.7 |
| 20 | 3603.5 | 1039.0 | 6100.7 | 20 | 4343.8 | 1460.4 | 6923.0 |
| 30 | 3615.1 | 1045.2 | 6114.8 | 30 | 4356.9 | 1468.4 | 6936.2 |
| 40 | 3626.8 | 1051.4 | 6128.9 | 40 | 4370.1 | 1476.4 | 6949.5 |
| 50 | 3638.5 | 1057.7 | 6143.0 | 50 | 4383.3 | 1484.4 | 6962.8 |
| **65°** | 3650.2 | 1063.9 | 6157.1 | **75°** | 4396.5 | 1492.4 | 6976.0 |
| 10′ | 3661.9 | 1070.2 | 6171.1 | 10′ | 4409.8 | 1500.5 | 6989.2 |
| 20 | 3673.7 | 1076.6 | 6185.2 | 20 | 4423.1 | 1508.6 | 7002.4 |
| 30 | 3685.4 | 1082.9 | 6199.2 | 30 | 4436.4 | 1516.7 | 7015.6 |
| 40 | 3697.2 | 1089.3 | 6213.2 | 40 | 4449.7 | 1524.9 | 7028.8 |
| 50 | 3709.0 | 1095.7 | 6227.2 | 50 | 4463.1 | 1533.1 | 7041.9 |
| **66°** | 3720.9 | 1102.2 | 6241.2 | **76°** | 4476.5 | 1541.4 | 7055.0 |
| 10′ | 3732.7 | 1108.6 | 6255.2 | 10′ | 4489.9 | 1549.7 | 7068.2 |
| 20 | 3744.6 | 1115.1 | 6269.1 | 20 | 4503.4 | 1558.0 | 7081.3 |
| 30 | 3756.5 | 1121.7 | 6283.1 | 30 | 4516.9 | 1566.3 | 7094.4 |
| 40 | 3768.5 | 1128.2 | 6297.0 | 40 | 4530.4 | 1574.7 | 7107.5 |
| 50 | 3780.4 | 1134.8 | 6310.9 | 50 | 4544.0 | 1583.1 | 7120.5 |
| **67°** | 3792.4 | 1141.4 | 6324.8 | **77°** | 4557.6 | 1591.6 | 7133.6 |
| 10′ | 3804.4 | 1148.0 | 6338.7 | 10′ | 4571.2 | 1600.1 | 7146.6 |
| 20 | 3816.4 | 1154.7 | 6352.6 | 20 | 4584.8 | 1608.6 | 7159.6 |
| 30 | 3828.4 | 1161.3 | 6366.4 | 30 | 4598.5 | 1617.1 | 7172.6 |
| 40 | 3840.5 | 1168.1 | 6380.3 | 40 | 4612.2 | 1625.7 | 7185.6 |
| 50 | 3852.6 | 1174.8 | 6394.1 | 50 | 4626.0 | 1634.4 | 7198.6 |
| **68°** | 3864.7 | 1181.6 | 6408.0 | **78°** | 4639.8 | 1643.0 | 7211.6 |
| 10′ | 3876.8 | 1188.4 | 6421.8 | 10′ | 4653.6 | 1651.7 | 7224.5 |
| 20 | 3889.0 | 1195.2 | 6435.6 | 20 | 4667.4 | 1660.5 | 7237.4 |
| 30 | 3901.2 | 1202.0 | 6449.4 | 30 | 4681.3 | 1669.2 | 7250.4 |
| 40 | 3913.4 | 1208.9 | 6463.1 | 40 | 4695.2 | 1678.1 | 7263.3 |
| 50 | 3925.6 | 1215.8 | 6476.9 | 50 | 4709.2 | 1686.9 | 7276.1 |
| **69°** | 3937.9 | 1222.7 | 6490.6 | **79°** | 4723.2 | 1695.8 | 7289.0 |
| 10′ | 3950.2 | 1229.7 | 6504.4 | 10′ | 4737.2 | 1704.7 | 7301.9 |
| 20 | 3962.5 | 1236.7 | 6518.1 | 20 | 4751.2 | 1713.7 | 7314.7 |
| 30 | 3974.8 | 1243.7 | 6531.8 | 30 | 4765.3 | 1722.7 | 7327.5 |
| 40 | 3987.2 | 1250.8 | 6545.5 | 40 | 4779.4 | 1731.7 | 7340.3 |
| 50 | 3999.5 | 1257.9 | 6559.1 | 50 | 4793.6 | 1740.8 | 7353.1 |
| **70°** | 4011.9 | 1265.0 | 6572.8 | **80°** | 4807.7 | 1749.9 | 7365.9 |
| 10′ | 4024.4 | 1272.1 | 6586.4 | 10′ | 4822.0 | 1759.0 | 7378.7 |
| 20 | 4036.8 | 1279.3 | 6600.1 | 20 | 4836.2 | 1768.2 | 7391.4 |
| 30 | 4049.3 | 1286.5 | 6613.7 | 30 | 4850.5 | 1777.4 | 7404.1 |
| 40 | 4061.8 | 1293.7 | 6627.3 | 40 | 4864.8 | 1786.7 | 7416.8 |
| 50 | 4074.4 | 1300.9 | 6640.9 | 50 | 4879.2 | 1796.0 | 7429.5 |
| **71°** | 4086.9 | 1308.2 | 6654.4 | **81°** | 4893.6 | 1805.3 | 7442.2 |

**41. Medidas Inglesas y Métricas.** Fig. 13. Comparación entre las curvas en sistema inglés y métrico (u otras medidas). Sea, otra vez

$C = c$ = una cadena de $n$ metros ($\Delta = D$).

$C' = c' =$ — — de 100 pies (30.48 m) ($\Delta = D'$).

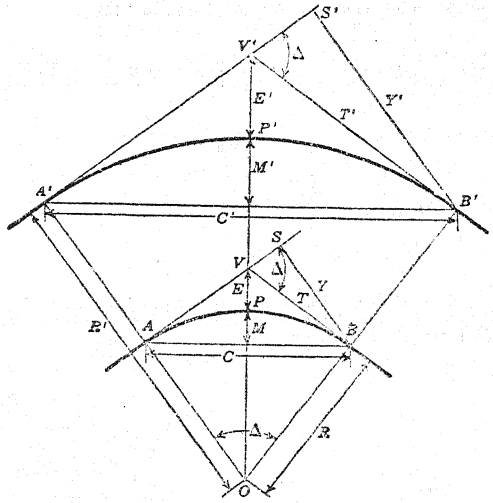



**Fig. 13. (Repetida).**

Entonces las dos curvas, en Fig. 13, aunque de diferente agudeza, son *citadas* por el mismo numero de grados; porque, en cada una, la cadena adoptada subtiende el mismo ángulo, $D = D'$.

Por ejemplo, de la ecuacion (43), tenemos : —

$$\frac{\text{R metros}}{R' \text{ pies}} = \frac{c \text{ metros}}{c' \text{ pies}} = \frac{n}{100} ; y$$

$$R \text{ metros} = R' \text{ pies} \frac{c \text{ metros}}{c' \text{ pies}} = R' \text{ pies } n/100. \text{ O, en general : —}$$

$$F = F' \; c/c' = 0.01 \; n \; F' \ldots\ldots\ldots\

Entonces, donde se use la cadena « *disminuida* » (¶ 20), tenemos ($D$ en grados).

$$D = F_1/F\,d;\ F\,d = F_1/D;\ F_1 = D\,F\,d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (45)$$

Pero, donde se use la cadena completa, estas ecuaciones son solamente aproximadas. Para la relacion, $q$, entre el verdadero valor, $F\,d$, y el valor aproximado, $F_1/D$, tenemos; $R_1 = 50/\text{sen } 0°30' = 5729.65$; $\log R_1 = 3.758\,1281$.

$$q\left(=\frac{F\,d}{F_1/D}=\frac{R\,d}{R_1/D}\right)=\frac{50}{\text{sen }(D/2)}\cdot\frac{D\text{ sen }0°30'}{50}=D\,\frac{\text{sen }0°30'}{\text{sen }(D/2)} \quad \ldots\ldots \quad (46)$$

y, como (ecua 41, ¶ 38) $Q = \frac{a}{c} = \frac{\pi}{360}\times\frac{D}{\text{sen }(D/2)}$; tenemos : —

$$\frac{Q}{q}=\frac{\pi}{360}\cdot\frac{D}{\text{sen }(D/2)}\cdot\frac{\text{sen }(D/2)}{D\text{ sen }0°30'}=\frac{\pi}{360}\cdot\frac{1}{\text{sen }0°30'}=1.000\,013\ldots\ldots \quad (47)$$

$$\text{Log }(Q/q)=0.000\,0055.$$

En otras palabras, el valor de $Q$ (= arco/cuerda), en la tabla, ¶ 37, para cualquier valor dado de $\Delta$, puede tomarse practicamente identico al valor de $q$ [$= F\,d/(F_1/D)$], para el mismo valor de $D$.

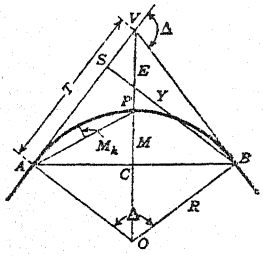



**Fig. 14a.**

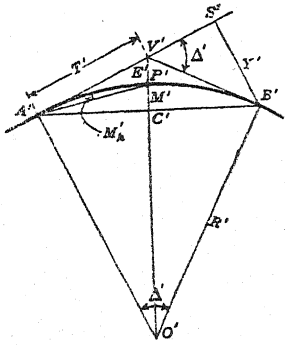



**Fig. 14b.**

*43.* Figs 14*a*, 14*b*. Entre dos curvas de **igual largo**, $L_C$ o $L_c$, la mas aguda (Fig. 14*a*) tiene por supuesto, el radio mas corto, $R$; pero tiene un ángulo, mayor, $\Delta$, y *mayores* valores para $T$, $M$, $E$, $M_h$ y $Y$. (Comparese ¶ 40.) Estos valores pueden encontrase por medio de las ecuaciones pag 967 ¶ 34.

*44.* Fig. 15. **Sub-cadenas**, $c_s$, $c_i$, $c_f$*. Vease ¶ 24. Supongamos que la curva empiece en $A$, $w$ pies despues de $y$, y que termine en $B$.

---

* Donde es necesario distinguir entre la inicial y la sub-cadena final, usamos los índices, $i$ y $f$, respectivamente. De otro modo, usamos el índice, $s$.

**Si a b = una cadena disminuida** = casi de 100 pies, y *a m b* = una unidad de arco = 100 pies (vease ¶ 20), tenemos : —

Sub-arco inicial, $A\ a = 100 - w$ ;

Sub-angulo inicial, $d_i = D\ \frac{100 - w}{100}$ ;

Sub-cadena inicial, $c_i = A\ a = 2\ R \operatorname{sen}(d_i/2) = c\ \frac{\operatorname{sen}(d_i/2)}{\operatorname{sen}(D/2)}$ ..... (48)

Habiendo determinado $b$, tenemos : —

Sub-angulo final, $d_f = b\ O\ B = 2\ b\ A\ B$ ;

Sub-arco final, $b\ B = 100\ \frac{d_f}{D}$ ;

Sub-cadena final, $c_f = b\ B = 2\ R \operatorname{sen}(d_f/2) = c\ \frac{\operatorname{sen}(d_f/2)}{\operatorname{sen}(D/2)}$ ;

$x = B\ z = 100 -$ (sub-arco final, $b\ B$) ..... (49)

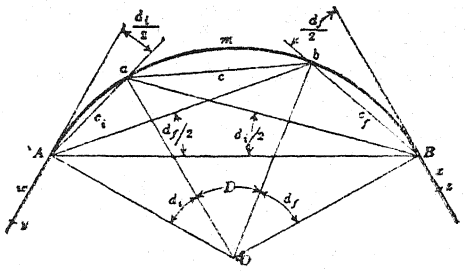



**Fig. 15.**

**45.** Fig. 15. **Con la cadena completa** ($c = a\ b = 100$ pies; unidad de arco, $a = a\ m\ b$, $> 100$ pies) tenemos : —

Sub-cadena inicial, $c_i = A\ a = 100$ pies $- w$ ;

Para sub-angulo inicial;

$$\operatorname{sen}\frac{d_i}{2} = \frac{c_i}{2\ R} = c_i\ \frac{\operatorname{sen}(D/2)}{100} \quad \ldots\ldots\ldots\ldots\ldots \quad (50)$$

Habiendo determinado $b$, tenemos, como antes : —

Sub-angulo final, $d_f = b\ O\ B = 2\ b\ A\ B$ ; y

Sub-cadena final, $c_f = b\ B = 2\ R \operatorname{sen}(d_f/2) = 100\ \frac{\operatorname{sen}(d_f/2)}{\operatorname{sen}(D/2)}$ ;

Pero $x = B_z = 100$ pies $-$ sub-*cadena* final, $c_f$ ..... (51)

**46.** Fig. 15. **Valores « nominales » o aproximados** de $c_s$ * y de $d$. **Con unidad de cadena completa o disminuida**, sea :

$c_s$ = el *verdadero* valor de cualquiera de las dos sub-cadenas, como se encuentra arriba;

$d$ = al *verdadero* valor del sub-angulo correspondiente;

$c$ = la cadena usada completa o disminuida, como sea el caso;

$c_n = c\ d/D$ = el valor aprox o « nominal » de $c_s$ ;

$d_n = D\ c_s/c$ = el valor aprox o « nominal » de $d$.

---

* Donde es necesario distinguir entre la sub-cadena inicial y final, usamos los índices, *i* y *f* respectivamente. De otro modo, usamos el índice, *s*.

Entonces:

$$c_s > c_n\,;\; d_n > d\,;\; \frac{c_s}{c_n} = \frac{d_n}{d} \qquad (52)$$

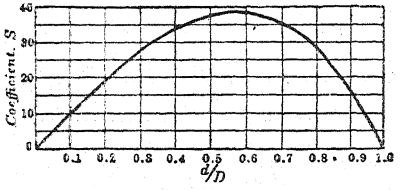
y, para la cadena de 100 pies o unidad de arco de 100 pies, aprox,

$$c_s - c_n = S\,(Q - 1) \qquad (53)$$

Donde $Q$ = valor del $\frac{\text{arco}}{\text{cuerda}}$ (tabla, ¶ 37) para el valor dado de $D$,

y $S$ = un coeficiente dado por la Fig. 16, abajo.

**Fig. 16.**




Coeficiente, $S$, para deducir el verdadero valor de $c_s$, de la sub-cadena, de su valor « nominal », $c_n = cd/D$.

**47.** Para cualquier valor, $c$, de la cadena, o unidad de arco distinto de la de 100 pies, tenemos: —

$$c_s - c_n = S\,(Q - 1)\,c/100 \qquad (54)$$

**48.** Del ¶ 27, tenemos, aprox: —

Sen 0°1′ : sen $(d/2)$ = 1 min : $d/2$ en min o bien $d/2$ en min = sen $(d/2) \div$ 0.000 2909 = 3437 $\frac{\text{sub-cadena.}}{2R}$ ..... (55)

**49.** Si, en una curva circular dada, la cadena completa y la sub-cadena se dividen en el mismo numero de partes iguales, sus ordenadas estan aproximadamente como los cuadrados de los largos de cadena.

## CURVAS COMPUESTAS

Comparense las Curvas Invertidas, ¶ ¶ 70, etc.

### Definiciones.

**50.** Figs 17, 18. Curvas compuestas. Cuando dos curvas consecutivas, $A\ P$ y $P\ B$, de radios desiguales, $R_g$ y $R_s$, se encurvan en la misma direccion (ambas a la derecha o a la izquierda), se dice que estan compuestas o combinadas, y la curva entera, $A\ P\ B$, se llama curva compuesta.

**51. Ramas.** Las dos porciones, $A\ P$ y $P\ B$, de la curva compuesta, se llaman sus ramas. Ellas caen sobre el mismo lado de la tangente comun, $v_g\ v_s$. Su punto de encuentro, $P$, o punto de tangencia comun, es el punto « C. C. » de cambio de *curva* a *curva*. Tambien se llama el punto « P. C. C. » compuesto de curvatura. Vease ¶ 2.

**52. Radios.** Los centros, $O_g$ y $O_s$ de las dos ramas, estan necesariamente en linea recta con el punto compuesto de curvatura $(P)$: *i. e.*, los dos radios son normales a la tang comun $V_g\ V_s$, en $P$ coincide en $O_s\ P$. En $A$ y en $B$ el radio es normal respectivamente a las semitangentes, $A\ V$, $V\ B$, de la curva entera.

**53. El vertice, V**, de la curva entera compuesta, no se encuentra (como está en la Fig. 17) generalmente en $O_g\ P$ prolongada. Vease ¶ ¶ 57 y 146.

**54. Indices.** $R_g$ es el radio *mayor;* y el desarrollo de su curva, semitangente, etc., correspondientes estan designadas con $\Delta_g$ (o simplemente $g$), $T_g$, etc., respectivamente; y del mismo modo para los *radios* mas cortos, $R_s$, con $\Delta_s$ (o solo $s$) semitangente, $T_s$, etc., etc.

**55. Las semitangentes** de la curva entera, $A\ P\ B$, son $T_g$ y $T_s$; respectivamente perpendiculares, en $A$ y en $B$, o *vice versa*, al radio, $R_g$ y $R_s$. Las semitangentes de las dos ramas son respectivamente, $t_g = v_g\ P$, y $t_s = P\ v_s$.

**56.** Si (como es costumbre y como está en las figs 17, 18) $\Delta < 180°$, encontramos $T_g > T_s$; pero, cuando $\Delta > 180° < 360°$, $T_g < T_s$. Vease ¶ 69. Si $\Delta = 180°$, $T_g$ y $T_s$ son infinitas, y no hay vertice, $V$.

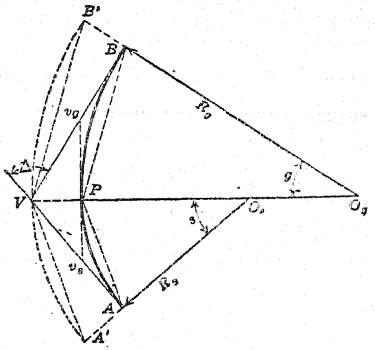



**Fig. 17.**

## Ecuaciones.

**57.** Fig. 17. En el **caso especial** en que el vertice, $V$, queda en la linea radial comun, $O_g\ P$, prolongada, como en fig. 17, tenemos :

$$V\ P = R_g \text{ exsec } g = R_s \text{ exsec } s.$$

Por tanto :

$$\frac{R_g}{R_s} = \frac{\text{exsec } s}{\text{exsec } g} = \frac{1 - \cos s}{\cos s} \cdot \frac{\cos g}{1 - \cos g} \quad \ldots\ldots\ldots \quad (56)$$

Y, en este caso especial, tenemos $s > g$, o el radio *mas corto*, $R_s$, corresponde la mayor *extension*, $s$, y *vice versa*.

Tambien en este caso, tenemos :

$$T_g = R_s \text{ tang } g\,; \qquad T_s = R_s \text{ tang } s\,;$$

$$T_g \text{ tang } (g/2) = T_g \text{ tang } V\ B\ P = T_g \text{ tang } B'\ V\ B$$
$$= B'\ B = V\ P = A'A$$
$$= T_s \text{ tang } (s/2); \text{ o}$$

$$\frac{T_g}{T_s} = \frac{\text{tang } (s/2)}{\text{tang } (g/2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (57)$$

**Desarrollos (ó amplitud ó ángulos del centro), etc.,** $\Delta_g$, $\Delta_s$, $\Delta$.

**58.** Figs 18. Los desarrollos, $\Delta_g$ (ó $g$) y $\Delta_s$ (o, $s$) de las dos ramas, pueden ser iguales o desiguales; y, en general cualquiera puede ser el mayor; pero véase el caso especial del ¶ 57.

Como ambas ramas se encurvan en la misma direccion (derecha o izquierda), tenemos, para, $\Delta$, de la curva entera compuesta, $A\ P\ B$ :

$$\Delta = g + s\,; \qquad g = \Delta - s\,; \qquad s = \Delta - g \ldots\ldots\ldots\ldots\ldots\ldots \quad (58)$$

Vease tambien ec (60).

| En la fig. 18 *a*. | En la fig. 18 *b*. |
|---|---|
| Trácese $Puw \perp O^s B$. Entonces sen $s$. | Tracese $Puw \perp O_g A$. Entonces sen $g$. |

$$= \frac{P\,w}{R_g} = \frac{P\,u}{R_s} = \frac{u\,w}{O_g\,O_s} = \frac{P\,w - P\,u}{R_g - R_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (59)$$

**59. Angulos, A y B.** Sea $V\,A\,B = A$; $V\,B\,A = B$. Entonces:
$\Delta\ (= 180^o - A\ V\ B) = A + B$; $A = \Delta - B$; $B = \Delta - A$.. ... (60)
Vease tambien ec (58).

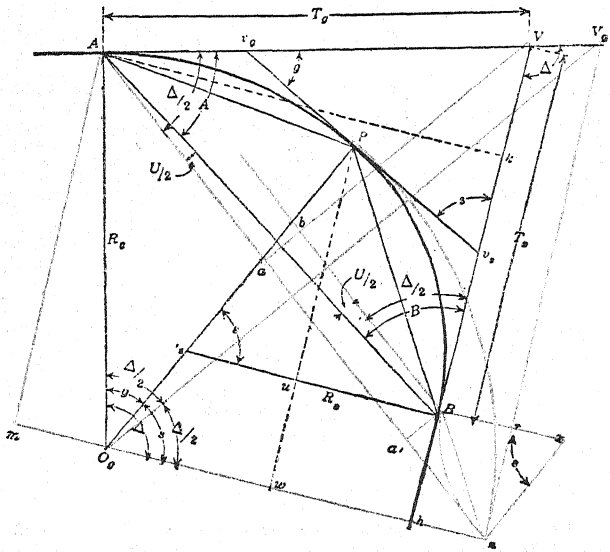



**Fig. 18 a.**

**60. Angulos V A P y V B P.**

$V\,A\,P = g/2$; $V\,B\,P = s/2$;
$V\,B\,P + V\,A\,P = s/2 + g/2 = (s + g)/2 = \Delta/2$
$V\,B\,P - V\,A\,P = s/2 - g/2 = (s - g)/2 = \Delta/2 - g$..... (61)

**61. Cuerda larga,** *C delta* = **A B.** Vease tambien ¶ 64.

| En fig. 18 *a*. | En fig. 18 *b*. |
|---|---|
| Trase $A\,k \perp B\,V$. Entonces: $A\ k = C$ *delta* sen $B = T_g$ sen $\Delta$. | Trácese $B\,k \perp A\,V$. Entonces: $B\ k = C$ *delta* sen $A = T_s$ sen $\Delta$. |

y $C$ *delta* $= T_g$ sen $\Delta$/sen $B = T_s$ sen $\Delta$/sen $A$..... (62)
Tambien vease ecs (69) y (67).

| Fig. 18 *a*. | Fig. 18 *b*. |
|---|---|
| **62.** Prolonguese la rama $A\ P$. hasta $n$, haciendo. $O_g n \parallel O_s B$. Entonces $P$, $B$ y $n$. | Prolonguese la rama $B\ P$. hasta $n$, haciendo. $O_s n \parallel O_g A$. Entonces $P$, $n$ y $A$. |

Estan en linea recta.

**Angulo, U/2.**

| | |
|---|---|
| Unase $A\ n$, y prolónguese $n\ V_g \parallel B\ V$. Trácese $B\ b \parallel A\ n$. Sea $B\ A\ n\ (= A\ B\ b) = U/2$. | Unase $B\ n$, prolónguese $n\ V_s \parallel A\ V$. Trácese $A\ a \parallel B\ n$. Sea $A\ B\ n\ (= B\ A\ a) = U/2$. |

Entonces..... $A = \Delta/2 - U/2 = (\Delta - U)/2$

$$B = \Delta/2 + U/2 = (\Delta + U)/2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (63)$$

Sumando, tenemos $A + B = \Delta$, como en ec (60);
Restando, tenemos:

$$B - A = U\ ;\ B = A + U\ ;\ A = B - U\ ;$$

$$U = \Delta - 2\ A = 2\ B - \Delta \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (64)$$

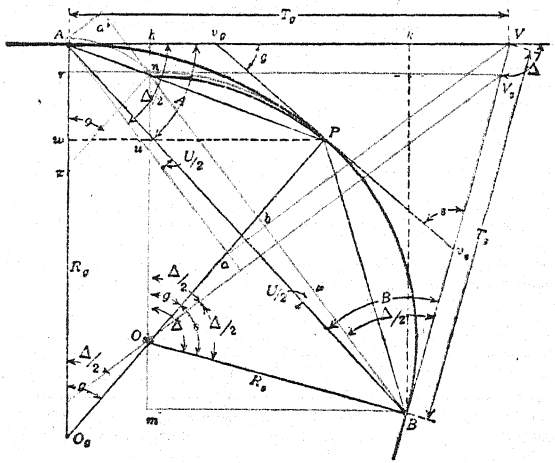



**Fig. 18 b.**

**63.** Entonces.

| En la Fig. 18 *a*. | En la Fig. 18 *b*. |
|---|---|
| $O_g\ V_g$, $\perp A\ n$, divide $A\ n$ en partes iguales; $A\ O_g\ n = \Delta$; y $A\ O_g\ V_g = V_g\ O_g\ n = \Delta/2$. | $O_s\ V_s$, $\perp B\ n$, divide $B\ n$ en partes iguales; $B\ O_s\ n = \Delta$; y $B\ O_s\ V_s = V_s\ O_s\ n = \Delta/2$..... (65) |

**64. Cuerda larga,** $Cdelta = \mathbf{A\,B}$. Vease tambien ¶ 61.

| Fig. 18 *a*. | Fig. 18 *b*. |
|---|---|
| Tracese $V\,a$ y $B\,a' \parallel O_g V_g \perp A\,n$.<br>Entonces $Cdelta$ sen $(U/2) = B\,a'$. | Tracese $V\,a$ y $A\,a' \parallel O_s\,V_s \perp B\,n$.<br>Entonces $Cdelta$ sen $(U/2) = A\,a'$. |

$$= V\,a - V\,b$$
$$= T_g \text{ sen } (\Delta/2) - T_s \text{ sen } (\Delta/2)$$
$$= (T_g - T_s) \text{ sen } (\Delta/2).$$

Por lo tanto:

$$Cdelta = (T_g - T_s)\,\frac{\text{sen } (\Delta/2)}{\text{sen } (U/2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (66)$$

Vease tambien ecuaciones (62) y (67).

Tenemos, tambien

| | |
|---|---|
| $Cdelta = \sqrt{A\,k^2 + B\,k^2} =$<br>$\sqrt{(T_g \text{ sen } \Delta)^2 + (T_s - T_g \cos \Delta\,\dagger)^2}$ | $Cdelta = \sqrt{B\,k^2 + A\,k^2} =$<br>$\sqrt{(T_s \text{ sen } \Delta)^2 + (T_g - T_s \cos \Delta\,\dagger)^2}$<br>$\ldots\ldots$ (67) |

Vease tambien ecuaciones (62) y (66).

**65. Semitangentes,** $T_g$ y $T_s$. Tracese $n\,x \parallel = O_g O_s = R_g - R_s$.

| | |
|---|---|
| Encontrandose $O_s\,E$ prolongadas, en $x$.<br>Entonces $B\,x\,n = s$.<br>Tracese $n\,r$ y $A\,m \parallel B\,V \perp O_s\,x$.<br>Entonces $A\,k = T_g$ sen $\Delta$.<br>$= m\,n - k\,n$.<br>$= m\,n - B\,r$.<br>$= R_g$ senoverso $\Delta - (R_g - R_s)$ senoverso $s$. | Encontrandose $O_g\,A$ en $x$.<br>Entonces $A\,x\,n = g$.<br>Tracese $n\,r$ y $B\,m \parallel A\,V \perp O_g\,x$.<br>Entonces $B\,k = T_s$ sen $\Delta$.<br>$= m\,n + h\,n$.<br>$= m\,n + A\,r$.<br>$= R_s$ senoverso $\Delta + (R_g - R_s)$ senoverso $g$. $\ldots\ldots\ldots\ldots$ (68) |

**66. Semitangentes de ramas,** $t_g$ y $t_s$; y **tangentes comunes,** $v_g\,v_s$.

$$t_g\ (= v_g\,P) = R_g \tan (g/2)$$
$$t_s\ (= v_s\,P) = R_s \tan (s/2)$$

Tangente comun, $v_g\,v_s = t_g + t_s$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (69)

| | |
|---|---|
| $B\,n = 2\,(R_g - R_s)$ sen $(s/2)$. | $A\,n = 2\,(R_g - R_s)$ sen $(g/2)$. (70) |

**67.** Las Figs 18 *a*, 18 *b* y las Ecs (58) y (68) nos permiten encontrar los elementos necesarios de las curvas compuestas, dandose otros elementos. Veanse los ejemplos a continuacion:

**Ejemplos.**

| Dandose.... | **Buscar** los elementos indicados en tipo **grueso.** |
|---|---|
| $R_g\ R_s\ g\ s$... | $\Delta$ se encuentra inmediatamente por la Ec (58) y $\mathbf{T}_g$ y $\mathbf{T}_s$ por Ec (68). |
| $R_s\ T_s\ \Delta\ s$... | $\mathbf{g = \Delta - s}$, ec (58).<br>$\mathbf{R}_g$ senoverso $g = T_s$ sen $\Delta + R_s$ senoverso $g - R_s$ senoverso $\Delta$. (71)<br>$\mathbf{T}_g$ sen $\Delta = R_g$ senoverso $\Delta - (R_g - R_s)$ senoverso $s$ .... (72) |

† Vease ¶ 69.

| | |
|---|---|
| $R_g\ T_g\ \Delta\ g$.. | $\mathbf{s} = \Delta - g$, ec (58)<br>$\mathbf{R_s}$ senov $s = R_g$ senov $s + T_g$ sen $\Delta - R_g$ senov $\Delta$ .... (73)<br>$\mathbf{T_s}$ sen $\Delta = R_s$ senov $\Delta + (R_g - R_s)$ senov $g$..... (74) |
| $R_g\ R_s\ T_g\ \Delta$.. | $(R_g - R_s)$ senov $\mathbf{s} = R_g$ senovs $\Delta - T_g$ sen $\Delta$..... (75)<br>$\mathbf{g} = \Delta - s$, ec (58)<br>$\mathbf{T_s} = R_g$ sen $\Delta - (R_g - R_s)$ sen $s - T_g$ cos $\Delta^*$ ..... (76) |
| $R_s\ T_g\ T_s\ \Delta$.. | Tan $(\mathbf{g}/2) \times (T_g + T_s \cos \Delta^* - R_s$ sen $\Delta)$<br>$= T_s$ sen $\Delta - R_s$ senov $\Delta$..... (77)<br>$\mathbf{s} = \Delta - g$, ec (58)<br>$\mathbf{R_g}$ sen $g = R_s$ sen $g + T_g + T_s \cos \Delta^* - R_s$ sen $\Delta$..... (78) |
| $R_g\ T_g\ T_s\ \Delta$.. | Tan $(\mathbf{s}/2) \times (R_g$ sen $\Delta - T_g \cos \Delta^* - T_s)$<br>$= R_g$ senov $\Delta - T_g$ sen $\Delta$..... (79)<br>$\mathbf{g} = \Delta - s$, ec (58)<br>$\mathbf{R_s}$ sen $s = R_g$ sen $s - R_g$ sen $\Delta + T_g \cos \Delta^* + T_s$ .... (80) |

**68.** Figs 18. Dando los angulos, $A$ y $B$, cuerda larga $C$ *delta* $= A\ B$, y $R_g$ o $R_s$, encontrar $\mathbf{g}$, $\mathbf{s}$ **y los otros radios.** Encontrar $\Delta = A + B$ ; $T_g$ y $T_s$ por la ec (62).

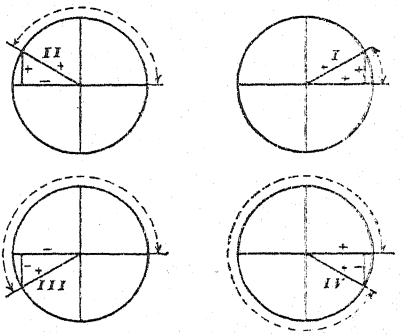



**Fig. 19.**

Entonces teniendo $R_g$ (o $R_s$), $T_g$, $T_s$ y $\Delta$, buscar $g$, $s$ y $R_s$ (o, $R_g$) por ecs (77) a (80).

**69. Angulos mayores de 90°.** Cuando, debido a la considerable extension de la curva, o a otras causas, cualquier angulo aludido exceda 90°, debe atenderse al hecho de que los signos algebraicos ( $+$ $y$ $-$ ) varian con los varios cuadrantes del circulo. Vease « Signos positivos y negativos », pag 104.

Asi, fig. 19.

| **En cuadrante.** | I | II | III | IV |
|---|---|---|---|---|
| Incluyendo angulos de..... | 0° a 90° | 90° a 180° | 180° a 270° | 270° a 360° |
| Seno y cosecante son..... | mas | mas | menos | menos |
| Tangente y cotangente son..... | mas | menos | mas | menos |
| Secante y coseno son..... | mas | menos | menos | mas |

★ En nuestras figs 18, $\Delta$, $> 90°$, y cos $\Delta$ es por consiguiente *negativo*. Vease ¶ 69.

## CURVAS INVERTIDAS

### Definiciones.

**70.** Figs 20-23. Comparese Curvas Compuestas, ¶ ¶ 50-69. Cuandos dos curvas circulares consecutivas, $A\ P$ y $P\ B$, de igual o desigual radio, $R_a$ y $R_b$, se encurvan en direcciones *opuestas* (una que vaya a la derecha, y la otra a la izquierda) se dice que son invertidas, y la combinacion se llama una curva *invertida*. Las curvas invertidas algunas veces son inevitables, como en algunos enlaces y desvios; pero en otras partes deben evitarse.

**71. Las dos ramas**, $A\ P$ y $P\ B$, descansan en lados *opuestos* de la tangente comun, $v_a\ v_b$. Su punto de encuentro, $P$, o el punto comun de tangencia, es el punto, « C. C. » de cambio de *curva* a *curva*; o el punto de curvatura invertido. « P. R. C. » Vease ¶ 2.

**72. Los centros**, $O_a$ y $O_b$, de las dos ramas, están necesariamente en linea recta con el C. C. $(P)$, **los dos radios** son normales a la tangente comun, $v_1\ v_b$, en $P$, En $A$ y en $B$, los radios son normales respectivamente a las tangentes, $A\ V$, $V\ B$. de la curva entera. El vertice, $V$, está hacia el lado de *la curva de mayor extension*.

**73. Las semitangentes** de la curva entera invertida son $T_a = A\ V$ y $T_b = B\ V$. Vease ¶ 77. Las semitangentes de las dos ramas respectivamente, son $t_1 = v_1\ P = v_a\ A$, y $t_b = v_b\ B$. **Tangente comun**, $v_1\ v_b = t_1 + t_b$.

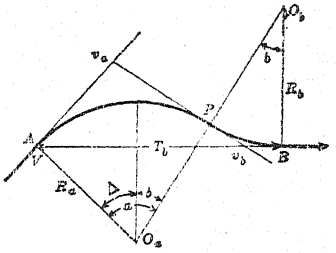



**Fig. 20 a.**

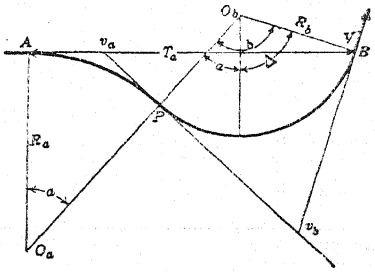



**Fig. 20 b.**

**74.** Figs 20.

Fig. 20 *a*. Si $V$ coincide con $A$, tenemos :

$$T_a\ (= A\ V) = 0,\ \text{y}\ T_b\ (= V\ B) = A\ B.$$

Fig. 20 *b*. Si $V$ coincide con $B$, tenemos :

$$T_b\ (= B\ V) = 0,\ \text{y}\ T_a\ (= V\ A) = A\ B.$$

**75. Los desarrollos,** $\Delta_a$ y $\Delta_b$ (o, simplemente, $a$ y $b$) de las dos ramas, pueden ser iguales o desiguales, y cualquiera puede ser el mayor.

**76.** Como las dos ramas se desvian en direcciones opuestas, la desviacion resultante, $\Delta$, debido a la inversion de la curva en conjunto, es :

$$\Delta = \textit{diferencia} \text{ entre } a \text{ y } b \ldots\ldots\ldots\ldots\ldots \quad (81)$$

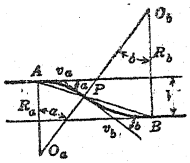



**Fig. 21.**

**77.** fig. 21. Cuando las tangentes son paralelas, tenemos $a = b$ y $\Delta$ ( $= a - b$ ) $= 0$. Entonces $T_a$ ( $= A\ V$) y $T_b$ ( $= B\ V$) son infinitos, no habiendo vértice, $V$. Véase ¶ 79.

También, $P$ está en la linea $A\ B$.

## Ecuaciones.

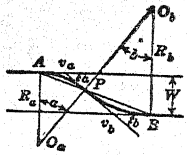
**78.** Figs 22 y 23. **Semitangentes,** $A\ v_a$ y $v_b\ B$ **no paralelas.**
El « P. R. C. » ($P$) no esta en $A\ B$.

| **Radios desiguales.** Fig. 22. | **Radios iguales.** Figs 23. $R_a = R_b = R$. |
|---|---|
| De la semitangente, $A\ v_a$, en $A$, trácese $X\ A\ D = Z\ D\ A = \Delta/2$. | |
| Midase la cuerda. $A\ D = 2\ R_a \operatorname{sen} (\Delta/2)$. | Midase la cuerda. $A\ D = 2\ R \operatorname{sen} (\Delta/2)$. |

Entonces $D$ es un punto de la curva $A\ P$ (prolongado, figs 22 $b$, 23 $b$). Dibujese $O_a\ D \parallel O_b\ B$. Entonces $A\ O_a\ D = \Delta$. Por $D$, tracese la tangente, $Z\ N \parallel B\ V$, distante $D\ B' = W$, de $B\ V$. De $D$ tracese arco $D\ P$ ($D\ O_a\ P = b$), determinando $P$.

| | |
|---|---|
| Midanse las cuerdas. $D\ P$ y $P\ B$. Entonces : $D\ B = D\ P + P\ B$ $= W/\operatorname{sen}\ (b/2)$ $= 2\ (R_a + R_b) \operatorname{sen}\ (b/2)$. (82) | Midanse las cuerdas. $D\ P = P\ B$. Entonces : $D\ B = D\ P + P\ B$ $= 2\ D\ P = 2\ P\ B$ $= W/\operatorname{sen}\ (b/2)$ $= 4\ R \operatorname{sen}\ (b/2) = 2\sqrt{W\ R}$ ... (82′) |
| $R_b = R_a\ P\ B/D\ P$ $= \dfrac{D\ B}{2 \operatorname{sen}\ (b/2)} - R_a$..... (83) | $R = R_a = R_b$ $= \dfrac{D\ P}{2 \operatorname{sen}\ (b/2)} = \dfrac{P\ B}{2 \operatorname{sen}\ (b/2)}$ ... (83′) |
| Tangente comun : $v_a\ v_b = t_a + t_b$ $= R_a \tan\dfrac{a}{2} + R_b \tan\dfrac{b}{2}$. (84) | Tangente comun : $v_a\ v_b = t_a + t_b$ $= R\left(\tan\dfrac{a}{2} + \tan\dfrac{b}{2}\right)$. (84′) |

$$\text{Sen } D\ B\ v_b = \operatorname{sen}\ (b/2) = W/D\ B \ldots\ldots\ldots\ldots \quad (85$$



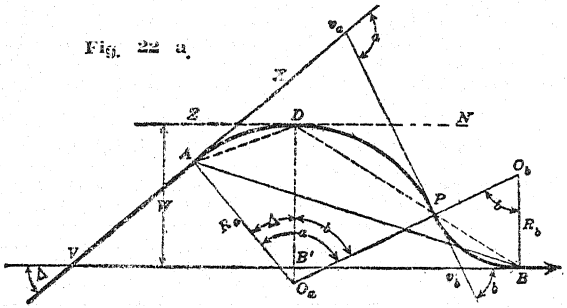

**79.** Fig. 21. **Semitangentes, $Av_a$, $v_bB$, paralelas.**

$A\,B$ corta á la tangente comun, $v_a\,v_b$, en el C. C. $(P)$. $a = b$; $\Delta = a - b$ $(o,\ b - a) = 0$; $T_a$ y $T_b$ infinito; $B\,A\,v_a = A\,B\,v_b = a/2 = b/2$.

**Fig. 21 (Repetida).**

| **Radios desiguales**, fig. 21. | | **Radios iguales**, $R_a = R_b = R$. | |
|---|---|---|---|
| $W = (R_a + R_b)$ senov $a$ | | $W = 2\,R$ senov $a$ | |
| $= (R_a + R_b)$ senov $b$ | | $= 2\,R$ senov $b$ | |
| $= A\,B$ sen $(a/2)$ | | $= A\,B$ sen $(a/2)$ | |
| $= A\,B$ sen $(b/2)$..... | (86) | $= A\,B$ sen $(b/2)$..... | (86') |
| $A\,P = 2\,R_a$ sen $(a/2)$ | | $A\,P = P\,B = A\,B/2$ | |
| $= 2\,R_a\,W/A\,B$..... | (87) | $= 2\,R$ sen $(a/2)$ | |
| $P\,B = 2\,R_b$ sen $(b/2)$ | | $= 2\,R\,W/A\,B$..... | (87') |
| $= 2\,R_b\,W/A\,B$..... | (88) | $= 2\,R$ sen $(b/2)$ | |
| | | $= 2\,R\,W/A\,B$..... | (88') |
| $A\,B = A\,P + P\,B$ | | $A\,B = 2\,A\,P = 2\,P\,B$ | |
| $= 2\,(R_a + R_b)$ sen $(a/2)$ | | $= 4\,R$ sen $(a/2)$ | |
| $= \sqrt{2\,W\,(R_a + R_b)}$ ... | (89) | $= \sqrt{4\,W\,R} = 2\sqrt{W\,R}$. | (89') |
| | | $R = A\,B^2/4\,W$..... | (90) |

Fig. 22 a.



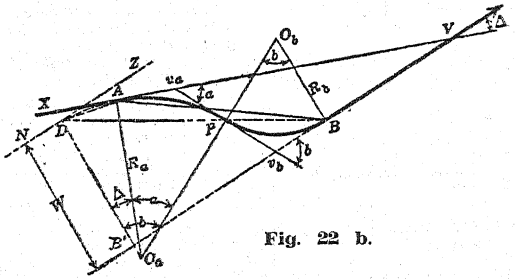




Fig. 22 b.

**80.** Figs 22 y 23. Como en las curvas **compuestas, dando ciertos** elementos de los siete, $R_a$, $R_b$, $T_a$, $T_b$, $a$, $b$, $\Delta$; **los otros** pueden encontrarse.

Por ejemplo :

| Dando. | Las cantidades **requeridas** estan indicadas en tipos gruesos. |
|---|---|
| $R_a$ $R_b$ $T_a$ $\Delta$. | En todos los casos, $\Delta = a - b$ ó $b - a$.<br>**senov b** $= \dfrac{R_a \text{ senov } \Delta + T_a \text{ sen } \Delta}{(R_a + R_b)}$ ..... (91)<br>$T_b = T_a \text{ cos } \Delta + R_a \text{ sen } \Delta \pm (R_a + R_b) \text{ sen } b$ ..... (92) |
| $R_a$ $R_b$ $T_b$ $\Delta$. | **senov a** $= \dfrac{R_b \text{ senov } \Delta + T_b \text{ sen } \Delta}{(R_a + R_b)}$ ..... (93)<br>$T_a = T_b \text{ cos } \Delta + R_b \text{ sen } \Delta \pm (R_a + R_b) \text{ sen } a$ ..... (94) |

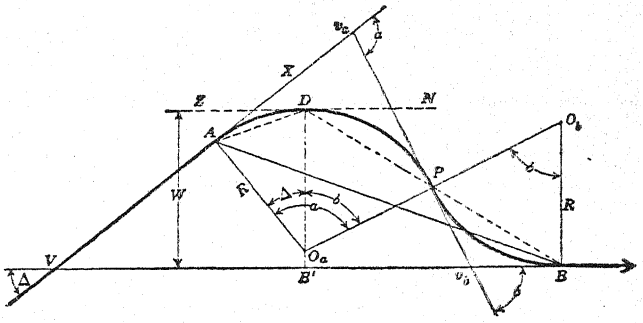



**Fig. 23 a.**

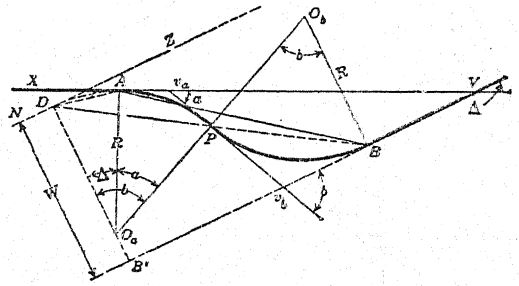



**Fig. 23 b.**

## TRAZADO DE LA CURVA

**81. Fig. 24. Por Angulos de Deflexion ó Periféricos.** Vease ¶ ¶ 4, etc. Ejemplo. Instrumento en el T. C. (*A*) o en el C. T. (*B*). Para otros puntos, véase ¶ ¶ 84, etc.)

**(1) Si la cadena empieza en A.**

| Para determinar. | Inst en A. Trácese | Inst en B. ángulo. | | Cadenas † |
|---|---|---|---|---|
| | —— | $V\ B\ A$ * = | $\Delta/2 = d_i/2 + 3\ D/2 + d_f/2$ | * |
| $a$..... | $V\ A\ a$ | $= A\ B\ a = A\ O\ a/2$ | $= d_i/2$ | $c_i = A\ a.$ |
| $b$..... | $V\ A\ b$ | $= A\ B\ b = A\ O\ b/2$ | $= d_i/2 + 1\ D/2$ | $c = a\ b.$ |
| $c$..... | $V\ A\ c$ | $= A\ B\ c = A\ O\ c/2$ | $= d_i/2 + 2\ D/2$ | $c = b\ c.$ |
| $d$..... | $V\ A\ d$ | $= A\ B\ d = A\ O\ d/2$ | $= d_i/2 + 3\ D/2$ | $c = c\ d.$ |
| $B$..... | $V\ A\ B$ | —— = | $\Delta/2 = d_i/2 + 3\ D/2 + d_f/2$ | $c_f = d\ B.$ |

**(2) Si la cadena empieza en B.**

| Para determinar. | Inst en A. Trácese | Inst en B. ángulo. | | Cadenas. |
|---|---|---|---|---|
| $B$..... | $V\ A\ B$ * | —— = | $\Delta/2 = d_f/2 + 3\ D/2 + d_i/2$ | * |
| $d$..... | $B\ A\ d$ | $= V\ B\ d = B\ O\ d/2$ | $= d_f/2$ | $c_f = B\ d.$ |
| $c$..... | $B\ A\ c$ | $= V\ B\ c = B\ O\ c/2$ | $= d_f/2 + 1\ D/2$ | $c = d\ c.$ |
| $b$..... | $B\ A\ b$ | $= V\ B\ b = B\ O\ b/2$ | $= d_f/2 + 2\ D/2$ | $c = c\ b.$ |
| $a$..... | $B\ A\ a$ | $= V\ B\ a = B\ O\ a/2$ | $= d_f/2 + 3\ D/2$ | $c = b\ a.$ |
| $A$..... | —— | $V\ B\ A$ = | $\Delta/2 = d_f/2 + 3\ D/2 + d_i/2$ | $c_i = a\ A.$ |

**82. Para la tangente**, en cualquier estacion dada, como $B$, tenemos:

Mirando atrás al T. C., $A$;

$z\ B\ x = V\ B\ A = V\ A\ B$ = suma de los ángulos trazados, en $A$, desde la tang, $A\ V$, a la estacion dada, $B$;

Mirando hacia atras a otra estacion, como $b$;

$z\ B\ y = V\ B\ b = e\ b\ B$ = suma de los ángulos trazados en $b$, desde la tangente, $b\ e$, a la estacion dada, $B$, etc., etc.

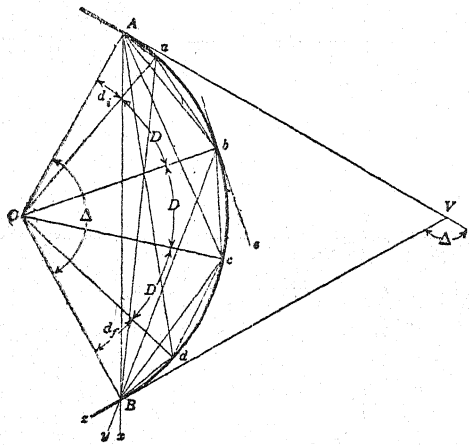



**Fig. 24.**

* Mirando de cualquier extremo de la curva, al otro extremo, donde la cadena empieza, el punto deseado se determina por la interseccion de la línea visual con la tangente, $B\ V$ o $A\ V$.

† Las cadenas dadas, $ci$, $c$, etc, se refieren a sus varios *ángulos de cadena*, $V A a$, $a\ A\ b$, $b\ A\ c$, etc. El ángulo total, $V\ A\ c$, por ejemplo, requiere cadenas $ci + c + c$.

## Cambio del Instrumento. Puntos de Tránsito *.

**33.** Fig. 24. Cuando la suma, $V\,A\,d$, de los angulos, trazados o marcados desde la tangente con el instrumento en un punto como $A$, excede, digamos de 15° o 20°, entonces, con el extremo de atrás de la cadena sostenida en la estacion que preceda a la ultima, como, $c$, el extremo libre de la cadena puede estar en una posicion distinta de la que le corresponde, en $d$, sin que lo pueda comprobar el operador del tránsito; y esta dificultad aumenta con el mayor largo de la visual.

**34.** En este caso, o cuando un obstáculo impide el trazado del próximo angulo, el instrumento se cambia a otro punto (« punto de transito » *), en la curva, y el trabajo se continua desde alli.

**35.** La posibilidad de errar esta mas reducida, observando en todo su curso, el método sistemático siguiente:

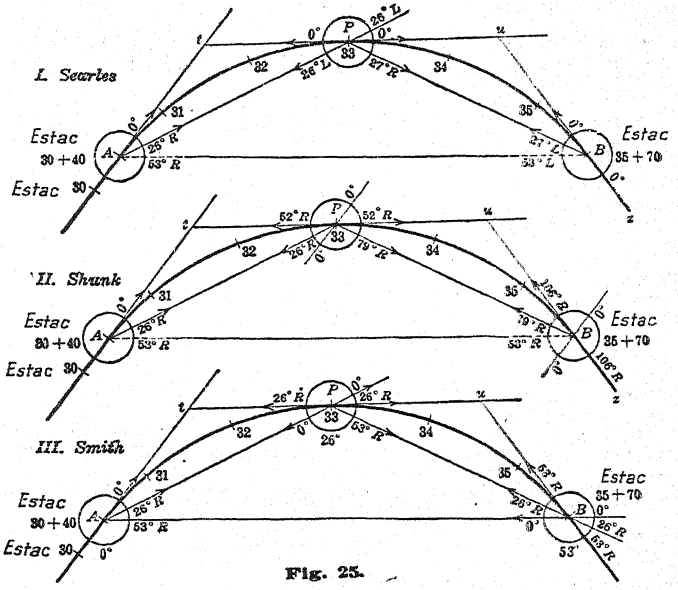



**Fig. 25.**

Figs 25. Ejemplo. En la curva de 20° $A\ P\ B$, marquemos las tres estaciones nos 31, 32 y 33 ($P$) desde la T. C. ($A$, estacion 30 + 40); entonces determinemos las tres estaciones, nos 34, 35 y 35 + 70 ($B$), desde $P$; y luego fijemos la tangente final, $Bz$, desde la C. T. ($B$, estacion 35 + 70).

Los circulos, en $A$, en $P$ y en $B$, en cada Fig. indican la orientacion del instrumento, los angulos girados y la lectura que resulta del vernier, o nonio.

En cada una de las tres figs (vease Figs 25, I) sea:

$$\text{Angulo } t\ A\ P = 0.6 \times 10° + 10° + 10° = 26°;$$
$$\text{—} \quad u\ P\ B = 10° + 10° + 0.7 \times 10° = 27°.$$

* Tales puntos se llaman á menudo « puntos de cambio » pero esto es propenso a confusion con los « puntos de cambio » en nivelación.

**Tres metodos** están en uso comun en coneccion con el cambio del tránsito a un punto nuevo. Estos pueden llamarse: —

I. Searles. Vease « Field Engineering », pp 53, 57, etc.
II. Shunk. Vease « The Field Engineer », pp 66, etc *.
III. Smith. Vease « Eng News, 1876 Aug 26, p 277 reimpreso 1888 sept 29, p 245; * tambien « Engineer Field Book » de *Cross*, pp 56, etc.

**Metodo I (Searles)** esta ya suficientemente descrito en la primera de las tres secciones de la tabla comparativa en el ¶ 86.

**Metodo II (Shunk)** puede hacerse mas claro por la siguiente aplicacion de este metodo al mismo caso. Vease Figs 25, II. Aqui todos los angulos son a la derecha (*R*) del cero.

| | | | |
|---|---|---|---|
| Instrumento en.. | $A$ | $P$ | $B$ |
| Visual al punto viejo..... | $t$ | $A$ | $P$ |
| Lectura del vernier en el punto.. viejo..... | 0° | $\left\{\begin{array}{c} t\,A\,P = \\ t\,P\,A = \\ 26° \end{array}\right\}$ | $\left\{\begin{array}{c} 2\,t\,A\,P + u\,P\,B \\ = 52° + 27° \\ = 79° \end{array}\right\}$ |
| Agregar..... | 0° | $\left\{\begin{array}{c} A\,P\,t = \\ t\,A\,P = \\ 26° \end{array}\right\}$ | $\left\{\begin{array}{c} P\,B\,u = \\ u\,P\,B = \\ 27° \end{array}\right\}$ |
| Visual a la tang.. | $A\,t$ | $t\,u$ | $u\,z$ |
| Lectura del vernier en la tang.. | 0° | $\left\{\begin{array}{c} t\,A\,P + A\,P\,t \\ = 2\,t\,A\,P - 0° \\ = 2 \times 26° - 0° \\ = 52° \end{array}\right\}$ | $\left\{\begin{array}{c} 79° + 27° = \\ 2 \times 79° - 2\,t\,A\,P \\ = 158° - 52° \\ = 106° \end{array}\right\}$ |
| Agregar..... | $t\,A\,P = 26°$ | $u\,P\,B = 27°$ | |
| Visual al nuevo punto..... | $P$ | $B$ | |
| Lectura del vernier en el nuevo punto..... | $\left\{\begin{array}{c} t\,A\,P = \\ 0° + 26° \\ = 26° \end{array}\right\}$ | $\left\{\begin{array}{c} 2\,t\,A\,P + u\,P\,B \\ = 52° + 27° \\ = 79° \end{array}\right\}$ | |
| Cambio al nuevo punto..... | $P$ | $B$ | |

Despues de cambiar el instrumento de un punto viejo, $p$ (no indicado), a un punto nuevo, $p'$, la lectura del vernier, $C$, que colocará al anteojo en la tangente nueva, se determina asi: — Sea.

$A$ = lectura del vernier mirando el punto nuevo, $p'$, desde $p$;
$B$ = lectura del vernier mirando a lo largo de la tangente vieja, en $p$;
$C$ = lectura del vernier mirando a lo largo de la tangente nueva, en $p'$.
Entonces:

$$C = 2\,A - B.$$

Método III (Smith) se describe en ¶ ¶ 89 a 91.

**86.** La siguiente tabla indica el procedimiento por los tres metodos respectivos. M. S. = movimiento superior del tránsito, M. I. = movimiento inferior del transito.

$R$ = derecho; $L$ = izquierda.

* En su 9ª edicion, 1890, M. Shunk describe el metodo « Smith » (III), atribuyéndoselo a M. Robert Burgess, C.E., quien lo describe en Eng News, 1888, Sep 22.

† En obsequio de la uniformidad, aqui supondremos que, al usar el metodo III (Smith), movemos el instrumento lo mismo a $P$ que a $B$, y que, despues de cambiarlo asi, miramos a las mismas estaciones ($A$ y $P$, respectivamente) como en los metodos I y II; pero, de hecho, con el metodo III, la condicion descrita en ¶ 90, se obtendra sin que importe a qué estacion nos cambiemos, o a qué estacion miremos desde él. Véase 91.

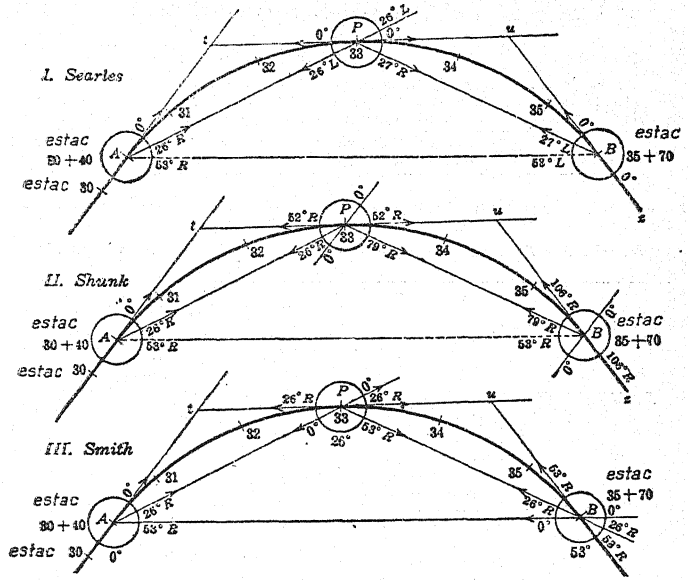



Fig. 25. (Repetida.)

| 1 Mirando desde. | 2 M. S. lectura Vernier. | 3 Cambio a Estacion | 4 M. S. con lectura Vernier. | 5 M. I. visual a Estacion | 6 M. S. Vernier Puesto a | 7 Instrumento en la tangente |
|---|---|---|---|---|---|---|
| | **Metodo I, Searles.** | | | | | |
| *A* a *t*.. | 0° + 0° = 0° | | | | | |
| *A* a *P*. | 0° + 26° = 26° | *P* | 26° *L* | *A* | 0° | *P* |
| *P* a *B*. | 0° + 27° = 27° | *B* | 27° *L* | *P* | 0° | *B* |
| | **Metodo II, Shunk.** | | | | | |
| *A* a *t*.. | 0° + 0° = 0° | | | | | |
| *A* a *P*. | 0° + 26° = 26° | *P* | 26° *R* | *A* | 52° *R* | *P* |
| *P* a *B*. | 52° + 27° = 79° | *B* | 79° *R* | *P* | 106° *R* | *B* |
| | **Metodo III, Smith†.** | | | | | |
| *A* a *t*.. | 0° + 0° = 0° | | | | | |
| *A* a *P*. | 0° + 26° = 26° | *P* * | 0° | *A* * | 26° *R* | *P* |
| *P* a *B*. | 26° + 27° = 53° | *B* * | 26° *R* | *P* * | 53° *R* | *B* |

* Vease † nota al pie, pag. 1000.
† Vease ¶¶ 89 a 91.

**87.** Fig. 25. Se notará que, cuando el **vernier está en cero.**

| En método. | El anteojo está : |
|---|---|
| I. Searles.... | En la tangente por el punto ocupado; |
| II. Shunk..... | Paralelo a la tangente original $A\ t$; |
| III. Smith..... | Mirando atras hacia $A$, cuyo angulo (¶ 89) es cero. |
| | Cuando el instrumento esta en $A$, estas tres lineas de colimacion coinciden. |

**88.** Para las figs 25, **las lecturas del vernier**, por los tres métodos respectivos comparanse como sigue :

(*N. del T.* — En las figs *R* (right) indica Derecha ; *L* (left) indica Izquierda.)

| Mirando. | Método. I Searles. | Método. II Shunk. | Método. III Smith. * |
|---|---|---|---|
| | Lectura del vernier. $D$ = derecha; $I$ = izquierda. | | |
| Desde estacion 30 + 40 (*A*)..... | | | |
| a — 30 + 40 (*A*) (tan). | 0° | 0° | 0° |
| a — 31..... | 6° *D* | 6° *D* | 6° *D* |
| a — 32..... | 16° *D* | 16° *D* | 16° *D* |
| a — 33 (*P*)..... | 26° *D* | 26° *D* | 26° *D* |
| a — 35 + 70 (*B*)..... | 53° *D* | 53° *D* | 53° *D* |
| Desde estacion 33 (*P*)..... | | | |
| a — 30 + 40 (*A*)..... | 26° *I* | 26° *D* | 0° |
| a — 33 (*P*) (tan)..... | 0° | 52° *D* | 26° *D* |
| a — 34..... | 10° *D* | 62° *D* | 36° *D* |
| a — 35..... | 20° *D* | 72° *D* | 46° *D* |
| a — 35 + 70..... | 27° *D* | 79° *D* | 53° *D* |
| Desde estacion 35 + 70 (*B*)..... | | | |
| a — 30 + 40 (*A*)..... | 53° *I* | 53° *D* | 0° |
| a — 33 (*P*)..... | 27° *I* | 79° *D* | 26° *D* |
| a — 35 + 70 (*B*) (tan). | 0° | 106° *D* | 53° *D* |

**89. En el Metodo III (Smith)**, figs 25, III, antes de empezar a correr la curva, calcúlese y anótese en la libreta de campo (como en la ultima columna de la tabla, ¶ 88), opuesta a cada estacion (como 30 + 40, 31, etc.), y opuesta a cualquiera otros puntos, en la curva, que por cualquier razon, pueda desearse determinarlos, el ángulo total (como $t\,A\,t = 0°$, $t\,A$ 31, $t\,A\,P$, $t\,A\,B$, etc.), entre la tangente *original*, $At$, y la cuerda, $A$-31, $A$-$P$, $A$-$B$, etc.) desde $A$ a tal estacion, como si cada uno de los puntos, en toda la curva entera, fuera a ser determinado desde el punto $A$ de la curva.

**90.** Entonces, despues de hacer estacion en cualquier punto que sea de la *curva*, póngase el vernier con el ángulo asi anotado en la libreta de campo opuesto a cualquier otro punto conveniente, $x$, en la curva, y vease hacia $x$, con el instrumento asi orientado. Entonces, si el vernier esta puesto para el movimiento superior, a la lectura registrada, como arriba, opuesto a cualquier otro punto, $y$, en la curva, el instrumento será mirado en $y$. Con el vernier en el ángulo registrado opuesto al *punto ocupado*, el instrumento esta en la *tangente que pasa por ese punto.*

* Buscando la uniformidad, aqui suponemos que, al usar el III metodo (Smith), cambiamos el instrumento ya sea a $P$ ó a $B$, y que despues de cambiarlo, miramos las mismas estaciones ($A$ y $P$, respectivamente, como en los metodos I y II, pero, de hecho, con el metodo III, la condicion, descrita en ¶ 90 se obtendrá sin que importe a qué estacion nos cambiamos, o de qué estacion miramos desde él. Vease ¶ 91.

**91.** Si la agudeza de la curva está expresada por un numero entero de grados, si la curva empieza con una sub-cadena, y si se van a poner muchas estaciones, puede convenir **hacer una adicion arbitraria a cada angulo,** para dar, a las estaciones completas, angulos expresados por un numero entero de grados (o en grados y medios grados); simplificando asi los calculos. † Por ejemplo, en una curva de 3°, si los angulos actuales son como en la linea *A*, por debajo, podiamos añadir 0°18′ a cada angulo, haciendolos leer como en la linea *B*.

| Estación | 30 + 20 (*A*) | 31 | 32 | 33 | 34 | 34 + 40 (*B*) |
|---|---|---|---|---|---|---|
| *A*. | 0° | 1°12′ | 2°42′ | 4°12′ | 5°42′ | 6°18′ |
| *B*. | 0°18′ | 1°30′ | 3°0′ | 4°30′ | 6°0′ | 6°36′ |

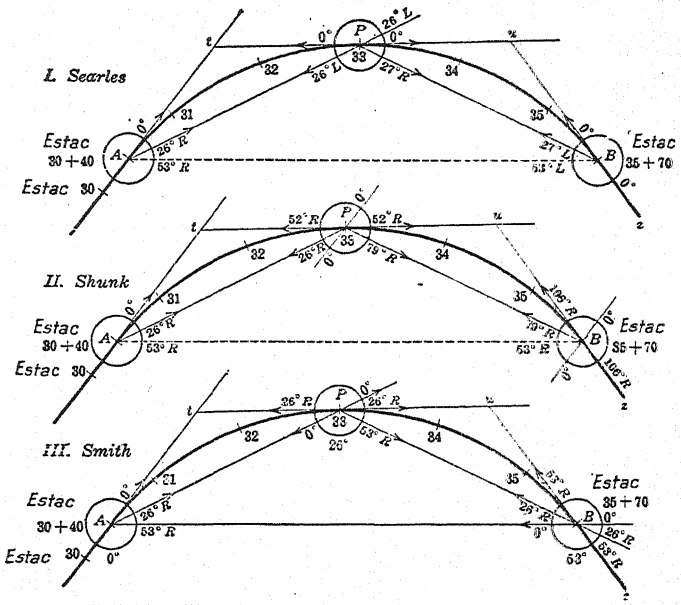



**Fig. 25. (Repetida.)**

## Miscelánea de Métodos.

**92. Trazar una curva sin el tránsito.** Fig. 26. Sean $a\ b$, $b\ c$, cadenas; $a\ b = b\ c = c$; sean $a\ t$ y $b\ n$ las tangentes en $a$ y en $b$, respectivamente. Prolónguese la cuerda $a\ b$ hasta $f$, y hágase $a\ t = b\ f = a\ b = c$. Entonces :

$$t\ b = 2\ c \operatorname{sen} (D/4);\ f\ c = 2\ c \operatorname{sen} (D/2) = 2\ c \frac{c}{2\ R} = \frac{c^2}{R} \ldots\ldots\ldots\ldots \quad (95)$$

† T. Appleton, Revista de Assen Engng Socs, Marzo, 1883.

**93.** Las distancias, $t\ b$ y $f\ c$, pueden usarse para determinar un curva sin el transito. Asi:

(1) Habiendo trazado $a\ t\ (=c)$ a lo largo de la tangente, $a\ t$, para determinar $b$; desde $a$, trácese $a\ b\ (=c)$ para encontrar $t\ b$ trazada desde $t$.

(2) Habiendo trazado $b\ f\ (=c)$ a lo largo de la cuerda $a\ b$ prolongada para determinar $c$; desde $b$, trace $b\ c\ (=c)$ para encontrar $f\ c$ trazada desde $f$.

(3) Habiendo trazado $c\ e\ (=c)$ a lo largo de la cuerda $b\ c$ prolongado para encontrar la tangente, $e\ t'$, en $e$; desde $c$, trácese $c\ t'\ (=c)$ para encontrar $e\ t\ (=t\ b)$ trazado desde $e$.

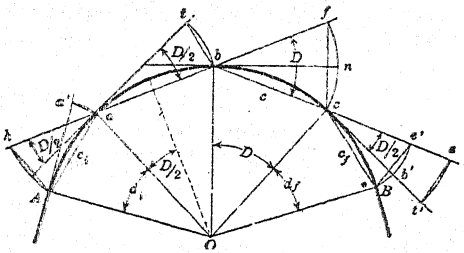



**Fig. 26.**

**94. Para las sub-cadenas**, $A\ a = c_i$, y $c\ B = c_f$, **sub-angulos**, $d_i$ y $d_f$, etc., tenemos las siguientes ecuaciones:

| Distancias. | Angulos. | Aproximado. |
|---|---|---|
| $a'\ a = 2\ c_i$ sen $(d_i/4)$ | sen $(d_i/2) = \dfrac{c_i}{2\,R}$ | $a'\ a = t\ b\ \dfrac{c_i^2}{c^2}$ ... (96) |
| $B\ b' = 2\ c_f$ sen $(d_f/4)$ | sen $(d_f/2) = \dfrac{c_f}{2\,R}$ | $B\ b' = e\ t'\ \dfrac{c_f^2}{c^2}$ ... (97) |
| $A\ h = 2\ c_i$ sen $(A\ a\ h/2)$ | $A\ a\ h = \dfrac{D}{2} + \dfrac{d_i}{2}$ | $d_i = D\ \dfrac{c_i}{c}$ .... (98) |
| $B\ e' = 2\ c_f$ sen $(B\ c\ e'/2)$ | $B\ c\ e' = \dfrac{D}{2} + \dfrac{d_f}{2}$ | $d_f = D\ \dfrac{c_f}{c}$ .... (99) |

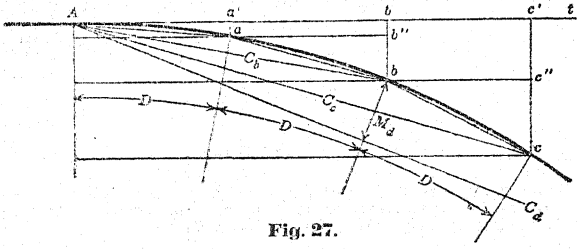



**Fig. 27.**

**95. Por ordenadas desde la tangente, A t.** Fig. 27. Sea $R$ = radio; $c$ =, largo de cadena, $A\ a$, $a\ b$, etc; $C_b$, $C_c$, $C_d$, etc = cuerda larga $A\ b$, $A\ c$, $A\ d$, etc, desde $A$ a estacion 2ª, 3ª, 4ª etc; $M_b$, $M_c$, $M_d$, etc = media ordenada para $C_b$, $C_c$ $C_d$, etc; $D/2 = a'\ A\ a$; $3\ D/2 = b''a\ b$; $5\ D/2 = c''\ b.c$, etc. Entonces:

*Distancias en la tangente.*

$$A\,a' = c\cos(D/2) = R\,\text{sen}\,D \ldots\ldots = C_b/2 \ldots\ldots (100)$$

$$A\,b' = c\,[\cos(D/2) + \cos(3\,D/2)] = R\,\text{sen}\,(2\,D) \ldots\ldots = C_d/2 \ldots\ldots (101)$$

$$A\,c' = c\,[\cos(D/2) + \cos(3\,D/2) + \cos(5\,D/2)] = R\,\text{sen}\,(3\,D) \ldots\ldots = C_f/2 \ldots\ldots (102)$$

*Ordenadas desde la tangente.*

$$a'\,a = c\,\text{sen}\,(D/2) = c^2/(2\,R) = R\,\text{senoverso}\,D = M_b \ldots\ldots (100\,a)$$

$$b'\,b = c\,[\text{sen}\,(D/2) + \text{sen}\,(3\,D/2)] = C_b\,\text{sen}\,b'\,A\,b = C_b\,\text{sen}\,D = (C_b)^2/(2\,R).$$
$$= R\,\text{senoverso}\,2\,D = M_d \ldots\ldots (101\,a)$$

$$c'\,c = c\,[\text{sen}(D/2) + \text{sen}\,(3\,D/2) + \text{sen}\,(5\,D/2)].$$
$$= C_c\,\text{sen}\,(3\,D/2) = (C_c)^2/(2\,R).$$
$$= R\,\text{senoverso}\,3\,D = M_f \ldots\ldots (102\,a)$$

Así, sea $c = 100$ pies; $D = 12°$. Entonces:
(Vease *N. del T.* ¶ 11, pag 926.)

| Para la dist $A\,c'$, sobre la tang. | Para la ordenada, $c'\,c$ desde la tang. |
|---|---|
| Cos $(D/2)$ = cos 6° = 0.99452 | Sen $(D/2)$ = sen 6° = 0.10453 |
| Cos $(3\,D/2)$ = cos 18° = 0.95106 | Sen $(3\,D/2)$ = sen 18° = 0.30902 |
| Cos $(5\,D/2)$ = cos 30° = 0.86603 | Sen $(5\,D/2)$ = sen 30° = 0.50000 |
| Suma de cosenos = 2.81161 | Suma de los senos = 0.91355 |
| $A\,c' = 2.81161\,c = 281.16$ pies. | $c'\,c = 0.91355\,c = 91.355$ pies. |

**96.** Primero, desde $A$, aliniense los puntos $a'$, $b'$, $c'$, etc., en la tangente, $A\,t$ Entonces, desde $a'$, $b'$, $c'$, etc., trácese $a'\,a$, $b'\,b$, $c'\,c$, etc., normal a $A\,t$; o trácese, $a'\,a$, $b'\,b$, etc., al azar, a cruzarse con $A\,a$, $a\,b$, $b\,c$, etc., cada una $= c$.

**97.** Si, como es costumbre, la curva comienza o termina con una sub-cadena, pudiera ser preferible dividir la curva (desarrollo $= \Delta$) en un numero conveniente, $n$, de partes iguales, cada una $= \Delta/n$, y substituir $\Delta/n$ por $D$, y $2\,R\,\text{sen}\,/(\Delta 2\,n)$ por $c$, etc.

A medida que el trabajo avanza, y los angulos $a\,b\,b''$, $b\,c\,c'$, etc., se vuelven mas agudos, y el trabajo resulta menos exacto.

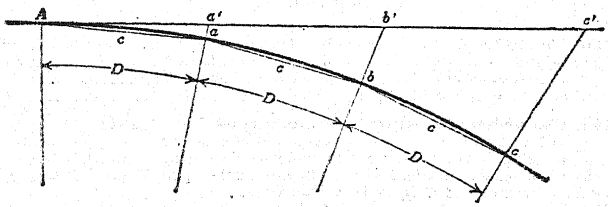



**Fig. 28.**

**98. En la Fig. 28, las dists,** $a\,a'$, $b\,b'$, etc., estan **en linea con los radios** en $a$, en $b$, etc. Aquí:

$$A\,a' = R\,\text{tang}\,D;\ A\,b' = R\,\text{tang}\,2\,D;\ A\,c' = R\,\text{tang}\,3\,D,\ \text{etc.} \ldots\ldots (103)$$

$$a\,a' = R\,\text{exsec}\,D;\ b\,b' = R\,\text{exsec}\,2\,D;\ c\,c' = R\,\text{exsec}\,3\,D,\ \text{etc.} \ldots\ldots (104)$$

Desde $A$, aliniese $a'$, $b'$, $c'$, etc. Desde $A$ y desde $a'$, simultaneamente, trácese $a'\,a$, respectivamente, determinando el punto $a$. Entonces, desde $a$ y desde $b'$, respectivamente, trácese simultáneamente, $c$, y $b'\,b$, determinando el punto $b$, y asi sucesivamente. Vease ¶ 97.

**99. Dando dos estaciones consecutivas.** Fig. 29. Teniendo dos estaciones consecutivas, $z$ y $A$, y la direccion de la tangente, $m\ n$, en $A$, determinar una tercera estacion, $a$.

Desde $A$, en la tangente, $m\ n$, tracese $A\ n = c \cos (D/2)$. Entonces tracese $A\ a = c$, para encontrar $n\ a = c$ sen $(D/2) = R$ senoverso $D = 2\ R$ sen$^2$ $(D/2)$, trazado desde $n$; o, desde $A$, $\perp\ m\ n$, tracese $A\ A' = c$ sen $(D/2)$; y desde $A'$ trace $A'\ a = c \cos (D/2)$, para encontrar $A\ a = c$ trazado desde $A$, o bien, $n\ a = c$ sen $(D/2)$, trazado desde $n$.

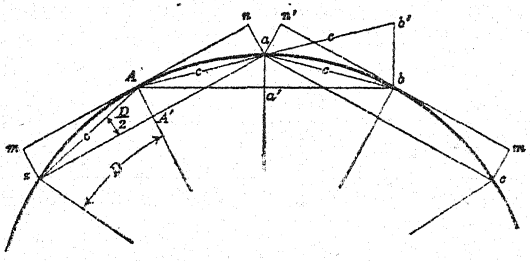



**Fig. 29.**

Para determinar la proxima estacion, $b$; en $A\ a$ prolongado, hágase $a\ b' = c$, y búsquese $b$ por medio de ¶ ¶ 92, 93, donde $f\ c = b'\ b$, Fig. 29.

Para la direccion de la tangente, $n'\ m'$, en $b$; determinese la próxima estacion $c$, como arriba; desde $a$ y desde $c$, $\perp\ a\ c$, trácese $a\ n'$ y $c\ m'$ respectivamente; $a n' = c\ m' = c$ sen $(D/2)$.

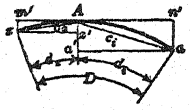



**Fig. 30.**

**100.** Fig. 30. En el caso de una sub-cadena, $c_i = A\ a$, sea $d_i =$ el sub-angulo: $d_z = D - d_i$; $c_i\ (= A\ a) = 2\ R$ sen $(d_i/2)$; $c_z\ (= z\ A) = 2\ R$ sen $(d_z/2)$. Entonces;

$a\ a' = A\ n' = c_i \cos (d/2)$; $A\ a' = n'\ a = c_i$ sen $(d!/2)$ ..... (105)

$z\ z' = m'\ A = c_z \cos (d_z/2)$; $A\ z' = m'\ z = c_z$ sen $(d_z/2)$ ..... (106)

**101. Por ordenadas sobre cuerdas largas.** Fig. 31. Sea:

$c = 1$ cadena $= a\ b = b\ c =$ etc., que subtiende $D$;

$c_i =$ sub-cadena inicial, $A\ a$, subtendiendo el sub-angulo inicial, $A\ O\ a = d_i$;

$c_f =$ sub-cadena final $h\ B$, subtendiendo el sub-angulo final $h\ O\ B = d_f$;

$\Delta =$ desarrollo, $A\ O\ B$, de la curva entera.

Entonces, para las abscisas, $A\ a'$, $A\ b'$, $A\ c'$, etc., tenemos:

$$A\ a' = c_i \cos a\ A\ a' = c_i \cos \frac{\Delta - d_i}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (107)$$

$$a'\ b' = a\ b'' = c \cos b\ a\ b'' = c \cos \frac{\Delta - 2d_i - D}{2}$$

$$b'\ c' = b\ c'' = c \cos c\ b\ c'' = c \cos \frac{\Delta - 2d_i - 3D}{2}$$

etc. y

$A\ c' = A\ a' + a'\ b'$;

$A\ c' = A\ b' + b'\ c'$, etc ..... (108

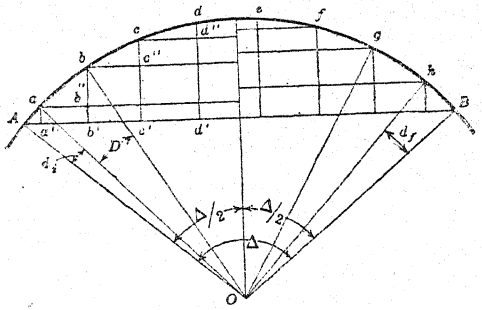



**Fig. 31.**

De modo análogo para las ordenadas, $a'\,a$, $b'\,b$, $c'\,c$, etc., tenemos :

$$a'\,a = c_i \text{ sen } \frac{\Delta - d_i}{2};$$

$$b''\,b = c \text{ sen } \frac{\Delta - 2\,d_i - D}{2};$$

$$c''\,c = c \text{ sen } \frac{\Delta - 2\,d_i - 3\,D}{2}; \text{ etc., y}$$

$$b'\,b = a'\,a + b''\,b\,;\; c'\,c = b'\,b + c''\,c, \text{ etc} \ldots\ldots\ldots\ldots\ldots\ldots \quad (109)$$

**102.** Fig. 32. *Para eliminar las sub-cadenas* (vease ¶ 97), sea la curva. $A\,P\,B$, divida en un numero conveniente de arcos iguales; y sea :

$c_e$ = la cuerda, $A\,a = a\,b$, etc., subtendiendo uno de estos arcos;
$D_e$ = el angulo central, $A\,O\,a$, subtendido por $c_e$;
$C$, $C'$, $C''$, etc = las cuerdas largas, $A\,B$, $a\,h$, $b\,g$, etc., respectivamente;
$M$, $M'$, $M''$, etc = las medias ordenadas, $E\,P$, $E'\,P$, $E''\,P$, etc. de $C$, $C'$, $C''$, etc., respectivamente.

Entonces :

Abscisas.

$C/2 = A\,E = R$ sen $(\Delta/2)$;
$C'/2 = a\,E' = R$ sen $(\Delta/2 - D_e)$;
$C''/2 = b\,E'' = R$ sen $(\Delta/2 - 2\,D_e)$; etc.

$$A\,a' = C/2 - C'/2 = c_e \cos a\,A\,a' = c_e \cos \frac{\Delta - D_e}{2};$$

$$a'\,b' = a\,b'' = C'/2 - C''/2 = c_e \cos b\,a\,b'' = c_e \cos \frac{\Delta - 3\,D_e}{2};$$

$$b'\,c' = b\,c'' = C''/2 - C'''/2 = c_e \cos c\,b\,c'' = c_e \cos \frac{\Delta - 5\,D_e}{2};$$

etc.; y

$$A\,b' = A\,a' + a'\,b';$$
$$A\,c' = A\,b' + b'\,c'; \text{ etc} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (110$$

Ordenadas.

$M = E\ P = R$ senoverso $(\Delta/2)$;
$M' = E'\ P = R$ senoverso $(\Delta/2 - D_c)$;
$M'' = E''\ P = R$ senoverso $(\Delta/2 - 2\ D_c)$; etc.

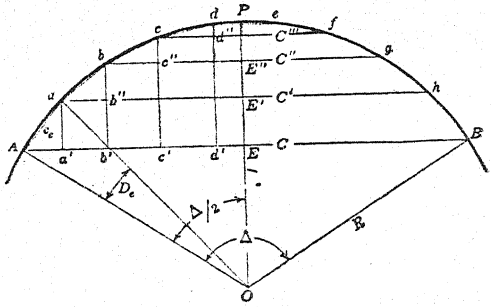



Fig. 32.

$$a'\ a = M' - M' = c_e \text{ sen } \frac{\Delta - D_c}{2};$$

$$b''\ b = M' - M'' = c_e \text{ sen } \frac{\Delta - 3\ D_c}{2};$$

$$c''\ c = M'' - M''' = c_e \text{ sen } \frac{\Delta - 5\ D_c}{2}; \text{ etc.; y}$$

$b'\ b = a'\ a + b''\ b = M - M''$;
$c'\ c = b'\ b + c''\ c = M - M'''$; etc..... (111)

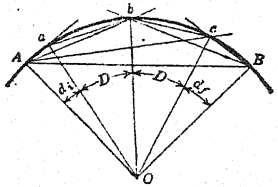



Fig. 33.

**103. Por medio de Dos Tránsitos, sin medidas lineales.** Fig. 33. Es útil cuando es imposible usar la cadena, como en pantanos, etc.

Dando cualquiera de las dos estaciones, *A* y *B*, cada una visible de la otra;

De cada una de las dos estaciones, *A* y *B*, vease a la otra y trácense los ángulos encontrados, como abajo, determinando los puntos, *a*, *b*, etc., por medio de las intersecciones de las visuales.

Así, sea *a b*, *b c*, las cadenas; *A a*, *c B*, las sub-cadenas. Entonces para determinar el punto *b*, en la interseccion de *A b* y *B b*; tenemos:

$$b\ A\ B = b\ O\ B/2 = (D + d_f)/2;$$
$$b\ B\ A = b\ O\ A/2 = (D + d_i)/2 \ldots\ldots\ldots\ldots\ldots (112)$$

Similarmente, para el punto *c* (interseccion de *A c* y *B c*), tenemos:

$$c\ A\ B = c\ O\ B/2 = d_f/2;$$
$$c\ B\ A = c\ O\ A/2 = (2\ D + d_i)/2 \ldots\ldots\ldots\ldots\ldots (112')$$

**104. Por Cuerdas Largas desde el Instrumento. Fig. 34.** Sea $a\,b = b\,c = c = 1$ cadena; $A\,a = c_i$ = sub-cadena inicial; $c\,B = c_f$ = sub-cadena final.

Entonces en la fig. 34 :

$$t\,A\,a = d_i/2; \qquad A\,a = 2\,R \operatorname{sen} t\,A\,a;$$
$$t\,A\,b = (d_i + D)/2; \qquad A\,b = 2\,R \operatorname{sen} t\,A\,b;$$
$$t\,A\,c = (d_i + 2\,D)/2; \qquad A\,c = 2\,R \operatorname{sen} t\,A\,c;$$
$$t\,A\,B = (d_i + 2\,D + d_f)/2 = \Delta/2; \qquad A\,B = 2\,R \operatorname{sen} t\,A\,B. \ldots\ldots \quad (113)$$

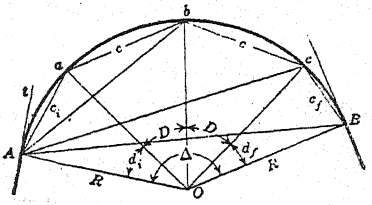



**Fig. 34.**

**105. Plantillas auxiliares.** La determinacion de curvas sobre el plano se facilita grandemente con el uso de *plantillas*, en escalas adecuadas.

**106.** El transportador de curva transparente de Wm. F. Shunk suministra un número de curvas en escala de 1 pulg = 400 pies.

**107.** En la revista Eng News 1902 jun 19, p. 500-501, Chas H. Quimby Jr. sugiere un « *proyectador de curvas y escalas* » (del cual se ha solicitado patente) que es un transportador de curvas transparente en el cual cada curva esta perforada a cada 100 pies de su longitud por la escala, a fin de que, con un instrumento de punta, los puntos correspondientes puedan marcarse en el plano que está debajo.

**108. Ordenadas.** En curvas agudas pueden determinarse puntos adicionales, intermedios de las estaciones, por medio de ordenadas a una cuerda que una dos estaciones consecutivas. Generalmente se estira un cordel entre las dos estaciones consecutivas, y las ordenadas se miden, tan cerca como se pueda a angulo recto con él por medio de una vara graduada o lienza. Los puntos a lo largo del cordel desde el cual se han de medir las ordenadas (especialmente el punto del medio y los dos puntos a la mitad entre este y los extremos), se distinguen por nudos o de otro modo.

**109.** La distancia entre las ordenadas por su puesto que depende de las condiciones del trabajo y de la agudeza de la curva. Para guiar los movimientos de tierra 50 pies (15 m) es bastante cerca, o 25 pies (7,5 m) para radios menores de 1.000 pies (digamos 300 m) para tendido de via, la distancia puede variar de 25 pies (7,5 m) en curvas fáciles a 10 pies (digamos 3 m) y aun 5 pies (digamos 1,5 m) en las mas agudas.

Vease ecs (32) y (34) a (39), y tabla de ordenadas, pags 974 á 977.

## OBSTÁCULOS

Otros métodos, ademas de los que se han indicado aqui pueden idearse para acomodarse a estos u otros casos.

**110. Visual interceptada.** Las siguientes indicaciones se refieren a casos que surgen en el método usual de determinacion por angulos perifericos, ¶ ¶ 81, etc.; pero la dificultad puede algunas veces evitarse abandonando ese metodo y usando en su lugar uno u otro de los metodos de determinacion mencionados en ¶ ¶ 92-104. Véase tambien en Agrimensura p. 291 1ª linea « caso » 5°.

**111.** Fig. 35. Teniendo, determinados $a$ y $b$, desde $A$, por medio de ángulos periféricos, $t'\,A\,a$ y $t'\,A\,b$, y cadenas, $A\,a$ (sub-cadena) y $a\,b$, encontramos que un obstáculo obstruye la visual, $A\,c$, a la próxima estacion, $c$.

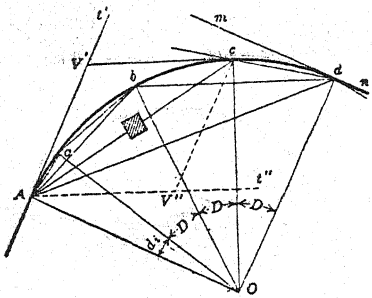



Fig. 35.

**112.** Determínese la próxima estacion, $d$, despues de $c$, trazando $t'\,A\,d = t'\,A + D$, y trazando $b\,d = 2\,R$ sen $D$, o, $A\,d = 2\,R$ sen $t'\,A\,d$ (vease tabla, p. 970). En curvas para F. C. de vapor, podemos tomar aprox, $b\,d = 2\,c$; y $A\,d = 3\,c + ci$; donde $c$ = a la unidad de cadena; y $ci$ = sub-cadena inicial, $A\,a$.

Para la tangente, $m\,n$, en $d$, tenemos $A\,d\,m = t'\,A\,d = \Delta/2$.

Para determinar $c$, entonces tenemos $m\,d\,c = D/2$; y $d\,c$ = cadena, $c$.

**113. O, podíamos determinar c directamente,** así: en la tang, $A\,t'$, trazar $A\,V' = R$ tang $t'\,A\,c = R$ tan $(di/2 + D)$. En $V'$ trazar $t'\,V'\,c = 2\,(t'\,A\,c)$, y $V'\,c\,(= A\,V')$ hacia $c$. Si $V'$ es inaccesible, podemos en su lugar determinar $V''$ (haciendo $b\,A\,V'' = t'\,A\,d = (D/2) + t'\,A\,c$; y $A\,V'' = A\,V'$); trazando $t''\,V''\,c = t'\,V'\,c = 2\,t''\,A\,c$, y hacer $V''\,c = V'\,c = A\,V''$.

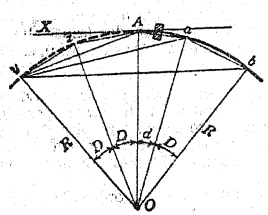



Fig. 36.

**114.** Fig. 36. ($A\,a$ = sub-cadena inicial.) Si un obstáculo impide la visual de $A$ hacia $a$, podemos mirar atras desde $A$ a lo largo de la tangente, $A\,X$, y extender la curva hacia atrás desde $A$, una o mas cadenas, como sea necesario, digamos hasta $z$ ó $y$. Entonces, con el instrumento puesto en $y$, por ejem., podemos mirar hacia $A$, trazando $A\,y\,a = \frac{1}{2}\,A\,O\,a = d/2$, y la cuerda larga, $y\,a = 2\,R$ sen $(y\,O\,a/2)$ = (en la fig. 36) $2\,R$ sen $(D + d/2)$. Para $b$, tenemos $a\,y\,b = \frac{1}{2}\,a\,O\,b = D/2$, y $a\,b$ = unidad de cadena; o, $y\,b = 2\,R$ sen $(y\,O\,b/2)$; etc., etc.

**Posición de un punto inaccesible necesario.**

**115. Vertice, V, inaccesible.** Fig. 37. Dando $\Delta$ y $R$ o $D$, buscar los puntos de la curva, $A$, $B$. Midase la dist, $a\ b$, entre cualquiera de los dos puntos accesibles sobre las dos tangentes respectivamente. Midanse los angulos $z\ b\ a$ e $y\ a\ b$, y búsquese $V\ b\ a$ ( $= 180° - z\ b\ a$) y $V\ a\ b$ ( $= 180° - y\ a\ b$). Entonces $V\ b\ a + V\ a\ b$ ( $= 180° - a\ V\ b$) $= \Delta$, ó $a\ V\ b = 180° - \Delta$.

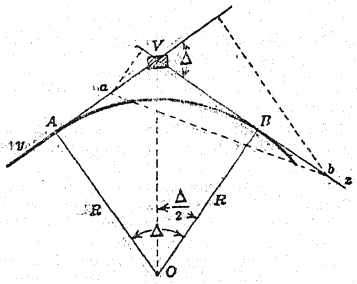



Fig. 37.

Encuéntrese, $V\ a = a\ b$ sen $V\ b\ a$/sen $\Delta$;
y $V\ b = a\ b$ sen $V\ a\ b$/sen $\Delta$.

Entonces :

$$A\ a = A\ V - V\ a = R \text{ tang } (\Delta/2) - V\ a;$$
$$B\ b = B\ V - V\ b = R \text{ tang } (\Delta/2) - V\ b.$$

Nótese que, donde (como en el ultimo caso) $B\ b$ es negativo, $b$ está *despues* de $B$ y $B\ b$ debe marcarse desde $b$ *hacia* el vertice, $V$, para determinar $B$.

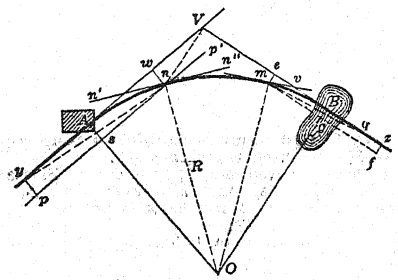



Fig. 38.

**Cualquiera de los puntos, A o B de la curva inaccesibles.**

**116. T. C. (A) inaccesible.** Fig. 38. Se necesita determinar un punto accesible, $n$, en la curva, y una tangente, $n'\ n''$, en el punto $n$. El punto, $n$, (se prefiere una estacion) debe dominar una visual, $n\ p$, paralela a la tangente, $A\ V$.

Sea $A\ O\ n$ la extension del arco, $A\ n$, (aprox, $A\ O\ n^\circ = D^\circ \times A\ n/100$). Sea $w\ n$ normal a la tangente $A\ V$, y sea $n\ s = p\ s$. Entonces $n\ s = p\ s = w\ A = y\ A = R$ sen $A\ O\ n$; $w\ n = A\ s = y\ p = R$ senoverso $A\ O\ n$; y $V\ w = V\ A - A\ w = T - A\ w = T - n\ S$.

Buscar la linea $n\,p$ (o, la $n\,p'$), y el punto, $n$, por uno de los dos métodos alternativos siguientes :

(1) Desde $p$, trazar $p\,n \parallel A\,V$, haciendo $p\,n = 2\,p\,s = 2\,R_s$ en $A\,O\,n$. O bien :

(2) Desde $V$, trazar $V\,w$ a $w$ ; midase $w\,n$ hasta $n$ ; y desde cualquier otro punto, $y$, sobre la tang $A\,V$, midase $y\,p = w\,n$ ; o desde $n$, trácese $y\,n\,p$ (tang $y\,n\,p = w\,n/w\,y$); o, desde $n$, tracese $V\,n\,p'$ (tang $V\,n\,p' = w\,n/V\,w$).

Entonces, para la tang, en $n$, tracese $n'\,n\,s = n''\,n\,p' = A\,O\,n$.

**117. C. T. (B) inaccesible.** Fig. 38. Se requiere determinar un punto, como $z$ ó $q$, en la tangente, $V\,B$, mas allá de $B$.

Damos tres metodos alternativos, como sigue :

Desde cualquier punto, como $m$, en la curva, a una distancia conocida desde $B$, tirese $m\,f \parallel$ a la tangente, $V\,B$. Entonces :

(1) Desde $m$, y desde cualquier otro punto, como $f$, en $m\,f$, tracense ordenadas $= m\,e = f\,z = R$ senoverso $B\,O\,m$, determinando la tangente, $V\,B$, y el punto, $z$. O bien :

(2) Desde $m$, trácese cualquier angulo, $f\,m\,q$, a una linea, $m\,q$, intersectando la tang $V\,B$, y trácese $m\,q = e\,m/\text{sen}\ f\,m\,q$, a $q$ que estará en la tang $V\,B$. Entonces $B\,q = q\,e - e\,B = m\,e \cot f\,m\,q - R$ sen $B\,O\,m$. O bien :

(3) Desde $m$, trácese $f\,m\,v = B\,O\,m$, determinese la tang, $m\,v$, y trácese $m\,v = R$ tang $(B\,O\,m/2)$. Entonces $v$ sera un punto en la tang $V\,B$. Desde $v$ vease hacia $m$, y tracese $m\,v\,V = B\,O\,m$. Pongase el anteojo, mirando a lo largo de la tang, $V\,B$. Entonces un punto, $z$ ó $q$, despues de $B$, puede encontrarse por los métodos dados en ¶ 110, etc., para obstáculos en las lineas rectas.

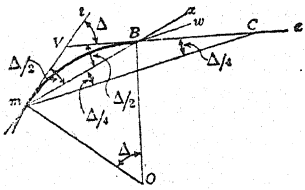



**Fig. 39.**

**118. Fig. 39. La C. T. (B) aunque accesible, no es ap ropiada**pa a punto del tránsito. Se necesita encontrar la tangente, $B\,z$.

Fijese cualquier punto conveniente, $m$, en la curva. Midase la cuerda, $m\,B$. Sea $\Delta$ = el angulo, $m\,O\,B$ = extension de la curva entre $m$ y $B$.

Desde $m$, tracese una tang, $m\,t$. Mirese hacia $B$. Entonces el angulo $t\,m\,B = \Delta/2$. Tracese $B\,m\,C = \Delta/4 = t\,m\,B/2$.

Tracese $B\,C = B\,m$, hasta encontrar la linea $m\,C$. Entonces $C$ es un punt en la tangente, $B\,z$.

Demostracion.

Como $m\,B\,C$ es isosceles, tenemos $B\,C\,m = B\,m\,C = \Delta/4$.

Extiendase $m\,B$, como hasta $x$, y tracese $B\,w \parallel m\,C$.

Entonces, del paralelismo de los lados :

$$x\,B\,w\ (= B\,m\,C) = \Delta/4;$$
$$w\,B\,C\ (= B\,C\,m) = \Delta/4;$$
$$\text{y}\ x\,B\,C\ (= x\,B\,w\ + w\,B\,C) = \Delta/2.$$

Sea $B\,z$ una tangente a la curva en $B$.

Entonces $x\,B\,z$ ( = el angulo opuesto $V\,B\,m$) $= \Delta/2 = x\,B\,C$, y $B\,C$ por consiguiente coincide con la tangente, $B\,z$.

## PROBLEMAS DEL TRAZADO DE CURVAS

(Vease tambien : Replanteo, ¶ ¶ 155, etc.)

### Soluciones Gráficas.

**119.** Gran parte de los problemas de curvas pueden resolverse con facilidad gráficamente, con suficiente aproximacion, y con la ventaja adicional de que el dibujo muestra las relaciones de las distintas líneas con mas claridad que el cálculo, y asi reduce la posibilidad de un error serio. La escala que se adopte depende de la naturaleza del problema y del grado de aproximacion que se requiera. Rara vez ser amenor de 1 pulg = 20 pies (digamos $^1/_{250}$). Lo que sigue se da como indicacion. Véase también ¶ 128.

**120. Ejemplo.** Fig. 46. Se desea resolver graficamente el problema del ¶ 131, a saber : Dadas dos curvas, $A\ B'$ y $B\ G$ de diferentes radios, $R_g$ y $R_s$, y diferentes centros, $O_g$ y $O_s$, y dando las tangs, $A\ V$ y $V\ B$, con su extension o desarrollo, $\Delta$; se desea buscar la linea radial comun $O_g\ E$, de las dos curvas; sus respectivos desarrollos, $g$ y $s$, y la dist mas corta, $d$ (que estara en la linea $O_g\ E$) entre ellos; donde se desee unir tambien las dos curvas por una espiral.

Desde $A$, tracese $A\ O_g \perp A\ V$ y ademas $= R_g$.
Desde $B$, tracese $B\ O_s \perp B\ V$ y ademas $= R_s$.
Por $O_g$ y $O_s$ tracese $O_g\ E$, como radio a ambas curvas.
Midanse los angulos, $g$ y $s$, y la dist, $d$.

Vease carta de C. M. Estabrook, Eng Record 1906 abril 7, p. 466.

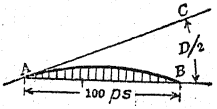



**Fig. 40.**

**121. Trazar una curva circular de radio grande.** Fig. 40. Tracese una cuerda, $A\ B$, que represente 100 pies en la escala. Divídase $A\ B$ en 20 espacios iguales. En cada punto de division, levantese una ordenada normal a la cuerda, dándoles el largo de las 19 ordenadas que correspondan en la tabla, pp. 976-7 para la agudeza dada, $D$. Unanse los extremos de las ordenadas.

**122.** Si la curva $A\ B$ es la linea de centro, las curvas para los carriles, pueden trazarse a una dist de $A\ B$ igual a la mitad del ancho de via. Ordinariamente es bastante exacto medir estas distancias en las direcciones de las ordenadas, ya trazadas. De lo contrario las distancias deben medirse radialmente desde la curva, $A\ B$; i. e., a angulos rectos con las tangentes trazadas (vease ¶ 124) en los extremos de las 19 ordenadas.

**123.** El angulo, $C\ A\ B$, entre la cuerda de 100 pies, $A\ B$, y una tangente, $A\ C$ en cada extremo, $A$ ó $B$, de la cuerda, es $D/2$, donde $D$ = la agudeza de la curva = desarrollo (angulo central) subtendido por la cuerda de 100 pies.

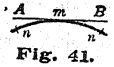



**Fig. 41.**

**124. Trazar una tangente a una curva**, en un punto dado, $m$, fig. 41, Desde $m$, tracense dos distances iguales, $m\ n$, $m\ n$, a la curva. Por $m$, tracese $A\ B$. paralela a $n\ n$. Entonces $A\ B$ sera tangente a la curva en $m$.

## Soluciones en el campo.

**125.** Muchos problemas son generalmente, con suficiente aproximacion y facilidad, resueltos por **tanteo en el campo**, ayudados, donde es necesario, por plantillas o transportadores de curva (vease ¶ ¶ 105-107) y un tablero de croquis de campo.

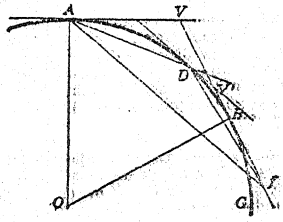



**Fig. 42.**

**126. Ejemplo.** Fig. 42. En una curva dada, $A\ G$, encontrar el punto $B$, desde el cual podamos trazar la tang. $B\ f$ por un punto dado, $f$.

Esto se resuelve trigonometricamente en el ¶ 129, fig. 44.

Con el instrumento en cualquier punto como $A$, en la curva, vease a lo largo de la tangente, $A\ V$; midase el angulo, $V\ A\ f$, y tracese el angulo, $V\ A\ D = V\ A\ f/2$. Con el instrumento en $D$, donde la linea $A\ D$ corta a la curva, repitase el procedimiento, mirando a lo largo de la tangente, $D\ V'$, y a $f$, midiendo el angulo $V'\ D\ f$, y trazando $V'\ D\ f/2$ (no señalado) a la curva en un punto cerca de $B$, y asi lo demas aproximandose a $B$. Cuando $f$ se ve en la tangente, $B\ f$, el instrumento está en el punto, $B$ que se busca.

## Soluciones Geométricas.

A continuacion se dan soluciones geometricas de unos pocos problemas tipico de frecuente uso.

**Advertencia.**

**127.** Cuando $\Delta$ es grande, tengase cuidado con los **cambios necesarios en los signos** + y −. Vease ¶ 69.

Asi, en la fig. 46, ¶ 131, si $\Delta > 90^\circ < 180^\circ$, tenemos sen $\Delta$ positivo pero cos $\Delta$ negativo; haciendo $B'\ I$ positivo, pero $V\ i$ negativo.

En general, el uso de los signos, + y −, en nuestro texto, se refiere a nuestras figs, en las cuales, por conveniencia, la extension $\Delta$ se hace generalmente $< 90^\circ$

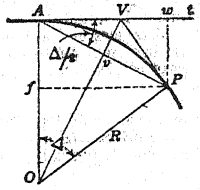



**Fig. 43.**

**128. Pasar una curva por un punto dado**, $P$. Fig. 43.

Dando, direccion de la tangente, $A\ t$; la T. C., $A$; y las co-ordenadas, $A\ w$ y $w\ P$, del punto dado, $P$;

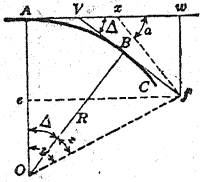
Se desea conocer, $\Delta$ y $R$ (o $D$), para la curva, $A\ P$. Aqui : tang $(\Delta/2) = w\ P/A\ w$; $R = f\ P/\text{sen}\ \Delta = A\ w/\text{sen}\ \Delta = w\ P/\text{senoverso}\ \Delta$.

Dando $R$ en lugar de $A\ w$; se requiere, $\Delta$ y $A\ w$. Aqui : senoverso $\Delta = w\ P/R$; $A\ w\ (= P\ f) = R$ sen $\Delta$.

Graficamente, únase $A\ P$, midase angulo, $w\ A\ P = \Delta/2$, divídase la cuerda $A\ P$ en $v$, tírese una línea en la dirección $O\ v\ V$, perpendicular a $A\ P$, que corta $A\ t$ en $V$, y midase $A\ V$ y $V\ P$. Entonces $V$ es el vértice de la curva $A\ P$ que une $A$ y $P$ y es tangencial a $A\ t$ en $A$; $A\ V$ y $V\ P$ son sus semitangentes; y tenemos:

$$\Delta = 2\ w\ A\ P; \qquad R = A\ V/\tan\ (\Delta/2) = V\ P/\tan\ (\Delta/2).$$

Fig. 44.




**129.** Fig. 44. En una curva dada, $A\ C$, **buscar el punto B, desde donde una tangente, B f, pase por un punto dado, f.** Para solucion con el instrumento y aproximada, vease ¶ 126. Fig. 42.

Dando, el radio, $R$, punto de la curva, $A$, la dirección de la tangente inicial, $A\ V$, y las co-ordenadas, $A\ w$ y $w\ f$, del punto dado, $f$.

(Si las co-ordenadas, $A\ w$ y $w\ f$, no son dadas, vease abajo.) Se desea el angulo, $\Delta$. Por $f$, tírese $e\ f \perp A\ O$. Entonces:

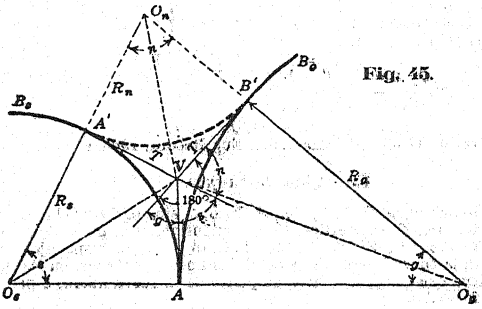
$$A\ e = w\ f; \qquad e\ f = A\ w;$$

$$\text{y tang } y = \frac{e\ f}{e\ O} = \frac{A\ w}{R - w\ f}; \qquad \cos z = \frac{R}{O\ f} = \frac{R}{A\ w/\text{sen } y}; \quad \Delta = y - z.$$

**Si las co-ordenadas, A w y w f, no son dadas,** y si no pueden medirse facilmente; tómese cualquier punto conveniente, como $x$, en la tang inicial, $A\ V$; midase $x\ f$ y $A\ x$, y el angulo, $A\ x\ f$, ó su suplemento, $a = 180° - A\ x\ f$. Entonces:

$$A\ w = A\ x + x\ w = A\ x + x\ f \cos a;$$
$$w\ f = x\ f \text{ sen } a = x\ w \text{ tang } a.$$

Fig. 45.




**130.** Fig. 45. **Teniendo dos curvas,** $A\ B_\beta$ y $A\ B_g$, empezando desde un punto comun de la curva, $A$, se desea **conectarlas por medio de una tercera curva,** $A'\ B'$, tangente a ambas curvas y que empieza desde un punto dado, $A'$ en una de las dos curvas.

Dando, ambos radios, $R_\beta$ y $R_g$, y el desarrollo, $\Delta_\beta$, (o $\beta$) $= A'\ O_\beta\ A$, de la curva que contiene el punto dado, $A'$;

Se busca la semitangente comun, $T$; la extension, $\Delta_g$ (o $g$) de $A\ B'$, y el desarrollo $\Delta_n$ (o $n$) y el radio, $R_n$, para la nueva curva, $A'\ B'$.

Aqui tenemos :

$T = A\ V = A'\ V = B'\ V = R_s \text{ tang } (s/2) = R_g \text{ tang } (g/2) = R_n \text{ tang } (n/2).$

Por lo tanto :

$\text{Tang } (g/2) = T/R_g\ ; \qquad n = 180° - (s + g); \qquad R_n = T/\text{tang } (n/2).$

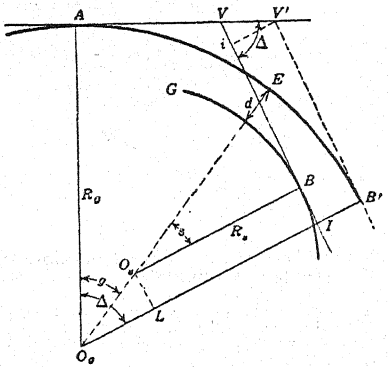



**Fig. 46.**

**131. Fig. 46, Dando dos curvas.** *A E* y *B G*, con radios diferentes dados, $R_g$ y $R_s$, y centros diferentes, $O_g$ y $O_s$, y dandose las semitangentes, *A V* y *V B*, con sus angulos centrales o desarrollos, Δ; se requiere buscar la linea radial comun, $O_g$ *E*, de las dos curvas, sus ang centrales, $\Delta_g$ (o *g*) y $\Delta_s$ (o *s*) y la distancia mas corta, *d*, entre ellas (*d* estará en la linea $O_g$ *E*, que une los centros, $O_g$ y $O_s$, de las dos curvas). Para la solucion gráfica, vease ¶ 120.

Tracese $O_g B' \perp V B$, y extiéndase la curva *A E* que intercepta $O_g B'$ en *B'*. Tracese $V' B' \parallel V B$. Desde *V'* tracese $V' i \perp V E$. Entonces :

$B'\ V' = A\ V' = R_g \text{ tang } (\Delta/2); \qquad V\ V' = A\ V' - A\ V;$

$B'\ I\ = V'\ i = V'\ V \text{ sen } \Delta; \qquad V\ i = V'\ V \cos \Delta.$

Trácese $O_s L \perp O_g B'$. Entonces :

$O_s L = B\ I\ = B'\ V' + V\ i - V\ B;$

$O_g L = O_g B' - L\ I\ - B' I = R_g - R_s - B' I;$

$\text{Tang } s = O_s L/O_g L;$

$\Delta_g = \Delta - \Delta_s;$ y

$d = R_g - R_s - (O_g L) \text{ sec } s.$

Véase carta de Wm. R. Dunham, Jr., Eng Record, 1906, abril 7, p. 466.

## Cambios de Lugar.

**132.** En lo que sigue, *R*, *T*, *E*, etc., *A*, *V*, *B*, Δ, etc., indican : el radio dado, semitan, dist ext, etc., T. C., vertice, C. T., extension, etc., para la curva *existente*; y *R'*, *T'*, *E'*, etc., *A'*, *V'*, *B'*, Δ', etc., indican largos correspondientes, puntos y angulos para la curva *nueva*. En general, las lineas *continuas*, en las figs, se refieren a la curva existente; las lineas de punto de la curva *nueva*, o para la demostracion de formulas.

### Con Tangentes Inalterables.

**133.** Fig. 47. Cambio de curva que une dos tangentes dadas (Δ y *V* constante); se requiere el cambio de *R*, *T* y *E* (*E* = *V P*) dados.

Aqui :

$T\ (= A\ V) = R \text{ tang } (\Delta/2); T'\ (= A'\ V) = R' \text{ tang } (\Delta/2); T - T' = A\ A';$

$E\ (= V\ P) = R \text{ exsec } (\Delta/2) = T \text{ tang } (\Delta/4); E'\ (= V\ P') = R' \text{exsec } (\Delta/2) = T' \text{ tang } (\Delta/4); E - E' = P\ P'; R - R' = (O\ O') \cos (\Delta/2).$

Por consiguiente :

1. Dando $T'$; buscar $R'$ y $E'$.

$$R - R' = (T - T') \cot (\Delta/2);$$
$$E - E' = (R - R') \operatorname{exsec} (\Delta/2) = (T - T') \tan (\Delta/4).$$

2. Dando $E'$; buscar $R'$ y $T'$.

$$R' = R\,E'/E; \; R - R' = \frac{E - E'}{\operatorname{exsec} (\Delta/2)};$$

$$T - T' = (R - R') \tan (\Delta/2) = (E - E') \cot (\Delta/4).$$

3. Dando $R'$; buscar $T'$ y $E'$.

$$T - T' = (R - R') \tan (\Delta/2);$$
$$E - E' = (R - R') \operatorname{exsec} (\Delta/2) = (T - T') \tan (\Delta/4).$$

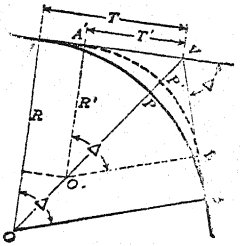



**Fig. 47.**

**34. Cambios en las tangentes.** Donde solamente una de las tangentes se cambia, suponemos por razon de uniformidad que esta sea la tang *final*, la tang *inicial* permanece inalterable; pero con los cambios en las letras, las instrucciones dadas se aplican igualmente al caso contrario; por tanto, donde la tangente *inicial* se cambia, la tangente *final* permanece inalterable.

**Una tangente movida paralela** a si misma ($\Delta$ constante). Cambiado el punto de la tang. $B$.

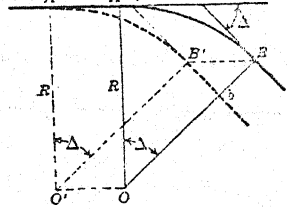



**Fig. 48.**

**135.** (1) Fig. 48. El radio, $R$, constante; punto de la curva, **A, corrido.** $A\,B\,B'$, $O\,O'$, $V\,V'$ iguales y paralelos.

Dando, la dist, $B\,b$, entre tangs. Buscar la dist movida, $A\,A'$.

$$A\,A' = B\,B' = \frac{B\,b}{\operatorname{sen} \Delta}.$$

**136.** (2) Fig. 49. Punto de la curva, $A$, constante; radio, **R, cambiado.** $A$, $B$ y $B'$, estan en linea recta; $B'$ es la interseccion de la nueva tang $V'B'$, con la cuerda larga, $A\,B$. $B\,b$ = dist entre las tangs nueva y vieja, $T = A\,V$; $T' = A\,V'$.

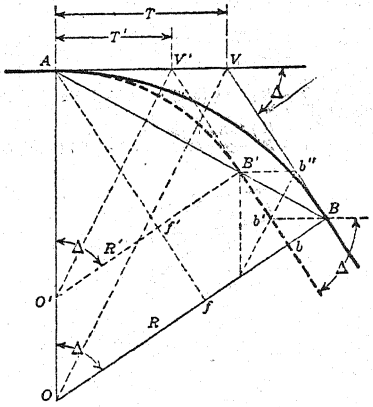



**Fig. 49.**

Se desean, $R'$, y $V\,V'$.
Desde $A$, trácese $A\,j$, $\perp O\,B$.
a. Dando, $B\,b$.
$B\,b = B\,f - b\,f = B\,f - B'\,f' = R\,\text{senov}\,\Delta - R'\,\text{senov}\,\Delta = (R - R')\,\text{senov}\,\Delta$.

Por tanto $R - R' = \dfrac{B\,b}{\text{senoverso}\,\Delta}$; $\qquad R' = R - \dfrac{B\,b}{\text{senoverso}\,\Delta}$;

b. Dando, $B\,B'$.

$$R - R' = \frac{B\,B'}{2\,\text{sen}\,(\Delta/2)}; \qquad R' = R - \frac{B\,B'}{2\,\text{sen}\,(\Delta/2)};$$

c. Dando $B\,B'$ y $A\,B$,

$$R' = R\,\frac{A\,B - B\,B'}{A\,B}.$$

En cada caso:
$V\,V' = T - T' = (R - R')\,\text{tang}\,(\Delta/2) = B'\,b'' = B\,b' = B\,b/\text{sen}\,\Delta$.

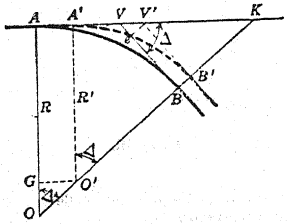



**Fig. 50.**

**137.** (3) Fig. 50. El punto **A, movido; R, cambiado.** El nuevo punto de la tang $B'$, opuesto al punto viejo $B$ de la tang.

Dando, $B\,B'$. Se desea $R'$, $A\,A'$ y $V\,V'$. Trácese $O'\,G \parallel A\,K$; y $V'\,e \parallel B\,B'$. Prolonguese el radio $O\,B$ hasta encontrar la tang $A\,K$ en $K$.

$$B\,B' = B\,K - B'\,K = R \operatorname{exsec} \Delta - R' \operatorname{exsec} \Delta = (R - R') \operatorname{exsec} \Delta.$$

Por tanto $R - R' = \dfrac{B\,B'}{\operatorname{exsec} \Delta}$; y $R' = R - \dfrac{B\,B'}{\operatorname{exsec} \Delta}$;

$$A\,A' = G\,O' = G\,O \operatorname{tang} \Delta = (R - R') \operatorname{tang} \Delta = \frac{B\,B'}{\operatorname{exsec} \Delta} \operatorname{tang} \Delta = \frac{B\,B'}{\operatorname{tang} (\Delta/2)};$$

$$V\,V' = \frac{V'\,e}{\operatorname{sen} \Delta} = \frac{B\,B'}{\operatorname{sen} \Delta}.$$

**138. Cada tang movida, paralela** a si misma. $\Delta$ constante. $R$ constante. Punto $A$, de la curva, y punto $B$, de la tang, cambiados.

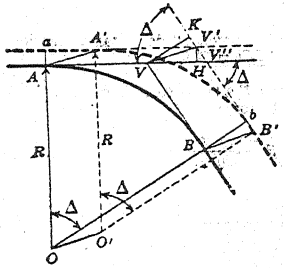



**Fig. 51.**

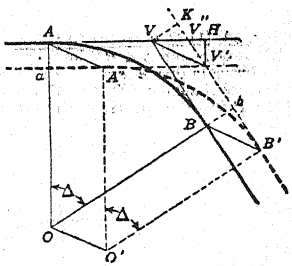



**Fig. 52.**

(1) Fig. 51. **Ambas tangs movidas** hacia afuera, como se indica, o ambas hacia adentro.

Supongamos la tang $A\,V$ movida la dist $A\,a$; la tang $V\,B$ la dist $B\,b$.

Entonces $A\,A'$, $B\,B'$, $O\,O'$ y $V\,V'$ son iguales y paralelas.

Se desea $a\,A'$ y $b\,B'$.

Sea V'' la interseccion de $A\,V$ y $V'\,B'$.

Trácese $V''\,K \parallel B\,b$, y $V'\,H \parallel A\,a$. Entonces :

$$a\,A' = V\,H = V\,V'' - V''\,H = \frac{B\,b}{\operatorname{sen} \Delta} - \frac{A\,a}{\operatorname{tang} \Delta}.$$

$$b\,B' = K\,V' = K\,V'' - V''\,V' = \frac{B\,b}{\operatorname{tang} \Delta} - \frac{A\,a}{\operatorname{sen} \Delta}.$$

**139.** (2) Fig. 52. **Una tang movida hacia adentro, la otra hacia afuera.**

Sea $A\,a$, una de las movidas y considerese como *negativa*. Entonces $V''\,H$ y $V''\,V'$ son negativas; y

$$a\,A' = V\,V'' \textit{ plus } V''\,H; \qquad b\,B' = K\,V'' \textit{ plus } V''\,V'.$$

**Tangentes divergentes, $V\,z$ y $V'\,z'$, formando un angulo $a$.**

**140. Figs 53, 54, 55. Tangentes que se cortan en el punto B, de la tangencia.**

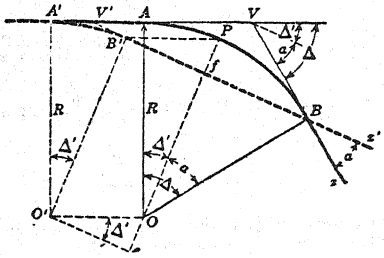



**Fig. 53.**

Fig. 53. (1) **R constante** ; $A$ movida; punto nuevo, $B'$ de la tang. Dando el radio $R$ de la curva $A\,B$ y los angulos, $\Delta$ y $a$, $\Delta' = \Delta \pm a$ *.

Se busca la dist $A\,A'$.

Hágase el angulo $A\,O\,P = \Delta'$, y por $O$, trácese $e\,P$ a la curva $A\,B$ en $P$, haciendo $O\,e = P\,f = R$ senoverso $a$. Unase $O\,O'$ y $e\,O'$. Entonces :

$$\mathbf{A\,A'} = O\,O' = \frac{O\,e}{\text{sen}\,\Delta'} = R\,\frac{\text{senoverso}\,a}{\text{sen}\,\Delta'}.$$

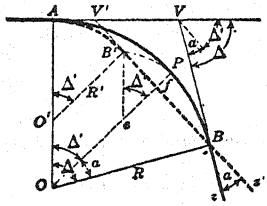



**Fig. 54.**

**141.** Fig. 54. (2). **A constante**; $R$ cambiada; punto nuevo. $B'$, de la tang.

Se busca, $R'$.

*Nota.* — A fin de que las tangs se corten en $B$, debemos tener $R' < R$. Con e objeto de que $A$ permanezca constante, $V'\,B'$ debe encontrar $A\,V$ en un punto, $V'$, delante de $A$. $a$ = angulo, $z\,B\,z'$, entre las dos tangs. $\Delta' = \Delta - a$.

Hagase $A\,O\,P = \Delta'$, y trácese $O\,P$, cortando la curva, $A\,B$, en $P$. Trácese $B'\,e \parallel O\,A$. Entonces :

$$\mathbf{R - R'}\ (= O\,O') = B'\,e = \frac{P\,f}{\text{senoverso}\,\Delta'} = R\,\frac{\text{senoverso}\,a}{\text{senoverso}\,\Delta'};$$

$$R' = R\left(1 - \frac{\text{senoverso}\,a}{\text{senoverso}\,\Delta'}\right).$$

---

* En la fig 53, $a$ es negativo, y $\Delta' = \Delta - a$. Si, como en la fig 55, $V$ está entre $A$ y $V$ $a$ es positivo, y $\Delta' = \Delta + a$.

**142.** Fig. 55 (3). **B constante**; $A$ movido; $R$ cambiado.
Se busca, $R'$ y la dist, $A\,A'$.

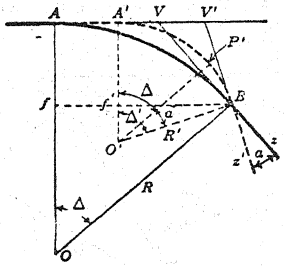



**Fig. 55.**

Trácese $B\,f \perp O\,A$, y $O'\,P' \parallel O\,B$. Entonces:

$$A\,f = A'\,f' = R \text{ senoverso } \Delta = R' \text{ senoverso } \Delta.$$

$$\therefore \mathbf{R'} = R\,\frac{\text{senoverso } \Delta}{\text{senoverso } \Delta'};$$

$$\mathbf{A\,A'} = B\,f - B\,f' = R \text{ sen } \Delta - R' \text{ sen } \Delta'.$$

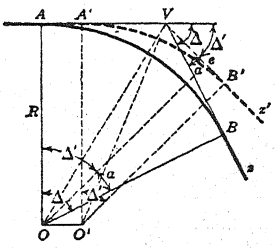



**Fig. 56.**

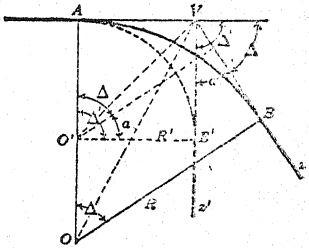



**Fig. 57.**

Figs 56 y 57. **Tangentes que se cortan en el vértice, V.**

**143.** Fig. 56 (1). **R constante**; $A$ movido; $B'$ punto nuevo, de la tang.
Dando, $R$, $\Delta$ y $a$.
Se desea correr $A\,A'$, para $A$.

$$A\,V = R \text{ tang } (\Delta/2); \qquad A'\,V = R \text{ tang } (\Delta'/2);$$

$$\therefore \mathbf{A\,A'} = A\,V - A'\,V = R\,[\text{tang } (\Delta/2) - \text{tang } (\Delta'/2)].$$

**144.** Fig. 57 (2). **A constante**; $R$ cambiado; $B'$, punto nuevo, de la tang $R'$ puede ser mayor o menor que $R$.
Dando, $R$, $\Delta$ y $a$.
Se busca, $R'$.

$$A\,V = R \tan (\Delta/2) = R' \tan (\Delta'/2);$$

$$\therefore \mathbf{R'} = \frac{A\,V}{\tan\ (\Delta'/2)} = R\,\frac{\tan\ (\Delta/2)}{\tan\ (\Delta'/2)}.$$

## Problemas de Curvas Compuestas.

**145. El punto P inaccesible.** Fig. 58. Buscar la tang comun $v_s\ v_g$, asi: $A\ v_s = v_s\ P = R_s$ tang $(s/2)$; y $B\ v_g = v_g\ P = R_g$ tang $(g/2)$; o, desde cualquier punto conveniente, como $K$ o $K'$, en cualquier ramal de la curva, trácese $K\ L$, o, $K'\ L'$ paralelo a $O_g\ P$; $K\ L = M\ P = R_s$ senoverso $K\ O_s\ P$; y $K'\ L' = M'\ P =$

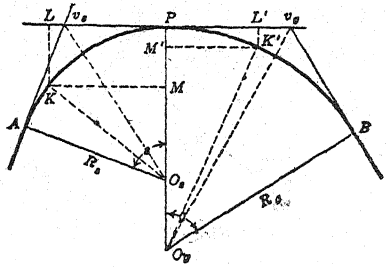



Fig. 58.

$R_g$ senoverso $K'\ O_g\ P$. Entonces $L\ P = K\ M = R_s$ sen $K\ O_s\ P$; $L'\ P = K'\ M' = R_g$ sen $K'\ O_g\ P$. Desde la tangente, $v_s\ v_g$, asi encontrada, las ramas de la curva pueden determinarse por el metodo dado en ¶ ¶ 95-98.

Si la tang comun, $v_s\ v_g$, esta obstruida, podiamos usar, en la misma forma, una linea paralela. $K\ M$ o $K'\ M'$, encontrada como se expresa arriba, dejando margen para la distancia, $M\ P$ o $M'\ P$, entre ella y $v_s\ v_g$.

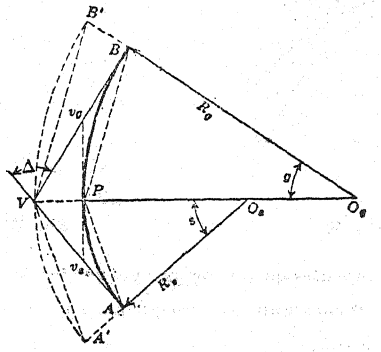



Fig. 59.

**146.** Fig. 59. Caso especial. El vértice, $V$, viene a caer en la linea radial comun $O_g\ P$, prolongada. Vease ¶ 57.

Dando, $\Delta\ (= s + g)$; $T_s = A\ V$, y $T_g = V\ B$.

Se busca, $s$, $g$, $R_s$ y $R_g$.

$$\text{Sen}\ \frac{s-g}{2} = \text{sen}\left(\frac{s}{2} - \frac{g}{2}\right) = \text{sen}\ \frac{\Delta}{2}\left(\frac{T_g - T_s}{T_g + T_s}\right) *;$$

* Véase nota al pie, pág 1023.

$s = \Delta/2 + (s - g)/2$; $\qquad g = \Delta - s$;

$\mathbf{R}_s = T_s/\tan s$; $\qquad \mathbf{R}_g = Tg/\tan g$.

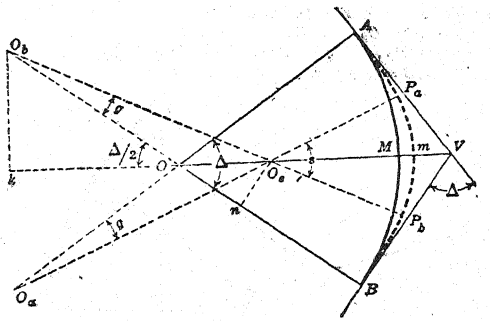



**Fig. 60.**

**147.** Figs 60-61. Sea, una curva simple, $A\ M\ B$, que une dos tangentes, $A\ V$ y $V\ B$. Conservando las mismas tangs, se desea reemplazar aquella por **una curva compuesta de tres centros.**

**148.** Fig. 60. **A. Conservando A y B.** Curva compuesta $A\ P_a\ m\ P_b\ B$.

Sea $R = O\ A = O\ M = O\ B =$ radio de la curva simple, $A\ M\ B$;

$R_s = O_s\ P_a = O_s\ m = O_s\ P_b < R$;

$R_g = O_a\ A = O_b\ B > R$;

$s = P_a\ O_s\ P_b$; $\qquad g = A\ O_a\ P_a = B\ O_b\ P_b$;

$\Delta = A\ O\ B = s + 2\ g =$ desarrollo o angulo central de la curva simple, $A\ M\ B$;

$M\ m =$ dist entre los puntos medios de las dos curvas.

Prolonguese $V\ O$ hasta $k$, y trácese $O_b\ k \perp V\ K$. Entonces :.

$O_b\ k = O_b\ O \operatorname{sen}(\Delta/2) = (R_g - R) \operatorname{sen}(\Delta/2) = O_b\ O_s \operatorname{sen}(s/2) = (R_g - R_s) \operatorname{sen}(s/2)$;

De donde :

$$\operatorname{Sen} s/2 = \frac{(R_g - R) \operatorname{sen}(\Delta/2)}{R_g - R_s}; \qquad g = \frac{\Delta - s}{2};$$

$$\mathbf{R}_g = \frac{R \operatorname{sen}(\Delta/2) - R_s \operatorname{sen}(s/2)}{\operatorname{sen}(\Delta/2) - \operatorname{sen}(s/2)}.$$

Trácese $O_s\ n \perp O\ B$. Entonces :

$$O\ O_s = O_s\ n/\operatorname{sen}(\Delta/2) = (R_g - R_s) \operatorname{sen} g/\operatorname{sen}(\Delta/2);$$

y $$\mathbf{M\ m} = O\ O_s + O_s\ m - O\ M = O\ O_s - (O\ M - O_s\ m)$$

$$= (R_g - R_s) \frac{\operatorname{sen} g}{\operatorname{sen}(\Delta/2)} - (R - R_s).$$

---

* De las ecuaciones para $\operatorname{sen}(A + B)$, $\cos(A + B)$, $\operatorname{sin}(A - B)$ y $\cos(A - B)$, ¶¶ 15, 16, pag 106 se tiene que

$$\operatorname{sen} \frac{s - g}{2} = \operatorname{sen} \frac{\Delta}{2} \times \frac{\tan(s/2) - \tan(g/2)}{\tan(s/2) + \tan(g/2)},$$

y de $Tg = T_s \dfrac{\tan(s/2)}{\tan(g/2)}$, ¶ 57 y pag 39, ¶ (5), tenemos

$$\frac{Tg - T_s}{Tg + T_s} = \frac{\tan(s/2) - \tan(g/2)}{\tan(s/2) + \tan(g/2)}$$

**149. Fig. 61. B. Conservando el mismo punto medio, M.** Curva compuesta, $A' P_a M P_b B'$.

Sea $R = O\ A = O\ M = O\ B$ = radio de la curva simple, $A\ M\ B$;

$R_s = O_s\ P_a = O_s\ M = O_s\ P_b < R$;

$R_g = O_a\ A' = O_b\ B' > R$;

$= P_a\ O_s\ P_b$; $\quad g = A'\ O_a\ P_a = B'\ O_b\ P_b$;

$= A\ O\ B = s + 2\ g$.

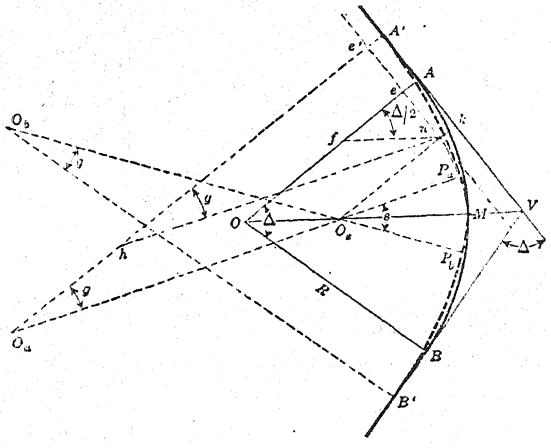



**Fig. 61.**

En la fig., prolonguese la curva, $P_b\ M\ P_a$, hasta encontrar en $n$ la linea $O_s k$, $\| O\ A$; y tracese $n\ h \| P_a\ O_a$, y $nf = O_s\ O$. Entonces $n$ es un punto en la cuerda larga, $M\ A$ (no dibujada), y en la cuerda larga, $P_a\ A'$ (no dibujada); la tang, $e\ n$, por $n$, es $\| A\ V$; y

$n\ k = A\ e = n\ f$ senoverso $(\Delta/2) = (R - R_s)$ senoverso $(\Delta/2)$

$= A'\ e' = n\ h$ senoverso $g \quad = (R_g - R_s)$ senoverso $g$;

de donde tenemos:

$$\text{senoverso } \mathbf{g} = \frac{n\ k}{R_g - R_s} = \frac{(R - R_s) \text{ senoverso } (\Delta/2)}{R_g - R_s};$$

$$\mathbf{R}_g = R_s + (n\ k/\text{senoverso } g);$$

$$\mathbf{R}_s = R - [n\ k/\text{senoverso } (\Delta/2)].$$

$$\text{Cot } (\mathbf{g}/2) = \text{cot } (\Delta/4) + (A\ A'/n\ k); \qquad \mathbf{s} = \Delta - 2\ g;$$

$$\mathbf{A\ A'} = \mathbf{B\ B'} = k\ A' - k\ A = n\ k\ [\text{cot } (g/2) - \text{cot } (\Delta/4)].$$

**150. Correr C. T. a lo largo de su tangente.** Fig. 62. Dando una curva compuesta, $U\ P\ E$, que une dos tangs, $U\ V$ y $V\ E$. Conservando la misma direccion de las tangentes, y el radio inicial dado, $R_u$, se desea que la segunda rama una la misma tang final, $V\ E'$, como antes pero en $E'$, en lugar de $E$.

Se busca, el nuevo radio final, $R_e'$, y la nueva distribucion del desarrollo, $\Delta$, entre las dos ramas; *i, e.*, los nuevos valores ($u'$ y $e'$), de los desarrollos, $u$ y $e$. El cambio envuelve la corrida del C. C, desde $P$ a un punto nuevo, como $P'$.

Sea $y$ ( $= E\ E'$) *positivo* cuando se mide hacia *adelante* desde $E$, como en la fig. 62.

Como en la fig. 62, supongamos, que el indice, $e$ se refiere a la rama cuyo radio $R_e$, se vaya a cambiar, y el indice $u$ a la otra rama.

Como las direcciones de las tangentes son inalterables, tenemos $\Delta = u + e = u' + e'$; y $u' = u + e - e'$.

En la Fig. prolonguese la rama $u$, $U P$, hasta encontrar, en $n$, la linea $O_u n$, paralela a $O_e E$ ($n$ esta en cualquier linea recta $P E$, $P' E'$, que una C. C. con el correspondiente punto de tangencia). Trácese $n i$, $\parallel$ y ademas $= O_u O_e = R_e - R_u = i E$. Trácese $n k'$, $\perp O_e E$ y $\perp O_e' E$.

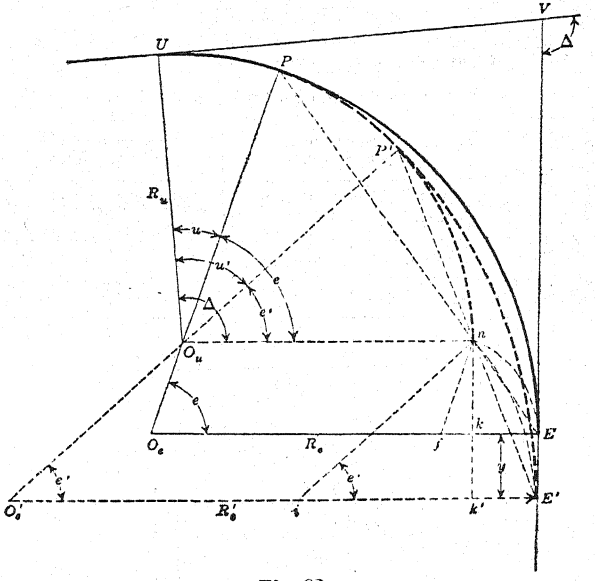



**Fig. 62.**

(1) Dando el cambio ($= P O_u P' = u' - u = e - e'$) en los desarrollos o angulos centrales; se busca $R_e'$ y ($= E E'$). Aqui tenemos $e' = P' O_e' E' = P' O_u n = \Delta - (u + P O_u P')$. En la Fig. trácese $n i \parallel$ y tambien $= O_u O_e' = R_e' - R_u$. Entonces:

$E' k' = (R_e' - R_u)$ senoverso $e' = E k = (R_e - R_u)$ senoverso $e$. Por tanto,

$$\mathbf{R}e' (= O_e' E') = O_e' i + \frac{E k}{\text{senoverso } e'} = R_u + (R_e - R_u) \text{ senoverso } e/\text{senov } e';$$

$$\mathbf{y} (= \mathbf{E E}' = n k' - n k) = E' k' \cot (e'/2) - E k \cot (e/2)$$
$$= E k [\cot (e'/2) - \cot (e/2)].$$

(2) Dando $R_e'$; se busca $P O_u P' (= u' - u = e - e')$ y ademas $y (= E E')$. Aqui tenemos:

$$\text{Senoverso } e' = \frac{E' k'}{n i} = \frac{E k}{n i} = \frac{(R_e - R_u) \text{ senoverso } e}{R_e' - R_u}; \text{ y}$$

$$\mathbf{P O_u P'} = e - e'; \; u' = u + P O_u P'.$$

$\mathbf{y} (= \mathbf{E E}')$ como se expresa arriba.

(3) Dando $y$ ( $= E E'$ ); se busca $P O_u P'$ ( $= e - e'$ ) y $R_e'$.

Como (vease arriba) $y$ ( $= E E'$ ) $= E k$ [cot $(e'/2)$ − cot $(e/2)$], tenemos:

$$\frac{y}{E k} = \cot (e'/2) - \cot (e/2); \text{ y } \cot (e'/2) = \cot (e/2) + \frac{y}{E k}$$

$$= \cot (e/2) + \frac{y}{(R_e - R_u) \text{ senoverso } e}. \text{ Entonces:}$$

**P $O_u$ P'** y **R'** pueden encontrarse como arriba.

**Signos mas y menos.** Vease pag 993, ¶ 69.

| | $E'$ despues de $E$ ($y$ positivo). | | $E'$ entre $E$ y $V$ ($y$ negativo). | |
|---|---|---|---|---|
| | $R_e > R_u$ (Fig. 62). | $R_e < R_u$ | $R_e > R_u$ | $R_e < R_u$ |
| $PO_uP'$..... | Positivo. | Negativo. | Negativo. | Positivo. |
| $e'$..... | $< e$ | $> e$ | $> e$ | $< e$ |
| $R_e'$..... | $> R_e$ | $> R_e$ | $< R_e$ | $< R_e$ |

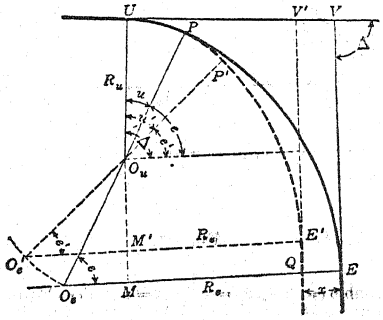



**Fig. 63.**

**151.** Fig. 63. Dando una curva compuesta, $U P E$, que una las tangs $U V$ y $V E$. Se desea **correr la tangente final**, $V E$, paralela a si misma, a la nueva posicion, $V' E'$, conservando el radio original, $R_u$ y $R_e$. El cambio envuelve la corrida del C. C. desde $P$, a un punto nuevo, como $P'$.

Como en la Fig. 63, sea el indice, $e$, el que se refiere a esa rama que una la tang, $V E$, que se va a correr; y el indice, $u$, a la otra rama.

Como $V E$ y $V' E'$ son paralelas, tenemos $\Delta = u + e = u' + e'$, o $u' = u + e - e'$.

Dando $e$, $R_e$, $R_u$ y la corrida, $x$ ( $= E Q$) de la tang (llamase *x positiva* cuando esté medida hacia dentro desde $E$, como en la Fig. 63);

Se busca la nueva distribucion del desarrollo o angulo central, $\Delta$, entre las dos ramas; y ademas los nuevos valores ($u'$ y $e'$), de los desarrollos, $u$ y $e$.

Trácese $O_u M \perp O_e E$ y $\perp O_e' E'$. Entonces:
(1) $O_e' M' = R_e - E' M' = O_e' O_u \cos e' = (R_e - R_u) \cos e'$;
(2) $O_e M = R_e - E M = O_e O_u \cos e = (R_e - R_u) \cos e$.
Restando (2) de (1), tenemos.
$E M - E' M' (= E Q) = x = (R_e - R_u)(\cos e' - \cos e)$ y.

$$\cos e' = \cos e + \frac{x}{R_e - R_u}; \quad u' = \Delta - e'.$$

**Signos mas y menos** ($\Delta < 90°$). Vease ¶ 69.

| $V' E'$ interior de $V E$ ($x$ positivo). | | $V' E'$ exterior de $V E$ ($x$ negativo). | |
|---|---|---|---|
| $R_e > R_u$ (Fig. 63). | $R_e < R_u$ | $R_e > R_u$ | $R_e < R_u$ |
| $e' < e$..... | $e' > e$ | $e' > e$ | $e' < e$ |

**152.** Figs 64. Dando una curva compuesta, $U P M E$, que una las tangs, $U V$ y $V E$; se desea que la **direccion de la tangente V E se cambie** a $V' E$, formando $V' E$, con $V E$, el angulo, $a = V E V'$, y pasando por $E$.

Conservando el primer radio, $R_u$, y el punto de la curva, $U$, se busca el radio nuevo, $R_e'$, que reemplaza $R_e$, y los cambios en $u$ y $e$. El cambio envuelve la corrida del C. C. desde $P$ al punto nuevo, $P'$.

$\Delta$, $\Delta'$ = extension o ang central de la curva compuesta para curva vieja y nueva respectivamente.

$R_u$ = radio inicial (invariable en este problema).

$R_e$, $R_e'$ = radio final para curvas viejas y nuevas respectivamente.

$u$, $u'$ = desarrollo o ang central de $R_u$ para curvas viejas y nuevas respectivamente.

$e$ = desarrollo o ang central de $R_e$.

$e'$ = desarrollo o ang central de $R_e'$.

$a = V E V'$.

Entonces $\Delta = u + e = \Delta' \pm a$; $u = \Delta - e = \Delta' \pm a - e$.
$e = \Delta - u = \Delta' \pm a - u$

$\Delta' = u' + e' = \Delta \pm a$; $u' = \Delta' - e' = \Delta \pm a - e'$
$e' = \Delta' - u' = \Delta \pm a - u'$.

Se busca, $R_e'$, $\Delta'$, $e'$, y $u'$.

**153. (1) Dando $\Delta$, a, u, e, $R_u$ y $R_e$.** Metodo grafico.

Desde $E$, trácese una linea en la direccion, $E O_e' \perp V' E$. En esta linea midase $E H = R_u$. Una $H O_u$. Dividase $H O_u$ en $h$, y trácese $h O_e' \perp H O_u$, hasta encontrar $E H$ (prolongando si fuera necesario), en $O_e'$. Entonces, $R_e' = R_u \pm H O_e' = R_u \pm O_u O_e' = E H \pm H O_e' = E O_e'$.

Desde $O_e'$, por $O_u$, trácese $O_e' P'$, hasta tocar la rama $U$, $U P$ (prolongada si fuera necesario), en el C. C. nuevo, en $P'$. Entonces:

$$e' = P' O_e' E; \; u' = U O_u P'.$$

| Cuando. | Fig. | Tenemos $R_u$. | $O_e$ y $O_e'$ en. | Focos. | Eje mayor |
|---|---|---|---|---|---|
| $R_e > R_u$. | $a$, | $= O_e' E - O_e' O_u$<br>$= O_e E - O_e O_u$<br>$= R_e - (R_e - R_u)$ | Hiperbola. | $O_u$, $E$ | $m n = R_u$ |
| $R_e < R_u$. | $b$, | $= O_e' E + O_e' O_u$<br>$= O_e E + O_e O_u$<br>$= R_e + (R_u - R_e)$ | Elipse. | $O_u$, $E$ | $m n = R_u$ |

**154. Figs 64. (2) Dando $\Delta$, a, u, e, $R_u$, $R_e$, U V y E V.**

O, si $U V$ y $E V$ no son dados, tenemos:

**U V** sen $\Delta = R_u$ senoverso $\Delta + (R_e - R_u)$ senoverso $e$; vease ec (74).

**E V** sen $\Delta = R_e$ senoverso $\Delta - (R_e - R_u)$ senoverso $u$; vease ec (72).

$\mathbf{E\ V'} = E\ V$. sen $\Delta$/sen $\Delta'$; $\mathbf{V\ V'} = E\ V$, sen $a$/sen $\Delta'$;

$\mathbf{U\ V'} = U\ V \pm V\ V'$;

$$\text{Tang}\ \frac{\mathbf{e'}}{2} = \frac{U\ V' \text{ sen } \Delta' - R_u \text{ senoverso } \Delta'}{E\ V' + U\ V' \cos \Delta' - R_u \text{ sen } \Delta'}.$$ Vease ec (77);

$$\mathbf{u'} = \Delta' - e' = \Delta \pm a - e';$$

$\mathbf{R_e}$ sen $e' = E\ V' + U\ V' \cos \Delta' - R_u$ sen $\Delta' + R_u$ sen $e'$ vease ec (78);

$\mathbf{P\ O_u\ P'} = \pm (e - e' \pm a) = \pm (u' - u)$.

Usar + (como en las Figs 64) $U\ V' > U\ V$, entonces $\Delta > \Delta'$, y $Re' < Re$ · y vice-versa. Esto afecta los signos (+ y −) de los otros valores.

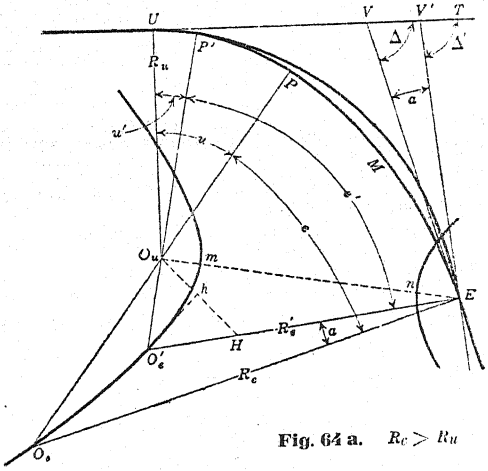



**Fig. 64 a.** $R_e > R_u$

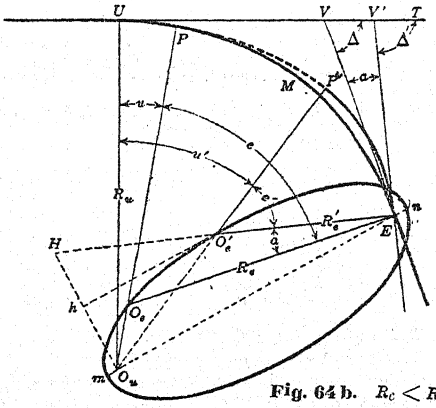



**Fig. 64 b.** $R_e < R_u$

## RETRAZADO DE CURVAS

### Buscar la agudeza, D, de una curva existente.

**155. Fig. 65. Vertice, V, accesible,** y se dan las direcciones de las tangs $V A$ y $V B$. Se busca el radio, $R$ (o la agudeza, $D$); $A$ y $B$.

Con el instrumento en $V$, midase $\Delta$; y desde cualquier tang tracese el ángulo, $B\ V\ P$ o $A\ V\ P = (180^o - \Delta)/2$. Midase el exterior, $E = V\ P$, desde $V$ a la línea de centro de la via existente. Entonces tenemos :

$$R\ (= O\ A = O\ B) = E/\text{exsec}\ (\Delta/2);$$

$$T\ (= A\ V = V\ B) = E/\text{tang}\ (\Delta/4) = R\ \text{tang}\ (\Delta/2);$$

$$\text{Sen}\ (D/2) = 50/R.$$

Aprox, tenemos $D = 5730/R$.

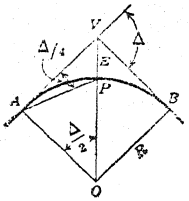



Fig. 65.

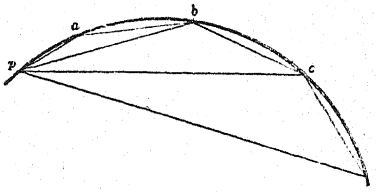



Fig. 66.

**156. Fig. 66. Vertice, V** (no indicado), **inaccesible.**

Levantese sobre la curva un punto, como $p$, en la linea de centro de la curva (preferible en, o cerca de cualquier extremo); y tracense y midanse los angulos periféricos, $a\ p\ b$, $b\ p\ c$, etc., cada uno subtendido por una unidad de cadena (como $a\ b$, $b\ c$, etc.) en la curva. El promedio de estos angulos pueden tomarse (aproximadamente) como el angulo periferico, $D/2$, de la curva; y dos veces $(D/2) = D$ como la agudeza *.

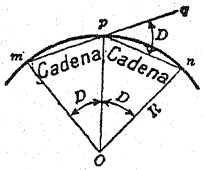



Fig. 67.

**157.** Fig. 67. O bien levántese sobre la curva cualquier punto, $p$, en la línea de centro de la via. Midase una cadena, $p\ m = p\ n$, en cada direccion, a $m$ y a $n$, tambien en la linea de centro. Midase el angulo $q\ p\ n = D$ *.

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* En todo este trabajo, la curva entera debe recorrerse tanto como sea posible. El metodo se repite, sobre diferentes partes de la curva, y se toma un promedio; porque en curvas que se han usado por algun tiempo, se encuentra frecuentemente que la curvatura varia de un punto a otro; la via se ha corrido por el trafico y quizas ha sido alineada de nuevo a simple vista sin tener en cuenta las estacas de centro.

**153. Por ordenadas medias.** Fig. 68. En cualquier círculo :

$$(2\ R - m)\ m = (c/2)^2.$$

Aproximadamente, $2\ R\ m = (c/2)^2$;

$$\text{o aprox., } R = \frac{c^2/4}{2\ m} = \frac{c^2}{8\ m}.$$

Entonces, si una cuerda, de cualquier largo dado, $c$, subtendiendo un ang cent pequeño, se tiende entre dos puntos, $n$ y $p$, en el lado interior de la cabeza del carril exterior, y si su media ordenada, $m$, se mide en la cabeza del carril, tenemos:

$$R = (c^2/8\ m) - \text{la mitad del ancho de vía.}$$

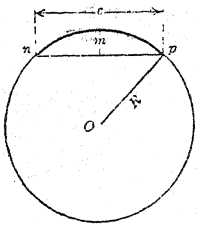


**Fig. 68.**

Usualmente la mitad del ancho de vía puede omitirse.

Entonces sen $(D/2) = 50/R$; o, aprox, $D = 5730/R$.

Si la cuerda es de 100 pies de largo (30,48 m) ( = un largo de cadena), y la ordenada media $m$, medida en pies, $D$, puede tomarse de la tabla, pags 974-5 *.

Vease *N. del T.* ¶ 11, pag. 926.)

**159. Reciprocamente, para encontrar la ordenada media, M** en pulgs, o $m$ en pies, correspondiendo a cualquiera cuerda de largo, dado $c$, en pies, y para cualquiera agudeza dada, $D$, o radio, $R$, tenemos (aprox).

$$M, \text{ pulgs} = 12\ m = 12\ c^2/8\ R = 1.5\ c^2/R.$$

Asi, con un carril de 30 pies ($c = 30$ pies aprox), en una curva de 3° ($R = 1910$ pies), tenemos, aprox : $M = 1.5 \times 900/1910 = 0{,}707$ pulgs.

Asimismo (aprox) $c = \sqrt{8\ m\ R} = \sqrt{8\ m\ 5730/D} = 214\ \sqrt{m/D}$.

Si, ahora, hacemos $M$ = la ordenada media en pulgs $= 12\ m = D$, en grados, o bien $m = D/12$, tenemos, aprox :

$$c = \sqrt{\frac{8\ D}{12} \cdot R} = \sqrt{\frac{8\ D}{12} \cdot \frac{5730}{D}} = \sqrt{3820} = 61.81 \text{ pies.}$$

Entonces, con una cuerda, $c$, 61.81 (o 62) pies largo, tenemos :

$M$ = ordenada media en pulgs = agudeza, $D$, de la curva en grados *. Compárese ¶ 178.

**160.** Fig. 69. Desde cualquier punto, como $a$ (preferible una union de carril), en el lado interior del carril exterior, diríjase una visual a otro punto tal, como $b$, de tal modo que la visual, $a\ b$, sea tangente el carril interior, como en $p$.

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* En todo este trabajo, la curva entera debe recorrerse tanto como sea posible. El método se repite, sobre diferentes partes de la curva, y se toma un promedio ; porque en curvas que se han usado por algun tiempo, se encuentra frecuentemente que la curvatura varia de un punto a otro, la vía se ha corrido por el tráfico y quizás ha sido alineada de nuevo a simple vista sin tener en cuenta las estacas de centro.

Entonces, para la media ordenada de la cuerda, $a\ b$, tenemos $p\ p' = G =$ ancho de via. (Vease *N. del T.* ¶ 11; pag. 926.)

Sea $l =$ largo del carril, en pies; $N =$ numeros de largos de carril en el arco, $a\ p'\ b$. Entonces, $N\ l =$ arco, $a\ p'\ b$, en pies.

Sea $\delta =$ angulo, $a\ O\ b$; $C =$ cuerda, $a\ b$; $R' =$ radio del carril exterior; $R =$ radio de la linea de centro; $D =$ agudeza de la linea de centro.

Midase $C = a\ b$; o cuentese $N$, búsquese $N\ l$, y (generalmente bastante aproximado) supongase la cuerda, $a\ b =$ arco, $N\ l$. Entonces:

Tang $(\delta/4) =$ tang $p'\ a\ p = 2\ G/C$; $R' = C/2$ sen $(\delta/2)$;

$R = R' - (G/2)$; sen $(D/2) = 50/R$; o, aprox, $D = 5730/R$.

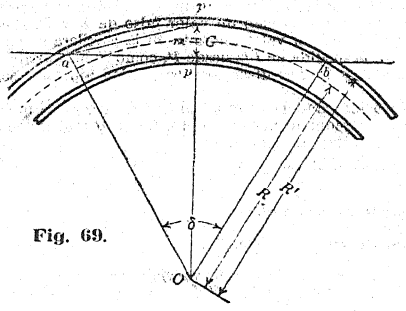


Fig. 69.

## Clasificacion de los Casos de replanteo de Curva.

**161.** Segun el método elegido los casos de replanteos, pueden clasificarse como sigue:

*A.* Vertice, $V$, accesible; curvas cortas;

*B.* Vertice, $V$, inaccesible.

1. Curvas de largo moderado;
2. Curvas largas.

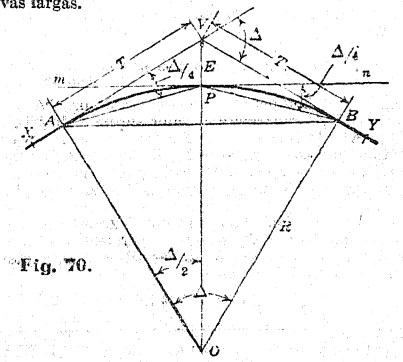


Fig. 70.

## Procedimiento.

### A. Vertice, V, accesible; curvas cortas.

**162.** Fig. 70. Fijado el vertice, $V$, así; con el instrumento, en un punto, $X$, sobre una tangente, digamos 50 pies atras del punto $A$ de la curva, mirese hacia atras por la tangente, y corrase el anteojo, mirando hacia $V$. Haciendo lo mismo en $Y$, en la otra tang, $V\ B$, encontramos la interseccion, $V$, de las dos tangs.

Encontrar $\Delta$, $E$, $R$, $D$ y $T$ como en el ¶ 155.

Entonces :

$L$ ( = largo de la curva, $A\ P\ B$, en cadenas de unidad de longitud) $= \Delta/D$.

Fíjense los puntos de la curva, $A$ y $B$, trazando $T = V\ A = V\ B$, desde $V$, o trazando $L/2$, en pies, a lo largo de la linea de centro de la curva, desde $P$. $L/2$, en cadenas de longit de unidad $= \Delta/(2\ D)$; $L/2$, en pies, $= 50\ \Delta/D$. (Vease *N. del T.*, ¶ 11, pag. 926.)

Con el instrumento en $P$, trácense los angulos rectos, $V\ P\ m$ y $V\ P\ n$, determinando la tang, $m\ n$ ; y comprobando los angulos $m\ P\ A = n\ P\ B = \Delta/4$; o bien :

Con el instrumento en $A$ y en $B$, comprúebese el angulo $V\ A\ B = V\ B\ A = \Delta/2$, o $V\ A\ P = V\ B\ P = \Delta/4$.

Si la carrilera se ha corrido en $P$, dando un valor falso de $E$, las ecuaciones anteriores daran por supuesto tambien un valor falso de $R$ y $D$. Comparese el valor de $D$, asi encontrado, con el valor anotado. Si la carrilera se ha corrido entre $P$ y $A$ o entre $P$ y $B$, la medida de $L/2$ desde $P$, como se hizo arriba, puede dar posiciones falsas para $A$ y $B$. Las discrepancias pueden ajustarse segun las circunstancias.

Trácese la curva marcándola provisionalmente. Observese qué espacio (juego), y en qué direccion, se requiere en cada punto. Reajustese si es necesario, y métanse las estacas para marcar el trazado final.

### B. Vértice, V, inaccesible.

### Curvas de largo moderado.

### a. Por Curvas de Prueba. Fig. 71.

**163.** Se da, la agudeza, $D$, de una curva vieja, cuya posicion original era $A\ B$, y las direcciones de sus tangs, $A\ V$ y $V\ B$. Se busca $\Delta$ y las posiciones de la T. C, $(A)$ y C. T. $(B)$.

Sea $A''$ la posicion supuesta de la T. C., o un punto cerca de la T. C., sobre la tang o en la tang prolongada. Situese en $A''$, y empiecese a marcar una curva de prueba, $A''\ m$, de la agudeza dada, $D$, marcando los puntos de cadena de 100 pies provisionalmente.

Si $A''$ esta distante de la T. C. $(A)$, como en la fig. 71 *, la curva de prueba, $A''\ m$, pronto se encontrara considerablemente desviada de la carrilera vieja $A\ B$ como en $m$. Midase la distancia, $m\ s$, entre la curva de prueba, $A''\ m$, y la linea de centro, $A\ B$, de la carrilera vieja. Entonces, para la movida $A''\ A$, que se busca para el punto T. C., tenemos :

$$A''\ A = m\ n = (\text{aprox})\ m\ n' * = \frac{m\ s}{\text{sen}\ \Delta_m}\ \dagger$$

donde $\Delta^m$ (o bien, $m$ †) = ang del cent de la curva de prueba entre $A''$ y $m$ *.

Supongamos que $A''\ A'$ represente el valor aproximado asi encontrado para la movida $A''\ A$. Muevase el instrumento a $A'$, y recorrase la curva de prueba, con $D$ como antes, tomando puntos de transito, $T_1$, $T_2$, etc., (con preferencia a distancias de, digamos 500 pies (como 150 m). De cada punto de tránsito, fíjese una linea corta, como $T_3\ x$, paralela a la tang original, $A\ V$. Marcando dicha linea con una aguja, como en $x$.

Tracese la curva de prueba, $A'\ T_3$, en una extension, $s$ †, hasta un punto, como $T_4$, donde su tang final, $v'\ T_4$, se supone paralela a la tang $V\ B$, de la curva vieja, $A\ B$. Por $T_4$ trácese una linea, $T_4\ u$, paralela a $V\ B$, y midase el angulo, $t$ †, entre $T_4\ u$ y $v'\ T_4$ prolongada. Entonces :

$$\Delta = s \pm t\ \dagger.$$

---

* Para facilitar la ilustracion, los angulos centrales en la fig 71, estan grandemente exagerados. En la práctica, las tangs de las dos curvas, en $m$ y en $s$, son aproximadamente paralelas.

† En la fig 71, los angulos, $\Delta_m$, $\Delta_s$ y $\Delta_t$ estan designados como $m$, $s$ y $t$, respectivamente para evitar el amontonamiento de letras.

Determinese el término, $B'$, de la curva de prueba, $A'\ B'$. La tang, $V'\ B'$, es entonces paralela a la tang $V\ B$, por $B$, y el ángulo $F\ V'\ B' = F\ V\ B = \Delta$.

Midase la dist, $B'\ e$, desde $B'$ a la linea de centro de la carrilera vieja *. Entonces, para la movida de $A'\ A = B'\ B$, que todavia se busca para traer $A'$ y $B'$ a sus propias posiciones, $A$ y $B$, respectivamente, tenemos :

$$B'\ B = \frac{B'\ e}{\text{sen}\ \Delta}\ *.$$

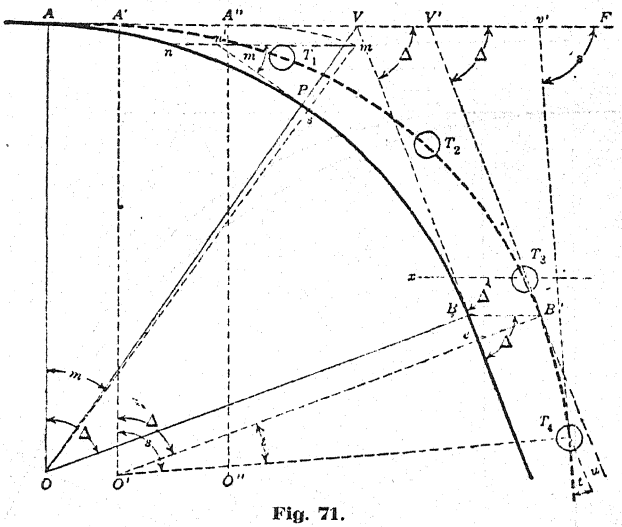


**Fig. 71.**

Cada uno de los puntos de tránsito, $T_1$, $T_2$, etc., tambien se mueve una dist $= B'\ B$, en la direccion, $T_3\ x$, paralela a la tang original, $A\ V$; la curva entera se mueve asi, en esa direccion en una dist $= B'\ B$.

Si la curva, $A'\ B'$, hubiera caido dentro (en lugar de afuera) de la carrilera existente, esta movida de la curva de prueba se hubiera hecho, por supuesto en la direccion opuesta.

De cada punto de tránsito, asi movidos, y desde $A$, la curva puede ser ahora recorrida en cada direccion, coincidiendo casi con la carrilera existente. La dist máxima desde el instrumento a la estaca, será la mitad de la dist entre dos puntos de tránsito.

---

* Para facilitar la ilustración, los angulos centrales, fig. 71, estan grandemente exagerados. En la práctica, las tangs de las dos curvas, en $m$ y en $n$, son aproximadamente paralelas.

**164.** Fig. 72. Pero si una curva de prueba, $A\ B$, requiere una movida demasiado grande de la carrilera existente (no mostrada); busquese, por un trazado, otra curva, $A_1\ B_1$, de diferente agudeza, (dentro ó afuera de $A\ B$) que reduzca la movida de la carrilera. Sobre el trazado midase la distancia, $E = V\ P$, de la curva de prueba, $A\ B$, y la movida, $P\ P_1$ que la linea preferida, $A_1\ B_1$, necesita en el punto del medio, $P$, de la curva de prueba. Entonces busquese el radio, $R_1$ de la curva preferida, $A_1 B_1$, por medio de la ecuacion :

$$\text{Radio de la curva preferida} = R_1 = \frac{E \pm P\ P_1}{\text{exsec } (\Delta/2)}.$$

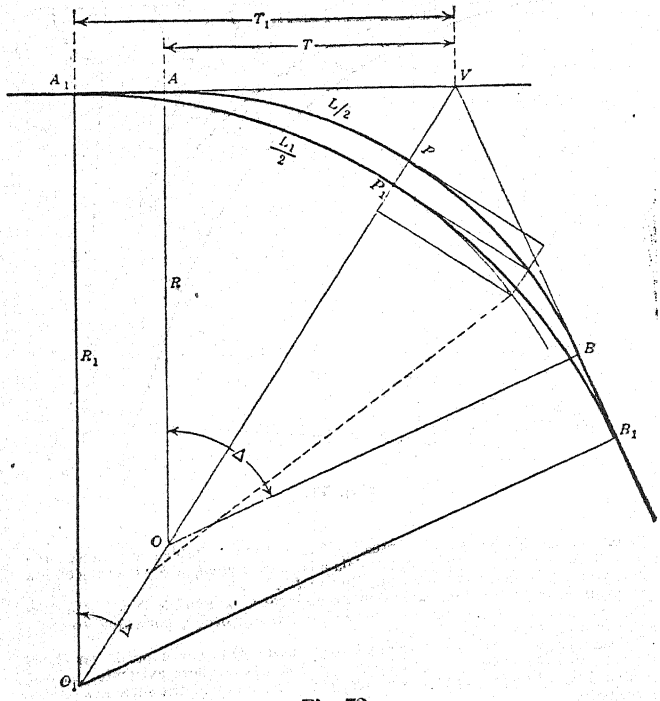


**Fig. 72.**

Entonces, para correr los puntos de la curva $A$ y $B$, hacia $A_1$ y $B_1$ respectivamente, como en ¶ 163, tenemos :

$$A\ A_1 = B\ B_1 = T - T_1 = \text{(aprox) } L/2 - L_1/2\text{; donde :}$$

$$T_1 = R_1 \text{ tang } (\Delta/2); \quad T = R \text{ tang } (\Delta/2);$$

$$T - T_1 = (R - R_1) \text{ tang } (\Delta/2); \quad L/2 \text{ en piés} = 50\ \Delta/D;$$

$$\text{y } L_1/2 \text{ en pies} = 50\ \Delta/D_1.$$

(Vease *N. del T.*, ¶ 11, pag. 926.)

Trácese la curva preferida, $A_1\ B_1$, prefiriendo hacerlo hacia adelante desde $A_1$, hacia atras desde $B_1$, y en ambas direcciones desde $P_1$.

**165.** Fig. 73. Los puntos en la curva final, $A_1$ $B_1$, pueden determinarse **por simples medidas desde la curva, A B**, así:

Se desea buscar (aprox) la dist $x\,x_1$, entre las dos curvas, $A$ $B$ y $A_1$ $B_1$, medida ‖ con el radio central, $O_1$ $P_1$.

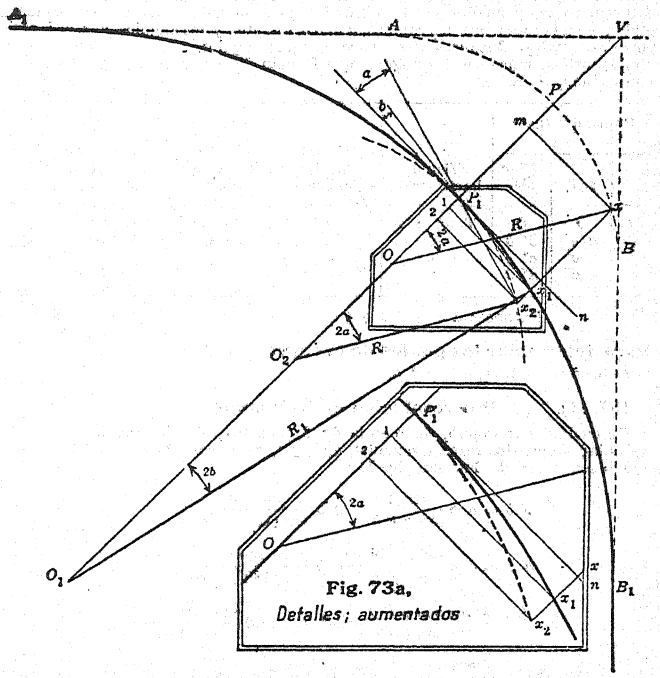


**Fig. 73.**

En la fig. 73 (muy exagerada; vease detalles; fig. 73 *a*), y supongamos que la curva de puntos, $P_1$ $x_2$ represente una curva que tenga el mismo radio, $R$, que la curva, $A$ $B$, pero pasando por el punto medio, $P_1$, de la curva, $A_1$ $B_1$; y sea $x_2\,x_1$ la dist (paralela a $P\,P_1$) de un punto dado, $x_2$ en la curva $P_1\,x_2$, a un punto requerido $x_1$, en la curva $A_1\,B_1$. Trácese $x_1$ 1 y $x_2$ 2, paralelos a la tang, $P_1\,n$, por $P_1$. Entonces:

$$x_2\,x_1 = P_1\,2 - P_1\,1 = P\,m - P_1\,1 = R \text{ senoverso } 2\,a - R_1 \text{ senoverso } 2\,b;$$

$$\text{donde } a = n\,P_1\,x_2, \qquad \text{y } b = n\,P_1\,x_1;$$

y la dist, $x\,x_1$, desde la curva, $A$ $B$, en la misma dirección al punto deseado, $x_1$ en la curva final, $A_1$ $B_1$, es

$$x\,x_1 = x\,x_2 - x_2\,x_1 = P\,P_1 - (R \text{ senoverso } 2\,a - R_1 \text{ senoverso } 2\,b).$$

Este metodo es suficientemente aprox en la práctica, a pesar de que las medidas, dadas por las ecuaciones, son paralelas a $P\,P_1$, en lugar de ser radiales en cualquiera de las dos curvas. A medida que esta divergencia aumenta (con la dist del punto, $x_1$, a $P_1$) la dist, $x\,x_1$, entre las dos curvas, disminuye.

**b. Por Transversales.**

**166.** Fig. 74. Sea $x\ A\ B\ y$, la linea de centro de la carrilera existente. Fíjese un punto, como $a$, sobre una tangente, atrás del punto supuesto de la curva, $A$; y trácese una línea como $a\ b\ c\ d$, seleccionando los puntos transversales, $b$, $c$, $d$, etc., prefieranse puntos (sobre o cerca del afirmado de la via) que la nueva curva deba evitar o tocar, y no demasiado separados para medidas exactas con cadena. Midanse cuidadosamente los rumbos, $a\ b$, $b\ c$, $c\ d$, etc., y los angulos, $\alpha$, $\beta$, $\gamma$, $\delta$, etc., entre los rumbos. Busquense las sumas de estos angulos, como estan en la tabla abajo; dichas sumas son los ángulos que forman dichos rumbos respectivamente con la tang, $A\ V$.

| Punto del Instr. | | | Visual hacia atras. | Visual hacia adelante. | Angulo entre la visual hacia adelante | |
|---|---|---|---|---|---|---|
| | Lat. | Desviación. | | | y hacia atras. | y $A\ V$. |
| $a$ | $O$ | $O$ | Tang $a\ x$ | $b$ | $+\alpha$ | $A = a$ |
| $b$ | $a\ b'$ | $b'\ b$ | $a$ | $c$ | $+\beta$ | $B = A + \beta$ |
| $c$ | $a\ c''$ | $c''\ c$ | $b$ | $d$ | $-\gamma$ | $C = B + (-\gamma)$ |
| $d$ | $a\ d''$ | $d''\ d$ | $c$ | Tang, $d\ y$ | $+\delta$ | $\Delta = C + \delta$ |

Entonces $\Delta = \alpha + \beta + (-\gamma) + \delta$ = suma algebraica.

**Para determinar los puntos de la curva, A y B.**

Se buscan las dists, $a\ A$ y $d\ B$.

**167.** Fig. 74. **1. Por resolucion de triángulos.**

En cualquier triangulo, como en la fig. 9, pag. 107, sean los angulos $A$, $B$ y $C$, y los lados opuestos respectivamente, $a$, $b$, y $c$. Entonces (vease pag. 107, ¶ 20) : $a/b$ = sen $A$/sen $B$. Entonces fig. 74 :

$$F\ b = a\ b\ \frac{\operatorname{sen} a}{\operatorname{sen} a\ F\ b} = a\ b\ \frac{\operatorname{sen}\ a}{\operatorname{sen}\ (180^\circ - B)};$$

$$a\ F = a\ b\ \frac{\operatorname{sen} a\ b\ F}{\operatorname{sen} a\ F\ b} = a\ b\ \frac{\operatorname{sen} \beta}{\operatorname{sen}\ (180^\circ - B)};$$

$$G\ c = F\ c\ \frac{\operatorname{sen} a\ F\ b}{\operatorname{sen} c\ G\ F} = (F\ b + b\ c)\ \frac{\operatorname{sen}\ (180^\circ - B)}{\operatorname{sen}\ (\Delta - \delta)};$$

$$F\ G = F\ c\ \frac{\operatorname{sen} \gamma}{\operatorname{sen}\ (\Delta - \delta)}; \qquad G\ d = G\ c + c\ d;$$

$$V\ d = G\ d\ \frac{\operatorname{sen}\ (\Delta - \delta)}{\operatorname{sen}\ (180^\circ - \Delta)}; \qquad G\ V = G\ d\ \frac{\operatorname{sen}\ \delta}{\operatorname{sen}\ (180^\circ - \Delta)};$$

$$F\ V = G\ V - F\ G.$$

Búsquese $\Delta$ como en el ¶ 166. Entonces :

$$A\ V = V\ B = R\ \text{tang}\ (\Delta/2) = \text{aprox}\ \frac{5730}{D}\ \text{tang}\ (\Delta/2).$$

Entonces :

$$\mathbf{a\ A} = a\ V - A\ V = a\ F + F\ V - A\ V;$$

$$\mathbf{d\ B} = V\ d - V\ B.$$

**168.** Fig. 74. **2. Por Latitudes y Desviaciones.**

*Latitudes* (dists paralelas con una tang como $A\ V$); para buscar la dist $a\ A$ y situacion del punto $A$ :

Lat de $d = a\,d' = a\,b' + b\,c' + c\,d'$
$= a\,b \cos A + b\,c \cos B + c\,d \cos C$;
$a\,A = a\,d'' - d''\,V - V\,A$
$= a\,d'' - d\,d''/\tan \Delta - R \tan (\Delta/2)$.
(Para $d\,d''$, vease abajo. $R$ = radio de la curva existente).

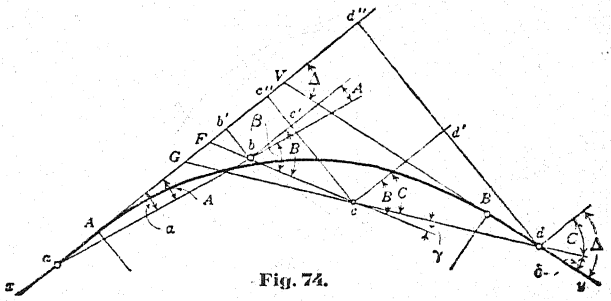


Fig. 74.

*Desviaciones* (dists normales a las latitudes); para buscar la dist $d\,B$ y la situacion del punto, $B$:
Desv de $d = d\,d'' = b\,b' + c\,c' + d\,d'$
$= a\,b \operatorname{sen} A + b\,c \operatorname{sen} B + c\,d \operatorname{sen} C$;
$d\,B = d\,V - V\,B$
$= d\,d''/\operatorname{sen} \Delta - R \tan (\Delta/2)$.

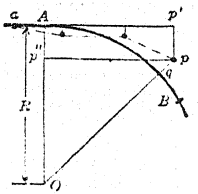


Fig. 75.

**Por puntos de las transversales.**

**169.** Fig. 75. (Para curvas con espirales vease el final de este ¶). Habiendo así encontrado, por medio de la transversal, ¶ ¶ 166-168, Fig. 74, el punto de la curva, $A$, y el punto de la tang $B$, los varios puntos transversales pueden usarse para determinar por simples medidas otros tantos puntos en la curva; asi, Fig. 75: Para encontrar la distancia radial, $p\,q$, de un punto transversal dado, $p$, a la curva, $A\,B$;
Sea $a\,p'$ = lat de $p$ { referido al origen, $a$, de la transversal,
$p'\,p$ = diverg de $p$ { y encontrado como en el ¶ 168.
$R$ = radio, $O\,q$.
Entonces:

$$\text{Tang } A\,O\,p = \frac{p\,p''}{O\,p''} = \frac{a\,p' - a\,A}{R - p'\,p};$$

$$O\,p = \frac{p\,p''}{\operatorname{sen} A\,O\,p} = \frac{a\,p' - a\,A}{\operatorname{sen} A\,O\,p};$$

$$p\,q = R - O\,p.$$

Si, como en la Fig. 75, $R < O\,p$, entonces $p$ debe moverse hacia dentro.
Si $R > O\,p$, entonces $p$ debe moverse hacia afuera.

Fig. 76. Para un punto, $q$, en la porción circular, $B\ B_f$, de una curva espiral, (para $A$, en lo anterior, lease, $A'$; y, para $R - p'p$, lease $R + H - p'p$). Entonces, como arriba, $p\ q = R - O\ p$.

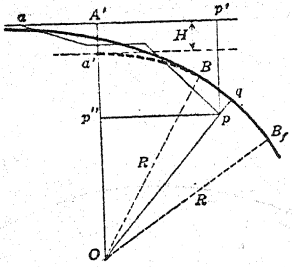


**Fig. 76.**

## Curvas largas, que se desea combinar.

**170.** En curvas muy largas, especialmente donde la carrilera se ha movido mucho fuera de su posicion, algunas veces se hace impracticable situarla satisfactoriamente con una curva simple sencilla.

Entonces se recurre a combinaciones de curvas de otros radios. Como las curvas largas son rara vez agudas, el cambio requerido de agudeza es tambien, muy rara vez, de mas de unos pocos minutos.

El instrumento se pone sobre un punto tan cerca como se pueda al punto medio de la curva, y se corre en cada direccion una curva de prueba, con la agudeza, $D$, de la curva, y se corre en cada direccion una curva de prueba, con la agudeza, $D$, de la curva existente, como se encuentra en las anotaciones, ó planos o como se calcule, o se suponga.

Cuando cualquier curva de prueba comienza a desviarse considerablemente de la carrilera existente, la agudeza se aumenta o disminuye ligeramente, como sea necesario. Esto puede tenerse que hacer varias veces durante el trazado de la curva de prueba.

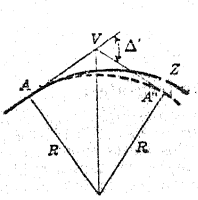


**Fig. 77.**

Fig. 77. Supongase que una de estas curvas de prueba, acercandose a la derecha en la Fig., ha llegado asi a un punto, $A'$, digamos 500 pies (digamos 150 m) desde su origen que es el punto, $A$, de la curva existente, $Z\ A$.

Se desea, determinar el punto $A$, y la agudeza, $D$, de la curva, $A'\ A$, que se unirá tangencialmente a la tangente, $A\ V$, en $A$. Tracese una tang, $A'\ V$, que se une en, $V$, con la tang, $A\ V$, de la curva existente, $Z\ A$. Midase el angulo de interseccion, $\Delta'$, en $V$, y la semitangente, $T = A'\ V = V\ A$.

Entonces, para la agudeza, $D$, de la curva corta, $A'\ A$, que completará la linea, tenemos :

$$R = T \cot (\Delta'/2);\ D = \text{aprox } 5730/R;$$

y $V\ A\ (= T) = A'\ V$, medido desde $V$, da el punto de la curva, $A$.

**171. Cuando se hace de doble vía una vía sencilla existente**, Fig. 78, *y A B a b z* (centros en *O* y en *o*), la agudeza de la curvatura puede reducirse (si da lugar la zona de propiedad) colocando la segunda vía, *y' A' B'*, *B" a' b'*,

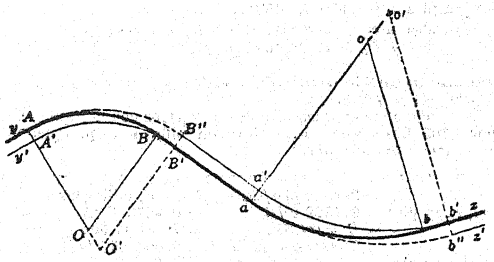


Fig. 78.

*b" z'* (centro en *O'* y *o'*), alternativamente sobre uno y otro lado de la vía sencilla existente, construyendo tambien las curvas nuevas, *A B"* y *a b"* (centros en *O'* y en *o'*), y eventualmente cambiando las curvas antiguas, *A B* y *a b*. (Eng News, 1913, oct 23, p 802.)

## SUPERELEVACION (PERALTE) del carril exterior en las curvas.

(Para curvas de transición, vease ¶ ¶ 194, etc.)

### Teoría.

**172.** Sea *W*, un bloque sin friccion deslizándose hacia adelante, sobre una vía recta, en la cual los dos carriles estan en el mismo nivel, y supongamos que encuentra una curva, con el *peralte*, *e*, como se ve en la Fig 79.

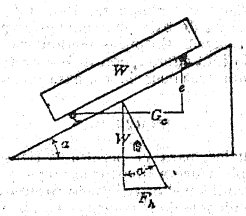


Fig. 79.

**173.** Sobre la curva, su fuerza centrífuga (horizontal), transversal a la vía, es $W\ v^2/R\ g$ (pag 373); donde $W$ = peso del bloque; $v$ = su velocidad, en pies por segundo; $R$ = rad de la curva, en pies; y $g$ = aceleración de la gravedad = 32,2 pies (9,81 m) por segundo.

**174.** La componente horizontal que lo hace descender por el plano es (fuerza centripeta): $F_h = W$ tang $a = W\ e/G_c$; donde $e$ = superelevacion (peralte) en pies, y $G_c$ = dist hor, en pies, entre centros de carril.

(*N. del T.* Puede emplearse el Sistema Métrico, tomando medidas en metros y para aceleracion de la gravedad = 9,81 m.)

**175** *. Con el objeto de que el bloque no se corra lateralmente, debemos de tener estas dos fuerzas iguales; $W\ v^2/R\ g = W\ e/G_c$; de aqui:

Peralto, $e$, en pies $= G_c\ v^2/R_g = 0.03106\ G_c\ v^2/R$ ..... (114)

Para veloc., $V$, en millas por hora, tenemos:

Peralto, $e$, en pies $= 0.06680\ G_c\ V^2/R$ ..... (115)

Aprox, $e$, en pies $= G_c\ D\ v^2$ * *$/5730\ g = G_c\ D\ v^2/184{,}500$

$= G_c\ D\ V^2/85{,}768$ (†) ..... (116)

**176. Con ancho de via** (*standard*). Modelo ( = 4 pies, 8.5 pulgs = 4.708 pies. tenemos, aprox, $G_c = 4.9$ pies, y

(*N. del T.* Cuando se tengan las medidas en metros conviértanse en pies multip. aquellos por 0,305. Los kilometros por hora en millas multip. aquellos, por 0,6214 así pueden usarse estas fórmulas.)

| | Para $v$, pies/seg. | Para $V$, millas/hora. | |
|---|---|---|---|
| Superel, $e$, en pies | $= 0.1522\ v^2/R$ | $= 0.2273\ V^2/R$ | |
| | $= D\ v^2/37{,}652$ | $= D\ V^2/17{,}504$ | |
| | $= 0.000\ 026\ 56\ D\ v^2$ | $= 0.000\ 057\ 2\ D\ V^2$ ..... | (117) |
| Superel, $E$, en pulgs | $= 1.826\ v^2/R$ | $= 3.928\ V^2/R$ | |
| | $= D\ v^2/3{,}137.7$ | $= D\ V^2/1{,}458.6$ | |
| | $= 0.000\ 319\ D\ v^2$ | $= 0.000\ 686\ D\ V^2$ ..... | (118) |

**177. Para otros anchos de via**, como el angulo, $a$, Fig. 1, es independiente del ancho de via, el peralto, $e$, para un radio y velocidad dada, es proporcional al ancho de la via.

**178. Cuerda, $C_0$, cuya media ordenada = peralte.** Comparese ¶ ¶ 159-186. En cualquier arco circular, tenemos aprox:

$$\text{Cuerda} = \sqrt{8\ R \times \text{med ord.}}$$

De aqui (Ec 117) tenemos:

$$C_0 = \sqrt{8\ R \times 0.000\ 057\ D\ V^2} \quad \ldots\ldots\ldots\ldots \quad (119)$$

y, como $R\ D$ = aprox 5730, tenemos aprox:

$$C_0 = 1{,}62\ V = \sqrt{1.219}\ v = 1.1\ v \quad \ldots\ldots\ldots\ldots \quad (120)$$

**179.** Asi, sea la velocidad 40 millas/hora. Para este caso, Ec (120) da $C_0 =$ 64.8 pies. Estirese una cuerda de 64.8 pies, entre dos puntos cualquiera sobre el lado concavo de cualquier carril. Entonces cualquiera que sea la agudeza de la curvatura, la media ord. de esta cuerda, con la cabeza del carril, sera aproximadamente el peralto que dan nuestras ecuaciones, para una velocidad de 40 millas/hor

## Práctica.

**180.** En los trenes actuales, para el peralte se tiene en cuenta, no solamente la posicion de los troles (plataforma que forman ejes y ruedas) transversalmente a la carrilera (¶ 175), sino tambien el **equilibrio de los cuerpos en los carros.** El peralto neutraliza la tendencia de los cuerpos en los carros a oscilar hacia fuera bajo la acción de la fuerza centrifuga.

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* (Vease *N. del T.*, ¶ 11, pag. 926).

** (*N. del T.* — Con un radio de 5730 pies (aprox) la cuerda unidad de 100 pies (una cadena) subtiende un arco de un grado (1°) o de otro modo $D$ (en grados) $= 5\,730/R$ (vease ¶ 29, pag. 959) y por tanto $1/R = D/5\,730$. Para que una cuerda de 20 metros subtienda un arco de (1°) un grado, es necesario que el radio sea de 1146 m; (vease ¶ 29, pag. 959), asi pues en el sistema métrico $D$ (en grados) $= 1146/R$. Naturalmente $R$ en metros.).

† (*N. del T.* — La fórmula para el peralte en sistema metrico es $e = \dfrac{Gc\ v^2}{9.81\ R}$; por supuesto dando a $Gc$ = ancho via; $v$ = velocidad par seg y $R$ = Radio en metros.)

**181.** Las ecuaciones anteriores estan basadas sobre las **condiciones ideales** de la Fig. 79; pero un carro de ferrocarril, y, todavia mas aun, un tren, es un cuerpo complicado, armado de muchas partes sobre las que obran de diverso modo diferentes fuerzas; y las condiciones son marcadamente distintas de las de la Fig. 79. La friccion, entre la llanta de la rueda y la parte superior de la cabeza del carril, hace que la primera rueda exterior de un carro se balancea hacia afuera y su pestaña ejerce presion contra la cabeza del carril exterior; y la resistencia de ese carril al movimiento hacia adelante de la rueda origina una fuerza centripeta, adicional a la de $F_h$ debida a la gravedad (¶ 174), deficiente en la Fig. 79; y la acción de las dos fuerzas principales, sobre las ruedas siguientes, es complicada por la tracción ejercida sobre ellas por las partes, que le preceden en el tren. Vease ¶ 36, pag 1133. Aun si se pudiera encontrar una fórmula idealmente exacta comprendiendo la velocidad, su utilidad estaria limitada por el hecho de que solo un peralte puede darse en un pedazo de la via; ademas, en la mayoria de los ferrocarriles, los trenes tienen que viajar con notables diferencias de velocidades, no obstante estas ecuaciones son generalmente usadas en la práctica, con modificaciones para ajustarlas a casos especiales.

**182. El valor y detalles del peralte** se determinan generalmente por un criterio individual (¶ ¶ 183 a 187), basado en las condiciones locales, por la naturaleza del tráfico, etc. En vista de la seguridad y para la comodidad de los pasajeros, el peralte se calcula para las velocidades *mayores* posibles; pero esto puede producir un trabajo fuerte para trenes largos y pesados; y, donde un tren se pare sobre una curva aguda, resulta una inclinacion inconveniente para la carga de los carros.

**183.** Es costumbre tomar el peralte de 0.5 a 1.0 pulg (digamos 12 a 25 mm) (en algunos casos 1.25, 1.5 y aun 2 pulgs) (digamos 30 a 50 mm) **por grado de agudeza**; con un limite total maximo de 4 a 6 (y aun 8) pulgs (digamos 10 a 15 y aun 20 cm); la velocidad se reduce donde sea necesario. La Ec (114) da un peralto de 0.5 pulg por grado de agudeza en cerca de 27 millas/hora, 1 pulg en cerca de 38 m/h, y 2 pulgs en cerca de 55 m/h. (1)

**184.** La practica comun (¶ 183) de hacer el peralte proporcional a la agudeza de la curva, descarta el hecho de que **sobre las curvas agudas se disminuyen las grandes velocidades.** (Vease ¶ 187.)

**185. En el F. C. New York Central** (cuatro vias) $E$ max = 6.5 pulgs (162 mm). Para trenes de pasajeros en curvas de mas de 1° de agudeza,

$E$ = 2 pulgs (50 mm) por grado;

para trenes de pasajeros en curvas de 1° de agudeza y mas agudas,

$E$ = 1 pulg (25 mm) por grado + 1.5 pulgs (37 mm);

para trenes de carga,

$E$ = 0.75 pulgs (18 mm) por grado.

Esto da:

| Para agudeza. | 0.25° | 0.50° | 0.75° | 1.00° | 1.25° | 1.50° | 1.75° |
|---|---|---|---|---|---|---|---|
| $E$, pulgs, pasajeros | 1/2 | 1 | 1 1/2 | 2 1/2 | 2 3/4 | 3 | 3 1/4 |
| Carga | 3/16 | 3/8 | 9/16 | 3/4 | 15/16 | 1 1/8 | 1 5/16 |

| Para agudeza. | 2.00° | 3.00° | 4.00° | 5.00° | 6.00° | 8.00° | 10.00° |
|---|---|---|---|---|---|---|---|
| $E$, pulgs, pasajeros | 3 1/2 | 4 1/2 | 5 1/2 | 6 1/2 | 6 1/2 | 6 1/2 | 6 1/2 |
| Carga | 1 1/2 | 2 1/4 | 3 | 3 3/4 | 4 1/2 | 6 | 6 1/2 |

**186. El F. C. Phila. & Reading** hace $E$ o $e$ = media ordenada de la cuerda cuyo largo, $C_0 = 1.466\ V$, es la dist corrida por trenes expresos en un segundo; $E$ max = 8 pulgs (20 cm). Comparense ¶ ¶ 178, 179.

**187. Otras líneas** usan $E$ = 1 pulg (25 mm) por grado, mas una cantidad empezando con 1 pulg para una curva de 1° y disminuyendo 1/8 de pulg (3 mm) en cada grado.

Esto da:

| Para $D$ = | 0 | 1° | 2° | 3° | 4° | 5° | 6° | 8° | 10° |
|---|---|---|---|---|---|---|---|---|---|
| $E$ en pulgs | 0 | 2° | 2 7/8 | 3 3/4 | 4 5/8 | 5 1/2 | 6 3/8 | 8 1/8 | 10 |
| $E$ en mm (aprox) | 0 | 50 | 61 | 93 | 115 | 140 | 160 | 203 | 250 |

(1) *N. del T.* — Esto equivale a un peralte de 12 mm. *por grado de agudeza* (cada grado del val. de D) en cerca de 43 kilometros por hora; 25 mm en cerca de 60 km/h y 50 mm. en cerca de 88 km/h.

Con esto resulta que los recorridos mas rápidos pueden ocurrir en las curvas mas fáciles. Vease ¶ 184.

Para la practica ordinaria, la **Asen Am de Ings F. C.**, Manual de 1915, p. 158, recomienda peralte $E = 0.00066\ D\ V^2$. Compárese nuestra ec (118) ¶ 176; $E$ max = ordinariamente 8 pulgs (20 cm).

**Seccion Transversal.**

**188. En via sencilla el peralte puede hacerse de varios modos :**

| | | |
|---|---|---|
| (1) Subir el carril exterior; | Carril interior permanece en rasante; | Centro de gravedad del carro levantado $e/2$. |
| (2) Bajar el carril interior; | Carril exterior permanece en rasante; | Centro de gravedad del carro levantado $e/2$. |
| (3) Combinacion de (1) y (2); | Linea central permanece en rasante; | Centro de gravedad del carro conserva la elevación. |

**La Asen Am de Ings F.C.**, Manual, 1915, p. 159, recomienda el metodo (1 .

*Plano.* *Dentado.* *En escala.*

**Fig. 80.**

**189. Fig. 80. En doble via**, se usan tres metodos :

**Plano.** Favorable para el desagüe, y para colocar cruceros y cruces de ferrocarril y carreteras.

**Dentado:** Usado generalmente. Requiere desagües cruzados bajo una de las carrileras;

**En escala.** Permite el desagüe sin desagües cruzados.

**Peralte gradual.**

**190.** Donde no se use (¶ ¶ 194) curva de transición, el peralte **generalmente empieza en la tangente,** a una distancia (graduando el peralte) desde cada punto de la curva = 30 a 60 pies (9 a 18 m) por cada pulgada (25 mm) de peralte. La cantidad depende de las velocidades que se supongan y el peralte completo se obtiene cuando se llega a las curvas, pero algunas veces el tercio del peralte se deja para darlo en la misma curva.

**191.** Cualquiera que sea el método adóptado, es un compromiso poco deseable estar forzado por la necesidad a pasar inmediatamente de una tangente a una curva circular, o viceversa. En la tangente la via debe estar a nivel, transversalmente, y el peralte gradual (que requiere el peralte en la tang) solamente se adopta por ser (atendiendo a la seguridad) menos inconveniente que tener la via a nivel transversalmente en la curva.

**192.** Algunas lineas adoptan un **largo fijo** como 120 pies (36 m) para el peralte gradual para todas las curvas. El New York Central usa 120 pies por pulgada (36 m por 25 mm) de peralte y, hasta $E = 3$ pulg (75 mm) y 360 pies (como 110 m) (total) para $E > 3$ pulgs. La Asen Am Ings F. C. en su Manual de 1915, p. 159, recomienda una proporcion de peralte gradual de 1 pulg en $1.75 \times V$ pies; donde $V$ = vel en millas/hora. En doble via, algunos ferrocarriles usan en cada via, un peralte gradual mas largo en el extremo donde los trenes entran a la curva que en el otro extremo.

**193.** El peralte gradual es generalmente **un plano inclinado;** asi, un perfil de cualquier carril es una *linea recta* que forma, en cada extremo, un angulo vertical con el perfil del mismo carril en la tang o en la curva respectivamente. **Teoricamente,** sin curvas de transicion, el perfil del peralte gradual debe ser una curva vertical invertida que consiste en dos curvas verticales de transicion, de radios infinitos en sus extremos. En algunos casos en la práctica esto es solo aproximado,

## CURVAS DE TRANSICIÓN

## O ESPIRALES

**194.** Cuando un tren deja una tangente y entra en una curva circular, la direccion de su movimiento cambia repentinamente. Cuando deja la curva y pasa a una tangente, queda repentinamente aliviado de la compulsion ejercida por los carriles en curva. En cada caso, recibe un choque lateral, que aumenta las resistencias, y que es perjudicial a la via y al material rodante y posiblemente a la carga, ademas desagradable o peligroso para los pasajeros. Análogos efectos se producen cuando un tren pasa directamente de una curva a otra de muy diferente agudeza, o que va en direccion opuesta. Ademas (vease ¶ 191) es imposible distribuir la superelevacion satisfactoriamente donde solo se usa una curva circular.

**195.** Estas dificultades se obvian por medio de las curvas de « transicion » o « espirales » porque, el cambio, entre cada tang y la agudeza completa de la curva, o entre una curva circular y otra, ambos cambios el respectivo a la agudeza y al peralte se hacen exclusivamente en la misma espiral; la agudeza de la curvatura y el peralte aumentan gradualmente desde cero, al comienzo de la espiral (hasta el que corresponde a la curva circular), al extremo de la espiral.

**196. La Asen Am Ings F. C.** (Manual, 1915, p. 133) recomienda el uso de espirales en todas las curvas que requieran un peralte de 2 pulgs (50 *mm*) o mas para la mas alta velocidad permitida.

**197.** Figs 81 *a*, 81 *b*. Asi, supongamos la curva circular $a'\ a\ f'$ que conecta las dos tangs, $A\ a'$ y $a f''\ A\ f$, formando la linea, $A\ a'$, $a f''\ A f$. Esta linea pudiera reemplazarse por la linea, $A\ B\ B f'\ A f$, compuesta de una espiral inicial, $A\ B$, una curva central circular, $B\ B f'$, y una espiral final, $B f'\ A f$. Aqui la curva circular original, $a'\ a f''$, con su centro en $O'$, se mueve hacia dentro a $a\ a'$, con el centro en $O$, conservando su radio original, $R$, pero solamente una porcion central, $B\ B f'$, se conserva como parte de la linea nueva.

En la fig. 81 *a*, la espiral inicial y final son semejantes. Donde, como en la fig. 81 *b*, no sea este el caso, vease ¶ 226.

**198.** En la curva *ideal* de transición, el radio disminuye proporcionalmente con la dist desde el comienzo de la espiral, (*medido a lo largo de la misma espiral*) aumenta. En una curva tal el peralte en cada punto, es el que corresponde a la agudeza en ese punto, a la velocidad que se supone. Esta curva fue descrita por Mr. Ellis Holbrook, en la revista Railroad Gazette, 1880 dec 3, p 639. Fue estudiada por el Prof. A. N. Talbot, en Tecnografia (Universidad de Illinois) n.º 5, 1890-91, y despues detallada por él mismo en el articulo « La Espiral de transicion de ferrocarril », New-York, Eng News Pub Co, 1904. En « La Curva de transicion », New-York, John Wiley & Sons, 1899, el Prof. C. L. Crandall desarrolla métodos exactos para la aplicacion de esta curva en casos de grandes desarrollos. Esta espiral fue recomendada por la Comision de Via de la Asen de Ingo de F. C. y Mantenimiento de Via, Boletin 73, marzo 1906, pp 58, etc.

**199.** Muchas formas de curvas de transicion se han propuesto y trazado. Entre las que hay en uso comun, la eleccion se hace segun la facilidad de calcularla y de trazarla mas que por cualquier diferencia mecánica entre el modo de obrar unas y otras.

### La espiral de 10 cuerdas.

**200.** Buscando la simplificacion de las fórmulas necesarias, la **Asociacion Americana de Ingenieros de ferrocarril** (« Manual », 1915, pag 132) recomienda el uso de una curva, identica practicamente con la espiral ideal descrita. La curva recomendada está subtendida por diez cadenas de espiral iguales (gene-

ralmente menores de 100 pies), y se llama «la espiral de diez cuerdas». En esta curva, la agudeza aumenta con la dist desde el comienzo de la espiral, medida en cadenas de 100 pies y no a lo largo de la curva misma, como en la espiral verdadera. Véase ejemplo, ¶ 233.

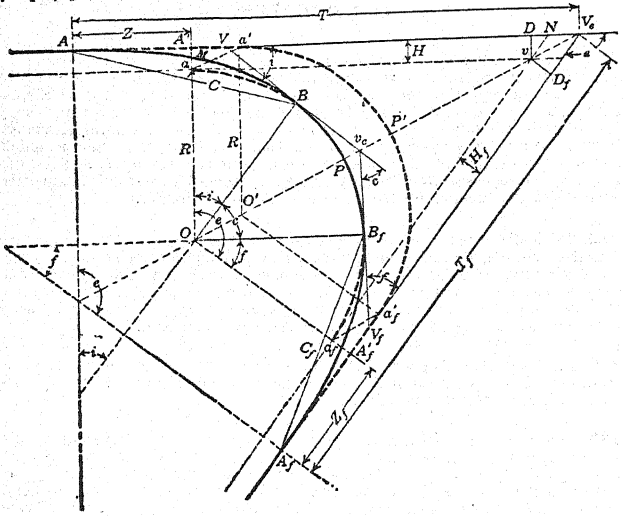


**Fig. 81 a.**

**Propiedades geométricas.** Véase reglas de uso en el campo, ¶ ¶ 231, etc.

## Símbolos.

**201.** Figs 81 *a*, 81 *b*. Sean los puntos, *A*, *B*, *Bf*, *Af*, etc., de cambio designados como sigue (Manual de la Ascn Americana de Ingo de F. C., 1915, p 135).

(*A*) T. E. De tang a espiral (hasta aquí llamado P. de E., punto de espiral);
(*B*) E. C. De espiral a curva circular;
(*Bf*) C. E. De curva circular a espiral;
E. T. De espiral a tangente;
E. E. De — a espiral (de una espiral a la otra).

**202. Lista alfabética de símbolos.** En general las funciones de espiral *final* se distinguen por el índice, subscrito, *f*.

| Figs. | Símbolos. | Significados. |
|---|---|---|
| 81, 82..... | $C$ | = la cuerda, *A B*, en pies, de T. E. a E. C.; |
| | $C_n$ | = la — en pies, de T. E., a cualquier punto *P*, en la espiral; |
| | $c$ | = la cadena espiral usada, en pies; |
| | $c = \Delta_c$. | Vease $\Delta_c$ abajo. |
| 81..... | $D$ | = la agudeza de la curva central circular, *B Bf* = (prácticamente) la agudeza de cualquier espiral en *B* ó *Bf*, donde se une a la curva circular; |
| | $d$ | = la agudeza de la espiral en cualquier punto dado; |

81 $a$.....$E$ = la distancia exterior $P\ V_c$ de la curva entera, $A\ B\ Bf\ Af$, cuando las dos espirales son iguales. Asi definida por la Ascn Am Ingo F. C. Compare ¶ ¶ 238, 239.

$e = \Delta_e$. Vease $\Delta_e$ abajo.

82, 83.....$F$ = el angulo entre la tangente inicial, $A\ V$, y la cuerda, $P'\ P''$, uniendo cualquiera de los dos puntos, dados de la espiral;

83.....$f$ = el angulo, $m\ P'\ P''$, en cualquier punto $P'$ dado de la espiral entre la tang $P'\ m$, en ese punto, y la cuerda, $P'\ P''$, a cualquier otro punto dado, $P''$;

$f = \Delta f$. Vease $\Delta f$ abajo.

81, 82, 84, 86....$H$ = la dist, $A'\ a$, entre la tang, $A\ V$ o $A'\ V_e$, y la tang paralela, $a\ v$, a la curva circular $a\ B$ ($B\ Bf$, prolongada);

= la ordenada del punto, $a$, de la curva circular prolongada referida al T. E. (abscisa = $Z$);

$i = \Delta_i$. Vease $\Delta_i$ abajo.

$k$ = el aumento en agudeza de la espiral por 100 pies de cuerda;

$L$ = la suma, en pies, de los largos de las 10 cadenas de espiral;

$l$ = la suma, en pies, de los largos de la cadena de espiral entre el T. E., en $A$, y cualquier punto dado en la espiral;

$n$ = el número (1, 2, 3..... 10) que indica cualquiera de los 10 puntos de la cadena en la espiral, contando el T. E. como cero;

= el número de cadenas espirales entre el T. E. y cualquier punto dado en la espiral;

$q$ = el número de cadenas de 100 pies entre cualquiera de los dos puntos dados de la espiral, $P'$ y $P''$;

81, 82, 84, 86... $R$ = el radio de la curva circular central;

$S$ = el número de cadenas de 100 pies del T. E. al E. C.;

$s$ = el número de cadenas de 100 pies entre el T. E. y cualquier punto dado de la espiral;

81, 82..... $T_a$, $T_b$ = las semitangs iniciales y finales, $A\ V$ y $V\ B$, de la espiral;

$t_a$, $t_b$ = las semitangs iniciales y finales para cualquier porcion de la espiral;

81, 84..... $T$, $T_f$ = las semitangs iniciales y finales, $A\ V_e$, $V_e\ A\ f$, de la curva entera;

82..... $X$, $Y$ = la abscisa y ordenada del E. C. ($B$), referido al T. E. ($A$);

$x$, $y$ = la abscisa y ordenada de cualquier punto dado de la espiral, referido al T. E. ($A$);

81, 82..... $Z = A\ A'$ = abscisa del punto $a$, de la curva circular prolongada, referida al T. E.; (ordenada = $H$);

81, 82..... $\Delta$ o $\Delta_i$ o $i$. Vease $\Delta_i$ abajo.

81, 86..... $\Delta_c$ o $c$ = angulo central de la curva central circular, $B\ Bf$ *;

81, 84, 86... $\Delta_e$ o $e$ = — — entera, $A\ B\ Bf\ Af$;

81..... $\Delta f$ o $f$ = — de la espiral final, $Bf\ Af$ *;

81, 82..... $\Delta_i$ o $i$ = — — inicial, $A\ B$ *;

83..... $\grave{\upsilon}$ = la abertura o desarrollo de cualquier porcion dada de la espiral empezando en la T. E., ($A$);

= el angulo entre la tang inicial, $A\ V$, y la tang por cualquier punto dado de la espiral;

Vease tambien la prox pagina.

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* Cuando unicamente la espiral inicial (o final) esté en discusion, es conveniente designar la extension por $\Delta$; pero cuando las espirales iniciales y finales se discuten juntas, puede ser necesario distinguir sus extensiones respectivas, y los indices se usan entonces con o sin la letra, $\Delta$.

81, 82..... $\Theta$ = el ángulo periférico o « deflexion » $V\ A\ B$, en la T. E. $(A)$, entre la tang inicial, $A\ V$, y la cuerda, $A\ B$, de la espiral;
$\theta$ = el ángulo periférico o « deflexion » entre la tang, $A\ V$, en $A$, y la cuerda desde $A$, a cualquier punto dado de la espiral;

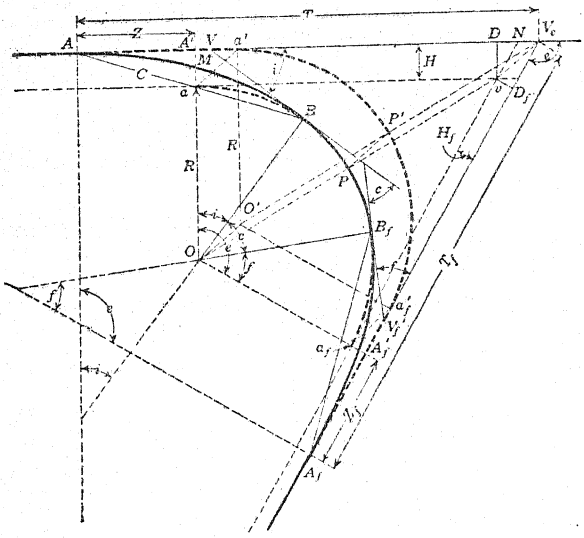


**Fig. 81 b.**

81, 82..... $\Phi$ = el ángulo periférico o « deflexion », $V\ B\ A$, en la E. C., $(B)$, entre la tang final $V\ B$, por la E. C. y la cuerda, $A\ B$;
$\phi$ = el ángulo periférico o « deflexion », en cualquier punto dado de la espiral, entre la tang por ese punto y la cuerda de la T. E.

## Ecuaciones.

**203.** Figs 82, 82'. **Relaciones entre L, I, S, s, c, n y q.**

$L = 100\ S = 10\ c$; $\quad S = 0.01\ L$; $\quad c = 0.1\ L = 10\ S$;
$l = 100\ s = n c$; $\quad n = l/c = 100\ s/c$;
$q = s'' - s' = 0.01\ c\ (n'' - n')$;
$s = 0.01\ l = d/k = 0.01\ c\ n = 0.001\ n\ L$..... (121)

**204. Agudeza, D o d.**

$D = k S = 0.01\ k L = 2\ \Delta/S = 200\ \Delta/L$..... (122)
$d = k\ s = 0.01\ k\ l = 2\ \delta/s = 200\ \delta/l = n\ \Delta/5\ S = 0.01\ k\ c\ n$..... (123)

**205. Proporcion k, del cambio de agudeza.**

$k = D/S = 100\ D/L = d/s = 100\ d/l = 2\ \Delta/S^2 = (\text{aprox})\ 6\ \Theta/S^2 = 2\ \delta/s^2$
$= (\text{aprox})\ 6\ \theta/s^2$..... (124)

Vease tambien ec (133).

**206. Angulo central ó desarrollo $\Delta$ ó $\delta$.**

$$\Delta_e = \Delta + \Delta_c + \Delta f \quad \ldots\ldots (125)$$

$$\Delta = D\,S/2 = D\,L/200 = k\,S^2/2 = k\,L^2/20{,}000 = (10/n)^2\delta = (S/s)^2\delta$$
$$= (L/l)^2\delta = (\text{aprox})\ 1.854\sqrt{D\,H} \quad \ldots\ldots (126)$$

$$\delta = d\,s/2 = d\,l/200 = d^2/2\,k = k\,s^2/2 = k\,l^2/20{,}000 = (n/10)^2\,\Delta$$
$$= (s/S)^2\,\Delta = (l/L)^2\,\Delta \quad \ldots\ldots (127)$$

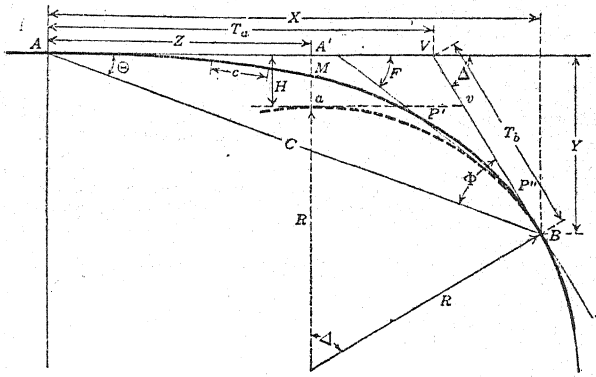


**Fig. 82.**

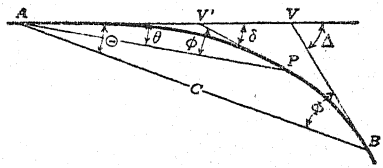


**Fig. 82'.**

**Angulo periferico o deflexion, Θ o θ.**

El angulo entre una tang y una cuerda.

**Para las ecuaciones de uso en el campo,** vease ¶ 231.

**Valor del angulo de deflexion, θ, en T. E., o al empezar (A) la espiral,** entre la tang, $A\ V$, y la cuerda que une $A$ con cualquier punto $P$ de la espiral fig. 82'.

**207.** Sea $\delta$ = la extension de la espiral entre $A$ y el punto dado, $P$, fig. 82'. Entonces :

$$\theta \text{ aprox } * = \delta/3 \quad \ldots\ldots (128)$$

* Grado de aproximacion. Error = 0.00297 $(\delta^\circ)^3$.

| Cuando $\delta$ = | 1° | 2° | 3° | 4° | 5° |
|---|---|---|---|---|---|
| Error = | 2.97 | 23.8 | 80.2 | 190 | 371 segundos. |
| 1000 × error/$\delta$ = | 0.08 | 0.33 | 0.74 | 1.32 | 2.06 |

Para el valor verdadero de $\theta$, tenemos, bastante aprox,

$\delta/3 - \theta$, en segundos, $= 0.00297$ ($\delta$ en grados)$^3$.

Asi, sea $\delta = 30^o$; $\theta$ aprox $= 30^o/3 = 10^o$. Entonces :

$\delta/3 - \theta$, en segundos, $= 0.00297 \times 30^3$

$= 0.00297 \times 27{,}000$

$= 80.19$ segundos,

o, $\theta = (\delta/3) - 0.00297\ (\delta^o)^3 = 10^o - 80.19$ segundos

$= 36{,}000'' - 80{,}19'' = 35{,}919.81'' = 9^o 58' 39.81''$.

Tomando $\theta$ aprox $= \delta/3$, tenemos :

$\theta$ *aprox* en grados $= \delta^o/3 = k^o\ s^2/6 = d^o\ s/6$ ..... (129)

$\theta$ *aprox* en minutos $= 10\ k^o\ s^2 = 10\ d^o\ s$ ..... (130)

Por lo tanto. Figs 81, 82, para el angulo $\Theta = V\ A\ B$, entre la tang $A\ V$, y la cuerda, $A\ B$, desde $A$ al extremo, $B$, de la espiral, tenemos :

$\Theta$ aprox en minutos $= 10\ k^o\ S^2 = 10\ D^o\ S$ ..... (131)

**Valor del angulo de deflexion**, $\theta_1$, en la T. E. ($A$), entre la tang $A\ V$, y la cuerda que une $A$ con el **primer punto de la espiral**, $n = 1$.

**208.** Sea $s_1$, ( $= 0.01\ c$), $\theta_1$ y $\delta_1$, los valores de $s$, de $\theta$ y de $\delta$, respectivamente para el punto $n = 1$, al extremo de la primera de las diez cadenas espirales.

Si $\Delta$ es menor de $45^o$ (como sucede siempre en la práctica), tenemos de la ec (127), para la extension, $\delta_1$, subtendida por la primera cadena espiral : $\delta_1 < 0.45^o$; y, de la ec (128) practicamente :

$\theta_1 = \delta_1/3 = d_1\ s_1/6 = s_1^2\ k/6 = (0.01\ c)^2\ k/6$ ..... (132)

de donde :

$k = 6\ \theta_1/s_1^2 = 6\ \theta_1/(0{,}01\ c)^2$ ..... (133)

De la ecuacion (127), tenemos :

$\delta_1 = (1/10)^2\ \Delta = \Delta/100$ ..... (134)

y

$\delta_1 = \delta_1/3 = \Delta/300$ ..... (135)

De la ecuacion (130), tenemos :

$\theta_1$, en minutos $= 10\ k^o\ (S/10)^2 = k^o\ S^2/10 = D^o\ S/10 = D^o\ S_1$ ..... (136)

**Valores de los angulos de deflexion F y f, entre una tang y la cuerda, P' P''**, uniendo dos puntos cualquiera de la espiral, figs 83 *a* y 83 *b*.

Sea :

$F$ = el ángulo entre la tang inicial, $A\ V$, y la cuerda, $P'\ P''$;

$f$ = el ángulo, en $P'$, entre la tang, $P'\ m$, y la cuerda $P'\ P''$.

**209.** Sea:

$d'$, $d''$ = la agudeza en $P'$ y en $P''$, respectivamente;

$\delta'$ = la extension entre $A$ y $P'$ = el angulo, en $V'$, entre la tang inicial, $A\ V$ por $A$, y la tang $V'\ m$, por $P'$;

$s'$, $s''$ = el número de cadenas de 100 pies entre $A$ y $P'$, y entre $A$ y $P''$, respectivamente;

$q = s'' - s'$ = el número de cadenas de 100 pies entre $P'$ y $P''$.

Se buscan, $F$ y $f$; ($f = F - \delta'$).

**210.** Fig. 83 *a* *. De cualquier punto de la espiral, $P'$, corramos una curva *circular* $P'\ c$, con la agudeza, $d'$ de la espiral en $P'$. Esta curva circular diverge de la tang, $P'\ m$, en $P'$ en la proporcion *constante* de $d'$ grados por cada 100 pies de cuerda; pero la *espiral*, $P'\ P''$, diverge de la misma tang, $P'\ m$, en la proporcion constante en que aumenta en $(d' + k)^o$ por 100 pies de cuerda, donde $k$ = el aumento en agudeza de la espiral por 100 pies de cuerda *. Por tanto, la espiral, $P'\ P''$, diverge de la curva *circular*, $P'\ c$, en la proporcion constante de $k^o$ de aumento por cada 100 pies de cuerda *; pero esta $k$ es tambien la proporcion en que, la espiral $A\ P''$, diverge de la tang inicial, $A\ V$ en $A$. O (como cualquier punto en la espiral puede tomarse como $P'$) supongase a $P'$ tomado en $A$ *. Entonces, en $A$,

---

* En la fig. 83*b*, la proporcion del aumento proporcional $k$ de la agudeza de la curva es *negativo* ; asi, la agudeza *disminuye* de $P'$ hacia $P''$, y la espiral diverge de la curva circular en una proporcion *decreciente* de $(d' - k)^o$ por cada 100 pies de cuerda. Tambien, en la fig. 83*b*, si $P'$ se toma en $A$, entonces $P''$ tambien esta en $A$.

la tang inicial, $A\ V$, la tang por $P'$ (ahora tomado en $A$), y la curva circular (aquí una linea recta; agudeza, $d' = 0°$) coincide, y tenemos (con $P'$ en $A$) : $d' = 0$; y

proporcion de la divergencia de la espiral con la tangente por $A$
= proporcion de la divergencia de la tangente por $P'$ (tomado en $A$)
= proporcion de la divergencia de la espiral de la curva circular empezando en $P'$ (tomado en $A$) = $(d' + k)° = k°$.

**211.** En $q$ cuerdas de 100 pies de $P'$, la curva circular, $P'\ c$, describe una extension de $q\ d'$ grados, y un ángulo periférico o de « deflexion » = mitad de $q\ d'$ ; o bien

$$m\ P'\ c = q\ d'/2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (137)$$

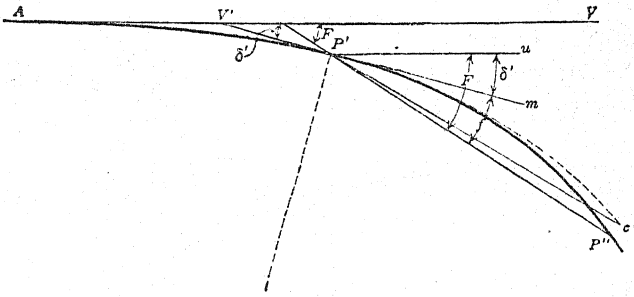


**Fig. 83 a.**

**212.** En la misma distancia, $q$, la espiral aumenta su agudeza, sobre la de la curva circular, $P'\ c$, en $q\ k°$ ; su agudeza *media* aumenta $q\ k°/2$; y su aumento de extension sobre la de la curva circular, es $q\ (q\ k°/2) = q^2\ k°/2$; pero la divergencia, $c\ P'\ P''$, de la *cuerda* espiral, $P'\ P''$, con la cuerda de la curva circular, $P'\ c$, es solamente un tercio de $q^2\ k°/2$ (vease ec 128); o

$$c\ P'\ P'' = q^2\ k/6 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (138)$$

**213.** Ahora :

$$f = F - \delta' = m\ P'\ P'' = m\ P'\ c + c\ P'\ P''$$
$$= q\ d'/2 + q^2\ k/6$$
$$\text{Ec (137)} \quad \text{Ec (138)}$$

$$= \frac{q}{6}\ (2\ d' + d' + q\ k) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (139)$$

$$= \frac{q}{6}\ (2\ d' + d'') \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (140)$$

y

$$F = \delta' + q\ d'/2 + q^2\ k/6 = \delta' + \frac{q}{6}\ (2\ d' + d'') \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (141)$$

**Valor de f entrando $\theta_1$.**

**214.** Sea $m = f/\theta_1$. De la ec (140) y ec (132) tenemos :

$$\left.\begin{aligned} m &= \frac{f}{\theta_1} = q\ \frac{2\ d' + d''}{d_1\ s_1}\ ; \\ \text{o bien } f &= m\ \theta_1 = \theta_1\ q\ \frac{2\ d' + d''}{d_1\ s_1} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (142)$$

**215.** Así, sea $c = 20$ pies; $k = 2°$.
Entonces, $s_1 = 0.01\ c = 0.2$ y $d_1 = s_1\ k = 0.4°$; $d_1\ s_1 = s_1^2\ k = 0.08$; $\vartheta_1 = d_1\ s_1/6 = 0.0133\ldots$; y, por ejemplo:

| Visuales. | | $q =$ | $d' =$ | $d'' =$ | $m = f/\vartheta_1 =$ | $f =$ |
|---|---|---|---|---|---|---|
| De est. | a est. | $0.01\ c \times (n'' - n')$ | $0.01\ k\ c\ n'$ | $0.01\ k\ c\ n''$ | $q\ \frac{2\ d' + d''}{d_1\ s_1}$ | $m\ \vartheta_1$ |
| 5 | 8 | 0.6 | 2.0 | 3.2 | 54 | 0.72 |
| 8 | 5 | − 0.6 | 3.2 | 2.0 | − 63 | − 0.84 |

Para $\vartheta_1$, vease ec (132).

**216.** Para los valores del coef, $m = f/\vartheta_1$, usado para obtener el angulo de deflexion, $f$, para determinar cualquier punto de la espiral, $P''$, desde cualquier punto del instrumento, $P'$, vease tabla, ¶ 233.

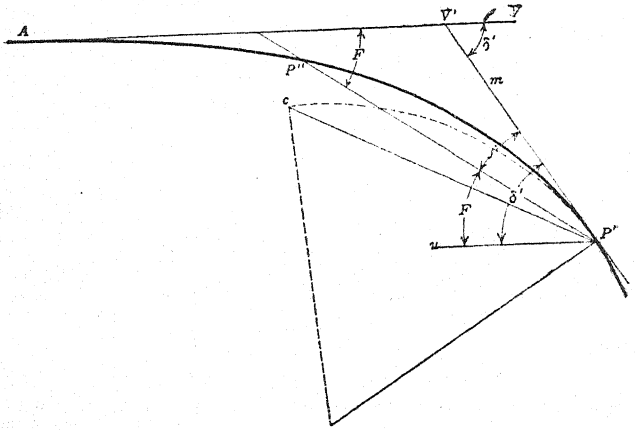


**Fig. 83 b.**

**Valores de los angulos de deflexion $F_a$ y $f_L$** entre una tang y la cuerda $P'\ P''$ ( = 1 cadena espiral) que une dos puntos de cadena *adyacente* de espiral.

**217.** Figs 83. Supongamos los puntos, $P'$ y $P''$, que sean ahora dos puntos *adyacentes* de cadena de espiral, $n$ y $n + 1$. Entonces, de la ec (140) y (123), tenemos (desde ahora $q = S/10$):

$$f_a = \frac{S}{60} \times \frac{2\ n + n + 1}{5\ S}\ \Delta = \frac{3\ n + 1}{300}\ \Delta \quad \ldots\ldots\ldots \quad (143)$$

y, de la ec (127), tenemos:

$$\delta' = \frac{n^2}{100}\ \Delta = \frac{3\ n^2}{300}\ \Delta \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (144)$$

De aqui:

$$F_a\ (= f_a + \delta') = \frac{3\ n^2 + 3\ n + 1}{300}\ \Delta \quad \ldots\ldots\ldots \quad (145)$$

Sea $m_a = 3\,n^2 + 3\,n + 1 = 300\,F_a/\Delta$. Entonces para los nueve puntos, de $n = 0$ a $n = 9$, mirando de $n$ a $n + 1$, o viceversa, tenemos, respectivamente :

| | | | | | |
|---|---|---|---|---|---|
| $n =$ | 0 | 1 | 2 | 3 | 4 |
| $n + 1 =$ | 1 | 2 | 3 | 4 | 5 |
| $m_a = \frac{300}{\Delta} F_a =$ | 1 | 7 | 19 | 37 | 61 |
| $m_a/300 = F_a/\Delta =$ | 0.0033 | 0.0233 | 0.0633 | 0.1233 | 0.2033 |

| | | | | | |
|---|---|---|---|---|---|
| $n =$ | 5 | 6 | 7 | 8 | 9 |
| $n + 1 =$ | 6 | 7 | 8 | 9 | 10 |
| $m_a = \frac{300}{\Delta} F_a =$ | 91 | 127 | 169 | 217 | 271..... (146) |
| $m_a/300 = F_a/\Delta =$ | 0.3033 | 0.4233 | 0.5633 | 0.7233 | 0.9033 |

del cual, y de $F_a = \frac{m_a}{300}\,\Delta$ los varios valores de $F_a$ para los puntos de 10 cadenas.

se encuentran facilmente, para cualquier valor dado de $\Delta$.

**218.** Asi, en estacion $n = 7$, mirando a la estacion $n + 1 = 8$, el angulo, $F_a$, entre la visual 7-8, y la tangente inicial, $A\ V$, es :

$$F_a = \frac{m_a}{300}\,\Delta = \frac{169}{300}\,\Delta = (0.5633.....)\,\Delta.$$

**219.** Figs. 81, 82. **Valor del angulo de deflexion,** $\Theta$ en $A$, entre la tang $A\ V$ y la cuerda $A\ B$. Sean los valores sucesivos de $F_a$, asi encontrados :

$$F_1 = \frac{\Delta}{300}; \qquad F_2 = 7\,\frac{\Delta}{300}; \qquad F_3 = 19\,\frac{\Delta}{300}; \text{ etc.;}$$

y sea $x_1, x_2, x_3$, etc., $y_1, y_2, y_3$, etc., las coordenadas para los puntos $n = 1$, $n = 2$, $n = 3$, ... $n = 10$ (punto $B$). Sea $L$ = la suma de las cadenas de espiral, en pies; $c = L/10$.

Entonces :

$$x_1 = c\,\cos\,F_1\,; \qquad y_1 = c\,\text{sen}\,F_1\,;$$

$$x_2 = c\,(\cos\,F_1 + \cos\,F_2); \qquad y_2 = c\,(\text{sen}\,F_1 + \text{sen}\,F_2) \ldots\ldots \quad (147)$$

etc. as etc.

$$\text{Tan}\,\Theta = \tan\,V\,A\,B = \frac{Y}{X} = \frac{\text{sen}\,F_1 + \text{sen}\,F_2 + \ldots \text{sen}\,F_{10}}{\cos\,F_1 + \cos\,F_2 + \ldots \cos\,F_{10}} \ldots\ldots\ldots \quad (148)$$

De los valores de las funciones de $\Theta$, asi encontrados, se deriva el valor :

$$\left(\frac{\Delta}{3} - \Theta\right)\text{en segundos} = 0.002\ 97\ (\Delta \text{ en grados})^3$$

o bien $\Theta = \Delta/3 - [0.002\ 97\ (\Delta^\circ)^3]$ segundos..... (149)

**Valor del angulo de deflexión,** $\Phi$, en la E. C. ($B$), entre la tan $V\ B$ y la cuerda $A\ B$.

**220.** Figs 81, 82. Necesariamente, $\Delta = \Theta + \Phi$; o bien :

$$\Phi = \Delta - \Theta = \Delta - \left[\frac{\Delta}{3} - (0.002\ 97\ (\Delta \text{ en grados})^3 \text{ segundos }^*)\right].$$

De aqui :

$$\Phi = \frac{2}{3}\Delta + \left(0.002\ 97\ (\Delta \text{ en grados})^3 \text{ segundos }^*\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (150)$$

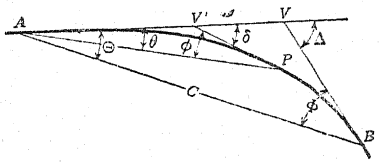


**Fig. 82' (Repetida).**

Practicamente * : $\Phi = 2\ \Delta/3 = 2\ \Theta$ ..... (151)

Similarmente, fig. 82', en cualquier porción, *A P*, de la curva espiral *A P B*, dicha porcion empezando en la T. E. (*A*), tenemos practicamente :

$$\delta = \theta + \Phi;$$
$$\theta = \delta/3;$$
$$\Phi = 2\ \delta/3 = 2\ \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (151')$$

**221. Relaciones entre H, L y Z**, fig. 82.

$$H = \text{aprox}\ \frac{L^2}{24\ R} = \frac{L^2}{24} \cdot \frac{D}{5730} = 0.0727\ S^2/D \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (152)$$

$$L = \sqrt{24\ R\ H} = 4.9\sqrt{R\ H} = 4.9\sqrt{5730\ H/D} = 371\sqrt{H/D} \quad \ldots\ldots \quad (153)$$

$$\Delta = 0.005\ DL \text{ (ec 126)} = 0.005\ D\ 371\sqrt{H/D} \text{ (ec 153); o } \Delta = 1.854\sqrt{DH} \quad (154)$$

**222. Bisecciones en M aproximadas.** Fig. 82. Practicamente :

$$L/2 = A\ M = M\ B = A\ A' = Z = a\ B \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (155)$$

$$A'\ M = M\ a = H/2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (156)$$

o, practicamente, $A'\ a$ divide en dos partes iguales y es dividida por la espiral *A B*, en *M*.

Mas exactamente (Boletin 73, p. 67, 1906 de la Asen Am Ings F. C.) :

$$\frac{L}{2} - Z\left(= \frac{L}{2} - A\ A'\right) = \frac{L^3}{1280\ R^2} = \frac{L^3}{1280}\left(\frac{D}{5730}\right)^2 \quad \ldots\ldots\ldots\ldots \quad (157)$$

**223. Ordenadas.**

$$y = \frac{l^3}{6\ R\ L} = \frac{l^3\ D}{34.380\ L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (158)$$

Multiplicando por $4\ L^2/4\ L^2$

$$y = \frac{4\ L^2\ l^3}{24\ R\ L^3}\text{; y, (ec 152) } y = 4\ H\ (l/L)^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (159)$$

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* Vease * nota, pag 1018.

**224. Otras relaciones.** Fig. 82.

$$C = \sqrt{X^2 + Y^2} = X \sec \Theta \quad (160)$$

$$C = \text{aprox } L\,[\cos 0.3\,\Delta + 0.004 \operatorname{exsec}(3\,\Delta/4)] \quad (161)$$

$$X = C \cos \Theta; \qquad Y = C \operatorname{sen} \Theta \quad (162)$$

$$T_a = C \operatorname{sen} \Phi/\operatorname{sen} \Delta; \qquad T_b = C \operatorname{sen} \Theta/\operatorname{sen} \Delta = Y/\operatorname{sen} \Delta \quad (163)$$

$$t_a = C_n \operatorname{sen} \Phi/\operatorname{sen} \delta; \qquad t_b = C_n \operatorname{sen} \theta/\operatorname{sen} \delta \quad (164)$$

$$Z = X - R \operatorname{sen} \Delta \quad (165)$$

$$H = Y - R \operatorname{senoverso} \Delta \quad (166)$$

**225.** Fig. 81 *a*. Cuando las **dos espirales**, *A B* y *Af Bf*, son **iguales y semejantes**, tenemos :

$$T = a\,v + D\,V_e + A\,A' = (R + H) \operatorname{tang}(\Delta_e/2) + Z \quad (167)$$

Distancia externa, $E = P\,V_e = (R + H) \operatorname{exsec}(\Delta e/2) + H$ (168)

**226.** Cuando, como en la fig. 81 *b*, las **dos espirales**, *A B* y *Af Bf*, son **desemejantes**; sea *a' a* (igual y paralela a *P' P*, $V_e\,v$, $a_f'\,a_f$ y *O' O*) la dist a que la curva circular original *a' a*$_f''$ es movida a *a*$a_f'$; y sea $\alpha$ = el angulo *A' a a'*; $a_f$ = el angulo $A_f' a_f a_f''$.

Entonces, dando la dist movida = *a' a*, y los angulos, $\alpha$ y $a_f$, tenemos :

$$H = a'\,a \cos \alpha; \quad H_f = a_f'\,a_f \cos a_f = a'\,a \cos a_f, \text{ y } H_f/H = \cos a_f/\cos \alpha \quad (169)$$

Dando *H* y $\Delta_e$, tenemos : $\dfrac{A'\,a'}{H}\left(= \dfrac{D\,V_e}{H}\right) = \dfrac{N\,V_e}{H} - \dfrac{N\,D^*}{H}$; o bien :

$$\text{tang } \alpha = \frac{H_f}{H \operatorname{sen} \Delta_e} - \cot \Delta_e^* \quad (170)$$

$$a_f = \Delta_e - \alpha \quad (171)$$

Tenemos, tambien :

$$T\ (= A\,V_e) = A\,A' + a\,v + D\,N\dagger + N\,V_e = Z + R \operatorname{tang}(\Delta_e/2) + H \cot \Delta_e + H_f \operatorname{cosec} \Delta_e \quad (172)$$

Una ecuacion análoga da los valores de la otra semitang, $T_f = A_f\,V_e$.

$Z$ = practicamente $L/2 = 2.45\sqrt{R\,H}$ (vease ec 153) (173)

## Eleccion de la espiral.

**227.** Fig. 82. La eleccion del largo, *L*, de la espiral puede **restringirse por el valor de H**, determinado topograficamente o por otras condiciones. Tenemos entonces (ec 153), para el largo de la espiral :

$$L = \sqrt{24\,R\,H};$$

donde *R* = radio de la curva circular.

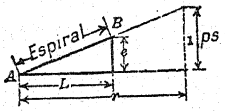



**Fig. 83½.**

**228.** Fig. 83 ½. Donde la espiral elegida no esta asi restringida, el largo, *L*, se **determina por el peralte, e**, en pies (o *E* en pulgs) en la curva circular, y por la relacion, *r*, del peralte gradual por pies de linea (¶ ¶ 190-193); asi (ec 117) :

* Cuando, como en la Figs 81, $\Delta e > 90° < 180°$, *N D* y cot $\Delta e$ son *negativas*.

† Cuando $\Delta e < 90°$, *D H* y *Df Hf* son *negativas*.

$L = e\,r = 0.3273\ V^2\ r/R = 0.1522\ v^2\ r/R = 0.1522\ v^2\ t/R$ ..... (174)
donde $t = r/v$ = tiempo, en segundos, durante el cual la rueda exterior en el peralte gradual, se levanta 1 pie.

La regla de la Ascn Am Ings F. C., ec (175), da $t \leqslant 5.45$ cuando $V = 45$ millas por hora (digamos 70 kilom/hora), y el peralte = 8 pulgs (20 cm) = 2/3 pie.

**229. La Ascn Am Ings F. C.** (Manual, 1915, pag 131-132) recomienda:
En curvas que no limitan la velocidad de los trenes:
$L \leqslant 360\ e_u$; o $L \leqslant 8\ V_u\ e_u$ ..... (175)
En curvas que limitan la velocidad de los trenes:
Cuando $D < 6°$, $L \leqslant 16\ V_s/3$; cuando $D \leqslant 6°$, $L \leqslant 240$ ..... (176)
donde
$L$ = largo de la espiral, en pies;
$V_u$ = velocidad maxima, en millas/hora; $e_u$ = superel, en pies, para $V_u$;
$V_s$ = velocidad en millas/hora calculada para una elevacion de 8 pulgs (como 20 cm).

$$= \sqrt{\frac{8\ R}{3.928}} \text{ (vease ec 118).}$$

Con $V = 45$ millas/hora, y superel, $E$, = 8 pulgs, o $e = 2/3$ pie, cada una de estas reglas da $L \leqslant 240$ pies (digamos 72 ms).

**230. En la práctica**, y con objeto de evitar el uso de sub-cadenas en la espiral puede usarse una longitud aproximada a la $L$, de la espiral, calculada.
Por ejemplo, con $H = 9.6$ pies, curva circular 5° ($R = 1146.3$ pies), tenemos (ec 153):

$$L = \sqrt{24\ R\ H} = 513.9 \text{ pies.}$$

Con $V = 50$ m/h, en una curva de 6° ($R = 955.4$ pies), relacion, $r$, del peralte gradual = 600, tenemos (ec 174):

$$L = r\ 0.3273\ V^2/R = 513.9 \text{ pies.}$$

En cualquiera de los dos casos, pudieramos usar 500 pies (10 cadenas de 50 pies) o 510 pies (10 cadenas de 51 pies) o 525 pies (10 cadenas de 52.5 pies), etc.

## Trazado.

Resultados condensados, para usarlos en el campo.

### Por angulos de deflexión.

**231.** La espiral puede determinarse convenientemente con angulos de «deflexion» desde una tang como si fueramos a determinar una curva circular; pero en las espirales, los incrementos sucesivos, en el angulo de deflexion total marcado desde la tang en un punto dado, aumenta regularmente, en lugar de ser constantes como en las curvas circulares.

**232.** Las derivaciones de las ecuaciones para tales angulos se dan en ¶ ¶ 207 a 220. Los siguientes (para uso en el campo) dan la deflexion necesaria para determinar cualquier punto, en cualquier espiral dada de 10 cuerdas, cuando el instrumento está en cualquier otro punto en la espiral. Asi:

**233.** El largo de cadena espiral, $c$ (largo de espiral, $L = 10\ c$) y el aumento, $k$, de agudeza por 100 pies de cuerda, determinan completamente la espiral. Vease el ejemplo abajo.
Dando $c$ y $k$, **se busca el angulo de deflexion**, $i$, Figs 83 $a$, 83 $b$, necesario para fijar cualquier punto dado, $P''$, desde la tang, $P'\ m$, desde cualquier punto dado, $P'$.

Sea:

$c$ = el largo dado de la cadena espiral en pies;

$k$ = el aumento dado de la agudeza, $d$, por 100 pies de cuerda, en grados;

$n$ = el numero de cadenas espirales entre la T. E. ($A$) y cualquier punto dado de la espiral;

$s_1$, $s'$, $s''$ ( = 0.01 $c\ n$) = el numero de cuerdas de 100 pies entre la T. E. ($A$) y los puntos $n = 1$, $P'$ y $P''$, respectivamente;

$d_1$, $d'$, $d''$ (= $Ks_1$, $Ks'$, $Ks''$ respeetivamente) = a la agudeza de la espiral en los puntos $n = 1$, $P'$ y $P''$ respectivamente;

$q = s'' - s'$ = el número de cuerdas de 100 pies entre $P'$ y $P''$;

$\vartheta_1$ = deflexion de..... { la tang inicial, $A\ V$, } en $A$, a { la cuerda ( = 1 cadena espiral, $c$) de $A$ al primer punto de la espiral donde $n = 1$;

$f$ = deflexion requerida de { la tangente $P'\ m$, } { en cualquier punto dado de la espiral, $P'$, } a { cuerda, $P'\ P''$, de $P'$, a cualquier otro punto de la espiral $P''$;

$m = f/\vartheta_1$ = un coeficiente.

De la ec (132) tenemos $\vartheta_1 = (0.01\ c)^2 k/6$.

De la ec (142) tenemos

$$m\ (= f/\vartheta_1) = q\ \frac{2\ d' + d''}{d_1\ s_1};\ \text{y}\ f = m\vartheta_1.$$

La siguiente tabla da los valores de $m\ (= f/\vartheta_1)$ cuando se mira desde cualquier punto dado de la espiral, $P'$ hacia adelante (Fig. 83 *a*) o hacia atrás (Fig. 83 *b*) a cualquier otro punto $P''$ de la espiral.

**Valores de m** $(= f/\vartheta_1)$ para ángulos de deflexion de tangentes a puntos de cadena. Figs 83 *a*, 83 *b*.

| Punto del instrumento $P'$. | Numero, $n$, del punto requerido, $P''$. | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | T. E. $A$ 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | E. C. $B$ 10 |
| | *m* | *m* | *m* | *m* | *m* | *m* | *m* | *m* | *m* | *m* | *m* |
| 0 $A$ ($T$ E)... | 0 | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 |
| 1..... | 2 | 0 | 4 | 10 | 18 | 28 | 40 | 54 | 70 | 88 | 108 |
| 2..... | 8 | 5 | 0 | 7 | 16 | 27 | 40 | 55 | 72 | 91 | 112 |
| 3..... | 18 | 14 | 8 | 0 | 10 | 22 | 36 | 52 | 70 | 90 | 112 |
| 4..... | 32 | 27 | 20 | 11 | 0 | 13 | 28 | 45 | 64 | 85 | 108 |
| 5..... | 50 | 44 | 36 | 26 | 14 | 0 | 16 | 34 | 54 | 76 | 100 |
| 6..... | 72 | 65 | 56 | 45 | 32 | 17 | 0 | 19 | 40 | 63 | 88 |
| 7..... | 98 | 90 | 80 | 68 | 54 | 38 | 20 | 0 | 22 | 46 | 72 |
| 8..... | 128 | 119 | 108 | 95 | 80 | 63 | 44 | 23 | 0 | 25 | 52 |
| 9..... | 162 | 152 | 140 | 126 | 110 | 92 | 72 | 50 | 26 | 0 | 28 |
| 10 $B$ ($E$ $C$)... | 200 | 189 | 176 | 161 | 144 | 125 | 104 | 81 | 56 | 29 | 0 |

**Ejemplo. Dando :**

$c$ = largo de la cadena espiral, = 20 pies;

$k$ = aumento de agudeza de la espiral, $d$, por 100 pies de cuerda, = 3°.

Entonces :

Ec (121) Largo, $L$, de la espiral, $A\ B$, en pies, medida a lo largo de las cadenas de espiral = $10\ c = 10 \times 20 = 200$ pies;

Numero, $S$, de cuerdas de 100 pies en $L$ = aprox $0.01\ L = 2$;

Ec (122) Agudeza, $D$, de la espiral en la E. C. ($B$), = aprox $k\ S = 3° \times 2 = 6°$;

Ec (126) Extension, $\Delta$, de la espiral, $A\ B$, = aprox $D\ S/2 = 6° \times 1 = 6°$;

Ec (129) Angulo de deflexion, $\Theta = V\ A\ B$, = aprox $\Delta/3$ = aprox 2°;

Angulo de deflexion, $\Phi = V\ B\ A$, = aprox $2\ \Delta/3$ = aprox 4°.

**Se busca el ángulo de deflexion, f**, de la tangente, $P'\ m$, en el punto $P'$ a otro punto de la espiral, $P''$ :

Aquí ec (132) :

$$\vartheta_1 = (0.01\ c)^2\ k/6 = 0.04 \times 3°/6 = 0.02°;$$

y (de la tabla arriba), fig. 83 *a*, mirando de :

$P'$, $(n = 5)$ a $P''$, $(n = 8)$, $m = 54$.

Por lo tanto, en este caso,

$$f\ (= m\ \mathfrak{d}_1) = 54 \times 0.02^o = 1.08^o.$$

Ademas, fig. 83 *b*, mirando de $P'$, $(n = 8)$ a $P''$, $(n = 5)$ tenemos :

$$f\ (= m\ \mathfrak{d}_1) = 63 \times 0.02^o = 1.26^o.$$

**234. Valores de m, etc., en casos especiales.** Figs. 82, 83.

(*a*) Cuando el punto $P'$ del instrumento esta en la T. E. $(A)$ o punto $n = 0$ (Linea de arriba de la tabla ¶ 233.)

Figs 82, 83 *a*. Cuando se mira (hacia adelante) de $A$ a cualquier otro punto de la espiral $P''$, de numero $n$, la ec (142) se convierte :

$$m = q\ \frac{2\ d' + d''}{d_1\ s_1} = 0.01\ c\ n\ \frac{k\ s''}{0.01\ k\ c.\ 0.01\ c} = n\ \frac{s''}{0.01\ c} = n^2;$$

y $\mathfrak{d}''$ ($\mathfrak{d}$ para el punto $P''$) $= m\ \mathfrak{d}_1 = n^2\ \mathfrak{d}^1$.

Por lo tanto, cuando $P''$ esta en la E. C. $(B$, o $n = 10)$, tenemos $m\ (= n^2) = 100$, y $\Theta\ (= V\ A\ B) = 100\ \mathfrak{d}_1 =$ aprox $\Delta/3$ *.

(*b*) Fig. 83 *b*. Cuando el punto, $P''$, mirado (hacia atras), esta en la T. E. $(A)$ o punto $n = 0$ (Columna en cabezada T. E.).

Cuando mirando de cualquier otro punto, $P'$, de numero $n$, hacia $A$, tenemos, aprox, ec (151'), $\Phi = 2\ \mathfrak{d}$.

Por lo tanto, cuando el punto $P'$ del instrumento, esta en la E. C. $(B$, ó $n = 10)$, tenemos $\Phi\ (= V\ B\ A) = 200\ \mathfrak{d}_1 =$ aprox $2\ \Theta = \frac{2}{3}\ \Delta$ *.

**Por ordenadas desde la tangente inicial, A V.**

**235.** Fig. 82. Las ordenadas, $x$ e $y$ (ecs 147, 158, 159) a cada uno de los puntos en toda la espiral $A\ B$, pueden medirse desde la tang inicial, $A\ V$.

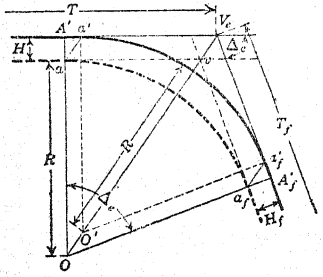



Fig. 84.

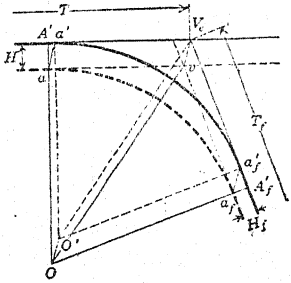



Fig. 85.

O, hacer la ordenada, $A'\ M$, por el punto medio, $M$, de la espiral $= A'\ a/2 = H/2$ (¶ 222), y el resto de las ordenadas proporcionales a los cubos de sus distancias desde $A$ y de $B$ respectivamente, midiendo las ordenadas por la primera mitad de la espiral hacia adentro a angulos rectos desde la tang $A\ V$, y el resto radialmente hacia afuera de la curva circular, $a\ B$.

**Por perpendiculares de la cuerda larga A B.**

**236.** Fig. 82. Sea :

$h$ = la ordenada normal desde la cuerda larga a cualquier punto dado en la espiral;

$H$ = dist entre tangentes paralelas, $A\ V$ y $a\ v$;

$n$ = numero de cadenas espirales entre la T. E. $(A)$ y el punto dado;

$N$ = numero de cadenas de espiral en la espiral.

---

* Vease nota al pie *, p. 1048.

Entonces, **en cualquier espiral** :

$$\frac{h}{H} = 4\,n\,\frac{N^2 - n^2}{N^3} \qquad (177)$$

Esta ecuacion, es bastante aproximada para la espiral de 10 cuerdas de la Asen Am Ings F. C.

**En la espiral de 10 cuerdas**, $N = 10$; asi que :

$$\frac{h}{H} = 4\,n\,\frac{100 - n^2}{1000} = 0.4\,n - 0.004\,n^3 \qquad (178)$$

Por lo tanto tenemos :

Cuando

| $n =$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $h/H =$ | 0 | 0.396 | 0.768 | 1.092 | 1.344 | 1.500 | 1.536 | 1.428 | 1.152 | 0.684 | 0 |

$h$ es un maximo ( $= 1.54\ H$) cuando $n = N/\sqrt{3} = 10/1.7321 = 5.77$.

## Insercion de espirales en vias existentes.

Vease tambien ¶ 169.

**237 (a)**. Figs 81, 84, 85. La curva circular original, $a'\,af'$, corrida **sin cambiar de radio**, en una dist dada, $a'\,a$, (igual y paralela a $V_e\,v$ y a $af'\,af$) a una nueva posicion, $a\,af$.

Se buscan las semitangentes, $T$ y $T_f$, a la curva entera, $A\,B\,B_f\,A_f$, fig. 81.

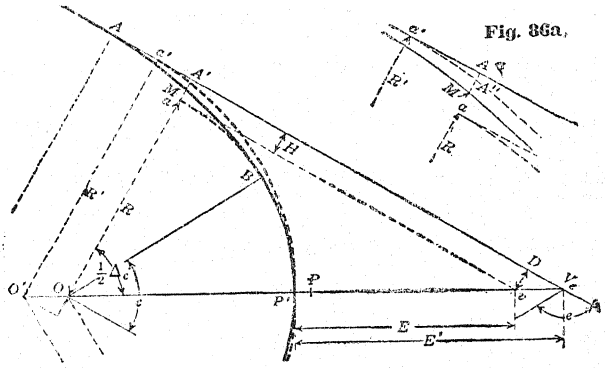



Fig. 86.

(1) Figs 81 $a$, 84. Las dos espirales similares. Corrase, $a\ a$, paralela a la bisectriz comun $O\,v$, de $a'\,af'$ y de $a\,af$ ;

$Tf = T$; $Hf = H$. $R =$ rad de la curva circular.

Aqui $H = H_f = a'\,a \cos(\Delta_e/2)$ ..... (179)

$T = T_f = (R + H)\tan(\Delta_e/2) + Z$ ..... (180)

De las ecs (153 y 157), tenemos :

$$Z = \frac{L}{2} - \frac{L^3}{1280\,R^2} = \text{prácticamente } \frac{\sqrt{24\,R\,H}}{2} = 2.45\,\sqrt{R\,H}.$$

(2) Figs 81 *b*, 85. Las dos espirales desemejantes. Corrase $a'\ a$ (igual y paralela a $V_e\ v$ y á $a_f'\ a_f$), no paralela a la bisectriz $O\ v$ ó, $O'\ V_e$.
$T_f$ y $T$ desiguales $H_f$ y $H$ desiguales.
Para $T$ y $T_f$, vease ec (172).
Para $H$ y $H_f$, vease ec (169).

**238 (b).** Fig. 86. **Conservando el punto medio original**, $P'$ y acortando el radio de $R'$ a $R$. Se cambia la linea de $A\ a'\ P'$ a $A\ B\ P'$. Habiendo determinado (¶ ¶ 227 etc.) la longitud, $L$, y $A'\ a = H = \frac{L^2}{24\ R}$;

Sea:

$\Delta_e$ = ángulo central entero, = 2 arc $a\ P'$ = 2 angulo $a\ O\ P'$;
$A''\ M$ (Fig. 86 *a*) = movida máxima de la linea de centro de la via;
$a'\ A'$ = movida del punto $a'$ de la curva circular, hacia la interseccion, $V_e$; y

| Radio. | Semitang. | Dist ext. | Agudeza. |
|---|---|---|---|
| $R' = O'\ a'$ | $a'\ V_e$ | $E' = P'\ V_e$ | $D'$ de la curva circular existente $a'\ P'$; |
| $R = O\ a$ | $a\ v$ | $E = P'\ v$ | $D$ de la curva circular aguda, $a\ P'$) |

Entonces:

$$E = E' - V_e\ v = E' - H \sec (\Delta_e/2); \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (181)$$

$$R = \frac{E}{\text{ex sec } (\frac{1}{2}\ \Delta_e)}; \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (182)$$

$$D = \frac{E \text{ de una curva de 1.º, de una extension dada, } \Delta_e}{E} \quad \ldots\ldots\ldots\ldots \quad (183)$$

$$a'\ A' = (R' - R - H) \text{ tang } (\Delta_e/2) = a'\ V_e - (R + H) \tan (\Delta_e/2) \ldots\ldots \quad (184)$$

Practicamente (Fig. 86 *a*), $A''\ M = \frac{H}{2} - \frac{7}{8}\left(\frac{a'\ A'}{100}\right)^2 D$ ..... (185)

**239 (c).** Fig. 86. **Con la corrida mínima de la linea.** Corrase el punto medio, $P'$ a $P$, haciendo $P'\ P = H/2$. Entonces:

$$E = E' - H \sec (\Delta_e/2) - H/2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (186)$$

Úsese este nuevo valor de $E$ en Ecs (182) y (183).

# SEÑALES

## GENERALIDADES

**1. El objeto** de las señales de ferrocarril es facilitar el movimiento de trenes, y evitar los choques dandole al maquinista informaciones con respecto a la via o las necesarias instrucciones para seguir o parar. La expresion « señales » se extiende hasta incluir los cambios de agujas, para correr los trenes en diferentes vias segun sea necesario.

**2.** El asunto lo tratamos aqui con poca extension, él en sí y el trazado de vía estan considerablemente relacionados sobre todo donde el tráfico puede ser fuerte. Nos hemos desviado un poco de la nomenclatura acostumbrada y en el modo de tratar el asunto con el propósito de abreviar el artículo, y facilitar su comprension a los que no estan versados con la práctica de señales.

**3. Las divisiones principales** del tema pueden hacerse así; (*a*) manipulacion; (*b*) trasmision (mecanismo); (*c*) señales.

(*a*) *Manipulacion* es la operacion (manual, eléctrica, o neumática) de las palancas u otros medios, que, por « trasmision » (*b*), hacen las señales (*c*).

(*b*) *Trasmision* son los medios (tales como barras, tubos, alambres, etc.) por los cuales la manipulacion (*a*) hace las señales (*c*). Los medios pueden ser mecánicos, eléctricos, o neumáticos.

(*c*) *La señal* es el aparato que el personal de la locomotora observa y del cual directamente recibe sus instrucciones o informacion. La señal puede ser visible (semáforos, discos o luces) o auditivos (campanas o pitos, etc.). En la expresion « señal » puede incluirse tambien la Parada Automática, ¶ 64, que para el tren si el maquinista no atiende a una señal de parada. Estas tres divisiones se tratan aqui en orden invertido, con una division en los « Sistemas » insertada entre « Trasmision » y « Manipulacion ».

**Fig. 1** *Parar*

**Fig. 2** *Seguir*

**Fig. 3** *Retroceder*

**Fig. 4** *Aflojar Frenos*

**Fig. 5** *Aplicar Frenos*

**Fig. 6** *Tren Partido.*

## SEÑALES

**4. Las señales a Mano** se intensifican a menudo de dia con el uso de una bandera, y de noche con faroles. Tres de las principales señales de mano se indican en las Figs 1 a 6. Un movimiento al través de la via con cualquier color u objeto significa « Parar ». Siempre que sea practicable se emplean banderas de colores y faroles, pero los colores no son esenciales.

**5. Hachones** se usan, generalmente, por la noche y en las neblinas; y algunas veces de dia, su humo sirve de señal. Son fuegos de artificio, conocidos como « fuego de color », con un extremo aguzado para enterrarlos en el terreno o en una traviesa. Cuando se encienden dan un resplandor continuo brillante (cinco o diez minutos); aun con viento o lluvia. Indican a un tren que sigue, que delante de él ha pasado un tren por el lugar, solamente unos minutos antes. Pueden arrojarse (encendidos) desde un tren en marcha a la via.

**6. Torpedos** son de tamaño pequeño en forma de cazoleta, se colocan en la cabeza del carril del lado derecho, y detras del tren que se va a proteger. El torpedo explota por las ruedas del primer tren que sigue, produciendo un ruido que el

maquinista seguramente tiene que oir. Se usan mucho durante las neblinas, nevadas, tormentas, etc., cuando las señales de vista no se pueden ver bien. Se usan sobre todo como auxiliar para hacer señales de bandera « atrás », para proteger el tren delantero mientras el abanderado vuelve al tren, o en caso de que el maquinista que sigue no pueda ver el abanderado.

### Señales Fijas.

**7. Los discos** de señales dan su indicacion, de dia tanto como de noche, solamente por medio del color, y no por la forma o posición. Cada disco de señal esta incluido en una caja. Moldeada como una guitarra (de aquí el nombre señal de « guitarra »), provista de un frente de cristal. Para usarla de dia, la señal consiste esencialmente en un disco de color opaco. Si este disco se mira al traves de la cara de cristal, indica « parar » cuando es rojo, o « precaución » * cuando es verde. Para indicar que se « siga », el disco se hace girar a un lado, dejando un fondo blanco que se ve al traves de la cara de cristal. De noche, un disco de cristal de color frente a una lampara indica « parar » o « precaucion »; o bien « seguir » al ser quitados aquellos dejando ver la lampara por un cristal claro.

**8. Semáforos.** Para señales se usa generalmente el tipo semáforo, vease figs 7 a 12, incl. Cada una consistente en un brazo movible o « tablero », de cerca de 4′ (1,20 m) de largo y 10″ (25 cm) de ancho, giratorio y montado en un poste u otro objeto apropiado.

**9. Señal Definitiva.** Una señal definitiva es aquella segun la cual el tren debe parar cuando la señal esta puesta para » parar ». Se coloca generalmente a la entrada de un (« block ») tramo de via, o inmediatamente antes de llegar a un desvío, descarrilador, crucero, puente giratorio, u otro objeto del que se va a proteger. Cuando está horizontal indica « parada », y por la noche la posicion horizontal se indica por un vidrio rojo, que aparece al frente de un farol. Cuando esta inclinado, la señal indica « seguir » y por la noche enseña una luz de algun color que no sea rojo. Vease tambien « Aspectos de las señales », ¶ ¶ 17, 32 y 33.

**10. Señal Distante.** Cuando se instala una señal « Distante » o de « Precaución * », se la coloca a alguna distancia (generalmente a varios millares de pies o centenares de m.) « atras de » (antes de llegar) a la señal « definitiva » a que se refiera y asi indica ademas la posición de la señal definitiva, que puede quizás estar oculta. De este modo se le anticipa al maquinista la informacion con respecto a la señal definitiva y hace posible el recorrido mas rápido y seguro. Las indicaciones de las señales definitivas y distantes simultaneamente siempre corresponden. A menos que el tren esté en la via entre las dos, en cuyo caso la señal distante indica « precaucion » * aunque la señal definitiva pueda estar en la posicion de « seguir ».

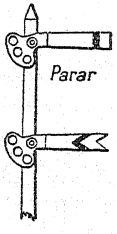



Fig. 7.

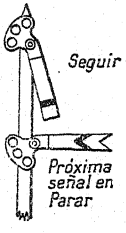



Fig. 8.

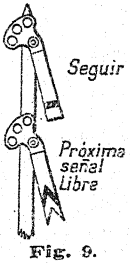



Fig. 9.

**11. Disposición** de las señales. La señal definitiva y la señal distante para el bloque o tramo de via adelante, están usualmente montados en el mismo poste, como en las figs 7, 8 y 9; la de arriba es la señal definitiva, y la de abajo, la señal distante.

---

*"Precaución" es una expresion usada aquí algo vaga, aunque indica su usual o roximado significado; « siga preparado para parar a la próxima señal ».

**12.** Cuando dos o mas señales definitivas, fig. 15, están montadas en el mismo poste, la de arriba (cuando indica libre) relativamente permite movimientos de alta velocidad, o movimientos sobre via recta. La señal de abajo gobierna movimientos mas lentos, o aquellos que van con rumbo para vias laterales.

**13. Señales enanas** son señales de semáforo, pequeñas, cerca de 9″ (como 23 cm) de largo y 3″ (como 8 cm) de ancho. Usualmente colocadas cerca del terreno, y se aplican a los trenes que vayan en contra de la direccion normal del tráfico.

**14. Las señales en conjunto** están formadas de luces y discos, etc., dando frente a direcciones diferentes, y montadas de modo que giren en un eje vertical. Se usan sobre todo en conexión con los cambios.

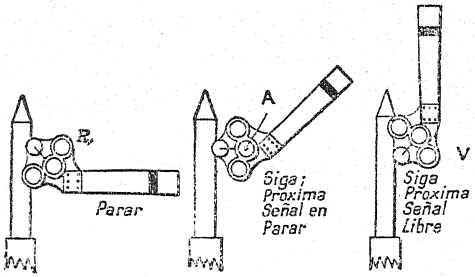



**Fig. 10.** R = *Rojo*.

**Fig. 11.** A = *Amarillo*

**Fig. 12.** V = *Verde*

**15. Señal de tres-posiciones**, figs 10, 11 y 12. Una señal de tres posiciones combina en un semáforo, las indicaciones de una señal definitiva y de una señal de distancia.

**16. Señal de dos luces** son aquellas (generalmente de tres-posiciones) en la cual una segunda ó luz simulada o « marcador », (o luz y semáforo) está debajo de la señal activa. Este marcador muestra rojo, (o « parar ») pero su indicacion es la que predomina cuando la señal activa indica otra cosa. Las ventajas son : que, siendo de color, el marcador ayuda de noche a determinar e identificar la señal activa, y reduce la posibilidad de que se deje de notar la señal propia en el caso de que cualquier luz se haya extinguido.

**17. Indicaciones o aspectos de señales.** Véase también ¶ ¶ 32 y 33. Aunque hay muchas variaciones en la práctica, los posiciones y colores indicados en las figs 10, 11 y 12 son las mas adoptadas. Las indicaciones de *dia* son dadas por la *posicion* del semáforo, y el color del semáforo no es entonces esencial, aunque generalmente es rojo o amarillo en la cara. Las indicaciones de *noche* estan dadas por las luces de color, — como sigue :

| | « Parar. » | « Precaucion *. » | « Seguir. » |
|---|---|---|---|
| | — | — | — |
| Práctica nueva..... | Roja. | Amarilla. | Verde. |
| Práctica antigua..... | Roja. | Verde. | Blanca. |

(El « amarillo », por algunos se llama anaranjado; mientras que el « blanco » luz descolorida tiene generalmente un tinte amarilloso.)

---

*"Precaución" es una expresion usada aquí algo vaga, aunque denota aproximadamente su significado usual; « siga preparado a parar en la señal próxima ».

**18.** Se está generalizando el uso de levantar el brazo al cuadrante de arriba (lado *izquierdo*) en los ferrocarriles eléctricos, tiene la ventaja de colocar la señal en los mismos ángulos como en el conocido sistema antiguo o sea en el cuadrante de abajo (lado derecho), y así queda mas facilmente colocado para ser visible entre los postes del telégrafo y las pértigas de los troles.

**19. Luces de relámpago**, para usarlas de noche. Se estan probando especialmente en el extranjero. El iluminante es generalmente gas acetileno, y cada lámpara esta preparada para lanzar destellos periodicamente (por el estilo de ciertos faros) alumbrando generalmente cerca de 0.1 de segundo y, extinguiendose por serca de 0,3 a 0,9 de segundo. Parecen adecuadas y economicas, y ayudan a distinguir las luces de señales, de otras luces extrañas al ferrocarril, y tambien para establecer diferencias entre diversas clases de señales.

**20. Las señales por luces** que no se usan con semáforo, son economicas en su costo primitivo, y no tienen partes movibles, fuera de las piezas que las ligan, Se usan en vias subterraneas y túneles, donde un semáforo no se puede ver bien; y hasta cierto punto en ferrocarriles eléctricos. Cuando se usan a la luz del dia, cada luz debe estar acompañada de una cubierta u otros medios para evitar que los rayos de sol caigan sobre los lentes, y por una pantalla u otro objeto grande obscuro, para llamar la atencion sobre la señal, y hacerla mas visible reduciendo asi la posibilidad de que se pase por alto. Vease tambien « Señales de dos vias », ¶ 16.

**21.** Se estan haciendo experiencias para combinar las señales de luces y de relámpagos.

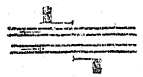


Fig. 13
*Doble via*

Fig. 14
*Cruce*

Fig. 15
*Dos a destino y una distancia*

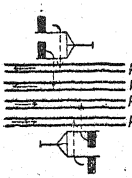



Fig. 16

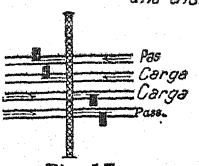



Fig. 17

*Cuatro vias*

**22. Situacion de las señales.** Las señales fijas visibles van colocadas generalmente a la derecha, o directamente sobre la via a que se aplican. Las figs 13 a 17, incl, muestran ciertas disposiciones típicas de señales, seleccionadas de las que trae el « Diccionario de Señales » de la Asen de Señales de F. C.

## MECANISMOS DE TRASMISIÓN

**23.** Las señales y los cambios estan operados convenientemente desde un punto central, como una torre de señal; y deben proveerse de los medios para connectarse a la torre, y para manejarlos desde la misma.

### Operados a mano.

**24. Las conexiones mecánicas** para la manipulacion se hacen generalmente con tubos (usados como barras) ó con alambres.

**25. El tubo** es generalmente de hierro dulce o acero, de una pulgada (25 mm) de diámetro, y como puede obrar lo mismo por tension que por compresion, solamente se necesita un tubo (en general) para cada señal o chucho. Las lineas de tuberia se ponen sobre rolletes soportados en marcos. Cuando hay que darle vueltas rápidas, los extremos del tubo estan conectados por medio de brazos de cigüeña, *B*, fig. 18 (no esta en escala), o por medio de barras encorvadas, *D* que se deslizan entre rolletes :

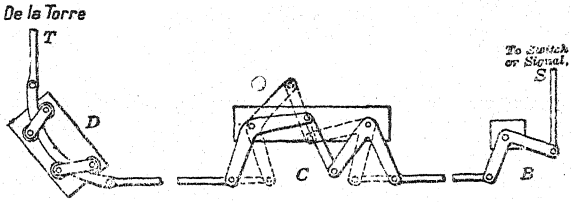



Fig. 18.

**26. Compensadores.** Como los cambios, aun los moderados, de temperatura alteran el largo de la tuberia, y alteran el ajuste, las lineas de tuberia estan divididas en secciones y estan conectadas por medio de « compensadores », *C*, que neutralizan los efectos del cambio de temperatura sin afectar los movimientos de las señales. La fig. 18, representa una linea de tuberia, que arranca de la torre de señal, pasando por una barra de desvio, *D*, un « compensador », *C*, y un brazo de cigüeña, *B*, a un chucho o a una señal *S*. Las lineas de punto indican las posiciones de las partes del compensador cuando la linea de la tuberia se expande por el calor.

## Fuerza.

**27.** La fuerza puede ser aire comprimido llevado por los tubos a los cilindros en que actua sobre los émbolos conectados con las señales o chuchos, y tambien la electricidad conducida por alambres a motores eléctricos (o a solenoides para pequeñas señales). El aire comprimido, o electricidad, que *opera* las señales, esta dirigida generalmente por medio de circuitos eléctricos relativamente ligeros, o, en los aparatos que son todos neumáticos, por presion de aire relativamente baja. En cualquiera de los dos casos la fuerza es aplicada por medio de palancas, etc.

**28.** El uso de sistemas **todo-eléctrico** se está generalizando : tienen la ventaja de que no necesitan planta especial para el aire comprimido; y se eliminan las varias dificultades del aire comprimido.

## SISTEMAS

**29. Intervalo de tiempo.** Con este sistema, se le debe dar un adelanto al tren de 5, 7 ó 10 minutos, antes de que al tren que le siga se le permita continuar, con el objeto de que, si el primer tren pasa inesperadamente, tener tiempo suficiente para permitir al abanderado hacerle señales al tren que le sigue; pero el sistema, en las mejores condiciones está lejos de ser seguro.

**30. Espacio de intervalo o bloque.** Bajo este sistema, el largo de la via está dividido en secciones llamados bloques; de largo adecuado y donde se practica, no se permite que dos trenes lleguen a estar en un mismo bloque al mismo tiempo. El largo del bloque puede variar, desde cerca de una milla (como 1.600 m) o menos hasta 5 ó 10 milles (9 a 16 kiloms) segun la densidad y caracter del tráfico.

**31. Paso permitido.** La expresion paso ó bloque permitido, significa que a un tren se le permite pasar una señal de bloque que indique « parar » con la condicion de que debe correr a una velocidad disminuida de modo que pueda parar en el primer punto que no pueda pasar, con seguridad evidente. Esto permite a un tren operar en un largo de bloque mayor; pero es menos seguro que el bloque absoluto.

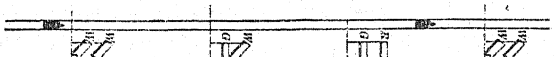


Fig. 19

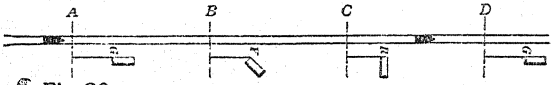



Fig. 20.

**32. Aspectos de señales.** Se aplica frecuentemente a las señales de bloque. Las figs 19 y 20 ilustran, para condiciones dadas en una linea determinada, las posiciones de señales con los sistemas de semáforo respectivamente de « dos posiciones » y « tres posiciones ». *A B*, *B C* y *C D* representan bloques. Bajo cualquier sistema, el tren en el bloque *C D*, está protegido por la señal definitiva en *C*, detrás de él, puesta en « parar »; pero al mismo tiempo, un tren puede entrar en el bloque *B C* en *B*, bajo la señal de « precaución » que está ahi; o un tren puede entrar en el bloque *A B*, en *A*, sin restriccion; la señal, en *A*, indica no solamente « seguir », sino también que la « próxima señal adelante esta franca ».

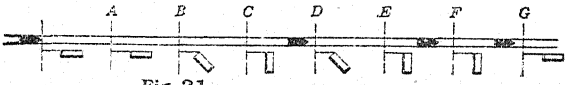



Fig. 21.

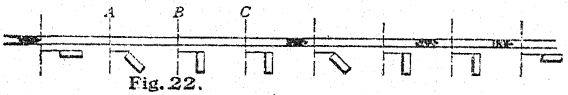



Fig. 22.

**33.** Las figs 21 y 22 ilustran, para condiciones dadas, en una linea dada, las posiciones de semáforo en el sistema de « tres posiciones » bajo los sistemas respectivos « **franco normal** » y « **peligro normal** ». Bajo el sistema « franco normal » la señal que gobierna un bloque, siempre muestra franco cuando el bloque está franco, aunque el tren esté o no proximo a entrar al bloque. Bajo el sistema « peligro normal », la señal permanece en peligro (si el bloque está ocupado o no) hasta que un tren se aproxima a la señal, para entrar en el bloque; y entonces, por supuesto, la señal se franquea solamente si el bloque esta actualmente franco. Estas diferencias se indican en los bloques *A B* y *B C*.

**34. Bloque por telegrafo ó bloque a mano.** En este sistema se estaciona un hombre al final de cada bloque; y, cuando un tren pasa de un bloque al otro, comunica el hecho eléctricamente, a los hombres situados en los otros puntos. Los hombres avisados de este modo, despliegan entonces las señales necesarias.

**35. El teléfono** está sustituyendo mucho al telégrafó para tales señales, y para despachar trenes. Esta usado comunmente en conjuncion con otros sistemas. Vease ¶ 71.

**36. Sistema manual restringido**, o de Cierre y Bloque. Para reducir la posibilidad de un error en la operacion en los sistemas de bloque manual y telegráfico, se pueden instalar, dispositivos que requieren la acción simultanea de ambos hombres en quitar y poner los contactos eléctricos antes de que se pueda poner la señal de « seguir ».

**37. Cierre automático.** Para evitar un acuerdo entre los dos hombres para abrir el bloque cuando aun no está franco, pueden disponerse los « aparatos de vía, o el circuito de vía » (véase ¶ 50) de tal modo que este acuerdo no se pueda practicar mientras que el tren no haya salido del bloque.

**38. Sistema de Pértigas.** Este método es aplicable especialmente a ferrocarriles de vía sencilla. En cada extremo del bloque hay un receptáculo que contiene un número de duelas o barras de metal. Los dos receptáculos estan dispuestos y conectados eléctricamente, de tal modo, que solamente *una* pértiga puede sacarse de los receptáculos; y esta pértiga debe devolverse a uno de los receptáculos antes de que otra pueda sacarse. La pértiga es el permiso del tren para viajar por el bloque, y la lleva el mismo tren. Cuando el tren deja el bloque (en cualquier extremo), la pértiga debe colocarse en el receptáculo de ese extremo ; haciendo otra vez posible sacar una pértiga del receptáculo en cualquier extremo del bloque.

**39. Requisitos especiales.** Pueden establecerse requisitos especiales, para operar las máquinas en bloque permitido « y tomar previsiones para las máquinas « empujadoras » (de servicio) que regresan; pero esto todavía no permite que dos pértigas completas estén fuera del receptáculo al mismo tiempo.

## MANIPULACIÓN

**40.** Las manipulaciones pueden ser lo mismo « *iniciales* » o « *entrelazadas* » o ambas (Vease ¶ 3 *a.*)

**41.** La manipulacion **inicial** puede ser efectuada por *empleados*, o (mecánicamente o por electricidad) por los mismos *trenes.*

**42.** La manipulacion **entrelazada** (parte de la cual puede estar en « Trasmision ») es la de dispositivos automáticos, ideados para evitar operaciones impropias. Esto puede hacerse por movimientos puramente mecánicos, por electricidad o por aire comprimido. Cuando unas cuantas señales y desvios estan operados desde un punto, pueden ocurrir choques de trenes causados por errores de los operadores. Con el propósito de evitar esto, las palancas y barras están agrupadas de tal modo, y provistas de piezas corredizas adicionales, llamados « perros », o con contactos eléctricos trancadores, etc., tales que (en lo posible) las combinaciones o posiciones con las cuales pudieran peligrar los trenes, están evitadas mecánicamente.

**43. Mecánica.** Aquí consideramos el entrelazado mecánico como un ejemplo : Así pues en un cruce de doble via cuando todas las señales indican « parar », cualquier otra señal puede hacerse indicando « seguir », pero, al hacer esto, la barra conectada con su palanca mueve los perros de modo que trancan las palancas de las señales que gobiernan las vias que cruzan por aquella vía. Así, si la señal para un tren rumbo al Oeste es libre, las dos señales para los movimientos rumbo Norte y Sur están por consiguiente trancadas en « parar »; pero la señal para la via rumbo Este queda franca; porque ambos trenes, el de rumbo Este y Oeste pueden pasar por el crucero simultaneamente mientras las vias rumbo Norte y Sur están trancadas en « parar ».

**44.** A la inversa, a los trenes con rumbo Norte y Sur puede darsele señal franca solamente despues que las señales rumbo Este y Oeste hayan sido puestas en « parar » y trancadas en esa posición.

**45.** El principio fundamental, aqui ilustrado por un ejemplo sencillo, se lleva a cabo con gran cuidado.

**46. Desvíos para descarrilar.** Para evitar choques debido a que un maquinista inadvertidamente pase una senal de « parar », se suele emplear un « descarrilador » donde es practicable. Esto consiste generalmente en un medio desvio, que cuando « abre » es suficiente para sacar un tren fuera de los carriles sobre las traviesas; y cuando es para tráfico de pasajeros, puede ser un desvio completo que vaya a un desviadero enterrado en arena, o provisto de otros medios para detener

el tren con un mínimo de daño. Este descarrilador esta entrelazado con los desvíos y señales de modo de evitar que se « cierre » para el recorrido normal, a menos que los otros desvíos y señales referidos estén puestos de tal modo que sea seguro el hacerlo así. El descarrilador produce un efecto muy ventajoso sobre cualquier tendencia del maquinista a riesgos indebidos ó á olvidar una señal puesta en peligro

**47. Bloque Escocés** (Scotchblock). Otros descarriladores comunmente usados en vías laterales de mercancías, etc., consisten en una pieza movible de metal, de tal forma que cuando se coloca sobre el carril, levanta y dirige la pestaña de la rueda por sobre el carril y fuera de la vía.

**48. Cierre de Chuchos,** * fig. 23 (no está en escala y muestra lo esencial solamente). Ademas de la palanca del chucho, generalmente se provee este de una palanca para trancar, que puede forzar un pasador, *A*, por uno o dos agujeros en la barra, *B*, solamente cuandos las puntas del chucho, *C* y *D*, han hecho su movimiento completo en cualquier direccion. Si por una acumulación de basura o de nieve, las puntas del chucho no se han llevado a su lugar, ningun agujero en la barra, *B*, viene frente al pasador, *A*, y el pasador no puede entrar en el agujero; y como la palanca de trancar a su vez se entrelaza con las señales, estas no pueden ser francas. Conectada a la palanca de trancar, hay tambien una barra indicadora.

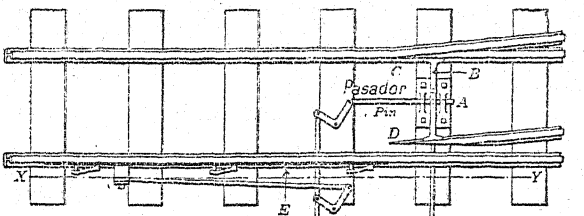



Fig. 23. *Plan*

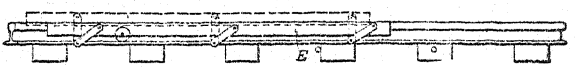



**Fig. 24.** *Sección por X Y.*

**49. Barra indicadora,** *E*, fig. 24, que está conectada de tal modo con la barra de cierre que la barra debe levantarse sobre la cabeza del carril cuando la palanca de trancar se mueve. Esto no se puede hacer si hay ruedas en el chucho. Así resulta practicamente imposible cambiar la posición de un chucho mientras un tren está pasando por él.

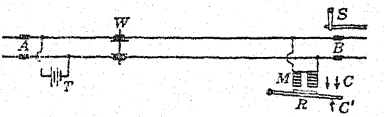



**Fig. 25.**

**50. El circuito de la via** es un recurso que facilita el modo de llevar a cabo muchas operaciones automáticas de señales, y muchas combinaciones ó su equivalente descartando muchas torres de señales, y plantas complejas. El principio fundamental del circuito de la via se ve en la fig. 25, que muestra los dos carriles de una via sencilla en un « bloque » u otra seccion, *A B*. Esta sección está aislada eléctricamente de las secciones adyacentes por mordazas de madera pesada, o de fibra aisladora, indicadas en *A* y *B*.

* Véase *N. del T.*, pag. 858.

En un extremo de la sección, hay una bateria, *T*, uno de cuyos alambres está conectado con un carril, y el otro alambre con el otro carril. En el otro extremo de la sección hay un electro magneto *M*, conectado de modo análogo. La armadura, *R*, de este magneto, lleva uno ó varios contactos, algunos de los cuales, *C*, están hechos, y otros *C'*, se interrumpen cuando la armadura es atraida por el magneto.

**51.** Supongamos que el circuito de la vía, *A B*, esté completo, y supongamos que no haya carros en la vía. Entonces la corriente de la bateria, *T*, obra sobre el magneto, *M*; la armadura es llamada hacia el magneto y el contacto superior *C* queda cerrado, dejando franca la señal *S* en *B*. Pero si un tren o cualquier otro par de ruedas, *W*, está sobre los carriles entre *A* y *B*, la corriente de la bateria, *T*, estará en corto circuito por las ruedas, hacia la bateria, robando de este modo corriente al magneto *M*, y haciendo que su armadura, *R*, caiga, y la señal, *S*, marque « parada ». Tambien, si por accidente o de otro modo el circuito se rompe en alguna parte o si ocurre algun corto circuito, o si la bateria falla, o si cualquier objeto metálico toca ambos carriles entre *A* y *B*, la señal marcará « parar ».

**52. La disposicion** es tal que la corriente, que acciona el magneto, *M*, es suficiente para atraer la armadura, *R*, a pesar de las pérdidas por el balasto, agua, nieve, y hielo, pero siempre el corto circuito total, causado por un par de ruedas. *W*, entre *A* y *B*, hará con seguridad caer la armadura, y poner la señal en « parar ».

**53. Las pérdidas debidas a las corrientes directas** de ferrocarriles eléctricos próximos a la via o el retorno directo de la corriente del mismo ferrocarril eléctrico (cuando se usan los carriles para circuitos de la vía), pudieran cambiar la señal cuando no se debiera cambiar, pero el uso de las corrientes alternas, para hacer señales, evita practicamente tal peligro.

**54. El Relevo**, *M C*, fig. 25, conectado con el circuito de via está hecho generalmente para servir simultaneamente a varios propósitos, por medio de sus varios contactos. Cuando la armadura cae, puede romper varios circuitos y poner varias señales en « parar », y puede tambien formar parte de él:

**55. Entrelazado eléctrico.** Donde se usa la electricidad para el manejo, y se usa electricidad o aire para la fuerza que opera, el chucho* y las palancas de señal simplemente hacen y deshacen los contactos electricos y operan una relativamente pequeña máquina de entrelazado; y son generalmente de menos de un pie de largo. En distintos tipos, las palancas se mueven en sentidos distintos. Pero las características principales son comunes a todos los tipos.

**56. Entrelazado de trenes.** La mera presencia de un tren, en cualquier sección de la via provista de un circuito en la línea, puede arreglarse por medio de su relevo, cortando los circuitos de cualquier señal o señales que guíen esa via. Esto suprime la necesidad de las « barras indicadoras » que son tan toscas en los chuchos.

**57. El entrelazado todo-automático** se ha usado hasta cierto punto y en él las válvulas sustituyen los contactos eléctricos, y los diafragmas ó émbolos sustituyen a los magnetos y armaduras.

## Operación de señales para manejo de trenes.

**58. Las señales automáticas** han demostrado ser muy útiles. Dependen principalmente del « circuito de la via », parrafo 50. Cuando la cabeza del tren entra en un bloque, ó tramo, la señal que ha pasado marca inmediatamente « parar », a cualquier otro tren que pueda venir. Cuando todo el tren haya pasado este bloque, el segundo bloque y la señal detras de él se « borra ». Esto sucesivamente se produce de modo automático y se continua indefinidamente de bloque en bloque, operando señales distantes.

Ademas, la mera presencia de cualquier tren, y aun, de un par de ruedas, en cualquier parte de un bloque de una sección de circuito, conservará (en cualquier señal o señales que gobierne cualquier entrada a tal seccion) la indicacion de « parar ». En la mejor práctica como se indica en el « Entrelazado Electrico », ¶ 55, los circuitos de via están entrelazados, por relevos, etc., con cualquier chucho* o

* Vease *N. del T.*, pág. 858.

cruceros en que estén envueltos. Si un tren ha entrado en un bloque, ningun chucho (1) de desviadero que conecte con ese bloque puede abrirse; é inversamente, una vez que el bloque este despejado, y un chucho (1) de desviadero se abre, todas las señales que vayan a la sección marcarán « parar ».

**59. Via sencilla automática.** Las señales automáticas tales como fueron aplicadas primero en operaciones de via sencilla, permitirian que dos trenes en extremos opuestos de un tramo de via sencilla, dejando sus desviaderos, corrieran uno contra el otro, siendo por supuesto la via sencilla bastante larga para contener un numero de bloques ó secciones. No obstante, no resultaria choque, porque cada tren pone en « parar », no solamente las señales que pasa sino tambien bastantes de las señales *delante* de él, para evitar choque. Pero un tren o el otro eventualmente quizas tendria que retroceder al desviadero que dejó.

**60. Amplitud-permitida.** En este sistema, no obstante, un tren entrando por cualquier extremo de un largo de via sencilla mantendrá en la señal de « parar » *todas* las señales delante de él en ese tramo. Otros trenes pueden seguirlo, como en las lineas corrientes de doble via con señales automáticas.

## MISCELÁNEA

### Seguridad.

**61.** La seguridad debida a las señales de ferrocarril han llegado a un alto grado. Todos los accesorios, hasta donde ha sido posible están dispuestos de tal modo que cualquier falta o descuido hará desplegar la señal de « parar ».

### Indicadores.

**62.** Se usan varios tipos de indicadores, y con varios propósitos, generalmente son modelos en miniatura de señales, o chuchos (1) o vias, e indican a los hombres de señales de patio la condición o posición de los objetos representados, pero, que no pueden ser visibles.

### Otros Métodos.

**63. Métodos nuevos** de gobierno de trenes, por medio de aparatos de señal se han desarrollado, con principios radicalmente diferentes a los expuestos hasta aqui, basados sobre otros principios fundamentales. Estos envuelven en conjunto, varias combinaciones o modificaciones de la parada automática (¶ 64), control de tiempo o velocidad, y señales de casilla. Algunos de estos métodos estan en uso con gran éxito; pero, como esta parte del arte de señales está en estado de transicion, nos abstenemos, por el presente, de discutirla mas en detalle.

**64. Parada automática.** El efecto de la parada automática sobre el motorista, es análoga a la que hace el descarrilador (¶ 46). Si aquel no para antes de llegar a cualquiera de estos dispositivos, se le llama seguramente la atencion y se disciplina; mientras que, de otro modo podria con frecuencia pasar una señal, sin darse cuenta, hasta que el hábito llegue á un grado peligroso. El alto costo de la parada automática y la duda de su seguridad bajo las severas condiciones del tiempo en los ferrocarriles de vapor han demorado su adopción para tal servicio.

**65. Señales « Rompibles ».** Se usan algunas veces en las proximidades de puentes giratorios y en otros puntos particularmente peligrosos. Una señal rompible consiste en un brazo o tabla, que se extiende sobre el paso de un carro en una posicion tal que franquee los trenes cuando marca « via libre »; pero, cuando está puesto en posición de « parar », chocará con alguna parte de la locomotora o del carro. El golpe rompe o desbarata (de aqui su nombre) la señal y deja una marca en la locomotora que sirve para identificar al maquinista culpable.

**66. Señales de tranvias y ferrocarriles eléctricos.** El aumento de tamaño y velocidad del material rodante eléctrico de calle e interurbano ha hecho deseable el cambio de métodos primitivos por aparatos de alta perfección. No obstante, el uso de los carriles para el regreso de la corriente, hace el circuito de señales de via, inseguro ó costoso : por esto se utiliza el paso de la rueda del trole por debajo de un contacto especial colocado sobre el alambre del trole para poner las señales.

(1) Véase *N. del T.*, pág. 838.]

**67.** La fig. 26 representa un tramo de linea de vía sencilla, con una señal y un cruce ó pase en cada cabeza, cuando el tramo está libre de carros, el aparato está « neutral ». Supóngase ahora un carro que este entrando por la izquierda. Pasando bajo el « contacto », en *B*, este carro pone la señal, *E*, en « seguir », y al mismo tiempo poniendo la señal, *F* (en el extremo lejano del tramo) en « parar » avanza un paso o diente una « rueda contadora », en cada señal.

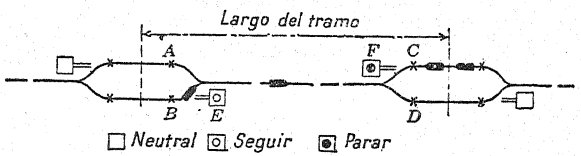



**Fig. 26.**

**68.** Estando la señal *F* en « parar », ningun carro puede entrar en el tramo por la derecha; pero un carro que le sigue, entrando por la izquierda puede entrar en el tramo pasada la señal de « seguir » en *E*, el motorista no obstante, la considera como una señal de precaución y observa que « destella », indicándole que su carro ha sido « contado ». Y asi sucesivamente, con cualquier otro carro que siga, entrando por la izquierda.

**69.** Pero, a medida que cada carro deja el tramo, pasando el contacto, *D*, la rueda contadora en cada señal echa para atras un diente o paso, hasta que el último carro haya dejado el tramo. Las ruedas entonces se han puesto en cero, y ambas señales, *E* y *F*, vuelven automáticamente a ponerse en « neutral ».

**70.** Entre los perfeccionamientos recientes hay aparatos para evitar que aparezcan las indicaciones de « seguir » en ambos extremos de un tramo simultaneamente, aun cuando dos carros, en extremos opuestos del tramo hagan sus contactos respectivos simultaneamente. Hay también aparatos para asegurar la contada exacta aun cuando se hagan movimientos inversos; para evitar trastornos de las ruedas contadoras al desconectar o conectar la corriente motriz; para aplicar el sistema a señales en via sencilla de tramo; y para la extensión del sistema incluyendo movimientos desde un tercer punto : digamos, un desviadero.

**71. Señales seleccionadas.** Estas permiten que un despachador de trenes ponga una señal cualquiera o un gran número de señales, visible ú oibles y asi, ponerse en comunicación con uno o mas jefes de Estación o con el personal de trenes.

**72.** El despachador está provisto de aparatos con los que él puede mandar, por el alambre (que conecta con todas las señales) inducciones eléctricas a intervalos exactos predeterminados o combinaciones de aquellas como en los sistemas de alarmas en los incendios. Cada señal está arreglada de modo diferente para responder solo a una clase dada de induccion ó combinación.

**73.** Si la señal es visible cuando se pone para que marque « parar », esta manda una « indicacion » al despachador de modo que puede este asegurarse que la señal está puesta, y esta se tranca automáticamente en esa posición. Solamente puede ponerse en « libre » con la cooperación del despachador. El conductor de un tren que ha sido parado por la señal, telefonea al despachador; y el despachador, cuando queda satisfecho de que sus instrucciones han sido entendidas, mueve sus aparatos de modo que la señal no quede trancada, permitiendo asi al conductor « borrarla ».

**74. Alarmas en Cruceros de Carreteras.** La alarma en si mismo puede ser o una campana u otro aparato que suene, o luces o señales ú ambas. Las alarmas son generalmente manejadas eléctricamente por medio de circuitos de vía, y varias disposiciones complejas de relevos a menudo entrelazadas unas con otras.

**75.** Una señal cruzada, utilizando la flexibilidad vertical de los carriles, elimina, la mayor parte de las dificultades debidas a los movimientos irregulares del tren, etc., y tiene fuerza propia. El mecanismo es *sensible* « direction sensitiv » y necesita un movimiento de tren en la direccion correcta para dar el alarma y esta termina automáticamente cuando el tren se para.

**76. Uso de señales.** Enero 1, 1911 y 1915, tal como la dá

La Comision de Comercio entre Estados :

(Una « milla de *linea* », o milla de *camino* = a una milla de *carretera*, si está ocupada por una vía, o por dos o mas vias. Una « milla de vía » = una milla de *vía* sencilla. De modo, que hay *dos* « millas de vía « en una » milla de linea » de *doble* vía) (1).

| Clase de señal. | En uso, en 1911, en | | En uso, en 1915, en | |
|---|---|---|---|---|
| | Millas de linea. | Millas de vía. | Millas de linea. | Millas de vía. |
| Disco expuesto | 323 | 537 | 257 | 391 |
| Disco cubierto (« Guitarra ») | 1,921 | 3,866 | 1,356 | 2,961 |
| Semáforo ; | | | | |
| Electro-neumático | 434 | 1,391 | 433 | 1,392 |
| Electro-gas | 919 | 2,018 | 891 | 1,890 |
| Electro-motor | 14,168 | 21,339 | 26,575 | 42,409 |
| Total | 15,521 | 24,748 | 27,899 | 45,691 |
| Franco normal | | 23,059 | | 41,667 |
| Peligro normal | | 6,093 | | 7,753 |
| Total de señales automáticas de secciones | 17,710* | 29,152 | 29,864 | 49,442 |
| Total de señales no automáticas de secciones | 53,558 | 63,506 | 66,745 | 74,673 |
| Total auto y no auto de secciones. | 71,269 | 92,708* | 96,609 | 124,115 |
| Total de líneas de pasajeros en operación | 172,390 | 195,922 | 193,180 | 223,081 |
| Por ciento de secciones con señal. | 41.4* | 47.3* | 50.0 | 55.6 |
| Telégrafo | 38,613 | 44,542 | 37,938 | 41,174 |
| Teléfono | 12,199 | 15,038 | 28,364 | 32,851 |
| Campana eléctrica y gobierno manual | 3,212 | 4,357 | 2,883 | 3,622 |
| Personal de tren eléctrico | 346 | 347 | 388 | 407 |
| Nº. de estaciones de señal en secciones | | 9,912 | | 11,496 |
| Nº. de estaciones de señal cerradas parte del tiempo | | 3,751 | | 5,799 |
| Número de secciones cerradas | | 29,881 | | 51,690 |

**77. Costo** de señales, antes de 1911, tal como lo da la comisión sobre Relaciones de Operacion de F. C. en Legislacion, 1911, nov. 14, como resultado de las preguntas y repuestas recibidas de los ferrocarriles que operan cerca del 80 0/0 de toda la via en los E. U. equipada con señales de secciones. Tomado de la revista Railway Age Gazette, 11 nov. 1917.

Instalación, por milla de vía;

Automática, $1,146 ($710 por kilometro); No automática, $248 ($154. por kilometro).

Mantenimiento automático por año;

$169 por milla ($105 por km) de vía; $ 69 por paleta de señal.

Los costos de instalación durante 1911 fueron de 10 a 40 0/0 mayores que lo expresado; probablemente es debido a la introducción de mayores perfeccionamientos.

(1) *N. del T.* — Para convertir millas en kilómetros, multipliquense aquéllas par 1,609.

* Substancialmente exacto, á pesar de la discrepancia insignificante entre tablas distintas en los Informes de la comisión de comercio entre Estados.

# ESTACIONES Y PATIOS

**1. Generalidades.** Los patios, estaciones y terminales * consisten en vias adicionales á las vías principales, edificios y varias facilidades, para el movimiento y manejo de locomotoras, carros, mercancias, pasajeros, equipajes, etc. Puede simplemente consistir en una pequeña estacion, un desvío sencillo, un almacén de mercancias; o puede incluir grandes almacenes y estaciones de pasajeros; y docenas de vias, y cubrir un area de 1 ½ kilómetro o más de largo.

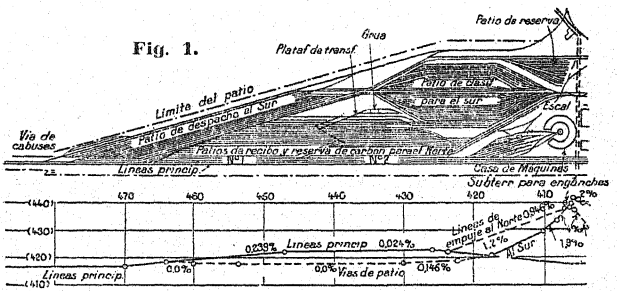
## PATIOS

**2. Definición** (Asen Am Ings F. C.). « Patio. — Un sistema de vias, dentro de límites definidos, provisto para preparar trenes, depositar carros, y otros propósitos, sobre el cual se hacen movimientos no autorizados por itinerarios o por el orden de trenes; sujetos a señales prescritas y reglamentadas. »

**3. Generalidades.** Un patio se compone generalmente de :
(a) un *Patio de Recibo*, ¶ 10, sobre el cual los trenes que vienen de la linea principal, pueden quedarse hasta ser distribuidos;
(b) los *Patios de Servicio*, ¶ 11, en los cuales las locomotoras toman carbon, los carros refrigeradores se proveen de hielo, etc., y el material rodante se repara;
(c) el Patio adecuado para *Separar*, *Sortear*, o *Clasificar*, ¶ 46; y
(d) un *Patio de Entrega*, o *Vias de salida*, ¶ 79, en el cual los trenes *formados* recientemente pueden esperar antes de pasar a la linea principal.

**4. El trazado** del patio, dependerá, en gran parte del carácter, dirección y cantidad de tráfico, del área y forma del terreno disponible, de la proximidad de los materiales varios y demas facilidades, y de otros factores. Los patios grandes

Fig. 1.




deben trazarse teniendo en cuenta una buena información sobre aquellos puntos pertinentes, y despues de considerar cuidadosamente el tráfico futuro y su crecimiento.

**5. Ejemplo.** El patio del F. C. Missouri Pacific en Dupo, Illinois, fig. 1, es una buena muestra del patio ideal corriente para un tráfico bien equilibrado.

**6. Dos direcciones.** Generalmente, en todo caso habrá virtualmente dos patios, uno para cada direccion de tráfico, pero habrá un solo patio de *servicio* utilizable en ambas direcciones.

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* Véase nota de la llamada entre párrafos 81-82.

Los dos patios pueden ser aproximadamente iguales, o de tamaño muy desigual, esto depende de las cantidades relativas de tráfico en las dos direcciones.

**7. Las Vias de linea principal**, en general, especialmente en patios grandes, deben estar situadas fuera del patio, una a cada lado, si es posible, o ambas sobre el mismo lado si es necesario; para que las facilidades (patios de servicio, etc.) se agrupen cerca del centro del patio.

**8. Conexión con la Via Principal.** El número de chuchos (1) que van directamente de la linea principal debe ser un mínimo, con un crucero si es via doble en un patio pequeño, las lineas distintas de *patio* se conectan solamente *entre sí*. Cualquier chucho (1) debe, donde sea posible, no dar el frente al tráfico, de modo que los trenes de vía principal no entren en ellos, reduciendo asi el riesgo de « precipitarse en un chucho abierto ». El chucho (1) debe tambien estar entrelazado con cualquier señal de la via principal, y tener un descarrilador, pp. 900-2 para evitar que los carros pasen sin querer a la via principal.

**9. Uso.** Los patios de carga se usan (1) para el almacenaje del material rodante que no se usa, (2) para retener los carros mientras se cargan o se descargan, y (3) para la distribución y clasificación de carros, cuando se usan para esto, se llaman comunmente « patios de clasificación » y son probablemente los mas importantes, especialmente en los grandes centros, y no solamente incluyen por lo regular los otros dos tipos, sino que sirven sobre todo para reducir su tamaño. Varios escritores afirman la importancia de considerar un patio como lugar para *manejar* carros, y *no* como lugar para *almacenarlos*.

## Patios de Recibo.

**10.** El patio de recibo debe ser bastante largo para recibir el tren mas largo, y debe tener un número suficiente de vias para recibir todos los trenes que lleguen hasta que puedan manejarse. De ahi la loco de cada tren se saca para el patio de servicio para reponer su carbon, etc., y el tren se empuja al patio adecuado de clasificación.

## Patios de Servicio.

**11.** Los patios de servicio en general deben estar situados en el centro; provistos con vias y otros medios para cambiar los carros del conductor y locomotoras, para quitar y retener los carros en « mal estado », y con vias de tránsito para permitir

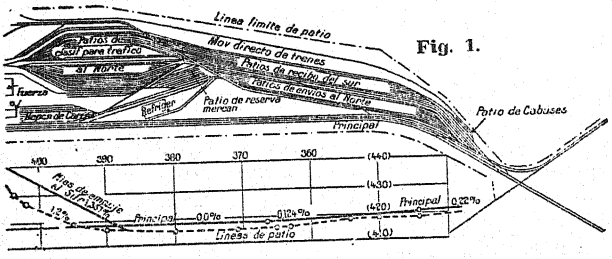


**Fig. 1.**

que los carros y las locos trafiquen libremente alrededor o por los patios. Deben proveerse facilidades; para quitar cenizas de las loco; para tomar agua, carbón y arena; para limpiar, aceitar, para darles alojamiento y hacerles reparaciones menores; para inspeccionar y reparar carros y cajas de chumaceras calientes; para limpiar los coches de pasajeros; para proveer de hielo a los carros refrigeradores; para proveer de agua y alimentar animales vivos; y para otras necesidades que puedan existir.

**12. Los carros averiados** del tipo de tolva o descarga por abajo, y su contenido algunas veces se manejan convenientemente haciéndolos pasar por una via elevada sobre caballetes y descargando su contenido en carretones por debajo de esta via elevada.

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(1) Véase *N. del T.*, pág. 858.

**13. Vias de Reparación** (Manual 1915 de la Asen Am Ings F. C.). Estas deben tener una capacidad de 15 carros cada una, permitiendo 50 pies (como 15 m) para cada carro alternando las vias con espacios de 16 y 24 pies (4,80 a 7,20 m) de centro a centro. Los patios de reparación deben estar liberalmente equipados con tuberias de agua y aire, con salidas de aire cada 50 pies (15 m). Las vias para reparaciones grandes deben estar bajo techo. Otras especificaciones exigen vias separadas para reparaciones grandes y pequeñas, con el objeto de que los carros que necesiten reparaciones pesadas, levantarlos, etc., no puedan atrasar el trabajo de los otros carros.

**14. Patios de carga y trazado de talleres**, Mr. Walter G. Berg, Ingeniero Jefe del F. C. Lehigh Valley Gaceta de F. C. de marzo 13 á abril 10, urge la importancia de : — un trazado tal que reduzca al mínimo el traslado innecesario de hombres y materiales; la distribucion apropiada de espacios entre los distintos talleres; medios amplios para comunicar y transferir : facilidades entre un departamento y otro para las aglomeraciones; espacio amplio, para salones, pasajes y corredores; buena luz, calefacción, ventilación, servicio sanitario, abasto de agua, protección para incendios, construcciones a prueba de incendio; facilidades para la fuerza motriz : y previsiones necesarias para el aumento o ensanche por (digamos) unos 10 años.

**15. Electricidad :** ésta ha hecho posible la realización de cualquiera disposición de los talleres y de las vias que se deseen, mientras que sin ella, los edificios tenian que agruparse mas o menos, o instalar plantas de fuerza motriz separada.

**16. Una grua movediza elevada**, de suficiente resistencia y espacio para levantar o izar una locomotora entera sobre las otras, evita la necesidad de usar plataformas de transferencia, mesas giratorias, vias de enlace y chuchos (1).

**17. Plataforma de Transferencia**, es una plataforma estrecha, con una via usualmente lo bastante larga para una loco y alijo o ténder. La plataforma corre sobre una via ancha en una fosa a angulo recto con su propia vía y la del taller, y asi transfiere su carga de una via a otra y a la via paralela en el taller. Para vencer el obstáculo ofrecido por la fosa profunda al paso de hombres y materiales, Mr. Henry V. Miller (Eng News, 1909, julio 15) ha patentado una plataforma de transferencia « sin fosa », cuyas ruedas con colgantes en forma de pescuezo de pato, estan por arriba de la misma mesa, « y permiten que su via esté abajo *casi* a nivel con el fondo del espacio sobre el cual rueda ».

**18. Mesas giratorias**, fig. 2. *En general*, la fosa de la mesa giratoria está rodeada por la pared circular, W, la parte superior de la cual se llama parapeto : y el marco consiste en dos alas; cada ala comprende un par de vigas armadas, *G*, con tirantes cruzados apropiados. Las dos alas estan unidas formando una doble viga de contrapeso, continuo sobre el centro, *C*, y el asiento del pivote, *P*.

**19.** *Equilibrio.* Normalmente, al girar la mesa, con su carga viva, se equilibra sobre el pivote central, *P*, al cual se trasmite toda la carga por rolletes cónicos o discos (¶ 39); pero, cuando una locomotora entra o sale de la mesa (y algunos veces cuando gira, véase párrafo 29), la carga descansa *parcialmente tambien sobre el carril circular*, *R*, por medio de ruedas con ejes radiales, colocadas en los extremos de las alas. Las ruedas están alojadas de tal modo que se extienden poca cosa debajo de las pestañas bajas de las vigas.

**20.** *Vigas armadas.* Las vigas *cantilever* (2) para locomotoras modernas pesadas, que requieren mesas giratorias mayores de 75 pies (digamos 22 m), deben ser muy altas; necesitando al mismo tiempo, fosas muy profundas, que pasan algunas veces de 11 pies (3,30 m) Tales fosas son difíciles de desaguar bien. Para reducir las alturas de las vigas se sustituyen algunas veces por vigas (pony) o armadas. Para permitir con seguridad el paso de hombres entre ellas y la loco, a cada lado, las vigas deben estar separadas cuando menos 4,50 m de centro a centro.

**21.** O bien fig. 3, las dos alas, *A* y *B*, son independientes, cada una descansa sobre el centro, *C*, por un extremo, y sobre el circulo de carril, *R*, en el otro extremo. Estas mesas no se inclinan. Al ser sus luces mas cortas, sus vigas y la fosa se hacen mas planas.

**22.** O fig. 4, se usa una viga baja y continua, y está parcialmente soportada por miembros que trabajan en tension, *D*, *D*, desde una torre, *T*, que descansa sobre el centro, *C*.

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(1) Véase *N. del T.*, pág. 858.
(2) Véase nota ** *N. del T.* al pie de la pág. 458.

**23.** *Largo.* Muchas mesas antiguas varian en su longitud desde 60 a 70 pies (18 a 21 m); pero, de 57 ferrocarriles importantes interrogados por una comision de la Asen Am de Puentes y Edificios de F. C.* en 1912, 54 usaban mesas variando en largos de 75 a 90 pies (23 a 27 m); dos, 30 m; y uno, 31,50 m.

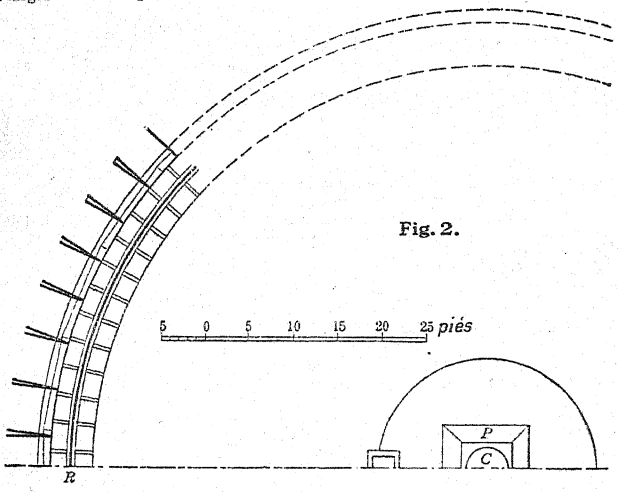
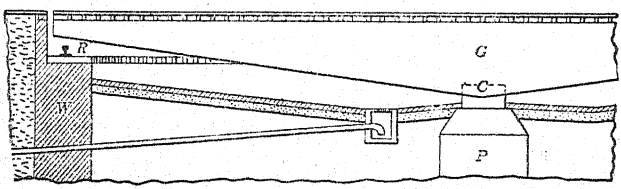



Fig. 2.




**24.** *Exceso de longitud.* Las mesas giratorias ordinariamente deben considerarse mas largas que la base de ruedas combinadas del ténder y la loco, especialmente donde tienen que « balancear » ténder vacios con sus locos (descansando solamente sobre un pivote central). Comparese figs 5.

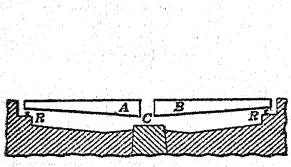



Fig. 3.

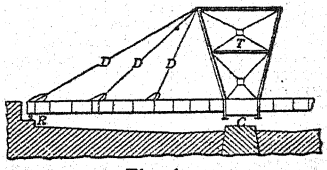



Fig. 4.

* Informe de la Asen Am de Puentes y Edificios de F. C., 1912. A esta comisión que redactó su informe detallado, adoptado en oct. 1912, le estamos agradecidos por muchas de las informaciones y recomendaciones que se dan aquí.

| | Informe pp. | Pesos. | | |
|---|---|---|---|---|
| | | | Ténder. | |
| | | Peso locomotoras kg. | Peso cargados kg. | Peso vacios kg. |
| Mogul Am Loco Co..... | 2 | 84,816½ | 65,226½ | 25,084 |
| Baldwin Mikado..... | 4 | 138,390 | 80,152 | 22,680 |
| Santa Fé Mallet 24 ruedas. | 5 | 279,411 | 106,140 | 47,627 |

y 6, que indican los largos de las mesas necesarias para una loco del tipo « Mogul » (Material Rodante, párrafo 65), con ténder cargado, fig. 5, y vacio, fig. 6.

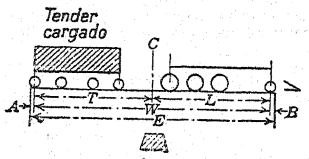


Fig. 5.

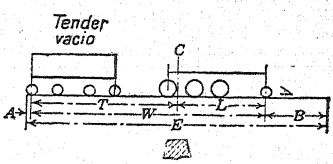


Fig. 6.

**25.** La tabla de arriba indica los pesos y distancias horizontales (con dif de 2 á 3 cm) para este y los otros dos tipos. Los dos primeros son los mas pesados de los tipos hechos por los talleres de la American y Baldwin respectivamente, hasta 1912. La comision * recomienda un largo mínimo de unos 23 m para la práctica ordinaria; y 27,50 m para las máquinas mas pesadas; prefiriendo las mesas ó placas giratorias en forma de Y cuando son de mas de 27,50 m de largo.

**26.** *Mecánica, Momentos y cizallamiento.* Esfuerzos y deflexiones. Cuando una mesa giratoria esta « equilibrada », con o sin carga, el marco debe considerarse como formado por dos *cantilevers* (1) (vigas de pie de amigo), cada una consistiendo de dos vigas paralelas armadas, *G*, los dos cantilevers están conectadas sobre el soporte central. Para este caso, solo una posición de la carga viva es posible. Pero, cuando la carga está colocada de modo que una ala del marco de la mesa descansa en un extremo, sobre el círculo del carril, *R*, y el otro extremo sobre el soporte central, *C*, (como ocurre cuando una loco entra o sale de la mesa), esa ala debe considerarse como una viga simple, y la otra como un cantilever (1), y este es siempre el caso con la mesa que no se inclina, fig. 3. En cada caso, los momentos y cizallamientos, para la carga muerta, para la carga viva, y para ambas combinadas, se buscarán (para distintas posiciones de la carga viva) como indican las pp. 461 a 475; los esfuerzos correspondientes y deflexiones como estan en la pp. 488, etc. y 503 á 504. Para la mesa equilibrada, las deflexiones en el extremo no deben exceder de cerca de 1 ½ cm y no deben llevar la mesa hasta descansar en el círculo del carril, *R*.

**27.** Segun la comision * en la práctica se usan *unidades de esfuerzo* de 1125 kg/cm cuad en tension, y 703 en cizallamiento, cuando se tiene ademas en cuenta el choque (vease vigas armadas pgs 821-2); la comision recomienda 703 y 422 respectivamente cuando no se hace tal prevision; a menos que, en los extremos, los esfuerzos de carga viva deban duplicarse teniendo en cuenta los choques.

**28.** *Carga sobre ruedas.* Al diseñar una mesa giratoria, las locos que puedan usarla, deben estudiarse y el diseño debe basarse sobre *la loco que pueda producir los mayores esfuerzos.* Por razon de su base de rueda corta, las *Cargas de Cooper* (loco + ténder = 14.64 m para todas las clases; vease Especificaciones de Armaduras, p. 818) son inadecuadas para mesas giratorias modernas pesadas; pero sin embargo se

* Informe de la Asen Am de Puentes y Edificios de F C., 1912. A esta comisión in forme detallado, adoptado en oct. 1912) le estamos agradecidos por las muchas informaciones y recomendaciones que nos han suministrado.

(1) Véase *N. del T.* **, pág. 458.

Largos con una pulg de aproximación.

| W | Fig. 5. (Ténder cargado.) A | T | L | B | E | Fig. 6. (Ténder vacio.) A | T | L | B | E |
|---|---|---|---|---|---|---|---|---|---|---|
| ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ |
| 56-10 | 0-7 | 28-05 | 28-5 | 0-7 | 58 | 0-8 | 35-4 | 21-06 | 14-6 | 72 |
| 67-01 | 0-6 | 36-00 | 31-1 | 5-5 | 73 | 0-9 | 43-9 | 23-04 | 21-2 | 89 |
| 108-02 | 2-2 | 57-10 | 50-4 | 9-8 | 120 | 2-3 | 65-3 | 42-11 | 24-7 | 135 |

usan algunas veces; su separación de ruedas se aumenta con dicho objeto. En la tabla siguiente, las tres loco mas pesadas al distribuir su carga sobre una porción mayor de la luz, producirán, si se coloca sobre un *puente*, aproximadamente los mismos esfuerzos que indica la fórmula mas ligera de Cooper E 50; pero, en una *mesa giratoria* equilibrada (donde cada ala actúa como un cantilever (1) sus largos mayores y pesos producen, en el centro, mayores esfuerzos de cizallamiento y mayores momentos negativos : (2)

| Carga. | Loco lbs. | Ténder lbs. | Base de rueda loco+ténder | Momentos y cizallamiento en en el centro, de la *mesa giratoria*. Momento neg pies-lbs. | Debido a Cooper. | Cizallamiento lbs. |
|---|---|---|---|---|---|---|
| | 225,000 | 130,000 | 48′00″ | 2,149,260 | E 50 | 225,000 |
| Am Loco Co (ténder cargado). | | | | | | |
| Mik.... | 315,000 | 169,700 | 67′10½″ | 4,349,000 | E 100 | 270,000 |
| Pac.... | 317,000 | 175,700 | 71′ 5⅝″ | 4,650,000 | E 110 | 248,300 |
| Mal.... | 483,000 | 186,400 | 88′ ½″ | 7,228,000 | E 170 | 346,900 |

**29.** *Rotación.* Las mesas pequeñas se hacen girar a mano, para cuyo propósito hay en cada extremo de la mesa una palanca de (2,5 a 3 m) de largo, y a una altura conveniente. Las mesas mas pesadas se hacen girar con fuerza neumática, o con gasolina o maquina de vapor. La fuerza eléctrica se prefiere si se puede conseguir. El aire comprimido se usa cuando hay posibilidad de congelaciones, necesitandose siempre el uso previo de fuerza manual. El motor se coloca algunas veces cerca del centro, *C*, otras cerca de un extremo de cualquier ala, ó, al extremo de una ala ligera a ángulo recto con las alas principales. Con los motores, las mesas por lo regular se giran sin estar en equilibrio, con la carga viva descansando parcialmente sobre el carril circular, *R*. Donde haya que girar mesas corrientes que no están equilibradas y que no se inclinan es costumbre dar a los extremos de las alas y a sus alas peso excesivo para sostener la parte de la carga viva que obra sobre ellos. Entonces las mesas pueden girarse « equilibradas » cuando se viran las loco mas cortas, y « sin estar equilibradas » con loco mas grandes. Las mesas que no se tumban, debido a las grandes palancas que tiene la resistencia en sus soportes extremos, son difíciles de girar, y costosas en su sostenimiento por desgaste de sus soportes.

**30.** Las mesas giratorias se *mantienen en posicion*, para el paso de locos que entren y salgan empleando diversos sistemas como frenos (especialmente donde se usa la fuerza para girarlas); o barras que giran verticalmente sobre una bisagra asegurada a las traviesas de la mesa, y alojando aquella, cuando giran, en una entalladura, ligada a las traviesas entre los carriles de la vía mas próxima; o por pestillos corredizos, que se deslizan en bujes sobre las traviesas de la mesa y en encajes asegurados entre los carriles de las vías mas próximas. Estas últimas se conectan y se mantienen en posicion de cerrarlas por medio de muelles, y se desconectan por medio de una palanca de mano, que puede retirar o desconectar simultaneamente los pestillos en ambos extremos de la mesa. El sistema de cierre está frecuentemente conectado a una señal que indica su posición.

(1) Véase *N. del T.* ** pág. 438.

(2) *N. del T.* — Nos ha parecido más conveniente dejar esta tabla en medidas inglesas. Para tenerlas en sistema métrico recuérdese que las libras se convierten en kilogramos mult por 0,453; pies × 0,305 = metros; pulgs × 0,025 = metros; pies libras × 0,138 = kilogrametros.

**31.** *El cruce de vias radiales adyacentes*, y cerca de la mesa, empleando ranas, etc., se necesita frecuentemente para economizar espacio.

**32.** *Piso.* Ahora se cubre toda la fosa solamente en casos especiales; donde se necesite un paso para parejas de caballo, etc.; o donde el peligro de las congelaciones sea serio. En este último caso puede montarse una estufa sobre las vigas giratorias. Algunas veces se pone un tubo de vapor alrededor de la fosa, junto al carril, circular *R* (fig. 2). Cuando se usa el piso, éste está soportado por vigas ligeras armadas y radiales. Comunmente la mesa se hace lo bastante ancha para que quepa un *pasillo* en uno o ambos lados; el pasillo se sostiene sobre traviesas de largo extra, colocadas a intervalos; y se le pone algunas veces un pasamanos. Con frecuencia se provee con una á dos alas adicionales ligeras a angulo recto con las alas principales para llevar el motor o la estufa, etc., o para otros propósitos; formando todo una especie de cruz. Para darle rigidez, los extremos de las cuatro alas pueden conectarse por medio de riostras.

**33.** *Altura del carril.* La mesa se colloca por lo regular a tal altura que cuando esté sin carga sus carriles esten $^{3}/_{4}$ pulgs (como 2 cm) sobre los carriles de las vias mas cerca, de modo que deje $^{1}/_{4}$ pulg (6 mm) de espacio entre el carril circular y las ruedas de los extremos de la mesa cuando los extremos de las alas cedan ½ pulg (12 mm) bajo una carga equilibrada. La parte superior de los carriles de la mesa deben quedar parejos con los de las carrileras mas próximas al extremo por el cual una loco pueda entrar ó salir de la mesa. Se han empleado muelles de acero para amortiguar los choques ocasionados por las locomotoras que entran y salen.

**34.** *Los cimientos* de ambos, el del asiento ó descanso central y pared circular son usualmente de concreto; y por supuesto que deben construirse con cuidado (especialmente el asiento central) en vista del servicio fuerte a que están sometidos. El asiento central comunmente se cubre con un sillar. Cuando el fondo de roca no es accesible se meten pilotes debajo del asiento central, bajo el carril circular y bajo la pared del parapeto. Los que van bajo el asiento central son generalmente cuadrados, y de 4 a 7 o pilotes por lado cubriendo un area de 3,6 × 3,6 á 5 × 5 ms.

**35.** *La pared del circulo* es por lo regular de concreto (rara vez de madera), con remate de madera para soportar los extremos de las carrileras de las vías, y traviesas radiales de madera para soportar el carril circular. Cuando los carriles de las vias o el carril circular descansan directamente sobre el concreto, este último puede romperse con facilidad. La pared circular es generalmente de (1,8 a 2,1 m) en su mayor *ancho*, y la pared del parapeto de 0,5 a 0,6 m. Debido a su forma circular actúa como si fuera un arco horizontal.

**36.** *Un nicho* se deja en la pared circular en algun punto, conveniente de acceso al extremo de la mesa, para inspección, etc.

**37.** *Mesas giratorias de madera.* Se usan algunas veces, por motivos de economia, especialmente para uso provisional. En ciertos casos se hacen sin rolletes ó discos en el centro y entonces, al girar descansan sobre las ruedas del extremo ó sobre una serie de ruedas dispuestas en un círculo no distante del centro. En el último caso, la carga viva descansa parcialmente sobre las ruedas del extremo cuando entra ó sale de la mesa. Generalmente se necesitan dos hombres con un mecanismo de cigüeña para hacerlas girar. La economia hecha así, en el costo primitivo, puede hacer mayor el costo de reparacion ó sostenimiento.

**38.** *Fosa de desagüe.* Es siempre muy importante sobre todo con las fosas profundas que son necesarias para las locomotoras modernas pesadas. El agua en las fosas oxida las chumaceras y obliga a parar los trabajos para levantar con gatos para poder limpiar y aceitar. El agua congelándose en la fosa, puede paralizar completamente las operaciones. Donde el fondo de la fosa sea demasiado bajo para efectuar un desagüe conveniente, puede hacerse menos profunda por medio de uno u otro de los sistemas mencionados en ¶ ¶ 20-22.

**39.** Figs 7 y 8. * El centro es en síntesis una caja de acero circular, la tapa de la cual (que lleva los marcos) descansa sobre ella generalmente por medio de rolletes

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cónicos, collar de bolas, o discos, de acero endurecido; *los rolletes cónicos*, fig. 7, son los que más se usan. Las mesas mejor hechas, están sostenidos en su lugar en general y radialmente por » anilos vivos (*live-rings*) que los rodean en su parte interior y parte exterior, y separados de ellos por collares de bolas o arandelas sin fricción. Los rolletes cónicos son comunmente de 17 a 30 cm de largo, y de 10 a 20 cm de diám en su extremo mayor. Generalmente descansan, arriba y abajo sobre *placas de via* anulares y delgadas relativamente, las que, cuando se gastan pueden cambiarse, dejando la caja y la tapa intacta. El trazado del centro se deja a menudo a juicio del fabrciante.

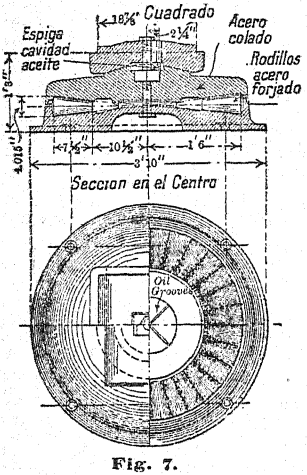


Fig. 7.

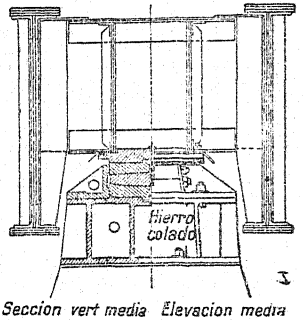


Fig. 8.

(18 7/8" = 47,94 cm; 2 1/4" = 5,71 cm; 1'3" = 38,12 cm; 4,015" = 10,31 cm; 7 ½" = 19,05 cm; 10 ½" = 26,67 cm; 1'6" = 45,74 cm.)

**40.** No obstante el uso con éxito de los *centros de disco*, fig. 8, en puentes giratorios, donde soportan cargas mas pesadas, no son aun de uso comun para las mesas giratorias. La comision * recomienda que sean estudiados seriamente.

**41.** Centros « Hydraulicos » (se usa aceite o glicerina en lugar de agua debido al temor de que se congele) se han indicado. Serían de fácil ajuste en su altura.

**42.** Los centros ó espigas son *muy importantes*. Deben usarse los mejores que puedan obtenerse. Tres cuartas partes de los F. C. usan rolletes cónicos o chumaceras de bolas, de tipos fijados por los manufactureros. Muchas de las dificultades anteriores con las chumaceras de rolletes, parece que se han debido al diseño defectuoso y a su tamaño pequeño, con el consiguiente descuido en su mantenimiento.

**43.** *El mecanismo debe aceitarse* cuando menos anualmente, y *anegarlo* tan a menudo como sea necesario, ó cuando la mesa se pone dura para girarla. Los marcos, o bastidores, etc., deben pintarse con frecuencia. *Para facilitar la inspección*, reparaciones, etc., la mesa se puede levantar con gatos; estos descansan sobre dos asientos de concreto, colocados diametralmente y levantados por medio de grampas de acero, remachadas á la mesa.

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**44.** El costo de las mesas giratorias varía mucho con muchos factores, pero los siguientes pueden tomarse como aproximados. Costo de la cubierta de acero de la *mesa giratoria completa*, con tractor, incluyendo fosa, etc., $100 por pie lineal; mesas giratorias continuas $150. Mesas giratorias de madera pueden costar tan solo como $15 o $20 por pie (30,5 cm) lineal. *Fosa*, forrada, incluyendo cimiento (del estribo central desde $0,50 a $1.30 por pie cuadrado ($5,4 a $14,00 por métro cuad); *pavimentado* solamente a menudo se omite), desde $1,70 á $2,7 por m cuadrado.

**45.** Triangulos de via. Véase desvios, ¶ ¶ 18, 19, pag. 900.

## Patios de clasificación.

**46.** El objecto principal del patio de clasificacion es recibir, desde uno ó mas puntos ó lineas, como *A*, *B*, *C*, *D*, etc., trenes en cada uno de los cuales, puede haber carros para cualquiera ó todos los otros puntos ó lineas, como *M*, *N*, *O*, *P*, etc., y así disponer los carros para preparar trenes nuevos, cuyos carros pueden ir directamente (ó con un mínimo de maniobras) a sus destinos respectivos, *M*, *N*, *O*, *P*.

**47.** Otros trabajos de menor importancia son la reclasificación similar (ó «Transferencia») del *contenido* de los carros, y las renovaciones generales y reparaciones.

**48. Empuje y tiro.** Cuando la gravedad sola no se puede utilizar para dar los cortes a los carros (¶ 50), el movimiento de estos debe hacerse con locomotora. A los carros se les puede dar un *empuje* por la loco, y entonces se deja que vayan adonde se quiera y, algunas veces ayudados por una ligera pendiente. Deben estar gobernados por los retranqueros o guardafrenos. También los carros pueden arrastrarse o *tirarse* « haciendo cortes corridos », asi : con el tren en movimiento, se desengancha la loco y sigue corriendo adelante, a velocidad creciente por un desvio o via desocupada; y el desvio se cambia rápidamente a tiempo para que los carros que vienen entren en el desviadero que se quiera. Esto exige cooperación hábil del personal del tren y del patio. Estos métodos aunque censurados y costosos durante la operacion, son no obstante, con frecuencia mas económicos en patios pequeños o de poca importancia, que cualquiera de los métodos siguientes, que son mas costosos para instalarlos y mantenerlos, pero que remuneran bien su costo en patios grandes de mucho tráfico.

**49. Corte con Pértigas.** Por medio de pértigas gruesas, de dos a tres veces el largo del espacio que hay entre dos carros, la loco (algunas veces provista con un carro construido especialmente), y corriendo hacia adelante y hacia atrás en una via paralela) empuja desprendiendo del tren uno ó mas carros cada vez y los manda bajo su propio impulso por los cambios a su destino.

**50.** Patio de clasificacion de **loma ó gravedad.**

En este tipo de patio de clasificacion, el tren se empuja sobre una loma en la via y en ésta se desenganchan los carros, uno ó más a un tiempo, y descienden por gravedad, al lugar que se desee.

**51. Operación.** Un tren entra por las vias de recibo, su loco se desengancha y se manda a una via lateral para proveerse de carbon, etc. Una loco especial de patio viene entonces detrás del tren y lo empuja lentamente hacia adelante, sobre una loma. A medida que un carro, ó juego de carros pasa sobre la loma, se desenganchan. Entonces, sigue por gravedad (generalmente va primero a una báscula de via, en el cual se puede pesar cada carro) y entra en « la via de distribución » que los conduce por desvios a las diferentes vias de clasificación. A medida que cada carro o juego de carros empieza a moverse, puede marcarse con un número o simbolo, indicando su destino, a medida que el carro ó juego de carros desciende por la via de distribución, es anotado por el chuchero* y este le abre el chucho* que le corresponde. Estos carros llevan sus guardafrenos que regulan su velocidad. Las cuestas o lomas y la via de clasificación están dispuestas de tal modo que aseguran el envio de los carros al lugar mas remoto que se desee. El guardafrenos entonces regresa por otro carro o juego de carros. Cuando es hora de preparar un tren desde una o mas vias de clasificacion la « locomotora de camino » retrocede del extremo mas bajo del patio, engancha la coleccion de carros deseada, y entonces sigue al patio de entrega o sigue a la linea principal.

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* Véase *N. del T.*, pág. 858.

**52. Velocidad de la operacion.** Segun Mr C. L. Bardo, Jour N. Y. R. R. Club, 1903, Dic, un tren de 60 carros, con 50 cortes necesita 2 horas, empujando y tirando : con pértiga, 1 hora, 15 mins; por lomas 30 mins.

**53. Las pendientes** de las lomas en los patios, dependen sobre todo del promedio y máxima resistencia de los carros (generalmente mayor en los carros vacios que en los cargados) por unidad de peso; que depende mucho a su vez del tiempo que los carros han estado parados; de la temperatura (la resistencia es mayor en tiempo frio); de la direccion del viento; de su velocidad máxima probable; del estado de la vía; de las curvas, etc. Vease Resistencia del tren, pp. 1129.

**54.** En la siguiente tabla, la primera linea dá las recomendaciones del manual de la Asen Am Ings de F. C. de 1911. Las otras (de la revista Railway Age Gazette, 1912 Aug 9, pp. 236-9) dán los valores max, promedio, y min informado por Mr. Shelby Saufley Roberts, y representa cerca de treinta patios de lomas ó por simple gravedad. Las pendientes están dadas en ms por cada 100 metros.

| | Primera pendiente desde la cumbre. | Via de distribucion | Patio de clasificacion. |
|---|---|---|---|
| | — | — | — |
| Asen Am Ings F. C..... | 3.00 | 1.00 | 0.50 |
| S. S. Roberts Máxima... | 4.00 | 1.75 | 1.00 |
| — Promedio.. | 2.66 | 0.97 | 0.30 |
| — Mínimo... | 1.00 | 0.50 | 0.00 |

**55.** *Cambios debidos a las estaciones.* Con el objeto de compensar las marcadas diferencias en resistencias, debidas a la temperatura, las pendientes de las lomas o planos inclinados se cambian algunas veces, modificandolas dos veces al año o disponiendo dos lomas una al lado de la otra : una más pendiente para usarla en invierno, y una mas suave para el verano; pero tambien la loma de mas pendiente se puede usar con ventaja en verano para los carros vacios en recorridos dificiles. Véase en el próximo ¶ otro método.

**56.** La *loma mecánica*, trazada por Mr. A. W. Epright, inspector de básculas del F. C. de Pennsylvania consiste esencialmente en un puente formado de dos luces de vigas cortas, el soporte central o del medio, puede ser bajado o levantado por medio de gatos, y entonces se asegura a la altura adecuada con bloques. En uno instalado en el enlace de West Brownsville, Div de Monongahela, cada luz es de cerca de 6 ms, el movimiento del soporte central es de cerca de 20 cm.

**57.** Después de haber pasado la pendiente, es corriente hacer pasar el carro por una

**58. Báscula de via.** Hasta hace poco, muy poca atención se daba al diseño de básculas de via porque han sido mas bien aparatos delicados propensos á descomponerse y á dar lecturas erroneas. No obstante, ultimamente se han producido aparatos mucho mas satisfactorios. Son esencialmente aparatos de grandes dimenciones para pesar grandes masas, en cuyas plataformas se instala la via.

**59.** *Las dificultades* encontradas se han puesto en lista por la comision de la Asen Am de Puentes y Edificios de F. C. como sigue :

Descuido del pesador; equilibrio impropio, otro carro descansando en parte sobre la báscula.

Miscelanea : plataforma trabada, carriles trabados, soportes y piezas rotas, soportes embotados, puntos de apoyo sostenidos en los angulares, espacio insuficiente entre los pies y maderas de la báscula, palancas fuera de linea o flojas, y materias extrañas en fricción con las palancas ó partes de la báscula. Vease tambien ¶ 63.

Muchas de estas dificultades son originalmente debidas á fundaciones insuficientemente firmes.

**60.** *Carriles muertos.* Para evitar el desgaste innecesario de las básculas las locos y carros que no necesiten pesarse pueden, cuando se aproximan a la báscula enchucharse y pasar la fosa de la báscula sobre un par de « carriles muertos » tendidos paralelamente a los carriles de pesar y separados unos 15 cm de ellos, pero, soportados rigidamente sobre postes que pasan por la báscula sin tocarla y descansan sobre los cimientos de la fosa de la báscula; pero las básculas modernas son suficientemente fuertes para soportar todo el tráfico sin dañarse; y los carriles muertos están en desuso, todas las locos y carros pasan sobre las vias de pesar. Los carriles

muertos hacen peligrar la vida del personal de vía, que pueden cogerse los pies entre los carriles de pesar y los muertos, y sus soportes obstruyen el acceso al mecanismo de la báscula.

**61.** « *Carriles-Puente* ». Los carriles de pesar están conectados, algunas veces con los carriles fijos, en cada extremo de la báscula, por pequeños tramos cortos de carriles embisagrados. Esto no solamente evita el golpe causado cuando una rueda salta el espacio entre los extremos del carril fijo y los carriles de pesar, sino que transfiere mas gradualmente la carga a la báscula.

**62.** *Accesorio para desconectar.* El F. C. de Pennsylvania ha empleado un sistema patentado por Mr. A. W. Epright, que consiste en unas coyunturas y pistones, operados por aire o agua, regulados por el pesador usando una válvula de tres pasos, por medio del cual, en un segundo, en el intervalo del paso de dos carros cualquiera, la carga de la báscula y de cualquier carro que siga puede a voluntad quitarse por medio de las coyunturas, o volver a ponerla en la escala de las palancas de la báscula.

**63.** *Trabada de las plataformas.* Esta ha sido causa de frecuentes dificultades y pudiera evitarse haciendo una construccion cuidadosa, evitando que se deforme hacia adentro (a consecuencia de las heladas) de los costados de la fosa; a las caras opuestas se les hace un declive mas ancho abajo que arriba, de modo que los objetos que caigan no se acuñen y se tranquen. Los extremos del carril, aun cuando estén asegurados contra el deslizamiento son propensos a trabarse, bajo los cambios de temperatura; esto puede evitarse insertando puntas o agujas de chucho en los carriles cerca de los extremos de la báscula.

**64.** El *largo de la báscula*, si cada carro que se va a pasar se va a parar en la báscula, debe ser cerca de la longitud del carro que sea mas largo. Si los carros se van a pesar en movimiento, cuando pasen por la báscula, el largo de la báscula debe ser cerca de un tercio mayor.

**65.** *Pruebas.* Las básculas deben probarse periodicamente, digamos cada dos meses, corriendole por encima y parandolos en varios puntos, carros especiales, llamados de prueba, cargados con pesos conocidos. Es preferible que los pesos se puedan variar facilmente con el objeto de ver si las indicaciones de la báscula son exacta y verdaderamente proporcionales a la carga. Muchos ferrocarriles tienen carros especiales con este objeto.

**66.** Es necesario el *desagüe, calefacción* y *alumbrados* de la fosa de la báscula para evitar los perjuicios del agua y las heladas, y para facilitar la inspección y ajuste.

**67.** *Velocidades.* La velocidad de los carros sobre las básculas mientras se pesan, pueden variar entre cero y 9 o 13 kilómetros/hora; cerca de 7 kilóm/hora es lo usual. Tambien se pueden pesar varios carros por minuto; pero el tiempo que se requiere por carro, puede variar, desde 8 segundos hasta varios minutos, aunque aquella rapidez rara vez puede sostenerse.

**68. Clasificación.** A medida que los carros se desconectan del tren, y cuando pasan por la pendiente, los chucheros deben saber para qué linea está destinado cada grupo de carros que se desconecta, con el objeto de que puedan cambiar los chuchos (1) adecuadamente; y el retranquero o guardafrenos de cada grupo de carros debe saberlo también, para que pueda regular la velocidad al pasar por el chucho, y al tener que enganchar con algunos carros mas delante. Algunas veces el lugar a que van destinados se escribe con yeso sobre el cuerpo del carro, otras veces (especialmente por la noche) el guardafrenos informa al chuchero por medio de señales arbitrarias.

**69.** La « *lista de cortes* » recomendada por el Suplemento de 1913 de la Asen Am Ings F. C., Manual de 1911 la forman dos o mas listas duplicadas, dando (1) el número (1.°, 2.°, 3.°, etc.) del corte (2) el número de la via al cual se destina el grupo de carros, y (3) el número de carros en el grupo. Una copia de la lista se le da a cada chuchero interesado y (si se cree necesario) una copia al guardafrenos de cada grupo de carros.

**70.** *Los chuchos* (1) *de patio* (Asen Am Ings F. C. Manual 1911)p ara vias de conexión deben tener *ranas* no mas agudas que el n.° 8. Véase tambien ¶ 57 (Enlaces) en ***Desvios en escala,*** pág. 947.

(1) Véase *N. del T.*, pág. 858.

**71.** *Cambios.* La manipulación de cada cambio (en el patio), por una palanca colocada en el cambio, es barata en su instalación, pero costosa en su manipulación y deficiente en un patio grande, si se compara con el gobierno de numerosos cambios desde una sola torre por medio del equipo usual de cambios y señales. Véase Señales, pags 1059, etc., etc.

**72.** *Compensación de curvas.* Como las pendientes se disminuyen en las curvas, para facilitar las subidas, del mismo modo se aumentan en las curvas de los patios donde va a obrar solo la gravedad, para ayudar a pasar los carros por las curvas. Mr. H. M. North, del F. C. L. S. & M. S., recomienda un aumento de 1/2000 por cada grado de agudeza.

**73. Vias de enlace** son las que contienen los cambios que llevan las distintas vias de clasificación (¶ 74). En patios pequeños, una via de enlace es suficiente : generalmente se desperdicia espacio cuando se sacan mas de 8 o 10 vias de un enlace, y entonces es conveniente usar dos o mas enlaces. Vease tambien ¶ 57, pag. 947.

**74. Vias de clasificación.** Son vias largas paralelas, a las cuales se corren los carros que vienen de los enlaces. En general, cada via de clasificación representa una ruta diferente o diferente destino o dif objeto de tren. Asi, la via n.° 1 puede usarse para carros destinados al F. C. M., via n.° 2 para carros del F. C. N., y asi las demas; o la via n.° 1 para carros destinados a la estacion *P*, la via n.° 2 para la estacion *Q*, etc. y puede por consiguiente formarse un tren tomando carros sucesivamente de la via 1, 2, etc., en tal orden que puedan desconectarse en las estaciones *P*, *Q*, etc., a medida que el tren sigue, y sin dar cortes dobles : o la via n.° 1 puede reservarse para un tren regular o diario, que sale a una hora determinada; la via n.° 2 para un tren especial; via n.° 3 para vacios que van a devolverse cuando se hayan acumulado bastantes para formar un tren, etc., etc.

**75. Los patios de clasificación secundaria** se usan cuando la clasificación deseada, especialmente con respecto al orden de las estaciones, no se puede lograr en el patio principal de clasificación. Tales patios se colocan convenientemente fuera del patio de clasificación principal, y puede tener una « pendiente » para su uso. Estos patios tambien son llamados « patios de reclasificacion » o « patios de agrupacion ».

**76. Tamaño** de las vias de estos patios de clasificación. Ninguna via necesita ser mas larga que el tren mas largo que haya que preparar.

**77. Separación** de vias de patio. Recomienda la Asen Am de Ings de F. C., Manual, 1915; mínimo de centro a centro para vias maestras (vias principales de clasificación de patio) 4 a 4,20 m; via de enlace y primer cuerpo de via, 4,60 m de cualquier via adyacente. El espacio mínimo debe permitir que los hombres vean las señales y evitar que choquen con postes de luz eléctrica, etc.

**78. Recorrido de carros.** Un « corte « rara vez contiene mas de cinco carros, aun cuando mas de cinco carros esten destinados para el mismo lugar. Cada corte está (o debe estar) regulado por un guardafreno, desde la pendiente hasta cerca de su destino. El regreso de los hombres a la pendiente representa fatiga y pérdida de tiempo si caminan, y gasto si son llevados. Su transporte puede efectuarse por medio de una loco vieja ligera con un carro, o con un carro de inspección, de gasolina o eléctrico. Se ha pensado en aceras movibles.

## Patios de Entrega.

**79.** El patio de entrega se coloca fuera de los patios de clasificación. Cada una de sus vias debe ser tan larga como el tren mas largo. A menudo dos o mas trenes pueden acomodarse en una via, especialmente si se provee de enlaces. Deben proveerse tubos de aire para probar los frenos mientras se engancha la loco y el carro

del personal. *Las vias de tránsito*, por el patio ó al costado del patio, que permiten movimiento rápido y libre de locos y carros del personal; son muy necesarias y deben construirse.

### Iluminación.

**80.** La Asen Am de Ings de F. C., Manual, 1911, recomienda, para las vias de pendiente y de enlace en los patios lamparas de arco de 2.000 bujias cada una, a 9 ms o mas sobre el terreno, y separadas de 42 a 45 ms. Otras autoridades recomiendan luces de menos bujias, con menos separación. El uso de reflectores o lentes, y pantallas, etc., para dirigir la luz eficazmente requiere un estudio mas cuidadoso que el que hasta ahora se ha hecho.

**81.** Cuando el cuerpo principal de vias no esta iluminado, puede dejarse una luz detras de cada colección de carros, y correrla detras de cada nuevo « corte » que se haga.

### Estaciones de carga. *

**82. Estaciones de transbordo.** Como los patios de clasificacion sirven para preparar y distribuir los carros en los trenes, asi la estación de transbordo sirve primeramente para la distribución del contenido de los carros entre los distintos carros, transbordándolos desde un carro cualquiera a otros, sobre andenes o plataformas adyacentes a las vias en que corren los carros. Las plataformas se colocan con preferencia a nivel con el piso de los carros cubiertos, con objeto de que las carretillas puedan pasar facilmente sobre tablones colocados en el espacio entre el carro y el anden. Deben haber almacenes (cubiertos es mejor), sobre los andenes para recibir las mercancias para las cuales no haya carros dispuestos.

**83. Manipulación mecánica.** El uso de carritos eléctricos, plataformas viajeras, o transporte por cable, etc., es económico, en el transbordo y trabajo de carga, en estaciones grandes. « Donde haya que transbordar grandes cantidades de carga, se recomienda el uso de plataformas movibles con fuerza motriz y cubiertas. « (Asen de Ings de F. C. Manual de 1911, p. 401.)

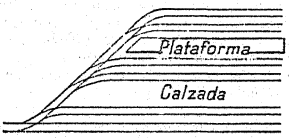


Fig. 9.

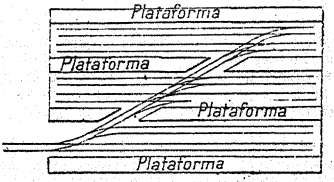


Fig. 10.

**84. Estaciones y Patios de carga.** El trazado de estas vias es simplemente el de una serie de vias paralelas, en pares, que entran en uno o mas vias de la linea principal del patio de clasificacion, con espacios suficientes entre ellas, para carretones o para andenes, como en la fig. 9; los patios son abiertos y las estaciones por lo regular cubiertas. Para economizar espacio, la via principal de tráfico, atraviesa a menudo diagonalmente el edificio, y las vias de cargar se sacan como ramales en ambas direcciones algo parecida a lo que se indica en la fig. 10. « Cuando se pueda los almacenes de recibo de carga deben tener un piso de 15 ms de ancho. Los de salida 7,50 ms. » (Asen Am de Ings de F. C.)

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* La Asen Am de Ings de F. C. define una *terminal* como « un conjunto de facilidades provistas por un ferrocarril en un término ó en puntos intermedios de su linea con el objeto de preparar o armar y separar trenes ». Para evitar confusión no obstante, nos abstenemos de usar la palabra « Terminal » (excepto en el sentido que se indica abajo), y hemos usado « *Estación* ». Una estación que no sea una estación *de cruce* la llamamos un « Términus » (plural « Termini »), o una « Estación Terminal ».

**85. Aparatos modernos para manejar cargas.** Estos permiten economizar espacio haciendo posible construir estaciones de dos o mas pisos; un piso para recibo de carga, otro para salida, y otros para almacenaje, etc.; la carga se transfiere entre los pisos, por medio de plataformas movibles, conductores de barriles, conductores de correa, elevadores, y (para los descensos) conductos en espiral o rectos para piezas que no son frágiles, todo esto ademas de los aparatos de manejar cargas que se mencionan en el ¶ 83.

El patio de cargas debe tener una grua viajera para piezas mui pesadas; y aparatos especiales (como conductores de cubo para carbon, granos, etc.) donde haya que mover mercancias de diversas formas, pesos y condiciones.

**86. Patios para entregas por carretones.** La Asen Am Igns F. C. (Manual 1911, p. 398) recomienda vias cortas por pares de 3,60 ms de centro a centro de vias, y si es practicable, no menos de 9 ms de centro a centro de los pares; vias de no mas de 20 carros de capacidad; entrada y salida para carretones en cada extremo del camino; gruas de fuerza motriz, básculas para carretones y para la via ferrea.

## Estaciones de Pasajeros *.

**87. Generalidades.** Algunos de los puntos importantes que se deben observar en el trazado de las estaciones son : que el pasaje, equipaje y expreso se maneje tanto a la entrada como a la salida con un mínimo de atraso y confusion, y en la terminales *, sacar la loco fuera, virarla, proveerla de agua y carbón, y reengancharla al tren; tambien hacer la limpieza de carros, etc. Alguna parte de la limpieza de los carros puede hacerse en la misma estación.

**88. Plataformas.** Para hacer expedito el movimiento de pasajeros y equipaje, se recomienda por la « Comision de Patios y Terminales » de la Asen Am de Ings de F. C., marzo de 1911 que, si no se pueden proveer andenes de equipajes además de los andenes de pasajeros y al mismo nivel, el equipaje puede llevarse de un nivel a otro por medio de ascensores, colocados de tal modo que logren mantener lo mas posible el equipaje fuera de los andenes.

**89. Las rampas,** o pasajes inclinados en sustitución de las escaleras facilitan mucho el movimiento de pasajeros en las estaciones y reducen la posibilidad de accidentes en las horas de mas movimiento. La comisión de Patios y Terminales, Asen Am de Ings de F. C., marzo 1911, recomienda una pendiente que no exceda del 1 %; pero se están usando mayores pendientes con éxito. La superficie debe estar preparada para evitar el peligro de los resbalamientos.

**90. Pasillos de tránsito.** A menudo se puede hacer mucho en el trazado para reducir la congestión en la estación y en los pasajes, evitando el cruce de los pasillos de tránsito de los pasajeros para lo cual se trazan en la mayor extensión posible, paralelos entre si.

**91. Necesidades para el futuro.** Debido a las dificultades y gastos que ocasiona la reconstrucción de una estación grande cuando el tráfico llega a exceder a su capacidad, las necesidades del futuro deben calcularse para 20 años, si es posible, y de acuerdo con esto preparar el diseño de la estación.

**92. Las estaciones pequeñas y numerosas** aventajan a las estaciones grandes y únicas en las ciudades grandes. El manejo, en una sola estación de todo o casi todo el tráfico de pasajeros en una gran ciudad necesita por lo regular, un viaje largo para llegar o salir de la estación, y caminar mucho en la estación necesariamente grande en si misma para llegar a los trenes o salir; y, por eso la construcción de varias estaciones pequeñas en la misma ciudad, a lo largo de la linea ha sido sugerida (por Mr. Fred A. Delano, Pres del F. C. Wabash), aun cuando esto obligue a invertir un tiempo adicional en las paradas de los trenes expresos.

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* Véase nota *, ¶ 81.

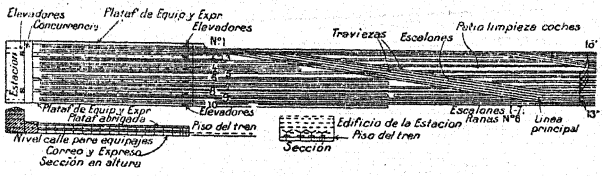


**Fig. 11.**

**93. Plan.** La fig. 11 es un trazado para una estacion terminal de pasajeros tal, como lo recomienda la comisión de la Asen Am de Ings F. C., marzo 1911; y la

**Fig. 12.**

fig. 12 tipo de cruces en la estación. En la estacion terminal, se proponen dos niveles, uno para pasajeros y otro para equipaje y expreso, junto con un sistema comprensivo de vias de enlace, con patio adyacente para limpieza de carros. Esta disposición ideal tiene por supuesto, que alterarse muchas veces debido a restricciones locales. En el plano para estación de cruces, se notará que dos vias principales se dividen en 6 permitiendo dos vias de cruce en el medio (sin desviacion en su curso), y dos vias con anden (con un anden aislado eutre ellos) a cada lado, dando asi amplias facilidades de carga y descarga, sin demorar los trenes que no tengan parada reglamentaria en la estación.

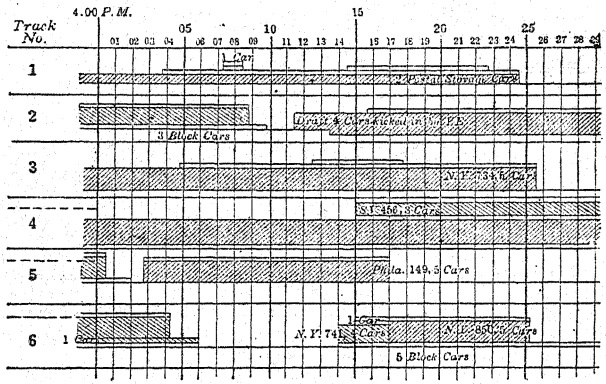


**Fig. 13.**

**94. Estudio.** La fig. 13 muestra parte de un diagrama que registra la ocupacion de varias vias del F. C. de Pennsylvania en la Estación de la calle Broad, en Philadelphia (Mr. E. B. Temple Auxiliar del Ings Jefe) durante un periodo de tiempo daoa.

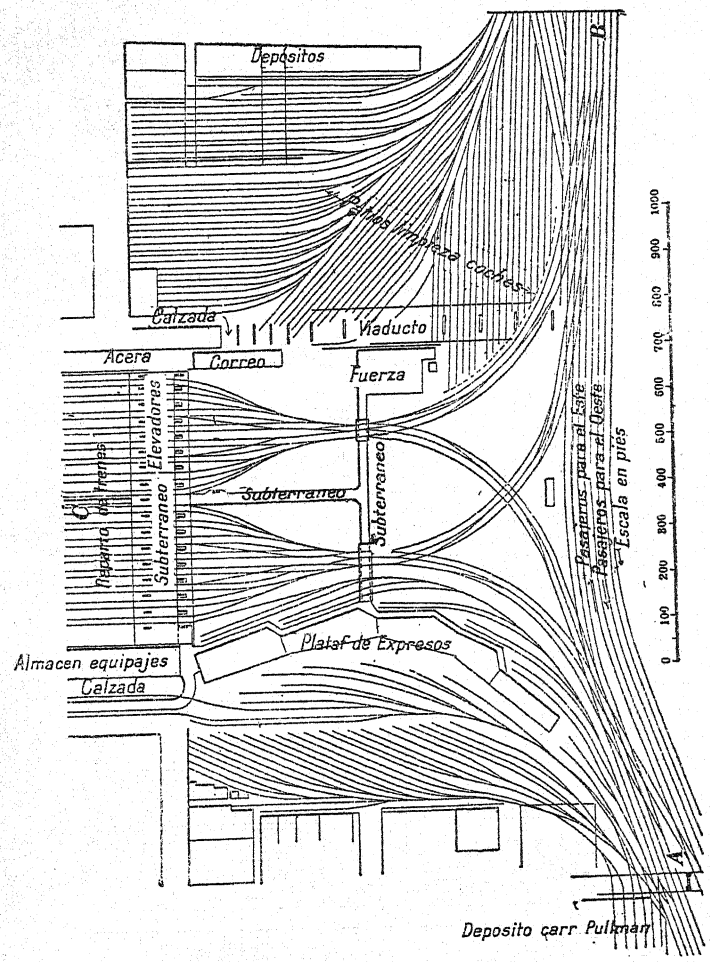


Fig. 14.

(4:00 a 4:30 p m, enero 18, 1912; solamente se enseñan 6 vias). El diagrama entero incluye 16 vias de la estación, y comprende un periodo de tiempo de las 4 a las 6 p m. Diagramas análogos mostraban las condiciones existentes durante el mismo periodo en partes del patio. Las distintas vias estan trazadas en ellos en escala vertical, de extremo a extremo. La extensión vertical, asignada a cada via representa el largo que puede ocupar sin obstruir mas de una via. Los carros y

loco están marcados con areas rectangulares. Las *dimensiones* verticales de los rectángulos representan los largos de la via ocupada por carros, etc; las *posiciones* verticales de los rectángulos representan las *posiciones* de los carros, etc., en las distintas vias; y sus *dimensiones* y posiciones horizontales representan los periodos durante los cuales las distintas vias estuvieron asi ocupadas. En la práctica, los trenes que llegan y salen se distinguen por los diversos modos de dar sombra en el dibujo a estos rectángulos; y las clases de carros y loco por iniciales, como L. C. para « locos de camino ». Diagramas análogos se han usado en los F. C. del Estado Belga en y la estación Camden del F. C. de Baltimore & Ohio.

**95**. La fig. 14, p. 1086, representa una *estación* y muestra el *retroceso* de un tren con rumbo a la estación que viniendo por *A*, sigue primero a *B*, y entonces retrocede a la estacion por ej en *C*, permitiendo asi que la loco sea rápidamente sustituida.

**96. Velocidad en el manejo de trenes.** Dicha velocidad en las terminales depende sobre todo de las facilidades realizadas para el rápido manejo de locos, pasajeros, y equipajes y para limpiar carros; tambien de la naturaleza del tráfico y de la fuerza motriz. Comparando 26 estaciones grandes de pasajeros Americanos, la Comision de Patios y Terminales, de la Asen Am de Ings de F. C. (Eng New 1911, abril 6, p. 414), encontró que el número de trenes manejados actualmente durante la hora de mas tráfico, varian de 1 a 3,5 como promedio por via (incluyendo movimiento en *todas* las vias), y desde 2 a 8 en la via de *mas tráfico;* mientras que por calculos hechos debian variar de 2 a 12. Algunas de las cifras altas fueron alcanzadas en estaciones de cruce.

**97. Operación de trenes con electricidad.** Asi se facilita mucho su rápido movimiento en las terminales, tanto mas que la fuerza motriz no necesita salir de la estación, o cambiar su posicion, para invertir la loco o el tren, o para aprovisionarse. Las vias y patios quedan de este modo en su totalidad libres para movimientos completos de trenes.

**98. Terminal de tránsito rápido.** Se disponen con frecuencia con una o mas *vias de volteo*, alrededor de las cuales los trenes pueden correr sin invertir la locomotora o el motorista. Esto es practicable a menudo con equipo de tránsito rápido, que pueda correr fácilmente en curvas agudas (fuertes) y es por tanto innecesario invertir mucho terreno para construir la via de volteo.

# ESTACIONES DE AGUA

**Las estaciones de agua** son ciertos puntos de la línea de un ferrocarril en donde las máquinas se paran á tomar agua. **La distancia de una á otra** varía (lo mismo que las estaciones ó depósitos de combustibles, que están junto de aquéllas), más ó menos, de 9 ó 10 kilómetros en las líneas de gran tráfico, á 24 ó 32 km sobre las que corren pocos trenes. Depende en mucho, sin embargo, de los lugares donde se consiga el agua. Algunas veces hay que conducirla en tubos desde 3 á 5 km ó más. El objeto de tenerlas próximas unas de otras es para evitar retardos cuando muchas máquinas están obligadas á tomar agua en la misma estación. Para evitar interrupciones en el tráfico, se colocan, á menudo, al lado de un desvío. En las estaciones se guardan grandes cantidades de agua, generalmente en tinas grandes de madera ó estanques, abrigados *en casas de estanques*. La casa del estanque se sitúa cerca de la línea, dejando solamente un espacio de .60 á 1.20 m libre para los carros.

Es de 2 pisos; se coloca el estanque en el piso superior, con su fondo de 3 á 3.60 m más ó menos, sobre carriles. En el piso bajo se halla generalmente la bomba para elevar el agua al estanque, y una estufa para impedir que el agua se congele en el invierno *.

Los estanques son generalmente de forma circular y de diámetro algunas pulgadas mayor en el fondo que en la parte superior para que se aprieten más los aros. **Su capacidad** varía generalmente de 23 á 150 m cúb (raras veces 300 m cúb ó más) según el número de máquinas que toman agua.

**El estanque de un tender** contiene de 11 á 26 m cúb; **y una máquina evapora ó gasta** de 50 á 350 lit por km, según la clase de máquina, peso del tren, pendientes, etc. Quizás 90 lit es un término medio regular para máquinas de trenes de pasajeros, y 180 para los de carga. La tabla que sigue da **la capacidad de estanques de diferentes diámetros** interiores, y profundidades de agua.

| Diámetro. | Profundidad. | Capacidad. | Diámetro. | Profundidad. | Capacidad. |
|---|---|---|---|---|---|
| Metros. | Metros cúb. | Metros. | Metros. | Metros. | Metros cúb. |
| 3.66 | 2.44 | 25.613 | 7.32 | 3.66 | 153.697 |
| 4.27 | 2.74 | 39.224 | 7.92 | 3.96 | 195.412 |
| 4.88 | 2.74 | 51.230 | 8.53 | 4.27 | 244.060 |
| 5.49 | 3.05 | 72.043 | 9.14 | 4.57 | 300.188 |
| 6.10 | 3.05 | 88.943 | 9.75 | 4.88 | 364.317 |
| 6.71 | 3.35 | 118.383 | 10.36 | 5.18 | 436.982 |

**La madera de ciprés ó de cualquier clase de pino** sirve muy bien para estanques. Las duelas pueden ser de un espesor de 6 cm para los estanques pequeños y de 10 á 12 cm para los grandes. A los fondos puede dárseles el mismo espesor. Las duelas deben acepillarse en máquinas para ajustarlas exactamente á las curvas. Entonces no es preciso interponer nada entre ellas para que cierren. Una espiga ó diente sencillo se inserta entre cada dos duelas, cerca de su parte superior, para mantenerlas en su puesto mientras se están armando. El fondo se ensambla y se coloca simplemente dentro de un canal hecho con mucha exactitud, como de 2.5 cm de profundidad, en la circunferencia interior de la tina, á algunos centímetros del fondo de las duelas.

Un estanque de 6 m de diámetro y 3.60 m de profundidad debe tener 9 aros de buen hierro, colocados algunos cms más próximos unos de otros en el fondo que en la parte superior del estanque.

El ancho de los aros 7½ cm, y el espesor de los dos más bajos de 6 mm, dismi-

* Una casa de estanque de armadura (madera) de 5.4 m en cuadro, con cimientos de mampostería para la casa y el estanque, cuesta de $400 á $600. Una de ladrillos ó mampostería cuesta algo más.

nuyendo de aquí gradualmente hasta el más alto, que tiene sólo la mitad del espesor. Los dos más bajos se colocan pegados uno al lado del otro. Estas dimensiones permiten la ejecución de los agujeros de remaches para remachar las extremidades sobrepuestas y resistir á los esfuerzos regulares que se ejercen al apretar los aros para llevarlos á su lugar *. Tres remaches de 12 mm de diámetro, colocados á 7½ cm de distancia entre sí, en línea, son suficientes para el empate de un aro inferior. Un estanque de 10.36 m de diámetro y 5.12 m de profundidad, puede tener 12 aros, los inferiores de 10 cm × 13 mm, con 3 remaches de 2 cm para la junta baja del fondo.

Los tablones del fondo deben descansar firmemente sobre sus vigas de apoyo.

Un estanque debe tener un **tubo de entrada** por el cual el agua penetra; un **tubo de desagüe** para evitar el derrame, y un **tubo de descarga** de 18 á 20 cm de diámetro en, ó cerca del fondo, por el cual pasa el agua al tender. La extremidad interior del tubo de descarga está provista de una válvula que abre el maquinista á voluntad por medio de una cuerda y una palanca exterior.

A su extremidad exterior se fija, generalmente, un tubo flexible de lona de 18 á 20 cm de diámetro, y de 2.40 á 3 m de largo, por el cual el agua entre en el depósito de agua del tender. O en lugar de una manguera puede el tubo de descarga prolongarse con un tubo metálico, suficientemente largo para que llegue hasta el tender, dispuesto de tal modo que pueda echarse hacia un lado cuando no esté en uso, ó enviarle á una posición vertical para que no estorbe á los trenes que pasan. El mismo estanque puede alimentar dos locomotoras que estén sobre diferentes vías, al mismo tiempo. Los estanques son muy durables.

**El estanque de patente á prueba de heladas de John Burnham,** Batavia, Illinois, es simplemente un estanque ordinario en el cual se impide que el agua se congele : 1.º, por medio de un techo circular que protege á un cielo raso de planchas, entre las cuales hay una capa de mortero; 2.º, por un espacio de aire obtenido por un método igual debajo de las maderas ó vigas sobre las cuales descansa el estanque. Aunque los lados están sin protección alguna, no necesitan casa, sino solamente postes y vigas fuertes sobre unos cimientos de mampostería, para sostener el estanque. Los tubos surtidores están en cajas hechas de madera y papel alquitranado.

**Los estanques se hacen frecuentemente rectangulares** con lados verticales de postes forrados con tablas y ligados en ambas direcciones por medio de barras de hierro. Son más expuestos á filtraciones que los estanques circulares. También se han hecho de hierro, pero parece que se prefieren los de madera.

**El agua para surtir los estanques** puede elevarse con bombas de mano, de vapor, fuerza animal, viento, ariete hidráulico, ó de otro modo, de un río, de un pozo hecho por una represa en un arroyo pequeño é irregular, de una cisterna ó aljibe que se halla debajo del estanque, ó de una fuente ordinaria. Muchas líneas con un tráfico de 10 á 12 trenes diarios en cada dirección, se surten de aljibes y con bombas de mano; generalmente dos hombres para una bomba.

Las líneas de un tráfico grande, cuando no se pueden surtir de agua con caída natural, generalmente emplean fuerza de vapor. **El molino de viento** ** es la fuerza más económica, y cuando está bien construído se descompone muy poco. Por supuesto, que no trabaja cuando hay calma, pero este inconveniente puede evitarse en la mayor parte de los casos, teniendo estanques suficientemente grandes que contengan agua para varios días. Sin embargo, la fuerza de vapor es la más segura.

La tabla que sigue dará una idea de **la fuerza que requiere una máquina de vapor para elevar agua.** Al encargar una máquina no se especifique el número de caballos de fuerza, sino el número de m cúb de agua que debe elevar en un número de horas dadas, á una altura y con una presión de vapor dadas (digamos de 4 á 6 kg por cm cuad). La bomba debe ser suficientemente potente para que no haya que trabajar de noche, y debe ser capaz de ejecutar por lo menos 25 por ciento más de lo que se requiere.

**Un caballo regular** puede, por término medio, elevar en 8 horas las cantidades indicadas en las primeras dos columnas, á la altura de la tercera columna, ó la suficiente para alimentar el número de locomotoras que se expresan en la cuarta columna, con cerca de 7½ m cúb para cada máquina. Dos hombres harían, más ó menos, la tercera parte de este trabajo.

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* Un estanque como éste, armado en su lugar, cuesta $400, más ó menos.

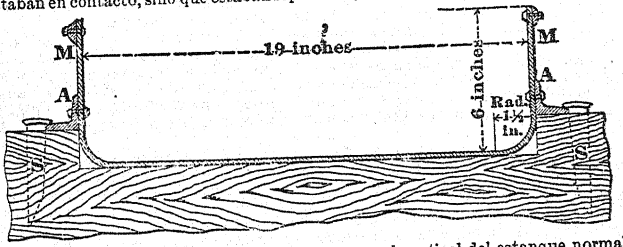
** El costo de un molino de viento sólo para estaciones de ferrocarril varía, más ó menos, de $450 para 5.4 m de diámetro, á $1,500 para 11 m de diám. Precio en fábrica.

| Metros cúbicos. | Kilogramos. | Metros altura. | Locomotoras. | Metros cúbicos. | Kilogramos. | Metros | Locomotoras. |
|---|---|---|---|---|---|---|---|
| 45.31 | 45,310 | 30.48 | 6 | 126.60 | 126,600 | 10.67 | 17 |
| 56.63 | 56,630 | 24.38 | $7^1/_2$ | 151.01 | 151,010 | 9.14 | 20 |
| 75.52 | 75,520 | 18.29 | 10 | 181.23 | 181,230 | 7.62 | 24 |
| 90.61 | 90,610 | 15.24 | 12 | 226.53 | 226,530 | 6.10 | 30 |
| 100.76 | 100,760 | 13.72 | $13^1/_4$ | 302.08 | 302,080 | 4.57 | 40 |
| 113.27 | 113,270 | 12.19 | 15 | 453.10 | 453,100 | 3.05 | 60 |

**Un estanque ó depósito de agua, con un tubo fijo de alimentación,** es preferible á una tina ó depósito ordinario con bomba cuando la localidad lo permite, por estar aquél menos expuesto á descomponerse que esta última, resultando al fin más económico. El depósito se supone que se llena entrando el agua con caída natural, y que tiene el fondo, por lo menos, á 2.40 m sobre los carriles, ó á cualquiera otra elevación que el terreno ó la altura del agua requieran. El depósito puede hacerse en el suelo, revestido con mampostería de ladrillos ó piedras y cemento, con fondo de concreto, ó construirse sobre el terreno, según la localidad. Puede tener un tubo, si se quiere, y puede estar cerca de los carriles, ó á distancia considerable, según las circunstancias *. Desde su fondo se lleva un tubo de hierro de 20 á 30 cm de diámetro (generalmente por debajo de la tierra) hasta cerca de la vía.

En este punto tiene un codo vertical y un tubo de 2.5 á 3 m de altura sobre la vía, formando un **tubo vertical de alimentación, es decir, una columna de agua,** de cuya parte superior sale el agua (por una manguera) como en el caso, del estanque. Varios de estos tubos, ó uno más grueso, pueden colocarse para alimentar dos ó más locomotoras al mismo tiempo, por sendos tubos de alimentación. En el punto en donde el tubo se dobla y se vuelve vertical, hay una válvula para abrirlo y cerrarlo, que puede actuarse por medio de una rueda de mano colocada á una altura que el maquinista pueda alcanzarla fácilmente.

En algunas de las líneas más importantes los ténderes de trenes rápidos **toman el agua á toda marcha por medio de una larga artesa ó largo acueducto ó estanque de canal,** lleno de agua, colocado entre los rieles. Estos estanques tienen generalmente 400 m de longitud. Deben, por supuesto, estar á nivel, y por consiguiente requieren que la vía esté también á nivel. Cuando se introdujeron por primera vez en Inglaterra por Ramsbottom, el estanque ó acueducto era de hierro fundido, en secciones de 1.80 m, más ó menos. Éstas se unían por medio de pernos, que pasaban por rebordes que había en sus extremidades, los cuales no estaban en contacto, sino que estaban separados por una tira de caucho vulcanizado.




Nuestra figura representa una sección transversal vertical del estanque normal. Está construído con planchas de hierro de 5 mm de espesor y 1.57 m de longitud. Las secciones están sobrepuestas 5 cm, dejando 1.52 m de longitud útil. Las planchas están cortadas un poco cónicas, de modo que la extremidad de cada sección sea 5 mm más honda que la otra, y de este modo sus partes superiores están rasas en toda su longitud. Las juntas están remachadas con una hilera doble de remaches de 10 mm, y distantes, más ó menos, 4 cm de centro á centro, y no están en

* Un depósito ó estanque sin techo de 12.25 m y 3.66 m de profundidad, revestido de ladrillos ó de mampostería, cuesta, generalmente, de $2,500 á $3,500, según las circunstancias.

direcciones opuestas uno al otro, sino alternados. En cada extremidad del estanque el fondo está doblado hacia arriba en una longitud de 1.83 m, llegando al nivel de las orillas superiores de los lados. Como se ve, las traviesas tienen un corte para alojar el estanque, el cual está sujeto á ellas flojamente por medio de dos clavos SS, colocados en cada traviesa. Las cabezas de los clavos ajustan sobre los rebordes horizontales de hierro angular de 4×4 cm AA. M y M son molduras de barras de hierro de 3×1.2 cm; los hierros angulares de los rebordes y lás molduras están en secciones de 4.5 m de longitud y remachados á los lados del estanque en toda su extensión.

El cucharón del ténder se **baja** al estanque, y se **levanta** por medio de una palanca situada en la plataforma del fogonero, y no se le deja tocar el fondo del estanque. El estanque entre carriles se surte por medio de tubos de un depósito próximo y lo regula un encargado.

Para impedir que el agua se cóngele en el invierno, se hace pasar vapor de la caldera de la bomba al estanque por tubos de hierro colocados, debajo de la tierra á lo largo en un lado de la vía. Estos tubos están provistos de ramales, que introducen el vapor en el estanque á cada 12 m de longitud. Los tubos de vapor están protegidos por cajones de madera, y provistos de válvulas para regularizar la cantidad de vapor entrante.

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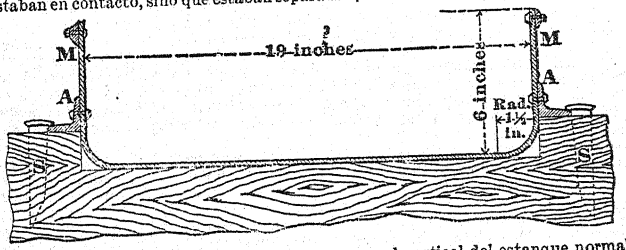


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# COSTO DEL MOVIMIENTO DE TIERRA

**Art. 1.** Es conveniente estipular el pago de esta especie de trabajo por metros cúbicos *de excavación solamente*, en lugar de fijar precios diferentes para la excavación y el terraplén. De este modo nos desembarazamos de algunas dificultades en la cubicación, así como de las controversias y litigios, que se suscitan á menudo al determinar lo que haya de descontarse por el asiento del terraplén.

Es además nuestra opinión que, para proceder con equidad, debe emplearse la práctica inglesa de pagar los trabajadores por metro cúb y no por día. La experiencia demuestra claramente que cuando hay escasez de brazos y los jornales están subidos, apenas se puede conseguir que los hombres hagan las tres cuartas partes del trabajo que hacen cuando los salarios son bajos, y nuevos obreros están esperando ser empleados en caso de que algunos de los viejos se despidan. El autor ha palpado los resultados más satisfactorios empleando el sistema de tareas, acompañado de primas ó remuneraciones amplias para todo trabajo hecho á más de la tarea. De este modo se identifican los intereses de los obreros con los del contratista, y cada uno tiene cuidado de que los otros hagan la parte del trabajo que les corresponde.

El Sr. Ellwood Morris, I. C., de Filadelfia, fué, según creemos, la primera persona que investigó propiamente los elementos que entran en el costo del movimiento de tierra, y quien los redujo á una forma que nos facilita el cálculo del costo total, con un grado considerable de exactitud. Sus resultados se publicaron en el *Journal of the Franklin Institute* en 1841. Esta publicación forma la base sobre la cual, con algunas variaciones, consideraremos la materia; y la que usaremos tanto para el estudio del transporte en carretillas como en carretas. En todo este artículo, al hablar de una unidad cúbica, la consideraremos en su lugar antes de ser excavada para su transporte. No es necesario advertir que todo esto debe considerarse solamente como una aproximación tolerable ó término medio. Como se dijo, los trabajadores ejecutan menos trabajo, cuando los jornales están subidos, y más en el caso contrario. En gran parte depende esto, además, de la habilidad, inspección y energía del contratista y de sus encargados. Es frecuente ver dos contratistas trabajando á los mismos precios y con el mismo material, y estar uno de ellos ganando y el otro perdiendo, por falta de tacto en la distribución propia de las fuerzas, conservando los caminos en buen estado, haciendo conducir bien las carretas y carretillas, etc. Una temporada larga de tiempo lluvioso puede afectar seriamente el costo del movimiento de tierra, haciendo más difícil la aflojada, carga y descarga del material; además de mantener los caminos en mal estado para el tráfico de las carretas.

El costo total de excavar y acarrear la tierra se compone de las partidas siguientes, á saber :

1.ª *Aflojar la tierra hasta que esté en disposición de palearla.*
2.ª *Echarla en las carretillas ó carros por medio de palas.*
3.ª *Acarrearla incluyendo la descarga y el retorno de los carros.*
4.ª *Extender el material en capas sucesivas en el terraplén.*
5.ª *Mantener en buen estado el camino carretero ó la vía de tablones para los carros de mano ó carretillas.*
6.ª *El desgaste, la composición y depreciación de la herramienta y el interés sobre el costo de ella.*
7.ª *Superintendencia y aguadores.*
8.ª *Ganancia del contratista.*

Consideraremos todas estas partidas un poco detalladas, basando nuestros cálculos en la suposición de que el trabajo ordinario cuesta $1 por día de 10 horas

de labor. El resultado de nuestras tablas debe, por consiguiente, aumentarse ó disminuirse más ó menos en la misma proporción en que los jornales ordinarios aumentan ó disminuyen con relación al supuesto *.

**Art. 2. Aflojada de la tierra hasta dejarla en disposición de palearla.** Esto se hace generalmente por medio de arados ó de picos; menos costoso por medio de los primeros. Un arado de dos caballos, con dos hombres que lo manejen, á $1 por día de trabajo, $.75 por día cada caballo, y $.37 por día para el arado; incluyendo aperos, desgaste, reparaciones, etc., ó un total de $3.87, aflojará en terrenos duros ó pisados de 50 á 220 m cúbicos por día, á razón de 2.5 á 1.7 centavos por metro, ó en terreno ordinario de 300 á 450 m cúbicos, á razón de 1.26 á .83 de un centavo por m cúbico. Por consiguiente, podemos suponer comotérmino medio ordinario, que el costo real de aflojar la tierra con el arado es para el contratista el siguiente: En terreno duro y pesado, 2 cent; tierra ordinaria, 1cent; terreno arenoso, liviano, .5 cent por m cúbico: arcilla ó greda pura muy ríg da ó granzón compactado fuertemente, puede calcularse á razón de 3.25 cent; se necesitan de 3 á 4 caballos.

Con picos, el trabajo regular de un día es más ó menos de 0 á m cúb de greda dura ó granzón compacto; 19 m cúb de terreno duro y pesado; 30 m de tierra ordinaria; 45 en terreno arenoso liviano — todos medidos en el terreno mismo antes de excavar; — lo cual da, á razón de $ por día de trabaj o, para greda dura, 9 centavos; terrenos pesados, 5.2 cent; tierra ordinaria, 3.2 cent; terreno arenoso liviano, 2.16 cent por m cúbico. La arena pura no requiere sino poco trabajo para aflojarla, su costo no será más de .6 cent por m.

**Art. 3. Echada de la tierra suelta en las carretas por medio de palas.** La cantidad del material paleado por día depende en parte del peso del material, pero más de la relación que haya entre el número de picadores y de carretas al de paleadores, de modo que estos últimos nunca tengan que esperar ni las carretas ni el material. En las cuadrillas bien organizadas, los paleadores no se ocupan realmente en palear más que seis décimas partes del tiempo, estando desocupados sólo cuatro décimos de dicho tiempo, mientras que, mal organizados, pierden considerablemente más de la mitad del tiempo. Un paleador puede fácilmente echar en una carreta la cuarta parte de un m cúbico medido en su lugar (lo que es más ó menos la carga media de una carreta) de tierra arenosa en 5 minutos; de tierra ordinaria, en 6 minutos; y de cualquiera de las tierras pesadas, en 7 minutos. Esto daría, para un día de 10 horas de trabajo, 120 cargas ó como 30 m cúbicos de arena; 100 cargas ó como 25 m cúbicos de tierra ordinaria; ú 86 cargas ó 22 de las tierras pesadas. Pero de estas cantidades debemos deducir cuatro décimas partes por el tiempo perdido necesariamente; reduciendo de este modo la cantidad de trabajo efectivo á 18 m cúb de tierra liviana arenosa; á 15 m cúb de tierra ordinaria y 13 m cúb de tierra pesada.

Cuando los paleadores hacen menos que esto, es porque hay alguna desorganización en la dirección del trabajo. Suponiendo equitativas estas cantidades, tenemos que, á $1 por jornal, el costo efectivo para el contratista por palear un m cúbico de material, medido en su lugar antes de excavar, será más ó menos: en terrenos arenosos, 5.4 cent; en tierra ordinaria, 6.5 cent; tierras pesadas, arcilla, etc., 7.5 cent por m cúbico.

En la práctica, las carretas no se llenan menos de tierras pesadas que de tierra liviana. Ni hay tampoco necesidad de hacerlo, por cuanto la diferencia que existe entre el peso de una carreta y la cuarta parte de un m cúb de varias tierras es demasiado pequeña para que se tome en cuenta, especialmente si el camino se mantiene en buen estado, como lo mantendrá el contratista que conozca su propio interés. Tampoco es necesario modificar la carga por causa de las *pequeñas inclinaciones* que puedan resultar al dar las pendientes á los caminos. Una carreta de cargar tierra, por sí sola, pesa media tonelada más ó menos.

**Art. 4. Transporte de la tierra, vaciada y regreso.** Un caballo camina, por término medio, más ó menos 4 kilómetros por hora, ó 66 m por minuto; lo que equivale á 33 m de viaje en cada dirección, ó á 33 m de avance, como se acos-

* *N. de T.* — Dejamos el dólar como unidad, en todos los estudios y cálculos de costos que van á seguir, pues es una unidad monetaria muy conocida y además de muy simple relación con las que usan todos los pueblos que hablan español. Véase tabla de monedas, pág. 231.

tumbra decir en el transporte de la tierra*. Además, en cada viaje hay una pérdida como de 4 minutos que se invierten en esperar á que se cargue, en vaciar la carreta, etc. Por tanto, cada viaje durará tantos minutos cuantas porciones de 33 m haya en cada viaje, y 4 minutos más. Por consiguiente, para hallar el número de viajes por día á cualquiera distancia media de transporte, dividamos el número de minutos empleados en un día de trabajo por 4 más el número de porciones de á 33 m contenidas en la distancia total á que debe acarrearse la tierra, es decir:

$$\frac{\text{El número (600) de minutos de un día de trabajo}}{4 + \text{el número de porciones de 33 m comprendidas en la distancia de transporte}} = \text{número de viajes ó carretadas hechos por día y por carreta}$$

y como $^1/_4$ parte de m cúbico, antes de aflojarla, da por término medio una carretada, el número de cargas, dividido por 4, dará el número de m cúbicos transportados por día por cada carreta, y el gasto total de cada carreta por día, dividido por el número de m cúbicos transportados, dará el costo del transporte por m cúbico.

*Nota.* Cuando se transporte piedra suelta, que exige más tiempo para echarla en los carros, tendremos:

$$\frac{\text{Número de minutos (600) en un día de trabajo}}{6 + \text{número de porciones de 33 m comprendidas en la distancia de transporte}} = \text{número de carretadas transportadas por día y por carreta}$$

A distancias comunes de transporte, un carretero puede atender á cuatro carretas, lo cual, á $1 por día, sale á 25 centavos por carreta. Cuando los jornales están á $1, el gasto de un caballo es generalmente 75 centavos; y el de la carreta, incluyendo apero, lubricantes, reparaciones, 25 centavos; lo que da un costo total por día y por carro de $1.25.

El gasto de un caballo es el mismo en domingos y días de lluvia que cuando está trabajando, y estas consideraciones están incluídas en los 75 centavos. Algunos contratistas emplean un número mayor de carreteros, que también ayuden á cargar las carretas, de modo que el costo es más ó menos el mismo en cualquier caso.

Ejemplo: ¿Cuántos metros cúbicos de tierra ordinaria medida en el banqueo puede transportar un caballo con su carreta en un día de 10 horas de trabajo (600 minutos) á la distancia total de 330 m (á 10 porciones de á 33 m), y cuánto costará al contratista el transporte del m cúbico, suponiendo en $1.25 el costo total de carreta y caballo?

$$\text{Tendremos: } \frac{600 \text{ minutos}}{4 + 10 \text{ porciones de á } 33 \text{ m}} = \frac{600}{14} = 43 \text{ carretadas;}$$

$$\text{y } \frac{43 \text{ carretadas}}{4} = 10.75 \text{ m cúbicos;}$$

$$\text{y } \frac{125 \text{ cent}}{10.75 \text{ m cúb}} = 11.6 \text{ centavos por metro cúb.}$$

De este modo se han calculado las columnas segunda y tercera de las tablas que siguen.

**Art. 5. Colocación y arreglo de la tierra sobre el terraplén en capas regulares á nivel.** Un hombre puede extender de 38 á 76 m cúbicos de tierra ordinaria, ó de cualquiera de las tierras más pesadas ó gredas, según el estado de sequedad, etc.; esto, á razón de $1 por día, sale de 2.6 á 1.3 centavos por m cúbico; podemos calcular como término medio 2 cent el costo medio de las tierras de esta clase, mientras que $1\,^1/_2$ cent es suficiente para tierras livianas arenosas. Este gasto se evita cuando la tierra simplemente se echa en la extremidad del terraplén para vaciarla. Sin embargo, debe calcularse como $^1/_2$ cent por m cúb para mantener el lugar donde se vacía, limpio y en buen orden.

**Art. 6. Conservación del camino carretero en buenas condiciones para el tráfico de las carretas.** Los carriles ó canjilones formados por el tráfico de las ruedas y los hoyos deben siempre rellenarse; el agua de lluvia debe sacarse por medio de desagües de poca profundidad y conservarse el camino en buenas condiciones. De otro modo, el trabajo de los caballos y el desgaste de las carretas

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* Cuando todo el material del banqueo se convierte en terraplén, la distancia media de transporte es la distancia entre los centros de gravedad del banqueo y del terraplén.

aumentará mucho. Generalmente se destina tanto por m cúbico para las reparaciones del camino, pero nosotros proponemos que sea $^1/_{10}$ de cent por m cúbico y por 33 m de dist de transporte.

**Art. 7. Desgaste, composición y depreciación de los picos y de las palas.** La experiencia ha demostrado que $^1/_3$ cent más ó menos por m cúb es suficiente para esto.

**Superintendencia y aguadores.** Estos gastos varían según las circunstancias locales; pero convenimos con el Sr. Morris que 2 cent por m cúb es, en circunstancias ordinarias, lo suficiente para cubrir los gastos á este respecto. Puede destinarse, en justicia, $^1/_3$ cent más ó menos por el trabajo extraordinario de desagües laterales, nivelación y aparejo del terraplén conforme á la pendiente, etc. En banqueos pequeños esto puede aumentarse á ½ cent por m cúbico.

**Art. 8. Utilidad del contratista.** Ésta puede en general calcularse de 6 á 15%; según la magnitud de la obra, los riesgos posibles y varias circunstancias accidentales. Fuera de esto, el contratista generalmente tiene que pagar dependientes, almaceneros y otros agentes, así como los gastos de ranchos, etc.; aunque esos gastos generalmente están resarcidos por las ganancias de los almacenes y por el precio á que pagan los trabajadores á los contratistas el alojamiento y posada.

**Art. 9 *. El conocimiento de los particulares que anteceden nos permite calcular el costo del movimiento de tierra con tolerable exactud.** Por ejemplo, tratemos de averiguar el costo por m cúb de excavación en tierra ordinaria, medida en su lugar primitivo, y de su transporte al terraplén, á una distancia media de 330 m, siendo el jornal de los trabajadores $1 por día de 10 horas de trabajo; costo de un caballo 75 cent; y de un carro 25 cent. Un carretero para 4 carretas.

| | Cent. |
|---|---|
| Tenemos costo de la aflojada ó picada de la tierra con picos, art. 2, m cúbico | 3.25 |
| Cargada de las carretas, art. 3 | 6.50 |
| Acarreo á 330 m, como se calculó anteriormente en el ejemplo del art. 4 | 11.60 |
| Su disposición en capas, art. 5 | 2.00 |
| Conservación del camino carretero, art. 6, 10 porciones á 33 m | 1.00 |
| Varias partidas mencionadas en el art. 7 | 2.60 |
| Costo total para el contratista | 26.95 |
| Más utilidad del contratista 10% | 2.69 |
| Costo total para la Compañía por m cúb | 29.64 |

* Hemos hecho la conversión necesaria en cada ejemplo para que resulte el costo del m cúb y no de la yarda cúb que es la unidad del texto inglés. Estas tablas y cálculos son sumamente útiles á los contratistas de obras. (*N. del T.*)

Es fácil construir una tabla como la que sigue, del costo por unidad cúbica para diferentes porciones de la distancia de acarreo. Las columnas 2 y 3 se obtienen primero por la Regla del art. 4; luego á cada cantidad de la columna 3 se agrega la cantidad *variable* de $^1/_{10}$ de centavo para cada porción de 33 m de la distancia de acarreo, para la conservación del camino, y la cantidad *constante* (para cualquiera clase de terreno dado) compuesta de los precios por yarda cúb, por picar ó aflojar, echarla en los carros, colocación en capas, bote ó desperdicios, etc., tomándolos ó de los artículos que preceden, ó modificados según circunstancias especiales. De esta manera han sido preparadas las tablas.

**Por carretas. Jornales á $1 por día de 10 horas de trabajo.**

| Distancia á que se hace el bote de tierra en metros. | Número de metros cúbicos medidos en su lugar botados por día, por cada carro. | Costo por metro cúbico, medido en su lugar por acarrear y vaciar solamente. | Tierra común. COSTO TOTAL POR METRO CÚBICO, NO INCLUÍDA LA GANANCIA DEL CONTRATISTA. | | | | Tierra dura pesada. COSTO TOTAL POR METRO CÚBICO, NO INCLUÍDA LA GANANCIA DEL CONTRATISTA. | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Picada y extendida. | Picada y tirada. | Arada y extendida. | Arada y tirada. | Picada y extendida. | Picada y tirada. | Arada y extendida. | Arada y tirada. |
| 7.62 | 35.91 | 3.46 | 17.80 | 16.17 | 15.59 | 13.96 | 20.80 | 19.17 | 17.55 | 15.92 |
| 15.24 | 33.92 | 3.65 | 18.02 | 16.39 | 15.81 | 14.18 | 21.02 | 19.40 | 17.77 | 16.15 |
| 22.86 | 32.16 | 3.86 | 18.26 | 16.64 | 16.05 | 14.43 | 21.27 | 19.64 | 18.02 | 16.39 |
| 30.48 | 30.56 | 4.06 | 18.49 | 16.86 | 16.28 | 14.65 | 21.49 | 19.86 | 18.24 | 16.64 |
| 45.72 | 27.81 | 4.46 | 18.95 | 17.33 | 16.74 | 15.12 | 21.96 | 20.33 | 18.71 | 17.08 |
| 60.96 | 25.44 | 4.87 | 19.43 | 17.81 | 17.22 | 15.60 | 22.44 | 20.81 | 19.19 | 17.56 |
| 91.44 | 21.85 | 5.68 | 20.37 | 18.75 | 18.16 | 16.53 | 23.37 | 21.75 | 20.12 | 18.50 |
| 121.92 | 19.10 | 6.50 | 21.32 | 19.69 | 19.11 | 17.48 | 24.32 | 22.70 | 21.07 | 19.45 |
| 152.40 | 16.96 | 7.32 | 22.27 | 20.64 | 20.06 | 18.43 | 25.27 | 23.65 | 22.02 | 20.40 |
| 182.88 | 15.28 | 8.12 | 23.20 | 21.58 | 20.99 | 19.37 | 26.21 | 24.58 | 22.96 | 21.33 |
| 213.36 | 13.90 | 8.93 | 24.14 | 22.52 | 21.93 | 20.31 | 27.14 | 25.52 | 23.89 | 22.27 |
| 243.84 | 12.76 | 9.72 | 25.06 | 23.44 | 22.85 | 21.23 | 28.07 | 26.44 | 24.82 | 23.19 |
| 274.32 | 11.77 | 10.56 | 25.90 | 24.27 | 23.69 | 22.06 | 28.90 | 27.27 | 25.65 | 24.02 |
| 304.80 | 10.92 | 11.36 | 26.96 | 25.34 | 24.75 | 23.13 | 29.96 | 28.34 | 26.71 | 25.09 |
| 335.28 | 10.16 | 12.22 | 27.15 | 26.32 | 25.74 | 24.11 | 30.95 | 29.33 | 27.70 | 26.08 |
| 365.76 | 9.55 | 13.00 | 28.86 | 27.20 | 26.65 | 25.02 | 31.86 | 30.21 | 28.61 | 26.99 |
| 396.24 | 9.01 | 13.78 | 29.77 | 28.14 | 27.56 | 25.93 | 32.77 | 31.15 | 29.52 | 27.90 |
| 426.72 | 8.48 | 14.52 | 30.68 | 29.05 | 28.47 | 26.84 | 33.68 | 32.06 | 30.43 | 28.81 |
| 457.20 | 8.02 | 15.47 | 31.72 | 30.09 | 30.51 | 27.88 | 34.72 | 33.10 | 31.47 | 29.85 |
| 487.68 | 7.64 | 16.25 | 32.63 | 31.00 | 30.42 | 28.79 | 35.63 | 34.00 | 32.38 | 30.76 |
| 518.16 | 7.27 | 17.03 | 33.54 | 31.91 | 31.33 | 29.70 | 36.54 | 34.92 | 33.29 | 31.67 |
| 548.64 | 6.94 | 17.81 | 34.45 | 32.82 | 32.24 | 30.61 | 37.45 | 35.83 | 34.20 | 32.58 |
| 579.12 | 6.65 | 18.72 | 35.49 | 33.86 | 33.28 | 31.65 | 38.49 | 36.87 | 35.24 | 33.62 |
| 609.60 | 6.36 | 19.50 | 36.40 | 34.77 | 34.19 | 32.56 | 39.40 | 37.78 | 36.15 | 34.53 |
| 635.80 | 5.76 | 21.58 | 38.80 | 37.18 | 36.59 | 34.97 | 41.81 | 40.48 | 38.56 | 36.93 |
| 732.00 | 5.27 | 23.53 | 41.08 | 39.45 | 38.87 | 37.24 | 44.08 | 42.46 | 40.83 | 39.21 |
| 804.65 | 5.03 | 24.70 | 42.43 | 40.81 | 40.22 | 38.60 | 45.43 | 43.84 | 42.18 | 40.56 |
| 914.40 | 4.49 | 27.56 | 45.76 | 44.43 | 43.55 | 41.92 | 48.76 | 47.44 | 45.51 | 43.89 |
| 990.60 | 4.19 | 29.64 | 48.16 | 46.54 | 45.95 | 44.32 | 51.17 | 49.54 | 47.92 | 46.29 |
| 1066.80 | 3.92 | 31.59 | 50.44 | 48.81 | 48.23 | 46.60 | 53.44 | 51.82 | 50.19 | 48.57 |
| 1143.00 | 3.68 | 33.67 | 52.84 | 51.22 | 50.63 | 49.01 | 55.85 | 54.22 | 52.60 | 50.97 |
| 1219.20 | 3.47 | 35.75 | 55.25 | 53.62 | 53.04 | 51.41 | 58.25 | 56.63 | 55.00 | 53.38 |
| 1295.40 | 3.28 | 37.83 | 57.65 | 56.03 | 55.44 | 53.82 | 60.66 | 59.03 | 57.41 | 55.78 |
| 1371.60 | 3.12 | 39.78 | 59.93 | 58.30 | 57.72 | 56.09 | 62.93 | 61.31 | 59.68 | 58.07 |
| 1447.80 | 2.96 | 41.86 | 62.33 | 60.71 | 60.12 | 58.50 | 65.34 | 63.71 | 62.09 | 60.46 |
| 1524.00 | 2.83 | 43.94 | 64.74 | 63.11 | 62.53 | 60.90 | 67.74 | 66.12 | 64.49 | 62.87 |
| 1609.30 | 2.69 | 46.45 | 67.31 | 65.69 | 65.10 | 63.48 | 70.32 | 68.69 | 67.07 | 65.44 |
| 2011.62 | 2.18 | 56.94 | 79.82 | 78.19 | 77.61 | 75.98 | 82.82 | 81.20 | 79.57 | 77.95 |
| 2413.95 | 1.83 | 67.73 | 92.33 | 90.70 | 90.12 | 88.49 | 95.33 | 93.70 | 92.08 | 90.45 |
| 2816.27 | 1.58 | 78.52 | 104.83 | 103.21 | 102.62 | 101.00 | 107.83 | 106.21 | 104.58 | 102.96 |
| 3218.70 | 1.39 | 89.31 | 117.34 | 115.71 | 115.13 | 113.50 | 120.34 | 118.72 | 117.09 | 115.47 |

**Por carretas. Jornales á $1 por día de 10 horas de trabajo.**

| Distancia del bote de tierra. | Número de metros cúbicos, medidos en su lugar acarreado por día, por cada carro. | Costo por metro cúbico, medido en su lugar por acarreo y vaciada solamente. | Greda dura ó granzón compacto. COSTO TOTAL POR METRO CÚBICO, NO INCLUÍDA LA GANANCIA DEL CONTRATISTA. | | | | Tierra arenosa liviana. COSTO TOTAL POR METRO CÚBICO, NO INCLUÍDA LA GANANCIA DEL CONTRATISTA. | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Picada y extendida. | Picada y tirada. | Arada y extendida. | Arada y tirada. | Picada y extendida. | Picada y tirada. | Arada y extendida. | Arada y tirada. |
| 7.62 | 35.91 | 3.46 | 24.70 | 23.07 | 18.85 | 17.22 | 14.98 | 14.00 | 13.32 | 12.35 |
| 15.24 | 34.92 | 3.65 | 24.92 | 23.30 | 19.07 | 17.45 | 15.20 | 14.22 | 13.54 | 12.57 |
| 22.86 | 32.16 | 3.86 | 25.17 | 23.54 | 19.32 | 17.69 | 15.44 | 14.47 | 13.79 | 12.82 |
| 30.48 | 30.56 | 4.06 | 25.39 | 23.76 | 19.54 | 17.91 | 15.66 | 14.69 | 14.01 | 13.04 |
| 45.72 | 27.81 | 4.46 | 25.86 | 24.23 | 20.00 | 18.38 | 16.13 | 15.16 | 14.48 | 13.51 |
| 60.96 | 25.44 | 4.87 | 26.34 | 24.71 | 20.49 | 18.86 | 16.61 | 15.64 | 14.96 | 13.99 |
| 91.44 | 21.85 | 5.68 | 27.27 | 25.65 | 20.42 | 19.80 | 17.55 | 16.57 | 15.90 | 14.92 |
| 121.92 | 19.10 | 6.50 | 28.22 | 26.60 | 22.37 | 20.75 | 18.50 | 17.52 | 16.20 | 15.87 |
| 152.40 | 16.96 | 7.32 | 29.17 | 27.55 | 23.32 | 21.70 | 19.45 | 18.47 | 17.80 | 16.82 |
| 182.88 | 15.28 | 8.12 | 30.11 | 28.48 | 24.26 | 22.63 | 20.38 | 19.41 | 18.73 | 17.76 |
| 213.36 | 13.90 | 8.93 | 31.04 | 29.42 | 25.19 | 23.57 | 21.32 | 20.34 | 19.67 | 18.69 |
| 243.84 | 12.76 | 9.72 | 31.97 | 30.34 | 26.12 | 24.49 | 22.24 | 21.27 | 20.59 | 19.62 |
| 274.32 | 11.77 | 10.56 | 32.80 | 31.10 | 26.95 | 25.32 | 23.07 | 22.10 | 21.42 | 20.45 |
| 304.80 | 10.92 | 11.36 | 33.86 | 32.24 | 28.01 | 26.39 | 24.14 | 23.17 | 22.49 | 21.51 |
| 335.28 | 10.16 | 12.22 | 34.85 | 33.23 | 29.00 | 27.38 | 25.13 | 24.15 | 23.48 | 22.50 |
| 365.76 | 9.55 | 13.00 | 35.76 | 34.14 | 29.94 | 28.29 | 26.04 | 25.06 | 24.39 | 23.41 |
| 396.24 | 9.01 | 13.78 | 36.67 | 35.05 | 30.82 | 29.20 | 26.95 | 25.97 | 25.30 | 24.32 |
| 426.72 | 8.48 | 14.52 | 37.58 | 35.96 | 31.73 | 30.11 | 27.86 | 26.88 | 26.21 | 25.23 |
| 457.20 | 8.02 | 15.47 | 38.62 | 37.00 | 32.77 | 31.15 | 28.90 | 27.92 | 27.27 | 26.27 |
| 487.68 | 7.64 | 16.25 | 39.53 | 37.91 | 33.68 | 32.06 | 29.81 | 28.83 | 28.16 | 27.18 |
| 518.16 | 7.27 | 17.03 | 40.44 | 38.82 | 34.59 | 32.97 | 30.72 | 29.74 | 29.07 | 28.09 |
| 548.64 | 6.94 | 17.81 | 41.35 | 39.73 | 35.50 | 33.88 | 31.63 | 30.65 | 29.98 | 29.00 |
| 579.12 | 6.65 | 18.72 | 42.39 | 40.77 | 36.54 | 34.92 | 32.67 | 31.69 | 31.02 | 30.04 |
| 609.60 | 6.36 | 19.50 | 43.30 | 41.68 | 37.45 | 35.83 | 33.58 | 32.60 | 31.93 | 30.95 |
| 685.80 | 5.76 | 21.58 | 45.71 | 44.08 | 39.86 | 38.23 | 35.98 | 35.01 | 34.33 | 33.36 |
| 762.00 | 5.27 | 23.53 | 46.18 | 46.36 | 42.43 | 40.51 | 38.26 | 37.28 | 36.61 | 35.63 |
| 804.65 | 5.03 | 24.70 | 49.33 | 47.71 | 43.48 | 41.86 | 39.61 | 38.64 | 37.96 | 36.98 |
| 914.40 | 4.49 | 27.56 | 52.66 | 51.04 | 46.81 | 45.19 | 42.94 | 41.96 | 41.29 | 40.31 |
| 990.60 | 4.19 | 29.64 | 55.07 | 53.44 | 49.22 | 47.69 | 45.34 | 44.37 | 43.69 | 42.72 |
| 1066.80 | 3.92 | 31.59 | 57.34 | 55.72 | 51.49 | 49.87 | 47.62 | 46.64 | 45.97 | 44.99 |
| 1143.00 | 3.68 | 33.67 | 59.73 | 58.12 | 53.90 | 52.27 | 50.02 | 49.05 | 48.37 | 47.40 |
| 1219.20 | 3.47 | 35.75 | 62.15 | 60.53 | 56.30 | 54.68 | 52.43 | 51.45 | 50.78 | 49.80 |
| 1295.40 | 3.28 | 37.83 | 64.56 | 62.93 | 58.71 | 57.08 | 54.83 | 53.88 | 53.21 | 52.23 |
| 1371.60 | 3.12 | 39.78 | 66.83 | 65.21 | 60.98 | 59.36 | 57.11 | 56.13 | 55.46 | 54.48 |
| 1447.80 | 2.96 | 41.86 | 69.24 | 67.61 | 63.39 | 61.76 | 59.5 | 58.54 | 57.86 | 56.89 |
| 1524.00 | 2.83 | 43.94 | 71.64 | 70.02 | 65.79 | 64.17 | 61.92 | 60.95 | 60.27 | 59.29 |
| 1609.30 | 2.69 | 46.45 | 74.22 | 72.59 | 68.37 | 66.74 | 64.49 | 63.52 | 62.84 | 61.87 |
| 2011.62 | 2.18 | 56.94 | 86.98 | 85.10 | 80.87 | 79.25 | 77.01 | 76.02 | 75.35 | 74.37 |
| 2413.95 | 1.83 | 67.73 | 99.23 | 97.60 | 93.38 | 91.75 | 89.50 | 88.53 | 87.85 | 86.88 |
| 2816.27 | 1.58 | 78.52 | 111.73 | 110.11 | 105.88 | 104.26 | 102.01 | 101.03 | 100.36 | 99.38 |
| 3218.70 | 1.39 | 89.31 | 124.24 | 122.62 | 118.39 | 116.77 | 114.52 | 113.54 | 112.87 | 111.90 |

**Art. 10. En carretillas.** El costo, empleando carretillas, puede estimarse del mismo modo que por carretas. Véase art. 1, etc. Los hombres tirando carros de mano caminan más ó menos la misma distancia durante el mismo tiempo que un caballo con carreta, es decir, 4,000 metros por hora, ó como 66 m por minuto, ó un minuto por cada 33 m de distancia.

El tiempo empleado en cargar, descargar, etc. (cuando, como se acostumbra, el trabajador llena su propio carro), es más ó menos 1.25 minutos, sin tener en cuenta la distancia de transporte; además de lo cual, el tiempo que se pierde descansando á ratos, ajustando los tablones del camino, y otras causas incidentales, equivale más ó menos á $^1/_{10}$ parte del tiempo total. De modo que, en la práctica, no debemos sino considerarle como trabajando 9 horas de las 10 de trabajo. Por consiguiente,

$$\frac{\text{El número de minutos de un día de trabajo} \times .9}{1.25 + \text{número de porciones de á 33 m comprendidos en la distancia de transporte}} = \text{el número de viajes ó cargas transportadas por día y por carro}$$

Véase la nota próxima.

El número de cargas dividido por 18½ dará el número de metros cúbicos, porque un m cúbico medido en su lugar primitivo antes de excavar, da por término medio 18½ cargas de carro de mano. Y el costo de un hombre y un carro de mano por día (digamos $1 para el hombre y 5 cent para el carro) dividido por el número de m cúb, da el costo por m cúb, de cargar, rodar y vaciar.

Ejemplo: ¿Cuántos metros cúbicos de tierra ordinaria medidos en su lugar antes de excavar, puede un hombre cargar, acarrear y vaciar, por día de 10 horas de trabajo (ó 600 minutos), siendo la distancia de transporte 330 m (ó 10 porciones de á 33 m), y cuál será el costo por metro cúb, suponiendo el jornal del hombre con su carretilla á razón de $1.05 por día?

$$\text{Tendremos}: \frac{600 \text{ minutos} \times .9}{1.25 + 10 \text{ porciones}} = \frac{540}{11.25} = 48 \text{ viajes ó cargas por día;}$$

$$\text{y } \frac{48}{18.5} = 2.6; \quad \text{y } \frac{105 \text{ centavos}}{2.6 \text{ m cúbicos}} = 40.4 \text{ cent.}$$

por m cúb de tierra echada en la carretilla, acarreada y botada y regresando con la carretilla vacía. El gasto en palas aumentaría este costo en una cantidad casi inapreciable; sin embargo, está incluído en las tablas que siguen, en el costo de la herramienta.

*Nota.* Para piedras que requieren más tiempo para cargar las carretillas, tenemos

$$\frac{\text{Número de minutos en un día de trabajo} \times .9}{1.6 + \text{número de porciones de 33 m}} = \text{número de cargas transportadas por día y por carretilla.}$$

**En carretillas. Jornales á razón de $1 por día de 10 horas de trabajo.**

| Distancia á que se hace el bote, ó á que se lleva la tierra en carretilla, en metros. | Número de metros cúbicos en su lugar cargados y acarreados, por día y por cada carretilla. | Costo por metro cúbico, medido en su lugar por carga, acarreo y vaciada. | Tierra común. Costo total por metro cúbico, no incluída la ganancia del contratista. | | | | Tierra dura pesada. Costo total por metro cúbico, no incluída la ganancia del contratista. | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Picada y extendida. | Picada y tirada. | Arada y extendida. | Arada y tirada. | Picada y extendida. | Picada y tirada. | Arada y extendida. | Arada y tirada. |
| 7.62 | 19.63 | 5.32 | 13.15 | 11.53 | 10.95 | 9.32 | 15.41 | 13.48 | 11.86 | 10.23 |
| 15.24 | 16.88 | 6.17 | 14.04 | 12.42 | 11.83 | 10.20 | 15.9 | 14.36 | 12.74 | 11.11 |
| 22.86 | 14.74 | 7.07 | 14.97 | 13.35 | 12.77 | 11.14 | 16.93 | 15.30 | 13.68 | 12.05 |
| 30.48 | 13.06 | 7.98 | 15.91 | 14.29 | 13.70 | 12.08 | 17.86 | 16.24 | 14.61 | 12.99 |
| 45.72 | 10.70 | 9.75 | 17.74 | 16.12 | 15.54 | 13.91 | 19.69 | 18.07 | 16.44 | 14.82 |
| 60.96 | 9.09 | 11.47 | 19.53 | 17.90 | 17.32 | 15.69 | 21.48 | 19.85 | 18.23 | 16.60 |
| 76.20 | 7.87 | 13.22 | 21.38 | 19.76 | 19.18 | 17.55 | 23.34 | 21.71 | 20.08 | 18.46 |
| 91.44 | 6.93 | 15.08 | 23.27 | 21.65 | 21.06 | 19.44 | 25.22 | 23.60 | 21.97 | 20.34 |
| 106.68 | 6.22 | 16.77 | 25.02 | 23.40 | 22.82 | 21.19 | 26.98 | 25.35 | 23.73 | 22.10 |
| 121.92 | 5.62 | 18.59 | 26.91 | 25.29 | 24.70 | 23.07 | 28.86 | 27.24 | 25.61 | 23.98 |
| 137.16 | 5.13 | 20.28 | 28.67 | 27.04 | 26.46 | 24.83 | 30.62 | 28.99 | 27.37 | 25.74 |
| 152.40 | 4.71 | 22.10 | 30.55 | 28.93 | 28.34 | 26.71 | 32.50 | 30.88 | 29.25 | 27.63 |
| 182.88 | 4.06 | 25.61 | 34.19 | 32.57 | 31.98 | 30.36 | 36.14 | 34.51 | 32.89 | 31.27 |
| 213.36 | 3.57 | 29.25 | 37.96 | 36.34 | 35.75 | 34.13 | 39.91 | 38.29 | 36.66 | 35.04 |
| 243.84 | 3.19 | 32.76 | 41.60 | 39.98 | 39.39 | 37.77 | 43.55 | 41.93 | 40.30 | 38.68 |
| 274.32 | 2.87 | 36.27 | 45.24 | 43.66 | 43.03 | 41.40 | 47.19 | 45.57 | 43.94 | 42.32 |
| 304.80 | 2.62 | 39.78 | 48.88 | 47.22 | 46.67 | 45.05 | 50.83 | 49.20 | 47.58 | 45.96 |
| 365.76 | 2.22 | 46.93 | 56.29 | 54.66 | 54.08 | 52.46 | 58.24 | 56.62 | 54.99 | 53.37 |
| 426.72 | 1.93 | 53.95 | 63.57 | 61.95 | 61.36 | 59.74 | 65.52 | 63.90 | 62.27 | 60.65 |
| 487.68 | 1.71 | 60.97 | 70.85 | 69.49 | 68.64 | 67.02 | 72.80 | 71.17 | 69.55 | 67.93 |
| 548.64 | 1.53 | 68.25 | 78.39 | 76.77 | 76.18 | 74.56 | 80.34 | 78.72 | 77.09 | 75.47 |
| 609.60 | 1.38 | 75.40 | 85.80 | 84.18 | 83.59 | 81.97 | 87.75 | 86.13 | 84.50 | 82.88 |
| 670.56 | 1.27 | 82.29 | 92.95 | 91.33 | 90.74 | 89.12 | 94.90 | 93.27 | 91.65 | 90.03 |
| 731.52 | 1.17 | 89.18 | 100.10 | 98.48 | 97.89 | 96.27 | 102.05 | 100.42 | 98.80 | 97.18 |
| 804.60 | 1.06 | 98.15 | 109.38 | 107.76 | 107.17 | 105.54 | 111.33 | 109.71 | 108.08 | 106.46 |

**Art. 11.** Las tablas anterior y posterior han sido calculadas como las hechas para el transporte en carretas, y calculando lo mismo las columnas 2 y 3, por medio

e la regla del art. 10, y luego agregando á cada suma de la columna 3.ª la cantidad variable de .1 centavo por metro cúbico y por cada 33 metros de distancia, para mantener en orden los tablones del camino, y los gastos de picar ó aflojar y extender la tierra y los de superintendencia, aguadores, etc., por metro cúbico, como se ha indicado en los artículos 2, 5 y 7 que anteceden.

**En carretillas. Jornales $1 por día de 10 horas.**

| Distancia en m á que se hace el bote, ó á que se lleva la tierra en carretilla. | Número de m cúbicos, medidos en su lugar cargados y acarreados, por día, por cada carretilla. | Costo por metro cúbico, medido en su lugar por acarreo y vaciada. | Greda dura ó granzón compacto. COSTO TOTAL POR METRO CÚBICO, NO INCLUÍDA LA GANANCIA DEL CONTRATISTA. | | | | Tierra arenosa liviana. COSTO TOTAL POR METRO CÚBICO, NO INCLUÍDA LA GANANCIA DEL CONTRATISTA. | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Picada y extendida. | Picada y tirada. | Arada y extendida. | Arada y tirada. | Picada y extendida. | Picada y tirada. | Arada y extendida. | Arada y tirada. |
| 7.62 | 19.63 | 5.32 | 19.00 | 17.38 | 13.15 | 11.53 | 11.43 | 10.45 | 9.78 | 8.80 |
| 15.24 | 16.88 | 6.17 | 19.89 | 18.27 | 14.04 | 12.42 | 12.31 | 11.34 | 10.66 | 9.69 |
| 22.86 | 14.74 | 7.07 | 20.82 | 19.20 | 14.97 | 13.35 | 13.25 | 12.27 | 11.60 | 10.62 |
| 30.48 | 13.06 | 7.98 | 21.76 | 20.14 | 15.91 | 14.29 | 14.18 | 13.24 | 12.53 | 11.56 |
| 45.72 | 10.70 | 9.75 | 23.60 | 21.97 | 17.74 | 16.12 | 16.02 | 15.04 | 14.37 | 13.39 |
| 60.96 | 9.09 | 11.47 | 25.38 | 23.75 | 19.53 | 17.90 | 17.80 | 16.82 | 16.15 | 15.17 |
| 76.20 | 7.87 | 13.22 | 27.24 | 25.64 | 21.38 | 19.76 | 19.66 | 18.68 | 18.00 | 17.03 |
| 91.44 | 6.93 | 15.08 | 29.12 | 27.50 | 23.27 | 21.65 | 21.54 | 20.57 | 19.89 | 18.91 |
| 106.68 | 6.22 | 16.77 | 30.88 | 29.25 | 25.02 | 23.40 | 23.30 | 22.32 | 21.65 | 20.67 |
| 121.92 | 5.62 | 18.59 | 32.76 | 31.14 | 26.91 | 25.29 | 25.18 | 24.20 | 23.53 | 22.56 |
| 137.16 | 5.13 | 20.28 | 34.52 | 32.89 | 28.67 | 27.04 | 26.94 | 25.96 | 25.28 | 24.31 |
| 152.40 | 4.71 | 22.10 | 36.40 | 34.78 | 30.55 | 28.93 | 28.82 | 27.85 | 27.17 | 26.19 |
| 182.88 | 4.06 | 25.61 | 40.04 | 38.42 | 34.19 | 32.57 | 32.46 | 31.49 | 30.81 | 29.84 |
| 213.36 | 3.57 | 29.25 | 43.84 | 42.19 | 37.96 | 36.34 | 36.23 | 35.26 | 34.58 | 33.61 |
| 243.84 | 3.19 | 32.76 | 47.45 | 45.83 | 41.60 | 39.98 | 39.87 | 38.90 | 38.22 | 37.24 |
| 274.32 | 2.87 | 36.27 | 51.09 | 49.47 | 45.24 | 43.62 | 43.51 | 42.54 | 41.86 | 40.89 |
| 304.80 | 2.62 | 39.78 | 54.73 | 53.10 | 48.88 | 47.26 | 47.15 | 46.18 | 45.50 | 44.53 |
| 365.76 | 2.22 | 46.93 | 62.14 | 60.51 | 56.29 | 54.66 | 54.56 | 53.59 | 52.91 | 51.87 |
| 426.72 | 1.93 | 53.95 | 69.42 | 67.80 | 63.57 | 61.95 | 61.84 | 60.87 | 60.19 | 59.22 |
| 487.68 | 1.71 | 60.97 | 76.70 | 75.08 | 70.85 | 69.23 | 69.12 | 68.15 | 67.47 | 66.50 |
| 548.64 | 1.53 | 68.25 | 84.24 | 82.62 | 78.39 | 76.77 | 76.66 | 75.69 | 75.01 | 74.04 |
| 609.60 | 1.38 | 75.40 | 91.65 | 90.02 | 85.80 | 84.18 | 84.07 | 83.10 | 82.42 | 81.45 |
| 670.56 | 1.27 | 82.29 | 98.80 | 97.18 | 92.95 | 91.33 | 91.22 | 90.25 | 89.57 | 88.60 |
| 731.52 | 1.17 | 89.18 | 105.95 | 104.33 | 100.10 | 98.48 | 98.37 | 97.40 | 96.72 | 95.75 |
| 804.60 | 1.06 | 98.15 | 115.23 | 113.61 | 109.38 | 107.76 | 107.65 | 106.68 | 106.00 | 105.03 |

**Art. 12. Por medio de rastrillos* de ruedas y de tracción.** El cuerpo del rastrillo de ruedas es una caja de planchas de acero lisas, de 1.06 m en cuadro, más ó menos × 40 cm = de hondo, que contiene más ó menos 400 litros de tierra cuando se llena á nivel. La caja está abierta por delante (en algunas máquinas se cierra por medio de una compuerta cuando está llena) y puede suspenderse, bajarse y hacerse girar sobre un eje horizontal. Para llenar la caja, se baja ésta y se sostiene mientras que la pareja de caballos tira del aparato hacia adelante. Cuando está llena, se suspende más ó menos 3 dcm del suelo, y, al llegar al lugar de la descarga, se vacía volteándola sobre su eje. Todos los movimientos de la caja se ejecutan por medio de palancas y sin parar la pareja, la que, de este modo, camina constantemente. Las ruedas tienen llantas anchas para impedir que penetren en el suelo.

En el rastrillo de tracción, la caja sometida á mayor resistencia, se hace mucho más pequeña. Contiene más ó menos 115 á 190 lit, y está siempre abierta por delante. El manejo de este rastrillo* es semejante al de ruedas, con la excepción de que la caja, al estar llena, se descarga en el suelo y es arrastrada por la pareja.

Cada rastrillo * (de cualquier clase) requiere el empleo constante de una pareja de caballos y un carretero. Además se requiere en la excavación y el bote de la tierra, un número de hombres que depende de la distancia del transporte y del

* *Wheeled scrapers* (rastrillo, trailla, rastra). (*N. del T.*)

número de rastrillos. Excepto cuando sea arena ó tierra muy ligera, hay economía en emplear un arado antes de hacer uso del rastrillo.

En las distancias cortas se usan rastrillos más pequeños que en las grandes.

Nuestros cálculos están basados sobre las cargas siguientes:

| | |
|---|---|
| Para rastrillos de tracción (empleados sólo en distancias cortas) ..... | 156 litros. |
| Para rastrillos con ruedas á distancia menor de 33 m..... | 255 — |
| dist de 33 á 100 m..... | 300 — |
| — de 133 á 165 m..... | 380 — |
| — más de 165 m..... | 460 — |

El gasto diario de un rastrillo por el jornal del carretro y de la pareja de caballos es de $2.50 más ó menos. Para distancias de 130 m ó más, podemos agregar 50 centavos por día para una pareja de caballos auxiliar, para ayudar á cargar los rastrillos más grandes, que entonces se usan. Una pareja auxiliar sirve generalmente para varios rastrillos.

Debido al hecho de que las parejas están constantemente en movimiento sin descargar, caminan algo más despacio que con carretas. Calculamos 46 m por minuto (ó sean 23 m de transporte por minuto) por término medio.

Al cargar y descargar las parejas de caballos, no solamente tienen que salirse del camino, sino que también caminan más despacio que cuando están simplemente halando. Para compensar esta diferencia, haremos una adición de 33 m á cada porción de la distancia de transporte, sea corta ó larga.

Agregamos 1 centavo por m cúbico para la carga y descarga y estimamos el costo aproximado de las otras partidas como sigue:

Reparación del camino carretero $^1/_{10}$ centavo por m cúb para cada 33 m de distancia de transporte.

| Picada ó aflojada. | Tierras ligeras cent por m cúb. | Tierras pesadas cent por m cúb. |
|---|---|---|
| Por medio de p'cos..... | * | 6.5 |
| Por medio de palas..... | * | 2.6 |
| Extender..... | 1.3 | 2.0 |
| Superintendencia, desgaste, etc..... | 1.3 | 1.3 |

Repetimos que nuestros números no pueden considerarse sino como aproximaciones, y que están sujetos á grandes variaciones según la habilidad del contratista y del superintendente, resistencia de la pareja de caballos, calidad del material que ha de transportarse, estado del tiempo, etc.

$$\text{Número de viajes por día por rastrillo} = \frac{\text{Número de minutos (600) en un día de trabajo}}{\text{Número de distancias de 23 m en (dist + 33 m)}}$$

$$\text{Número de metros cúbicos medidos en su lugar antes de excavar, transportados por día por cada rastrillo} = \text{Número de viajes por día por rastrillo} \times \text{Número de metros cúbicos medidos en su lugar antes de excavar, por rastrillo y por viaje.}$$

$$\text{Costo por metro cúbico medido en su lugar antes de excavar; con carga, acarreo, vaciada y retorno} = \frac{\text{Gasto diario en una raspadora}}{\text{Número de metros cúbicos medidos en su lugar, transportados por día por cada rastrillo}} + \text{1 centavo para cargar y descargar}$$

$$\text{Costo total por metro cúbico medido en su lugar excluyendo utilidad del contratista} = \text{Costo por metro cúb medido en su lugar; carga, acarreo vaciada y retorno} + \text{.1 centavo por metro cúb en su lugar por cada 33 m de dist por reparación de camino} + \text{Costo por metro cúb medido en su lugar por la picada, extendida y botada, y superintendencia, etc.}$$

---

* Las tierras livianas las afloja el mismo rastrillo.

**Con rastrillos de ruedas.** Jornales á $1 por día de 10 horas de trabajo.

| (a) | (b) | (c) | (d) | | | | | |
|---|---|---|---|---|---|---|---|---|
| Distancia del bote de la tierra. | Metros cúbicos, medidos en su lugar, acarreados por día por cada, rastrillo. | Costo por metro cúbico, medidos en su lugar, por carga, acarreo, vaciada y retorno. | Costo total por metro cúbico, sin incluir la ganancia del contratista. | | | | | |
| | | | TIERRA BLANDA. | | TIERRA DURA. | | | |
| | | | Extendida. | Tirada. | Picada y extendida. | Picada y tirada. | Arada y extendida. | Arada y tirada. |
| metros. | m cúb. | cts. | cts. | cts. | cts. | cts. | cts. | cts. |
| 15.24 | 76.40 | 5.85 | 8.58 | 7.28 | 15.73 | 13.78 | 11.83 | 9.88 |
| 30.48 | 68.76 | 6.37 | 9.10 | 7.80 | 16.25 | 14.30 | 12.35 | 10.40 |
| 45.72 | 53.48 | 7.80 | 10.66 | 9.36 | 17.81 | 15.86 | 13.91 | 11.96 |
| 60.96 | 45.84 | 8.97 | 11.83 | 10.53 | 18.98 | 17.03 | 15.08 | 13.13 |
| 91.44 | 34.38 | 11.44 | 14.43 | 13.13 | 21.58 | 19.63 | 17.68 | 15.73 |
| 121.92 | 34.38 | 12.87 | 15.99 | 14.69 | 23.14 | 21.19 | 19.24 | 17.29 |
| 182.88 | 29.03 | 14.95 | 18.33 | 17.03 | 25.48 | 23.53 | 21.58 | 19.63 |
| 243.84 | 22.92 | 18.59 | 22.23 | 20.93 | 29.38 | 27.43 | 25.48 | 23.53 |
| 304.80 | 18.34 | 23.01 | 26.91 | 25.61 | 34.06 | 32.11 | 30.16 | 28.21 |

**Por rastrillos de tracción.** Jornales $1 por día de 10 horas de trabajo.

| (a) | (b) | (c) | (d) | | | | | |
|---|---|---|---|---|---|---|---|---|
| Distancia del bote de la tierra. | Metros cúbicos, medidos en su lugar, acarreados por día por cada rastrillo. | Costo por metro cúbico, medidos en su lugar, por carga, acarreo, vaciada y retorno. | Costo total por metro cúbico, sin incluir la ganancia del contratista. | | | | | |
| | | | TIERRA BLANDA. | | TIERRA DURA. | | | |
| | | | Extendida. | Tirada. | Picada y extendida. | Picada y tirada. | Arada y extendida. | Arada y tirada. |
| metros. | m cúb. | cts. | cts. | cts. | cts. | cts. | cts. | cts. |
| 15.24 | 45.84 | 8.97 | 11.7 | 10.4 | 18.85 | 16.9 | 14.95 | 13.00 |
| 22.86 | 38.20 | 10.40 | 13.13 | 11.83 | 20.28 | 18.33 | 16.38 | 14.43 |
| 30.48 | 34.38 | 11.44 | 14.17 | 12.87 | 21.32 | 19.37 | 17.42 | 15.47 |
| 45.72 | 27.50 | 14.04 | 16.9 | 15.60 | 24.05 | 22.10 | 20.15 | 18.20 |
| 60.96 | 22.92 | 16.51 | 19.37 | 18.07 | 26.52 | 24.57 | 22.62 | 20.67 |

**Art. 13. Por medio de carros y locomotora** en una vía horizontal. Hemos basado nuestros cálculos sobre las suposiciones siguientes: Trenes de 10 carros conteniendo cada carro (1½ yd cúb) 1.15 m cúb medido en su lugar primitivo. Velocidad media de los trenes, incluyendo la salida y la llegada, pero ninguna parada, 16 kilómetros por hora=8 km de transporte por hora. Jornales á $1 por día de 10 horas. Aflojar ó picar la tierra, palearla, extenderla; desgaste de la herramienta, superintendencia, etc., lo mismo que con carretas, arts. 2, 3, 5 y 7. Pérdida de tiempo en cada viaje, en cargar, descargar, etc., 9 minutos=.15 hora. Por tanto, tenemos:

$$\text{Número de viajes por día y por tren} = \frac{\text{10 horas de un día de trabajo}}{.15 + \text{número de porciones de 8 km en la dist de transporte}}$$

$$\text{Número de m cúbicos medidos en su lugar primitivo por día y por tren} = \text{Número de viajes por día y por tren} \times \text{Número (10) de carros en un tren} \times \text{Número de m cúb (1.15) medidos en su lugar primitivo en cada carro}$$

$$\text{Costo por m cúb en su lugar primitivo por acarreo, descarga y retorno} = \frac{\text{Gastos del tren en un día} + \text{costo de la vía en un día}}{\text{Número de m cúb medidos en su lugar primitivo y por tren}}$$

**Gastos del tren en un día :**

| | |
|---|---|
| Precio de 10 carros á $100 | $1,000 |
| Precio de locomotora | 3,000 |
| | $4,000 |

| | |
|---|---|
| Intereses de un día al 6% sobre el precio del tren | $ .67 |
| Pago del maquinista, que también es fogonero | 2 |
| Caporal para la descarga | 2 |
| 3 hombres para descargar | 3 |
| Combustible | 2 |
| Agua | 1 |
| Reparación de locomotora y carros | 2.33 |
| Gasto diario total de un tren | $ 13 |

Depreciación (considerando la duración de material rodante, 10 años)
dígase $100 por año por $1,000
= $400 — tren
= $ 4 por día (suponiendo 100 de labor por año). 4
Gasto diario y depreciación de un tren ..... $17

Suponiendo el costo de la vía puesta á razón de $2,500 por milla, y su duración de 5 años, **el gasto diario de la vía**, con intereses, depreciación, manejo y reparaciones, se puede calcular en $6 por cada milla (1.609 m) de transporte.

Por consiguiente,

$$\text{Costo por m cúb, medido antes de excavarlo transporte, botada y vuelta} = \frac{\$17 + (\$6 \text{ por milla de transporte (como \$4 por km)})}{\text{Número de viajes por día y por tren} \times \text{Número de carros por tren (10)} \times \text{Número de m cúb en cada carro (1.15)}}$$

$$\text{Costo total por m cúb medido en su lugar primitivo, sin contar utilidad del contratista} = \text{Costo por m cúb (medido en su lugar) del acarreo, descarga y retorno} + \text{Costo por m cúb (medido en su lugar), por aflojar, ó picar, extender ó botar, y superintendencia, etc. (Arts. 2, 3, 5 y 7.)}$$

**Por medio de carros y locomotoras.** Jornal $1 por día de 10 horas.

| (a) | (b) | (c) | (d) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Distancia del bote de la tierra. | Metros cúbicos, medidos en su lugar acarreados por día, por cada tren. | Costo por metro cúbico, medido en su lugar por acarreo, vaciada y retorno. | Costo total por metro cúbico, medido en su lugar, sin incluir la ganancia del contratista. | | | | | | | |
| | | | TIERRA BLANDA. | | | | TIERRA DURA. | | | |
| | | | Picada y extendida. | Picada y tirada. | Arada y extendida. | Arada y tirada. | Picada y extendida. | Picada y tirada. | Arada y extendida. | Arada y tirada. |
| metros. | m cúb. | cts. | cts. | cts. | cts. | cts. | cts. | cts. | cts. | cts. |
| 402.25 | 573.42 | 3.21 | 14.69 | 13.39 | 13.05 | 11.75 | 20.50 | 18.55 | 17.25 | 15.30 |
| 804.50 | 458.73 | 4.33 | 15.81 | 14.51 | 14.17 | 12.87 | 21.62 | 19.67 | 18.37 | 16.42 |
| 1,206.75 | 378.48 | 5.64 | 17.12 | 15.82 | 15.48 | 14.18 | 22.93 | 20.98 | 19.68 | 17.73 |
| 1609.00 | 321.11 | 7.12 | 18.60 | 17.30 | 16.96 | 15.66 | 24.41 | 22.46 | 21.16 | 19.21 |
| 3218.00 | 206.43 | 13.06 | 25.44 | 24.14 | 23.80 | 22.50 | 31.25 | 29.30 | 28.00 | 26.05 |
| 4827.00 | 149.09 | 23.33 | 34.81 | 33.51 | 33.18 | 31.88 | 40.62 | 38.67 | 37.37 | 35.42 |
| 6436.00 | 114.68 | 35.53 | 47.01 | 45.71 | 45.37 | 44.07 | 52.82 | 50.87 | 49.57 | 47.62 |

Cuando se ejecutan obras en grande escala, el **excavador de vapor, draga de tierra ó pala de vapor** economiza generalmente tiempo y dinero. Cuando la altura del banqueo es menos de 3 m, se pierde tanto tiempo mudando el excavador de un lugar á otro, que no se puede usar con ventaja. En terrenos duros pueden hacerse banqueos de más ó menos 5 á 6 m de altura sin cambiar el nivel de la máquina. Para alturas mayores en terrenos como éstos, el trabajo se hace á dos niveles, porque el cubo ó sacador no alcanza á más altura. Pero en arena y granzón suelto pueden hacerse banqueos mucho más altos á un solo nivel.

El excavador se parece á una draga en su apariencia y en el modo de operar. Un cubo grande de planchas de acero, semejante al de la draga, con un fondo chato de charnelas y provisto de dientes de acero cortantes, penetra en la tierra arrastrado por debajo de ella, con fuerza de vapor. La descarga del material se hace por el fondo movedizo del cubo, sobre carros que lo transportan á otra parte, ó lo botan.

Cada máquina está montada sobre un carro de vía normal, que puede engancharse á un tren ordinario de carga. El carro se hace de hierro ó de madera, y está provisto de un accesorio de locomotora, por medio del cual puede ser movido de un punto á otro, según avanza el trabajo. Estas máquinas también pueden usarse como **carro auxiliar** ó **carro de grúa ó aparejo**. Cada máquina tiene un estanque de agua (que contiene de 1,100 á 2,000 lit), para alimentar su caldera. Antes de principiar la excavación, la punta del carro más próxima á la obra se levanta sobre la vía por medio de gatos hidráulicos ó de tornillos, sobre los cuales descansa durante el trabajo.

En los terrenos duros el excavador deja los lados de la cortada casi verticales y el talud deseado se hace después con picos y palas. Cuando el suelo es duro ó está muy helado, puede aflojarse antes de excavarlo por medio de explosivos.

El excavador tiene que mudarse hacia adelante (á medida que la obra avance) 2.40 m más ó menos á la vez. Como generalmente se hace, puede excavar á una distancia de 5 m horizontalmente desde el centro del carro, en cualquiera dirección, y puede vaciarse á 3.6 m sobre la vía. En arena ó granzón, la máquina saca por minuto durante la excavación efectiva 3 cubos llenos (de 3½ á 4½ m cúb el cubo, =2.80 á 3.80 m cúb en su lugar); en greda dura 2 cubos rasos por minuto (=2.2 á 3 m cúb en el cubo y 1.9 á 2.5 m cúb en su lugar). El trabajo medio de un día (10 horas) de una máquina « N.º 1 » es 380 m cúb más ó menos en tierra dura, y de 912 á 1,140 m cúb en arena ó granzón. En esta cantidad están incluídas las dilaciones inevitables y ordinarias para tener listos los carros para el excavador.

Los excavadores usan más ó menos de 5 á 6 atmósferas de presión, queman de 46 á 70 kg de carbón y requieren un maquinista, un fogonero, un hombre para atender á la grúa y de 5 á 10 trabajadores, incluyendo el capataz. Los trabajadores atienden á los gatos, colocan la vía para el excavador y los carros de descarga, ayudan á mover estos últimos, traen ó bombean el agua, etc.

Después de llegar al lugar del trabajo, se necesitan más ó menos 30 minutos para poner el excavador en condición de trabajar y un tiempo igual después de terminar el trabajo, para ponerlo en estado de transportarlo.

Los números que siguen se han tomado de los datos sacados de un trabajo hecho por una máquina « N.º 1 », desde mayo hasta noviembre de 1883. El material era greda dura con depósitos de arena. Los gastos hechos por día de trabajo de 12 horas á razón de $1.50 por día de dicho trabajo, fueron :

| | | |
|---|---|---|
| Agua suministrada sin restricción alguna | $ 5 | |
| Carbón bituminoso, 1½ tonel | 10 | |
| Salario del maquinista | 4 | |
| — de un fogonero | 1.50 | |
| — de un peón para atender á la grúa y la excavadora | 2.50 | |
| — de un capataz | 3 | |
| — de 8 trabajadores á $1.50 | 12 | |
| Aceite, desgastes, reparaciones (estimados) | 5 | |
| Interés sobre el costo de la máquina ($7.500) | 1.25 | |
| | $44.25 | |

Reduciendo esto á nuestro jornal tomado por modelo, de $1 por día de 10 horas esta suma daría más ó menos $30.

Reducida al mismo modelo, teniendo en cuenta la pérdida de tiempo, relativamente mayor al principiar en la mañana y parar en la tarde, la cantidad media excavada por día, medida en su lugar ó antes de excavada, fué, en banqueos de poca altura, de 403 m cúb; en banqueos profundos, 912 m cúb; término medio de toda la operación, 608 m cúb.

Esto daría como costo de la aflojada y echada á los carros del m cúb medido en su lugar ó antes de excavado, 7.4 centavos, 3.3 centavos y 4.9 centavos respectivamente; mientras que su costo por ararla y palearla, en terreno duro y pesado, es más ó menos, según los arts. 2 y 3, de 9.6 centavos, y picando y paleando como 13 cent.

**Art. 14. Transporte de piedra de excavación por medio de carretilla.** Un metro cúbico de piedra dura, medida en su lugar primitivo, ó antes de ser volada, pesa más ó menos 2.3 toneladas si es piedra arenisca ó roca conglomerada, ó 2.6 toneladas si es granito bueno compacto, gneis ó piedra caliza. De modo que es suficiente aproximación en la práctica para nuestro caso, tomar el peso de cualquiera de ellos, á razón de 2.3 toneladas por m cúb más ó menos, medida en su lugar ó sea antes de volarla.

Ahora bien, **un metro cúbico sólido, convertido en pedazos** por medio de explosivos, para su transporte en carretillas ó carretas, ocupará más ó menos un volumen de 1.8 ó $1^1/_5$ m cúb; mientras que la tierra ordinaria, después de aflojada, no aumenta sino á 1.2 ó $1^1/_5$ de su volumen primitivo, aunque después de empleada en terraplenes, se reduce á menos de su primitivo volumen. Al hacer el presupuesto para el transporte de tierra, se supone que $^1/_{18}$ parte de m cúbico medida en su lugar ó antes de excavada, es carga regular para una carretilla. Un metro cúbico de dicha tierra, medida antes de ser aflojada, pesa por término medio 1.42 toneladas, por consiguiente : $\frac{1,420}{18} = 79$ kg es el peso de una carga de carretilla, ó 65 lit de tierra floja. Calculando que una carretilla de piedra suelta pesa más ó menos lo mismo que una de tierra, podemos tomarla por $^1/_{32}$ de metro cúbico, lo que da $\frac{2,300 \text{ kg}}{32} = 72$ kg por carga de piedra suelta que ocupa un espacio de 57 lit.

En la tabla que sigue, las columnas 2 y 3 se hicieron siguiendo el mismo principio que las hechas para el cálculo de transporte de tierra, como se indicó en el art. 10. La columna 4 se prepara agregando á cada cantidad de la columna 3, .2 centavo por cada 33 m más ó menos de dist de transporte, por conservación de los tablones del camino, etc., como 58 cent por m cúb medido antes de excavar, como costo efectivo por aflojar, incluyendo herramienta, taladros, explosivos, etc., lo mismo que un desagüe moderado, y cualquiera otra contingencia ordinaria, no incluída en la columna 3. Las utilidades del contratista no están por supuesto incluídas.

Amplias experiencias demuestran que con jornales de á $1, los 58 centavos asignados anteriormente por m cúb son un precio bastante suficiente por aflojar roca dura en circunstancias ordinarias. En la práctica, el costo es generalmente de 39 á 78 centavos según la posición de las capas, su dureza, tenacidad, agua y otras consideraciones. La arcilla esquistosa blanda y otras rocas semejantes, pueden muchas veces extraerse por medio del pico ó del arado, á un costo tan bajo como de 20 á 26 centavos, mientras que, por otra parte, desmontes de poca altura, en roca muy tenaz, con las capas en una posición desfavorable, especialmente en el fondo de una excavación, pueden costar $1.3 por m, y aun considerablemente más. Estos, sin embargo, son casos excepcionales, que relativamente se presentan rara vez.

Para volar roca dura ordinaria, se requiere más ó menos de 112 á 180 gramos de pólvora por m cúb medido en su lugar, antes de excavar, pero la naturaleza de la roca, la posición de las capas, etc., pueden aumentarla hasta 225 gr ó más.

La roca blanda necesita á menudo más pólvora que la dura. Un hombre práctico en el manejo de la chompa puede perforar por día de 2.4 á 3 metros de profundidad haciendo taladros de 75 cm de hondo, más ó menos, y de 5 cm de diámetro, en roca dura ordinaria y á razón de 49 á 60 centavos por metro. Los perforadores reciben jornales mayores que los trabajadores ordinarios.

**Transporte de piedra dura en carretillas.**

Jornales á $1 por día de 10 horas de trabajo.

| Distancia del bote. | Metros cúbicos, medidos en su lugar, acarreados por día, por cada carretilla. | Costo por metro cúbico, medido en su lugar, por carga, acarreo y vaciada. | Costo total por metro cúbico, medido en su lugar, sin incluir la ganancia del contratista. | Distancia del bote. | Metros cúbicos, medidos en su lugar, acarreados por día, por cada carretilla. | Costo por metro cúbico, medido en su lugar, por carga, acarreo y vaciada. | Costo total por metro cúbico, medido en su lugar, sin incluir la ganancia del contratista. |
|---|---|---|---|---|---|---|---|
| metros. | m cubs. | cts. | cts. | metros. | m cubs. | cts. | cts. |
| 7.62 | 9.32 | 14.23 | 69.41 | 182.88 | 2.26 | 46.15 | 106.21 |
| 15.24 | 8.17 | 42.75 | 71.31 | 213.36 | 2.00 | 52.13 | 112.45 |
| 22.86 | 7.32 | 44.30 | 73.06 | 243.84 | 1.79 | 58.24 | 118.82 |
| 30.48 | 6.62 | 45.73 | 74.49 | 274.32 | 1.62 | 64.35 | 125.19 |
| 45.72 | 5.55 | 48.85 | 77.74 | 304.80 | 1.48 | 70.33 | 131.43 |
| 60.96 | 4.77 | 21.84 | 80.36 | 365.76 | 1.26 | 82.68 | 149.50 |
| 76.20 | 4.19 | 24.83 | 83.98 | 426.72 | 1.10 | 94.77 | 155.54 |
| 91.44 | 3.74 | 27.15 | 87.23 | 487.68 | .978 | 106.86 | 169.52 |
| 106.68 | 3.37 | 30.94 | 90.35 | 548.64 | .879 | 118.95 | 182.13 |
| 121.92 | 3.07 | 33.93 | 93.47 | 609.60 | .794 | 131.04 | 194.74 |
| 137.16 | 2.82 | 37.05 | 96.72 | 670.56 | .728 | 143.26 | 207.48 |
| 152.40 | 2.60 | 40.04 | 99.84 | 731.52 | .671 | 155.35 | 220.09 |

**Art. 15. Transporte de piedra, de excavación, en carretas.** Una carretada de piedra puede calcularse en $^1/_6$ m cúbico medido en su lugar, antes de ser volada. Pesará por término medio 383 kg, ó solamente como 18 kg más que una carretada de tierra ordinaria. Como la carreta misma pesa más ó menos ½ tonelada, el peso total es más ó menos igual en ambos casos. Las columnas 2 y 3 de la tabla que sigue se calcularon bajo el mismo principio que para el transporte de tierra, como se indicó en el art. 4. La columna 4 se preparó agregando á la columna 3 las partidas siguientes :

Por volar la piedra (y por todo con excepción de las partidas de la columna 3. carga y conservación del camino carretero), 58 centavos por metro cúbico medido en su lugar, antes de volarla; y por reparaciones del camino, $^1/_3$ de centavo por cada 3 m de bote, no incluída la utilidad del contratista.

### Transporte de piedra dura, en carretas.

Jornales á $1 por día de 10 horas de trabajo.

| Distancia del bote. | Número de metros cúbicos medido en su lugar, acarreados por día, por cada carreta. | Costo por metro cúbico, medido en su lugar, por acarreo y vaciada. | Costo total por metro cubico, medidos en su lugar, sin incluir la ganancia del contratista. | Distancia del bote. | Número de metros cúbicos medidos en su lugar, acarreados por día, por cada carreta. | Costo por metro cubico, medido en su lugar, por acarreo y vaciada. | Costo total por metro cúbico, medido en su lugar, sin incluir la ganancia del contratista. |
|---|---|---|---|---|---|---|---|
| metros. | m cúbs. | cts. | cts. | metros. | m cúbs. | cts. | cts. |
| 7.62 | 14.67 | 8.46 | 77.48 | 548.64 | 3.82 | 32.50 | 86.08 |
| 15.24 | 14.13 | 8.80 | 77.87 | 579.12 | 3.67 | 33.80 | 107.64 |
| 22.86 | 13.59 | 9.14 | 78.26 | 609.60 | 3.53 | 35.23 | 109.33 |
| 30.48 | 13.06 | 9.48 | 78.65 | 685.80 | 3.22 | 38.61 | 113.36 |
| 45.72 | 12.22 | 10.15 | 79.43 | 762.00 | 2.96 | 41.99 | 117.39 |
| 60.96 | 11.46 | 10.83 | 80.21 | 804.50 | 2.83 | 43.81 | 119.60 |
| 91.44 | 10.16 | 12.18 | 81.90 | 914.40 | 2.54 | 48.75 | 125.45 |
| 121.92 | 9.17 | 13.52 | 83.46 | 990.60 | 2.38 | 52.13 | 129.48 |
| 152.45 | 8.33 | 14.95 | 85.15 | 1066.80 | 2.23 | 55.64 | 133.64 |
| 182.88 | 7.64 | 16.25 | 86.71 | 1143.00 | 2.11 | 58.81 | 137.51 |
| 213.36 | 7.05 | 17.68 | 88.40 | 1219.20 | 1.99 | 62.27 | 141.57 |
| 243.84 | 6.55 | 18.98 | 89.92 | 1295.40 | 1.89 | 65.78 | 145.73 |
| 274.32 | 6.11 | 20.28 | 91.52 | 1371.60 | 1.79 | 69.16 | 149.76 |
| 304.80 | 5.73 | 21.71 | 93.21 | 1447.80 | 1.71 | 72.54 | 153.79 |
| 335.28 | 5.39 | 23.01 | 94.77 | 1524.00 | 1.61 | 75.92 | 157.82 |
| 365.76 | 5.09 | 24.31 | 96.33 | 1600.00 | 1.56 | 79.56 | 162.24 |
| 396.24 | 4.83 | 25.74 | 98.02 | 2011.25 | 1.27 | 97.50 | 183.56 |
| 426.72 | 4.58 | 27.04 | 99.58 | 2413.50 | 1.08 | 115.44 | 204.88 |
| 457.20 | 4.34 | 28.47 | 101.27 | 2815.75 | 0.932 | 133.25 | 226.60 |
| 487.68 | 4.16 | 29.77 | 102.83 | 3218.00 | 0.825 | 151.19 | 247.52 |
| 518.16 | 3.99 | 31.20 | 104.52 | 3620.25 | 0.735 | 169.00 | 268.84 |

**Piedra ó roca suelta** costará como 39 centavos menos por metro cúbico, y aun la roca *sólida* costará por término medio 13 centavos menos de lo que indican las tablas.

**Art. 16. Transporte de piedra (excavada) en carros y locomotoras,** por una vía á nivel. Nuestros cálculos están basados en las suposiciones siguientes: Un tren de 10 carros que contenga cada uno $^3/_4$ m cúb de roca (medido en su sitio). Velocidad media del tren incluyendo paradas, salidas, llegadas, etc., pero no dilaciones: 16 kilómetros por hora. Aflojada de la piedra, 58 centavos (en su sitio)

Para cargar la piedra en las carretas, 10½ centavos por metro cúbico medido antes de volada. Velocidad media del tren, incluyendo salida y llegada, pero no paradas, 16 kilómetros por hora=5 millas de transporte por hora.

Aflojar la roca, 58 centavos por m cúbico, medido en su lugar. Costo de la vía, por intereses y conservación, $3 por día y por milla (1,609 m) de distancia de transporte. Los cálculos se han hecho bajo el mismo principio que los del art. 13.

### Transporte de roca dura en carros y en locomotoras.

Jornales á $1 por día de 10 horas.

| | | | | | |
|---|---|---|---|---|---|
| Dist á la cual se transporta la roca...km | 1.6 | 5.8 | 8 | 11.2 | 16 |
| Número de m cúb medidos antes de ser volados, transportados por día por cada tren. | 2,200 | 990 | 600 | 460 | 300 |
| Costo del transporte, vaciada y retorno por m cúb medido en su lugar antes de ser volado.....centavos | .8 | 2.2 | 4.5 | 7.4 | 14 |
| Costo total por m cúbico medido en su lugar antes de ser volado, sin contar la utilidad del contratista.....centavos | 70 | 71 | 73.5 | 76 | 83 |

# TÚNELES

Los túneles para ferrocarriles deben ser, **si es posible, rectos**, especialmente en los de una sola vía, por cuanto las colisiones ú otros accidentes que ocurren en un túnel son esencialmente desastrosos. Raras veces es conveniente hacer un túnel en tanto que la cortada no excede de una altura de 18 m. La roca firme de una dureza mediana y naturaleza durable **es el material más favorable para túneles**, especialmente si está libre de manantiales y si las capas están dispuestas horizontalmente. En la roca blanda ó esquistosa (aunque dura y firme al principio) ó en tierra, se hace necesario dotarlos de un revestimiento de ladrillos ó piedras con cemento. Un túnel debe tener una **pendiente ó inclinación** en una dirección, para facilitar más tarde el desagüe y la ventilación. No se requiere una disposición especial para la **ventilación** durante la construcción ó después de ella, cuando la longitud del túnel no excede de 300 m; pero excediendo de esta longitud, se recurre generalmente durante la construcción á pozos ó respiraderos, ó se proveen de los medios para introducir aire por tubos desde sus bocas. Pero después de concluída la obra, excepto en circunstancias especiales, nada de esto se necesita. Los pozos respiraderos á menudo arrastran el aire hacia adentro, y frecuentemente, aun ayudados por una pendiente escarpada y uniforme, no aseguran la ventilación. El túnel del Monte Cenis, debajo de los Alpes, concluído en 1871, tiene 12 kilómetros de longitud y no tiene respiraderos, aunque tiene pendientes en subida en cada extremidad, que es lo más desfavorable para la ventilación sin respiraderos.

Fué hecho así para facilitar el desagüe. Su ventilación se mantiene haciendo entrar el aire por las extremidades por medio artificial. El túnel de Hoosac, en Mass., de 7,200 m de longitud, tiene respiraderos, uno de ellos de 314 m de profundidad, pero fueron hechos para facilitar el trabajo. **Los respiraderos cuestan generalmente** de 1½ hasta 3 veces más el m cúbico que el túnel mismo, debido á la mayor dificultad para excavar, extraer el material y deshacerse del agua; todo lo cual se hace izando el material. Cuando los respiraderos se hacen en tierra, deben revestirse lo mismo que los túneles, y este revestimiento requiere generalmente que se apuntale por debajo. O bien puede construirse dicho revestimiento sobre el respiradero en ejecución, y después hacerse descender aquél minando por debajo gradualmente. El área de la sección varía generalmente de 4 á 9 metros cuadrados. Tienen la gran ventaja de acelerar la obra, aumentando el número de puntos donde puede trabajarse; pero si se colocan demasiado próximos unos á otros, su costo perjudica. El aire en algunos túneles, durante la construcción, se vicia más que en otros, de modo que, después de comenzado el trabajo, es conveniente establecer respiraderos de aire comprimido, cuando no se hayan hecho con anticipación. Al excavar el túnel mismo, se hace una galería ó pasaje de 1.5 á 2.4 m de altura y de 1 á 3.6 m de ancho, como avance ó adelanto, y se mantiene siempre á una distancia corta (de 3 á 30 m ó más, según la firmeza del material) delante de la obra. En túneles abiertos en roca, la galería se hace justamente debajo de la parte superior del túnel, de modo que los trabajadores puedan hacer las perforaciones en el fondo para volar la roca; pero en tierra la galería se hace en el fondo ó piso del túnel, como más conveniente para agrandar la abertura hasta el tamaño del túnel, minando por debajo y dejando caer la tierra. Cuando se hacen túneles en tierra, el techo y los lados de la galería como los del túnel deben estar sujetos, para que no se puedan hundir antes de la construcción del revestimiento, y esto se hace por medio de hileras de puntales verticales de madera y dinteles ó durmientes horizontales, entre los cuales y la tierra se colocan pedazos de tablas ordinarias para formarle techo y soportes laterales provisionales á la excavación. Los puntales y dinteles se colocan primero, y después se meten entre ellos y las caras de la excavación las tablas á fuerza de golpes de martillo. Esto se va quitando gradualmente á medida que progresa el revestimiento. Este **revestimiento**, cuando se hace de ladrillos, se ejecuta generalmente del espesor de 2 ó 3 ladrillos (40 á 50 cm), en el pie, y de 1½ á 2½ ladrillos de espesor en la parte superior, y cuando se hace de piedra bruta con cemento, se le da la mitad más de grueso. Es importante que los ladrillos ó piedras sean de una calidad excelente y dura y asentados en buen cemento. Los ladrillos deben hacerse de la forma del arco. A medida que se termina el resvestimiento por partes, y antes de quitar las cimbras, debe rellenarse completamente

y con cuidado **cualquier cavidad ó vacío** que haya entre el revestimiento y la tierra. Aun en roca se hace necesario el revestimiento si aquélla no es de naturaleza durable, como el esquisto ordinario, ó si tiene grietas. **La sección transversal** de un túnel para una sola vía, hecha en cualquier punto para carros de un ancho extremo de 3.30 m, no debe ser menor de 4.5 m más ó menos, por 5.4 m de alto; ni, en una vía doble, menos de 8 m de ancho por 7.4 m de altura; á menos que en último caso el material sea roca dura, en el cual no se requiere un arco alto para revestirlo. El techo puede entonces ser mucho más rebajado, de modo que la altura de 6 m es suficiente. Para carros de un ancho total de 3 m, el ancho del túnel puede reducirse á 7.5 m; para carros de 2.7 m de ancho, á 6.9 m. Muchos se han construído de 6.6 m solamente. El túnel del Monte Cenis es de 7.9 m de ancho por 7.60 m alto. **La relación de la marcha ó progreso diario** desde cada boca de un túnel, varía de .45 á 2.70 m de longitud en 24 horas, relevándose los trabajadores 3 veces. En el túnel del Monte Cenis, las cantidades extremas del avance ó progreso de la obra fueron más ó menos de 1.2 á 2.7 m por día durante un año entero y de cada lado del túnel. Se emplearon en la ejecución de las galerías taladros actuados por aire comprimido, teniendo dichas galerías 3.6 m de ancho por 2.40 m de alto. Ordinariamente pueden tomarse de .45 á 1 m como término medio. La diferencia de la proporción del progreso ó avance en la construcción, entre un túnel de una sola vía y uno de vía doble, no es grande como podría suponerse; por cuanto en un túnel ancho se pueden emplear más y mejor las fuerzas que en uno angosto. Cuando el túnel se construye en tierra, la construcción del revestimiento iguala más ó menos al tiempo empleado para la excavación de uno en roca. Si se hace en roca, con jornales á $1, **el costo** varía ordinariamente con la naturaleza de la roca, de $2.6 á $6.5 por metro cúbico en el túnel mismo, y de $3.9 á $13 en las galerías, mientras que los respiraderos cuestan generalmente un 50% más que las galerías. El costo de un túnel de una sola vía, cuando los jornales están á razón de $1 por día, es generalmente de $100 á $246 por metro de longitud. Sin embargo, los trabajos de un túnel están expuestos á serias contingencias que no pueden preverse. Como el techo y las paredes del túnel quedan imperfectos después de volada la roca, deben calcularse la altura y el ancho para el contratista como de .45 á .60 m mayores que el espacio neto establecido. De cualquiera manera, el modo de medir ó cubicar debe expresarse claramente en las estipulaciones del trabajo. Cuando se construye un túnel con una pendiente uniforme, el trabajo progresa en general más satisfactoriamente en la extremidad más baja, porque el descenso favorece la salida de las aguas de manantial, que generalmente se encuentran; mientras que en la boca superior dicha agua debe sacarse por medio de baldes ó bombas. La boca superior tiene, sin embargo, la ventaja de desembarazarse más pronto del humo que resulta de los barrenos. Antes de principiar un túnel, y aun antes de resolverse á hacerlo, deben practicarse **pozos ó excavaciones de prueba**, para cerciorarse de la naturaleza del material. En los túneles largos es necesario tener el mayor cuidado y exactitud en conservar la línea de dirección, á fin de que el trabajo que se ejecuta por ambas bocas se encuentre precisamente en el centro.

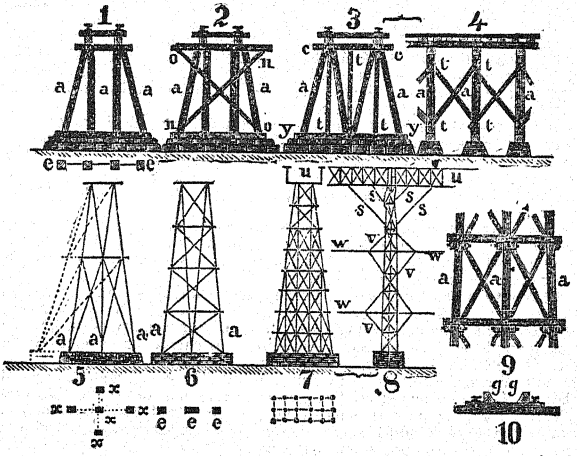
En la galería del túnel de Vosburg (Pensilvania) del ferrocarril de Lehigh Valley, construído en 1884 con una sección transversal de 2.29×7.9 m, el **progreso medio** por día de 24 horas con dos remudas de á 12 horas cada una, fué como sigue: taladrando á mano, .84 m á .72 m respectivamente en cada extremo; con taladros demáquina (dos taladros rivales en competencia), 1.7 á 2.40 m. El material consistía en piedra arenisca aplomada dura. Para todo el túnel, el progreso fué de .60 m más ó menos por día.

Para más informes respecto á túneles, recúrrase al tratado muy amplio sobre la materia del Sr. H. S. Drinker, publicado por los Sres. Wiley.

# PILAS

## (CABALLETTES, TORRES, etc.)

Las figs. 1, 2, 3, 5, 6, 7, son **elevaciones de pilas**, vistos á través de la vía ó camino. Podemos considerar la fig. 1 como adaptada á una altura de 3 á 6 m más ó menos; las figs. 2 y 3, á alturas de 6 á 9 m; la fig. 5, de 9 á 12 m; la fig. 6, de 12 á 18 m aproximadamente. Estas armazones por supuesto admiten muchas modificaciones. Están generalmente apoyadas en mampostería, como se representa en las figuras, y así se evita á las maderas inferiores el contacto con la tierra, que produce más pronto su destrucción. Es conveniente hacer estas bases suficientemente altas para impedir que las bestias y vehículos que pasen las destruyan. Hasta la altura de 12 á 15 m más ó menos, es suficiente una sola hilera de




postes ó puntales, *a*, *a*, *a*, figs. 1 á 9, como se indica en *ee*, debajo de las fig. 1 y 6; pero cuando aumenta la altura, deben introducirse más puntales, como se ve en *x*, *x* debajo de la fig. 5; ó dos hileras completas, ó tres hileras, como se indica debajo de la fig. 7 y también en la fig. 8, que es una vista lateral de la fig. 7. Las figs. 7, y 8 se parecen mucho á los caballetes de 58 m de altura, con bases de mampostería de 9 m de alto (S. Seymour, I. C.) que sostenían el ferrocarril de Erie (ahora el ferrocarril de Nueva York, Erie, etc.) sobre **el río Genesee, en Portage, N. Y.** En dicho puente cada pila tenía 21 puntales de 35 cm en cuadro en su base, y 15 postes de 30 × 30 cm en su parte superior. Las otras maderas eran de 15 × 30 cm, muchas de ellas en pares abrazando los postes. Este viaducto de una sola vía se principió el 1.º de julio de 1851 y se concluyó el 14 de agosto de 1852. Contenía 3,790 metros cúb de madera y 49 toneladas de hierro. En sus cimientos había 7,000 m cúb de mampostería. Su costo total fué de $140,000 más ó menos. Se quemó en 1875, y fué reemplazado, en menos de tres meses. por un **viaducto de pilas armadas de hierro** para una sola vía que contenía por todo 603 tons de hierro y 309 metros cúbs de madera; costando por todo, exceptuada la mampostería, $95,000 más ó menos. A menudo se colocan los postes de las pilas en pares; y las otras maderas pasan por entre ellos, todo asegurado con pernos.

En la fig. 4 los postes *a*, *a*, *a*, son vistas de extremo de los tres postes, seme-

jantes al de la fig. 3, y *tt*, son brazos diagonales que se prolongan de un poste á otro. Pueden colocarse ó en el intermedio de los puntales, como en la fig. 3, con las cabezas de los dos exteriores terminando en el dintel, *cc*, de una pila, y sus pies fijos al durmiente, *yy*, del que sigue; ó todos ellos pueden clavarse ó asegurarse por medio de pernos á los puntales mismos, como en la fig. 4. Esto último es lo mejor, porque sirve además para reformar los puntales, como lo hacen también las crucetas *oo*, *nn*, fig. 2; pero éstas se omiten á menudo. Durante el pasaje de un tren, la presión del vapor hacia atrás, ejercido por medio de las ruedas motrices contra la vía, produce un empuje serio en el sentido de la vía, que tiende á volcar las pilas, y la aplicación repentina de los frenos á un tren en marcha produce un esfuerzo ó empuje semejante en la dirección opuesta. Estos esfuerzos son más peligrosos con el aumento de altura de las pilas ó caballetes. Ordinariamente los puntales exteriores pueden tener una inclinación de 12 á 20 por ciento.

Los puntales ó postes no deben tener menos de 30 cm en cuadro, excepto en pilas muy bajas, y aun entonces no menos de 25 × 25 más ó menos. Las diagonales pueden ser tan anchas como los postes y más ó menos de la mitad de su espesor. La distancia de una pila á otra, cuando la calzada está sostenida simplemente por vigas longitudinales, no debe exceder de 3 á 3.6 m en los viaductos para ferrocarriles; pero si estas vigas están soportadas también por tornapuntas por debajo, como *ss*, en la fig. 8, ó por tirantes armados de hierro, puede aumentarse la distancia á 4.5, 6 ó más metros. Pero cuando las pilas son muy altas, y contienen una gran cantidad de madera, es más económico ponerlos más distantes unos de otros, digamos de 9 á 18 metros; y apoyar el ferrocarril sobre vigas armadas regulares como en *uu*, fig. 7 y 8, como en un puente con pilas de mampostería. En el viaducto de Genesee, las pilas armadas estaban colocadas á una distancia de 15 m de centro á centro.

Cuando estos caballetes, como el de la fig. 8, son demasiado delgados en proporción de su altura, se puede aumentar su estabilidad agregándoles vigas *ww* que pasan de una á otra; y aún más, poniéndoles refuerzos diagonales *vv*, como en el antiguo viaducto de Genesee.

Arréglense las piezas, en los posible, para que se puedan cambiar por nuevas al deteriorarse.

**En las curvas** se debe aumentar la resistencia hacia el lado convexo, como lo indican las líneas de puntos en la fig. 5. En las pilas muy altas especialmente (así como en los puentes) no se deben omitir jamás las guardarruedas, tanto al interior como al exterior de los carriles.

**En terrenos pantanosos**, se pueden hacer estacadas para sostener las pilas, ó dejarse fuera del suelo para que ellas mismas formen los postes. Estas pilas se pueden emplear á menudo ventajosamente, aun cuando después queden cubiertas por los terraplenes. De esta manera sostienen los carriles en su propio nivel hasta que el terraplén ha alcanzado su asiento final.

Se emplean generalmente para evitar los gastos de terraplén, especialmente cuando la tierra se encuentra lejos. Aun cuando es tan conveniente la tierra como la madera, rara vez cuestan más de la mitad que los terraplenes, hasta con una altura de 9 m. Pero debido á su fácil deterioro deben emplearse tan sólo en casos de necesidad, ó como expediente provisional.

# MATERIAL RODANTE

## GENERALIDADES

**1. Características generales** y detalles comunes a ambos, locomotoras y carros. Bajo esta denominacion tratamos de dar solamente una información general tal, que pueda ser útil a los ingenieros *civiles* a quienes interesa sobretodo la planta *fija* de los ferrocarriles.

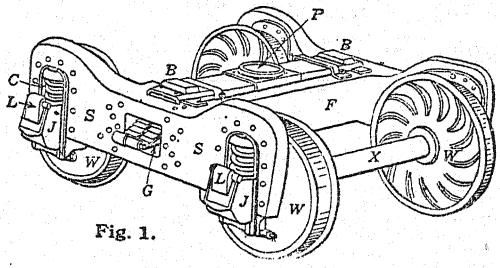



Fig. 1.

**2. Carretillas.** Una carretilla, fig. 1, es un marco en el cual están colocadas las cajas de las chumaceras, *C*, de los ejes y que soportan el cuerpo del carro. En los E. U., se hace rara vez que la carretilla soporte el vehículo directamente, de modo rígido, sino que casi siempre está conectada al carro o locomotora por medio de un pivote, *P*, que permite un juego para adaptarse a los cambios en la direccion de la via. Las piezas laterales, *S S*, de la carretilla, se hacian anteriormente armadas de fuertes planchuelas de acero, formando una armadura simple; hay todavia muchas en uso. En equipos mas recientes se usan con frecuencia armaduras de acero prensado (que actúan como vigas) y también armaduras de acero fundido, cuya plantilla está hecha por las antiguas armaduras de planchuelas.

**3. El pivote,** *P*, está en el centro de la carretilla, excepto en tipos especiales de carretillas de locomotoras, y en algunas carretillas de carros eléctricos donde se usa un solo motor y un par de ruedas y donde el pivote se coloca casi cerca de las voladoras, para darle mas « adhesion ».

**4. Soportes laterales.** La carga es generalmente soportada por platos en el pivote, y solamente las cargas debido al movimiento del carro son soportadas por las placas, *B, B;* pero en algunos tipos que se usan ahora, todas las cargas son soportadas por los laterales, mientras que el pivote no soporta ninguna.

**5. Muelles.** Se colocan elípticos, planos, o espirales, entre el marco principal, *F*, y el puente, *B P B*, que soporta el plato del pivote. Para carros de pasajeros, se coloca cuando menos un muelle espiral entre cada caja de chumacera y el marco de la carretilla. Los muelles espirales se hacen en mazos de dos o más, uno dentro del otro, y estos mazos se usan tambien en pares, lado a lado, o en grupos de cuatro o más, especialmente cuando se usan entre el marco de la carretilla y el cuerpo del carro. Los muelles semi-elípticos se usan en locomotoras.

**6. Cajas de Chumaceras.** Los muñones giran en las cajas de chumacera. El asiento se hace por su puesto sobre la parte superior del muñon. Cada caja debe ser bastante grande para contener un aprovisionamiento de estopa engrasada para lubricacion. Una tapa, *L*, permite el acceso, y evita mucho la entrada de la basura. Las chumaceras de rodillos se han usado en la tracción eléctrica.

**7. Muñones.** Los muñones están en los extremos de los ejes, fuera de las ruedas en las carretillas de arrastre de dos ruedas de las locos; en los ejes de las voladoras de las locos, están entre las ruedas voladoras. Los muñones varian 3 $^3/_4$ a 6 plgs (95 á 152 mm,) en diam y de 7 á 11 pulgs (178 á 279 mm.) en largo. Los muñones de carretillas de arrastre son hasta de 8 × 14 pulgs (203 á 356 mm.).

## Ruedas.

**8. Materiales.** Las ruedas de hierro fundido en las que la llanta y la pestaña han sido enfriadas para aumentar la dureza de la superficie, se usan todavia mucho en el servicio de carga; las de acero ensunchadas, casi se usan exclusivamente en el servicio pesado de pasajeros.

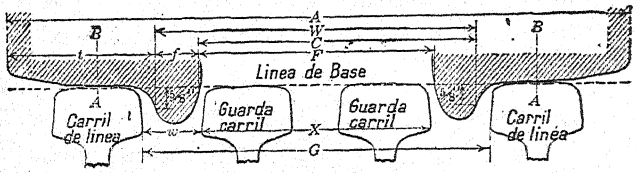



**Fig. 2.**

**9. Llanta y pestaña.** La fig. 2 muestra las ruedas de hierro fundido tipicas de los maestros constructores de carros, en relacion con la cabeza del carril, incluyendo los guardacarriles como en fig. 2.

$A$ = cartabón total = 5'4 $^{3}/_{8}$" (1,635 m);
$C$ = cartabón de comprobación = 4'6 $^{29}/_{32}$"; (1,382 m);
$f$ = grueso de pestañas = 0'1 $^{15}/_{32}$" (0,031 m);
$X$ = cartabón de guardacarril = 4'5" (1,346 m);
$G$ = cartabón de vía = 4'8 $^{1}/_{2}$" (1,435 m);
$W$ = cartabón de rueda = 4'7 $^{11}/_{16}$" (1,413 m);
$F$ = cartabón de pestaña = 4'5 $^{7}/_{32}$" (1,351 m);
$t$ = llanta = 0'4 $^{11}/_{32}$" (0,110 m);
$w$ = vía de pestaña = 0'1 $^{3}/_{4}$" (0,044 m);

El diámetro de la rueda se mide en la linea de centro, $A$ $B$, del carril.

**10. El tipo o diám usual**, para todas las ruedas de carros (0,788 m) 33 pulgs. Peso de cada rueda 281.22 kg (620 lbs) para carro de 30 ton de capacidad, á 326.60 kg (720 lbs) para 50 ton.

**11. Conicidad de la llanta.** Con el objeto de disminuir las resistencias que presentaria una rueda de llanta cilindrica, cuando pasa por las curvas (véase Resistencia de Tren, ¶ 36, p. 000), la transición, desde la llanta a la pestaña de la rueda se hace por medio de una curva, de cerca de una pulg (25 mm) de radio, que usualmente mantiene el lado de la pestaña separado del lado de la cabeza del carril; y la llanta de la rueda se hace en forma cónica (con su diámetro mayor junto a las pestañas), con el objeto de que las ruedas *exteriores* (con tendencia a resbalar siempre hacia fuera del carril) puedan asi girar sobre sus diámetros *mayores*, y las ruedas *interiores* sobre sus diámetros mas *pequeños* : pero debido al tráfico el cono se va reduciendo, y aun llega a invertirse; el diam de la rueda, próximo a la pestaña, algunas veces llega a ser menor que el mas distante de la pestaña, antes de 1907 el cono tipo de la Asociacion de Maestros Constructores era $^{1}/_{38}$. En 1907 el cono tipo se aumentó a $^{1}/_{20}$. El F. C. de Pennsylvania encontró que la rotura de pestañas de rueda disminuyó mucho con el aumento del cono. (Véase Ry Age Gaz, 1912 Jul 19, p. 92.)

**12. Zunchos.** El objeto del zuncho es dar mayor resistencia a las ruedas y resistir el desgaste. Los zunchos van : o forzados sobre las ruedas por reducción de temperatura, o asegurados en una u otra forma de los muchos y diferentes modos que hay de hacerlo, o de ambos modos.

## Frenos.

**13. Un zapato de Freno** es una pieza de metal que por un lado conforma con la llanta y pestaña de la rueda. Ordinariamente, se provee para cada rueda un zapato de freno; pero los frenos de « abrazadera » provistos de dos zapatos de freno, uno a cada lado de la rueda, presentan grandes ventajas. Los dos (o cuatro)

zapatos para un par de ruedas están conectados por *vigas de freno* (con frecuencia armadas para darle mayor resistencia) colocadas paralelamente al eje, entre los dos pares o afuera, segun convenga; o de ambos modos como en el freno de « abrazadera ». Los *contrapesos colgantes de los frenos* están conectados a las vigas de los frenos y carretillas de tal modo que recojan las fuerzas producidas por la friccion de los zapatos contra las ruedas y sostengan los zapatos en su lugar. Un sistema de *Palancas de Freno* distribuye debidamente la presión del freno a todas las ruedas, y conecta las vigas del freno con el origen de la fuerza activante; por ej. : con el eje de la rueda de mano o con el piston del cilindro de aire, o con ambos.

**14. Frenos de aire.** Para la acción de los frenos por aire comprimido, cada carro está provisto de un *cilindro de Freno*, colocado con un émbolo conectado con el sistema de palancas de freno. Cada carro va tambien equipado con un *Tubo de Freno* que corre a todo su largo. El tubo del freno tiene una manguera flexible en cada extremo, y conecciones por donde toda la linea puede unirse, y también con uno o dos *Compresores de aire*, movidos por vapor, que van en la locomotora. Una *válvula de Freno* colocada en la casilla de la locomotora, permite al maquinista manejar los frenos de la locomotora y de todo el tren.

**15. Freno de aire « Directo. »** En este freno, el aire comprimido pasa desde el compresor (o de un recipiente de aire comprimido) en la locomotora, por la válvula de freno del maquinista, y el tubo de freno, directamente a los cilindros de los frenos y los aplica. Su acción, no obstante, no es bastante rápida para trenes largos, y tiene la desventaja mayor de que cualquier falla en la provisión de aire, como una manguera rota, resulta un fracaso de todo el sistema; y esto tanto mas propenso a ocurrir cuanto mas se le exige al sistema.

**16. El freno de aire « automático »** prácticamente elimina estas objeciones. Cada carro tiene un *Recipiente Auxiliar* de aire comprimido, y está provisto con una válvula especial de muchos detalles, conocida como la « *Válvula Triple* ». Obra por diferencias de presión entre el tubo del freno y el recipiente auxiliar del carro. La característica esencial es que una reducción de presión en el tubo del freno (causada por la acción de la valvula de freno del maquinista, por una válvula de emergencia en el carro, o por alguna falla en la linea del tren) obra sobre cada válvula triple de modo que el aire pase por ella desde el recipiente auxiliar de cada carro al cilindro del freno. Para soltar los frenos, el maquinista pone su válvula de freno en posición tal que restablezca la presion normal en la linea del tren; asi que esto sucede la válvula triple permite el escape del aire de cada cilindro de freno, y los muelles apropiados aflojan los zapatos de freno de las ruedas. La valvula del maquinista tiene varias posiciones, por medio de las cuales los frenos pueden (1) aflojarse gradualmente (2), sostenerse todos como se hayan aplicado (3), aplicarse gradualmente, o (4) aplicarse instantaneamente.

**17. « Freno de alta velocidad ».** En éste las presiones empleadas son casi completamente suficientes para calzar las ruedas á pesar del relativamente bajo coeficiente de fricción de los frenos a alta velocidad (vease « Friccion », p. 433). Pero, a medida que la velocidad del tren se reduce, el aire se deja escapar gradualmente, de modo que disminuye el rozamiento de las ruedas a medida que el coeficience de fricción aumenta.

**18. El control eléctrico** de las válvulas, se usa mucho y ha sido empleado para trenes de vapor muy largos donde aun el freno de « aire automático » suele ser lento en su operación. En los carros eléctricos obra un compresor de aire en cada carro accionado por un motor eléctrico en ese carro.

**19. Los frenos electricos** se han usado mucho en tranvías electricos. En el freno eléctrico Westinghouse se hace que los motores del tranvía generen electricidad que magnetiza un zapato que va directamente sobre cada carril. El zapato entonces se pega al carril, y el arrastre que resulta fuerza los zapatos de los frenos contra las ruedas. Los frenos eléctricos, que dependen de la rotación de los motores, no pueden sostener un carro que esté parado en una pendiente.

**20. Nuevo freno de alta velocidad.** La Cia de Frenos de Aire Westinghouse junto con el F. C. de Pennsylvania (S. W. Dudley, auxiliar del Ing Jefe Soc Am de Ings Mec, N. Y. Seccion, 1914 Feb 10) probaron en todas formas un freno nuevo de aire, con los siguientes resultados:

Tren de 12 carros de pasajeros, de acero y loco moderna; largo total cerca de 300 m; peso total cerca de 1000 tons.

| | | | | |
|---|---|---|---|---|
| Velocidad en kilómetros por hora... | 48 | 96 | 128 | |
| Freno ordinario paro el tren en..... | | 480 a 540 | | metros. |
| — nuevo — — ..... | 60 | 300 | 600 | — |

**21. Frenos de vacio.** No se han empleado mucho. Su modo de obrar es casi el mismo que el del « freno de aire » (comprimido). Como es por consiguiente imposible obtener presiones de mas de 1 kg por cm cuad, se necesitan cilindros de freno muy grandes; pero el aparato tiene la ventaja de que, en caso de accidente del servicio de aire, los frenos se aplican automáticamente, sin el uso de la « valvula triple ».

## Miscelánea.

**22. Las señal de aire y el aprovisionamiento de vapor** en los trenes de pasajeros se hacen por medio de lineas de tubos separadas y tendidas a lo largo del tren, cada una conectada entre los carros y locomotoras, por conexiones de manguera. El principio fundamental de la señal de aire es parecida a la del freno de aire automático, en el que la presion del aire se mantiene normal en la tuberia, y el trabajo (en este caso el soplido del pito, en la casilla), se hace por el escape de aire, (1) efectuado por una válvula sobre la que obra una cuerda de señal en el carro; o (2) de un salidero, como el que puede ocurrir por la rotura del tren en dos.

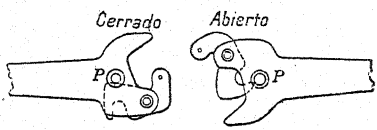



Fig. 3

**23. Enganches.** Se dispone uno en cada extremo del carro, simetricamente con respecto al eje vertical por el centro del carro (la quijada abierta con la cara a la izquierda cuando uno está parado en el terreno de frente al carro), asi que carros y locos pueden siempre engancharse juntos sin invertir el carro ni el enganche. La fig. 3 muestra dos enganches casi enganchados; uno abierto y uno cerrado. Las *muelas* van trancadas por medio de los pasadores, *P*, *P*, de tal modo, que con cualquiera *muela* abierta, los carros se conectarán cuando se junten, sin posterior atención para el personal del tren, tambien, cualquier enganche puede abrirse, y los carros desengancharse, por medio de una palanca en una esquina del carro, sin necesidad, para el personal del tren de estar entre los carros. No obstante las mangueras del tubo del tren deben conectarse y desconectarse por un hombre que si tiene que estar entre los carros; pero se han introducido mejoras por las cuales esto puede hacerse automaticamente sobretodo usando tracción eléctrica de unidad multiple. Los enganches son movibles en dos planos para poder girar en las curvas.

**24. Estos enganches** forman una combinación, en cada extremo del carro, de una barra longitudinal corrediza, con muelles para soportar el choque cuando arranca. Los muelles hacen posible también aproximar, el arranque del tren a la distancia de un carro a un tiempo lo que es importante, porque las resistencias de arranque son mayores que las del recorrido normal. En el *Enganche de Friccion*, se usan accesorios de fricción que sirven para absorber el rechazo de los muelles, evitando así la acumulación de los choques rítmicos repetidos.

## Sección libre.

**25. En los puentes,** véase pag 807. En túneles, la Asen Am de Ings de F. C. ha recomendado las secciones indicadas en la fig. 4, entre el centro de la via y el costado del túnel.

**26. En las curvas** se necesita una sec libre adicional y puede ser calculada por las siguientes fórmulas dadas por Jorge Paaswell, Ingeniero auxiliar, de la Comisión del Servicio Público de la ciudad de Nueva York, en la revista Ingeniería y Contratos, 1914, marzo 25, p. 367. En la fig. 5, sea :

$a$ = dist entre centros de carretillas;
$L$ = largo del carro;
$c$ = dist desde el medio de la linea de centro del carro al centro de la curva de la via;
$e$ = dist desde la linea de centro en el extremo del carro, al centro de la curva de la via;
$R$ = radio de la curva.

Son preferibles todas las unidades en pies.
Entonces, $c = a^2/8\ R$ ; y $e = (L^2 - a^2)/8R$.
Vease también esta materia en curvas, pag. 807.

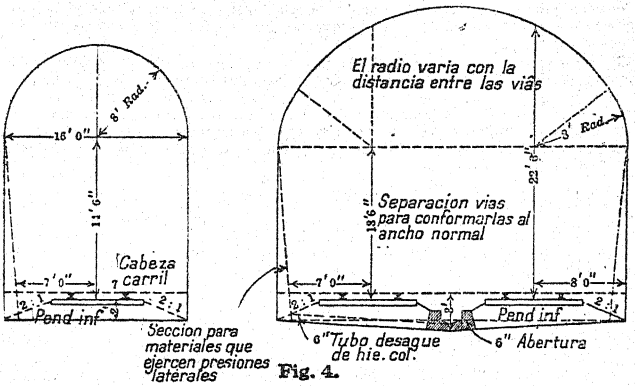



Fig. 4.

**27.** Where

**27.** Donde se usen **curvas de transición o « espirales »**, el radio puede tomarse, como el de la curva en el medio del carro. Para el radio de la curva de transición, $R_t$, en cualquier punto, tenemos $R_t = R_c\ T/t$ ; donde $R_c$ = radio de la curva circular; $T$ = largo de la curva de transición; $t$ = dist del punto desde el comienzo (T. E.) de la curva de transicion.

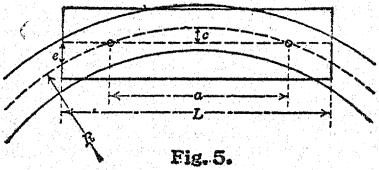



Fig. 5.

**28. Tumba del carro.** En adición a esto, debe darse margen, en el lado interior de la curva, cuando menos, para el ladeo del carro debido al peralte del carril. Esto, por supuesto, puede calcularse facilmente, dando el peralte, ancho de via, y altura del alero del techo u otro dato de influencia.

**29. Unas fórmulas mas precisas** y diagramas detallados para estos cálculos los da Mr. Frank H. Carter en Ingenieria y Contratos, 1913, dec 10, pp. 650-2.

## LOCOMOTORAS

**30.** En la siguiente descripción de una loco hemos seguido aproximadamente, el orden en que la energia se trasmite. Véase la fig. 6, tipo de una loco moderna donde se muestran las relaciones generales y proporcion de sus diversas partes.

**31. El tender** (no mostrado) es un carro corto de dos carretillas, con estanques para agua, y espacio para carbon (o estanques para aceite combustible). Se usan placas divisorias dentro del estanque para evitar movimientos violentos del agua. Algunas veces, especialmente en el servicio de carga y cortes, el agua y el combustible se llevan directamente sobre la locomotora, donde sirven para aumentar con su peso la « adhesión ». Los tender llevan algunas veces un cucharón embisagrado que puede bajarse casi a nivel de las traviesas, dentro de una canal tanque, véase, pag. 1090, y entonces, por virtud de la velocidad del tender, hace que el agua se levante desde el estanque canal por el tubo que lleva el agua al tender del estanque. En todos los casos, el agua se lleva a la loco por una manguera de conexión.

**32. Hogar.** El carbón pasa por la puerta de fuego (a veces movida por la misma máquina) al hogar donde cae sobre las parrillas (promedio de superficie 37 pies cuad *) (como 3,50 m cuad) que están hechas de barras de hierro fundido. Un cenicero está suspendido bajo las parrillas, para evitar que las cenizas caigan sobre la via. **Los alimentadores mecanicos de carbon** se están usando en las locos pesadas. Su principal ventaja es que ponen el carbon en el fuego mucho mas rápidamente que el fogonero.

**33.** Los gases calientes del fuego pasan por **los tubos** (no mostrados) y que practicamente llenan la caldera y van a lo largo de ella. (Promedio de superficie de calefaccion 2,422 pies cuad *. En el extremo del frente, los gases calientes se descargan de los fluses en la « **caja de humo** », y de ahi hacia arriba por la **chimenea.**

**34. La caldera** se extiende desde la caja de humo en el frente, a la caja de fuego o fogon atrás, y abarca los costados y parte de arriba del hogar. El fogon u hogar y los tubos estan rodeados enteramente de agua, dando asi una gran superficie de calefacción. El agua se introduce en la caldera por un **Inyector** (no mostrado). La altura del agua en las calderas, puede observarse por el **Nivel**, probarla y registrarla por medio de las **Llaves de prueba** (no mostradas). También hay un **manómetro** y una **válvula de Seguridad, Tarugos Fusibles,** etc. (no mostrados) todo se usa lo mismo que en las calderas fijas.

**35. La cúpula de vapor** está en la parte superior de la caldera, y en ella se aloja el vapor mas caliente y seco. De la cúpula, el vapor se puede hacer pasar por la Válvula de Cuello (manejada por una palanca en la casilla y por el « *Tubo Seco* ») grande (indicado en la fig. con lineas de puntos) hacia las cajas de vapor en los cilindros.

**36. Supercalentadores** se usan ahora en todas las locomotoras nuevas. Despues de dejar el vapor la cúpula, pasa en la caja de humo por muchos tubos pequeños, que presentan una gran superficie externa de calefaccion, y reciben los gases calientes que se escapan en los extremos del frente de los tubos. El vapor asi es recalentado a una temperatura mucho mas alta que la que corresponde a su presión cuando está en presencia del agua. La economia estriba principalmente en que no se hace la condensación en los cilindros, a pesar de la caida de una parte de la presión debido al paso por tantos tubos pequeños. La economia llega, en máquinas simples, hasta a un 25 %, con temperaturas de cerca de (600° F) 315° C.

**37. Un recalentador** es practicamente un supercalentador colocado entre los cilindros de alta y baja presión de una maquina compuesta.

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* Promedio ó total para las locos en los E. U., junio 30, 1914 (excepto las loco en servicio de los llamados pequeños ferrocarriles y de compañias terminales, Mallet y locos sin clasificar), segun la Comision Interior de Comercio « Estadisticas de F. C. en los E. U., 1914.

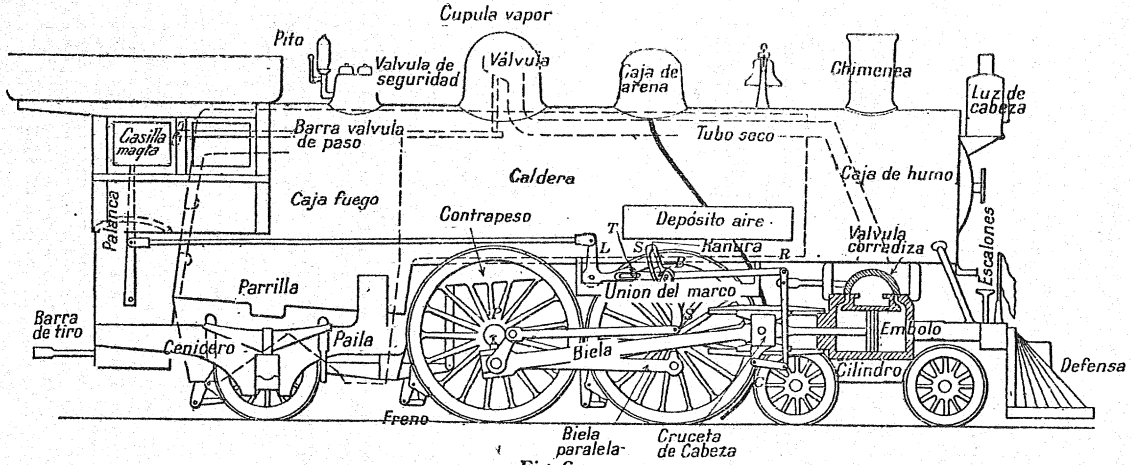


**Fig. 6.**

El aparejo de válvula de distribución Walschaerts está indicado arriba en esquema. Dos movimientos « simples, armónicos » se resuelven en uno, y se trasmite a la válvula de distribucion, por la barra vertical que tiene su parte superior en *R*. (1) El movimiento horizontal en *C* viene directamente desde la cruceta. (2) El movimiento horizontal en *R* está determinado (*a*) por el movimiento del cuadrante oscilante ranurado, *S S*, y (*b*) por la *posicion* de la palanca, *L*. El bloque del cuadrante, *B*, está pivotado en la barra *T R*, y puede deslizarse hacia arriba o hacia abajo en la ranura *S S*, y puede ser sostenida a cualquier nivel por la palanca de cambio. Entonces la barra *T R* recibe del cuadrante *S S*, un movimiento, hacia alante o hacia atrás, grande o pequeño, con cualquier movimiento dado y que depende de la posición del bloque *B*. Como está dibujado arriba, la posicion de *B* es opuesta al pasador de *S S*, y no tendria por consiguiente movimiento horizontal. En la práctica, la ranura y pasador *T*, están a veces sustituidos por un cuandrante vertial, no indicado conectando el extremo hacia adelante de la biela (mucho más alto de lo que se ve), con el extremo de atrás de la barra horizontal *TR*.

**38. Cilindros.** De el « tubo seco » o desde el supercalentador el vapor pasa por las *cajas de vapor* y las *Válvulas de Corredera* a los cilindros, donde empuja los émbolos y sus barras. Hay una válvula pequeña llamada *de purga* del cilindro (no mostrada), en cada extremo de cada cilindro, que se maneja desde la casilla permitiendo al maquinista botar el agua que se deposite en ellos. También hay válvulas atmosféricas en los cilindros, que permiten el libre movimiento de los émbolos atrás y adelante cuando la válvula del cuello está cerrada.

**39. Válvula de distribución.** Las valvulas que admiten vapor en los dos extremos de cada cilindro alternativamente son planas o cilíndricas. Varios sistemas se emplean tanto para equilibrar las presiones del vapor contra las válvulas como para reducir la fricción. Las válvulas han sido hasta ahora generalmente movidas por el movimiento « *Stephenson* » de excéntrica y cuadrante muy corriente en máquinas fijas. En el aparejo Walschaerts, que está generalizándose, las principales ventajas son debidas a su situacion enteramente fuera de las motrices o voladoras que lo hace accesible para aceitarlo, etc., y que permite trabarlo y cruzarlo mas con mas firmeza del marco o bastidor de la loco, véase p. 1.117.

**40. El cambio y corte** esta manejado por la palanca de cambio en la casilla. En locos pesadas se están generalizando los aparejos de rosca operados a mano, y también por fuerza artificial. Son de varios tipos diferentes.

**41. Las barras de los émbolos** son guiadas por las « **crucetas** », y van conectadas a la bielas de las motrices (voladoras) por « **barras de conexion** » (bielas). Las bielas, en el lado derecho e izquierdo de la loco, están colocadas a ángulo recto entre si, sobre el eje; de modo que cuando una biela esté en « punto muerto », e incapacitada de hacer fuerza y girar la otra produce su efecto máximo. Los varios pares de motrices van conectados entre sí por « **barras paralelas** », a cada lado de la loco.

**42. Contrapesos.** Los contrapesos van colocados en las motrices opuestas al pasador o muñon de la biela, para equilibrar el peso de las partes que giran. En locos de dos cilindros, se agrega un exceso al contrapeso, para neutralizar el efecto de las partes recíprocas, que de otro modo producen en la loco esfuerzos peligrosos, cuando corre a alta velocidad, en un plano horizontal. Algunas veces se usan cuatro cilindros, y cada émbolo va a una biela separada, y las dos bielas en el mismo lado de la loco van colocadas a 180° de separación, de modo que sus partes recíprocas se equilibren mas con las otras; así no se necesita exceso de peso.

**43. Pestañas.** Anteriormente, en locos de tres o mas pares de motrices solo las llantas del frente y los de atrás en cada lado tenian pestañas; las voladoras (motrices) intermedias se dejaban sin pestaña o « planas », para facilitar el paso por las curvas : pero se está generalizando ponerle pestañas a todas las llantas de las motrices, y se facilita el paso por las curvas dándole a las motrices de atras y delante mayor « juego », entre la pestaña y la cabeza del carril.

**44. El marco o bastidor** (no mostrado) lleva la caldera, fogon, casilla, cilindros y la mayor parte de los accesorios. El peso se trasmite del bastidor por medio de los muelles y por un sistema de palancas, que distribuyen la carga, como se desee, sobre los ejes y cajas de las voladoras y sobre las ruedas de la carretilla.

**45. Auxiliares.** Ademas de los aparatos usados en las máquinas fijas, la loco tiene un *Pito* de vapor; una *Campana* (algunas veces movida por aire comprimido), y una *Farola* de aceite acetileno o eléctrica. La farola eléctrica, a pesar de su fuerza para alumbrar la vía, es de dudosa ventaja debido a la brillantez de sus reflejos blancos sobre las cristales de señales de color, y al efecto de encandilar al personal del tren que llega en direccion opuesta. Algunas veces las farolas van en pivotes y giran automáticamente cuando pasan por las curvas; o a voluntad del maquinista. También hay *Escalones* y *Pasamanos* para subir a la locomotora y para llegar a varias partes de ella; *Grampas* para bandera y farol : luces de casilla y otros accesorios; una « *Defensa* » (para apartar obstrucciones accidentales); y una *Caja de Arena* con válvula y tubo, para llevar la arena sobre el carril directamente en frente de las ruedas motrices para aumentar la adhesion.

**46.** En locos de **expansion** (1.333 4 cyl de expansion, y 659 2 cilindros de expansion, entre el total de 63,510 locos *), la expansion se efectua generalmente al través de dos o, cuatro cilindros. Las areas de alta y baja presión de los émbolos son de 1 a 2.5 aprox. Donde se empleen dos cilindros, el cilindro de alta presión va colocado a un lado de la loco, y el de baja presión en el otro; y los dos estan conectados por un gran « *Tubo Colector* », provisto de una « Valvula de Incomunicar » por la cual el vapor a alta presión puede admitirse a voluntad directamente de la caldera al cilindro de baja presión, para hacer arrancar la loco cuando el cilindro de alta esté en « punto muerto » o para hacer temporal y anormalmente un gran esfuerzo. Donde se usen cuatro cilindros van colocados un cilindro de alta y otro de baja presión a cada lado de la loco, dispuestos uno detrás del otro accionados por la misma barra de émbolo o apareados uno al lado del otro, el cilindro de alta va dentro, o bien uno encima del otro. Cuando van apareados uno al lado del otro, cada émbolo actúa sobre una biela separada, cada biela va colocada a diferente cuadrante. Esto logra casi un equilibrio perfecto entre las partes recíprocas. La economia en combustible y agua lograda por la expansión dispuesta de este modo varia de 10 a 25 por ciento, segun el servicio que se haga. La economia se pierde a menudo, por lo menos en parte por el aumento del costo de reparaciones, debido a la mayor complejidad de esta disposición.

## Datos sobre las Locomotoras.

**47. Las siguientes tablas** dan datos extremos aproximados de la construcción de loco alrededor de 1913 segun los catalogos de los manufactureros.

**48. Práctica general.** Como lo indican las listas de tipos de locos en uso por el F. C. de Pennsylvania, varian desde un mínimo alrededor de la mitad entre nuestro tipo maximo y mínimo hasta un maximo igual o que ligeramente excede a nuestros máximos en las tablas. La loco corriente en uso, por consiguiente, cae bajo nuestro máximo, alrededor de $^1/_4$ (o $^1/_5$ si incluimos los F. C. pequeños) de la diferencia entre nuestros maximos y mínimos. Promedio de peso de las loco, excluyendo el tender, cerca de 166.000 * lbs (como 75.300 kg.).

**49.** La columna encabezada « **Clase** » da (1) el número de ruedas bajo la loco, segun el sistema Whyte, el primer número indica el número de ruedas de la carretilla al frente, el segundo número es el de las motrices, y el tercero el número de ruedas que siguen; y (2) la nomenclatura de los Talleres de Locomotoras « Baldwin ».

**50.** Bajo « **Servicio y Tipo** » se da (1) la clase de servicio y (2) el nombre popular comercial del tipo.

**51. Base de rueda** es la distancia a lo largo del carril, entre los puntos de contacto con los carriles de las ruedas del frente y de atrás.

**52. La Fuerza de tracción**, dada en cada caso, es la fuerza horizontal que la loco puede ejercer, siempre que las ruedas no resbalen. La que se calcula por la fórmula siguiente para locos simples, es aproximada. Es buena para velocidades hasta cerca de diez millas (16 kilometros) por hora.

$$\text{Fuerza de tracción} = \frac{d^2\, S\, (0{,}85\ P)}{D} \ (\dagger) = \text{usualmente entre } 0{,}22 \text{ y } 0{,}25 \text{ del}$$

peso en todas las motrices.

$d$ = diámetro, de un cilindro, en pulgs;
$S$ = carrera del émbolo, en pulgs (en un sentido);
$P$ = presión de la caldera (a la cual estan adheridas las válvulas de seguridad), lbs en por pulg cuad;
$D$ = diámetro de las motrices en pulgs.

Para locos de expansión de dos cilindros, úsese 0,60 en lugar de 0,85, y hágase:
$d$ = diámetro del cilindro de alta presion.

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* Promedio o total para todas las locos en los E. U., 1914, junio 30 (excepto locos en el servicio de los llamados pequeños ferrocarriles y los de compañias terminales, Mallet y locos sin clasificar, según la comision de comercio entre estados. « Estadísticas de F. C. en los E. U. » 1914.

(†) (*N. del T.* — Poniendo el diám del cilindro, la carrera del émboly el diam de las motrices en centímetros y la presión d. la caldera en kilogramos por centímetros cuad la fuerza de tracción viene en kilogramos.)

Para las loco de cuatro cilindros de expansion, la formula se convierte en :

Fuerza de tracción $= \frac{d_l^2\, S\, (1.6\, P)}{(d_l^3/d_h^2 + 1)\, D}$ en la cual $d_l$ y $d_h$ son los diametros en pulgs de los cilindros de alta y baja presión respectivamente.

**53. Fuerzas de tracción a altas velocidades.** Los siguientes datos los suministra la American Locomotive Co en la tabla 9 de su Boletin n.º 1.002. Multipliquese el esfuerzo de traccion, calculado por la fórmula, por el correspondiente factor dado en la siguiente tabla :

| Velocidad del embolo en pies por min | 300 | 350 | 400 | 450 | 500 | 550 | 600 | 650 | 700 | 750 | 800 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Factor..... | .954 | .908 | .863 | .817 | .772 | .727 | .680 | .636 | .590 | .550 | .517 |
| Velocidad del embolo en pies por min..... | 850 | 900 | 950 | 1000 | 1100 | 1200 | 1300 | 1400 | 1500 | 1600 | |
| Factor..... | .487 | .460 | .435 | .412 | .372 | .337 | .307 | .283 | .261 | .241 | |

**54. Pesos Máximos y Mínimos de Locomotoras.**

| Clase. | Servicio y tipo. | Peso en orden de servicio en libras. | | | | Capacidad del téndor. | | |
|---|---|---|---|---|---|---|---|---|
| | | De Locomotora. | | Tender cargado | Total Loco y tender. | Carbon, tons | Agua galones. | |
| | | En todas las motrices. | Total. | | | | | |
| **Ancho de via, Tipo, 4′ 8 ½″.** | | | | | | | | |
| 4-4-0 | Pasajeros..... | 92,000 | 136,000 | 100,000 | 236,000 | 8 | 5000 | En cada par de lineas; 1.ª linea, máx.; 2.ª linea, mín. |
| 8-*C* | American..... | 24,000 | 38,000 | 38,200 | 76,200 | 3 | 1400 | |
| 4-6-0 | Pasjs y Carga. | 160,000 | 203,000 | 120,000 | 323,000 | 9 | 6000 | |
| 10-*D* | 10 ruedas.... | 29,300 | 38,000 | 25,800 | 63,800 | 1.5 | 1100 | |
| 2-8-0 | Carga..... | 210,000 | 237,000 | 143,500 | 380,500 | 11 | 7400 | |
| 10-*E* | Consolidada... | 37,000 | 41,500 | 25,800 | 67,300 | 1.5 | 1100 | |
| 0-6-0 | Patio..... | 165,000 | 165,000 | 104,950 | 269,950 | 9 | 5000 | |
| 6-*D* | | 22,600 | 22,600 | *20,000 | 45,300 | 1.5 | 800 | |
| **Via Estrecha, 3′ 0″.** | | | | | | | | |
| 4-4-0 | Pasajeros..... | 47,000 | 74,000 | 58,000 | 122,000 | 5 | 2500 | |
| 8-*C* | American..... | 20,000 | 30,000 | 28,000 | 58,000 | 2 | 1200 | |
| 2-8-0 | Carga..... | 120,000 | 131,000 | 80,000 | 211,000 | 9 | 4000 | |
| 10-*E* | Consolidada... | 34,000 | 39,000 | 25,800 | 64,800 | 1.5 | 1100 | |
| 0-6-0 | Patio..... | 110,000 | 110,000 | 80,000 | 190,000 | 4.5 | 4000 | |
| 6-*D* | | 14,000 | 14,000 | *20,000 | 38,000 | 1.5 | 600 | |

**55.** Vease Tabla de **Dimensiones** de Locomotoras en la prox página.

* Aproximado.

**56. Locomotoras Mallet articuladas.** Se usan algunas veces para servicio de carga pesada. La loco Mallet tiene sus motrices divididas en dos grupos, cada grupo está contenido en un bastidor separado y movido por un par de cilindros separados; cada par de cilindros tiene su aparejo de movimiento de válvula distinto, etc. Los bastidores de atrás están asegurados en rígida alineacion con la caldera. Los bastidores del frente están embisagrados o articulados al frente del bastidor de la máquina de atrás. El frente de la caldera (generalmente se dedica a supercalentadores y accesorios similares, por lo excesivamente largos que resultarian los tubos) va soportado por el bastidor del frente de la máquina, que es siempre mas o menos facil prolongarlo por debajo. Algunas veces la caldera está construida de modo que se doble en una union llamada de « acordeon ». La Mallet tiene la necesaria flexibilidad para pasar curvas y permite el uso de un doble numero de motrices en una loco de bastidor rígido. Las dos máquinas van invariablemente dispuestas para trabajar por expansión, los cilindros de atrás obran a alta presión y descargan su vapor de escape, por conexiones flexibles a la máquina de adelante, que trabaja a baja presión. Los resultados dan un gran aumento de fuerza de tracción por locomotora y personal dado, y, economia de combustible para un tonelaje dado de tren.

**Dimensiones Máximas y Mínimas de Locomotoras.**

| Clase. | Cilindro. Diam pulgs. | Cilindro. Carrera pulgs. | Ruedas motrices. Diam pulgs. | Base de Ruedas de Locomotora. Motrices pies | Motrices pulgs | Total pies | Total pulgs | Largo extremo loco y tender. pies | Largo extremo loco y tender. pulgs | Ancho extremo. pies | Ancho extremo. pulgs | Altura desde la chimenea sobre carril. pies | Altura desde la chimenea sobre carril. pulgs | Esfuerzo aproximado de traccion en libras. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Ancho de via Tipo, 4′ 8 ½″.** | | | | | | | | | | | | | | | |
| 4-4-0 | 20 | 26 | 72 | 9 | 11 | 24 | 4 | 63 | 11 | 10 | 0 | 15 | 0 | 25,000 | En cada par de lineas : 1.ª linea max; 2.ª linea mínim. |
| 8-*C* | 10 | 16 | 48 | 5 | 6 | 16 | 1 | | | 8 | 11 | 12 | 3 | 6,000 | |
| 4-6-0 | 22 | 28 | 68 | 15 | 0 | 26 | 2 | 67 | 6 | 10 | 6 | 15 | 0 | 21,000 | |
| 10-*D* | 10 | 16 | 33 | 7 | 6 | 14 | 3 | 36 | 10 | 8 | 6 | 10 | 6 | 6,800 | |
| 2-8-0 | 24 | 32 | 63 | 17 | 0 | 26 | 6 | 72 | 4 | 10 | 6 | 15 | 10 | 49,740 | |
| 10-*E* | 10 | 16 | 30 | 9 | 0 | 14 | 2 | 33 | 4 | 8 | 6 | 10 | 3 | 7,480 | |
| 0-6-0 | 22 | 28 | 56 | 11 | 6 | 11 | 6 | 57 | 0 | 10 | 4 | 15 | 0 | 35,000 | |
| 6-*D* | 8 | 14 | 28 | 6 | 8 | 6 | 8 | 31 | 8 | 7 | 2 | 10 | 2 | 5,500 | |
| **Via Estrecha, 3′ 0″.** | | | | | | | | | | | | | | | |
| 4-4-0 | 15 | 18 | 46 | 8 | 6 | 20 | 11 | 53 | 0 | 8 | 4 | 12 | 0 | 11,970 | |
| 8-*C* | 9 | 16 | 42 | 5 | 9 | 15 | 3 | | | | | | | 4,190 | |
| 2-8-0 | 19 | 24 | 48 | 14 | 0 | 21 | 8 | 58 | 8 | 9 | 6 | 13 | 6 | 90,000 | |
| 10-*E* | 10 | 16 | 30 | 9 | 0 | 14 | 2 | 33 | 4 | 8 | 0 | 10 | 3 | 7,970 | |
| 0-6-0 | 18 | 24 | 48 | 10 | 9 | 10 | 9 | 50 | 6 | 9 | 4 | 14 | 0 | 22,000 | |
| 6-*D* | 7 | 12 | 26 | 5 | 3 | 5 | 3 | 31 | 8 | 8 | 0 | 10 | 8 | 2,880 | |

**57.** La siguiente tabla da aproximadamente las variaciones de dimensiones, etc., de las Mallets Americanas mejor conocidas. La mayor parte de los mínimos son de locos de *via estrecha.*

| | Pulg máx. | Pulg mín. |
|---|---|---|
| Diam cilindro alta presión..... | 28 (71,12 cm) | 12 (30,48 cm) |
| Diam cilindro baja presión..... | 44 (111,76 cm) | 19 (48,26 cm) |
| Largo de la carrera..... | 32 (81,28 cm) | 18 (45,72 cm) |
| Número de motrices..... | 16 | 8 |

(La tabla concluye en la próxima página.)

(*N. del T.* — Los equivalentes métricos van entre paréntesis.)

Tabla de Mallets (concluye).

| | Pulg máx. | Pulg mín. |
|---|---|---|
| Diametro de motrices..... | 56 (142,24 cm) | 37 (93,98 cm) |
| | Max. | Min. |
| Peso en motrices, lbs..... | 475.000 (215.457 kg) | 80,000 (36.287 kg) |
| Peso en loco, total..... | 540.000 (244.940 kg) | 104,000 (47.137 kg) |
| Base de rueda de todas las motoras..... | 42' 1" (12,83 m) | 16' 2" (4,93 m) |
| Base de rueda, incluyendo ruedas de carretilla..... | 57' 4" (17,47 m) | 28' 0" (8,53 m) |

## Locomotoras en el trabajo.

**53. Combustible y agua usada.** Una loco poderosa típica, que lleve un tren corriente de pasajeros evapora de 25,900 a 30,000 libras (11,300 a 13,600 kg) de agua por hora, quemando de 3,500 a 5,000 libras (1,590 a 2,270 kg) de carbon; o sea de 75 a 100 lbs/milla (21 a 28 kg por kilometro). (Catecismo de Locomotora, Forney, por George L. Fowler.)

**59.** Las locos de pasajeros generalmente llevan combustible y agua suficiente para 60 u 80 kilometros; algunas de 100 a 110. En los trenes de carga, es suficiente para 30 ó 38 kilometros. Vias o divisiones, con fuertes pendientes exigen estaciones de combustible y agua mas cerca unas de otras que cuando las pendientes son mas suaves.

**60. Capacidad de arrastre.** Las siguientes tablas dan las cargas en toneladas (sin contar la loco y tender) que las locos arriba descritas pueden arrastrar, a velocidades usuales, sobre una via recta y en diferentes pendientes. Las cargas están basadas, considerando que la llamada « adhesion » de la loco no es mas del 25 % del peso sobre todas las motrices, y que las condiciones de la via y carros es tal que la resistencia a la friccion de los carros no excede alrededor de 7 u 8 lbs por tonelada de 2.000 lbs (o sea de 0,35 a 0,40 %) de su peso. Estos son ordinariamente condiciones favorables. La adhesion es rara vez menor de un quinto ni mayor de un tercio, del peso sobre las directrices.

### 61. Ancho de via Tipo; 4 pies 8 ½ pulgs (1,435 m).

Tabla de cargas, en toneladas de 2.000 lbs (907 kg) que arrastrarán las locomotoras en diferentes pendientes.

| Clase. | Servicio y Tipo. | | En una pendiente de: 0 por ciento = 0 pies/milla | ½ por ciento = 26.4 pies/milla. | 1 por ciento = 52.8 pies/milla. | 2 por ciento = 105.6 pies/milla. | 3 por ciento = 158.4 pies/milla. |
|---|---|---|---|---|---|---|---|
| 4-4-0 | Pasajeros | Min.. | 575 | 265 | 160 | 80 | 45 |
| 8-*C* | American | Max.. | 2657 | 1181 | 678 | 348 | 213 |
| 4-6-0 | Pasajeros y Carga | Min.. | 1014 | 380 | 227 | 114 | 70 |
| 10-*D* | 10-Ruedas | Max.. | 4020 | 1900 | 1165 | 595 | 385 |
| 2-8-0 | Carga | Min.. | 1117 | 419 | 248 | 127 | 79 |
| 10-*E* | Consolidado | Max.. | 5370 | 2555 | 1575 | 840 | 535 |
| 0-6-0 | Patio | Min.. | 487 | 203 | 122 | 62 | 37 |
| 6-*D* | | Max.. | 4952 | 2098 | 1283 | 671 | 420 |

### 62. Via Estrecha; 3 pies 0 pulg (0,914 m).

Tabla de cargas, en toneladas de 2.000 lbs (907 kg) que arrastrarán las locos en diferentes pendientes.

| Clase. | Servicio y Tipo. | | En una pendiente de : 0 por ciento = 0 pies/milla. | ½ por ciento = 26.4 pies/milla. | 1 por ciento = 52.8 pies/milla. | 2 por ciento = 105.6 pies/milla. | 3 por ciento = 158.4 pies/milla. |
|---|---|---|---|---|---|---|---|
| 4-4-0 | Pasajeros | Min.. | 445 | 205 | 120 | 60 | 30 |
| 8-*C* | American | Max.. | 1380 | 550 | 330 | 165 | 100 |
| 4-6-0 | Pasajeros y Carga | Min.. | 1000 | 380 | 227 | 114 | 70 |
| 10-*D* | 10-Ruedas | Max.. | 3718 | 1407 | 840 | 438 | 278 |
| 2-8-0 | Carga | Min.. | 860 | 400 | 245 | 127 | 79 |
| 10-*E* | Consolidado | Max.. | 3100 | 1460 | 900 | 480 | 310 |
| 0-6-0 | Patio | Min.. | 310 | 145 | 85 | 45 | 25 |
| 6-*D* | | Max.. | 3718 | 1407 | 840 | 438 | 278 |

**63. La eleccion** de una loco dependerá del servicio y sus requisitos, y estará limitada por las secciones de paso de los puentes, túneles, etc., resistencia de puentes, y peso del carril; este último se considera suponiendo que cada 10 lbs por yarda (4,96 kg por metro) de carril soportara una carga de 3.000 lbs (1.361 kg) sobre cada rueda; con esto para carriles que pesen de 90 a 100 lbs/yarda (44.64 a 49,61 kg/metro) permite cargas de 50.000 (22.680 kg) a 60.000 lbs (27.215 kg) por *par* de ruedas motrices.

**64. Máquinas de Patio** (9,881 en uso *). Son de tipos 0-4-0, 0-6-0 o 0-8-0; usualmente 0-6-0. Porque estas locos rara vez corren a altas velocidades, el cabeceo no es fácil que ocurra (véase ¶ 67) las ruedas de carretilla no son necesarias, y sin ellas, al concentrar el peso de la loco sobre las motrices, resulta el esfuerzo de tracción máxima.

**65. Locos de Carga** (37,405 en uso *) son de los siguientes tipos 4-4-0 « *American* » para servicio ligero; 2-6-0 « *Mogul* » y 4-6-0 « *Diez-ruedas* » para servicio mixto : 2-8-0 « *Consolidado* » y 2-8-2 « *Mikado* » para pendientes fuertes y trenes pesados; tambien se estan generalizando los tipos 2-10-2 « *Santa-Fe* » : y « *Mallet Articulado* » para servicio pesado y pendientes fuertes.

**66. Locos de pasajeros** (14,090 en uso *) sus tipos son 4-4-0, 4-4-2 « *Atlantic* » 4-6-0, 4-6-2 « *Pacific* »; y recientemente, 4-8-2 « *Montaña* ».

**67. Las ruedas de carretilla** evitan el cabeceo a altas velocidades y ayudan a guiar la loco al pasar las curvas. El tipo 2-8-2 « Mikado » está especialmente adaptado para correr en cualquiera dirección.

**68. El diam de las directrices (voladoras)**, en pulgs no debe en general ser menor que la velocidad max en milla/hora.

**69.** La fuerza y velocidad de las locos, y su consumo de combustible y agua, varia mucho con las circunstancias, tales como : pendientes, curvas, condicion de la via

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* Promedio ó total para todos las locos en los E.U., 1914, junio 30 (excepto locos en el servicio de los llamados pequeños ferrocarriles y tambien en los de las compañías terminales, Mallet y locos sin clasificar, según la comisión de comercio Interior « Estadísticas de F. C. en los E. U. », 1914.

y material rodante; numero de carros en el tren; diámetros, número y dist (aparte), de las ruedas de carro; sistema de enganches; habilidad del corredor de la loco y de su fogonero, etc., etc.

**70. Recorrido.** Las siguientes cifras están tomadas y deducidas del Informe de la Comision de comercio entre Estados de los E. U. en 1914:

| | Millaje de Locomotoras. Pasajeros. | Carga. | Todas clases. |
|---|---|---|---|
| Kilómetros total..... | 1,000,000,000 | 1,119,395,000 | 2,720,494,000 |
| Numero de Locomotoras... | 14,612 | 38,752 | 64,760 |
| Kilómetros por locomotora por año..... | 68,543 | 28,962 | 42,027 |
| por dia..... | 187.7 | 79.3 | 115.2 |

**Recorridos Rápidos.**

**71.** Como parece evidente de que los manufactureros han llegado al limite de la resistencia de sus carriles, el aumento de peso del material rodante ha tendido a restringir las velocidades. El F. C. de Pennsylvania limita la velocidad de sus trenes de pasajeros en general a 70 millas (como 112 kilómetros) por hora, con limites menores en ciertas localidades, y el F. C. de Baltimore & Ohio, tiene limites de 60 millas (como 96 km) por hora.

**72.** Entre los **itinerarios notables mantenidos con regularidad** están los que siguen:

| Ferrocarril. | Tren y Ruta. | | Dist en kilometros. | Promedio velocidad km/hora. |
|---|---|---|---|---|
| Phila & Reading. | Cada Hora. | Phila. | Jersey City. | 145.1 | 88.8 |
| | | Camden. | Atlantic City. | 88.8 | 97.5 |
| Pennsylvania.... | | N. Phila. | Harrisburg. | 167.2 | 81.9 |
| | ...Broad'y Ltd. | New York. | Chicago. | 1,462.4 | 73.1 |
| N. Y. Central... | 20th C Ltd. | New York. | Chicago. | 1,570.7 | 78.5 |

**73. Para apreciar la velocidad** de un tren ú otro vehiculo, en el que se viaja, se cuentan espacios conocidos, tales como, largos de carril, dist entre postes telegráficos, o revoluciones de una rueda en un tiempo dado. Los largos de carril pueden, a veces, contarse por el sonido de las ruedas al pasar por las bridas (juntas); y las revoluciones por cualquier irregularidad de la rueda.

**74.** Sea $L$ = el largo de uno de los espacios iguales conocidos;

Entonces:

Velocidad V por segundo = numero de tales largos pasados en $n$ segundos, (digamos $N$ largos) multip. por $L$ y dividido por $n$ es decir $V = \frac{N \times L}{n}$.

## CARROS

**75.** En uso * 53,466 carros de pasajeros, 2,325,647 de carga, 124,709 de miscelánea. Véase también « carretillas », etc., ¶ 2. Practicamente todos los carros Americanos, excepto algunos carros de cola para personal, y de descarga de minerales, están construidos con dos carretillas cada uno, fig. 7; cada carretilla tiene 2 o 3 pares de ruedas. En carretillas de 6 ruedas, los muelles y balancines distribuyen la carga por igual, o aproximadamente, entre los tres ejes. Sobre los platos de centro de ambas carretillas descansa un marco o « cama » rigida, larga para formar el cuerpo del carro. Esto fué anteriormente diseñado para soportar todos los esfuerzos de tracción de la carga longitudinal y verticalmente; pero los costados de los carros están hechos ahora, hasta donde es posible, como vigas armadas, aligerando asi y reforzando a la vez el carro. Las camas o marcos anteriormente se hacian de madera, generalmente atirantadas con sopandas de hierro; pero ahora se hacen casi universalmente todas de acero.

**76. Acero.** Los cuerpos de carro son de madera ó acero, o mas generalmente, y sobretodo en los carros modernos de carga, combinaciones de armazón de acero con pisos y cubiertas de madera. Los carros de carga, todos de acero se han usado mucho tiempo antes que los coches de pasajeros todos de acero. El uso del acero

* Véase nota al pie, página 1123.

en todo, excepto para cierto amueblado interior, se ha establecido prácticamente como tipo para todo el equipo de carros de pasajeros en lineas principales importantes, comenzando alrededor de 1905 a 1910. En peso, los carros de acero tienen prácticamente el mismo promedio *por pasajero* que los carros de madera, aunque tienen generalmente de 10 a 20 por ciento mas de peso total y de capacidad para pasajeros. Son mucho mas fuertes, y estan considerados como mucho mas seguros que los carros de madera en caso de accidente y en caso de fuego. Debido a la alta conductividad del calor por al acero son casi siempre desagradables cuando hace mucho calor o frio, a pesar de los trabajos de aislamiento hechos con células de dobles tabiques.

**77. Datos.** La siguiente tabla da aproximadamente las dimensiones maximas y minimas y también peso, etc., de carros de acero.

Dimensiones aproximadas, etc., de Carros de acero.
Pasajeros y Carga, hasta 1908.

| Clase. | **Largo** del cuerpo pies. | **Ancho** Pies. | **Altura** maxima sobre el carril incluyendo el freno, pies. | **Peso** vacio, lbs limites usuales. | **Capacidad** máxima. |
|---|---|---|---|---|---|
| Pasajeros.... | 60 a 80<br>18,29 a 24,38) | 10<br>(3,05) | 12 a 14<br>(3,66 a 4,27) | 90,000 a 110,000<br>(40,823 a 49,895) | 60 a 90 pasajeros. |
| | | | | | Libras. |
| Equipaje, Correo, Expreso..... | 60 a 75<br>(18,29 a 22,86) | 10<br>(3,05) | 13 a 14<br>(3,96 a 4,27) | 106,000 a 128,000<br>(48,080 a 58,060) | 35,000 a 60,000<br>(15,876 a 27,215) |
| Cubierto, Ganado, Auto. | 35 a 45<br>(10,67 a 13,72) | 8 a 10<br>(2,44 a 3,05) | 12 a 15<br>(3,66 a 4,57) | 35,000 a 50,000<br>(15,876 a 22,680) | 66,000 a 110,000<br>(29,937 a 49,985) |
| Góndola..... | 35 a 50<br>(10,67 a 22,68) | 10<br>(3,05) | 6 a 10<br>(1,83 a 3,05) | 30,000 a 50,000<br>(13,607 a 22,680) | 88,000 a 125,000<br>(39,916 a 56,669) |
| Descarga..... | 30 a 42<br>(9,14 a 12,80) | 10<br>(3,05) | 8 a 12<br>(2,44 a 3,66) | 33,000 a 50,000<br>(14,969 a 22,680) | 88,000 a 121,000<br>(39,916 a 54,884) |
| Mineral..... | 22<br>(6,71) | 8 a 10<br>(2,44 a 3,05) | 9 a 11<br>(2,74 3,35) | 31,000 a 33,000<br>(14,061 a 14,969) | 88,000 a 120,000<br>(39,916 a 54,431) |
| Plancha..... | 40 a 42<br>(12,19 a 12,80) | 10<br>(3,05) | | 29,000 a 40,000<br>(13,154 a 18,144) | 88,000 a 110,000<br>(39,916 a 49,895) |
| Tanque..... | 30 a 38<br>(9,14 a 11,58) | 8 a 10<br>(2,44 a 3,35) | 12 a 15<br>(3,66 a 4,57) | 35,000 a 55,000<br>(15,876 a 24,948) | 80,000 a 100,000<br>(36,287 a 45,359) |
| Mina..... | 8 a 13<br>(2,44 a 3,96) | 2 ½ a 7<br>(0,76 a 2,13) | 3 a 5<br>(0,91 a 1,52) | 1,750 a 4,725<br>(794 a 2,143) | 56 a 120 pies c.<br>(1,586 a 3,398 mets cub.) |

(*N. del T.* — Entre paréntesis y *debajo* de cada cita estan los piés convertidos en metros, y las libras en kilogramos.)

**78. Carros de Carga.** Como puede verse en una tabla, publicada en la Revista de F. C. é Ingenieria, feb 10, 1912, donde se recopilan datos de 11 grandes ferrocarriles que poseem sobre 450,000 carros de carga y por un periodo de tres años, el promedio de duración, costo, etc., de los carros de carga es el siguiente :

Promedio de duración de todos los carros, alrededor de 10 años. Cada carro es reparado por término medio una vez al mes, con un costo de $6.26 cada vez. El promedio de recorrido de cada carro por año, 10,400 millas (como 16.700 kilómetros) de lo que resulta en reparaciones $0.007/milla ó sean $0.0043 por kilómetro.

**79. Carros de pasajeros.** El promedio de duración es alrededor de 16 años. Promedio anual de reparaciones, incluyendo pintura, $300 a $700; para carros de correo y expreso $150 a $300. El promedio del peso de un carro de pasajeros es unas diez veces el de su carga completa de pasajeros.

**80. Carros de descarga.** Los hay de varios diseños. Pueden descargar por el fondo, por uno o mas « tolvas » (usualmente transversal, pero algunas veces longitudinalmente), o por los costados, en cuyo caso los costados se levantan o abren hacia afuera, o tambien los costados permanecen practicamente estacionarios mientras todo el piso del carro se inclina, formando asi una abertura bajo el costado y una rampa para que el material resbale. Los distintos fabricantes de carros por medio de varias combinaciones de costados embisagrados y piezas de fondo hacen posible la descarga del material entre los carriles, o, a un lado ó a ambos lados de la via. Los carros de descarga por el fondo se operan ordinariamente a mano, las puertas o postigos se sneltan rápidamente para abrirlas, y se cierran enrollando las cadenas por medio de maniguetas. Los carros con fondo de tumba son frecuentemente operados por aire comprimido, y, en algunos casos, el manejo es algo laborioso; a veces es posible también tumbar toda la carga simultaneamente lo mismo a la derecha que a la izquierda, o a ambos lados, derecha e izquierda en carros alternados, pero siempre bajo el cuidado de un operador, en cualquier punto, del tren.

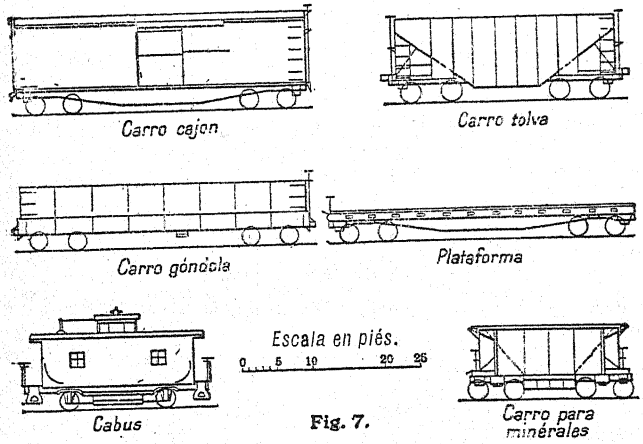


**Fig. 7.**

**81. Carros para minerales** (véase fig. 7). Son hechos anormalmente cortos. y, relativamente de pequeña capacidad volumétrica, porque los carros mas largos resultan ineficaces llevando minerales, que son mucho mas pesados que el carbón y los otros materiales que se llevan mas comunmente.

**82. Carros gruas.** Son usados sobre todo para despejar descarrilamientos y choques, para excavaciónes y manejo de tierra, para mover la via, y para dar carbón a las locomotoras.

**83. Gruas locomóviles.** Parecen ser sumamente útiles para estos trabajos. Tienen movimiento propio y rápido de un lugar a otro.

**84. Las paleadoras de vapor, y martinetes** montados en carros en varios estilos son fabricados por muchos manufactureros.

**85. Los extendedores de balasto** están mencionados en la pag. 894.

**86. Los arados de nieve** echan la nieve fuera de la via (a ambos lados para via sencilla, y a la derecha para via doble); o son del tipo » *rotativo* », en el que una rueda grande cortante gira en el extremo del frente, en un eje longitudinal horizontal, por medio de máquinas poderosas, a menudo movidas por una caldera de locomotora de tamaño completo, y que remueve la nieve, sobretodo, por la fuerza centrífuga desarrollada en la rueda cortante. En lineas ferreas por vapor, el arado se empuja en la nieve por una o mas locomotoras, empujando detras del arado; mientras que, en tranvias eléctricos, el arado o *barredor* (que realmente barre sin emplear la fuerza centrífuga), se mueve hacia adelante por su propia fuerza, las escobas o cepillos se hacen girar con motores independientes. Algunos arados están dispuestos de tal modo que las superficies desviadoras pueden forzarse hacia afuera por el aire comprimido (cuando hay espacio suficiente), lanzando así la nieve mas lejos de la via, y por consiguiente con menor posibilidad de volver a la via.

**87. Limpiadores.** Son piezas, generalmente de planchas de hierro o acero, formadas de tal modo que pueden bajarse sobre los carriles directamente en frente de las ruedas, para quitar unas pocas pulgadas de nieve o hielo que puede haber sobre el carril o a lo largo de su cara interior. Algunas veces se ponen en las locomotoras, y generalmente se agregan a los arados.

**88. Carros de « despejo ».** * Estan diseñados para determinar la proximidad de los objetos en toda la via. Los carros de *despejo* construidos por el F. C. de Pennsylvania y Baltimore & Ohio tienen montados sobre una de las carretillas un marco transversal al carro, cuyo ancho es algo menor que el probable ancho mínimo libre de la via. A este marco están fijados por medio de pivotes, a intervalos de cerca de medio pie, brazos movibles, que se proyectan cerca de dos pies fuera del marco. Cada brazo está provisto de una escala, graduada de tal modo que indica directamente el numero de pulgadas que se han desviado hacia adentro de su posicion extrema. El carro corre por la via, cerca de 4 millas por hora, donde hay que pasar varios objetos, como puentes, estaciones, túneles, etc.; los brazos son desviados hasta que den paso y, los que se desvian queda marcado y anotado. Por supuesto que se deben hacer correcciones para tomar en cuenta el grado de curvatura si lo hay, y por la distancia entre los platos de centro de las carretillas de cualquier carro que tenga que circular sobre la via. Ademas del marco dicho con sus brazos una de las carretillas esta conectada con un puntero marcador y su esfera de modo tal que marque directamente el grado de curvatura, mientras que un péndulo que puede girar libremente al traves del carro va conectado a otro puntero y esfera para indicar directamente la sobreelevacion (peralte) de cualquier carril.

**89. Carros de inspeccion y de mano.** Estos varian, en su construcción, desde simples plataformas con palancas de mano engranadas, hasta la combinación de carro y locomotora, usados por el personal directivo del ferrocarril. No obstante muchos de estos carros llevan ahora motores de gasolina : que si es verdad que requieren mas conocimiento para manejarlos, que los carros de mano, pueden alcanzar velocidades muy altas, y pueden generalmente mantenerse fuera de la via de los trenes. Algunos están construidos de tal modo que sus ruedas y ejes forman un circuito metálico que opera las señales automáticas y se protege asi de los trenes en movimiento en la misma via : pero muchos de ellos tienen las ruedas aisladas, y necesitan por consiguiente orden de via para circular.

**90. Carros registradores de via.** Vease este epigrafe en construcción de via y conservacion, pag. 896.

**91. Carros dinamómetros,** vease epigrafe Resistencia del Tren ¶ ¶ 56, etc. pag. 1139.

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* *N. del T.* — Estos carros van, en verdad, a despejar la via. No sabiendo cómo se llaman en español, hemos resuelto llamarlos de « *despejo* ».

**92. En los equipos de auxilio** se tiene generalmente una poderosa grua de vapor montada en un carro, y cierto número de carros, que llevan un equipo variado con medios para comer y dormir. Llevan carriles, traviesas y partes de desvíos, tornillos surtidos, tuercas y tornillos; herramientas de muchas clases : equipo para izar aparejos, cadenas, sogas, cables, ganchos, bloques de madera, también luces portátiles potentes, extinguidores de incendio, hombres de linea con teléfono, botiquines, encarriladores, etc., etc.

**93. Encarriladores** son hechos en pares, y son piezas fundidas formadas de ranuras o pestañas de tal modo, que cuando se colocan sobre las traviesas y adyacente a los carriles, guian cada par de ruedas hacia los carriles a medida que el carro es empujado por la via. A menudo van provistos con tacos, para encajar, o para clavarlos en la traviesa, y ayudar a su fijacion.

**94. Tipos de carga.** Tal como están establecidos por la Asociacion de Maestros constructores de carros, se determinan en un folleto que puede obtenerse del Secretario en el Edificio Old Colony, Chicago Ill. El propósito es por supuesto, cargar los carros lo mas igual posible, para poder sostener la carga en su lugar, y tomar precauciones, cuando la carga ocupa mas de un carro teniendo en cuenta las curvas y desigualdades de la via. Cuando la carga es lo suficiente larga para dos o mas carros, puede soportarse en dos pivotes improvisados, uno en el frente y otro atras del carro. Si el material debe sostenerse por el medio, se provee de placas sostenedoras, de metal, que permitan un deslizamiento lateral fácil. Cuando se tengan que usar mas de dos carros, el carro extra generalmente no soporta peso y actúa simplemente como un separador. Algunas cargas que van en tres largos de carro, como vigas armadas de acero, etc., pueden cargarse completamente sobre el carro central, los otros dos son separadores. Las pilas de listones, barriles, tubos, tejas, etc., deben asegurarse adecuadamente en su lugar con amarres laterales y verticales.

# RESISTENCIAS EN LOS TRENES

## GENERALIDADES

**1. Las fuerzas** (o fuerzas componentes) que actúan sobre un tren en dirección paralela a la vía, pueden clasificarse como :
(a) fuerzas de « *impulso* »; tienden a poner en movimiento, a acelerar el tren;
(b) resistencias; tienden a detener o retardar el tren.

**2.** La fuerza de tracción de la locomotora, la gravedad, el viento, la inercia, pueden obrar como fuerzas de impulso (a) o como resistencias (b) según las circunstancias; pero la fricción (externa e interna), las curvas, y el aire en reposo producen necesariamente *resistencia* (b).

**3.** En cualquier caso dado, la resistencia del tren, $F$, es la suma de las resistencias que sobre él actúan.

**4.** La fuerza de tracción que el vapor trasmite por medio de la locomotora tiene que mover a ésta y al ténder, lo mismo que a los coches; pero frecuentemente se toma la « resistencia » como la de los coches solamente, o sea la que se opone a la tracción ejercida por la barra de tiro, detrás del ténder.

**5. Componentes de la resistencia de un tren.**

$F_n$ = res. normal = la res. total en una vía recta y a nivel; tren en reposo o a una velocidad uniforme, en aire en reposo, y a temperaturas normales; = la resistencia a que todo tren se halla necesariamente sometido;

$F_s$ = resistencia de las pendientes;
$F_c$ = resistencia de las curvas;
$F_w$ = resistencia del viento;
$F_i$ = resistencia de la inercia, debida a cambios de velocidad

} Resistencias incidentales.

Por tanto, para la resistencia total, tenemos :

$$F = F_n + F_s + F_c + F_w + F_i \ldots\ldots\ldots\ldots\ldots (1)$$

**6. Unidad de resistencia.** Supongamos que $W$ = al peso del tren. Entonces el cuociente, $f = F/W$, es la resistencia *por unidad de peso*, o la unidad de resistencia. De modo semejante, $f_1$, $f_2$, etc., representan respectivamente la resistencia de la unidad de pendiente, y la resistencia de la curva tomada por unidad. Cualquier resistencia puede ser expresada por una pendiente *equivalente*. Véase ¶ 33.

## RESISTENCIA NORMAL

**7.** La resistencia normal, $f_n$, es la resistencia, por unidad de peso, en una vía recta, a nivel, a una velocidad uniforme, en aire quieto. De la resistencia total, es esa la porción a que todo tren en movimiento se halla siempre sujeto. Para resistencias incidentales véase ¶ 31, etc.

### Componentes de la resistencia normal de un tren, $f_n$.

**8.** (a) fricción entre las cabezas del carril y las llantas y las pestañas de las ruedas;
(b) resistencia debida a la ondulación de la vía bajo un tren en movimiento;
(c) fricción interior de los coches y de la locomotora;
(d) resistencia del aire en reposo.

### Fricción entre las ruedas y las cabezas de carril.

**9.** Cuando las pestañas de las ruedas corren separadas de la cabeza de los carriles, tenemos, normalmente, entre los carriles y las llantas de las ruedas, tan sólo la fricción del mov sobre la vía (véase pág. 435, ¶ 193); y esto es de insignificante importancia cuando la vía y el material rodante se hallan en buena disposicion; pero la fricción por deslizamiento entre la cabeza del carril y las pestañas de la rueda, puede ser de considerable importancia, particularmente cuando hay oscilación o viento lateral (véase ¶ 44), y va a menudo acompañada del roce lateral de las llantas sobre las cabezas. En las curvas, los dos carriles son de diferentes longitudes, y tenemos una fricción longitudinal entre llanta y cabeza. Las condiciones varían tanto, en diferentes vias, y de un momento a otro en el mismo tren, que es imposible establecer preceptos útiles sobre este particular, ni sobre lo que sigue.

**10. La resistencia debida a las ondulaciones de la vía** bajo un tren en movimiento es, por supuesto, mayor sobre carriles y vias dúctiles.

**11. La fricción interna del material rodante** consiste principalmente (la de locomotoras en parte) en la fricción *en los muñones*. Su valor, en libras, por tonelada, es :

$$f_1 = 2000\ c\ d/D \qquad (2)$$

donde $c$ = al coeficiente de fricción del muñón girando en su muñonera, $d$ = al diámetro del muñón, y $D$ = al diámetro de la rueda. Esto supone $d/D$ constante; y en la práctica, puede de ordinario calcularse un valor (término medio) de $d/D$, para cada tren.

**12. Resistencias de rueda y muñón combinadas.** Es difícil determinar estas resistencias, independientemente de las otras resistencias normales. Los experimentos que se han hecho con ese fin no parecen ofrecer ninguna confianza. De los experimentos sobre cuestas, Wellington deducía de 4 a 6 libras por tonelada de 2.000 libras.

**13.** En las locomotoras (Am Loco Co., Bulletin 1.001, feb, 1910), la fricción de las ruedas motrices, pistones, válvulas, cabeceos, etc., se calcula en 22.2 libras por cada 2.000 libras de peso sobre las motrices. La *fricción* en los *muñones* de la locomotora, y de las ruedas del tènder, se considera igual a las de los coches.

**14. Resistencia del aire normal,** $F_1$. Resistencia con aire quieto. (Véase también resistencia del viento, $f_w$, ¶ 44). Los datos son contradictorios. El profesor W. F. M. Goss, *Western Ry Club Proces*, 1898, abril 25, página 347; relata experimentos hechos en una caja rectangular ancha y cerrada, en la cual las corrientes de aire de velocidades conocidas tropezaban directamente contra la extremidad del frente de trenes fijos de 1 a 25 coches modelo, de cerca de 3,5 pulg de ancho, 4,5 pulg de alto, 12 pulg de largo (8,89 × 11,43 × 30.48 cm) cada uno de ellos conectados por medio de un dinamómetro, a una base fija. El profesor Goss deriva las siguientes fórmulas, en las cuales supone que las dimensiones lineales del material rodante efectivo son 32 veces las de los modelos en cada dimension lineal; que los vagones de carga tienen 33 pies (10,06 m) de largo, y los de pasajeros 66 pies (20,12 m).

Supongamos que :

$F_h$ = a la resistencia atmosférica total, en lbs.;
$V$ = a la velocidad, en millas por hora;
$L$ = a la longitud, en pies, de cualquier tren, incluyendo la locomotora y el ténder;
$l$ = a la longitud, en pies, de cualquier tren, excluyendo la locomotora y el ténder;

luego, para el tren entero : $F_t = 0.0003\,(L + 347)\,V^2$..... (3)
para los coches solamente : $F_t = 0.0003\,(l + 53)\,V^2$..... (4)
para la loco y ténder : $F_t = 0.125\,V^2$..... (5)

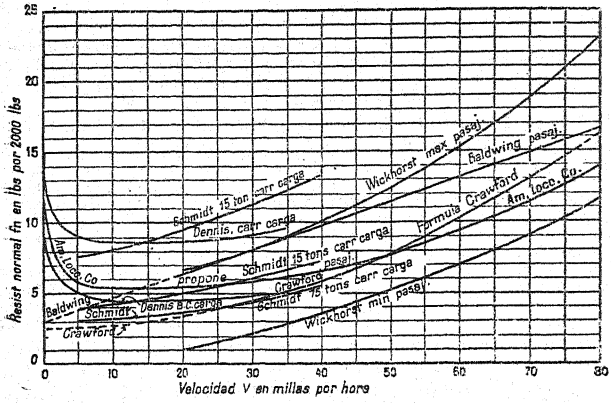



**Fig. 1.**

El profesor Goss considera que estas fórmulas dan resultados que no varían más de 15 a 20 por ciento de la verdad en la práctica.

Unidad de resistencia aérea, $f_a = F_a/W$, donde $W$ = al peso del tren en toneladas de 2.000 lbs.

**15.** Aumentan la resistencia aérea *los espacios entre coche y coche ;* las *puertas laterales abiertas*, y la *diferencia de altura en los vagones*, y tambien cuando en un mismo tren alternan furgones y plataformas. « Se pagan con carbón las desventajas y la tardanza en que se incurre al no poner juntos los furgones cerrados. Eso mismo permite al propio tiempo, al guardafreno moverse en el tren con mas rapidez. « (Angus Sinclair, en « Conducción y Manejo de locomotoras », 1891, pág. 375.).

**16. La resistencia aérea de frente de la locomotora** se calcula como de $0{,}002\,V^2A$; donde $V$ = vel., en millas por hora, $A$ = superficie transversal de la locomotora, generalmente calculada en 120 pies cuadrados (11,15 m²). Boletín 1.001, febrero de 1910, de la Cia Americana de Locomotoras.

## Factores que afectan la resistencia normal.

### Impulso inicial (salida).

**17.** Al arrancar el tren (véase figura 1), el valor de $f_n$ es mucho mayor que cuando está en marcha. La iniciación de la marcha puede ser facilitada retrocediendo lo bastante para aflojar los enganches, de modo que el tren arranque como si fuera carro por carro.

**18. A. C. Dennis,** Am. Soc. C. E. Trans., junio 1903, vol. 50, p. 1, encuentra que « después de que el tren se ha detenido por algún tiempo, las resistencias son de cerca de 2 lbs por tonelada sobre lo normal hasta que el tren ha corrido lo suficiente para que los muñones se calienten y se aceiten bien ».

**19. A. K. Shurtleff,** Am. Ry. Eng. & M. of W. Assn, Boletín 84, febrero de 1907, pág. 98, etc., opina que sobre la temperatura de congelación la mayoría de los trenes de carga puede echarse a andar, en una vía a nivel, con una fuerza de tracción de 13 lbs por tonelada de 2.000 lbs; pero una temperatura glacial, o un arranque descuidado, pueden aumentar esta proporción a 25 ó 30 libras por tonelada. En cambio, donde la parada es solo de un instante, los trenes arrancan con una fuerza en la barra de tracción de 6 lbs por tonelada.

**En marcha.**

**20. Efecto de la distancia recorrida.** A medida que la distancia recorrida aumenta, después de una parada, los muñones se calientan, es mejor su lubricación, y la resistencia disminuye en consecuencia. Durante la primera milla o dos, aun a velocidad uniforme y temperatura aerea constante, la resistencia normal, $f_n$, puede ascender hasta 50 % sobre el mínimo, que no se alcanza hasta que el tren lleva recorridas de 8 a 10 millas (14 a 16 km) en tiempo cálido, y alrededor de 35 millas (56 km) si hace frío. Con coches pesados y altas velocidades, la distancia requerida para alcanzar el mínimo $f_n$, es menor que con coches ligeros y bajas velocidades; pues el peso sobre las chumaceras y el aumento de velocidad ayudan a calentar aquellas y a licuar y distribuir bien el lubricante. (Prof. Eduardo C. Schmidt, Central Ry Club; Ry Age Gazette, enero 12 de 1912.)

**21. Aumento de velocidad,** fig. 1. La alta resistencia normal, experimentada al arrancar, disminuye muy rápidamente en cuanto la velocidad aumenta digamos en 5 millas por hora. De 5 o 10 (8 a 16 km) hasta 30 o 35 millas (48 a 56 km) por hora (velocidades usuales de los trenes de carga) $f_n$ aumenta ligeramente, y luego más rápidamente a medida que aumenta la velocidad.

La fig. 1 muestra las curvas de $f_n$ conforme a los experimentos y fórmulas que siguen :

## Experimentos.

**22. A. C. Dennis,** 1902. Am. Soc. C. E. Trans, 1903 junio, volumen 50, p. 1 Carrera total, más de 3.000 millas. Curvas *A* y *B-C*. Carreras emprendidas con :

(*A*), 105 furgones vacíos, vía sólida (frío), buen carril;
(*B*), 47 — cargados, vía sólida (frío), buen carril; neto/tara = 2.
(*C*), 52 — vía suave; neto/tara = 63/27.

Los resultados (*B*) y (*C*) son prácticamente idénticos.

**23.** Las resistencias (compensadas por los cambios de velocidad, véase ¶ ¶ 45-52) parecían ser mayores que las normales al aumentar la velocidad, y menores cuando disminuye. Mediante pruebas en cuestas, la resistencia de la locomotora, por unidad de peso, resultó ser casi igual a la de los carros *vacíos*.

**24. Max H. Wickhorst,** 1900, en el Ferrocarril de Chicago, Burlington Quincy, entre Chicago y Burlington, III (206 millas). Eng News, 1901, octubre 31. Cinco carreras, con vagón dinamómetro, 2 de equipaje y 2 de correo; tres carreras cargado, 180 toneladas, promedio 55 millas (88,5 km) por hora; dos carreras ligeras, 160 toneladas, promedio 27.5 millas por hora. Temperatura media, 70° Fahr (21,1 C); viento muy suave : muy numerosas observaciones. Las dos curvas muestran, respectivamente, los mínimos y los máximos aproximados.

**25. Prof Eduardo C. Schmidt,** 1910, Boletín de la Universidad de Illinois, vol. VI, n.° 39. Muy minuciosas pruebas de 32 trenes de carga ordinarios, con pesos de 747 a 2.908 toneladas por tren, y de 26 a 89 carros en cada tren, y con un promedio de cerca de 15 a 70 toneladas por carro; vía en buenas condiciones; temperatura atmosférica de 34° a 82° Fahr. (1,1 a 27,8 C). Véase tres curvas escogidas, figura 1; velocidades de 5 a 40 millas (8 a 64 km) por hora.

## Fórmulas.

**26. Cia Am. de Locom.** Boletín 1.001, feb. 1910, pág. 10; R. R. Age Gaz, 1909, sep. 10, p. 455 :

$$f_n = 5.4 + 0.002\ (V - 15)^2 + 100/(V + 2)^2 \ldots\ldots\ldots\ldots\ (6)$$

en la cual $V$ = velocidad en millas por hora *.

---

* (*N. del T.*) — Para convertir kilómetros en millas se multip. aquéllos por 0.621. Para conv millas en kilómetros se multip. las millas por 1.609.

**27. J. G. Crawford**, Eng. News, 1901, oct. 31. Basados en los experimentos hechos por Wickhorst en el Chicago, Burlington y Quincy, ¶ 24. Para velocidades entre 25 y 75 millas (40 y 121 km) por hora:

$$f_n = 2.5 + V^2/468 = 2.5 + 0.0021\ V^2 \ldots\ldots\ldots\ldots\ (7)$$

**28. Baldwin Locomotive Works**, fórmula de línea recta. R. R. Gaz 1899, marzo 17. Basada en experiencias hechas en la red de Filadelfia y Reading, entre Camden y Atlantic City, 55.5 millas (89,3 km) sin paradas.

$$f_n = 3 + V/6 = 3 + 0.167\ V \ldots\ldots\ldots\ldots\ (8)$$

## Tiempo Frío.

**29.** La Am. Loco. Co. (Boletín 1001, 1910, feb.) hace las siguientes reducciones del tonelaje arrastrado con temperaturas normales:

| Temperatura, Fahr. y C. | Deducción. |
|---|---|
| 45° a 25° (7.2 a − 3.9 C)..... | 8 por ciento. |
| 25° a 0° (− 3.9 a − 17.8 C)..... | 16 — |
| Bajo 0° (− 17.8 C)..... | 25 — |

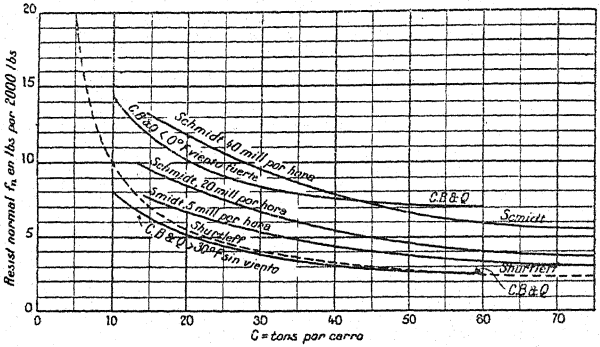



Fig. 2.

## Efecto del peso de los vagones.

**30. La fig. 2** muestra el efecto del peso de los vagones de carga, y de una manera incidental, hasta cierto punto, de la velocidad, sobre la resistencia normal, $f_n$; como sigue:

Experimentos en el C. B. & Q., R. R. Age Gazette, 1909, agosto 27, sep. 3;

Prof. Eduardo C. Schmidt, sus experimentos en el Illinois Central R. R. Véase también ¶ 25.

La curva de puntos representa la fórmula de A. K. Shurtleff, American Ry. Eng. & M. W. Assn., Boletín 84, febrero 1907, p. 99:

$$f_n = 1 + 90/C \ldots\ldots\ldots\ldots\ (9)$$

donde $C$ = peso medio del vagón cargado, en toneladas de 2.000 lbs. Esta fórmula está basada en experimentos hechos en dos redes ferroviarias. Total recorrido, más de 3,000 millas (4.828 km), 19 a 41 toneladas por vagón. La vía variaba desde «balasto con fango» con línea y superficies medianas hasta líneas de primera clase bien embalastadas con líneas y superficies buenas desde líneas en valles hasta líneas de montañas y en pendientes. Temperatura, de 0° a 60° Fahr (− 17.8 a 21.1 C).

## RESISTENCIA INCIDENTALES. Véase ¶ 5.

**31.** Además de la resistencia normal del tren, debemos ordinariamente tomar en cuenta las resistencias incidentales :

$F_p$, debida a las pendientes; $F_w$, debida al viento;
$F_c$, — las curvas; $F_i$, — a inercia.

### Resistencia debida a las pendientes.

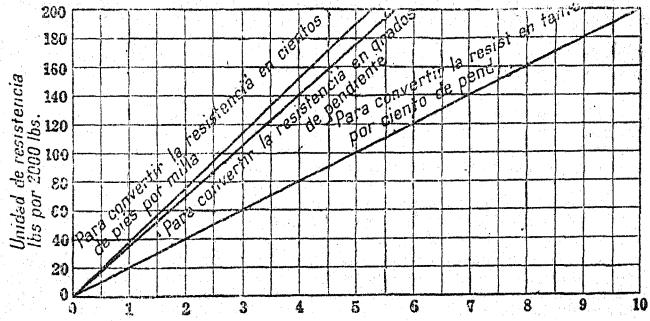
**32.** En toda pendiente, supóngase $a$ = al ángulo con la horizontal. Entonces, para la resistencia, paralela a la inclinación y necesaria para evitar el movimiento hacia abajo de un peso, $W$ ; tenemos :

$F_s = W \text{ sen } a$ ; y, como $f_s = F_s/W$ ; $f_s = \text{sen } a$..... (10)

Pero, con excepción de las lineas muy pendientes, la inclinación es tan ligera que, aproximadamente, sen $a$ = tang $a$. Por consiguiente, si $L$ = a la distancia horizontal entre dos puntos, y $h$ = a su diferencia de nivel, tenemos :

$F_s = W \text{ tang } a$, y $f_s = \text{tang } a = h/L$..... (11)

**33. La pendiente equivalente**, $s_e$, es la pendiente que produce la misma tendencia a lo largo de la vía que la debida a una resistencia dada; o bien la pendiente donde la tendencia a resbalar hacia abajo se halla precisamente equilibrada por la resistencia dada. En consecuencia, toda unidad de resistencia $f_s = F/W$, puede ser expresada como una pendiente. Para las varias conversiones, véase la fig. 3.




**Pendiente** (*Dada en tanto por ciento; pies (o bien metros por cien metros) por 100 pies; Cientos de pies por milla; o en Grados*).

**Fig. 3.**

**34. Unidades** de resistencia por la pendiente. Digamos :
$s$ = pendiente, en pies por cada 100 pies (uno por ciento = 1 %);
$S$ = pendiente, en pies por milla = 52.80 $s$.

**35.** Luego, por la **resistencia debida a la pendiente**, en lbiras por ton. de 2.000 lbs, tenemos :

$f_s = 20\ s = 0.3788\ S$..... (12)

y, por cualquiera unidad de resistencia $f$, debida a cualquier causa, tenemos :

**pendiente equivalente**, en pies por 100 pies = $f/20$..... ..... (13)

Pendientes reglamentarias; véase « Costo de Explotación », pag. 1156.
Pendiente de impulsión; véase pag 1149, ¶ 40.
Trabajo hecho y perdido en las pendientes; véase pag. 1147, ¶ 30.
Efecto de la longitud de la pendiente en la fuerza de tracción; véase pag. 1148, ¶ 37.

### Resistencia de las Curvas.

**36.** En cualquier punto de una curva, un eje, con ruedas cilíndricas fijas de diámetros iguales, tiende a rodar en dirección perpendicular a sí mismo y en una tangente con la línea central de la vía, conservándose paralelo consigo mismo.

Por consiguiente, la pestaña de la rueda exterior tiende a ejercer presión hacia afuera contra la cabeza del carril exterior, originando una fricción de deslizamiento entre los costados de la pestaña y de la cabeza del carril; y esta fricción se opone a la rotación de la rueda. Además, la cabeza del carril externo, mediante su reacción horizontal sobre la pestaña de la rueda, empuja la rueda externa (con su eje y la otra rueda) hacia el centro de la curva, compeliendo entrambas ruedas a resbalar lateralmente sobre sus respectivas cabezas de carril, causando así fricción de deslizamiento entre la cara de la rueda (llanta) y la superficie de la cabeza del carril. Esto, por supuesto, se combina (conforme a la resultante de los movimientos y las fuerzas) con cualquier resbalamiento longitudinal, y a medida que estos van ocurriendo; (¶ 37).

Todo esto acontece principalmente con las primeras ruedas de un vagón o tren. En las carretillas siguientes, la dirección de la tracción coincide, en cada carretilla o trole, con la cuerda de una porción de la curva; y así tiende a reducir la presión de dichos troles ulteriores contra el carril exterior.

**37.** Además, los ejes deben oscilar, en sentido horizontal, sacándolos de posiciones paralelas a ellas y haciéndolos entrar en posiciones más aproximadas a direcciones radiales a la curva; y esto exige que una rueda o la otra, o ambas ruedas resbalen longitudinalmente, a la vía. Véase también Corrugación, bajo el título de « vía » pag. 867, ¶ 91.

**38.** A fin de evitar o disminuir las resistencias así originadas, las caras de las ruedas se « hacen cónicas ». Véase Material rodante, pag. 1112, ¶ 11.

**39.** La resistencia, $F_c$, debida a las curvas está influenciada por muchas circunstancias; tales como el diámetro de las ruedas; la estrechez de la vía; la forma de la llanta de las ruedas (según sean más o menos cónicas), la longitud de las bases rígidas de las ruedas (véase ¶ 43), el sistema de acopladura, condición de la vía, elevación del carril externo (peralte), largo del tren (por lo que se refiere a la oblicuidad de tracción, véase fin del ¶ 36).

**40. Unidad de resistencia debida a las curvas.** Se considera generalmente que varía, — en igualdad de otras circunstancias, — en relación directa con la agudeza de la curvatura; es decir que, por resistencia de la curva, en libras por 2.000 lbs, en una curva de D° de agudeza, se tiene :

$$f = D\, f_1 \qquad (14)$$

donde $D$ = agudeza de la curvatura, en grados;
$f_1$ = resistencia de la curva, en libras por ton de 2.000 lbs, en una curva de 1° de agudeza.

**41. $f_1$ se calcula generalmente** que varía (según la naturaleza y las condiciones de la vía y del material rodante) entre 0.5 y 1.5 lbs por cada 2.000 lbs. Para compensar la curvatura, en las pendientes (véase pag. 1150, ¶ 51), es costumbre dar un margen entre 0.6 y 0.8 lbs por tonelada. Véase fig. 4.

**42. La fig. 4,** muestra la unidad de resistencia de las curvas, $f_c$, por Ec. (14), en varias unidades, que corresponde a las agudezas de 0° a 20°, y para valores de $f_1$, de 0.5 a 1.5. La línea radial punteada indica, en los diferentes grados de agudeza el desarrollo, en grados, de una milla de línea en curva de una agudeza dada; y la curva punteada indica los pies de línea por grado de desarrollo.

$$\text{Pies por grado de desarrollo} = 100/D \qquad (15)$$

$$\text{Grados de desarrollo por milla} = 5\,280\ D/100 \qquad (16)$$

Estos dos diagramas punteados representan las condiciones puramente geométricas, sin ocuparse de la *resistencia.*

**43. Efecto de la dimensión de la base de la rueda.** El prof. Wm. G. Raymond, en la Gaceta del F. C., (R. R. Gaz.), 1906, agosto 17, formula la resistencia en las curvas :

$$f_c = 0.4 + D\,(0.205 + 0.035\ B) \qquad (17)$$

donde $D$ = agudeza de la curva, en grados;
y $B$ = longitud de la base de la rueda, en pies.

**43 a. Efectos de la velocidad.** Es probable que la resistencia de la curva, $f_1$, por grado de agudeza (como res. norm.), varíe con la velocidad, en algo como lo indica la fig. 1. En otros términos, podemos esperar que sea (1) alta a velocidades muy bajas (porque el valor de la fricción se aproxima al de la fricción estática), (2) reducida a un mínimo a velocidades moderadas, y (3) aumentando, otra vez a altas velocidades.

### Resistencia del viento, $f_w$.

Para resistencia *normal*, $f_a$, debida al aire en reposo, véase ¶ 14.

**44.** Los vientos de frente se oponen por supuesto, — y los vientos en dirección contraria ayudan, — a los movimientos de avance de los trenes; y los vientos laterales, empujando las pestañas de la rueda contra la cabeza del carril, aumenta las resistencias; pero es poco lo que se sabe acerca de la importancia de estas influencias; y como su acción no puede ser verificada o prevista, su efecto es practicamente incalculable. Debe dejarse sin embargo un margen por tal concepto, según el criterio, al calcular la capacidad de las máquinas para mover cargas dadas en condiciones dadas. Donde la velocidad de los vientos de frente, o vientos de atrás pueden calcularse, la velocidad de viento puede agregarse o deducirse según corresponda, a la velocidad del tren y la suma o diferencia puede usarse en conexión con el ¶ 14. Por vientos « fuertes », la Am. Loco. Co. deduce un 8 % del tonelaje arrastrado con aire tranquilo, Boletín 1.001, feb. 1910. Véase también dos diagramas relativos al C., B. & Q., en la fig. 2.

### Inercia.

**45.** El acto de acelerar un tren exige una « fuerza de impulsión »; como el retardarlo requiere una resistencia. A la inversa, un tren que sufre aceleración puede decirse que presenta una resistencia, debida a la inercia; y uno que se retarda puede decirse que ejerce una fuerza, debida a la inercia. A fin de computar la resistencia total del tren, cuando se cambia la velocidad, hay que incluir la resistencia debida a la inercia. A fin de computar la resistencia total del tren, cuando se cambia la velocidad, hay que incluir la resistencia debida a la inercia; y para esto, es menester que sepamos o supongamos el peso, y (a) el grado de aceleración o (b) la velocidad, y la distancia dentro de la cual se realiza el cambio dado de velocidad.

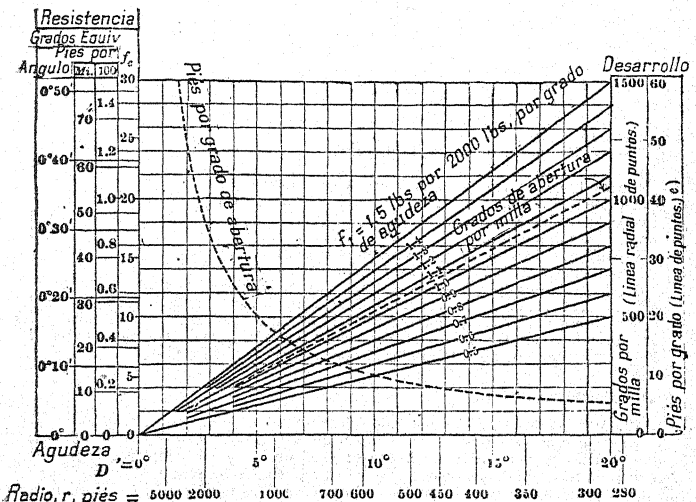



Fig. 4.

**46.** (a) Cuando un peso cae libremente, la aceleración de la velocidad, debida a la fuerza, $W$, de la gravedad, es $g = 32.2$ pies (9,81 m) por seg.; y la aceleración, $A$, de cualquier cuerpo, es directamente proporcional a la fuerza no equili-

brada $F_i$, que obra sobre él; o, $g/A = W/F_i$. Por tanto, para la fuerza o resistencia, $F_i$, debida a la inercia (y que produce aceleración o retardo), $A$, tenemos :

$$F_i = \frac{W}{g} A = M A \ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

(b) Si un vagón se mueve en una vía horizontal, y :

donde : $M = W/g$ = a la masa del cuerpo.
$W$ = al peso del vagón; $M = W/g$ = masa del vagón;
$V$ = a la velocidad del vagón;
$F_i$ = a la fuerza hor. no equilibrada, que actúa sobre el vagón;
$L$ = a la dist y $T$ = al tiempo dentro del cual $F_i$ puede producir o destruir $V$;
$K$ = la energía cinética del vagón = $F_i L$ = al trabajo correspondiente.

Entonces (Mecánica, pág. 359, etc., Art. 19) :

$$K = M V^2/2 = F_i L \ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

$$F_i = K/L = \frac{\frac{1}{2} M V^2}{\frac{1}{2} T V} = M \frac{V}{T} = M A \ldots\ldots\ldots\ldots \quad (20)$$

**47. Valores.** Lo siguiente puede tomarse como guía de la escala de aceleraciones, $A$, y de los valores de $fi$, que pueden esperarse en la práctica :

$A$ = aceleración en pies/seg en cada seg = 0.0161 $fi$;

$fi$ = fuerza en libras por 2.000 lbs = 62.1 $A$.

Aceleraciones de salida;

Pruebas de acelerómetro por Harry Egerton Wimperis, Inglaterra; Mins Proc Inst C. E., 1911-12, II Parte, lámina 8.

Trenes eléctricos de unidad múltiple, al salir de las estaciones.

| | $A$ | $fi$ |
|---|---|---|
| Promedio | 1.5 | 93.2 |
| Máximo | 2.5 | 155.3 |
| Tren de vapor | 0.5 | 31.0 |
| Una sola locomotora americana, pesada, que arrastre un tren compuesto tan solo de 2 vagones de pasajeros, puede esperarse que logre (hasta velocidades moderadas) | 1.0 | 62.1 |
| Retardo de freno; | | |
| Trenes eléctricos y de vapor, Inglaterra | 4.0 a 4.5 | 248.4 279.4 |
| Varios tipos de frenos neumáticos Westinghouse modernos de alta velocidad. Velocidades 25 a 50 millas (40 à 80 km) por hora, Ferroc. Pensylvania. | 3.0 | 186.3 |
| maxim (exactamente antes de detenerse) | 8.0 | 496.8 |
| Suponiendo el coeficiente de fricción, $c = 0.2$ lb/lb; $A = 2.000 \times 0.0161\ c = 32.2\ c = g\ c$; y dando por sentado que todos los frenos estén debidamente ajustados | 6.4 | 397.4 |

Con trenes muy livianos para el servicio de arrabales, donde son esenciales las altas aceleraciones, las fuerzas o resistencias necesarias para contrarrestar la inercia, pueden exceder mucho a todas las demás fuerzas o resistencias.

## Inercia de rotación.

**48.** Además de la inercia resultante del movimiento del vagón, en general, a lo largo de la vía, tenemos, en la rotación de ruedas y ejes, una inercia *adicional*, que resulta de su rotación únicamente; y, como la inercia lineal del vagón, esta inercia de rotación de las ruedas y los ejes produce una resistencia durante la aceleración, y una « fuerza de impulsion » durante el retardo. Tenemos que tomar aquí en cuenta el « radio de giro » (pag. 369) de las partes rotatorias. Este es menor que el radio de la llanta de la rueda.

Supóngase :

$R$ = radio de la llanta de la rueda; $r$ = radio de giro de las partes rotatorias;

$V$ = velocidad rectilínea del vagón; $v$ = velocidad rotativa del punto al extremo de $r$;

= velocidad rotatoria de un punto en la llanta de la rueda alrededor del eje de la rueda;

$A$ = aceleración correspondiente; $a$ = acel. rotat. corresp.;

$K$ = energía del vagón; $k$ = energ. rot. de las ruedas, etc.;

$F_i$ = fuerza hor., aplicada en, y normal al eje de la rueda, que produce $A$; $F_r$ = fuerza tangencial al extremo de $r$ que produce $a$;

$F_t$ = fuerza hor., en el eje, que produce $a$.

$W$ = peso del vagón; $w$ = peso de las partes rotativas;

$M$ = masa del vagón $= W/g$; $m$ = masa de las partes rotativas $= w/g$.

Luego (puesto que $V/v = A/a = R/r$) tenemos :

$$k = \tfrac{1}{2}\, m\, v^2\,; \qquad K = \tfrac{1}{2}\, M\, V^2\,; \qquad \frac{k}{K} = \frac{m\, v^2}{M\, V^2} = \frac{m\, r^2}{M\, R^2} = \frac{m}{M}\left(\frac{r}{k}\right)^2;$$

$$F_t = F_r\, \frac{r}{R} = m\, a\, \frac{r}{R}\,; \qquad F_i = M\, A\,; \qquad F_r = m\, a\,;$$

$$\frac{F_t}{F_i} = \frac{m\, a\, r}{M\, A\, R} = \frac{m\, r^2}{M\, R^2} = \frac{m}{M}\left(\frac{r}{R}\right)^2 = \frac{k}{K} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

**49.** En virtud de quedar las masas de los ejes junto a sus ejes de rotación, su inercia rotatoria es tan insignificante que de ordinario es despreciada. Pero las ruedas de vagones y los motores eléctricos son de un peso y un diámetro considerables y su inercia rotatoria constituye una porción importante de la inercia total del vagón.

**50. Valores.** En ruedas de vagón ordinarias, $r/R$ ( $= v/V$), es generalmente alrededor de 0.7, variando algo con el dibujo de la rueda; y la relación, $w/W$, entre el peso, $w$, de las ruedas, y el peso total, $W$, del vagón y la carga, varía (principalmente de acuerdo con la cantidad de carga que el vagón contenga), generalmente entre 0.17 y 0.06.

De consiguiente $\dfrac{k}{K} = \dfrac{m\, r^2}{M\, R^2}$, puede esperarse que varíe de $0.7^2 \times 0.17$ a $0.7^2 \times 0.06$; es decir, de 0.085 a 0.03; o, como término medio, digamos $k/K = 0.05$: y $K + k = 1.05\ K$.

**51.** Como quedó determinado por la *Westinghouse Mfg. Co.*, y computado con datos proporcionados por la *General Electric Co.*, $k/K$, *en motores eléctricos*, varía entre límites muy amplios. En general, sin embargo, se calcula entre 0.05 y 0.15 en vagones de servicio urbano o interurbano, despreciando las ruedas y los engranajes, por los cuales, en semejantes vagones, $k/K$ parece valer alrededor de 0.10.

Con locomotoras eléctricas, la variación es mucho más amplia; ciertas locomotoras eléctricas sin engranajes, tienen $k/K$ menor que 0.10, incluyendo las ruedas; mientras que en otras locomotoras de carga de bajo engranaje, $k/K$ puede llegar hasta 0.60.

## Carga de velocidad.

**52.** Supongamos :

$H = V^2/2\, g$ = carga debida a la velocidad hor. del vagón;

$h = v^2/2\, g$ = carga debida a la velocidad curvilínea del punto al extremo del radio de giro, $r$.

Para una masa dada, $h/H = k/K$. Por consiguiente, tomando $k/K = 0.05$, o $K + k = 1.05\ K$, tendríamos :

$$H + h = 1.05\, H.$$

De la ec (20), ¶ 46, substituyendo $(K + k) = 1.05\ K$, por $K$, tenemos :

$F_i = 1.05\ K/L.$ Por tanto, $F_i = 1.05\ W\ H/L$ ; y :

$$f_i = \frac{2.000}{W} F_i = \frac{2.100\ H}{L} = \frac{2.100\ V^2}{2\ g\ L};$$

donde $V$ = velocidad hor. del vagón, en pies por sec. = $(1.466..) \times V\ m$;
donde $V m.$ = velocidad en millas por hora. Por consiguiente tenemos :

$$f_i = 70.15\ V\ m^2/L \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

## SUMARIO

**53.** La fig. 5 da una idea general de las relaciones que existen entre la resistencia normal, $f_n$, y las tres resistencias incidentales calculables, $f_s$, $f_c$ y $f_i$, debidas a pendientes, a las curvas, y a aumentos de velocidad, respectivamente.

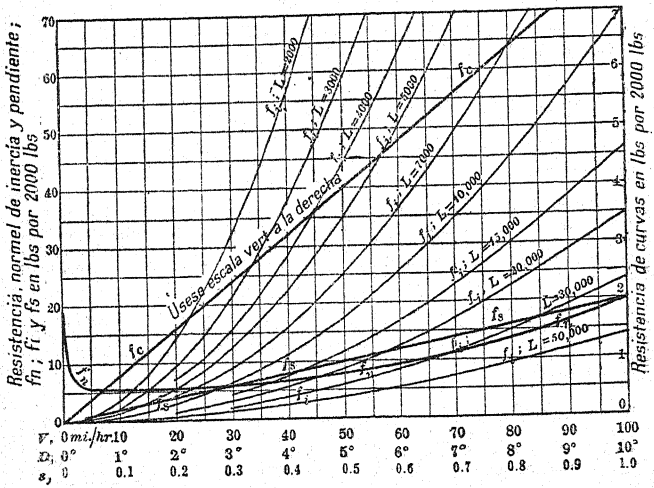



**Fig. 5.**

La escala $V$ se refiere solamente a los diagramas $f_n$, y $f_i$; mientras que las escalas $s$ y $D$ se refieren a los diagramas de $f_s$ y $f_c$ respectivamente.

Obsérvese que la escala vertical de la derecha se relaciona con el diagrama $f_c$ únicamente.

El diagrama $f_n$ representa la fórmula ec (6) ¶ 26 de la Am. Loco Co. :

$$f_n = 5.4 + 0.002\ (V - 15)^2 + \frac{100}{(V + 2)^2},$$

donde $V$ = velocidad en millas por hora.

El diagrama $f_s$ representa ec (12), ¶ 35,

$$f_s = 20\ s,$$

donde $s$ = pendiente en pies por 100 pies ó metros por 100 mets.

El diagrama $f_c$ representa ec (14), ¶ 40 :

$$f_c = D\ f_1,$$

donde $D$ = agudeza de la curva, y $f_1 = 0.8$.

**54.** Las diversas curvas $f_i$ representan la suma de las resistencias de inercia lineal y rotatoria, donde la suma = 1.05 × res. lineal, y $L$ = a la distancia, en pies, en la cual se efectúa el cambio de velocidad. Al usar estas curvas, es necesario deducir las dos resistencias, una por el movimiento inicial y otra por la velocidad final (cada cual por la distancia $L$ en cuestión). Luego, resistencia de inercia = *únicamente a la diferencia* entre estas dos resistencias, cada una de las cuales representa por sí solo la resistencia cuando una de las velocidades es *cero*.

$$f_i = 70.15\ Vm^2/L \ldots\ldots\ldots\ldots \text{ec. (22), ¶ 52}$$

**55. Ejemplo.** Supongamos que el peso, $W$, del tren sea 1.500 toneladas; que la pendiente ascendente o terraplen, $s$ = 0.05 pies por 100 pies; la agudeza de la curva, $D$ = 3°; y supongamos que la velocidad, $V$, sea aumentada, durante una distancia, $L$, de 10,000 pies, de 30 a 40 millas por hora (vel. media, 35 millas por hora).

Entonces, en la fig. 5, tenemos :

| | Lbs por tons. |
|---|---|
| Resistencia normal, $f_n$, Am. Loco Co. ($V$ = 35 millas por hora)... | 6.2 |
| — de la pendiente, $f_s$ (= 20 × 0.05)..... | 1.0 |
| — de la curva, $f_c$ (= 0.8 × 3)..... | 2.4 |
| — de la inercia, $f_i$ (= 11.2 − 6.3)..... | 4.9 |
| Unidad total de resistencia, $f$, ..... | 14.5 |

Resistencia total, $F = f\ W$ = 14.5 × 1.500 = 21,750 lbs.

De las cuales son debidas :

a resistencias con velocidad uniforme 9.6 × 1.500 = 14,400 lbs.

a resistencias por aumento de velocidad 4.9 × 1.500 = 7,350 lbs.

## VAGONES DE DINAMÓMETRO

**56. La función principal** de un vagón de dinamómetro es el de medir (y registrar) la *fuerza de arrastre o tracción*. El vagón dinamómetro se coloca entre el ténder de la locomotora y el primer coche o furgón del tren, y la suma del arrastre de la barra de tracción indica entonces el esfuerzo de tracción ejercido sobre el tren con exclusión del ténder.

**57. Contador o registro.** El vagón se halla de ordinario dotado de aparatos destinados a registrar o trazar los diversos detalles que habían de obtenerse. Este aparato consiste esencialmente de una mesa o marco ancho, colocado en el vagón, con una larga y continua banda o tira de papel que va registrando los datos. Esta banda se mueve longitudinalmente sobre la mesa y bajo numerosos lápices o plumas, cada uno de los cuales es susceptible de mayor o menor movimiento lateral a través del papel.

**El movimiento del papel,** podrá hacerse proporcional a la distancia recorrida a lo largo de la vía, o al tiempo consumido. Si el movimiento del papel ha de ser proporcional a la distancia recorrida, el carrete donde va envuelto el papel, o que lo guía, va generalmente encajado en un eje; dejándolo sin encajar, toda vez que la acción del freno pueda hacer que las ruedas se resbalen. Es preferible arreglar este encaje de modo que el papel se mueva en una sola dirección, sea que el vagón avance o retroceda, pues de otro modo las inscripciones se sobrepondrían al retroceder y volver a arrancar el tren. Si se quiere que el movimiento del papel sea proporcional al *tiempo*, el carrete o tambor puede ser movido mediante un mecanismo de relojería, o por medio de un mecanismo en abanico bien construído o si no, con magnetos movidos por contactos operados a cortos intervalos por un sistema de relojería.

**58. Plumas.** Los movimientos laterales de los diversos lápices y plumas se gobiernan, unas veces por medio de magnetos eléctricos, y otras valiéndose de correspondencias mecánicas de varias clases, según las necesidades.

**59. La fuerza de tracción de la barra de tiro,** se medía en otros tiempos por medio de un mecanismo semejante al de las básculas, puesto a un costado, de manera que mida las fuerzas horizontales en vez de las verticales. Es costumbre general ahora, sin embargo, conectar la barra de tiro directamente con un émbolo

que se desliza en un sólido cilindro que contenga aceite. Hay un tubo que va de este cilindro a un cilindro menor colocado en el aparato registrador, y dotado de un émbolo y un resorte, como en los « indicadores » ordinarios de máquinas de vapor. Entonces este registra, así, directamente sobre el papel, moviendo su pluma más o menos de un lado, según la intensidad de la fuerza trasmitida a través del aceite, desde la barra de tracción. Por otro método se hace de la barra de tiro un « indicador » de cilindro principal, ajustándolo con resortes ó muelles precisos, pero pesados, y transmitiendo directamente su movimiento mecánicamente a la pluma del registrador. Al usar ambos métodos deberá compensarse el escape de aceite a causa del paso del émbolo en el cilindro. Esto se hace generalmente valiéndose de una bomba especial.

**60. Aceleración.** Los acelerómetros dependen ordinariamente de la desviación hacia adelante o hacia atrás de un péndulo u otra masa excéntricamente suspendida, bajo la acción de la fuerza resultante de la aceleración o del retardo. Se ha experimentado alguna dificultad en evitar las oscilaciones violentas y engañosas que producen indebidamente retardo en el movimiento. El acelerómetro de Wimperies (*Mins.* Proc. Inst. C. E. (Inglaterra), 1911-12, Parte II, página 420), parece dar buenos resultados, amortiguando los movimientos de las masas magnéticamente. Wilfredo Lewis de Filadelfia ha empleado un instrumento parecido al tubo nivelador corriente; y la marcha o funcionamiento de la ampolla parece pasablemente satisfactorio; pero este artificio no registra. Otro instrumento, con que ha hecho experimentos la Compañía Trautwine, no puede sobreindicar, por repentinos que sean los cambios de fuerza aceleradora. El aparato (transportado en el vagón), depende de la caída constante de una corriente de partículas, tales como arena, bolitas de acero, o gotas de agua, desde un punto dado del aparato, y desde determinada altura, sobre una báscula colocada en el fondo del aparato y colocada en la dirección en que la aceleración ha de medirse. Las partículas caerán en rápida sucesión, y cada una de ellas, cayendo libremente, deberá continuar cayendo con la velocidad horizontal que tenía al ser soltada. Por consiguiente, cualquier cambio de velocidad del vagón ocurrido durante la caída de cualquiera de las partículas, remueve más o menos la báscula que está debajo, de modo que la partícula cae sobre la báscula a una distancia mayor o menor del cero (que se halla inmediatamente debajo del punto desde donde las partículas se sueltan), en proporción directa a la aceleración media durante el lapso de la caída.

**61.** Estrictamente hablando, un acelerómetro puede medir la aceleración *únicamente* si *se corre a nivel.* Con el tren en una *pendiente*, el efecto de la pendiente aumenta o disminuye las indicaciones, y, a menos que se puedan hacer correcciones, en este sentido, las indicaciones de las aceleraciones son engañosas. En un vagón o tren que corra sin fuerza motriz ni frenos (en cualquier pendiente), las indicaciones de un acelerómetro dan entonces la medida de la resistencia total del vagón, o del tren, en conjunto.

**62. Las fuerzas laterales,** debidas a las curvas o a desigualdades de la vía, son frecuentemente medidas por un acelerómetro colocado a ángulos rectos con la vía, particularmente en los vagones de prueba.

**63. El tiempo** deberá registrarse cuando el movimiento del papel está en proporción con la distancia recorrida; y una indicación de tiempo es útil aun cuando el papel se mueva a intervalos de tiempo. Esto se realiza generalmente con una pluma que se desvía por medio de un electromagneto accionado cada cinco segundos mediante contactos con un reloj.

**64. La distancia** deberá registrarse si el papel se hace correr en proporción con el tiempo, y una indicación de distancia es útil aun cuando el papel se mueva en proporción a *la distancia.* Se hace con una pluma desviada por medio de un electromagneto, o mediante engranaje con las ruedas, cada vez que se haya recorrido cierta distancia. En todo caso, la distancia se registra generalmente también por un vigilante sirviéndose de un botón de presión, mediante el cual puede desviar otra pluma al pasar cada poste de milla u otro objeto.

**65. Velocidad.** La velocidad puede deducirse de los *records* de distancia si el papel se mueve en proporción dada del tiempo, o *viceversa.* Sin embargo, se la traza con preferencia con una pluma especial que, a medida que la velocidad cambia

va desviándose de su línea mediante una correspondencia mecánica con un aparato semejante a una palanca centrífuga, o por algún otro sistema de medidor de velocidad; tambien puede arreglarse la pluma de manera que sea desviada de su línea por medio de un contacto eléctrico o engranaje con las ruedas, a intervalos proporcionales a la distancia recorrida, soltándola al término de intervalos de tiempo dados, para dejarla que vuelva a la línea de partida, obteniendo así un *record* de velocidades medias a cortos intervalos sucesivos de tiempo. Usando de un mejoramiento adicional solamente el mecanismo contador o de marcha ascendente regresa a cero, en tanto que la pluma permanece en el punto anteriormente alcanzado, trazando por consiguiente una curva en vez de una serie de ordenadas.

**66. La acción del freno neumático** se registra generalmente por medio de un émbolo en un cilindro pequeño conectado con el cilindro del freno del vagón.

**67. La presión de la caldera** se ha registrado de modo semejante, valiéndose de un tubo de vapor conectado con la locomotora.

**68. Velocidad del viento.** — Esta ha sido registrada (al menos en el vagón de pruebas de la Universidad de Illinois) con el anemómetro Robinson en forma de copa que ordinariamente se usa en la Oficina de Señales Meteorológicas, produciendo y rompiendo el contacto eléctrico cada 0.2 milla de viento.

**69. Dirección del viento** : ésta se registra (en el mismo vagón) por medio de una conexión mecánica con una veleta neumática por la cual la desviación de la pluma se conserva = al seno del ángulo de la veleta, derecha o izquierda, mientras que la posición de avance o retroceso de la veleta es indicada por otra pluma.

**70. Las curvas** pueden registrarse por una conexión mecánica con una de las carretillas o troles del vagón.

**71. Los datos mixtos o diversos** son registrados las más veces a mano, por medio de botones de presion colocados en el vagón o en la locomotora; este método se emplea cuando se trata de medir las paladas de carbón que se ponen en el hogar de una locomotora; cuando se abren y cierran las puertas de éste; cuando se abre y cierra la válvula de paso, para la posición de la palanca de contravapor, etc., etc.

# DINÁMICA DE LOS TRENES

## GENERALIDADES

### Fuerza de tracción, Resistencias, Velocidad, Energía.

**1.** Los movimientos de un tren se rigen por la relación que existe entre la fuerza de tracción y las resistencias. Esta relación se halla parcialmente bajo el gobierno del maquinista. Véase ¶ 4.

**2. Fuerza de tracción.** La fuerza de arrastre de una locomotora (pag. 1128) aumenta a medida que entra mas vapor en los cilindros (como cuando se aumenta el rendimiento de la caldera o la abertura de la válvula de paso, o se retarda la acción del cortavapor), y aumentando la « adhesión » entre las ruedas motrices y los carriles (como cuando se riega arena en ellos). La fuerza de tracción se disminuye reduciendo el vapor en los cilindros y por el fácil deslizamiento entre motrices y carriles.

**3. La resistencia** se aumenta aplicando los frenos o invirtiendo las máquinas de la locomotora; y se disminuye soltando los frenos.

**4. En la práctica**, no obstante, estas operaciones suelen complicarse con otros fenómenos. Así, un positivo *aumento de velocidad* (como el que resulta de admitir más vapor en los cilindros) aumenta la dificultad en mantener los aumentos de presión de los cilindros; en cambio, a medida que *el tren disminuye su marcha* se va facilitando más y más el mantenimiento de las expresadas presiones.

**5.** Además, en ciertas condiciones (Resistencia, páginas 1129, etc.) la resistencia atmosférica y la resistencia de fricción aumentan si se aumenta la velocidad. Como resultado de esto, cada pendiente tiene su límite de velocidad, que un tren en descenso y a su propio impulso no puede exceder. Véanse pág. 1159-60, ¶¶ 37-38.

### Efecto de la inercia.

**6. Energía.** Al aumentar la velocidad se almacena energía dinámica en el tren, y tal acumulación (salvo cuando la produce un movimiento en pendiente hacia abajo) exige un trabajo adicional de la locomotora. Al ir disminuyendo la velocidad, la energía motriz, acumulada ya en el tren, coopera en el movimiento de avance.

**7.** Para todo cambio de velocidad, como de $v_1$ a $v_2$, el cambio, en energía motriz, es

$$W \frac{v_1^2 - v_2^2}{2g} = m \frac{v_1^2 - v_2^2}{2} \text{ ; donde :}$$

$W$ = al peso, y $m$ = la masa, del tren; y $g$ = aceleración de gravedad = digamos, a 32.2 pies (9.81 m) por sec en cada seg.

**8. La altura de caida** = a la altura de la cual un cuerpo debe caer, para cambiar su velocidad de $v_1$ a $v_2$, es :

$$h_v = \frac{v_1^2 - v_2^2}{2g}. \text{ Por consiguiente,}$$

$$\text{Cambio en la energía motriz} = W \frac{v_1^2 - v_2^2}{2g} = W\ h_v.$$

Donde si $v_1$ o $v_2$ = cero, tenemos, velocidad = $v$, y cambio de energía = $W\ v^2/2\ g$ = $W\ h_v$.

## PENDIENTES

**9.** Cuando se trata de establecer las pendientes, es de vital importancia *su efecto sobre las velocidades del tren.*

### Perfil virtual.

**10. Altura de caida virtual; pendiente virtual; perfil virtual.** En las figuras 1 *a*, 1 *b* y 1 *c*, supóngase que *A B C... G* sea el perfil de un ferrocarril dado; y en cada fig., supóngase que un tren, viniendo de la izquierda, llega a *A* con una velocidad de *v* pies por seg. Imaginemos que *A a* representa en la escala del perfil efectivo, la altura de caida correspondiente, $h_v = v^2/2\,g$. Para mayor simplicidad, despreciamos aquí la velocidad rotativa de las ruedas; véase pag. 1136. En otros puntos, de ambas figuras, establezcánse los ordenados, *B b*, *C c*, *D d*, etc., que representan, según la misma escala, las alturas de caida en esos puntos, respectivamente. Luego, por lo que hace a las condiciones de estos perfiles, véase ¶ 13; en cualquiera de las figuras, *a b c... g* es el perfil virtual que corresponde al efectivo, *A B C... G*.

**11.** Supóngase que :

| | | |
|---|---|---|
| $T$ = la fuerza de arrastre (tracción) de la locomotora..... | paralela a la vía. | |
| $F$ = la resistencia total o resultante..... | — | — |
| $t = T - F$ = la fuerza resultante..... | — | — |
| = la fuerza que produce la aceleración..... | — | — |
| $F_g$ = la componente de la gravedad..... | — | — |
| $F_r$ = la resistencia que no obra..... | — | — |
| (« + » = a favor $T$ ; « − » = contra $T$)..... | — | — |
| $W$ = peso del tren..... | obrando verticalmente | hacia abajo. |
| $g$ = aceleracion debida a la gravedad... | — | — |

**12.** Como será demostrado, la marcha del tren se gobierna mediante la concordancia (que en parte depende de la dirección del maquinista) entre $T, F, F_g$ y $F_r$.

| Si $T$ es | $F_g$ es | $T + F$ es | $F_r$ es | $F =$ | $t\ (= T - F) =$ |
|---|---|---|---|---|---|
| $< F_g$ | − | − | + | $F_g - F_r$ | $T - (F_g - F_r)$ |
| | | | | | $= T - F_g + F_r$ |
| $> F_g$ | − | + | − | $F_g + F_r$ | $T - (F_g + F_r)$ |
| | | | | | $= T - F_g - F_r$ |
| → | 0 | + | − | $F_r$ | $T - F_r$ |
| ↘ | + | + | − | $F_r - F_g$ | $T - (F_r - F_g)$ |
| | | | | | $= T + F_g - F_r$ |

**13. Las figuras 1 a, 1 b y 1 c** representan tres casos típicos, y supuestos como sigue :

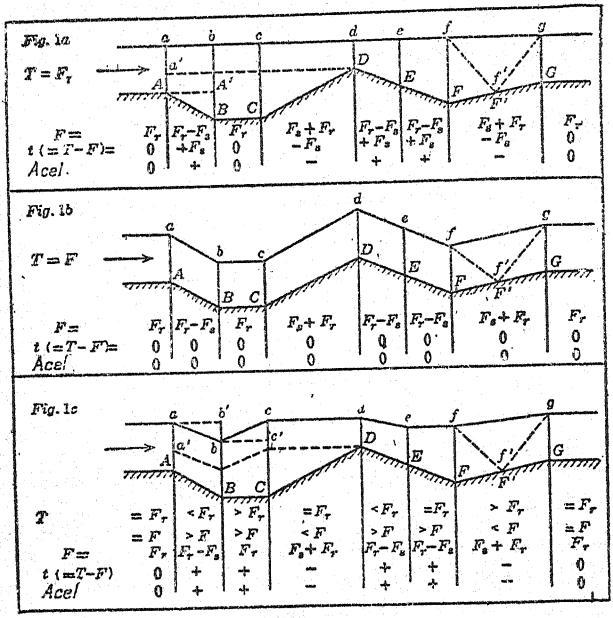
Fig. 1*a*. $T = F_r$. Perfil virtual horizontal;
Fig. 1*b*. $T = F$. Perfil virtual paralelo al perfil real;
Fig. 1*c*. Ninguna de las dos condiciones anteriores satisfecha.

**14. Fig. 1 a. « Tren sin fricción ».** Vamos a suponer ahora que la fuerza de tracción o arrastre en todo el tren se mantiene igual a la resistencia que no obra; o bien $T = F_r$. El tren se halla entonces mecánicamente en la condición de un cuerpo libre de toda fricción; es decir, todos los cambios de velocidad son los debidos a la gravedad solamente.

**15.** Luego, pasando de *A* a *B*, el tren pierde *A' B* en elevación, pero gana una suma igual en la altura de caida debido al aumento de velocidad, haciendo su alt de caida total, en *B*, $h_v = B\,b = B\,A' + A'\,b$. Por consiguiente, el perfil virtual, *a b*, entre *A* y *B*, es horizontal, como lo es también (en la fig. 1 *a*) el perfil virtual,

*a b c... g*, en la línea entera, *A B C... G*, suponiendo (como arriba) que $T$ y $F_r$ se mantengan iguales. Esto puede suceder, al menos aproximadamente, en los trenes ligeros y rápidos de pasajeros en pendientes cortas y moderadas. Tal condición continuaría existiendo indefinidamente (suponiendo que $T$ se mantenga $= F_r$), con la excepción de que si, en cualquier punto, la pendiente en cuestión, *A... G*, asciende al nivel *a... g* el tren se para, y no puede volver a arrancar sin un aumento de $T$ sobre $F$. Véase ¶ 21.

**16. Fig. 1 b. Velocidad uniforme.** Si la fuerza de tracción se mantiene igual a la resistencia total (que incluye la componente debida a la gravedad), es decir, si $T = F = \pm F_r \pm F_s$; de modo que $t = T - F =$ cero; entonces la velocidad (y por consiguiente la alt de caída, $h$, representada por *A a*, *B b*, etc.) permanece constante, e igual a la del tren al llegar a *A*; y cada tramo o extensión, *a b*,




Figs. 1.

*b c*, etc., del perfil virtual, *a b c... g* es paralelo al tramo correspondiente, *A B*, *B C*, etc., del perfil real, *A B C... G*. En tramos *a nivel*, como *B C*, tenemos $F_s =$ cero, $F = F_r$, y, en la fig. 1 *b* (como en toda la fig. 1 *a*), $T = F_r$, y *b c* horizontal.

**17. Fig. 1 c. Caso común.** En general (y especialmente con trenes de carga, pesados) la relación entre $T$, $F_s$, $F_r$ y $F$, *graduada* por el maquinista (véase ¶ ¶ 1, 2, 3) con arreglo a las condiciones existentes, no llena ni los requisitos de la fig. 1 *a* ni las de la fig. 1 *b*.

**18.** En el supuesto caso, representado por la fig. 1 *c*, tenemos :

En la fig. 1 *c*.

| Sección. | Pendiente real. | Pendiente virtual. | Fuerza de tracción, $T$. | Pendientes reales y virtuales. | $t\ (= T - F)$ y aceleración. |
|---|---|---|---|---|---|
| ..... *A* | → | → | $= F_r = F$ | *Paralelo* | 0 |
| *A* — *B* | ↘ | ↘ | $< F_r > F$* | *divergente* | + |
| *B* — *C* | → | ↗ | $> F_r > F$ | " | + |
| *C* — *D* | ↗ | → | $= F_r < F$ | *convergente* | — |
| *D* — *E* | ↘ | ↘ | $< F_r > F$* | *divergente* | + |
| *E* — *F* | ↘ | → | $= F_r > F$ | " | + |
| *F* — *G* | ↗ | ↗ | $> F_r < F$ | *Convergente* | — |
| *G* .... | → | → | $= F_r = F$ | *Paralelo* | 0 |

En general; figs 1 *a*, 1 *b*, 1 *c*,

| Si la pendiente virtual es : | La fuerza de tracción, $T$, es : | Si las pendientes virtual y real son : | La velocidad es : |
|---|---|---|---|
| en subida, | $> F_r$ | divergentes, | acelerada. |
| a nivel, | $= F_r$ | paralela, | uniforme. |
| en bajada. | $< F_r$ | convergente, | retardada. |
| | | coincidente, | cero. |

**19. El perfil de puntos**, *f f' g*, en cada una de las tres figuras, representa un caso donde, en lugar de proceder continuadamente de *F* a *G*, el tren se detiene, como en *F'* ; es decir, el caso en que la velocidad se reduce a cero, y las dos pendientes coinciden.

**20. Perfiles diferentes** *para trenes diferentes.* Puesto que los trenes difieren en cuanto a velocidad, etc., cada tren requiere separadamente su propio perfil virtual; así, la fig. 1 *a* representa aproximadamente las condiciones probables para trenes rápidos de pasajeros; la fig. 1 *c* las de trenes de carga, etc.

**21.** El perfil virtual muestra las velocidades, y los cambios de velocidad, que pueden esperarse de un tren dado en un perfil dado. Así : fig. 1 *a* o 1 *c*, si, en *A*, la velocidad fuera tal que la altura de caída fuera tan solo *A a'*, el tren se atascaría en *D* siempre que las demás condiciones permanezcan como queda indicado; pues el perfil virtual, *a d*, en ese caso desciende verticalmente y paralelo, a sí mismo en la distancia *a a'* = *d D*. Véase ¶ 15.

* En una pendiente de descenso, la resistencia *total* se convierte en *Fr* menos *Fs* (véase ¶ 12); y puede por tanto ser muy pequeña, ó cero, ó una cantidad negativa. Por consiguiente, la fuerza de tracción de la locomotora, *T*, aun siendo menor que *Fr*, puede aun ser mayor que la resistencia « total », *F*.

**22. Altura da caída ganada.** Fig. 1 *c*. En cualquier distancia dada, como *B C*, la alt de caida ganada que no obra, $h_v$ (o la alt de caida perdida segun el caso), representada por *c' c*, es directamente proporcional a la fuerza no equilibrada que no obra $t_r$ ($= T - F_r$ o $F_r - T$), obrando sobre el tren en toda esa distancia; y la proporción, entre dicha fuerza no equilibrada, $t$, y el peso del tren, $W$, es igual a la relación entre $h_v$ y la distancia, $L$, representada por *B C*. En otros términos,

$$\frac{t_r}{W} = \frac{h_v}{L}; \text{ o } t_r = W \frac{h_v}{L}.$$

**23. Las pendientes del perfil miden el exceso de fuerza.** Fig. 1 *c*. De un modo semejante, en la distancia, representada por *A B*, hay un exceso ($= t_r = F_r - T$) de resistencia que no obra sobre la fuerza de tracción ; y el aumento de velocidad y de altura de caida es en consecuencia menor, en la figura 1 *c*, que en la misma extensión, *A B*, en la fig. 1 *a*, donde $T = F_r$. La pérdida correspondiente, $h_r$, de velocidad inicial, en *A B*, fig. 1 *c*, comparada con la de la fig. 1 *a*, se halla representada por *b' b*. Aquí también :

$$t_r = W \frac{h_v}{L}.$$

La pendiente de las líneas en el perfil virtual dan pues la medida del exceso de la fuerza de tracción sobre la resistencia que no obra ó viceversa.

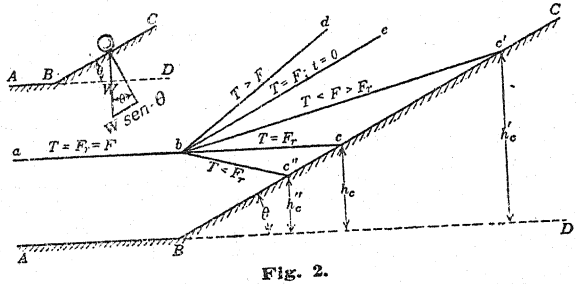



**Fig. 2.**

## Altura a la cual un tren subirá por una pendiente.

**24. Fig. 2.** Supóngase que un tren corre, a velocidad uniforme, sobre una vía a nivel, *A B*. Entonces (siendo uniforme la velocidad), tenemos, **en A B, T = F**; o bien, $t\ (= T - F)$ = cero (¶ 18); y (con *A B* estando a nivel), tenemos : componente de la gravedad, $F_s$ = cero; y fuerza de tracción, $T\ (= F) = F_r + F_s = F_r + 0$ = la resistencia que no obra, $F_r$.

**25. En B**, supongamos que el tren encuentra una subida, *B C* ; y supongamos
$\theta$ = al ángulo, *C B D*, entre la pendiente, *B C*, y la horizontal;
$t = T - F$ = fuerza neta de tracción.
= fuerza de tracción de la loco menos la resistencia total o resultante.
$W$ = peso del tren y la locomotora;
$F_s = W$ sen $\theta$ = a la componente de la gravedad;
$g$ = aceleración de la gravedad = 32.2 pies (9.81 m) por sec en cada seg;
$v$ = velocidad del tren en *B*.

**26. Y si, en la pendiente, B C** (a pesar de que $F_s$ se agrega, a la resistencia) $T$ es mantenida de modo que tengamos :
**T > F** (la pendiente virtual, *b d*, es *divergente* de la pendiente real, *B C*), entonces (véase ¶ 18) la velocidad es acelerada, y la aceleración = $g\ t/W$.

Si :

$\mathbf{T} = \mathbf{F}$, $t = 0$ (pendiente virtual, $b\ e$, paralela a la pendiente real ó efectiva, $B\ C$), entonces la velocidad es uniforme (aceleración $= g\ t/W =$ cero). En cualquiera de estos casos, el tren asciende la pendiente indefinidamente.

**27.** Pero si, en la pendiente, $B\ C$, tenemos :

$\mathbf{T} < \mathbf{F}$ (la pend virtual $b\ e$, $b\ e'$, o, $b\ e''$, y la pend. real, $B\ C$, convergen), entonces. la velocidad es retardada, con aceleración negativa $= -\ g\ t/W$, y la fuerza retardatriz no equilibrada, $-\ t = T - F$, llegará a detener el tren :

á una altura, $h_e$, $h_e'$ o $h_e''$, y

á una distancia, $h_e/\text{sen}\ \theta$, ó $h_e'/\text{sen}\ \theta$, ó $h_e''/\text{sen}\ \theta$ determinadas como sigue :

**28.** (1) Si, en la pendiente, $B\ C$, tenemos :

$\mathbf{T}\ (< \mathbf{F}) = \mathbf{F}_r$ (pend. virt., $b\ e$, *horizontal*),

entonces el tren (véase ¶ 14) se mueve bajo la acción de la componente de la gravedad, $t = F_s = W$ sen $\theta$, sola, con aceleración negativa $= -\ g\ t/W = -\ g\ W$ sen $\theta/W = -\ g$ sen $\theta$. Por consiguiente, la fuerza retardatriz no equilibrada, $-\ t = F_s = W$ sin $\theta$, llegará a parar el tren.

á una altura, $h_e = v^2/2g$, y

á una distancia, $B\ e = h/\text{sin}\ \theta$.

**29.** (2) Pero si, en la pendiente, $B\ C$, tenemos :

(a) $\mathbf{T}\ (< \mathbf{F}) > \mathbf{F}_r$ (pend. virtual, $b\ e'$, ascendente), o

(b) $\mathbf{T}\ (< \mathbf{F}) < \mathbf{F}$ ( — $b\ e''$, descendente):

entonces la fuerza retardatriz no equilibrada, $-\ t = T - F$ llegará a parar el tren

á una altura, (a) $h_e' = h_e\ W$ sin $\theta/t'$, o

(b) $h_e'' = h_e\ W$ sin $\theta/t''$ ;

á una distancia, (a) $B\ e' = h_e\ W/t' = h_e'/\text{sin}\ \theta$; o

(b) $B\ e'' = h_e\ W/t'' = h_e''/\text{sin}\ \theta$.

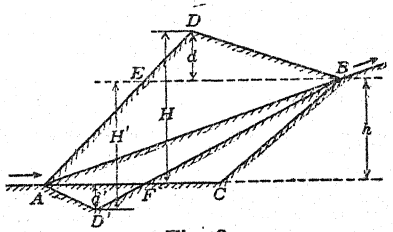



**Fig. 3.**

## Trabajo que resulta de las diferencias de alturas.

**30. Fig. 3. Teoría.** Al mover un tren, de peso, $W$, de $A$ a $B$ (sea por la pendiente uniforme, $A - B$, o por la vía $A - C - B$, o $A - D - B$, o $A - D - B$), el trabajo neto o resultante hecho debido a la diferencia de elevación, solamente, y despreciando la fricción, es teóricamente $W\ h$, es decir el trabajo de levantar el peso, $W$, a la altura vertical $h$.

**31.** Al atravesar $A - B$, o $A - C - B$, el tren no sube más allá de $B$, o desciende bajo $A$ ; y el trabajo *total*, debido a diferencia de elevación, es igual al trabajo *neto, resultante*, $W\ h$.

**32.** Al moverse por $A - D - B$ ó $A - D' - B$, el peso del tren, $W$ debe en efecto ser elevado a una altura mayor, $H$ o $H'$, respectivamente, haciendo el trabajo total debido a la diferencia de elevación $= W\ H$, o $W\ H'$, respectivamente; pero teóricamente, el trabajo adicional, $W\ d$ o $W\ d'$, debido a la elevación, $d = H - h$, o $d' = H' - h$, respectivamente, es compensado por el que realiza el tren cuesta abajo, después de trasponer la cumbre, $D$, o antes de alcanzar el punto más bajo, $D'$.

**33. Práctica.** Por tanto, el trabajo resultante, debido á la diferencia de nivel, queda, mecánicamente = $W\ h$; y esto se realiza con frecuencia, con más o menos aproximación, en la práctica; y la energía, debida a los descensos, se utiliza en una extensión que depende de las longitudes y de lo fuerte de las pendientes, y de las resistencias en ellas; dependiendo también de la pendiente posterior a $B$, y de si hay una parada en $B$ o al pie ($D'$) de la pendiente en bajada.

**34.** Toda subida, $E - D$ (o descenso, $A - D'$), que requiere un trabajo mayor que $W\ h$, va acompañada de un descenso, $D - B$ (o subida, $D' - F$) de igual altura vertical. Pero la « subida y bajada », en $E - D - B$ ($A - D' - F$) es $d$ ($d'$); y no 2 $d$ (2 $d'$).

## Consideraciones prácticas.

**35. Las pendientes aumentan el costo de explotación.**

En la subida de una pendiente, $A\ B$ o $C\ B$, fig. 3, pag. 1147, el ascenso, $h$, implica un gasto adicional de combustible. Reduciendo la velocidad, en la pendiente, se disminuirá la normal resistencia total (compárese la fig. 1 relativa a Resistencia del Tren, pág. 1130); pero la reducción de velocidad significa tiempo perdido y por consiguiente un aumento en el costo de la explotación.

**36. Pendientes en descenso.** En el descenso de toda pendiente se observará siempre una reducción en lo que hace al consumo de combustible, sobre la cantidad requerida generalmente en una vía á nivel; asi como en el costo de salarios por haber recorrido una distancia dada en menos tiempo; pero el descenso de pendientes fuertes resulta, sin embargo, incómodo y costoso, por el desgaste de frenos y ruedas.

**37. Efecto de la longitud de la pendiente sobre la fuerza de tracción.** En una subida larga, a menos que se reduzca la velocidad, el *exceso* de presión con que la caldera haya entrado en la pendiente, se emplea y puede agotarse a cierta distancia según la carga y la inclinación de la pendiente. Así, la fuerza de tracción está limitada por las propiedades generadoras de la caldera; y debe contarse con que la fuerza media de tracción, en toda la pendiente disminuirá a medida que aumente la longitud de la pendiente. Comparando diversos casos, el señor Beverley S. Randolph encuentra, aproximadamente :

| Longitud de una subida continua (kilómetros): | 0 | 8 | 16 | 24 | 32 y más |
|---|---|---|---|---|---|
| Fuerza media de tracción = | 31 | 24 | 19 | 17 | 16 % de peso en las motrices. |

La inclinación parece no afectar estos números considerablemente. Am. Soc. C. E., Trans Vol 70, dic. 1910, p. 323.

**38. Subidas y bajadas.** No obstante lo dicho, las pendientes menores (y aún más fuertes que las reglamentarias) pueden á menudo entrar debidamente en el perfil de un ferrocarril. Su adopción puede ocasionar grandes economías en la construcción, evitando desmontes y rellenos importantes. Para lo concerniente á « pendientes reglamentarias », véase ¶ 11, pag. 1156.

(Véase tambien *N. del T.* al pie de esta pagina).

**39. Pendiente de aceleración** es la pendiente dada con el fin de ayudar a parar o dar impulso a los trenes en una estación situada en la cumbre de la pendiente, o de acelerar o retardar su velocidad en un punto dado. Tales pendientes pueden reducir no solamente el deterioro de las llantas de las ruedas y de las calzas de los frenos sino tambien el esfuerzo necesario para reanudar la velocidad, y además el tiempo que ambas operaciones exigen. Su empleo es frecuente en las líneas de cambios rápidos, aun cuando implique mayores excavaciones para subterráneos, o pronunciadas subidas, para los pasajeros, a estaciones elevadas.

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(*N. del T. al* ¶ 38. — Además de lo dicho por el autor, suele ser decisivo para hacer pendiente parte de una línea ó su totalidad, cuando se logra con aquellas, como resulta muchas veces, acortar la longitud de la línea. En este caso debe calcularse si el mayor costo de explotacion por kilómetro del trayecto en pendieute multiplicado por sú longitud, es menor que el producto de la mayor longitud de la linea más suave por el menor costo kilométrico de su explotación. Si a esto se agregan los intereses del capital de costo en cada una, se tendrán los mas importantes datos para la elección; pero es muy cierto que hay muchas circunstancias locales que tambien influyen sobre la adopción de una ú otra vía.)

**40. Pendiente de impulsión** se llama asi á la que, aunque muy fuerte, es tan corta que cualquier tren, con velocidad razonable, puede pasarla sin perder demasiada velocidad. Es de ordinario tan fuerte que un tren no podría pasarla sin « cobrar impulso », ó volver a arrancar si está parado en ella.

**41.** Las pendientes de impulsión deben, por consiguiente, adoptarse solamente después de considerar todas las circunstancias probables o posibles que tiendan a parar el tren; tales como cruceros de pendiente, curvaturas, desviaderos, depósitos de agua, estaciones, etc., en, ó cerca de la pendiente; o la posible instalación futura de tales cosas. Si el peligro de las paradas es pequeño; o si un tren, cuando está parado, puede prontamente retroceder, y emprender la marcha otra vez, y si el retardo para el tráfico subsiguiente no es serio; entonces, la pendiente de impulsión está permitida.

**42. Limitaciones de pendientes en subida.** *Arranque en subida.* Cuando los trenes deben arrancar hacia arriba, la pendiente debe ser tal (Webb) que permita una aceleración cuando menos de 24 kilometros por hora en 300 ms. (Alaska Central Ry.)

**43.** En las cortadas, aun donde existen otrasc ircunstancias que permitan el uso de una linea horizontal, es costumbre establecer una pendiente no menor de 2 por mil, con objeto de facilitar los desagües.

**44.** Un *ferrocarril especifica* que, en las vías laterales, ninguna pendiente debe pasar de 5 %, en una tangente, ó de 3 % en una curva, ó de 1 % donde los carros se cargan.

**45. Limitaciones de pendientes en bajada.** La velocidad adquirida al bajar una pendiente, depende en parte de su desnivel y en parte de la proporción de la pendiente. Véase Clasificación de pendientes, pag. 1159, ¶ 37. El declive no deberá nunca ser tal que los trenes de carga (abandonados a su propio impulso) adquieran velocidades que excedan digamos de 48 a 56 km por hora; ni deberá tampoco reducir la velocidad de trenes ascendentes a menos de 16 km por hora.

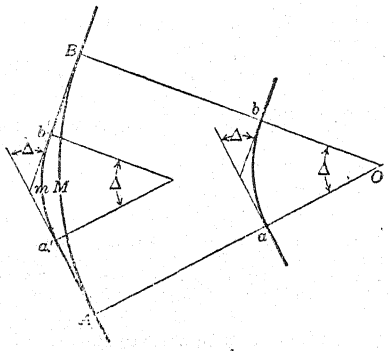


Fig. 4.

**46. Curvas verticales.** Deben emplearse donde quiera que haya un cambio de pendiente marcado. Evitan los choques tanto en el avance como en los retrocesos, debido á que se aflojan ó se aprietan los enganches entres los carros y evitan el peligro consiguiente de que el tren se parta en dos.

**47.** En la fig. 4, considerando la resistencia de la curva como proporcional a la agudeza, $D$, o aproximadamente inversa al radio, $R$ (véase Resistencia de trenes, pag. 1134, ¶ 40), supongamos :

| | Para la curva $A\ M\ B$. | Para la curva $a\ b$ |
|---|---|---|
| $n$ = ..... | $R/r$ | $R/r$ |
| Agudeza = aprox..... | $d = D/n$ | $D = n\ d$ |
| Radio = ..... | $R = n\ r$ | $r = R/n$ |
| Resistencia de la curva = aprox... | $f_c = F_c/n$ | $F_c = n\ f_c$ |
| Longitud de la curva, en cadenas de 100 pies..... | $L = n\ l$ | $l = L/n$ |

Extension o desarrollo o áng del centro... $\Delta = Ld = n\ l\ D/n = l\ D$.

**48. Energía.** Luego, por la energía, $Ec$, gastada (= trabajo hecho) en equilibrar la resistencia de la curva, solamente en cualquiera de estas dos curvas de idéntica abertura, $\Delta$, tenemos :

$$E_c = F_c\ l = n\ f_c\ L/n = f_c\ L.$$

En otros términos, el trabajo debido a la curva es el mismo en ambos casos; y, si en cualquiera de estas curvas fuera acortada o aumentada su amplitud $\Delta$ el trabajo debido a la curvatura variará en proporción.

**49. Trabajo proporcional a la curvatura.** Por consiguiente, suponiendo (como antes) $F_c = n\ f_c$, encontramos que el trabajo, debido tan solo a la curvatura, en *cualquier* curva, es proporcional a, $\Delta$, y es independiente de su agudeza, $D$, o de su radio, $R$.

**50. Pero, comparando las dos líneas,** $A\ M\ B$ y $A\ m\ B$ de la misma $\Delta$ (esta última incluyendo las tangentes, $A\ a'$ y $b'\ B$), tenemos : longitud $A\ m\ B >$ longitud $A\ M\ B$ ; y por consiguiente el trabajo en partes no curvas en $A\ m\ B >$ que en $A\ M\ B$ ; y como hemos visto (¶ 49), el trabajo debido a la curvatura es *igual* en las dos líneas de idéntica $\Delta$, es claro que el trabajo total en $A\ m\ B$ (= trabajo en la parte no curva, en $A\ m\ B$ + trabajo de la parte curv. en $a'\ m\ b'$) > trabajo total en $A\ M\ B$.

## Compensación de las curvas por las pendientes.

**51.** En virtud del aumento de resistencia, debido a las curvas, es costumbre (cuando es posible) reducir las pendientes en las curvas, a fin de que su resistencia total (pendiente + curv.) no exceda a la de la *tangente* con la pendiente original.

**52. El tipo de compensación (tipo de reducción de pendiente)** es la reducción de pendiente por cada grado de agudeza, $D$. De modo que « 0.04 » significa una reducción de pendiente de 0.04 por ciento de pendiente por cada grado de agudeza, $D$ ; así es que, por una curva de 5°, con 0.04 de compensación, en una pendiente de 0.7 por ciento, tendríamos :

Pendiente reducida o compensada :

$$= 0.7\ \% - (0.04 \times 5)\ \%$$
$$= 0.7\ \% - 0.2\ \% = 0.5\ \%.$$

**53. Las pendientes que llegan al límite** (Véase ¶ 11, pag. 1156.) deberán ser siempre compensadas, si es posible; puesto que la pendiente límite, en una *tangente*, ofrece la resistencia máxima que la locomotora normal puede vencer; y cualquier resistencia *adicional* (por ejemplo : la de una curva) tendería a parar los trenes.

**54. Las pendientes que no llegan al límite** (véase p. 1156, ¶ ¶ 12-14) no necesitan ser compensadas más allá de una amplitud en que la pendiente compensada llegue a ser equivalente a la pendiente límite. En muchos casos no se necesitará compensación en las pendientes que no llegan al límite; pero, en toda pendiente no compensada, la existencia de las curvas implica un aumento en el costo, debido unas veces a la reducción de velocidad, otras al aumento del gasto de combustible. La compensación resulta, por tanto, deseable siempre, aun en los casos en que no sea realmente necesaria.

**55.** La compensación, llevada a tal grado que resulte una pendiente en bajada, es desfavorable al tráfico en la dirección contraria, pero esto no implica necesariamente que no deba hacerse. Por ejemplo, el tráfico de regreso puede ser con trenes muy livianos como cuando se trata de carros de carbón o de minerales que regresen vacíos.

**Práctica.**

**56. Proporción.** Algunas autoridades consideran una proporción compensatoria excesiva la de 0.03 por ciento, por grado de agudeza, mientras otras creen que 0.04 es insuficiente. El promedio de éstas, = 0.035, es generalmente aceptado como una justa proporción, con modificaciones, como las anotadas anteriormente, para circunstancias especiales.

**57. Las diferencias entre las diversas vías,** como la estrechez del cartabón, es decir, la falta de juego en la vía, el peralte, etc., entran por mucho en la práctica en esta materia.

**58. Proporción mayor.** Los factores que tienden a aumentar la proporción o tipo de compensación son : la preponderancia de trenes pesados de carga a baja velocidad; la probabilidad de que los trenes tengan que disminuir su velocidad y, si es necesario, hasta detenerse permaneciendo por algún tiempo en la curva; la existencia de un sitio de paradas inmediatamente *antes* de una curva, en cuyo caso se ha recomendado (Webb) una compensación hasta de 0.10 por ciento; humedad probable en túneles, etc.

**59. Proporción menor.** Por el contrario, pueden usarse proporciones menores de compensación donde predominan los trenes de pasajeros a alta velocidad; y cuando hay un sitio normal de paradas inmediatamente *despues* de la curva, caso este, en que la resistencia de curva puede utilizarse, junto con toda la pendiente, para ayudar a detener el tren.

**60. Cambio en la proporción.** Algunas autores sugieren que la proporción o tipo de compensación debe disminuir a medida que aumenta la agudeza; otros sostienen que el tipo de compensación debe ser proporcional a ella.

## Agudeza máxima.

**61. Teóricamente,** la agudeza máxima permitida es la que, a la mayor velocidad permitida, requiere el peralte máximo permitido.

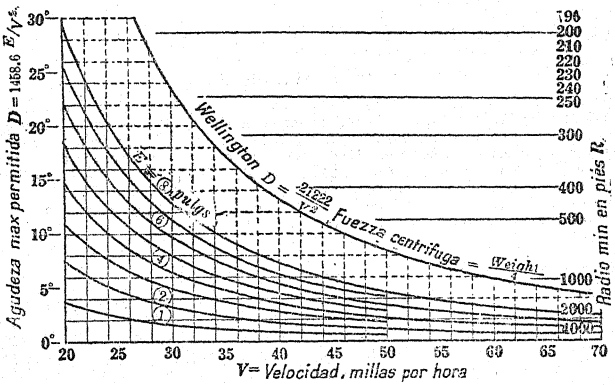


**Fig. 5.**

**62.** En la fig. 5, los ocho diagramas inferiores muestran la agudeza máxim permitida para diferentes velocidades, y para peraltes ó superelevaciones dea

carril exterior desde 1 hasta 8 pulgadas (2,5 á 20 cm en vía normal = 4 pies 8.5 pulg. (1.435 m), conforme a la ecuación 118, Curvas, Superelevación, pag. 1040, ¶ 176; $E$ pulg. = $V^2$ $D$/1458,6, de donde resulta $D$ = 1458,6 $E/V^2$. El diagrama superior indica la agudeza exigida por el señor A. M. Wellington (Trazados de Ferrocarriles, p. 270; Am. Ry Eng. & M. W. Assn., Procs., 1910, vol. II, parte I, p. 681). Fuerza centrífuga $>$ Peso/4.

**63.** Prácticamente, la agudeza máxima se rige también segun las condiciones del terreno y del tráfico. Camp (« Vía », p. 257) indica lo siguiente como práctica corriente en lo que hace al servicio de vía principal y ancho normal de vía.

| $D$. | $R$, piés. | |
|---|---|---|
| $>$ 4° $<$ | 1433 | considerada como aguda; |
| 10° | 574 | — — muy aguda; |
| 15° | 383,1 | límite material para velocidad completa; |
| 20° | 287,9 | usado, como obligatorio, con reducción de velocidad; |
| 22° | 262,0 | — — — — |

Nosotros añadimos :

| | | |
|---|---|---|
| 12° | 478,3 | agudeza máxima normal (con espirales) Alaska Central Ry., 1909; |
| 14° | 410,3 | agudeza máxima normal (con espirales) N. Pacific Ry., 1903. |

Curvas de agudeza hasta de 67° (90 pies (27.45 m) de radio) se han usado en los ferrocarriles elevados de la ciudad de Nueva York.

**64. Para vías secundarias ó derivadas** se hallan en uso las siguientes agudezas :

| $D$. | $R$, piés. | |
|---|---|---|
| 17° | 338 | cuando se trata de grandes locomotoras para colocar vagones en vías derivadas; |
| 29° | 200 | para vagones acoplados de muebles; |
| 39° | 150 | — carboneros acoplados, y para vagones encajonados empujados por locomotoras de transporte de 4 ó de 6 ruedas; |
| 95° | 60 | para vagones tirados por caballos o por locomotoras pequeñas de transporte en vías derivadas de almacenes, etc., con muy bajas velocidades. |

**65. Guarda carriles.** Se usan generalmente en curvas de 16° ($R$ = 359.3 pies = 109,51 m.) y de mayor agudeza.

## Agudeza mínima, etc.

**66.** A veces se especifica que la agudeza *no* deberá ordinariamente, ser *menor de* 1° (para $R$ = 5730 pies) ($R$ = 1746,5 m) excepto cuando el desarrollo ó áng central $\Delta$, $<$ 3°; que la longitud de la curva debe ser $\not<$ 300 pies (como 90 m); y que una **tangente**, $\not<$ 500 pies (150 m) de largo, deberá usarse entre dos curvas en la *misma* dirección, y una tangente, $\not<$ 600 pies (como 180 m) de largo; entre dos curvas en direcciones opuestas.

Véase asímismo Guardacarriles, ¶ 65; Ancho de vía en las curvas, pag 886; Superelevación o peralte, pag. 1039, etc.

# COSTO DE EXPLOTACIÓN

## GENERALIDADES

**1. El proyecto** y la construcción de un ferrocarril y de su material rodante (como su rendimiento) dependen grandemente (1) de la cantidad y la índole del tráfico que se espera tener, (2) de los ingresos resultantes, (3) de los fondos disponibles para construcción, y (4) de los gastos (gastos fijos más el costo de explotación). Las primeras tres partidas las calculan generalmente los promotores; pero el cálculo de la construcción y funcionamiento son las más de las veces de la incumbencia del ingeniero.

**2. Los costos de construcción y de explotación**, dependen principalmente de :

(1) La topografía del terreno;

(2) Las facilidades para disponer de las necesarias cantidades de material y mano de obra;

(3) El carácter y la importancia del tráfico posible;

(4) El carácter y la importancia del material rodante propuesto (fuerza motriz y vagones);

(5) La cantidad de dinero disponible para construcciones.

**3. El problema.** Informado de todo lo que concierne a estos puntos, el ingeniero se ocupa particularmente de hacer lo mas adecuada posible a la topografía del terreno, la línea que mejor responda a las necesidades, teniendo como finalidad (desde el punto de vista de los interesados) el rendimiento o ganancia que produzca el dinero invertido.

**4. La comparación entre dos líneas en proyecto**, *A* y *B*, puede basarse (1) en sus costos respectivos de construcción, más los valores capitalizados de sus respectivos costos de explotación; o (2) en sus respectivos costos de explotación, más la valorización anual de sus respectivos costos de construcción. Podemos, pues, suponer un caso donde los costos por milla de las dos líneas, *A* y *B*, se comparen así : (capitalización al 6 %).

| | (1) Capitalizado. | | (2) Anual. | |
|---|---|---|---|---|
| | *A*, $. | *B*, $. | *A*, $. | *B*, $. |
| Construcción..... | 25,000 | 20,000 | 1,500 | 1,200 |
| Explotación ..... | 100,000 | 125,000 | 6,000 | 7,500 |
| Total..... | 125,000 | 145,000 | 7,500 | 8,700 |

En este caso, para un rédito dado, la línea *A* es preferible, no obstante su mayor costo de construcción. Para Presupuestos sobre el costo de construcción ferroviaria, véase pág. 1166.

## MILLA DE TREN, Y COSTO POR MILLA DE TREN

**5. La milla de tren, « m — t ».** Cuando un tren de cualquier longitud, peso o carácter) recorre (en cualesquiera condiciones) una milla se llama a eso : una milla de tren y se indica así : $(m - t)$.

**6. Costo, m, de la milla de tren.** Por cualquier serie dada de operaciones (como en las operaciones de cualquier línea en un día, o en todas las operaciones de una región en un año) el **costo medio de m — t** es :

$$m = \frac{\text{costo total de las operaciones}}{\text{número de millas de tren recorridas}}$$

De modo semejante, el costo de $m - t$, debido a cualquier partida o partidas dadas de operaciones (exploitación), es :

$$\frac{\text{Costo de operaciones debidas a dicha partida o partidas.}}{\text{Número de } (m - t) \text{ recorridas}}$$

**7. Costo medio de « m − t » en los EE. UU.** En el 24º informe anual de la Comisión de Comercio Interior (véase Tabla 1, a continuación), encontramos como promedio total del costo de « $m - t$ », para las tres clases de líneas consideradas, el año de 1911 :

| Clases. | Costo anual de explotación en bruto. | $E$ = total costo anual de explotación. (Declaración 41). | $T$ = total recorrido anual en millas de tren. (Declaración 36, p. 49.) | $m = E/T = (m - t)$ costo. |
|---|---|---|---|---|
| I..... < | $1,000,000 | $1,844,065,958 | 1,185,632,129 | $1.555 |
| II..... < | $1,000,000 | $ 57,092,361 | 42,315,853 | $1.349 |
| II..... > | $ 100,000 | | | |
| III..... > | $ 100,000 | $ 13,472,094 | 9,552,156 | $1.410 |
| Totales y promedio.. | | $1,914,630,413 | 1,237,500,138 | $1.547 |

Los informes de I. C. C. relativos a 1890-9 dan valores de $m$ que fluctúan solamente entre $0,918 y $0,984; promedio, $0,952.

En nuestras discusiones subsecuentes, suponemos $m$ = $1,50.

**8. Salida.** — El costo $m$ de milla de tren ($m - t$), varía considerablemente, no tan solo (de un tiempo a otro) con la fluctuación de la unidad de costo sino (en un tiempo dado) entre líneas diferentes. Véase ¶ 7. No obstante, dado el costo por unidad, el costo de $m - t$, en una línea determinada, es prácticamente constante, porque el trabajo requerido para mover un tren sobre la línea es fijo dentro de límites reducidos, por el hecho de que la economía prohibe el uso de una carga menor del máximum, y esto a su vez está determinado por las condiciones de la línea. Véase ¶ 10.

## Distribución del costo por milla de tren.

**9. Tabla 1.** Esta tabla contiene los percentajes del costo de $m - t$, atribuidos a las diferentes partidas que forman ese costo, según lo determinó la Comisión de Comercio Interior. Los presupuestos o cálculos de estos tantos por cientos varían mucho; pero los tantos por cientos contenidos en la tabla 1 representan el promedio de la práctica americana, constituyendo así una guía útil subordinada a modificaciones en los casos en que las condiciones especiales obligan a desviarse de las condiciones normales.

Costo de explotación.
Tabla 1.

Sacada del Informe anual vigésimo cuarto de la Comisión de Comercio Interior (para el año que termina en 30 de Junio de 1911) de la Estadística de Ferrocarriles en los Estados Unidos, publicada por la Imprenta del Gobierno, en Washington, 1913.

| Ferrocarriles de..... | Clase I*. | Clase II*. | Clase III*. |
|---|---|---|---|
| Millas de línea..... | 215,146 | 19,120 | 9,167 |
| Millas de vía..... | 328,800 | 22,989 | 11,036 |

Análisis de los gastos de explotación.

| | $a$, = % de $m$. | | $a$, = % de $m$. | $a$, = % de $m$. |
|---|---|---|---|---|
| Mant. de vías y Construcciones : | | | | |
| Administración..... | 0,963 | 0,963 | 1,218 | 1,508 |
| Caminos y vía : | | | | |
| Balasto..... | 0,423 | | | |
| Traviesas..... | 2,992 | | | |
| Carriles..... | 0,897 | | | |
| Otros materiales de vía..... | 1,021 | | | |
| Caminos y vía †..... | 7,240 | 12,573 | 17,801 | 22,40 |

(Esta tabla concluye en la página siguiente.)

* Véase ¶ 7, arriba.
† Véase * nota, pág. 1155.

## Gastos de explotación.

(Viene de la página anterior.)

| | Clase I. | | II. | III. |
|---|---|---|---|---|
| | $a_1$ = % de $m$. | | $a_2$ = % de $m$. | $a_3$ = % de $m$. |
| Construcciones de vía : | | | | |
| Túneles, puentes, etc. | 1.682 | | | |
| Cruceros, signos, cercas, etc. | 0.369 | | | |
| Señales, telégrafo, etc. | 0.759 | 2.810 | 3,373 | 3,323 |
| Edificios, muelles y embarcaderos... | 1.943 | 1.943 | 1.679 | 0.920 |
| Miscelánea | 0.583 | 0.583 | 0.660 | 0.804 |
| Total, Mant. de vías y obras. | | 18.872 | 24.731 | 28.961 |
| Mantenimiento del Equipo : | | | | |
| Administración | 0.661 | | 0.865 | 0.722 |
| Reparaciones de locomotoras | 8.058 | | 6.386 | 5.836 |
| — vagones | 9.178 | | 6.381 | 4.720 |
| — del Equipo flotante. | 0.050 | | 0.024 | 0.008 |
| — — de trab. | 0.223 | | 0.210 | 0.136 |
| Renovaciones del Equipo | 0.859 | | 0.304 | 0.538 |
| Miscelánea | 0.806 | | 0.793 | 0.463 |
| Depreciación del Equipo | 2.702 | 22.537 | 3.751 | 3.109 |
| Total, Mant. del Equipo | | 22.537 | 18.714 | 15.532 |
| Tráfico (Agencias, publicidad, etc.) | 3.116 | 3.116 | 2.540 | 1.882 |
| Transporte : | | | | |
| Administración y despacho | 2.183 | | | |
| Empleados de estación | 7.062 | 9.245 | 9.373 | 8.248 |
| Conductores y guardafrenos de patio. | 2.869 | | | |
| Maquinistas de patio | 1.618 | | | |
| Locomotoras de patio, combustible | 1.601 | | | |
| Miscelánea de explot. en patios y estaciones | 3.085 | 9.173 | 5.461 | 1.962 |
| Locomotoras de camino; combustible. | 10.454 | 10.454 | 11.935 | 12.240 |
| Maquinistas de camino | 6.256 | | | |
| Locomotoras de camino; otros gastos. | 2.969 | 9.225 | 9.094 | 9.720 |
| Jornaleros de tren | 6.642 | | | |
| Gastos y suministros de trenes | 1.782 | | | |
| Explotación de desvíos, cambios y señales | 0.509 | | | |
| Miscelánea, trenes y señales | 0.415 | 9.348 | 7.749 | 7.818 |
| Miscelánea | 4.307 | 4.307 | 3.998 | 3.441 |
| Total, transporte | | 51.752 | 47.610 | 43.429 |
| En general (Administr., seguro, etc.). | 3.723 | 3.723 | 6.405 | 10.196 |
| Total | | 100.000 | 100.000 | 100.000 |

* Referente a balasto, traviesas, carriles y otros materiales de vía ; conservación de vía, de su lecho ; limpieza general ; recorridos y vigilancia ; cambio de alineamientos y pendientes ; muros de protección ; terraplenes ; servicio de trenes, etc.

## COMPARACIÓN DE LÍNEAS

### Factores determinantes y no determinantes.

**10.** Al comparar líneas alternativas, los factores por considerar, que afectan principalmente los costos de manipulación: son *diferencias de pendiente*, de *curvatura* y de *longitud*. Considerados colectivamente y con relación a su efecto sobre el costo de explotación, las pendientes y las curvas se dividen en dos clases generales, que es menester distinguir con cuidado, a saber: curvas y pendientes compensadas y no compensadas, o reglamentarias y no reglamentarias, o « mayores » y « menores ».

**11. La curva o pendiente compensada ó reglamentaria** o la combinación compensada de pendiente y curvatura, en una sección de locomotoras dada, es la que afecta el costo de manipulación *limitando la longitud y el peso del tren* que, en condiciones normales de trabajo y sin « impulsión », puede una locomotora transportar a través de la sección; y de ese modo, limita el número mínimo de trenes (e igualmente el trabajo mínimo) necesario para un tráfico dado.

**12. Trabajo total.** Con una fuerza motriz dada, *el peso máximo de un tren*, que puede ser transportado en una línea dada (aunque prácticamente independiente de la longitud de la línea), se ve así determinado por condiciones limitadas de pendiente o de curvatura o por entrambas combinadas; pero el trabajo total requerido, para transportar este peso máximo de tren por la línea, puede ser afectado por no limitar las condiciones en cuanto a longitud, como lo fué por causas de pendientes o curvaturas.

**13. Por consiguiente**, supongamos que, con una locomotora dada, una curva larga de 12° (inevitable en cada una de las dos líneas alternativas, *A* y *B*) hace que sea 500 ton. el peso máximo de un tren que puede ser transportado en cualquiera de las dos líneas. Esa curva constituye por tanto un factor limitador ó « mayor ». Pero, si la línea *B* tiene también una o más curvas de 4°, o pendientes cortas, que no existen en la línea *A*, o si la línea *B* es más larga que la línea *A* (siendo las dos líneas por lo demás, semejantes), entonces la línea *B* requiere mayor trabajo que la línea *A* para arrastrar el expresado tren de 500 ton. sobre la vía; y este exceso de esfuerzo contribuye a hacer más costosa la explotación de la línea *B*.

**14.** En consecuencia tenemos:

| | Que afectan. | Que no afectan. |
|---|---|---|
| Pendientes y curvas: | | |
| Que limitan..... | peso máx. del tren. | trab. del tren máx. |
| Que no limitan..... | trab. del tren máx. | peso máx. del tren. |
| Diferencias de longitud..... | — | — |
| Pendientes de empuje..... | — | — |

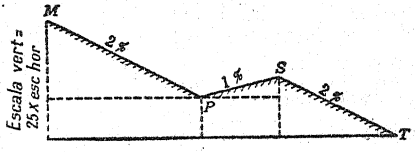


**Fig. 1.**

**15. Pendientes muy fuertes. Pendiente de limite.** El límite de la pendiente no es necesariamente la más fuerte de la sección. Así:

(1) Una pendiente corta, más fuerte que la pendiente ordinaria puede ser pasada por los trenes y locomotoras reglamentarios, mediante una « impulsión ». (Véase ¶ 40, pag. 1149.)

(2) Fig. 1. Cuando hay, por ejemplo, vagones cargados que se mueven de *M* a *T*, regresando vacíos de *T* a *M*, la pendiente *más ligera y más corta*, *P S*, contraria al *tráfico más pesado*, puede ser la pendiente ordinaria o de límite.

(3) El límite puede ser una combinación de pendiente moderada con curvatura fuerte.

**16. Límite de agudeza.** De modo semejante, la curva de límite, de una sección, puede ser larga y relativamente suave. Además, el efecto limitativo de una curva dada puede ser eliminado por la reducción de la pendiente. Véase pag. 1150 ¶ 51, etc.

Para el análisis del efecto de las pendientes y curvas límites, sobre el costo de explotación, véase ¶¶ 19 a 24.

**17. No son factores de limitación** :

(1) Las diferencias de longitud;
(2) Las pendientes de empuje;
(3) Curvas..... } no limitadoras.
Pendientes..... }

## Cálculos.

### Métodos de Procedimiento.

**18.** Cuando se trata de comparar los costos de explotación de dos líneas, afectadas por diferencias limitantes o no limitantes de pendientes, de curvatura y de longitud, se calcula (compárese Tabla 2) :

1. El costo normal por milla de tren ($m - t$) de cada operación;
2. El tanto por ciento, $a$, con que cada uno de estos costos parciales contribuye a formar el costo total de $m - t$, $m$ (véase Tablas 1 y 2);
3. El grado (tanto por ciento, $b$, de $a$) en que el costo de cada partida se halla afectada por la diferencia de pendiente, la diferencia de curvatura o la diferencia de longitud en cuestión;
4. El cambio consiguiente ($a' = b\,a$, percentaje de $m$) en el valor $a$, del tanto por ciento de cada partida, debido a la diferencia en cuestión;
5. La suma, $A'$, de los valores de $a'$, obtenidos en el término « 4 »; es decir, el costo total de $m - t$ de la diferencia en cuestión, expresado como un tanto por ciento del costo $m$ total y normal de $m - t$;
6. $m' = A'm$ = al costo de $m - t$ de la diferencia en cuestión, expresado en dólares;
7. $C_t$ = al costo anual de la diferencia en cuestión; ¶ 43.
8. $V_c$ = al valor capitalizado de la diferencia en cuestión.

### Ejemplo del Método.

Construcción de la primera columna (« Tren adicnl. ») de la Tabla 2.

**Factores limitadores.**

**19. Ejemplo.** — Por el concepto de Mant. de Vías y Estructuras, supongamos :

$t$ = número normal de viajes redondos diarios;
$m_s$ = costo $m - t$ supl de viajes normales, $t$;
$a_s = 100\ m_s/m = m_s$ como percentaje de $m$;
$S$ = costo supl normal, total y diario;
$t'$ = número de viajes redondos diarios adicionales;
$m_s'$ = costo $m - t$ supl de viajes adicionales, $t'$;
$a_s' = 100\ m_s'/m = m_s'$ como percentaje de $m$;
$s$ = aumento en $S$, debido al tren adicional;
$L$ = longitud de la línea, en millas.

Vamos a suponer que, con un alineamiento y un perfil dados, 10 viajes redondos al día ($t = 10$) se necesiten para manipular un tráfico dado, y que un aumento de pendiente o curvatura, o ambos, debido a algún cambio propuesto en la línea, redujera el peso máximo del tren a tal grado que se requieran 11, en vez de 10, viajes redondos para el mismo tráfico.

Entonces :

$$t' = 11 - 10 = 1; \quad \text{y} \quad t'/t = {}^1/_{10}.$$

**20. Para encontrar b.** Supongamos que este aumento de $^1/_{10}$, en $t$, aumenta el costo total de administración, $S$ (primera partida de la Tabla 2) en $^1/_{100}$ o **1 %** solamente.

Entonces :

$$\frac{s}{S} = \frac{L}{L}\,\frac{t'}{t}\,\frac{m_s'}{m_s} = \frac{t'}{t} \times \frac{m_s'}{m_s} = \frac{1}{100}\text{; y (¶ 18,4) :}$$

$$b_s = \frac{a_s'}{a_s} = \frac{m_s'}{m_s} = \frac{s}{S} \times \frac{t}{t'} = \frac{1}{100} \times \frac{10}{1} = \frac{1}{10} = 10\ \%;$$

o, en este ejemplo, el costo $m - t$, $m_s'$, de suplemento, por el tren adicional, es **10 %** del costo suplementario normal, $m_s$.

**21. Efecto sobre los demás elementos del costo.** Entonces, calculando un promedio de los valores de $a$ para las tres clases de vía en Tabla 1, ¶ 9, usando números redondos para mayor facilidad, y aplicando percentajes supuestos, $b$ (sujetos, por supuesto, a amplias variaciones en diferentes circunstancias), a los elementos restantes del costo de manipulación, obtenemos (tan solo a manera de ilustración del método) los demás valores de $a'$ ($= b\ a$) indicados bajo el rubro de « tren adicional » en la Tabla 2.

**22. Sumando** estos valores de $a'$, obtenemos $A'$, = 43.8, como el costo $m - t$ del tren adicional, expresado como tanto por ciento del costo $m - t$ normal, $m$. Entonces :

$$m' = A'\ m,$$

donde $m$ = al costo $m - t$ normal total, considerado aquí en $1.50.

**23.** En este caso, por consiguiente, tenemos, como costo $m - t$ del tren adicional requerido por los factores determinantes :

$$m' = A'\ m = 0.438 \times \$1.50 = \$0.657.$$

**24.** A, $m'$, así encontrado, deberá añadirse una pequeña suma (digamos de 1 a 1.5 centavos) para cubrir los intereses sobre el costo de cada locomotora y ténder adicionales requeridos.

**Otros factores (no determinantes).**

**25.** En sus columnas restantes, la Tabla 2 ofrece análisis semejantes en relación con los demás factores del costo de manipulación (no determinantes) : a saber, — longitud, pendientes de empuje, altos y bajos, y curvatura. Véase abajo.

**26. Diferencia de longitud.** Con un peso máximo dado de tren (determinado por la pendiente limitativa, por la curvatura, o por ambas), un aumento de longitud aumenta el costo de manipulación, ocasionando un trabajo adicional en cada tren, a saber : el trabajo de transportar el peso máximo del tren sobre la longitud adicional.

**27.** *Diferencias de longitud grandes y pequeñas.* En cualquier diferencia de longitud (grande o pequeña), el costo de vía y mantenimiento por $m - t$ equivaldrá aproximadamente al costo normal de esas partidas; pero el costo de salarios de trenes por $m - t$ (que, en una gran diferencia de longitud, puede igualmente equivaler al costo normal), son despreciables en diferencias de longitud pequeñas. El efecto de estosobre el presupuesto puede ser ilustrado con un caso, asi : se acepta (para mayor facilidad) que la vía y el mantenimiento de vía asciendan normalmente a 15 %, y los jornales de trenes a 13 % (compárese Tabla 1) del costo total de manipulación, suponemos los valores probables de $b$, encontramos los valores consiguientes de $a'$ y comparamos así los efectos de las diferencias de longitud grandes y pequeñas por lo que se refiere a la suma de los costos de estos dos elementos únicamente.

| | Normal en toda la línea. | En gran dif de long. | | En pequeña dif de long. | |
|---|---|---|---|---|---|
| | $a$ % de $m$. | $b$ % de $a$. | $a'$ % de $m$. | $b$ % de $a$. | $a'$ % de $m$. |
| Mant. de vías y caminos.... | 15 | 100 | 15 | 100 | 15 |
| Jornales de trenes..... | 13 | 100 | 13 | 0 | 0 |
| % $m$, del costo total $m - t$, por estos dos elementos únicamente..... | 28 | | 28 | | 15 |

**28.** Las diferencias de longitud se clasifican a veces como sigue :

| | Diferencia de longitud. Clases. | | |
|---|---|---|---|
| | *A.* | *B.* | *C.* |
| Jornales de trenes..... | *No afectado.* | **Afectado.** | **Afectado.** |
| Número de estaciones o andenes..... | *No afectado.* | *No afectado.* | **Afectado.** |

Los efectos de las diferencias de longitud sobre el costo de manipulación se hallan indicadas en la Tabla 2.

**29.** *Efecto de la diferencia de longitud sobre los ingresos.* De dos vías de competencia, entre dos puntos dados, cualquiera de ellas perderá fletes si cobra más caro que la otra. Por consiguiente, en los casos de competencia, las tarifas, en ambas líneas, entre dos puntos dados, son generalmente (en igualdad de otras circunstancias) las propias a la línea más corta, y esta es una ventaja importante; puesto que la línea más corta es la de más bajo costo de manipulación.

**30.** Pero, no habiendo competencia, las tarifas de carga y pasajeros se basan de ordinario en la distancia recorrida; y el producto de una línea, entre dos puntos dados, es en consecuencia absolutamente proporcional a la longitud de la línea; mientras que el costo total de manipulación en general aumenta con menos rapidez que el de la longitud de la línea. Por tanto, no existiendo competencia, la línea más larga es en general la más remunerativa.

**31. Es pendiente de empuje** la que, por ser tan inclinada, y tan larga, necesita regularmente dos o tres locomotoras (en vez de una) en cada tren que la sube. Por consiguiente, cuando las locomotoras de empuje son de igual potencia que las locomotoras ordinarias, la pendiente de empuje se hace generalmente dos o cerca de tres veces más inclinada que la pendiente límite en la sección, según que cada tren requiera dos locomotoras o tres.

**32.** En teoría, por consiguiente, la primera locomotora proporciona, al igual de cada una de las de empuje, tan solo la potencia necesaria para transportar el tren máximo por la pendiente ordinaria de límite; y el trabajo de cada equipo de empuje, es teóricamente solo el necesario para arrastrar o transportar el tren máximo en un *trecho adicional* (= longitud de pendiente de empuje) de la pendiente ordinaria de límite.

Para el análisis del costo de explotación en pendientes de empuje, véase Tabla 2.

**33.** Véase Tabla 2. *Las pendientes de empuje* afectan principalmente las partidas de :

Mantenimiento de Vías y Caminos.
Reparaciones y renovación de locomotoras, carros, etc.
Combustible y gastos de locomotoras.

**34.** El costo total anual de explotación, de una *locomotora de empuje y su tênder* varía ordinariamente entre $7,500 y $18,000, con $10,000 a $15,000 como límites más probables.

**35. La pendiente menor o no limitada,** es menos inclinada que la pendiente de límite (¶ 11), o tan corta que puede ser subida, por « impulso adquirido » (pag. 1156, ¶ 40), por una locomotora normal que arrastre el tren máximo. Afecta, pues, el costo de explotación porque exige un esfuerzo adicional para arrastrar el tren máximo. Los problemas relacionados con las pendientse que no son de límite se llaman generalmente problemas de « **subidas y bajadas**».

Comparando las definiciones de $s_e$, $h_t$, $h_e$ y $L_s$, ¶¶ 39 y 43, se verá que el esfuerzo adicional, ocasionado por una pendiente menor, se iguala con el de transportar el tren máximo en un *trecho adicional* de vía a nivel, cuya extensión es tal que el trabajo de este transporte, en ella, es igual al de levantar el peso del tren a la altura vertical de la pendiente.

Para el análisis de problemas relativos a pendientes que no son de límite véase ¶ 39).

**36. De modo semejante,** una curva menor que no es límite es a veces menos aguda que la curva límite, y otras tan corta que una locomotora ordinaria puede atravesarla con el tren máximo. Afecta el costo de explotación, aumentando la resistencia que la locomotora, que arrastra el tren máximo, debe vencer. Aumenta así el trabajo necesario para mover el tren máximo en la línea.

Comparando las definiciones de $D_e$, $\Delta d$, $\Delta_e$ y $L_c$, ¶¶ 39 y 43, se verá que el esfuerzo adicional, ocasionado por una curva, se iguala con el de transportar el tren máximo sobre el *trecho adicional* de la vía recta, cuya longitud es tal que el trabajo de ese transporte, en ella, es igual al que resulta de la curva.

Para análisis de problemas relacionados con curvas menores, más suaves, véase ¶ 39.

**37. Clasificación de pendientes que no son límites,** fig. 2, pág. 1163. De acuerdo con la marcha de los trenes de descenso, las pendientes que no son límites se hallan clasificadas como en la tabla de la pág. 1160, ilustrada por la fig. 2, pág. 1163. Compárese J.-B. Berry, Am. Ry Eng. M. W. Assn., Boletín 49, marzo 1904, pag. 21; y A. M. Wellington. Trazado de vías, págs 330 y 374, y Tabla 122, págs 372-3.

| | Clase de pend. Véase fig. 2, pág. 1163. | | |
|---|---|---|---|
| | *A.* | *B.* | *C.* |
| Caída (altura de).. | > 30 pies (9,15 m). | > 30 pies (9,15 m); pero > caída indicada para una curva sólida. | > caída indicada para una curva sólida. |
| Observaciones..... | Efecto desprec. | No sobrepujará locom. de subida. | Las locom. de subida deben usar arena. |

De ordinario una velocidad inicial de 10 mill/h no alcanzará 30 mill/h si :

| | *A* | *B.* | *C.* |
|---|---|---|---|
| El vapor está..... | Abierto. | Cortado. | Cortado. |
| Los frenos están... | Sueltos. | Sueltos. | Aplicados. |

**38.** En una pendiente de 0.6 %, el efecto de gravedad longitudinal iguala poco más o menos a la resistencia a 32 mill/h. Por consiguiente, sin vapor y sin frenos, esa velocidad no excederá ordinariamente, en pendientes de esta especie ni en pendientes más ligeras, por larga que sea la pendiente o por grande que sea la altura de caída.

**39. Efecto de las pendientes y curvas no límites sobre el costo de explotación.**

Supongamos :

| | |
|---|---|
| $s_e$ = la pendiente, en pies por 100 pies*. | $D_e$ = la agudeza de curva, en grados de desarrollo por cadena de 100 pies, ¶§. |

que produzcan una resistencia adicional, debida a :

| | |
|---|---|
| pendiente tan solo. | curvatura tan solo. |

igual a la resist. normal, $f_n$ (véase pag. 1129) en una tangente a nivel.

Entonces, para el *trabajo* hecho en un tren dado, al equilibrar esta resistencia adicional solamente, tenemos :

| | |
|---|---|
| En una milla (5.280 pies) de pend. $s_e$, trabajo = levantar el tren 52.8 $s_e$ pies, en cualquier pendiente; * | En una milla (5.280 pies de curva $D_e°$, trabajo = al necesario para equilibrar la resist. de una curva de cualquier agudeza § en 52.8 $D_e°$ de desarrollo. |

i esto es igual también al trabajo para equilibrar la resistencia normal, $f_n$, en una milla de tangente a nivel. Pero el costo del trabajo, necesario para :

| | |
|---|---|
| la elevación de un tren 52.8 $s°$ pies. | para equilibrar la resistencia solamente de una curva, en 52.8 $D_e°$ de desarrollo. |

es mucho *menor* que el *costo* normal, $m$ (digamos § 1.50; véase ¶ 7) de una milla de tren en una tangente a nivel; porque (continúa en la pag. 1161).

* $s_e$ es comunmente 0.5 % = 26.4 pies por milla; o 52.8 $s_e$ = 26.4 pies elevación.

† En vez de 60 % y 65 %, Mr. Berry (*Am. Ry Eng. y M. W. Sssn, Boletín* 49, marzo 1904, pag. 22) toma 25 % y 45 % respectivamente. La diferencia nos indica como cambian tan ampliamente estas apreciaciones según el criterio de cada cual.

¶ $D°$ se toma generalmente por 10°; o bien 52.8 $D°$ = 528° de desarrollo.

§ Pongamos $f_1$ = resist. debida a la curva en una curva de 1°.
Entonces $D_e = f_n/f_1$.
$f_n$ se supone generalmente que varía entre 5 y 10 lbs. y $f_1$ de 0.5 1.5 lb., por ton. de 2.000 libras.

(1) el *gasto* de energía, en el trabajo de subida de los trenes, es a menudo compensado (al menos parcialmente) por una *ganancia* de energía en el *descenso* correspondiente; y :

(2) el costo normal por milla de tren, $m$, incluye elementos que no se hallan directamente afectados por la subida o por la curvatura. Así :

Excepto por la partida relativa a combust., el efecto de subida del tren sobre el costo de explotación es materialmente despreciable. Del costo total por combust. para locom. (= digamos a 10 % del costo normal m-t, $m$, véase Tabla 1), una considerable proporción; un promedio digamos de 45 %, es debida a pérdidas en el fuego, en detenerse, en parar y arrancar y en vencer la resistencia de curvas o pendientes, dejando solamente 55 % como costo de transporte o arrastre en una tangente a nivel, cuyo trabajo en una milla, es igual al necesario para 52.8 $s_p$ pies de subida. A este 55 % el Prof. Webb agrega, en las clases. *B* y *C* (véase ¶ 37) 5 % por combustible en las pendientes de bajada; y en la clase *C*, otro 5 % por pérdidas por los frenos; haciendo un total medio, $b = 60$ % para la clase B, y $b = 65$ % para la clase *C* *. Las pendientes en la clase A son despreciadas, pues no afectan de un modo material el costo de explotación.

Tenemos, por tanto, para todos los componentes (haciendo suposiciones generales por los gastos que no se refieran a combustible de locom. (Véase Tabla 2.)

| Clase B. | $a$. | $b$. | $a$. |
|---|---|---|---|
| Combust..... | 11 | 60 | 6.6 |
| Varios..... | 8 | .. | 2.0 |
| Por pendientes... | $A'$ | = | 7.6 |

$m' = A'\,m$ ;
$= 0.076 \times \$1.50$;
$= \$0.114$.

| Clase C. | $a$. | $b$. | $a'$. |
|---|---|---|---|
| Combust..... | 11 | 65 | 7.1 |
| Varios..... | 8 | .. | 2.0 |
| Por pendientes... | $A'$ | = | 9.1 |

$m' = A'\,m$;
$= 0.091 \times \$1.50$;
$= \$0.136$.

La curvatura afecta principalmente los componentes de mantenimiento de vías y caminos, las reparaciones de locom. y vagones, y el combustible para locom.

Supongamos (véase Tabla 2) :

| | $a$. | $b$. | $a'$. |
|---|---|---|---|
| Mant. de v. y caminos..... | 16 | 75 | 12 |
| Reparaciones : | | | |
| Locom..... | 7 | 100 | 7 |
| Vagones..... | 8 | 100 | 8 |
| Combustible loco.. | 11 | 50 | 5.5 |
| Varios..... | 8 | 25 | 2 |
| Curvatura.... | $A'$ | = | 34.5 |

Por consiguiente, para curvatura :

$m' = A'\,m$.
$= 0.345 \times \$1.50 = \$0.5175$.

Por suplemento de caminos y vías, Mr. J. B. Berry, Am. Ry Eng. y M. W. Assn., Boletín 49, marzo 1904, pag. 20, da (en vez de nuestro $a = 16$ %) $a = 14.76$ %, obtenido como sigue :

| | $a$. | $b$. | $a'$. |
|---|---|---|---|
| Carriles..... | 2.28 | 300 | 6.84 |
| Traviesas..... | 3.01 | 50 | 1.50 |
| Balasto..... | 0.28 | 50 | 0.14 |
| Clavos, etc., etc.. | 0.86 | 100 | 0.86 |
| Varios..... | 8.33 | 50 | 4.16 |
| Supt..... | 14.76 | 91 | 13.50 |

* Véase † nota, pág. 1.160.

**Tabla 2, deter-**

| Factores. | | Tren Adl. | | Dif. de longitud para clases *A*, *B*, *C*, véase ¶ 28. | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | *A.* | | *B.* | | *C.* | |
| Materias. | *a.* | *b.* | *a'.* | *b.* | *a'.* | *b.* | *a'.* | *b.* | *a'.* |
| Mant. de cam. y const. : | | | | | | | | | |
| Administración..... | 1 | 10 | 0.1 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| Camino, vía..... | 16 | 50 | 8.0 | 80 | 12.8 | 90 | 14.4 | 100 | 16.0 |
| Estructuras..... | 4 | 00 | 0.0 | 5 | 0.2 | 10 | 0.4 | 70 | 2.8 |
| Miscelánea..... | 1 | 00 | 0.0 | 100 | 10.0 | 100 | 1.0 | 100 | 1.0 |
| | 22 | | | | | | | | |
| Equip. : | | | | | | | | | |
| Superint..... | 1 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| Rep. Locoms..... | 7 | 70 | 4.9 | 40 | 2.8 | 40 | 2.8 | 50 | 3.5 |
| — Carros..... | 8 | 70 | 5.6 | 40 | 3.2 | 40 | 3.2 | 60 | 4.8 |
| Miscelánea..... | 1 | 00 | 0.0 | 30 | 0.3 | 40 | 0.4 | 50 | 0.5 |
| Depreciación..... | 3 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| | 20 | | | | | | | | |
| Tráfico..... | 2 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| Transp. : | | | | | | | | | |
| Superint etc..... | 9 | 20 | 1.8 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| Patios, Estac..... | 7 | 20 | 1.4 | 00 | 0.0 | 00 | 0.0 | 80 | 5.6 |
| Locoms, combust.... | 11 | 80 | 8.8 | 50 | 5.5 | 50 | 5.5 | 60 | 6.6 |
| — gastos..... | 9 | 80 | 7.2 | 10 | 0.9 | 70 | 6.3 | 80 | 7.2 |
| Operaciones..... | 8 | 50 | 4.0 | 20 | 1.6 | 50 | 4.0 | 60 | 4.8 |
| Miscelánea..... | 4 | 50 | 2.0 | 60 | 2.4 | 60 | 2.4 | 60 | 2.4 |
| | 48 | | | | | | | | |
| General..... | 8 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| Total..... | 100 | $A' = 43.8$ * | | $A' = 30.7$ † | | $A' = 40.4$ † | | $A' = 55.2$ † | |
| Con $m = \$1.50$, tenemos $m' = A' m =$ ..... | | \$0.657 | | \$0.460 | | \$0.606 | | \$0.828 | |

**40.** La tabla 2 supone valores para $a$ (% de $m$), y para $b$ (% de $a$), y deduce valores de $a$ ($= b\ a$), $A'$ (suma de $a'$) y $m'$ ($= A'\ m$).

**41.** La tabla 2 ilustra la aplicación del método del ¶ 18, para encontrar el costo $m'$ de m-t modificado, debido a cada uno de los diversos factores que afectan el costo de operación, así :

1. Pendientes y curvas limites (tren adicional).
2. Diferencias de longitud.
3. Pendientes de impulsión.
4. { Pendientes no limitadas (subida y caída).
   { Curvas no limitadas.

**42. Advertencia.** Los valores de $b$ (% de $a$) dados aquí, representan simples valores probables, y se usan para ilustrar el método del ¶ 18, mas que para representar verdaderos percentajes. Los verdaderos valores de $b$ varian ampliamente con diferentes condiciones, tambien con la clasificación del costo, y con el criterio del que hace la apreciación. Véase Nota † al pie de la pag. 1160. Debe se restimado en cada caso.

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* Para trenes adicionales (1.ª columna) J. B. Berry Am Ry Engs M. W. Assn, Boletín 49, marzo 1904, pag. 12, encuentra $A' = 43.29$ % de $m$.

† Usando otra clasificación para las diferencias de longitud J. B. Berry. U. P. Ry A. R. E. M. W. A. Boletín 49, marzo 1904 y E. H. Mc Henry. N. P. Ry 1903, adoptan los siguientes valores de $A'$ :

| | | Clase A. | Clase B. | Clase C. |
|---|---|---|---|---|
| Berry U. Pac..... | $A' =$ | 32 % | 46 % | 59 % |
| Mc Henry N. Pac..... | $A' =$ | 22 % | 52 % | 100 % |

**minación de m'.**

| Factores. | | Impulsión. | | Altura y Caída. B. | | Altura y Caída. C. | | Curv. | |
|---|---|---|---|---|---|---|---|---|---|
| Materias. | *a.* | *b.* | *a'.* | *b.* | *a'.* | *b.* | *a'.* | *b.* | *a'.* |
| Mant. de Com. y Const : | | | | | | | | | |
| Administración..... | 1 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| Camino, vía..... | 16 | 50 | 8.0 | 00 | 0.0 | 00 | 0.0 | 75 | 12.0 |
| Estructuras..... | 4 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| Miscelánea..... | 1 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| | 22 | | | | | | | | |
| Equipo : | | | | | | | | | |
| Superint..... | 7 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| Rep. Locoms..... | 7 | 80 | 5.6 | 00 | 00. | 00 | 0.0 | 100 | 7.0 |
| — Carros..... | 8 | 80 | 6.4 | 00 | 0.0 | 00 | 0.0 | 100 | 8.0 |
| Miscelánea..... | 1 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| Depreciación..... | 3 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| | 20 | | | | | | | | |
| Tráfico..... | 2 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| Transp : | | | | | | | | | |
| Superint..... | 9 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| Patios, Estac..... | 7 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| Locoms, combust.... | 11 | 80 | 8.8 | 60 | 6.6 | 65 | 7.1 | 50 | 5.5 |
| — gastos..... | 9 | 50 | 4.5 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| Operaciones..... | 8 | 20 | 1.6 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| Miscelánea..... | 4 | 20 | 0.8 | 00 | 0.0 | 00 | 0.0 | 00 | 0.0 |
| | 48 | | | | | | | | |
| General..... | 8 | 00 | 0.0 | .. | 1.0* | .. | 2.0* | 25 | 2.0 |
| Total,..... | 100 | $A' = 35.7$ | | $A' = 9.1$ | | $A' = 9.1$ | | $A' = 34.5$ | |
| Con $m = \$1.50$ Tenemos $m' = A'\ m =$ | | \$0.535 | | \$0.114 | | \$0.136 | | \$0.517 | |

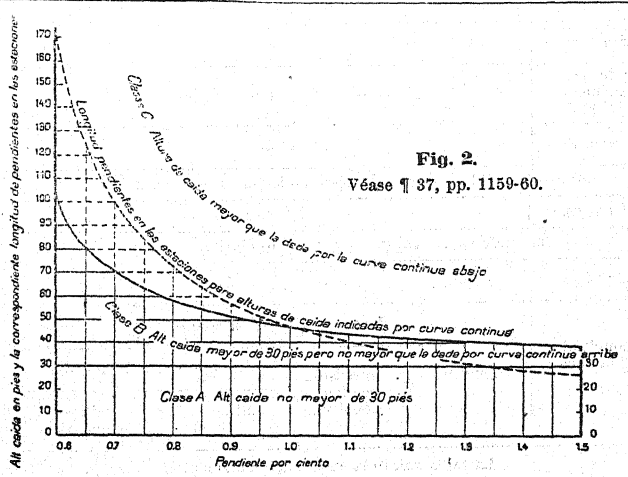


**Fig. 2.**

Véase ¶ 37, pp. 1159-60.

* Cubre todos los capítulos, excepto el combustible para Loco.

**43. Deducción del costo anual y su capitalización.** Teniendo, de acuerdo con los ¶ ¶ 18 a 42, el costo modificado $m'$ de la milla de tren $(m - t)$; determinado por un factor dado en un caso dado; se procede, como se indica a continuación para deducir de $m'$, el costo anual $C_a$, de una diferencia dada (en condiciones limitadas ó no) entre dos lineas, y el valor capitalizado $V_c$, de dicho costo anual.

En el inventario que sigue un « aumento » y sus efectos pueden ser negativos.

Sea.

| | |
|---|---|
| $s_e$ = la pendiente en pies por 100 pies (tanto por ciento), | $D_e$ = la agudeza en grados de desarrollo por cadena de 100 pies, |

que produce una resistencia adicional, a la resistencia normal, $f_n$, en una tangente a nivel;

| | |
|---|---|
| $h_d$ = dif en cuanto a, altura de caida entre dos lineas;<br>$h_e = 52.8\ s_e$<br>= 1 « equivalente a pendiente por milla »<br>= la altura para la cual el trabajo requerido es : | $\Delta_d$ = dif de desarrollo angular entre dos lineas;<br>$\Delta_e = 52.8\ D_e$<br>= 1 « equivalente a una milla en curva »<br>= desarrollo ang en el cual el trabajo debido a la curva es : |

equivalente a la fuerza de arrastre del tren en una milla de tangente a nivel.

Entonces :

| | |
|---|---|
| $L_s = \frac{h_d}{h_e}$ = la dif en cuanto a altura y caida, entre las dos lineas expresado en términos del « equivalente a pendiente por milla », $h_e$. | $L_c = \frac{\Delta_d}{\Delta_e}$ = dif en cuanto a desarrollo angular entre las dos lineas expresado en términos del «equivalente a una milla en curva », $\Delta_e$. |

$C_a$ = costo anual de una dif dada entre dos lineas;
$V_c$ = valor capitalizado de $C_a$;
$d$ = el número (generalmente 365 ó 313) de dias de tren en un año.

| (1). | (2). | (3). | (4). | (5). |
|---|---|---|---|---|
| Sea $t'$ = | $t$ = | | | |
| Nº de viajes adicionales ¶ 11. | Número de viajes redondos por dia. | | | |
| Entonces $d\ t'$ = | $d\ t$ = | | | |

número de millas de tren sobre cualquier milla en un año por :

| | |
|---|---|
| $t'$ trenes, | $t$ trenes, |

haciendo cada uno un viaje redondo diario.

| Sea $L$ = | $L_d$ = | $L_p$ = | $L_s$ = | $L_c$ = |
|---|---|---|---|---|
| longitud de la linea. | dif entre dos lineas en cuanto a : | | | |
| | longitud | impulsión pendiente longitud. | número equivalente de | |
| | | | millas pendientes. | millas en curva. |

(La tabla continúa en la página siguiente.)

(Continuación de la tabla de la página anterior.)

| (1). | (2). | (3). | (4). | (5). |
|---|---|---|---|---|
| **Entonces para un tráfico dado, un aumento en :** | | | | |
| el límite pendientes curvas. | longit linea. | impulsión pendiente longitud. | No limitadas : | |
| | | | Pendientes. | Curvaturas. |
| aumenta el costo de operaciones por : | | | | |
| aumento $t'$ al nº de viajes redondos por dia en $L$; | haciendo que cada uno de los $t$ trenes corra $L_t$ millas, mas en cada viaje sencillo; | requiriendo impulsión del tren en cada viaje sencillo en $L_p$; * | aumentando la resisten en las | |
| | | | pendientes; | curvas; |
| aumentando asi, el trabajo normal con un trabajo equivalente al requerido para arrastrar, diariamente : | | | | |
| $t'$, trenes. | $t$, trenes. | | | |
| sobre una distancia igual a : | | | | |
| $L$. | $L_d$. | $L_p$ *. | $L_s$. | $L_c$. |
| **$m' = A' m$ = costo de la milla-tren de este trabajo adicional.** | | | | |
| **Número de millas trenes equivalentes, adicionales por año** debido a las dif entre las dos lineas = | | | | |
| $d\,t\,L$. | $d\,t\,L_d$. | $d\,t\,L_p$ *. | $d\,t\,L_s$ †. | $d\,t\,L_c$. |
| $C_a$ = **costo anual adicional** = | | | | |
| $d\,t'\,L\,m'$. | $d\,t\,L_d\,m'$. | $d\,t\,L_p\,m'$ *. | $d\,t\,L_s\,m'$ †. | $d\,t\,L_c\,m'$. |
| **$V_c = C_a \div$ la rata de los intereses.** | | | | |

Debe advertirse que, en los cinco casos, el trabajo adicional, es el del arrastre del tren o es igualado al (y establecido en términos del) arrastre. En la columna 1 (diferencias en las condiciones de límites) dicho arrastre es el de los *viajes adicionales* sobre *la misma longitud L*, de linea, en ambos casos; por el contrario, en cada una de las otras cuatro columnas el número de viajes no está afectado por la diferencia entre las dos lineas comparadas y el arrastre es el de dichos viajes (*el mismo* para ambas líneas) sobre una *longitud adicional*, *Ld*, *Lp*, *Ls* o *Lc*, de línea. Véanse ¶ ¶ 32, 35 y 36.

* Esto supone que la impulsion de un equipo en milla, es equivalente a una milla de tren, y que el costo de impulsión del equipo es el mismo en ambas direcciones. Al desviarnos de estas suposiciones, debe dejarse un margen al determinar el valor de *b*, ¶ 18 (8). Véase tambien ¶ 32.

† Esto supone un total igual de altura, en pendientes no limitadas, en ambas direcciones; y desprecia la ventaja de las bajadas por las pendientes.

# PRESUPUESTOS SOBRE EL COSTO DE CONSTRUCCIONES

## PRELIMINARES

### Variaciones y límites.

*N. del T.* — Este capítulo contiene en su mayor parte datos estadísticos que sólo pueden ser útiles a los Ingenieros en trabajos de ferrocarriles en los Estados Unidos, pero, no obstante, hemos extractado todos los datos, informes y observaciones que por ser más generales pueden ser útiles a cualquier ingeniero de este ramo.

Como es natural, al tomar los datos muchas veces salteados, que aparecen entre las págs 1166 a 1179 suelen no tener conexión inmediata, pero siempre son comprensibles y útiles.

En las partes donde hemos salteado algunas líneas ó párrafos ponemos líneas de punto.

**1. Factores que afectan el costo.** El costo de las construcciones ferroviarias en general varía mucho. Depende principalmente de las condiciones topográficas y geológicas de las regiones que la línea atraviese; del valor intrínseco de ellas y del importe de las concesiones o derechos de paso, consiguientes; de la extensión y el precio de los terrenos que se necesiten para patios y estaciones; del de los diversos elementos de construcción; del costo de la mano de obra; de los métodos de construcción que se empleen; de las necesidades generales en cuanto a pendientes y alineamiento de las vías; de la importancia y el carácter del tráfico a que la línea se destine; de que haya o no competencia; de la naturaleza de ésta ; y de los fondos disponibles.

**2. Costo inicial y costo definitivo.** El costo de construcción de un ferrocarril dado, si se basa en los libros de la compañía, aumenta según sea el período de tiempo que la obra requiera; porque el costo original (a menudo mínimo a causa del anhelo económico de poner cuanto antes la vía en condiciones de producir ganancias) puede ser mucho menor que el costo total subsecuente (« Costo original hasta la fecha ») una vez hechas las mejoras deseables. Véanse ¶ ¶ 4 y 11.

**3.** Por otra parte; hay construcciones, incluídas en el costo original, que a menudo son abandonadas; pero que su costo, sin embargo, entra en el costo total de la construcción.

**4. Cuentas.** En cualquier costo efectivo de una construcción dada, las bases del costo son siempre modificadas por la diferencia en el sistema de contabilidad. Por ejemplo, hay partidas que siendo propiamente aplicables a Construcción, con frecuencia se cargan a Mantenimiento de Vías; como cuando se trata de una obra provisional de un caballete de madera (parte de una construcción original) que después se va terminando gradualmente y el servicio de trenes que ella origina se carga a Mantenimiento.

**5.** Al deducir el costo de construcción de una línea en proyecto del de una línea existente, deberá, por supuesto, dejarse un margen por la diferencia de condiciones entre ambas líneas; pues aun tratándose de costos de unidad determinadas, sucede que el costo por milla es a menudo distinto; y por consiguiente :

$$\text{El costo de construcción aparente por milla} = \frac{\text{costo total}}{\text{longitud}}$$

de una parte de la línea en cuestión, puede resultar mucho más costosa, y el de otra parte mucho más baja, que el promedio del costo de toda la línea.

**6. Ejemplo.** Así, una línea principal que corra rumbo al Este desde el Pacífico, atravesando diversos Estados, y que se halle gravada además de un fuerte costo por terrenos en sus estaciones, amén de fuertes gastos de construcción propiamente dicha en la porción de línea que pasa por las montañas, puede pasar también por regiones de más favorable topografía que requieran muy pocos gastos po terrenos. Al usar, por tanto, a manera de comparación, las cifras del costo de construcción de una línea así, deberá tenerse siempre en cuenta la índole particular de la vía que se desea construir. Si, por ejemplo, la línea proyectada pasa por terrenos favorables y de poco costo tan sólo, ni exige terminales costosos, sería descaminado aplicar a ella, sin más modificación, la cifra de costo por construcción referente a toda la línea principal incluyendo sus secciones más costosas.

**7. Tendencia al alza.** Cuando se usen datos primitivos como base para el presupuesto de costos presentes o futuros, téngase en cuenta que la tendencia de todo costo de construcción ferroviaria, considerada en conjunto, es a aumentar; porque la reducción del costo de unidad, en algunos respectos debida a los adelantos de fabricación y a los sistemas de trabajo y perfección de los aparatos, se ve de ordinario más que compensada por el aumento de peso en el material rodante; por la incesante demanda de precisión por parte del público tanto en relación con el tráfico de carga como con el de pasajeros; por el aumento de tráfico, que justifica lo costoso de la construcción en beneficio de la reducción de los gastos de explotación; y por la creciente insistencia de la comunidad para que se eviten las molestias y los accidentes.

**8. De acuerdo con estas ideas,** cuando existen cruces de pendientes, se hace necesario dotarlos con aparatos perfeccionados de señales y de uniones; mas sucede con frecuencia que se necesita suprimir tales crucero en pendientes y la operación puede costar $100,000 o más.

**9.** La amplitud de las variaciones, tratándose del costo total, aumenta igualmente y por idénticas razones.

**10. Y del mismo modo,** en una línea de regular longitud que una varios centros populosos entre sí, el costo de las estaciones, de estilo moderno, puede igualar al costo del resto de la construcción; mientras que en una línea muy larga que pase por muchos poblados de menor importancia o que dependa principalmente del tráfico industrial o de minerales, el costo de las estaciones puede resultar relativamente insignificante.

**11. Comisión del Comercio Interior.** — Por decreto aprobado en marzo de 1913 en el Congreso, se mandó fijar, respecto de todas las propiedades ferrocarrileras :

(1) Su costo original, hasta la fecha;
(2) Su costo de reproducción, nueva;
(3) Su costo de reproducción, nueva, menos la depreciación.

**12. Diferencias de presupuesto.** — Como puede suponerse, se nota generalmente una sensible diferencia entre (a) presupuestos preparados por una compañía con el objeto de mostrar un valor elevado, como en casos de probable venta o cuando se desea justificar las tarifas de carga o pasajeros, y (b) presupuestos preparados por la misma compañía con el fin de mostrar un valor bajo, como cuando se trata por ejemplo de pagar contribuciones, etc. Véase ¶ 139.

**13. « Gastos Generales ».** — Además del costo de construcción propiamente dicho, existen partidas de costo por administración, negociaciones, servicios legales, seguros, impuestos, deterioros, etc. (Véase III. Gastos Generales. ¶ 18.), que afectan el avalúo de toda propiedad, del mismo modo que cuando se trata de impuestos. Semejante avalúo puede debidamente comprender las ganancias que la instalación rinde, que como es natural pueden ser a su vez afectadas por consideraciones completamente ajenas al costo de construcción. Por ejemplo, las propiedades comerciales del territorio atravesado bastarían a dar un alto valor a una vía de reducido costo; o pueden, en cambio, ser tales que la vía más costosa pierda todo su valor.

**14.** Lo único que aquí puede ensayarse como útil, es proporcionar algunos informes o datos (sacados de documentos oficiales) que tiendan a facilitar el cálculo del costo preliminar de una línea en proyecto, teniendo en cuenta las indicaciones y precauciones que preceden.

**15. Costo de unidad.** — Al calcular el costo de las construcciones ferroviarias, deben considerarse :

(1) Los costos de unidad (por yarda cúbica de declive; por acre de limpia; por pie lineal de túnel, etc.) y
(2) los costos por milla, costos por milla de línea o vía sencilla.

**16. Milla de línea contra milla de vía.** — Cuando se trata de calcular el costo por milla, es necesario distinguir entre :

(1) El costo por milla de línea (o milla de camino) (costo de milla lineal)

$$= \frac{\text{costo total}}{\text{número de millas de línea principal y ramales}}$$

(cualquiera que sea el número de vías ó desvíos) y
(2) El costo por milla de vía.

$$= \frac{\text{costo total}}{\text{número de millas de la vía}}$$

**17.** Por consiguiente un camino de doble vía, que tenga $L$ millas de largo, sin vías derivadas, tiene $L$ millas lineales y $2\,L$ millas de vía.

*En cualquier caso*, la milla de vía simple $\leqq$ que la milla de línea ó camino.

Para una línea de vía simple, sin ramales ni desvíos tenemos : costo de vía simple = costo de la línea.

Para cualquier vía doble sin ramales ni desvíos tenemos :

Costo de milla de línea o camino = 2 × costo de vía simple; pero el costo de una línea de ferrocarril de dos vías (en igualdad de otras circunstancias) es *menos* del doble de una línea de una sola vía.

## Clasificaciones de la C. C. I.

**18.** Condensamos a continuación la Clasificación de los gastos de instalación y equipos de los caminos de hierro a vapor, prescritos por la Comisión del Comercio Interior de acuerdo con la Sección 20 del Acta para Regular el Comercio; hecha efectiva en julio 1.º de 1914.

**Cuentas Generales.**

I. Vía o camino. « Costo del terreno, mejoras fijas, máquinas y útiles para el servicio de trasporte. »

II. Equipo. « Costo de las diversas clases de equipos necesarios al tráfico ó necesarios para la explotación del negocio. »

III. Gastos Generales. « A éstos van las erogaciones relativas a las adquisiciones y construcciones de la vía original y su equipo, y de ampliaciones, adiciones y mejoras de la vía y del equipo, cuandos estas erogaciones no pueden debidamente ser incluidas en algunas de las partidas siguientes, como una parte del costo de alguno de los trabajos especificados. »

**Primeros gastos.**

**I. Vía.**

1. Ingeniería.

Comprende : « El pago de los gastos de ingenieros, asistentes, escribientes, empleados en el trazado y construcción de nuevas líneas y extensiones de las subsistentes, ó en adiciones y mejoras incluyendo andenes y muelles.

2. Terrenos para trasportes.
Comprende : « El costo de terrenos para dar ancho a la via; para las estaciones, oficinas, tiendas y otros terrenos; para dar entrada y salida a ciertos lugares; para tener la propiedad de los pozos; sitios de desechos, palizadas para nieves, para arena y otras necesidades; para almacenes de materiales, derechos de la via; el costo de terrenos para apeaderos y muelles y el costo de terrenos ribereños ó con derechos al agua; el costo del derecho de paso y de remover las propiedades de otros a otros lugares y el derecho a estos nuevos terrenos, cuando estos gastos se calculan en el trasporte. »

3. Nivelación.
Comprende : « El costo de limpieza, despejo y nivelacion de la via y el de construccion y protección de caracter permanente, de la calzada, via, terraplenes y desmontes. »

4. Tuberia para fuerza, bajo tierra.

5. Túneles y vias subterráneas.
Comprende : « El costo de tuneles y vias subterraneas, incluyendo el trabajo de ventilación y alumbrado y trabajos de seguridad, distintos al de señales. »

6. Puentes, pilas (torres, etc.) y alcantarillas.
Comprende : « El costo de la sub y superestructura de puentes, pilas y alcantarillas para el paso sobre cursos de agua, barrancos, quebradas, caminos públicos y privados y sobre otros ferrocarriles. »

7. Construcciones elevadas.
Comprende : « El costo de las construcciones elevadas y las fundaciones de las estructuras para ferrocarriles elevados, que no sean trabajos de tierra y que están, para el propósito de vias elevadas sobre la pendiente de las calles y que no son propiamente clasificables como puentes ó torres.

8. Traviesas.

9. Carriles.

10. Otros materiales de via.
Comprende : « Placas, etc., contra resbalamientos, postes y sistemas de sujeción para choques, cruceros, descarriladores, contracarriles, abrazaderas, bridas (uniones), placas de asiento, clavos, desvios, etc. »

11. Balasto.

12. Colocación y preparacion de la via.

13. Derecho de palizadas.

14. Palizadas para nieve y arena. Sostén de terraplenes.

15. Cruceros y señales.

Exclusivos « para cruces de linea en pendientes »; para esto, véase número 10.

16. Estaciones y otros edificios.

17. Edificios en la via.

18. Estaciones de agua.

19. Estaciones carboneras.

20. Tiendas y casa de locomo.

21. Elevadores de granos.

22. Almacenes y depósitos.

23. Andenes y muelles.

24. Andenes para carbón y minerales.

25. Plantas generadoras de gas.

26. Lineas de telégrafos y teléfonos.

27. Señales y cruces.

28. Diques, canales, y tuberias.

29. Planta generadora de fuerza.
Comprende : « El costo del edificio... » Véase nº 45... adelante.

30. Subestación de fuerza.
Comprende : « El costo de los edificios para la transformación de la fuerza... » Véase n.º 46, mas adelante.

31. Sistemas de transmisión de fuerza.
Comprende : « El costo de los sistemas de transmisión a alta tensión... usados para transferir la fuerza que produce la planta al lugar donde se va a transformar... »

32. Sistemas de distribución de fuerza.
Comprende : « El costo de los sistemas de distribución... para llevar la fuerza eléctrica a baja tensión de las plantas que las producen ó de las estaciones donde se transforman y para llevar el vapor ó el aire comprimido de las plantas productoras al lugar donde se emplean... »

33. Pértigas, postes, etc.

34. Conductos subterráneos.

35. Estructuras varias (Miscelánea).

36. Pavimentacion.

37. Maquinaria de via.

Comprende : « calderas, carros (carros de mano; palancas, motores de inspección, de empuje, etc.), mezcladoras de concreto, escavadoras, dragas, máquinas portátiles, niveladoras, maquinaria hidráulica, martinetes, clavador de estacas, descargadores, perforadoras de roca, cilindros de vapor (pisones), etc. »

38. Herramienta menor de via.

39. Apartado para mejoras.

40. Ingresos y gastos durante la construccion.

41. Costo de caminos.

42. Reconstruccion de caminos.

43. Otros gastos. Caminos.

44. Almacen de materiales para máquinas.

45. Planta para maquinaria para la fuerza.

46. Subestacion de aparatos para fuerza.

47. Materiales de construccion y repuestos sin aplicación.
(Los números 48 al 50 en blanco.)

**II. Equipo.**

51. Locomotoras de vapor.

52. Otras locomotoras.

53. Carros de carga.

54. Carros de pasajeros.

55. Equipo de motores para carros.

56. Equipo flotante.

57. Equipo de trabajo.

58. Miscelánea (equipo).

(Nos 59 a 70 incl. en blanco.)

**III. Gastos Generales.**

71. Gastos de organización.
72. Oficiales y empleados.
73. Legislación (parte jurídica).
74. Efectos de escritorio é imprenta.
75. Impuestos.
76. Intereses durante la construccion.
77. Otras erogaciones. Generalidades.

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# DATOS DEL COSTO

## CUENTAS GENERALES, I. VIA O CAMINO

### Primeros gastos. Cuenta 1
### Ingeniería.

**Levantamiento.**

**19.** En « levantamiento » incluimos reconocimientos, levantamientos preliminares y trazado y algunos otros trabajos de ingeniería que pueden necesitarse antes de empezar la construccion.

**20. Importante.** — La adopcion de una linea en condiciones de inferioridad, implica aumento de gastos de construcción, ó aumento del costo de explotación por cada tren, durante la existencia de la línea, ó ambos. El costo del levantamiento (trabajo de agrimensura) será generalmente mucho $<$ 1 % del costo total de la linea y es una economia muy insignificante (sobre todo en líneas de importancia) restringir los gastos del levantamiento.

**21. Reconocimientos.** — Estos consisten en una exploración de las *áreas*, mas que de las *líneas*. La comision se compone generalmente de un ingeniero en jefe, un ingeniero para el tránsito (trazador), un topógrafo, un dibujante, un auxiliar para la mira, uno ó mas hacheros y un cocinero, si se necesita y el gasto mensua será entre $750 (dólares) y $1,500 segun la importancia de la linea; pero el tiempo necesario para los reconocimientos, varia entre amplios límites con la importancia de la linea y la naturaleza y condiciones de los terrenos, sobre todo con el número de las líneas que parezcan prestarse a estudio. El costo de los reconocimientos varia por tanto ampliamente; pero probablemente, rara vez excederá de $25 a $50 por milla de linea construida ó digamos de 0.10 % a 0,25 % del costo de construcción.

**22. Levantamiento preliminar y localización.** — La comisión se compondrá generalmente de :

| | Sueldos $/mes. |
|---|---|
| Ingeniero localizador | 125 a 175 |
| — auxiliar | 100 a 125 |
| — del Tránsito | 85 a 100 |
| Nivelador | 80 a 90 |
| Topógrafo | 70 a 100 |
| Dibujante | 70 a 90 |
| Auxiliar para la mira | 50 a 65 |
| Cadeneros; 2 a $40 ó $50 | 80 a 100 |
| Peon para bandera atrás | 30 a 40 |
| — la cinta; 1 ó 2 de $30 a $45 | 30 a 90 |
| — estacas (estaquero) | 30 a 35 |
| Hacheros, 2 á 5, de $25 a $30 uno | 50 a 150 |
| Total : 14 a 18 hombres | |
| Sueldos de la comision por mes cuando el jornal vale $1,50/hombre/dia | $800 a $1,600 |

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**23. Los trabajos (topográficos), de levantamiento por milla de linea construida** pueden llegar a valer tres y cuatro veces los del trazado de la linea ya indicados.

**24.** El costo de la milla de linea de levantamiento depende de las condiciones de los terrenos y de las limitaciones impuestas como máximo de pendientes y máximo de agudeza en las curvas.

**25.** Una planicie poco accidentada ó una linea faldeando un rio que ofrezca pocas dudas en la eleccion de una linea necesitará pocas millas de estudio preliminar en relación con las millas de construccion definitiva comparado con las que necesitará una linea que atraviese una region escabrosa, quebrada que exija el estudio de varias lineas favorables para encontrar la mejor. En esta clase de terrenos, por supuesto que las restricciones en las pendientes y en la agudeza de las curvas aumenta el gasto de trabajos topográficos por milla construida.

**Ingenieria.**

**26. Costo probable** de este ramo deducido de la comparación de diez y ocho costos en diversas lineas :

| | $/milla de linea. | % del costo total de constn. |
|---|---|---|
| Gastos preliminares y estudio de la linea.. | 150 a 400 | ½ a 1 |
| Gastos de ingenieria durante la construcción (vigilancia trabajos, superintendencia, etc.)..... | 400 a 1,300 | 1.5 a 3 |
| Total..... | 550 a 1,700 | 2 a 4 |

## Primeros gastos. Cuenta 2.
## Terrenos para Transportes.

**Bienes raices.**

**27.** Quizás este es el capitulo en que se requiere mas previsión cuando se quiere deducir el costo de una linea del de otras existentes y al tratar de los derechos y propiedad de las vias, estaciones, terminales, derechos de agua, etc., etc.

**28. El precio por acre** puede variar de $10 y hasta menos en tierras sin valor a $20,000 ó mas en terminales de ciudades; y el costo en estos sitios puede crecer rápidamente de acuerdo con el desarrollo de las ciudades. Indudablemente que, muchas veces los terrenos *se ceden gratuitamente* a las compañias por el gobierno ó por los interesados en el desarrollo del comercio bajo la acción de una linea que atravicee el pais; y por otra parte tambien se dictan previsiones para contrarrestar la demanda de precios excesivos por algunos dueños llenos de avaricia; otras veces las compañias suelen pagar estos precios antes que recurrir a una expropiación que trae litigios y que obligan a la compañia a restricciones, en cuanto al uso exclusivo de los terrenos para asuntos del ferrocarril y asi suelen costarle a la compañia de 2 á 5 veces el valor comercial de las propiedades adyacentes.

**29. El ancho de la via propia** puede variar de 50 pies (como 15 m) ó menos para una linea económica de una sola via en terrenos llanos a 200 pies (como 60 m) ó mas para una linea principal de poca pendiente, de dos a cuatro vias, desmontes profundos y terraplenes altos. El area y los precios de los terrenos necesarios, para patios, estaciones, etc., varia ampliamente con la naturaleza del tráfico.

1 acre por milla = 8.25 pies (como 2.5 m) ancho linea;
66 pies (como 20 m) ancho = 8 acres por milla (2.008 hectareas por kilom);
100 pies (30,5 m) ancho = 12.12 acres por milla (3.042 hectareas por kilom).

**30. La relación**, entre el costo del terreno para la Cia de Ferroc y el valor en plaza del mismo para usos ordinarios, se llama el « *múltiplo del costo del terreno para la via* ».

**31. Adquisiciones.** — Esta materia es muy complicada por el hecho de que las compañias de ferrocarriles frecuentemente compran una zona ó faja mucho mayor de la que necesitan para su linea, esperando vender el excedente a precios muy superiores como consecuencia de la construccion de la via y otras veces obligados a ello como concesiones a propietarios recalcitrantes que no venden lotes menores.

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## Construcción del lecho de la vía.

**32.** Esta construcción (la preliminar para situar la vía) puede considerarse incluida en C. C. I., cuentas 3 á 7 inclusive (véase ¶ 18). Se compone principalmente de la 3, Nivelacion (incluyendo limpieza, desyerbo); Cuenta 5, Túneles y vias subterráneas; Cuenta 6, Puentes, etc.

**33.** Dando el costo por unidad, de materiales y obra de mano, el costo por milla de trazado y de vía de cada una de las materias incluidas en « Lecho para la vía » varía entre muy amplios límites. Así una linea en llano, no necesita túneles, puentes, pilas y poca ó ninguna nivelacion, mientras que en terrenos dificultosos casi toda la linea puede necesitar rozas fuertes, desmontes, terraplenes y quizas túneles, alcantarillas, y puentes con alguna frecuencia; especialmente donde la tendencia moderna a reducir el costo de explotación, suavizando las pendientes y el alineamiento, obliga a desmontes mas profundos y hace mas frecuentes los puentes, alcantarillas, etc.

## Primeros Gastos. Cuenta 3.
## Nivelación.

**34.** La nivelación incluye, desyerbos, rozas, represas, desmontes, terraplenes, en puentes, calzadas y sobre alcantarillas, pilas, trabajos con la pala de vapor, muros de revestimiento y de sostenimiento, fosas, compuertas, etc.

### Desyerbos y rozas.

**35.** Estos trabajos estan a veces incluidos en los de nivelación; pero los desyerbos y rozas se calculan a veces aparte, porque frecuentemente solo en una parte del terreno se necesitan. Los troncos y raíces se dejan debajo de los terraplenes, a menos que éstos sean poco profundos.

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**36. Elementos que afectan el costo.** — El costo por unidad de area, de las rozas, depende de la densidad del monte, de la especie y tamaño de los árboles y del monte bajo, de la naturaleza y accesibilidad del terreno y de las facilidades para mover y sacar los materiales. Este costo puede ser disminuido ó compensado y hasta excedido por el valor de la madera que se saque. Tambien depende aquel costo del método que se emplee en los trabajos de nivelacion. Así por ejemplo una pala de vapor (en tierra) ó una perforadora en roca, pueden hacer gran parte del trabajo que corresponde a las rozas y tumbas. Por otra parte el uso de los arados en desmontes exige tumbas y rozas cuidadosas.

**37.** El costo de la milla de linea por roza y tumba, depende del área por milla de linea y ésta, a su vez del ancho de la zona donde va la linea y de la parte de ésta ocupada por árboles y monte.

| | | | | | |
|---|---|---|---|---|---|
| 1 | 5 | 8.25 | 10 | 16.5 | pies de ancho = |
| 0.1212 | 0,606 | 1 | 1.212 | 2 | acres por milla. |
| 0.30 | 1.52 | 2.59 | 3.05 | 5.03 | metros de ancho = |
| 0.0303 | 0,151 | 0,251 | 0,303 | 0,50 | hect por kilómetro. |
| 20 | 24.75 | 30 | 33 | 40 | 41.25 | pies de ancho = |
| 2.424 | 3 | 3.636 | 4 | 4.848 | 5 | acres por milla. |
| 6.10 | 7.55 | 0.14 | 10.06 | 12.19 | 12.57 | metros de ancho = |
| 0.606 | 0.75 | 0.909 | 1 | 1,212 | 1.25 | hect por kilómetro. |
| 49.5 | 50 | 82.5 | 100 | 165 | 200 | pies de ancho = |
| 6 | 6.06 | 10 | 12,12 | 20 | 24.24 | acres por milla. |
| 15.08 | 15.24 | 25.14 | 30.48 | 50.29 | 60.96 | metros de ancho = |
| 1.50 | 1.515 | 2.50 | 3.03 | 5 | 6.06 | hect por kilómetro. |

**38. Generalmente** de 1 a 6 acres por milla (0.25 a 1.50 hect por kilómetro) ó sea una faja de (8.25 a 49.5 pies ó de 2.59 a 15,08 metros) requieren limpia y tumba.

**39.** El sitio de Brockton, Mass, tiene 480 árboles de (15 a 50 cm de diámetro) por acre = un árbol por cada 91 pies cuad = un arbol por cada 8.50 met cuad. Es muy denso.

**40.** El costo de limpia y tumba puede ser generalmente el siguiente :

| | Tumba. | Roza. | Ambas cosas. |
|---|---|---|---|
| Por arbol ó tronco..... | $1 á 2 | $0.05 á 1 | $1 á 3 |
| Por acre (0.40 hect)..... | $25 a 150 | $25 a 150 | $50 a 300 |
| Generalmente calculada por. | ..... | ..... | $50 |
| Por milla de linea..... | $50 a 600 | $50 a 300 | $100 a 1000 |

### Movimiento de tierra.

**41.** La « nivelación » propiamente incluye (ademas de la limpia y tumba) los « trabajos de tierra » ó sea la excavación, trasporte, depósito y apisonado de tierra, en sus varias formas y de cascajos, capas consistentes, cimientos cascajosos, piedra suelta y rocas sólidas. Cuando los desmontes no dan tierras suficientes para los rellenos y terraplenes se toma el material en fosas y zanjas y este trabajo se acostumbra clasificarlo como « terraplenes ».

**42. Elementos que influyen en el costo.** — El valor de la unidad cúbica de los movimientos de tierra varia mucho con la naturaleza del material, con su accesibilidad, con las cantidades, con las facilidades para manejarlo con la pericia del contratista, con el costo de los salarios, pero sobre todo con la distancia a que debe ser trasportado. Veanse pags 1092 á 1106. En cualquier trabajo de consideración, sinembargo los trasportes largos y cortos se considera que se compensan y es costumbre contratar a un precio fijo el metro ó yarda cúbica de las diferentes clases de material que se encuentren, con un límite especificado para la longitud del arrastre, mas allá del cual el contratista paga (generalmente un centavo por yarda cúbica (1.30 por met cúb) por cada 100 pies (como 30 m) de exceso en la distancia.

**43. Costo por milla.** — Para un precio por unidad cúbica el costo por milla de los movimientos de tierra varía entre límites muy separados, dependiendo de la cantidad que se necesita por milla, la que a su vez depende de las condiciones del terreno y de la necesidad de pendientes y trazados favorables en la linea final; una región dada, naturalmente requiere mas movimientos de tierra para una linea de pendientes y curvaturas limitadas que para otra donde las pendientes fuertes y las curvas agudas y largas, son permitidas.

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### Primeros Gastos. Cuenta 5.
### Túneles y vias subterráneas.

**Túneles.**

**44. Elementos que influyen en el costo.** El costo de los túneles por unidad cúbica depende de la naturaleza del material que hay que extraer y de las facilidades para extraerlo; de la necesidad de revestirlos y la naturaleza del revestimiento; de la cantidad de agua que se encuentra y la facilidad de sacarla, de lo accesible de su situación para los trabajos; del costo del jornal; de la pericia del director de los trabajos, etc. Se hacen menos por contrato que por salarios, auxiliados por las compañías que proporcionan las fuerzas. En los túneles largos, debido a los largos arrastres, a la necesidad de hacer chimeneas de tiro por la dificultad de la ventilacion y al mayor costo (por sus condiciones) de la planta necesaria, resulta generalmente (en igualdad de otras condiciones) el costo mayor que en los túneles cortos. Es menor para *la mesa* (parte baja) que para la galeria (parte cilíndrica); y, por la misma razón (especialmente en secciones pequeñas) disminuye cuando se aumenta el area de la sección.

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## Primeros Gastos. Cuenta 6.
## Puentes, Pilas, Alcantarillas.

**45. Peso del acero.** — La fórmula que sigue de la linea recta, para pesos del acero en la superestructura de puentes de ferrocarril de una via, incluyendo los sistemas de pisos, está extractada de la de los Sres J. B. Johnson y H. G. Tyrrell G. 1471, 1474. Para puentes de ferrocarriles de doble via agregúese el 90 %. Véanse muestras pags 790, 797 y 798.

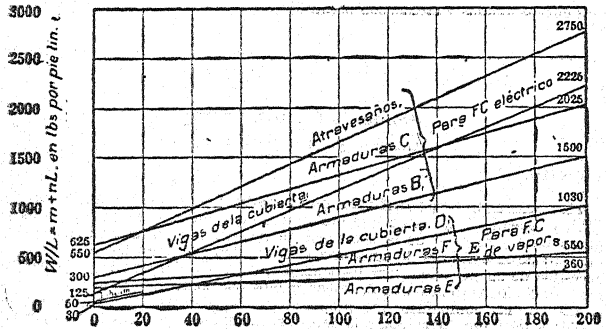
$$W = L\ (m + n\ L)$$

en donde $W$ = peso de la estructura en libs; $L$ = luz en pies; y $m$, $n$ = coeficientes como sigue.

| Puentes para | Diferencias notas abajo. | $m$ | $n$ |
|---|---|---|---|
| Carreteras..... | *A* | 50 | 2 |
| Ferroc. de vapor : | | | |
| Vigas laminadas de la cubierta..... | | 100 a 150 | 9 a 12 |
| Traviesas..... | | 500 a 600 | 10 a 12 |
| Armadura de puentes..... | *B* | 200 a 400 | 5 a 7 |
| | *C* | 600 a 650 | 7 |
| Ferroc eléctricos : | | | |
| Vigas en I (luz de 5 a 20 pies = 1.52 a 6.10 m)..... | | 50 | 5 |
| Vigas laminadas de la cubierta..... | *D* | 30 | 5 |
| Armaduras remachadas..... | *E* | 200 | 0.8 |
| | *F* | 250 | 1.5 |

*A.* Carga viva 100 lbs/pie cuad (488 kg/m cuad). Agréguese 15 lbs/pie de lu (23 kg por metro de luz) por cada 0 m. 60 de variacion desde 5 m de ancho de l carretera.

*B.* Para 2 loco de 100 ton y 4.000 lbs por pie lineal (5952 kg por m lineal) de tren.




*Las lbs se convierten en kgs multiplicándolas por 0.453 y las lbs por pié en kgs por metros multiplicandolas por 1.486.*

**Fig. 1.**

*C.* Para luces de 30 a 230 pies (9.15 a 70.00 m). Carga Cooper E-50 (pag. 818); dos loco 177.5-ton y 5000 lbs por pie lineal (7440 kg por m lineal) de tren.

*D*. Carga viva, 2.000 lbs/pie lineal (2976 kg/m lineal).
*E*. — 1.000 — (1488 kg/m lineal); carros de 15 ton; luz de 40 a 200 pies (12 a 60 m).
*F*. Carga viva; 2.000 lbs/pie lineal (2976 kg/m lineal); carros de 30 ton; luces de 20 a 180 pies (6 a 54 m).

**46.** La fig. 1 muestra, aproximadamente, para cada clase el valor medio de $W/L = m + n\,L$ = peso/pies de luz, en la superestructura de puentes de ferrocarril, como lo da esta ecuacion.

**47. Para pilas, arcos y estribos** para puentes de ferrocarriles a vapor, agréguense 9 lbs/pie cuad (44 kg/m cuad) del perfil del area del terreno a la base del carril; para ferroc eléctricos las $^2/_3$ partes.

**46.** Cantidad de madera en las pilas incluyendo 164 pies de tabla por pie lineal (1.270 m cúb de madera por metro lineal) y en las pilas de 4 pilas arqueadas 16 pies (4.88 m) de *c* a *c*, *G*966.

$$F = L\,(m\,H + n) = m\,A + n\,L\,;$$
$$P = L\,(H + 20)/4.$$

donde :

$F$ = total de pies de madera en la construcción;
$L$ = longitud de la pila en pies;
$A$ = perfil del area de la pila en pies cuad desde el terreno hasta 4 pies (1.22 m) bajo la base del carril;
$H = A/L$ = altura media.
$P$ = pies lineales de pilotaje en las pilas;
$m$, $n$ = coeficientes, como sigue :

| | En las partes de la pila | | En las pilas. | | | |
|---|---|---|---|---|---|---|
| Para alturas. | hasta 15′ (4.5 m) | 15′ (4.5 m) a 25′ (7.5 m) | 25′ (7.5 m) | 25′ (7.5 m) a 50′ (15 m) | 50′ (15 m) a 75′ (22.5 m) | 75′ (22.5 m) a 125′ (37.5 m) |
| Coeficientes. | | | | | | |
| $m$ = | 0 | 0 | 6 | 8 | 9 | 10 |
| $n$ = | 185 | 200 | 220 | 240 | 240 | 240 |

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## Primeros Gastos. Cuentas 8 a 12 incl.
## Via.

**49.** Los trabajos de via, pueden ser tomados como parte de C. C. I. Primero gastos 8 a 12 inclusive, como sigue :

8. Traviezas.
9. Carriles.
10. Otros materiales de via (incluyendo cambios).
11. Balasto.
12. Colocacion de via y arreglo de la superficie.

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**59. Separacion de las traviezas.**

| Dist, de cent a cent. | Trav por carril de 30 pies (9.15 m). | Trav por carril de 33 pies (10.06 m). | Trav por milla. | Trav por kilómetro. |
|---|---|---|---|---|
| 1′ 8″ (0.508 m) | 18 | 19 1/5 | 3168 | 1964 |
| 1′ 9″ (0.533 m) | 17 1/7 | 18 6/7 | 3017 | 1870 |
| 1′ 10″ (0.559 m) | 16 4/11 | 18 | 2880 | 1785 |
| 1′ 11″ (0.584 m) | 15 10/23 | 17 5/23 | 2755 | 1708 |
| 2′ (0.610 m) | 15 | 16 1/2 | 2640 | 1637 |
| 2′ 1″ (0.635 m) | 14 2/3 | 15 21/25 | 2534 | 1571 |
| 2′ 2″ (0.661 m) | 13 11/13 | 15 3/13 | 2437 | 1511 |
| 2′ 3″ (0.686 m) | 13 1/3 | 14 2/3 | 2347 | 1455 |
| 2′ 4″ (0.711 m) | 12 6/7 | 14 1/7 | 2263 | 1403 |
| 2′ 5″ (0.737 m) | 12 12/29 | 13 19/29 | 2185 | 1355 |
| 2′ 6″ (0.762 m) | 12 | 13 1/5 | 2112 | 1309 |

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**Costo de la milla de via.**

**51.** Prácticamente cada milla de via requiere 5280 pies lineales (1609 m) de balasto de traviezas, de colocación y arreglo de superficie y 10,560 pies (3.218 m) lineales de carriles, con sus bridas y demas piezas. Solo el capítulo de desvios está sometido a notables variaciones en el costo de la milla, aun en distintas lineas del mismo carácter general; y estas variaciones son de relativa insignificancia, pues su valor es generalmente de 1 a 3 % del costo total de la via.

**52.** Por tanto, para un costo dado y una construcción dada, el costo por milla de la linea (carriles, etc.) es practicamente constante.

**53. Elementos que afectan el costo.** — Con un material dado el costo de la milla de balasto depende de la seccion transversal de éste; el de las traviezas de su sección y separación el de los carriles y piezas menudas, del peso por unidad lineal de aquéllos; y el de los desvios, de la calidad de su construccion y de su número por milla.

**54. El costo sube.** — Los adelantos en la manufactura (particularmente la hecha en grande escala, la union, consolidacion de sistemas de lineas pequeñas a grandes lineas para su dominio (control) en conjunto y el creciente esfuerzo en el sentido de mayor eficiencia) han producido una reduccion en el costo de la unidad de carril; pero, no ha resultado una grande y progresiva reducción desde 1887; y el costo de la unidad de otros elementos (sobre todo el de las maderas, debido a la escasez) va en aumento. Ademas, el aumento de peso del material rodante y de las cargas han obligado a aumentar la solidez de la construcción. Así pues la tendencia del costo de la via (carriles, traviezas, etc.) es de aumento.

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## GASTOS GENERALES III

**55.** En este capítulo deben ponerse los gastos, como los relativos a la rganización de las compañias, incluyendo los jurídicos ; los necesarios para obtener los certificados de conveniencia y necesidades públicas (exigidos en algunos estados); los necesarios para obtener franquicias y privilegios; los de compensaciones a los promotores (muchas veces sometidos a graves abusos); los requeridos para emitir y dar curso comercial a las acciones y bonos, incluyendo

autorizaciones oficiales y los descuentos de esos valores que pueden tener que venderse bajo la par. Estos totales varian mucho. Mr Walter Loring Webb (« Construcciones de Ferrocarriles », 1905 pag 394 dice : « Se calcula que mas ó menos 2 % (sobre el valor de los titulos) del capital de los ferrocarriles en la Gran Bretaña, se han invertido en gastos « Parlamentarios ».)

**Cuenta 73 (Legislación) y 76 (Intereses** durante la construcción).

**56.** Las partidas de intereses y de gastos de legislación generalmente alcanzan le 2.5 a 5 % del costo total de construccion, sin entrar el material rodante; pero en estimados y valuaciones de peritos, tratándose de largos periodos en que han trascurrido épocas en condiciones no remunerativas, se ha llegado algunas veces a cifras mucho mayores. Asi en el caso del « Gt Northn Ry Sistem. G 1734, en que el costo, sin equipo, llegó a $56,330/milla lineal, el Ing en Jefe estimó los intereses en $5690/milla lineal y los gastos legales en $563/milla; total $6253/milla lineal.

## CONSERVACIÓN DE LA MADERA

**Art. 1.** (a) **La destrucción de la madera** la causa, el crecimiento y la actividad de los hongos. Los pequeños gérmenes de uno de estos hongos, al desarrollarse en un pedazo de madera, produce unos filamentos delgados, que penetran en sus celdas y á poco producen un fermento que destruye ciertas partes de la fibra de la madera. Esta fibra disuelta sirve de alimento á los hongos. Los filamentos van echando ramas y ramales, y muy pronto la madera se encuentra invadida por una masa de aquéllos, cuyo crecimiento produce más fermento, alterando las propiedades químicas y físicas de la madera y convirtiéndola, á veces, en una especie de carbón parduzco, otras, toda blanca ó blanca y fibrosa, diciéndose entonces que la madera se ha podrido. Por fin algunos de los filamentos salen á la superficie y forman hongos al aire, y otras excrecencias; bajo éstas existen cavidades que contienen millares de gérmenes, á los cuales los arrastra el aire y caen sobre otras maderas, en donde se repite el procedimiento. La humedad y el calor son propicios al crecimiento de los hongos, así como también las féculas, azúcares y aceites que se encuentran en las células de la albura de la madera y que no existen en el corazón. Si se la protegiese de la acción de estos hongos la madera duraría indefinidamente. Por esta razón se debe evitar la acumulación de maderas cortadas. Si se excluye el aire, como cuando se conserva la madera constante y enteramente sumergida en agua salada ó dulce, los hongos no pueden prosperar. La savia conservada en la madera con el aire se fermenta produciendo la *carcoma*, como sucede cuando las vigas quedan herméticamente encerradas en la mampostería de ladrillos, etc., y cuando las maderas verdes se pintan ó barnizan, ó se creosotan, etc.; pues entonces la savia no sólo impide la completa penetración del aceite, etc., sino que hace que la mayor parte de la madera se pudra aun cuando en su superficie presente una apariencia de solidez. (b) Por lo tanto se debe destruir la savia **sazonándola**, ya sea secándola al aire á una temperatura normal ó más alta, ó sometiendo antes la madera á una presión de vapor para evaporar la savia y luego eliminarla por el vacío. Para sazonar completamente grandes trozos de madera al aire seco y á temperaturas ordinarias puede requerirse años. La secada rápida en hornos raja y debilita la madera. Es dudoso si la acción del vapor y del vacío eliminan la savia tan completamente como el procedimiento de seca lenta. (c) La exposición alternada al agua y al aire es muy destructora. Causa la putrefacción húmeda.

**Art. 2. Gusanos de mar.** La *limnoria terebrans* trabaja casi desde el nivel del agua hasta un poco más abajo de la superficie del lodo del fondo; el *teredo navalis* entre límites un poco menores. Se dice que el teredo se hace menos activo con la presencia de las aguas de cloacas.

**Art. 3.** (a) Los mejores procedimientos para conservar la madera son prácticamente **inútiles á menos que sean llevados á cabo al pie de la letra.** Si la ganancia en durabilidad no compensa el gasto de tiempo y dinero que se requieren para esto, es más económico emplear la madera en su estado natural. (b) **Las maderas más apropiadas al tratamiento son aquellas** de fibra abierta y porosas que absorben el aceite, etc., mejor que las maderas más compactas, y su baratura hace la aplicación del tratamiento más productiva. (c) La mayor parte de los sistemas que se emplean más frecuentemente parecen hacer á la madera menos combustible. (d) Después del tratamiento de la madera por cualquier sistema, debe secarse bien antes de usarla.

**Art. 4.** (a) **El aceite de creosota** (*ó dead oil*) es el mejor preservativo conocido. Contra los gusanos de mar es eficaz de 15 á 25 años y es la única protección conocida. (b) Como remedio temporal se cubren las estacas algunas veces con láminas de metal ó con clavos de cabezas anchas muy unidos. Éstos se oxidan ó se destruyen en pocos años. Estacas de roble cortadas en enero y clavadas dejándoles la corteza han resistido á los teredos por 4 ó 5 años; y estacas de ciprés bien chamuscadas, por 9 años. (c) Se necesitan como 144 kg por m cúb para su protección ordinaria en la tierra. Para proteger la madera de los gusanos de mar se gastan 130 kg por m cúb en climas como los de la Gran Bretaña y los Estados Unidos; pero en aguas más cálidas, en que el teredo es muy activo, se emplean 220 kg por m cúb. En trozas grandes puede no necesitar la madera saturación completa y requerir, por tanto, menor cantidad. Pero véase (i) y el final del art. 1 (a). (d) Dicho aceite pesa como 1.04 kg el litro (8.8 lbs. gal). (e) Las piezas deben reducirse á las dimensiones finales que se desee y armarse (si fuese necesario hacerlo) antes del tratamiento; especialmente si van á exponerse al *teredo*, pues éste atacará con seguridad cualquier punto que por corte posterior haya quedado sin protección. (f) Las traviesas creosotadas han soportado sanas hasta 22 años de exposición. La creosota protege los clavos de la oxidación. (g) El abeto y el alerce no se pueden creosotar por razón de su irregular densidad. (h) La creosota endurece la madera y la hace un poco más quebradiza. En época de calor se exuda un poco y descolora. Su olor la excluye de las habitaciones. (i) No se sale de la madera, pero á menudo sucede que no penetra hasta el corazón, y por consiguiente si ha quedado alguna savia comienza á podrirse en el centro. Véase el final del art. 1. (a) Por mucho tiempo se ha tenido como el mejor preservativo, tratarla interiormente con el sistema de Burnett y usar una capa de creosota por fuera (véase art. 7). Ésta es la mayor ventaja que presenta el procedimiento de **Allardyce**. Es más barato que creosotar por completo. En el **procedimiento de Rutgers**, que se ha empleado con éxito en Alemania para las traviesas, desde 1874, se inyectan simultáneamente la creosota y una solución de cloruro de cal. (j) En el **procedimiento de creosotar** *, el fluido preservativo consiste de 38 por ciento de creosota, 2 por ciento de *formaldehyde* y 60 por ciento de resina fundida.

**Art. 5.** (a) **Las soluciones minerales** son inferiores á la creosota, aun en tierra, y completamente inútiles en aguas dulces ó contra el gusano de mar, pero duplican casi la duración de las maderas de calidad inferior colocadas en tierra; su bajo precio permite usarlas en donde la creosota es demasiado cara. (b) Hacen más dura á la madera, y se vuelve quebradiza si la solución es demasiado fuerte. Están expuestas á ser lavadas por la lluvia, etc., y por consiguiente, la capa exterior se pudre primero. Véase art. 4 (i) y art. 8 (b), (c), (d). (c) Una comisión de la Sociedad Americana de Ingenieros Civiles**, después de comparar un gran número de experimentos, recomendaron el sistema de Burnett, **para exposiciones á la humedad** (art. 7), como en traviesas, en pisos húmedos, etc.; y el sistema de Kyan (art. 6) **para exposiciones relativamente secas** con exposición al aire y al sol (como en las vigas de un puente), para las cuales es más apropiado que el sistema de Burnett, porque parece debilitar menos la madera. En semejantes casos conserva la madera de 20 á 30 años.

**Art. 6.** (a) El sistema de Kyan consiste en sumergir la madera en una solución de 1 por cien de agua de **bicloruro de mercurio (sublimado corrosivo)**. (b) Generalmente se hace que la madera se remoje un día por cada 25 mm de la menor dimensión de la pieza, y un día más cualquiera que sea el tamaño. (c) El general Cram encontró el procedimiento muy *malsano* porque producía « salivación » á los obreros, pero B. Francis, en Lowell, y H. Bissell, del ferrocarril del Este de Massáchusets, encontraron poco ó ningún inconveniente á este respecto. Sin embargo, el sublimado, que es muy venenoso, tiende á esflorecer, y el uso de la madera se hace, por tanto, peligroso. (d) El procedimiento es valioso para las maderas colocadas en lugares moderadamente húmedos, pero la sal está expuesta á ser lavada por el agua corriente. El abeto tratado por el sistema de Kyan empleado en postes de cercas, clavados á 1.20 m de profundidad, en Lowell, Mass., en 1850, fueron examinados en 1891, y la mayor parte se encontraron en buen estado tanto sobre como debajo de la tierra.

**Art. 7.** (a) El sistema de Burnett consiste en sumergir la madera durante algunas horas en una solución de 2 por ciento de *cloruro de cinc*, bajo una presión como de 7 á 21 kg por cm cuad.

* Véase « Método propuesto para la conservación de la madera », por F. A. Kummer rans. Am. Soc. C. E., vol. XLIV, diciembre 1900.

T** Véase Trans. Am. Soc. C. E., julio, agosto y septiembre 1885.

**Art. 8. Otros preservativos.** (a) Se ha usado mucho el de sumergir la madera en una solución de **sulfato de cobre** (**vitriolo azul**), pero parece que no ha dado siempre buenos resultados. El vitriolo azul lo suelta fácilmente. (b) En el procedimiento de Barschall ó Hasselmann §, introducido en Alemania en 1887, y en los Estados Unidos en 1899, se hierve la madera á una temperatura de 100° á 140° C y bajo una presión de 1 á 3 kg por cm cuad, en una solución de sulfatos de hierro, cobre, aluminio y de « Kainit », un sulfato de magnesia y potasa, extraído de las minas de Stassfurt, en Alemania. Se dice que la solución arrastra la savia (la madera se trata más fácilmente por este procedimiento cuando está verde), y que el cobre destruye los hongos, y el hierro forma un compuesto insoluble con la celulosa ó fibra leñosa. También se asegura que el procedimiento endurece mucho la madera, especialmente las clases blandas, haciéndolas útiles para traviesas, sin disminuir su resistencia, elasticidad y flexibilidad. (c) En el **procedimiento de Wellhouse** se inyecta primero una solución de cloruro de cinc con cola y luego una de tanino (ambas bajo presión), á fin de disminuir el consiguiente arrastre del cloruro. En una modificación posterior, las soluciones de cinc, cola y tanino se inyectan separadamente. De esta manera se han tratado algunos millones de traviesas No se recomienda este procedimiento para maderas que se emplean bajo el agua. (d) Los procedimientos en los cuales se trata la madera por **pintura** ó **inmersión** *, son : Carbolineum America (C. A. preservador de Maderas) y Carbolineum Avenarius (esencia de alquitrán, cloro, etc.), Ligni Salvor (esencia de alquitrán, etc.), Woodiline y Spiritine (soluciones químicas) y un destilado de pino, empleado por el ferrocarril de Pensilvania en la construcción de vagones. (e) Parece que los postes de cercas, etc., se conservan hasta cierto punto con sólo sumergir su extremo inferior en **alquitrán** bien hervido, para quitarle el amoníaco que destruye la madera. El extremo superior se deja sin alquitranar para que se evapore la savia. (f) Los experimentos hechos para conservar la madera por medio del *vapor de la creosota* han resultado ineficaces. (g) Mientras la madera se conserve completamente saturada de **petróleo** no se pudre, pero hay que petrolizarla á menudo, pues al evaporarse la deja sin protección. (h) Las traviesas de madera de algodonero colocadas en un **terreno** que contenga un 2 por ciento de carbonato de cal, 1 por ciento de sal y .5 por ciento de potasa y óxido de hierro, en el ferrocarril Union Pacific, en 1868, se encontraron en 1882 « tan sanas y mucho más fuertes que cuando se colocaron », y las mismas traviesas en otros terrenos tan sólo duraron de 2 á 5 años. (i) El empleo de soluciones de **cal y sal**, así como el chamuscar la superficie, es á veces útil en lugares húmedos.

§ Railroad Gazette, febrero 9, 1900.

* Véase el Informe de O. Chanute á la Am. Ry. Eng. and Maint of Way. Assotion, de marzo 1901.

## Art. 4. Término medio de la resistencia máxima á la tensión, ó cohesión de las maderas.

Esas son las cargas más pequeñas en kg que, colocadas en el extremo inferior de una varilla vertical de 6.45 cm cuads de sección, y bien fija en el extremo superior, la romperían separándola ó rasgándola en pedazos. Para largas piezas de madera recomendamos reducir estas constantes en ¼ á ⅓ parte.

(*Obs. del T.* — Hemos convertido la tabla del autor al sistema métrico.)

| Las resistencias en todas estas tablas pueden ser una tercera parte mayor ó menor que nuestros términos medios. | Kg por cm cuad. | Las resistencias en todas estas tablas pueden ser una tercera parte mayor ó menor que nuestros términos medios. | Kg por cm cuad. |
|---|---|---|---|
| Aliso | 984 | Caoba, Honduras | 562 |
| Fresno inglés | 1,125 | — Española | 1,125 |
| — americano (autor) como | 1,160 | Mangle blanco, Bermuda | 703 |
| Abedul | 1,055 | Morera | 844 |
| — negro americano | 492 | Roble blanco americano | |
| Laurel | 844 | — americano de cesta | |
| Haya inglesa | 808 | — — rojo | 703 |
| Bambú | 422 | — seco de Dantzic | |
| Boj | 1,406 | — de Riga | |
| Cedro, Bermuda | 534 | — inglés | |
| — Guadalupe | 668 | — vivo, americano | 703 |
| Castaño | 914 | Peral | |
| — de Indias | 703 | Pino americano, blanco, rojo, y pitchpine, Memel, Riga * | 703 |
| Burato | 422 | Ciruelo | 773 |
| Sauco | 703 | Alamo temblón | 492 |
| Olmo | 422 | Membrillo | 492 |
| — del Canadá | 914 | Pruche ó abeto | 703 |
| Abeto ó pinabete | 703 | Sicomoro | 844 |
| Espino blanco ó albar | 703 | Teca ó encina de Africa | 1,054 |
| Avellano | 1,265 | Nogal | 562 |
| Acebo común | 1,125 | Tejo (parecido al abeto) | 562 |
| Madera de hierro | 1,406 | **Transv á la fibra**, roble | 1,617 |
| Nogal americano | 773 | — álamo temblón | 126 |
| Guayaco americano | 773 | — alerce á | 119 |
| Palo de lanza | 1,617 | — abeto, etc., pino | 39 |
| Alerce, Escocia | 492 | | |
| Locust (algarrobo, acacia, etc.) | 1,265 | | |
| Arce | 703 | | |

**Estos son términos medios.** Las resistencias varían mucho con la edad del árbol, el lugar en que crece, según la pieza sea del corazón ó de las capas exteriores, con el grado de sequedad, rectitud de sus fibras, nudos, etc., etc. Además, como las constantes se deducen de ensayos hechos con buenos materiales de tamaño pequeño, mientras que las vigas grandes son más ó menos defectuosas por los nudos, por la sinuosidad de sus fibras, etc., es conveniente, en la práctica, reducir estas constantes como se recomienda arriba.

* **Efecto en los árboles de la extracción de la trementina.** Los experimentos preliminares hechos por la Sección de Selvicultura, del Departamento de Agricultura de los Estados Unidos, en pino de hojas grandes de Alabama, indican que (contra la opinión general) « la madera de las terebintáceas » ó sea la madera de los árboles á que se les ha extraído la trementina, aunque tienen ligeramente menos resistencia á la tensión y al esfuerzo cortante, presentan de 20 á 30 por ciento más de resistencia á la compresión (ya sea en el sentido ó transversalmente á las fibras) y bajo la acción de esfuerzos transversales. En dicha madera, sin embargo, la resina se acumula en ciertos puntos, y embotando ó engomando los instrumentos, se hacen más difíciles para labrarlas que las que provienen de árboles que no se les ha quitado la trementina. Los trozos probados se cortaron, en su mayor parte, á alturas desde 1.13 m hasta 10 m sobre el suelo (circular n.º 8 publicada en 1892). Las maderas á que se les ha extraído ó no la resina se llaman frecuentemente « sangradas » ó « sin sangrar ».

**Art. 1. Resistencia á la compresión de las maderas americanas,** *cuando han sido secadas lenta y cuidadosamente.* Promedios aproximados deducidos de muchos ensayos hechos con la máquina de probar maderas del Gobierno de los E. U., en Watertown, Mass, por S. P. Sharples, para el censo de 1880. **Las maderas secas** resisten mucho más que las verdes la trituración; en muchos casos hasta el doble. Esto debe tenerse presente al construir puentes, etc., con maderas recién cortadas. Diversas muestras de la misma madera se diferencian mucho entre sí, frecuentemente como 5 á 8, 9 ó más.

(*Obs. del T.* — Hemos convertido la tabla que trae el autor al sistema métrico.)

| La resistencia en todas estas tablas puede variar fácilmente hasta 1/3 más ó menos de nuestro promedio. | Por los extremos*, kg por cm cuad. | Por los lados**, kg por cm cuad. | |
|---|---|---|---|
| | | .25 mm. | 2.5 mm. |
| Fresno, rojo y negro | 478 | 91 | 211 |
| Alamo temblón | 309 | 56 | 98 |
| Haya | 492 | 77 | 134 |
| Abedul | 562 | 91 | 176 |
| Castaño de Indias | 309 | 42 | 98 |
| Nogal blanco americano | 386 | 49 | 112 |
| Plátano (sicomoro) | 422 | 91 | 176 |
| Cedro, rojo | 422 | 49 | 70 |
| — blanco (arbor vitæ) | 309 | 35 | 63 |
| Catalpa (haba india) | 351 | 49 | 91 |
| Cerezo, silvestre | 562 | 119 | 176 |
| Castaño | 372 | 63 | 112 |
| Cafeto, Kentucky | 365 | 91 | 176 |
| Ciprés, pelado | 422 | 35 | 84 |
| Olmo, americano ó blanco | 478 | 91 | 176 |
| — rojo | 541 | 91 | 176 |
| Pinabete | 372 | 42 | 77 |
| Nogal americano | 562 | 141 | 281 |
| Lignum vitæ | 703 | 112 | 913 |
| Tilo, americano | 351 | 35 | 63 |
| Algarrobo, blanco y amarillo | 689 | 133 | 309 |
| — miel | 492 | 112 | 176 |
| Caoba | 633 | 119 | 373 |
| Meple (ó arce) de hojas anchas de Oregón | 373 | 98 | 176 |
| — de azúcar ó negro | 562 | 134 | 302 |
| — blanco ó rojo | 478 | 91 | 203 |
| Roble, blanco, de hierro, blanco de pantanos, rojo y negro | 492 | 112 | 281 |
| — achaparrado y para cestos | 422 | 119 | 295 |
| — castaño y vivo | 527 | 112 | 316 |
| — de aguja | 457 | 91 | 210 |
| Pino, blanco | 386 | 42 | 84 |
| — rojo ó de Noruega | 443 | 42 | 98 |
| — de tea y de Jersey | | | |
| — achaparrado | 351 | 70 | 141 |
| — de Giorgia | 598 | 91 | 176 |
| Alamo | 351 | 42 | 77 |
| Sasafrás | 351 | 91 | 148 |
| Abeto, negro | 400 | 49 | 91 |
| — blanco | 316 | 42 | 85 |
| Sicomoro | 421 | 91 | 176 |
| Nogal, negro | 562 | 91 | 176 |
| — blanco | 387 | 49 | 112 |
| Sauce | 309 | 49 | 98 |

* Muestras de 4 cm en cuadro, 32 cm de largo.

** Muestras de 4 cm en cuadro, 16 cm de largo, puestas en la plataforma de la máquina de prueba. La presión aplicada en mitad de su longitud, por medio de una perforación de un gancho de hierro de 4 cm en cuadro, es decir, lo suficiente para cubrir todo el ancho de la muestra y un cuarto de su longitud. La primera columna de la tabla enca-

De lo anterior aparece que el pino blanco y el amarillo, el abeto y el roble común, **secos**, que son las maderas más usadas en los Estados Unidos para puentes, techos, etc., se trituran longitudinalmente con 350 á 490 kg por cm cuad, **en pequeños bloques**, dando un promedio de 420.

Pero es conveniente recordar que en la práctica es raro obtener presiones perfectamente distribuídas. En algunos ensayos de compresión lateral, hechos con bloques de pino blanco, de 15 cm de alto, 12½ de largo y dos de ancho, se encontró que bajo una presión, igualmente distribuída, de un total de 2,265 kg ó 350 kg por cm cuadrado, comprimían de 3 á 6 mm, lo que es igual, más ó menos, á $^1/_{48}$ á $^1/_{24}$ de la altura, siendo el promedio de unos 3 por ciento de la altura. Bajo 4,535 kg de carga total, ó 70 kg por cm cuadrado, se astillaron por completo, y en algunos casos saltaron grandes pedazos.

La resistencia á la tensión ó la cohesión del pino y del roble es de un promedio de 703 kg por cm cuad, ó la mitad del promedio del hierro colado, ó casi el doble de su resistencia á la trituración. La resist á la tensión no varía con el largo de la pieza; de manera que en la práctica se puede tomar como de seguridad una fuerza como de 70 á 140 kg por cm cuad, dependiendo de la naturaleza de la construcción, etc., sin tomar en cuenta la longitud, excepto cuando ésta es tan grande que hay que empatar una ó más piezas para formarla, debilitando mucho la pieza de esta manera.

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**Tabla de cargas de seguridad, estática só en reposo, de vigas rectangulares horizontales de pino blanco ó abeto de 25 mm de ancho apoyadas en ambos extremos y cargadas en el centro, y de la flexión, producida por dicha carga.**

Estas cargas de seguridad son una sexta parte de las cargas de ruptura.

**Para obtener la carga neta**, dedúzcase ½ del peso de la misma viga. La *flexión*, no obstante, es la *verdadera;* el peso de las vigas se ha introducido al calcularla.

**Las cargas aplicadas repentinamente** duplicarán las flexiones dadas en la tabla, como cuando, por ejemplo, se sostiene un peso en la mano, de modo que se halle en *contacto* con una viga, y se suelta repentinamente.

**Advertencia.** Como esta tabla se basa en piezas de fibras derechas bien secas, sin nudos ni otros defectos, no debemos tomar en la práctica más de dos tercios de las cargas dadas en la tabla para cargarlas con un coeficiente de seguridad igual á 6 en las construcciones ordinarias de madera de buena calidad; y con estas cargas reducidas *no* se deben reducir las flexiones *.

**Obsérvese también que** nuestra tabla es para cargas de seguridad **centrales**; pero es claro que en la práctica no podemos aplicar siempre el término con sumo rigor, pues de otra manera la carga tendría que estar sostenida por el filo de una cuchilla para que se hallase aplicada en el *verdadero centro* de la viga; ahora bien, en el ejemplo de la nota pág. 1055, si pretendiéramos sostener la carga central de 6,075 lbs (2,756 kg) en un filo de cuchilla, en el acto cortaría la viga en dos. Si siquiera la aplicamos en una extensión de 8 á 10 cm de su longitud, siempre penetraría en la viga cortándola, y no tendríamos un coeficiente de seguridad contra la trituración igual á 6 en la parte superior de la viga, hasta tanto que, como en el caso de los extremos, distribuyamos la carga en 117 cm de longitud ó en 80 cm para una seguridad de 4.

**La carga de seguridad** es en este caso $^1/_6$ de la de ruptura, y ésta es de 204 kg en el centro de una viga que tenga 25 mm de sección en cuadro y .30 m de luz. Para construcciones simplemente temporales puede agregarse 50 % á las cargas de la tabla, convirtiéndolas así en ¼ parte de la carga de ruptura. Pero en construcciones de importancia, sujetas á vibraciones, debe sustraerse 25 % á las cargas tubulares, reduciéndolas así á ⅛ de la carga de ruptura. Sobre todo si la madera no está bien seca.

**Con las cargas de seguridad de esta tabla una viga puede doblarse demasiado** para muchas aplicaciones prácticas. Cuando así sea, podemos, por la reducción de las cargas, reducir las flexiones casi en la misma proporción.

Todas las cargas de la tabla son cargas de **seguridad excesiva contra el esfuerzo cortante, contra la trituración en los extremos**, etc.; véanse las « Advertencias » que siguen á continuación de la tabla.

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bezada .25 mm da los pesos que producen una mella de .25 mm. La segunda columna encabezada 2.5 mm da los que producen una mella de 2.5 mm.

★*N. del T.* — Este consejo no debe olvidarse, pues realmente esta tabla está hecha con maderas en muy buenas condiciones, porque las cargas comparadas con las de otras varias tablas son siempre mayores.

(*Obs. del T.* — Hemos convertido al sistema métrico la tabla **original** del autor. Como se dijo arriba, á las vigas en esta tabla se les supone 2.5 centímetros de ancho. Con esta tabla puede calcularse la carga para cualquier viga de ancho, altura ó luz distintas á las de la tabla si se recuerda que la resistencia varía en proporción del ancho, en proporción al cuadrado de la altura y en razón inversa de la luz. No sabemos con qué fórmula ni con cuáles experiencias formó el autor esta tabla; pero llamando B el ancho de la viga; D, su altura, y L, la luz, tomando las primeras en centímetros y la última en decímetros; calculando la resistencia segura de la madera á la tensión en 10 kg por cm cuad y empleando la fórmula muy conocida $\frac{RI}{T} = \frac{I}{n} Pl$; en que $\frac{I}{T} = \frac{BD^3}{6}$ (pág. 491, para vigas de sección rectangular); $n = 4$, para vigas apoyadas en los extremos y cargadas en el centro, y R = 10 kg, como ya dijimos, resultan para P valores muy aproximados á los de la tabla.)

| Altura de la viga. cm. | Luz 1.20 m. | | Luz 1.80 m. | | Luz 2.40 m. | | Luz 3.00 m. | | Luz 3.60 m. | | Luz 4.20 m. | | Luz 4.80 m. | | Peso de 3 m de viga kg. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Carga kg. | Flex mm. | Carga kg. | Flex mm. | Carga kg. | Flex mm. | Carga kg. | Flex mm. | Carga kg | Flex mm. | Carga kg. | Flex mm. | Carga kg. | Flex mm. | |
| 2.5 | 8.6 | 10.01 | 5.8 | 23.07 | 4.53 | 45.72 | 3.63 | 76.24 | 2.72 | 111.76 | ..... | ..... | ..... | ..... | .9 |
| 5.0 | 34 | 5.69 | 22 | 11.03 | 17.23 | 20.83 | 13.60 | 33.02 | 11.34 | 48.26 | 9.52 | 68.58 | 8.62 | 93.98 | 1.8 |
| 7.5 | 77 | 3.30 | 51 | 8.02 | 38.55 | 13.46 | 30.28 | 21.34 | 25.85 | 33.02 | 21.77 | 43.18 | 19.05 | 58.42 | 2.7 |
| 10.0 | 136 | 2.54 | 90 | 5.69 | 68.02 | 9.91 | 54.48 | 16 | 45.35 | 23.37 | 39 | 33.02 | 34.01 | 43.18 | 3.6 |
| 12.5 | 212 | 2.03 | 141 | 4.67 | 106.12 | 7.87 | 84.80 | 13.70 | 70.74 | 18.29 | 60.77 | 25.40 | 53.06 | 33.02 | 4.5 |
| 15.0 | 306 | 1.52 | 204 | 4.81 | 152.83 | 6.60 | 122.44 | 10.41 | 102.04 | 15.24 | 87.52 | 21.08 | 76.19 | 28.94 | 5.4 |
| 17.5 | 416 | 1.52 | 277 | 3.05 | 208.61 | 5.59 | 166.43 | 9.89 | 138.77 | 12.95 | 118.82 | 18.78 | 104.30 | 23.62 | 6.3 |
| 20.0 | 544 | 1.37 | 362 | 3.79 | 272.10 | 4.82 | 217.62 | 7.47 | 181.40 | 11.43 | 155.55 | 15.49 | 136.05 | 20.57 | 7.2 |
| 22.5 | 689 | 1.01 | 459 | 2.54 | 344.66 | 4.32 | 275.27 | 7.86 | 229.92 | 10.16 | 197.02 | 13.72 | 172.33 | 18.29 | 8.1 |
| 25.0 | 850 | 1.01 | 566 | 2.28 | 424.93 | 4.06 | 340.12 | 6.09 | 283.44 | 8.89 | 243.08 | 12.45 | 212.24 | 16.26 | 9.0 |
| 27.5 | 1029 | 1.01 | 686 | 2.03 | 514.72 | 3.55 | 411.32 | 5.59 | 343.30 | 8.13 | 293.87 | 11.18 | 257.13 | 14.73 | 9.9 |
| 30.0 | 1224 | .76 | 816 | 1.88 | 612.22 | 3.30 | 489.78 | 5.08 | 408.15 | 7.36 | 350.10 | 10.16 | 306.11 | 13.46 | 10.8 |
| 35.0 | 1666 | .76 | 1111 | 1.52 | 833.08 | 2.79 | 666.64 | 4.32 | 555.54 | 6.35 | 476.17 | 8.63 | 416.31 | 11.43 | 12.7 |
| 40.0 | 2176 | .51 | 1451 | 1.37 | 1088.40 | 2.54 | 870.72 | 4.81 | 725.60 | 5.59 | 622.20 | 7.62 | 544.20 | 10.16 | 14.5 |
| 45.0 | 2755 | .51 | 1836 | 1.37 | 1377.28 | 2.28 | 1102.0 | 3.55 | 918.34 | 5.08 | 787.28 | 6.86 | 688.41 | 9.89 | 16.3 |
| 51.0 | 3401 | .51 | 2267 | 1.01 | 1700.62 | 2.03 | 1360.50 | 3.05 | 1133.75 | 4.57 | 972.76 | 6.09 | 849.31 | 8.87 | 18.1 |
| 56.0 | 4115 | .51 | 2743 | 1.01 | 2057.53 | 1.78 | 1646.20 | 3.79 | 1371.84 | 4.06 | 1175.92 | 5.59 | 1028.54 | 7.36 | 19.9 |
| 61.0 | 4897 | .51 | 3265 | 1.01 | 2448.90 | 1.52 | 1959.12 | 2.54 | 1632.60 | 3.81 | 1400.41 | 5.08 | 1224.45 | 6.60 | 21.7 |

(Continúa en la pág. 1187.)

| Altura de la viga. | Luz 5.40 m. | | Luz 6.00 m. | | Luz 7.50 m. | | Luz 9.00 m. | | Luz 10.50 m. | | Luz 12.00 m. | | Peso de 3 m de viga. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| cm. | Carga kg. | Flex mm. | Carga kg. | Flex mm. | Carga kg. | Flex mm. | Carga kg. | Flex mm. | Carga kg. | Flex mm. | Carga kg. | Flex mm. | kg. |
| 15.0 | 68 | 35 | 61 | 46 | 49 | 73 | 41 | 114 | 35 | 165 | 30 | 233 | 5.4 |
| 17.5 | 92 | 30 | 83 | 38 | 67 | 63 | 55 | 99 | 47 | 147 | 41 | 193 | 6.3 |
| 20.0 | 121 | 25 | 108 | 33 | 87 | 53 | 72 | 81 | 62 | 116 | 54 | 162 | 7.2 |
| 22.5 | 153 | 23 | 137 | 30 | 110 | 48 | 91 | 71 | 79 | 101 | 68 | 140 | 8.1 |
| 25.0 | 189 | 21 | 170 | 25 | 136 | 43 | 113 | 63 | 97 | 89 | 85 | 124 | 9.0 |
| 27.5 | 229 | 19 | 206 | 23 | 164 | 38 | 136 | 56 | 117 | 81 | 102 | 109 | 10.0 |
| 30.0 | 272 | 17 | 244 | 21 | 196 | 35 | 163 | 50 | 139 | 73 | 122 | 99 | 10.9 |
| 35.0 | 370 | 15 | 333 | 18 | 267 | 30 | 222 | 43 | 190 | 61 | 166 | 81 | 12.7 |
| 40.0 | 483 | 13 | 435 | 16 | 348 | 25 | 290 | 38 | 248 | 53 | 217 | 71 | 14.5 |
| 45.0 | 612 | 11 | 551 | 14 | 441 | 22 | 367 | 33 | 315 | 45 | 275 | 63 | 16.3 |
| 50.0 | 755 | 10 | 680 | 13 | 544 | 20 | 453 | 30 | 388 | 40 | 340 | 56 | 18.1 |
| 55.0 | 914 | 9 | 823 | 11 | 658 | 18 | 548 | 27 | 470 | 38 | 411 | 51 | 19.9 |
| 61.0 | 1088 | 8 | 979 | 10 | 783 | 16 | 653 | 24 | 559 | 33 | 489 | 46 | 21.8 |
| 66.0 | 1277 | 8 | 1145 | 10 | 915 | 15 | 763 | 22 | 657 | 30 | 572 | 40 | 33.6 |
| 71.0 | 1481 | 7 | 1333 | 9 | 1067 | 14 | 888 | 20 | 761 | 27 | 666 | 38 | 25.4 |
| 76.0 | 1700 | 6 | 1530 | 8 | 1224 | 13 | 1020 | 19 | 874 | 27 | 765 | 35 | 27.2 |
| 81.0 | 1935 | 6 | 1741 | 7 | 1393 | 11 | 1160 | 18 | 994 | 25 | 870 | 33 | 29.0 |
| 86.0 | 2184 | 6 | 1965 | 7 | 1573 | 11 | 1310 | 17 | 1123 | 23 | 982 | 30 | 30.8 |
| 91.0 | 2448 | 6 | 2204 | 7 | 1763 | 10 | 1469 | 16 | 1259 | 21 | 1103 | 28 | 32.6 |

**El roble blanco y el mejor pitchpine* del sur** soportarán cargas ¼ parte mayores.

**Para el hierro colado** multiplíquense las **cargas** de la tabla por 4.5, y para el **hierro forjado**, por 5.3. Para estas nuevas cargas multiplíquense las **flexiones** por .4 para el hierro colado, y por .3 para el forjado.

**Si la carga está uniformemente** distribuída sobre **la luz** puede ser el doble de la central, y las flexiones serán 1¼ veces las de la tabla. **Si las cargas de la tabla** están distribuídas uniformemente á lo largo de **toda** la viga, las flexiones no serán sino $^5/_4$ de las de la tabla. Cuando se requiera mayor exactitud, la mitad del peso propio de la viga debe sustraerse de la carga central; y todo él cuando la carga está uniformemente distribuída. El peso de la viga, en la última columna, está calculado suponiendo que la madera está regularmente seca y por tanto que pesa 461 kg por m cúb.

**Usos de la tabla anterior.** Ejemplo 1.° ¿Cuál deberá ser el ancho de una viga horizontal, rectangular, de pino blanco, que tiene 35 cm de altura, apoyada en ambos extremos y con 6 m de longitud entre sus apoyos, para que soporte con seguridad una carga de 5 toneladas (5,000 kg) en su centro? En este caso, frente á la altura de 35 cm de la tabla y en la columna de 6 m de abertura, encontramos que una viga con un espesor de 2.5 cm soporta 333 kg; por tanto $^{5000}/_{333} = 15$ cm, es el ancho buscado, porque la resistencia está en la misma proporción que el ancho.

Ejemplo 2.° ¿Cuál será la carga de seguridad en el centro de una viga para suelo ó durmiente de pino blanco, que tenga 5.40 m de largo, 7.5 cm de ancho y 30 de altura? En este caso, en la columna en cabeza da 5.40 m y frente á 30 cm de altura encontramos que la carga de seguridad para un ancho de 2.5 cm (supuesto en la tabla) es de 272 kg; por tanto $272 \times 3 = 816$ kg es la carga buscada.

*Nota.* **Advertencia en el uso de la tabla anterior.** Por ejemplo, al colocar cargas muy pesadas sobre vigas cortas pero altas y resistentes, debemos tener cuidado de que las vigas descansen en una longitud suficiente sobre sus soportes, de modo que se evite todo peligro de *ruptura* por trituración en los extremos. Así, si colocamos una carga de 2,756 kg en el centro de una viga que tenga de luz 1.20 m, de alto 45 cm y sólo 2½ de espesor, cada extremo de la viga soporta una fuerza de compresión vertical de $\frac{2{,}756}{2} = 1{,}378$ kg, que obra **transversalmente á las**

* *N. del T.* — Traduciremos á *pitchpine*, por *pichipen*, término muy usado en Sur-América.

**fibras.** En este caso, el pino blanco, el abeto y el pinabete se rompen por término medio bajo 56 kg por cm cuad, y para que presente una resistencia de seguridad de 6, es necesario que la presión se reduzca á cosa de 9.35 kg por cm cuad. Por tanto, nuestra viga, para que presente una resistencia de seguridad 6 contra la trituración (ruptura por compresión) de sus extremos, debe descansar en cada soporte en una superficie de $\frac{1,378}{9.35} = 147$ cm cuad ó para una carga seguridad de 4 como 98. Cuando una presión está igualmente distribuída en sentido transversal (esto es, perpendicularmente á la dirección de las fibras) sobre toda la superficie comprimida de un bloque ó viga (para lograr esto la superficie opuesta debe estar apoyada en todos sus puntos), la compresión que resulta puede escapar fácilmente á la observación, á menos que se la mida realmente. Pero cuando se aplica una presión considerable á una parte de la superficie, como en los topes ó pies de los postes, ó en los extremos de viguetas ó durmientes cargados, la compresión resalta á la vista porque las partes comprimidas se hunden más que las no oprimidas por causa de la flexión ó ruptura de las fibras adyacentes. Lo que en el primer caso (especialmente si es ligera) se llamaría **compresión**, sería llamada en el segundo caso **trituración**; aun cuando ninguna de ellas sea tan considerable como para hacerla insegura. Debido á la resistencia que dichas fibras adyacentes oponen á la flexión ó ruptura, es claro que una presión dada **por pulgada cuadrada, por cm cuad ó por pie cuadrado**, etc., causará una compresión ó trituración algo menor cuando se aplica á una sola parte de la superficie que cuando se la aplica á toda ella.

**El autor ha visto** 40 soportes de pinabete, medio secos, de 30 cm en cuadro cada uno, apoyados á intervalos de 1.52 m de centro á centro, sobre umbrales semejantes de pinabete de 30 × 30 cm, á los cuales estaban fijos por espigas, y que descansaban en todo su largo sobre peldaños de piedra. Cada poste fué cargado gradualmente con 32 toneladas ó 35.15 kg por cm cuad; y sus pies todos se introdujeron en los umbrales de 6 á 12 milímetros. Sus cabezas se introdujeron á la misma profundidad. **En la práctica**, la presión en las cabezas y bases de los postes, rara vez y quizás nunca, es uniforme; lo mismo sucede con los extremos de las viguetas y vigas cargadas, etc., en las cuales un ligero doblez causa un aumento de presión en los bordes inferiores de sus apoyos.

**Art. 31. Tabla de las mayores cargas centrales para vigas horizontales de pino blanco, amarillo ó abeto, de sección rectangular, y de 2½ cm de ancho, sostenidas en ambos extremos, y sin que presenten una flexión mayor de $^1/_{180}$ de la luz. En la práctica, por nudos, etc., tómense los $^2/_3$ solamente.**

Los pesos en esta tabla incluyen el peso de la viga en la luz : $^6/_4$ de este peso debe ser deducido de las cargas de la tabla, cuando la viga está cargada en el centro. Cuando lo esté uniformemente, las cargas serán 1.6 veces las de la tabla, pero en este caso se deducirá todo el peso de la viga. En la práctica rara vez se hace esta deducción.

**Luz en metros** (Original). (*Obs. del T.* — Hemos convertido la tabla del autor a lsistema métrico.)

| Altura | .91 | 1.22 | 1.52 | 1.83 | 2,13 | 2.44 | 2.74 | 3.05 | 3.66 | 4.27 | 4.88 | 5.49 | 6.10 | 7.62 | 9.15 | 10.68 | 12.20 | Altura. | Peso de 3m viga. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| cm. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | cm. | kg. |
| 2.5 | 3.81 | 2.48 | 1.36 | .95 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 2.5 | .9 |
| 3.8 | 13.01 | 7.35 | 4.72 | 3.26 | 2.40 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 3.8 | 1.3 |
| 5.1 | 31.02 | 17.42 | 11.06 | 7.75 | 5.62 | 4.35 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 5.0 | 1.8 |
| 6.3 | ..... | 34.01 | 21.77 | 14.96 | 10.88 | 8.62 | 6.35 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 6.3 | 2.2 |
| 7.6 | ..... | 58.96 | 37.64 | 26.30 | 19.05 | 14.51 | 11.79 | 9.52 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 7.6 | 2.7 |
| 8.9 | ..... | ..... | 59.42 | 41.72 | 30.38 | 23.13 | 18.14 | 14.96 | 10.43 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 8.9 | 3.4 |
| 10.2 | ..... | ..... | 88.89 | 62.13 | 45.35 | 34.92 | 27.21 | 22.22 | 15.42 | 11.34 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 10.1 | 3.6 |
| 11.4 | ..... | ..... | ..... | 87.07 | 65.30 | 48.98 | 39.00 | 31.74 | 21.77 | 16.33 | 12.24 | ..... | ..... | ..... | ..... | ..... | ..... | 11.4 | 4.0 |
| 12.7 | ..... | ..... | ..... | 121.08 | 88.89 | 68.02 | 52.61 | 43.54 | 29.93 | 22.22 | 16.78 | 13.15 | ..... | ..... | ..... | ..... | ..... | 12.7 | 4.5 |
| 14.0 | ..... | ..... | ..... | ..... | 117.91 | 90.70 | 70.75 | 58.05 | 40.36 | 29.48 | 22.67 | 17.69 | 14.61 | ..... | ..... | ..... | ..... | 14.0 | 4.9 |
| 15.2 | ..... | ..... | ..... | ..... | 152.38 | 117.46 | 92.51 | 75.28 | 52.15 | 38.09 | 29.02 | 23.13 | 18.59 | 11.79 | ..... | ..... | ..... | 15.2 | 5.4 |
| 16.5 | ..... | ..... | ..... | ..... | ..... | 150.56 | 117.91 | 93.87 | 67.12 | 49.43 | 37.64 | 29.48 | 23.58 | 14.96 | 10.43 | ..... | ..... | 16.5 | 5.8 |
| 17.8 | ..... | ..... | ..... | ..... | ..... | 186.84 | 146.93 | 119.26 | 83.00 | 60.77 | 46.26 | 36.73 | 29.93 | 19.05 | 13.15 | 9.52 | ..... | 17.8 | 6.3 |
| 19.0 | ..... | ..... | ..... | ..... | ..... | 231.74 | 182.76 | 146.93 | 102.04 | 75.28 | 57.14 | 45.35 | 37.19 | 23.58 | 16.33 | 11.79 | ..... | 19.0 | 6.8 |
| 20.3 | ..... | ..... | ..... | ..... | ..... | ..... | 219.49 | 178.68 | 123.80 | 91.15 | 69.84 | 54.87 | 44.44 | 28.57 | 19.95 | 14.51 | 10.88 | 20.3 | 7.2 |
| 21.6 | ..... | ..... | ..... | ..... | ..... | ..... | 265.30 | 215.44 | 149.65 | 109.29 | 83.44 | 65.76 | 53.06 | 33.56 | 23.58 | 17.23 | 13.15 | 21.6 | 7.7 |
| 22.9 | ..... | ..... | ..... | ..... | ..... | ..... | 313.82 | 253.96 | 176.44 | 129.70 | 99.32 | 78.45 | 63.49 | 40.81 | 28.12 | 20.41 | 15.87 | 22.8 | 8.1 |
| 24.1 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 300.22 | 208.07 | 152.38 | 116.55 | 91.61 | 74.37 | 47.46 | 32.65 | 23.58 | 18.14 | 24.1 | 8.6 |
| 25.4 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 348.74 | 242.47 | 177.77 | 136.05 | 107.48 | 87.07 | 55.78 | 38.55 | 28.57 | 21.77 | 25.4 | 9.0 |
| 26.7 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 280.72 | 205.89 | 157.82 | 124.26 | 100.22 | 63.94 | 44.44 | 32.20 | 24.49 | 26.7 | 9.5 |
| 27.9 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 322.44 | 236.73 | 180.95 | 143.31 | 116.40 | 73.92 | 51.70 | 37.64 | 29.02 | 27.9 | 9.9 |
| 29.2 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 368.69 | 270.74 | 206.80 | 163.71 | 131.97 | 84.00 | 58.50 | 42.47 | 32.20 | 29.2 | 10.4 |
| 30.5 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 418.58 | 307.47 | 235.37 | 185.93 | 150.56 | 96.14 | 67.12 | 48.98 | 37.64 | 30.5 | 10.8 |
| 33.0 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 396.36 | 303.39 | 239.45 | 194.47 | 123.73 | 85.26 | 62.58 | 47.62 | 33.0 | 11.7 |
| 35.6 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 487.51 | 373.06 | 295.23 | 239 00 | 152.83 | 106.12 | 78.00 | 59.86 | 35.6 | 12.7 |
| 38.1 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 453.50 | 363.25 | 295.68 | 186.39 | 131.54 | 96.44 | 73.47 | 38.1 | 13.6 |
| 40.6 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 557.80 | 440.35 | 356.90 | 228.56 | 158.72 | 116.55 | 89.34 | 40.6 | 14.6 |
| 43.1 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 532.86 | 428.40 | 273.11 | 190.47 | 140.43 | 107.03 | 43.1 | 15.4 |
| 45.7 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 628.40 | 508.37 | 325.46 | 225.84 | 165.53 | 126.98 | 45.7 | 16.3 |
| 50.8 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 697.48 | 446.24 | 310.49 | 227.66 | 174.14 | 50.8 | 18.1 |
| 55.9 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 593.63 | 412.68 | 302.94 | 232.49 | 55.9 | 19.9 |
| 61.0 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 774.40 | 535.58 | 393.18 | 301.42 | 61.0 | 21.7 |
| 66.0 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 681.61 | 500.66 | 383.21 | 66.0 | 23.5 |
| 71.1 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 850.77 | 624.92 | 478.44 | 71.1 | 25.4 |
| 76.2 | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 768.68 | 588.19 | 76.2 | 27.2 |

Del lado acá de la linea gruesa las cargas de seguridad de la tabla pág. 1054 no llegarán á flexar las vigas de madera en 1/480 de su luz.

**Hierro y acero.** — Con el hierro colado, y para igual flexión, soportará 11 1/2 veces lo que soporta el pino blanco, el amarillo ó el abeto. El hierro forjado 19 veces. Como se ha dicho para la madera, debe deducirse la misma proporción del peso de la viga. El promedio para el acero es de 29 veces el del pino.

# COLUMNAS DE MADERA

(*Obs. del T.* — Algunas fórmulas y tablas, en que el autor trata esta materia, resultan, al convertirlas, poco prácticas para su uso en sistema métrico. Además, están muy recargadas de teorías. Las hemos reemplazado por 3 tablas muy prácticas tomadas del « Aide-Mémoire des Conducteurs des Ponts et Chaussées, etc. », por J. Eug. Petit, con su autorización.)

## Indicaciones generales.

Las únicas experiencias que se tienen sobre esta materia son experiencias de ruptura.

| Según las experiencias de Rondelet, tenemos : | Encina. | Pino. |
|---|---|---|
| Carga práctica por metro cuadrado..... | $.4\times10^5$ | $.4\times10^5$ |
| Carga de ruptura por metro cuadrado..... | $4.5\times10^5$ | $4.5\times10^5$ |

Para la resistencia al aplastamiento, Rondelet prescribe no dar á las piezas una longitud superior á diez veces el diámetro de la base y tomar como medida una carga de 50 kilos por centímetro cuadrado de la base.

Si el poste en lugar de ser redondo es rectangular, es necesario tomar la raíz cuadrada de la superficie de la base, para obtener el lado medio que debe servir de guía para la longitud. Rondelet agrega que la carga práctica permanente no debe pasar de $^1/_5$ de la carga de aplastamiento.

## Cargas prácticas de los postes de madera.

M. Hodgkinson ha dado también las fórmulas para calcular las cargas prácticas que se pueden hacer soportar á los postes de encina ó de pino.

Estas fórmulas son las siguientes :

1.° Postes cuadrados de encina :

$$P=256.5\times\frac{c^4}{l^2}\left\{\begin{array}{l}c=\text{lado del poste;}\\ l=\text{largo del poste.}\end{array}\right.$$

2.° Postes rectangulares de encina :

$$P=256.5\times\frac{ab^3}{l^2}\left\{\begin{array}{l}a=\text{lado pequeño del rectángulo;}\\ b=\text{lado grande del rectángulo;}\\ l=\text{largo del poste.}\end{array}\right.$$

3.° Postes cuadrados de pino :

$$P=180\times\frac{c^4}{l^2}.$$

4.° Postes rectangulares de pino :

$$P=180\times\frac{ab^3}{l^2}.$$

*Nota.* $a$, $b$, $c$, están expresadas en centímetros y $l$ en decímetros.

El número 180 es un término medio. Se pueden tomar 214 para el pino rojo y 160 para el pino poco resistente.

Estas fórmulas son aplicables desde $\frac{l}{c}=1$ hasta $\frac{l}{c}=35$.

*Problema* 1. ¿Qué peso se puede hacer sostener á un poste de sección cuadrada, de encina fuerte, de .22 m de lado y 2.50 m de altura?

La fórmula es :

$$P=256.5\times\frac{22^4}{25^2}$$
$$=256.5\times375$$
$$=96187\text{ kg.}$$

*Problema* 2. Determinar la escuadría de un poste de sección cuadrada, de encina fuerte, de 5 metros de alto y destinado á sostener una carga de 12,000 kilogramos.

La fórmula en tal caso es:

$$P = 256.5 \times \frac{c^4}{l^2},$$

en la cual:

$P = 2{,}000^k$, $l = 50$ decímetros.

Da:

$$c^4 = \frac{12{,}000^k \times 50^2}{265.5} = 116{,}960;$$ de donde $c = .185$ m.

## Tabla que da las cargas prácticas de los postes de madera de sección cuadrada para alturas de 2 á 10 metros.

(Tomadas del tratado de carpintería del Sr. Oslet.)

### Tabla n.° 1.

| Lado del cuadrado en cm. | Altura de los postes (encina y pino flojos, 40 kg por cm cuadrado). | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 m. | 3 m. | 4 m. | 5 m. | 6 m. | 7 m. | 8 m. | 9 m. | 10 m. |
| | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg |
| 10 | 2300 | 1650 | 1000 | » | » | » | » | » | » |
| 11 | 3000 | 2200 | 1450 | » | » | » | » | » | » |
| 12 | 3900 | 2800 | 1950 | 1250 | » | » | » | » | » |
| 13 | 4900 | 3500 | 2600 | 3001 | » | » | » | » | » |
| 14 | 5000 | 4400 | 3300 | 2400 | » | » | » | » | » |
| 15 | 7100 | 5400 | 4000 | 3100 | 2300 | » | » | » | » |
| 16 | 8300 | 6500 | 5000 | 3800 | 2900 | » | » | » | » |
| 17 | 9700 | 7700 | 5950 | 4600 | 3600 | » | » | » | » |
| 18 | 11000 | 8900 | 7000 | 5500 | 4300 | 3500 | » | » | » |
| 19 | 12700 | 10300 | 8200 | 6400 | 5100 | 4200 | » | » | » |
| 20 | 14300 | 11800 | 9500 | 7600 | 6200 | 5000 | » | » | » |
| 21 | 15850 | 13400 | 11000 | 9000 | 7200 | 6000 | 5000 | | » |
| 22 | 17900 | 15100 | 12500 | 10300 | 8400 | 6900 | 5700 | » | » |
| 23 | » | 16800 | 14100 | 11600 | 9600 | 8000 | 6700 | » | » |
| 24 | » | 18500 | 15800 | 13100 | 10900 | 9100 | 7700 | 6000 | » |
| 25 | » | » | 17800 | 14900 | 12400 | 10400 | 8800 | 7500 | » |
| 26 | » | » | » | 16800 | 14000 | 11800 | 10000 | 8400 | » |
| 27 | » | » | » | 19000 | 16000 | 13300 | 11300 | 9600 | 8500 |
| 28 | » | » | » | » | 17700 | 15000 | 12800 | 10800 | 9400 |
| 29 | » | » | » | » | » | 16700 | 14500 | 12200 | 10700 |
| 30 | » | » | » | » | « | 18700 | 16000 | 14000 | 12000 |

*Nota.* Esta tabla da la carga práctica que pueden sostener, con entera seguridad, os postes de encina y pino flojos, de 2 metros á 10 metros y escuadrías de .10 m á .30 m cuadrados. Se han adoptado 40 kilogramos por centímetro cuadrado para carga práctica.

Esta tabla debe emplearse para construcciones que no necesiten maderas de primera calidad.

## Tabla n.º 2.

| Lado del cuadrado en cm. | Alto de los postes (encina fuerte, 60 kg por cm cuadrado). | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 m. | 3 m. | 4 m. | 5 m. | 6 m. | 7 m. | 8 m. | 9 m. | 10 m. |
| | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. |
| 10 | 3500 | 2400 | 1500 | » | » | » | » | » | » |
| 11 | 4500 | 3200 | 2100 | » | » | » | » | » | » |
| 12 | 5900 | 4200 | 2900 | 1850 | » | » | » | » | » |
| 13 | 7400 | 5300 | 3800 | 2700 | » | » | » | » | » |
| 14 | 8900 | 6600 | 4900 | 3500 | » | » | » | » | » |
| 15 | 10600 | 8000 | 6000 | 4500 | 3400 | » | » | » | » |
| 16 | 12500 | 9700 | 7350 | 5600 | 4400 | » | » | » | » |
| 17 | 14500 | 11500 | 8800 | 6800 | 5400 | » | » | » | » |
| 18 | 16500 | 13300 | 10500 | 8100 | 6400 | 5000 | » | » | » |
| 19 | 19000 | 15400 | 12200 | 9600 | 7700 | 6200 | » | » | » |
| 20 | 21500 | 17600 | 14300 | 11400 | 9200 | 7500 | » | » | » |
| 21 | 23800 | 20000 | 16500 | 13400 | 10800 | 8800 | 7350 | » | » |
| 22 | 26900 | 22600 | 18700 | 15400 | 12500 | 10300 | 8500 | » | » |
| 23 | » | 25200 | 21100 | 17500 | 14500 | 12000 | 10000 | » | » |
| 24 | » | 28000 | 23800 | 19700 | 16400 | 13700 | 11500 | 9600 | » |
| 25 | » | » | 26700 | 22300 | 18500 | 15600 | 13200 | 11200 | » |
| 26 | » | » | » | 25200 | 21000 | 17700 | 15000 | 12700 | » |
| 27 | » | » | » | 28000 | 23900 | 20000 | 17000 | 14500 | 12700 |
| 28 | » | » | » | » | 26600 | 22500 | 19300 | 16300 | 14200 |
| 29 | » | » | » | » | » | 25000 | 21600 | 18400 | 16000 |
| 30 | » | » | » | » | » | 28000 | 24000 | 21000 | 18000 |

*Nota.* Esta tabla deberá servir para maderas de encina de muy buena calidad pudiendo resistir 60 kilogramos por centímetro cuadrado.

**Tabla n.º 3.**

| Lado del cuadrado en cm. | Altura de los postes (pino fuerte, 50 kg por cm cuadrado). | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 m. | 3 m. | 4 m. | 5 m. | 6 m. | 7 m. | 8 m. | 9 m. | 10 m. |
| | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. |
| 10 | 2900 | 2100 | 1300 | » | » | » | » | » | » |
| 11 | 3700 | 2700 | 1800 | » | » | » | » | » | » |
| 12 | 4900 | 3500 | 2500 | » | » | » | » | » | » |
| 13 | 6200 | 4400 | 3200 | 2300 | » | » | » | » | » |
| 14 | 7400 | 5550 | 4100 | 3000 | » | » | » | » | » |
| 15 | 8800 | 6700 | 5000 | 3750 | 2900 | » | » | » | » |
| 16 | 10400 | 8200 | 6200 | 4700 | 3700 | » | » | » | » |
| 17 | 12100 | 9600 | 7400 | 5700 | 4500 | » | » | » | » |
| 18 | 13750 | 11150 | 8800 | 6800 | 5400 | 4200 | » | » | » |
| 19 | 15850 | 12900 | 10200 | 8000 | 6400 | 5200 | » | » | » |
| 20 | 17900 | 14700 | 11900 | 9500 | 7700 | 6300 | » | » | » |
| 21 | 19800 | 16200 | 13800 | 11200 | 9500 | 7400 | 6200 | » | » |
| 22 | 22400 | 18900 | 15600 | 12800 | 10500 | 8600 | 7100 | » | » |
| 23 | » | 21000 | 17500 | 14600 | 12000 | 10000 | 8300 | » | » |
| 24 | » | 24000 | 19800 | 16400 | 13700 | 11400 | 9500 | 8000 | » |
| 25 | » | » | 22200 | 18600 | 15500 | 13000 | 11000 | 9300 | » |
| 26 | » | » | » | 21000 | 17500 | 14800 | 12500 | 10600 | » |
| 27 | » | » | » | » | 19900 | 16700 | 14200 | 12100 | 10600 |
| 28 | » | » | » | » | 22200 | 18800 | 16100 | 13600 | 11700 |
| 29 | » | » | » | » | » | 20850 | 18000 | 15400 | 13300 |
| 30 | » | » | » | » | » | 23300 | 20000 | 17500 | 15000 |

*Nota.* Los elementos de esta tabla deberán emplearse para maderas de pino d muy buena calidad pudiendo resistir 50 kilogramos por centímetro cuadrado

## Aplicación de las tablas anteriores.

*Problema* 1. Hallar las dimensiones de un poste de madera de encina floja que pueda sostener una carga de 5,000 kg, teniendo una altura de 4 metros.

Se busca en la columna 4 *metros*, de la tabla n.º 1, el número 5,000 kg y hallamos que esta cifra corresponde á una escuadría de .16/.16.

*Problema* 2. ¿Cuál es el peso que puede soportar un poste cuadrado de encina fuerte de .18/.18, teniendo una longitud de 6 metros?

Se busca en la primera columna de la tabla n.º 2 el número 18 y se sigue la línea horizontal hasta la columna 6 m. Hallamos que el peso que corresponde es de 6,400 kg.

*Primera observación.* Si el poste es rectangular, se busca primero la carga de un poste cuadrado cuya escuadría será el lado más pequeño de la sección, y en seguida se multiplica el resultado por la relación de las dos dimensiones de la sección transversal. Ejemplo. Supongamos un poste de encina floja de 4 metros de altura, cuya escuadría es 20/25.

Se busca la carga de un poste cuadrado de 20/20 y de 4 metros y hallamos 9,500 kg. Se multiplica en seguida 9,500 kg por la proporción 25 : 20 = 1.25.

$$9{,}500 \times 1.25 = 11{,}875 \text{ kg.}$$

Este resultado nos da la carga que puede sostener un poste rectangularde 20/25 teniendo 4 metros de altura.

*Segunda observación.* Si tenemos un poste superior á .30/.30, cuya escuadría no se encuentra en las tablas, se procede entonces de la manera siguiente:

Supongamos un poste de .36/.36 y de 8 metros de altura, de encina floja. Si reducimos á la mitad las dimensiones del poste (altura y lado de su sección transversal), la carga total queda reducida á la cuarta parte. Se trata pues, en este caso, de buscar la resistencia de un poste cuadrado de .18 de lado y de 4 metros de altura, lo que, según la tabla n.º 1, da 7,000 kg.

Por consiguiente, la carga correspondiente al poste de 8 metros de altura y de $.36 \times .36$ es el cuádrupie de la anterior. Así pues: $7{,}000 \times 4 = 28{,}000$ kg.

## REQUISITOS PARA EL HIERRO Y EL ACERO

(Véase también en Especificaciones para puentes.)

**Resumen de especificaciones** adoptadas, con sujeción á votación por letras en la 4.ª Reunión Anual de la **Sección Americana** de la **Asociación internacional para ensayo de materiales**, el 29 de junio de 1901. Adoptadas por votación por letras, en agosto de 1901, excepto para el hierro forjado, respecto del cual se difirió la decisión.

### Proceso de manufactura.

Hierro forjado; pudelado, ó laminado de pilas de recortes de hierro forjado, solo ó agregándoles hierro de primera laminación. Fundiciones de acero. Procedimiento Martín Siemens, de crisol ó de Bessemer.

Forjaduras de acero. Procedimiento Martín Siemens, de crisol ó de Bessemer.

Carriles de acero. Bessemer ó Martín Siemens.

Los lingotes se tendrán verticales en hornos de pozo. No se usarán lingotes sangrados. Se descartará suficiente material de los topes de los lingotes para asegurar la solidez de los carriles.

Barras de unión de acero. Bessemer ó Martín Siemens.

Planchas de caldera y acero de remache. Martín Siemens.

Acero de construcción para puentes y buques. Martín Siemens.

Acero de construcción para edificios. Martín Siemens ó Bessemer.

### Piezas de prueba.

Para planchas chatas se usará la muestra expuesta en la fig. J.

*La pieza ha de ser del mismo espesor que la plancha.*

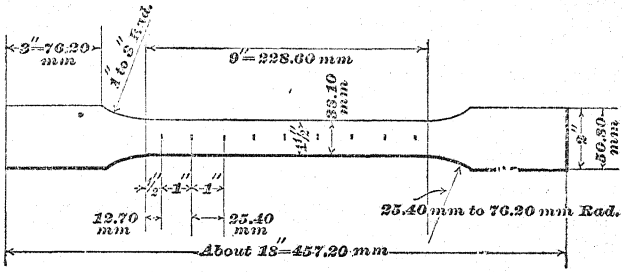


**Fig. J.**

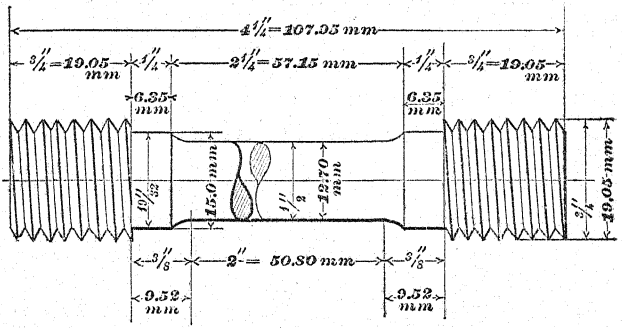


**Fig. K.**

Para grandes baquetas, muestra de prueba señalada en la fig. K. El centro de la muestra estará á media distancia entre el centro y el exterior de la baqueta.

Siempre que sea posible, el hierro se probará en tamaño entero, como se laminó.

Las muestras de prueba se cortarán de la barra como se hayan laminado.

## Pruebas.

**Pruebas de tarja.** La muestra será ligera y uniformemente tarjada de un lado y doblada hacia atrás en este punto hasta un ángulo de 180° dándole una serie de golpes ligeros.

**Pruebas de flexión al fuego.** Las piezas de prueba serán calentadas á un rojo brillante y dobladas por presión y por una sucesión de golpes ligeros, sin martillar en la cabeza.

**Pruebas de flexión al frío.** Las muestras deberán doblarse por presión ó por sucesivos golpes ligeros.

**El punto donde empieza á ceder** (*yield point*) será determinado por cuidadosa observación de la caída del balancín ó por el escantillón de la máquina de pruebas.

**Pruebas de precisión.** Esta máquina para probar carriles tendrá una maza de 2,000 libras (906 kg), cuya superficie golpeante tendrá un radio de no más de 5 pulgadas (12.7 cm); y el carril de prueba, de no más de 6 pies (1.83 m) de largo, se colocará con la cabeza hacia arriba sobre sólidos apoyos distantes 3 pies (.91 m). La masa del yunque, que debe pesar por lo menos 20,000 lbs (9,060 kg), y los apoyos formarán parte del yunque ó estarán firmemente ligados á él. La caída de la masa de 4.57 m para carril de 22.5 kg por m á 5.79 m para carril de 42.5 kg ó más. De cada quinta hornada se escoge una pieza de prueba.

**Pruebas de homogeneidad para acero de las cajas de fuego.** Una parte de la muestra rota en la prueba de tensión, se tarja ó se estría á $^1/_{16}$ de pulg (1½ mm) de profundidad en tres lugares distantes como 2 pulgs (5 cm) y en lados opuestos. Entonces se sujeta en un torno y se rompe con ligeros golpes de martillo, doblándola sucesivamente por cada estría. La muestra no debe presentar ni una sola rasgadura ó cavidad de más de $^1/_4$ de pulg (6 mm) de largo.

## Notas al cuadro que sigue.

(*a*) Enteramente doblado.

(*b*) La muestra ha de doblarse sobre una barra de diámetro igual á su propio diámetro ó espesor.

(*c*) La muestra ha de doblarse sobre una barra de dos veces su diámetro.

(*d*) Alargamiento mín por ciento, en secciones menores de .654 lbs, por pie lineal (.97 kg por metro), clase A, 19; B, 15; C, 12.

(*e*) Prueba de tarja. Máx por ciento de superficie granular, clase A, 10; B, 10; C, 15.

(*f*) Prueba de flexión al fuego. La barra ha de doblarse sin rasgarse en la parte exterior del doblez. Cada clase debe doblarse: 180° en las clases A y B y rigurosamente á 90° en C. La clase A al calor amarillo y súbitamente apagado en agua, entre 80 y 90° F. (26.7 y 32.2 C) debe doblarse á 180°. También, calentado al rojo brillante rasgado en el extremo y doblada cada parte hacia atrás á 180°. Punzado para taladrar lo menos .9 del diámetro ó ancho de la barra.

(*g*) Fósforo, piezas para prueba física, .05 para cada clase.

(*h*) Azufre, piezas para prueba física, .05 para cada clase.

(*i*) Flexión. Muestra una pulgada × ½ pulg (25 × 13 mm) ha de doblarse en frío sobre una barra de 1 pulgada de diámetro (25 mm) sin fractura en el exterior del doblez.

(*j*) Flexión. Muestra 1 pulgada × ½ pulgada (25 × 13 mm) para doblarse en frío, sin fractura en el exterior del doblez sobre una barra de ½ pulgada (12½ mm).

(*k*) Lo mismo, alrededor de un diámetro de 1½ pulgs (38 mm).

(*l*) Lo mismo, alrededor de un diámetro de 1½ pulgs (38 mm), si no tiene menos de 20 pulgs (51 cm) de diámetro, alrededor de un diámetro de 1 pulg (25 mm), si tiene menos de 20 pulgs (51 cm) de diám.

(*m*) Lo mismo, sobre un diámetro de 1 pulgada (25 mm).

(*n*) Lo mismo, sobre un diámetro de ½ pulg (12½ mm).

(*o*) Lo mismo, sobre un diámetro de 1 pulg (25 mm).

(*p*) Dedúzcase 1 por ciento por cada $^1/_4$ pulg (3 mm) en espesores mayores de

$^{3}/_{4}$ de pulg (19 mm), y 2½ por ciento por cada $^{1}/_{16}$ de pulg (1½ mm) en menos de $^{5}/_{16}$ de pulg (8 mm).

(*q*) Flexión. Las barras para hacer remaches han de probarse enteras, como salen del laminador. Las muestras de las planchas serán de 1½ pulg (38 mm) de ancho. Las planchas de no más de $^{3}/_{4}$ de pulg (19 mm) de espesor, tendrán el mismo ancho que la plancha y la muestra tendrá, cuando sea posible, la superficie laminada en ambas caras. Para planchas más gruesas de $^{3}/_{4}$ de pulg (19 mm), la muestra podrá ser de ½ pulg (12½ mm). Serán sometidas á las pruebas de flexión en frío y apagada. Para apagar, el material ha de calentarse al rojo cerezo claro (visto en la obscuridad) y apagarse en agua á temperatura, entre 80° y 90° Fahr (26.7 y 32.2 C). Las muestras deberán doblarse sin fractura en el exterior. La flexión puede hacerse por presión ó por golpes.

(*r*) Para pasadores, el alargamiento será de 5 por ciento menos. El centro de la muestra de prueba á 1 pulg (25 mm) de la superficie.

(*s*) Las barras de ojo serán de acero mediano. Pruebas de tamaño entero darán 12½ por ciento de alargamiento en 15 pies (4.57 m). Tensión mín, 55,000 lbs por pulg cuad (3,850 kg por cm cuad). Por lo menos $^{2}/_{3}$ de las barras de ojo probadas se romperán en el cuerpo de la pieza.

(*t*) Lo mismo que (*q*), pero omitiendo la prueba de la apagada.

(*u*) Véase Prueba de homogeneidad, en el texto, arriba.

Véanse notas, p. 1194 y 1195.

**Requisitos para**

| | Metal. | Percentaje permitido de | | | | |
|---|---|---|---|---|---|---|
| | | Carbón. | Fósforo. | Azufre | Manganeso. | Níquel. |
| | Hierro Forjado. | Máx. | Máx. | Máx. | Máx. Mín. | Máx. Mín. |
| | Clase A | ..... | ..... | ..... | ..... | ..... |
| | Clase B | ..... | ..... | ..... | ..... | ..... |
| | Clase C | ..... | ..... | ..... | ..... | ..... |
| 2. | Fundiciones de Acero | | | | | |
| | Duro | 0.40 | 0.08 g | 0.05 h | ..... | ..... |
| | Medio | 0.40 | 0.08 g | 0.05 h | ..... | ..... |
| | Dulce | 0.40 | 0.08 g | 0.05 h | ..... | ..... |
| | Forjaduras de Acero. | | | | | |
| | Carbón blando ó bajo | ..... | 0.10 | 0.10 | ..... | ..... |
| | Carbón, no templado | ..... | 0.06 | 0.06 | ..... | ..... |
| | Carbón, templado | ..... | 0.04 | 0.04 | ..... | ..... |
| | Carbón, templado al aceite | ..... | 0.04 | 0.05 | ..... | ..... |
| | Níquel, templado | ..... | ..... | ..... | ..... | 1.00 3.00 |
| | Níquel, templado al aceite | ..... | ..... | ..... | ..... | 4.00 5.00 |
| | Carriles de Acero. | | | | | |
| | Kilogr. por metro lineal. | Máx. Mín. | | | | |
| | 24,803 á 29,267 | 0.45 0.35 | 0.10 | 0.20 | 1.00 0.70 | ..... |
| 4. | 29,763 » 34,228 | 0.48 0.38 | 0.10 | 0.20 | 1.00 0.70 | ..... |
| | 34,724 » 39,188 | 0.50 0.40 | 0.10 | 0.20 | 1.05 0.75 | ..... |
| | 39,684 » 44,149 | 0.53 0.43 | 0.10 | 0.20 | 1.10 0.80 | ..... |
| | 44,645 » 49,606 | 0.55 0.45 | 0.10 | 0.20 | 1.10 0.80 | ..... |
| 5. | Barras de Acero para Bridas | Máx. 0.15 | 0.10 | ..... | 0.60 0.30 | ..... |
| | Planchas Martin Siemens para calderas y Acero de remaches. | | básico ácido. | | | |
| | Acero de reborde, ó caldera | ..... | 0.06 0.04 | 0.04 | 0.60 0.30 | ..... |
| 6. | Acero de caja de fuego | ..... | 0.04 0.03 | 0.04 | 0.50 0.30 | ..... |
| | Acero extra dulce para remaches de caldera | ..... | 0.04 0.04 | 0.04 | 0.50 0.30 | ..... |
| | Acero para puentes y buques. | | básico ácido. | | | |
| | Acero de remaches | ..... | 0.08 0.06 | 0.06 | ..... | ..... |
| 7. | Acero dulce | ..... | 0.08 0.06 | 0.06 | ..... | ..... |
| | Acero mediano | ..... | 0.08 0.06 | 0.06 | ..... | ..... |
| | Acero para edificios. | | Máx. | | | |
| | Acero de remaches | ..... | 0.10 | ..... | ..... | ..... |
| 8. | Acero mediano | ..... | 0.10 | ..... | ..... | ..... |

**Hierro y Acero.** Véanse notas, págs. 1194 y 1195.

| Pruebas de tensión. | | | | | Pruebas de flexión en frío. | | Observaciones sobre otras pruebas. |
|---|---|---|---|---|---|---|---|
| Kg por centímetro cuadrado. | | Límite de elasticidad y punto cedente *. kg por centímetro cuadrado. | Alargamiento Percentaje en 8 pulgs (20,3 cm). | Contracción del área. Percentaje. | Angulo del doblez. | Como dobla. | |
| Máx. | Mín. | Punto cedente *. Mín. | Mín. | Mín. | ° | | |
| ..... | 3500 | 1750 | 25 d | ..... | 180 | a | e, f |
| ..... | 3360 | 1750 | 23 d | ..... | 180 | b | e, f |
| ..... | 3360 | 1750 | 20 d | ..... | 180 | c | e, f |
| ..... | 5950 | 2677 | | 20 | ..... | i | |
| ..... | 4900 | 2205 | | 25 | 90 | i | |
| ..... | 4200 | 1890 | | 30 | 120 | i | |
| | Término medio. | Término medi | Término medio. | Término medio. | | | |
| ..... | 4060 | 2030 | 28 | 35 | 180 | j | |
| ..... | 5250 | 2625 | 18 | 30 | 180 | k | |
| ..... | 5250 | 2625 | 23 | 32.5 | 180 | l | |
| | | Límite de elasticidad. | | | | | |
| | 5950 | ..... | 21.5 | 42.5 | 180 | m | |
| ..... | 5600 | 3325 | 24.5 | 42.5 | 180 | n | |
| ..... | 6300 | 4200 | 22.5 | 47.5 | 180 | o | |

Vease Prueba de percusión, en el texto, arriba.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 4480 | Mín. 3780 | Mín. 2240 | Mín. 25 | ..... | 180 | a | |
| | | Máx y Mín. | | | | | |
| 4550 | 3850 | | 25 p | ..... | 180 | a, q | |
| 4340 | 3640 | 1/2 fuerza | 26 p | ..... | 180 | a, q | u |
| 3850 | 3150 | de tensión. | 28 p | ..... | 180 | a, q | |
| | | | r. s. | | | | |
| 4200 | 3500 | 1/2 fuerza | 26 p | ..... | 180 | a, t | |
| 4340 | 3640 | de tensión. | 25 p | ..... | 180 | a, t | |
| 4900 | 4200 | | 22 p | ..... | 180 | b, t | |
| | | | r. | | | | |
| 4200 | 3500 | 1/2 fuerza | 26 p | ..... | 180 | a, t | |
| 4900 | 4200 | de tensión. | 22 p | ..... | 180 | b, t | |

* *N. del T.* — Véase *N. del T.* (§) al pie de la pág. 482 *b*.

**Resumen de especificaciones que sirven de norma á los manufactureros del acero**, adoptadas por la Asociación Americana de Manufactureros de Acero el 9 de agosto de 1895 y revisadas el 6 de febrero de 1901. En vigencia el 31 de julio de 1908.

Las **palabras entre paréntesis** en tipo grueso (como **puede ser Martín Siemens ó Bessemer**) se aplican sólo á acero de construcción. Las palabras entre paréntesis *en bastardilla* como (*cuatro*), (*dos*) se sustituirán con las palabras precedentes (**dos**), (*uno*), para acero especial Martín Siemens de plancha y remaches. De otro modo, y sólo cuando se diga especialmente, las indicacions se aplicarán á ambos aceros.

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(Pueden ser **Martín Siemens ó Besseimer.**)

Las **pruebas é inspecciones** se harán en el lugar de la manufactura antes del embarque.

### Piezas de prueba.

**Piezas de prueba modelo para planchas cizalladas.** (Véase fig. J, pág. 1193.)

**Piezas de prueba de otro material**, ya como se dice arriba, ya acepilladas ó torneadas paralelas á la longitud de la pieza.

En todo caso, los lados opuestos de la pieza de prueba han de ser laminados, si fuese posible.

« **Barretas y barras para remaches** deben probarse en su longitud completa como salen del laminador. »

« **Dos** (*cuatro*) **piezas de prueba** se tomarán de cada fundición (ú hornada) de material acabado, una (*dos*) para tensión y una (*dos*) para flexión; pero en caso de que en cualquier prueba se encuentren defectos, ó que la pieza de prueba de tensión se rompa fuera del tercio medio de su longitud, podrá desecharse y sustituirse por otra. »

« Cuando un material haya de ser **templado ó tratado de otro modo** antes de emplearse, su muestra será tratada del mismo modo antes de probarla. » Es decir, el material « será probado en la condición en que salga de los cilindros ».

« En toda pieza de acero será **estampado** el número de fundición (ó de hornada), y el acero para pasadores llevará dicho número estampado en los extremos. El acero para remaches (y las piezas pequeñas para planchas de pasadores y refuerzos pueden embarcarse en haces bien atados con alambre, con el número de la hornada ó de la fundición en una placa atada de metal. »

### Máximum de fósforo y azufre por ciento.

| Be construcción | Fósforo. | Azufre. |
|---|---|---|
| para edificios, tinglados, puentes de camino real, etc..... | .10 | ... |
| para puentes de ferrocarril..... | .08 | ... |
| Especial | | |
| acero de calderas..... | .06 | .04 |
| acero extradulce y para cajas de fuego..... | .04 | .04 |

### Propiedades mecánicas.

| | Acero de construcción. | | | Acero especial. | | |
|---|---|---|---|---|---|---|
| | Remache. | Puentes de ferrocarril. | Mediano. | Extradulce *. | Caja de fuego. | Para caldera. |
| Prueba de flexión § 180° sin fractura en el exterior del doblez..... | 0 | t | t | 0** | 0** | 0** |
| Máxima resistencia á la tensión en miles de kgs por cm. cuad..... | 3.36 á 4.06 | 3.85 á 4.55 | 4.2 á 4.9 | 3.15 á 3.85 | 3.64 á 4.34 | 3.85 á 4.55 |
| Alargamiento (para deducciones, etc., véase abajo)..... | 98,434 ÷ Máx resistencia. | | | | | |
| Límite de elasticidad..... | No menos de la mitad del máximo de resistencia. | | | | | |

* Los remaches de caldera han de ser de acero « extradulce ».

** Flexión en frío ó apagada.

= espesor de la pieza de prueba.

**El alargamiento**, en barras de $^5/_8$ pulg (16 mm), ó menos, medidas en una longitud = 8 × diám de sección probada; en otros casos, medidas en 8 pulgs (20.3 cm). De los alargamientos especificados, han de hacerse las siguientes **deducciones: *por cada* aumento** de espesor de $^1/_8$ pulg (3 mm) **en más de $^3/_4$ pulg** (19 mm) 1%; (mín 20% para material de barras de ojo, 18% para otra material de construcción); para **cada disminución** de $^1/_{16}$ pulg (1½ mm) en menos de $^5/_{16}$ pulg (8 mm), 2.5%; para pasadores, 5%; el alargamiento debe medirse « en una pieza de prueba, cuyo centro esté á una pulg (25 mm) de la superficie de la barra ».

**Percentaje de variación permitida en peso ó sección transversal de planchas cizalladas.**

**Cuando se ordenan por peso.**

| Peso por m cad. | | < 61 kg | | < 61 kg | |
|---|---|---|---|---|---|
| | cm. | cm. | cm. | cm. | cm. |
| Ancho en cm... | < 190.5 | 190.5 á 254 | < 256 | < 254 | < 254 |
| Variación %.... | ± 2.5 | + 5, — 3 | + 10, — 3 | ± 2.5 | ± 5 |

**Cuando se ordenan por escantillón**, percentaje de exceso en planchas rectangulares. (Planchas con > .01 pulgada (.25 mm) en menos se consideran dentro del escantillón.) Al acero laminado se le supone una densidad de 7.82.

| Espesor en milímetros. | Ancho en cm. <127 | á 177.8 | > 177.8 | < 190.5 á 254 | 254 á 292.1 | 292.1 |
|---|---|---|---|---|---|---|
| 3.175 á 3.9687 | 25.40 | 38.10 | 50.8 | | | |
| 3.9687 á 4.762 | 21.59 | 31.75 | 43.18 | | | |
| 4.762 á 6.350 | 17.78 | 25.40 | 38.10 | | | |
| 6.350 | ..... | | | 25.4 | 35.56 | 45.72 |
| 7.937 | ..... | | | 20.32 | 30.48 | 40.64 |
| 9.525 | ..... | | | 17.78 | 25.40 | 33.02 | 43.18 |
| 11.112 | ..... | | | 15.24 | 20.32 | 25.40 | 33.02 |
| 12.700 | ..... | | | 12.70 | 17.78 | 22.86 | 30.48 |
| 14.287 | ..... | | | 11.43 | 16.51 | 21.59 | 27.94 |
| 15.875 | ..... | | | 10.16 | 15.24 | 20.32 | 25.40 |
| > 15.875 | ..... | | | 8.89 | 12.70 | 16.51 | 22.86 |

En otros casos, una variación > 2.5% en peso ó sección transversal será motivo suficiente de rechazo.

**El frío intenso debilita el hierro.**

La creencia (que tuvo origen en Styff, de Suecia) va ganando terreno. Se cree que el hierro y el acero no se hacen *más quebradizos por el frío intenso*, sino que, el gran número de rupturas de carriles, ruedas, ejes, etc., en el invierno, se debe al mayor efecto de los golpes por causa de la helada y falta de elasticidad de la tierra en aquella estación del año. Pero los experimentos de Sandberg demuestran terminantemente que, aunque estos metales pueden tal vez resistir en invierno como en verano la misma fuerza *gradualmente* aplicada, su resistencia á los golpes ó *fuerza súbita*, no es más de ½ ó ⅓ en el frío riguroso; porque éste los hace menos flexibles y menos dilatables. Es probable que no se atienda á esta circunstancia con todo el interés que merece al calcular puentes de hierro, etc.

Algunos experimentos con buen hierro forjado han demostrado que aun á 23° F (—5° C) ó tan sólo 9° F (5° C) debajo del punto de congelación, se produce una pérdida de resistencia de 2½ á 4 por ciento.

**Hierro fundido maleable.** Los experimentos de D. L. Barnes, de Chicago, en gran número de muestras de una misma fábrica de hierro fundido « maleable », dieron en la mayor parte de los casos tensiones que variaron de 24,000 á 32,000 lbs por pulg cuadrada (1,680 á 2,240 kg por cm cuad) con un promedio de 28,000 (1,960). Las cifras más altas fueron obtenidas generalmente con las barras más pequeñas (como de 3 × ¼ pulg (7½ × .6 cm), y las más bajas con las barras más gruesas (como de 3 × 1 pulg (7½ × 2.5 cm). Piezas acepilladas en los cuatro lados dieron sólo un promedio de 24,000 lbs por pulg cuad (1,680 kg por cm cuad). Esto puede

explicar la diferencia en favor de las secciones más pequeñas, en las cuales la cubierta original forma una gran parte de la sección transversal.

## HIERRO FUNDIDO

| | |
|---|---|
| Resistencia á la tensión | 980 á 1,400 kg por cm cuad *. |
| Resistencia á la compresión (promedio como 7,000) | 6,300 á 9,100 — — |
| Resistencia transversal, barra 1 pulg cuad (6.45 cm cuad), 1 pie (.305 m) de luz, carga central (1,134 kg) 2,500 lbs. Flexión, mínima, .15 pulgada (3.8 mm). | |
| Límite de elasticidad, unos | 420 kg por cm cuad. |
| Módulo de elasticidad | 1,050,000 — — |

### Especificaciones.

| | |
|---|---|
| Resistencia á la tensión. | |
| Oficina del Agua, Filadelfia | 1,120 á 1,400 kg por cm cuad. |
| Departamento del Agua, St. Louis, Mo | 1,260 — — |

Resistencia transversal.

Oficina del Agua, Filadelfia.

(6.45 cm cuad). 1 pulg cuad. 56 pulg (1.42 m) de luz, carga central 500 lbs (226.5 kg).

1 pulg cuad (6.45 cm cuad) 36 pulgs (.915 m) de luz, carga central (340 kg) 750 lbs. Flexión, mínima (10 á 15 mm) .4 á .6 pulg.

Departamento del Agua, St. Luis, Mo.

3 pulgs × ½ pulg (7½ × 1.3 cm) (puesta de plan) 18 pulgs (45.7 cm) de luz, carga central 1,000 á 1,250 lbs (453 á 566 kg). Flexión mínima .3 á $^5/_8$ pulg (7.6 á 15 mm).

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* Hierros superiores en alto grado, pueden llegar hasta 2,100 y 2,800 kg por cm cuad.

(*N. del T.* — Hemos reemplazado las tablas del autor por otras en medidas métricas.)

## PESO DE LOS TUBOS

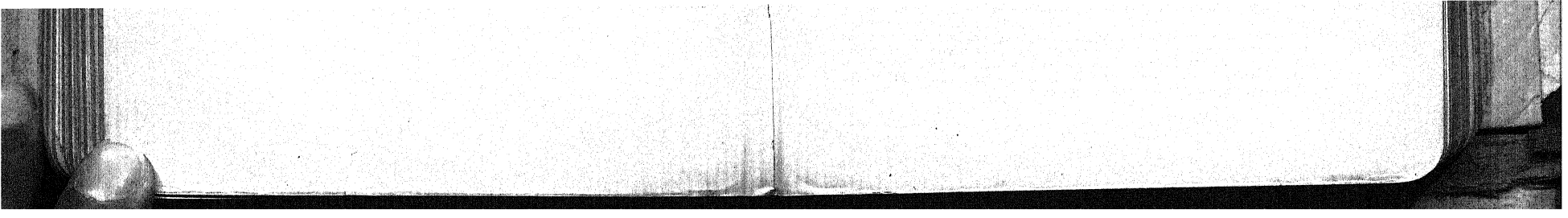
Peso en kg por 1 metro de longitud.

### Tubos de hierro forjado.

| Diámetro interior mm. | Espesor de las paredes en milímetros. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 10 | .59 | .95 | 1.37 | 1.82 | 2.34 | 2.90 | 3.50 | 4.16 | 4.87 |
| 15 | .83 | 1.32 | 1.85 | 2.44 | 3.07 | 3.75 | 4.48 | 5.26 | 6.09 |
| 20 | 1.07 | 1.68 | 2.34 | 3.05 | 3.80 | 4.60 | 5.45 | 6.35 | 7.30 |
| 25 | 1.32 | 2.05 | 2.83 | 3.65 | 4.53 | 5.45 | 6.43 | 7.45 | 8.52 |
| 30 | 1.56 | 2.41 | 3.31 | 4.26 | 5.26 | 6.30 | 7.40 | 8.55 | 9.74 |
| 40 | 2.05 | 3.14 | 4.29 | 5.48 | 6.72 | 8.01 | 9.35 | 10.73 | 12.18 |
| 50 | 2.53 | 3.87 | 5.26 | 6.70 | 8.18 | 9.72 | 11.30 | 12.93 | 14.61 |
| 60 | 3.02 | 4.59 | 6.23 | 7.92 | 9.64 | 11.42 | 13.25 | 15.12 | 17.05 |
| 70 | 3.50 | 5.33 | 7.20 | 9.13 | 11.10 | 13.12 | 14.20 | 17.31 | 19.48 |
| 80 | 4.00 | 6.06 | 8.18 | 10.35 | 12.57 | 14.83 | 17.14 | 19.50 | 21.92 |

### Tubos de hierro fundido.

| Diámetro interior mm. | Espesor de las paredes en milímetros. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
| 25 | 3.417 | 7.975 | 13.67 | 20.50 | 28.47 | 37.58 | 47.83 | 59.22 |
| 30 | 3.987 | 9.113 | 15.38 | 22.78 | 31.32 | 41.00 | 51.81 | 63.77 |
| 35 | 4.557 | 10.15 | 17.08 | 23.61 | 34.17 | 44.41 | 55.80 | 68.32 |
| 40 | 5.126 | 11.39 | 18.79 | 27.33 | 37.01 | 47.83 | 59.78 | 72.88 |
| 45 | 5.695 | 12.53 | 20.50 | 29.61 | 38.86 | 51.24 | 63.77 | 77.43 |
| 50 | 6.254 | 13.67 | 22.21 | 31.89 | 42.70 | 54.66 | 67.75 | 81.98 |
| 60 | 7.402 | 15.94 | 25.62 | 36.44 | 48.39 | 61.49 | 75.74 | 91.12 |
| 70 | 8.540 | 18.22 | 29.04 | 40.99 | 54.10 | 68.34 | 83.71 | 100,2 |
| 80 | 9.679 | 20.50 | 32.46 | 45.56 | 59.79 | 75,16 | 91.68 | 109.3 |
| 90 | 10.82 | 22.78 | 35.88 | 50.11 | 65.49 | 82.00 | 99.65 | 118.4 |
| 100 | 11.96 | 25.06 | 39.29 | 54.66 | 71.17 | 88.83 | 107.6 | 127.5 |
| 125 | 14.80 | 30.75 | 47.83 | 66.04 | 85.40 | 105.9 | 127.5 | 150.3 |
| 150 | 17.65 | 36.45 | 56.38 | 77.44 | 99.65 | 123.0 | 140.5 | 173.1 |
| 175 | 20.50 | 42.14 | 64.91 | 88.83 | 113.8 | 140.0 | 167.4 | 195.9 |
| 200 | 23.34 | 47.82 | 73.45 | 100.2 | 128.1 | 157.1 | 187.3 | 218.7 |
| 225 | 26.19 | 53.53 | 82.00 | 111.6 | 142.3 | 174.2 | 207.3 | 241.4 |
| 250 | 29.04 | 59.22 | 90.53 | 122.8 | 156.6 | 191.3 | 227.2 | 264.2 |
| 275 | 31.89 | 64.92 | 99.8 | 134.3 | 170.8 | 208.4 | 247.2 | 287.0 |
| 300 | 34.73 | 70.61 | 107.6 | 145.7 | 185.0 | 225.5 | 267.0 | 309.7 |
| 325 | 37.58 | 76.30 | 116.1 | 157.2 | 199.3 | 242.5 | 287.0 | 332.6 |
| 350 | 40.42 | 82.00 | 124.7 | 168.5 | 213.5 | 259.7 | 307.0 | 355.3 |
| 375 | 43.28 | 87.69 | 132.2 | 179.9 | 227.8 | 276.6 | 326.8 | 378.2 |
| 400 | 46.11 | 93.38 | 141.8 | 191.3 | 241.9 | 293.8 | 346.4 | 400.8 |



## Peso en kilogramos de hierros redondos y cuadrados.

Densidad 7.788.

| Diám. ó lado. | Cuadrado. | Redondo. | Diám. ó lado. | Cuadrado. | Redondo. | Diám. ó lado. | Cuadrado. | Redondo. |
|---|---|---|---|---|---|---|---|---|
| mm. | | | mm. | | | mm. | | |
| 2 | .031 | .024 | 46 | 16.479 | 12.948 | 90 | 63.088 | 49.563 |
| 3 | .070 | .055 | 47 | 17.204 | 13.517 | 91 | 64.486 | 50.271 |
| 4 | .125 | .098 | 48 | 17.944 | 14.098 | 92 | 65.918 | 51.791 |
| 5 | .195 | .158 | 49 | 18.699 | 14.692 | 93 | 67.358 | 52.923 |
| 6 | .280 | .220 | 50 | 19.470 | 15.296 | 94 | 68.815 | 54.607 |
| 7 | .382 | .300 | 51 | 20.257 | 15.916 | 95 | 70.287 | 55.224 |
| 8 | .498 | .392 | 52 | 21.059 | 16.546 | 96 | 71.774 | 56.393 |
| 9 | .631 | .496 | 53 | 21.876 | 17.188 | 97 | 73.262 | 57.574 |
| 10 | .779 | 612 | 54 | 22.710 | 17.843 | 98 | 74.776 | 58.644 |
| 11 | .942 | .740 | 55 | 23.559 | 18.510 | 99 | 76.330 | 59.970 |
| 12 | 1.121 | .881 | 56 | 24.423 | 19.189 | 100 | 77.880 | 61.190 |
| 13 | 1.316 | 1.034 | 57 | 25.303 | 19.881 | 102 | 79.59 | 62.48 |
| 14 | 1.526 | 1.199 | 58 | 26.199 | 20.584 | 104 | 82.74 | 64.75 |
| 15 | 1.752 | 1.377 | 59 | 27.110 | 21.300 | 106 | 85.96 | 67.48 |
| 16 | 1.994 | 1.566 | 60 | 28.036 | 22.028 | 108 | 89.23 | 69.94 |
| 17 | 2.251 | 1.768 | 61 | 28.979 | 22.769 | 110 | 92.56 | 72.42 |
| 18 | 2.523 | 1.983 | 62 | 29.937 | 23.521 | 115 | 101.17 | 79.42 |
| 19 | 2.811 | 2.209 | 63 | 30.911 | 24.286 | 120 | 110.16 | 86.48 |
| 20 | 3.115 | 2.448 | 64 | 31.900 | 25.063 | 125 | 119.53 | 93.81 |
| 21 | 3.435 | 2.698 | 65 | 32.884 | 25.853 | 130 | 129.28 | 101.44 |
| 22 | 3.769 | 2.962 | 66 | 33.925 | 26.654 | 135 | 139.42 | 109.47 |
| 23 | 4.120 | 3.237 | 67 | 34.960 | 27.468 | 140 | 149.94 | 117.70 |
| 24 | 4.486 | 3.525 | 68 | 36.012 | 28.294 | 145 | 160.84 | 126.01 |
| 25 | 4.868 | 3.824 | 69 | 37.079 | 29.133 | 150 | 172.12 | 135.06 |
| 26 | 5.265 | 4.136 | 70 | 38.161 | 29.983 | 160 | 199.84 | 153.73 |
| 27 | 5.677 | 4.461 | 71 | 39.259 | 30.846 | 170 | 221.08 | 173.49 |
| 28 | 6.106 | 4.797 | 72 | 40.373 | 31.721 | 180 | 247.86 | 194.57 |
| 29 | 6.550 | 5.146 | 73 | 41.502 | 32.548 | 190 | 296.46 | 216.79 |
| 30 | 7.009 | 5.507 | 74 | 42.647 | 33.508 | 200 | 306.00 | 240.21 |
| 31 | 7.484 | 5.880 | 75 | 43.806 | 34.119 | 210 | 337.37 | 264.75 |
| 32 | 7.975 | 6.266 | 76 | 44.983 | 35.343 | 220 | 370.26 | 290.66 |
| 33 | 8.481 | 6.664 | 77 | 46.176 | 36.288 | 230 | 404.69 | 317.59 |
| 34 | 9.003 | 7.074 | 78 | 47.382 | 37.228 | 240 | 440.64 | 346.64 |
| 35 | 9.540 | 7.496 | 79 | 48.605 | 38.189 | 250 | 478.13 | 364.33 |
| 36 | 10.093 | 7.930 | 80 | 49.843 | 39.162 | 260 | 517.14 | 485.95 |
| 37 | 10.662 | 8.377 | 81 | 51.097 | 40.147 | 270 | 557.69 | 437.88 |
| 38 | 11.246 | 8.836 | 82 | 52.367 | 41.144 | 280 | 599.76 | 470.81 |
| 39 | 11.806 | 9.307 | 83 | 53.632 | 42.154 | 290 | 643.37 | 504.64 |
| 40 | 12.461 | 9.790 | 84 | 54.952 | 43.176 | 300 | 688.50 | 540.24 |
| 41 | 13.092 | 10.286 | 85 | 56.208 | 44.210 | 350 | 937.12 | 736.01 |
| 42 | 13.738 | 10.794 | 86 | 57.600 | 45.256 | 400 | 1,224.00 | 960.84 |
| 43 | 14.400 | 11.314 | 87 | 58.947 | 46.315 | 450 | 1,549.12 | 1,216.68 |
| 44 | 15.078 | 11.846 | 88 | 60.310 | 47.386 | 500 | 1,912.50 | 1,497.33 |
| 45 | 15.771 | 12.391 | 89 | 61.689 | 48.469 | | | |



(*N. del T.* — Hemos reemplazado dos tablas del autor por las siguientes.)

## Tabla comparativa de los pesos de diversos metales.

| Espesor en mm. | Peso del metro cuadrado de planchas en: | | | | | Espesor en mm. | Peso del metro cuad de planchas en: | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Palastro. | Cinc. | Estaño. | Plomo. | Cobre rojo. | | Palastro. | Cinc. | Estaño. | Plomo. | Cobre rojo. |
| | kg. | kg. | kg. | kg. | kg. | | kg. | kg. | kg. | kg. | kg. |
| 1/4 | 1.95 | 1.72 | 1.83 | 2.84 | 2.20 | 10 | 77.88 | 68.61 | 73.00 | 113.52 | 87.88 |
| 1/2 | 3.90 | 3.43 | 3.65 | 5.68 | 4.40 | 11 | 85.67 | 75.47 | 80.30 | 124.87 | 96.67 |
| 1 | 7.79 | 6.86 | 7.30 | 11.35 | 8.79 | 12 | 92.46 | 82.33 | 87.60 | 136.22 | 105.46 |
| 2 | 15.58 | 13.72 | 14.60 | 22.70 | 17.58 | 13 | 100.23 | 89.19 | 94.90 | 147.58 | 114.24 |
| 3 | 23.36 | 20.58 | 21.90 | 34.06 | 26.36 | 14 | 109.03 | 96.05 | 102.20 | 158.93 | 123.03 |
| 4 | 31.15 | 27.44 | 29.20 | 45.41 | 35.15 | 15 | 116.82 | 102.92 | 109.50 | 170.28 | 131.82 |
| 5 | 38.94 | 34.30 | 36.50 | 56.76 | 43.94 | 16 | 124.61 | 109.78 | 116.80 | 181.63 | 140.61 |
| 6 | 46.73 | 41.17 | 43.00 | 68.11 | 52.73 | 17 | 132.40 | 116.64 | 124.10 | 192.94 | 149.40 |
| 7 | 54.52 | 48.03 | 51.10 | 79.46 | 61.52 | 18 | 140.18 | 123.50 | 131.40 | 204.38 | 158.18 |
| 8 | 62.30 | 54.89 | 58.40 | 90.82 | 70.30 | 19 | 147.97 | 130.36 | 138.70 | 215.69 | 166.97 |
| 9 | 70.09 | 61.75 | 65.70 | 102.17 | 79.09 | 20 | 155.76 | 137.22 | 146.10 | 227.04 | 175.76 |

## Hierros chatos (sección rectangular).

Peso por metro en kg.

| Espesor. | Ancho de la barra en milímetros. | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 20 | 25 | 35 | 45 | 55 | 65 | 75 | 85 | 95 | 100 |
| 5 | .78 | .975 | 1.36 | 1.75 | 2.14 | 2.53 | 2.92 | 3.31 | 3.7 | 3.9 |
| 6 | .94 | 1.17 | 1.64 | 2.10 | 2.57 | 3.04 | 3.51 | 3.97 | 4.45 | 4.7 |
| 8 | 1.25 | 1.56 | 2.18 | 2.80 | 3.43 | 4.05 | 4.68 | 5.30 | 5.9 | 6.2 |
| 10 | 1.56 | 1.95 | 2.73 | 3.51 | 4.29 | 5.06 | 5.85 | 6.62 | 7.4 | 7.8 |
| 12 | 1.87 | 2.34 | 3.27 | 4.21 | 5.14 | 6.08 | 7.01 | 7.95 | 8.9 | 9.35 |
| 14 | 2.18 | 2.73 | 3.92 | 4.91 | 6.00 | 7.09 | 8.20 | 9.30 | 10.4 | 10.9 |
| 15 | 2.34 | 2.92 | 4.20 | 5.26 | 6.42 | 7.60 | 8.77 | 9.94 | 11.1 | 11.7 |
| 16 | 2.50 | 3.12 | 4.46 | 5.61 | 6.86 | 8.10 | 9.35 | 10.6 | 11.8 | 12.5 |
| 18 | 2.80 | 3.50 | 5.00 | 6.31 | 7.72 | 9.15 | 10.5 | 11.95 | 13.3 | 14 |
| 20 | 3.12 | 3.90 | 5.45 | 7.00 | 8.57 | 10.15 | 11.7 | 13.27 | 14.8 | 15.6 |
| 22 | 3.43 | 4.29 | 6.00 | 7.71 | 9.43 | 11.15 | 12.8 | 14.6 | 16.3 | 17 |
| 25 | 3.90 | 4.87 | 6.82 | 8.76 | 10.7 | 12.7 | 14.6 | 16.6 | 18.5 | 19.5 |
| 26 | 4.05 | 5.06 | 7.10 | 9.15 | 11.2 | 13.2 | 15.2 | 17.2 | 19.2 | 20.2 |
| 28 | 4.42 | 5.53 | 7.74 | 9.92 | 12 | 14.2 | 16.4 | 18.6 | 20.7 | 22.1 |
| 30 | 4.68 | 5.85 | 8.20 | 10.53 | 12.9 | 15.2 | 17.6 | 19.9 | 22.2 | 23.4 |

**Pesos por metro cuadrado de láminas de hierro galvanizado.** Lista modelo adoptada por la Asociación Americana de hierro galvanizado en Pittsburgh, abril 1884.

(*Obs. del T.* — Hemos convertido la tabla que trae el autor al sistema métrico.)

| N.º | Kg por m cuad. | Metros cuadrados por tonelada de 2,240 lbs. | N.º | Kg por m cuad. | Metros cuadrados por tonelada de 2,240 lbs. | N.º | Kg por m cuad. | Metros por tonelada de 2,240 libs. |
|---|---|---|---|---|---|---|---|---|
| 29 | 3.6619 | 277.79 | 24 | 5.1879 | 195.83 | 19 | 10.0711 | 100.89 |
| 28 | 3.9671 | 256.12 | 23 | 5.7983 | 175.23 | 18 | 11.5971 | 87.60 |
| 27 | 4.2723 | 237.82 | 22 | 6.4087 | 158.49 | 17 | 13.1231 | 77.38 |
| 26 | 4.5775 | 221.94 | 21 | 7.3243 | 138.70 | 16 | 14.6491 | 69.30 |
| 25 | 4.8827 | 208.10 | 20 | 8.5451 | 118.91 | 14 | 18.3110 | 55.46 |

**El galvanismo es simplemente una capa delgada de cinc en** ambos lados de la plancha; como lo que se conoce por «hojalata» ó planchas estañadas, que son en realidad hierro en planchas cubierto con estaño. El cinc así como el estaño resiste á la corrosión por la influencia atmosférica ordinaria mucho más que el hierro, y de aquí el uso de estos metales para proteger el hierro. Un techo de buen declive, bien galvanizado, sólo sufrirá un poco, en 5 ó 6 años, sin haber sido reparado. Coge fácilmente la pintura y debe pintarse de vez en cuando. Es mejor, sin embargo, pintarlos de una vez con estaño.

**La pintura no se adhiere bien al cinc nuevo** y esta es la razón principal por la cual no se pintan los techos nuevos galvanizados; pero esto puede remediarse dándole una mano al cinc con la siguiente preparación: disuélvase 1 parte de cloruro de cobre, 1 parte de nitrato de cobre y otra de sal amoníaco, en 64 partes de agua; agréguese entonces 1 parte de ácido clorhídrico del comercio. Cuando se le da la mano con esta solución, se pone negro el cinc. Séquese de 12 á 24 horas y después puede pintarse.

Se emplea generalmente la pintura con óxidos minerales de color castaño y le aplican una mano en ambos lados en el taller y la otra después de haber sido colocado en el techo. Bastará después pintarlos de nuevo cada 3 ó 4 años. El hierro no galvanizado (llamado hierro negro, por distinción), es también muy resistente para techos, si se pinta bien cada 1 ó 2 años. La ventaja principal del techo galvanizado es que no necesita ser pintado tan á menudo como el de hierro negro. El galvanismo aumenta como .8 kg por metro cuadrado de superficie cubierta ó como 1.60 kg por m cuadrado de plancha, por ambos lados, sin tomar en cuenta el grueso de la plancha. La pintura para techos no debe contener mucho secante. Véase el capítulo Pintura.

Los humos sulfurosos del carbón corroen mucho el hierro galvanizado y el negro: como puede verse en los talleres, puentes de ferrocarril ó edificios para máquinas techados con uno de los dos, y cuando no se emplean medios eficaces para alejar el humo. Lo mismo sucede con otros metales. Se dice que el ácido del roble destruye el cinc del hierro galvanizado. Véase el capítulo titulado Estaño y Cinc más adelante. El hierro en planchas se clava generalmente sobre tablas próximas; pero la resistencia del hierro acanalado evita la necesidad de esto y permite que se coloquen planchas separadas de 1.50 á 1.80 metros de viga á viga sin sostén intermedio. Las planchas de hierro acanalado se fijan al techo por medio de remaches de alambre galvanizado como de 3 mm de grueso de á 660 en kg (y bien clavados á fin de impedir que pase el agua de lluvia) con ocho ó diez cms de distancia en todo el rededor. Los agujeros de los remaches se hacen de antemano por medio de taladros á fin de asegurarse de la coincidencia de las diferentes planchas. Los remaches son clavados por dos hombres, uno por arriba y otro por debajo del techo. Para el hierro negro y para techar con tablitas delgadas ó empizarrar, se usan comúnmente, como un preservativo parcial contra la corrosión, los clavos sin galvanizar, hervidos en aceite de linaza. Los clavos galvanizados son sin embargo mejores en todos estos casos y aun los de cobre para empizarrar, porque una buena pizarra dura mucho más tiempo que la tabla y el hierro y viene á ser, por tanto, una verdadera economía el usar metales durables para clavarlos. En ninguno de estos casos están los clavos expuestos completamente á la intemperie.

**Las planchas de hierro planas** se colocan doblando las orillas más ó menos como lo demuestra la figura, pág. 1251.

Las juntas en el sentido de la corriente se hacen como en *ss*, y las horizontales como en *tt;* excepto que, como quiera que éstas no están soldadas en las planchas de hierro, la junta está hecha como de ¾ á 1 pulgada (19 á 25 mm) de ancho en lugar de ¼ de pulgada (6 mm) que es lo mejor para evitar filtraciones. Se usan

lañas como en el estaño, con dos clavos para cada laña. Las planchas de hierro se colocan mejor sobre entarimado de tablas; pero en los tinglados, etc., se colocan algunas veces directamente sobre los pares, separadas no más de 18 pulgadas (45 cm) dejando que las planchas sobresalgan un poco de las vigas á fin de formar ramblas que den salida al agua. En semejantes casos será bueno disponer los extremos superiores de los pares como en esta figura.

**Una seria objeción sobre el hierro como cubierta de un techo** es la rápida condensación de la humedad atmosférica : que cae del hierro en gotas, en forma de lluvia y puede ser perjudicial á los cielos rasos, pisos, ó á los efectos que están en los departamentos debajo del techo. La pintura no disminuye apreciablemente este efecto; puede evitarse sin embargo enyesando.

**Planchas de hierro acanalado.** El tamaño ordinario de las planchas acanaladas es de 30 pulgadas de ancho por 96 de largo (.76 m × 2.44 m). El acanalado reduce la anchura á 27½ pulgadas (.70 m). Cuando las planchas acanaladas se colocan sobre el techo, la superposición de 6 cm en todo el largo de los lados y 10 cm en sus extremos disminuye el área cubierta de techo por una plancha como á siete octavos de la superficie de la plancha entera acanalada; y el peso por metro cuadrado de techo cubierto es como una séptima parte mayor que el de la plancha acanalada por metro cuadrado. El peso de hierro acanalado por metro cuadrado de techo cubierto es como una quinta parte mayor que el de las planchas planas con que se hacen las acanaladas.

Se calcula generalmente como en 15 cm el vuelo de los aleros.

Los pesos por pies cuadrados correspondientes á los diferentes números del calibrador Birmingham varían un tanto entre los diferentes fabricantes. Los dos sistemas de acanalado, dados en la tabla que sigue, 5 × 1¼ y 2½ × ⅛, son los más frecuentemente usados.

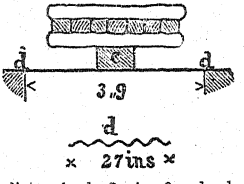
| N.º del calibrador Birmingham. | Espesor en pulgs. | Peso en libras por pie cuadrado de planchas. | | Peso en libras por pie cuadrado de techo. | |
|---|---|---|---|---|---|
| Negro. | Negro. | Negro. | Cubierto con plomo ó galvanizado. | Negro. | Cubierto con plomo ó galvanizado. |
| 20 | .035 | 1.84 | 2.00 | 2.12 | 2.30 |
| 22 | .028 | 1.50 | 1.60 | 1.73 | 1.84 |
| 24 | .022 | 1.20 | 1.25 | 1.38 | 1.44 |
| 26 | .018 | 1.00 | 1.12 | 1.15 | 1.29 |

(*Obs. del T.* — La misma tabla en sistema métrico.)

| N.º del calibrador Birmingham. | Espesor en milímetros. | Peso en kg por metro cuadrado de plancha. | | Peso en kg por metro cuadrado de techo. | |
|---|---|---|---|---|---|
| Negro. | Negro. | Negro. | Cubierto con plomo ó galvanizado. | Negro. | Cubierto con plomo ó galvanizado. |
| 20 | 8.88 | 8.997 | 9.720 | 10.367 | 11.247 |
| 22 | 7.11 | 7.335 | 7.823 | 8.460 | 8.997 |
| 24 | 5 59 | 5.868 | 6.112 | 6 748 | 7.042 |
| 26 | 4 57 | 4.890 | 5.477 | 5.623 | 6.308 |

## Resistencia del hierro acanalado. Experimentos del autor.

**Primero. Una plancha de hierro** *dd* **n.º 16** (como de $^1/_{17}$ de pulgada (1.69 mm) de grueso) 27 pulgadas de ancho (.686 m), por 4 pies (1.219 m) de largo, con 5 canales completos de 5 pulgadas por 1 pulgada (.127 m × .025 m), fué




colocada sobre apoyos á distancia de 3 pies 9 pulgadas (1.14 m). Un trozo de madera *c* de 9 pulgadas de ancho por 7 pulgadas de espesor y 30 pulgadas de largo (.23 m × .18 m × .76 m), se colocó en el centro y se cargó gradualmente con hierros que pesaban 1,600 lbs (725.75 kg).

Esto produjo una flexión en el centro precisamente de ½ pulgada (13 mm). Al remover el peso después de una hora, no era apreciable ningún descenso permanente. La prueba se ratificó de propósito, aplicando los pesos de una manera brusca y sacudiéndolo todo lo más posible *. El área suspendida de la plancha era de 8.44 pies cuadrados (.78 metro cuadrado), y como la carga actual del centro de (725.75 kg) equivale como á 1,361 kg, *igualmente distribuídos*, tenemos 1,361 = $\frac{1,361}{.78}$ = 1,745 kg por metro cuadrado; pero 1,361 kg igualmente distribuídos sólo producirían una flexión como de 6 mm. Además, 1,745 kg por metro cuadrado es como 4 veces el peso de la mayor carga que se puede acumular sobre un piso.

Por consiguiente, esta plancha á 3'9" (1.14 m) de luz resiste prácticamente cualquiera sobrecarga ordinaria. Sin embargo, dicha sobrecarga producirá una flexión central de sólo ¼ parte de ¼ de pulgada, ó $^1/_{16}$ de una pulgada (1.59 mm), ó $^1/_{720}$ de toda la luz, lo cual es sólo las ⅔ partes del límite de Tredgold de $^1/_{480}$ de la abertura.

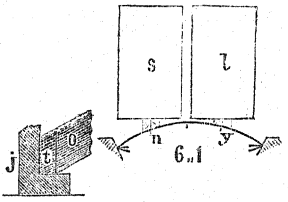
En uno de los experimentos los extremos de la plancha descansaban sobre apoyos preparados, de modo que presentan ondulaciones correspondientes muy próximamente á la forma del acanalado; pero en el otro los apoyos eran planos y cada extremo de la plancha descansaba solamente sobre los puntos inferiores del acanalado. No se observó ninguna diferencia apreciable en los resultados.

**Segundo. Una bóveda de hierro acanalado n. 18** ($^1/_{20}$ de pulgada (1.27 mm)) como el anterior; pero, aumentando la profundidad del acanalado á 1 ¼ pulgada (32 mm) por el procedimiento de arquear la plancha; con luz de 6 pies 1 pulg (1.85 m); elevación, 10 pulgadas (.25 m); ancho, 27 pulgadas

* Sin permitir que la flexión excediera de $^1/_2$ pulgada (12 mm), lo cual se impidió por medio de un apoyo colocado debajo de la plancha.

(.68 m) (de las cuales, sin embargo, sólo 25 pulgadas (.63 m) descansaban sobre los estribos).

Cada pie derecho *o* de la bóveda descansaba sobre una fundición *j*, cuya parte interior *t* estaba undulada en la parte superior, para corresponder á los acanalamientos del arco que descansaba sobre ella. En *y* (cuarta parte de la abertura) se colocaron dos trozos de madera que ocupaban una extensión de .23 m y que se prolongaban á lo largo de la bóveda; sobre ellos se acumuló una carga, *l*, de fundición de 4,480 libras ó 2 toneladas. Bajo esta carga, la bóveda, descendió como 12½ mm en *y* aplanándose en ese lado y formando una curva hacia arriba en




el lado sin carga *n*. Dos bloques parecidos fueron colocados entonces en *n* y sobre ellos 2 toneladas de carga *s* además de las 2 toneladas de *l*. formando un total de 4 toneladas. Esto llevó á la bóveda casi á su forma anterior, pero ligeramente aplanada en *n* é *y;* y un poco más curva en el centro. Entonces se aumentó la carga á 4,536 kg y se la dejó por algunos días. Se encontró que dos tirantes de hierro, cada uno de 12×42 mm, usados para impedir que los estribos se abrieran, se habían estirado cerca de 6 mm. Se añadieron otras cargas y se aumentó el peso á un total de 6 toneladas, parte de él en *s* y *l*, y parte en forma de largas y anchas barras de hierro en el centro del arco debajo de las cargas *s* y *l* y entre *n* é *y*. Según lo que se puede juzgar á la simple vista, la forma de la bóveda era casi perfecta. *Las cargas s* y *l no se tocaban*. Después de haber permanecido allí por más de una semana, la carga fué volteada accidentalmente deformando la bóveda. La carga era como de 4,882 kg por metro cuadrado de bóveda. Dichas bóvedas son desde entonces de uso común **para pisos contra el fuego** en lugar de las de ladrillo.

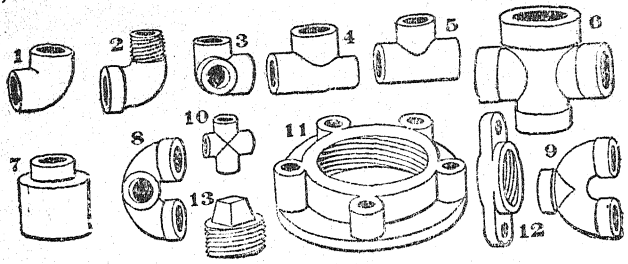
**Los techos curvos de 7 á 11 metros de abertura** y con una elevación como de ¼ de dicha abertura, pueden hacerse de hierro acanalado ordinario de los n.° 16 al 13, remachados como de costumbre y sin otros accesorios que tirantes de hierro á pocos metros de distancia y ángulos de hierro en los arranques, y pendolones de hierro para impedir que los tirantes se doblen.

**Tubos soldados de hierro forjado,** para vapor, gas y agua, con longitudes como de 18 pies (5.491 m).

(*Obs. del T.* — Hemos convertido la tabla del autor al sistema métrico.)

| Diám. interior. | | Espesor. | Peso por metro (nominal.) | Filetes por c. m. de sornillo. | Precio por metro lineal. | Diam. interior. | | Espesor. | Peso por metro nominal. | Filetes por c. m. de sornillo. | Precio por metro lineal. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Nominal. | Efectivo. | | | | | Nominal.* | Efectivo. | | | | |
| m. m. | m. m. | m. m. | kgr. | | $ | m. m. | m. m. | m. m. | kgr. | | $ |
| 3.175 | 6.858 | 1.727 | 0.360 | 10.5 | 0.18 | 88.9 | 90.42 | 5.74 | 13.440 | 3.1 | 3.42 |
| 6.350 | 9.245 | 2.235 | 0.625 | 7 | — | 101.6 | 102.56 | 6.02 | 15.883 | — | 3.54 |
| 9.525 | 12.547 | 2.311 | 0.834 | — | — | 114.3 | 114.5 | 6.25 | 18.610 | — | 4.26 |
| 12.700 | 15.824 | 2.768 | 1.252 | 5.5 | 0.28 | 127.0 | 128.14 | 6.578 | 21.605 | — | 4.76 |
| 19.050 | 20.93 | 2.870 | 1.669 | — | 0.38 | 152.4 | 154.05 | 7.112 | 27.952 | — | 6.17 |
| 25.400 | 26.62 | 3.403 | 2.488 | 4.5 | 0.54 | 177.8 | 178.38 | 7.645 | 34.672 | — | 7.71 |
| 31.75 | 35.05 | 3.556 | 3.338 | — | 0.74 | 203.2 | 202.74 | 8.178 | 41.988 | — | 9.25 |
| 38.10 | 40.92 | 3.683 | 3.9 3 | — | 0.88 | 228.6 | 228.62 | 8.737 | 50.243 | — | 11.15 |
| 50.80 | 52.50 | 3.911 | 5.379 | — | 1.18 | 254.0 | 254.48 | 9.296 | 59.600 | — | 13.94 |
| 63.50 | 62.69 | 5.181 | 8.553 | 3.1 | 1.89 | 279.4 | 279.4 | 9.525 | 67.050 | — | 15.58 |
| 76.20 | 77.90 | 5.542 | 11.235 | — | 2.48 | 304.8 | 304.8 | 9.525 | 73.040 | — | 17.06 |

**Accesorios para tubos de hierro forjado.** 1, codos; 2, codo de servicio; 3, codo con orificio de salida á un lado; 4, T de reducción; 5, T; 6, cruz de reducción; 7, conexión de reducción; 8, conexión para revolverse con orificio de salida á un lado; 9, la misma con orificio de salida por detrás; 10, cruz; 11, conexión de reborde; 12, reborde ovalado; 13, tapón.



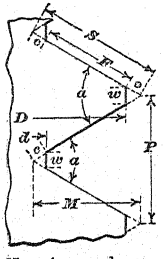

**Tubos de hierro de fundición al carbón vegetal, soldados, para calderas,** en longitudes hasta de 20 pies (6.10 m).

(*Obs. del T.* — Hemos convertido la tabla del autor al sistema métrico.)

| Diám. exterior. | Espesor. | Peso nominal por metro. | Precio por metro. | Diám. exterior. | Espesor. | Peso nominal por metro. | Precio por metro. | Diám. exterior. | Espesor. | Peso nominal por metro. | Precio por metro. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| m. m. | m. m. | kgr. | $ | m. m. | m. m. | kgr. | $ | m. m. | m. m. | kgr. | $ |
| 21.1 | 24.13 | 1.344 | 0.984 | 76.20 | 2.77 | 4.962 | 1.448 | 203.2 | 4.191 | 20.338 | 4.92 |
| 31.75 | — | 1.744 | 0.918 | 82.55 | 3.048 | 5.900 | 1.312 | 228.6 | 4.572 | 24.972 | 5.576 |
| 38.10 | — | 2.086 | 0.886 | 88.90 | — | 6.377 | 1.443 | 254.0 | 5.156 | 31.290 | 6.888 |
| 44.45 | — | 2.473 | 0.722 | 95.25 | — | 6.874 | 1.640 | 279.4 | 5.588 | 37.250 | 8.20 |
| 50.80 | — | 2.846 | 0.656 | 101.60 | 3.404 | 8.150 | 1.804 | 304.8 | 5.817 | 42.465 | 9.512 |
| 57.15 | — | 3.218 | 0.787 | 114.30 | — | 9.193 | 2.034 | 330.2 | 6.045 | 47.769 | 10.496 |
| 63.50 | 2.77 | 4.097 | 0.918 | 127.00 | 3.76 | 11.294 | 2.460 | 355.6 | 6.299 | 53.640 | 11.972 |
| 69.85 | — | 4.530 | 1.115 | 152.40 | 4.14 | 15.128 | 3.280 | 381.0 | 6.577 | 60.494 | 13.448 |
| — | — | — | — | 177.80 | — | 17.734 | 3.936 | 406.4 | 6.86 | 67.348 | 15.088 |

## Roscas de tornillo, pernos, tuercas y arandelas.

**Rosca de tornillo.** $a$=ángulo entre dos lados de una rosca; P=*paso* del tornillo; $w$=ancho del tope ó base plana de cada rosca; todo medido en un plano que



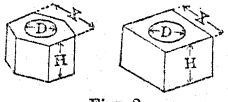

contenga el eje del tornillo; N=número de roscas por pulgada (25.4 mm)= 25.4 mm ÷ P. En los modelos Sellers, ó del Instituto Franklin, propuestos por

Mr. William Sellers y adoptados por el Instituto en 1864, $a=60°$; $S=P$; $w=c=P/S$; $F=.75\,P$; $M=P\cos a/2=.8660\,P$; D (diámetro) $=d+2\times.866\times.75P=d+1.299P$. Bajo el título de **Modelos de los Estados Unidos**, el Departamento de Marina adoptó en 1868 el sistema de Sellers, excepto para las cabezas y tuercas bien acabadas, las cuales hizo que fuesen iguales á las cabezas y tuercas bastas *.

| D | d | w | Número de filetes en 25 mm. | D | d | w | Número de filetes en 25 mm. | D | d | w | Número de filetes en 25 mm. | D | d | w | Número de filetes en 25 mm. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| mm. | mm. | mm. | | mm. | mm. | mm. | | mm. | mm. | mm. | | mm. | mm. | mm. | |
| 6.35 | 4.63 | .157 | 20 | 25.4 | 21.26 | .396 | 8 | 50.8 | 43.48 | .703 | 4½ | 101.6 | 90.60 | 1.049 | 3 |
| 7.93 | 6.40 | .188 | 18 | 28.57 | 23.88 | .432 | 7 | 57.15 | 49.83 | .703 | 4½ | 107.5 | 96.47 | 1.105 | 2⅞ |
| 9.52 | 7.46 | .198 | 16 | 31.75 | 27.05 | .432 | 7 | 63.50 | 55.27 | .792 | 4 | 114.30 | 102.31 | 1.153 | 2¾ |
| 11.11 | 8.74 | .226 | 14 | 34.93 | 29.46 | .528 | 6 | 6 .35 | 61.62 | .792 | 4 | 120.65 | 108.10 | 1.209 | 2⅝ |
| 12.70 | 10.46 | .244 | 13 | 38.10 | 32.64 | .528 | 6 | 76.20 | 66.78 | .907 | 3½ | 127.00 | 113.79 | 1.270 | 2½ |
| 14.28 | 11.53 | .264 | 12 | 41.27 | 35.28 | .576 | 5½ | 82.55 | 73.13 | .907 | 3½ | 133.35 | 120.14 | 1.270 | 2½ |
| 15.87 | 12.88 | .287 | 11 | 44.45 | 37.87 | .635 | 5 | 88.90 | 78.74 | .975 | 3½ | 139.70 | 125.81 | 1.336 | 2⅜ |
| 19.05 | 15.75 | .317 | 10 | 47.62 | 41.08 | .635 | 5 | 95.25 | 84.25 | 1.042 | 3 | 146.45 | 132.16 | 1.336 | 2⅜ |
| 22.22 | 18.57 | .350 | 9 | — | — | — | — | — | — | — | — | 152.40 | 137.74 | 1.440 | 2¼ |

## Dimensiones de las cabezas y tuercas.

| | Bastas. | Acabadas. |
|---|---|---|
| X | 1½ D+3 mm | 1½D+1½ mm |
| H (en la cabeza) | ½ X | D—1½ mm |
| H (en la tuerca) | D | — — |




Figs. 2.

En la rosca del modelo **Whitworth** (inglés), el ángulo $a$, fig. 1, es de 55°. Los topes y bases de las roscas están redondeados, en vez de ser planos como en los modelos americanos. El número (N) de roscas por pulgada (por 25.4 mm) es el mismo que el anterior para diámetros de pernos hasta de tres pulgadas (76 mm), excepto para D= ½ pulgada (12½ mm) en que N=12.

En la **rosca métrica internacional**, adoptada en Zurich, en octubre de 1898, se hizo uso del perfil de la rosca de Sellers. Sus dimensiones son como sigue (en milímetros):

| Diám. | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 | 48 | 52 | 56 | 60 | 64 | 68 | 72 | 76 | 80 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Separación entre roscas. | 1.0 | | 1.25 | | 1.5 | | 1.75 | | 2.0 | | 2.5 | | 3.0 | | 3.5 | | 4.0 | | 4.5 | | 5.0 | | 5.5 | | 6.0 | | 6.5 | | 7.0 |

Los diámetros intermedios deben ser de un número completo de milímetros, y á la misma separación que el próximo diámetro menor de la tabla. Así, para diám de 65 ó 69 mm, la separación=6.0 mm.

* *N. del T.* — Hemos convertido estas tablas del autor á las equivalentes en medidas métricas.

Con las anteriores dimensiones un perno falla generalmente por quebradura entre la cabeza y la tuerca, donde el diámetro se ha disminuído en el corte de la rosca. **El diámetro D de la rosca** debe, naturalmente, ser mayor que el que se necesitá para soportar con seguridad la tensión que se desea, en una cantidad igual al *doble* del espesor de la rosca. El desperdicio de hierro que resulta al hacer *todo el perno* de aquel diámetro mayor, se evita haciendo el perno de una barra que tenga tan sólo las dimensiones necesarias para soportar el esfuerzo con seguridad, **y haciendo sus extremos más gruesos**, como se ve en la fig. 3 donde está aumentado su diámetro lo suficiente para el corte de la rosca.

En la carpintería así como en las traviesas para mamposterías, se colocan *arandelas ww* de hierro colado ó fundido entre la madera ó piedra y la tuerca y cabeza, á fin de distribuir la presión en una superficie mayor é impedir que se maltrate la superficie de contacto especialmente en la madera.

**Cuando hay mucha presión contra la madera**, el lado de una arandela cuadrada de hierro fundido, ó el diámetro *ww* de una circular no debe ser menos de 4 veces el diámetro del *tornillo* (como en la figura), y su espesor *tw* ½ diámetro por lo menos.

Dos arandelas cuadradas como éstas pesarán juntas como una longitud de 1.8 diámetros de una barra redonda del mismo diámetro que el tornillo. En cualquiera de los dos casos una tuerca y la cabeza cuadradas pesarán como 6 diámetros. Siendo as arandelas de hierro colado más propensas á romperse bajo presiones fuertes, deben hacerse dos veces más gruesas que las de hierro forjado. Cuando el esfuerzo es muy grande, el diámetro de la arandela debe ser 5 ó 6 veces el del tornillo, y su espesor igual al diámetro; pero 4 diámetros bastarán en la mayor parte de los casos, ó aun 2.5 cuando el esfuerzo es pequeño; el grueso puede ser entonces de .14 .2 del diámetro del tornillo.

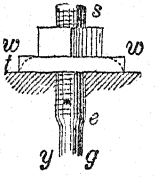


Fig. 3.

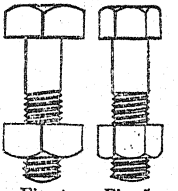


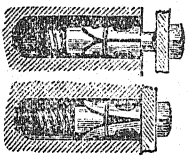
Fig. 4. Fig. 5.

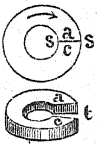
**Tabla de tornillos ó pernos de máquinas y de carros**, cabeza y tuerca cuadradas y hexagonales, figs. 4 y 5, hechas por Hoopes y Townsend, Filadelfia. Todos sus tornillos son hechos de filetes **modelo de los Estados Unidos**, según la tabla pág. 1209, á menos que expresamente sean pedidos de otra forma.

| Diám. D, fig. 1. | Longitud sin la cabeza en mm. | | Peso en kilogramos de 100 tornillos. | | Precio en dólares por cada 100. | |
|---|---|---|---|---|---|---|
| | Mín. | Máx. | Mín. | Máx. | Mín. | Máx. |
| | mm. | mm. | kg. | kg. | | |
| 6.25 | 38.10 | 203.2 | 4.769 | 5.986 | 1.70 | 2.74 |
| 7.937 | — | — | 2.812 | 9.206 | 2.00 | 3.55 |
| 9.525 | — | 304.8 | 4.390 | 19.727 | 2.40 | 5.75 |
| 11.112 | — | — | 6.666 | 26.439 | 2.80 | 7.00 |
| 12.700 | — | 508.0 | 9.251 | 35.398 | 3.60 | 13.22 |
| 14.286 | — | — | 11.793 | 68.492 | 5.20 | 19.25 |
| 15.875 | — | 609.6 | 16.783 | 101.604 | — | 22.30 |
| 19.05 | — | — | 26.308 | 149.686 | 7.20 | 29.70 |
| 22.225 | 50.8 | — | 44.307 | 213.189 | 11.20 | 42.00 |
| 25.40 | — | — | 65.771 | 283.495 | 16.00 | 55.60 |

**Pernos de expansión**, para fijar planchas, maderas, etc., á muros de ladrillos ó mampostería ordinaria. La tuerca cuneiforme que corre hacia arriba del perno, á proporción que éste da vuelta, comprime las alas contra los lados del hueco, el cual en la práctica se perfora con ancho suficiente para admitir la tuerca y las alas,

y para impedir que gire la tuerca junto con el perno. Si se hace el hueco más grande, como se ve, la tuerca debe sujetarse por medio de una pequeña cuña.






**Arandelas para tuercas de seguridad** (*lock-nut*). Cuando los pernos están sujetos á movimientos bruscos como en las bridas de los carriles, etc., las tuercas están *expuestas á aflojarse y á destornillarse por sí mismas*. En los ferrocarriles este es motivo de grandes inconvenientes y se han empleado multitud de invenciones para impedirlo. La tuerca de seguridad **Verona** es una simple arandela circular hecha de acero con una ranura *ss*, practicada á través de ella y de bordes afilados. En un lado *a* de la ranura se comprime el metal hacia arriba como 12 mm y el del otro lado *c* la misma distancia hacia abajo, de modo que aparezca como en *t*. Cuando la tuerca se atornilla hacia abajo sobre la arandela, en la dirección de la flecha, la ranura no ofrece impedimento; pero si luego tiende á destornillarse, el borde afilado superior de la ranura en *a* presenta un frotamiento contra la parte inferior de la tuerca que tiende á mantenerla en su lugar. Además, la arandela por su elasticidad tiende á recobrar su forma primitiva y comprime de este modo los filetes de la tuerca contra los del perno ó tornillo, y el frotamiento adicional producido así, también contribuye á sujetar la tuerca. Otro medio empleado es el de hacer en el extremo del perno algunos filetes de tornillo de menor diámetro que el principal *y en dirección opuesta*. La tuerca se atornilla entonces sobre el diámetro mayor y después la tuerca de retención se atornilla en la otra dirección sobre el diámetro menor, hasta que se ponga en contacto con la tuerca principal. En el perno con tuerca de seguridad **Smith**, esta segunda tuerca es solamente de ¼ de pulgada (3 mm) de grueso; después de haber sido atornillada, una de sus esquinas se plega sobre el borde de la tuerca principal.

Las arandelas de seguridad **Atwood** adquieren mayor elasticidad en la tuerca misma.

Se pretende que **si los filetes** de un perno ordinario y de una tuerca se hacen cuidadosamente de modo **que estén entre sí en perfecto contacto en toda su extensión**, no se necesita ningún arbitrio de seguridad, porque el rozamiento entre los dos filetes se distribuye sobre una superficie mayor y no se ejerce un rozamiento tan grande como si los filetes se tocaran entre sí sólo en algunos puntos, y que, por tanto, las tuercas están menos expuestas á aflojarse con sacudimientos repetidos.

Debido á la dificultad de obtener pernos y tuercas perfectos á causa del deterioro de los instrumentos cortantes que se emplean en su manufactura, se han hecho pernos y tuercas en los cuales los filetes del perno se diferencian un poco en la forma de los de la tuerca. También se hacen tuercas en las que los filetes, en lugar de ser uniformes en toda su extensión, son gradualmente más profundos y más gruesos por tener el ángulo de sus lados más agudo y su parte superior truncada. Estas tuercas se usan con pernos que tengan el filete uniforme usual. El perno entra en la tuerca por el lado donde el filete es de la misma forma que el suyo, pero sus filetes se encuentran y están forzados á un paso más angosto y más profundo por entre los filetes de la tuerca. En ambos casos la coincidencia forzada de los dos filetes produce el perfecto contacto deseado. La mayor fuerza que se requiera para atornillar la tuerca, aumenta también el rozamiento entre los filetes.

## LÁMINAS CONVEXAS

Las láminas convexas son generalmente de acero de ¼ á ½ pulg de espesor (6 ¼ á 12½ mm) y de 3 á 4 pies cuadrados (27.6 á 37 decím cuad); algunas veces son largas con varias curvaturas. La convexidad de 2 á 3 pulgadas (51 á 76 mm). De orillas planas ó con filetes, de 2 á 4 pulgadas (51 á 102 mm). Se emplean en los pisos de edificios y para puentes de calzada.

Carga total permitida, en libras, sobre una sola plancha cuadrada convexa de cualquier tamaño y espesor. Carga $= 4\,kth$, en que $k$ = la unidad permitida de resistencia en el metal, lbs por pulg cuad, unas 6,000; $t$ = espesor del metal, en pulgadas, y $h$ = flecha de la curvatura, en pulgadas. (*Obs. del T.* — Esta fórmula puede usarse en sistema métrico con aproximación suficiente poniendo en lugar de 6,000 lbs por pulg cuadrada su equivalente $6{,}000 \times .070307 = 421.842$ kg por cm cuad y las medidas de $h$ y $t$ en centímetros. Entonces viene el producto en kg.)

Las láminas convexas son más fuertes y requieren menor cantidad de concreto, etc., para su relleno, cuando se colocan con el lado convexo hacia abajo. Pesan muy poco más que las láminas planas, ó sean 10 lbs por pie cuadrado (48.8 kg por m cuad) con espesor de ¼ de pulg (6 ¼ mm).

## PESOS Y RESISTENCIA DE LOS PERNOS DE HIERRO

(Original.)

Diámetros, pesos y esfuerzo aproximado de ruptura, **para pernos redondos**; el esfuerzo de ruptura por centímetro cuadrado, se supone como sigue: Hasta 25 mm de diámetro, 20 toneladas; de 25 á 50 mm en cuadro ó de diámetro, 19 toneladas; 50 á 75 mm, 18 toneladas; 75 á 100 mm, 17 toneladas; 100 á 125 mm, 16 toneladas; 125 á 150 mm, 15 toneladas.

Una barra larga recalcada de rosca ensanchada no es más fuerte que una que no lo sea, en casos de *aplicación lenta* de cargas, ó esfuerzos. Ambas se rompen entonces hacia la mitad de su longitud, bajo igual tensión. En tales casos hágase uso de las columnas 4 y 5.

**Barras cuadradas.** Resistencia ó peso = 1.273 × resistencia ó peso de la barra redonda.

**Barras de cobre.** Resistencia = .8 × resistencia de una barra semejante de hierro.
Peso = 1.14 × peso de una barra semejante de hierro.

(*N. del T.* — Hemos convertido la tabla del autor al sistema métrico.)

| Extremos recalcados de embutido roscado. | | | Extremos no embutidos. | | Extremos recalcados de embutido roscado. | | | Extremos no embutidos. | |
|---|---|---|---|---|---|---|---|---|---|
| Diám del fuste. | Peso por metro. | Esfuerzo de ruptura. | Diám del fuste. | Peso por metro. | Diám del fuste. | Peso por metro. | Esfuerzo de ruptura. | Diám del fuste. | Peso por metro. |
| mm. | kg. | tons. | mm. | kg. | mm. | kg. | tons. | mm. | kg. |
| 3.175 | .061 | .245 | | | 44.45 | 12.06 | 45.7 | 54.36 | 17.88 |
| 4.762 | .138 | .553 | | | 46.03 | 12.94 | 49.0 | 56.39 | 19.22 |
| 6.350 | .246 | .983 | 8.89 | .47 | 47.62 | 13.85 | 52.5 | 58.42 | 20.56 |
| 7.937 | .384 | 1.53 | 10.92 | .67 | 49.21 | 14.79 | 56.0 | 60.45 | 21.90 |
| 9.525 | .554 | 2.21 | 12.70 | .97 | 50.80 | 15.79 | 59.7 | 62.23 | 23.39 |
| 11.112 | .754 | 3.00 | 14.73 | 1.33 | 53.97 | 17.88 | 63.8 | 65.79 | 26.07 |
| 12.700 | .985 | 3.93 | 16.76 | 1.69 | 57.15 | 19.96 | 71.6 | 69.34 | 29.05 |
| 14.287 | 1.247 | 4.97 | 18.54 | 2.10 | 60.32 | 22.20 | 79.7 | 73.15 | 32.18 |
| 15.875 | 1.535 | 6.14 | 20.32 | 2.48 | 63.50 | 24.58 | 88.4 | 76.71 | 35.61 |
| 17.462 | 1.862 | 7.42 | 22.35 | 3.02 | 66.67 | 27.11 | 97.4 | 80.26 | 38.88 |
| 19.050 | 2.220 | 8.83 | 24.38 | 3.59 | 69.85 | 29.80 | 106.9 | 83.82 | 42.46 |
| 20.637 | 2.607 | 10.40 | 26.42 | 4.18 | 73.02 | 32.63 | 116.8 | 87.63 | 46.33 |
| 22.225 | 3.025 | 12.00 | 28.45 | 4.85 | 76.20 | 35.46 | 127.2 | 91.44 | 50.51 |
| 23.812 | 3.472 | 13.80 | 30.48 | 5.61 | 82.55 | 41.57 | 141.0 | 98.04 | 58.25 |
| 25.40 | 3.948 | 15.70 | 32.26 | 6.36 | 88.90 | 48.27 | 163.6 | 104.65 | 66.15 |
| 26.99 | 4.455 | 16.80 | 34.29 | 7.10 | 95.25 | 55.42 | 187.7 | 112.01 | 75.99 |
| 28.575 | 4.991 | 18.90 | 36.07 | 7.86 | 101.60 | 63 02 | 213.6 | 119.38 | 86.12 |
| 30.162 | 5.558 | 21.10 | 37.85 | 8.65 | 107.95 | 71.22 | 227.0 | 126.49 | 97.14 |
| 31.75 | 6.154 | 23.30 | 39.37 | 9.52 | 114.30 | 79.86 | 254.5 | 133.35 | 108.62 |
| 33.337 | 6.794 | 25.70 | 41.66 | 10.49 | 120.65 | 88.95 | 283.5 | 140.46 | 119.94 |
| 34.925 | 7.450 | 28.20 | 43.69 | 11.53 | 127.00 | 98.48 | 314.2 | 147.32 | 131.26 |
| 36.512 | 8.150 | 30.80 | 45.72 | 12.63 | 133.35 | 108.62 | 324.7 | 154.43 | 144.53 |
| 38.100 | 8.865 | 33.60 | 47.50 | 13.70 | 139.70 | 119.20 | 356.4 | 161.54 | 157.94 |
| 39.687 | 9.625 | 36.40 | 49.28 | 14.72 | 146.05 | 130.37 | 389.5 | 168.40 | 172.84 |
| 41.275 | 10.415 | 39.40 | 50.80 | 15.79 | 152.40 | 141.84 | 424.1 | 175.26 | 187.74 |
| 42.862 | 11.220 | 42.50 | 52.58 | 16.83 | | | | | |

El **calibrador de alambre Birmingham** es el que se usa más generalmente para el hierro. Para el peso muy aproximado del acero, multipliquese el del hierro por 1.01; para el **plomo**, por 1.46; para el **cinc**, por .9; para el **bronce**, por 1.06; para el **cobre**, por 1.134.

**Pesos de alambres de hierro** * y tabla comparativa de los calibres usados en varios paises.

| Calib milimétrico. | | Peso de | Calib Bir- | Calib | Calib | Calib de West- |
|---|---|---|---|---|---|---|
| N.º | Espesor mm. | 1 000 m | mingham. | Halifax. | de París. | phalia. |
| 100 | 10 | 600.00 | | 3/0 | 30 | 29 |
| 94 | 9.4 | 530.40 | 2/0 | 2/0 | 29 | 28 |
| 88 | 8.8 | 464.86 | 0 | 0 | 28 | 27 |
| 82 | 8.2 | 403.59 | » | » | 27 | » |
| 76 | 7.6 | 346.68 | 1 | 1 | 26 | 26 |
| 70 | 7.0 | 294.00 | 2 | 2 | 25 | 25 |
| 65 | 6.5 | 253.50 | 3 | 3 | 24 | » |
| 60 | 6.0 | 216.00 | 4 | 4 | 23 | 24 |
| 55 | 5.5 | 181.50 | 5 | 5 | 22 | 23 |
| 50 | 5. | 150.00 | 6 | 6 | 21 | » |
| 46 | 4.6 | 126.96 | 7 | 7 | 20 | 22 |
| 42 | 4.2 | 105.84 | 8 | 8 | 19 | 21 |
| 38 | 3.8 | 86.64 | 9 | 9 | » | 20 |
| 34 | 3.4 | 69.36 | 10 | 10 | 18 | 19 |
| 31 | 3.1 | 57.66 | 11 | 11 | 17 | 18 |
| 28 | 2.8 | 47.04 | 12 | 12 | 16 | 17 |
| 25 | 2.5 | 37.50 | 13 | 13 | 15 | 16 |
| 22 | 2.2 | 29.04 | » | » | 14 | 15 |
| 20 | 2.0 | 24.00 | 14 | 14 | 13 | 14 |
| 18 | 1.8 | 19.44 | 15 | 15 | 12 | 13 |
| 16 | 1.6 | 15.36 | 16 | 16 | 11 | 11 |
| 14 | 1.4 | 11.76 | 17 | 17 | 9 | 10 |
| 13 | 1.3 | 10.17 | » | » | 8 | 9 |
| 12 | 1.2 | 8.64 | 18 | 18 | 7 | 8 |
| 11 | 1.1 | 7.26 | » | 19 | 6 | 7 |
| 10 | 1.0 | 6.00 | 19 | 20 | 5 | 6 |
| 9 | .9 | 4.86 | 20 | 21 | 4 | 5 |
| 8 | .8 | 3.84 | 21 | 22 | 3 | 4 |
| 7 | .7 | 2.94 | 22 | 23 | 2 | 2 |
| 6 | .6 | 2.16 | 23 | 25 | 1 | 1 |
| 5 | .55 | 1.81 | 24 | » | » | » |
| 5/5 | .5 | 1.50 | 25 | 26 | P. | » |
| 4/5 | .45 | 1.26 | 26 | 27 | P. 1 | » |
| 4 | .4 | .96 | 27 | 28 | P. 2 | » |
| 3/7 | .37 | .82 | 28 | 29 | P. 3 | » |
| 3/4 | .34 | .69 | 29 | 30 | P. 4 | » |
| 3/1 | .31 | .58 | 30 | 31 | P. 5 | » |
| 2/8 | .28 | .47 | » | 32 | P. 6 | » |
| 2/0 | .26 | .41 | 31 | 33 | P. 7 | » |
| 2/1 | .24 | .35 | » | 34 | P. 9 | » |
| 2/2 | .22 | .29 | 32 | 36 | P. 10 | » |
| 2 | .2 | .24 | 33 | 37 | P. 11 | » |

**Calibrador ó hilera americana** para láminas y planchas de hierro y acero (**1893**). Omitimos las columnas del peso en kilogramos por pie cuadrado y en libras por metro cuadrado, y simplificamos los encabezamientos de las columnas restantes.

* La concordancia de los números no es sino aproximada.

El Congreso estableció un calibrador modelo para láminas y planchas de hierro y acero, en esta forma:

**El Senado y Cámara de representantes de los Estados Unidos, reunidos en Congreso, decretan**: Con el objeto de asegurar la uniformidad se establece el siguiente calibrador como medida modelo única de las láminas y planchas de hierro y acero, en los Estados Unidos de América, á saber:

| N.º | Espesor aproximado. | | Peso. | | |
|---|---|---|---|---|---|
| | Pulgadas. | Milímetros. | Por pie cuad en avoirdupois. onzas. | libras. | Por m cuad en kg. |
| 7-0 | 1-2 =.5 | 12.7 | 320 | 20.00 | 97.65 |
| 6-0 | 15-32 =.46875 | 11.90625 | 300 | 18.75 | 91.55 |
| 5-0 | 7-16 =.4375 | 11.1125 | 280 | 17.50 | 85.44 |
| 4-0 | 13-32 =.40625 | 10.31875 | 260 | 16.25 | 79.33 |
| 3-0 | 3-8 =.375 | 9.525 | 240 | 15. | 73.24 |
| 2-0 | 11-32 =.34375 | 8.73125 | 220 | 13.75 | 67.13 |
| 0 | 5-16 =.3125 | 7.9375 | 200 | 12.50 | 61.03 |
| 1 | 9-32 =.28125 | 7.14375 | 180 | 11.25 | 54.93 |
| 2 | 17-64 =.265625 | 6.746875 | 170 | 10.625 | 51.88 |
| 3 | 1-4 =.25 | 6.35 | 160 | 10. | 48.82 |
| 4 | 15-64 =.234375 | 5.953125 | 150 | 9.375 | 45.77 |
| 5 | 7-32 =.21875 | 5.55625 | 140 | 8.75 | 42.72 |
| 6 | 13-64 =.203125 | 5.159375 | 130 | 8.125 | 39.67 |
| 7 | 3-16 =.1875 | 4.7625 | 120 | 7.5 | 36.62 |
| 8 | 11-64 =.171875 | 4.365625 | 110 | 6.875 | 33.57 |
| 9 | 5-32 =.15625 | 3.96875 | 100 | 6.25 | 30.52 |
| 10 | 9-64 =.140625 | 3.571875 | 90 | 5.625 | 27.46 |
| 11 | 1-8 =.125 | 3.175 | 80 | 5. | 24.41 |
| 12 | 7-64 =.109375 | 2.778125 | 70 | 4.375 | 21.36 |
| 13 | 3-32 =.09375 | 2.38125 | 60 | 3.75 | 18.31 |
| 14 | 5-64 =.078125 | 1.984375 | 50 | 3.125 | 15.26 |
| 15 | 9-128 =.0703125 | 1.7859375 | 45 | 2.8125 | 13.73 |
| 16 | 1-16 =.0625 | 1.5875 | 40 | 2.5 | 12.21 |
| 17 | 9-160 =.05625 | 1.42875 | 36 | 2.25 | 10.99 |
| 18 | 1-20 =.05 | 1.27 | 32 | 2. | 9.765 |
| 19 | 7-160 =.04375 | 1.11125 | 28 | 1.75 | 8.544 |
| 20 | 3-80 =.0375 | .9525 | 24 | 1.50 | 7.324 |
| 21 | 11-320 =.034375 | .873125 | 22 | 1.375 | 6.713 |
| 22 | 1-32 =.03125 | .793750 | 20 | 1.25 | 6.103 |
| 23 | 9-320 =.028125 | .714375 | 18 | 1.125 | 5.493 |
| 24 | 1-40 =.025 | .635 | 16 | 1. | 4.882 |
| 25 | 7-320 =.021875 | .555625 | 14 | .875 | 4.272 |
| 26 | 3-160 =.01875 | .47625 | 12 | .75 | 3.662 |
| 27 | 11-640 =.0171875 | .4365625 | 11 | .6875 | 3.357 |
| 28 | 1-64 =.015625 | .396875 | 10 | .625 | 3.052 |
| 29 | 9-640 =.0140625 | .3571875 | 9 | .5625 | 2.746 |
| 30 | 1-80 =.0125 | .3175 | 8 | .5 | 2.441 |
| 31 | 7-640 =.0109375 | .2778125 | 7 | .4375 | 2.136 |
| 32 | 13-1280=.01015625 | .25796875 | 6½ | .40625 | 1.983 |
| 33 | 3-320 =.009375 | .238125 | 6 | .375 | 1.831 |
| 34 | 11-1280=.00859375 | .21828125 | 5½ | .34375 | 1.678 |
| 35 | 5-640 =.0078125 | .1984375 | 5 | .3125 | 1.526 |
| 36 | 9-1280=.00703125 | .17859375 | 4½ | .28125 | 1.373 |
| 37 | 17-2560=.006640625 | .168671875 | 4¼ | .265625 | 1.297 |
| 38 | 1-160 =.00625 | .15875 | 4 | .25 | 1.221 |

Y en lo sucesivo, desde el primero de julio de 1893, inclusive, ésta y no otra debe ser usada para determinar los derechos é impuestos fijados por los Estados Unidos de América, á las láminas y planchas de hierro y acero.

## MEDIDAS CIRCULARES

Usadas para comparar las secciones transversales de los alambres, etc.

La unidad circular es el área de un círculo cuyo diámetro sea una unidad lineal. Así, una pulg circular es el área (= .7854 pulgs cuads) de un círculo cuyo diámetro es una pulg. (*Obs. del T.* — De igual modo un metro circular es el área (.7854 met cuad) de un círculo cuyo diámetro es un metro.)

Hemos extractado la tabla que sigue, con autorización del autor de las importantes tablas de equivalencias de unidades de medidas de Mr. Carl Hering, Nueva York, 1888. Como nosotros tomamos 1 metro=39.37 pulgs, en lugar de 39.37079 pulgs, nuestros valores difieren algo de los de él.

| | | | Logaritmos. |
|---|---|---|---|
| 1 ○ mil *..... | = | .78540 □ mil *..... | 1.895 0899 |
| | = | .00064516 ○ milímetro..... | 4.809 6692 |
| | = | .00050671 □ milímetro..... | 4.704 7591 |
| 1 □ mil *..... | = | 1.2732 ○ mils *..... | .104 9101 |
| | = | .00082145 ○ milímetro..... | 4.914 5793 |
| 1 ○ milímetro..... | = | 1550.0 ○ mils *..... | 3.190 3308 |
| | = | 1217.4 □ mils *..... | 3.085 4207 |
| | = | .78540 □ milímetros..... | 1.895 0899 |
| 1 □ milímetro..... | = | 1973.5 ○ mils *..... | 3.295 2409 |
| | = | 1.2732 ○ milímetros..... | .104 9101 |

**Ninguna estupidez comercial** es más insensata que la rutina de seguir con los varios calibrajes de Birmingham, etc., en vez de expresar de una vez el espesor y diámetro de hojas, alambre, etc., en milímetros ó pulgadas como desde hace tiempo se ha insinuado. Para evitar errores, que suelen ocurrir muy á menudo, por el número de escantillones en uso y por la absurda práctica de aplicar el mismo número á diferentes espesores de diferentes metales, en diferentes ciudades, lo mejor es desecharlos todos; y al hacer pedidos, determinar el diámetro del alambre y el espesor de hojas de metal en milímetros. O empléese el peso p or cien ó mil metros de alambre ó por metros cuadrados para hojas. Nosotros *creemos* que el escantillón precedente de Birmingham se aplica al cinc, cobre, bronce y plomo, aunque generalmente se dice que es para hierro y acero solamente. Usase otro escantillón de Birmingham para bronce en hojas, oro, plata y algunos otros metales; pero nunca hemos oído decir cuáles son esos otros. Hay diferentes escantillones, aun para alambres, que han de usarse con diferentes *fines;* y varias firmas tienen escantillones suyos propios, ni siquiera acordes entre sí.

Como el Sr. Stubs hace varios escantillones ingleses, el término « **escantillón Stubs** » por sí mismo no significa nada. Generalmente, sin embargo, en nuestros talleres de máquinas se aplica al escantillón de Birmingham.

**Escantillón de Birmingham para hojas de bronce, plata, oro** y todos los metales excepto hierro y acero?

(*N. del T.* — Las siguientes son tres tablas del autor convertidas al sistema métrico.)

| N.º | Espesor en mm. | N.º | Espesor en mm. | N.º | Espesor en mm. | N.º | Espesor en mm. | N.º | Espesor en mm. | N.º | Espesor en mm. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | .1016 | 7 | .381 | 13 | .914 | 19 | 1 626 | 25 | 2.413 | 31 | 3.378 |
| 2 | .127 | 8 | .406 | 14 | 1.041 | 20 | 1 702 | 26 | 2.616 | 32 | 3.632 |
| 3 | .203 | 9 | .483 | 15 | 1.194 | 21 | 1 829 | 27 | 2.870 | 33 | 3.683 |
| 4 | .254 | 10 | .610 | 16 | 1.295 | 22 | 1. 880 | 28 | 3.048 | 34 | 3.759 |
| 5 | .305 | 11 | .737 | 17 | 1.448 | 23 | 1 956 | 29 | 3.150 | 35 | 4.013 |
| 6 | .330 | 12 | .864 | 18 | 1.549 | 24 | 2 083 | 30 | 3.200 | 36 | 4.242 |

★ Un mil = 1/1000 pulgada

**Las fábricas que laminan hierro en los Estados Unidos** usan generalmente el siguiente calibraje, que difiere ligeramente del de Birmingham.

| N.º | Kg por m cuad. | N.º | Kg por m cuad. | N.º | Kg por m cuad. | N.º | Kg por m cuad. |
|---|---|---|---|---|---|---|---|
| 1 | 61.025 | 8 | 33.490 | 15 | 13.718 | 22 | 6.102 |
| 2 | 58.589 | 9 | 30.464 | 16 | 12.205 | 23 | 5.468 |
| 3 | 53.707 | 10 | 27.437 | 17 | 10.643 | 24 | 4.882 |
| 4 | 48.824 | 11 | 24.410 | 18 | 9.080 | 25 | 4.394 |
| 5 | 42.717 | 12 | 21.383 | 19 | 8.299 | 26 | 3.906 |
| 6 | 39.642 | 13 | 18.307 | 20 | 7.518 | 27 | 3.515 |
| 7 | 36.615 | 14 | 15.232 | 21 | 6.835 | 28 | 3.124 |

**Cuando se pide alambre, metal en hojas, etc.,** por un número de escantillón, y no se especifica el escantillón, los comerciantes en los Estados Unidos despachan el pedido como sigue :

Latón, plata alemana en hojas, alambre de plata alemana, latón bronceado tubería de bronce, cobre ó cinc, por el escantillón de Brown y Sharpe (ó « Americano » ).

Cobre en hojas; alambre de cobre; latón sin empate, tubería de bronce ó cobre; y pequeñas barras de latón ó bronce, por el escantallón de Stubs (ó de Birmingham).

Alambre de latón no recocido ó duro tiene como $\frac{3}{4}$ de las resistencias del cuadro que sigue, y cosa de $^1/_9$ más de peso. Si es recocido, sólo la mitad de la fuerza.

**Alambre de cobre duro** puede estimarse en $\frac{2}{3}$ de las resistencias tabulares y $^1/_7$ más de peso.

**Tabla de alambre de hierro al carbón vegetal, fabricado por la Trenton Iron Co.,** de Trenton, N. J. Las cifras de la primera columna son las del **calibrador de la Trenton Iron Co.** Los diámetros correspondientes en la segunda columna se verá que son algo menores que los del calibrador de Birmingham.

| N.º | Diámetro en mm. | Metros en cada kg. | Resist, tensión aprox en kg. | N.º | Diámetro en mm. | Metros en cada kg. | Resist, tensión aprox en kg. | N.º | Diámetro en mm. | Metros en cada kg. |
|---|---|---|---|---|---|---|---|---|---|---|
| 00000 | 11.43 | 1.25 | 5,713 | 11 | 2.98 | 18.34 | 458.03 | 26 | .457 | 781.506 |
| 0000 | 10.16 | 1.58 | 4,514 | 12 | 2.66 | 22.96 | 367.41 | 27 | .432 | 876.104 |
| 000 | 9.14 | 1.95 | 3,684 | 13 | 2.34 | 29.58 | 286.21 | 28 | .406 | 990.979 |
| 00 | 8.38 | 2.32 | 3,120 | 14 | 2.03 | 39.53 | 215.00 | 29 | .381 | 1,126.260 |
| 0 | 7.74 | 2.72 | 2,687 | 15 | 1.77 | 51.65 | 168.73 | 30 | .356 | 1,291.890 |
| 1 | 7.23 | 3.11 | 2,370 | 16 | 1.54 | 68.09 | 132.44 | 31 | .330 | 1,498.110 |
| 2 | 6.73 | 3.60 | 2,073 | 17 | 1.33 | 92.04 | 100.69 | 32 | .305 | 1,758.427 |
| 3 | 6.22 | 4.21 | 1,790 | 18 | 1.14 | 125.03 | 76.65 | 33 | .279 | 2,092.911 |
| 4 | 5.71 | 5.00 | 1,530 | 19 | 1.01 | 157.74 | 62.14 | 34 | .254 | 2,532.075 |
| 5 | 5.20 | 6.02 | 1,288 | 20 | .88 | 206.72 | 48.53 | 35 | .241 | 2,806.463 |
| 6 | 4.82 | 7.01 | 1,123 | 21 | .78 | 263.55 | ..... | 36 | .229 | 3,125.335 |
| 7 | 4.44 | 8.26 | 969 | 22 | .71 | 322.90 | ..... | 37 | .216 | 3,503.985 |
| 8 | 4.06 | 10.88 | 822 | 23 | .63 | 405.19 | ..... | 38 | .203 | 3,956.315 |
| 9 | 3.68 | 12.04 | 683 | 24 | .57 | 500.37 | ..... | 39 | .190 | 4,511.999 |
| 10 | 3.30 | 14.98 | 559 | 25 | .50 | 633.01 | ..... | 40 | .178 | 5,165.528 |

(*N. del T.* — Esta tabia es la del autor convertida en unidades del sistema métrico.)

**En esta tabla el alambre** se supone duro, brillante y no templado.

Las cifras de la columna de resistencias á la tensión están basadas en experiencias practicadas con buen alambre de hierro al carbón vegetal, hechos de trozos de hierro bruto de Trenton de primera forjadura.

| | | | | |
|---|---|---|---|---|
| La resistencia á la tensión de alambres hechos de buen hierro refinado..... | es como de 15 | por 100 | menos. | que el del alambre brillante al carbón, que se da en la tabla arriba. |
| Hierro de Suecia de fundición al carbón vegetal..... | 10 | — | — | |
| Acero Bessemer dulce..... | 10 | — | más. | |
| Acero ordinario de crisol..... | 25 | — | — | |
| Acero especial ..... | 30 á 120 | — | — | |

El temple hace al alambre más dúctil y flexible, pero menos elástico y reduce la resistencia á la tensión en un 20 á un 25 por ciento.

**Para encontrar aproximadamente el número de alambres rectos que entran en un cable de un diámetro dado**, divídase el diámetro del cable por el diámetro del alambre (usando cualquier unidad); elévese al cuadrado el cociente y multiplíquese este cuadrado por .77. El resultado será exacto con sólo diferencias de 4 á 5 por ciento á lo sumo, en un cable *cilíndrico*.

**La solidez ó área metálica de todos los alambres de un cable** es al área del cable como 1 es á 1.3 próximamente. En otras palabras, el área de los vacíos es como ¼ parte del área del cable, en tanto que la de los alambres es ¾ del área de éste. Todo aproximadamente.

(*Obs. del T.* — Hemos creído más conveniente reemplazar con las seis tablas siguientes las análogas que trae el autor, porque éstas son de un solo tipo especial de vigas, que vende la Casa Carnegie de los Estados Unidos, y por tanto las que damos á continuación, tomadas de la obra Notas y Fórmulas del Ingeniero, por Laharpe, son más generales.)

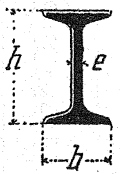
**Hierros laminados en doble T y en U.** Según el ancho relativo de las alas, se distinguen tres series de hierros, á saber :

1.° Hierros de *alas ordinarias* (abreviando AO).
2.° Hierros de *alas anchas* (AA), que se emplea menos.
3.° Hierros de *perfil normal* (PN) ó de alas medianas. El más usado.

Los dos primeros se fabrican, salvo algunas excepciones, en hierro y en acero, los PN en acero solamente Las tablas siguientes contienen los tipos corrientes. Los espesores indicados para las almas, son los espesores mínimos á los cuales se laminan los hierros. Se pueden obtener espesores más fuertes, hasta en 5 ó 6 mm de más; los anchos de las alas se aumentan entonces en lo mismo.

Dividiendo por $10^4$ los números de la columna de las I, y por $10^3$ los de la columna de las $\frac{I}{n}$, se tienen los valores de estas cantidades, expresados en metros. Los momentos resistentes ó los valores de $R\frac{I}{n}$, están en kilogrametros. Se ha admitido una carga de seguridad de 10 kg. Para valores de R = 6, 7, 8... kg, se tendrán los valores correspondientes de $R\frac{I}{n}$, multiplicando los números de la tabla respectivamente por .6, .7, .8.

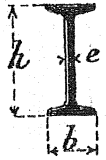
## Hierros doble T de alas ordinarias (AO).




Dimensiones en mm.

| Altura *h*. | Ancho alas *b*. | Espesor del alma *e*. | Sección en mm cuad $\Omega \times 10^{3}$. | Peso aproximado en kg por metro. | Resistencia máx á la flexión — I — Momento de inercia $I \times 10^{9}$. | Resistencia máx á la flexión — I — Módulo de resistencia $\frac{I}{n} \times 10^{6}$. | Resistencia máx á la flexión — I — Para una carga de seguridad R=10 kg por mm cuad. — Momento resistente $R\frac{I}{n}$. | Resistencia mín á la flexión — H — Momento de inercia $I \times 10^{9}$. | Resistencia mín á la flexión — H — Módulo de resistencia $\frac{I}{n} \times 10^{6}$. | Resistencia mín á la flexión — H — Para une carga de seguridad R=10 kg por mm cuad. — Momento resistente $R \times \frac{I}{n}$. |
|---|---|---|---|---|---|---|---|---|---|---|
| 75 | 31.5 | 3.5 | 570 | 4.50 | 496 | 13.22 | 132 | 32 | 2.00 | 20 |
| 80 | 40 | 4 | 770 | 6.50 | 784 | 19.60 | 196 | 67 | 3.35 | 34 |
| 100 | 42 | 4.5 | 960 | 8.00 | 1,482 | 29.64 | 296 | 82 | 3.91 | 39 |
| 120 | 45 | 5 | 1,160 | 9.30 | 2,510 | 41.83 | 418 | 107 | 4.77 | 48 |
| 140 | 47 | 5.5 | 1,476 | 11.50 | 4,311 | 61.59 | 616 | 149 | 6.33 | 63 |
| 160 | 50 | 5.5 | 1,680 | 13.50 | 6,449 | 80.61 | 806 | 189 | 7.58 | 76 |
| 180 | 55 | 6 | 2,088 | 16.30 | 10,165 | 112.94 | 1,129 | 278 | 10.31 | 103 |
| 200 | 60 | 6.5 | 2,370 | 19.00 | 14,675 | 146.75 | 1,468 | 364 | 12.16 | 122 |
| 220 | 64 | 7 | 2,811 | 22.00 | 20,007 | 181.88 | 1,819 | 484 | 15.26 | 153 |
| 260 | 75 | 8.5 | 3,806 | 30.00 | 37,008 | 284.68 | 2,847 | 856 | 22.82 | 228 |

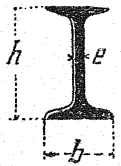
## Hierros doble T de alas anchas (AA).




Dimensiones en mm.

| Altura $h$. | Ancho alas $b$. | Espesor del alma $e$. | Sección en mm cuad $\Omega \times 10^{3}$. | Peso aproximado en kg por metro. | Resistencia máx á la flexión — Momento de inercia $I \times 10^{9}$. | Resistencia máx á la flexión — Módulo de resistencia $\frac{I}{n} \times 10^{6}$. | Resistencia máx á la flexión — Para una carga de seguridad R=10 kg por mm cuad. — Momento resistente $R \frac{I}{n}$. | Resistencia mín á la flexión — Momento de inercia $I \times 10^{9}$. | Resistencia mín á la flexión — Módulo de resistencia $\frac{I}{n} \times 10^{6}$. | Resistencia mín á la flexión — Para una carga de seguridad R=10 kg por mm cuad. — Momento resistente $R \times \frac{I}{n}$. |
|---|---|---|---|---|---|---|---|---|---|---|
| 80 | 55 | 3.5 | 1,001 | 7.90 | 1,113 | 27.82 | 278 | 194 | 7.07 | 71 |
| 100 | 60 | 4.0 | 1,268 | 10.00 | 2,184 | 43.68 | 437 | 279 | 9.32 | 93 |
| 120 | 70 | 5.0 | 1,770 | 14.00 | 4,332 | 72.20 | 722 | 515 | 14.73 | 147 |
| 140 | 80 | 6.0 | 2,320 | 18.00 | 7,637 | 109.10 | 1,091 | 855 | 21.39 | 214 |
| 160 | 80 | 7.5 | 2,868 | 22.50 | 11,771 | 147.14 | 1,471 | 986 | 24.65 | 247 |
| 160 | 90 | 6.5 | 2,543 | 20.50 | 10,796 | 134.95 | 1,350 | 1,097 | 24.37 | 244 |
| 180 | 100 | 7.0 | 3,446 | 27.10 | 19,609 | 217.88 | 2,179 | 1,762 | 35.24 | 352 |
| 200 | 100 | 8.0 | 3,808 | 29.00 | 24,870 | 248.70 | 2,488 | 2,007 | 40.15 | 402 |
| 220 | 100 | 8.0 | 3,968 | 31.20 | 31,007 | 281.88 | 2,819 | 2,008 | 40.17 | 402 |
| 220 | 110 | 7.5 | 4,008 | 31.50 | 32,302 | 293.66 | 2,937 | 2,453 | 46.51 | 465 |
| 250 | 110 | 8.0 | 4,601 | 37.00 | 47,053 | 376.42 | 3,764 | 2,838 | 51.60 | 516 |
| 260 | 118 | 9.0 | 5,328 | 46.00 | 65,172 | 501.32 | 5,013 | 4,395 | 74.50 | 745 |
| 300 | 130 | 10.0 | 6,360 | 50.20 | 91,253 | 608.42 | 6,084 | 5,149 | 79.22 | 792 |
| 350 | 140 | 11.0 | 8,494 | 68.40 | 167,307 | 956.55 | 9,566 | 8,267 | 118.10 | 1,181 |
| 406.4 | 151.4 | 13.0 | 11,234 | 87.60 | 296,672 | 1,460.00 | 14,600 | 10,495 | 139.00 | 1,390 |
| 457.2 | 176.8 | 13.0 | 13,724 | 106.50 | 469,316 | 2,053.00 | 20,530 | 18,829 | 213.00 | 2,130 |
| 508 | 189 | 14.25 | 16,204 | 126.30 | 697,484 | 2,746.00 | 27,460 | 26,366 | 279.00 | 2,790 |

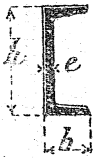
## Hierros doble T de acero de perfil normal (PN).




Dimensiones en mm.

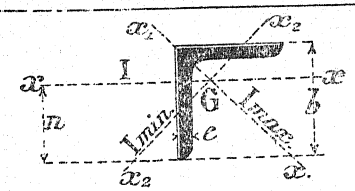
| Altura $h$. | Ancho alas $b$. | Espesor del alma $e$. | Sección en mm cuad $\Omega \times 10^{1}$. | Peso aproximado en kg por metro. | Resistencia máx á la flexión — I — Momento de inercia $I \times 10^{1}$. | Resistencia máx á la flexión — I — Módulo de resistencia $\frac{I}{n} \times 10^{1}$. | Resistencia máx á la flexión — I — Para una carga de seguridad R=10 kg por mm cuad. — Momento resistente $R\,\frac{I}{n}$. | Resistencia mín á la flexión — H — Momento de inercia $I \times 10^{0}$. | Resistencia mín á la flexión — H — Módulo de resistencia $\frac{I}{n} \times 10^{1}$. | Resistencia mín á la flexión — H — Para una carga de seguridad R=10 kg por mm cuad. — Momento resistente $R \times \frac{I}{n}$. |
|---|---|---|---|---|---|---|---|---|---|---|
| 80 | 42 | 3.9 | 758 | 6.00 | 785 | 19.62 | 196 | 73 | 3.49 | 35 |
| 100 | 50 | 4.5 | 1,063 | 8.30 | 1,721 | 34.42 | 344 | 142 | 5.69 | 57 |
| 120 | 58 | 5.1 | 1,418 | 11.10 | 3,307 | 55.11 | 551 | 251 | 8.67 | 87 |
| 140 | 66 | 5.7 | 1,824 | 14.30 | 5,787 | 82.66 | 827 | 414 | 12.55 | 126 |
| 150 | 70 | 6.0 | 2,037 | 16.00 | 7,421 | 98.95 | 990 | 517 | 14.76 | 148 |
| 160 | 74 | 6.3 | 2,280 | 17.90 | 9,444 | 118.05 | 1,180 | 644 | 17.42 | 174 |
| 170 | 78 | 6.6 | 2,519 | 19.80 | 11,813 | 138.94 | 1,389 | 787 | 20.17 | 202 |
| 180 | 82 | 6.9 | 2,787 | 21.90 | 14,600 | 162.22 | 1,622 | 960 | 23.42 | 234 |
| 200 | 90 | 7.5 | 3,343 | 26.20 | 21,617 | 216.17 | 2,162 | 1,379 | 30.65 | 307 |
| 220 | 98 | 8.1 | 3,951 | 31.00 | 30,895 | 280.86 | 2,809 | 1,902 | 39.23 | 392 |
| 240 | 106 | 8.7 | 4,608 | 36.20 | 42,870 | 357.25 | 3,573 | 2,612 | 49.29 | 493 |
| 260 | 113 | 9.4 | 5,329 | 41.90 | 56,140 | 401.00 | 4,010 | 3,034 | 55.10 | 551 |
| 280 | 119 | 10.1 | 6,101 | 47.90 | 57,980 | 446.00 | 4,460 | 3,407 | 60.29 | 603 |
| 300 | 125 | 10.8 | 6,899 | 54.10 | 76,573 | 546.95 | 5,470 | 4,290 | 72.11 | 721 |
| 320 | 131 | 11.5 | 7,771 | 61.00 | 98,884 | 659.22 | 6,592 | 5,301 | 84.82 | 848 |
| 340 | 137 | 12.2 | 8,668 | 68.00 | 126,219 | 788.87 | 7,889 | 6,518 | 99.51 | 995 |
| 360 | 143 | 13.0 | 9,698 | 76.10 | 158,265 | 930.97 | 9,310 | 7,888 | 115.16 | 1,152 |
| 380 | 149 | 13.7 | 10,698 | 83.90 | 197,659 | 1,098.10 | 10,981 | 9,562 | 133.74 | 1,337 |
| 400 | 155 | 14.4 | 11,775 | 92.30 | 242,074 | 1,274.07 | 12,741 | 11,375 | 152.68 | 1,527 |
| 500 | 185 | 18.0 | 17,935 | 140.50 | 294,462 | 1,472.31 | 14,723 | 13,495 | 174.13 | 1,741 |
| | | | | | 692,445 | 2,769.78 | 27,698 | 28,709 | 310.37 | 3,104 |

## Hierros en U.



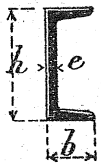

Dimensiones en mm.

| Altura *h*. | Ancho de las alas *b*. | Espesor del alma *e*. | Sección en mm cuad $\Omega \times 10^3$. | Peso aproximado en kg por metro. | Resistencia máx á la flexión — Momento de inercia $I \times 10^4$. | Resistencia máx á la flexión — Módulo de resistencia $\frac{I}{n} \times 10^3$. | Resistencia máx á la flexión — Para una carga de seguridad R=10 kg por mm cuad. Momento resistente $R\frac{I}{n}$. | Resistencia mín á la flexión — Momento de inercia $I \times 10^4$. | Resistencia mín á la flexión — Posición del eje que pasa por el centro de gravedad en mm, *n*. | Resistencia mín á la flexión — Módulo de resistencia $\frac{I}{n} \times 10^3$. | Resistencia mín á la flexión — Para una carga de seguridad R=10 kg por mm cuad. Momento resistente $R \times \frac{I}{n}$. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 80 | 40 | 6.0 | 905 | 7.30 | 960 | 24.00 | 240 | 137 | 29.5 | 4.63 | 46 |
| 80 | 50 | 7.0 | 1,291 | 10.10 | 1,237 | 30.93 | 309 | 313 | 31.8 | 9.66 | 97 |
| 100 | 40 | 7.0 | 1,228 | 9.50 | 1,733 | 34.65 | 347 | 171 | 28.2 | 6.07 | 61 |
| 100 | 50 | 7.0 | 1,431 | 11.20 | 2,118 | 42.36 | 424 | 339 | 33.1 | 10.26 | 103 |
| 110 | 50 | 6.0 | 1,517 | 11.00 | 2,646 | 48.10 | 481 | 332 | 34.4 | 9.68 | 97 |
| 120 | 60 | 7.0 | 1,741 | 13.50 | 3,814 | 63.56 | 636 | 606 | 41.0 | 14.77 | 148 |
| 140 | 45 | 7.0 | 1,628 | 13.00 | 4,746 | 67.80 | 678 | 279 | 33.0 | 8.45 | 85 |
| 140 | 60 | 7.0 | 1,881 | 14.70 | 5,501 | 78.59 | 786 | 637 | 42.2 | 15.11 | 151 |
| 160 | 60 | 7.5 | 2,250 | 17.60 | 8,475 | 105.94 | 1,059 | 751 | 42.3 | 17.75 | 177 |
| 175 | 60 | 8.0 | 2,440 | 19.10 | 10,660 | 121.83 | 1,218 | 779 | 43.7 | 17.84 | 178 |
| 200 | 70 | 8.0 | 2,902 | 22.60 | 17,034 | 170.34 | 1,703 | 1,305 | 50.3 | 25.94 | 259 |
| 200 | 80 | 6.5 | 2,770 | 22.35 | 17,612 | 176.12 | 1,761 | 1,788 | 58.0 | 30.79 | 308 |
| 220 | 70 | 10.0 | 3,580 | 28.00 | 23,886 | 217.15 | 2,172 | 1,435 | 51.6 | 27.81 | 278 |
| 235 | 85 | 10.0 | 4,150 | 32.40 | 32,214 | 282.67 | 2,827 | 2,704 | 61.5 | 43.90 | 439 |
| 250 | 80 | 10.0 | 4,110 | 32.00 | 35,934 | 287.47 | 2,875 | 2,245 | 59.4 | 37.81 | 378 |
| 300 | 75 | 10.0 | 4,495 | 35.00 | 53,625 | 357.50 | 3,575 | 1,955 | 57.6 | 33.93 | 339 |




| Dimensiones en milím | | | Peso aproximado en kg por metro. | $x_1 x_1$ (bisectriz) máximum : I máx $\times 10^4$. | $x_2 x_2$ perpendicular á $x_1 x_1$ mínimum : I mín $\times 10^4$. | $xx$, paralelo á los brazos I $\times 10^4$. | Posición del eje que pasa por el centro de gravedad, en mm, $n$. | Módulo de resistencia relativa al eje $xx$ $\frac{I}{n} \times 10^1$. |
|---|---|---|---|---|---|---|---|---|
| Ancho $b$. | Espesor $e$. | Sección Ω. | | | | | | |
| 60 | 6 | 684 | 5.33 | 372 | 90 | 231 | 42.8 | 5.4 |
| | 7 | 791 | 6.20 | 421 | 107 | 264 | 42.4 | 6.2 |
| | 8 | 896 | 7.00 | 470 | 123 | 296 | 42.1 | 7.0 |
| | 9 | 999 | 7.80 | 515 | 138 | 327 | 41.7 | 7.8 |
| | 10 | 1,100 | 8.60 | 559 | 152 | 356 | 41.4 | 8.6 |
| 65 | 7 | 861 | 6.72 | 544 | 139 | 342 | 46.3 | 7.3 |
| | 8 | 976 | 7.61 | 605 | 160 | 383 | 45.9 | 8.4 |
| | 9 | 1,089 | 8.49 | 668 | 180 | 424 | 45.6 | 9.3 |
| | 10 | 1,200 | 9.36 | 725 | 201 | 463 | 45.2 | 10.2 |
| | 11 | 1,309 | 10.19 | 779 | 222 | 501 | 44.9 | 11.1 |
| 70 | 7 | 931 | 7.3 | 687 | 176 | 432 | 49.9 | 8.6 |
| | 8 | 1,056 | 8.2 | 768 | 199 | 484 | 49.4 | 9.7 |
| | 9 | 1,179 | 9.2 | 847 | 221 | 534 | 49.2 | 10.8 |
| | 10 | 1,300 | 10.1 | 921 | 242 | 583 | 48.9 | 11.9 |
| | 11 | 1,419 | 11.0 | 991 | 264 | 627 | 48.4 | 12.9 |
| | 12 | 1,536 | 11.9 | 1,057 | 284 | 671 | 48.2 | 13.9 |
| 75 | 8 | 1,136 | 8.86 | 921 | 283 | 602 | 53.5 | 11.3 |
| | 9 | 1,269 | 9.90 | 1,030 | 300 | 664 | 53.1 | 12.2 |
| | 10 | 1,400 | 10.92 | 1,134 | 316 | 725 | 52.6 | 13.8 |
| | 11 | 1,529 | 11.90 | 1,227 | 339 | 783 | 52.4 | 14.9 |
| | 12 | 1,656 | 12.90 | 1,317 | 361 | 839 | 52.0 | 16.2 |
| 80 | 8 | 1,216 | 9.48 | 1,178 | 296 | 737 | 57.1 | 12.8 |
| | 9 | 1,359 | 10.60 | 1,300 | 330 | 815 | 56.7 | 14.5 |
| | 10 | 1,500 | 11.70 | 1,417 | 363 | 890 | 56.3 | 15.8 |
| | 11 | 1,639 | 12.84 | 1,531 | 393 | 963 | 56.0 | 17.2 |
| | 12 | 1,776 | 13.80 | 1,643 | 421 | 1,032 | 55.5 | 18.5 |
| 85 | 10 | 1,600 | 12.50 | 1,713 | 445 | 1,079 | 60.1 | 18 |
| | 11 | 1,749 | 13.60 | 1,851 | 483 | 1,167 | 59.7 | 19.5 |
| | 12 | 1,896 | 14.80 | 1,984 | 520 | 1,252 | 59.4 | 21.1 |
| | 13 | 2,041 | 15.90 | 2,111 | 559 | 1,335 | 59.0 | 22.6 |
| 90 | 9 | 1,539 | 12.00 | 1,882 | 478 | 1,180 | 64.1 | 18.3 |
| | 10 | 1,700 | 13.30 | 2,054 | 528 | 1,291 | 63.8 | 20.2 |
| | 11 | 1,859 | 14.50 | 2,222 | 573 | 1,399 | 63.5 | 22.1 |
| | 12 | 2,016 | 15.70 | 2,381 | 618 | 1,503 | 63.1 | 23.8 |
| | 13 | 2,171 | 16.90 | 2,538 | 652 | 1,595 | 62.8 | 25.6 |
| 95 | 10 | 1,800 | 14.00 | 2,437 | 627 | 1,532 | 67.6 | 22.7 |
| | 11 | 1,969 | 15.36 | 2,639 | 681 | 1,660 | 67.2 | 24.7 |
| | 12 | 2,136 | 16.70 | 2,837 | 733 | 1,785 | 66.9 | 26.7 |
| | 13 | 2,301 | 17.95 | 3,020 | 792 | 1,906 | 66.5 | 28.7 |
| 100 | 10 | 1,900 | 14.80 | 2,870 | 730 | 1,800 | 71.5 | 25.1 |
| | 11 | 2,079 | 16.20 | 3,110 | 796 | 1,953 | 71.0 | 27.5 |
| | 12 | 2,256 | 17.60 | 3,340 | 864 | 2,102 | 70.6 | 29.7 |
| | 13 | 2,431 | 18.90 | 3,559 | 928 | 2,244 | 70.2 | 32.0 |
| | 14 | 2,604 | 20.30 | 3,775 | 991 | 2,383 | 69.9 | 34.1 |
| | 15 | 2,775 | 21.60 | 3,970 | 1,070 | 2,520 | 69.5 | 36.7 |

## Hierros en U de perfiles normales.




Dimensiones en mm.

| Altura $h$. | Largo de los brazos $b$. | Espesor del alma $e$. | Sección en mm cuad $\Omega \times 10^{-1}$. | Peso aproximado por metro lineal : kg. | Resistencia máx á la flexión — Momento de inercia $I \times 10^{6}$. | Resistencia máx á la flexión — Módulo de resistencia $\frac{I}{n} \times 10^{5}$. | Resistencia máx á la flexión — Para una carga de seguridad R=10 kg por mm cuad. Momento de resist $R \frac{I}{n}$. | Resist mín á la flexión — Momento de inercia $I \times 10^{-1}$. | Resist mín á la flexión — Posición del eje que pasa por el centro de gravedad en mm, $n$. | Resist mín á la flexión — Módulo de resistencia $\frac{I}{n} \times 10^{-1}$. | Resist mín á la flexión — Para carga de seguridad R=10 kg por mm cuad. Momento de resistencia $R \times \frac{I}{n}$. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 80 | 45 | 6.0 | 1,100 | 8.60 | 1,060 | 26.50 | 265 | 236 | 29.3 | 8.04 | 80 |
| 100 | 50 | 6.0 | 1,350 | 10.50 | 2,055 | 41.10 | 411 | 330 | 33.1 | 9.96 | 100 |
| 120 | 55 | 7.0 | 1,700 | 13.30 | 3,642 | 60.70 | 607 | 431 | 37.5 | 13.10 | 131 |
| 140 | 60 | 7.0 | 2,040 | 15.90 | 6,043 | 86.40 | 864 | 707 | 40.9 | 17.29 | 173 |
| 160 | 70 | 7.5 | 2,400 | 18.70 | 9,280 | 116.00 | 1,160 | 1,189 | 44.9 | 21.14 | 211 |
| 235 | 90 | 10.0 | 4,240 | 33.10 | 34,310 | 292.00 | 2.920 | 3.374 | 64.8 | 52.31 | 523 |

### Observaciones sobre las tablas precedentes.

1.ª Para los hierros en doble T y en U, en el caso de su resistencia máxima á la flexión, el módulo de resistencia $\frac{I}{n}$, cuando el espesor $e$ del alma aumenta en $\Delta e$, crece la cantidad : $\Delta \frac{I}{n} = \Delta e . \frac{h^2}{6}$ ; siendo $h$ la altura del hierro y quedando el perfil de las alas el mismo.

2.ª Para los hierros en doble T, en el caso de su resistencia mínima á la flexión, y para el mismo aumento de espesor $\Delta e$ del alma, corresponde un aumento del módulo de resistencia igual : $\Delta \frac{I}{n} = h . \frac{(\Delta e)^2}{6}$.

3.ª El cuadrado del radio de giro mínimo $r^2$, que figura más adelante en la fórmula de Rankine, se obtiene para los hierros en doble T y en U de estas tablas multiplicando por $n$ el módulo $1/n$ relativo á la resistencia mínima á la flexión, y dividiendo por la sección $\Omega$.

Para las doble T, $n$ es igual al semiancho del ala.

4.ª Para un pequeño aumento (epsilón) del espesor de un ángulo de brazos iguales de longitud $b$, le corresponde al módulo de resistencia $1/n$, relativo á un eje paralelo á uno de los brazos un incremento

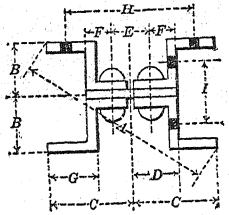
$$\Delta \frac{I}{n} = \frac{5}{18} b\ (b - 2\ e).$$

5.ª El módulo de resistencia $n$ para un eje paralelo á uno de los brazos de un ángulo, no se considera, en general, sino en el caso de una pieza formada de dos ángulos yuxtapuestos siguiendo el otro brazo y unidos solidariamente.

## COLUMNAS CARNEGIE DE BARRAS DE ACERO EN FORMA DE Z.

### Tabla de dimensiones, en mm.

Diámetro del perno ó remache = ¾ pulg (19 mm).
Con respecto al área de la sección, peso por metro, menor radio de giro, y carga de seguridad, véanse tablas, págs. 1228 á 1231.




(*N. del T.* — Hemos convertido la tabla del autor al sistema métrico.)

| | Espesor del metal. | Véase arriba la figura. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | A | B | C | D | E | F | G | H | I |
| Columnas de 150 mm. | 6.35 | 312.74 | 79.375 | 134.94 | 73.025 | 63.50 | 41.275 | 68.262 | 215.90 | 82.55 |
| | 7.937 | 314.32 | 81.756 | 134.94 | 73.025 | 63.50 | 41.275 | 69.85 | 212.72 | 85.72 |
| | 9.525 | 309.56 | 80.962 | 131.76 | 73.025 | 63.50 | 41.275 | 68.262 | 209.55 | 85.72 |
| | 11.112 | 311.15 | 83.347 | 131.76 | 73.025 | 63.50 | 41.275 | 69.85 | 206.37 | 88.90 |
| | 12.70 | 304.8 | 82.55 | 128.59 | 73.025 | 63.50 | 41.275 | 68.262 | 203.2 | 88.90 |
| | 14.287 | 306.39 | 84.93 | 128.59 | 73.025 | 63.50 | 41.275 | 69.85 | 200.02 | 92.07 |
| Columnas de 200 mm | 6.35 | 373.06 | 104.77 | 153.99 | 82.55 | 76.20 | 44.45 | 77.787 | 241.30 | 107.95 |
| | 7.937 | 374.65 | 107.156 | 153.99 | 82.55 | 76.20 | 44.45 | 79.375 | 238.12 | 111.12 |
| | 9.525 | 376.29 | 109.54 | 153.99 | 82.55 | 76.20 | 44.45 | 80.962 | 234.95 | 114.30 |
| | 11.112 | 368.35 | 107.16 | 149.22 | 82.55 | 76.20 | 44.45 | 77.787 | 231.77 | 112.71 |
| | 12.70 | 369.94 | 109.54 | 149.22 | 82.55 | 76.20 | 44.45 | 79.375 | 228.60 | 115.89 |
| | 14.287 | 371.53 | 111.92 | 149.22 | 82.55 | 76.20 | 44.45 | 80.962 | 225.42 | 119.06 |
| | 15.875 | 362.0 | 109.54 | 144.46 | 82.55 | 76.20 | 44.45 | 77.787 | 222.25 | 117.47 |
| | 17.462 | 363.59 | 111.92 | 144.46 | 82.55 | 76.20 | 44.45 | 79.375 | 219.07 | 120.65 |
| | 19.05 | 365.18 | 114.30 | 144.46 | 82.55 | 76.20 | 44.45 | 80.962 | 215.90 | 123.82 |
| Columnas de 250 mm | 7.937 | 419.1 | 130.97 | 163.51 | 88.90 | 82.55 | 47.625 | 82.55 | 257.17 | 134.94 |
| | 9.525 | 420.69 | 133.35 | 163.51 | 88.90 | 82.55 | 47.625 | 84.137 | 254.0 | 138.11 |
| | 11.112 | 422.28 | 135.73 | 163.51 | 88.90 | 82.55 | 47.625 | 85.725 | 250.82 | 141.29 |
| | 12.7 | 415.93 | 133.35 | 158.75 | 88.90 | 82.55 | 47.625 | 82.5 | 247.65 | 139.70 |
| | 14.287 | 417.51 | 135.73 | 158.75 | 88.90 | 82.55 | 47.625 | 84.137 | 244.47 | 142.87 |
| | 15.875 | 419.1 | 138.11 | 158.75 | 88.90 | 82.55 | 47.625 | 85.725 | 241.3 | 146.05 |
| | 17.462 | 411.16 | 135.73 | 153.99 | 88.90 | 82.55 | 47.625 | 82.55 | 238.12 | 144.46 |
| | 19.05 | 412.75 | 138.11 | 153.99 | 88.90 | 82.55 | 47.625 | 84.137 | 234.95 | 147.64 |
| | 20.637 | 414.34 | 140.49 | 153.99 | 88.90 | 82.55 | 47.625 | 85.725 | 231.77 | 150.81 |
| Columnas de 300 mm | 19.525 | 479.43 | 157.16 | 180.97 | 101.6 | 101.6 | 50.8 | 88.90 | 285.75 | 161.92 |
| | 11.112 | 481.01 | 159.34 | 180.97 | 101.6 | 101.6 | 50.8 | 90.487 | 282.57 | 168.27 |
| | 12.7 | 482.6 | 161.92 | 180.97 | 101.6 | 101.6 | 50.8 | 92.075 | 279.40 | 171.45 |
| | 14.287 | 474.66 | 159.64 | 176.21 | 101.6 | 101.6 | 50.8 | 88.90 | 276.22 | 166.69 |
| | 15.875 | 476.25 | 161.92 | 176.21 | 101.6 | 101.6 | 50.8 | 90.487 | 273.05 | 169.86 |
| | 17.462 | 477.84 | 164.31 | 176.21 | 101.6 | 101.6 | 50.8 | 92.075 | 269.87 | 173.04 |
| | 19.05 | 471.49 | 161.92 | 171.45 | 101.6 | 101.6 | 50.8 | 88.90 | 266.70 | 171.45 |
| | 20.637 | 473.08 | 164.31 | 171.45 | 101.6 | 101.6 | 50.8 | 90.487 | 263.52 | 174.62 |
| | 22.225 | 474.66 | 166.69 | 171.45 | 101.6 | 101.6 | 50.8 | 92.075 | 260.35 | 177.80 |

## COLUMNAS CARNEGIE DE BARRAS DE ACERO EN FORMA DE Z.

**Tabla de cargas de seguridad** de columnas de extremos cuadrados, como las suministran Carnegie, Steel Co. Coeficiente de seguridad = 4. Las cargas dadas están basadas en los siguientes esfuerzos en lbs por pulg cuad ó kgs por cm cuad.

Para longitudes no mayores de 90 radios de giro, 12,000 (843.715 kg por cm cuad).

Para longitudes mayores de 90 radios de giro, en lbs pulg cuad :

$$17{,}100 - 57\frac{\text{longitud}}{\text{radio}} \text{ ó bien, } 1202.13 - 4.007\frac{\text{longitud}}{\text{radio}} \text{ ; en kg por cm cuad.}$$

Cada columna está formada de cuatro barras en forma de Z y una plancha que le sirve de alma (todo de espesor uniforme), remachadas ó unidas entre sí por medio de pernos ó remaches, como se ve en la figura anterior.

## Columnas de barras de acero en forma de Z, de 6 pulgs (152 mm).

Compuestas de cuatro barras en forma de Z como de 3 pulgs (76 mm) de altura y de una plancha ó alma de 5 ¾ pulgs (146 mm) de ancho.

| | | | | | | |
|---|---|---|---|---|---|---|
| Espesor del metal en pulgs. | 1/4 | 5/16 | 3/8 | 7/16 | 1/2 | 9/16 |
| Espesor del metal en mm. | 6.3 | 7.9 | 9.5 | 11.1 | 12.7 | 14.3 |
| Area de la sección en pulgs cuads. | 9.31 | 11.7 | 13.6 | 16 | 17.6 | 20 |
| Area de la sección en cm cuad. | 60.06 | 75.48 | 87.74 | 103.22 | 113.54 | 129.03 |
| Peso en lbs por yarda. | 95.1 | 119.4 | 138.6 | 162.9 | 179.7 | 203.7 |
| Peso en kg por metro. | 47.18 | 59.23 | 68.75 | 80.81 | 89.14 | 101.95 |
| Radio menor de giro en pulgs. | 1.86 | 1.90 | 1.88 | 1.93 | 1.90 | 1.95 |
| Radio menor de giro en cm. | 4.72 | 4.82 | 4.77 | 4.90 | 4.82 | 4.95 |

| Long de la columna | | Carga de seguridad en libs. (*N. del T.* — Multiplicando estas cargas en lbs por .4536 se obtendrán en kg). | | | | | |
|---|---|---|---|---|---|---|---|
| en metros. | en pies. | | | | | | |
| 3.658 ó menos. | 12 ó menos. | 111800 | 140600 | 163200 | 191600 | 211400 | 239600 |
| 4.267 | 14 | 111400 | 140600 | 163200 | 191600 | 211400 | 239600 |
| 4.877 | 16 | 104600 | 133000 | 153200 | 182600 | 199800 | 229600 |
| 5.486 | 18 | 97600 | 124600 | 143400 | 171200 | 187200 | 215600 |
| 6.096 | 20 | 90800 | 116200 | 133400 | 159800 | 174400 | 201600 |
| 6.706 | 22 | 84000 | 107800 | 123600 | 148600 | 161800 | 187600 |
| 7.315 | 24 | 77200 | 99400 | 113800 | 137200 | 149200 | 173600 |
| 7.925 | 26 | 70400 | 91000 | 103800 | 126000 | 136400 | 159600 |
| 8.534 | 28 | 63400 | 82600 | 94000 | 114600 | 123800 | 145600 |
| 9.144 | 30 | 56600 | 74200 | 84000 | 103400 | 111000 | 131600 |

## Columnas de barras en forma de Z, de 8 pulgs (203 mm).

Compuestas de cuatro barras en forma de Z como de 4 pulgs (102 mm) de altura y de una plancha ó alma de 6½ pulgs (165 mm) de ancho.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Espesor del metal en pulgs..... | 1/4 | 5/16 | 3/8 | 7/16 | 1/2 | 9/16 | 5/8 | 11/16 | 3/4 |
| Espesor del metal en mm..... | 6.3 | 7.9 | 9.5 | 11.1 | 12.7 | 14.3 | 15.9 | 17.5 | 19 |
| Area de la sección en pulgs cuads.... | 11.3 | 14.1 | 17.1 | 19. | 21.9 | 24.8 | 26.3 | 29. | 31.9 |
| Area de la sección en cm cuads..... | 72.9 | 90.96 | 110.32 | 122.57 | 141.28 | 160. | 169.67 | 187.09 | 205.80 |
| Peso en lbs por yarda..... | 114.9 | 144.3 | 174. | 194.1 | 221.1 | 252.3 | 267.6 | 296.4 | 325.2 |
| Peso en kg por metro..... | 56.997 | 71.58 | 86.32 | 96.286 | 109.68 | 125.02 | 132.78 | 147.03 | 159.35 |
| Radio menor de giro en pulgs..... | 2.47 | 2.52 | 2.57 | 2.49 | 2.55 | 2.60 | 2.52 | 2.58 | 2.63 |
| Radio menor de giro en cm..... | 6.27 | 6.40 | 6.52 | 6.32 | 6.48 | 6.60 | 6.40 | 6.55 | 6.68 |

| Longitud de la columna | | Carga de seguridad de columnas en lbs. (*N. del T.* — Multiplicando esta carga en lbs por .4536 se obtiene en kg.) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| en metros. | en pies. | | | | | | | | |
| 5.486 ó menos. | 18 ó menos. | 135000 | 169600 | 204800 | 228400 | 262400 | 297000 | 315000 | 348600 | 382400 |
| 6.096 | 20 | 130000 | 165000 | 201000 | 221000 | 256400 | 292800 | 306600 | 342600 | 379200 |
| 6.705 | 22 | 123800 | 157400 | 191800 | 210600 | 244800 | 279800 | 292400 | 327000 | 362600 |
| 7.315 | 24 | 117600 | 149600 | 182600 | 200200 | 233000 | 266800 | 278200 | 311600 | 346000 |
| 7.925 | 26 | 111400 | 142000 | 173600 | 189600 | 221200 | 253800 | 264000 | 296200 | 329400 |
| 8.534 | 28 | 105200 | 134200 | 164600 | 179200 | 209400 | 240600 | 249600 | 280800 | 312800 |
| 9.144 | 30 | 98800 | 126600 | 155400 | 168800 | 197600 | 227600 | 235400 | 265400 | 296400 |
| 9.753 | 32 | 92600 | 119000 | 146400 | 158400 | 186000 | 214600 | 221200 | 250000 | 279800 |
| 10.363 | 34 | 86400 | 111200 | 137400 | 148000 | 174200 | 201600 | 207000 | 234600 | 263200 |
| 10.973 | 36 | 80200 | 103600 | 128200 | 137400 | 162400 | 188600 | 192800 | 219200 | 246600 |
| 11.582 | 38 | 74000 | 96000 | 119200 | 127000 | 150600 | 175600 | 178800 | 203800 | 230000 |
| 12.192 | 40 | 67800 | 88200 | 110000 | 116600 | 139000 | 162600 | 164400 | 188400 | 213400 |

## COLUMNAS DE BARRAS EN FORMA DE Z DE ACERO CARNEGIE.
## TABLA DE CARGAS DE SEGURIDAD (CONTINUACION).

### Columnas de barras de acero en forma de Z, de 10 pulgs (25.4 mm).

Compuestas de cuatro barras en forma de Z como de 5 pulgs (127 mm) de altura y de una plancha ó alma de 7 pulgs (178 mm) de ancho.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Espesor del metal en pulgs..... | 5/16 | 3/8 | 7/16 | 1/2 | 9/16 | 5/8 | 11/16 | 3/4 | 13/16 |
| Espesor del metal en mm..... | 7.9 | 9.5 | 11.1 | 12.7 | 14.3 | 15.9 | 17.5 | 19. | 20.6 |
| Area de la sección en pulgs cuads.... | 15.8 | 19. | 22.3 | 24.5 | 27.7 | 30.9 | 32.7 | 35.8 | 39. |
| Area de la sección en cm cuad..... | 101.93 | 122.58 | 143.87 | 158.06 | 178.70 | 199.35 | 210.96 | 230.96 | 251.60 |
| Peso en lbs por yarda..... | 161.1 | 194.1 | 227.4 | 249.9 | 282.6 | 315.6 | 330.0 | 368.4 | 397.8 |
| Peso en kg por metro..... | 79.916 | 96.286 | 112.805 | 123.966 | 140.2 | 155.558 | 163.701 | 182.750 | 197.334 |
| Radio menor de giro en pulgs..... | 3.08 | 3.13 | 3.18 | 3.10 | 3.15 | 3.21 | 3.13 | 3.18 | 3.25 |
| Radio menor de giro en cm..... | 7.82 | 7.95 | 8.08 | 7.87 | 8.00 | 8.15 | 7.95 | 8.08 | 8.25 |

| Longitud de la columna | | Carga de seguridad, de la columna en lbs. (*N. del T.* — Multiplicando esta carga en lbs por .4536 se obtendrá en kg.) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| en metros. | en pies. | | | | | | | | | |
| 6.705 ó menos. | 22 ó menos. | 189400 | 228400 | 267800 | 294000 | 332400 | 371200 | 392000 | 429800 | 468000 |
| 7.315 | 24 | 185600 | 225200 | 266200 | 289200 | 329600 | 370600 | 387200 | 427800 | 468000 |
| 7.925 | 26 | 178600 | 217200 | 256600 | 278400 | 317400 | 357400 | 373000 | 412400 | 453200 |
| 8.534 | 28 | 171600 | 208800 | 247000 | 267600 | 305400 | 344200 | 358600 | 397000 | 436800 |
| 9.144 | 30 | 164600 | 200400 | 237400 | 256800 | 293400 | 331000 | 344400 | 381600 | 420400 |
| 9.753 | 32 | 157600 | 192200 | 227600 | 246000 | 281400 | 317800 | 330000 | 366200 | 404000 |
| 10.363 | 34 | 150600 | 183800 | 218200 | 235200 | 269400 | 304600 | 315800 | 350800 | 387600 |
| 10.973 | 36 | 143600 | 175600 | 208600 | 224400 | 257400 | 291400 | 301400 | 335600 | 371200 |
| 11.582 | 38 | 136600 | 167200 | 199000 | 213600 | 245400 | 278200 | 287200 | 320000 | 354800 |
| 12.192 | 40 | 129600 | 158800 | 189400 | 202800 | 233400 | 265000 | 273000 | 304600 | 338200 |
| 12.801 | 42 | 122600 | 150600 | 179800 | 192000 | 221200 | 251800 | 258800 | 289200 | 321800 |
| 13.411 | 44 | 115400 | 142200 | 170200 | 181200 | 209200 | 238600 | 244400 | 273800 | 305400 |
| 14.021 | 46 | 108400 | 134000 | 160600 | 170400 | 197200 | 225400 | 230200 | 258400 | 289000 |
| 14.680 | 48 | 101400 | 125600 | 151000 | 159600 | 185200 | 212200 | 215800 | 243000 | 272600 |
| 15.240 | 50 | 94400 | 117200 | 141400 | 148800 | 173200 | 199000 | 201600 | 227600 | 256200 |

## Columnas de barras de acero en forma de Z, de 12 pulgs (305 mm).

Compuestas de 4 barras en forma de Z de 6 pulgs (152 mm) de altura y de una plancha ó alma de 8 pulgs (203 mm) de ancho.

| Espesor del metal en pulgs | 3/8 | 7/16 | 1/2 | 9/16 | 5/8 | 11/16 | 3/4 | 13/16 | 7/8 |
|---|---|---|---|---|---|---|---|---|---|
| Espesor del metal en mm | 9.5 | 11.1 | 12.7 | 14.3 | 15.9 | 17.5 | 19. | 20.6 | 22.2 |
| Area de la sección en pulgs cuads | 21.4 | 25. | 28.8 | 31.2 | 34.8 | 38.5 | 40.5 | 44.1 | 47.7 |
| Area de la sección en cm cuad | 138.06 | 161.28 | 185.80 | 201.28 | 224.51 | 248.38 | 261.28 | 284.51 | 307.73 |
| Peso en lbs por yarda | 218.1 | 255.6 | 293.4 | 318.6 | 355.5 | 392.7 | 413.4 | 449.7 | 486.3 |
| Peso en kg por metro | 108.192 | 126.794 | 145.545 | 158.046 | 176.350 | 194.804 | 205.073 | 223.079 | 241.236 |
| Radio menor de giro en pulgs | 3.67 | 3.72 | 3.77 | 3.70 | 3.75 | 3.73 | 3.68 | 3.66 | 3.64 |
| Radio menor de giro en cm | 9.32 | 9.45 | 9.58 | 9.40 | 9.52 | 9.47 | 9.35 | 9.30 | 9.25 |

| Longitud de la columna | | Carga de seguridad de la columna en lbs. (*N. del T.* — Multiplicando esta carga en lbs por .4536 se obtendrá en kg.) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| en metros. | en pies. | | | | | | | | | |
| 7.925 ó menos. | 26 ó menos. | 256600 | 300600 | 345200 | 374600 | 418200 | 462000 | 486000 | 529000 | 572200 |
| 8.534 | 28 | 254000 | 299400 | 345000 | 372000 | 417800 | 460600 | 481600 | 522800 | 564200 |
| 9.144 | 30 | 246000 | 290200 | 335200 | 360400 | 405000 | 446600 | 466400 | 506400 | 546400 |
| 9.753 | 32 | 238000 | 281000 | 324800 | 349000 | 392200 | 432600 | 451400 | 490000 | 528400 |
| 10.363 | 34 | 230200 | 271800 | 314400 | 337400 | 379600 | 418400 | 436400 | 473400 | 510400 |
| 10.973 | 36 | 222200 | 262600 | 304000 | 325800 | 366800 | 404200 | 421200 | 456800 | 492600 |
| 11.582 | 38 | 214200 | 253400 | 293600 | 314200 | 354000 | 390200 | 406200 | 440400 | 474600 |
| 12.192 | 40 | 206200 | 244200 | 283000 | 302800 | 341400 | 376000 | 391200 | 423800 | 456600 |
| 12.801 | 42 | 198200 | 235000 | 272600 | 291000 | 328800 | 361800 | 376000 | 407400 | 438800 |
| 13.411 | 44 | 190200 | 225800 | 262200 | 279600 | 316000 | 347800 | 361000 | 391000 | 420800 |
| 14.021 | 46 | 182400 | 216600 | 252400 | 268000 | 303200 | 333600 | 345800 | 374400 | 402800 |
| 14.630 | 48 | 174400 | 207200 | 241400 | 256400 | 290600 | 319600 | 330800 | 358000 | 384800 |
| 15.240 | 50 | 166400 | 198200 | 231000 | 244800 | 277800 | 305400 | 315800 | 341400 | 367000 |

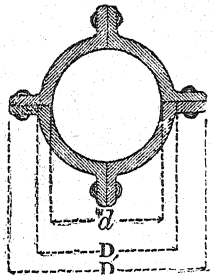



**Tabla de columnas en segmentos,**

**de hierro laminado de la Phœnix Iron Co,**

410 Walnut St., Filadelfia.

Las **dimensiones** dadas están sujetas á ligeras variaciones que son inevitables en la laminación de modelos ó figuras de hierro. Los **pesos** de columnas dados son los de los 4, 6 ú 8 segmentos de que están formadas. Las **espigas** de los remaches que se usan para unirlas, de seguro que compensan la cantidad de metal extraída por el taladro al hacer los agujeros; pero las *cabezas* de los remaches aumentan el peso dado de 2 á 5 por ciento. **Los remaches se separan** 3, 4 ó 6 pulgs (7½, 10 ó 15 cm) de centro á centro.

Pueden suministrarse de **cualquier espesor que se desee.** Damos las dimensiones, peso, etc., correspondientes al espesor principal. Las columnas G son de 8 segmentos; las E, de 6. Todas las otras de 4.

(*N. del T.* — La tabla del autor en sistema métrico.)

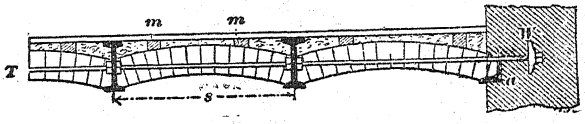
| Marca. | Espesores mm. | **Diámetros** en milím. | | | Una columna. | | | Tamaño de los remaches. |
|---|---|---|---|---|---|---|---|---|
| | | *d* | D | D'. | Area de la sección transversal en cm² | Peso por kg en metros. | Radio menor de giro en mm. | mm. |
| A | 4.762 | 92.075 | 101.6 | 153.99 | 24.51 | 18.774 | 36.83 | 28.575 × 9.525 |
| » | 6.35 | » | 104.77 | 157.16 | 30.96 | 23.840 | 38.10 | 31.75 × » |
| » | 7.937 | » | 107.95 | 160.34 | 37.41 | 28.760 | 39.37 | 34.925 × » |
| » | 9.525 | » | 111.12 | 163.51 | 43.86 | 33.675 | 40.39 | 38.10 × » |
| B¹ | 6.35 | 122.24 | 134.94 | 204.79 | 41.28 | 31.740 | 48.77 | 41.275 × 12.7 |
| » | 9.525 | » | 141.29 | 209.55 | 59.34 | 45.595 | 51.31 | 44.45 × » |
| » | 12.7 | » | 147.64 | 214.31 | 77.42 | 59.600 | 53.59 | 47.625 × » |
| » | 15.875 | » | 153.99 | 219.07 | 95.46 | 73.460 | 55.88 | 53.975 × » |
| B² | 6.35 | 150.81 | 163.51 | 231.77 | 47.73 | 36.654 | 59.44 | 41.275 × » |
| » | 9.525 | » | 169.86 | 236.54 | 68.37 | 52.597 | 61.72 | 44.45 × » |
| » | 12.7 | » | 176.21 | 241.3 | 89.01 | 68.540 | 64.01 | 47.625 × » |
| » | 15.875 | » | 182.56 | 246.06 | 109.68 | 84.335 | 66.29 | 53.975 × » |
| C | 6.35 | 182.56 | 195.26 | 293.69 | 64.52 | 49.617 | 71.12 | 47.625 × 15.875 |
| » | 12.7 | » | 207.96 | 303.21 | 116.13 | 89.400 | 75.69 | 57.150 × » |
| » | 19.05 | » | 220.66 | 309.56 | 162.54 | 125.160 | 80.26 | 66.675 × 19.05 |
| » | 25.4 | » | 233.36 | 319.09 | 214.14 | 164.795 | 84.84 | 76.2 × » |
| » | 31.75 | » | 246.06 | 328.44 | 265.74 | 204.577 | 89.41 | 82.55 × » |
| E | 6.35 | 279.4 | 292.10 | 392.11 | 108.36 | 83.440 | 106.17 | 50.8 × » |
| » | 12.7 | » | 304.8 | 403.22 | 170.28 | 131.120 | 110.74 | 60.325 × » |
| » | 19.05 | » | 317.5 | 414.34 | 243.81 | 187.740 | 115.57 | 69.85 × » |
| » | 25.4 | » | 330.2 | 425.45 | 321.21 | 247.340 | 120.14 | 76.2 × » |
| » | 31.75 | » | 342.9 | 436.56 | 398.61 | 306.940 | 124.71 | 82.55 × » |
| G | 7.937 | 365.125 | 381.0 | 485.77 | 154.84 | 119 200 | 138.43 | 47.625 × 15.875 |
| » | 12.7 | » | 390.53 | 493.71 | 232.26 | 178.800 | 141.99 | 57.150 × » |
| » | 19.05 | » | 403.23 | 504.82 | 335.48 | 258.220 | 146.56 | 66.675 × 19.05 |
| » | 25.4 | » | 415.93 | 517.53 | 438.71 | 337.634 | 151.13 | 76.2 × » |
| » | 34.925 | » | 434.98 | 533.4 | 593.55 | 456.835 | 158.24 | 85.725 × » |

## Conversión de pulgadas y dieciseisavos en milímetros.

| Pulgs. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | » | 25.40 | 50.8 | 76.2 | 101 | 127 | 152 | 177 | 203 | 228 | 254 | 279 |
| 1/16 | 1.58 | 26.98 | 52.3 | 77.8 | 103 | 128 | 154 | 179 | 204 | 230 | 255 | 281 |
| 1/8 | 3.17 | 28.57 | 53.9 | 79.3 | 104 | 130 | 155 | 181 | 206 | 231 | 257 | 282 |
| 3/16 | 4.76 | 30.16 | 55.5 | 81.0 | 106 | 131 | 157 | 182 | 208 | 233 | 258 | 284 |
| 1/4 | 6.35 | 31.74 | 57.1 | 82.5 | 108 | 133 | 158 | 184 | 209 | 235 | 260 | 285 |
| 5/16 | 7.94 | 33.33 | 58.7 | 84.1 | 109 | 134 | 160 | 185 | 211 | 236 | 261 | 287 |
| 3/8 | 9.52 | 34.92 | 60.3 | 85.7 | 111 | 136 | 161 | 187 | 212 | 238 | 263 | 288 |
| 7/16 | 11.11 | 36.51 | 60.9 | 87.3 | 112 | 138 | 163 | 188 | 214 | 239 | 265 | 290 |
| 1/2 | 12.70 | 38.09 | 63.4 | 88.8 | 114 | 139 | 165 | 190 | 215 | 241 | 266 | 292 |
| 9/16 | 14.28 | 39.68 | 65.0 | 90.4 | 115 | 141 | 166 | 192 | 217 | 242 | 268 | 293 |
| 5/8 | 15.87 | 41.27 | 66.7 | 92.0 | 117 | 142 | 168 | 193 | 219 | 244 | 269 | 295 |
| 11/16 | 17.46 | 42.86 | 68.2 | 93.6 | 119 | 144 | 169 | 195 | 220 | 246 | 271 | 296 |
| 3/4 | 19. | 44.44 | 69.8 | 95.2 | 120 | 146 | 171 | 196 | 222 | 247 | 273 | 298 |
| 13/16 | 20.64 | 46.04 | 71.4 | 96.8 | 122 | 147 | 173 | 198 | 223 | 249 | 274 | 300 |
| 7/8 | 22.23 | 47.62 | 73.0 | 98.4 | 123 | 149 | 174 | 200 | 225 | 250 | 276 | 301 |
| 15/16 | 23.81 | 49.21 | 74.6 | 100.1 | 125 | 150 | 176 | 201 | 227 | 252 | 277 | 303 |

## Pisos á prueba de fuego, de vigas en I y arcos de ladrillo.

Los arcos son por lo común de « cuatro pulgs » (10 cm) ó « de medio ladrillo » de alto; luz, *s*, de 4 á 6 pies (1.22 á 1.83 m); flecha, cosa de un dozavo á un dieziseisavo de la luz. Barras T, de trabazón, de 20 mm á 25 mm de diámetro, separadas de 1.20 á 1.80, á 2.40 m y ancladas en cada muro por una placa gruesa, W. En cada muro se usa ordinariamente un ángulo de hierro, *a*, en vez de una viga. Las enjutas

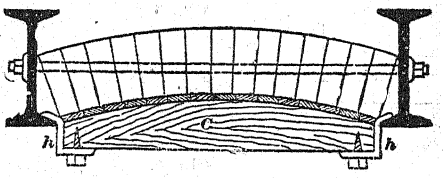



se llenan y nivelan con concreto, incluyendo listones de madera, *mm*, de 2½ á 5 cm; dos sobre cada arco. En estos listones se clava el piso.

El peso de un arco de 10 cm, con su relleno de concreto y su piso de madera, pero sin comprender las vigas, es como de 342 kg por m cuad de piso.

Una densa multitud de personas apenas pesará más de 390 kg por m cuad.

Cada centro, C, tiene adherido en cada extremo una banda de hierro, *h*, que forma un gancho por el cual está el centro suspendido de las alas inferiores de las vigas.



## LA COLUMNA GRAY

**La columna Gray**, ideada y patentada por el Sr. J. H. Gray, consiste, en su forma primitiva, en ángulos, conexionados á intervalos D (generalmente de 2 pies 6 pulgadas) (.76 m) por planchas transversales encorvadas T, ordinariamente de $9 \times {}^{3}/_{8}$ pulgadas (238 mm). Esta construcción hace las partes de la columna fácilmente accesibles, para la pintura, etc.; pero bajo fuerzas transversales tiene la columna que obrar como un marco rectangular sin diagonales. Para remediar esto se ha ideado una forma más reciente, la columna de « doce ángulos », fig. 5, teniendo, en la columna cuadrada, figs. 1 y 2, en lugar de las planchas transversales encorvadas T, cuatro ángulos adicionales, que corren longitudinalmente como se indica. Estos ángulos suplen á la columna dos almas, que se cruzan en ángulos rectos.

Las figs. 1 y 2 hacen ver la columna cuadrada, usada en el interior de los edificios; y se emplea principalmente en los tamaños de 355, 381 y 408 mm. La fig. 2 muestra planchas remachadas á los rebordes exteriores, como se hace en algunas de las columnas más pesadas. Las figs. 3 y 4 son columnas de pared y de ángulo ó rincón respectivamente. La fig. 5 muestra la columna de « doce ángulos » y la fig. 6 una de muchas formas á prueba de fuego, envuelta en bloques de « terra cotta ». Los remaches son ordinariamente de ${}^{3}/_{4}$ de pulg (19 mm) de diámetro.

La carga de seguridad, en libras por pulgada cuadrada, de la columna ordinaria, se calcula como en $17{,}100 - 57\,\frac{L}{r}$, ó sea $1202.13 - 4.007\,\frac{L}{r}$ *kg por cm cuad donde

L=largo de la columna y r=su menor radio de giro.

El costo de la columna Gray, en el taller, es de 1 á 15 centavos por lb (2.2 á 3.3 por kg) más el costo de los ángulos.

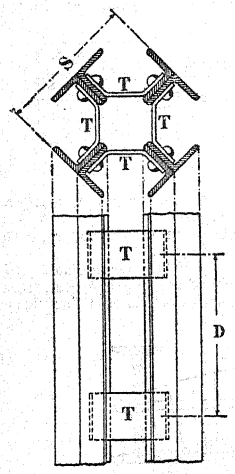



Fig. 1.

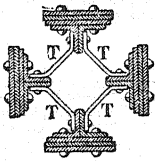



Fig. 2.

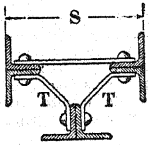



Fig. 3.

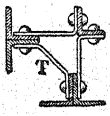



Fig. 4.

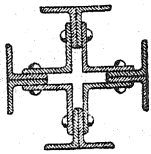


Fig. 5.

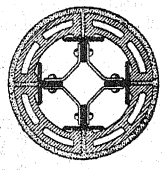


Fig. 6.

* *N. del T.* — Convertida la anterior al sistema métrico.

## Columna Gray. Lista de tamaños escogidos.

(Véase la *Obs. del T.* después de la tabla.)

| Tamaño S. pulgs. | Angulos. pulgs. | Area pulgs cuad. | I momento de inercia | $r$ radio mínimo de giro pulgs. | $r^2$. | Cargas de seguridad * para columnas de una longitud de 12 pies. | Cargas de seguridad * para columnas de una longitud de 30 pies. |
|---|---|---|---|---|---|---|---|
| **Columnas cuadradas.** Figs. 1 y 2. | | | | | | | |
| 9 | 2 ×2½×¼ | 8.48 | 64 | 2.7 | 7.3 | 115 | 80 |
| » | 2 ×3 ×¼ | 18.00 | 119 | 2.6 | 6.8 | 250 | 165 |
| 10 | 2½×2½×¼ | 9.52 | 95 | 3.1 | 9.6 | 135 | 100 |
| » | 2½×3 ×½ | 20.00 | 179 | 3.0 | 9.0 | 285 | 205 |
| 12 | 3 ×3 ×⅜ | 16.88 | 241 | 3.8 | 14.4 | 250 | 195 |
| » | 3 ×5 ×½ | 30.00 | 327 | 3.3 | 10.9 | 435 | 325 |
| 13 | 3 ×3 ×⅜ | 16.88 | 285 | 4.1 | 16.8 | 255 | 205 |
| » | 3 ×5 ×¾ | 43.52 | 552 | 3.6 | 13.0 | 645 | 495 |
| 14 | 3 ×3 ×⅜ | 16.88 | 336 | 4.5 | 20.2 | 255 | 210 |
| » | 3 ×3½×⅝ | 29.36 | 526 | 4.3 | 18.5 | 445 | 360 |
| » | 3 ×4 ×½ | 26.00 | 444 | 4.2 | 17.6 | 390 | 315 |
| » | 3 ×5 ×½ | 30.00 | 463 | 4.0 | 16.0 | 450 | 355 |
| » | 3½×3½×⅝ | 31.92 | 597 | 4.4 | 19.4 | 485 | 395 |
| » | 3½×5 ×⅝ | 39.36 | 624 | 4.0 | 16.0 | 590 | 470 |
| » | 3½×6 ×½ | 36.00 | 526 | 3.8 | 14.4 | 535 | 420 |
| 15 | 3 ×3 ×½ | 22.00 | 496 | 4.8 | 23.0 | 335 | 280 |
| » | 3½×3½×⅝ | 31.92 | 693 | 4.7 | 22.1 | 490 | 405 |
| » | 3½×5 ×⅝ | 39.36 | 731 | 4.4 | 19.4 | 600 | 490 |
| » | 4 ×4 ×½ | 30.00 | 653 | 4.7 | 22.1 | 440 | 380 |
| » | 4 ×5 ×⅝ | 41.84 | 817 | 4.4 | 19.4 | 635 | 520 |
| » | 4 ×6 ×⅝ | 46.88 | 828 | 4.2 | 17.6 | 710 | 570 |
| 16 | 3 ×3 ×½ | 22.00 | 570 | 5.1 | 26.0 | 340 | 285 |
| » | 4 ×4 ×⅝ | 36.88 | 912 | 5.0 | 25.0 | 570 | 480 |
| » | 4 ×6 ×¾ | 55.52 | 1,134 | 4.6 | 21.2 | 850 | 700 |
| 18 | 3 ×3 ×½ | 22.00 | 746 | 5.8 | 33.6 | 345 | 300 |
| » | 4 ×4 ×⅝ | 36.88 | 1,182 | 5.7 | 32.5 | 575 | 495 |
| » | 5 ×5 ×⅝ | 46.88 | 1,465 | 5.6 | 31.4 | 730 | 630 |
| 20 | 4 ×4 ×⅝ | 36.88 | 1,485 | 6.4 | 41.0 | 580 | 510 |
| » | 6 ×6 ×¾ | 67.52 | 2,588 | 6.2 | 38.4 | 1,065 | 930 |
| 30 | 5 ×4 ×⅝ | 41.88 | 4,147 | 9.9 | 98.0 | 680 | 630 |
| » | 6 ×6 ×1** | 264.40 | 22,688 | 9.2 | 84.6 | 4,285 | 3,930 |
| **Columnas de pared.** Fig. 3. | | | | | | | |
| 12 | 3 ×4 ×⅜ | 14.88 | 94 | 2.5 | 6.2 | 205 | 130 |
| 14 | 3 ×3½×½ | 18.00 | 160 | 3.0 | 9.0 | 255 | 185 |
| » | 3½×5 ×⅝ | 29.52 | 241 | 2.9 | 8.4 | 420 | 295 |
| 15 | 3½×4 ×½ | 21.00 | 217 | 3.3 | 10.9 | 305 | 225 |
| » | 4 ×5 ×⅝ | 31.38 | 317 | 3.2 | 10.2 | 455 | 335 |
| 16 | 3 ×3 ×½ | 16.50 | 200 | 3.5 | 12.2 | 240 | 185 |
| » | 4 ×6 ×⅝ | 35.16 | 375 | 3.3 | 10.9 | 510 | 380 |
| 18 | 4 ×4 ×⅝ | 27.66 | 434 | 4.0 | 16.0 | 415 | 330 |
| 20 | 4 ×5 ×⅝ | 31.38 | 562 | 4.2 | 17.6 | 475 | 380 |
| 30 | 5 ×4 ×⅝ | 31.38 | 1,408 | 6.8 | 46.2 | 495 | 440 |
| » | 6 ×6 ×1** | 198.30 | 8,937 | 6.7 | 44.9 | 3,150 | 2,785 |
| **Columnas de rincón.** Fig. 4. | | | | | | | |
| 10½ | 3½×3½×13/16 | 23.48 | 272 | 3.4 | 11.6 | 345 | 260 |
| 12½ | 3½×3½×½ | 15.75 | 288 | 4.3 | 18.5 | 240 | 195 |
| 13 | | | | | | | |
| 13 | 4 ×5 ×⅝ | 24.91 | 425 | 4.2 | 17.6 | 375 | 305 |
| 14 | | | | | | | |

* En millares de lbs, por la fórmula : carga segura $= 17{,}100 - 57\,\frac{L}{r}$ en lbs por pulg cuad. Véase la misma fórmula en sistema métrico, pág. 981.

** Tres planchas de 1 pulg remachadas á cada par de ángulos. Fig. 2.

(*Obs. del T.*— Como la tabla anterior da los elementos de un tipo especial de columna, la hemos dejado en medidas inglesas, pero damos á continuación todas las conversiones necesarias (para las diversas unidades que aquélla trae) en ambos sistemas.

Nos permitimos recordar al lector que como el momento de inercia es un producto de cuatro dimensiones (véase § 14, pág. 490, en Resistencia de Materiales) y, si están éstas tomadas en una unidad, por ejemplo en metros, queremos hallar su valor en otra unidad, se multiplicará, ó dividirá, según el caso, por 10,000 $n$, indicando $n$ la diferencia en el orden de unidades de medida, así para pasar á milímetros se multiplicará por $10{,}000^4$.

Como una pulgada equivale á 2.54 cm ó á 25.4 mm, si queremos pasar del momento de inercia en pulgs al momento de inercia en centímetros, multiplicaremos á aquél por la cuarta potencia de $2.54 = 41.6231$, y si lo queremos en mm, por la cuarta potencia de $25.4 = 416{,}231.4256$. Para pasar de cm ó mm á pulgs se hará lo contrario.)

# COLUMNAS DE HIERRO Y DE ACERO

**Para columnas en general,** véanse págs. 527, etc.
**Para columnas de madera,** véanse págs. 1183, etc.

### Lista de referencias.


(1) Acero en construcción, Pencoyd Iron Works, 12.ª ed, 1900.
(2) Cambria Steel Co, Manual, 1907.
(3) Carnegie Steel Co, Compañero de Bolsillo, 1903.
(4) Phœnix Iron Co, Manual, 1906.
(5) Bethlehem Steel Co, Acero de construcción, 1907.
(6) Passaic Steel Co, Manual, 1903.
(7) Actos y Resoluciones de la Legislatura de Massáchusets, 1907.
(8) Trans. Am. Soc. C. E., vol. 15, pág. 530, julio 1886.
(9) Trans. Am. Soc. C. E., vol. 20, pág. 258, junio 1889.
(10) Trans. Am. Soc. C. E., vol. 54, junio 1905.
(11) New York Building Code, Aprobado en 24 de octubre de 1899 con modificaciones hasta abril 12 de 1906.
(12) Eng. News, 13 de enero, 1898, págs. 27, etc.
(13) Eng. News, 30 de junio, 1898, pág. 424.
(14) Modern Framed Structures, por J. B. Johnson, Nueva York, John Wiley and Sons, 1893.
(15) Ciudad de Nueva York. Departamento de Puentes, Especificaciones para el Diseño de Puentes Municipales, 1907.
(16) Comunicaciones del Instituto de Examen de Materiales á la Politécnica de Suiza en Zurich, Entrega VIII, 1896.
17) « Pruebas de Metales, etc. » Watertown Arsenal. Año terminado el 30 de junio de 1888.
(18) « Pruebas de Metales, etc. » Watertown Arsenal. Años terminados en 30 de junio de 1886, 1890, 1894.


(*N. del T.*— Hemos dejado estas referencias para los lectores que traducen inglés ó alemán y desearen más detenido estudio de la materia.)

**1.** Para economizar material, las columnas de hierro y acero son casi siempre **huecas.** La columnas de hierro fundido son de ordinario redondas ó rectangulares, mientras que las columnas de construcción de acero son por lo común de alguna forma especial ó hechas de piezas; véanse figs. 1 á 12 y págs. 1237.

**2.** Debido al desplazamiento del molde en la fundición, las columnas de hierro fundido resultan á veces mucho más delgadas de un lado que de otro. Son susceptibles de fuerzas iniciales, debidas á la contracción desigual en el enfriamiento, y son objetables también á causa de la fragilidad y relativa inseguridad del material y su baja resistencia de tensión. Cuando el metal, para columnas muy largas, se vierte en ambos extremos del molde, puede endurecerse de tal manera que las dos porciones no se unan perfectamente en el centro; quedando así una región débil precisamente donde obra el momento máximo de flexión. El aire arrastrado por el

metal fundido produce cavidades y rebolliduras; y las impurezas, que se reunen en el fondo del molde, debilitan la fundición. (Véase también el § 14).

Véanse Columnas en General, §§ 9, 10, 13 y 35 á 43, págs. 527, etc.

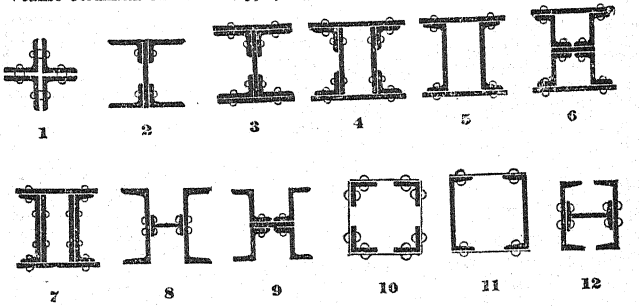



**Fig. 1 á 12.**

**3. Evítense** los diseños que, como los en *g* y en *aa*, fig. 13, sacan parte del fuste de la columna de la línea de presión produciendo **momentos de flexión** en el metal.

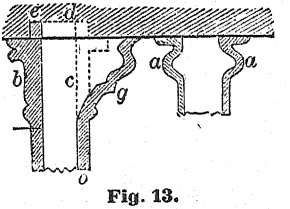



**Fig. 13.**

**4.** Usese con preferencia una forma más sencilla, como en *b*, ó una columna derecha, *ocd*, á la cual pueden adherirse cualesquiera adornos como los que se ven en *g*.

**5.** Las figs. 1 á 12 muestran **formas comunes de columnas** hechas de ángulos, planchas, piezas en U y barras en Z, de acero. Las **secciones cerradas**», figs. 4 á 7, no pueden repintarse interiormente. Sólo son convenientes, por tanto, para sitios secos en donde los cambios son ligeros. En las figs. 8, 10, 11, 12 los rebordes deben colocarse bastante distantes para permitir el remachado á máquina.

**6.** En las figs. 14 á 17, de la «Cambria Steel Co», se ven pormenores de **conexiones** entre una columna y otra y entre columnas y vigas, como se encuentran ya hechas. Las figs. 18 á 20, de la «Cambria Steel», muestran disposiciones para **bases de columnas**, que también se encuentran ya hechas.

**7. Acostúmbrase estipular que $K = L/r$ no exceda de más ó menos** 125; pero en estructuras ligeras, K excede á veces considerablemente de ese límite. Así, en once torres de transmisión, K varió de 111 á 300. Donde llegó á 300, la pieza consistía en 1 ángulo de $2 \times 2 \times {}^{1}/_{4}$ pulg ($50 \times 50 \times 3$ mm), L=120 pulgs (3050 mm); $r$=.4 pulgs (10 mm). (R. D. Coombs, Am. Soc. Civ. Engrs., Trans., vol. 61, Dic. 1908, pág. 1202.)

**8. En columnas compuestas de dos vigas en U**, enrejadas, fig. 11, las vigas están ordinariamente colocadas á tal distancia, que la tendencia de la columna será á flexarse en el plano de las almas, esto es, de modo que el mínimo radio de giro de la columna es el radio máximo de giro de la viga en U.

**9.** Siendo asimétricas las secciones de ángulo, la carga es generalmente más ó menos excéntrica. En consecuencia, la fuerza permitida para columnas de ángulo debe ser menor que la permitida para las de secciones simétricas.

**10. Los remaches están por lo común separados** >7½ cm de centro á

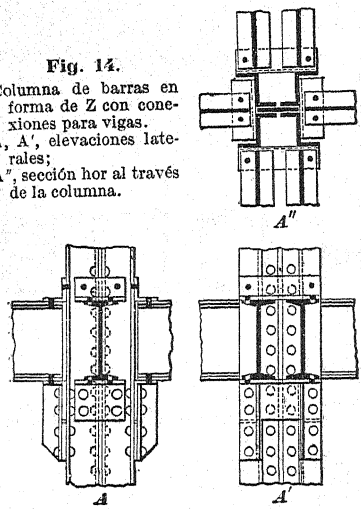



**Fig. 14.**

Columna de barras en forma de Z con conexiones para vigas.
A, A', elevaciones laterales;
A'', sección hor al través de la columna.

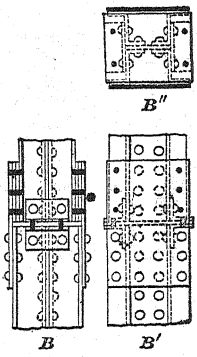



**Fig. 15.**

Empate entre 2 columnas de barra en Z de diferentes tamaños.
B, B' elevaciones laterales;
B'', sección hor al través de la columna.

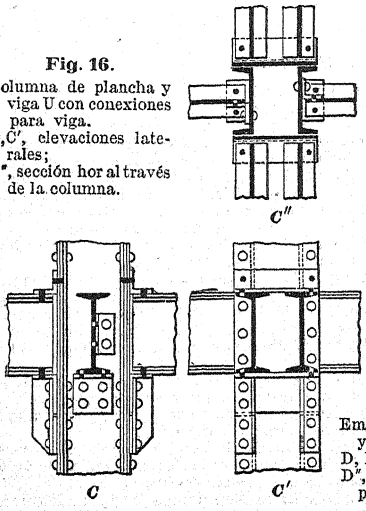



**Fig. 16.**

Columna de plancha y viga U con conexiones para viga.
C, C', elevaciones laterales;
C'', sección hor al través de la columna.

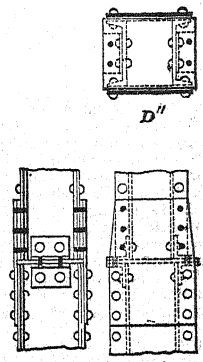
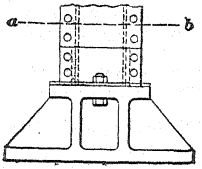


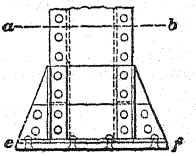

**Fig. 17.**

Empate entre 2 columnas de plancha y viga U de dif tamaños.
D, D', elevaciones lateral;
D'', sección hor al través de los empates.



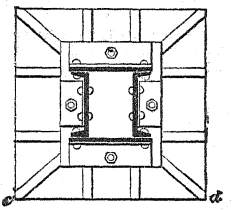

*Elevación c-d.*

*E*




*Elevación g-h.*

*F*

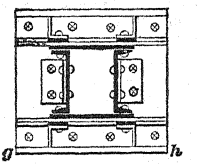



*Sección a-b.*

*E'*

**Fig. 18.**

Base de hierro fund para col. de plancha y de viga U.

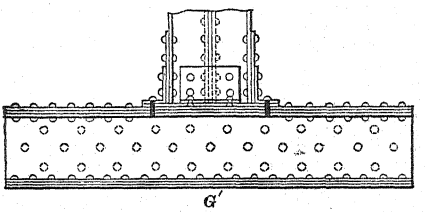



*Sección a-b.*

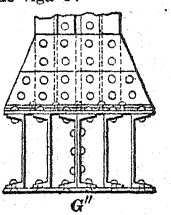
*F'*

**Fig. 19.**

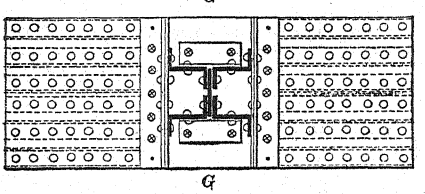
Base de acero para col de plancha y de viga U.



*G'*



*G''*



*G*

**Fig. 20.**

Apoyos de vigas en U y planchas, para columnas de barras en Z.
G, sección horizontal al través de la columna; G', elevación lateral; G", elevación en el extremo.

centro, cerca de los extremos de las columnas, en una distancia igual á dos veces el ancho de la columna. La distancia entre centros de remaches, en la línea de los esfuerzos, $>16$ veces el menor espesor de metal de las partes unidas. La distancia entre remaches, perpendicular á la línea de los esfuerzos. $>32$ veces el espesor del metal.

**11. Las barras de celosía, las planchas de tablillas y conexiones** pesan juntas el 30% ó más de las columnas pesadas, y 50 ó 60 % del peso de las columnas más livianas.

**12.** En la construcción de edificios, las **columnas pueden considerarse como de extremos cuadrados. Factor** habitual **de seguridad**=4. Véanse §§ 14 y 18.

**13.** Las columnas de ángulos ó vigas en I son generalmente de **acero blando**; las de planchas y ángulos, vigas en U, planchas ó barras en Z, se calculan por lo común para **acero mediano.**

**14. Las columnas de hierro fundido** no permiten conexiones tan rígidas, con otras columnas ó con vigas, como las columnas de acero laminado y las remachadas. No deben usarse en edificios, con un factor de seguridad menor de 8. (Véanse también §§ 2, 12, 18.)

**15. Á fin de evitar el efecto debilitante de los agujeros de los remaches,** en las columnas de acero remachadas, se emplea la columna sólida en forma de H (cuya sección transversal se ve en la fig. 21).

**Fig. 21.**

**16.** (*Obs. del T.* — La importante fórmula de Rankine, con sus coeficientes, etc., que trae el autor en esta parte, la hemos planteado y discutido en sistema métrico (siguiendo á Marvá) y hemos dejado (aunque también convertidos en unidades métricas) los coeficientes y en general los datos que sirven de base á las casas constructoras y á muchas obras importantes.)

Se aplica la siguiente fórmula de Rankine á piezas de sección cualquiera; con estos elementos :

P=carga permanente de trabajo ordinario;
R'=coeficiente de trabajo por compresión, del hierro, en kg por mm cuad empleado en modelos cortos=6;
$\omega$=sección en mm cuad cuando está en el numerador;
$l$=longitud de la pieza;
I'=menor de los momentos de inercia de la sección recta;

B=un coeficiente numérico cuyo valor es :

| | | |
|---|---|---|
| Caso 1. | Para dos bases planas (ó empotradas) | .00003 |
| — 2. | Una base plana y otra redondeada ó articulada | .000045 |
| — 3. | Dos bases redondeadas ó articuladas | .00006 |
| — 4. | Un extremo empotrado y otro libre | .00008 |

La fórmula es :

$$P = \frac{R'\omega}{I + B\frac{\omega l^2}{I'}} \text{ ; ó bien } R' = \frac{P}{\omega}\left(1 + B\frac{\omega l^2}{I'}\right).$$

El cociente $\frac{\omega l^2}{I'}$, como tiene cuatro dimensiones en el numerador y cuatro en el denominador, dará el mismo valor siempre que $\omega$, $l$, ó I' se refieren á la misma unidad de medida cualquiera que sea ésta.

Para el acero dulce de construcción se puede hacer R'=9 kg por mm cuad.

**17.** Según Rankine, en el caso 1 (bases planas ó empotradas), B=.000033. Bauschinger encontró experimentalmente que B=.000027 para hierros en que la carga de ruptura R'=31.5 kg por mm cuad. Preandeau acepta B=.00002.

**18.** En el caso 3 (dos bases redondeadas ó articuladas), Bauschinger dedujo como promedio del resultado obtenido en 27 experiencias con hierros en que el coeficiente de fractura por aplastamiento era R'=22.7 kg por mm cuad, que B=.000058.

Puede, pues, hacerse uso para el cálculo de piezas laminadas ó compuestas, de sección cualquiera, de las siguientes fórmulas :

$$P = \frac{6\omega}{1 + B\,\frac{\omega l^2}{I'}} \text{ (para el hierro)};$$

y la misma reemplazando el 6 por 9 para el *acero;* dando por supuesto á B los valores establecidos anteriormente.

**19.** M. Lebel encuentra defectuosa la fórmula de Rankine, porque no tiene en cuenta la excentricidad del esfuerzo de compresión, siendo muy frecuente que el centro de presión ó punto de aplicación de la fuerza P, no coincida con el centro de gravedad de la base, y propone para el caso de sólido de bases planas ó empotradas la fórmula siguiente :

$$P = \frac{R\omega}{.8 + .005\,\frac{dl\omega}{I'}};$$

siendo *d* la excentricidad de la compresión, ó sea la distancia del punto de aplicación de esta fuerza al centro de gravedad de la base.

(*Obs. del T.* — En los datos que siguen, aunque damos siempre su equivalencia en las unidades usadas en el sistema métrico, dejamos además los valores en medidas inglesas, porque como son los empleados por fabricantes y Compañías americanas, pueden ser de mucha utilidad para cualquier pedido, ó para cualquier estudio que pueda hacerse entre ingenieros, habituados unos á emplear solamente el sistema inglés y los otros sólo el métrico.)

## Valores de la presión máxima (s) por unidad de superficie y del coeficiente B para columnas de acero y hierro *.

Para referencias, véase lista pág. 1236.

**TS** = Máxima fuerza de tensión; $\mathbf{S}_e$ = límite de elasticidad; **PC** = punto cedente**, todo en miles de libras por pulg **(y entre paréntesis y debajo de cada cifra su equivalente en kg por cm cuad.** *N. del T*).

---

* *N. del T.* — Hemos convertido el coeficiente 1/m que trae el autor al equivalente B de la fórmula y lo damos en cien milésimas.

** *N. del T.* — Este punto lo llaman los americanos « Yield point », y como hemos consultado muchos diccionarios y varios ingenieros notables sin encontrar cómo se expresa en una sola palabra su significado, hemos resuelto llamarlo « Punto cedente », porque es precisamente el punto en que la barra sigue alargándose *sin que se aumente el esfuerzo á que está sometida* y puede considerarse como el tercer período elástico (Marva, *Mecánica aplicada*), caracterizado además porque el alargamiento se localiza y se « forma el huso » ó región de la gran contracción transversal ó *estrechamiento de sección*.

## ACERO Y HIERRO FORJADO

### Para edificios, cargas máximas (ó de ruptura).

Cambria Steel Co. Acero mediano y dulce (2). págs. 194–7.
Carnegie Steel Co. (3), págs. 143–4 y Phoenix Iron Co. (4), pág. 88.
Factores de seguridad; carga muerta, F=4;
carga viva, F=5.

| Acero. | S Dulce. | S Mediano. | B en cien milésimas. |
|---|---|---|---|
| Extremos redondeados ó articulados | 45,000 (3,150) | 50,000 (3,500) | 5.5 |
| Un extremo redondeado ó articulado | 45,000 (3,150) | 50,000 (3,500) | 4.0 |
| Extremos cuadrados | 45,000 (3,150) | 50,000 (3,500) | 2.7 |

Passaic Steel Co., **hierro forjado** (6), pág. 153.

| | S | B |
|---|---|---|
| Extremos redondeados ó articulados | 40,000 (2,812) | 5.0 |
| Extremos cuadrados | 40,000 (2,812) | 3.3 |
| Extremos fijos | 40,000 (2,812) | 5.0 |

### Para edificios, cargas permitidas.

| | | | |
|---|---|---|---|
| Boston building code * (7), pág. 415. (Código de construcción de Boston.) | **Acero** | 16,000 (1,125) | 5.0 |
| | **Hierro forjado** | 12,000 (843) | 5.0 |

### Especificaciones para puentes, cargas permitidas.

| Osborn Engineering Co., 1903–4. | TS. | $S_e$ | PC. | S Vapor. | S Eléctrico. | S Caminos. | B en cien milésimas. |
|---|---|---|---|---|---|---|---|
| **Hierro forjado.** | 48 (3.36) | — | 25 (1.75) | 13,000 (914) | 15,000 (1,054) | 18,000 (1,260) | |
| **Acero dulce.** | 52–62 (3.64)–(4.34) | 32 (2.24) | — | 15,000 (1,054) | 17,000 (1,195) | 20,000 (1,406) | |
| **Acero mediano.** | 60–70 (4.20)–(4.90) | 35 (2.45) | — | 17,000 (1,195) | 19,000 (1,336) | 22,000 (1,540) | |
| Ambos extremos articulados | | | | | | | 5.5 |
| Un extremo cuadrado | | | | | | | 4.0 |
| Ambos extremos cuadrados | | | | | | | 2.7 |

| **Acero de construcción.** | S | B en cien. milésimas |
|---|---|---|
| Phila and Reading RR, 1906; TS=60 (4.2) | 15,000 (1,054) | 8.3 |
| Pensylvania RR, revisado hasta enero 1.º de 1907; Dulce; TS=52 á 62; $S_e$ = 28 (3.6 — 4.3) (1.96) | 16,000 (1,125) | 8.3 |
| Erie Railroad, 1900; revisado hasta junio 1.º de 1905; Dulce; TS=56 á 64; $S_e$ = .58 TS. (3.9 — 4.5) | | |

Ambos extremos articulados; Un extremo fijo; Ambos extremos fijos;

$$S^{**} = 8{,}000\left(1 + \frac{\text{fuerza mín}}{\text{fuerza máx}}\right)$$

B: 5.5, 4.0, 2.7

* El código de Boston usa la fórmula de Rankine para columnas de acero y de hierro forjado; el método de J. R. Worcester para madera, y (aparentemente) la fórmula de la línea recta de T. H. Johnson para hierro fundido.

** Véase *N. del T.* al comienzo pág. siguiente.

(*N. del T.* — Para calcular está última presión en kg por cm cuad, reemplácese simplemente el factor 8,000 por 560.)

| | s | | |
|---|---|---|---|
| | Carga muerta. | Viva. | |
| New York Central RR, 1904; | | | |
| TS=56 á 64; $S_e$=33 | 16,000 | 8,000 | 5.5 |
| (3.9 — 4.5) | (1,123) | (562) | |
| Del Lacka and W R R, nov. 1903; | | | |
| Dulce; TS=54 á 62; $S_e$=.5 TS. | | | |
| Extremos articulados | 12,500 | 8,500 | 5.5 |
| | (875) | (597) | |
| Uno ó ambos extremos fijos | 12,500 | 8.500 | 4.0 |
| | (875) | (597) | |
| Consulting Engineer, 1907, Puentes de camino real. | | | |
| **Hierro forjado** | 16,000 | 8,000 | 12.5 |
| | (1,125) | (562) | |
| **Acero mediano** | 20,000 | 10,000 | 12.5 |
| | (1,406) | (703) | |
| **Acero de níquel** | 30,000 | 15,000 | 12.5 |
| | (2,100) | (1,050) | |
| El profesor Mansfield Merriman cree que, para columnas de **acero**, los mejores valores para B son próximamente los siguientes : | | | |
| Extremos redondos | | | 16.6 |
| Extremos articulados | | | 12.5 |
| Extremos cuadrados | | | 5.5 |
| Extremos fijos | | | 4.0 |

## HIERRO FUNDIDO

### Para edificios, cargas máximas (de ruptura).

| | | |
|---|---|---|
| Cambria Steel Co. (2), pág. 284; F=8 | 80,000 | 12.5 |
| | (5,600) | |
| Cargenie Steel Co. (3), pág. 148; extremos articulados.. | 80,000 | 25 |
| | (5,600) | |
| Passaic Steel Co. (6), pág. 206; un extremo articulado.. | 80,000 | 18 |
| | (5,600) | |
| Extremos cuadrados.. | 80,000 | 12.5 |
| | (5,600) | |

### Para edificios, cargas permitidas.

| | | |
|---|---|---|
| Chicago building code (6), pág. 119 | 10,000 | 6,000 |
| | (703) | (16.6) |

### Ensayos.

## ACERO DE CONSTRUCCIÓN

**p**=carga media por unidad de superficie en la columna;
**s**=esfuerzo máximo por unidad de superficie en la sección transversal;
$s_s$=la misma anterior para bloques cortos;
**TS**=tensión máxima;
$S_e$=límite de elasticidad;
**K**=$L/r$=longitud dividida por el radio mínimo de giro.

(*Obs. del T.* — En lo que sigue están convertidos los resultados que trae el autor al sistema métrico. Así damos los valores de $p$, $s$, etc., en kilogramos por cm cuad; y aunque en este capítulo el autor no indica en qué unidades están TS y $s_e$ los hemos convertido suponiéndolos en miles de lbs por pulg cuad (porque así vienen hasta ahora), y los damos en miles de kg por cm cuad. Como K es un cociente que expresa la altura de la columna en *radios de giro*, es el mismo para cualquier sistema de medida.)

## Resistencia máxima de las columnas articuladas.

**Jas G. Dagron.** (Am. Soc. C. E. Trans., June 1889, vol. 20, pág. 254). 8 cols enrejadas, de sección rectangular, armadas de planchas y ángulos. Carbón .26 á .27%; TS = 5.86 á 5.91; $s_e$ = 3.58 á 3.77;
6 cols de 2 planchas de 203×6 mm, 4 áng de 57×57×6 mm; 4.88 á 7.32 m.
2 — — 228×9.5 mm, 4 — 70×70×8 mm; 7.81 m.

+ **C. P. Buchanan.** (Eng. News., dec. 26, 1907.) 7 cols de puente como sigue: Bessemer (Bess) y Martín Siemens (M. S.)

| N.° | | | | Carbón % | K | $p$. |
|---|---|---|---|---|---|---|
| 3, Postes, | 4 barras Z, | 76 mm, | Almas 150× 9.5 mm, | Bess — | 83 | 2399 |
| 4, Postes, | 4 barras 2, | 76 mm, | Almas 150× 9.5 mm, | Bess — | 83 | 2303 |
| 9, Postes, | 4 áng, | 150×89 mm, | Almas 273× 9.5 mm, | Bess — | 97 | 1944 |
| 16, Cordones, | 2 áng, | 76×76 mm, | 2 almas 406× 9.5 mm, | | | |
| | 2 áng, | 101×76 mm, | tapa, 558× 9.5 mm, celosía 127×11.0 mm, | (M. S.) .21 | 46 | 2145 |
| 17, Postes, | 4 áng, | 76×76 mm, | 2 almas 254× 9.5 mm, Celosía (M. S.) | .23 | 45 | 2,218 |
| 18, cordón<br>19, cordón | 4 áng 76×76 mm | | 1 alma 457×9.5 mm<br>1 cubierta 508×9.5 mm<br>celosía 57×11 | (M.S.). 15 | 34<br>35 | 2,23<br>2,27 |

**J. A. L. Waddell,** Eng. News, enero 16, 1908. 6 columnas de puente de níquel acero y 6 de carbón acero. Cada una de 4 ángulos, 76×76×9.5 mm, 2 almas planas, 305×9.5, y celosía 63×9.5. Largos 3.05 y 9.15 m, $r$ = 113 mm, $a$ = 112.5 cm cuad. Níquel acero, TS = 7 — 8.05; $s_e$ ≮ 4.2. Níquel, 3.5%; carbón, .38%; manganeso, .30%, Carbón acero. TS = 4.2 — 4.9; $s_e$ ≮ 2.45.

| | | | | | Término medio. |
|---|---|---|---|---|---|
| Níquel acero, | K = 27, | $p$ = 4,795 | 4,795 | 4,844 | 4,809 |
| — — | K = 81, | $p$ = 3,108 | 3,304 | 2,975 | 3,129 |
| Carbón acero, | K = 27, | $p$ = 2,723 | 2,723 | 2,786 | 2,744 |
| — — | K = 81, | $p$ = 2,072 | 2,072 | 2,268 | 2,135 |

**Comisión del puente «Quebec».** (Eng. News, abril 23 de 1908). Dos cordones de acero mediano, á saber: N.° 1, un tercio de tamaño de «A9L» (véase § 26) y de sección semejante, y n.° 2, con 2, en vez de 4 piezas; las piezas idénticas á las exteriores del n.° 1; pero los ángulos de la celosía eran 50% más pesados; los remaches en las conexiones de la celosía duplicados; las intersecciones de los ángulos de la celosía reforzadas con planchas angulares. Pasadores, 305 mm de diámetro, como en «A9L».

| N.° | Area cm cuad. | Largo c. á c. | L/$r$ * | ** | Carga de *** ruptura en indicada § | kg por cm cuad neta §. |
|---|---|---|---|---|---|---|
| 1 | 558.0 | 579 cms | 42 | 35 | 1,879 | 1,550 |
| 2 | 275.0 | 346 cms | 25 | 15 | 2,590 | 2,136 |

* Eje paralelo al pasador.
** Eje paralelo al alma.
*** Carga neta recibida = 82.5 0/0 de la carga indicada, á causa de error de la máquina de pruebas. Tanto la carga indicada como la carga neta están proyectadas, con L/$r$ para eje paralelo al pasador, habiendo cedido ambas columnas de prueba en planos perpendiculares á los pasadores.
§ Excéntrica respecto al eje. El espesor, en el fondo, varió de 20 á 27 mm.

## HIERRO FUNDIDO

### Ensayos.

### Columnas huecas.

+ Pruebas sobre 10 columnas cilíndricas en Phoenixville para el Departamento de construcción de Nueva York, como sigue :

| Diám exterior mm. | Espesor. mm. | Largo. metros. | K. | $p$ máxima kg por cm cuad. |
|---|---|---|---|---|
| 381 | 25 | 4.826 | 38.3 | 2,156 |
| 381 | 28 | 4.826 | 38.7 | 1,939 |
| 381 | 28 | 4.826 | 38.7 | 1,743 |
| 381 | 28 | 4.826 | 38.3 | 1,764 |
| 381 | 29 | 4.826 | 38.5 | 2,247 |
| 381 | 30 | 4.826 | 38.8 | >2,828 |
| 203 | 25 | 4.064 | 64.0 | 2,233 |
| 203 | 26 | 4.064 | 64.4 | 1,876 |
| 152 | 29 | 3.048 | 67.2 | 1,589 |
| 152 | 28 | 3.048 | 66.5 | 1,841 |

✱ Pruebas de 5 columnas cilíndricas de 4 m más ó menos de largo, en el arsenal de Watertown ·

| Diámetro, mm exterior D | interior $d$ | K | $p$ máxima. |
|---|---|---|---|
| 221 | 152 | 60 | >1,800 |
| 201 | 135 | 65.9 | >2,127 |
| 183 | 122 | 72.4 | 1,783 |
| 163 | 104 | 83 | 1,905 |
| 145 | 102 | 90.6 | 1,757 |

### Columnas macizas.

■ Resistencia media, generalmente aceptada, para aplastamiento de bloques cortos, 7,000 kg por cm cuadrado.

● Pruebas de 6 cilindros pequeños en el Arsenal de Watertown, (18).

| N.º | Diám. mm. | $r$ mm. | Largo. mm. | K. | $p$ Carga máxima. kg por cm cuad. |
|---|---|---|---|---|---|
| 1 | 20.3 | 5.1 | 102 | 20 | 6,734 |
| 2 | 20.3 | 5.1 | 127 | 25 | 6,273 |
| 3 | 20.3 | 5.1 | 127 | 25 | 6,034 |
| 4 | 19.0 | 4.7 | 139 | 29.3 | 5,365 |
| 5 | 19.0 | 4.8 | 139 | 29 | 5,337 |
| 6 | 28.7 | 7.2 | 267 | 37.5 | 4,410 |

▲ Pruebas de 5 cilindros con extremos cuadrados cortos, en el Arsenal de Watertown (17), págs. 737-742. Diámetro=51 mm. $r$=12½ mm. L=largo de la porción redonda=228 mm. K=18. $p$, máx de 4,326 á 5,235 kg por cm cuad.

**24.** Los experimentos en cilindros pequeños sólidos de hierro fundido, arriba registrados, y muchos otros por Tetmajer (16), indican que la fórmula de Rankine es correcta en su forma; pero los experimentos prácticos con columnas cilíndricas huecas demuestran que aquellas columnas fallan bajo cargas inferiores á las dadas por la fórmula, especialmente en el caso de columnas cortas. En columnas huecas delgadas la excentricidad de la carga, que siempre ocurre, aun en ensayos cuidadosos, y mucho más en la práctica de las construcciones, concentra la mayor parte de la presión en un lado, y hace que ese lado obre sin el apoyo debido del resto de la sección. Impídese así que la columna obre como un cuerpo enterizo.

**25. Tetmajer** (16). pág. 77. encuentra que las **columnas remachadas** de acero de construcción **se conducen lo mismo que simples columnas de hierro laminado** con tal que :

(1) La distancia entre los centros de los remaches no exceda de $70 \times$ el espesor del reborde;

(2) Los remaches llenen completamente sus huecos;

(3) El debilitamiento de la sección por los huecos de los remaches $>$ como de 12 por ciento.

**26. Fracaso del puente de Quebec.** El hundimiento de la porción meridional del gran puente de *cantilever* de acero sobre el río San Lorenzo, cerca de Quebec, el 29 de agosto de 1907, durante su construcción, parece haberse debido á la falla de un elemento principal de compresión, « A9L », en el brazo de anclaje. Esta pieza, fig. 25, de 17.38 m de largo, estaba compuesta de cuatro almas, cada una de 89 á 95 mm de espesor (hecha de cuatro planchas de más ó menos 22 mm de grueso, remachadas entre sí) y de 1.37 m de ancho, provistas de ángulos de reborde de $150 \times 200$ mm y $90 \times 200$ mm, separados como en la fig. 25 y unidos entre sí por enrejado de ángulos de $100 \times 75 \times 10$ y ángulos de $90 \times 75 \times 10$ mm en la parte superior y en la base, siendo reemplazados los ángulos por planchas de $12\frac{1}{2}$ mm por espacio de varios pies en cada extremo. El radio de giro de las cuatro almas solas, inclusive los brazos más largos de los ángulos del reborde, obrando conjuntamente alrededor del eje *ab*, es de 495 mm. Esto da, para la sección entera, obrando conjuntamente, $L/r = 17{,}380/495 = 35.1$.

**27.** Según *la hoja* (*sheet*) de resistencia, aquel miembro estaba **destinado á soportar**, en servicio :

| | | |
|---|---|---|
| Carga muerta..... | 5,101,942 kg | |
| Carga viva..... | 1,823,018 | |
| Carga muerta y viva..... | 6,924,960 | $= 1{,}397.7$ kg por cm cuad. |
| Viento..... | $\pm$ 3,342,80 | |

**28.** Para miembros de compresión en que $L/r$ es (como en este caso) $< 50$, la **especificación permitía** una carga, en kg por cm cuad, de

$$840\left(1 + \frac{\text{fuerza mínima}}{\text{fuerza máxima}}\right)^{*} \text{ ó, en este miembro, de}$$

$$840 \times 1.7317^{*} = 1{,}454 \text{ kg por cm cuad.}$$

**29. La pieza parece haber sostenido**, al tiempo de su caída, una presión media de (más ó menos) 1,120 kg por cm cuad.

**30. Cada una de las cuatro almas**, inclusive el brazo más largo de su reborde, tiene, respecto de su propio eje neutro, *cd*, un radio de giro tan sólo de 25 mm (más ó menos), haciendo su $L/r > 650$. Para columnas en que $L/r$ excede de 50, la especificación permite una carga de sólo

$$.07\ (12{,}000 - 50\ L/r)\ (1 + \frac{\text{resistencia mín}}{\text{resistencia máx}}),$$

que, si se aplicara á una de estas almas, permitiría tan sólo una carga menor de 2,450 kg por cm cuad (más ó menos); pero, naturalmente, se confió en el enrejado angular y en las planchas, obligando á las cuatro almas á obrar conjuntamente.

**31.** Después del hundimiento de la estructura, sin embargo, se encontró torcido este miembro en la forma de una S y la flexión tuvo lugar en ángulos rectos á los planos de las cuatro almas, ó en la dirección en que habría ocurrido si se hubieran omitido los brazos de refuerzo. « La falla del cordón inferior A9L sirve de ejemplo como sistema de celosía ó enrejado insuficiente. » (Informe de la Comisión del Puente de Quebec.)

## Esfuerzos sobre las vigas de celosías.

**32.** « La índole poco satisfactoria de las fórmulas para columnas es muy conocida de los ingenieros, pero **las fórmulas de columnas pueden conside-**

---

* Resistencia mín = carga muerta — carga viva mín.
$= 5{,}101{,}942 - 20{,}385^{**} = 5{,}081{,}557$ kg.
Resistencia máx = carga muerta + carga viva máx + carga viva mín.
$= 5{,}101{,}942 + 1{,}823{,}018 + 20{,}385 = 6{,}745{,}000$ kg.

** *N. del T.* — El autor trae para este caso 45,000 lbs como carga viva mínima = 20,385 kg.

**rarse como fórmulas exactas comparadas con las fórmulas para celosías.** Las fórmulas de celosía fijan, de cierto modo, un valor de la resistencia máxima de las fibras en la barra del enrejado; pero sólo las pruebas y los ensayos pueden determinar si éstas dan resultados económicos y seguros. » (Informe de la Comisión del Puente de Quebec.)

**33.** El siguiente **método aproximado** (véanse comunicaciones del **prof. Clyde T. Morris**, Universidad del Estado de Ohio, Eng. News, 7 de noviembre de 1907, pág. 487; febrero 27, 1908, pág. 44) supone (1) que el diagrama de los momentos, en las columnas, es una línea recta, es decir: que el incremento de resistencia, por unidad de longitud de columna, es constante. Esta suposición hace que el esfuerzo máx, en las planchas para un mom máx dado, sea menor de lo que probablemente es; pero, por otra parte, supónese también (2) que el esfuerzo en las fibras extremas de las planchas ó vigas está uniformemente distribuído sobre la sección transversal de cada plancha ó viga. Esta suposición (porque cada porción de la plancha ó viga, entre conexiones de enrejado, puede doblarse en una ú otra dirección) hace el promedio del esfuerzo mayor de lo que es.

**34.** En una columna de celosía compuesta de dos vigas ligadas por barras que se cruzan, fig. 26, sean

$s$ = compresión permitida por unidad en la sección transversal de la columna;
= compresión permitida por unidad en trozos cortos del material dado;
P = carga total máx permitida sobre la columna;
$a$ = área de la sección transversal de la columna;
$p = P/a$ = compresión máx permitida por unidad en la sección transversal de la columna;
L = longitud de la columna;
$r$ = menor radio de giro;
V = esfuerzo total transmitido por las barras de una á otra plancha;
= mitad de la diferencia del esfuerzo entre las dos planchas;
$l$ = distancia á que se transmite V;
$v = V/l$ = incremento del esfuerzo total V, por unidad de longitud de columna;
$w$ = dist entre las líneas de remaches entre las dos planchas;
$\theta$ = ángulo entre las barras de la celosía y el eje de la columna;
$x = w.\ \cot\theta$ = proyección longitudinal de una barra de celosía;
$n$ = número de barras en cada tramo enrejado;
$vx/n$ = incremento del esfuerzo á lo largo de $x$ en una barra;
$c$ = coeficiente de la fórmula $p = s - c\,L/r$;
= un esfuerzo pequeño de la misma naturaleza que $s$;
$P_L$ = esfuerzo total longitudinal en una barra de la celosía.

**35.** Tenemos:

$$V = (s-p)(a/2);$$
= dif de la compresión por unidad × área de una de las planchas ..... (1)

$$v = V/l = (s-p)\,a/2l \ldots\ldots (2)$$

$$= \left(s - s + c\frac{L}{r}\right)a/2l = cLa/2lr \ldots\ldots (3)$$

$$vx/n = vw\cot\theta/n \ldots\ldots (4)$$

$$P_L = (vx/n)\sec\theta = \frac{v}{n}\cdot\frac{w}{\text{sen. }\theta}$$

$$= \frac{cLa}{2lrn}\cdot\frac{w}{\text{sen. }\theta} \ldots\ldots (5)$$

**36. En las columnas redondeadas ó articuladas,** la curva de elasticidad es sencilla, y tenemos, para la distancia, $l$, dentro de la cual se ha transportado V, $l = L/2$; y $L/2l = 1$. En las columnas de extremos cuadrados y fijos, la curva se voltea con dos puntos de flexion opuestas y $l = L/4$ y $L/2l = 2$. Por tanto:

$$\textbf{Con extremos redondeados, } P_L = \frac{c\,a}{r\,n}\cdot\frac{w}{\text{sen }\theta} \ldots\ldots (6)$$

$$\textbf{Con extremos cuadrados, } P_L = \frac{2\,ca}{r\,n}\cdot\frac{w}{\text{sen }\theta} \ldots\ldots (7)$$

**37.** El siguiente **método aproximado** (véase **W. C. Armstrong**, Western Soc. of Engrs, Journal, junio 1908, pág. 337) da el esfuerzo total $P_L$, en una barra

de celosía, **cuando la columna está expuesta á fallar en cualquier dirección.**

Se supone que las placas de empate soportan ¼ y la ligazón ¾ del esfuerzo cortante entre las planchas ó vigas. Sea:

$d$ = espesor de la plancha;
I = momento de inercia de la sección transversal;
$M = 2sI/d$ = momento de resistencia á la flexión en ambas direcciones; los otros símbolos como en el § 34.

**38.** Luego

$$V = M/v = 2sI/dw \quad (1)$$

$$v = V/l = 2sI/dwl \quad (2)$$

$$vx/n = 2sIx/dwln = 2s\,\mathrm{I}\cot\theta/dln \quad (3)$$

$$\mathbf{P_L} = \tfrac{3}{4}\cdot\frac{vx}{n}\cdot\sec\theta = \frac{1.5sI}{dln\,\mathrm{sen}\,\theta} \quad (4)$$

**Con extremos redondos**, $l = \mathrm{L}/2$; y $\mathbf{P_L} = \dfrac{3sI}{dLn\,\mathrm{sen}\,\theta}$ ..... (5)

**Con extremos cuadrados**, $l = \mathrm{L}/4$; y $\mathbf{P_L} = \dfrac{6sI}{dLn\,\mathrm{sen}\,\theta}$ ..... (6)

**39.** Suponiendo que las mismas barras de la celosía sean suficientemente fuertes, la integridad del sistema de celosías queda aún **limitado por la posibilidad de que se resbalen los remaches**; la resistencia contra esto, en cualquier unión, es = número de remaches × el área de la sección del remache × unidad de resistencia de los remaches á la fricción.

El prof. J. B. Johnson (Materiales de construcción, págs. 526-7) da 840 kg por cm cuadrado de la sección del remache como la **resistencia á la fricción** para los remaches de acero; 700 kg para los remaches de hierro.

**40.** « **El tamaño y la resistencia del pasador** empleado tiene un efecto apreciable en los resultados obtenidos, pero no se ha determinado el monto de este efecto. » (Informe de la Comisión del Puente de Quebec.)

**41.** « No puede esperarse que una **pieza de compresión**, de diseño y dimensiones corrientes, desarrolle un máximo de esfuerzo mucho mayor que la mitad, más ó menos, de una **pieza de tensión** hecha con el mismo material. » (*Ibid.*)

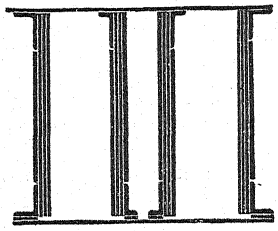


**Fig. 25.**

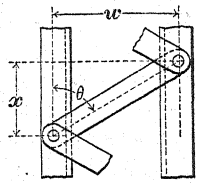



**Fig. 26.**

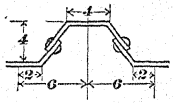
## SECCIONES, « PENCOYD », DE PISO

(*Obs. del T.* — Hemos convertido las fórmulas, etc., al sistema métrico.
L = luz, en metros;
C = coeficiente;
W = carga distribuída, en kg, por metro de ancho de piso.

$$W = .453^5 \frac{C}{L}.$$

**Pisos undulados** para puentes y edificios.
W = carga que produce un esfuerzo de fibra de 1,050 kg por cm cuadrado.

SECCION 210 M.

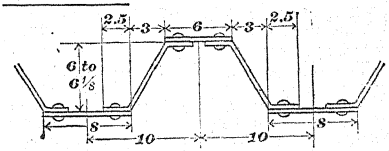



(*Obs. del T.* — Damos en seguida los equivalentes, en mm, de los números de la figura que indican pulgs: 4 = 102; 6 = 152; 2 = 51.)

Espesor en mm.

| Alma. | Reborde. | Peso en kg por m cuad. | C. |
|---|---|---|---|
| 4.8 | 6.3 | 72.22 | 44,000 |
| 5.9 | 7.9 | 89.79 | 55,000 |
| 7.1 | 9.5 | 106.87 | 66,000 |
| 8.3 | 11.1 | 124.44 | 77,400 |
| 9.5 | 12.7 | 142.00 | 88,800 |

SECCION 260 M.

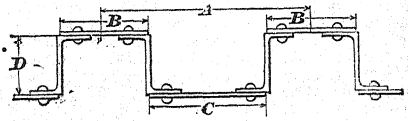



(*Obs. del T.* — Damos en seguida los equivalentes, en mm, de los números de la figura que indican pulgadas: 2.5 = 63.5; 3 = 76; 6 = 152; 6¼ = 155; 8 = 203; 10 = 254.)

Espesor en mm.

| Alma. | Reborde. | Peso en kg por m cuad. | C (en miles). |
|---|---|---|---|
| 6.3 | 6.3 á 15.9 | 97.6 á 149.8 | 105 á 186 |
| 7.9 | 9.5 á 19.0 | 129.3 á 185.5 | 134 á 224 |
| 9.5 | 9.5 á 19.0 | 143.5 á 195.7 | 153 á 237 |

**Barras en Z para pisos.**
(Hierros zorés)
W = carga de seguridad.




| Sección N.º | Dimensiones en cm. A | B | C | D | Barras Z, mm. | Planchas, (espesores) mm. | Peso en kg por m cuad.* |
|---|---|---|---|---|---|---|---|
| 1 | 38.10 | 15.24 | 22.86 | 10.16 | 6.35 | 6.35 á 12.7 | 126.39 á 176.17 |
| | | | | | 7.937 | | 142.00 á 191.78 |
| | | | | | 9.525 | | 157.62 á 207.4 |
| 2 | 45.72 | 20.32 | 25.4 | 12.7 | 7.937 | 7.93 á 14.28 | 156.65 á 206.42 |
| | | | | | 9.525 | | 171.78 á 221.55 |
| | | | | | 11.112 | | 187.4 á 237.17 |
| 3 | 53.34 | 22.86 | 30.48 | 15.24 | 9.525 | 9.52 á 15.87 | 191.78 á 241.56 |
| | | | | | 11.112 | | 206.91 á 256.69 |
| | | | | | 12.7 | | 222.04 á 271.82 |

* Los valores de C varían para estas tres primeras de : 93,400 á 167,000 ; para las tres siguientes, de 143,000 á 233,000; para las últimas, de 203,400 á 307,200. Los valores intermedios pueden calcularse por simple proporción.

## PESO Y RESISTENCIA DE LAS CADENAS DE HIERRO

### Cuadro de la resistencia de las cadenas.

Las cadenas de hierro superior resisten de ⅓ á ¼ más. (Original.)

| Diám de la barra de que están hechos los eslabones. | Peso de la cadena por metro. | Fuerza de ruptura. | Diám de la barra de que están hechos los eslabones. | Peso de la cadena por metro. | Fuerza de ruptura. |
|---|---|---|---|---|---|
| mm. | kg. | kg. | mm. | kg. | kg. |
| 4.762 | .745 | 785.161 | 25.4 | 15.943 | 22,353 |
| 6.35 | 1.192 | 1,392.0 | 28.575 | 18.625 | 26,864 |
| 7.937 | 1.490 | 2,174.53 | 31.75 | 23.84 | 33,164 |
| 9.525 | 2.533 | 3,139.75 | 34.925 | 27.267 | 40,053 |
| 11.112 | 2.980 | 4,267.40 | 38.1 | 32.333 | 47,754 |
| 12.7 | 3.725 | 5,588.25 | 41.275 | 38.74 | 56,025 |
| 14.287 | 4.768 | 7,071.52 | 44.45 | 41.72 | 64,997 |
| 15.875 | 6.407 | 8,717.54 | 47.625 | 47.68 | 74,617 |
| 17.462 | 7.45 | 10,556.88 | 50.8 | 56.62 | 84,890 |
| 19.05 | 8.642 | 12,558.62 | 57.15 | 80.46 | 101,807 |
| 20.637 | 9.983 | 14,651.53 | 63.5 | 105.79 | 125,685 |
| 22.225 | 11.92 | 17,069.57 | 69.85 | 131.12 | 152,102 |
| 23.812 | 13.41 | 19,630.14 | 76.2 | 156.45 | 180,958 |

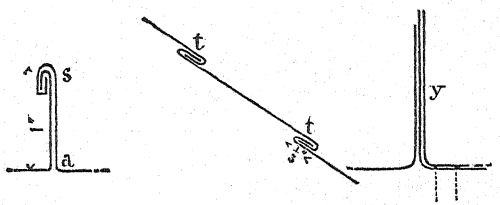
**Los eslabones de cadenas ordinarias de hierro** se hacen generalmente tan cortos como lo permita el fácil juego de ellos, á fin de que no se doblen cuando se enrollan en un tambor ó cilindro, etc., y para que sean más manuables en la suspensión de grandes trozos de piedra, etc. Experimentos hechos por orden del Gobierno de los Estados Unidos, en 1878, prueban que los clavos ó pernos debilitan los eslabones.

Cuando se hacen así, su peso por metro es aproximadamente 3½ veces el de la barra de hierro redondo de que se hacen. Desde luego que cada eslabón se compone de dos gruesos de barra; se puede suponer que una cadena tiene como el doble de la resistencia de una barra sencilla; pero la resistencia de la barra se reduce como en $^3/_{10}$ al convertirse en eslabones; de modo que la cadena tiene realmente tan sólo $^7/_{10}$ de la resistencia de las barras. Como las barras de hierro gruesas no sostienen una carga proporcionalmente mayor á la que soporta una barra delgada, de igual modo las cadenas gruesas son relativamente menos resistentes que las delgadas. En la tabla anterior se calcula 3.10 toneladas por **centímetro cuadrado** el esfuerzo medio de ruptura de una barra recta de hierro laminado ordinario de 2½ cm de diámetro ó en cuadro; en 2.94 toneladas por cm cuad el de una de 2½ á 5 cm, y en 2.79 ton por cm cuad el de una de 5 á 8 cm cuad. Deduciendo $^3/_{10}$ por cada una de éstas, tenemos como esfuerzo de ruptura de las dos barras que componen cada eslabón, lo siguiente: 2.17 ton por cm cuad hasta 2½ cm de diámetro; 2.06 ton por cm cuad, de 2½ á 5 cm, y 1.95 ton de 5 á 8 cm de diámetro. Sobre estas suposiciones está basada la tabla. Los pesos son aproximados, dependiendo de la exactitud del diámetro del hierro y de la forma del eslabón.

## ESTAÑO Y CINC

**El metal puro se vende en tejos y se llama «estaño en tejos».** Cuando está perfectamente puro (lo que rara vez sucede, pues frecuentemente lo adulteran mucho con los metales más baratos, plomo y cinc), tiene una densidad de 7.29. Es suficientemente maleable para reducirlo á láminas de sólo 0.025 mm de espesor. Su resistencia no es sino como de 323 kg por cm cuad, ó como 490 kg por cm cuad cuando se transforma en alambre. Entra en fusión á la temperatura moderada de 227° C. El estaño puro no se emplea en construcciones ordinarias, sino láminas delgadas de palastro cubiertas de estaño por ambas caras. Éstas son las que constituyen las **láminas estañadas** ó la *hojalata*, y se usan para techos, canales, etc. Para techos, se colocan sobre tablas.

**Las láminas de hojalata se disponen** como se ve en esta figura. En el taller se unen primero varias láminas, extremo con extremo, como en *tt*, doblándolas primero, luego se amartillan hasta ponerlas planas y después se sueldan. Para colocarlas en el techo se hace un rollo de ellas suficientemente grande que alcance á cubrir desde el caballete hasta el alero. Cuando se han extendido diferentes




rollos en el sentido de la inclinación del techo, se unen lateralmente doblando simplemente sus orillas, como se ve en *a* y *s*, por medio de un instrumento especial. Los maestros techadores llaman al doblez en S *doble empate*, y los más sencillos en *t*, *empate sencillo*.

Para fijar bien la plancha en las tablas, se clavan en sus bordes pedazos de hojalata de ocho á diez cm de largo por 5 cm de ancho, de 45 en 45 cm á lo largo de las juntas que han de cogerse y que están ya plegadas en S. Esto se comprenderá en la figura, en la cual la pieza del medio es el pedazo dicho antes de haber sido doblado. Los clavos deben ser de empizarrar, de cabezas más anchas que los ordinarios. Como no están expuestos á la intemperie, pueden ser de hierro ordinario.

Se usa mucho lo que llaman **estaño emplomado** (*ternes*) para techos. Es simplemente hierro en planchas cubierto con plomo en lugar de estaño que es un metal más costoso. No es tan durable como las estañadas, pero es más barato.

Las mejores láminas para estañar y emplomar son hechas de hierro de fundición al carbón vegetal (*charcoal*), que, como es más duro, soporta mejor el doblez. Las de fundición de cok, ó con hulla, se usan para planchas más baratas y son inferiores en lo que se refiere al doblez. Al hacer pedidos es importante especificar si se desean láminas de palastro al cok ó al carbón vegetal, así como también si se quieren estañadas ó de estaño emplomado.

Se usan mucho las láminas estañadas y emplomadas de acero Bessmer y otros aceros baratos. Se venden próximamente al precio de las planchas de palastro al cok estañadas y emplomadas.

También se usan para techos ciertos metales compuestos que resisten al deslustre mejor que el plomo, estaño ó cinc; pero son tan fusibles que están expuestos á derretirse con cualquier chispa ó pedazo grande de carbón encendido que cayese sobre el techo.

Un techo con cubierta de hojalata ú otro metal no debe tener una inclinación menor de cinco grados, ó sea $^1/_{12}$, y en los aleros una caída violenta al canal para impedir que el agua retroceda y se eleve por encima de la junta produciendo filtraciones.

Donde se emplea el carbón mineral como combustible, debe darse á los techos de hojalata dos manos de pintura al colocarlos y después una mano cada 2 ó 3 años. Donde sólo se usa leña no es necesario esto, y un techo de hojalata con una buena inclinación durará 20 ó 30 años.

Dos buenos obreros pueden colocar y pintar exteriormente, en un día de 8 horas de trabajo, de 23 á 28 m cuad de techo de hojalata.

Las láminas de hierro estañado se venden por cajas.

(*Obs. del T.* — La siguiente tabla se ha deducido de la del autor y en ella se expresan todas las medidas en sistema métrico.)

### Tabla de láminas estañadas (hojalata) y láminas ú hojas de estaño emplomado (ternes).

**Advertencia.** Las cajas contienen á menudo considerablemente menos peso de láminas estañadas que el que la tabla indica, pues las láminas se hacen delgadas y se estañan delgado, á fin de hacer que se pague á los mecánicos por más material que el que suministran.

Las marcas indican el espesor aproximadamente como sigue :

| Marca. | N.º del calibrador de Birmingham. | Milímetros. | Kg por m cuad. | Marca | N.º del calibrador de Birmingham. | Milímetros. | Kg por m cuad. |
|---|---|---|---|---|---|---|---|
| IC | 30 | .30 | 2.344 | DC | 27 | .41 | 3.125 |
| IX | 28 | .36 | 2.734 | DX | 25 | .51 | 3.906 |
| IXX | 27 | .41 | 3.125 | DXX | 23 | .63 | 4.883 |
| IXXX | 26 | .46 | 3.515 | DXXX | 22 | .71 | 5.517 |
| IXXXX | 25 | .51 | 3.906 | DXXXX | 21 | .81 | 6.299 |

| Dimensiones en cm. | Marca. | N.º de láminas que contiene una caja. | Peso por caja en kg. | Dimensiones en cm. | Marca. | N.º de láminas que contiene una caja. | Peso por caja en kg. |
|---|---|---|---|---|---|---|---|
| 22.9×45.7 | IC | 225 | 58.967 | 35.6×61 | IX | » | 76.204 |
| » | IX | » | 73.482 | » | IXX | » | 87.543 |
| 25.4×25.4 | IC | » | 36.287 | 35.6×63.5 | IC | » | 63.503 |
| » | IX | » | 45.359 | » | IX | » | 79.379 |
| 25.4×35.6 | IC | » | 50.802 | » | IXX | » | 91.172 |
| » | IX | » | 63.503 | 35.6×66 | IXXX | » | 107.501 |
| » | IXX | » | 73.028 | 35.6×71.1 | IC | » | 71.214 |
| » | IXXX | » | 82.554 | » | IX | » | 88.904 |
| » | IXXXX | » | 92.079 | » | IXX | » | 102.058 |
| 25.4×50.8 | IC | » | 72.575 | 35.6×76.2 | IXX | » | 109.316 |
| » | IX | » | 90.719 | 35.6×78.7 | IX | » | 98.430 |
| 27.9×27.9 | IC | » | 43.999 | » | IXX | » | 112.944 |
| » | IX | » | 54.885 | 38.1×38.1 | IX | 225 | 102.058 |
| 27.9×55.9 | IC | 112 | 43.999 | » | IXX | » | 117.480 |
| » | IX | » | 54.885 | » | IXXX | » | 147.871 |
| » | IXX | » | 63.049 | 40.6×40.6 | IC | 225 | 92.986 |
| 30.5×30.5 | IC | 225 | 50.802 | » | IX | » | 116.120 |
| » | IX | » | 63.503 | » | IXX | » | 133.356 |
| » | IXX | » | 73.028 | 40.6×48.3 | IX | 112 | 68.946 |
| 30.5×61 | IC | 112 | 52.163 | 43.2×43.2 | IX | » | 63.957 |
| » | IX | » | 65.317 | » | IXX | » | 75.296 |
| » | IXX | » | 75.296 | 43.2×63.5 | DX | 100 | 104.305 |
| » | IXXX | » | 84.822 | » | DXX | » | 133.356 |
| 31.8×43.2 | DC | 100 | 44.452 | 45.7×45.7 | IX | 112 | 73.482 |
| » | DX | » | 54.431 | » | IXX | » | 81.647 |
| » | DXX | » | 66.678 | » | IXXXX | » | 106.594 |
| » | DXXX | » | 76.204 | 50.8×50.8 | IX | » | 90.719 |
| » | DXXXX | » | 85.729 | » | IXX | » | 104.326 |
| 33 ×33 | IC | 225 | 61.235 | » | IXXX | » | 117.934 |
| » | IX | » | 76.657 | » | IXXXX | » | 131.542 |
| 33 ×33 | IXX | 225 | 87.997 | 50.8×71.1 | IC | » | 101.604 |
| 33 ×66 | IC | 112 | 61.235 | » | IX | » | 127.006 |
| » | IX | » | 76.657 | » | IXX | » | 146.057 |
| » | IXX | » | 87.997 | » | IXXX | » | 165.107 |
| 35.6×35.6 | IC | 225 | 70.761 | » | IXXXX | » | 184.159 |
| » | IX | » | 88.904 | | | | |
| » | IXX | » | 102.058 | **Láminas de estaño emplomado.** | | | |
| » | IXXX | » | 115.212 | | | | |
| 35.6×50.8 | IC | 112 | 50.802 | 25.4×50.8 | IC | 112 | 36.287 |
| » | IX | » | 63.503 | » | IX | » | 45.359 |
| » | IXX | » | 73.028 | 35.6×50.8 | IC | » | 50.802 |
| » | IXXX | » | 82.554 | » | IX | » | 63.503 |
| » | IXXXX | » | 92.079 | 50.8×71.1 | IC | » | 101.600 |
| 35.6×55.9 | IX | » | 69.853 | » | IX | » | 127.006 |
| » | IXX | » | 80.286 | | | | |

Láminas de mayores dimensiones pueden fabricarse para pedidos especiales; las de hierro estañado se hacen en Inglaterra; pero las de estaño emplomado se hacen también en Filadelfia y en otro lugares.

Una caja de 225 láminas 34.9×25.4 cm contiene 19.96 m cuad; pero, teniendo en cuenta lo que se llevan los dobleces, sólo cubrirá como 14 m cuad de techo. Sin embargo de que no se hace ninguna rebaja por las pérdidas que ocasionen los cortes que indispensablemente exige el ajuste de las láminas en las esquinas.

Para calcular el área cubierta por una lámina cualquiera dedúzcase 5 cm de su ancho y 2.5 cm de su longitud.

**El cinc en láminas**, colocadas del mismo modo que la pizarra, se emplea mucho para techos en algunas partes de Europa. Expuesto á la intemperie, se cubre pronto de una capa de óxido blanco, que lo protege en lo sucesivo y lo hace durable También se usan láminas acanaladas de cinc. Véase Hierro galvanizado en láminas.

Las láminas de cinc son generalmente de .91×2.13 m ó por 2.44 m. Su espesor difiere del de hierro, así : el n.° 13 es .813 mm de espesor ó 5.75 kg m cuad; el n.° 14, .889 mm y 6.59 kg por m cuad; el n.° 15, 1.067 mm, y 7.27 kg por m cuad; el n.° 16, 1.245 mm y 7.91 kg por m cuad. Cualquiera de estos números puede usarse para techos, pero debe ser de buena calidad.

Se dice que es perjudicial á la salud **el agua conservada en envases de cinc**, y recientemente se ha protestado contra los tubos de hierro galvanizado para el servicio doméstico. Sin embargo se han empleado durante muchos años en Nueva Inglaterra, en Filadelfia y en otras partes, sin ningún efecto pernicioso. Esto se debe probablemente al hecho de que siendo cortos los tubos del servicio, el agua se renueva en ellos varias veces al día y por consiguiente no permanece en contacto con el cinc y el plomo el tiempo suficiente para hacerse venenosa. Al tomar posesión de una casa debe botarse de los tubos del servicio el agua que haya permanecido estancada por algún tiempo, porque su uso hace daño á la salud.

**El cobre para techos** está generalmente en láminas de .76 m × 1.52 m =1.16 m cuad, y pesa de 4.54 á 6.35 kg cada lámina. Se colocan sobre tablas. No se usan soldaduras en las juntas horizontales como en los techos de hoja de lata; pero las juntas horizontales ásí como las del declive se forman sobreponiendo y doblando las láminas solamente, como se ve en las figs. de la pág. 998, excepto las horizontales que están dobladas ó entrelazadas, y luego muy aplanadas (la misma figura).

**Plomo en láminas.** Lista de **pesos modelos** en kg por m cuad y espesores en centímetros. (*Obs. del T.* — Hemos convertido la tabla del autor.)

| Peso en kg por m². | Espesor en cm. | Peso en kg por m². | Espesor en cm. | Peso en kg por m². | Espesor en cm. |
|---|---|---|---|---|---|
| 12.21 | .107 | 29.30 | .259 | 48.83 | .432 |
| 14.65 | .130 | 34.18 | .302 | 58.59 | .516 |
| 19.53 | .173 | 39.06 | .345 | 68.36 | .602 |
| 24.41 | .216 | 43.94 | .389 | 78.12 | .688 |

**Los tubos de plomo para el servicio** de las casas de habitación sencillas de Filadelfia son comúnmente de un diámetro interior desde 12½ mm, con peso por m lineal de 1.49 á 3.72 kg, hasta un diám de 16 mm y un peso de 2.23 á 4.46 kg por m, según la carga. **Rara vez se revientan** al cerrar repentinamente las llaves de retención, pero algunas veces sucede esto cuando se hiela el agua que contienen.

## Tubos de plomo. — Dimensiones modelos.

| Diámetro interior en mm. | Espesor en mm. | Peso por m en kg. | Diámetro interior en mm. | Espesor en mm. | Peso por m en kg. | Diámetro interior en mm. | Espesor en mm. | Peso por m en kg. | Diámetro interior en mm. | Espesor en mm. | Peso por m en kg. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 9.525 | 2.03 | 0.93 | 19.05 | 2.54 | 1.86 | 38.1 | 3.56 | 5.215 | 76.2 | 4.76 | 13.410 |
| » | 3.05 | 1.49 | » | 3.05 | 2.60 | » | 4.32 | 6.330 | » | 6.35 | 17.880 |
| » | 4.06 | 1.86 | » | 4.06 | 3.35 | » | 4.83 | 7.450 | » | 7.94 | 23.840 |
| » | 4.83 | 2.23 | » | 5.08 | 4.47 | » | 5.84 | 9.685 | » | 9.52 | 29.800 |
| 12.7 | 2.29 | 1.11 | » | 5.84 | 5.21 | » | 6.86 | 11.920 | 88.9 | 4.76 | 14.155 |
| » | 2.79 | 1.49 | » | 7.62 | 7.07 | 44.45 | 3.3 | 5.960 | » | 6.35 | 22.350 |
| » | 3.3 | 1.86 | 25.4 | 2.79 | 2.98 | » | 4.32 | 7.450 | » | 7.94 | 26.565 |
| » | 4.06 | 2.61 | » | 3.56 | 3.72 | » | 5.33 | 9.685 | » | 9.52 | 32.780 |
| » | 4.83 | 2.98 | » | 4.32 | 4.84 | » | 6.86 | 12.665 | 101.6 | 4.76 | 18.625 |
| » | 6.35 | 4.47 | » | 5.33 | 5.96 | 50.8 | 3.81 | 7.077 | » | 6.35 | 23.840 |
| 15.875 | 2.29 | 1.49 | » | 6.10 | 7.07 | » | 4.57 | 8.940 | » | 7.94 | 31.290 |
| » | 3.3 | 2.23 | 31.75 | 2.54 | 2.98 | » | 5.59 | 10.430 | » | 9.52 | 37.250 |
| » | 4.06 | 2.98 | » | 3.05 | 3.72 | » | 6.86 | 13.410 | 114.3 | 4.76 | 20.860 |
| » | 5.08 | 3.72 | » | 3.56 | 4.47 | 63.5 | 4.76 | 11.920 | » | 6.35 | 26.820 |
| » | 5.59 | 4.09 | » | 4.06 | 5.58 | » | 6.35 | 16.390 | 127.0 | 6.35 | 29.800 |
| » | 6.35 | 5.21 | » | 4.83 | 7.07 | » | 7.94 | 20.860 | » | 9.52 | 46.190 |
| » | » | » | » | 6.35 | 8.94 | » | 9.52 | 25.330 | » | » | » |

## PLOMO LAMINADO, COBRE Y BRONCE; PLANCHAS Y BARRAS

| Espesor, ó diám, ó lado. | PLOMO | | | COBRE | | | BRONCE | | |
|---|---|---|---|---|---|---|---|---|---|
| | En planchas por mcuad. | En barras cuad por metro. | En barras redondas por m. | En planchas por mcuad. | En barras cuad por m. | En barras redondas por m. | En planchas por mcuad. | En barras cuad por m. | En barras redondas por m. |
| mm. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. |
| .794 | 9.077 | .0074 | .006 | 7.027 | .006 | .0045 | 6.637 | .006 | .0045 |
| 1.587 | 18.154 | .0283 | .0223 | 14.103 | .0223 | .018 | 13.225 | .021 | .0162 |
| 2.381 | 27.230 | .0655 | .0506 | 21.130 | .0506 | .0402 | 19.813 | .048 | .0374 |
| 3.175 | 36.307 | .1162 | .0909 | 28.158 | .0894 | .070 | 26.450 | .0834 | .066 |
| 3.968 | 45.384 | .180 | .142 | 35.136 | .140 | .110 | 32.940 | .131 | .103 |
| 4.762 | 54.656 | .259 | .204 | 42.261 | .201 | .158 | 39.675 | .189 | .149 |
| 5.556 | 63.440 | .353 | .279 | 49.288 | .274 | .215 | 46.360 | .258 | .203 |
| 6.35 | 72.312 | .462 | .364 | 56.120 | .358 | .282 | 52.704 | .337 | .264 |
| 7.937 | 90.763 | .723 | .568 | 70.272 | .560 | .440 | 65.880 | .526 | .413 |
| 9.525 | 108.824 | 1.040 | .817 | 84.424 | .806 | .633 | 79.544 | .757 | .594 |
| 11.112 | 126.880 | 1.415 | 1.112 | 98.576 | 1.097 | .861 | 92.720 | 1.030 | .809 |
| 12.7 | 145.424 | 1.848 | 1.451 | 112.728 | 1.433 | 1.125 | 105.896 | 1.345 | 1.056 |
| 14.287 | 163.490 | 2.340 | 1.833 | 126.880 | 1.818 | 1.423 | 118.584 | 1.699 | 1.341 |
| 15.875 | 181.536 | 2.891 | 2.265 | 141.032 | 2.235 | 1.758 | 132.248 | 2.100 | 1.639 |
| 17.462 | 199.192 | 3.487 | 2.742 | 154.696 | 2.712 | 2.131 | 145.424 | 2.533 | 1.997 |
| 19.05 | 217.648 | 4.157 | 3.263 | 168.848 | 3.218 | 2.533 | 158.600 | 3.025 | 2.384 |
| 20.637 | 235.764 | 4.872 | 3.829 | 183.000 | 3.800 | 2.965 | 171.776 | 3.546 | 2.786 |
| 22.225 | 254.248 | 5.662 | 4.440 | 197.152 | 4.381 | 3.442 | 184.952 | 4.112 | 3.233 |
| 23.812 | 273.280 | 6.511 | 5.096 | 211.304 | 5.036 | 3.948 | 198.128 | 4.738 | 3.710 |
| 25.4 | 290.360 | 7.390 | 5.811 | 225.456 | 5.737 | 4.500 | 211.304 | 5.379 | 4.232 |
| 28.575 | 326.072 | 9.342 | 7.331 | 253.760 | 7.256 | 5.692 | 237.657 | 6.809 | 5.364 |
| 31.75 | 363.022 | 11.547 | 9.074 | 281.576 | 8.955 | 7.033 | 264.496 | 8.404 | 6.600 |
| 34.925 | 399.184 | 13.961 | 10.981 | 309.880 | 10.847 | 8.523 | 290.848 | 10.162 | 8.001 |
| 38.1 | 435.784 | 16.688 | 13.067 | 338.184 | 12.889 | 10.132 | 317.200 | 12.099 | 9.506 |
| 41.275 | 471.896 | 19.519 | 15.347 | 366.488 | 15.198 | 11.890 | 343.552 | 14.200 | 11.160 |
| 44.45 | 507.520 | 22.648 | 17.731 | 394.304 | 17.582 | 13.783 | 370.392 | 16.539 | 12.933 |
| 47.625 | 546.560 | 26.075 | 20.413 | 422.608 | 20.115 | 15.794 | 396.744 | 18.923 | 14.855 |
| 50.8 | 580.720 | 29.502 | 23.244 | 450.424 | 22.946 | 18.029 | 423.096 | 21.456 | 16.837 |

(*Obs. del T.* — Hemos convertido la del autor al sistema métrico.)

## Promedio de las máximas tensiones de los metales.

| El máximo de tensión por cm cuad de cualquier material, se llama frecuentemente su *invariable, coefiriente* ó *módulo* de tensión. | kg por cm cuadrado. |
|---|---|
| Antimonio fundido | 70 |
| Bismuto fundido | 225 |
| Bronce fundido | 1,652 |
| — alambre, sin temple ó duro, 5,600. Templado | 3,445 |
| — al fósforo, alambre, duro 10,500. Templado | 4,429 |
| Cobre fundido, 1,260 á 2,100 | 1,687 |
| — láminas | 2,109 |
| — pernos, 1,960 á 2,660 | 2,320 |
| — alambre (templado) | 4,214 |
| Estaño inglés en tejos | 323 |
| — alambre | 490 |
| Hierro fundido, inglés, 938 á 1,568 | 1,258 |
| — — de fundición al coke | 1,019 |
| El hierro fundido americano da un promedio de una cuarta parte más que el anterior. | |
| El hierro fundido corriente, en buen estado, estira como .00018, ó sea 1 m en 5,555 m por cada 70 kg de tensión por cm cuad hasta su límite de elasticidad, que es más ó menos la mitad de su esfuerzo de ruptura. La amplitud de su estiramiento varía mucho, sin embargo, según la calidad del hierro, como sucede con el hierro forjado. | |
| Hierro y acero laminados, véanse págs. 813, etc., 941, etc. | |
| — fundido, maleable, templado | 3,386 |
| Metal de cañones de cobre y estaño, 1,110 á 3,850 | 2,742 |
| — — hierro fundido. Artillería de los E. U | 2,672 |
| Oro fundido | 1,406 |
| — alambre, 1,750 á 2,100 | 1,933 |
| Plata fundida | 2,882 |
| Platino, alambre, templado, 2240. Sin templar | 3,937 |
| Plomo fundido, 119 á 168. Por el autor | 144 |
| — alambre | 115 |
| Cinc fundido, 210 á 259 (El último por el autor) | 236 |

**Las barras grandes de metal** soportan menos por cm cuad que las pequeñas. Las barras de hierro relaminadas en frío aumentan su tensión de 25 á 50 por ciento sin aumento de densidad. Se dice que pierden esta fuerza al recalentarlas.

**Empleo del plomo en las juntas de la mampostería.** Véase pág. 692. Bajo presiones generalmente menores que la fuerza de trituración de la piedra, el plomo se sale lateralmente, y, por medio de su fricción con la piedra, ejerce sobre ésta una acción desgarrante normal á la presión y tendente á rajar la piedra en pedazos cuyos ejes forman ángulos muy agudos con la línea de presión. Esto, naturalmente, debilita mucho la piedra.

**Algunas veces se coloca en las juntas de las columnas de piedra plomo en láminas**, con el objeto de igualar la presión y de esta manera aumentar la resistencia de la columna. Pero la experiencia ha demostrado que el efecto es directamente contrario y que la columna se **debilita materialmente** por este procedimiento.

## Promedio de las cargas de ruptura por aplastamiento.

Debe recordarse que estas son las cargas para piezas que sólo tienen de altura dos ó tres veces su menor dimensión. A medida que aumenta la altura, disminuye la carga de ruptura.

(*Obs. del T.* — La tabla que sigue es la del autor convertida al sistema métrico.)

| La carga de ruptura por cm cuad de cualquier material, se llama frecuentemente *constante, coeficiente* ó *módulo* de ruptura ó de compresión. | kg por m cuad. |
|---|---|
| **Hierro fundido**, generalmente..... | 5,976 á 8,788 |
| Ordinariamente se supone á razón de 7,030 kg por cm cuad. Su resistencia á la ruptura por compresión es generalmente 6 á 7 veces la de su resistencia á la tensión. Entre su límite medio de elasticidad, como de 2,362 kg por cm cuad, el hierro fundido corriente se acorta en la relación de 1 en 5555 ó 1 milímetro en 5.555 m bajo la carga de cada tonelada (2,240 lbs) por pulg cuad, ó 157 kg por cm cuad, ó como el doble del hierro forjado. De aquí que, á razón de 2,362 kg por cm cuad, se acortará como 1 en 370 ó 1 cm en 3.7 m. Los diferentes hierros fundidos pueden, sin embargo, variar de 10 á 15 por ciento en uno ú otro sentido. | |
| **El metal de cañones de la artillería de los E. U.**, como. | 12,304 |
| **Hierro forjado** dentro de sus límites de elasticidad..... | 1,574 á 2,519 |
| Su límite de elasticidad, bajo presión, da un promedio de 2,047 kg por cm cuad..... | 2,047 |
| Empieza á acortarse perceptiblemente bajo una presión de 1,260 á 1,575 kg por cm cuad, pero recobra su tamaño al quitarle dicha carga. Con 2,835 á 3,150 kg por cm cuad se acorta *permanentemente* como $^1/_{30}$ de su longitud; y con 4,252 á 4,725 kg por cm cuad, como $^1/_{15}$ por término medio. Las cargas de ruptura de la tabla, por consiguiente, no son las que absolutamente deforman el hierro forjado por completo, sino simplemente aquellas con las cuales cede demasiado para las aplicaciones más prácticas en las construcciones. Unos 630 kg por cm cuad se considera como su carga de seguridad media, en piezas no más largas de 10 diámetros; y se acortan á razón de 1 en 2,880. **El bronce** se reduce en $^1/_{44}$ parte de su longitud bajo una carga de 3,586 kg por cm cuad y la mitad bajo una de..... | 11,601 |
| **El cobre** (fundido) se desmenuza, destriza (*crumbles*)..... | 8,226 |
| — (forjado) se reduce en $^1/_8$ de su longitud con..... | 7,242 |
| **El estaño** (fundido) se reduce en $^1/_{16}$ de su longitud con 619 kg por cm cuad y $^1/_2$ con..... | 1,090 |
| **El plomo** (fundido) reduce á ½ de su longitud con 492 á 541 kg por cm cuad..... | 517 |
| Por el autor. Un pedazo de 25.4 milímetros en cuadro, 2 pulgs (51 mm) de alto, con una carga á razón de 84 kg por cm cuad, la compresión fué de $^1/_{300}$ de su altura; con una á razón de 140 $^1/_{20}$; una de 218 $^1/_5$; con una de 350, $^1/_3$; con una de 490 por cm cuad, ½ de la altura. | |
| **Peltre ó cinc** (fundido). Por el autor. 1 pieza de una pulg en cuadro (6.45 cm cuad), 4 pulgs de altura (10.16 cm) con una carga de 2,000 lbs, es de cir de 140 kg por cm cuad, se comprimió $^1/_{400}$ de su altura; con 4,000 lbs ó sean 280 kg por cm cuad, $^1/_{200}$; con 6,000 lbs (420 kg cm cuad), $^1/_{100}$; con 10,000 lbs (700 kg cm cuad), $^1/_{28}$; con 20,000 lbs (1,400 kg cm cuad), $^1/_{15}$; con 40,000 lbs (2,800 kg /cm cuad) cedió rápidamente y se rompió en pedazos. | |
| **Acero** Una carga de 15,749 kg por cm cuad lo acorta de .2 á .4. | |
| — americano. (Blach Diamond Steelwarks, Pittsburg, Penn, experimentos de Lient W. H. Shock, armada | |

de los E. U.) En piezas de 12.7 mm en cuadro y 89 mm ó de una longitud igual á 7 veces el lado.

| | | |
|---|---|---|
| **Acero** | no templado, 7,038 á 7,312 kg por cm cuad..... | 7,175 |
| — | caldeado al rojo claro cereza y sumergido después en aceite 27°7 C, 12,177 á 14,005 kgs por cm cuad..... | 13,091 |
| — | caldeado al rojo cereza claro y sumergido después en agua á 20°56 C, luego templado en una plancha caliente de 22,878 á 23,961 kgs por cm cuad..... | 23,420 |
| — | caldeado al rojo cereza claro, después sumergido en agua á 20°56 C, 19,377 á 28,123 kg por cm cuad.... | 23,750 |
| — | **Límite de elasticidad**, 2,362 á 4,252 kg por cm cuad..... | 3,307 |
| — | **La compresión** dentro del límite de elasticidad, da un promedio como de 1 parte en 13,300 con 3,300 kg por cm cuad. | |

**Los filos de las cuchillas del mejor acero** para grandes romanas de ferrocarril, se consideran seguros sometidos á una presión de 1,250 kg por cm lineal de filo, y los **rodillos** cilíndricos de acero macizo que se colocan debajo de los puentes, y que *resbalan sobre acero*, tienen una carga de seguridad de $\sqrt{\text{diám en pulgs}} \times 3100\ 000$, en lbs por pulg lineal de rodillo paralelas al eje, ó bien (*del T.*) en kg por cm lineal:

$197.33\ \sqrt{\text{diám en cms.}}$

(*Obs. del T.* — Basado en el mismo principio que produjo el radical penúltimo, en medidas inglesas, da el autor cinco fórmulas, tomadas del *Specifications for Iron Drawbridge at Milwaukee*, por Don J. Whittmore, I. C.

Estas fórmulas las hemos convertido al sistema métrico y simplificado, dando no obstante resultados con diferencias (de ocho ó diez kgs) despreciables en la práctica. Todas dan kg por centímetro lineal paralelo al eje del rodillo.)

| | |
|---|---|
| **Ruedas sólidas de hierro fundido** que corren sobre hierro forjado..... | $66.37 \times \sqrt{\text{diám en cm}}$ |
| **Ruedas sólidas de hierro fundido** que corren sobre hierro fundido..... | $52.82 \times \sqrt{\text{diám en cm}}$ |
| **Ruedas sólidas de acero** que corren sobre acero..... | $126.77 \times \sqrt{\text{diám en cm}}$ |
| **Ruedas sólidas de acero** que corren sobre hierro forjado..... | $113.54 \times \sqrt{\text{diám en cm}}$ |
| **Ruedas sólidas de acero** que corren sobre hierro fundido..... | $103.25 \times \sqrt{\text{diám en cm}}$ |

## Promedio de resistencia de las piedras á la tensión, etc.

| Las resistencias efectivas pueden fácilmente discrepar en $^1/_3$ más ó menos de las de nuestra tabla. | Kilogramos por cm cuadrado. |
|---|---|
| Ladrillo, 2.8 á 28 | 15.5 |
| Piedra de Caen, 7 á 14 | 10.5 |
| Cemento y concreto, véanse los artículos sobre cemento y concreto. | |
| Vidrio, 177 á 630 | 404.3 |
| La cola adhiere las maderas, 25 á 56 | 38.7 |
| El cuerno de buey | 632.7 |
| El marfil | 1,124.9 |
| Correas de cuero, 105 á 350. Buenas | 210.9 |
| Mortero corriente, de 6 meses, .7 á 1.4 | 1.06 |
| Mármol fuerte, blanco, de Italia | 72.7 |
| — de Champlain, jaspeado * | 117.1 |
| — de Glenn's Falls. N. Y., negro * | 62.7 |
| — de Montgomery Co, Pa., gris * | 82.6 |
| — — — blanco * | 51.6 |
| — de Lee, Mass., blanco | 61.5 |
| — de Manchester, Vt. * 38 á 56 | 47.5 |
| — de Tennessee, jaspeado * | 72.7 |
| Olitas, 7 á 14 | 10.5 |
| Yeso bien asentado | 4.9 |
| Cuerda de Manila, la mejor | 843.6 |
| — de henequén, la mejor | 1,054.6 |
| Arenisca de Ohio * | 7.4 |
| — de Picton, N. S. * | 30.5 |
| — de Conn, roja * | 41.5 |
| Pizarra, Lehigh * | 174.0 |
| — rosada | 268.0 |
| Piedra artificial de Ransome | 21.1 |
| Barba de ballena | 534.3 |

* Hechos por el autor. Sección de ruptura 9.67 cm cuad.

**Promedio de las cargas de trituración en toneladas por metro cuadrado**, para piedras, etc. Se suponen las piedras colocadas *de plan en hiladas* y que sus alturas sean de **1.5 á 2** veces su lado menor. Las piedras comienzan generalmente á rajarse ó á romperse con la mitad, más ó menos, de su carga de trituración. En la práctica, ni las construcciones de piedra, ni las de ladrillo, deben someterse con confianza á más de $^1/_6$ á $^1/_{10}$ de la carga de trituración, según las circunstancias. Cuando se **hallan completamente mojadas** algunas areniscas absorbentes pierden íntegra la mitad de su resistencia.

(*Obs. del T.* — Damos la tabla que sigue en **kg por metro cuad**, deducida de la del autor, que está en medidas inglesas; cuando se desee obtener dichas cargas de ruptura expresadas en kg por cm cuad no hay más que dividir por 10,000 las que esta tabla suministra.)

| | Kg por m cuad. | Término medio en kg por m cuad. |
|---|---|---|
| Granitos y sienitas | 3281113 á 13124450 | 8202781 |
| Basalto | | 7655929 |
| Piedra caliza y mármoles * | 2434260 á 10937042 | 6885651 |
| Olitas buenas | 1093704 á 2734260 | 1913982 |
| Caliza roja de Connecticut : | | |
| De « construcción » | 6234114 á 10608931 | 8421522 |
| De « puentes » | 4374817 á 6890336 | 5632576 |
| Ladrillo * | 437482 á 3281113 | 1859297 |
| La mampostería ordinaria de ladrillo se raja con | 218741 á 328111 | 273426 |
| Buena, con cemento * | 328111 á 437482 | 382796 |
| Mampostería de ladrillo de primera clase con cemento | 546852 á 765593 | 656222 |
| Pizarra | 4374817 á 8749634 | 6562225 |
| Piedra de Caen ** | 765593 á 2187408 | 1476500 |
| — — para romperse | | 765593 |
| Yeso duro | 218741 á 328111 | 273426 |
| Yeso de París de 1 día | | 437482 |
| **Cemento.** Portland, puro, de los E. U., ó extranjero, 7 días en el agua | 820278 á 1640556 | 1230617 |
| Cementos comunes de los E. U., puros, 7 días en agua | 164056 á 328111 | 246083 |
| **Concreto de cemento** Portland, arena y granzón ó piedra picada, en las proporciones convenientes, apisonado, de 1 mes | 131245 á 196867 | 164056 |
| De 6 meses | 524978 á 787467 | 656222 |
| De 12 meses | 809341 á 1312445 | 1060893 |
| **Con buenos cementos hidráulicos corrientes,** como de .2 á .25 de lo anterior. | 1093704 á 1640556 | 1367130 |
| **Concreto Coignet** de 3 meses | 164056 á 382796 | 273426 |
| Mampostería de **cascajo,** mortero ordinario grueso | 14218154 á 25155196 | 19686675 |
| Vidrio, verde, *crown glass* y *flint glass* | ó casi 3 veces la del granito | |
| Hielo duro *** | 131245 á 196867 | 164056 |

* **Ensayos hechos en el puente de San Luis**, por orden del capitán James B. Eads, I. C., demostraron que algunas piedras calcáreas que contienen magnesia, no se rompían bajo una presión menor de 12,031 tons por m cuad. Una columna de 203 mm y de 51 mm de diám se acortó en (.0025 pulg) = .063 mm sometida á presión y recobró su longitud al cesar dicha presión.

**Los ensayos hechos con la máquina de prueba del Gobierno en Watertown, Massáchusets, de 1882 á 1883**, dieron como carga máxima de trituración para el mármol blanco y el azul, procedente de Lee, Mass, 15,342 tons por m cuad; para el mármol azul del condado de Montgomery, Pa, 7,656 tons por m cuad; para la piedra calcárea de Conshohocken, Pa, 10,500 tons por m cuad; para la misma de Indiana, 5,468.21 kg por m cuad; para la piedra arenisca roja de Hummelstown, Pa, 9,187 tons por m cuad; para la piedra arenisca amarilla del Ohio, de 2,843 á 10,937 tons por m cuad. Ladrillos de Filadelfia, colocados de plan, fuertes, hechos en máquina, de 3,827 á 7,655 tons por m cuad; hechos á mano, de 7,655 á 14,218 tons por m cuad; prensados, en máquina, de 4,921 á 6,343 tons por m cuad. Columnas de mamp de ladrillo, de 33 cm en cuadro y 33 cm de altura, 100 toneladas, con cal; con cemento, 150 toneladas.

** Los ensayos hechos por el coronel Wm. Ludlow, E. U. de A., con las máquinas de ensayos del Gobierno, en 1881, dieron de 229 á 700 tons por m cuad para el hielo puro y duro; y de 174 á 207 tons por m cuad para las clases inferiores. Los trozos de ensayo (cubos de 15 á 30 cm) se redujeron de 12 á 25 mm antes de su trituración.

### Altura de trituración del ladrillo y de la piedra.

Si suponemos que la mampostería de ladrillo pesa, en número redondo, á razón de 1,800 kg el m cúb y que se rompe bajo una carga de 329 tons por m cuad, una columna vertical uniforme de dicho material de 183 m se rompería en su base por la acción de su propio peso. La piedra de Caen *, que pesa 2,082 kg el m cúb, necesitaría una columna de 419 m para su trituración. Las areniscas corrientes del peso de 2,322 kg el m cúb necesitarían una de 1,267 m, y siendo el peso del granito corriente de 2,643 kg el m cúb, requeriría una altura de 2,482 m. Pero las piedras empiezan á romperse y á astillarse con la mitad de la carga máxima de trituración, y en la práctica no se considera conveniente someterlas á una carga mayor de $^1/_6$ á $^1/_{10}$ de dicha carga de trituración, especialmente en trabajos de importancia, porque los cimientos y la imperfección de la obra de mano causan á menudo esfuerzos indebidos que obran en ciertas partes solamente.

La torre donde se hacen municiones en Baltimore tiene de altura 74.98 m y su base soporta una presión de 71,091 kg por m cuad. La base de la pila de granito de puente de Saltash (hecha por Brunel), de mampostería maciza hasta 29.26 m del altura, que soporta los extremos de dos tramos de hierro de 138.68 m cada uno, recibe una presión de 102 tons por m cuad. La base de una chimenea de ladrillo en Glasgow, Escocia, de 142.64 m de altura, soporta 99 tons por m cuad, y el profesor Rankine considera que con viento fuerte, el lado de sotavento debe soportar como 164 tons. La pila más alta del acueducto de piedra de Rocquefavour, en Marsella, es de 92.96 m y soporta en su base una presión de 142 tons por m cuad.

## VIGAS DE PIEDRA

**Tabla de las cargas netas estáticas de seguridad para vigas de granito bueno de construcción**, de 25 mm de ancho, apoyadas en ambos extremos y cargadas en el centro, suponiendo que la carga de seguridad es un décimo de la carga de ruptura y ésta de 138.4 kg para una viga de 25 mm en cuadro y 1 decímetro de luz. Se ha deducido ya la mitad del peso de las vigas, á razón de 2.72 kg por dcm cúb.

(*Obs. del T.* — Hemos convertido la tabla del autor al sistema métrico. Con el objeto de hacer más uniformes los datos de entrada á la tabla, hemos calculado las pulgadas para espesores, etc., á 25 mm en vez de 25.4, lo que produce pequeñas diferencias sin importancia en la práctica.)

| Altura de la viga. | LUZ EN METROS | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | .30 | .60 | .90 | 1.20 | 1.50 | 1.80 | 2.10 | 2.40 | 3.05 | 3.66 | 4.58 | 6.10 |
| | Carga de seguridad en el centro. | | | | | | | | | | | |
| mm | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. |
| 25 | 4.5 | 2.2 | | | | | | | | | | |
| 50 | 18.1 | 9.0 | 5.8 | 4.5 | | | | | | | | |
| 75 | 40.8 | 20.4 | 13.1 | 9.5 | 7.7 | | | | | | | |
| 100 | 72.5 | 35.8 | 23.5 | 17.6 | 14.0 | 11.7 | 9.5 | | | | | |
| 125 | 113.3 | 56.2 | 37.1 | 27.6 | 21.7 | 18.1 | 15.4 | | | | | |
| 150 | 163.2 | 81.1 | 53.9 | 40.3 | 31.7 | 26.3 | 21.7 | 19.0 | 14.5 | | | |
| 175 | 222.2 | 110.6 | 73.4 | 54.4 | 43.5 | 35.8 | 30.3 | 26.3 | 20.4 | 16.3 | 12.2 | 7.2 |
| 200 | 289.8 | 144.6 | 96.1 | 71.6 | 57.1 | 47.1 | 39.9 | 34.4 | 26.7 | 21.3 | 16.3 | 9.9 |
| 250 | 453.1 | 226.3 | 150.1 | 112.4 | 89.3 | 73.9 | 63.0 | 54.4 | 42.6 | 34.4 | 26.3 | 17.2 |
| 300 | 652.7 | 325.6 | 216.8 | 161.9 | 128.8 | 107.0 | 91.1 | 78.9 | 62.1 | 50.3 | 38.5 | 26.3 |
| 350 | 888.5 | 443.6 | 294.8 | 220.9 | 176.0 | 146.0 | 124.2 | 107.9 | 85.2 | 69.4 | 53.5 | 36.7 |
| 400 | 1160.7 | 579.6 | 385.5 | 288.4 | 229.9 | 190.9 | 162.8 | 141.5 | 111.5 | 91.1 | 71.2 | 49.4 |
| 450 | 1469.2 | 733.9 | 488.5 | 365.6 | 291.6 | 241.8 | 206.3 | 179.6 | 141.9 | 116.5 | 90.7 | 63.9 |
| 500 | 1814.0 | 906.2 | 602.8 | 451.3 | 360.1 | 299.3 | 255.3 | 222.2 | 176.0 | 144.6 | 112.9 | 79.8 |
| 550 | 2195.0 | 1096.3 | 729.8 | 546.5 | 435.9 | 362.8 | 309.3 | 269.4 | 213.1 | 175.5 | 137.4 | 97.9 |
| 600 | 2610.8 | 1305.0 | 869.0 | 650.4 | 519.3 | 431.3 | 368.7 | 321.1 | 254.9 | 210.0 | 164.2 | 117.9 |
| 675 | 3305.8 | 1651.9 | 1099.9 | 823.2 | 657.7 | 546.5 | 467.2 | 407.3 | 323.4 | 266.7 | 209.5 | 150.5 |
| 775 | 4081.4 | 2039.3 | 1358.5 | 1017.4 | 812.3 | 675.4 | 577.4 | 503.4 | 400.0 | 330.2 | 259.9 | 188.2 |
| 850 | 4938.7 | 2468.0 | 1643.8 | 1231.0 | 983.3 | 817.8 | 699.4 | 610.0 | 484.3 | 400.5 | 315.7 | 229.0 |
| 925 | 5877.7 | 2937.7 | 1956.8 | 1465.5 | 1170.7 | 973.8 | 832.7 | 727.1 | 578.3 | 478.0 | 377.3 | 274.8 |

* Es una piedra calcárea color de crema, para edificar, traida de Caen, Francia.

(*N. del T.* — Recordando que la carga en el centro varía en proporción del ancho de la viga *b*; del cuadrado de la altura *d*, y en razón inversa de la longitud *l*, se puede calcular dicha carga por medio de esta tabla para valores de *b*, *d*, *l*, que no figuren en ella.)

**Si la carga está uniformemente distribuída en la luz,** la carga de seguridad neta será doble de la que da la tabla.

**Para pizarra de buena calidad, colocada de plan,** las cargas de seguridad pueden calcularse como triples; **para la piedra arenisca de buena clase, colocada de plan,** como la mitad, y para el mármol bueno ó piedra caliza colocados de plan, próximamente lo mismo que la de la tabla.

### Arcilla. Densidad *.

La densidad de los minerales (sobre todo los silicatos de alúmina) de que se componen las arcillas, varían generalmente entre 2, 5 y 3; la de las partes sólidas de las arcillas entre 2, 3 y 2.9 **.

Pero el peso específico de la arcilla, considerado como un material poroso, varía entre límites mucho mayores, á causa de su porosidad, la cantidad de agua que contiene, etc. Véase pág. 222, §§ 11 á 13. De esta manera encontramos los valores dados de 1.66 á 2.64 ***. Arcilla con granzón, 2.48§; arcilla de alfarería, 1.8 á 2.1; arcilla seca en terrones, suelta, 1.0.

# MORTERO, LADRILLOS, ETC.

## MORTEROS DE CAL

**Art. 1. Mortero.** La proporción de uno de cal viva en pedazos irregulares ó en polvo para 5 de arena, es más ó menos la proporción usada para el mortero común por los buenos constructores de nuestras ciudades principales del Atlántico; y si ambos materiales son buenos, y bien mezclados (dando *el punto*) con agua limpia, el mortero es tan bueno como puede exigirse para los trabajos ordinarios que no requieren una adición de cemento hidráulico. El volumen del mortero ya mezclado excede generalmente al de la arena seca y suelta en $^1/_4$ parte, más ó menos.

**Cantidad requerida.** 566 litros de arena y 113 lit de cal viva, sacudiendo ligeramente las medidas (para que se compacten), componen, más ó menos, 637 litros de mortero; lo suficiente para poner en obra 1,000 ladrillos del tamaño ordinario de 20×10×5 cm, con las juntas gruesas de mortero de 10 á 12 mm, que se dejan en los muros interiores de las casas. Con estas juntas 1,000 ladrillos de estas dimensiones hacen 1.528 m cúb de obra maciza. Casi la tercera parte del volumen es de mortero. Para las juntas exteriores, que están á la vista, donde se requiere un mortero más blanco y de mejor apariencia, los constructores aumentan la proporción de la cal de 1 en 4, ó 1 en 3. Para mortero de granzón fino cernido para muros de sótanos de piedra bruta, ú obra ordinaria de ladrillos, se usa 1 de cal para 6 ú 8 de granzón, y el mortero es bueno. En mamposterías corrientes ordinarias de piedra bruta, ó de ladrillos como se dijo antes, una tercera parte del volumen es de mortero; por consiguiente, en un m cúbico entran, más ó menos, 650 ladrillos; 368 lit de arena, y 74 lit de cal viva. Una mampostería de primer orden, de piedra bruta bien desbastada, bien hecha, contiene solamente $^1/_3$ parte de su volumen de mortero, 200 lit de arena y 40 lit de cal por m cúbico.

Para obras públicas de ingeniería, especialmente si son pesadas ó expuestas á *la humedad*, debe agregarse á los anteriores morteros una cantidad de cemento hidráulico igual á $^1/_3$ parte de la cal, ó mejor aún omitir $^1/_3$ parte de la cal sustituyéndola con igual cantidad de cemento. **Si la obra recién construída ha**

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* Véase «Clays», por Heinrich Ries, Nueva York, John Wiley and Sons, 1906.

** Exploración Geológica de New Jersey, Informe final, 1904. Vol. IV, pág. 114; Exploración Geológica de Iowa, 1904. Vol. XIV, pág. 116.

*** Exploración Geológica de New Jersey. Informe sobre Arcillas, 1878; Expl. Geol. de Misurí, 1906. Vol. XI, págs. 562, etc. Segunda Expl. geog. de Pensilvania. Vol. B, pág. 3

§ L. M. Haupt. Especificaciones de Ingeniería y Contratos.

**de estar expuesta al agua,** debe emplearse muy poca ó ninguna cal en el exterior.

Las que siguen son **las cantidades de mortero y de ladrillos que entran en un metro cúbico de mampostería maciza** hecha con ladrillos de 20×10×5 cm.

| Espesor de las juntas. | Proporción de mortero que entra en toda la masa. | N.º de ladrillos por m cúbico. |
|---|---|---|
| 3 mm | $^1/_9$ más ó menos | 830 |
| 6 mm | $^1/_4$ — — | 746 |
| 10 mm | $^3/_{10}$ — — | 679 |
| 13 mm | $^1/_3$ — — | 617 |
| 15 mm | $^4/_{10}$ — — | 563 |

Al hacer el cálculo de los ladrillos que entran en una obra maciza, **cuéntese con 2 ó 3% de desperdicio,** y en edificios comunes 5 % ó más. La mayor parte del desperdicio proviene del recorte de los ladrillos para ajustarlos á los ángulos, etc. En Filadelfia se calcula 1 barril de cal sin apagar para 1,000 ladrillos; ó para 2 perchas (708 lit cada una) de piedra bruta en muros de sótanos. En muros delgados entra algo menos de mortero para cada 1,000 ladrillos, que en las construcciones macizas de ingeniería, porque los primeros tienen, relativamente, más superficie que no se cubre con mortero; pero en muros delgados hay más desperdicios durante la construcción, así es que ambos requieren, más ó menos, la misma cantidad de materiales. Ensayos hechos con cuidado prueban que el mortero se endurece más y se adhiere mejor al ladrillo ó á la piedra, si se aumenta la proporción de cal. Por esta razón en nuestras obras públicas se estipula generalmente la proporción de 1 de cal viva para 3 de arena, pero probablemente no se usa nunca.

**La cal se vende generalmente en terrones por barriles** de más ó menos 104 kg netos, ó 113 kg brutos.

**Observaciones generales acerca del mortero y de la cal.** En una proporción demasiado grande de nuestras obras públicas puede verse cómo el mortero de cal se pudre y se pierde expuesto á la humedad, llevada por la acción capilar de la tierra hasta varios pies más arriba de la superficie natural, ó mucho más abajo de la superficie de los rellenos depositados detrás de los estribos, muros de contención, etc. Lo mismo se observará frecuentemente en el intradós y arranque de los arcos debajo de un relleno. *El mortero de cal, expuesto así á una humedad constante, jamás se endurecerá bien.* Aun siendo muy viejo y duro, absorbe el agua libremente. *El cemento hace lo mismo, pero se endurece.*

**El polvo de ladrillo,** ó greda quemada, mejora el mortero y lo hace hidráulico. En lugares donde no se pueda obtener arena, puede emplearse en su lugar greda quemada y molida, la cual da, generalmente, un mortero mejor.

Es absolutamente esencial **proteger la cal viva contra la humedad** y aun contra el aire; de lo contrario, se apaga espontáneamente, ó sufre el proceso llamado apagamiento por el aire, por el cual se reduce á polvo, como si fuera apagada con agua, como se acostumbra, pero sin producir calor y con poco aumento de volumen. Como el apagamiento por el aire se hace desde unos meses hasta un año ó más, según la calidad y la exposición, da lugar á que la cal absorba del aire una cantidad de ácido carbónico suficiente á dañar y destruir su eficacia. **Pero la cal viva** molida y guardada herméticamente en barriles se **conserva en buen estado por mucho tiempo.** Al molerla se reducen también á polvo las partículas refractarias que se encuentran en toda cal y que perjudican al mortero, pues no se apagan sino después de hecho y empleado. Por la misma razón es preferible no usar nunca la cal inmediatamente después de apagada, sino dejarla por un día ó dos, y aun varios, resguardada del sol, de la lluvia y del polvo, antes de hacer el mortero.

**La cal viva apagada en gran cantidad puede chamuscar y hasta quemar la madera.**

**La pasta de cal y el mortero duran años** y mejoran si se entierran bien en el suelo. Y duran meses si están bajo techo y cubiertos simplemente por una capa de arena. La pasta se contrae y agrieta al secarse, pero la arena en el mortero lo impide.

**Como un término medio aproximado,** que varía mucho según las condiciones de la piedra y el grado de calcinación, y según la finura ó grosor de la arena, si se mezcla una parte de cal viva buena en terrones ó en polvo, con una cuarta parte de agua ó menos, dará en menos de una hora hasta 2 partes más ó menos de

cal seca en polvo. Y si á este polvo se agregan 3 partes de arena seca y $^3/_4$ partes más de agua, y se mezcla bien, el resultado dará 3 ⅓ partes, más ó menos, de mortero. La misma cantidad de cal en polvo apagada, mezclada con 1 parte, más ó menos, de agua y 5 de arena, dará como 5 ⅔ partes de mortero. En ambos casos el volumen del mortero será, más ó menos, una octava parte mayor que el de la arena sola. Si para apagarla se le echan ¾ de una medida de agua, el resultado en lugar de un polvo seco, será una pasta dura de ⅛ más de volumen; ó si se le echa una medida entera de agua para apagarla, el resultado dará ¼ más de una pasta delgada, que tiene más ó menos la consistencia adecuada para mezclarla con la arena. Las cales *grasas*, muy puras, se apagan prontamente y producen de 2 á 3 tantos de polvo, mientras que las cales *magras* requieren más tiempo y aumentan menos. La lentitud para apagarse y el poco aumento de volumen, en caso de que la piedra de cal haya sido bien quemada, no son generalmente malas propiedades, sino que, por el contrario, indican generalmente que la cal es hasta cierto punto hidráulica. En este caso produce un mortero mejor, especialmente para obras expuestas á la humedad ó á la intemperie. Las cales muy puras son *cales impropias* para la intemperie y no deben emplearse sin cemento en obras importantes.

**La cal de caracoles** parece ser casi de la misma calidad que la de piedra caliza más pura; pero la cal hecha de yeso es todavía inferior y no soporta más de 1½ partes de arena, su mortero jamás se endurece mucho. Parece que las madréporas (generalmente llamadas coral) dan una cal intermedia entre la producida por el yeso y la caliza. No requieren sino calcinación mediana.

**El peso medio** del mortero endurecido es, más ó menos, de 1,680 á 1,840 kg por m cúbico.

**El mortero delgado** se prepara tan líquido que tiene la fluidez de la crema. Se usa para llenar los intersticios dejados en las juntas por el mortero común en la mampostería bruta; pero, á menos que contenga una gran cantidad de cemento, es probable que sea enteramente inútil, porque la gran cantidad de agua afecta las propiedades de la cal; y además, sus ingredientes se separan unos de otros, asentándose la arena debajo de la cal. Además, nunca se endurece completamente en el interior de gruesas masas de mampostería, y lo mismo puede decirse, probablemente, de cualquier mortero de cal. En algunos lugares se ha hallado perfectamente blando, aun después de muchos años.

Tanto la arena como el agua para el mortero de cal **deben estar libres de arcilla y de sal**. La arcilla puede quitarse lavándola, pero es en extremo difícil desembarazar de la sal á las arenas de las playas del mar aunque se laven repetidas veces. Generalmente le queda suficiente sal para conservar húmeda la obra y para producir eflorescencias de nitro en la superficie, tanto con el mortero de cal como con el cemento. Apagando la cal con agua salada da menos pasta que con agua dulce.

El mortero no debe hacerse sobre la superficie de un suelo gredoso, sino sobre una plataforma de tablones, de ladrillos ó lajas. La arena de fosos cernida y sacada del gneis descompuesto, ó de otras piedras semejantes, es excelente para el mortero, porque sus ángulos agudos hacen con la cal una masa más compacta que los granos redondos de arena de río ó playa. El mortero debe aplicarse más húmedo en tiempo cálido que en tiempo frío, especialmente en mamposterías de ladrillos; de lo contrario la mampostería absorbe demasiada agua y daña el mortero.

**La resistencia á la tensión** del mortero de buena cal en las proporciones acostumbradas de arena y cal, á los seis meses es más ó menos de 1 á 2.10 kg por cm cuad. Con menos arena ó con más tiempo, será más fuerte.

**La resistencia á la trituración** de un buen mortero de seis meses, es de 10.5 á 21 kg por cm cuadrado.

**La resistencia al deslizamiento,** ó la que el mortero común opone á cualquiera fuerza que tienda á hacer que una capa de mampostería se deslice sobre otra, según afirma Rondelet, es solamente de .35 kg por cm cuad.

**Resistencia transversal** de un buen mortero de 6 meses. Una barra de 1 cm en cuadro con una luz de 10 cm, se rompe con una carga central de .35 á .70 kg.

**La cal del mortero destruye la madera rápidamente**, sobre todo en lugares húmedos y encerrados. Sin embargo, la inmersión de la madera por una ó dos semanas en una solución de cal viva y agua, parece obrar como un preservativo.

**El hierro** incrustado completamente en el mortero, y de este modo aislado del aire y de la humedad, se ha encontrado en perfecto estado después de 1,400 años, pero si la humedad penetra en el mortero, el hierro se destruye, y es probable que lo mismo suceda con otros metales.

**El término medio de la adhesión del mortero al ladrillo ordinario**

**ó á la piedra bruta** al cabo de cualquier tiempo, será más ó menos las ¾ partes de la resistencia de cohesión del mortero para la misma época, ó sean de .84 á 1.68 kg por cm cuadrado en mortero de 6 meses. Si se tiene el cuidado de eliminar por completo el polvo, mojando bien el ladrillo en agua antes de colocarlo, ó echándole agua á la piedra con una manguera, etc., se aumentará la adhesión, pues el mucho polvo puede impedir toda adhesión. La precaución de mojar es especialmente necesaria en tiempo muy cálido para evitar que los ladrillos ó piedras calientes dañen el mortero por la absorción y evaporación rápida del agua. La adhesión á los ladrillos lisos muy prensados, ó á las piedras talladas muy lisas ó aserradas, es considerablemente menor.

## LADRILLOS

**Art. 2. Peso, tamaño, etc., de los ladrillos.** Un ladrillo de 20×10×5 cm contiene un litro; entran mil en el m cúb.

Al hacer pedidos de un gran número de ladrillos, debe especificarse el límite mínimo de sus dimensiones, para evitar engaños. Un ladrillo que tenga 6 mm menos en todo sentido que la medida dada, no tiene sino como .80 lit; necesitándose, por consiguiente, como 25% más de ladrillos para ejecutar la misma obra, y además 25% más de gasto por la obra de mano, que generalmente se paga por millar.

**El peso** de un buen ladrillo corriente de 20×10×5 cm es, por término medio, 1.875 kg ó 1.875 ton por metro cúb. Un buen ladrillo prensado del mismo tamaño tiene, por término medio, 2.08 kg. Como el peso del mortero endurecido es poco menos que el de un buen ladrillo común, podemos suponer para cálculos ordinarios el **peso de la mampostería** de ladrillos comunes, en 1.87 toneladas por m cúbico.

**Sumergidos en agua**, cualquiera de ellos absorbe, en algunos minutos, de 226 á 340 gramos de agua.

**Colocación en obra de los ladrillos.** Un albañil con un oficial que le suministre el material, puede poner en obra un promedio de 1,500 ladrillos por día de 10 horas en muros de casas comunes. En los muros exteriores (de más esmero), de 1,000 á 1,200; en los frentes bien hechos de las casas, de 800 á 1,000; en los frentes muy ornamentados de pisos bajos, de 150 á 300, según el número de ángulos, etc. En obras de ingeniería macizas y sencillas, un albañil puede colocar unos 2,000 ladrillos por día, es decir, hacer como 3 m cúb; en arcos grandes, unos 1,500 ó 2.3 m cúbicos.

Como los ladrillos se contraen al secarse y quemarse $^1/_{13}$ parte, más ó menos, de sus dimensiones, los moldes deben ser de $^1/_{12}$ parte más grandes en todos sentidos que el ladrillo quemado que se desea hacer. Los ladrillos buenos y bien quemados suenan como campanas al chocar unos con otros. En las fábricas de ladrillos cerca de Filadelfia, el trabajo de un moldeador de ladrillos es de 2,333 ladrillos por día, ó 14,000 por semana. Lo ayudan dos muchachos; uno le da la greda preparada, la arena de moldear y el agua, y el otro se lleva los ladrillos ya moldeados. Otra persona los coloca en hileras para que se sequen. Se calcula en ¾ de cuerda (montón de leña de 2.44×1.22×1.22 m) la leña que consume la quema de 1,000 ladrillos. Cuando se emplea carbón de piedra los hornos se encienden con antracita, y se concluye el cocimiento con carbón bituminoso. Con una tonelada de carbón se hacen 4,500 ladrillos.

**Pavimento de aceras con ladrillos.** Se ponen los ladrillos sobre una capa de granzón de 15 cm de espesor, libre de greda y bien consolidado. En un m cuadrado entran, empleando ladrillos de 20×10×5 cm, con juntas de 3 á 6 mm de ancho, colocados de plan, 45 ladrillos; de canto, 88, y de cabeza, 180. Un albañil regular con un peón que le pase los ladrillos y el granzón, colocará en 10 horas unos 2,000 ladrillos. Concluída la colocación se llenan las juntas con arena.

**Art. 3. La resistencia de los ladrillos á la trituración varía mucho, naturalmente.** Un ladrillo algo blando se tritura bajo una presión de 31 á 42 kg por cm cuad; mientras que uno muy bueno prensado requiere de 215 á 430 kg por cm cuadrado, ó sea, más ó menos, el máximo de trituración de la mejor piedra arenisca, ⅔ del de los mejores mármoles y piedras calizas, ó la mitad del mejor granito ó pizarra para techos. Pero los bloques de mampostería de ladrillos se trituran bajo cargas mucho más pequeñas que los ladrillos solos. En algunos ensayos hechos en Inglaterra con pequeñas masas cúbicas de sólo 23 cm por lado, asentadas con cemento, se trituraron bajo 30 á 43 kg por cm cuad. Otros hechos con pilares de 23 cm en cuadro y 69 cm de alto en cemento con sólo dos días de construídos, necesitaron de 47 á 66 kg por cm cuad para triturarlos. Otro pilar hecho

de ladrillos prensados, con el mejor cemento Portland, se dice que resistió 217 kg por cm cuadrado; y en mortero corriente de cal no resistió sino ¼ parte de dicha presión.

Debe, sin embargo, recordarse que principian á rajarse y astillarse, generalmente, bajo la mitad de la carga de trituración. Para mayor seguridad, la carga nunca debe exceder de ⅛ de la que produce la trituración; lo mismo con respecto á la piedra. Además, estos ensayos se hicieron sobre bloques bajos de mampostería y la resistencia disminuye aumentando la proporción de la altura al espesor.

La presión en la base de una torre donde se hacen municiones en Baltimore, como de 75 m de altura, se estima en 7.3 kg por cm cuad; y en una chimenea de ladrillos en Glosgow, Escocia, de 143 m (más ó menos) de elevación, en 10.1 kg por cm cuadrado. El profesor Rankine calcula que durante un temporal fuerte esta presión aumenta á 16.1 kg hacia el lado de sotavento. Con nuestros conocimientos tan imperfectos sobre esta materia, no debemos suponer que la mampostería, *con cemento*, de ladrillos prensados, aun de la mejor calidad, trabaja con seguridad bajo una carga mayor de 13 á 16 kg por cm cuadrado; ó la mampostería de ladrillos buenos, hechos á mano, á más de ⅔ partes de esta presión.

**La resistencia á la tensión** de los ladrillos es de 2.8 á 28 kg por cm cuadrado.

**La pértica inglesa** (*rod*, medida inglesa) de mampostería de ladrillos, tiene 305 pies cúbicos, ú 8.6366 m cúb, y entran en ella, más ó menos, 4,500 ladrillos del tamaño del modelo inglés; con 2.124 m cúbs, más ó menos, de mortero. *El quinta inglés de cal* es una yarda cúbica = .764 m cúb.

**Mortero helado.** Es peligroso usar el mortero común en tiempo frío. Si el frío continúa por bastante tiempo para que el mortero helado pueda fraguar bien, la obra *puede* permanecer firme ó segura; pero si hace un día de calor entre la helada y la fragua del mortero, dándole sol á un lado del muro, puede deshelarse el mortero de aquel lado y permanecer del otro lado helado y duro. En este caso el muro está expuesto á caerse, y si no sucede esto, por lo menos quedará siempre débil, porque el mortero que ha fraguado parcialmente durante la helada, al deshelarse no recobrará nunca su resistencia. Según los ensayos del autor, el cemento hidráulico parece no sufrir por la helada.

**Ensayos para hacer la mampostería de ladrillos impermeables al agua.** Extracto de una Memoria leída ante la Sociedad de Ingenieros Americanos, mayo 4, 1870, por William L. Dierborn, ingeniero civil y miembro de dicha Sociedad. Los muros del frente de la Oficina de Compuertas de Bach Bays, en el nuevo depósito del acueducto de Croton, situado al norte de la calle 86, en el Parque Central, fueron construídos con ladrillos de la mejor calidad, bien cocidos, y puestos en la obra con un mortero compuesto de cemento hidráulico de Nueva York y arena en la proporción de una parte de cemento para dos de arena. El espacio comprendido entre los muros es de 1.22 m y se rellenó con concreto.

Los muros de frente fueron hechos con sumo cuidado, y se tomaron todas las precauciones necesarias para que las juntas quedasen bien llenas y el trabajo bien ejecutado. Tienen un espesor de 30.5 cm y una altura de 12.2 m, y cuando está lleno el depósito, contiene generalmente 11 m de agua.

Cuando el estanque se llenó por primera vez y se le dió entrada al agua, se notó una filtración considerable por entre los muros. Tan pronto como se descubrió esta filtración se vació el agua con la intención de tratar de remediarla ó impedirla. Después de considerar cuidadosamente diferentes modos de lograr el objeto deseado me decidí á probar el « Procedimiento de Sylvester para evitar la humedad de los muros exteriores ».

El método consiste en usar dos soluciones ó capas para cubrir la superficie de los muros de ladrillos, la una compuesta de jabón de Castilla y agua, y la otra de alumbre y agua. Las proporciones son : 450 gramos de jabón para 5 litros de agua, y 75 gramos de alumbre para 5 lit de agua, ambas substancias perfectamente disueltas antes de usarlas.

Los muros han de estar perfectamente limpios y secos, y la temperatura del aire no debe ser menor de 10° C cuando se aplican las composiciones.

La primera solución, la del jabón, debe aplicarse á la temperatura del agua hirviendo con una brocha chata, teniendo cuidado de que no se forme espuma sobre la mampostería. Esta mano debe dejarse por 24 horas para que seque y se endurezca antes de aplicar la de alumbre, lo cual se hace del mismo modo que la primera. La temperatura de esta última solución puede ser de 15° á 21° C al aplicarse, y también se debe dejar transcurrir 24 horas para poner una segunda de jabón; las manos de dicha solución deben repetirse hasta que los muros se hayan hecho impermeables al agua.

El alumbre y el jabón así combinados, forman una composición insoluble que llena los poros de la mampostería, é impide por completo que el agua penetre en los muros.

Antes de aplicar estas composiciones á los muros del estanque, se hicieron algunos ensayos para probar la absorción del agua por los ladrillos bajo presión después de cubiertos con estas soluciones, y determinar cuántas manos podían necesitarse para hacer los muros impermeables al agua.

Para efectuar esto se hizo un cajón fuerte de madera, unido con tornillos y suficientemente grande para contener dos ladrillos, y en la parte superior se insertó un tubo de 25 mm de diámetro y 3 m de longitud. En este cajón se colocaron dos ladrillos, después de estar perfectamente secos; luego se cubrieron con una mano de cada solución, como se dijo arriba, y se pesaron. Se les sometió á la presión de una columna de agua de 12 m de altura, y después de dejarlos así durante un tiempo suficiente, se sacaron y se pesaron otra vez para saber la cantidad de agua que habían absorbido. Después de secar los ladrillos se les aplicaron otras dos manos de las soluciones, se pesaron y sometieron á la misma presión que antes, repitiendo esta operación hasta que los ladrillos no absorbieron más agua. Cuatro manos hicieron los ladrillos impermeables bajo la presión de una columna de agua de 12 m de altura.

El peso medio de los ladrillos (secos) antes de aplicar las soluciones era del .585 kg y el promedio de agua absorbida = .226 kg. Se usó un hidrómetro para probar las soluciones. Como este experimento se hizo durante el otoño ó invierno (1863), después de haberse puesto los techos provisionales á la Oficina de Compuertas, hubo que usar un calor artificial para secar los muros y mantener el aire á una temperatura adecuada. El costo fué de 10.06 centavos por pie cuadrado. Tan luego como se hubo endurecido la última mano, se hizo entrar el agua en el estanque, y los muros resultaron perfectamente impermeables; así estaban todavía en 1870, después de unos 6 años y ½.

El **arco de ladrillos** del paso del puente « High Bridge », se compone : de un arco de 9 m de radio, y de 30.5 cm de espesor; el ancho de la parte superior es de 5.18 m y la extensión cubierta fué de 128 m cuad.

Las primeras dos hiladas de ladrillos del arco son de los mejores ladrillos, bien cocidos, colocados de canto, en un mortero compuesto de una parte en volumen, de cemento hidráulico de Nueva York, y dos de arena. La parte superior de estos ladrillos y el interior de la albardilla ó coronamiento de granito, contra los cuales se apoyan las partes superiores de las dos hiladas de ladrillos, se cubrieron, después de estar perfectamente secas, con una capa de asfalto de 12 mm de grueso, la cual se colocó á una temperatura de 182° á 270° C.

Sobre ésta se colocó una hilada de ladrillos de plan sumergidos en asfalto, y colocados cuando el asfalto estaba caliente; las juntas se llenaron también de asfalto caliente.

Sobre esta capa se asentó otra de ladrillos de plan, prensados, colocados con mortero de cemento hidráulico, formando la calzada y piso del puente. El asfalto era de Trinidad y se le mezcló con un 10 % (por volumen) de alquitrán mineral y 25 % de arena. Se hicieron algunos ensayos para probar la resistencia de este alfalto empleado para unir los ladrillos; de estos ensayos se mencionan dos más abajo.

Se comprimieron juntos seis ladrillos de plan, con pega de asfalto, y se rompieron después de seis meses de colocados. La distancia entre los soportes era de 30 cm; la carga de ruptura, 405 kg; área de cada junta, 184 cm cuad. El asfalto estaba adherido á los ladrillos tan fuertemente, que en algunas partes al desprenderlo arrancó parte de aquéllos.

Dos ladrillos de punta, unidos con asfalto y comprimidos, se fracturaron después de seis meses.

La distancia entre los soportes era de 25 cm; área de la junta, 55 cm cuadrados; peso que produjo la fractura, 68 kg.

El área del puente cubierta con ladrillos asfaltados era de 2,143 m cuadrados. Se emplearon 42,728 kg de asfalto, 33 barriles de alquitrán mineral, 7.6 metros cúbicos de arena y 93,800 ladrillos. El tiempo invertido fué 109 días de albañiles, y 148 días de peones. Dos albañiles y dos peones pueden mezclar y extender en la primera mano 153 m cuadrados por día. El costo total de cada mano fué 56½ centavos por m cuadrado, fuera del derecho de importación sobre el asfalto. Atravesando enteramente el área de ladrillos é inmediatamente debajo de la primera capa de asfalto, se hicieron tres canales de 5 cm de ancho por 10 cm de profundidad, que dividieron

el arco en 4 partes iguales. Estos canales se llenaron de un cemento de pintura elástica.

Esto se hizo para contrarrestar los malos efectos de la contracción del arco en el invierno, porque se esperaba que podía ceder ligeramente en estos puntos y no en otros, y que el cemento elástico impediría cualquiera filtración.

El ensayo todo fué un éxito completo y el arco ha permanecido sin filtración alguna.

Al proponer el sistema anterior de aplicación del asfalto en la mampostería de ladrillos, el objeto fué evitar el empleo de una gran superficie continua de asfalto, como se hace generalmente para cubrir los arcos que á menudo se rajan por ser la contracción del asfalto mayor que la de la mampostería con la cual está en contacto, siendo la extensión de asfalto en esta obra solamente como de 1.6 cm cuad para cada ladrillo. Esto se cree un elemento esencial para el buen éxito de una cubierta impermeable.

Este es un método barato y muy eficaz para impedir la infiltración del agua por los arcos de acueductos y aun de los puentes. Muchas pruebas costosas con composiciones resinosas han fallado.

El cemento hidráulico parece que sólo disminuye el mal. Muchos de los inconvenientes son probablemente ocasionados por los cambios de temperatura.

**La eflorescencia blanca**, tan común en los muros, especialmente en los de ladrillos, es debida á la presencia de sales solubles en los ladrillos y en el mortero. Estas sales se disuelven y salen á la superficie del muro por la lluvia ú otra causa de humedad. El sulfato de magnesio (sal de Epsom) parece ser la causa más frecuente de este defecto. En mucho lugares se hace la cal para mortero de delomita, ó piedra caliza, *magnésica*, que muchas veces contiene 30%, ó más, de magnesio; ésta se encuentra también muchas veces en la arcilla de ladrillos. El carbón de piedra, generalmente, contiene azufre, más á menudo en combinación con hierro, formando las bien conocidas « piritas de hierro ». La combustión del carbón al quemar la piedra caliza ó greda, en las fábricas, en las cocinas, etc., convierte el azufre en gas ácido sulfúrico, el cual cuando se pone en contacto con el magnesio y el aire, como en los hornos de cal ó de ladrillos, ó en los muros ó chimeneas, se convierte en ácido sulfúrico, y se une con el magnesio, formando el sulfato soluble. No conecemos medio alguno que impida su presencia en tales circunstancias, pero la formación del sulfato puede evitarse con el uso, para ladrillos, de la piedra caliza y de la greda libres de magnesio.

# MORTERO DE CEMENTO

## Cemento.

**Para experiencias y práctica**, véase pág. 1346.

**Para especificaciones**, véanse págs. 1278, 1280, 1401.

**Para concreto**, véanse págs. 1295, etc.

**1. Hidraulicidad** es la propiedad de fraguar y endurecerse bajo el agua; los **cementos** que se endurecen en el agua se llaman **cementos hidráulicos** ó por brevedad, **cementos**. Para los fenómenos que se producen **al mezclar cementos con agua**, lleven ó no arena, véase « Mortero », pág. 1287.

## Materiales.

**2. Las substancias que ejercen mayor influencia** en los morteros de cal y cemento son:

| | | |
|---|---|---|
| Calcio, | Ca | Oxígeno, O. |
| Aluminio, | Al | |
| Carbón, | C | |
| Sílice, | Si | |
| Hidrógeno, | H | |

**3. El oxígeno se combina con cada uno de los** otros para formar **óxidos**, así :

Oxido de calcio CaO, que es cal;
Sexquióxido de aluminio, $Al_2O_3$, es alúmina;
Bióxido de carbón, $CO_2$, es ácido carbónico;
Bióxido de sílice, $SiO_2$, que es ácido silícico;
Oxihidrógeno, $H_2O$, que es agua.

**4. Los elementos** más usados en la fabricación de cementos son : (a) *Calcáreos*, (b) *substancias arcillosas*, ó (c) ambos, calcáreos y arcillas.

(a) **Calcáreos** (ricos en carbonatos de cal).

**Piedra caliza**, carbonato de cal, ó una combinación de cal y ácido carbónico, $CaO+CO_2=CaCO_3$. El mármol es una caliza.

**Dolomita, ó caliza magnésica** que contiene un 45% de carbonato de magnesia, $MgO.CO_2$. Cuando las capas de caliza y dolomita están juntas, la roca varía en su composición entre ambas, conteniendo proporciones de carbonato de magnesia variables entre 0 y 45.

**Una caliza blanda** (como fango consistente), formada por restos de conchas marinas.

**Marga** (*marl*), un carbonato de cal hidratado, blando é impuro, precipitado en aguas tranquilas, se encuentra en los lechos y orillas de lagos existentes ó extintos.

**Residuos alcalinos**, carbonato de cal, precipitado como un residuo en la fabricación de soda cáustica.

**Coral**. (Véase el § 5.)

(b) **Substancias arcillosas** (ricas en silicatos de alúmina).

**Arcilla** (incluyendo las substancias arcillosas minerales en general), un silicato de alúmina ó combinación de alúmina con ácido silícico, $Al_2O_3+SiO_2$.

**Esquistos arcillosos**, arcilla solidificada por proceso geológico.

**Puzolana**, escoria volcánica que se encuentra en Puzzuoli, cerca del Vesubio, es un silicato de alúmina impuro.

**Escorias de fundición**, que son como puzolanas artificiales.

**Polvos de ladrillo**. (Véase el § 6.)

(c) Ricos en **carbonato de cal y silicato de alúmina**.

**Las rocas propias para cemento** son calizas arcillosas. Contienen generalmente de 13 á 35% de silicato de alúmina; y el carbonato de magnesia en proporción hasta de 25%.

**5. Roca blanda de coral** de los cayos cerca de Colón (Panamá), mezclada con arcilla y fango que trae el río Chagres, ó con tobas que se encuentran en el istmo ó con ambas molidas, quemadas y ensayadas en el « Lehigh Valley Testing Laboratory », en Allentown, Pensilvania; dieron un cemento uniforme comparable á las marcas corrientes de los cementos de Lehigh. La roca de coral es un carbonato de cal notablemente puro. La arcilla y el fango del Chagres son pobres en sílice, pero contienen buena cantidad de hierro en proporción á la alúmina. Las tobas tienen más ó menos la misma composición que las substancias arcillosas empleadas en Lehigh.

**6.** El Sr. Ernest McCullough « mezcló polvo fino de ladrillo con cal hidratada y produjo un cemento satisfactorio para un pequeño trabajo de concreto en un sitio donde no pudo fácilmente obtener cemento Portland ».

**7. Cal**. Cuando se queman las calizas (sin arcilla) se va el $CO_2$ y el producto que es **cal viva** tiene gran afinidad por el agua, y la absorbe con tal avidez, que desarrolla calor suficiente para producir vapor, y éste desintegra y aumenta el volumen de la masa. Combinada así con el agua, se forma un hidrato de cal, $CaOH_2O=CaH_2O_2$. Esto es **apagar** la cal y se la llama **cal apagada**. Cuando se emplea en los morteros, absorbe gradualmente ácido carbónico del aire y se forma carbonato de cal, evaporándose el agua. Los morteros de cal endurecidos pueden considerarse como una caliza artificial.

## Fabricación.

**8. Cemento**. Cuando se quema un silicato de alúmina, como la arcilla, en cantidades suficientes con un carbonato de calcio, como una caliza, el producto, llamado cemento, aparece aún sin **fraguar** (sin apagarse); pero por otra parte los elementos combinados de la cal, la alúmina, la sílice y el agua formados durante la quema, son tales, que luego en el mortero reaccionan rápidamente bajo, la acción del agua. Los químicos no están de acuerdo sobre la naturaleza de estas combina-

ciones, excepto en que constituyen un proceso de cristalización, producido principalmente por la formación del silicato y el aluminato de cal hidratada, cuyos componentes constituyen la mayor porción de casi todos los cementos.

## Cemento natural y Portland.

**9.** En la fabricación del cemento **natural** las rocas de cemento quebradas en terrones se calcinan entre 1000 y 1400° C (1800° á 2500° F) en un horno fijo empleando como combustible capas alternadas de carbón triturado al tamaño de un guisante; luego se muele, muy fino, y este polvo se mezcla bien para lograr la mayor uniformidad.

**10. La calidad** de los cementos naturales varía mucho con las diversas composiciones de las rocas empleadas.

**11.** El nombre de **Rosendale**, original y propiamente restringido á los cementos naturales fabricados en el condado de Ulster, Nueva York, se ha aplicado indistintamente á todos los cementos naturales americanos en general.

**12.** En Europa llaman **cemento romano** á los naturales de fragua rápida.

**13.** El **cemento Portland** es llamado así por su semejanza, cuando ha fraguado, con las piedras calizas de Portland, en Inglaterra.

**14.** El **cemento Portland** se hace por las diferentes combinaciones de calcáreos y substancias arcillosas mencionadas en el § 4, y esto requiere diversos procedimientos preliminares. Así, las rocas duras se trituran; las blandas y las arcillas se muelen; las arcillas se mezclan húmedas y á veces cuando están fangosas se bombean para llevarlas á la fábrica. Siempre el producto se seca y se muele muy fino, se mezcla, y luego se calcina á una temperatura de 1450° C á 1500° C, hasta que se inicia la vitrificación, que consiste en la combinación química de la sílice, alúmina y cal, en escorias vítreas (*clinker*) que son silicatos dobles de alúmina y cal. Estas escorias (*clinker*) se vuelven á moler hasta convertirlas en un polvo impalpable que constituye el producto definitivo.

**15. Las proporciones** de los diversos elementos se gradúan cuidadosamente. Éstas son generalmente de 74 á 77.5% de carbonato de cal y un 20% de silicato de alúmina y óxido de hierro. (Véase § 32.)

**16. Manipulación.** Con el material bruto se forman á veces ladrillos que se queman en hornos fijos; pero ya se usa más llevarlo molido en polvo fino á hornos rotatorios de 6 á 8 pies (1.80 m á 2.40 m) de diámetro y de 60 á 100 pies (18 m á 30 m) ó más de largo. El carbón en polvo como combustible es impelido por el otro extremo y por medio de un inyectador de aire. El aire requerido para la combustión entra libremente por otras aberturas.

**17.** Como sucede con la cal, **la combustión** expele el ácido carbónico y el agua y completa la oxidación del hierro.

**18. El mayor costo del cemento Portland** se debe á la esmerada selección de los materiales y á la costosa elaboración, todo lo que permite obtener mayores resistencias y uniformidad que en los cementos naturales.

**19. Las mejoras** alcanzadas en la fabricación del cemento Portland ha disminuído el empleo de las otras clases. Debido á la mayor cantidad de arena que soporta los constructores lo usan aun en los casos en que les es permitido usar los naturales.

**20.** Si el material es deficiente en cal, puede **quemarse demasiado** (exceso de arcilla). La falta de quema (crudeza) produce un *clinker* blando color moreno, de fragua muy rápida que se calienta á la acción del agua. Algunos cementos de fragua lenta se hacen más rápidos después del almacenaje.

**21.** El **cemento Portland** se usa en construcciones sometidas á fuertes ó frecuentes resistencias, en casos en que se requieren elevadas resistencias en breve tiempo, para construcciones de concreto, cuando la obra va á estar en contacto con el agua, en muros delgados sometidos á la presión de las aguas, y para obras expuestas á altas temperaturas; mientras que los **cementos naturales** pueden emplearse en fundaciones secas y abrigadas y que no vayan á resistir presiones mayores de 75 lbs por pulg cuad (5.27 kilogramos por cm cuad) y no aplicables sino tres meses después de construídas, para sostener y rellenar muros de piedra y de concreto donde sólo influyen el peso y el volumen, y para cimientos (bases) de calzadas y de cloacas.

## Puzolanas.

**22. Cementos de escoria** (llamados á veces cementos de puzolana ó simplemente *puzolana*). Son mezclas íntimas de cal apagada y escorias de horno, ambas bien molidas, sin calcinar. A medida que las escorias van saliendo del horno, se

enfrían y desintegran echándolas en el agua. A veces se le agrega un poquito de soda para precipitar la fragua. No se confunda el cemento de escoria con los Portland en que se emplean las escorias como ingredientes.

**23.** En el aire seco, los sulfuros, contenidos en los cementos de puzolanas, se oxidan y producen grietas superficiales. Fraguan más despacio que los Portland á menos que se les ponga soda. Tratados así, la soda se convierte en carbonato después de prolongado almacenaje y el cemento vuelve á hacerse de fragua lenta. Como los cementos de puzolanas, bien hechos, no contienen cal libre ni anhídrica, no se deforman ni se hinchan, y requieren menos agua que los Portland; pero para su estabilidad es necesario que la construcción se mantenga húmeda. Se los recomienda para ser empleados en agua salada, solos ó mezclados con Portland. El mortero hecho con ellos es duro, pero nunca alcanzan la resistencia de los Portland. No deben estar expuestos á frotamientos ó golpes.

**24.** Se dice que los **cementos de puzolana** dan buen resultado empleados con 2 ó 3 partes de arena y no expuestos á temperaturas bajo 0° C. Los ingredientes deben estar muy bien molidos é íntimamente ligados. Se usan cuando no se requiere gran resistencia y dureza.

## Cementos silícicos.

**25. Cementos silícicos, ó arenosos.** Se hicieron en su origen mezclando al cemento Portland arena de cuarzo (sílice) y moliendo la mezcla hasta la extrema finura. Se asegura que el cemento así preparado se puede moler más fino y que estos cementos silícicos, en proporciones de una parte de cemento Portland y tres de arena, pueden soportar en el mortero casi tanta arena como el cemento puro; también se asegura que los morteros de estos cementos son menos permeables al agua que los ordinarios de cemento puro.

**26.** Debido al gran costo de la molienda de arenas cuarzosas, se las sustituye ahora con materiales menos duros, como las **piedras calcáreas**. El producto así obtenido se llama todavía « cemento silícico », aunque contiene menor proporción de sílice que el mismo cemento Portland.

**27. Los cementos silicados** dan morteros que se prestan más suavemente al trabajo de cuchara que los otros.

**28.** En la construcción de una esclusa de concreto en San Pablo (Minnesota) se trató de usar 1.5 volumen de cemento silícico, como equivalente á 1 volumen del cemento Portland marca « Saylor »; pero las pruebas demostraron que, á los 6 meses, el concreto hecho con cemento silícico era **tan fuerte** como el de Portland.

## Otros cementos.

**29. Cemento Portland blanco;** se obtiene haciendo ciertas modificaciones en la fabricación y es casi incoloro. Sirve para imitar mármol, etc., y se presta para recibir un colorido artificial. Es más caro que los Portland corrientes. (Véase § 44.)

**30. Cemento de minerales de hierro** ( « Erz-cement » ) Krupp steel Co. En este cemento las substancias arcillosas del Portland están casi en su totalidad reemplazadas por el óxido de hierro. Los elementos se queman y se muelen como para el Portland. Véanse §§ 13, etc. Densidad 3.31, fragua más lenta que el Portland. Su resistencia al principio es baja, pero luego supera la del Portland. No tiene expansión, ni se agrieta en agua salada bajo 15 atmósferas. (Wm. Michaelit Jr. Western Soc. of Engeneers, vol 12 n.° 4, agosto 1907; S. B. New-Berry, « Cemens Age », enero 1907.)

**31. Cal hidráulica** es el nombre dado á algunos cementos (muy usados en Europa), los cuales, aunque hidráulicos hasta cierto punto, no contienen suficientes elementos de hidraulicidad para impedir la efervescencia; ésta, no obstante, es más lenta, y menor el aumento de volumen que en la cal corriente.

## Composición.

**32. Análisis de los cementos.** Percentaje.

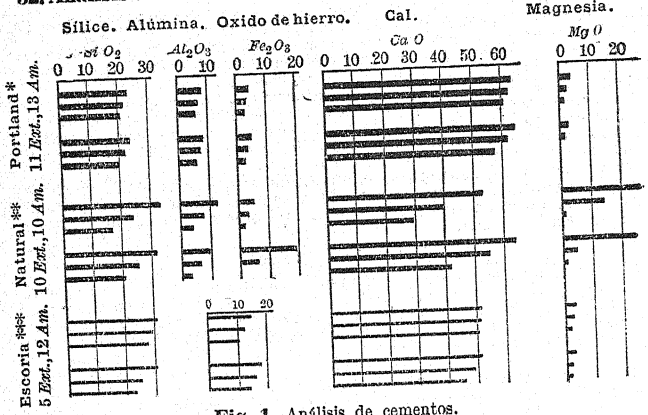


**Fig. 1.** Análisis de cementos.

(*Obs. del T.* — Los números 0, 10, 20, que están *dentro* de la figura debajo de la segunda y tercera columnas de rayas horizontales, se refieren á la combinación de alúmina y óxido de hierro. En cada grupo de 3 líneas, la superior indica el máximo percentaje; la intermedia, el mediano, y la más baja, el mínimo. Ext significa extranjeros; Am, americanos.)

**33.** La proporción en peso entre el silicato de alúmina y la cal en un cemento, es su **índice de hidraulicidad**.

**34.** Así, un cemento que contenga 30% de silicato de alúmina y 60% de cal, tendrá como índice de hidraulicidad $^{30}/_{60} = .50$.

**35. El módulo hidráulico** es aproximadamente el **inverso** del índice de hidraulicidad, es decir: el módulo es la proporción en peso de la cal para la sílice, la alúmina y el óxido de hierro. A veces se especifica que el módulo en el cemento Portland debe ser 1.7.

**36.** En los cementos naturales el módulo varía generalmente de .667 á 1.667.

**37.** Mr. Spencer B. Newberry emplea la siguiente proporción: (cal — alúmina) ÷ sílice, la cual llama él coeficiente de cal y éste varía en el material bruto de 2.7 á 2.8, y en los mejores cementos comerciales de 2.5 á 2.6.

**38.** Mr. Edwin C. Eckel (Cement Limes and Plasters, pág. 170) indica el siguiente,

$$\text{« Índice de cementación »} = \frac{2.8\,s + 1.1\,a + .7\,hi}{c + 1.4\,m}$$

en donde $s$, $a$, $hi$, $c$, $m$, son el percentaje en peso de sílice, alúmina, óxido de hierro, cal y magnesia respectivamente.

**39.** Con lo que generalmente se adulteran los cementos, es con calizas molidas, cal, pizarra, escorias y cenizas; y al Portland lo adulteran con cementos naturales. La mayor parte de los adulterantes son inertes y por tanto sólo debilitan el cemento, pero la cal viva hace muchos más graves daños. Véase Morteros de cemento, §§ 28, etc., pág. 1289.

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* Richard K. Mende, « Portland Cement », 1906, págs. 16, 17.

** Edwin C. Eckel, « Cements Limes and Plasters », 1907, págs. 253, etc.

*** 16 análisis del « cemento de escoria de acero » fabricado por la « Illinois steel Co », al sur de Chicago, según informes de la comisión de ingenieros del Gobierno americano, 1900, dieron el mismo promedio, pero, generalmente con mayor uniformidad: sílice, 29.9 á 27.8; alúmina y hierro, 12.1 á 11.1; cal, 52.1 á 50.3; magnesia, 3.0 á 1.6.

## Propiedades.

### Fineza.

**40. Fineza.** Aun en los cementos de fineza normal, la parte interior de los granos parece que permanece inerte. Mientras más molido está el cemento, más arena resiste en los morteros, y no obstante éstos alcanzan la fuerza requerida. Pero en en todo caso hay un límite en el cual el mayor costo para darle mayor fineza destruye la ventaja dicha.

**41.** De aquí que la fineza es menos importante en los naturales que en los Portland; pues la baratura de los naturales hace que se puedan emplear éstos en mayor proporción, en vez de gastar en molerlos más finos, para darles más fuerza.

**42.** Los cementos molidos á una **exagerada fineza**, para alcanzar mayores resistencias que la de los cementos comerciales, fraguan tan rápidamente que tienen que usarse inmediatamente que se les agrega el agua.

**43. La fineza del cemento y de la arena son las siguientes**, donde los números grandes representan los grados de los cedazos, los pequeños á la izquierda de aquéllos representan el percentaje que queda en los cedazos, y los números pequeños á la derecha representan el percentaje pasado por el último cedazo. Los números pequeños suman=100. Así, $^{5}20\ ^{15}30\ ^{35}40^{15}$ significa que en el cedazo n.º 20 quedó 5% en el 30, 15% y en el 40 quedó 35% y el último 45% pasó por el cedazo n.º 40.

### Color.

**44.** Los silicatos y aluminatos de cal que propiamente constituyen los cementos, son incoloros, cuando están puros (véase cemento blanco, § 29). Por consiguiente, el color de los cementos se debe á otras materias cuya presencia es inevitable; en especial, los óxidos de hierro, que pueden ser afectados por ingredientes, benéficos, dañinos ó neutrales. De aquí que el color en sí no es un indicio para la calidad; pero **las diferencias de tinte**, en un cemento dado, pueden indicar diferencias en las rocas empleadas ó en el grado de cocción. Así, en los cementos naturales, un color claro indica casi siempre rocas de calidad inferior ó incompleta cocción. Un cemento de grano grueso, de color claro y liviano, es muy sospechoso.

**45.** « En los cementos Portland el color gris ó gris verdoso se considera como el mejor; gris azulado indica probablemente exceso de cal, y color pardo un exceso de arcilla. Los cementos naturales son generalmente pardos, pero varían de claro á obscuro. Los cementos de escoria tienen un tinte morado como lila suave. » (Prof. Ira O. Baker, « Tratado de construcciones de mampostería », pág. 55.)

### Peso.

**46. Densidad y peso.** Sobre densidad, véase el Resumen de especificaciones págs. 1278 y 1280. La densidad de las partículas sólidas del cemento no varía con el grado de fineza, pero disminuye por la absorción del agua y del ácido carbónico al aire libre y por tanto se aumenta al secarse. La densidad de los Portland puede variar entre 2.9 y 3.25, ordinariamente de 3.0 á 3.2; los cem naturales, entre 2.7 y 3.2; los puzolanos, entre 2.7 y 2.9.

**47.** El peso del cemento en polvo varía con la exposición al aire libre ó la secada, como ya se dijo, y aumenta con la compresión por ej. : cuando es embalado. Se reduce con la fineza, pues las partículas pequeñas al embalarlo se compactan menos. Faija encontró una pérdida en peso de 6% pocos días después de molido; 17% en seis meses y 21% en un año.

**48.** En un cem Portland alemán, Eliot C. Clarke encontró 1.44 kg por litro, cuando el 40% había quedado en el cedazo n.º 120, y 1.20 kg cuando todo pasó por el mismo cedazo.

**49.** Aproximadamente el cemento Portland se calcula en 100 lbs y los naturales en 75 lbs por pie cúbico (ó sea 1.61 kg por litro los primeros y 1.20 kg los naturales).

### Embalaje.

**50.** Debido á las variaciones de la densidad de los cementos, varían también el tamaño y peso de los bultos en que se embalan. La práctica comercial es vender un barril de cem Portland de 400 lbs bruto (181 kg) incluyendo el barril; el natural 300 lbs bruto (136 kg).

**51. Un barril de cem Portland** tiene de 2 á 2.2 pies (.61 á .67 m) de alto entre las tapas y 1.38 á 1.46 pies (.42 á .45 m) de diámetro medio. Pesa de 21 á 29 lbs (9.5 á 13 kg) y está forrado interiormente en papel. Su volumen es

de 87.7 á 99 litros, pero el cem comprimido en él ocupa de 106 á 121 litros suelto y pesa de 168 á 177 kg.

**52. Un barril de cem natural** pesa como 20 lbs (9 kg). En los Estados del Oeste contiene 120 kg, en los del Este 136 kilg.

**53, 54, 55.** (*N. del T.* — Estos números no tienen sino importancia muy local.)

**56.** El cem que no se exporta se embala generalmente en sacos de papel fuerte ó de tela.

**57, 58** y **59.** (*N. del T.* — Sólo tienen importancia local.)

## Edad del cemento.

**60.** La edad del cemento se traduce por el mayor ó menor apagamiento de la cal que le queda después de la quema. El buen cem Portland mejora dejándolo por algunas semanas expuesto al aire seco; no obstante, si se mantiene demasiado seco se deteriora á la larga con lentitud; pero los cementos naturales sufren generalmente expuestos al aire, y, en general, todos los cementos, compuestos como están por materiales dotados de gran afinidad por el agua, se deterioran expuestos á la humedad. Por lo tanto es conveniente preservarlos hasta de la excesiva humedad del aire. Con esta precaución, aunque se retarde su fragua, no se deteriora apreciablemente en muchos meses.

**61.** El almacenaje con presión lo apelmaza, pero esto no es signo de deterioro.

**62. Restauración por requema.** Los cem que se deterioran por el aire pueden en gran parte restaurarse recalentándolos al rojo.

**63.** Si el cem **se almacena en lugares calientes**, puede adquirir la propiedad de estallar (*flash*) al contacto del agua, y además fragua con mucha más rapidez.

## Pruebas.

Véase en los capítulos subsiguientes todos los Ensayos.

**64.** Aunque los **ensayos químicos** de los cementos se hacen con más precisión en los laboratorios, los ingenieros pueden hacer la siguiente sencilla prueba. Tratado por el ácido clorhídrico « el cem Portland puro hace una ligera efervescencia, produce un gas acre, picante, y forma gradualmente una como gelatina de color amarillo brillante sin sedimento. La piedra caliza ó roca con que se fabrica el cem mezclada con él, produce fuerte humo, hasta que se haya descompuesto todo el carbonato de cal y luego se forma una especie de gelatina amarilla. La arena de cuarzo queda sin disolverse. Deséchense los cem que contengan estos adulterantes ». Judson, « City Roads and Pavements ». La presencia de escorias se descubre generalmente por el azufre que contienen y que da un aspecto lechoso si se agita el cem en una solución de ácido clorhídrico en agua.

**65.** Fuller y Thompson encontraron que los cem que no soportaban este ensayo no tenían la fragua debida y los que lo resistían también sufrían con buen resultado otros ensayos químicos más concluyentes.

**Propiedades y ensayos del cemento. Informe de la comisión de ingenieros nombrada por el Gobierno de los E. U.** Propiedades y ensayos de los cementos Portland, naturales, y puzolanas *. Extracto de un informe de los mayores W. L. Marshall y Smith S. Leach y el capitán Spencer Cosby, ingenieros nombrados para hacer ensayos de cementos, 1901.

Desgraciadamente, las pruebas para la aceptación ó rechazo se hacen en los productos que aún no han alcanzado su final evolución. Cuando un cemento entra á formar parte de una mampostería está durante varios meses sufriendo alteraciones químicas, mientras que pocas veces es posible hacer ensayos con él por más de algunas semanas á lo sumo.

Pocas pruebas bien hechas valen más que muchas hechas á la ligera.

Los cem almacenados por mucho tiempo deben ser cuidadosamente probados antes de emplearlos, para apreciar su deterioro. Se debe rechazar todo cem sin tomar en cuenta la proporción de fallas en las pruebas hechas, si las muestras comprueban alguna peligrosa variación en la calidad por defectos de manufactura que quitan al producto su uniformidad.

La costumbre **de ofrecer primas** á los cementos de extraordinaria resistencia **es muy objetable**, pues esto hace que se produzcan cem con defectos difíciles de averiguar.

Para los cementos Portland ó puzolanos háganse **pruebas** de (**1**) fineza; (**2**) densidad; (**3**) constancia de volumen; (**4**) tiempo de la fragua; (**5**) resistencia á la tracción. En los cem naturales omítanse las (2) y (3).

(**1**) **Fineza.** La cualidad de adhesión del cem reside sobre todo en la mayor fineza alcanzada en la molienda. Usese un cedazo n.º 100 hecho con alambre de cobre, calibre n.º 40 Stubs; **zarandéese** el cedazo hasta que no pase más cem. El tanto por ciento colado se determina pesando el que no ha pasado.

El cedazo debe examinarse con frecuencia para ver si los alambres no se han separado más.

(**2**) **Densidad.** La prueba de la densidad es importante para conocer si un cem Port ha sido adulterado. Mientras más elevada sea la temperatura de la quema hasta el límite de la vitrificación, mejor será el cem y mayor su densidad. Si no ha sido bien quemado, la densidad del Port puede bajar de 3; si se ha pasado del punto de quema requerido, la densidad puede llegar á 3.5. Los cem naturales tienen una densidad de 2.5 á 2.8 y los puzolanos de 2.7 á 2.8.

La temperatura puede variar entre 17° y 27° C. Puede usarse cualquiera de los volumenómetros, por ej. : la botella de medir densidades, calibrada en cm cúb y fracciones. Llénese el aparato hasta el cero de la escala con bencina. Tómense 100 gram de cemento cernido, secado previamente por exposición de 20 minutos en una plancha metálica á una temperatura de 212° Fah (100° C), y déjesele pasar lentamente á la bencina, teniendo cuidado de que el polvo no se quede sobre el líquido en las paredes del tubo y que el embudo empleado no toque el líquido. La densidad será igual á 100 dividido por el volumen que ha desplazado la bencina en centímetros cúbicos. Esta operación requiere cuidado.

(**3**) **Constancia de volumen** y (**4**) **tiempo de fragua.** La temperatura no debe variar más de 5° ó 6° C alrededor de los 17° C. Para los Portland úsese 20% de agua en peso; para los nat, 30%, y para los puzolanos, 18%. Mézclese bien durante 5 minutos. En láminas de vidrio háganse dos torticas de siete centímetros de diám más ó menos y uno y medio cm ± de grueso en el centro, adelgazándolas hacia las orillas y cúbranse con una tela húmeda. Al cabo del menor tiempo establecido para el comienzo de la fragua, aplíquese una aguja de 2 milímetros de diám con un peso de ¼ lb (113 gramos); si penetra, el cem tiene las condiciones para la fragua inicial exigida. De otra manera la fragua es demasiado rápida. Al final del tiempo requerido para el término de la fragua, úsese la aguja de 1 mm de diám con peso de 1 lb (453 gramos); si no penetra es porque tiene las condiciones necesarias para la fragua requerida. De lo contrario la fragua es muy lenta.

En términos generales, la humedad prolonga ambos tiempos de fragua y se acortan aumentando la temperatura.

Cuando se van á probar los Portland en tiempo húmedo, las muestras deben

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* En este Informe se entiende por cem Portland el producto obtenido por la calcinación de mezclas íntimas de substancias arcillosas y calcáreas naturales ó artificiales, hasta que se inicia la fusión. Por cem naturales, los fabricados con rocas naturales calcinadas á una temperatura inferior á la de su fusión y luego moliendo el producto; y por puzolanos se entiende el producto obtenido por la molienda de escorias y cal apagada, sin subsiguiente calcinación.

secarse bien antes de agregarles el agua. Se obtendrá bastante uniformidad en la temp si el cuarto de ensayos se mantiene á confortable calor en invierno y si las muestras se conservan á la sombra en un cuarto fresco en verano y bajo un paño húmedo hasta que fraguen. La temp puede variar entre 16 y 27 C, sin afectar el resultado de las observaciones más allá de lo aceptable.

**Prueba á la ebullición**. Colóquense las dos tortas durante 24 horas bajo una tela húmeda. Una de ellas en su plancha de vidrio, se deja en agua durante 28 días; la otra sumérjase en agua á 21°, pero manteniéndola en un soporte sobre el fondo de la vasija; caliéntese el agua gradualmente hasta que hierva, manteniendo el calor por seis horas y luego déjesela enfriar. La torta hervida no debe deformarse, ni presentar grietas de expansión. Si la torta sumergida en agua fría sólo presenta aumento de volumen, se puede emplear el cemento en trabajos ordinarios al aire ó en agua dulce en mezclas pobres; pero si aparecen grietas ó deformaciones, debe rechazarse el cemento.

**Pruebas rápidas** no son recomendables, pero cuando se tiene que hacer una prueba en corto tiempo, la de ebullición es la mejor. No sólo suministra datos en breve tiempo, sino que manifiesta la presencia de ingredientes que pueden producir la desintegración. Por otra parte puede obligar á rechazar un cem que daría buenos resultados en la práctica y que soportaría las pruebas ordinarias después de estar algún tiempo expuesto al aire. El sulfato de cal que ayuda á los cem á soportar la prueba de la ebullición, es un elemento peligroso.

**(5) Las pruebas de tensión** son preferibles á las de flexión ó compresión. Las que se hacen ya mezclado con arena el cem, son las más importantes y deben siempre hacerse; las de cem puro pueden hacerse si hay tiempo.

Un cemento que da un resultado medianamente alto en 7 días y presenta un aumento importante durante los 28 días, es más probable que alcance el máximum de resistencia paulatinamente y lo conserve indefinidamente, que otro cemento que dé en las pruebas una resistencia exagerada á los 7 días, sin presentar ninguno ó muy poco aumento en los 28 días. Usense briquetas de la forma recomendada por la Sociedad Americana de Ingenieros Civiles, que tiene en la sección media de ruptura una pulgada cuadrada (6.45 cm cuad), sostenidas por piezas de metal bien ajustadas. Las pruebas deben hacerse al sacar las briquetas del agua.

**Pruebas de cem puro**. Usense cementos sin cernir. Para cem Portland empléese 20% de agua (en peso), 30% para el natural y 18% para los puzolanos. Colóquese el cemento en una plancha lisa no absorbente; hagásele en el centro un cráter para contener el agua, échesele casi toda el agua de una vez y el resto á medida que la vaya necesitando; mézclese muy bien, moviéndolo con una cuchara y por 5 minutos amásese vigorosamente.

Colóquese el molde de la briqueta sobre una plancha de vidrio ó de pizarra. Llénese el molde con varias capas de cemento como de 6 mm de espesor cada una y apriétese. Désele á cada capa como 30 golpes con un martillo de cobre ó bronce del peso de 1 lb, y de un diámetro de ¾ pulg (19 mm) y cayendo de una altura de ½ pulg (12 mm).

Después de llenar el molde y golpear la última capa, alísese con una paleta, golpéese el molde suavemente en un lado para separarlo de la plancha, quítese ésta y déjese por 24 horas cubierto con una tela húmeda. Luego sáquese la briqueta del molde y sumérjase ésta en agua dulce, que debe ser renovada constantemente ó por lo menos 2 veces por semana durante el tiempo especificado.

**Prueba de tensión con arena**. Para Portland y puzolanos úsese una parte de cem para tres de arena; para los naturales (ó Rosendale), partes iguales. Usese la arena de cuarzo que pase por cedazo n.° 20 y no por el 30.

Después de pesarlos cuidadosamente, mézclense secos el cem y la arena hasta la uniformidad, agréguese el agua como en las pruebas de cem puro y mézclese durante 5 minutos. La mezcla debe amasarse bien.

Para la máxima resistencia en las briquetas de prueba los cem Portland requieren de 11 á 12½ por ciento de agua (en peso de la arena y del cm); los naturales, de 15 á 17, y los puzolanos, de 9 á 10.

Es preferible emplear una máquina que aplique la fuerza gradual y automáticamente, y no una manejada á la mano. La fuerza debe aumentarse á razón de 180 kg por minuto; si se la aplica en una proporción muy distinta á ésta, dará resultados diversos. La mayor resistencia á la tracción, de cada grupo de briquetas hechas al mismo tiempo, debe considerarse como guía de la prueba.

**Otras pruebas** que se recomiendan, aunque se hayan hecho las anteriores. Con cem de marcas conocidas, para apreciar su peso, fineza, su textura y resistencia en las obras, y para saber si el cem es legítimo, si está en buen estado, se hacen

frecuentemente torticas y bolas del cem puro y del mortero probando en ellos con la uña del pulgar la solidez de la fragua, y la resistencia, rompiéndolas con la mano y dejándolas caer sobre una superficie dura. También puede usarse la prueba de la ebullición. Si estas pruebas de tanteo no satisfacen ó dan resultados sospechosos, entonces habrá que hacer los ensayos minuciosos ya explicados.

Se puede rechazar un cem que no tenga las siguientes

## Condiciones.

| | Portland. Lento | Portland. Rápido | Nat. | Puzolana. |
|---|---|---|---|---|
| Fineza. Percentaje que debe pasar por cedazo n.º 100 como se dice en el (1).. | 87 á 92 | | 80 | 97 |
| Densidad entre..... | 3.10 | 3.10 | No se | 2.7 |
| y..... | 3.25 | 3.25 | da | 2.8 |
| Tiempo de fragua. Inicial, no menos de. | 45 min | 20 min | 20 min | 45 min. |
| ni más de.. | ..... | 30 | ..... | ..... |
| Final, no menos de | ..... | 45 | ..... | ..... |
| no más de.. | 10 h | 2.5 h | 4 h | 10 h |
| Resist á la tracción, puro, | | | | |
| lbs por pulg cuad.. 7 días *.... | 450 | 400 | 90 | 350 |
| lbs por pulg cuad.. 28 días *.... | 540 | 480 | 200 | 500 |
| kg por cm cuad..... 7 días..... | 31.64 | 28.12 | 6.33 | 24.60 |
| kg por cm cuad..... 28 días..... | 37.97 | 33.75 | 14.06 | 35.15 |
| Resist á la tracción con arena como (5) | | | | |
| lbs por pulg cuad.. 7 días *.... | 140 | 120 | 60 | 140 |
| lbs por pulg cuad.. 28 días *.... | 220 | 180 | 150 | 220 |
| kg por cm cuad .... 7 días..... | 9.84 | 8.44 | 4.22 | 9.84 |
| kg por cm cuad .... 28 días..... | 15.47 | 12.65 | 10.54 | 15.47 |

(*N. del T.* — Para encontrar los kg por centímetro cuadrado que equivalen á un número de libras por pulgada cuadrada, divídanse éstas por 14.22.)

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* Rechácese todo cem que á los 28 días no haya aumentado la resistencia que tuvo á los 7.

## RESUMEN DE ESPECIFICACIONES

### Condiciones.

### Sociedad americana para prueba de materiales.

**Condiciones** adoptadas el 14 de nov. 1914 y Modificaciones 1908 *.

**1. Embalaje.** La marca y nombre del fabricante deben escribirse con claridad. El saco debe contener 42.64 kg netos, el barril de cem Portland=4 sacos; el nat=3 sacos.

**2. Las pruebas** (de acuerdo con las estipulaciones de la Comisión de Ings Civiles) deben ser: « Los cem que á los 7 días no satisfacen á las condiciones estipuladas, deben retenerse hasta las pruebas á los 28 días antes de rechazarse. »

| **3. Cualidades.** | **Natural.** | **Portland.** |
|---|---|---|
| Densidad. Los cem de superior calidad secados á 100° C *..... | mín 2.8* | mín 3.1 ...* |
| Pérdida de peso al fuego..... | ... | ...* |
| Fineza. Percentaje por peso : | | |
| Residuo en cedazo n.° 100..... | máx 10 | máx 8 |
| — — n.° 200..... | máx 30 | máx 5 |
| Tiempo de fragua, minutos inicial..... | mins 10 | mins 30 |
| — final..... | { mins 30 / { máx 180 | { mins 60 / { máx 600 |

Resistencia á la tracción,

Condiciones mínimas *, *kgs por cm cuad;* sección de la probeta, 6.45 cm cuad. Las probetas no deben disminuir sus resistencias durante el tiempo señalado.

Un día al aire húmedo en todos los casos.

| Cem puro. | Natural, kg por cm cuad. | Portland, kg por cm cuad. |
|---|---|---|
| 24 horas..... | 3.52 á 7.03 | 10.55 á 14.06 |
| 7 días..... | 7.03 á 14.06 | 31.64 á 38.67 |
| 28 días..... | 14.06 á 21.09 | 38.67 á 45.70 |
| 1 cem para 3 de arena (con las condiciones dichas para las pruebas) | | |
| 7 días..... | 1.76 á 5.27 | 10.55 á 14.06 |
| 28 días..... | 5.27 á 10.55 | 14.06 á 21.09 |

Constancia de volumen :

| (Para las pruebas normales y rápidas, véase pág. 1282.)..... | Deben soportar las pruebas normales. | Deben soportar las pruebas normales y rápidas. |
|---|---|---|
| Acido sulfúrico anhídrico..... | | máx 1.75% |
| Magnesia..... | | máx 4.00% |

(*N. del T.* — Para encontrar los kilogramos por centímetro cuadrado equivalentes á lbs por pulg cuad divídanse éstas por 14.22.)

### Condiciones oficiales adoptadas en nov. 23 de 1904 por la Junta de Ingenieros de la Gran Bretaña.

**1. Para las consignaciones** de 100 á 250 toneladas deben hacerse las pruebas prácticas y el análisis químico. Para las menores de 100 ton, los fabricantes darán en cada caso, al pedírselos, certificados de que los cem satisfacen aquellas condiciones.

**2. Muestras.** Las muestras deben tomarse del cem, en la fábrica ó ya en el trabajo, á opción del consumidor. Las muestras deben tomarse por lo menos en 12 diversas partes de la masa, mezclarlas y extenderlas, con 7½ cm de espesor por 24 horas, á una temperatura entre 14° y 18° C.

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*Modificaciones adoptadas por la Sociedad ya citada en sept. de 1908.

Resistencia: Los promedios de las cifras dadas deben tomarse como condición mínima cuando ésta no ha sido especificada.

Cem natural. Omítase la condición de la densidad. Cem Portland. Densidad. Para éstos léase « quemados á un rojo bajo » donde dice « secados á 100°C »

Pérdida de peso al fuego no más de 4 0/0.

**Condiciones oficiales adoptadas en nov. 23 de 1904 por la Junta de Ingenieros de la Gran Bretaña.** (Continuación.)

**3. Fineza.**

| Mallas por pulg lineal. | Mallas por pulg cuad. | Alambre, diám en pulgs. | Residuo máximo, |
|---|---|---|---|
| 76 | 5776 | .0044 | 5.0% |
| 180 | 32400 | .0018 | 22.5% |
| por centím lineal. | por centím cuad. | diám en mm. | |
| 30.4 | 924 | .11 | 5.0% |
| 72 | 5184 | .045 | 22.5% |

La tela de alambre no debe ser cruzada.

**4. Resistencia á la tracción.**

El cuarto de ensayos debe estar á una temperatura de 14° á 18° C.

El agua dulce, renovarse cada 7 días; y su temperatura de 14° á 18° C. La pasta debe ser suave, de fácil amasijo, que salga de la cuchara ó paleta en una masa compacta y nítida.

Debe llenarse el molde de la probeta sin golpearla, dejándolas descansar en una plancha de hierro hasta que el cemento fragüe. Las probetas deben quedar por 24 horas en una atmósfera húmeda y luego en agua hasta que se vayan á romper. Garfios; véase fig. 1.

(*N. del T.* R=15 mm; $r$ = 10 mm; W = 44 mm; $w$= ancho= espesor = 25 mm; H=76 mm; $h$=51 mm. Las figuras están en una escala de 2/5).

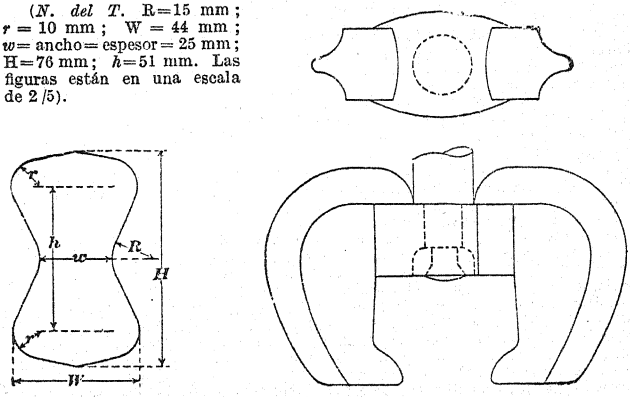



**Fig. 1.** Probeta y garfios.

*Carga;* comiéncese en 0; y agréguensele como 46 kg cada 12 segundos.

Ensayo de cem puro. Seis probetas en 7 días y 6 en 28 días. Se acepta el resultado medio de las 6 como la medida de la resistencia. Siete días no menos de 28 kg por cm cuad; á los 28 días no menos de 35 kg por cm cuad.

Cuando la prueba á los 7 días está entre :

| Kilg por cm cuad. | El aumento entre 7 y 28 días no debe ser menor de |
|---|---|
| 28.02 y 31.60 | 25% |
| 31.60 y 35.20 | 20% |
| 35.20 y 38.70 | 15% |
| 38.70 y más | 10% |

Prueba con la arena. Por peso se mezclan 1 de cem y 3 de arena normal, lavada y seca. La arena debe pasar por un tamiz n.° 20 de alambres de .4 de milímetro y que no pase por el tamiz n.° 30 de alambre de .28 mm. Mézclese y humedézcase sin exceso de agua. A los 7 días 8.44 kg por cm cuad; á los 28 días 15.82 kg. Aumento de 7 á 28 días no menos de 20%.

**Condiciones oficiales adoptadas en nov. 23 de 1904 por la Junta de Ingenieros de la Gran Bretaña.** (Continuación.)

| | Tiempo en minutos máx | mínimo. |
|---|---|---|
| **5. Fragua.** | | |
| Rápida | 30 | 10 |
| Mediana | 120 | 30 |
| Lenta | 300 | 120 |

La fragua se ha efectuado cuando la aguja con 1.13 kg de peso, teniendo la punta chata y de 1.6 mm en cuadro, no deja marca.

**6. Variación de volumen.** Le Chatelier: La expansión no debe exceder de 12 milímetros después de 24 horas al aire; 6 mm después de 7 días.

**7. Densidad.** No debe ser menor de 3.15 al tomar las muestras en fábrica; no menos de 3.10 al entregarlo al consumidor.

**8. Análisis.** Agua no más de 2%, ya sea agregada ó absorbida del aire.

**Sulfato de calcio**: no más de 2% del peso del cem, calculado como sulf de calcio anhídrico.

**Cal**: no más de la necesaria para saturar el sílice y la alúmina.

Residuo insoluble: no más de 1.5%. **Magnesia**: no más de 3%. *Anhidro sulfúrico*: no más de 2.5%.

## Pruebas.
## Sociedad americana de ingenieros civiles.

Extracto del Informe de la Comisión sobre pruebas uniformes de cem, de enero de 1903, y revisado en enero de 1904 y enero de 1908.

**1. La elección de las muestras** queda á discreción del ingeniero. El número de muestras y la cantidad que debe tomarse de cada lote, dependen de la importancia de la obra, del número de ensayos que se piensen hacer y de las facilidades para hacerlas. Cuando sea posible tómese la muestra para probar de cada 10 barriles uno. Las muestras aisladas pueden mezclarse y probar su promedio; pero cuando sea posible pruébense separadamente.

**2. El cem en barril** debe probarse tomando por un agujero el cem á mitad ó en la tapa del barril. El cem, en sacos, se debe probar de la superficie hacia el centro.

**3. Ciérnanse las muestras** en tamiz n.° 20.

**4. El análisis químico** puede demostrar la adulteración en cem ricos en materia inerte; pero esto no es concluyente para apreciar la calidad. La Comisión recomienda el método propuesto por la « Comisión de uniformidad, etc., de la Sociedad de Industria Química de la sección de Nueva York ».

**5. Prueba de la densidad.** Se recomienda el método Le Chatelier, fig. 1 (pág. 1281). El frasco, D, de 120 cm cúb; cuello de unos 9 mm de diámetro y 20 cm de largo, con un ensanche, C, de 20 cm cúb entre F y E. El cuello está graduado en .1 de cm cúb de F hacia arriba. El embudo, B, entra en el frasco y llega hasta F. Usese bencina (62° Baumé) ó kerosene libre de agua. Durante la operación, para evitar cambios de temperatura en este líquido, se mantiene el frasco dentro del agua en un depósito. Hay dos métodos, que son:

(*a*) Llénese el frasco hasta la marca E. Pésense 64 gramos de polvo de cem á la temp del líquido. Introdúzcase gradualmente el polvo de cem por el embudo B, hasta que el líquido suba á la marca F. Entonces llámese *w* á los 64 grms, menos lo que pese el polvo no empleado y la densidad será *w* /20.

(*b*) Llénese con el líquido hasta la marca E como se hizo antes. Échense todos los 64 grms de cem y véase á qué división del cuello llega el líquido; lo que marque esta división más 20 cc es el volumen *v* de los 64 gramos de cem y su densidad $= \frac{64}{v}$.

**6. Fineza.** Los tamices deben ser circulares de unos 20 cm de diám, 6 cm de alto, con una profundidad libre de 5 cm y con tapa. Estos tamices son de tela de alambre.

N.° 100, 96 á 100 mallas por pulg lineal (38 á 40 por cm); diám del alambre .0045 pulg (.11 mm);

N.° 200, 188 á 200 mallas por pulg lineal (75 á 80 por cm); diám = .0024 = .06 mm.

Empléense 50 á 100 gramos de cem secado á 100° C. Debe preferirse tamizar á mano. Usese el n.° 200 durante un minuto y dando 200 vaivenes por minuto, no pasa más de $^4/_{10}$%. Pésese el residuo y hágase lo mismo con el n.° 100. Una pequeña cantidad de municiones gruesas de acero ayuda la operación.

**Pruebas. Soc amer de ing civ.** (Continuación.)

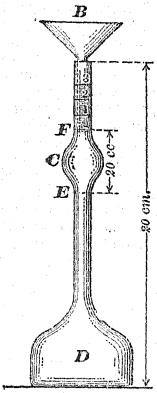
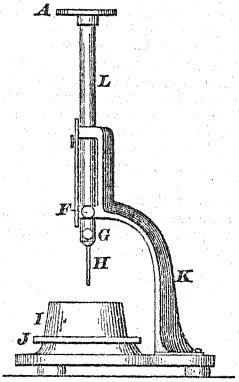



**Fig 1.**
Frasco de densidad.



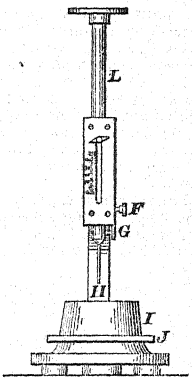




**Fig 2.**
Aguja de Vicat.

I, anillo de goma; J, plancha de vidrio; H, aguja Vicat; L, cilindro ó barra que resbala; F, tornillo que puede comprimirla.

**7. Consistencia normal.** El percentaje de agua empleado para hacer las pastas en las pruebas de resistencia, variación de volumen y fragua, afecta en alto grado los resultados. La consistencia normal se determina así: La cantidad de cem que se va á usar en lo sucesivo en cada montón para hacer las probetas, y que no debe ser menor de 500 gramos, se amasa en una pasta (como se indica más adelante en el capítulo «Mezcla», § 12), haciendo con ésta rápidamente una bola, y pasándola de mano á mano seis veces, á 15 cm de distancia. Luego se fuerza la bola en la abertura del aparato de la aguja Vicat dentro del anillo de caucho (7 cm de diám por 4 cm hondo), se la alisa por debajo y se coloca en una lámina de vidrio. Se pule la parte superior con una cuchara y se la pone en contacto con la punta de la aguja dejando descender el cilindro. La pasta será de consistencia normal si la aguja penetra un centímetro.

**8. Fragua.** Esta se inicia cuando la aguja de Vicat de 1 mm de diám, con peso de 300 gramos, se detiene á más ó menos 5 mm de la plancha de vidrio, y termina cuando la aguja no penetra en la masa. Las muestras de prueba deben mantenerse húmedas durante aquélla, en una caja ó armario húmedo ó colocadas sobre el agua en un soporte y cubiertas con una tela húmeda que no las toque. Manténgase la aguja limpia. El tiempo de fragua cambia mucho con la temperatura del agua que se le pone, la temperatura y humedad del aire, por la proporción del agua empleada y por el trajín con que se trabaje la pasta al moldearla.

**9. Arena normal.** No es aceptable el cuarzo triturado, «especialmente á causa de la gran proporción de los vacíos, la dificultad de compactarlo en los moldes y su poca uniformidad». La arena debe pasar por el tamiz n.º 20, cuyos alambres tengan un diám=á la mitad del espacio que los separa; no menos del 99% debe quedarse en un tamiz análogo n.º 30; después de darle un minuto de continuo zarandeo á una muestra de 500 gramos.

**10. Probetas** normales. (Véase fig. 3.)

**11. Moldes.** Deben ser de cobre, ú otro metal no corrosible, cuyas paredes sean bastante fuertes para soportar el apelmazamiento. Se recomiendan los moldes en serie, fig. 4, por requerir más mortero; además, se obtiene mayor uniformidad. Los moldes se limpian, antes de usarlos, con una tela aceitada.

**12. Operación de mezclar.** Las proporciones mencionadas son por peso y la cantidad de agua es un tanto % del material seco. Se recomienda usar el sistema

## Pruebas. Soc amer de ing civ. (Continuación.)

métrico. La temperatura del cuarto y el agua para la mezcla deben ser lo más próximas posible á 21° C. La arena y el cem se mezclan secos, y esta operación debe hacerse en una superficie no absorbente, prefiriendo el vidrio. La cantidad de material que se mezcla de una vez, depende del número de piezas de prueba que se van á hacer; las mezclas deben hacerse de 1,000 grms más ó menos, sobre todo cuando se hacen á mano. Se recomienda la mezcla y el moldeo á mano. Pesado el material y colocado en la plancha, se le forma un cráter en el centro, en el que se echa el agua limpia, y el material de la orilla se va sucesivamente mezclando con el agua. Al quedar absorbida el agua, se completa la operación amasando vigorosamente la mezcla durante 1½ minutos. Pueden protegerse las manos con guantes de caucho.

Se llenan los moldes al concluir la mezcla, se comprime fuertemente el material con los dedos y se alisa la capa superior con una paleta sin presión á martillo; la paleta, al barrer el exceso de material, debe correr al contacto del molde y ejercer una ligera presión sobre el material. Inviértase el molde y repítase la operación por el otro lado. Pésense las briquetas antes de sumergirlas en agua ó al sacarlas de la caja húmeda y rechácense las que varían en más del 3% del peso medio.

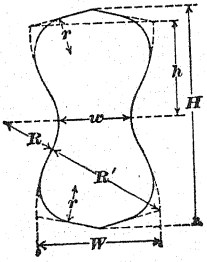



**Fig. 3.** Probeta.

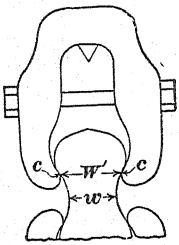



**Fig. 5.** Garfio.

Fig. 3. H=76 mm; $h$=33 mm; R=19 mm; $r$=13 mm; W=44 mm; $w$=ancho=espesor=25 mm.
La línea de puntos indica la forma recomendada últimamente.
Fig. 5. W'=31.7 mm; $w$=ancho=espesor=25 mm; $c$=contacto con la probeta=1.6 cm cuad.

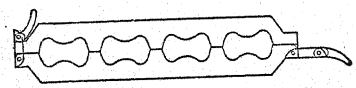


**Fig. 4.** Serie de moldes.

**13. Caja húmeda.** Ésta se hace de una piedra porosa como la esteatita, ó de pizarra, ó de metal, y éste forrado en fieltro mantenido siempre húmedo; el fondo debe contener agua y los lados provistos de apoyos para sostener las láminas angostas de vidrio que sustentan las probetas. Cuando no se dispone de una caja, se emplea un paño siempre húmedo que envuelva las probetas sin tocarlas.

**14. Inmersión.** Después de 24 horas de exposición al aire húmedo, las piezas de prueba deben sumergirse por mayor tiempo en agua lo más próxima á 21° C; se pueden conservar en estanques ó depósitos no corrosibles.

**15. Resistencia á la tracción.** Para esta prueba se recomiendan los garfios enterizos de metal, fig. 5. No debe haber cojines entre el garfio y la probeta, debe haber buen contacto. Las probetas deben ensayarse al sacarlas del agua colocándolas con cuidado en el garfio, para impedir tracciones transversales, y apliquese el peso á razón de 272 kg más ó menos por minuto. El promedio de los resultados de cada grupo de probetas ensayadas será el dato de la experiencia, excluyendo alguna que dé un resultado tan discordante que sea visiblemente erróneo.

**Pruebas. Soc amer de ing .civ.** (Continuación.)

**16. Constancia de volumen.** « En el estado actual de nuestros conocimientos no se puede rechazar un cem simplemente porque no satisface en las pruebas rápidas indicadas más abajo, ni aceptarlo como bueno por la sola razón de haber correspondido á ellas. »

Háganse tortas de consistencia normal (§ 7), de unos 7½ cm de diám y 1¼ de espesor en el centro, y adelgazándolo hacia las orillas, sobre láminas limpias de vidrio de 10 cm por lado; expóngase por 24 horas al aire húmedo antes de la prueba.

(1) Prueba normal. Sumérjase una de las tortas en agua, lo más próxima posible á 21° C, y la otra al aire á la temperatura ordinaria; y obsérvense ambas de vez en cuando por 28 días.

(2) Prueba rápida. Se expone una de las tortas á los vapores de agua hirviendo, en una vasija á medio tapar, por 5 horas.

Las tortas deben conservarse intactas y duras sin indicios de grietas, quebradura ó deformación. Se puede descubrir cualquier curvatura aplicando una arista recta á la superficie que tocaba el vidrio.

## ARENA*

### Composición.

**1.** La arena * que se usa en los morteros está compuesta generalmente de granos de **cuarzo (sílice)**, con algunas impurezas, en su mayor parte granos de minerales silícios. Para probar los cem en los laboratorios se emplea cuarzo triturado ó arena natural normal. (Véanse las especificaciones anteriores sobre el cem.)

**2.** La sílice del cuarzo en la arena, no sufre alteración química en el mortero; pero, el empleo de la arena disminuyendo la cantidad del cem requerido, reduce el costo de la obra. (Véanse la observaciones sobre resistencia, pág. 1292.

## TAMAÑO DE LOS GRANOS

**3. Tamizada.** La arena y el granzón se tamizan generalmente en un cedazo inclinado y fijo al que se lleva el material, por transporte mecánico ó paleado á mano, ó por medio de un cedazo giratorio cilíndrico ó prismástico al cual se va llevando el material.

**4. Cuarteo.** Para obtener un verdadero promedio de un montón de arena, granzón ó piedra, es útil el método de cuarteo. Se van tomando paladas de distintas partes del montón, se mezclan y se esparcen en círculo; este círculo se divide en 4 partes como una torta. Una de estas partes se separa del resto, se mezcla muy bien, se vuelve á esparcir y á dividir como antes, repitiendo esto hasta que la cantidad se reduce á lo necesario para la muestra.

### Análisis mecánico.

**5.** El análisis mecánico de las arenas consiste en determinar en una arena, granzón ó cascajo dados, la proporción de granos de diversos tamaños. Esto se logra por medio de tamices. A veces en las piedras trituradas basta escogerlos con la mano.

**6.** La fig. 1 sirve para ilustrar el examen de un granzón y una arena, hechos por Allen Hazen, de Massáchusets. Para representar ambos exámenes en un mismo diagrama hemos usado diferentes escalas para los diversos diáms de los dos materiales.

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* **Por arena y granzón** se entiende una mezcla de partículas minerales separadas por espacios llenos de aire ó de agua, ó de ambos. Por tanto, el volumen de una cantidad de arena ó granzón es el ocupado por las partículas sólidas y por el aire ó agua entre ellas.

**7.** En la fig. 1 el diagrama demuestra, para los dos materiales representados, que :

para la arena 10% era de granos menores y el 90% mayores de .0055 mm
— el granzón 10% — — — 90% — 34.5 —

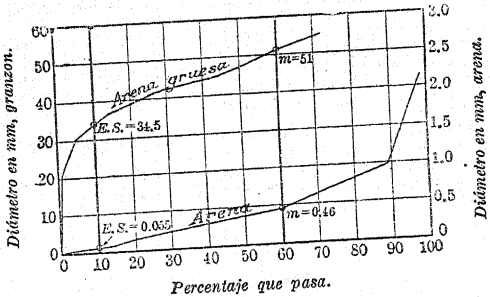



Fig. 1.

**8. El tamaño efectivo** (E S en el diagrama) de una arena ó de un granzón es aquel bajo el cual sólo hay un 10% (en peso) de granos en la masa y el 90% restantes son iguales ó mayores que el tamaño ó diám dicho. Así lo aprecia el Sr. Hazen, de Boston.

## Coeficiente de uniformidad.

**9. Coeficiente de uniformidad** (« c. u. ») De la misma manera sea $m$=al diámetro del grano bajo cuyo tamaño componen el 60% del peso de la masa siendo el 40% mayor. En la fig. 1 tenemos para la arena $m$=.46 mm; para el granzón $m$=51.00 mm.

El coeficiente de uniformidad es $m \div$ el tamaño efectivo; es decir, $m \div$ E S; lo que da para la arena, c. u. = .46 / .055 = 8.4; para el granzón, c. u. = 51.00 / 34.5 = 1.48.

**10.** Con $m$=E S el coeficiente de uniformidad (c. u.) tendrá el menor valor posible ó sea 1.

**11.** En los bancos de arena, el tamaño efectivo (E S) no varía mucho, por lo que el coeficiente de uniformidad, c. u. = $m$/E S, varía aproximadamente con $m$, diámetro éste superior al 60% del resto de los granos y sirve para indicar el grosor de la arena y lo que se aleja de la uniformidad.

## Método Feret.

**12.** El Sr. R. Feret (Anales de puentes y calzadas, 1892, 2.º semestre) ha hecho cuidadosos ensayos sobre los efectos de la fineza de la arena y de la mezcla de diferentes grados de fineza, sobre su densidad, etc., y sobre las diversas cualidades del mortero. Divide las arenas en los 3 grados de fineza siguientes :

Gruesa, $g$, más de 5.0 mm diám = 4 mallas /cm cuad = 5 mallas /pulg lineal.
Mediana, $m$, — 2.0 — — = 36 — / — = 15 — / — —
Fina, $f$, — .5 — — = 324 — / — = 46 — / — —

Los granos gruesos son detenidos en 2 mm diám; los « medianos » en .5 mm.

**13.** Los resultados obtenidos en determinado caso con las diferentes mezclas de estos 3 grados de fineza se ven demostrados en la fig. 2, la que se asemeja á los diagramas usados para las ligas de 3 metales.

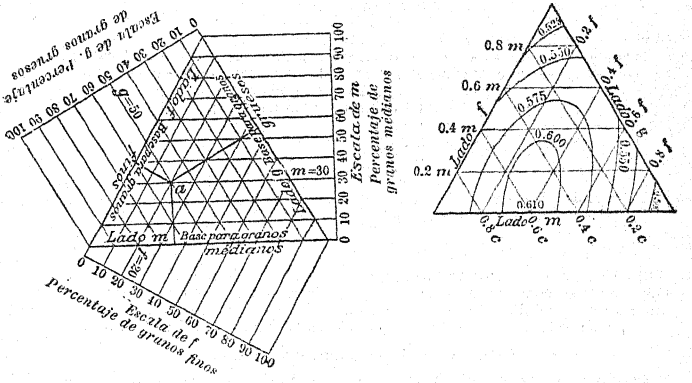
**14.** Después que se ha analizado una mezcla dada, y determinados así los percentajes de sus tres grados de fineza, se fija en el triángulo equilátero un punto situado de tal manera en él, que sus distancias á los lados sean las siguientes :

distancia al lado $g$ (arena gruesa ) = percentaje de granos gruesos;
— — $m$ ( — mediana) = — — medianos;
— — $f$ ( — fina ) = — — finos.

**15.** La fijación del punto y la medida de las distancias se facilita por medio de las paralelas á los tres lados.

**16.** Así el punto *a*, fig. 2, representa una arena que tiene 20% de granos finos, 30% medianos y 50% gruesos, como se ve en las distancias allí determinadas.

**17.** Cuando se ha hecho una serie de experimentos sobre cualquier cualidad dada (como densidad, porosidad, etc., para la cual son idénticas las otras condiciones) de la arena ó mortero, afectada por las diferencias de las mezclas de las 3 finezas, se fijarán los puntos que la midan en los lugares que les corresponda en el diseño alrededor del punto representativo de la mezcla y se trazarán **líneas de contorno** por los puntos que indican el mismo resultado sobre la cualidad que se estudia. Cada




una de esas líneas indica, por consiguiente, las diferentes mezclas que dan el valor (de la densidad, porosidad, etc.) que dicha línea representa.

**18.** Así, en la fig. 3, las cuatro líneas de contorno y el punto (.610) representan cinco mezclas de arena gruesa, fina y media que tienen las densidades de (véase § 20) .525, .550, .575, .600, .610, respectivamente.

## Densidad.

**19. Peso de la unidad de volumen.** El cuarzo sólido pesa ± 165 lbs por pie cúb = 2.643 grams por cm cúb; su densidad = de 2.64 á 2.67.

**20.** En mecánica (pág. 354, art. 14 *a*) **se define la densidad** por la masa bajo la unidad de volumen. En la arena las partes sólidas tienen prácticamente la misma densidad. Por tanto, en una arena dada, la densidad es la fracción del volumen de la parte exclusivamente sólida en la unidad de volumen de la arena, ó la relación entre la parte sólida y el volumen total. Esta relación se llama á veces el volumen real. Así, en la unidad de volumen de la arena, « densidad » = 1—volumen de los vacíos.

**21.** Mientras mayor es la densidad de una arena, menor cantidad de cemento requiere una cantidad dada de mortero.

**22. El peso por unidad de volumen** de una arena, de determinada densidad, varía directamente con esta densidad, y ésta, á su vez, depende de la forma de los granos, de la escala ó proporción de sus tamaños, de la compactación que se logre por sacudida, presión, golpe, etc., y de la sequedad.

## Efectos de la humedad.

**23. Los efectos de la humedad** sobre el volumen de una cantidad dada de arena varían con el volumen de aire introducido, con la cantidad de agua y con la forma de los granos. (Véase 29 á 31.)

**24.** No es posible medir el volumen de **aire introducido** y su presencia altera todas las observaciones. Cuando se dejan caer los granos de arena, uno por uno, en

agua, la mayor parte del aire que rodea los granos se queda en la atmósfera; pero, cuando se arroja la arena dentro del agua, en masa, ó cuando se remueve la arena húmeda ó mojada, arrastra una considerable y desconocida cantidad de aire que queda adherida al grano.

**25.** En la arena húmeda el volumen total ó « absoluto » de los vacíos está generalmente ocupado parte por el agua y parte por el aire.

**26.** Dentro de cierto límite, la **humedad** aumenta la adhesión entre los granos de la arena, se opone á que se deslicen unos sobre otros y estorba, por consiguiente, la compactación por contacto; pero, salvado aquel límite, obra como un lubricante y la facilita. (Véase 24-25.)

**27.** Sean

$V$ = volumen en litros de cuarzo seco en 1 litro de arena;
$v$ = — — — de los vacíos en 1 — — $V+v=1$ litro;
$W$ = peso en kg de 1 litro de cuarzo sólido = 2.643;
$w$ = — — de 1 — de la arena (seca ó húmeda) como sea el caso;
$d$ = — — de cuarzo seco en 1 lit de la arena (en arena seca $d=w$);
P = — — del agua agregada á 1 kg de arena seca;
= — — — en $(1+P)$ kg de arena húmeda;
$p$ = — — — en 1 kg — —
$m$ = — — — en 1 litro — —

Tenemos que $\frac{p}{P} = \frac{1}{(1+P)}$; y $p = \frac{P}{(1+P)}$;

$m = wp$; $d = w - wp = w(1-p)$;
$V = (w - wp) \div W = d/2.643$; $v = (1-V) = 1 - d/2.643$;
$w = W\frac{1-v}{1-p} = W\frac{V}{1-p}$.

**28. El grado de humedad** (kgs de agua en 1 kg de arena) se obtiene calculando un peso conocido de arena, á una temperatura no menor de 100° C hasta que no cese la pérdida de peso, y tomando nota del peso. Luego :

$p$ = pérdida de peso ÷ por el peso original de la porción calentada.

## Influencia de la forma y el tamaño.

**29. Granos esféricos.** Amontonando del modo más compacto posible un número de esferas de diámetro uniforme, D, la relación entre el volumen sólido y el volumen total es $\frac{\pi\sqrt{2}}{6} = \pm .74$; y los vacíos ($\pm 0.26 \times$ el volumen total) son de dos tamaños y tales que en ellos caben esferas de diáms = más ó menos .41 D y .22 D.

**30. Efecto de la graduación de tamaños.** La proporción de los vacíos se puede reducir indefinidamente mezclando á los granos originales otros más y más pequeños, ó más y más grandes, en la debida proporción, cada partícula irá ocupando los espacios vacíos de las partículas más gruesas. Por consiguiente, con partículas esféricas los vacíos son mayores y menor el peso por unidad de volumen cuando los granos son del mismo tamaño. Esto se aplica de un modo análogo á partículas no esféricas.

## Otras propiedades.

**31. Prueba de turbiedad á causa del fango.** Sepárese el fango de una cantidad considerable de arena y hágase una muestra especial que lleve la mayor cantidad relativa de fango permitido en la arena. En una vasija graduada colóquese dentro de determinada cantidad de agua limpia una porción conocida de arena. Agítese la vasija hasta lavar la muestra completamente. Insértese un alfiler horizontalmente en el extremo de una varilla, introdúzcase esta punta dentro de la vasija, bajándola hasta que deje de verse el alfiler dentro del líquido, y anótese la profundidad de aquél midiéndola en la graduación del vaso. Háganse varias pruebas y calcúlese la profundidad media en que deja de verse el alfiler. Si en las pruebas de las muestras el alfiler deja de verse á mayor altura que la normal, es porque la arena contiene más fango que el permitido y viceversa. (W. J. Dougias, E. N., dic. 20/1906, pág. 648.)

**32. La presencia de arcilla y barro en la arena** se puede comprobar frotando la arena húmeda en la mano y observando la mancha que deja, ó mezclándola con agua limpia para observar cuánto la ensucia.

**33. Lavado.** La arena sucia se puede lavar en un lavadero especialmente cons-

truído, ó en un depósito, con una manga de agua, ahogando la arena de manera que las impurezas más livianas floten y se derramen.

**34.** El lavado puede arrastrar las partículas más finas de una arena bien proporcionada, dejándola menos densa; sería conveniente ensayar una pequeña cantidad lavada y otra sin lavar, antes de lavarla para usarla. (Jas. C. Hain, E. R., enero 28/905.)

**35.** La bondad de una arena y la agudeza de sus granos se aprecia por el sonido que produce al friccionar con la mano unos granos contra otros, ú observarlos con una lente.

## MORTERO*

### Elementos.

**1. El mortero de cemento** consiste en cemento mezclado con agua, con ó sin materiales inertes granulados, como arena, granzón, cascajo, piedras ó escorias trituradas. Sin arena, etc., se llama **mortero puro** ó **pasta de cemento.**

**Cantidad de mortero requerida para la unidad de mampostería** **.

| Especies de mamposterías. | Por unidad de volumen, sea yarda ó metro cúbico. Mín. | Máx. |
|---|---|---|
| Mampostería de 18″=45 cm, juntas ¼″=6 mm | .03 | .04 |
| — de 12″=30 cm, — ¼″=6 mm | .06 | .08 |
| — de ladrillo (ladrillos de=21×10×5½ cm) : | | |
| — con juntas de 3 mm | .10 | .15 |
| — — de 9 á 6 mm | .25 | .35 |
| — — de 15 á 12 mm | .35 | .40 |
| — ordinaria de piedra pequeña en bruto | .33 | .40 |
| — — — grandes labradas toscamente á martillo | .20 | .30 |
| — — — cuadradas, muro de 45 cm y juntas de 20 mm | .12 | .15 |
| La misma anterior, pero de 30 cm de grueso | .20 | .25 |

**2. Efecto de la calcinación y la mojada subsiguiente.** Los materiales de que se hace el cem son mezclas inertes ó estables, que permanecen prácticamente sin cambiar en las condiciones ordinarias; pero, cuando se calcinan, los elementos calcáreos se someten á altas temperaturas, solos ó mezclados con las materias arcillosas, formándose entonces combinaciones relativamente instables, las cuales vuelven á convertirse en inertes cuando sus partículas se ponen en íntimo contacto al mezclarlas con agua, y ésta misma entra á formar nuevas combinaciones. La mezcla fragua pronto (pierde su plasticidad) y luego, por consiguiente, comienza á solidificarse y endurecerse. (Véase 8, Cemento, pág. 1269)

**3.** En el proceso de cristalización la alúmina parece que obra principalmente **como fundente** fomentando la formación del silicato de cal, de lo que depende el buen resultado de la operación. El óxido de hierro que se encuentra generalmente presente parece ejercer tan bien como la alúmina la acción fundente, requiriendo menor temperatura para la calcinación.

**4. La proporción de arena** que se debe emplear en un caso dado no se puede fijar con exactitud sin determinar la proporción entre sus vacíos y el volumen total; pero generalmente un buen cem Portland soporta dos ó tres veces su propio volumen de arena; los naturales de 1½ á 2 vol.

**5.** Cantidades **aproximadas** de cem Portland y arena suelta por metro cúbico de mortero :

| | Puro | 1:1 | 1:2 | 1:3 | 1:4 | 1:5 | 1:6 |
|---|---|---|---|---|---|---|---|
| Barriles de cem.... | 10.4 | 6 | 4 | 3 | 2.33 | 2 | 1.70 |
| Metros cúb de arena | 0 | .845 | 1.03 | 1.26 | 1.33 | 1.38 | 1.43 |

* Como la fuerza, permeabilidad, etc., de un hormigón de cemento depende sobre todo de que el mortero tenga aquellas condiciones, nos ocupamos en el capítulo « Mortero » de muchas de las propiedades tratadas en el capítulo « Hormigón de cemento ».

** Tomado de un tratado de « mamposterías » por el prof. Iro O. Baker. Nueva York, 1907.

## Cemento en el mortero.

(Véase también CEMENTO, pág. 1268.)

**6.** Debido al bajo precio con que hoy se fabrican los cem y la superioridad de los morteros hechos con ellos, éstos, recientemente y en gran parte, han reemplazado los morteros de cal, aun para trabajos ordinarios.

**7.** Al escoger un cemento, la **reputación** que haya adquirido en muchos años de prueba tiene más valor que el resultado de unas pocas pruebas; pero éstas tienen su valor para la exclusión de algún lote de inferior calidad.

**8.** Los cementos superiores resultan generalmente económicos aun costando más, pues soportan mayores cantidades de materiales baratos, como la arena, granzón y piedra picada.

**9. Cal libre.** El cem puede contener cal « libre » (sin combinar) como consecuencia de (1) insuficiente manipulación del material bruto, (2) de insuficiente calcinación, (3) de un exceso de carbonato de cal ($CaCO_3$) en la materia prima, ó (4) de adulteración después de la quema y molida.

**10.** Esta cal puede presentarse como cal viva, CaO, ó como cal apagada, $Ca(OH)_2$; cualquiera de las dos puede ser **lavada** (la CaO se convierte primero en $Ca(OH)_2$ por infiltración de agua. Esto, naturalmente, debilita el cem.

**11. La cal apagada** no influye en el proceso de endurecimiento, sino que permanece como material inerte de relleno.

**12. La cal viva** se apaga absorbiendo el agua empleada en la mezcla; y cuando se ha hecho la quema á alta temperatura, se demora la **apagada**. Si tiene lugar durante la fragua del cem, la hinchazón de la cal debilita á aquél haciéndolo poroso. Si la apagada se retarda hasta después del endurecimiento, y si la fuerza de expansión es bastante, se desintegra el cem.

**13. El exceso de cal** demora la fragua y daña la constancia de vol.

**14. Magnesia libre.** Hay mucha inseguridad respecto á los efectos de la magnesia libre en diferentes proporciones, sobre el cem. Como la cal, se expande al humedecerse, pero con mucha más lentitud, y su presencia, por tanto, permanece sin sospecharse mucho más tiempo. La **dolomita**, ó sea la caliza magnésica, contiene un 45% de magnesia. Antes 1.5% de magn libre en el cemento se consideraba peligroso; hoy se cree generalmente que más de 3 á 5% lo debilita y que 8% ó más lo agrieta. En cualquier proporción, es probablemente rechazable, por lo menos porque ocupa el puesto de la cal que es más útil.

## La arena en el mortero.

(Véase ARENA, pág. 1283.)

**15.** La calidad de los hormigones de cemento depende del vigor del mortero, y éste, á su vez, principalmente de las condiciones de la arena.

**16.** Para un peso dado, la mejor arena es la que produce el más pequeño volumen de mortero plástico.

**17. Peso.** Entre dos arenas de una misma especie, la más densa tiene naturalmente el menor vol de vacíos.

**18. Fineza.** Una arena fina bien surtida en el tamaño de sus granos y por tanto densa, puede producir un mortero mejor que otra gruesa con granos más uniformes y por consiguiente menos densa.

**19. La extrema fineza** impide la penetración de la pasta entre los granos y demora la fragua.

**20.** Los morteros de arena fina, aunque menos permeables que los de arena gruesa, se prestan más á la acción del **agua salada**.

**21. Contracción.** Los morteros de arena gruesa se contraen menos que los de arena fina.

**22. Agudeza de los granos.** Se ha acostumbrado insistir en esta cualidad de las arenas empleadas en los morteros, debido probablemente á la impresión de que los granos agudos se unen mejor al cem ó de que la tal agudeza es indicio de que la arena está pura; pero esto es dudoso. Las arenas de grano redondo se usan generalmente con satisfactorios resultados, y las pruebas de laboratorio indican en general que las de granos agudos no poseen ninguna notable superioridad. La redondez de los granos facilita el empaque y aumenta su densidad.

**23.** La Comisión de O. P. de Puerto Rico encontró en ensayos de morteros (de 1 : 2) 25% más de resistencia en los de arena lavada que en los de arena no lavada. La arena que contiene mucha materia extraña debe probarse antes de aceptarla.

**24.** En general, al juzgar sobre **los méritos relativos de la arena y de las cerniduras** (*screenings*), el fallo parece favorecer á éstas (véase « Experiencias »);

pero las opiniones divergen. La hidraulicidad del polvo en las cerniduras puede aumentar la fuerza del mortero.

**25.** Harry Taylor, capitán de ingenieros del ejército de los E. U., ensayó 1,650 probetas de morteros de 1 : 3, 1 : 4, 1 : 5 en 1, 3, 6 y 12 meses, de cuarzo normal triturado, arenas y **polvo de la trituración**; estos últimos produjeron probetas de 2.3 de mayor fuerza que las de arena y 72% más fuerte que las de cuarzo. Las de 1 : 5 hechas con polvo de piedra resultaron más fuertes que las de 1 : 3 de cuarzo.

**26.** El Sr. G. J. Griesenauer, de Chicago, hizo las siguientes 225 pruebas:

**Cerniduras de piedra caliza** 1 : 3 pasadas por cedazos n.° 12 y detenidas en el n.° 40 dieron un promedio de 74% superior á las arenas de pozos secos, 1 : 3 empleando todos los tamaños resultó 115% superior. Los morteros de 1 : 6 de cerniduras dieron 23% más fuerza que los de 1 : 3 de arena, **las cerniduras** de granzón no dieron mejor resultado que la arena.

**27.** En los caminos de Maryland las probetas hechas con **cerniduras de piedra** dieron de 34 á 62% más resistencia que las hechas con arena del río Potomac.

## La cal en los morteros.

**28.** El reemplazo de 10 á 20% de pasta de cal por igual volumen de pasta de cemento reduce el costo del mortero, y retarda un poco la fragua sin disminuir considerablemente su fuerza. Mayores cantidades lo debilitan. (Construcciones de mampostería, por Baker.)

**29.** Feret encontró que el efecto de la cal depende de la riqueza en cem del mortero. En morteros de 1 : 4 el aumento de 4 á 5% de cal apagada seca aumenta la fuerza; mientras que en morteros de cem de 1 : 1.25 disminuye la fuerza al agregarle cal. (« Chimie Appliquée », 1897, pág. 481.)

## Arcilla en el mortero.

**30.** Los ensayos de laboratorio indican que **pequeños aumentos de arcilla** aumentan en vez de disminuir la fuerza del mortero, y reducen su permeabilidad; pero, en la práctica, las partículas de arcilla conservan su compacticidad y forman como terrones de poca cohesión.

**31.** Las condiciones alcanzadas en los laboratorios en cuanto á sequedad, pulverización, etc., no se logran en la práctica.

**32.** Cuando la arena contiene arcilla no es posible una mezcla perfecta.

**33.** La arcilla y sus análogos producen más inconvenientes en las mezclas secas que en las húmedas.

## Consistencia.

**34. Resistencia relativa de los morteros secos y húmedos, 1 : 1.** Más de 5,000 ensayos hechos por Alfred Noble. Tomando como 100 á la resistencia del mortero seco.

| | Cem Portland. | | | | Cem natural. | | | |
|---|---|---|---|---|---|---|---|---|
| Edad | 30 días | 3 meses | 6 meses | 1 año | 30 días | 3 meses | 6 meses | 1 año. |
| Mortero seco..... | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| Medio espeso..... | 97 | 94 | 97 | 97 | 78 | 89 | 95 | 90 |
| Húmedo..... | 90 | 92 | 91 | 95 | 63 | 77 | 86 | 82 |

**35.** Usese el hormigón de cem (concreto) seco, cuando se vaya á someter inmediatamente á fuertes cargas. Los ensayos demuestran que los húmedos y los secos alcanzan la misma resistencia al año.

**36.** Los húmedos se adhieren mejor á las obras viejas que los secos. El exceso de agua aumenta la lechosidad y la eflorescencia.

**37. Regla para el percentaje, W, del agua.** (Cost Data, pág. 266, por H. P. Gillette.)

Siendo S = la parte de arena para 1 de cem, tenemos:

$$W = (8S + 24) \div (S + 1).$$

Esto da

| cuando S = | 1 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
|---|---|---|---|---|---|---|---|
| W = | 16 | 14.4 | 13.3 | 12.6 | 12.0 | 11.5 | 11.2 |

Falk encuentra que los morteros en estas proporciones se adhieren bien al acero.

**38.** El cem de escoria exige mucha agua para endurecerse bien. Por tanto, si se usan al aire los morteros de cem de escoria, deben mantenerse húmedos.

### Fragua y endurecimiento.

**39. La fragua**, ó pérdida de plasticidad, tiene lugar pocas horas (á veces pocos minutos) después de mezclar el cemento con el agua; mientras que el definitivo **endurecimiento** y **aumento de resistencia** (que aparecen como resultado de otros procesos químicos) se efectúan á menudo en meses y aun en años.

**40. Los bloques de cemento Portland** hechos en moldes, aun de 50 toneladas de peso, se pueden manipular y llevar á su destino, de 1 á 2 semanas después de hechos.

### Fragua inicial y final.

**41.** La fragua inicial y final son etapas del proceso general de la fragua, distinguidas arbitrariamente por razón de la resistencia que presenta el mortero á la penetración de alambres cilíndricos de diámetros y con pesos normales, haciendo que el extremo embotado descanse en una torta, hecha del mortero, en un molde cilíndrico chato, sobre una lámina de vidrio. (Véase 8, pág. 1281.)

### Determinación de la fragua.

**42. El general Totten** (Gen. Q. A. Gillmore, sobre « Cales, Cementos y Morteros », pág. 80) en Fort Adams, R. I., antes de 1830, empleaba un alambre de 2 mm de diám con peso de 100 grms, y de 1 mm de diám con peso de 450 grms tomando como fragua inicial y final el momento en que estos alambres no producían impresión en el mortero.

**43.** Vicat usaba un solo alambre ó « aguja ». La Asociación de Ingenieros civiles (véanse las Especificaciones pág. 1281) prescribe para esta aguja un diámetro de 1 mm y un peso de 300 gramos. La fragua inicial tiene lugar cuando la punta de la aguja, habiendo penetrado 4 cm dentro del mortero, no puede llegar á 5 mm de la lámina de vidrio; y la fragua final se ha hecho cuando la aguja no deja marca alguna en el mortero. El mortero para la prueba de fragua debe ser de « **consistencia normal** », ó tal que una varilla cilíndrica de 1 cm de diám, con peso de 300 gramos, penetre 1 cm en él.

### Modo de acelerar ó retardar la fragua.

**44.** Algunos de los mejores cem son de fragua lenta. Una capa de cem de fragua muy rápida puede fraguar parcialmente, sobre todo en época de calor, antes que el bloque de mampostería haya sido convenientemente situado y ajustado sobre la capa, y **todo lo que perturbe la fragua después de comenzada es perjudicial**. Por otra parte **los cem de fragua** rápida son, en ciertos casos, los mejores; como cuando están expuestos á aguas corrientes. Se puede retardar la fragua agregando pasta de cal en 5 y 15% de la pasta de cem sin que esto lo debilite seriamente. Los cem nat son en general de fragua rápida. Los de escoria, de fragua lenta.

**45.** En general **la fragua se acelera** por exceso de alúmina y por la presencia de soda y potasa en el cem, por la frescura y fineza de éste, por el uso de agua y arena caliente en la mezcla y por el tiempo cálido. La fragua se retarda por exceso de cal y sílice en el cem, por la presencia de arena, por la humedad de la mezcla, por el frío, por presencia de sal ó acido sulfúrico en el agua de la mezcla; por 1 ó 2% de sulfato de cal, hidratada ó anhídrica (yesos), ó cal apagada, en algunos casos por quema fuerte, y en general por la edad del cem, pero el almacenaje de cem nuevos en lugares calientes los acelera.

**45 a. Yeso.** Cas $O_4$. La duración de la fragua (inicial y final) aumenta rápidamente, agregándole hasta 2% de yeso, y se hace constante, ó aumenta poco empleando hasta el 4%. (E. Candlot, « Cements et Chaux hidrauliques ».)

**45 b.** El tiempo para comenzar y terminar la fragua aumenta agregándole hasta 1.5% de yeso, pero disminuye si se aumenta hasta 7%. (Kniskern and Gass, Sibley Journ of Engng, enero 1905.)

**45 c. Cloruro de cal,** $CaCl_2$. Una solución suave retarda, pero una concentrada ó fuerte acelera la fragua de los Portland, así: de 10 á 40 grams por litro, el tiempo de fragua es de 500 á 850 minutos; mientras que con 200 á 300 grms por litro, se reduce de 2 á 25 minutos. Los cem ricos ó pobres de alúmina se afectan poco con el cloruro de cal. Una solución suave (de 30 á 60 grms por lit) mejora mucho la calidad de un cem que contiene cal libre, facilitando la hidratación de ésta. (Obra citada de Candlot.)

**45 d.** Empleando ½ á 1½ % de cloruro de cal ($CaCl_2$) seco, molido con ***clinker*** de cem y amasado en tortas de consistencia normal (véase Pruebas, 7, pág. 1281),

aumenta el tiempo de la fragua inicial de 2 á 167 minutos y el de la fragua final de 52 á 275 minutos. Con 6% los tiempos de fragua son de 68 á 145 respectivamente, Kniskern and Gass, Sibley Journ. of Engng, enero 1905.

**46.** La fragua va acompañada de elevación de temperatura. En fraguas rápidas ésta puede alcanzar 10° C ó más.

**47.** Los cementos de fragua lenta alcanzan cierto grado de **endurecimiento** más rápidamente que los de fragua rápida.

**48.** En época de calor los cementos en fragua, al secarse, pierden la humedad de que depende el proceso de endurecimiento. Por lo cual **fraguan sin endurecer.** Se debe tomar toda clase de precauciones para impedir esto en época de calor.

**49.** Cementos de la misma clase difieren mucho en la **rapidez de su endurecimiento.** Al fin del primer mes uno de ellos puede alcanzar la mitad de lo que alcanza otro en un año, mientras que un tercero no llega á la sexta parte; sin embargo, al año pueden tener la misma fuerza. Por esta razón las pruebas de una semana ó de un mes no son nunca concluyentes sobre los méritos comparativos de los cementos.

**50. Se requieren muchos años para alcanzar el máximo de dureza:** pero después del primer año el aumento es muy pequeño y lento, especialmente en el caso de los cementos puros. Además, cualquier aumento subsiguiente es de poca importancia, porque, generalmente, para entonces, y á veces antes, se ha concluído la obra y se encuentra expuesta ya al máximo de su carga.

**51.** Cementos calculados de fragua lenta pueden en la fabricación **hacerse rápidos** (ó de « **relámpago** » « **flashing** ») por el almacenaje, sobre todo en lugares calientes, y si el cemento es pobre en cal. Esto se atribuye á la desintegración de las partículas y consiguiente aumento de fineza. El cambio es á veces muy rápido. Este inconveniente puede remediarse sin reducir la fuerza del cemento, almacenándolo en lugares frescos y agregándole de 1 á 2% de cal apagada. Para trabajos pequeños se pueden agregar algunos terrones de cal á cada barril de agua en la mezcla.

**52.** El requisito de que **aumente un tanto por ciento de su fuerza entre los 7 y los 28 días** provoca en el fabricante el deseo de moler su cemento grueso, ó adulterarlo con materiales inertes, para que no realice aquel aumento en los primeros 7 días.

## Otras propiedades.

### Constancia de volumen.

**53. La variación de volumen** en los morteros de cem es la tendencia á la expansión, contracción ó desintegración en el aire ó en el agua, ó bajo el calor ó frío. (Véanse Especificaciones.)

**54.** Los cementos de fábricas conocidas son rara vez deficientes en resistencia; pero sí pueden sufrir variaciones de volumen, de lo cual depende su durabilidad.

**55.** La variación de volumen **se debe** generalmente á un exceso de cal libre, producida por mala proporción, exceso de quema, falta de sazón, ó molienda gruesa : esta última impide la perfecta hidratación. La presencia de **sulfatos de cal** (yesos) es favorable á la conservación de volumen. Un cemento que varía de volumen se puede mejorar con el **almacenaje.**

**56. Variación de volumen durante el endurecimiento.** El hormigón de cemento expuesto al aire, **reduce** su volumen durante los primeros dos ó tres meses, mientras que se **expande en el agua** en el mismo período. Estos cambios son mayores en los hormigones que contienen mayor proporción de cemento.

**57. Contracción** de los morteros **al aire libre.**

| | por ciento. |
|---|---|
| Cem puro *..... | .132 á .140 |
| Mortero 1 : 1 *..... | .080 á .170 |
| Morteros pobres **..... | .030 á .050 |

La expansión **en el agua** es algo menor que la contracción en el aire. El cambio total de dimensiones es la suma algebraica de la debida á la fragua y á los cambios de temperatura.

**58.** Los hormigones se contraen menos cuando están bajo presión. La finura de la arena es causa de contracción.

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* Trabajos de la Asociación de Ingenieros civiles americanos, vol. XXVII, pág. 214.
** Considère. Investigaciones experimentales sobre el cem armado.

## Resistencia.

**59**. La resistencia á la tracción de los morteros de cem se prueba generalmente por medio de probetas.

**60. Factores que afectan las resistencias**. Influye mucho sobre la resistencia de las muestras ensayadas, la variación de temperatura del aire y del agua, y también la fuerza empleada para comprimir el cem dentro de los moldes; así como también por el grado de fragua al ponerlas en el agua y por el grado de sequedad al sacarlas, y aún más por la presión bajo la cual fragüen, lo que aumenta materialmente la resistencia; por esta razón, los cem empleados en obras de mampostería en circunstancias ordinarias pueden dar mejores resultados que en los ensayos de las muestras. Las causas antedichas, el grado de completa mezcla, la proporción de agua empleada, y otras consideraciones, pueden fácilmente afectar los resultados hasta en un **ciento por ciento** ó más. De aquí las divergencias en los informes sobre los distintos ensayos. Las muestras de diferentes cem, ensayados en condiciones aparentemente iguales, pueden dar muy diferentes resultados.

**61. Ecuación personal**. Durante la construcción del acueducto de « Croton », en Nueva York, un grupo de ensayadores, probando 835 probetas, obtuvieron un promedio de resistencia de 4.36 kg por cm cuad; mientras que otro grupo probando 2,434 probetas exactamente iguales, por los mismos métodos y en idénticas circunstancias, obtuvieron un promedio de 5.95 kg por cm cuad, ó sea 36% más.

**62**. Debido á estas inseguridades **deben hacerse las pruebas con un gran número de muestras** para que tengan valor y para poder obtener promedio de las diferencias.

**63**. Las diferencias en los resultados comparativos entre diversos materiales pueden ser producidas por una ó varias diferencias entre los materiales. Así, al comparar los morteros hechos con arenas limpias y sucias, la resistencia puede sufrir más por la diferencia de densidad de la arena que por su mayor ó menor limpieza.

**64. Efecto de la edad**. El diagrama *, fig. 1, demuestra aproximadamente la resistencia de los cementos Portland normales y de los naturales, puros y con 2 y 3 partes de arena, hasta la edad de dos años.

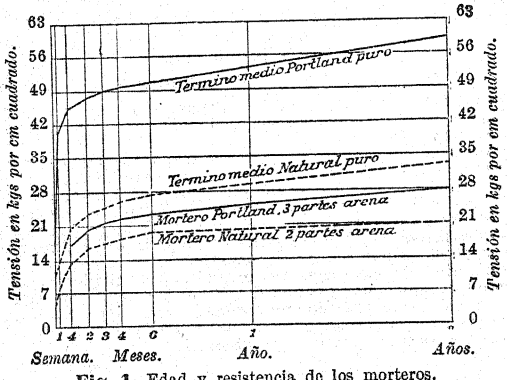


**Fig. 1**. Edad y resistencia de los morteros.

* Véase « Cement » de Richard L. Humphrey. Chicago, mayo de 1899.

**65. La fig. 2 demuestra, aproximadamente, los efectos de la arena,** en diversas proporciones, sobre la resistencia de los cementos Portland y naturales, en diversas edades, desde 1 semana hasta 1 año. Las cuatro curvas continuas representan los promedios de cementos Portland, y las cuatro líneas de puntos los pro-

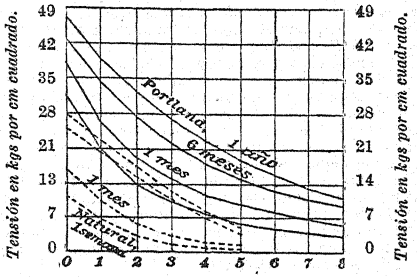


**Fig. 2.** Efecto de la arena sobre la resistencia.

medios de los cementos naturales. Para cada clase de cemento las curvas representan edades de 1 año, 6 meses, 1 mes y 1 semana, respectivamente, comenzando por la parte superior. Las curvas de los cementos naturales se llevan tan sólo hasta 5 partes de arena.

**66. La resistencia á la compresión** de los morteros de cemento, en cubos, parece ser de 8 á 10 veces la de su resistencia á la tensión; la resistencia transversal (esfuerzo cortante) es como ¼ de esta última.

**67.** La adhesión de los morteros de cem á los ladrillos ó á la piedra bruta, en diferentes edades, ya puros ó mezclados con arena, se puede calcular como los ¾ de la resistencia del mortero á la tensión y á la misma edad. Si los ladrillos y las piedras están húmedos y sin suciedades al colocarlos, la adhesión aumenta; mientras que, si están muy secos y sucios, sobre todo en época de calor, puede reducirse casi á cero. La adhesión á los ladrillos muy duros y á las piedras bien talladas, es menor que á las superficies ásperas y porosas.

**68.** El Dr. Bohme, de Berlín, encontró que la resistencia á la tensión, dividida por la de adhesión, era igual á 10 con morteros de 1 : 3 y 1 : 4 y de 6 á 8 con cem puro y mortero de 1 : 2.

## Otros detalles.

**69.** Los morteros de cal y cem, cuando se emplean en construcciones de ladrillos, á menudo desfiguran la obra, especialmente cerca del mar y en climas húmedos, con **eflorescencias** blancas, las que suelen extenderse sobre la superficie de la obra y dañar los ladrillos. También sucede, hasta cierto punto, con los cem Portland y en las juntas de las mamposterías de piedra, aunque en menor escala. Esto daña sólo á las piedras porosas. Generalmente es un carbonato de soda ó de potasa hidratado, ó un sulfato de magnesia unido á veces á otras sales. Como preventivo, el general Gillmore recomienda se agregue á cada 300 kg de cem en polvo, cien de cal viva, y de 8 á 12 de cualquier grasa animal barata, incorporando ésta muy bien á la cal viva antes que se apague y antes de agregarla al cem. Se dice que el aceite de linaza en proporción de 2 galones por cada 135 kg de cem seco, con cal ó sin ella, impide la eflorescencia en los casos dichos; pero, como la grasa, retarda mucho la fragua y debilita el cem. (Véase Ladrillos, pág. 1267.)

**70.** Para **recalzar** las juntas se debe emplear el mejor cem Portland puro, á menudo se usa mezclado con una ó dos partes de arena. Mézclese bajo techo y en cantidades como de dos litros cada vez, usando poca agua, de manera que el mortero, al usarlo, aparezca más bien incoherente y deficiente en plasticidad. Las juntas deben limpiarse y rasparse bien, hasta un centímetro ó más de profundidad, aplicando el mortero con cuchara adecuada; si la junta es recta, se mantiene debajo un instrumento de filo recto como auxiliar. Luego se calafatea bien el mortero hasta llenar la junta y se la pule con fuerza. Si son muy angostas, se ensanchan á

cincel lo suficiente. La junta se humedece bien antes de rellenarla y luego se la protege del sol para que no se seque demasiado pronto.

### Hormigón de cem bajo el agua.

**71. Lechosidad.** « Cuando los hormigones de cemento están bajo el agua, sobre todo en el mar, el cem exuda un fluido gelatinoso que flota en la superficie, dándole al agua un tinte lechoso, llamado por los franceses *laitance*. Como en estas condiciones á veces fragua de modo imperfecto, y algunos cem casi no fraguan, su interposición en las contrucción, aun en cantidades pequeñas, tiende á disminuir la continuidad y fuerza de la masa. Se acostumbra extraerlo de los espacios que ocupa por medio de bombas, trabajo que debe hacerse con cuidado para evitar daños en la construcción causados por las corrientes que se forman por la acción de las corrientes. La lechosidad se disminuye mucho reduciendo el área del hormigón expuesta al agua; también empleando cajas de 1 á 1½ metros cúbs de capacidad para sumergir el hormigón. » (Gillmore, « Cales, cementos hidráulicos y morteros ».)

**72.** Los autores no están de acuerdo sobre los **efectos del agua de mar.** H. Le Chatelier (International Assn for Testing Materials, Procs. 1906) encuentra que á los ingredientes activos del cem (cal, alúmina y sílice) los afectan las sales de magnesia del agua de mar, produciendo cloruro de calcio y sulfato de cal solubles. Este último, con el aluminato de cal, forma un compuesto cuya cristalización tiende á hinchar y agrietar el material.

**73.** Parece que las aguas, y en especial la de mar, con sus numerosas sales, y por su poder de infiltración en el cem, deberían pronto cerrarse ellas mismas la entrada al interior de la masa.

**74. La sustitución de hierro por alúmina**, en los cem, elimina uno de los más activos agentes de deterioro del agua de mar. (Véase Cem, § 30, pág. 1018.)

**75. Sobre la desintegración de estos hormigones en agua** (salada ó dulce) influye menos el agua misma que la alternada acción de las heladas, cuando el hormigón se encuentra expuesto á aquéllas por las variaciones del nivel de las aguas.

**76.** En algunos puertos de Italia existen morteros de puzolana y cal que se han conservado en perfecto estado y tienen ya de 15 á 20 siglos.

**77.** En el astillero de Kobe, en el Japón, se reemplazó dentro del dique el agua salada por agua dulce, para impedir los deterioros causados por aquélla, haciendo entrar la dulce por la superficie, mientras que la salada, más densa, se extrae por el fondo con bombas.

**Para concreto**, véanse págs. 1295, etc.

# CONCRETO *

**Para cemento, arena y mortero,** véanse págs. 1268, etc.

## AGREGADOS **

### Constituyentes.

**1. Orden de su valor.** (1) Trap (roca volcánica de la especie del basalto), (2) granito, (3) cascajo, (4) mármol, (5) piedra caliza, (6) escorias, (7) asperón, (8) pizarra, (9) esquisto, (10) ceniza gruesa.

**2. La resistencia del concreto,** con buen asperón, es como de .75 × resist con roca volcánica. Con pizarra, menos de la mitad de la misma. Buenas cenizas gruesas igualan á la pizarra y al esquisto. La dureza de los agregados aumenta en importancia con la edad del concreto, « porque, á medida que el cemento se endurece, hay mayor tendencia de las piedras mismas al esfuerzo cortante y así entra en juego la dureza de los agregados ». (Sanford E. Thompson, E. R., '06 /enero 27, pág. 109.)

**3.** La elección de los agregados es naturalmente cuestión de **costo** y de resistencia, etc., del producto. Así, con cascajo suficientemente barato, comparado con la piedra triturada, puede ser económico usar el cascajo, ó una mezcla de cascajo y piedra, y para obtener la resistencia total requerida se usa mayor masa de concreto. Esto es conveniente en cimientos, sobre terreno débil, porque distribuye la carga sobre un área mayor.

**4.** En muchos casos, la elección de la arena y agregados depende en gran parte del material que puede obtenerse y de su distancia.

**5.** Donde el cemento es barato, puede ser económico usar materiales más cercanos y conseguir la calidad aumentando el cemento.

**6.** La piedra que se rompe en fragmentos próximamente cúbicos, se aprieta mejor que la que se astilla en pedazos largos, y los fragmentos son menos susceptibles de romperse terminada la obra.

**7.** Buena piedra rota es mejor que cascajo. La aspereza de las partículas de piedra parece que da mejor adhesión.

**8.** A veces se usan cenizas gruesas para el agregado. Ellas son de ordinario las que resultan de la quemada de carbón bituminoso bajo las calderas. El material es en su mayor parte una ceniza fina, que contiene considerable cantidad de carbón no quemado.

**9.** Las cenizas gruesas de **antracita** se usan menos por ser menos abundantes.

**10.** El **conc** de ceniza gruesa, que sólo pesa de 1,280 á 1,600 kg por metro cúb, es ventajoso cuando se requiere poco peso. El conc de piedra rota ó cascajo pesa de 2,240 á 2,320 kg por m cúb.

**11.** La arcilla ó el barro que se adhiere á las partículas de cascajo destruye ó debilita la adhesión de los morteros á las piedras. La Comisión de Tránsito de Boston, Informe para 1901, pág. 39, encontró que la relación de las resistencias, entre conc con cascajo limpio y con cascajo sucio, es como de 60 á 45. Véase « Arcilla y Barro », en « Arena » é « Ingredientes accidentales », pág. 1346 y 1347.

### Tamaño.

**12.** En vigas, arcos, etc., el **tamaño del agregado** no debe exceder de 4 á 5 cm en ninguna de sus dimensiones; pero, si se limpia bien de polvo tamizándolo ó lavándolo, y si el mortero llena completamente los huecos, todos los tamaños, desde 1 á 10 cm, podrán usarse en obras de volumen, como cimientos, presas, muelles, etc.

**13.** Con agregados gruesos debe usarse arena gruesa y viceversa.

**14.** De ordinario se economiza cemento tamizando la arena del cascajo ó el material fino de la piedra triturada y luego remezclándolo en las proporciones requeridas.

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* *N. del T.* — Adoptamos esta expresión, cuyo uso se ha generalizado tanto, para reemplazar la de hormigón de cemento que es muy larga. La palabra concreto la abreviaremos así : *conc.*

** Por « agregado » significamos los materiales sólidos del concreto que no son el cemento y la arena. El término « agregado » se extiende también á veces á la arena.

### Densidad.

**15.** Cuando un cuerpo sólido se reduce á una masa que consiste en pedazos rotos separados por huecos, el aumento de volumen se debe solamente á los **huecos**, y es igual al espacio ocupado por ellos. En consecuencia, la razón entre el **aumento de volumen** y el volumen primitivo, es la de los huecos al volumen **primitivo**, y no al volumen final. Así, si un metro cúbico de piedra sólida, después de rota en pedazos, ocupa dos veces el espacio de antes, entonces el aumento de volumen, ó el espacio ocupado por los huecos, es=al ocupado por los pedazos sólidos=la mitad del ocupado por toda la masa rota.

**16.** En la piedra rota aflada y angular, que tenga todos sus pedazos de tamaño casi uniforme; cuando se mida suelta, como el 50 por **ciento** del volumen serán **huecos**. Si los tamaños de las piedras varían entre límites algo amplios, como desde 5 cm hasta ½ cm, el volumen ocupado por los huecos será menor, á menudo sólo de 28 á 30% del total.

**17. Las pruebas** hechas por el Sr. Wm. Hall (Trans. A S C E, vol 42, 1899, pág. 132) con piedra caliza azul de Río Verde triturada hasta 6 cm, tamizada y mezclada con cascajo muy limpio del río Ohio, de 4 cm, dieron el siguiente resultado:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Tanto por ciento de piedra..... | | | 100 | 80 | 70 | 60 | 50 | 0 |
| — | — | cascajo..... | 0 | 20 | 30 | 40 | 50 | 100 |
| — | — | huecos..... | 48 | 44 | 41 | 38½ | 36 | 35 |

Estos son términos medios de un número de pruebas de varias gabarras de materiales, pero se hicieron pocas variaciones entre las mezclas.

### Concreto ciclópeo.

**18-19.** El « concreto ciclópeo », que consiste en piedra bruta, grande, colocada en mortero de cemento, se usa en general, económica y ventajosamente, en obras de volumen, especialmente en represas donde se requiere mucho peso y esfuerzo cortante horizontal. No es necesario que las piedras sean chatas. Déjanse caer ordinariamente en el mortero mojado, asentándolas sólo por su caída. El concreto mojado facilita el asentamiento de las piedras, y se liga mejor con ellas que el seco.

**20. En la presa de la caída de agua de Chaudiere**, Canadá, los bloques se obtuvieron de capas duras en el lecho del río, con buena forma para asentarlas. El volumen del agregado varió entre 25 y 30% del volumen de la presa; máx, 40%.

**21.** En las obras de Desenvolvimiento en Transmere Bay (Procs. Inst. C. E., vol. 171, 1908, pág. 145) los bloques eran de asperón. Cerca de las bases de los muros, pesaban una tonelada ó más. La proporción de los bloques disminuyó, con el espesor del muro, desde 10 hasta 7% de la masa total.

**22. Las restricciones innecesarias** que suelen imponerse á los contratistas pueden suprimir la ventaja debida al uso de los bloques. (Véase § 19.)

## CONCRETO COMÚN

**1. El concreto de cemento** se compone con piedra rota, cascajo, ceniza gruesa, escoria, conchas, ú otro material duro é inerte* (que es el agregado) adheridos entre sí por mortero de cemento, compuesto de cemento y arena.

### Ventajas.

**2. Las ventajas principales** del concreto son: la comodidad con que puede colocarse, particularmente en situaciones de otro modo difíciles ó debajo del agua; su utilidad en trabajos bajo el agua; su baratura, debida en gran parte á la facilidad para colocarlo, y el hecho de poder usar piedra muy pequeña; sus cualidades de resistencia contra el fuego comparado con la piedra caliza (que se calcina) y con el granito (que se astilla).

**3.** El uso del concreto se ha extendido muchísimo, con el **cemento armado**, que permite su empleo (antes de ahora á menudo impracticable) en piezas sujetas á tensión lo mismo que á compresión, como en los *cantilevers*, presas, muros de sostenimiento, en columnas, y en arcos en que la altura es ó muy grande ó muy pequeña

* Sin afinidad química por otros materiales.

relativamente á la luz. El cemento armado permite el uso de secciones mucho menores que las empleadas antes con seguridad cuando sólo se hacía uso de la resistencia *compresiva* del material.

**Para cemento armado**, véase pág. 1320.

**4. Desventajas.** El concreto es algo más débil que la buena mampostería de piedra, y tiene sólo como la mitad de la resistencia de la mampostería de granito de primera clase con juntas delgadas de cemento. A semejanza de la piedra y del mortero, está sujeto á deterioro, especialmente en el agua de mar; pero esta dificultad se está eliminando por el esmero con que se hace la manufactura de cemento, alentados por su extenso uso y por su elaboración en grande escala. Como en toda obra humana, y notablemente en las construcciones de albañilería, es necesario mucho cuidado, pues de éste depende en gran parte el éxito de la construcción. La calidad de la obra acabada puede, sin embargo, probarse por taladros.

**5.** El concreto se usa para poner á nivel cimientos de niveles desiguales, antes de colocar las capas de mampostería. Por este medio se empareja el número de capas horizontales en la mampostería y se impide que se asienten desigualmente.

**6.** En trabajos de ferrocarril, el empleo del concreto puede **evitar el uso de grúas** (*derricks*), que son fuente de estorbo y de peligro para los trenes.

**7.** El concreto se usa ventajosamente para **reforzar y proteger la mampostería vieja de piedra**; pero, á menos que se tomen precauciones especiales, las dos construcciones son susceptibles (al cabo de cierto tiempo) de separarse, debido á que se asientan desigualmente, sobre todo si el apisonado no ha sido perfecto.

## Cemento natural.

**8. El cemento natural** rara vez se usa ahora en concreto, excepto en obras de volumen donde no está sujeto á la acción deteriorante del agua ó de la helada y en que no se requiere pronta resistencia. Es apropiado para bases y para muros bajos de sostenimiento no sujetos á vibraciones fuertes.

**9.** En presas, rompeolas, etc., la parte interior es frecuentemente de *cemento natural*, con una envoltura exterior fuerte de concreto de cemento Portland.

## Proporciones.

**10. Las proporciones de cemento, arena y agregado** deben **determinarse** teóricamente, ya todos por peso, ya todos por volumen y estando el polvo suelto; pero, en la práctica, el cemento se mide por el número de sacos ó barriles, etc., usados cuyo contenido se conoce; la arena y el agregado se miden sueltos.

## « Mezcla natural ».

**11. Acostúmbrase designar** las cantidades de cemento, arena y agregado, en un concreto, por proporciones. Así : 1 : 2 : 4 quiere decir 1 parte de cemento con 2 partes de arena y 4 partes de agregado. Tal designación es necesaria al dar instrucciones á los trabajadores; y, cuando las series de tamaños de las partículas son conocidas, ella indica la condición del concreto. Las proporciones se rigen naturalmente por la naturaleza de la obra; pero no deben hacerse distinciones entre trabajos de clases próximamente semejantes.

**12. Proporciones usadas para concreto de cemento Portland :**

En obras excepcionalmente sólidas (nivelación de cimientos, represas, rompeolas), 1 : 1.5 : 8 á 1 : 5 : 10; con cemento natural, 1 : 2 : 5.
Cimientos, generalmente, 1 : 3 : 6; á veces sólo 1 : 4 : 8.
Muelles, pedestales, estribos, 1 : 2½ : 5½ á 1 : 3½ : 7.
Estribos y bóvedas en los filtros, 1 : 2½ : 5½.
Muros y vigas armados, 1 : 3 : 6; secciones ligeras, 1 : 2½ : 5.
Cimientos, 1 : 2½ : 5½; muros de sostenimiento, 1 : 2½ : 5½ á 1 : 3 : 6.
Muros de enjutas ó riñones, 1 : 3 : 6.
Conductos, desagües, cloacas, 1 : 2½ : 5½ á 1 : 3 : 6.
Muros para estanques de agua y filtros, 1 : 1½ : 3½ á 1 : 2½ : 5½.
Obras bajo el agua, 1 : 2 : 3.
Sistemas de pisos (vigas, losas), 1 : 2 : 4 á 1 : 2½ : 5½.
Escaleras y techos, 1 : 2 : 4.
Arcos, 1 : 2½ : 5; secciones ligeras, 1 : 2 : 4.
Albardillas y asientos de puente, 1 : 1 : 2 á 1 : 2 : 4.

Pero el requisito esencial es que todos los huecos, entre las partículas de arena y el agregado, se llenen de mortero de cemento. En consecuencia, á menos que la gradación de los tamaños, de arena y de agregado, sea conocida, ó supuesta, el simple enunciado de las proporciones, de cemento, arena y agregado, en una mezcla, es sólo un dato poco útil en cuanto al valor del concreto.

**13.** En general, **en obras de cemento armado** deben emplearse mezclas más ricas que las que se usan en obras de grandes masas. A fin de conseguir una adhesión conveniente y firme, que es lo más importante, las barras deben estar completamente rodeadas por el cemento.

### Materiales requeridos.

**14. Materiales requeridos para un metro cúbico de concreto de cemento Portland apisonado.** $c$=cemento, bls; $s$=arena, metros cúbs; $a$=agregado, metros cúbs. Se suprime el polvo al cernirlo. Las piedras no más grandes de 2½ cm.

| Mezcla | $c$ | $s$ | $a$ |
|---|---|---|---|
| 1 : 2 : 4 | 1.89 | .57 | 1.16 |
| 1 : 2 : 5½ | 1.55 | .60 | 1.18 |
| 1 : 3 : 5 | 1.44 | .66 | 1.10 |
| 1 : 3 : 6 | 1.31 | .60 | 1.19 |
| 1 : 3 : 7 | 1.18 | .55 | 1.20 |
| 1 : 4 : 7 | 1.02 | .66 | 1.16 |
| 1 : 4 : 8 | 1.00 | .61 | 1.21 |

Con piedra de 6 cm, las cantidades de todos los materiales, por m cúb de concreto, fueron aumentadas de 2 á 5%. Con cascajo, >2 cm, fueron disminuídas como en 9%. (Chas. A. Matcham. Natl. Builders' Supply Assn., 1905.)

**15.** Sean

B=N.° de barriles de cemento requerido por m cúb de concreto
=N.° de veces .108 m cúb de cemento requerido por m cúb de concreto
P=partes de arena (ó agregado) para 1 parte de cemento.

Entonces

1/B=N.° de m cúb de concreto hechos con 1 barril de cemento;
.108 P=N.° de m cúb de arena (ó agregado) para 1 barril de cemento;
.108 PB=N.° de m cúb de arena (ó agregado) para 1 m cúb de concreto.

### Vacíos. Véase Peso, pág. 1314.

**16. Reducción de los vacíos.** Si se usan piedras que tengan 50% de huecos, y arena que tenga 50% de huecos, con cemento, en las proporciones:

Cemento, 1 parte = .25 m cúb;
Arena, 2 partes= .50 m cúb;
Piedra, 4 partes=1.00 m cúb;

el concreto resultante medirá algo más de un m cúb, y sin embargo contendrá huecos no llenados.

**17.** Estas proporciones, sin embargo, no son económicas. Eligiendo una arena que tenga una serie de tamaños, ó mezclando dos ó más arenas que tengan granos de diferentes tamaños, los huecos de la arena pueden reducirse á, digamos, 33%. De manera semejante, los huecos en la piedra pueden reducirse á 35%. Digamos pues, que tenemos:

Cemento, 1 parte = .12 m cúb;
Arena, 3 partes= .36 m cúb;
Piedra, 8 partes=1.00 m cúb;

con resultados tan buenos como con la mezcla precedente de 1 : 2 : 4, aunque usando sólo la mitad del cemento.

**18.** El Sr. Geo W. Rafter (Trans. A. S. C. E, dic. de 1899, vol. 42, pág. 106) recomienda que las proporciones se expresen por medio de la razón del volumen del mortero al volumen del agregado. Así: un concreto que contenga 75 volúmenes de agregado y 25 volúmenes de mortero, sería un concreto de 33⅓ %.

**19.** En condiciones usuales, los **huecos** en el **agregado deben llenarse** de

un mortero tan rico como lo exija la fuerza de la obra. Puede resultar un concreto mejor del uso de un mortero pobre que llene los huecos que del uso de un mortero más rico que no llene los huecos sino parcialmente.

**20.** El mortero no puede distribuirse uniformemente en todo el agregado, y algunos de los vacíos son demasiado pequeños para admitir los granos de arena. Además, la mezcla es susceptible de alteraciones al depositarse. En consecuencia, habrá huecos en el concreto, á no ser que haya un exceso de mortero sobre los vacíos calculados en el agregado.

**21.** En la práctica, el **exceso de volumen de mortero requerido**, sobre el de los vacíos medidos en el agregado, á fin de conseguir que queden llenos, es generalmente de 15 á 25% del volumen de los vacíos. Pero, por 15 experimentos con piedra caliza, encontró el prof. Baker que los huecos no se llenaban completamente, á no ser que el volumen del mortero excediese del volumen de los vacíos en 40%. (Tabla 13 *c*, pág. 112 *b*, *Baker's Masonry Construction*, 1907.)

**22.** El Sr. John Watt Sanderman (Procs., Instr. C. E., vol. 121, pág. 219, 1895) cree que, para **garantizar la impermeabilidad**, el volumen del mortero debe ser 50% del volumen del agregado que tenga 35% de vacíos; ó bien el exceso de mortero=43% de los vacíos.

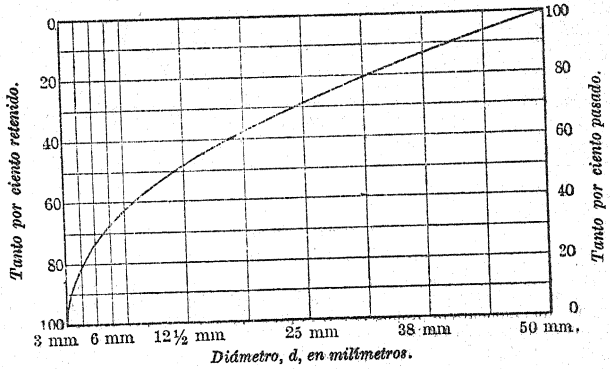


**Fig. 1.** Parábola de densidad máxima. Véase § 23.

**Densidad.** Véase Peso, pág. 1314.

**23.** El Sr. Wm. B. Fuller (T. and T. pág. 197) encuentra que se obtiene la **mayor densidad**, y en consecuencia la menor cantidad de cemento requerida, cuando el agregado y la arena están graduados de modo que los tantos por ciento, por peso, que pasan por los varios tamices, están como los indican las ordenadas de la parábola en la fig. 1, donde las abscisas son los diámetros, *d*, de las aberturas en los tamices; mientras las ordenadas *debajo* de la parábola representan los tantos por ciento *pasados* y las de *encima* los tantos por ciento *retenidos*, por estas aberturas respectivamente.

**24.** En esta parábola $d=P\sqrt{M}$; donde $d$=un diámetro dado; P=proporción de partículas más pequeñas que $d$; M=diámetro máx de piedra (=2 pulgs (50 mm) en la figura).

**25.** Los experimentos (Trans. A. S. C. E., vol. 59, págs. 67, etc., 1907) muestran que puede efectuarse un ahorro de 12% de cemento y obtenerse un producto más impermeable graduando así los tamaños de la arena y del agregado; pero el ahorro puede á veces ser destruído por el costo adicional de tal graduación, especialmente en obras pequeñas.

(*Obs. del T.* — Como siempre, en el ejemplo que sigue convertimos los datos del autor al sistema métrico.)

**26.** En el revestimiento del túnel para el acueducto de Sudbury :

| | |
|---|---|
| 1 barril de cemento Portland como venía del vendedor.... | = 97.0 litros |
| $2^1/_0$ barriles de arena suelta..... | =208.4 — |
| $5\frac{1}{2}$ barriles de piedra triturada suelta..... | =526.0 — |
| Total..... | =831.4 — |

Sacudiendo ligeramente la arena y la piedra, se hicieron prácticamente las proporciones 1 : 2 : 5.

Estos 831.4 litros produjeron de 566 á 595 litros de concreto, apisonado en el lugar; ó digamos, 1,400 litros de materiales para 1,000 lit, ó un m cúb de concreto.

**27.** El Sr. Wm. B. Fuller (Asociación Nacional de los que emplean cemento Procs., '07, pág. 95) probó vigas de concreto, de 30 días, de 1 : 2 : 6, 1 : 3 : 5, 1 : 4 : 4, 1 : 5 : 8, 1 : 6 : 2, 1 : 8 : 0 (todas 1 : 8). Las fuerzas comparadas como en la fig. 2 *.

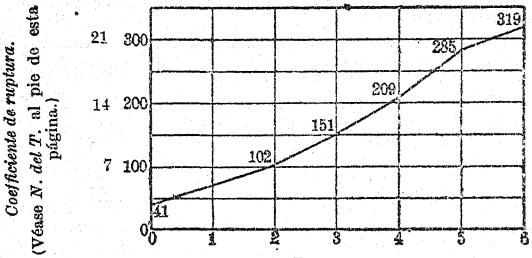


**Fig. 2.** Proporciones; resistencia.

**28.** De esto aparece que, cuando los huecos en el agregado están llenos de mortero, la fuerza de compresión del concreto parece más bien aumentar que disminuir á medida que aumenta la proporción de la piedra, y además, que depende en gran parte de la riqueza del mortero.

**29. Graduar las proporciones por mezclas de ensayo.** (Wm. B. Fuller, Trans. A. S. C. E., vol. 59, págs. 77, etc.)

Habiéndose determinado las clases de arena y piedra que han de emplearse en cualquier obra, úsese un cilindro corto, fuerte y rígido, de hierro forjado de 25 cm (forma de un tubo de agua), cerrado con un casquete en un extremo.

**30.** Sobre un pedazo de lámina de acero ú otro material no absorbente, pésense y mézclense todos los ingredientes, hasta la consistencia requerida para la obra. Échese la mezcla dentro del cilindro, apretándola cuidadosa y continuamente, y anótese la altura hasta donde se llena el cilindro. Antes de que la mezcla tenga tiempo de fraguar, vacíese y límpiese el cilindro.

**31.** Hágase otra cantidad de mezcla, usando los mismos pesos de cemento y de agua que antes, y el mismo **peso total** de arena y piedra, pero con una proporción ligeramente *diferente de los pesos* de la arena y de la piedra.

**32.** Nótese la altura alcanzada, en el cilindro, por esta segunda y por las subsiguientes mezclas. La mejor mezcla es la *que da la menor altura en el cilindro*, con tal que su aspecto en el cilindro demuestre que todas las piedras están cubiertas de mortero.

**33.** Este método pone al ingeniero en situación de escoger los mejores de los materiales disponibles en cualquier caso dado.

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* *N. del T.* — Frente á los números 100, 200, 300 del dibujo, que indican lbs por pulg cuad, están respectivamente 7, 14, 21, que son los kg por cm cuad, á que equivalen. Para valores intermedios sirve la misma escala de la figura. Los números 41, 102, 151, etc., del interior del clisé multiplicados por 0.07 dan los valores en kg por cm cuadrado de los puntos, respectivos en la curva.

**Consistencia.** Véase también Mortero, §§ 34, etc.

**34.** La habilidad y el cuidado, en la colocación, y la *uniformidad* en la consistencia son más importantes que la misma consistencia.

**35.** Los extremos de la **práctica** son : (1) Concreto con mortero apróximadamente tan húmedo como tierra húmeda; usar tan sólo el agua suficiente para que aparezca en la superficie después de atacarlo mucho y fuerte, (2) usar suficiente agua para hacer que el concreto tiemble cuando se pone por primera vez, y que sólo permita su colocación con las palas. La consistencia conveniente depende en gran parte de la naturaleza y objeto de la obra.

**36.** El concreto **seco** es preferible generalmente en obra grande abierta donde puede ser bien apisonado, y donde se necesita que adquiera pronto su resistencia, como en los arranques de los arcos. Cuando está bien apisonado desarrolla una resistencia á la compresión muy superior, y en poco tiempo, y puede llegar á tener una resistencia permanente algo mayor que las mezclas más húmedas; pero el apisonado imperfecto de estas mezclas puede dar por resultado un concreto muy débil, mientras que el apisonado perfecto puede hacer la obra más costosa de lo que justificaría el aumento de resistencia adquirido.

**37. Mediano.** La práctica actual favorece el uso, en general, de mezclas suficientemente húmedas para requerir sólo el paleo; pero, aun en este trabajo, puede necesitarse el apisonado de tiempo en tiempo para mezclas ocasionalmente secas.

**38.** El concreto **húmedo** se mezcla mejor, se coloca más pronta y económicamente y se hace entrar con más facilidad en los espacios estrechos entre las barras en el cemento armado. Se pone en más perfecto contacto con los moldes y armaduras, tomando así una superficie más lisa y más impermeable. Es, por consiguiente, preferible así (en edificios) en armaduras de forma complicada, ó en secciones delgadas, ó donde se requieren superficies lisas.

**39.** La **humedad** retarda la fragua, une mejor las capas sucesivas, da una masa compacta con menos pisón y suministra el aumento de agua requerida para suplir la absorción por las armaduras de madera. El concreto húmedo está menos expuesto que el seco á sufrir por mala mano de obra; pero un exceso de agua reduce la fuerza y aumenta la eflorescencia.

**40.** En el concreto « ciclópeo » pueden emplearse más piedras grandes cuando el concreto es húmedo; éste les permite asentarse bién y se une mejor con ellas.

**41.** Se usan con frecuencia las mezclas **suficientemente húmedas para echarlas** en las armaduras de las columnas de los pisos.

**42.** La humedad de la atmósfera reduce considerablemente la cantidad de agua requerida para una consistencia dada.

**43.** El agua sube al través de concreto colocado. En consecuencia, una proporción menor de agua en la mezcla puede bastar hacia el fin de un día de trabajo.

## MANIPULACIÓN Y MEZCLA

### Ingredientes.

**1.** Al proyectar una oficina (planta) para **elaborar** concreto deben tomarse en cuenta las cantidades que han de manipularse, las áreas sobre que deben distribuirse, las facilidades para conseguir y recibir los materiales, y el espacio de trabajo disponible; además, cada caso presentará peculiares circunstancias.

**2.** Las *disposiciones* de tales oficinas son tan variadas por su naturaleza como lo son las diversas clases de trabajo. En general, en cada obra debe estudiarse este punto muy especialmente, y el buen éxito y la economía dependen en gran parte de lo adecuado de esta oficina de manipulación.

**3.** Los materiales pueden llegar al sitio por carros, botes ó yuntas. Téngase cuidado de que no haya lodo ni esté sucio el fondo del vehículo. La arena y el agregado pueden dragarse del río.

**4.** Después de llegar á la obra, son llevados los materiales á los depósitos, por carretas, carretillas, carros pequeños, baldes, ó por cables. De los depósitos se llevan de ordinario por su propio peso, por medio de tolvas, al mezclador.

**5. Almacenaje.** El cemento se guarda comúnmente en tinglados ó almacenes y se manipula, separadamente de la arena y el agregado, en sacos ó barriles, á menudo por medio de conductores de cadena.

**6.** Para llevar los materiales de los **depósitos á los mezcladores**, y el concreto de los mezcladores á la obra, úsanse vagones, carretillas ó carros pequeños.

**7.** Cuando la obra cubre un área horizontal limitada, como en el caso de un edi-

ficio, pila ó estribo, **el mezclador** no necesita ser movido frecuentemente y su manejo es relativamente sencillo.

**8.** Cuando la obra cubre una gran área horizontal, como en un filtro de lenta filtración, ó donde cruza un valle, como en una presa, úsanse cables, con torres; ó se instalan una ó más plantas de mezclar, en sitios adecuados.

**9.** Cuando la obra se extiende á lo largo de una línea de considerable longitud, como en muros, cloacas ó acueductos, se tiende á lo largo de ella una vía de ferrocarril, á menudo vía ancha, con tres ó más líneas de carriles, y los materiales se manejan desde los carros de grúas; en general de modelos especialmente dispuestos para la obra que se tiene entre manos.

**10.** La obra se facilita teniendo los carros, carretillas, baldes, etc., de capacidad conocida, de modo que puedan servir de medidas al proporcionar la arena y el agregado. Así, los carros pueden contener arena ó agregado bastante para una mezcla y pueden voltearse en cajas más grandes, que contengan cada una arena y agregado en la proporción debida. El cemento se mide por lo común separadamente, contando los sacos ó barriles vaciados.

**11.** Cuando se usan **vagones**, pueden moverse por locomotora ó por cable, llegando á los depósitos por un plano inclinado.

**12.** En el caso de un **transmisor de correa ancha**, la arena y la piedra en debida proporción, y después el cemento, los toma ó se ponen en el cable desde los respectivos depósitos.

**13.** Comúnmente la **plataforma de medir** (ó la tolva de medir para máquinas de mezclar) se coloca directamente sobre el mezclador.

**14.** Para el mayor rendimiento, debe haber dos juegos de tolvas de medir, uno que esté volteándose en el mezclador mientras el otro está llenándose.

Para el lavado de la arena, véase ARENA, § 34.

**15. El agregado puede lavarse** en una criba cilíndrica giratoria, con un chorro de agua, bajo alta presión.

**16.** El trabajo se hace con frecuencia **de noche.**

**17. Planta portátil de mezclar concreto** (carro de plataforma). Se compone de dos maderos de 15 × 20 cm y 17 m de largo, á 1.20 m de distancia, colocados sobre el piso de un carro de plataforma de 11 m en ancho de vía corriente, proyectándose sus extremos 3 m fuera de cada extremo del carro, sostenidos por vientos á una armazón elevada en el centro del carro. Cada extremo lleva una tolva como de 1½ m cúb. La arena y el agregado se vierten con palas en esta tolva y se descargan en un conductor de cable que corre horizontalmente debajo de la tolva y que lo conducen hacia arriba á una de las tolvas (de 2 m cúb) situada 4.5 m por sobre el piso del carro y encima del centro de éste. Esta tolva elevada descarga la arena y el cascajo en un mezclador Smith de .57 m cúb, colocado en el centro del carro. El cemento se lleva al mezclador á brazo; el agua se toma de un tubo tendido á lo largo de la obra y provisto de conexiones de manguera. Un barril lleno de agua, y colocado sobre la armazón, sirve de depósito para casos imprevistos. La correa conductora, de 60 cm de ancho, consiste en dos cadenas de eslabones, unidas por una correa de lona fuerte de doble espesor. La correa está sostenida por travesaños de tubo de hierro forjado, á distancia de 45 cm. La correa forma senos entre las crucetas. Es movida por una máquina de vapor que hace girar un cilindro simple de 23 × 40 cm, montado en un extremo del carro. Capacidad media, 209 m cúb por día. Una tolva más baja es suficiente para abastecer al mezclador.

**18.** En la construcción de obras de forma circular, el concreto para pisos, columnas, vigas y techos, puede llevarse á las armaduras por medio de una torre central y un puente que atraviese la obra y llegue á una vía circular. El puente forma entonces una **grúa giratoria** que lleva mezcladores en su extremo.

## Mezcla.

**19. Generalidades.** Cada grano de arena debe ir cubierto de cemento; el mortero debe cubrir cada fragmento de piedra del agregado y debe estar igualmente distribuído en toda la masa. La piedra, si está seca, debe mojarse antes de agregarla al mortero.

**20. La perfección** de la mezcla es de la más grande importancia; especialmente cuando el concreto es pobre de cemento ó de consistencia seca.

**21.** La gran resistencia del concreto en el puente de Munderhingen se atribuye á su perfecta mezcla. Los materiales se mezclaron secos 2 minutos y 3 minutos mojados.

**22.** La variación de **color** de la mezcla indica cambio en las proporciones de los ingredientes.

**23.** Cuídese de que cualquier cemento, rechazado como defectuoso, sea reemplazado por otro bueno.

**24. Elevación del concreto.** Cuando la plataforma de mezclar no puede construirse cerca de la parte superior de la estructura, puede subirse el concreto por medio de un elevador á vapor hasta un nivel conveniente, y entonces se lleva por carriles á nivel. Para elevaciones pequeñas y cantidades pequeñas, se usan elevadores con fuerza de sangre; para elevaciones más altas y cantidades más grandes, se emplea una pequeña máquina de vapor ó de gasolina.

**25.** En algunos casos, el mezclador y su armazón son elevados completamente por la grúa que suministra materiales, y los deposita sobre la obra ó cerca de ella.

**26. La mezcla á brazo** es inaconsejable y antieconómica, excepto en trabajos pequeños.

**27.** En la mezcla hecha á brazo se acostumbra mezclar la arena y el cemento secos, volviéndolos con palas dos ó tres veces, hasta que la mezcla sea de color uniforme y cada grano de arena esté cubierto de cemento.

**28.** Agrégase entonces agua y se mezcla el mortero antes de añadir el agregado; ó el agregado puede esparcirse sobre la arena y el cemento mezclados en seco ó vertidos éstos sobre el agregado, mojando entonces el todo, mezclándolo con palas hasta que el agua se haya incorporado por completo.

**29.** La mezcla del cemento y la arena primero, como se dice arriba, reducen el trabajo total omitiendo la innecesaria manipulación del agregado.

**30.** La mezcla á brazo debe estar bien protegida del viento y la lluvia. El viento se lleva lo más fino (y por tanto lo mejor) del cemento, y la lluvia impide la conveniente mezcla (seca) del cemento y de la arena.

**31.** Para la sub-estación de la *Brooklyn Rapid Transit Co.* se emplearon dos armazones rectangulares sin fondo, una de las cuales tenía una capacidad de 400 lit y se llenó primero de arena. Vaciáronse entonces siete sacos de cemento encima de ella, y la masa se revolvió varias veces por cinco paleros hasta que el color se hizo uniforme. Nivelóse entonces, y la otra armazón (de 800 lit de capacidad) fué puesta encima llenándola de piedra triturada; se le echó agua con una manguera. Revolvióse entonces la masa cuatro veces, se paleó en carretillas y se depositó en las armaduras.

**32.** Con igual cuidado, la mezcla á máquina resulta mejor y más segura que la mezcla á brazo y es más económica en obras grandes.

**33. Debe inspeccionarse cuidadosamente el producto,** pues la obstrucción accidental y no sospechada de una tolva puede cambiar su naturaleza.

## Mezcladores.

**34.** Los mezcladores son **de dos tipos principales** : « continuos » y « por lotes ».

**35.** En los **mezcladores continuos** las materias primas son llevadas continuamente á la máquina por un extremo y el concreto mezclado sale continuamente por el otro extremo.

**36.** El mezclador (continuo) accionado sólo por la **gravedad** es una canal ó artesa estacionaria, colocada casi vertical y provista de aletas ó *estorbos* salientes contra los cuales choca el material en su caída y de los que depende la mezcla. Se da entrada al agua por un tubo de regadera en el tope de la canal. Sólo se necesita fuerza de vapor para elevar los materiales á la parte superior del mezclador, que está generalmente á 2.40 m de altura.

**37. Hay otros mezcladores continuos** en forma de artesas abiertas, casi horizontales y que tienen un eje giratorio longitudinal con aletas adheridas en forma de tornillo, que llevan el material, vertido en el extremo superior, á lo largo de la artesa, al extremo inferior ó de descarga. El agua se suministra por medio de tubos perforados á la largo de los lados de la artesa.

**38. Medida.** En los mezcladores continuos hay que graduar las proporciones de los ingredientes del concreto. Varios medidores automáticos han sido usados. A veces la arena, el cemento y el agregado se esparcen en capas, en la plataforma del mezclador, y se vierten con palas en el mezclador. A veces sólo se destina, por ejemplo, un palero para el cemento, tres para la arena y seis para la piedra, pero este método es demasiado primitivo para la generalidad de los casos.

**39.** Los mezcladores de cantidades fijas de mezcla dan el concreto en porciones, cuya magnitud depende de la capacidad del mezclador. Tienen un alcance más amplio que los mezcladores sólo movidos por la gravedad y dan mayor seguridad en la proporción de los ingredientes.

**40.** El mezclador de mezcla por lotes, más antiguo y más sencillo, consiste en una

caja **cúbica** de hierro, giratoria, lisa por dentro, sostenida por sus ángulos opuestos, y provista, por un lado, de una compuerta corrediza, para dar entrada á los materiales y descargar el concreto. Aplícase la fuerza del vapor por medio de un engranaje en el eje. Los ingredientes pueden mezclarse secos por un número de vueltas, y el agua se agrega después por medio de los huecos de los muñones, ó puede agregarse el agua antes de hacerse la mezcla. Los mezcladores cúbicos más antiguos tenían que detenerse tanto al cargar como al descargar el concreto.

**41.** En *Superior Entry*, Wis., el Gobierno de los Estados Unidos usó un mezclador cúbico de concreto, que cargaba y descargaba sin detenerse y sin variar la velocidad. Era movido por una máquina de vapor vertical simple de 18×25 cm y daba en 80 segundos un concreto perfectamente mezclado. El concreto se veía durante todo el proceso.

**42.** En más modernos mezcladores por lotes, la caja cúbica es reemplazada por **un tambor** (ó cilíndrico ó caja formada de dos conos), que se hace girar por medio de una cadena en un anillo que rodea el tambor, y provisto de aletas fijadas en su interior. Estas aletas elevan primero y después vierten el material, mezclándolo por la agitación que producen. La descarga se efectúa, en la máquina de doble cono de Smith, inclinando la máquina (como un convertidor de acero de Bessemer) sobre sus muñones, colocados en el centro de gravedad del tambor; y, en la máquina de tambor cilíndrico de Ransome, insertando una artesa de volteo, que, en la posición de descarga, toma el material como cae de las aletas.

**43.** Para remediar **las rupturas**, cada mezclador debe estar siempre provisto de **piezas de repuesto**.

**44. Montaje.** Los mezcladores son ó estacionarios, ó montados sobre rodillos ó carros con ruedas, con ó sin máquina de vapor, máquina de gasolina ó motor eléctrico.

**45.** El mezclador, con su armazón, es levantado á veces completamente desde su primera posición y llevado á otra, por una grúa ó vía de cable.

**46.** Se han usado **mezcladores de concreto con ruedas** y con tambores giratorios, en que se cargan los ingredientes, y se mezclan por el movimiento del vehículo al andar. La fuerza motriz puede darse á brazo, por fuerza de sangre ó por máquina de gasolina; y la relación entre la velocidad hacia adelante y la velocidad de rotación puede regularse por un engranaje.

**47.** Se venden pequeños mezcladores de cantidades fijas de mezcla para moverse á brazo, con capacidad del 13 m cúb por día.

**48.** En la **elección de un mezclador**, es de primera importancia la confianza, debida á que se haya usado con buen éxito, especialmente cuando se necesita un trabajo continuo.

**49.** La reducción del producto puede ser debida á **falta de fuerza** motriz del mezclador ó al mezclador mismo.

**50.** El mezclador **debe limpiarse** después de cada día de trabajo.

## COLOCACIÓN

**51.** El mejor concreto puede hacerse de muy poco valor por falta de cuidado ó método en su colocación.

**52.** Cuando el concreto se **descarga de una altura considerable**, parece que hay riesgo de que se altere la igual distribución de los materiales. En consecuencia, si se baja en baldes, éstos deben llevarse cerca del trabajo ya hecho, antes de voltearlos. Sin embargo, en la construcción de las pilas de concreto para un puente en Bethlehem, Pa., por Cramp and Co (E. R, '09/marzo/6, pág. 280), el concreto se descargaba por medio de una canal inclinada de madera, forrada con palastro, en un punto que quedaba verticalmente á 23 m debajo del mezclador; el método resultó económico, el concreto uniformemente bueno, y no hubo dificultad por separación de los ingredientes.

**53.** En obra que haya de ser de apariencia, las **capas** se limitan ordinariamente á cosa de 15 cm de grueso, debido á la dificultad de aplanar el paramento con el reverso de las palas cuando las capas son más gruesas; pero en cimientos y en obras pesadas, sobre el suelo, si ha de revestirse con mampostería, ó, si la apariencia no es de importancia, pueden usarse capas de concreto húmedo hasta de 60 cm de espesor.

**54.** Si el concreto, después de colocado, está **demasiado húmedo**, es mejor corregir el inconveniente colocando sobre él concreto más seco. Cuando se **le saca el agua excedente**, con ella se saca algún cemento que se desperdicia.

**55. El exceso de aplanamiento con las palas** hace subir el agua de abajo, y ésta lava el cemento del paramento.

**56. Las obras de considerable longitud**, tales como represas y muros, se construyen comúnmente en secciones alternativas, así: las secciones 1, 3, 5, etc., se construyen primero separadamente, y, cuando se han endurecido, se construye la sección 2 entre las secciones 1 y 3, la sección 4 entre las secciones 3 y 5, etc. Los lados de las secciones 1, 3, 5, etc., sirven así como parte de las hormas para las secciones 2, 4, etc. Este método facilita la trabazón entre las secciones, por medio de ranuras y ensambladuras verticales, formadas, por los moldes, en los lados de las secciones construídas primero. El concreto de las secciones restantes, colocado más tarde, entra en estas ranuras y las llena.

**57. En tiempo de helada** puede colocarse el concreto en grandes masas en agua ó debajo de la superficie del suelo. En las excavaciones, si se permite que el agua del suelo se eleve por sobre la obra durante la noche, ordinariamente impedirá que la helada llegue al concreto.

**58.** En la represa de la caída de agua, en Chaudière, se colocó concreto en **temperaturas muy bajas** (como — 29° C). Se erigió una casa para hacer mortero, y la temperatura interior se mantuvo, por medio de estufas, más alta que la de congelación. Los materiales se bajaban á la casa por grúas al través de escotillas hechas en el techo. Guardábase el agua en pipas y se mantenía tibia con chorros de vapor. La arena se calentaba fuera de la casa. La piedra, en pilas de 1 á 1.20 m de alto, se calentaba (pero no se secaba) por chorros de vapor de un tubo perforado que pasaba por debajo de las pilas. Después de colocado, se cubría el concreto suelto con lona, bajo la cual se introducía la boca de una manguera de vapor.

## Tapialeras y armaduras.

**59.** En los cimientos para muros, la zanja misma puede constituir la armadura; y en represas y arcos de bloques de concreto, los primeros bloques, colocados alternativamente, sirven á menudo como partes de las armaduras para los bloques restantes; pero de ordinario se requiere considerable cantidad de armazón de madera. Véase § 56.

**60.** La economía de la obra depende en tan gran parte del **diseño** de las armaduras, que á menudo es conveniente modificar el diseño de la obra misma, ó usar más concreto que el que de otro modo sería necesario, para conseguir economía en aquéllas. El diseño debe ser tal, que puedan usarse los tamaños corrientes de madera con un mínimo de desperdicio; y además que las armaduras puedan fácilmente levantarse y quitarse con el mínimo daño para ellas y ninguno para la obra, y usarse varias veces. Cuando sea posible, las armaduras se hacen en secciones, suficientemente pequeñas para ser movidas con comodidad y manejadas separadamente. El corte se hace económicamente con máquinas de aserrar.

**61.** Aun en trabajo de edificación, donde muchas veces hay que construirlas en su lugar, y donde sólo pueden mudarse quitándolas por partes, la madera puede usarse dos ó tres veces antes de ser desechada. Cuando las armaduras pueden reunirse en **secciones**, y éstas mudarse en un solo cuerpo pueden usarse muchas veces.

**62.** Las exigencias de diferentes obras, ejecutadas bajo diferentes condiciones, varían tanto, que, dentro de los límites de que disponemos, no podemos dar pormenores útiles sobre la construcción de las tapialeras, moldes, armaduras, etc., excepto para edificios. Véanse §§ 63, etc.

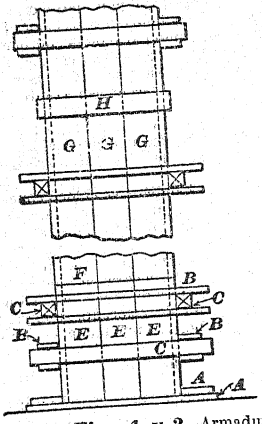
## Armaduras para edificios.

**63. En la construcción de edificios de cemento armado**, las armaduras son principalmente:

(*a*) Armaduras para columnas;
(*b*) Armaduras para vigas, planchas, pisos y techos;
(*c*) Armaduras para muros.

**64. Armadura** típica **para columna**, figs. 1 y 2. Las tablas, G, 3 cm de espesor, se sostienen por listones, H, de 3 12 cm y por abrazaderas, C, de piezas de 10 10 cm, y tablas, B, de 3 12 cm. Estas abrazaderas deben estar separadas para soportar la presion producida por el concreto. Al pie de una columna de 5.50 m de alto, deben estar a de 25 cm de centra a centro. En el fondo se ponen 4 tablas, A, para mantener la armadura vertical, y las tablas, G, se cortan, en un lado de la caja, en F, á 60 ó 90 cm del fondo, para formar una puerta los listones que cierran

la puerta no se ven) por la cual pueden barrerse todas las basuras. La puerta se mantiene entonces cerrada por las dos abrazaderas inferiores, y se llena la armadura. Usanse filetes triangulares, T, para hacer en bisel los ángulos de la columna.



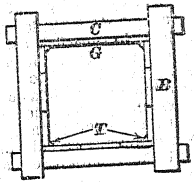




Fig. 2.

**Figs. 1 y 2.** Armadura para columna.

**65.** Las armaduras para columnas deben hacerse de manera que puedan quitarse sin perturbar las armaduras para las vigas. Así se pueden descubrir las armaduras de las columnas para inspeccionarlas antes de cargarlas.

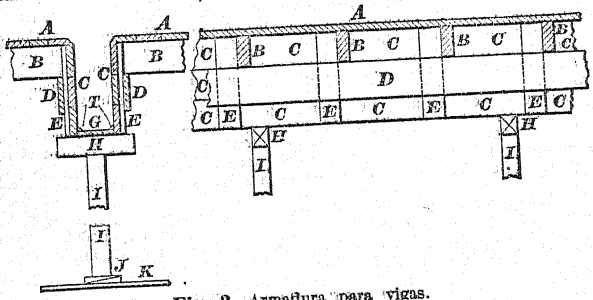



**Fig. 3.** Armadura para vigas.

**66. Armaduras típicas para vigas,** fig. 3. Las armaduras, ó cajas para vigas, se sostienen, entre columnas, por puntales temporales, I, de 10×10 cm, distantes entre sí como 1.80 m y descansando sobre cuñas, J, y tablas, K. Colócanse modillones, H, de 4×4 pulgadas, directamente debajo de los fondos, G (3 cm de grueso) y de los lados, C (3 cm de grueso) de las cajas de las vigas. Los lados, C, se mantienen unidos por listones de refuerzo, E de 3×12 cm y 60 cm distantes entre sí, á los cuales están clavados los listones, D (3×15 cm), sobre los cuales descansan las traviesas, B, de 5×15 cm, y más ó menos separadas 70 cm. Éstas sostienen la tablazón, A, de 3 cm de grueso, y ésta, á su vez, sostiene las planchas. Pequeños filetes triangulares, T, en los ángulos de las cajas de viga, cierran las rendijas de la caja y forman ángulos biselados. A las armaduras de viga debe dárseles una ligera curvatura.

**67. Las armaduras** típicas **para pisos** entre vigas de acero, figs. 4 á 6, varían con la luz y la carga. Las armaduras se cuelgan del reborde de la base de las vigas en I, por « pernos colgantes », A, figs. 4 y 6, de 15 mm de diám, con arandelas y tuercas. Estos pernos aseguran las piezas, E, de 5 × 10 ó 7½ × 10 cm, sobre las cuales descansan las tablas, HHH, sostenidas por traviesas de 5 × 15 ó 5 × 20 cm, D

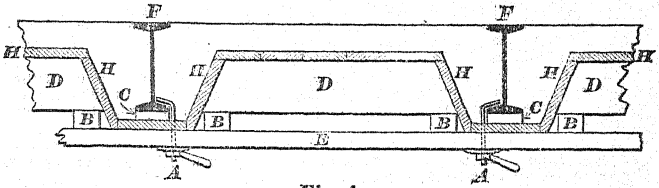



Fig 4.

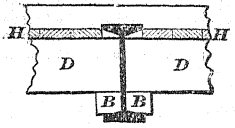



Fig 5.

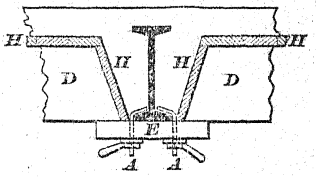



Fig 6.

**Figs. 4, 5 y 6.** Armaduras para pisos.

(colocadas como á 70 cm de centro á centro, para tablas de 2 cm). A veces se colocan bloques ó cuartones de madera, B, figs. 4 y 5, debajo de las traviesas para reducir su altura. Usanse bloques cortos de concreto, C, fig. 4, para mantener las armaduras separadas del reborde inferior de la viga de acero. Éstos quedan permanentemente en la obra. Para fomentar la adhesión entre los rebordes inferiores de las vigas en I y la masa delgada de concreto debajo de ellas, envuélvense á menudo los rebordes con tiras metálicas antes de colocar los bloques, etc.

**68. Las armaduras para muros** se hacen de ordinario en secciones, para que puedan usarse varias veces. Las secciones van ligadas entre sí por listones de refuerzo y de ordinario son de unos .90 × 3.60 m. Las secciones se mantienen separadas á la distancia conveniente por separadores de madera ó concreto, y se mantienen en posición por pernos ó ligaduras de alambre. Cuando se usan separadores de madera, deben retirarse precisamente antes de la colocación del concreto. A veces se usan separadores de bloques ó tubos de concreto. Éstos quedan en el muro. Cuando se emplean pernos que han de retirarse más tarde y usarse otra vez, deben aflojarse por medio de una llave inglesa, como 24 horas antes de colocar el concreto; de otro modo será difícil retirarlos.

**69.** En el sistema **Wiederholdt** de construcción de muros de cemento armado, se deposita el concreto dentro de bloques huecos hechos con pequeñas baldosas que van á formar el paramento exterior, y no se usan armaduras de madera ú otras temporales. Estas baldosas tienen la forma requerida para la obra. La colocación del concreto y la de las baldosas se hace simultáneamente.

**70.** Para reducir el costo de las armaduras en la construcción de edificios de cemento armado, pueden **modelarse en el suelo** las columnas, vigas, planchas, etc., y colocarlas luego como se desee; sacrificando, sin embargo, la rigidez producida por el carácter monolítico de las obras ordinarias de cemento armado.

**71. Armaduras de metal.** Cuando la estructura consta de secciones transversales pequeñas y uniformes, que permitan el uso repetido de las mismas armaduras, como en albañales, canales, túneles, etc., las cubiertas de las armaduras de madera pueden ser de láminas metálicas. En los túneles y obras semejantes, de considerable extensión, y en obras ornamentales pequeñas, pueden usarse armaduras compuestas enteramente de metal.

**72.** Ha de evitarse tanto el **alineamiento** descuidado como el demasiado

esmerado. El Sr. W. J. Douglas (E. N. '06 /Dic. /20, pág. 646) insinúa la concesión de 1 cm de discrepancia de las líneas establecidas en obras « acabadas », de 5 cm en las otras.

**73. Evítense detalles delicados** y los ángulos agudos. Los ángulos deben redondearse ó biselarse, para facilitar el curso del concreto y el retiro de las armaduras, y para que los ángulos estén menos expuestos á daños subsiguientes.

**74. Las armaduras de madera**, dentro de las cuales ha de ponerse el concreto, deben ser bastante impermeables, lisas y de suficiente fuerza y rigidez para conservar sus líneas bajo la presión del concreto fresco.

**75.** Las armaduras se hacen por lo común de madera de tamaños corrientes, cubiertas con tablas ó planchas acepilladas. La abertura de las juntas entre las planchas puede evitarse parcialmente empleando tablas ensambladas (machihembradas).

**76.** El mortero, que se exuda por las aberturas, deja huecos en la superficie. En consecuencia, en armaduras para obras de paramento, las **juntas deben hacerse impermeables**, si fuese necesario, con mortero, masilla, yeso, papel de forro ó metal delgado.

**77.** Si la madera está muy seca, su hinchazón, debída á la humedad, puede torcer las tablas y producir una obra fea. En tales casos, las tablas no deben ser ensambladas, sino que deben tener los bordes ligeramente biselados y los ángulos agudos de los bordes de las tablas adyacentes deben ponerse en contacto. La hinchazón aplastará los bordes antes que torcer la tabla.

## Madera para armaduras.

**78.** El pino blanco es el mejor para paramentos delicados y muy útil en construcciones ornamentales cuando se hacen con armaduras de madera.

**79.** El abeto, el pinabete, el pino de Noruega y las clases más blandas de pino del Sur son más susceptibles de cimbrarse que el pino blanco, pero son generalmente más rígidas y por tanto mejores para refuerzos y puntales.

**80.** La madera parcialmente seca es generalmente la mejor. La madera secada al horno es inapropiada, pues se hincha al contacto con el concreto húmedo. Con madera muy verde, especialmente el pino del Sur, pueden abrirse las juntas. La madera verde es pesada y no sostiene bien los clavos.

**81.** Para armaduras de tablero para muros, la madera machihembrada ó de bordes biselados es preferible á la de bordes cuadrados. La machihembrada da una superficie más lisa y menos aberturas de las juntas que la de bordes cuadrados ó biselados, pero es más costosa, á causa del desperdicio al labrarla, y hay más desgaste en las juntas si las armaduras se usan con frecuencia.

**82.** Aun para armaduras ordinarias, el acepillado por un lado puede ahorrar dinero reduciendo el costo de limpieza después del uso. Los puntales deben estar siempre acepillados por un lado para ajustarlos á las medidas.

**83. Espesor.** Para muros ordinarios, 4 cm; para construcciones pesadas, en que se emplean grúas, 5 cm. Para secciones de pisos son muy usadas las tablas de 2.5 cm; pero, en edificios altos, se deterioran mucho y producen un « mal acabado » en las caras inferiores de los pisos. Para lados de vigas bastan 2.5 á 4 cm, pero para fondos son mejor de 5 cm. Las armaduras para columnas son generalmente de tablones de 5 cm.

**84.** Los parales son generalmente de $7\times10$ á $10\times15$ cm; $10\times10$ es el tamaño más útil. La distancia es de ordinario 60 cm para tablas de 2.5 cm, 1.20 m para 4 cm (espesor), 1.50 m para 5 cm.

**85.** Puesto que las vigas y las columnas soportan fuerzas mayores que las planchas de piso, sus armaduras deben dejarse en su lugar mayor tiempo y por tanto deben ser independientes de las armaduras del pavimento, etc.

**86.** Los lados de las armaduras de las vigas deben calzarse entre sí para impedir que se separen de las tablas del fondo, bajo la presión del concreto.

**87.** Las cuñas de madera dura, en los topes y las bases de los puntales, facilitan la colocación y la remoción de aquéllos y las pruebas de flexión.

**88. Las piezas livianas** (digamos de $5\times20$ ó $5\times25$), con frecuentes puntales, son preferibles á tamaños más pesados, difíciles de manejar.

## Fuerza de las armaduras.

**89. La fuerza** requerida para las armaduras puede calcularse, cuando se usa concreto húmedo, suponiendo la presión del concreto igual á la de un líquido que

pese unos 2,400 kg por metro cúb *. Si se usa concreto seco y apisonado fuertemente, la acuñada de la piedra con el pisón aumentará considerablemente la presión.

**90. Cargas permitidas** en kg, sobre puntales de madera para construcción de pisos.

| Longitud sin soportes, metros. | Sección trasversal de puntales, centímetros. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 7.5×10=75 | | 10×10=100 | | 15×15=225 | | 20×20=400 | |
| | Por cm cuad. | Total. | Por cm cuad. | Total. | Por cm cuad. | Total. | Por cm cuad. | Total. |
| | kg. | kg. | kg. | kg. | kg. | kg. | kg. | kg. |
| 4 | | | 49 | 4900 | 63 | 14175 | 77 | 30800 |
| 3.50 | 42 | 3150 | 56 | 5600 | 70 | 15750 | 84 | 33600 |
| 3 | 49 | 3675 | 63 | 6300 | 77 | 17325 | 84 | 33600 |
| 2.50 | 59.5 | 4462 | 735 | 7350 | 84 | 18900 | 84 | 33600 |
| 1.80 | 70 | 5250 | 84 | 8400 | 84 | 18900 | 84 | 33600 |

**91.** En vigas de madera, calculadas para resistencia, la **resistencia de la fibra extrema** ha de calcularse en 52.5 kg por cm cuad.

**92. La carga viva durante la construcción**, mientras el concreto fragua, es de 366 kg por m cuad en las planchas, 244 por m cuad al calcular las vigas para las armaduras. Esto incluye el peso de los hombres, carretillas llenas de concreto y material de construcción depositado en el piso, pero no montones de cemento, arena ó piedra, que no se deben permitir á menos que se tenga en cuenta su peso.

**93. Las armaduras para pisos** deben basarse en la flexión **permitida** más que en la resistencia. Fórmula :

$$d = \frac{3WL^3}{384\,EI}; \qquad I = \frac{bh^3}{12}$$

en que

$d$=flexión;
W=carga total sobre tablazón ó vigas;
L=distancia entre soportes;
E=módulo elástico de la madera empleada;
I=momento de inercia de la sección transversal de la tablazón ó viguetas;
$b$=ancho de la tablazón ó vigueta;
$h$=altura de la tablazón ó vigueta.

En la fórmula usual para flexión (véase pág. 503) $^1/_{384}$ es el coeficiente para viga con extremos fijos, mientras $^5/_{384}$ es el coeficiente para extremos apoyados solamente.

Peso del concreto, inclusive refuerzo, 2,400 kg por m cúb.

(Sanford E. Thompson, Boletín 13 de la Asociación de Fabricantes de cemento.)

## Detalles de las armaduras.

**94.** Demasiados **clavos** aumentan la dificultad de desarmar las armaduras sin dañarlas. Los clavos punta de París pueden sacarse con menos daño para la madera que los clavos angulares.

**95.** Los tirantes de hierro ó acero, que se extienden al través del muro y mantienen las armaduras en posición, de ordinario se vuelven á usar si tienen $>$ 6 mm de diámetro. Si $\not>$ 6 mm de diámetro, se dejan generalmente; pero, si sus extremos llegan á la superficie exterior del muro, producen manchas feas de oxidación. Para impedir esto, se rebaja el concreto que rodea sus extremos y se recortan las puntas de las varillas. Los huecos que quedan se llenan después con mortero.

* El Sr. W. J. Douglas (E. N., '06/dic /20, pág. 646), considera al concreto como un líquido de la mitad de su propio peso, de 1,200 kg por m cúb.

**96.** A veces se emplean **separadores** (patentados por Wm. T. McCarthy, 1 Madison Ave., ciudad de Nueva York) hechos de cemento, en forma de cilindros huecos y de 10 á 15 cm de largo que rodean los pernos. Después de separado el perno, se llena con mortero el hueco del cilindro.

**97.** Las armaduras pueden sufrir por los golpes del balde de concreto, ó por el movimiento de las máquinas en contacto con ellas.

**98.** Todo concreto que se adhiera á una armadura debe quitarse antes de que ésta vuelva á usarse.

### Adhesión á las armaduras.

**99.** Si la madera es nueva, y si las armaduras se mojan perfectamente antes de colocar el concreto, éste, si está duro, no se adhiere á las armaduras cuando se retiran. Si han de retirarse las armaduras antes de que el concreto endurezca, deben, antes de colocarse el concreto, ser engrasadas con una substancia bastante delgada que corra y penetre en la fibra de la madera; se emplea el aceite crudo, el aceite de linaza, el jabón blando y otros lubricantes.

**100.** Una obra nueva puede adherirse á las maderas viejas, en que se ha adherido concreto anteriormente, aunque éste se haya limpiado.

**101.** El aceite, aplicado á las armaduras (para impedir su absorción de agua ó para facilitar su retiro, § 99), puede penetrar en las juntas entre la obra vieja y la nueva é impedir la formación de una adherencia satisfactoria. El **jabón** es por supuesto inocuo en este caso.

### Retiro de las armaduras.

**102.** La **prematura** remoción de las armaduras y los puntales ha causado **muchos fracasos** en edificios de concreto; pero el indebido retardo en su retiro significa retardo en la obra y aumento del número de armaduras.

**103.** La ley francesa requiere que los **bloques de prueba** y vigas de ensayo sean hechos para cada sección que se construye. Éstos permiten al ingeniero juzgar las condiciones de la obra.

**104.** Los **puntales** deben retirarse de **una sola viga** ó pieza á la vez y deben reemplazarse inmediatamente después que se han retirado las armaduras para esa viga. Esto permite descubrir y reparar los defectos.

**105.** Las **armaduras pueden retirarse más temprano** en tiempo cálido y seco que en tiempo frío y húmedo; más temprano cuando las cargas son ligeras que cuando son pesadas; con cemento de fragua rápida que con cemento de fragua lenta; con mezclas secas que con mezclas húmedas. (Véase Especificaciones, pág. 1409.)

**106.** Para aliviar **las cajas de vigas**, pueden apoyarse los postes sobre cuñas y ponerles una tabla arriba. Los postes y sus capiteles no se deben retirar de más de una viga á la vez. Después que se havan retirado las cajas de las vigas, deben reemplazarse los postes y las tablas antes de retirarse las armaduras de cualesquiera otras vigas; ó los postes pueden estar apoyados sólidamente y con una plancha de apoyo que forme el fondo y sostenga las tablas laterales de las cajas de la viga. Las tablas laterales pueden retirarse entonces, dejando sin mover los postes y planchas de apoyo.

**107. Los esfuerzos que obren como palancas contra el concreto**, al retirar las armaduras, pueden dañarlo.

### Juntas en el concreto.

**108. Dificultades.** En las obras grandes, las juntas, entre obras hechas en diferentes días, y aun antes y después de una hora de intervalo, pueden presentar inconvenientes, sobre todo donde se requiere impermeabilidad.

**109. Causas.** La dificultad parece deberse en parte á una nata ó película superficial del concreto endurecido, y en parte á la presencia de materiales aceitosos ó eflorescencias y suciedades, entre las dos superficies. El aceite empleado en las armaduras, ó contenido en la ropa de los trabajadores, puede llegar hasta las juntas. El aserrín es particularmente difícil de quitar. La unión es sobre todo débil si la superficie más vieja está helada.

**110. Remedios.** Muchos remedios se han propuesto, anunciado y usado, pero ninguno ha sido bien probado. Véase Especificaciones, pág. 1408. Los mejores preventivos son, probablemente, la limpieza de la superficie y el uso de mezclas húmedas. El agua, usada en lavar las juntas, debe ser limpia. Un chorro de vapor vivo de alta presión es muy eficaz, pues quita hasta el aserrín. Usase con éxito el

ácido hidroclórico. Los métodos patentados para asegurar la unión de las juntas, incluyen el uso de ligazones metálicas, saliendo sus extremos de la superficie más vieja, para trabar con la nueva. En otro método se emplea una capa de escoria esparcida sobre la superficie más vieja, cuando está aún blanda, quitándose la escoria suelta después del endurecimiento de la superficie más vieja, antes de colocar el material nuevo.

**111.** Cuando el concreto se usa para **reforzar** y proteger **una mampostería vieja**, debe quitarse una piedra aquí y allá de la mampostería vieja, y limpiarse y lavarse las juntas. Pueden también introducirse pernos grandes con grandes arandelas en las cabezas para que entren en el nuevo concreto. El concreto debe también penetrar bastante hacia el interior del muro para impedir que el agua entre en las juntas horizontales por encima de los muros laterales y de los muros principales.

## Apisonado.

**112.** El **apisonado** del concreto es sólo necesario en las mezclas relativamente secas. Cuando se hace bien, consolida la masa, cosa de 5 ó 6%, haciéndola menos porosa y considerablemente más fuerte. Para apisonadores, véase Especificaciones, pág. 1407. Los hombres que los usan, si están sobre el concreto, deben llevar zapatos de caucho.

**113. Debajo del agua** el apisonado sólo puede hacerse parcialmente y cuando el concreto está contenido en sacos. Puede usarse un rastrillo con suavidad para nivelar el concreto suelto depositado debajo del agua.

**114.** El apisonado debe suspenderse antes de empezar la fragua. El apisonado excesivo destruye la homogeneidad del concreto.

## Colocación debajo del agua.

**115. El concreto puede depositarse fácilmente debajo del agua.** Comúnmente se baja, poco después que se mezcla, en un balde de dragar ó en una caja de madera ó de palastro en forma de V, con una tapa que pueda cerrarse mientras desciende la caja. La tapa se omite á menudo. Esta caja está dispuesta de tal manera, que, al llegar al fondo, pueda halarse un perno por una cuerda que llega á la superficie, permitiendo así que uno de los lados inclinados se abra y deje caer el concreto. Entonces se eleva la caja para llenarse otra vez. En obras grandes la caja puede contener un metro cúbico ó más y debe estar suspendida de una grúa movible, por la cual pueda llevarse á cualquier punto de la obra. El concreto puede nivelarse suavemente, si fuere necesario, con un rastrillo, poco después que sale de la caja. Su consistencia y su fuerza desminuirán naturalmente si se sale de la caja y cae al través del agua; y además, no puede apisonarse debajo del agua sin dañarlo aún más. De la manera arriba expresada se ha depositado en profundidades de 15 m.

**116.** A veces se usa para depositar concreto debajo del agua una caja de madera ó de palastro, redonda ó cuadrada, abierta arriba y en el fondo, y de un largo apropiado á la profundidad del agua. Puede tener unos 45 cm de diámetro. Su parte superior, que siempre se mantiene sobre el nivel del agua, es en forma de tolva, para recibir más fácilmente el concreto. Se mueve lateral y verticalmente por una grúa ú otro aparejo apropiado. Al comenzar las operaciones, se apoya su extremo inferior en el fondo del río, llénase primero enteramente de concreto, el cual (para impedir que sea arrastrado en pedazos cayendo al través del agua en la caja) se baja en una cuba cilíndrica, con un fondo semejante á la caja descrita en el § 115, que puede abrirse cuando llega á lugar conveniente. Llena, la caja se mantiene así con el nuevo concreto, que se vierte en la tolva para ocupar el lugar del que gradualmente cae abajo, pues la caja se levanta siempre un poco para que el concreto vaya saliendo. El peso de la caja llena va compactando el concreto á medida que se deposita. Es mejor que la caja sea más ancha abajo para que facilite la salida del concreto.

**117.** El área sobre la cual se va á depositar el concreto debe rodearse previamente por alguna especie de valla, para impedir que el concreto se esparza fuera de sus límites, y para servir de molde á la forma deseada. Consiste generalmente en una caja de madera ó de palastro sin fondo que queda en la obra. Si es de madera puede requerir una fila de cajones adyacentes, exteriores, que se llenan de piedra ó cascajo para sumergirla. Debe evitarse el escape del concreto por espacios abiertos debajo de los lados de la caja ó valla. Para esto puede hacerse la caja de modo que se adapte á las desigualdades del fondo, cuando éste no pueda fácilmente nivelarse. A veces es mejor el empleo de láminas de palastro al interior, ó fijar afuera ó adentro una faja de encerado, en todo el contorno del borde inferior de la caja, pisándola a

con piedra ó cascajo para mantenerla en su sitio sobre el fondo. Piedra rota ó cascajo y aun tierra (las dos últimas donde no hay corriente) amontonados fuera de una caja débil, impedirán que se combe hacia afuera por la presión del concreto. Después que se haya subido el concreto hasta algunos pies del nivel más bajo del agua y ya nivelado, podrá comenzarse la mampostería sobre él por medio de un cajón (*caisson*) fijado ó empleando hombres con escafandras. O, si el concreto llega muy cerca del nivel menor del agua, puede echarse una primera camada gruesa de piedra, y ponerse así la obra desde luego sobre dicho nivel sin necesidad de aquellos recursos.

**118. El concreto debe extenderse** de .60 á 1.50 m (según el caso) fuera de la base de mampostería. Todo el lodo debe quitarse antes de depositar el concreto.

**119.** En algunos casos se usan **sacos llenos en parte de concreto** y simplemente arrojados en el agua. Si el tejido de los sacos es ligeramente abierto, parte de la pasta de cemento se sale y une el todo en una masa tolerablemente compacta. Tales sacos se emplean, con la ayuda de buzos, para coger filtraciones, apuntalar cimientos, y varias otras obras. Los sacos de cemento pueden apisonarse hasta cierto punto.

**120. Puede colocarse encerado sobre las grietas de la roca** para impedir la pérdida del concreto; y, en algunos casos, para impedir que sea arrastrado por la corriente.

**121.** El concreto debe colocarse en agua, en **grandes lotes**, á fin de que la proporción entre la superficie expuesta y el volumen sea pequeña. En agua corriente, desvíese el curso con tubos ó canales ó por medio de diques (para lo cual es apropiado el concreto en sacos). Si se extrae agua del pozo mientras se pone concreto, puede ella llevarse el cemento. Obsérvese el agua que sale de la bomba para saber si hay pérdida de cemento.

**122.** Los cimientos de diques de concreto sobre roca de 4 á 6 m debajo del menor nivel del agua, y si están cubiertos de lodo, se colocan con la ayuda de buzos, quitando el lodo por medio de chorros. No se nivela la roca. Se construyen las armaduras sobre la roca y se llenan los vacíos que queden debajo de aquéllas con sacos de concreto. Las armaduras se mantienen debajo del agua por medio de cajas llenas de piedra, y se anclan con cables de alambre cerca del fondo, en pilotes cercanos y reforzados arriba, con crucetas clavadas al dique existente. El concreto se baja por grúa, en baldes como de ½ m cúb que se descargan por el fondo cuando está cerca de la obra. El único cemento perdido es la poca cantidad que barre el agua en la parte superior del balde cuando se sumerge. La obra debe presentar caras lisas á lo largo de las armaduras y aparecer perfectamente homogénea. (E. R., '05/oct./21 pág. 468.)

**123.** Colocación de concreto, **bajo 27 m de agua**, en un pozo. Para detener la entrada del agua en el fondo de un pozo, se lleva el concreto, por una tolva, á un tubo de hierro forjado de 20 cm con uniones de tornillo, cuyo extremo inferior está cerrado con un tapón de madera y descansa en el fondo del pozo. Cuando el tubo se levantaba ligeramente, el tapón no salía y no descargaba el concreto. Retirado el tubo y desarmado, se vació cada sección. El tapón, que no estaba apretado, había dejado que la sección más baja se llenase de agua, y se desintegraba el concreto, dejando, en el tope de la sección más baja, un tapón de cemento puro que impedía, como se deseaba, al concreto de arriba expulsar el tapón de madera. Se repitió el ensayo con un tapón apretado. Dentro del tubo de 20 cm se colocó un tubo de 4 cm, por medio del cual se expulsaba el tapón de madera, dejando bajar el concreto. Se regularizó la descarga cambiando la distancia del pie del tubo sobre el fondo del pozo y se depositó una masa de concreto de 3 á 4 m de espesor. Los 15 ó 20 cm superiores nunca fraguaron; pero el resto apareció sólido y homogéneo. (Assn. C. E., Cornell Univ., Trans., 1898, pág. 74.)

**124.** En un caso en que los pilotes de hierro huecos, en fondo arenoso limpio, se llenaron de concreto, parte del mortero se salió y formó, con la arena circundante, masas de concreto que se adhirieron tenacísimamente á los pilotes, sugiriendo el empleo de **pilotes huecos perforados** *ad hoc*, en sus partes inferiores, con huecos pequeños, por los cuales se escapa hacia la arena el mortero delgado vertido por arriba. (Chas. List, Jour. Assn. Engg. Socs., marzo, 1903, vol. 30, n.° 3, pág. 124.)

**125.** *Superior Entry, Wis.* El mezclador se vacía en una tolva inferior provista de una canal de compuerta que á su vez descarga en los baldes que llevan el concreto y que corren en carriles situados debajo de la plataforma. Al llegar á la obra, los bajan á las armaduras sumergidas unas grúas movibles. Cada balde está provisto de dos cubiertas de lona, divididas en dos piezas, forradas en plomo y fijadas á los lados opuestos del balde. Cuando están en su posición, estas piezas se montan unas sobre otras en el medio de los baldes, cubriendo completamente el concreto. Cuando

el balde llega al fondo, se voltea por una aldaba especial provista de una cuerda que tiene el encargado de la grúa. Las cortinas de lona impiden que el agua arrastre el concreto. Un balde cargado pesa como 6 toneladas. El golpe del balde cargado sobre el concreto ya colocado parece compactarlo suficientemente. La descoloración del agua por el cemento, durante el descenso del balde cargado, se nota poco. (Informe del Ingeniero en jefe. E. U. de A., 1904, parte IV, pág. 3785.)

## ACABADO DE LAS SUPERFICIES

**126.** Después de retiradas las armaduras de madera, la superficie de concreto muestra las marcas de la fibra, nudos y juntas de las cubiertas. Esta apariencia puede ser ó no objetable.

**127.** El enlucido con mortero de cemento da un buen acabado en el interior de los edificios, donde la lluvia y la helada no pueden afectarlo, pero ordinariamente se desconcha cuando se aplica á superficies exteriores.

**128.** Las superficies exteriores **pueden ser lavadas con una mezcla delgada de cemento,** después de rellenar las juntas, donde sea necesario, con mortero de cemento. Esto debe hacerse mientras el concreto está nuevo, y, si fuere posible, inmediatamente después de retiradas las armaduras. Una mezcla delgada, compuesta de 1 parte de yeso y 3 partes de cemento, aplicada con brochas de lechada, da resultados satisfactorios.

**129.** Las superficies de concreto pueden ser labradas con un hacha dentada, dando una variedad de efectos. Si se pican cuando el concreto está algo blando, queda una superficie áspera, que muestra la piedra y luce como una obra ordinaria. Si la herramienta no es cortante, daña la superficie. Hanse usado eficazmente las herramientas de aire comprimido y la máquina de lanzar arena se ha empleado con éxito; las primeras en el Stadium de Harvard.

**130. Revestimientos de mortero especialmente preparado,** se colocan á veces al mismo tiempo que el cuerpo del concreto por medio de una lámina metálica puesta de canto. Esta lámina separa el revestimiento del relleno; y cuando el revestimiento y el relleno llegan á cierto nivel, se quita la lámina y vuelve á usarse en la capa que sigue. Después de quitada la armadura se unen el revestimiento semifluido y el relleno en el angosto espacio que ocupó la tapa.

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**131.** El revestimiento no debe ser más de 1 : 3, salvo para obras de ornamentación, para superficies sencillas, 1 : 4. Un revestimiento demasiado rico y excesiva pulitura tiende á cuartearse y es costoso. En el ferrocarril de Chicago Milwakee y San Pablo, en Chicago, el cemento empleado para poner un revestimiento de 3½ cm fué un mortero de 1 parte de cemento Portland para 2 de arena; se gastó en cemento el 9% del usado en toda la obra.

**132.** « En el caso de muros delgados, á veces no se puede concluir rápidamente la obra por la imposibilidad de hacer el revestimiento bastante ligero, y en todo caso se necesitan dos ó más hombres para mezclar y llevar mortero y atender á su colocación. » (W. A. Rogers, R. R. Gaz, '00 /julio /6, pág. 461.)

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**133.** Para **proteger el enlucido de la helada** puede colocarse una capa de papel alquitranado por fuera de los parales dejando un espacio de aire, del espesor de los parales, entre el papel y las cubiertas de la armadura. En este espacio la temperatura será de 4° á 5° mayor que la del aire exterior. Esta protección es por supuesto más necesaria en los lados expuestos al viento. (W. J. Douglas, E. N., '06 /dic. /20, pág. 650.)

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**134.** El cambio de operarios, mientras se ejecuta el trabajo del acabado, puede dar por resultado la pérdida de la uniformidad en el aspecto.

**135. Fregado antes de que fragüe el concreto.** El Sr. H. H. Quimby (Asociación de consumidores de cemento, Procs., 1907) friega la superficie de concreto fresco, antes que se endurezca, con un cepillo y agua, removiendo así la última película, y, con ella, toda impresión que hayan dejado las armaduras descubriendo y limpiando la piedra y la arena del concreto. Unos pases dados con una escoba ordinaria de las usadas en las casas, y una corriente de agua para lavar y enjuagar, bastan; pero una ligera friega adicional mejora el efecto. La necesidad de retirar ligero las armaduras, cuando se emplea este método, exige un esmero

especial para construirlas, aumentando su costo. Cuando se aplica á superficies que forman ángulos, las partículas de arena proyectadas le dan un aspecto áspero. Por esto debe cuidarse de no extender el sistema á aquellas esquinas.

**136.** Un efecto semejante al obtenido por el método del Sr. Quimby puede producirse, después de la fragua, **lavando con una solución ácida**, que se quita luego con un lavado alcalino, seguido de otro con agua. Este método ataca la piedra caliza del agregado.

**137. Los efectos de color** se producen del mejor modo usando agregado del color deseado.

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**138.** La dificultad para **hacer adherir la pintura al óleo** á las superficies de concreto fresco se debe á la humedad y á la cal libre. Un lavado de ácido diluído en agua neutraliza la cal, pero no es satisfactorio, pues el ácido muriático (hidroclórico) forma sales altamente higroscópicas, tales como cloruro de calcio, y el ácido sulfúrico sólo produce un efecto superficial. Disuélvanse 4.5 kg de carbonato de amoníaco (sales de cuerno-ciervo) en 170 lit de agua y apliquese una vez con una brocha, ó dense varias manos de una solución más débil, ó apliquese con regadera. El amoníaco se desprende y el ácido carbónico forma, con la cal libre, un carbonato insoluble que pronto se seca y endurece. Después de muchas pruebas se encontró que este método es el único que satisface todas las exigencias. El carbonato de amoníaco se conserva, indefinidamente, en vasijas bien tapadas. (Fred. J. Bosse. « Edad del Cemento », '09 /enero, pág. 48.)

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## PROPIEDADES DEL CONCRETO

**Peso.** Véase Vacíos, pág. 1298, y Densidad, pág. 1299.

**1. Pesos del concreto en kg por metro cúbico.**

**Concreto de piedra quebrada** ó de cascajo, 2,080 á 2,560; ordinariamente de 2,240 á 2,400.

Un pie BM = vol de un sólido de 1 pie en cuadro y 1 pulgada de espesor, = 144 pulgadas cúb = 1 pie cúb /12 = 2.358 litros.

| Peso, lbs /pies cúb.... | 100 | 110 | 120 | 125 | 130 | 140 | 150 | 160 |
|---|---|---|---|---|---|---|---|---|
| Kg /m cúb..... | 1,600 | 1,760 | 1,920 | 2,000 | 2,080 | 2,240 | 2,400 | 2,560 |

| | | |
|---|---|---|
| Concreto | de ceniza gruesa..... | 1,760 á 1,920 kg |
| — | de asperón..... | 2,288 kg |
| — | de piedra caliza..... | 2,368 — |
| — | de cascajo..... | 2,400 — |
| — | de trapa..... | 2,480 — |

Con cemento **natural** es de 64 á 80 kg más liviano por m cúb.

**2.** La unidad de peso varía no sólo con la naturaleza de los materiales, sino también con las proporciones, la consistencia, el grado de compactación, etc.

### Permeabilidad.

**3.** Aun cuando el principal objeto del concreto no es impedir la filtración por el agua, la impermeabilidad es de grande importancia para aumentar la durabilidad del concreto, y especialmente para proteger á la armadura metálica de la oxidación y de la pérdida de adhesión con el concreto.

**4.** El agua pasa á través del concreto, etc., tan lentamente, que la evaporación del exterior, más rápida, puede hacer que el muro aparezca **seco en el exterior** cuando aún se esté verificando la filtración en el interior.

**5.** Cuando se coloca en mortero endurecido y bien comprimido con la cuchara en todas las superficies que entren en contacto con agua, el **cemento puro** es tan **impermeable** como la mejor de las rocas naturales usadas para construcciones. Wm. B. Fuller, Trans., A. S. C. E., vol. 51, págs. 133-4, dic. 1903.)

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**6.** El mortero, ó concreto, proporcionado de modo que se obtenga **la mayor densidad posible**, y mezclado un tanto húmedo, es impermeable en condiciones ordinarias.

**7.** Se ha encontrado repetidas veces que pequeños bloques de concreto, hechos cuidadosamente de materiales de tal modo proporcionados que aseguren gran densidad, ó con un exceso de cemento, son tan impermeables como las mejores piedras naturales. Véase Experiencias y práctica, pág. 1349.

**8.** En **grandes masas**, es difícil producir una estructura absolutamente impermeable sin la adición de un revestimiento de material de mayor impermeabilidad que el concreto. Las variaciones en la mezcla, el descuido en la manipulación ó la colocación, ó en la unión de obras hechas en diversos días (la interrupción del trabajo de una hora, en el medio de un día caluroso), ha causado filtraciones. La insuficiencia de agua hará también permeable el concreto á pesar de la apropiada proporción teórica y la adición de cal. La mezcla debe ser por lo menos bastante húmeda para que se asiente con poco apisonado.

**9.** El concreto, impermeable en sí, puede presentar grietas por las cuales penetre el agua. Con refuerzos bien colocados se impiden las grietas.

**10.** El agua puede penetrar al través del mortero, de los elementos del agregado, ó entre el mortero y el agregado. Probablemente la mayor parte de las filtraciones se efectúan al través del mortero. Véase Mortero. Aquí tratamos de aquellos aspectos de la permeabilidad que pueden estudiarse mejor tratando el concreto como un material compuesto.

**11.** Cuando la filtración consiste en simple filtración al través de los **pequeños poros** del concreto, etc. (esto es : cuando no hay verdaderas grietas), disminuye generalmente con el tiempo, pues el agua (aunque aparentemente clara) obstruye su propio paso depositando en los poros del material, ya su propio sedimento natural, ya la cal y otros compuestos disueltos en el concreto mismo.

**12.** Esta acción depende de muchos factores, notablemente de la presión, de los tamaños y formas de los poros, de la dureza y solubilidad del material, y la naturaleza del sedimento arrastrado por el agua. Así, bajo alta presión, si el material es fácilmente lavable, ó si los poros son anchos y relativamente rectos, el derrame puede aumentar, en vez de disminuir, con el tiempo.

**13.** Cuando lo permite la naturaleza del caso, como en pisos, muros de sostenimiento, etc., es mejor desviar el agua por desagües apropiados que tratar de cerrarle el paso haciendo impermeable la estructura.

**14. Cuando se requiere impermeabilidad**, como en represas, deben proporcionarse cuidadosamente los materiales para el máximo de densidad; debe haber un exceso de mortero rico en cemento sobre el vol de los vacíos, y deben evitarse mezclas secas; la mezcla debe ser perfecta y la construcción, hasta donde sea posible, debe ser monolítica.

**15.** La aplicación de **materiales que producen** impermeabilidad puede ser ó (*a*) interna, mezclados con los ingredientes del concreto; (*b*) superficial, llenando los poros cerca de la superficie; (*c*) externa, impidiendo el contacto entre el agua y el concreto.

**16. Interna.** Para obra impermeable, el vol de mortero debe ser de 40 á 45% del vol de agregado, ó de 40 á 42% si el agregado se ha graduado. (Geo. W. Rafter, Trans., A. S. C. E., vol. 42, pág. 149, dic. 1899.)

**17.** Con agregado que tenga 35% de vacíos, el vol de mortero debe ser $<$ 50% del vol de agregado; el vol de arena seca y cemento $<$ ⅔ del vol de agregado; el vol de arena $>$ 2× vol de cem. Para cemento que deje $>$ 10% en tamiz n.º 120, arenas ordinarias, y agregado con 35% de vacíos, se dan las siguientes proporciones

| cemento | arena | agregado | (arena + agreg) ÷ cem. |
|---|---|---|---|
| 1 | 1.0 | 3.00 | 4.00 |
| 1 | 1.5 | 3.75 | 5.25 |
| 1 | 2.0 | 4.50 | 6.50 |

Véase Concreto común, § 22, pág. 1299.

**18.** Cada partícula de arena debe estar cubierta de cemento y cada partícula de piedra cubierta de mortero, de modo que las piedras ó los granos de arena no se toquen.

**19.** Para asegurar este resultado, hágase la mezcla con una de las máquinas más modernas, introduciendo primero la cantidad medida de agua y luego el cemento para hacer un mortero líquido que entre fácilmente en los vacíos más pequeños de la arena, y que después penetre en el menor tiempo posible. El mortero que resulta les todavía suficientemente líquido y entra en todos los vacíos de la piedra. (Wm. B. Fuller, Trans. A. S. C. E., vol. 51, pag. 135, dic. 1903.)

Para el uso de la cal, véase Experiencias, etc., 82 *a*, pág. 1392.

**20.** Para hacer planchas delgadas con un concreto de 2 partes de cemento para 5 de ceniza fina de carbón bituminoso reforzado con tejido de alambre, empleó el Sr. W. K. Hatt (Trans., A. S. C. E., vol. 51, pág. 120, dic. 1903) una solución de 5% de alumbre molido, en lugar de la mitad del agua indicada, y una solución de 7% de jabón, en lugar de la otra mitad. Esto fortaleció y endureció el concreto de cenizas en un 50%, y disminuyó su absorción en 50%. La sola **solución de jabón** disminuyó la absorción, pero no fortaleció el concreto. El mortero de arena no se fortaleció mucho por el tratamiento de jabón y alumbre, pero su absorción se disminuyó un 50%.

**21.** Si no se pueden evitar las juntas, pueden mojarse primero y luego cubrirse con pasta de cemento puro ó mortero de cemento 1 : 1 sobre el cual se coloca la nueva obra antes que se endurezca la pega.

**22.** La permeabilidad de los revestimientos de concreto de los acueductos, etc., puede disminuirse abriendo agujeros al través de ellos y metiendo á la fuerza mortero líquido detrás por medio de bombas adecuadas. El mortero líquido aparece á veces en muchos puntos, indicando que pasa no sólo por las grietas, sino también por la masa del concreto. Este método fué usado con éxito en el acueducto de agua filtrada de Torresdale, Filadelfia.

**23. Superficial.** Para revocar el interior de un aljibe cubierto de agua clara, usó el Sr. Edwd. Cunningham 0.56 kg de jabón blando para cada 5 baldes de agua y 1.35 kg de alumbre por saco de cemento. El mortero era fácil de manejar con la cuchara, pero tenía un olor nauseabundo. Se colocaron 2 capas de no más de 12 mm. Una pared divisoria de 45 cm no presentó ninguna filtración cuando á un lado contenía 5 m de agua. El jabón estaba hecho de grasas clarificadas, y costaba 16 cts. por kg; precio muy subido. Para 1 parte de cemento y 2 partes de arena, se necesitaron de 23 á 34 lit de agua y 6 kg de alumbre para cada barril de cemento. (Trans., A. S. C. E., vol. 51, págs. 127–8, dic. 1903.)

**24.** Como tratamiento **externo**, encontró el Sr. Richd. H. Gaines, Junta de Abasto de Agua de Nueva York (Trans., A. S. C. E., vol. 59, pág. 160, dic. 1907), « bastante eficaz, pero muy costoso en obras grandes », el procedimiento Silvester de jabón y alumbre (pág. 1266).

**25.** El **asfalto** puede aplicarse con éxito á superficies secas solamente. Hácese quebradizo y pierde su eficacia al oxidarse; pero con frecuencia impedirá derrames hasta que la obra se haya hecho impermeable á las infiltraciones. Véase §11, pág. 1315.

**26.** La superficie del concreto debe estar limpia y debe dársele primero una mano delgada de asfalto líquido, diluído con bencina. Éste entra en los poros del concreto y obra como una pega. Sin esto la capa de asfalto no se adherirá al concreto.

**27.** Las capas de asfalto deben hacerse continuas y protegerse del deterioro, de su tendencia á escurrirse y del desgaste colocándolas entre capas alternadas de concreto ó cubriéndolas con enladrillado ó mampostería.

**28.** Los túneles, subterráneos y cavas debajo del nivel del agua se han hecho enteramente impermeables con capas enterizas de papel grueso de techado bien impregnado de alquitrán ó asfalto y colocado entre los muros exteriores é interiores de concreto.

**29.** Los dos depósitos del acueducto de Queen Lane (Filadelfia), originariamente revestidos con concreto de cemento sobre una capa batida de arena arcillosa y que contienen 1,447 millones de litros de agua en 9 m de profundidad, fueron revestidos otra vez con asfalto de La Bermúdez en 1896–7. Al piso se le puso 5 cm de concreto de asfalto con una capa delgada, arriba, de asfalto líquido caliente; en los declives dos capas de asfalto líquido caliente con aspillera (cañamazo) entre ellas fijándose esta coleta arriba por medio de barras horizontales de hierro ó madera, á las cuales va arrollada é introducida en el pavimento de asfalto. Mientras se llevaba á cabo esta obra, se revestía de igual manera el estanque del sur del depósito da Roxborough (555 millones de litros y de 7.5 m de profundidad). En el estanque del norte, se empleó asfalto de Alcatraz (California), los declives y los lados fueron tratados con concreto de asfalto. Todos estos cuatro estanques han estado desde entonces en constante uso, sin filtración notable.

### Módulo de elasticidad, E. Véanse §§ 12 y 13, pág. 1321.

**30.** Cuando se somete el concreto á prueba de compresión su diagrama es generalmente una curva en toda su extensión; su módulo de elasticida d

$$E = \frac{\text{esfuerzo por unidad de superficie}}{\text{acortamiento por unidad de longitud}},$$

disminuye á medida que aumenta el esfuerzo.

## Resistencia.

**31.** Siendo el concreto frágil y de **tensión** débil, generalmente, y con razón, **no se aprecia** aquélla y se cuenta principalmente con su resistencia á la compresión; su resistencia á la tensión y el esfuerzo cortante se expresan como fracciones de su compresión.

**32. La resistencia á la compresión** se determina con preferencia experimentalmente por medio de cubos. La unidad de fuerza compresiva disminuye cuando la proporción largo/lado, aumenta, y, en muestras semejantes, cuando aumentan sus dimensiones.

**33. Prismas** de concreto, probados á la compresión por sus extremos, fallan de ordinario cortándose por planos oblicuos á los ejes de los prismas. Sobre estos planos oblicuos la unidad del esfuerzo cortante es como la mitad de la fuerza de ruptura por compresión.

**34.** La resistencia varía mucho con la naturaleza del concreto.

**35.** Para **cubos** de mezclas de cemento Portland de 30 cm que tengan de 6 á 18 volúmenes de (arena+agregado) para 1 volumen de cemento, deduce el Sr. **Edwin Thacher**, de los datos del Experimento 18 *a*, la fórmula de la línea recta (modificada por el traductor para el sistema métrico),

$$S = (M - NX) \ .0703$$

en que

S = á la fuerza máx de compresión en kg por cm cuad;
X = n.º de partes de arena para 1 parte de cemento;
M y N = valores como se expresan abajo:

| Edad = | 7 días | 1 mes | 3 meses | 6 meses |
|---|---|---|---|---|
| M = | 1,800 | 3,100 | 3,820 | 4,900 |
| N = | 200 | 350 | 460 | 600 |

El Sr. Thacher cree que para mezclas prácticas « la fuerza del concreto depende principalmente de la fuerza del mortero y no, en gran parte, de la cantidad de piedra ». En estos ensayos el volumen de piedra fué siempre dos veces el volumen de arena.

**36.** Pero pocos ensayos se han hecho para determinar la **resist á la tensión** del concreto. Generalmente se considera como de un décimo á un octavo de la de compresión, y **el esfuerzo cortante** como de 1.2 á 1.3 veces la resist á la tensión.

**37.** El prof. L. J. Johnson (Jour., Assn. Eng. Socs., vol. 38, n.º 6, pág. 310, junio, 1907) probó 25 vigas de cemento armado, .075 × .27 × 2.44 m, cargadas á 15 cm de cada apoyo; 19 de las vigas eran de 1 : 2 : 2¾ de escoria volcánica escamosa; 6 de 1 : 2,5 : 5. Todas las **vigas fallaron por resbalamiento, dislocación, de la armadura interior**; las vigas de 1 : 2 : 2¾, de 137 á 143 días de construídas, resistieron con éxito esfuerzos cortantes de 16.38 á 40.28 kg por cm cuad, término medio 33; y las vigas de 1 : 2.5 : 5, 34 á 53, término medio 44.

**38. En las vigas**, debido á la elevación del eje neutro, bajo carga, la **unidad máxima de la resistencia de la fibra**, ó módulo de ruptura, es como de 1.6 × la unidad de resistencia á la tensión.

## Fragua.

**39.** La fragua es, por supuesto, una función de la **pasta de cemento**. Véase Mortero. Aquí tratamos de la fragua en lo que se relaciona con el concreto como un cuerpo compuesto.

**40. Temperatura.** En tiempo cálido el concreto fragua mucho más ligero que en el frío, y la carga puede por tanto aplicarse más pronto; pero el tiempo requerido varía con la clase de construcción y del concreto.

**41.** Carga gradual. Cuando la carga es estática ó se aumenta gradualmente, el tiempo puede ser más corto que cuando se la aplica de súbito ó está sujeta á choque.

**42.** « Como regla general, los **estribos y pilas de puentes** de concreto hecho con cemento Portland deben dejarse fraguar por lo menos un mes antes de usarse, si se han construído durante tiempo cálido ordinario. Si se han construído durante el frío, su uso debe aplazarse, si fuese posible, hasta que comience el calor. » (W. A. Rogers, R. R. Gaz, '00/julio/27, pág. 514.)

**43. Se han colocado vigas** de planchas de acero sobre estribos de concreto de cemento Portland sin que hagan daño, 2 semanas después de terminados los estribos y en tiempo cálido; pero en otra obra de la misma naturaleza, acabada á principios de diciembre, se encontró que estaba poco sólida en su interior á principios del siguiente marzo.

## Efectos del calor y del frío.

**44.** La helada casi siempre daña el mortero ó concreto de cemento **natural** á tal extremo que hay que reemplazarlo con nuevo material.

**45.** Con concreto de cemento Portland la **helada suspende la fragua** y el endurecimiento del mortero mientras esté helado el material. La aparente pérdida de resistencia, en muestras heladas, puede obedecer simplemente á dicho retardo en la fragua.

**46.** Si bien la helada rara vez influye reduciendo de modo importante el máximo de resist del concreto de cemento Portland, **puede**, sin embargo, **producir graves resultados** dándole al concreto una dureza aparente, que ocasiona una prematura retirada de las armaduras ó la aplicación de cargas indebidas, que pueden producir la falla del concreto al deshelarse, si no ha fraguado lo suficiente antes de helarse.

**47.** Si poco después que se ha helado el mortero en todo el espesor de un muro, le da el sol sobre una de sus caras, lo suficiente para ablandar el mortero de esa cara, mientras que el mortero que está detrás permanece duro, es claro que el muro está expuesto á asentarse del lado calentado y por lo menos se inclinará hacia afuera si no se cae.

**48.** Si la helada se efectúa después que el cemento ha hecho su **fragua inicial**, hay poco peligro. No debe hacerse un muro delgado á $< 28°$ F (—2.2 C) en una temperatura ascendente, ni á $< 32°$ (0° C) en una descendente.

**49.** Cuando el concreto se ha helado antes de endurecerse en las aceras y muros, es posible que se cuartee **una costra delgada** y se desprenda. El acabado que queda granoso ó hecho á cuchara se astilla á veces al helarse.

## Protección.

**50. La protección contra la helada** es costosa é incierta. Por tanto, debe evitarse, hasta donde sea posible, el trabajo de concreto en tiempo de heladas.

**51. Abrigo y calefacción de la obra acabada.** Pueden usarse tiendas ó mamparas; pero los tinglados de madera son más eficaces.

**52.** El concreto debe cubrirse, tan pronto como se coloca, con lona, sacos de cemento vacíos ó papel alquitranado, ó con una gruesa capa de arena, paja, abono, serrín ú otros malos conductores del calor. La paja debe ser $< 30$ cm de espesor. El abono es lo mejor, pero mancha la obra. La lona, etc., debe mantenerse á una ó dos pulgadas distante del concreto, dejando un espacio de aire; ó pónganse dos telas.

**53. Calefacción de los materiales.** La piedra se caldea frecuentemente amontonándola sobre un tubo ú horno improvisado y prendiendo fuego dentro de él; ó sobre un serpentín provisto de numerosos agujeros pequeños, metiéndole vapor por dentro. El concreto debe usarse antes que el vapor se condense y congele. La arena se caldea sobre una estufa larga de palastro.

**54. Hágase bajar el punto de congelación del agua de mezcla** por la adición de substancias químicas.

**55.** La sal es el material más barato y más comúnmente usado. Ella baja el punto de congelación como $\frac{3}{4}°$ C por cada 1% de sal agregado al agua. Una solución de 10% (6 kg de sal por barril de cemento) baja el punto de congelación como 8° C y no daña la fuerza del concreto. Para 0° C disuélvase .45 kg de sal en 68 lit de agua; agréguense 3 onzas de sal por cada 3° bajo 0° C. (Jefe de Ingenieros, E. U. de A. Informe, 1895.) Mayores proporciones de sal parecen debilitar el concreto.

**56.** El cloruro de calcio en solución de 15%, baja el punto de congelación unos 11° C y no debilita el mortero. Absorbe rápidamente la humedad y es posible que, si se muele seco con la escoria del cemento Portland, aun hasta la cantidad de 0.5%, haría que el material absorbiese humedad. El cloruro se disuelve con extrema rapidez y puede agregarse al agua de la mezcla. (Prof. R. C. Carpenter, Cornell Univ., Sibley Jour. of Eng., jen 1905.)

**57.** La mayor parte de un montón de arena ó piedra puede estar en condición de usarse, aunque la superficie esté helada.

**58.** En invierno podemos **reducir las áreas** de las capas expuestas de la obra, colocando los tabiques divisorios interiores de las secciones del muro en construcción más cerca unas de otras. Un día de trabajo la llevará entonces á mayor altura y requerirá el uso de armaduras más fuertes.

**59.** Los morteros, colocados al aire libre, sufren más ó menos, **secándose** en vez de **fraguar**, cuando la temperatura excede de 18° á 23° C; pero si se mezclan sólo en pequeñas cantidades á la vez y se colocan pronto en la mampostería, de

piedra húmeda, de modo que estén abrigados del aire, el daño se reduce mucho. La arena y la piedra deben estar **húmedas**, no mojadas, en tiempo de calor, y puede usarse **un poco** más de agua en la pasta de cemento; también, si fuese posible, no sólo el mortero, mientras se mezcla, sino también la mampostería debe ponerse á la sombra.

## Expansión.

**60.** En los climas variables, los **cilindros de hierro colado llenos de concreto** son frecuentemente hendidos horizontalmente por expansiones y contracciones desiguales. En tales construcciones lo más seguro es considerar los cilindros como simples moldes del concreto y contar tan sólo con éste para soportar la carga.

**Para coeficientes de expansión**, véase Cemento armado, § 9, pág. 1321.

**61. Cuarteaduras y juntas.** En estribos ó alcantarillas de más de 18 m de largo, divídase el muro en secciones de unos 12 m y conclúyase una sección antes de empezar la otra. La contracción hará abrir la junta y así se evitarán grietas al través del cuerpo del muro. Pueden completarse sin parar, secciones cortas y evitarse así juntas horizontales. « Las pequeñas grietas, que sería difícil encontrar en una mampostería de piedra, se ven muy claramente en el concreto. » (W. A. Rogers, R. R. Gaz, '00/julio 6, pág. 461.)

**62. Efecto de altas temperaturas.** Durante la calcinación de los materiales para el cemento Portland, se expulsa el agua químicamente combinada. Cuando, en las mezclas, se devuelve esta agua al material, se efectúa el endurecimiento; pero al volver á aplicar temperaturas, suficientemente altas para expulsar otra vez el agua, invierte el proceso de endurecimiento y disgrega el material.

## Efectos químicos.

**63.** « La pérdida del agua de cristalización del concreto empieza probablemente á unos 260° C y se completa á unos 480° C; pero esto enfría las masas circundantes y aumenta así la resistencia del concreto al calor. » J. C.

**64. Rehidratación.** Probetas tenidas de 6 á 8 horas á la temperatura de 536° á 643° C (no en contacto con la llama) dejándolas enfriar, no presentaron prácticamente ninguna fuerza, pero una inmersión de 28 días en agua les devolvió la misma fuerza que tenían las probetas no calentadas.

**65. Resistencia al fuego.** En arena de cuarzo el coeficiente de expansión es dos veces el del feldespato, y la expansión en una dirección es el doble de la expansión en dirección perpendicular á ella.

**66.** En el incendio de Baltimore el daño producido al concreto expuesto á las llamas rara vez llegó á una profundidad mayor de 12 mm, aunque los ángulos salientes fueron en algunos lugares redondeados por las llamas con un radio de unos 5 cm.

**67. El agua de mar** aparentemente no afecta el concreto cuyas proporciones asegure una densidad máxima y que esté perfectamente mezclado. El daño ocasionado por el agua de mar que se dice tiene lugar á nivel del agua se debe probablemente, en gran parte, á las heladas. J. C.*

**68.** La acción destructora de la **electrolisis** sobre el concreto se debe aparentemente á condiciones anormales que rara vez ocurren en la práctica. J. C.

**69.** Los **ácidos** dañan el concreto fresco; al de primera calidad perfectamente endurecido, lo afectan sólo de modo apreciable los ácidos fuertes, que también dañan seriamente á otros materiales. J. C.

**70.** En la utilización de terrenos áridos, donde el suelo está muy cargado de **sales alcalinas**, sufren el concreto, la piedra, el ladrillo, el hierro y otros materiales bajo ciertas condiciones. Esta acción puede impedirse con el uso de una capa aisladora. J. C.

**71.** El concreto bien hecho y que tenga una superficie cuidadosamente acabada y endurecida, resiste á la acción del petróleo y de los **aceites** de máquina ordinarios. Los aceites que contienen ácidos grasos parece que lo dañan. J. C.

**72.** Los **gases** de ácido sulfuroso y de ácido sulfúrico, combinados con la humedad, corroen el concreto, especialmente si se calienta.

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* J. C. Informe de la Comisión Común, A. S. C. E., A. S. T. M., Am. Ry. Eng. y M. W. Assn., y Assn of Am Cem. Mfrs., enero '09.

### Pruebas del concreto en la obra.

**73. Las pruebas de concreto en la obra** pueden hacerse por análisis de una masa del interior del concreto, obtenido con un **taladro de penetración** *. Los huecos del taladro se llenan luego con mortero líquido**.

**74. La proporción del cemento y la arena**, en el mortero, se encuentra por medio de las cantidades que quedan sin disolver en ácido hidroclórico. Así se prueban separadamente la arena y el cemento, de las clases empleadas, y el mortero, tomado del interior. (Prof. R. L. Wales, en E. N., '08/enero 9, pág. 46.)

**75. La proporción del mortero y la piedra**, en el concreto, se encuentra (1) por separación efectiva, pesando la piedra y el mortero por separado ó (2) averiguando separadamente, y comparando los pesos específicos de la piedra el mortero y el concreto.

## CEMENTO ARMADO

**1.** La resistencia á la tensión y al esfuerzo cortante del concreto son bajas en comparación con su resistencia á la compresión. Por esto incrústanse varillas ó barras perfiladas en las estructuras de concreto, en aquellas partes sujetas á tensión y á esfuerzos cortantes, y en posiciones tales que soporten dichos esfuerzos.

**2. Usos.** La armadura ó armazón se usa principalmente en las partes de vigas y tirantes (inclusive losas de piso), columnas, muros, muros de sostenimiento, represas, etc., que soportan tensión; pero es también útil en muchos otros casos, como para impedir grietas en las superficies, para lo cual se coloca á pocos centímetros detrás de la cara un ligero tejido de metal (malla de alambre, metal desplegado, etc.); para impedir fracturas causadas por cambios súbitos inevitables en la secciones transversales, para unir muros que se encuentran en un ángulo y susceptibles de separarse uno de otro al asentarse; en alcantarillas para que puedan resistir la tensión horizontal debida á la presión del terraplén. Para este fin pueden usarse cadenas viejas, ó barras livianas, con pernos introducidos por huecos, para aumentar la adhesión.

**3. Seguridad.** Los edificios modernos de cemento armado son prácticamente monolíticos y por lo tanto más rígidos que las construcciones de armazón de acero.

**4.** En cambio, en un edificio de acero los pormenores son ejecutados con más exactitud y la obra es generalmente erigida por hombres hábiles á menudo empleados de los fabricantes de acero; de suerte que hay poca probabilidad de dañar el material en construcción, mientras que en obras de cemento armado, el mejor material puede dañarse al usarlo y hacer así insegura la obra.

**5.** Un buen concreto protege el acero incrustado de la **corrosión**, tanto sobre como bajo el nivel del agua dulce ó del mar; pero el agua puede penetrar el concreto poroso y oxidar el metal. El concreto colocado muy seco puede hacerse poroso.

**6.** El acero empleado en el hormigón armado tiene generalmente su máx de resistencia entre 3,500 y 4,900 kg por cm cuad, y su límite de elasticidad entre 1,750 y 2,450 kg por cm cuadrado; pero trabajado en frío puede elevar su límite de elasticidad á 2,800 ó 3,500. Para economizar acero se hacen barras llamadas « deformadas », de acero laminado, con un límite de elasticidad mucho más alto (se le calculan 3,500 á 4,550 kg por cm cuadrado); pero véase Armaduras de barras, págs. 1180, etc. Como en el hierro y el acero laminados en general, el módulo elástico puede suponerse que es por término medio de 2,100,000 kg por cm cuadrado. Véase § 11.

**7. Concreto.** En general, la necesidad de trabajar el concreto alrededor de las barras de la armadura exige que el agregado para el concreto en obras de cem armado sea menor de lo que sería permitido en obras ordinarias, y la importancia vital de la adhesión exige que todos los materiales para el concreto sean de los mejores y el mortero ni muy líquido ni muy seco.

### Expansión, contracción, etc.

**8.** La **contracción** del concreto, mientras fragua al aire, produce compresión en la armadura y tensión en el mismo concreto. Cuando fragua bajo el agua, la expansión del concreto produce los efectos contrarios.

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* Hecho por la Cyclone Drill Co., Orrville, O., inclusive taladros pequeños movidos á mano.

** B. G. Cope, en E. N., '08/enero/9, pág. 41.

**9.** El **coeficiente de expansión** lineal, *a*, de un material, es aquella fracción de su longitud original que una barra de dicho material gana ó pierde por cada cambio de 1° en su temperatura.

Aproximadamente :

| | | Para 1° centígrado. |
|---|---|---|
| En acero..... | 10,000 *a* = | .117 |
| En concreto..... | 10,000 *a* = | .108 |

**10.** El gran número de construcciones de cemento armado que han estado expuestas por muchos años á cambios extremos de temperatura, sin sufrir por diferencias de expansión, confirma los resultados de las experiencias arriba citadas, estableciendo que la diferencia entre los coeficientes de expansión de los dos materiales es despreciable.

## Módulo de elasticidad.

**11.** El **módulo de elasticidad, $E_s$, del hierro y del acero laminados,** de todas clases (pág. 482 *b*), es notablemente uniforme y constante, variando ordinariamente entre 1.9 á 2.2 (término medio, digamos 2.1) millones de kg por cm cuad.

**12.** Por el contrario, el **módulo de elasticidad, $E_c$, del concreto** varía mucho no sólo entre diferentes mezclas diversamente manipuladas, y entre diferentes muestras hechas bajo condiciones semejantes de materiales semejantes, sino en la misma muestra bajo diferentes intensidades de carga, de modo que al exponer los resultados de los experimentos se acostumbra especificar el campo de la variación de la unidad de resistencia á cuyo alrededor se hicieron las observaciones.

**13.** En el **concreto de piedras,** $E_c$, varía de 1.5 á 4 (término medio, digamos 3) millones de lbs por pulgada cuadrada = .1 á .28 (término medio, digamos .21 millones de kg por cm cuad. Véase Experimento 81 *a*, pág. 1387 En concreto de ceniza gruesa, $E_c$ es ordinariamente de 20 á 50 0/0 menos que en concreto de piedras. Véase § 30, pág. 1316.

**14. La relación,** $n$ (á veces llamada $r$ y $R$) $= E_s/E_c$, entre los módulos de elasticidad del acero y del concreto respectivamente, se supone generalmente entre 10 y 15 para concreto de piedras, con valores más altos para el concreto de ceniza gruesa. Véase Especificaciones, § 107, pág. 1414. Debido á la variabilidad de $E_c$ (véase § 12), no puede ser una cantidad constante, aun dentro de un solo experimento llevado de una carga cero á la carga de ruptura.

**15.** La relación, $n$, es, sin embargo, de uso constante é **importante** en todos los cálculos relativos á la conducta mutua del concreto y del acero.

**16.** Los experimentos de Considère (Experimento 16 *a*, pág. 1357, parecieron demostrar que el concreto cuando, está armado (obligado, por su adhesión al acero, á participar de sus movimientos) efectivamente sufría, sin fractura, **estiramientos** mucho mayores que cuando no estaba armado; pero experimentos posteriores (36, 38, 81 *e*, 81 *f*), observando la superficie del concreto más detenidamente, han indicado que el supuesto estiramiento del concreto se debía en realidad á la formación de grietas que antes habían escapado á la observación. Si la adhesión entre el concreto y el acero es uniforme, el agrietamiento debe estar igualmente distribuído sobre el área de contacto, y las grietas deben, en consecuencia, ser muy numerosas y muy delgadas, probablemente tan delgadas que no lleguen á poner en riesgo los materiales por la filtración del agua.

## Adhesión. Véase § 58, pág. 1337.

**17.** Con **mezclas ricas y húmedas,** tales como se usan en el cemento armado, el cemento se adhiere muy íntimamente al acero.

**18.** Después que ha sido vencida la adhesión propiamente dicha, la **remoción** de las piezas de acero del concreto encuentran todavía el obstáculo de la **fricción** entre los dos.

**19.** De esta resistencia de la adhesión y fricción á las fuerzas que tienden á vencerlas, depende, por supuesto, la seguridad de la construcción.

**20.** Tanto á la adhesión como á la fricción, y en particular esta última, las afecta considerablemente la naturaleza del concreto y su conducta bajo los esfuerzos y cambios de temperatura.

**21.** En las **pruebas** directas **para la adhesión** ya se ejerza tensión ó compresión en el acero, el concreto se halla siempre bajo compresión, que le produce alguna expansión lateral, y por tanto la presión aumenta sobre la armadura interior. En consecuencia, la adhesión puede encontrarse más alta (en igualdad de circunstancias) que en las vigas, donde no existe esta condición.

**22.** Por otra parte, cuando las barras horizontales de la armazón, en una viga, se doblan hacia arriba, cerca de los extremos, y pasan á la región de compresión y (como sucede á menudo) llegan á un punto por encima del apoyo, las altas presiones sobre la barra, en esas porciones, pueden darle mayor adhesión de la que podría tener una barra recta bajo prueba directa.

**23. Con grandes longitudes de empotramiento**, la tensión, en el acero, bajo grandes fuerzas, puede ser tal, que contraiga lateralmente el acero, lo bastante para reducir la adhesión. En consecuencia, las pruebas donde el acero se **empuja** en el concreto presentan adhesiones más altas.

**24. Adhesión máxima.** En general, los ensayos (véase Experiencias y práctica 64 *a*, *b*) dan, como máxima adhesión de un buen concreto á las barras redondas y lisas, de 14 á 21 kg por cm cuadrado en la superficie de contacto. Con barras redondas y lisas, en una viga, obtuvo Kleinlogel (Beton und Eisen, 1904, págs. 227 y sig.) 39 kg por cm cuadrado. Los resultados en la práctica difieren mucho, por lo general, de los que se obtienen en el laboratorio.

**25. Carga de trabajo de las uniones.** En vigas sujetas á choques, se conceden 3.5 kg por cm cuadrado: para carga quieta se concede á veces el doble de esto. Véase Especificaciones, §§ 113-115.

## COLUMNAS DE CEMENTO ARMADO

**1.** Una columna de concreto tiene de ordinario barras longitudinales de acero incrustadas cerca de la superficie en toda su extensión. Si no hay flexión ni resbalamiento entre el concreto y el acero, los dos materiales deben acortarse igualmente bajo la carga. En consecuencia (pág. 480, Ec /3), si L=longitud original, $l$=aumento ó disminución de la longitud, $a_s$ y $a_c$=áreas de secciones transversales; $s_s$ y $s_c$=esfuerzo por unidad de superficie, $E_s$ y $E_c$=módulos de elasticidad, del acero y del concreto, respectivamente, tenemos

$$s_c = E_s\, l/L; \quad s_c = E_c\, l/L; \ldots\ldots (1)$$

y como $l/L$ es necesariamente el mismo para ambos materiales,

$$s_s/s_c = E_s/E_c = n; \quad s_s = s_c\, n; \ldots\ldots (2)$$

y el esfuerzo total en el acero $= a_s\, s_s = a_s\, s_c\, n \ldots\ldots (3)$

— — en el concreto $= a_c\, s_c \ldots\ldots (4)$

— — en la columna $= P = a_s\, s_c + a_c\, s_c = s_c\,(a_c + a_s\, n) \ldots\ldots (5)$

$$a_c = P/s_c - a_s\, n. \ldots\ldots (6)$$

$$s_c = P/(a_c + a_s\, n) \ldots\ldots (7)$$

**2. Ejemplo.** Si una columna cuadrada de concreto de 40×40 cm y 3.60 m de largo tiene incrustada en cada esquina una barra redonda de acero de 2.5 cm de diámetro, tendremos que el área de la sección transversal de cada barra será=4.90 cm cuad; y la fuerza de compresión permitida, $s_c$, por unidad de superficie sobre el concreto=35 kg por cm cuadrado. Para saber el peso que puede soportar la columna, tenemos :

Area; $a_c$, del acero=4×4.906=19.624 cm cuad;
Area, $a_c$, de concreto=40×40 — 19.62=1,580.38 cm cuad;
$E_s$=2,100,000 kg por cm cuad;
$E_c$= 175,000 kg por cm cuad;
$n = E_s/E_c = 12$;

| | | |
|---|---|---|
| Carga total que soporta el concreto $= a_c\, s_c$ =1,580.38×35 kg=.. | | 55,312 kg |
| — — — el acero $= a_s\, s_c\, n$=19.62×35×12=..... | | 8,240 |
| — — — la columna..... | | 63,552 kg |

**3.** Aquí el acero toma 100×8,240 ÷ 63,552=12.9% de la carga total, que es una proporción segura. Esta proporción no debe exceder de 20% ó, cuando más, de 30%.

**4. Una regla que conviene** observar es la de contar cada centímetro cuadrado de acero, en columnas, como equivalente al valor de $n$ centímetros cuadrados de concreto.

**5.** Los constructores cuidadosos cargan las columnas de concreto aproximadamente como sigue :

| Largo | Mezcla. | | | |
|---|---|---|---|---|
| diám | 1:1.5:3 | 1:2:4 | 1:2.5:5 | 1:3:6 |
| | $p=P/a$=Carga en kg por cm cuad. | | | |
| <12..... | 42 | 35 | 24.5 | 24.5 |
| 12 á 18..... | 38.5 | 31.5 | 21 | 21 |

**6. En general se colocan** simétricamente **barras** longitudinales de refuerzo cerca de la superficie del concreto y se cubren con 4 á 5 cm de concreto. Las barras deben estar atadas unas á otras por varillas más delgadas ó por alambres, á intervalos que no excedan del diámetro de la columna.

**7.** Las especificaciones exigen de ordinario que el área en conjunto de la sección transversal de las barras de compresión no exceda de 2 á 3% del área de la sección transversal de la columna.

**8.** En edificios de tres ó cuatro pisos, las barras de cada sección se doblan hacia adentro, cerca de sus extremos, para formar un cilindro de 45 ó 50 cm de alto, de diámetro más pequeño que el cilindro principal de abajo, y la sección inmediata de arriba se ajusta sobre esta porción de modo que las dos secciones recubran la parte reducida.

**9.** Debido á sus áreas de sección transversal, mucho mayores, y á las más bajas unidades de resistencia en sus materiales, las columnas de cemento armado están menos expuestas á flexarse que las columnas de acero.

**10. Para cargas máximas sobre columnas de cemento armado longitudinalmente y sujetas á flexión,** tenemos la fórmula de Rankine :

$$p = \frac{P}{a} = \frac{s}{1 + mK^2} \qquad (8)$$

en que

$P$=carga máxima total sobre la columna;
$a$=área de la sección transversal de la columna;
$p=P/a$=carga máx por unidad de superficie;
$s$=resist máxima de compresión por unidad de sup en los cubos de concreto;
$K=L/r$=longitud /radio mínimo de giro.

El profesor Morsch hace $m=.0001$. Eisenbetonbau, '08, pág. 73.

## Columnas con sunchos.

**11.** Las columnas reforzadas con sunchos (ó espirales) de acero, ó con alma encorvada en forma cilíndrica, presentan grandes máx de resistencia y se usan profusamente; pero sufren considerables deformaciones antes de que se desarrolle la resistencia de los sunchos, pues éstos obran de modo semejante á un cilindro de acero, lleno de arena, que no puede desarrollar su fuerza hasta que la arena no está comprimida.

**12.** Los experimentos en Watertown (Ensayo de metales, 1905) demostraron que, cuando la columna está sujeta á cargas de 7 á 70 kg por cm cuadrado, la unidad de deformación lateral es menos de la cuarta parte de la unidad de deformación longitudinal. Así, si la columna se acortaba .004 de su largo, su diámetro aumentaba menos de .0001 de su dimensión original.

**13.** De pruebas hechas en la Universidad de Illinois (Am. Soc. Testg Matls., Procs., 1907, pág. 382) deriva el prof. A. N. Talbot las siguientes **fórmulas** para las resistencias máx de las columnas cilíndricas de concreto sunchadas 1 : 2 : 4 : de mezcla húmeda de 60 días; término medio; columnas de 30 cm de diámetro y 3.05 m de largo. Cubierta, sobre los sunchos generalmente 6 mm. Sunchos, 25 mm de ancho, tipos n.os 8, 12, 16, soldados eléctricamente y separados generalmente 5 cm de centro á centro. Sean

$p$=resistencia máx de la columna, kg por cm cuad;
$c$=proporción entre los aros y el alma del concreto;
112=resist á la compresión del concreto, kg por cm cuad.

Entonces,

Para acero dulce $p=112+4{,}550\ c$;..... (9)
— — mejor $p=112+7{,}000\ c$..... (10)

**14.** Suponiendo que la resistencia máx por unidad de superficie, en la armazón longitudinal de la columna, es 25 veces la del concreto, los sunchos produjeron un máx de fuerza adicional de 2 á 4 veces la producida por la armazón longitudinal.

**15.** Los Ensayos del Sr. Considère (« Génie Civil », nov. 1902), con columnas de concreto con armazones espirales, indican que las barras que forman los sunchos deben tener un diámetro aproximado de $^1/_{40}$ del diámetro de la columna; que el paso de las espiras (distancia entre los sunchos) debe ser de $^1/_5$ á $^1/_6$ del diámetro de la columna, y que el acero, en los aros ó espirales, agrega, á la última resistencia de la columna, 2.4 veces tanto como el mismo peso del metal empleado como armazón longitudinal. Da la fórmula :

$$\text{Máximo de la carga total en la columna} = 1.5\, a_c c + s_c (a + 2.4\, A) \ldots\ldots \quad (11)$$

en que

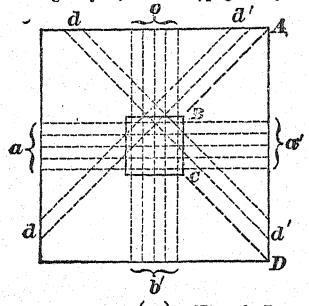
$a_c$ = área de la sección transversal de la columna dentro de la espiral;
$c$ = máxima resistencia á la comp por unidad de sup de concreto ordinario en bloques cortos;
$s_c$ = límite de elasticidad del acero;
$a$ = área de la sección transversal del refuerzo longitudinal existente;
$A$ = — — — — del refuerzo longitudinal de igual peso que la espiral.

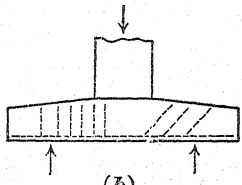
$1.5\, a_c$ se toma como representando el área de toda la sección transversal del concreto.

## Bases de columnas.

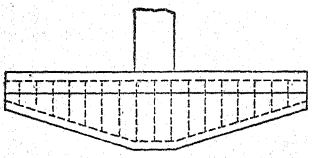
**16.** En las **bases de columnas** los esfuerzos son análogos á los de una plancha ó baldosa de piso que descansa sobre una columna; pero, debido á lo relativamente estrecho de la base, los momentos y esfuerzos cortantes son fuertes y requieren considerable profundidad. Los mayores esfuerzos obran debajo de los bordes de la columna. Las barras horizontales, en la base, son análogas á las barras cerca del tope de una viga sobre el soporte; esto es : sufren momentos negativos y algunas de ellas deben doblarse hacia arriba, ó estar provistas de estribos, precisamente fuera de los bordes de la columna.

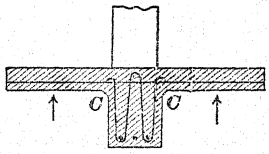
**17.** Figs. 1 y 2 (T. and M., págs. 261, 262). Fig. 1 : Dos series de barras de refuerzo principales, $aa'$, $bb'$, que se cruzan en ángulos rectos debajo de la columna, con barras diagonales, $dd'$, $dd'$. Fig. 2: Viga y baldosa combinadas. Las alas laterales de la baldosa tienden á doblarse hacia arriba, separándose de la viga en C y C.



(a) **Fig. 1.** Base de columna. (b)






(a) **Fig. 2.** Base de columna. (b)

En estas págs el autor usa la *sub s* y la *sub c* para distinguir respectivamente los coeficientes de una misma especie, en el acero ó el hierro, de los correspondientes en el concreto; así, por ejemplo, $E_c$ y $E_s$ son respectivamente los coeficientes de elasticidad en el concreto y en el acero. Lo mismo para los momentos, etc. Llamamos la atención del lector porque á veces la letrica *s* ó *c* aparece, por su pequeñez, un poco borrada.

## VIGAS DE CONCRETO ARMADO

**1.** La resistencia á la compresión del conc es de 8 á 10 veces su resistencia á la tensión. En consecuencia, en una viga de concreto armado de sección rectangular sometida á esfuerzos de flexión, ocurre la falla donde el esfuerzo obra como tensión.

**2.** La facilidad con que puede incrustarse el acero en el concreto, la igualdad práctica de los coeficientes de expansión de las dos substancias, la fuerte adhesión entre el concreto y el acero, y la facilidad de extender esta adhesión por prolongaciones ó proyecciones laterales, de la superficie del acero, facilitan combinaciones en que el principal servicio del concreto es resistir la compresión mientras que la del acero es resistir á la tensión.

**3.** El método de manufactura del concreto es tal, que su conducta, en un caso dado, es menos segura que la del acero.

Debido á esto y á la incertidumbre, en cuanto al grado de adhesión entre el concreto y el acero, de la que depende su acción combinada, la teoría de tales vigas es desde luego más complicada y menos exacta que la de las vigas de acero de secciones económicas. En el diseño de vigas de concreto armado debe hacerse la debida concesión por este respecto y todo cálculo de exagerada precisión está fuera de lugar.

### Teoría general.

**4.** Viga simple de concreto armado de sección rectangular, fig. 1.

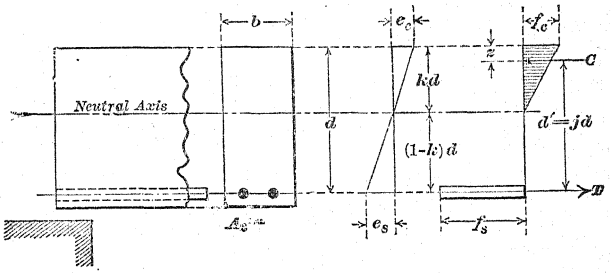



**Fig. 1.** Viga de concreto armado. Teoría.

### Hipótesis fundamentales.

1. Las secciones transversales planas antes de la flexión, permanecen planas bajo la flexión.

2. Las resistencias iniciales (por contracción, etc.) se desprecian.

3. No ocurren deslizamientos entre el concreto y el acero. Por tanto se deforman por igual.

4. La resistencia del concreto á la tensión se desprecia.

5. Los módulos de elasticidad $E_s$ y $E_c$, del acero y del concreto respectivamente, y en consecuencia su relación $n = E_s/E_c$, permanecen constantes.

**5. Observaciones** referentes á la figura 1, sean:

$b$ = ancho (perpendicular al papel) de la sección transversal de la viga;
$d$ = distancia, del lado de la viga en que se ejerce la compresión, al centro de gravedad del acero;
$kd$ = distancia del lado de la viga en que se ejerce la compresión al eje neutro;
$z$ = distancia del lado de la viga en que se ejerce la compresión á la resultante de las fuerzas de comp;
$(1-k)\,d$ = distancia del centro del acero al eje neutro;
$d' = jd$ = distancia del centro del acero á la resultante de las fuerzas compuestas = palanca del par resistente;
$j = d'/d$;
$E_s$ = módulo de elasticidad del acero; $E_c$ = módulo de elasticidad del concreto;

$e_s$=unidad de alargamiento del acero;
$f_s$=tensión por unidad de sup en el acero**;
$a_s$=área de la sección transversal del acero;
$e_c$=unidad de acortamiento del concreto*;
$f_c$=comp por unidad de sup en el concreto* **;
$a_c=bd$=área de la sección transversal del concreto por encima del centro del acero;
T=suma de los esfuerzos de tensión en el acero;
C=suma de los esfuerzos de compresión en el concreto;
$n=E_s/E_c$=relación entre los módulos de elasticidad del acero y del concreto;
$p=a_s/a_c$=relación entre el área del acero y la porción del área del concreto que está sobre el centro del acero †;
$M_s$=momento de resistencia, basado en el valor máximo permitido ‖ de $f_s$;
$M_c$= — — — — — de $f_c$;
M=momento de resistencia real, efectivo.
Entonces $a_s=pa_c=pbd$.

## Esfuerzos, momentos, diseño.

**6.** Las figs. 1 y 2 †† y los §§ 7 á 20 ilustran las **relaciones existentes entre los importantes factores**, $k$, $j$, $f_s$, $f_c$, $p$, $M_s$, $M_c$ y M; cuando ni $f_s$, ni $f_c$ exceden del límite de elasticidad. Cuando exceden de ese límite, véanse §§ 21, 22, pág. 1331.

**7.** En equilibrio, el **momento de flexión** de la carga (véase pág. 496) queda equilibrado por el **momento de resistencia** igual del par compuesto de las dos fuerzas horizontales iguales, T y C; pues estas fuerzas son respectivamente las resultantes de las fuerzas de tensión en el acero y de las fuerzas de compresión*** en el concreto.

**8. Los esfuerzos de tensión** $f_s$, en el acero, se suponen que están uniformemente distribuídos sobre toda su sección transversal, $a_s$; y su resultante, T, se toma en consecuencia como si obrase en el centro de gravedad del área del acero; pero las **fuerzas de compresión**, en el concreto, en cualquier sección transversal, decrecen uniformemente (véanse §§ 21 y 22) desde un máximum, $f_c$, en la superficie superior de la viga, hasta cero, en el eje neutro. Su resultante, C, se aplica, por tanto, en un punto distante $kd/3$ debajo de la parte superior de la viga, siendo $kd$ la distancia desde dicha parte sup de la viga hasta el eje neutro, y $d$ la distancia desde la misma parte sup de la viga hasta el centro de gravedad del acero.

**9. Valor de « j ». El brazo de palanca**, $d'$, **del par de resistencia** es por tanto

$$d'=jd=d-kd/3=d(1-k/3) \quad \ldots\ldots (1)$$

y tenemos

$$j=d'/d=1-k/3 \quad \ldots\ldots (2)$$

Para valores aproximados de $j$, véase § 12.

**10. Valor de « k ».** De la hipótesis 1, § 4, tenemos

$e_c/e_s=k/(1-k)$ (Véase triáng semejantes, fig. 1, que tienen por base $e$ y $e_s$. — *N. del T.*) ..... (3)

Del supuesto 5, tenemos

$$f_c=e_cE_c;\ f_s=e_sE_s \quad \ldots\ldots (4)$$

De aquí que

$$\frac{f_c}{f_s}=\frac{e_cE_c}{e_sE_s}=\frac{k}{1-k}\times\frac{E_c}{E_s}=\frac{k}{n(1-k)} \quad \ldots\ldots (4a)$$

---

* En las fibras más externas del lado de la compresión.
** $f_s$ y $f_c$ son las unidades de esfuerzos *reales*. Véase § 13, pág. 1327.
† Véanse §§ 15, 16, pág. 1327.
‖ Véase § 13, pág. 1327.
*** Debajo del eje neutro, el concreto está en *tensión*, pero su esfuerzo de tension se desprecia. Véase hipótesis 4, § 4, pág. 1325.
†† Las figs. 2 y 3 son hechas por el prof. A.W. French. A.S.C.E., Trans., vol. 56, 06, págs. 362, etc.

Para el equilibrio, C=T; pero

$$C = f_c\, bkd/2 \text{ (véase fig. 1)} = e_c\, E_c\, bkd/2 \quad \ldots\ldots (5)$$

$$y\ T = f_s\, a_s = f_s\, pbd = e_s\, E_c\, pbd \quad \ldots\ldots (6)$$

De aquí (igualando valores de C y T y despejando á $k$). (*N. del T.*)

$$k = 2p\frac{e_s E_c}{e_c E_c} = 2\,p\,n\,\frac{1-k}{k};$$

$$\text{ó } k = \sqrt{(p\,n)^2 + 2\,p\,n} - pn. \quad \ldots\ldots (7)$$

**11.** En consecuencia, la **posición del eje neutro** (dada por $k$) depende solamente de la razón, $p$, entre el área del acero y el área del concreto, y de la relación, $n$, entre la elasticidad del acero y la del concreto. Para valores aproximados de $k$, véase lo siguiente :

**12. Valores aproximados** de $j$ y $k$. Véase fig. 2.

| cuando | y | tenemos | y |
|---|---|---|---|
| $n=10$, | $p=.010$: | $j=.88$; | $k=.36$; |
| | $p=.015$: | $j=.86$; | $k=.42$; |
| $n=15$, | $p=.010$: | $j=.86$; | $k=.42$; |
| | $p=.015$: | $j=.84$; | $k=.48$. |

**13.** Cuando, como en el concreto armado, se usan conjuntamente dos materiales enteramente diferentes, sucede de ordinario que, debido á la imposibilidad de darle siempre á cada uno su área ideal de sección transversal, **el uno ó el otro** están inevitable y no económicamente sometidos á menos de su esfuerzo máximo permitido. Así, con un valor dado de $p = a_s/a_c$, si cargamos la viga hasta que ó bien $f_s$ ó $f_c$ llegue á su límite permitido, el otro, $f_c$ ó $f_s$, permanecerá generalmente por debajo de su límite permitido. Véase § 19 *f*. Sean $F_s$ y $F_c$ = respectivamente los **valores máximos permitidos de** $f_s$ y $f_c$.

**14. Momentos**. Para momentos de resistencia, basados sobre los **valores máximos permitidos**, $F_s$ y $F_c$, de $f_s$ y $f_c$ respectivamente, tenemos :

$$M_s = Td' = F_s a_s jd = F\, pjd^2 \quad \ldots\ldots (8)$$

$$M_c = Cd' = C jd = F_c\, bkdjd/2 = F_c\, kjbd^2/2 \quad \ldots\ldots (9)$$

**Para valores usuales**, podemos tomar (véase § 12) : $j = {}^7/_8$; $k = {}^3/_8$, $kj = {}^1/_3$. De aquí, **aproximadamente,**

$$M_s = 7F\, a\, d/8;$$
$$M_c = F_c\, bd^2/6.$$

Pero el **momento de resistencia real, M**, de la sección, en cualquier caso dado, no puede tener, por supuesto, sino *un valor;* y éste es el *menor* de los dos valores $M_s$ y $M_c$. Puesto que $jbd^2$ es común á los dos valores, M se determina investigando cuál es el menor entre $F_s p$ ó $F_c k/2$.

**15. Relación entre** $f_s$, $f_c$ y $p$. Puesto que C=T, ó $f_c bkd/2 = f_s pbd$, tenemos

$$\mathbf{f_s = \frac{kf_c}{2p}};\quad \mathbf{f_c = \frac{2pf_s}{k}};\quad \mathbf{p = \frac{kf_c}{2f_s}} \quad \ldots\ldots (10)$$

De la Ecuac (4 *a*) tenemos :

$$\frac{f_c}{f_s} = \frac{k}{n-nk}.$$

De aquí $k = \dfrac{nf_c}{nf_c + f_s}$;

$$y\ \mathbf{p} = kf_c/2f_s = \frac{.5}{\frac{f_s}{f_c}\left(\frac{nf_c+f_s}{nf_c}\right)} = \frac{.5}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c}+1\right)} \quad \ldots\ldots (11)$$

**Generalmente, p varía** de .010 á .015. Raras veces $< .005$ ó $> .020$.

**16.** Nótese que $f_s$, $f_c$ y $p$ no pueden escogerse arbitrariamente. Dados dos cualesquiera de ellos, el tercero depende de los dos así dados.

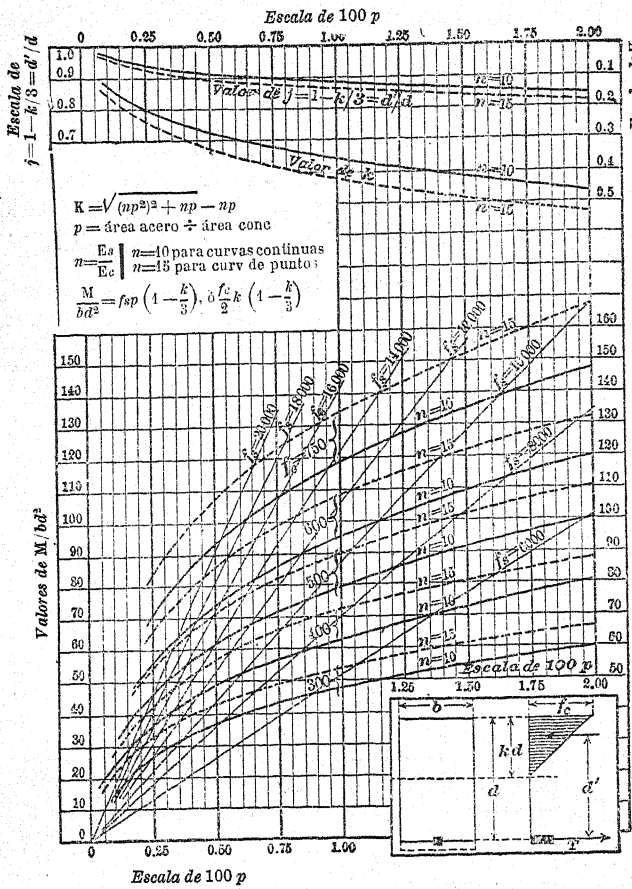



**Fig. 2.** Para esfuerzos de trabajo. (Para los máx ó de ruptura, véase fig. 3

$\mathbf{k} = \sqrt{(p\,n)^2 + 2\,p\,n} - pn$, $\mathbf{j} = d'/d$;

$\mathbf{f_s}$ = esfuerzo por unidad de superficie en el acero, $\mathbf{f_c}$ = esf por unidad de sup en el concreto en la parte sup de la viga;

$\mathbf{p} = a_s/a_c$; = proporción entre el área del acero y el área del concreto;

$\mathbf{M_s}$, $\mathbf{M_c}$ = momento de resistencia, basado sobre el valor permitido de $f_s$, $f_c$, respectivamente;

**M** = momento de resistencia, real.

$$\frac{\mathbf{M}}{bd^2} = f_s p\,(1 - \frac{k}{3}) \quad \text{ó} \quad \frac{f_c}{2}\,k\,(1 - \frac{k}{3}).$$

$n=E_s/E_c$. Las **curvas continuas** representan $n=10$; las **curvas de puntos**, $n=15$.

Las **líneas de acero** trazadas para $n=10$; aproximadamente para $n=15$.

**17. Valor de M/bd².** Sean $F_s$ y $F_c$ los valores *máximos permitidos* de las unidades de fuerzas, $f_s$ y $f_c$, en el acero y en el concreto respectivamente. Entonces, de las ecuaciones (8) y (9), § 14, tenemos (fig. 2, parte inferior):

$$M_s/bd^2=F_s pj=F_s p\ (1-k/3);$$
(líneas casi rectas para el acero)..... (12)

$$M_c/bd^2=F_c kj/2=F_c k\ (1-k/3)/2;$$
(líneas curvas para el concreto)..... (13)

Las líneas curvas de puntos y sólidas para el concreto representan $n=15$ y $n=10$, respectivamente. Las líneas casi rectas para el acero están proyectadas para $n=10$, pero son bastante aproximadas también para $n=15$.

**18.** La porción superior de la **fig. 2** da **valores de**

$$k=\sqrt{2pn+(pn)^2}-pn;$$

véase § 10) y de

$$j=1-k/3=d'/d,$$

correspondientes á valores dados de $p$, para $n=10$ y $n=15$. Nótese que $j$ varía muy poco con las variaciones de $p$.

## Ejemplos.

### I. Investigaciones.

### Se piden los momentos de resistencia $M_s$, $M_c$ y M.

(*Obs. del T.* — Para hacer uso de los, por su claridad, sencillez y utilidad práctica, admirables diagramas de las **figs. 2** y **3**, debemos convertir los datos que tengamos en metros y kg á unidades inglesas, usar el diagrama y luego convertir el resultado obtenido en medida inglesa al sistema métrico. En esta materia es facilísimo, pues nunca hay más de tres datos que convertir. Vamos á hacerlo así, en todos los ejemplos que trae el autor, para que el lector aprecie la sencillez de la operación; advirtiendo que, cuando la extremada exactitud en la conversión no sea de trascendencia en los resultados, haremos aquéllas simplemente aproximadas; convirtiendo la pulgada, por ejemplo, en 2.5 cm, cuando la verdad es 2.54, etc.)

**19 a. Dada una viga rectangular de concreto** armado de $b=20$ cm; $d=50$ cm convertidos en pulgs dan: $b=8''$; $d=20''$; $a_c=bd=8\times20=160$ pulgs cuads *; $n=E_s/E_c=15$. Sean $F_s=1{,}120$ kg por cm cuad$=16{,}000$ libs por pulg cuad, y $F_c=35$ kg por cm cuad$=500$ lbs por pulg cuad los valores *máximos permitidos* de las unidades de fuerzas, $f_s$ y $f_c$, en el acero y en el concreto respectivamente; y sea P el valor de $p$ basado sobre estas fuerzas máximas permitidas.

Entonces $F_s/F_c=32$; $\dfrac{F_s}{nF_c}+1=3.133$; y, de la ecuación (11), § 15, tenemos:

$$P=\frac{.5}{32\times3.133}=.004987,$$

como lo da la intersección, en la fig. 2, de la línea radial, para $f_s=16{,}000$ (1,120 kg por cm cuad) con la curva de puntos para $f_c=500$ (35 kg por cm cuad).

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* *N. del T.* — Para convertir pulgs en cm se multiplican las pulgs $\times$ 2.54; para convertir cm en pulgs se multip los cm por .3937. Las libras por pulgs cuad multiplicadas por .0703 dan kg por cm cuad. Los kg por cm cuad multiplicados por 14.2234 dan lbs por pulg cuad.

**19 b. (Caso 1) Reforzado con dos barras redondas** de 19 mm=3/4"=.75 diám.

$a_s = 2 \times \pi \times 0{,}375^2 = .884$ pulgs cuad;
$p = a_s / a_c = .884 / 160 = .005525 > P$;
$pn = 15 \times .0055 = .0825$;

$$k = \sqrt{(pn)^2 + 2pn} - pn$$
$$= \sqrt{.0825^2 + .1650} - .0825 = .3322;$$
$d' = dj = d\ (1 - k/3) = 20\ (1 - .1107) = 20 \times .89 = 17.8$ pulgs:
$C = F_c\, bkd/2 = 500 \times 8 \times .3322 \times 10 = 13{,}288$ lbs $= 6{,}027.33$ kg *;
$M_c = Cd'$ (fig. 1) $= 13{,}288 \times 17.8 = 236{,}526$ lbs-pulgs $= 2{,}720$ kg-metros **;
$T = F_s a_s = 16{,}000 \times .884 = 14{,}144$ lbs $= 6{,}407$ kg *;
$M_s = Td'$ (fig. 1) $= 14{,}144 \times 17.8 = 251{,}763$ lbs-pulg $= 2{,}900$ kg-metros **;
$M = M_c =$ 236,526 — $= 2{,}720$ kg-metros.

Nótese que cuando, como en este caso y en el caso 2, $P < p$, el momento, $M_c$, basado en la fuerza máxima permitida, $F_c$ en el *concreto*, es el momento real, M. Cuando $P > p$, $M_s$ es el momento real.

**19 c. Por la fig. 2.** La intersección de la línea vertical, sobre 100 $p = .55$, con la línea radial para $f_s = 16{,}000$ lbs por pulgada cuadrada, da $M_s / bd^2 = 78.7$; y $M_s = 78.7\ bd^2 = 78.7 \times 8 \times 20^2 = 251{,}840$ lbs-pulgs; pero la intersección de la línea vertical sobre 100 $p = .55$, con curva de puntos ($n = 15$) para $f_c = 500$ lbs por pulgada cuadrada, da $M_c / bd^2 = 74$; y $M = M_c = 74\ bd^2 = 74 \times 8 \times 20^2 = 236{,}800$ lbs-pulgs.

**19 d. (Caso 2) Reforzado con 3 barras redondas**, de 2.5 cm diám=1 diám;

$a_s = 3\pi\ 0.5^2 = 2.356$ pulgs cuad;
$p = a_s / a_c = 2.356 / 160 = .01473 > P$;
$pn = 15 \times .01473 = .2209$;

$$k = \sqrt{(pn)^2 + 2pn} - pn$$
$$= \sqrt{.22^2 + .44} - .22 = .48;$$
$d' = dj = d\ (1 - k/3) = 20\ (1 - .16) = 20 \times .84 = 16.8$;
$C = F_c\ bkd/2 = 500 \times 8 \times .48 \times 10 = 19{,}200$ lbs;
$M_c = Cd' = 19{,}200 \times 16.8 = 322{,}560$ lbs-pulgs $= 3{,}716$ kg-m **;
$T = F_s a_s = 16{,}000 \times 2.356 = 37{,}696$ lbs $= 17{,}098.9$ kg;
$M_s = Td' = 37{,}696 \times 16.8 = 633{,}293$ pulgs-lbs $= 7{,}295$ kg-m;
$M = M_c =$ 322,560 — $= 3{,}716$ kg-m.

**19 e. Por la fig. 2.** La intersección de la línea vertical sobre 100 $p = 1.473$, con la línea radial para $f_s = 16{,}000$ lbs por pulg cuadrada, daría (sobre un diagrama bastante exacto) $M_s / bd^2 = 198$; y $M_s = 198\ bd^2 = 198 \times 8 \times 20^2 = 633{,}600$ lbs-pulgs $= 7{,}299$ kg-m; pero la intersección de la línea vertical sobre 100 $p = 1.473$, con la curva de puntos ($n = 15$) para $f_c = 500$ lbs por pulg cuadrada $= 35$ kg por cm cuad **, da $M_c / bd^2 = 101$; y $M = M_c = 101\ bd^2 = 101 \times 8 \times 20^2 = 323{,}200$ pulgs-lbs $= 3{,}723$ kg-m.

**19 f.** Se notará que, en estos casos, **un aumento de 166.5 0/0**, en la cantidad de acero, ha **aumentado el momento de resistencia** (que todavía depende del concreto) en menos de 38 0/0; y el acero, en el caso 2, trabaja sólo como con 8,000 lbs por pulgada cuadrada $= 560$ kg por cm cuad ó la mitad de la fuerza máxima permitida (véase que la intersección de la vertical para 100 $p = 1.473$, con la curva de puntos para $f_c = 500$, es casi interceptada por la línea radial para $f_s = 8{,}000$). Véase § 13.

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* *N. del T.* — Para convertir lbs en kilogramos, se multiplican aquéllas por .4536, y á la inversa, kg en lbs, se mult aquéllos por 2.204. Para convertir *lbs por pulg cuad* en kg por cm cuad, se multiplican aquéllas por .0703.

** *N. del T.* — 236,526 lbs-pulgadas expresan el momento de una fuerza de 236,526 lbs que tiene una pulgada de brazo de palanca; ó bien un brazo de palanca de 100 pulgs á cuyo extremo obra una fuerza de 2,365.26 lbs; y así se pueden cambiar el brazo de palanca y la fuerza siempre que el producto de ellos dé el valor 236,526 del momento dicho. Para encontrar, pues, el número de kilogramos que obrando con un metro de palanca den un momento equivalente, es necesario convertir las pulgadas del brazo de palanca en metros multiplicándolas por .0254 y las libras en kg multiplicándolas por .4536, ó sea multiplicar el momento en (pulgs-lbs) por el producto $.4536 \times .0254$, es decir, por la fracción .01152. Para convertir, á la inversa, en lbs-pulgs, se multiplican los primeros por 86.805.

**19 g.** En ambos casos, (1) y (2), la intersección de la línea radial para $f_s = F_s = 16{,}000$, con la curva de puntos para $f_c = F_c = 500$, daría (sobre un diagrama bastante amplio, exacto), $p = P = .004987$); $M/bd^2 = 71.5$ y $M = 71.5\ bd^2 = 228{,}800$ lbs-pulgs $= 2{,}636$ kg-m, el momento real, para los $b$ y $d$ dados, en el **caso ideal** en que $f_s$ y $f_c$ = respectivamente $F_s$ y $F_c = 16{,}000$ y 500.

## II. Diagrama.

**20 a. Recíprocamente, dado el momento de flexión**, 2,724 kg-m $= 236{,}500$ lbs-pulgs; $F_s = 1{,}120$ kg por cm cuad $= 16{,}000$ lbs por pulg cuad; $F_c = 35$ kg por cm cuad $= 500$ lbs por pulg cuad; de donde $P = .004987$, como antes. **Se pregunta el valor de b y d.**

Sean K y J = los valores de $k$ y $j$ respectivamente, que corresponden á $f_s = F_s$ y $f_c = F_c$.

Aquí tenemos :

$$Pn = 15 \times .004987 = .075;$$

$$K = \sqrt{(Pn)^2 + 2Pn} - Pn$$

$$= \sqrt{.075^2 + .150} - .075 = .3193;$$

$$J = 1 - K/3 = 1 - .1064 = .8936;$$

$$bd^2 = \frac{M}{F_s PJ} = \frac{2M}{F_c KJ} = \frac{2 \times 236{,}500}{500 \times .3193 \times .8963} = 3315.$$

**20 b. Un número infinito de áreas de sección**, $bd$, que dan el mismo momento de resistencia, M, pueden deducirse de $bd^2$.

**20 c.** Así, en el ejemplo del § 20 *a*, con $bd^2 = 3315$, tenemos

| $b$ | $d^2$ | $d$ | |
|---|---|---|---|
| 6 | 552 | 23.5 | |
| 8 | 414 | 20.3 | |
| 10 | 331 | 18.2 | etc., etc. |

**20 d.** Puede demostrarse (T. and M., págs. 175-6) que, dados M, las unidades de resistencia, y la unidad de precios, el **costo** de una viga de concreto armado, por unidad de longitud, varía á la inversa de $d$ en razón directa de $\sqrt{b}$, y en razón directa de $\sqrt[3]{b/d}$. En consecuencia, para un $bd$ dado, mientras más alta sea la viga, menor será el costo; pero consideraciones prácticas (tales como los límites prácticos para la reducción de $b$, etc.), restringen á menudo el límite á que puede llevarse en la práctica esta economía.

**21. Dentro del límite de los esfuerzos de trabajo permitidos**, fig. 2, los esfuerzos y las deformaciones, en las varias fibras, se suponen (hipótesis 1, § 4) proporcionales á las distancias á que están las fibras del eje neutro, como las representa el triángulo sombreado en la pequeña figura de arriba en los diagramas (triángulo que representa aproximadamente la porción inferior del área parabólica que se ve en la fig. 3); y tenemos, ecuación (7), § 10,

$$k = \sqrt{(pn)^2 + 2pn} - pn.$$

**22. Para esfuerzos que excedan de los esfuerzos de trabajo permitidos**, fig. 3, la hipótesis 1 es inadmisible, y debemos emplear toda el área parabólica, cuyo vértice corresponde con el máx de fuerza compresiva del concreto; así tenemos

$$k = \sqrt{(3pn/2)^2 + 3pn} - 3pn/2 \qquad (14)$$

La fig. 3 da valores de $j$, $k$ y $M/bd^2$, para valores máx de ruptura de $f_s$ y $f_c$.

**23.** Nótese que, para esfuerzos en el acero, $f_s$, que no excedan del límite de elasticidad usual y con $f_c$ máx $\leqslant$ 2,000 lbs por pulg cuad, es decir, $\leqslant$ de 140 kg por cm cuad, el **momento de resistencia máx ó de ruptura aumenta directamente con la cantidad** de la armazón hasta que ésta llega á 2 por ciento ó más. Así, la fig. 3, con $f_s = 30{,}000$ lbs por pulg cuad $=$ 2,100 por cm cuad, $f_c$ máx $\leqslant$ 2,000 (ó sea $\leqslant$ de 140 kg por cm cuad) $p = 0$ á 2 por ciento, tenemos $M/bd^2$ = aproximadamente á $25{,}000\ p$.

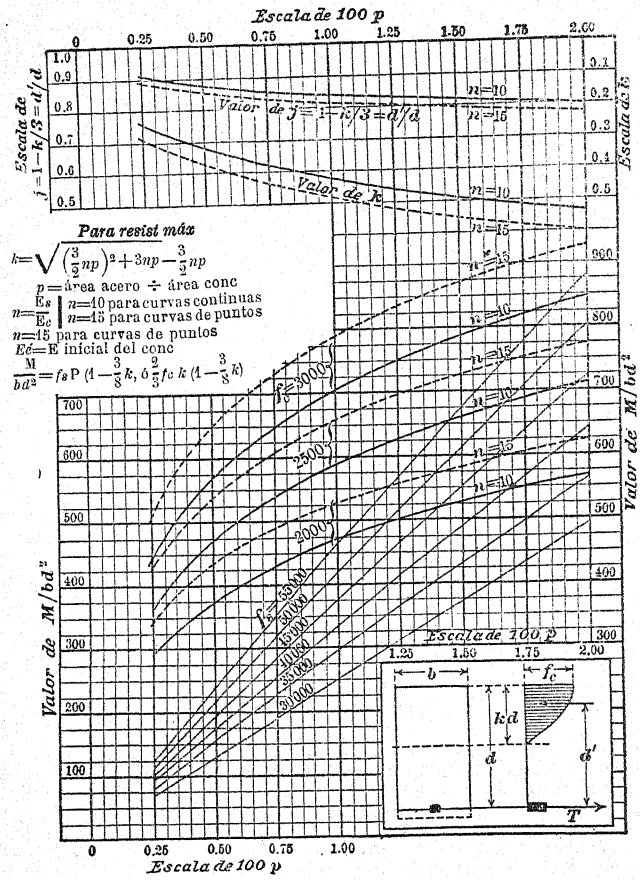


**Fig. 3.** Para esfuerzos de ruptura ó esf máximos. (Para fuerzas permitidas, véase fig. 2.)

$$k = \sqrt{\left(\frac{3}{2}pn\right)^2 + 3pn} - \frac{3}{2}pn, \qquad j = d'/d;$$

$f_s$=esfuerzo por unidad de superficie ó esf unitario en el acero, $f_c$=el mismo en el concreto y en la parte sup de la viga;

$p = a_s/a_c$=relación entre el área del acero y el área del concreto;

$M_s$, $M_c$=momento de resistencia, basado en el valor máximo permitido de $f_s$, $f_c$ respectivamente;

M=momento de resistencia, real

$$\frac{M}{bd^2} = f_s p\left(1 - \frac{3}{8}k\right) \text{ ó } \frac{2}{3}f_c k\left(1 - \frac{3}{8}k\right).$$

$n = E_s/E_c$. Las **curvas continuas** representan $n=10$; las **curvas de puntos**, $n=15$. Las **líneas de acero** para $n=10$; aproximadamente para $n=15$. $E_c = E$ *inicial* para el concreto.

## Secciones en T.

**24. Secciones en T** sencilla, fig. 4. $b$=ancho de la cabeza ó tabla; $b'$=ancho del alma; $t$=espesor de la tabla; $d$=diste de la parte sup de la viga al centro del acero; $kd$=altura del eje neutro; $d'=jd$=brazo de palanca de T y C.

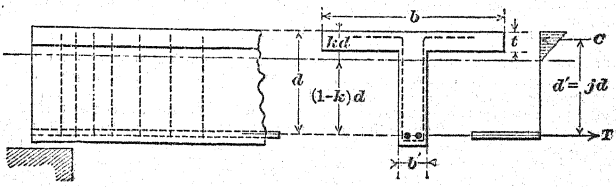


**Fig. 4.** Sección de una viga armada en forma de T simple. Teoría.

**25.** Cuando los topes de vigas rectangulares están unidos por losas, colocándose el todo de una vez y trabándose debidamente, el todo ó parte de la losa puede considerarse como una **tabla de compresión**, de cierto modo análogo á los ángulos y tablas de las vigas laminadas.

**26. El ancho de la losa,** $b$, fig. 4, que obra como tabla, se toma á veces igual á la distancia entre las vigas, pero no debe exceder de 1/3 de la luz de las vigas. Véase Especificaciones, §§ 168-178.

**27.** Un análisis exacto de tal sección es en extremo difícil, pero se cree que el siguiente **método** es racional y seguro.

**28.** Determínese la relación, $p=a_s/a_c$, del área de acero al área de concreto como si la viga fuese rectangular, con altura=$d$, y ancho=al ancho de la cabeza ó tabla, $b$. Con este valor de $p$, determínese la posición del eje neutro. Si éste cae dentro de la tabla ó cabeza ó cerca de su cara inferior, el momento de resistencia se encuentra exactamente como en el caso de cualquier sección rectangular. Véase Caso 1, § 19.

**29. Si el eje neutro cae por debajo** de la cara inferior de la tabla, la posición del eje neutro no será dada exactamente por la ecuación de las vigas rectangulares; pero la diferencia no será importante.

**30. El momento de resistencia** es $Cd'$ ó $Td'$, el que sea menor.

**31.** Ejemplos.

(1) **Eje neutro dentro de la tabla ó de la losa.**

Sea * $b = 1.50$ m $= 60$ pulgs; $b' = 20$ cm $= 8$ pulgs; $d = 50$ cm $= 20$ pulgs; $t = 12.5$ cm $= 5$ pulgs; esfuerzo máx permitido por unidad de superficie, $F_c = 35$ kg por cm cuad $= 500$ lbs por pulg cuad*; $F_s = 1{,}120$ kg por cm cuad $= 16{,}000$ lbs, etc., por pulg cuad; $E_c = 210{,}000$ kg, etc. $= 3{,}000{,}000$ lbs, etc.; $E_s = 2{,}100{,}000$ kg, etc. $= 30{,}000{,}000$ lbs, etc.; $n=10$. Supónganse 3 barras redondas de acero, diám 2.5 cm $= 1$ pulg.

Entonces

$$p = \frac{3 \times .785}{60 \times 20} = .002;$$

$$k = \sqrt{(pn)^2 + 2pn} - pn$$

$$= \sqrt{(10 \times 0.002) + 2 \times 10 \times .002} - 10 \times .002 = .18;$$

$kd = .18 \times 20 = 3.6$ pulgs;

$C = F_c bkd/2 = 500 \times 60 \times .18 \times 20/2 = 54{,}000$ lbs;

$T = 3 \times .785\ F_s$ = digamos 37,650 lbs.

Usando el valor más pequeño (el del acero) tenemos :

$M = Td' = T(d - dk/3) = 37{,}650\ (20 - 3.6/3) = 707{,}000$ lbs-pulgs $= 8{,}145$ kg-m .

* *N. del T.* — Véase *Obs. del T.*, § 18, pág. 1329, y *N. del T.* al pie de la misma página.

(2) **Eje neutro por debajo de la losa** **.

Sea $b = 1.5$ m $= 60$ pulgs; $b' = 25$ cm $= 10$ pulgs; $d = 75$ cm $= 30$ pulgs; $t = 10$ cm $= 4$ pulgs; $F_c$, $F_s$, $E_c$, $E_s$ y $n$ como en el ejemplo (1); 6 barras de acero redondas, diám $= 2.5$ cm $= 1$ pulgada. Entonces

$$p = \frac{6 \times .785}{60 \times 30} = .0026, \text{ y } k = .2; \; kd = .2 \times 30 = 6.$$

**32.** Puesto que la compresión unitaria *$= F_c = 35$ kg por cm cuad $= 500$ libs por pulg cuad, la fuerza, en el lado inferior de la tabla ó losa, es $500\ (kd - t)/kd = 500 \times 2/6 = 167$; y el **esfuerzo medio, en la tabla,** es $\frac{500 + 167}{2} = 333$ lbs por pulg cuad $= 23.3$ kg por cm cuad.

**33.** Las 2 pulgs (5 cm) del alma que están entre el eje neutro y el lado inferior de la tabla ejercen alguna resistencia á la compresión, pero ésta se desprecia, con un pequeño error del lado de la seguridad.

**34. La posición del centro de compresión** en la tabla puede encontrarse como si fuera el de un trapezoide; pero se acostumbra, y es seguro y suficientemente aproximado, suponer que está en el centro de la tabla, ó, en este ejemplo, á una distancia de $d - t/2 = 30 - 2 = 28$ pulgadas por encima del centro del acero. El momento de estas fuerzas es entonces $M = 333 \times 60 \times 4 \times 28 = 2{,}238{,}000$ lbs-pulgs $= 25{,}782$ kg-m **; pero, el momento de la resistencia tensiva del acero es solamente $M = 6 \times .785 \times 16{,}000 \times 28 = 2{,}110{,}000$ lbs-pulgs $= 24{,}307$ kg-m; y este momento, que es el menor de los dos, ha de tomarse como el momento real, M.

## Esfuerzo cortante.

**35. Esfuerzo cortante.** Además de los esfuerzos horizontales, á que resiste la compresión en el concreto y la tensión, en la armazón longitudinal de acero hay que atender á los esfuerzos cortantes verticales en vigas relativamente altas bajo cargas pesadas.

**36. Para el esfuerzo cortante total, V,** en cualquier sección vertical que dista $x$ de un apoyo, tenemos :

$$V = R - W \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (15)$$

donde R = á la reacción ascendente en el apoyo ;

W = el total de las cargas en la distancia $x$.

**37. El esfuerzo cortante vertical** se precave á veces empleando un gran factor de seguridad con el máx esfuerzo cortante del concreto, que se supone generalmente de 500 á 800 lbs por pulg cuad $= 35$ á 56 kg por cm cuad, mientras el esfuerzo cortante de trabajo está á menudo restringido á 30, ó 50 lbs por pulg cuad $= 2.10$ á 3.5 kg por cm cuad. Pero véase Estribos, §§ 38, etc.

## Armadura para el esfuerzo cortante. Estribos.

**38.** Cuando la carga produce un esfuerzo cortante que excede del límite supuesto para el concreto común, la viga se refuerza á menudo con **estribos** verticales, que consisten en barras ó varillas dobladas en forma de U, y que pasan por debajo de las barras horizontales y hasta cerca del tope de la viga; ó, en el caso de las vigas en T (fig. 4), dentro de las ramas horizontales ***.

**39. La distancia entre los estribos** se hace á veces tal, que dentro de una longitud horizontal $= d'$, haya en la sección un área agregada de barras de acero verticales suficiente para soportar el esfuerzo cortante vertical con la tensión unitaria permitida en el acero.

**40. Ejemplo.**

Considérese la viga en T del ejemplo (1), § 31, fig. 4; $b' = 20$ cm $= 8$ pulgs; $b = 1.50$ m $= 60$ pulgs; $d = 50$ cm $= 20$ pulgs; $k = .18$; $d' = 20 - kd/3 = 20 - 1.2 = 18.8$; momento seguro de resistencia, $M = 8{,}145$ kg-m $= 707{,}000$ lbs-pulgs. Sea la

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* *N. del T.* — Compresión por unidad de superficie.

** *N. del T.* — Véase *Obs. del T.*, § 18, pág. 1329, y *N. del T.* al pie de la misma página.

*** *N. del T.* — Dice Marvá que, en las vigas T, sirven también para transmitir los esfuerzos á la unión de la rama horizontal con la vertical y que en toda clase de vigas son un correctivo *á las hiendas inclinadas* 45° que tienden á producirse cerca de los apoyos, como consecuencia del esfuerzo cortante.

luz L=6.10 m=20 pies=240 pulgs. Entonces, para una carga uniforme, tenemos W=8M/L=8×707,000/240=23,600 lbs=10,705 kg.

Esfuerzo cortante en los extremos W/2=11,800 lbs=5,352 kg.

Con un esf cortante de seguridad por unidad de superficie de 3.5 kg por cm cuad=50 lbs por pulg cuad, tenemos para la resistencia de seguridad del esfuerzo cortante del concreto común en la sección=50 $b'd'$=50×8×18.8=7,500 lbs =3,402 kg.

Bajo carga uniforme, este esfuerzo cortante ocurre á una distancia de los extremos,

$$= \frac{(11,800 - 7,500)\ L}{2 \times 11,800} = 3.65 \text{ pies} = 1.11 \text{ m.}$$

Deste este punto hasta el centro de la luz, puede el concreto soportar el esfuerzo cortante, y allí no se necesitan estribos. Pero véanse §§ 41, 45.

Entre este punto y cada apoyo supongamos que los estribos sean de 3/8 de pulg (10 mm) de acero redondo; el área de la sección transversal total de los dos lados de cada estribo=.22 pulg cuad.=1.42 cm cuad.

Concediendo 1,120 kg por cm cuad=16,000 lbs por pulg cuad, un estribo sostendrá 16,000×.22=3,520 lbs=1,141 kg.

El esfuerzo cortante total, 11,800 lbs=5,350 kg, en el soporte, dividido por 3,520, da 3.3 como **número de estribos** requeridos, en 18.8 pulgs=47.7 cm de longitud de viga; ó la **distancia, próxima á los extremos**, deberá ser $\frac{18.8}{3.3}$ = 5.5 pulgs=14 cm. Supongamos la carga, W, = 23,600 lbs=10,704 kg, uniformemente distribuída. Entonces, en un punto á 3 pies=.91 cm del extremo, $V = \frac{10 - 3}{10} \times 11.800 = 8,260$ lbs; 8,260/3,520=2.35; y la distancia entre los estribos=18.8/2.35=8 pulgs=20 cm.

**41. La distancia** puede hacerse **variar uniformemente** entre estos límites; y sería conveniente para el refuerzo vertical extenderse más allá del punto final teórico (3.6 pies=1.06 m del extremo; véase § 40) con uno ó dos estribos separados entre sí un pie=.305 m de distancia. Véase § 45.

**42.** Sean

A=Area total de la sección transversal vertical de las barras horizontales, en pulgadas cuadradas *;

L=luz, en pies *;

$z$=distancia entre el extremo de la viga y el estribo, en pies;

S=Area total de la sección transversal **requerida en los 2 miembros del estribo**, en pulgs cuads.

Entonces, cuando los estribos están á 1 pie de distancia,

$$S = \frac{4A}{L}\left(1 - \frac{2z+1}{L}\right) \qquad (16)$$

(J. W. Schaub, E. N., 03/abril/16, pág. 348.)

**43.** En general, el espacio entre los estribos $> d'$.

**44.** El concreto, en cada sección, debe obrar como un medio de conexión entre el refuerzo ó armadura horizontal y el vertical. También está sujeto á fuerzas compresivas al transmitir el esfuerzo cortante de un estribo al inmediato. La acción aquí es compleja, y **debe emplearse un factor suficiente de seguridad.**

**45.** A fin de proveer contra cargas excesivas que puedan obrar temporalmente sobre las vigas durante la construcción, es conveniente emplear estribos, aun donde no se requieran realmente para los esfuerzos cortantes determinados teóricamente, como se dijo antes. Siendo livianos los estribos, el costo de su empleo está principalmente en el trabajo de mano; de modo que si se necesitan, deben emplearse con liberalidad. Véase § 41.

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* *N. del T.* — ara convertir cm cuadrados en pulgs cuad se multiplican aquéllos por .155; para convertir, á la inversa, pulgs cuad, en cm cuad se mult aquéllas por 6.45. Para convertir metros en pies se multip los m por 3.28, y para convertir pies en m se multip los pies por 0.3048.

## Esfuerzo cortante por unidad de superficie ó esfuerzo cortante unitario.

**46.** En cualquier sección horizontal de una viga, fig. 5, bajo carga uniforme ó central, las fuerzas horizontales de tensión ó de compresión aumentan desde los extremos, donde son cero, hacia el centro de la viga, donde alcanzan su máximo. En consecuencia, de dos secciones de plano vertical cualesquiera, 1 y 2, la sección 2, más próxima al centro de la viga, soportará los esfuerzos horizontales mayores.

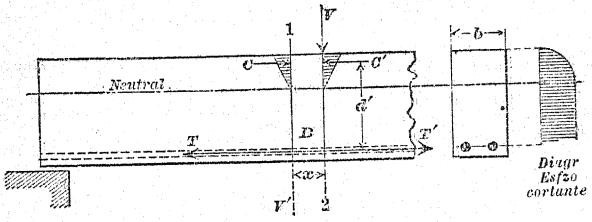


**Fig. 5.** Unidad de esfuerzo cortante.

**47.** Considérense las fuerzas como obrando sobre el cuerpo rectangular, B, entre las dos secciones 1 y 2.

**48.** En la sección izquierda, 1, el esfuerzo cortante vertical, V', procedente del soporte izquierdo, empuja á B hacia arriba; y la tensión, T, del acero hala á B horizontalmente hacia la izquierda; mientras la compresión total, C, que obra en el centro de las fuerzas de compresión, empuja á B hacia la derecha.

**49.** En la sección 2 de la derecha, el esfuerzo cortante vertical, V, empuja á B hacia abajo; mientras T' y C' están respectivamente en la misma línea que T y C, pero opuestas á ellas. Nótese que $T' > T$, y $C' > C$. Sea $T' - T = t$.

**50.** Supongamos que **no hay carga** sobre B. Entonces $V' = V^*$. Puesto que las fuerzas verticales distan $x$, su momento $= Vx = V'x$ *. El momento de $T' - T$ es $(T' - T)\ d' = td'$. En consecuencia, **para el equilibrio**, tenemos :

$$Vx = td'; \quad \text{ó} \quad t = Vx/d' \ldots\ldots (17)$$

**51.** En una viga de concreto armado, fig. 5, despreciamos la tensión del concreto. En consecuencia, la diferencia, $T' - T = t$, de tensión, entre las secciones 2 y 1, debe transmitirse, del acero al eje neutro, por un esfuerzo cortante total $= t$, uniforme * en cada sección horizontal; y, puesto que la sección horizontal del cuerpo, B, es $bx$, tenemos, para el **esfuerzo cortante** por unidad de superficie (ó esf cort unitario) ·

$$v = t/bx = Vx/d'bx = V/bd' = V'/bd' * \ldots\ldots (18)$$

### Esfuerzos diagonales.

**52.** Los esfuerzos de tensión longitudinales y los esfuerzos cortantes verticales y horizontales se combinan para formar esfuerzos **diagonales** que los reemplazan, y la armadura contra el esfuerzo cortante se determina más racionalmente encontrando, tan aproximadamente como se pueda, las direcciones é intensidades de estos esfuerzos diagonales resultantes (véase § 53), y colocando la armadura del mejor modo posible para resistirlos.

**53.** Del « Máximo esfuerzo por unidad de superficie en las vigas », pág. 523, tenemos, en vigas homogéneas, para **el ángulo**, A, entre el eje neutro y la normal

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* **Si hubiese una carga**, L, sobre B (como, por ejemplo, en el caso de cargas uniformes), tenemos $V' > V$, y $V' - V = L$; y tendremos *dos* pares de fuerzas verticales, con los momentos respectivos siguientes : $Vx$ y $(V' - V)x'$, donde $x' =$ distancia de la sección 1 al centro de gravedad de L. Aquí tenemos, para la sección 1, $v' = V'/bd$ ; y para la sección 2, $v = V/bd'$.

resultante (tensión y compresión) ó esfuerzos « principales », $sp$ en cualquier punto:

$$\tan 2A = 2v/s; \quad \ldots\ldots (19)$$

y, para el **esfuerzo máximo**:

$$sp = s/2 + \sqrt{(s/2)^2 + v^2}; \quad \ldots\ldots (20)$$

donde $v$=al esf cort vertical ú horiz por unidad de sup y $s$=al mismo, pero de tensión ó compresión horizontal en el punto dado.

**54. Pero, despreciando el esfuerzo de tensión del concreto**, tenemos, en vigas con armaduras de barras rectas para tensión, y para puntos entre el eje neutro y el acero, $s=0$; de donde

$$\tan 2A = \infty; \quad 2A = 90^\circ; \quad A = 45^\circ;$$

$$sp = \sqrt{v^2} = v = V/bd'. \quad \ldots\ldots (21)$$

**55.** En consecuencia, entre el eje neutro y el acero, deberíamos poner armadura contra el **esf unitario de tensión**, $sp = V/bd'$, obrando en direcciones paralelas y formando ángulos de 45° con el eje neutro.

**56.** Esto se hace con preferencia por medio de barras, **colocadas como barras de tensión** de un puente de armadura de Pratt, figs. 7 *b*, 8 *b*, 9 *b*, págs. 751 y 752, y formando ángulos de 45° con la horizontal.

**57.** Generalmente, las barras de tensión, en cada extremo, en una distancia horizontal casi igual á la altura de la viga, se doblan hacia arriba para formar aproximadamente un ángulo de 45°, con el eje de las vigas.

## Adhesión. Véase pág. 1321.

**58. Unidad de adhesión.** Sean

$x$=una porción dada de la longitud de la viga;
$t = T' - T$=el aumento, en tensión total, T, en el acero y en la longitud, $x$;
U=el esfuerzo cortante vertical total en la sección transversal;
$d'$=la distancia entre T y el centro de compresión del concreto;
$U = t'x$=la fuerza de unión (ligazón), por unidad de $x$;
$m$=el número de barras;
$a$=la circunferencia de una barra
=el área de contacto circunferencial de una barra, por unidad de $x$;
$u = U/ma$=á la fuerza de unión, por unidad de $a$

Entonces (véase § 50), $td' = Vx$; $t = Vx/d'$; $U = t'x = Vx'x/d'x = V/d'$; y

$$u = U/ma = V/d'ma. \quad \ldots\ldots (22)$$

**59.** Para valores dados de la fuerza de unión, U, por unidad de longitud, y de la fuerza de unión, $u$, por unidad del área de contacto circunferencial, el producto, $ma = U/u$ (=área circunferencial total por unidad de longitud) para un caso dado, es constante; pero el área de la sección transversal, peso y costo de las barras aumentan como el *cuadrado* de $a$. En consecuencia, para una adhesión total dada, **son más económicas varias barras pequeñas** que menor número de barras más grandes; pero hay naturalmente, para cada caso, un límite práctico en esta economía.

## Vigas continuas.

**60.** Los sistemas de pisos se componen generalmente de losas y vigas continuas sobre soportes; y, si los momentos de flexión negativos sobre los apoyos (que producen *tensión* en el *tope* de la viga) se proveen ampliamente, con armadura cerca de la parte superior, y si los apoyos son inflexibles, ó exactamente de igual flexión, aprovéchase de ordinario la reducción en los momentos positivos de flexión (en el centro de la luz y cerca de él) debido á su continuidad.

**61.** Cuando se colocan losas de piso continuas sobre las vigas de apoyo, es corriente suponer $WL/10 = wL^2/10$ como momento máximo de flexión, donde L=luz; W=carga total sobre la luz; $w = W/L$=carga por unidad de L. Las vigas, continuas sobre los apoyos, pueden tener un valor semejante al empleado en el diseño, si las vigas están ampliamente reforzadas en su parte superior y sobre los apoyos.

**62. En cuanto á la seguridad**, se especifica frecuentemente que las vigas

las losas, etc., se considerarán como no continuas sobre los soportes, obligando esta práctica á prever, en el centro de la luz, contra momentos (positivos) de flexión mayores que si la viga fuese continua sobre los apoyos; pero, por otra parte, muy pocas vigas son totalmente no continuas; esto es: aun cuando la viga se supone no continua, hay momentos de flexión negativos sobre los apoyos, debidos al ancho del apoyo y á la carga en la viga sobre el apoyo. Tales momentos requieren armadura en la parte superior, sobre los apoyos y cerca de ellos.

**63.** En consecuencia, si bien es conveniente, en el caso de vigas no continuas, calcular el momento positivo de flexión central en el supuesto de absoluta no continuidad, las condiciones, aun de vigas no continuas, sobre sus apoyos, debe investigarse cuidadosamente y debe proveerse para los momentos negativos que allí se encuentren.

**64. Doble armadura.** La necesidad, en ciertos casos, de crear refuerzos contra los momentos negativos y contra momentos positivos (§ 62), obliga á colocar armaduras (fig. 6) cerca de la parte superior é inferior de la sección. Para mayor brevedad, aquel lado que, bajo momento positivo, sufre compresión, se llamará « armadura de compresión ».

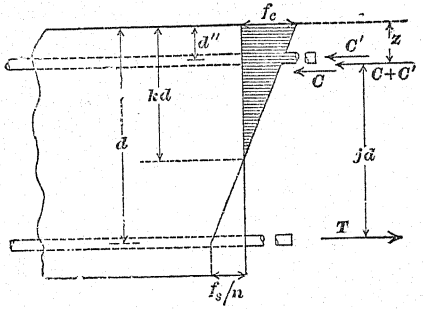


**Fig. 6.** Doble armadura.

**65.** Además de los símbolos del § 5, sean

$a_s'$ = área de la sección transversal del refuerzo de compresión;
$p' = a_s'/a_c = a_s'/bd$ = proporción del acero para la armadura de compresión;
$f_s'$ = esfuerzo unitario (por unidad de superficie) en la armadura de compresión;
$C'$ = esfuerzo total en la armadura de compresión;
$d''$ = distancia de la armadura de compresión á la cara más próxima de la viga:
$z$ = distancia de la resultante de las compresiones, $C+C'$, á la cara más próxima de la viga.

**66.** Entonces (despreciando la pequeña disminución de $a_c$; por la presencia de $a_s'$) para **la posición del eje neutro**;

$$k = \sqrt{2n\,(p+p'd''/d) + n^2\,(p+p')^2} - n\,(p+p'); \qquad (24)$$

**para la posición de la resultante de las compresiones:**

$$z = \frac{k^3 d/3 + 2p'nd''\,(k - d''/d)}{k^2 + 2p'n\,(k - d''/d)} \qquad (25)$$

**para brazo del par resistente:**

$$jd = d - z; \qquad (26)$$

**para esfuerzos de la fibra:**

$$f_c = \frac{6Mk/bd^2}{3k^2 - k^3 + 6p'n\,(k - d''/d)\,(1 - d''/d)} \qquad (27)$$

$$f_s = M/pjbd^2 = nf_c\,(1-k)/k \qquad (28)$$

$$f_s' = nf_c\,(k - d''/d)/k \qquad (29)$$

## Sistemas de armaduras.

**1.** La **teoría comúnmente aceptada** de vigas de concreto armado requiere armaduras de tensión longitudinal cerca del fondo * de la viga, y armadura de tensión diagonal á 45°, no sólo entre la armadura horizontal y el eje neutro, sino extendiéndose hacia arriba en la región de la compresión, á fin de aprovechar la adhesión superior debida á la compresión ejercida allí. Requiere también, generalmente, armadura de tensión cerca del tope *, en puntos situados sobre los apoyos ó cerca de ellos.

Véanse §§ 60, etc.

**2.** Se han ideado numerosos **sistemas de armazones** (pág. 1344) para atender á estas condiciones y se emplean mucho cuando las alturas de las vigas son suficientes y cuando la carga los exige.

**3.** Frecuentemente se sustituyen los **estribos verticales** por miembros diagonales ó se usan en combinación con ellos; ó se hace la armazón doblando simplemente algunas ó todas las barras horizontales del fondo * hacia arriba, de ordinario á 45° ó más ó menos.

**4.** Para **carga ligera** la armazón consiste simplemente en barras longitudinales cerca del fondo * de la viga.

**5.** Cuando la viga es á la vez delgada y ancha, como en las **planchas de pisos**, las pocas barras longitudinales, empleadas en la viga, se reemplazan (1) por barras numerosas y comparativamente delgadas, completadas por barras semejantes ó más ligeras, que las cruzan en ángulos rectos, soldadas ó atadas á ellas con alambre en sus intersecciones; ó (2) por un tejido, como la tela de alambre ó el « metal desplegado ». Véanse §§ 34, etc.

## Armaduras de barras.

**6.** Para un peso dado de metal, las **barras delgadas** dan mayor área de adhesión, y por tanto una adhesión total mayor que las barras más gruesas (§ 59, pág. 1337); y los esfuerzos se distribuyen sobre un área mayor de concreto. Además, empleando barras delgadas, una proporción mayor de metal puede bajarse hasta la distancia mínima permitida del fondo* de la viga. Dentro de ciertos límites, las barras delgadas se manejan más cómodamente que las gruesas. Las

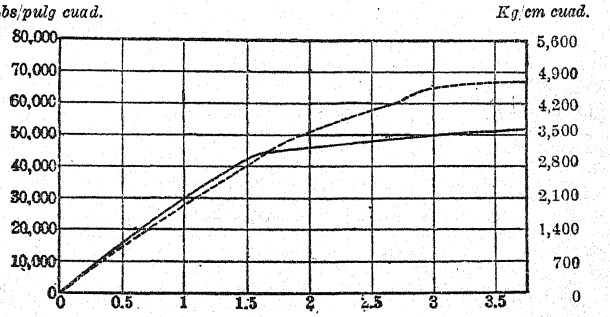


**Fig. 1.** La línea de puntos se refiere á barras torcidas, la continua á barras no torcidas.

barras usadas son rara vez $< 6$ mm ó $> 5$ cm de diámetro y varían por lo común entre 10 y 40 mm. En vigas altas, dos ó más hileras de barras delgadas son preferibles generalmente á una sola hilera de barras más gruesas.

**7.** En las armaduras verticales se doblan los **extremos libres de las barras** sobre la pieza, y los golpes accidentales sobre dichos extremos pueden transmitirse

*Los términos parte inferior ó « fondo » y parte superior ó tope se usan aquí refiriéndose á una viga sostenida en los extremos y cargada en la parte superior, y en la que la mayor porción del « fondo » está en tensión. En un *cantilever* es todo lo contrario.

á las partes ya incrustadas en el concreto, dañando su adhesión. Por este respecto son también preferibles las barras delgadas, porque transmiten menos los efectos de los golpes.

**8. Las barras de acero rico en carbón**, con sus altos límites de elasticidad, permiten el uso de secciones más pequeñas para un número de barras y un esfuerzo dado, pero son más frágiles (cuando son de calidad inferior) que las más blandas, y no se doblan fácilmente en frío para darles las formas deseadas. La pequeñez de las secciones comúnmente usadas y la protección que da el concreto hacen menos objetable la fragilidad en las obras de concreto armado que en casi toda otra obra en que se emplea el acero.

**9.** Puesto que el módulo de elasticidad del acero y del hierro laminados es casi el mismo (digamos 2,100,000 kg por cm cuad) para todas las clases, éstos se alargan todos lo mismo, por unidad de longitud, bajo unidades de esfuerzos iguales; pero el **acero con un límite elevado de elasticidad**, permitiendo el uso de secciones más pequeñas y á las que, por lo tanto, corresponden unidades de fuerza más altas, hace más probable el estiramiento, agrietándose en consecuencia el concreto; y además la contracción lateral del acero **pone en peligro la adhesión**. Por este motivo, se especifica á veces que, donde el límite de elasticidad excede de cierto mínimum (digamos 2,800 kg por cm cuad), se usarán barras torcidas, §§ 15, etc. A 2,100 kg por cm cuadrado, el acero se estira cosa de .10 por ciento; á 3,500, como .17 por ciento.

El trabajo en frío eleva el máximo de resistencia y el límite de elasticidad, pero baja un poco el módulo de elasticidad; véase fig. 1, que indica las pruebas hechas en el arsenal de Watertown (Pruebas de metales, 1904, pág. 397) sobre barras cuad de acero lisas y torcidas en frío, de 19 mm. Longitud estudiada 25 cm. La barra torcida tenía una vuelta en 20 cm. Resultados análogos se obtuvieron en las pruebas hechas en el arsenal de Watertown el 12 de julio de 1902 y publicadas por la Ransome Concrete Co. Véase § 21.

Las barras cuadradas de acero inferior se tuercen en caliente y son más frágiles.

**10.** Usanse muy generalmente en los E. U., para armaduras, las **barras de acero lisas redondas** y aún con más extensión en Europa. También se usan las **barras cuadradas**, pero son de manejo menos cómodo. Las barras chatas se han encontrado deficientes en la adhesión.

**11.** A fin de **aumentar la resistencia de las barras lisas** á su extracción del concreto, se doblan frecuentemente hacia arriba en ángulos rectos (ó á 180° de modo que formen un gancho) en sus extremos.

**12.** « El anclaje hecho por dobleces cortos en ángulo recto es menos eficaz que el de ganchos. » **J. C.**

**13.** Con el mismo fin (§ 11), pueden las barras aterrajarse en sus extremos y dotarse de **planchas de anclaje** de acero, aseguradas con tuercas. Estas planchas deben ser bastante grandes y gruesas para resistir los esfuerzos producidos por toda la fuerza de tensión de las barras. Al proyectar estas planchas supone el profesor L. J. Johnson una resistencia á la trituración, en el concreto, de 63 kg/cm cuad, y como resist de la fibra, en la plancha de anclaje, 1,750 kg /cm cuad. Varias barras juntas pasan á través de una gran plancha común, en cada extremo, que sirve también para mantener las barras en sus relativas posiciones mientras se coloca el concreto. Hay una importante previsión que consiste en colocar tuercas en el interior, que presentan á la plancha de anclaje un apoyo firme contra la acción de las tuercas exteriores. Se usan tales planchas en los muros, columnas, etc. De otro modo, y con el fin de darle cabida á la plancha de anclaje, la viga puede ahondarse en aquel lugar ó encorvar las barras hacia arriba cerca de sus extremos. Cuando se encorva hacia arriba, la barra ejerce una presión sobre el concreto, cerca del doblez. Esto aumenta la fricción en la porción doblada, y por ello reduce la tracción transmitida á la plancha de anclaje.

**14.** « Es preferible que la resistencia se deba á la conveniente adherencia en toda la longitud de una barra y no al anclaje del extremo. » **J. C.**

**15.** También para aumentar la adhesión (ó mejor dicho, para sustituirla por « una trabazón mecánica ») se usan « **barras deformadas** » de varios tipos.

**16.** La **principal ventaja** que se atribuye á las barras deformadas, es la de que la « trabazón mecánica » que producen, es la única seguridad de la armazón, después que la adhesión propiamente dicha se ha destruído, como sucede con una fuerza que exceda á la adhesión, por infiltración de agua, por golpes, durante ó después de la construcción, ó por constantes y rápidos cambios ó inversiones de carga en el trabajo. Las barras verticales especialmente, durante la construcción están expuestas

á golpes accidentales sobre sus extremos salientes superiores, y estos golpes pueden afectar la adhesión de las porciones ya incrustadas en el concreto.

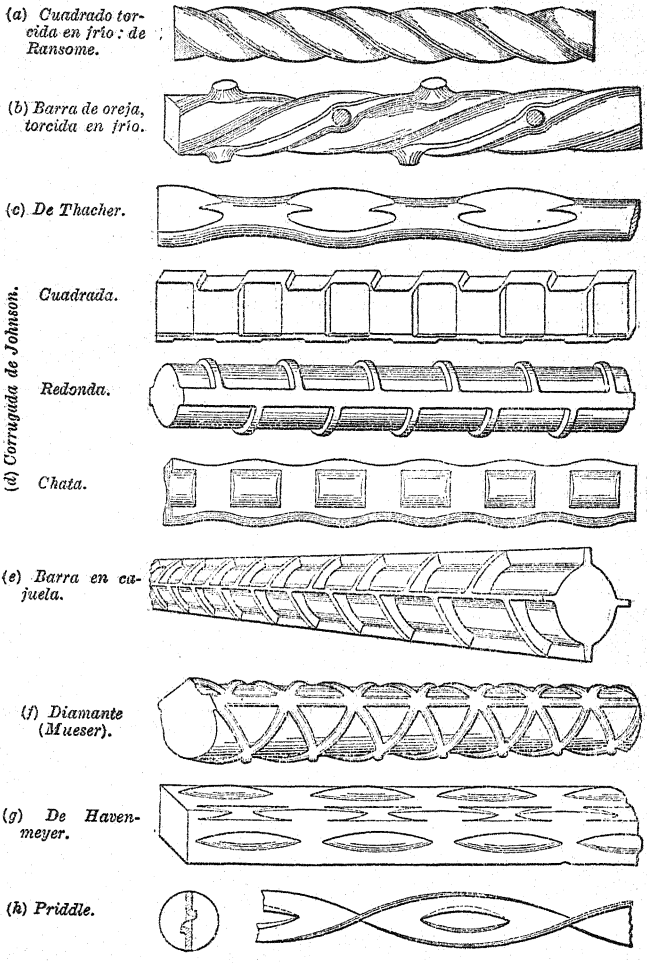


**Fig. 2.** Barras deformadas.

**17. Por otra parte,** se ha observado que numerosas construcciones hechas con barras lisas han resistido satisfactoriamente por años, en trabajos que producen aquellas vibraciones, y se pretende que cualquier ventaja que se derive de las barras deformadas está más que compensada por pequeño que sea el aumento de costo. Las

barras lisas están libres, por supuesto, de que se les pretenda patentar y siempre se encuentran de venta en el mercado.

**18.** Las proyecciones, en las superficies de algunas barras deformadas, pueden dañar la cubierta de concreto á menos que ésta sea de considerable espesor.

**19.** Al estudiar las pruebas comparativas de barras lisas y deformadas, debe prestarse atención á la riqueza de la mezcla del concreto; porque si éste no es bastante rico para lograr que cada barra quede completamente cubierta con cemento, la completa adhesión no se hará con regularidad y la prueba de extracción demostrará principalmente la diferencia con la « trabazón mecánica », en la cual, por supuesto, tendrán ventaja las barras torcidas.

**20.** « Las barras deformadas ofrecen medios apropiados para producir alta resistencia en las armazones. » **J. C.**

Las barras deformadas, figs. 2, tienen un uso más ó menos general :

**21. Ransome.** (*a*) Barras de acero cuadradas, torcidas en frío. Torcidas en el taller ó en el trabajo. (Son convenientes y económicas.)

**22. Barras de orejas torcidas en frío.** (*b*) Barra cuadrada, con ángulos redondos, para impedir la formación de grietas en el concreto, torcidas en frío. Las orejas están destinadas á resistir cualquier tendencia de la barra á destorcerse bajo tensión. Para efecto de trabajo en frío, véase § 9.

**23. Thacher.** (*c*) Barras redondas, deformadas achatándolas á cortos intervalos. El área de la sección transversal prácticamente constante. Cambios de forma hechos por medio de curvas graduales.

**24. Barras corrugadas**; (*d*) ordinariamente de acero con punto **cedente** (véase *N. del T.* § pág. 482*b*) 3,500 kg/cm cuad ó más. Cuadrada, redonda y chata.

**25. Barras de cajuelas** (*e*).

**26. Barra diamante.** (*f*) Laminada redonda, con dos proyecciones en espiral de igual paso y en direcciones opuestas (dividiendo la superficie en cuatro hileras de cajuelas con la forma de las facetas del diamante) y dos bordes longitudinales opuestos. El área de la sección transversal y el peso=los de barras cuadradas lisas de igual denominación. Ventajas que se le atribuyen : área de la sección transversal uniforme, estiramiento uniforme, distribución uniforme en la trabazón, los bordes proyectados ayudan á resistir la tensión; bordes redondos, sin tendencia á destorcerse bajo dicha acción.

**27. Barra Havenmeyer.** (*g*) Cuadrada con esquinas y proyecciones redondeadas.

**28. Barra Priddle** de trabazón interna. (*h*) Barra chata, perforada y torcida, y la ranura con rebordes, como se ve en la figura. Los tamaños pequeños trabajados en frío; los tamaños más grandes, en caliente. Puede formarse un tejido pasando barras más pequeñas, del mismo ó de otro dibujo, por las ranuras.

**29. La barra monolítica** consiste en una pieza de tensión horizontal con eslabones diagonales separados. En su sección la pieza horizontal semeja un carril pesado con dos cabezas en vez de una cabeza y una platina. Cada eslabón es una barra de acero redonda doblada en el extremo, formando así dos piernas diagonales paralelas, las que en la parte inferior se doblan horizontalmente, y estas partes horizontales se colocan en cada lado de la pieza horizontal que las agarra entre sus cabezas, retorcidas hacia adentro en esos puntos y con aquel objeto.

### Apoyos.

**30.** Es de la mayor importancia que las barras longitudinales de la armadura estén colocadas y se mantengan en sus **debidas posiciones**. Si al colocarlas definitivamente, quedan demasiado altas, su palanca de resistencia $d'$, y el momento de resistencia de la viga disminuyen. Si quedan demasiado bajas, tendrán debajo un espesor de concreto insuficiente para su protección. Varios sistemas se usan para mantener las barras en la debida posición.

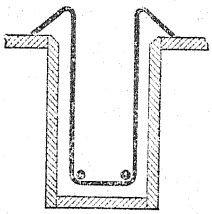

Fig 3.

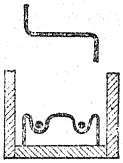

Fig 4.

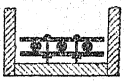

Fig 6.

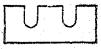

Fig 5.

Apoyo para las barras de la armazón.

**31. Los estribos**, fig. 3, obran como apoyos colgantes para las barras principales.

**32.** A veces se sujetan las barras livianas con apoyos de alambre, fig. 4, ó con bloques de concreto, de 3 á 5 cm de espesor, fig. 5.

**33.** Las barras más pesadas pueden sostenerse con **abrazaderas**, fig. 6, hechas de hierro acanalado 2 á 2.5 cm, sujetas entre sí con pernos de cabeza redonda, de 6 mm á 10 mm de diámetro, colocados en las armaduras á 1.5 ó 2.5 m de distancia.

### Armaduras tejidas.

**34.** Usanse éstas en losas, planchas, etc., anchas y delgadas, en muros delgados, en cloacas y tuberías, en columnas, etc.

**35.** La forma más sencilla consiste en varillas en ángulos rectos y sujetas con alambre ó soldadas en sus intersecciones. Las barras más pesadas, ó principales, se colocan naturalmente de tal manera, que soporten los mayores esfuerzos. Las barras transversales mantienen las barras principales en su posición durante la construcción, y después distribuyen su tensión al través del concreto, ofreciendo así una trabazón mecánica. La malla debe ser bástante grande para dar paso á las partículas del agregado empleado en el concreto

**36. Jean Monier**, de París, empleó dichos tejidos para reforzar los arcos.

**37. Metal desplegado**, fig. 7. El acero laminado se ranura y se despliega en cuadriláteros. Las láminas tienen de 30 á 183 cm de ancho, de 2.44 á 3.60 m de largo; con mallas de 1½ á 15 cm.

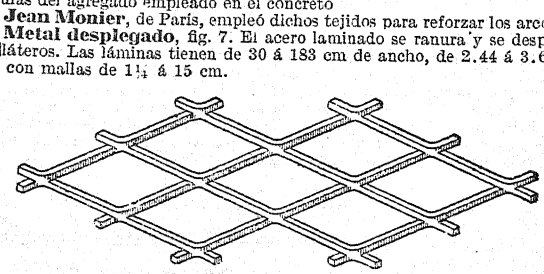

**Fig. 7.** Metal desplegado.

**38.** Cuando se emplea la armadura para losas en láminas cortas, éstas deben recubrirse lo suficientemente para transmitir la tensión de una lámina á la próxima. Este recubrimiento consume como el 10% del área del metal.

**39. Tiras metálicas de Clinton**, en cilindros de 30 á 60 m ó más, de alambres de acero estirados cruzándose en ángulos rectos, de 63 mm de malla, soldados eléctricamente, reforzados por cordones torcidos longitudinales, á 15 cm de separación y hechos cada uno de dos alambres engarzados y torcidos sobre cada cordón que cruza; y, cuando se desea, por atesadores transversales en forma de V de acero del escantillón n° 24, atados á los alambres á intervalos como de 20 cm. Se venden lisos, charolados ó galvanizados de 90 cm de ancho.

**40. Alambre soldado de Clinton**; alambre de acero estirado, n.° 3 á n.° 10 común ó galvanizado; malla, $7\frac{1}{2} \times 20$, $5 \times 30$, $7\frac{1}{2} \times 30$, $10 \times 30$ cm.

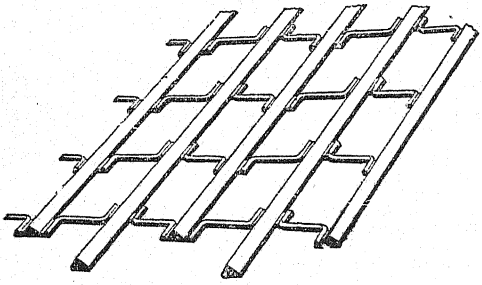

**Fig. 8.** Listoneado de metal.

**41. Listoneado de metal**, fig. 8; desplegado de planchas de acero especialmente laminado, con rebordes longitudinales. La malla varía de 2.5 en 2.5 cm. desde 5 hasta 20 cm. Las hojas son hasta de 4.80 m de largo.

**42. Otro tipo listoneado**, fig. 9.

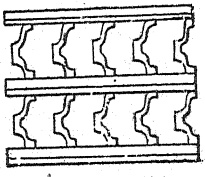

**Fig. 9.** Otro tipo listoneado.

## Armazones.

**43.** En general, las armazones son un poco más costosas que la simple armadura de barras comunes y, si se carga en piezas enterizas, sale más dispendioso su transporte y está más expuesto á sufrir en el viaje; pero tiene la gran ventaja de mantener las barras en su posición mientras se está colocando el concreto y de evitar el olvido ó la mala colocación de los estribos, etc., ya sea por accidente, ó voluntariamente. Las armazones pueden hacerse de barras comunes ó de barras torcidas. Deben estar provistas de medios de conexión por encima de los apoyos.

Sección transversal en el centro.

**Fig. 10.** Barra de Kahn.

**44.** En **las armazones de barras de Kahn**, fig. 10, las aletas laterales salientes están hendidas, en algunos lugares de su parte central, y dobladas hacia arriba, como se ve. La misma barra, invertida, se usa por encima de los apoyos.

**45.** La fig. 11 muestra la armazón llamada « Economy ».

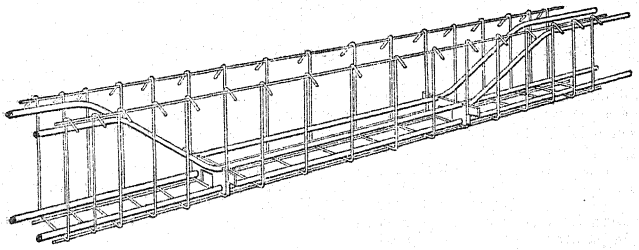

**Fig. 11.** Armazón « Economy

## Armazón con hierro de forma.

**46. El sistema Melan**, inventado por José Melan, de Austria Hungría en 1892, y patentado en los Estados Unidos en 1893, comprende un arco de concreto, en el cual van incrustadas vigas de hierro ó acero. Para luces pequeñas son de ordinario vigas laminadas en I, mientras que para luces considerables, consisten ordinariamente en cuatro ángulos unidos por celosías.

**47.** Cuando se incrusta en concreto una armadura de hierro de forma, de tamaño regular, para formar una viga, de modo que **el acero predomine** y suministre la mayor parte de la fuerza requerida, **el concreto obra principalmente como una cubierta protectora** para el acero; y el caso es apenas un problema de refuerzo propiamente dicho.

**48. Es difícil obtener un relleno perfecto**, con el concreto, en los espacios situados debajo de los rebordes de las formas laminadas ó armadas. En estos casos, cada día de trabajo debe terminar ó muy por encima ó muy por debajo del reborde, pues de otro modo la contracción, debajo de los rebordes, aumenta la dificultad.

## Armaduras para columnas.

**49.** Las columnas se arman por medio de **barras verticales**, colocadas cerca de la circunferencia y ordinariamente sujetas con alambre á intervalos, ó por **envoltura circular** (con sunchos espirales) ó con ambos.

Véase Columnas de concreto armado, pág. 1322.

**50.** En edificios altos las barras de las columnas se achatan á menudo en sus extremos para darles una buena superficie de apoyo y van unidas entre sí por dedales sueltos que mantienen á los extremos en el necesario contacto; debajo de los extremos de las barras, en la base, se coloca una plancha de hierro ó acero, para la distribución uniforme de la carga sobre el concreto de los cimientos.

**51.** En el sistema llamado **mushroom** (¿hongo?), del Sr. C. A. P. Turner, para columnas y pisos, las columnas se ensanchan en sus cabezas, para aumentar el área de apoyo, y la armadura del piso consiste esencialmente en piezas derechas (horizontales ó casi horizontales), que salen de las columnas y unidas, á intervalos, por piezas circulares ó poligonales, que cruzan las radiales generalmente en ángulos rectos. Se prescinde de las vigas y costillas y el piso es de espesor uniforme. Véase E. N., '09, febrero 18, pág. 178.

# EXPERIENCIAS Y PRÁCTICA

**Índice de los resultados seleccionados** de las págs. 1351, etc.

Las palabras en tipo de **letra negra**, que preceden á un punto y coma, se refieren á uno de los dos asuntos mencionados; las palabras en tipo común, que siguen á un punto y coma, al otro. Los numerales y letras se refieren á los informes de los ensayos, etc.

**Ejemplo.** Bajo **ARENA** (más abajo), « **Arena, naturaleza**; densidad del mortero, 8 *c*, *e*; 9 *d*, 86 *c* » se refiere á los Ensayos 8 *c*, etc., que informan sobre el efecto de (1) naturaleza de la arena, sobre (2) la densidad del mortero. Recíprocamente, en la pág. 1188, encontramos « **Mortero, densidad de** —; naturaleza de la arena, 8 *c*, *e*, 9 *d*, 86 *c* ».

## CEMENTO


**Cemento,**
**naturaleza de** —;
agua requerida, 61 *a*
**Portland y natural** —;
agua requerida, 4 *d*
resistencia, 14 *a*, 19 *a*
rozamiento, 4 *g*
permeabilidad, 65 *a*
electrolisis, 75 *a*
**sílica** —; aceite, 53 *d*
**mezcla típica**; 86 *f*
**edad de** —; solidez 29 *a*
**fineza de** —;
solidez, 29 *b*
fuerza del mortero, 4 *f*
agua requerida, 4 *d*
**cantidad requerida**; agregado, 79 *b*, *d*
**cantidad empleada**;
fuerza del mortero, 8 *a*
módulo de elasticidad, 70.5
**exposición**; 39 *a*, *b*
**ácido sulfúrico en** —; 49 *a*
**acción química de** —; 26 *a*, *b*, *c*


## ARENA


**Arena,**
**fineza de** —;
densidad de la arena, 2 *a*, 8 *h*, 8 *j*, 8 *k*
agua requerida, 61 *a*
densidad del mortero, 8 *c*, 9 *d*, 79 *e*
fuerza del mortero, 4 *e*, 8 *a*, 52 *b*, 79 *e*
permeabilidad del mortero, 8 *d*, 9 *e*
cal requerida para la impermeabilidad, 82 *b*
agua de mar, 8 *g*
**coeficiente de uniformidad**; 5 *a*
**graduación de** —;
mortero, 8 *c*, 86 *e*
**forma de granos**;
densidad de, arena, 8 *i*, 8 *l*, 94 *a*
**densidad de** —;
fineza, 2 *a*, 8 *j*, 8 *k*
coeficiente de uniformidad, 5 *a*
forma de granos, 8 *i*, 94 *a*
compactación, 2 *a*, 8 *h*, 8 *i*, 8 *k*, 45 *a*
naturaleza, 8 *l*
mica, 87 *a*
humedad, 2 *a*, 8 *h*, 8 *l*, 45 *a*
mortero, 86 *c*, *d*
**vacíos**;
esferas de diámetro uniforme 45 *b*
**compactación**;
densidad de la arena, 2 *a*, 8 *h*, 8 *i*, 8 *k*, 45 *a*
fineza de la arena, 8 *k*
**humedad en** —;
densidad de la arena, 2 *a*, 8 *h*, 8 *l*
agua requerida, 61 *a*
**naturaleza**;
densidad de la arena, 8 *l*
densidad del mortero, 8 *c*, *e*, 9 *d*, 86 *c*
resistencia, 19 *c*, 39 *g*, 50 *a*, 52 *a*, 62 *a*
absorción, 62 *a*
**impurezas en** —; 19 *c*, 52 *a*
**arcilla y marga en** —;
resist, 4 *a*, 34 *a*, 39 *g*, 50 *b*, 52 *a*, *b*, 56 *a*, 80 *a*
permeabilidad, 4 *a*
absorción, 56 *b*
**mica en** —; 79 *a*, 87 *a*
**fricción** —; 89 *a*
**percentaje de** —;
electrolisis, 91 *a*
rozamiento, 4 *g*
**punto de fusión**; 89 *b*
**vs cedazos**; 79 *a-j*
densidad, 79 *c*
permeabilidad, 79 *h*, *j*
absorción, 55 *a*
**vs piedra caliza triturada**; 50 *a*

**Índice de los resultados seleccionados**, págs. 1351-1400.

## INGREDIENTES ACCIDENTALES

**Arcilla en cemento**; 4 *a*
**Arcilla y marga**;
resistencia del mortero, 4 *a*, 34 *a*, 39 *g*, 50 *b*, 52 *a*, *b*, 56 *a*, 80 *a*
absorción, 56 *a*
plasticidad de la pasta, 4 *a*
densidad de la pasta, 4 *a*
permeabilidad, 4 *a*
mortero para revocar, 4 *a*
en concreto para columnas, 92 *a*
**Arcilla y alumbre**;
permeabilidad, 80 *a*
**Mica**; 79 *a*, 87 *a*
**Ácido sulfúrico**; 6 *a*, 49 *a*
**Sal**; 4 *c*, 19 *a*, 31 *a*
**Yeso**; 51 *a*
**Yeso y cal**; 51 *c*
**Cloruro de calcio**; 51 *a*, *b*
**Cal**; 80 *a*, 82 *d*
**Cal y yeso**; 51 *c*

## AGUA DE LAS MEZCLAS

**Agua, mezcla —,**
**sal en —**; 4 *c*, 19 *a*, 31 *a*
**evaporación de —**; 9 *a*
**cantidad requerida**;
cemento natural y de Portland, 4 *d*
cemento, naturaleza de — 61 *a*
tamaño y sequedad de los granos de arena, 61 *a*
mica, 87 *a*
**ácido sulfúrico en —**; resistencia, 6 *a*

## MORTERO

**Mortero,**
**puro y arena —**; 86 *i*
**consistencia del —**;
fineza del cemento, 4 *d*
ceniza gruesa, 83 *a*
proceso del fraguado, 4 *d*
volumen del concreto, 21 *a*
densidad, 61 *a*
resist, 39 *e*, 61 *a*, 83 *a*
módulo de elasticidad, 61 *b*, 81 *a*
permeabilidad, 33 *a*, 47 *c*, *f*, 61 *a*
lechosidad, 61 *d*; fuego, 46 *e*
preferible —, 61 *e*
agua de mar, 8 *g*
**riqueza de —**;
volumen del concreto, 21 *a*
densidad, 8 *c*, 9 *d*
permeabilidad, 8 *d*, 9 *e*
agua de mar, 8 *g*
**densidad de —**;
tanto por ciento de vacíos, 9 *b*
naturaleza de la arena, 8 *c*, *e*, 9 *d*, 86 *c*
riqueza, 8 *c*, 9 *d*
arcilla, 4 *a*
aire arrastrado, evaporación, 9 *a*
**resistencia de —**;
fineza del cemento, 4 *f*
proporción del cemento, 8 *a*
exposición del cemento, 39 *a*, 39 *b*
naturaleza de la arena, 4 *c*, 8 *a*, 86 *d*
arcilla, 4 *a*, 34 *a*, 39 *g*, 50 *b*, 52 *a*, *b*, 56 *a*, 80 *a*
sal, 4 *c*
ácido sulfúrico, 6 *a*
consistencia, 39 *e*
mezcla á mano y por máquina, 39 *c*
tratamiento de ladrillo, 39 *d*
**permeabilidad de —**;
naturaleza de la arena, 8 *d*, *e*, 9 *e*
riqueza, 8 *d*, 9 *e*
arcilla, 4 *a*
disminución de — con tiempo, 8
**plasticidad de —**; 4 *a*
**solidez de —**;
cemento, 29 *a*, *b*
**rozamiento**; 4 *g*
**expansión de —**; 4 *h*
**cal en —**; 82 *a*
**sal amoníaco en —**; 47 *l*
**ladrillos, tratamiento de —**:
resistencia, 39 *d*
**protección de metales por —**; 2 *b*
**en agua**; 4 *b*, 8 *f*
de mar —, 4 *b*, 7 *a*, 8 *g*
**para enlucidos**;
arcilla en —, 4 *a*
**aereación**;
**proceso del fraguado**, 84 *a*
**proporción de —, en el concreto**;
resistencia, 79 *f*
densidad, 79 *f*
permeabilidad, 13 *b*, 43 *a*, 79 *g*
volumen del concreto, 21 *a*.

## PROPORCIONES

**Proporciones**;
densidad del concreto, 9 *c*
módulo de elasticidad, 81 *a*
resistencia, 14 *a*, 15 *a*, 18 *a*, 19 *b*
esfuerzo cortante, 81 *b*
adhesión, 64 *b*
fuerza de columnas, 85 *a*
permeabilidad, 9 *f*, *g*, 13 *a*, *b*, 25 *a*, 43 *a*, 65 *a*

**Índice de los resultados seleccionados,** págs. 1351-1400.

## MORTERO

conductividad térmica, 46 *b*
electrolisis, 91 *a*
**Graduación;**
distribución, 47 *d*
cemento requerido, 79 *d*
densidad, 79 *d*
permeabilidad, 93 *a*
fuerza transversal, 72 *a*

## AGREGADO

**Agregado;**
fuego, 41 *d*
**proporción al mortero;**
volumen del concreto, 21 *a*
**adición de —;**
retardación del fraguado, 84 *a*
**suciedad en —;**
fuerza, 19 *e*
**peso de —;** 3 *a*
**densidad de —;** 3 *a*
cascajo y piedra rota, 8 *l*, 14 *a*
compactación, 21 *e*
**vacíos en —;**
esferas de diámetro uniforme, 45 *b*
**tamaño de —;**
cemento requerido, 79 *b*
permeabilidad, 79 *i*
densidad, 8 *l*, 79 *b;* resistencia, 79 *b*
módulo de elasticidad, 70.5
**clase de —;**
densidad, 8 *l*
proporciones, 17 *a*
permeabilidad, 79 *g*, 79 *j*
fuerza, 19 *b*, 35 *a*, 83 *a*
**cascajo;** 8 *l*, 79 *a*
resistencia, 39 *f*, 83 *a*
fuego, 41 *c*, 70 *f*
permeabilidad, 9 *g*
**piedra vs cascajo ;**
permeabilidad, 79 *j*
densidad, 79 *c*
resistencia, 14 *a*, 79 *c*
fuego, 41 *c*
**granito ;** 83 *a*
**piedra caliza ;**
agua, 69 *a*
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# EXPERIENCIAS Y PRÁCTICA

## Resultados seleccionados.

Véase Índice, págs. 1346, etc.

## Orden de las materias.

Los elementos y condiciones característicos en la manufactura del concreto y las cualidades y conducta de éste, son tan numerosos y están tan estrechamente ligados entre sí en los informes sobre las experiencias, que ha sido imposible agruparlos satisfactoriamente ni en el cuerpo del texto, ni en el resumen que sigue.

La mayor parte de nuestros « resultados escogidos » van, por consiguiente, colocados aquí, siguiendo un orden aproximado al de las fechas de su publicación, y acompañados de un Índice, págs. 1346, etc., por medio del cual puede encontrarse prontamente cualquier asunto especial. El Índice está dispuesto racionalmente (esto es, no alfabéticamente), y, hasta donde es posible, en el orden que se sigue en el texto (págs. 1268 á 1294; 1295 á 1345), que se refieren al cemento, arena, mortero, agregado y concreto, común y armado. Los asuntos á que se refiere cualquiera de los informes publicados, llevan un número común, y, bajo este número común, están indicados por letras los varios paragrafos. Estas letras también distinguen generalmente varias características que abarca el número común.

Así, bajo Experiencia 8, tenemos un número de conclusiones alcanzadas por R. Feret; bajo 8 *a*, conclusiones respecto de la fuerza del mortero afectada por la proporción del cemento y la fineza de la arena; bajo 8 *c*, conclusiones sobre la porosidad y permeabilidad afectadas por la fineza de la arena y la riqueza del mortero, etc.

En el Índice, los punto y coma, en general, se usan para distinguir dos materias diferentes pero relacionadas entre sí. Así, « **Resistencia** fineza de la arena » y « **Arena, fineza de** —; resistencia », se refieren á secciones que dan informes respecto del efecto de la fineza de la arena sobre la resistencia del mortero ó del concreto.

— 1 —

**1. Bouniceau**, Anales de Puentes y Calzadas, 1863, pág. 181.

**1 a. Coeficiente de expansión.**

| | | |
|---|---|---|
| Hierro en barras..... | .000 0123 5 por grado C; | .000 006 86 por grado F |
| Concreto de cemento Portland | .000 0137 0 — ; | .000 007 60 — |

— 2 —

**2. John C. Trautwine**, Cartera del Ingeniero Civil, 1872.

**2 a. Arena, densidad; humedad, compactación.**

**Muestras.** Arena pura ordinaria de la orilla del mar, tanto seca como húmeda (no mojada), véase tabla. La arena B era de grano mucho más fino que A. C se componía de los granos más finos cernidos de B.

**Tratamiento.** Las arenas secas eran compactadas por sacudimiento y trepidación perfectos; las arenas húmedas por apisonado en capas delgadas.

**Resultados.**

| | Arena A (gruesa) | | | | Arena B (más fina) | | | | Arena C (la más fina) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | seca | | | húmeda | seca | | | húmeda | seca | | |
| | kg por m cúb | sólido % | vacío % | kg por m cúb | kg por m cúb | sólido % | vacío % | kg por m cúb | kg por m cúb | sólido % | vacío % |
| Suelta..... | 1554 | 59 | 41 | 1378 | 1410 | 53.4 | 46.6 | 1105 | 1313 | 50 | 50 |
| Compactada... | 1794 | 68 | 32 | 1722 | 1627 | 61.6 | 38.4 | 1658 | 1578 | 60 | 40 |
| Aumento..... | 240 | 9 | -9 | 344 | 218 | 8.2 | -8.2 | 553 | 264 | 10 | -10 |
| Por ciento..... | 15.5 | 15.2 | 22 | 25 | 15.5 | 15.3 | 17.6 | 50 | 20.1 | 20 | 20 |

**2 b. Corrosión.** Diez años de prueba. La humedad excluída del todo después del fraguado. **Los cementos protegen** el hierro, el plomo, el cinc, el cobre, el bronce. El yeso protege á todos éstos, excepto al hierro no galvanizado.

**Para el índice**, véanse págs. 1346, etc.

— 3 —

**3. John Watt Sandeman.** Inst C. E., vol. liv, 1878, pág. 260.

**3 a. Agregados, densidad.**

**Resultados.**

| No, | Kgs por m cúb | Percentaje de vacíos. |
|---|---|---|
| 1. Piedra caliza, quebrada en su mayor parte de 7 cm..... | 1522 | 50.9 |
| 2. Granzón zarandeado, desde pequeñas partículas hasta 6 cm. | 1786 | 33.6 |
| 3. Partes iguales nos 1 y 2, bien mezclados ..... | 1818 | 34.0 |
| 4. Asperón quebrado, de 10 á 20 cm..... | 1185 | 50.0 |
| 5. » » desde arena hasta 10 cm..... | 1474 | 34.0 |
| 6. Partes iguales de los nos 4 y 5 mezcladas ..... | 1462 | 36.0 |

— 4 —

**4. Eliot C Clarke.** A. S. C. E. Trans, Apr, '85, vol. 14, pág. 163. Expts for Boston Main Drainage Works.

**Resultados.**

**4 a. Arcilla.** La agregación de arcilla sin exceder una parte para 2 de cemento dió una pasta « **más densa, plástica é impermeable**, propia para enlucir superficies ó para tapar las juntas que se filtren », y en general no influye notablemente en la **resistencia** de los cementos Portland y naturales. Las mezclas hechas con arena que contenga 10% de marga presentaron una resistencia normal á los 6 y 12 meses, aunque sólo dieron una resistencia de la mitad en el primer mes. La arcilla en el cemento es un « polvo casi impalpable cuyas partículas son bastante finas para llenar los espacios interiores del cemento ».

**4 b.** Un año de saturación en agua **dulce ó salada** en contacto con **roble, pino duro, abeto, ó haya**, no afectó los morteros.

**4 c. La sal**, ya sea en el agua que se usa para la mezcla, ó en la que se coloca el cemento, retarda un poco la **fragua**, pero no afecta notablemente la **resistencia**.

**4 d. Consistencia.** El exceso de agua retarda la **fragua**. Los cementos naturales necesitan más agua que el **Portland**; el de **polvo fino** más que el ordinario; el de **fragua rápida** más que el lento.

**4 e. Mientras más fina es la arena**, menor es la **resistencia**.

**4 f.** Con arena, **los cementos de grano fino** son más fuertes, los de **grano grueso** son más fuertes cuando están puros, especialmente los **Portland**.

**4 g. El Portland** resistió al rozamiento mejor cuando se mezcló con dos partes de arena, el natural con 1 parte. La resistencia disminuye rápidamente con pequeñas variaciones en aquel'as proporciones.

**4 h. En la fragua los morteros se dilatan** > 1 parte en 1,000.

— 5 —

**5. Allen Hazen**, Massáchusets, Junta de sanidad del Estado, Informe pág. 550. Arena de granos agudos.

| | | | |
|---|---|---|---|
| **5 a. Coeficiente de uniformidad** ((u. c.), pág. 947: | <2 | <3 | 6 á 8 |
| **Vacíos,** percentaje aproximado..... | 43 | 40 | 30 |

— 6 —

**6.** E. Carey, Inst. de Ing. Civil Procs, vol. 107, pág. 55.

**6 a. Ácido sulfúrico** : Resistencia. El cemento puro ligado con agua que contenía 5% de ácido sulfúrico, tenía á los 7 días solamente 27% de la **resistencia** del cemento puro mezclado con agua sin ácido.

— 7 —

**7. Dr. Wilhelm Michaelis**, Inst. de Ing. Civil. Procs., vol. 107, págs. 372, 375.

**7 a. La desintegración** del cemento poroso en **el agua de mar** se ha demostrado que se debe á la acción de los ácidos sulfúricos é hidroclórico (muriático), que contienen los sulfatos de magnesia y los cloruros del agua de mar. Estos ácidos dejan la base más débil, la magnesia (que se deposita en la forma de hidrato) se combina con la cal del cemento, dilatando y desintegrando el concreto.

— 3 —

**3. R. Feret.** Anales de Puentes y Calzadas, serie 7.ª, tomo IV, '92.

**3 a. Resultados. La resistencia** del mortero aumenta con la cantidad de cemento, y en general (especialmente al principio del endurecimiento), con el tamaño del grano de la arena.

**3 b.** Los morteros varían mucho con la **porosidad.** Compárense 9 *d*, 9 *e*.

**3 c. La porosidad aumenta**
con la finura de la arena;
con la riqueza del mortero.

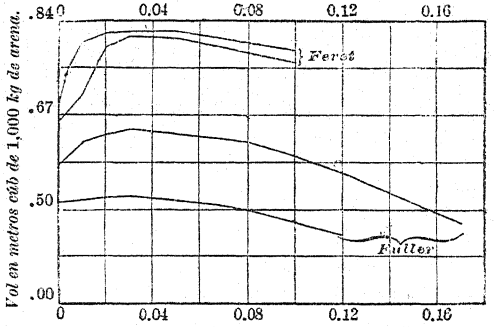
**3 d. La permeabilidad aumenta**
con el grosor de la arena;
con la riqueza del mortero.

**3 e.** Los morteros hechos con una mezcla **de arenas fina y gruesa** son menos porosos y menos permeables que los otros.

**3 f. La permeabilidad** de los morteros sujetos á la filtración continua de agua dulce ó de mar, **disminuye rápidamente,** pero en ciertos casos, el mortero se desintegra ó se agrieta.

**3 g.** Para evitar la desintegración en **el agua de mar,** úsese bastante arena gruesa y mucho cemento. Mézclense húmedos.

**3 h. Densidad de la arena.** Humedad y apisonado, fig. 1.




*Kilogramos de agua por kilogramo de arena seca.*

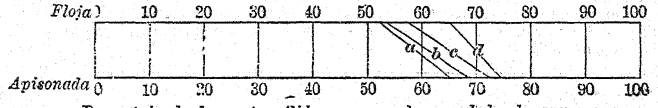
**Fig. 1.** Agua y apisonado.

(*N. del T.*— Llamemos (1) y (2) respectivamente, la curva superior é inferior de Ferret y lo mismo para Fuller.)

M. Feret empleó (1) una arena muy fina de duna y (2) una arena más gruesa de mar. Wm. B. Fuller, E. N., jul., 31, 1902, pág. 81, usó arena de ribera, (1) floja y (2) apisonada.

Según estos resultados parece que la agregación de agua afecta el volumen de la arena * de dos modos opuestos : (1) introduciéndose entre las partículas de la arena, aumentando de esta manera el volumen para un peso dado, (2) disminuyendo la fricción entre los granos y permitiéndoles que tomen con más facilidad la posición de mayor contacto, lo que disminuye el volumen.

Cuando se agregan pequeños volúmenes de agua solamente, parece que el primero de estos efectos prevalece, aumentando la masa hasta que la cantidad de agua llega á ser de 2 á 5% del volumen de arena seca. Con más agua prevalece el efecto lubricador, y el volumen disminuye.




*Percentaje de la parte sólida en un volumen dado de arena.*

**Fig. 2.**

**3 i. Figura del grano y compresión,** fig. 2.

Véase la nota al pie de la pág. 1283.

**Para el índice**, véanse pág. 1346, etc.

**Muestras.** De cuatro materiales como sigue :
*a*. Arena granítica, granos redondeados; *c*. Conchas quebradas;
*b*. Cuarzo molido, granos angulares; *d*. Residuo de *b*, granos laminares.
Cada uno de los cuatro materiales, cernido á la misma combinación granular, á saber, *c*, .5; *m*, .3; *f*, .2 **. Véase p. 1283.

**Resultados.** Véase la fig. 2.

**8 j. Efecto del tamaño del grano**, fig. 3.

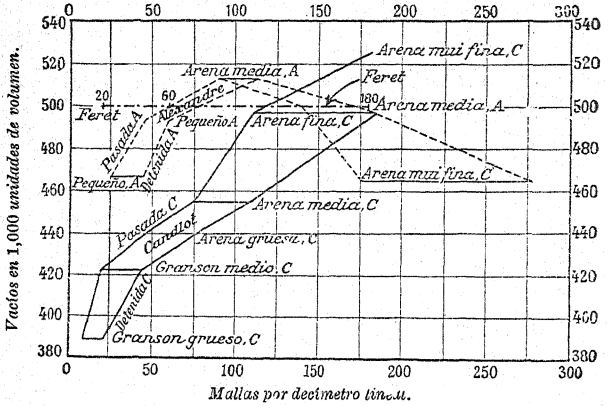



**Fig. 3.** Tamaño y densidad. **A** = Alexandre; **C** = Cand.ot.

Teóricamente, la densidad de una arena * ó granzón * compuestos de granos de tamaño *uniforme*, debería ser independiente del tamaño *absoluto* (§ 29, pág. 1286), pero los ensayos han dado resultados contradictorios, que demuestran variaciones sin importancia en la densidad en relación con el tamaño. Por ejemplo (T. and T., pág. 170), si la arena (excepto los tamaños muy finos que pasen por un cedazo de 30 mallas por cm lineal) y la piedra quebrada con partículas de tamaños irregulares de *forma* próximamente igual, se separan en porciones que contengan partículas de tamaño *uniforme*, estas diversas porciones darán un percentaje aproximadamente igual de vacíos. Esto está de acuerdo con los ensayos de R. Feret. (T. and T., págs. 171 y 142), fig. 3, según los cuales cada uno de los tres tamaños (grueso, mediano y fino) ** contenía 50% de vacíos. Los resultados del Sr. Feret están representados por la línea horizontal en la fig. 3. Por otra parte (fig. 3), M. Candlot (Feret, Anales de Puentes y Calzadas, 1892, 2.° sem) encontró que los vacíos aumentaban continuamente, y M. Alexandre (ídem) encontró que aumentaban primero, y disminuían después, al disminuir el tamaño.

**8 k. Efecto del tamaño de los granos, y de la cernida ó apisonada. La arena** suelta * tiene densidades que varían de .525 á .610, y su densidad es máxima cuando se mezcla 60% de arena gruesa ** con 40% de arena fina, sin arena mediana. En **la arena cernida, de manera completa** varían de .600 á .793, ocurriendo la mayor densidad en una mezcla de 55% de la gruesa con 45% fina, sin mediana.

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* Véase la nota al pie de la pág. 1283.
** Clasificación de los tamaños.

| | Pasa | Retenida por | |
|---|---|---|---|
| *c*. Gruesa..... | 20 | 60 | mallas por decímetro lineal |
| *m*. Mediana..... | 60 | 180 | » » » » |
| *f*. Fina..... | 180 | ... | » » « » |

**8 l. Densidades de las arenas y granzones (sueltos) sin cernir; forma y tamaño de los granos; humedad.**

| | Peso de los guijarros contenidos %. | Análisis mecánico de la arena propiamente dicha. | | | Arena seca, kg por m cúb. | Arena húmeda. | |
|---|---|---|---|---|---|---|---|
| | | Gruesa. | Mediana. | Fina. | | Humedad %. | Kg por m cúb. |
| Granítica granos redondeados... | 1.0 | .136 | .723 | .141 | 1,586 | .8 | 1,495 |
| Esquistosa..... | 25.4 | .359 | .293 | .348 | 1,753 | 1.2 | 1,650 |
| — ..... | 6.6 | .259 | .412 | .329 | 1,600 | 1.8 | 1,332 |

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**9. Luigi Luiggi y Valentino Cardi,** « Esperimenti sulle Calci, etc. », Genio Civile, Roma, '93.

**Porosidad, permeabilidad,** etc., **Cargas de seguridad.** Doce años de ensayos en relación con los trabajos del puerto de Génova, Italia.

**Resultados.**

**9 a.** En el mortero, **los vacíos se deben** parcialmente al aire adherido á las partículas de arena y del agregado y en parte á la evaporación del agua que se usa en la mezcla.

**9 b.** En el mortero, **el volumen de los vacíos** puede variar de 12 á 46% del volumen del mortero.

**9 c. Mínimo de vacíos** (5%) en concreto formado con 410 kg de cemento Portland, 1 metro cúbico de arena mezclada, 1.625 metros cúbicos de granzón pequeño.

**9 d. La porosidad aumenta**
con la finura de la arena;
con la riqueza del mortero;
al máx con cemento puro.
Compárense 8 *c* y 8 *d*.

**9 e. La permeabilidad aumenta**
con el grosor de la arena;
con la pobreza del mortero;
el mínimo con cemento puro.

**9 f. El concreto** de 676 kg de cemento Portland, 1 metro cúbico de arena mezclada, 1.625 m cúb de granzón pequeño, mezclado con cuidado con el agua suficiente (como $\frac{1}{3}$ de m cúbico) para reunirlo, fué **impermeable** bajo 12 metros de presión (1.2 kg por cm cuad).

**9 g. El concreto** de 410 kg de cemento Portland, 1 metro cúbico de arena mezclada, 1.625 m cúb de granzón pequeño, formando un cilindro hueco con espesor de 63 mm, resultó **impermeable** bajo 4 metros de presión (.4 kg/cm cuad) y apenas permeable bajo 8 m (.8 kg/cm cuad). Cilindros semejantes de la misma mezcla, **sin el granzón,** se filtraron algo bajo 4 m y con mucha facilidad bajo 8 m.

**9 h. Carga de seguridad en la compresión.** En los pisos de los astilleros : 1 : 2 : 3 concreto de cemento Portland, arena y granzón pequeño, soportan 7.5 kg/cm cuad con un factor de seguridad = 15.

— 10 —

**10. Dr. Keller,** Thonindustriezeitung '94, n.º 24.

**10 a. Coeficiente de dilatación.** Temperaturas desde —16° hasta —72° C Granzón (20 mm) y arena, en partes iguales.

| | Mezcla de arena y granzón. | | | |
|---|---|---|---|---|
| Proporciones (1 parte de cemento) para | 0 | 2 | 4 | 8 |
| Coeficiente por grado C..... | .0000126 | .0000101 | .0000104 | .0000095 |

— 11 —

**11. Geo. W. Rafter.** 2.º Informe sobre el Proyecto de Estanques de Depósito del río Genesee, '94. Véase E. R, '06, enero 27, pág. 109.

**11 a Concreto con piedra arenosa dura,** dió una **resistencia** 50% mayor que cuando se sustituyó con piedra **pizarrosa.**

**Para el índice**, véanse págs. 1346, etc.

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**12. Leibbrand.** E. R., '94, nov. 3.

**12 a. Resistencia á la compresión; edad.** Puente sobre el Danubio en **Munderkingen.** Concreto 1 : 2.5 : 5, húmedo. Cubos 20 cm. Muy bien mezclado en un cilindro de hierro que gira sobre un eje horizontal y que contenía 40 bolas de acero que pesaban en junto 300 kg. Mezclado durante dos minutos seco, 3 minutos húmedo.

| Edad en días | 7 | 28 | 150 | 970 | 3825 (=9 años |
|---|---|---|---|---|---|
| Resistencia á la compresión, kgs /cm cuad. | 202 | 254 | 332 | 520 | 570 |

**12 b. Presiones máximas existentes,** en el puente, 35 á 39 kg /cm cuad.

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**13. J. Watt Sandeman,** Inst. de Procs. de Ingeniería Civil, vol. 121, '95, pág. 220.

**13 a. Muros impermeables de concreto** (presión no especificada) hechos con 1 parte de cemento que dejó un residuo de 10% en el cedazo n.º 120.

2 partes de arena con 27% de vacíos.

4.5 partes de granzón grande y pequeño con ⅄ de 35% de vacíos.

**13 b.** Cuando el agregado tiene 35% de vacíos, **el volumen del mortero** debe ser 50% del volumen del agregado.

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**14. A. W. Dow.** Inspector de Asfalto y Cementos de los Estados Unidos. Informe del Comisionado de Ingeniería, Distrito de Columbia, '97, pág. 165.

**14 a. Resistencia á la compresión.**

**Muestras.** Cubos de concreto de 30 cm, secos; comprimidos en moldes de hierro fundido, humedecidos enteramente dos veces al día.

Los resultados en un año son el término medio de cinco cubos, el resto son el término medio de dos cubos. Dedúzcase de 3 á 8% por fricción.

Los materiales fueron los siguientes :

| **Cemento.** | Portland. | Natural. |
|---|---|---|
| Por ciento retenido en el cedazo de 100 mallas por 25 mm lineales | 8.5 | 14 |
| Lapso de la fragua inicial, minutos | 190 | 20 |
| — — dura, — | 305 | 36 |

La tensión como sigue, kg por cm cuadrado :

| | 1 día. | 7 días. | 1 mes. | 3 meses. | 6 meses. | 1 año. |
|---|---|---|---|---|---|---|
| Portland puro | 31 | 59 | | | | |
| Portland, 3 partes de cuarzo superior quebrado | | 17 | 30 | 28 | 30 | 33 |
| Natural puro | 6.7 | 12.6 | | | | |
| Natural, 2 partes de cuarzo superior quebrado | | 6.4 | 13 | 23 | 29 | 34 |

**Arena** que se usó en el concreto.

Sin residuo en un cedazo n.º 3; .5 por ciento, pasada por n.º 100. Vacíos 44 per ciento con 4.4 por ciento de agua.

**Piedra quebrada.** Gneiss. De los números 6 y 12 (Tabla más abajo) 3 por ciento retenido en un cedazo de mallas de 63 mm. Todas retenidas en uno de 31 mm. Otras, 0 retenidas en el de 63 mm, casi todas en uno de 2.5 mm. Para vacíos, véase la tabla más abajo.

**Granzón.** Cuarzo limpio, que pasa por un cedazo de 45 mm de mallas, 2 por ciento que pasa por uno n.º 10. Vacíos 29%.

**Agua.** Con el cemento Portland, .09 en volumen de concreto apisonado con cemento natural .12.

Para los resultados, véase la pág. 1198

**Resistencia á la trituración de cubos de concreto de 30 cm, en kg por cm cuadrado.**

Ensayos de A. W. Dow como anteriormente.

Partes por volúmenes. Cemento, 1; arena, 2; agregado, 6.

(*N. del T.* — Convertida al sistema métrico.)

| | Número. | Agregado. Partes de piedra quebrada | Agregado. Partes de grangzón. | Vacíos en el agregado. Percentaje del vol. | Vacíos en el agregado. Mortero en el percentaje de los vacíos. | Fuerza de trituración, kg/cm cuad, después, de : 10 días. | 45 días. | 3 meses. | 6 meses. | 1 año. |
|---|---|---|---|---|---|---|---|---|---|---|
| Portland | 7 | 6 | .. | 45.3 | 83.9 | 64 | 126 | 159 | 176 | 215 |
| | 8 | 3 | 3 | 35.5 | 107.0 | 67 | 130 | .. | 145 | 193 |
| | 9 | 4 | 2 | 37.8 | 100.6 | .. | .. | .. | .. | 200 |
| | 10 | 6 | .. | 39.5 | 96.2 | .. | .. | .. | .. | 190 |
| | 11 | .. | 6 | 29.3 | 129.1 | 49 | 114 | 189 | 129 | 198 |
| | 12 | 6 | .. | 45.7 | 83.9 | .. | .. | 115 | 107 | 130 |
| Natural | 1 | 6 | .. | 45.3 | 83.9 | 16 | 38 | 26 | 56 | 64 |
| | 2 | 3 | 3 | 35.5 | 107.0 | 7 | 26 | 42 | 44 | 59 |
| | 3 | 4 | 2 | 37.8 | 100.6 | .. | .. | .. | .. | 64 |
| | 4 | 6 | .. | 39.5 | 96.2 | .. | .. | .. | .. | 56 |
| | 5 | .. | 6 | 29.3 | 129.1 | 6 | 30 | 25 | 24 | 54 |
| | 6 | 6 | .. | 45.7 | 83.9 | .. | .. | 42 | .. | 58 |

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**15. Prueba de los metales**, '98, pág. 572.

**15 a. Concreto de ceniza** con cemento Portland; **resistencia máx de compresión.**

**Muestras.** Cubos de 30 cm. Agua 160 á 200 kg por m cúbico del concreto.

**Resultados :**

Proporciones por volumen. (*N. del T.* — Convertida al sistema métrico.)

| Cemento. | Arena. | Cenizas. | Edad en días. | N.° de ensayos. | Kg/cm cuad. |
|---|---|---|---|---|---|
| 1 | 1 | 3 | 30-38 | 18 | 108 |
| 1 | 1 | 3 | 90 | 18 | 144 |
| 1 | 2 | 3 | 39 | 3 | 77 |
| 1 | 2 | 3 | 102 | 3 | 115 |
| 1 | 2 | 4 | 38 | 3 | 63 |
| 1 | 2 | 4 | 98 | 3 | 93 |
| 1 | 2 | 5 | 30-38 | 15 | 51 |
| 1 | 2 | 5 | 90-99 | 15 | 77 |
| 1 | 3 | 6 | 29 | 3 | 37 |
| 1 | 3 | 6 | 91 | 3 | 55 |

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**16. Considère.** « Génie civil », '99.

**16 a. Ductibilidad.**

**Muestras y resultados :**

Cantilevers de concreto, 1 : 3, 6 cm cuad, 60 cm de largo, lado en tensión reforzado por 3 barras redondas de hierro de 4¼ mm de diámetro.

**Tratamiento.** Carga tal que el momento de flexión fué el mismo para toda las secciones transversales. En uno de los prismas, la carga aumentó hasta que la unidad de alargamiento=.002. Entonces cargas, =44 á 71 0/0 de esta carga original, se aplicaron 139,000 veces, volviendo el esfuerzo á 0 en cada vez.

**Resultados. Unidades de alargamiento,** .000545 á .00125, la resistencia se redujo muy poco. Pruebas semejantes de muestras no reforzadas dieron como unidad de alargamiento á la ruptura, tan sólo de .0001 á .0002; la armadura hizo, al parecer, que el cemento resistiese mucha mayor deformación que cuando no estaba armado. Pero véanse los Ensayos 36, 38.

**Para el índice**, véanse págs. 1346, etc.

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**17. C. E. Fowler.** A. S. C. E., Trans., '99, vol. 42, pág. 117.

**17 a. Resultados. Proporciones**, suponiendo que

1 barril de cemento Portland = 3.8 pies cúbicos = .108 m cúb;

34 metros cúbicos de concreto = 27 metros cúbicos más ó menos después de apisonado.

Aquellos concretos cuyos volúmenes de piedra aparecen en tipo negro como (**1.00**) tienen sus vacíos llenos, ó más que llenos, y en los de tipo corriente como (1.04) los vacíos no están llenos y el concreto es poroso y deficiente en resistencia. (*N. del T.* — La tabla que sigue la hemos convertido al sistema métrico.)

| Proporciones. | Cantidades en un metro cúb de concreto. | | | |
|---|---|---|---|---|
| | Barriles de cemento. | Arena, metros cúbicos. | Piedra con 40% de vacíos en m cúb. | Piedra (m cúb) con 50 0 0 de vacíos. |
| 1 : 2 : 3 | 2.30 | .51 | **.87** | 1.05 |
| 1 : 2 : 4 | 2.07 | .47 | **.95** | 1.15 |
| 1 : 2 : 5 | 1.81 | .42 | 1.04 | 1.26 |
| 1 : 3 : 4 | 1.69 | .57 | **.83** | **1.00** |
| 1 : 3 : 5 | 1.51 | .52 | **.92** | 1.11 |
| 1 : 3 : 6 | 1.35 | .48 | **1.00** | 1.20 |
| 1 : 4 : 6 | 1.30 | .55 | **.91** | 1.09 |
| 1 : 4 : 7 | 1.20 | .51 | **.97** | 1.17 |
| 1 : 4 : 8 | 1.08 | .47 | 1.03 | 1.25 |

Las cifras precedentes están de acuerdo con los resultados de la práctica. La columna para piedra con 40% de vacío representa muy aproximadamente la piedra caliza quebrada, que se quiebra en pedazos de varios tamaños, y la de 50% de vacío representa la roca volcánica, que se quiebra en pedazos de tamaño más uniforme.

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**18. Pruebas de metales**, '99.

**18 a. Fuerza de compresión de cubos** de 30 cm de **concreto de cemento Portland** seco, por Geo Kimball, ingeniero en jefe de la Compañía del ferrocarril elevado de Boston.

**Muestras :**

**Arena gruesa**, limpia, aguda. Vacíos, medida suelta y húmeda, 33%, medida después de asentarse saturándola con agua, 25%.

**Piedra**. Conglomerado de Roxbury, Massáchusets, vacíos, medidos sueltos 49.5%.

4.8% pasa por anillo 63 mm, detenida por anillo de 50 mm;
76.7% — — 50 — — — de 25 —
18.% — — 25 — — — de 12 —
.5% — — 12 —

**Tratamiento. Mezclada á mano.** El agua apareció apenas después de apisonada.

Los cubos, excepto los que se probaron á los siete días, se **enterraron** en la tierra húmeda hasta una semana antes de la prueba. En general, 5 cubos de las mezclas de cada marca se probaron en cada una de las edades.

**Resultados.** Las máx resistencias á la compresión, kg /cm cuad. Cada máxima ó mínima es el término medio de 5 ó más pruebas, con cubos hechos con una de las cuatro marcas de cemento, refiriéndose de esta manera al cemento que

dé la mayor ó menor resistencia bajo las condiciones especificadas. Los términos medios son los de estos resultados para las cuatro marcas.

| Edad. | 1 : 2 : 4 | | | 1 : 3 : 6 | | | 1 : 6 : 12 | | |
|---|---|---|---|---|---|---|---|---|---|
| | máx. | media. | mín. | máx. | media. | mín. | máx. | media. | mín. |
| 7 días..... | 155 | 107 | 63 | 108 | 86 | 60 | 53 | 40 | 29 |
| 1 mes..... | 185 | 171 | 159 | 152 | 145 | 127 | 85 | 73 | 61 |
| 3 meses..... | 219 | 206 | 183 | 178 | 170 | 165 | 88 | 74 | 59 |
| 6 meses..... | 310 | 274 | 253 | 222 | 208 | 193 | 111 | 92 | 57 |

**Para las fórmulas,** deducidas de estos resultados por E. Thacher, véase § 35, pág. 1317.

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**19. W. A. Rogers,** ferrocarril de Chicago, Milwakee y Saint Paul, Sociedad Occidental de Ingenieros, Diario, 1899, junio, vol. 4, n.º 3, pág. 262. La Gaceta de Ferrocarriles, junio 15, pág. 402, julio 27, 514.

**19 a.** Efecto del **frío,** y de la mezcla con **agua salada. Muestras; resistencia á la compresión** de cubos de 30 cm de concreto de cemento Portland y natural, 8 cubos de cemento Portland.

Portland Atlas, 1 de cemento, 3 de granzón (2 arena, 1 guijarros), 4 de piedra caliza dura pasada por triturador, 8 cubos de natural de Louisville, 1 de cemento, 3 de granzón, 3 de piedra.

Hechos como se han usado en las mamposterías elevadas de la línea del ferrocarril de Chicago Milwakee y St Paul.

**Tratamiento.** Todos los cubos fueron hechos por la misma persona en moldes de madera de tablas de 2.5 cm dejándolos en ellos hasta que se quebraron.

**Resultados.**

(*N. del T.* — Como siempre, hemos convertido todas estas tablas.)

| | | | Portland. | | Natural. | |
|---|---|---|---|---|---|---|
| | | | Temp C. | Kg-cm cuad. | Temp C. | Kg/cm cuad. |
| 1 cubo | (en la oficina) caliente | 28 días | 26°6 á — 7°7 | >91+ | 29.4 á 4.4 | 21 |
| 1 — | — — — | 28 — | » | >91+ | » | |
| 1 — | del lado fuera* (al aire) | 28 — | 13°9 á —31°1 | 63± | 13.9 á —23.3 | 14 |
| 1 — | — — — | 28 — | » | 48± | » | 18 |
| 1 — | — — — | 28 — | | | | |
| 1 — | (en la oficina) | 28 — | 29°4 á 0°0 | >91+ | 29.4 á 4.4 | 26 |
| 1 — | del lado fuera* — | 28 — | 13°9 á —31°1 | | 13.9 á —23.3 | |
| 1 — | (en la oficina) | 28 — | 29°4 á 0°0 | >91+ | 29.4 á — 4.4 | 25 |
| 1 — | del lado fuera* ** — | 28 — | 13°9 á —31°1 | >91+ | 13.9 á —23.3 | 16.6 |
| 1 — | — — — | 28 — | | >91+ | 13.9 á —23.3 | 17 |

* Durante la primera parte de los 28 días, la temperatura descendió á — 23.3° y — 29° C; después deshielo durante el día y helada en la noche.

\+ Ligeramente descascarado. Las resistencias excedieron la fuerza (12,'50 kgs/cm cuadrado) de la máquina.

± Se creyó que el frío había retardado la fragua.

** Mezclado con agua salada, uno para 2.5 de agua.

**Para el índice,** véanse págs. 1346, etc.

**19 b. Naturaleza del agregado, resistencia á la compresión.**
**Muestras.** Cubos de 30 cm de cemento, granzón y piedra. Granzón ⅔, ⅓ de guijarros de la arena hasta 38 mm. Cada resultado es el término medio de 3 cubos. Edad 28 días.

**Resultados.**

| | | kg/cm cuadrado. |
|---|---|---|
| 1 : 3 : 4½ | piedra caliza pasada por triturador..... | 89 |
| 1 : 3 : 4½ | blanda cernida..... | 82 |
| 1 : 3 : 4½ | granzón lavado de 10 á 50 mm..... | 73 |
| 1 : 4 : 7 | piedra caliza blanda cernida..... | 50 |
| 1 : 4 : 3½ | piedra » » » ..... | 45 |
| 3½ | granzón lavado de 10 á 50 mm..... | |

**19 c. Impurezas en la arena y en el agregado : resistencia á la compresión.**
**Muestras.** La arena y el granzón impuros tenían á la vista como 10% de « suciedades » y éstas « parecían contener gran cantidad de hierro ».

| **Resultados :** | Con arena, tensión, 90 días, kg/cm cuad. | | | Con granzón, comp, cubos 30 cm, 28 días, kg /cm cuad. | |
|---|---|---|---|---|---|
| | 1 : 1 | 1 : 2 | 1 : 3 | 1 : 2 : 5 | 1 : 2.5 : 5 |
| Limpia..... | 32 | 34 | 24 | 77 | 59 |
| Sucia..... | 44 | 38 | 30 | 69 | 65 |
| Más sucia..... | 36 | 36 | 28 | 71 | |

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**20. Edwin Thacher.** E. N., '99 septiembre 21.
**20 a.** « Varias marcas de cemento Portland mejoraron su resistencia á la tensión, con una **demora** de tres á cuatro horas entre la **mezcla** y la **colocación.** » Ransome.

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**21. Geo. W. Rafter.** A. S. C. E., Trans., dic. '99, vol. 42, pág. 104.
**21 a. Volumen**; consistencia, riqueza y proporción del mortero.
**Muestras**; 544 cubos de 30 cm quebrados en las máquinas de pruebas del Gobierno de los E. U. en Watertown, Mass., de cemento Portland, arena 1,384 á 1,544 kgs/m cúb, agregado piedra quebrada. Cubos como de dos años de edad.
« Secos », solamente un poquito más húmedos que la tierra húmeda;
« Plásticos », de la consistencia ordinaria que usan los albañiles;
« Exceso » (de agua). Bajo pisón moderado el concreto trepidaba como gelatina.
S=vol de arena en el mortero para 1 vol de cemento;
M=vol del mortero en el concreto para 1 vol de cemento;
A=vol de agregado en el concreto para 1 vol de cemento;
C=vol de concreto hecho con 1 vol de cemento.

**Resultados.**

| Consistencia * | Volumen. | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Mortero=33% del agregado. | | | | | Mortero=40% del agregado. | | | | |
| | Proporciones. | | | | Contracción. | Proporciones. | | | | Contracción. |
| | S | M | A | C | ** | S | M | A | C | ** |
| D. | 1 | 1.57 | 4.74 | 4.30 | 9.3 | 1 | 1.64 | 4.10 | 3.82 | 6.8 |
| P. | 1 | 1.82 | 5.51 | 5.01 | 9.1 | 1 | 1.66 | 4.14 | 3.82 | 7.7 |
| E. | 1 | 1.70 | 5.11 | 4.64 | 9.2 | 1 | 1.70 | 4.24 | 3.97 | 6.4 |
| D. | 2 | 2.42 | 7.29 | 6.74 | 7.4 | 2 | 2.44 | 6.12 | 5.89 | 3.8 |
| P. | 2 | 2.45 | 7.28 | 6.62 | 9.1 | 2 | 2.50 | 6.28 | 5.83 | 7.2 |
| E. | 2 | 2.35 | 7.02 | 6.36 | 9.4 | 2 | 2.60 | 6.47 | 5.97 | 7.7 |
| D. | 3 | 3.15 | 9.49 | 8.78 | 7.5 | 3 | 3.21 | 8.03 | 7.36 | 8.4 |
| P. | 3 | 3.30 | 9.92 | 8.89 | 10.4 | 3 | 3.31 | 8.23 | 7.62 | 7.4 |
| E. | 3 | 3.25 | 9.72 | 8.83 | 9.2 | 3 | 3.43 | 8.57 | 7.90 | 7.8 |
| D. | 4 | 4.18 | 12.69 | 11.75 | 7.4 | 4 | 4.24 | 10.71 | 9.84 | 8.1 |
| P. | 4 | 4.28 | 12.94 | 11.66 | 9.0 | 4 | 4.35 | 10.96 | 10.09 | 7.9 |
| E. | 4 | 4.37 | 13.14 | 11.78 | 10.4 | 4 | 4.33 | 10.84 | 9.64 | 11.1 |
| D. | 5 | 5.04 | 15.05 | 14.29 | 5.1 | 5 | 4.42 | 11.25 | ... | ... |
| P. | 5 | 5.00 | 15.00 | 13.66 | 9.1 | 5 | 5.00 | 12.50 | 11.56 | 7.5 |
| E. | 5 | 5.08 | 15.20 | 13.60 | 10.5 | 5 | 5.24 | 12.90 | ... | ... |

**21 b. Densidad del concreto**; enteramente apisonado,

| | |
|---|---|
| Vol de 1 : 1 mortero, | Vol de concreto apisonado, aprox. |
| .33 × vol del agregado, | .91 × vol del agregado, |
| .40 × — — | .93 × — — |

**21 c. Densidad del agregado, compactación.** Piedra quebrada para pasar por un anillo de 5 cm y que tenía 43.3% de vacíos al sacudirla ligeramente en la medida, conservó solamente 37.4 % de vacíos, como término medio en 5 pruebas, después de haberla comprimido en la medida con pisón de hierro, aplicando con la fuerza ordinaria con que se apisona el concreto.

— 22 —

**22. Pruebas de los metales**, '00, págs. 1109, etc. Para los contratistas de la Compañía de la Planta.

**22 a. Muestras**; cemento Portland, arena, piedra triturada, 1 : 5 : 5. Piedra pasada por un anillo de 63 mm; los pedazos que pasaban por un anillo de 13 mm se separaron cerniéndolos.

**A, mezclado á mano; B y C mezclado con un mezclador portátil** de 2.40 m de largo, que se compone de una cajuela de acero, con varias hileras de espigas de acero, que bambolean. El agua de un tubo de regadera golpea el mezclador como en la mitad de su largo. Por consiguiente, el concreto se mezcla seco en la mitad superior, y húmedo en la inferior.

Piedra regada por parejo en una plataforma frente del mezclador.
Arena regada por parejo por encima de la piedra.
Cemento regado por parejo por encima de la arena.

El material se introduce luego en el mezclador.

**B.** Se deja formar una pila cónica, acumulando las piedras alrededor de los bordes.

* Consistencia : D = seco; P = plástico, E = exceso.

** Contracción = $\frac{100\,(A - C)}{A}$.

**Para el índice**, véanse págs. 1346, etc.

**C.** El material, al descargarlo, se niveló con escardilla.

Cubos de 30 cm; las vigas de 10×15, 15×15; 75 cm de luz. Todos, 2 días al aire, 2 meses en agua, 1 mes en el aire.

| **Resultados**; | **Cubos Resistencia á la compresión**, kg/cm cuad. | | | **Vigas Módulo de ruptura**, kg/cm cuad. | | |
|---|---|---|---|---|---|---|
| | máx. | medio. | mín. | máx. | medio. | mín. |
| A..... | 247 | 224 | 206 | 32 | 29 | 26 |
| B..... | 313 | 299 | 284 | 40 | 37 | 32 |
| C..... | 308 | 290 | 282 | 38 | 32 | 24 |

— 23 —

**23. W. H. Henby**, Jour. Assoc. Eng. Socs., sept. 1900, pág. 153.

**23 a. El concreto de ceniza** pierde de ¼ á ⅓ de su **resistencia** por estar completamente húmedo, pero la recobra enteramente al secarse.

— 24 —

**24. E. Duryea, Jr.**, « Cemento », vol. 2, '01. Véase E. Thacher, en A. S. C. E., Trans., 05, vol. 54. Parte E, pág. 447.

**24 a. Acabado.**

Portales de túneles, Los Angeles, California, dos manos de 1 cemento : 4 arena; 1 pasta de cal.

Pedestales, ferrocarril de Chicago y E., Ill. R. R., 1 cemento : 1 arena. En buen estado.

Muelles, puente del río Arkansas, ciudad de Kansas, ferrocarril del Sur, dos manos 1 de cemento : 3 arena, una capa, 1 cemento : 1 arena : En buen estado.

1 de cemento : 3 arena : 1 pasta de cal, considerada como la mejor, debe evitarse excesivo cuchareo. El acabado debe conservarse húmedo durante dos semanas.

— 25 —

**25. Escuela Thayer.** Ensayos, '02. J. B. McIntire y A. L. True.

**25 a. Permeabilidad.** 97 ensayos, muestras de 25 cm de diámetro, 22½ de alto, tubo de 2 cm, se introdujo en 10 cm., Presiones (con agua) de 1.4, 2.8 y 5.6 kg/cm cuad durante dos horas. Todas las muestras de 30 á 45% 1 : 1 de mortero resultaron impermeables. Algunas con 40 á 45% de 1 : 2, y algunas de 1 : 2 : 4 y 1 : 2.5 : 4 fueron impermeables bajo presión de 5.6 kg. cm cuad 1 : 2 : 4 ó 1 : 2.5 : 4 recomendadas para presiones moderadas.

— 26 —

**26. Breuillé**, « Experimentos con el cemento armado », Anales de Puentes y Calzadas, '02, pág. 181.

**26 a. Corrosión y adhesión en el agua.**

**Muestras**; 4 planchas 90×99 cm, 30 cm de espesor, respectivamente de 599, 599, 798, 998 kg de cemento Portland, .327 m cúb de arena, .901 m cúb de guijarros, de 20 á 25 mm de diámetro. Barras de 5 mm de diámetro, colocadas á distancias diferentes de las superficies de las planchas.

**Tratamiento.** Las planchas se colocaron en agua y, de 12 á 15 m de profundidad, estas presiones se transmitieron sin disminución á los centros de los bloques. Las presiones se aligeraban de vez en cuando. El tratamiento se mantuvo por varios días. Después se dejaron los bloques al aire, expuestos á la intemperie.

**Resultados.** El metal se encontró perfectamente conservado, pero su superficie que era brillante, al colocarlo, se encontró mate al descubrirla después del ensayo, habiéndose destruído la adhesión donde había circulado el agua.

**26 b. Lustre.** Las barras de superficie brillante, colocadas en mortero de cemento durante varios días, tenían la superficie mate después que se les quitó el mortero, lo que indica una acción química entre el cemento y el hierro. Es probable que sea debido á esta acción que las barras pierden el moho cuando se colocan en mortero de cemento. La sal de hierro, que se forma por esta acción, la disuelve el agua que penetra hasta la superficie del hierro.

**26 c. Ganancia y pérdida en el peso.** Pedazos pequeños de hierro laminado, colocados en mortero de cemento, ganaron como .01% en el peso en 76 días. Puestas después en agua corriente, las láminas perdieron peso, lo que indica la solubilidad del compuesto cuya formación había aumentado el peso.

**26 d. Tiempo; adhesión.** Planchas de hierro, de 35×70×5 mm, fueron colocadas sobre concreto recientemente unido al mortero y éste (de 500 kg de cemento Portland por 1 metro cúbico de arena) fluía á la superficie. En varios períodos, estas placas presentaron un término medio de adhesión como sigue :

| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | 7 | 12 | 17 | 23 | 27 | días. |
| .278 | .636 | .946 | 1.132 | 1.295 | 1.316 | kg/cm cuadrado. |

Los resultados de la prueba 26 *d* no se modificaron materialmente cuando el mortero había estado expuesto al sol, ó se había mezclado caliente ó muy mojado.

— 27 —

**27. G. Y. Skeels**, auxiliar del ingeniero de la ciudad, Sioux, Iowa, E. N., '02, noviembre 6, pág. 382.

**27 a.** Término medio de 2 y 4 probetas, 1 día al aire, 14 días en el agua. Cemento Portland.

Bajo **mezcla continua** durante 8 ó 10 horas, el mortero de cemento puro perdió como $\frac{1}{8}$ de su resist á la tensión; 1 : 2 perdió como $\frac{1}{6}$.

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**28. Thomas S. Clark**, ingeniero residente, á cargo de la construcción de la Estación de Fuerza del ferrocarril de Manhattan, Nueva York. E. N., '02 julio 24, pág. 68.

**28 a. Retemple; resistencia.** El mortero de cemento natural puro, mezclado primero con 28 % de agua, mortero de cemento natural con arena con 14%. Retemplado una hora después de mezclado, « añadiéndole agua suficiente para volver á llevar la masa á su consistencia original ». Las muestras de un día, 3 horas al aire, las otras 24 horas. Las muestras retempladas presentaron, en general, más ó menos, la mitad de la resistencia normal.

Se obtuvieron resultados semejantes cuando se humedeció el cemento, cada 15 minutos durante una hora. En tales casos, en la práctica la resistencia aumenta algunas veces añadiendo un poco de cemento fresco.

Los morteros de cemento Portland retemplados después de una hora no presentaron deterioro notable, probablemente porque el cemento Portland se fragua más lentamente que el cemento natural.

— 29 —

**29. W. Purves Taylor**, A. S. T. M., vol. 3, pág. 376, '03.

**29 a. Edad, solidez. El envejecimiento** del cemento de polvo fino permite la hidratación de la cal libre, que casi siempre existe, haciéndolo inerte é impidiendo la expansión. Las muestras hechas con cemento de una semana, no estaban sólidas, pero al aumentar la edad del cemento, mejoró la solidez de las muestras hasta las 5 semanas, al cabo de las cuales estaban sólidas.

**29 b. Finura, solidez.** Las partículas más grandes del cemento molido grueso no se hidratan fácilmente. Un cemento cuyo 33% se retuvo en un cedazo n.° 200 y el 13% en uno n.° 100, se cuarteó y se agrietó en la prueba de ebullición, pero se hizo sólido al molerlo otra vez hasta que todo pasó por el cedazo n.° 100 dejándolo secarse durante dos semanas.

— 30 —

**30. La comisión del Gobierno francés.** Beton und Eisen, '03, vol. 5.

**30 a. Ductilidad.** Concreto 1 : 2 : 4. Los resultados fueron semejantes á los de Considère (véase ensayo 16 *a*). La ductilidad fué mayor cuando se endureció en el agua que en el aire.

— 31 —

**31. Chas. List**, Asociación de las Sociedades de Ingeniería, Diario, marzo '03, vol. 30, n.° 3, pág. 128.

**Para el índice,** véanse págs. 1346, etc.

**31 a. Efecto del agua de mar** en Guatemala, Centro América. Las pilas huecas, dentro del agua de mar se llenaron de concreto en que se había empleado el agua de mar para la mezcla. Una parte del mortero se filtró, y formó, con la arena circunvecina, masas de concreto que se adhirieron á las pilas. Al sacar las pilas se encontró el concreto en perfecto estado y fuertemente adherido á las pilas.

**31 b.** Cimientos de un puente de ferrocarril construído en 1895. Concreto ligero mezclado con agua salobre y permaneciendo en ella. De calidad excelente en 1903.

**31 c. Remolida.** El cemento se importó de Hamburgo (Alemania), en barriles. El buque hizo agua; considerándose el cemento como perdido, se devolvió su valor. Se almacenó debajo del piso de un depósito de mercancías con los lados abiertos, expuesto á la humedad del suelo y al salpique del mar. El cemento se endureció en bloques tan duros que se empleó como cimientos para postes de madera en los edificios. Este cemento en trozos, se trituró tan delgado como fué posible, mezclándolo con arena de playa menuda y agua salobre. El concreto estaba perfectamente duro á los tres días y se usó en los cimientos de puentes en agua salobre.

— 32 —

**32. Geo. W. Lee. Jr,** E. N., 1903, marzo 19, pág. 246.
**Acabado.**

**32 a.** Ferrocarril Central de Nueva York. **Armaduras** (pino de 5 cm con machiembrado) cubiertas con una mano de **jabón blando,** las aberturas en las juntas rellenas con **jabón duro.** El concreto se depositó y se sacó de las armaduras con una pala de punta cuadrada, mortero de 1 : 2 á lo largo de las armaduras. Después de quitar las armaduras, y mientras el concreto estaba todavía blando, se alisó la superficie con un movimiento circular, con pedazos de ladrillos blancos refractarios, ó de ladrillos de 1 cemento : 1 arena : se humedeció luego la superficie, y usando mortero líquido de 1 : 1 se la frotó y alisó con una regla de madera.

— 33 —

**33. Wm. B. Fuller,** A. S. C. E., Trans., 1903, junio, vol. 50, pág. 454.
**Acabado.**

**33 a. Estanque de concreto** armado en la planta de filtros, N. J., de 3.05 m de diámetro, 13 m de alto, muros de 38 cm de espesor en la parte inferior, 25 cm en la superior, construído en 8 horas; todo el concreto puesto desde arriba, descendiendo así 13 m. Se mezcló muy húmedo, colocando 140 lit (una carga de carretilla) á la vez, removiéndolo simplemente al colocarlo. Impermeable por dentro como por fuera. Se omitió el encalado interior que se había pensado hacer, por creerlo innecesario. Las superficies lisas; no se vieron piedras ó huecos.

— 34 —

**34. Profesor C. E. Sherman,** E. N., 1903, novbre, pág. 443.
**34 a. Arcilla y marga: resistencia.**

Los cementos Portland Dyckerhoff (alemán) y Lehigh (americano) con **arena** que contenía de 0 á 15% de **arcilla y marga.** La resistencia, en general, **aumentó materialmente** con la proporción de arcilla y de la marga. Con 10 y 15%, la resistencia, á los doce meses, era de 15 á 50% mayor que con la arena limpia.

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**35. Pruebas de los metales,** 1904, págs. 345, 387.

**35 a. Columnas de concreto, sencillas y armadas; máxima resistencia de compresión, s,** kg por cm cuadrado y **módulo de elasticidad,** E* kg por cm cuadrado.

* E tomada entre los límites de la resist á la comp como sigue, kg/cm cuad: Nos 1 y 17; 7 á 42 kg; 16, 42 á 70 kg; 19, 1 á 33 kg; todos los otros, 70 á 105 kg/cm cuad.

**Muestras.** Cemento Portland y arena; agregado, guijarros y piedra volcánica quebrada, de 1 á 4 cm, y cenizas. Las columnas eran de 32 × 32 cm × 2.44 m aproximadamente. Las barras de la armadura de 20 mm torcidas, « Tw »; 15 mm corrugadas, « Cr »; 20 mm Thacher, « Th ».

**Resultados.**

| N.° | Mezcla. | Agregado. | Agua ** | Edad. meses. | Edad. días. | Armadura. N.° y clase | Armadura. % ***. | 0.001 s | 0.001 E* |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1:1:2 | Guijarros | 42.5 | 8 | 0 | 4 Tw | 1.46 | 202 | 187 |
| 2 | » | — | » | 7 | 28 | Ninguno | Ninguno | 120 | 175 |
| 3 | 1:2:3 | — | » | 7 | 28 | 4 Tw | 1.44 | 141 | 159 |
| 4 | » | — | 53.1 | 7 | 25 | Ninguno | Ninguno | 124 | 151 |
| 5 | 1:2:4 | — | 56.7 | 3 | 13 | 4 Tw | 1.43 | 139 | 136 |
| 6 | » | — | » | 3 | 16 | 4 Cr | .97 | 153 | 155 |
| 7 | » | — | » | 3 | 14 | 4 Th | 1.03 | 139 | 162 |
| 8 | » | — | » | 3 | 15 | 8 Tw | 2.86 | 222 | 175 |
| 9 | » | — | » | 3 | 14 | 8 Cr | 1.94 | 198 | 214 |
| 10 | » | — | » | 3 | 12 | 8 Th | 2.09 | 194 | 216 |
| 11 | » | — | » | 7 | 26 | 4 TG | 1.45 | 127 | 167 |
| 12 | » | — | » | 3 | 17 | Ninguno | Ninguno | 120 | 165 |
| 13 | » | Piedra volc | « húmedo » | 5 | 10 | Ninguno | Ninguno | 123 | 197 |
| 14 | » | Ceniza | — | 5 | 16 | 4 Tw | 1.45 | 147 | 98 |
| 15 | » | — | — | 5 | 16 | Ninguno | Ninguno | 61 | 70 |
| 16 | 1:3:6 | Guijarros | 74.4 | 7 | 24 | 4 Tw | 1.44 | 96 | 72 |
| 17 | » | — | » | 7 | 23 | Ninguno | Ninguno | 32 | 101 |
| 18 | » | Piedra volc | » | 5 | 10 | 8 Cr | 1.94 | 160 | 216 |
| 19 | » | — | 57.6 | 0 | 7 | Ninguno | Ninguno | 33 | 155 |

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**36. F. E. Turneaure**, A. S. T. M., Trans, '04, pág. 504.

**36 a. Ductilidad.** Vigas de concreto armado reforzadas. Unidad de alargamiento del concreto, á la primera aparición del agrietamiento, .00010 á .00035, formado de muchas rajaduras pequeñas, que aparecieron cuando la presión sobre el acero era > 350 kg/cm cuad. Las vigas sencillas se quebraron (sin agrietamiento previo) con una unidad de alargamiento igual. Las grietas que correspondían á la unidad de alargamiento mínimo eran invisibles en el concreto seco, pero se percibieron en el concreto húmedo por la aparición de vetas angostas húmedas como de 3 mm de ancho. Un poco más tarde tenían la apariencia de hendeduras obscuras como cabellos.

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**37. Profesor Bauschinger**, « Beton und Eisen », '04, vol. IV, pág. 193.

**37 a. Corrosión; adhesión;**

Fragmentos de planchas de concreto armado, quebradas, en la prueba '87; **se expusieron á la intemperie,** hasta que se examinaron en '92. Adhesión; concreto quebrado á golpes de martillo, rompiéndose solamente cerca de los golpes. Corrosión; acero, libre de óxido aun junto á las superficies de fracturas descubiertas.

**37 b. Estanque dañado por maltrato;** agrietado: la armadura quedó descubierta en algunas partes. Hubo oxidación solamente en las partes así descubiertas. La adhesión como en 37 (*a*).

**37 c.** Fragmentos de planchas Monier de 6 á 8 centímetros de espesor. Expuestas á intervalos, durante 4 años, á **aguas sucias de cloaca.** El concreto permaneció duro, la armadura libre de oxidación á 1 centímetro de la superficie descubierta; la adhesión excelente.

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**38. A. Kleinlogel**, « Beton und Eisen », '04, vol. 2.

**38 a. Ductilidad.** Vigas de concreto armado de 15 × 30 cm de 220 centímetros de largo. 1 : 1 : 2, cemento, arena, piedra caliza. Conservados debajo de arena húmeda durante 6 meses. El momento de flexión constante en toda la parte que se midió. Unidad de alargamiento en el concreto; armado, .000148 á .000196; sin armar, .000143.

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* Véase nota al pie de la página anterior.

** 0/0 de cemento por peso.

*** 0/0 del área de la sección transversal.

**Para el índice**, véanse págs. 1346 etc.

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**39. Clarence Coleman**; Informe del jefe de ingenieros de los Estados Unidos, parte IV. Cemento Portland universal hecho de escoria de fundición.
(*N. del T.* — Hemos convertido la tabla del autor al sistema métrico.)

| | Arena *. | Mezcla. | Agua‡‡. | Promedio, tensión, kg/cm cuad. 7 días. | 28días. | 6 meses. | 1 año. | 3 años. |
|---|---|---|---|---|---|---|---|---|
| **39 a.** | | | | | | | | |
| Cemento en buen estado..... | Q | 1 : 3 | 12.5 | 12.37 | 21 | 30 | ... | ... |
| Cemento expuesto en sacos á la **humedad**..... | Q | 1 : 3 | 12.5 | 12.16 | 18 | 29 | ... | ... |
| Endurecido en bloques. No fraguado. Remolido..... | Q | 1 : 3 | 12.5 | 14 | 19 | 30 | ... | ... |
| **39 b.** | | | | | | | | |
| Cemento como se recibió en la obra. | Q | 1 : 3 | 12.5 | 1.00† | 1.00† | ... | ... | ... |
| Cemento de 4 á 6 meses en sacos en almacén..... | Q | 1 : 3 | 12.5 | 1.17† | 1.09† | ... | ... | ... |
| **39 c.** | | | | | | | | |
| Concreto mezclado duro en la plataforma‡..... | S | 1 : 10 | á capricho | 9 | 15 | 23 | 24 | ... |
| Concreto mezclado en **mezcladora** cúbica‡§..... | S | 1 : 10 | á capricho | 18 | 19 | 27 | 27 | ... |
| **39 d.** | | | | | | | | |
| **Como en el laboratorio**, 24 horas en un depósito húmedo, luego sumergido hasta quebrarlo ‖ .... | S | 1 : 10 | á capricho | 18 | 26 | 29 | 32 | 28 |
| **Como en la obra**, 10 días bajo tela húmeda, luego al aire hasta quebrarlo ‖ ..... | S | 1 : 10 | á capricho | 16 | 27 | 29 | 45 | 59 |
| **39 e.** | | | | | | | | |
| 8.25% de agua**..... | S | 1 : 3 | 8.25 | 18 | 20 | 27 | 28 | ... |
| 9.25% de agua**..... | S | 1 : 3 | 9.25 | 17 | 22 | 28 | 31 | ... |
| **39 f.** | | | | | | | | |
| Guijarros 2 á 6 mm..... | S | 1 : 10 | á capricho | 11 | 19 | 31 | 31 | ... |
| Guijarros 6 á 20 mm..... | S | 1 : 10 | á capricho | 13 | 22 | 32 | 32 | ... |
| **39 g.** | | | | | | | | |
| Arena limpia..... | S†† | 1 : 3 | 8.25 | 13 | 18 | 25 | 24 | ... |
| Arena con pequeño % de arcilla... | S†† | 1 : 3 | 8.25 | 13 | 19 | 27 | 25 | ... |

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**40. Prof. Chas L. Norton**, E. N., '02 oct. 23, '04, enero 14.
**Corrosión.** Algunos centenares de probetas de varias mezclas y consistencias, con acero incrustado, sujeto al aire, vapor y ácido carbónico.

* Q = Cuarzo de cristal normal.
S = Arena superior de Entry; cernida en cedazo... N.º 4 10 20 30 50
0/0 100 72.3 46.1 26.5 5.1

† Resistencias *relativas*.
‡ Probetas hechas de concreto tomado en la obra.
§ Un lote de concreto muy bien mezclado en 80 segundos.
‖ Concreto tomado de la plataforma de mezclar. Quitándole las piedras mayores de 2 cm.
** Á fin de acercarse en lo posible á las condiciones del trabajo en la práctica, se dejó reposar el mortero 30 minutos más que en el tratamiento ordinario.
†† Pasado por cedazo nº 10.
‡‡ Agua en la proporción del agregado seco.

**40 a. El acero limpio al incrustrarlo.** 3 semanas de exposición.
En el concreto se encontró óxido sólo en donde existían vacíos y otros defectos.
**40 b. El acero se oxidó incrustado,** de 1 á tres meses de exposición. Los cambios en el tamaño del acero tuvieron lugar solamente en las partes en que el cemento se aplicó mal.

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**41. John S. Sewell.** Acerca del Incendio de Baltimore, E. N., '04, marzo 24.
**41 a. Resultados.** « El concreto sufre más ó menos un **cambio molecular** en el fuego y se astilla ó desconcha. El cambio molecular fué muy lento. El material calcinado no se **desconchó** mucho, excepto en los ángulos cuadrados descubiertos. **La eficacia,** en conjunto, es grande. Es preferible á los **ladrillos** huecos comerciales para arcos de piso y planchas.
**41 b.** Columnas de concreto armado, vigas simples y laminadas, planchas para pisos, por lo menos tan buenas como las **piezas de acero** protegidas con los mejores ladrillos huecos comerciales.
**41 c.** El concreto de **piedra** se astilla más que cualquiera de otra clase, porque los pedazos de piedra contienen cavidades de aire y de humedad y los contenidos rompen la piedra al calentarse. **El granzón** es una piedra en que se ha eliminado la mayor parte de estas cavidades al quebrarlas. Es por consiguiente mejor que la piedra cuando el concreto deba soportar el « fuego ».
**41 d. « Los ladrillos quebrados, las escorias quebradas, las cenizas y las escorias del cemento** producen buen concreto resistente al fuego. »
**41 e. « Las cenizas** que contengan mucho carbón parcialmente quemado, son peligrosas, porque dichas partículas se quemarán por entero debilitando el concreto. Las cenizas de locomotora dañan el cemento además de ser combustibles. El concreto de cenizas es seguro solamente cuando se hace bajo una vigilancia muy rigurosa é inteligente. Pero cuando está propiamente hecho, con los materiales adecuados, es difícil que la mampostería de ladrillos lo supere en sus cualidades de resistencia al fuego, y nada lo supera en ligereza, en igualdad de circunstancias.»

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**42. Emile Low,** A. S. C. E., Trans., junio '04, vol. 52, pág. 96. Tajamar de Búffalo.
**42 a.** Contracción.

| | | | | |
|---|---|---|---|---|
| Cemento..... | 258 | metros | cúbicos | |
| Arena..... | 365 | — | — | |
| Guijarros..... | 1,175 | — | — | |
| Piedra quebrada..... | 972 | — | — | |
| Total de materiales..... | 2,770 | — | — | |
| Bloques hechos..... | 2,054 | — | — | |
| Contracción..... | 716 | — | — | =25.8% |

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**43. Alex. B. Moncrieff,** ing. en jefe, Gobierno del Sur de Australia. Carta á los autores, junio 7, '04.
**43 a. Permeabilidad.**
Muestras. Bloques de concreto, cubos de 60 cm, para los experimentos que se relacionan con la construcción del **dique Barossa.** Ingredientes, los mismos que se usaron en el dique. Agregado 3 á 50 mm, con vacíos variables. La preparación de agregado se vigiló muy cuidadosamente.
**Tratamiento.** El agua se trajo al centro del bloque por tubos de hierro forjado de 13 mm que terminaban en una pieza en T, envueltos en henequén que formaba un bulbo como de 10 cm de diámetro.
**Resultados. Todos los bloques se hicieron prácticamente impermeables.** El concreto que se usó en el dique « estaba basado en los resultados de los ensayos hechos principalmente con bloques n.° 7 y 8 ». No hubo « prácticamente ninguna infiltración » por el dique *.

* Véase § 4

**Para el índice,** véanse págs. 1346, etc.

Q = volumen de agua en la mezcla, % del volumen de concreto;

X = exceso de mortero = $100 \frac{\text{volumen del mortero} - \text{volumen de los vacíos}}{\text{volumen de los vacíos}}$;

A = edad del bloque, en semanas, al someterlo á presión;

I = intervalo, en minutos, entre la aplicación de la presión y la aparición del agua en la superficie del bloque.

Caída = 30.5 m = 3.1 kg/cm cuad. Bajo 61 m « el efecto es muy semejante á los resultados que se obtuvieron con la caída de 30.5 m ».

| N.° | Cemento. | Arena. | Agregado. | Q % | X % | A Semanas. | I Minutos. | Filtración observada *. Litros. | Término medio, litros por mes. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1.84 | 5.26 | 16.65 | 5 | 11 | ** | ** | ** |
| 2 | 1 | 1.84 | 5.26 | 15.45 | 5 | 11 | 34 | 3/8 en 7 sem. | 0.24 |
| 3 | 1 | 1.50 | 4.63 | 16.04 | 5 | 10 | 18 | 1/100 en 4 — | 0.02 |
| 4 | 1 | 2.00 | 4.50 | 16.04 | 15 | 10 | 14 | 7 en 2 — | 15.12 |
| 5 | 1 | 1.75 | 4.13 | 16.65 | 15 | 9 | 12 | 13 1/2 en 7 — | 8.88 |
| 6 | 1 | 1.50 | 4.12 | 16.04 | 10 | 8 | 35 | 1/100 en 2 — | 0.02 |
| 7 | 1 | 1.50 | 3.90 | 14.26 | 12.5 | 6 | 28 | 1/16 en 2 — | 0.14 |
| 8 | 1 | 1.50 | 3.70 | 13.68 | 15 | 5 | 30 | 1/100 en 2 — | 0.02 |

— 44 —

**44. Edwin Thacher,** A. S. C. E., Trans., '05, vol. 54, págs. 425, etc.

**44a. Efecto del frío. Puente de arco de Melan,** en Mishawaka, Indiana, 3 luces, de 33.55 m cada una, construídas á temperaturas que variaban de — 18° á 13° C. Se introdujo **agua caliente** en el mezclador. El concreto se colocó al calor de la sangre, suficientemente caliente para derretir la nieve 48 horas después. El arco central se completó á la temperatura de — 4° C, más ó menos. Al día siguiente la temperatura descendió á — 18° C. Dos semanas más tarde una balsa de hielos se llevó la cimbra y dejó el **arco sin apoyo.** No se observaron malos efectos. El asiento tan sólo un poco mayor que en los otros arcos, cuyas cimbras se quitaron más tarde como es costumbre.

**44b. Acabado.**

Puente en Economowoc, Wis. La cara de mortero, 1 cemento : 1 de residuos de cerniduras de granito : 1 arena. Al segundo día después de terminado, se quitaron los moldes y se alisó la superficie con piedra blanda y agua.

Arco de Inman, Hohenzollern. 1 cem : 5 piedra caliza quebrada. Después de fraguar 12 horas se quitó el cemento suelto con agua y brochas.

Fábrica de Bórax de la Compañía del Pacífico, Bayonne, N. J.

Acabado imitando sillería de mollejón, introduciendo listoncillos de madera en las armaduras y preparando las caras con un martillo neumático. Un hombre pudo preparar de 28 á 56 m cuadrados en 10 horas, con máquina; de 9 á 18, á mano. Muy buen efecto.

« El Sr. Cummings obtuvo un buen acabado repasando la superficie con una brocha de alambre mientras el cemento estaba todavía blando. »

Viaducto del ferrocarril de Utica y valle de Mohawk, en Herkimer, N. Y., y viaducto sobre vías férreas en Jacksonville. Fla. « Un acabado muy fino. » Para un muro duro, humedézcase la superficie y aplíquese con brocha una mezcla delgada de mortero de 1 : 2. Frótese la superficie con un pedazo de piedra de amolar, quitando las marcas de las tablas, rellenando los poros y produciendo una espuma en la superficie. Pásese sobre esta espuma, antes que se seque, una brocha mojada en

* Véase § 4, pág. 1314.
** Inseguro.

agua. Para una pared blanda todavía (habiéndose quitado las armaduras antes de 7 días) úsese mortero líquido de cemento puro, en lugar del mortero de 1 : 2. El resto del procedimiento se hará como se dice arriba.

Usense armaduras lisas, depositando el concreto húmedo directamente contra ellas. Después de quitar las armaduras alísese la sup con una regla de madera usando sólo el mortero suficiente para llenar los poros y darle una superficie lisa.

**44 c. Corrosión.**

Chicago. Las barras de hierro de las planchas de concreto de piedra caliza que cubrieron las bóvedas de las aceras durante 8 ó 10 años, estaban libres de moho. E. L. Ransome.

Obelisco en el Parque Central, Nueva York. Pequeño pedazo de hierro colocado en el mortero sacado de la base. Brillante después de 2,300 años. Pernos de seguridad de hierro, del lecho de concreto de un faro en el estrecho de Mackinac, libre de moho 20 años después de colocado. Wm. Sooy Smith.

Puente sobre el río Osage, Mo. Pilas cilíndricas de hierro rellenas con concreto de cemento de piedra caliza de Louisville. El hierro estaba absolutamente libre de moho después de 7 años de servicio. Albert A. Trocon, E. R., vol. 38, pág. 273.

Barras de acero, láminas de acero y metal dilatado, incrustado en bloques de concreto de 7.5×7.5×20 cm, y el acero sin protección, todo incluído en cajas de hojalata, y expuesto durante tres semanas una parte al vapor, al aire y al bióxido de carbono, otra al aire y al vapor, otra al aire y al bióxido de carbono, y otra á la atmósfera del cuarto de pruebas.

**Conclusiones :**

El concreto debe ser denso y mezclado húmedo. El cemento puro dió una protección perfecta.

En el concreto de ceniza, la corrosión se debió principalmente al óxido de hierro, no al azufre.

El concreto de ceniza, si está denso y bien apisonado, es casi tan bueno como el de piedra.

El acero debe estar brillante al incrustarlo.

Se debe dar una mano de cemento al acero antes de incrustrarlo. De otra manera quedará más moho que acero en el resultado. Prof. Chas. L. Norton, Informe n.º 2 del Inspector de la Estación de ensayos de Ingeniería de Boston.

Grenoble, Francia. Tubería de agua de cemento armado, Monier, de 30 cm de diámetro 4 cm de espesor, armadura de barras de acero de 6 y 1.5 mm. 15 años en la tierra húmeda. La adhesión era perfecta. El metal estaba absolutamente libre de moho.

Berlín. Muro de retención de concreto armado. Después de 11 años de uso, se encontró el metal libre de corrosión, « excepto en algunas partes en que las barras estaban á 8 ó 10 mm de la superficie ». El efecto del concreto, en preservar el metal, no se debió á la exclusión del aire. « Aunque el concreto sea poroso y no esté en contacto con el metal en todos los puntos, se filtrará y neutralizará el ácido carbónico, impidiendo la corrosión. » S. B. Newbury, E. N., vol. 47, '02, abril 24, pág. 335.

Los eslabones del anclaje de un puente de suspensión construído parcialmente por Rœbling en '55. Se quitaron en 75. Estaban perfectos. G. Bouscaren, E. R., vol. 38, pág. 253.

El anclaje del puente de suspensión del Niágara. No había moho en las partes en que la piedra caliza no había estado en contacto con el metal y donde no hubo movimiento estaba en perfecto estado después de 25 años. L. L. Buck.

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**45. Wm. B. Fuller,** « Un Tratado sobre el concreto », por T. y T., '05.

**45 a. Humedad; efecto del pisón :**

| Humedad..... | Seco. | 6% | Saturado. |
|---|---|---|---|
| Reducción de vol, % con el pisón..... | 9.6 | 18.8 | 8.8 |

**Volumen máximo** en las arenas, cuando hay agua es entre 5% y 8% por peso.

**45 b. Vacíos, entre las esferas** de diámetro uniforme ( « grandes masas de esferillas del mismo tamaño » ) no se pudieron reducir, echándolas en un envase y apisonándolas, á menos de 44% del vol. Véase § 30, pág. 1286.

**Para el índice**, véanse págs. 1346, etc.

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**46. Asociación nacional para la protección contra el fuego.** Informe de la Comisión, '05.

**46 a. Pruebas por fuego.**

**Muestras.** Vigas de 20×28 cm×1.83 m, cada una con tres barras redondas de acero lisas, de 2 in de largo, incrustadas á 2.5, 5, 7.5 cm de la parte inferior de la viga. Cemento Portland.

| Agregados. | | Mezclas. | | Vacíos, % |
|---|---|---|---|---|
| Granzón ordinario cernido..... | 1:2:3, | 1:2.5:5, | 1:3.5:7 | 35 |
| Piedra caliza < 32 mm..... | » | » | » | 42 |
| Granito rojo cernido < .40 mm. | » | » | » | 40 |
| Cenizas ordinarias..... | 1:2:5, | 1:2:6 | .... | .... |

Mezclados húmedos. Las muestras tenían de 45 á 48 días.

**Tratamiento.** Tres horas en el horno, temperatura 1038° á 1093° C.

**Resultados.**

**46 b. La conductividad** fué **más baja** en el concreto de **ceniza** y en los concretos más **ricos.** Por lo demás, los materiales no ejercieron efecto importante.

**46 c. La resistencia de las barras disminuyó** 25% á 410° C. Tiempo medio que se necesita para llegar á 410°; 2.5 cm de incrustación, 1 h; 5 cm, 2 h; 7.5 cm, 2.5 hs.

**46 d. El cemento no se quebró ni astilló** bajo el fuego, pero perdió prácticamente toda la resistencia á la profundidad de 10 cm de los lados y parte inferior, y todo él se ablandó perceptiblemente. El cemento y la mayor parte de la piedra estaban completamente calcinados en la superficie, disminuyendo su efecto en un espesor de 10 cm. En todos los casos, **apareció agua** en las grietas transversales de las vigas, especialmente en las mezclas más ricas y con la temperatura de 100° C.

**46 e. Recomendaciones.** Los materiales deben mezclarse bien, mojados por medio de máquina, y bien pisoneados. El incrustamiento debe ser < 5cm, en casos importantes 7.

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**47. John H. Quinton.** Agrimensor geológico de los E. U. « Ensayos » con tubos de acero-concreto, en plan de trabajo, abastecimiento de agua é irrigación de los E. U. Estudio 143, '05.

**47 a. Permeabilidad.** Determinar la utilidad bajo presión de aquellos tubos, para el servicio de cultivo de tierras de los Estados Unidos.

**Muestras.** Siete tubos de concreto armado mezclados á mano, de 1.50 m de diámetro, 15 cm de espesor, 6 m de largo, cada uno hecho en una sección, uno de las mismas dimensiones en cuatro secciones. Los obreros eran hábiles. En 3 de los 7 tubos, y en 3 de 4 secciones del octavo, se usó cal en la mezcla.

Los tubos variaban mucho de contextura. Uno « parecía de aspecto arenoso y habría sido fácil hacer un agujero en él ». Otro era « sumamente duro ».

**Tratamiento.** Los tubos se probaron con y sin forro interior de cemento y arena, etc., con y sin pasta de cal. La lechada de jabón y alumbre de Sylvester (pág. 1013), la pintura á prueba de fuego P y B; se probaron otras pinturas, y la arcilla se removió con agua dentro de los tubos. Las presiones hasta 4.9 kg/cm cuad.

**Resultados.**

**47 b.** A pesar de todas las precauciones, los tubos se filtraron especialmente á lo largo de las uniones. **La filtración disminuyó mucho con la presión,** pues el agua al colarse llenaba los poros de una lechosidad, pero entretanto la filtración era suficiente para dañar la base de los tubos.

**47 c. Las mezclas secas** dieron un concreto más permeable.

**47 d.** Con los granzones **graduados cuidadosamente** fué difícil obtener una **distribución** uniforme de los diferentes tamaños.

**47 e. Consérvese el concreto en la sombra** durante la mezcla y colocación.

**47 f.** Las interrupciones en el trabajo son menos peligrosas con las mezclas **húmedas**. Al apisonar ó calafatear evítese que se **mueva la armadura**.

**47 g. Hágase la armadura** bastante fuerte para proteger el concreto de a tensión.

**47 h.** La mixtura de **jabón y alumbre** es ventajosa al hacer el concreto; pero la mezcla con yeso (2 cm espesor) es conveniente para el interior, puesta en dos manos, la primera con pasta de cal, para retardar la fragua, la segunda (que se aplica al secarse la primera) se alisará con la cuchara. Al secarse aplíquese una lechada espesa de cemento puro.

**47 i.** Los tubos de concreto armado no se recomiendan para presiones mayores de 20 m. Para cortas distancias tómense precauciones cuando la presión sea de 30 m.

**47 k.** Los tubos de cemento son **propensos á agrietarse**, especialmente á lo largo de las juntas, pero aun agrietados son más secos y más durables que los de otras clases.

**47 l.** Al quebrar los tubos **apareció** el moho en una barra solamente y en una superficie de 4 cm de longitud en donde caía una filtración. El tubo había sido forrado con mezcla que contenía sal amoníaco y limaduras de hierro.

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**48. Considère**, « Beton und Eisen », '05 vol. 3.

**48 a. Ductilidad.**

**Muestras.** Mezcla 400 kg de cemento Portland, .4 de metro cúbico de arena, .8 de metro cúbico de cerniduras de piedra caliza. Vigas de 15 × 20 cm, 3 m de largo. El lado de la tensión reforzado con dos barras de hierro redondas de 16 mm y 3 de 12 mm. El momento de flexión fué constante en todo el largo medido.

**Tratamiento.** Una viga se conservó en agua, una debajo de arena húmeda por 6 meses.

**Resultados.** Unidades máx de alargamientos :

| | |
|---|---|
| Conservada debajo del agua..... | .00107 |
| — — de arena húmeda..... | .00050 |

No se descubrieron grietas, aunque la superficie se alisó con cemento.

**La resistencia no se había afectado.**

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**49. R. Feret,** « Un Tratado sobre el concreto, simple y armado », por Taylor y Thompson, '05.

**49 a.** La acción dañina del **agua de mar** se debe principalmente al **ácido sulfúrico** de los sulfatos disueltos, por consiguiente el cemento debe contener tan poco yeso (sulfato de cal) como sea posible. El cemento Portland debe estar fallo de aluminio y de cal. La existencia del material puzolánico es ventajosa. El cemento debe ser denso é impermeable.

— 50 —

**50. Profesor Ira H. Woolson.** Informe á la Compañía de luz, calor y fuerza de Astoria, '05.

**50 a. Naturaleza y resistencia.**

| | Resistencia en kg /cm cuad. | | | | | |
|---|---|---|---|---|---|---|
| | Tensión. | | | Compresión. | | |
| Cemento Portland 1 : 2 : 4. | Máx. | Prom. | Mín. | Máx. | Prom. | Mín. |
| Arena y caliza quebrada..... | 12 | 11 | 10 | 140 | 123 | 101 |
| Caliza triturada * y quebrada..... | 20 | 13.6 | 10 | 238 | 171 | 142 |

**50 b.** La arena contenía < 1% de **marga**; toda pasó por el cedazo de 3 mm; 75% pasó el cedazo de 8 mallas por cm. La piedra azul del río Hudson (caliza), que pasó por el cedazo de 30 mm, fué retenida en el cedazo de 10 mm. El concreto se apisonó húmedo en los moldes, 1 ó 2 días al aire, 5 ó 6 en agua. Se secó al aire de 4 á 7 semanas. **Resultados,** véase 50 *a*.

* $^3/_8$" (10 mm) cernidores del triturador; 87 % pasó el cedazo de $^1/_4$" (6.3 mm), 40% ó pasó el de $^1/_8$" (3.2 mm).

**Para el índice,** véanse págs. 1346, etc.

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**51. Profesor R. C. Carpentier,** Universidad de Cornell, Diario Sibley de Ingeniería, enero, '05.

**51 a. Retardo de la fragua; yeso** (sulfato de cal) $CaSO_4$, y **cloruro de calcio,** $CaCl_2$. Ambos molidos secos con el clinker.

**Fragua inicial;** la pasta carga una barra de 2 mm de diámetro, cargada con $^1/_4$ de libra (113 gr).

**Fragua final;** la pasta carga una barra de $^1/_{24}$ de pulg (1 mm) de diámetro, cargada con 453 gr.

**Tiempo,** en ambos casos, contado desde el momento de la mezcla y dado en minutos.

**Resultados.**

| | Percentaje por peso *. | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | .0 | .5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 5.0 | 6.0 | 7.0 |
| | Tiempo en minutos. | | | | | | | | | | | |
| $CaSO_4$ inicial.... | 2 | 6 | ... | 80 | 24 | 29 | 30 | 27 | 28 | 27 | 19 | 18 |
| $CaCl_2$ — .... | 2 | 115 | 160 | 167 | 127 | 103 | 45 | 97 | .. | 73 | 68 | .. |
| $CaSO_4$ final..... | 52 | 87 | ... | 157 | 114 | 79 | 69 | 72 | 45 | 59 | 37 | 59 |
| $CaCl_2$ — ..... | 52 | 274 | 272 | 234 | 212 | 180 | 182 | 185 | .. | 160 | 145 | .. |

**51 b.** E. Candlot (Cementos y Cales hidráulicas) encontró que las soluciones **concentradas** de $CaCl_2$ (tales como las de 100 á 400 gramos por litro), **aceleraban** la fragua y el endurecimiento.

**51 c. Agregado de cal apagada** á un cemento que contenía yeso, el cual, con el tiempo, había perdido su efecto retardador.

| | Inicial, mints. | Final, mints. |
|---|---|---|
| 2% de yeso (sin cal)..... | 12 | 15 |
| 2% — +5% cal..... | 120 | 300 |

De 2% á 5% de cal es útil con este objeto, pero no sin el yeso. La cal no disminuye la resistencia.

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**52. Jas. C. Hain.** Ferrocarril de Chicago, Milwakee y St. Paul, E. N., '04 /abril 28, pág. 413. E. R., '05, enero 28, pág. 103. **Arena; tamaño y limpieza.**

**Muestras.**

**52 a. Arenas impuras.**

1 : 3 mortero de cemento Portland, hechos con

(*a*) arena de cuarzo de granos redondos y lisos, mezclada con fragmentos más grandes de conchas de piedra caliza, el 92% pasó el cedazo n.° 24, el 28% pasó el n.° 50;

(*b*) « arena superior de St Paul », 54% pasó el n.° 24; el 11%, pasó el n.° 50;

(*c*) « arena superior de Otawa ».

**Resultados:**

Resistencias relativas de tensión : (*a*) 100, (*b*) 137, (*c*) 107.5.

La arena (*a*) dió un concreto excelente en una pila de centro, en un puente giratorio.

1 : 3 Mortero de cemento Portland, con arena que contenía de 3.2 á 15% de arcilla; las resistencias fueron < con arena limpia. Con cemento natural 1 : 2 y Portland 1 : 2, los resultados fueron generalmente favorables á la arena más limpia.

La arena con 6% de arcilla, dió morteros más fuertes antes del lavado que después.

Las arenas á las cuales se les había agregado artificialmente de 2 á 20% de marga rica, dieron un mortero que resultó algo irregular, pero en general más fuerte que los de arena limpia.

**52 b. Arena fina, con yeso.** Una arena que pasó toda por el cedazo n.°100, 93% pasó el n.° 200 (por consiguiente más fina que la mayor parte de los cementos, véanse las Especificaciones), y que contenía 12% de arcilla, dió una mezcla de

* 1 % = como 1,800 gr $CaCl_2$ para 1 barril de cemento Portland.

cemento Portland de 1 : 3, que mostró á los 6 meses y 1 año, casi la misma resistencia á la tensión que los morteros semejantes hechos con « arena superior de Otawa »; pero esta mezcla resultaba más débil á períodos más cortos.

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**53. Jas. C. Hain**, ingeniero de construcciones de mampostería del ferrocarril de Chicago Milwakee y St. Paul, E. N., 05, marzo 16.

**Aceites**. Pruebas por **Geo. J. Griesenauer.**

**53 a. Una probeta de cemento Portland puro**, de dos años, sujeta á inmersiones ocasionales en aceite de lámpara, empezó á desintegrarse á los 10 meses, pero no se encontraron estructuras de concreto recientes dañadas perceptiblemente por el aceite. **Un piso de concreto** sobre el cual se almacenaron aceites de alumbrar y de lubricar durante 6 años, no estaba aparentemente afectado. El aceite penetró como á 1.5 mm. Un pedazo de este piso puesto en aceite, durante 10 meses, estaba todavía sano.

**53 b. Cemento Portland**; puro, 1 : 3 arena, 1 : 3 de piedra caliza cernida, 18 probetas de cada una, 4 días al aire. Después se saturaban diariamente con aceite de lámparas, más tarde con menos frecuencia. Aparecieron grietas en las muestras de 1 : 3 á los 2 1/2 meses; en las muestras puras, á los 5 meses. Todas las probetas se desintegraron al fin.

**53 c. Cemento Portland: 54 probetas**, puras, 36 de arena 1 : 3. 7 días al aire. Después se saturaban diariamente con aceite, más tarde con menos frecuencia. Aceites que se usaban : extracto de manteca, aceite de ballena, de castor, de linaza hervido, petróleo crudo. El aceite de manteca desintegró la mayor parte de las probetas de 2 semenas á 2½ meses, pero algunas permanecieron sanas durante nueve meses. El aceite (animal y mineral, mezclados) produjo casi el mismo efecto. Los aceites de ballena y de castor afectaron solamente unas pocas probetas, mientras que el petróleo y el aceite de linaza cocido, no desintegraron ninguna. El petróleo disminuyó en algo la resistencia. El aceite de linaza cocido formó una cubierta de protección y no afectó la resistencia. Como regla general, las probetas puras cedieron primero. Las de piedra caliza y escoria cedieron más; las de piedra caliza y arcilla menos.

**53 d. Cemento de sílice**; puro, 1 : 1; 1 : 2; 1 : 3, arena. Una probeta de cada uno. 2 años en el agua, 20 días en aire caliente. En aceite durante 2 años. Las tres primeras probetas estaban sanas, 1 de (1 : 3) se desintegró.

**53 e.** El aceite de linaza, en el procedimiento de Sylvester (pág. 1266), la parafina y el silicato de sosa se aplicaron en **capas** á las probetas, pero **ninguna las protegió** de la acción de los aceites.

**53 f. Concreto rico**, bien hecho con materiales buenos, y bien fraguado y sazonado, es el que mejor resiste al aceite. En la práctica, las **estructuras de concreto** rara vez se saturan con aceite, como pasó con estas muestras.

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**54. Chas. A. Matcham.** Asociación nacional de suministros á los constructores, '05, abril 15, pág. 435.

**54 a. Corrosión.**

**Muestras y tratamiento.** Cubos de concreto de 150 mm, de 3 años, con cubos de acero de 75 mm.

Dos cubos, con cubos de acero de 75 mm **sin pintar** incrustados, expuestos á la intemperie en invierno y en verano, y algunas veces cubiertos de **hielo y nieve.**

**Resultados.**

**El acero no estaba dañado.** Presiones trituradoras de 204 kg y de más de 292 kg/cm cuad. Un cubo de 150 mm, con cubo de acero de 75 mm incrustado (pintado con pintura metálica), se puso en el fondo del río. El acero no sufrió. **La pintura desapareció.** Presión trituradora, 203 kg/cm cuad.

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**55. Profesor Ira H. Woolson.** E. N., '05 junio 1.

**55 a. Absorción.**

**Muestras.** Cubos de 20 cm, 1 : 2 : 4, de 3 semanas, secados al horno durante 13 días á 49° C.

Parte con arena de < 1% de marga; todo pasó al cedazo de 3 mm; 75% pasó el

**Para el índice**, véanse págs. 1346, etc.

cernidor de 8 mallas por cm. Parte con piedra caliza cernida en el triturador de 10 mm; el 87 0/0 pasó el cernidor de 6 mm; el 40 0/0 pasó el cernidor de 3 mm; arena y tierra suficiente para llenar los vacíos. La piedra pasó al anillo de 40 mm.

**Resultados.**

**Absorción media**; 4 horas, 2.87%; 24 horas, 2.95%; 48 horas, 3.33%.

No hubo diferencia notable entre **la arena y las piedras cernidas**.

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**56. W. C. Hoad,** Universidad de Kansas, E. N., '05, agosto 10.

**Arcilla y greda; resistencia y absorción.**

**56 a.** Cemento Portland con (*a*) arena normal de Otawa, 1 : 3; (*b*) 2 á 20% de la arena se reemplazó por arcilla ó marga. A los 90 días las resistencias relativas eran, en general: (*a*) 100; (*b*) de 94 á 125.

**56 b.** Hasta el 6 ú 8% de arcilla ó marga, no hubo aumento en la **absorción** con marga; y como 10% de disminución, con arcilla. Con **mayores porcentajes**, la absorción aumentó algo.

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**57. Eng News,** '05, septiembre 28.

**57 a. Permeabilidad.**

**Cisterna de cemento armado**, 341 m cúb. 1 : 2 : 4, cemento Portland, arena de río y granzón. Capa de 25 mm de **mortero en el fondo**. A los **muros** se les dió una lechada de tres manos con **cemento puro líquido**, con la consistencia de una crema, aplicada después de humedecer bien los muros. Cada mano se dejó secar 24 horas. Donde las paredes estaban demasiado húmedas, la lechada se cuarteó; donde estaban muy secas, la lechada no se adhirió. Durante pocos días después de llenarla, perdió 8 mm en profundidad por día. Está perfectamente impermeable desde entonces. La cisterna se construyó con el aire á una **temperatura menor de** — 7° C; pero se cubrió con tablas y se encendieron dos estufas de cok.

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**58. Profesor Ira Woolson,** E. N., '05, noviembre 2.

**58 a. Flujo (?)** (*Flow*). (*N. del T.* — Así llama el autor esta prueba.)

**Muestras.** Columnas de 10 cm de diámetro, 30 cm largo, modeladas en tubos de acero de 3 á 6 mm de espesor, y dejándolas fraguar y permanecer en ellas 17 días; al cabo de ellos el concreto estaba muy duro. El concreto permaneció en los tubos durante las pruebas.

**Resultados.** Bajo cargas de 68,000 kg, las columnas en los tubos más gruesos se acortaron solamente < 6 mm, pero bajo cargas de 54,400 á 68,000 kg, las columnas, en algunos de los tubos más delgados, estaban dobladas, deformadas y acortadas en 88 mm. aumentando su diámetro de 10 á 12 cm más ó menos. *Al quitar los tubos, se encontró el concreto sin quebrarse, sólido y perfecto!*

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**59. J. M. Braxton,** ingeniero asistente, de los Estados Unidos, Informes '05-6, E. N., '08, mayo 14, pág. 525.

**La corrosión en el agua de mar.**

**59 a.** Barras de acero de 12 mm incrustadas en 4 bloques de concreto hecho con arena de coral y ladrillo quebrado. 2 bloques en 1.20 m de agua de mar, 2 en un gabinete seco, ambos durante más de 1 año. La barra de uno de los bloques secos mostró señales de moho. Las otras estaban tan brillantes y lisas como al colocarlas.

**59 b.** 30 bloques, 30 × 30 × 15 cm, cemento Portland 1 : 3 : 5, ladrillo quebrado. Hechos bajo las condiciones ordinarias del trabajo. Barra de acero de 15 mm torcida, de 20 cm de largo, en el centro de cada bloque; 20 bloques con arena de coral, 10 con arena de cuarzo ordinaria. La mitad de cada grupo se colocó en el océano, la otra mitad estaba al aire libre. Se quebraron después de 1 año y 3 semanas. En todos los bloques que se colocaron en el océano se encontraron las barras en perfecto estado. Todas las otras estaban más ó menos enmohecidas.

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**60. Wm. R. Baldwin-Wiseman**, Instrucción en los Procedimientos de Ingeniería civil, '06, vol. 183, pág. 319.

**60 a.** Efectos del agua á presión por entre discos de concreto, de 33 cm de diámetro, 15 cm de espesor, 1 : 4 de cemento Portland, granzón triturado pasado por anillo de 25 mm. El peso específico del concreto era 2.23. En los moldes de madera, 10 semanas. El agua, para la presión, se bombeó de una formación cal-

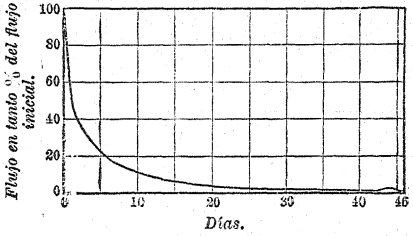


**Fig. 4.** Influencia del paso del agua á presión.

cárea. La temperatura del aire era de 12° á 15° C. Presiones, de 1.7 á 4.2 kg/cm cuad. La filtración fué la misma que en la fig. 4. Hacia el fin de los ensayos aparecieron pequeños **crecimientos estalactíticos** en el fondo de la pieza de prueba, y lo filtrado fué absorbido por la evaporación. Cerca de la superficie, el agua, bajo alta presión, disolvió un poco del material, pero la depositó en los poros un poco más lejos donde la presión había disminuído por su paso á través del bloque.

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**61. Sandford E. Thompson.** A. S. T. M., Procs., vol. VI, 1906, pág. 379.

**61 a. Consistencia; efecto sobre la densidad *, permeabilidad y resistencia á la compresión.**

Muestras para la densidad y la permeabilidad, de 21 días de edad; muestras para la resistencia á la compresión, de 5 $^1/_4$ meses.

**Muestras.**

Cemento Portland Atlas; arena de Newburyport, peso específico=2.65; piedra volcánica, peso específico=2.78. 1 : 2.3 : 4.6, por vol; 1 : 2 : 4 por peso.

| **Consistencia empleada.** | Agua, %** |
|---|---|
| **Seca.** Como tierra húmeda, el agua brillando en la superficie al apisonar duro | 5.4 |
| **Mediana.** Parecía mojada al mezclarla, no fluyó en la caja de mezclar. Trepidaba poco | 6.9 |
| **Húmeda** | 9.2 |
| **Muy húmeda.** Como sopa espesa, se asentó á nivel en la caja de mezclar. Se necesitaron cucharones para manejarla. Ligeramente más húmeda que la que se usa en los trabajos de edificios | 11.0 |
| **Sumamente húmeda** | 13.7 |

**Resultados.** (Véase fig. 5 y *Obs. del T.*)

Para una consistencia dada, el **percentaje de agua** depende de la naturaleza del cemento y del tamaño y sequedad de los granos de arena. Una arena fina, ó con muchos granos finos, puede necesitar dos veces tanta agua como la arena ordinaria.

(*Obs. del T.* — Los números verticales (fig. 5) á la izquierda, de 140 á 420 expresan kg/cm cuad. Los de 2,000 á 6,000, lbs/pulg cuad.)

* Densidad = vol de las partículas sólidas en la unidad de volumen del concreto.
** Percentaje del peso del cemento, arena y piedra

**Para el índice**, véanse págs. 1346, etc..

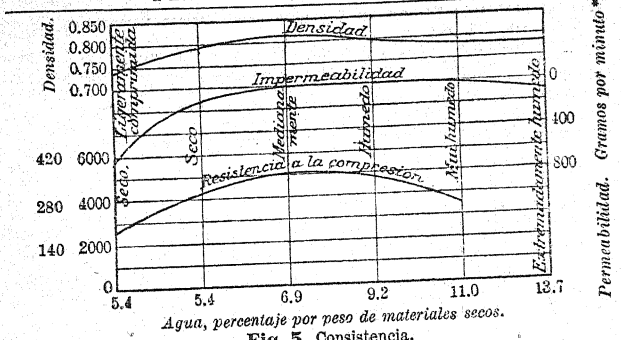


**Fig. 5.** Consistencia.

**61 b. Coeficiente de elasticidad.** 12 cubos de 30 cm, las deformaciones se midieron en un escantillón de longitud de 12.5 cm. Término medio de 4 muestras, 1 : 2 : 4, aproximadamente 1, 2, 6, y 17 meses de edad. Secas, 313,000 kg/cm cuad; medianas, 295,000; muy húmedas, 211,000. No hubo aumento apreciable con la edad en el coef.

**61 c. Edad; permeabilidad.** Los bloques se probaron á los 21 y 84 días, y mostraron permeabilidades, más ó menos de 2 : 1.

**61 d. El exceso de agua** arrastra el cemento fino, formando **lechosidades**, reduciendo la resistencia y aumentando la permeabilidad. **Espesor** de la lechosidad que se forma: 3 mm en las mezclas muy húmedas.

**61 e.** El **Sr. Thompson deduce** que en los trabajos de construcción y otros de cemento armado, el concreto debe estar suficientemente húmedo « para que fluya despacio por todas partes, incrustando muy bien el acero y haciendo que las superficies que dan contra los moldes sean lisas », y que el concreto mediano ó que trepida, es conveniente para las masas ordinarias de concreto, tales como cimientos, muros gruesos, arcos grandes, pilas y estribos.

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**62. A. Blach**, E. N., '06 agosto 30, pág. 236.

**62 a. Naturaleza de la arena; resistencias y absorción.**

| **Muestras.** | Que pasa el cedazo n.º 170. |
|---|---|
| A, gneiss triturado, cernido por mallas de 12 mm..... | 90.8% |
| B, arena de Cowe Bay, que se usa mucho en Nueva York y sus cercanías..... | 95.8% |
| C, arena silicosa, fina limpia..... | 95.8% |

**Resultados.** De 7 y 28 días, 1 : 2 y 1 : 3 morteros, A y B dieron, en general, de 20 á 25% de **resistencias** de tensión y compresión, mayores que C. En general, las mezclas más fuertes fueron las más absorbentes.

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**63. Alex. B. Moncrieff**, E. N., '06 agosto 30, pág. 227.

**63 a.** Probetas en el agua 2 años, en el aire 7 días y en aceite 6 meses. En general, el cemento puro perdió de 0 á 36% de su resistencia, mientras que el 3 : 1 ganó de 0 á 65 %. secándolo en el aire y sumergiéndolo en aceite.

**63 b.** Las probetas en el aire 7 días; luego 6 meses en aceite ó en agua. Las probetas de cemento puro en aceite resultaron de 0 á 55 % más débiles que las de cemento puro en agua y las probetas de 3 : 1 en aceite resultaron de 49 á 79 % más débiles que las que estaban en agua.

* Las pruebas de permeabilidad, á presión de agua de 5.6 kg/cm cuad.

**63 c.** Las probetas unas, 9 semanas en agua; otras 4 semanas; una semana al aire y 4 semanas en aceite. Con pocas excepciones, las probetas de cemento puro, en aceite, resultaron de 0 á 40% más fuertes que las que estaban en agua, mientras que las probetas de 3 : 1 resultaron de 0 á 63% más fuertes que las iguales que estaban en el agua. Muchas de las probetas tratadas por el aceite « se astillaron como pedernal ».

— 64 —

**64. Profesor Arthur N. Talbort.** Universidad de Illinois, Bull., vol. IV, n.º 1, '06 septiembre 1.

**64 a. Adhesión y fricción. '04.**

**Muestras y resultados.**

Mezcla, 1 : 3 : 6.

Tracción, en kg/cm cuad de sección neta;

Límite de elasticidad, en kg/cm cuad;

Adhesión, en kg/cm cuad de superficie incrustada:

(*N. del T.* Convertida al sistema métrico.)

| | Barras Johnson. | | Barras redondas. | | Barras cuadradas. | | |
|---|---|---|---|---|---|---|---|
| | ½" (12½mm) | ¾" (19mm) | ⅜" (10mm) | ¾" (19mm) | ⅜" (10mm) | ½" (12.5mm) | ¾" (19mm) |
| Tensión..... | 4,999 | 2,415 | 2,205 | 1,505 | 2,496 | 1,856 | 1,460 |
| Límite elasticidad. | 4,200 | 4,081 | 2,975 | 2,835 | 3,150 | 2,331 | 2,450 |
| Adhesión..... | 42 | 29 | 17 | 22 | 21 | 20 | 23 |

Con todas las **barras de Johnson**, las muestras se astillaron ó se quebraron. Todas las **barras lisas** se resbalaron. 6 de las 11 barras Johnson y 4 de las 11 barras cuadradas de 10 mm, « se golpearon 6 veces con una mandarria de 4.5 kg », reduciendo su adhesión en 5 á 20%.

**Muestras.**

**64 b.** '05-6. Cilindros de 15 cm de diámetro, 15 y 30 cm de largo, de 60 días. Mezcla de cemento Portland americano, resistencia á la tensión, puro, 51 kg/cm cuad á los 7 días; 1 : 3, 25 á 7 días, 37 á 75 días; arena gruesa de mortero; piedra caliza quebrada cernida por 25 mm. Metal, límite de elasticidad, kg/cm cuad. Acero dulce (D), redondo, 2,660; chato, 3,150. El cilindro de eje laminado en frío (C), 6,090. Acero de herramienta (T), 3,710.

**Resultados.**

| N.º de pruebas. | Acero. | Tamaño, mm. | Mezcla. | Largo incrustado, mm. | Kg/cm cuad de superficie incrustada. Adhesión. *a* | Fricción. *f* | *f*/*a* |
|---|---|---|---|---|---|---|---|
| 6 | D | 12.7 redondo. | 1 : 3 : 5.5 | 152 | 26.1 | 14.8 | .57 |
| 6 | » | » | 1 : 2 : 4 | » | 29.0 | 16.0 | .55 |
| 6 | » | 15.9 redondo. | 1 : 3 : 5.5 | » | 25.0 | 16.0 | .64 |
| 4 | » | » | 1 : 2 : 4 | » | 32.7 | 20.9 | .64 |
| 3 | » | 12.7 redondo. | 1 : 3 : 5.5 | 304 | 26.2 | 18.8 | .72 |
| 4 | » | » | 1 : 2 : 4 | » | 28.4 | 18.7 | .65 |
| 3 | » | 15.9 redondo. | 1 : 3 : 5.5 | » | 28.2 | 16.0 | .57 |
| 3 | » | » | 1 : 2 : 4 | » | 29.1 | 15.7 | .54 |
| 3 | » | 12.7 × 4.8. | 1 : 3 : 5.5 | 152 | 8.8 | 5.9 | .67 |
| 3 | C | 25.4 redondo. | » | » | 9.5 | 4.7 | .49 |
| 3 | » | 12.7 redondo. | » | » | 11.0 | 3.5 | .32 |
| 3 | T | 19.0 redondo. | 1 : 3 : 6 | » | 18.3 | ... | ... |

**La mezcla rica** es generalmente superior. **El cilindro de eje** laminado en frío y el **acero de herramientas** es generalmente inferior, debido á la uniformidad de sección y la lisura de la superficie.

**Para el índice**, véanse págs. 1346, etc.

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**65. Jos. W. Elms**, químico, comisionado de los trabajos hidráulicos de Cincinnati, E. R., '06, octubre 27, pág. 467.

**65 a. Permeabilidad.**

**Muestras.** Cemento Portland y natural (de Louisville); arena de cuarzo del río Ohio, limpia, más bien fina, bastante uniforme en tamaño; piedra caliza cernida con mucho polvo y muy fino.

**Cubos de 75 mm.**

Cemento Portland; (*a*) 1 cemento : 2 arena, 10% de agua; (*b*) 1 cem : 1 arena : 1 cerniduras, 11% de agua; (*c*) 1 cem : 2 cerniduras, 14% de agua.

Cemento natural; (*d*) 1 cem : 2 arena, 15% de agua; (*e*) 1 cm : 1 arena : 1 cerniduras, 15% de agua; (*f*) 1 cem : 2 cerniduras, 17% de agua.

**Cilindros huecos**, 15 cm de diámetro, 20 cm de largo, el hueco de 5 cm. Cemento Portland y arena, 1 : 1 y 10% de agua.

**Tratamiento.** Agua (clara) llevada al centro de las muestras. **Cubos**, 1 día al aire, 6 en agua. **Cilindros**, 1 día al aire, 27 en agua, 4 al aire.

**Resultados.** La filtración pasó á través del mortero, de 4 á 5 cm de espesor. **Cubos**; bajo 3.5 kg/cm cuad mantenidos durante 3 á 16 horas, poca ó ninguna agua (máx=6.5 lit por m cuad) pasó por entre los cubos de cemento Portland; de 11.6 á 98 lit/hora/m cuad á través de los cubos de cemento natural. En el Portland la filtración se hizo apreciable con presión de 4 á 5 kg/cm cuad; en el natural, á 1 kg. Los cubos con arena de 1 : 2 resultaron los más permeables. **Cilindros**, 1 á 2 kg/cm cuad; filtración .00920 á 50 lit/hora/m cuad.

La filtración disminuyó muy notablemente con el tiempo.

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**66. W. J. Douglas**, ingeniero á cargo de los puentes para Wáshington, E. N., '06, diciembre 20, pág. 649.

**66 a.** Un puente **pintado con un cemento rico en cal libre**, mostró después una masa de protuberancias de diferentes colores.

— 67 —

**67. Profesor C. von Bach.** Zeitschrift des Vereins Deutscher (*Gaceta de la Asociación de Ingenieros alemanes*), '95, '97.

**67 a. Relación entre la unidad de alargamiento y la unidad de esfuerzo.**

**Muestras.** Cilindros de concreto, de 25 cm de diámetro, 1 m de largo. Las deformaciones se midieron en un largo de 75 cm.

**Tratamiento.** Carga de 8 kilogramos/centímetro cuadrado, aplicada y quitada alternativamente hasta que la deformación no aumentaba. Después del mismo modo desde 16 kg/cm cuad hasta 40 kg/cm cuad.

**Resultados.** Desde el principio, las deformaciones aumentaron con más rapidez que las cargas. Sea :

$s$=á la unidad de esfuerzo=esfuerzo por unidad de área de la sección transversal;

L=largo original medido de 75 cm;

$l$=reducción de L bajo la compresión;

$e=l/L$=la unidad de deformación;

$c$=un coeficiente, que depende de la naturaleza del material;

$m$=exponente.

Entonces, $e=l/L=c\ s^m$.

| Mezcla. | | | | Valores aproximados. $1/c$ | |
|---|---|---|---|---|---|
| Cem. | Arena. | Granzón. | Piedra. | Para $s$ en kg/cm cuad. | $m$. |
| 1 | 2.5 | 5 | 0 | 298,000 | 1.14 |
| 1 | 2.5 | 0 | 5 | 457,000 | 1.16 |
| 1 | 3.0 | 0 | 0 | 315,000 | 1.15 |
| 1 | 1.5 | 0 | 0 | 356,000 | 1.11 |

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**68. R. C. Carpenter.** A. S. T. M., Procs., '07, vol. 7, pág. 398. **Aceite de linaza y de lubricar; solidez y resistencia á la tensión.** Probetas de cemento puro, unas con 2% de aceite de linaza ó de lubricar agregado al agua de la mezcla, las otras sin aceite. No se dió el número de probetas.

**68 a. Solidez.** 24 horas al aire húmedo. Las probetas mezcladas sin aceite, sólidas después de 8 días en cualquiera de los aceites. Las probetas mezcladas con aceite y sin él, permanecieron sólidas después de 3 horas de ebullición.

**68 b. Tensión.**

| Aceite en la mezcla. | Tensión, kg/cm cuad. | | |
|---|---|---|---|
| | 1 día. | 7 días. | 28 días. |
| Ninguno | 30.2 | 48.9 | 52.2 |
| 2% de aceite de linaza | 12.6 | 34.6 | 40.2 |
| 2% de aceite de lubricar | 23.3 | 48.4 | 48.4 |

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**69. M. R. Barnett,** Instituto de Ingeniería civil, Procs., '07, vol. 167, pág. 153.

**69 a. Acción del agua dulce sobre el concreto de piedra caliza.** Acueducto de Thirlmere, agua potable de Manchester, Inglaterra. La sección del acueducto, construida con concreto de piedra caliza. Piso 23 cm de espesor, se redujo como en 6 mm en espesor, con apariencia de panal, carcomido en muchas partes, y con mucha filtración.

**69 b.** Las muestras de las piedras calizas, con que se hizo el concreto, se conservaron durante seis meses en agua dulce corriente, en el acueducto, y perdieron en peso de 6.8 á 18.1% por año, mientras que los bloques de muestra de cemento puro y de mortero de cemento Portland de 1 : 1, ganaron 5.5 y 3.6% respectivamente.

— 70 —

**70. Profesor Ira H. Woolson,** A. S. T. M., Procs., '05, pág. 335, '07, pág. 404.

**Temperaturas altas y conductibilidad térmica.**

**70 a. Mezcla,** 1 : 2 : 4; con ceniza, 1 : 2 : 5. Cemento de una mezcla igual de 3 Portlands. Arena aguda, de calidad menuda buena, «no especialmente limpia»; 90% pasó el cedazo de 5 mallas por cm. El agregado de ceniza de caldera de buena calidad, habiéndosele quitado la mayor parte de las cenizas finas; granzón de cuarzo limpio de 19 mm; trapa (piedra volcánica) triturada. Mezclada moderadamente húmeda, apisonada en los moldes hasta que el agua fluyó á la superficie.

**70 b. Temperaturas altas.** '05, pág. 335. Fig. 6.

**Muestras.** Para la resistencia á la compresión, cubos de 10 cm; para la elasticidad, prismas de 15 × 15 × 35 cm. 3 cubos y 3 prismas se probaron sin calentar; 3 cubos y 2 prismas de cada agregado (trapa y caliza) á cada temperatura.

**Resultados.**

**70 c. Módulo de elasticidad, E.** Para E, las curvas de trapa (roca volcánica) y de caliza casi coincidieron.

**70 d.** Después de calentarlos hasta 1093° y 1371° C, los cubos de piedra caliza aparecían sanos mientras estaban calientes, pero se **desintegraron al enfriarse.**

**70 e. Después de enfriarse** bajo 400° C, los prismas de trapa y de caliza estaban cubiertos de grietas diminutas. Bajo temperaturas más altas, estas grietas aumentaron en número y en tamaño, y los prismas se cimbraron y desintegraron al enfriarse bajo 816° C.

**70 f.** Las muestras de concretos de **trapa** (piedra volcánica) **y de cenizas** permanecieron **sanas,** mientras que las de concreto de **granzón** se **agrietaron** y desmoronaron, debido, probablemente, al gran coeficiente de **dilatación** del dirección y al hecho de que este coeficiente en una dirección es doble del de la cuarzo, perpendicular.

**Para el índice**, véanse págs. 1346, etc.

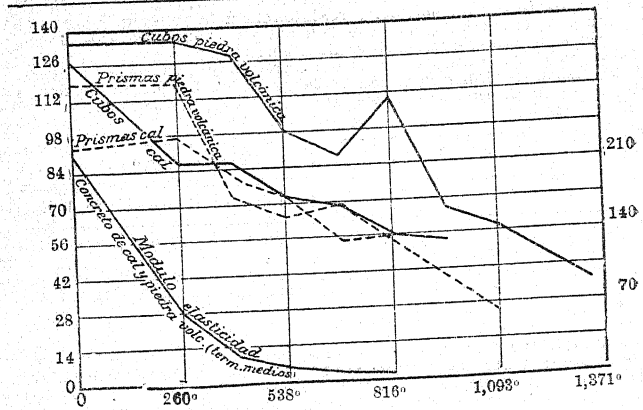



**Fig. 6.** Resistencia al calor; grados C.

(*N. del T.* Los números verticales á la izquierda, de 14 á 140, indican las cargas de trituración en kg/cm cuad. Los verticales de la derecha, de 70 á 210, el coeficiente de elasticidad en *miles* de kg/cm cuad.)

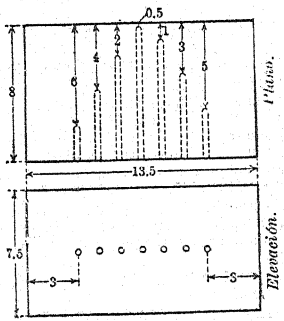


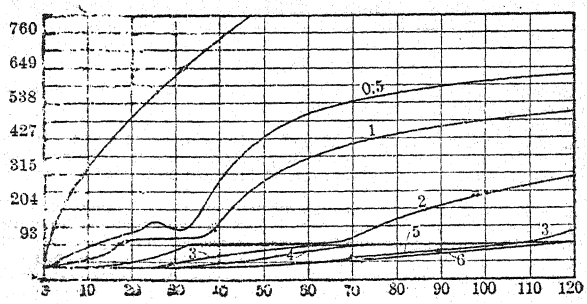

**Fig. 7.** Conductibilidad térmica.

(Equivalencias de los números de la fig. 7 en **centímetros** : .5=1.25; 1=2.5; 2=5; 3=7.5; 4=10; 5=12.5; 6=15; 7.5=18.75; 8=20; 13.5=33.75.)

**70 g. Conductibilidad térmica.** '07, pág. 404. Figs. 7 y 8.

**Muestras.** Bloques de concreto, con agujeros como en la fig. 7. La mezcla igual á la de la fig. 70 *a*.

**Tratamiento.** Las muestras estuvieron en los moldes 24 horas, en el agua 48 horas, se conservaron húmedas durante 2 ó 3 semanas, dejándolas secar bien. Edad, en la prueba, como de 2 meses. Los bloques se colocaron á la entrada de la hornalla.




*Tiempo en minutos.*
*Roca volcánica (basalto).*

**Fig. 8.** Conductibilidad térmica.

(**Explicación de la fig. 8.** Los números verticales á la izquierda, de 93 á 760, son temperaturas C. La curva primera á la izquierda, sin número, traza las temperaturas del horno. Las curvas numeradas corresponden á la conductibilidad en las líneas marcadas con los mismos números en la fig. 7.)

**Resultados.** La fig. 8 indica, para una de las muestras de piedra volc, los tiempos, en minutos, que se necesitaron **para transmitir las temperaturas de las hornallas** por los diferentes espesores del concreto. Las curvas de descenso, en ó cerca de 200° F (m. ó m. 93° C), se atribuyen á la **generación del vapor.**

**70 h. El concreto** de 5 á 7 cm (si permanece en su lugar) **protegerá** el metal de la armazón en un incendio ordinario.

**70 i. El metal de la armazón descubierto** no conduce calor perjudicial á la parte incrustada.

**70. 5. Wm. B. Fuller y Sanford E. Thompson,** « Leyes para las proporciones del concreto », A. S. C. E., Trans., '07/dic., vol. 59, págs. 139-143.

**Módulo de elasticidad, E.**, bajo compresión.

**Muestras.** Prismas de concreto de 15 cm en cuadro, 45 cm de largo, edad como de 140 días. Cemento Portland gigante, agregado : arena de Cowe Bay (CS), cerniduras del Parque Jerome (JSc). Agregado : granzón de Cowe Bay (CG), piedra del Parque Jerome (JSt).

**Resultados.**

**Efecto del tamaño máximo de la piedra.**

| Mezcla..... | 1 : 9* | 1 : 3 : 6 | 1 : 2.81 : 5.62 | 1 : 2.92 : 5.88 |
|---|---|---|---|---|
| Piedra. | Módulo de elasticidad, E, en miles de kg por cm cuad. | | | |
| 5.70 cm..... | 147 | 168 | 231 | 210 |
| 2.50 — .... | 119 | 126 | 217 | 182 |
| 1.25 — .... | 98 | 63 | ... | 154 |

**Efecto de la cantidad de cemento,** en % de material total seco *.

Módulo de elasticidad, E, en miles de kg por cm cuad.

| | Con (JSc) y (JSt). | | | | Con CS y CG. | | | | Con (JSc) y CG. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Cem..... | 8 | 10 | 12.5 | 15 | 8.5 | 10.6 | 13.25 | 15.9 | 10.2 | 72.75 | 15.3 |
| E..... | 126 | 147 | 161 | 329 | 161 | 273 | 259 | 301 | 245 | 266 | 245 |

* El material mayor en 5 mm de diámetro (como de 62 á 68 %) del total) graduado de acuerdo con las recomendaciones de los autores. Véase Concreto, §§ 23 á 25, pág. 1299.

**Para el índice**, véanse págs. 1346, etc.

— 71 —

**71. Richard L. Humphrey**, U. S. G. S. Bull, n.° 324, '07. Informe sobre el incendio de San Francisco, del 18 de abril, '06.

**Resultados.**

**71 a. El concreto** es probablemente el **mejor material** para las columnas á prueba de fuego. Su rigidez soporta el acero interior, al ablandarse por el calor.

**71 b. El concreto** resultó **superior al ladrillo** como medio á prueba de fuego.

**71 c.** A altas temperaturas el concreto **pierde su agua de cristalización.**

**71 d.** El concreto, especialmente donde estaba armado, resistió el **terremoto** y el **fuego. El dique de concreto** de San Mateo, aunque á pocos centenares de metros de la catástrofe, permaneció sano. Los **pisos de concreto** sólido, aunque de muy mala calidad, resultaron satisfactorios. El **concreto de ceniza** que se usó, en los pisos y otras partes, era rico en sulfatos, y perjudicial á la armazón.

— 72 —

**72. William B. Fuller.** Asociación nacional de los que usan concreto. Procs '07, págs. 95-7.

**Graduación y proporciones.**

**72 a.** Pruebas con 6 vigas, de 15 cm en cuadro, 1.80 m de largo; 1 de cemento á 8 de arena y piedra; **coef de ruptura** en kg/cm cuad : 1 : 2 : 6, 22.3; 1 : 3 : 5, 20; 1 : 4 : 4, 14.6; 1 : 5 : 3, 10.6; 1 : 6 : 2, 7.1; 1 : 8 : 0, 0.03.

**72 b.** Con un percentaje dado de cemento, la mezcla **más densa** de arena y agregado, da el concreto más fuerte, el menos permeable y, por consiguiente, más duradero, y el que se maneja con más facilidad, llenando por consiguiente mejor los vacíos y los ángulos.

— 73 —

**73. Comisión del cemento armado**, París, '07.

**73 a. Contracción y expansión.** El concreto se contrae al endurecerse al aire y se expande debajo del agua.

— 74 —

**74. T. L. Condron**, de la Compañía Condron y Sinks, representante de la Compañía de metal dilatado y barras corrugadas, Diario de las Sociedades occidentales de Ingenieros, '07, febrero, vol. 12, n.° 1. Ensayos por el profesor C. E. de Puy, Instituto Lewis, Chicago.

**74 a. Adhesión; barras deformadas y sencillas.**

**Muestras.**

Cilindros de concreto, de 15 cm de diámetro, 20, 30, 40, 50, 60 cm de largo. Mezclados á mano, proporcionados con exactitud, 1 : 2 : 4, de cemento Portland, arena gruesa, piedra caliza quebrada, de 12 mm y menos, sin polvo. Bastante humedad, para que entraran en los moldes con facilidad removiéndolos con una varilla. Todo el concreto se mezcló en un lote. Los bloques de 20 y 40 cm tenían 25 días cuando se probaron, los otros 31 días. Las barras pasaron enteramente por los bloques.

**Resultados.**

| | Diám en mm. | Esfuerzo kg/cm cuad de superficie incrustada. Resbalamiento > .25 mm. — Incrustados. | | Resbalamiento > .8 mm. — Incrustados. | |
|---|---|---|---|---|---|
| | | 30 cm. | 60 cm. | 30 cm. | 60 cm. |
| | | | | Adhesión kg/cm cuad. | |
| Redondo..... | 17.5 | 18.9 | 12.5 | 20.3 | 13.4 |
| Cuadrado..... | 15.9 | 22.2 | 16.1 | 24.0 | 17.0 |
| Torcido, de Búffalo. | » | 23.5 | 20.5 | 25.1 | 21.5 |
| Torcido, Ransome*. | » | 22.8 | 23.3 | 25.8 | 24.6 |
| Johnson**, nuevo. | » | 33.3 | 33.1 | 43.0 | 35.6 |
| Johnson, viejo *... | 19 | 45.8 | 37.6 | 55.2 | 37.6 |

— 75 —

**75. A. A. Knudson**, Instituto americano de Ingenieros electricistas, Procs. 07', febrero, vol. 26, parte 1.ª, pág. 231, E. N., '07, marzo 21, pág. 328.

**75 a. Electrolisis.**

**Muestras.**

1 : 1 cemento y arena, Portland y Rosendale. Los bloques se moldearon en baldes de metal para agua. El electrodo positivo fué un tubo de hierro forjado corto de 5 cm en el eje del bloque, sumergido como 20 cm.

**Tratamiento.** Los bloques se pusieron en agua (uno en agua dulce, el otro en agua salada) en un estanque; el electrodo negativo era un pedazo de lámina de hierro, sumergida en el estanque. La corriente de .1 amperio.

**Resultados.** Después de 30 días, los **bloques de Portland** (que se habían agrietado bajo la corriente) **se quebraron fácilmente**, mostrando depósitos amarillentos (moho del hierro al parecer) y se **ablandó el concreto** en las juntas. **Los tubos perdieron** más de 2 % por corrosión. **La resistencia eléctrica final** = 10 × la resistencia inicial, =m. ó m. la resistencia del concreto seco. Las grietas en el **Rosendale** aparecieron á los 6 días. Uno de los tubos estaba enteramente carcomido.

— 76 —

**76. J. L. Van Ornum**, Sociedad americana de Ingenieros civiles, Trans., vol. 51, pág. 443, '03/diciembre, y vol. 58, pág. 294, '07/junio.

**76 a. Fatiga.** Bloques de cemento puro en compresión. Las cargas repetidas producen fracaso si la carga es > que la mitad de la que se requiere para la trituración en una sola aplicación.

Vol. 58, pág. 294.

**76 b. Fatiga.** Como 600 pruebas.

**Muestras.**

Bloques de 12.5 × 12.5 cm y 30 cm de largo, en compresión, y vigas de 10 cm de ancho, 15 de alto, 1.80 m de luz, armadas con dos barras de acero lisas, de 12 mm en cuadro. Cada lote produjo 8 bloques ó 4 vigas. Mezcla de 1 : 3 : 5 en volumen. Cemento Portland americano corriente, probado según la especificación de la Sociedad americana de Ingenieros civiles (pág. 942). Arena del río Misisipí, gastada por el agua, más bien fina, 1,584 á 1,760 kg por m cúb; vacíos 30 á 38%. Piedra caliza quebrada, de las cercanías de San Luis, de 1,280 á 1,520 kg/m cúb, que pasó el cernidor de mallas de 4 cm; casi la mitad de las piedras eran mayores de 2.5 cm, como la décima parte menores que 1 cm. Vacíos 42 á 48 %. Vacíos, en 3 de arena + 5 de agregado, de 16 á 19%.

* Cubiertos con una capa delgada de moho, pero sin escamas. Las otras se tomaron frescas de los cilindros y estaban libres de moho.

** Johnson, fig. 2 (*d*), pág. 1341.

**Para el índice,** véanse págs. 1346, etc.

**Tratamiento.** Las muestras para la compresión se dejaron 1 día al aire en los moldes, las vigas por 2 días; después todo en agua por 2 semanas, después al aire, protegidas contra las corrientes, hasta que se probaron.

Las muestras para la compresión eran de 1 mes y de 1 año de edad, cargadas de 4 á 8 veces por minuto; las vigas, de 1 mes, 6 meses, y 1 año, cargadas de 2 á 4 veces por minuto.

**Resultados. El efecto en la proporción de la repetición** fué insignificante; pero se cree que aumenta rápidamente cuando pasa de 10 veces por minuto.

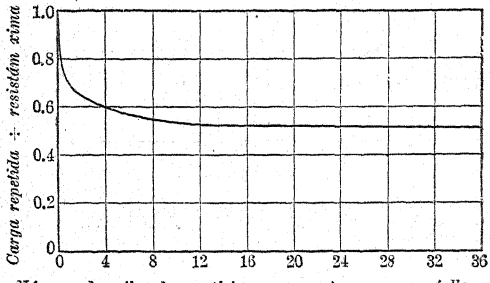



**Fig. 9.** Fatiga.

**Fatiga.** La curva de la fig. 9 representa bien los resultados que se obtienen en estas condiciones variables.

**76 c.** Al concreto repetidamente sometido á esfuerzos menores que el límite de fatiga es decir : á menos de la mitad del máximo del esfuerzo, véase figura), « se le comunicó un **límite definido de elasticidad**, dentro del cual las resistencias son proporcionales á los esfuerzos » (es decir : dentro de los cuales el **coef de elasticidad, E,** es constante).

**76 d. Fatiga y adhesión.**

**Muestras.** Barras de acero cuadradas sencillas de 15 mm, incrustadas en concreto como anteriormente. Las muestras se hicieron con gran cuidado, muy bien apisonadas.

**Tratamiento.**

2 días en los moldes, 7 días en agua, al aire 3 semanas. 30 muestras para la prueba de fatiga se sometieron á un « golpe y presión combinados, con su vibración consiguiente »; 150 golpes por minuto, cada golpe=8.18 kg-metro. Término medio, 50,000 golpes á cada muestra.

**Resultados. Adhesión** inicial media, 8.7 kg/cm cuad de superficie incrustada; **fricción** (después de deslizamiento), 6.3 kg/cm cuad. En **las muestras no fatigadas,** 10.5 y 7 kg/cm cuad respectivamente.

**76 e. Fatiga bajo carga continua.** 2 prismas de concreto permanecieron intactos durante 1 mes bajo el 90% de la resistencia á la trituración. « Unos pocos bloques de concreto fallaron en la prueba de compresión en pocas horas bajo una presión constante de un percentaje mayor. » Una viga de concreto armado falló en 10 meses bajo el 90% de su carga de ruptura.

— 77 —

**77. Henry S. Spackman.** Asociación de Fabricantes de cemento Portland americano, Nueva York, '07, diciembre.

**77 a. Mortero remolido después de endurecido.**

Las probetas de cemento Portland se quebraron en la prueba. Se remolieron y se hicieron nuevas probetas con ellas. Éstas presentaron, en general, como **la mitad de las resistencias de tensión** de las probetas originales. Del cemento

original, el 91.5% pasó un cedazo **n.º 100, el 76.2%** el n.º 200. El material remolido tenía la misma finura, más ó menos.

— 78 —

**78. R. Feret,** Asociación americana de Ingenieros civiles, Trans., '07, diciembre, vol. 59, pág. 152.

**78 a. Permeabilidad.** « **Los experimentos dan,** en general, **resultados inciertos.** No es raro ver muchos bloques del mismo concreto que, aunque tratados de manera idéntica, dejan filtrar cantidades muy diferentes de agua. »

| **78 b. Edad del bloque,** días | 5 | 29 | 30 | 365 |
|---|---|---|---|---|
| | **Flujo,** en grams /minuto por kg de presión /cm cuad. | | | |
| Presión de 5 á 20 kg /cm cuad; promedio..... | .554 | .044 | .159 | .294 |
| Después de permanecer bajo 20 kg /cm cuad 2 horas.. | .349 | .034 | .133 | .278 |

**78 c. Filtración** « **casi proporcional á la presión** ».

| **78 d. 3 bloques de 1 año.** Bloque | A | B | C |
|---|---|---|---|
| | **Flujo,** en grams /minuto por kg de presión cm cuad. | | |
| A 20 kg /cm cuad..... | .067 | .111 | .108 |
| Aumentada á 29 kg /cm cuad..... | .077 | .114 | .126 |
| Reducida á 20 kg /cm cuad..... | .068 | .114 | .111 |

« como si el efecto del aumento momentáneo de la presión hubiera sido abrir nuevos pasajes al agua, ó limpiar parcialmente pasajes que ya existían ».

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**79. Wm. B. Fuller y S. E. Thompson,** Sociedad americana de Ingenieros civiles, Trans., '07, diciembre, vol. 59, pág. 67.

**Resistencia, densidad y permeabilidad.** Como las afectan las **proporciones y la naturaleza de la arena y del agregado.** Ensayos en el estanque del Parque Jerome, Nueva York.

**79 a. Muestras.** Cemento Portland como se recibió para usarlo en el estanque, agregado (1) piedra y cerniduras de mica esquistosa en los trituradores en el estanque, 35% de mica, que en el mortero ó concreto « no forma planos que afecten la resistencia seriamente ». (2) Granzón y arena de Cowe Bay, sacados con draga del río (« arena y granzón redondeados por el agua, completamente limpios y que se componían casi enteramente de partículas de cuarzo »; peso específico como 2.65). Tamaño máximo de la piedra, 6, 2.5 y 1 cm.

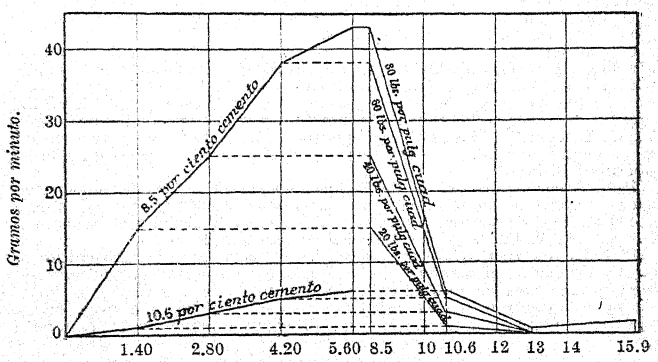



**Fig. 10.** Permeabilidad.

**Para el índice,** véanse págs. 1346, etc.

(Explicación complementaria de la fig. 10. Los números horizontales, de 1.40 á 5.6, son presiones en kg/cm cuad. En las líneas inclinadas que marcan presiones así: 80 lbs/pulg cuad=5.6 kg/cm cuad; 60 lbs/pulg cuad=4.2 kg/cm cuad; 40 lbs/pulg cuad=2.8 kg/cm cuad; 20 lbs/pulg cuad=1.40 kg/cm cuad.)

Las pruebas se hicieron con « **mezclas graduadas** » (dando con las proporciones la densidad mayor del agregado) y « **mezclas naturales** » (1 : 2.5 : 6.5, 1 : 3 : 6, 1 : 3.5 : 5.5).

**Resultados.**

**79 b. Tamaño del agregado; resistencia y densidad.**

| Tamaño mayor de la piedra..... | 5.7 cm. | 2.5 cm. | 1.2 cm. |
|---|---|---|---|
| **Resistencia relativa.** | | | |
| Compresión..... | 1.00 | .83 | .72 |
| Transversal..... | 1.00 | .91 | .77 |
| **Cemento que se necesita** para igual resistencia, relativa..... | 1.00 | 1.17 | 1.33 |
| **Densidad** relativa..... | 1.00 | .96 | .93 |

**79 c. Clase de agregado. La arena y las cerniduras.** Resistencia y densidades relativas.

| | Compresión. | Resistencia transversal. | Densidades. |
|---|---|---|---|
| Arena y piedra..... | 100 | 100 | 100 |
| — y granzón..... | 94 | 89 | 102 |
| Cerniduras y piedra.. | 67 | 85 | 98 |

**79 d.** La mezcla graduada dió la **densidad**=1.14×densidad con la mezcla natural; para la igual resistencia, las mezclas graduadas necesitaron .88×el **cemento que se necesita** con las mezclas naturales.

(Esto significa un promedio de **economía** de un 25% **por m cúbico** del concreto, más ó menos. Allen Hazen, Sociedad americana de Ingenieros civiles, vol. 59, pág. 150, diciembre, '07.)

**79 e.** El exceso de arena fina ó mediana, ó la deficiencia de arena fina en el concreto menudo, disminuye la resistencia y la densidad.

**79 f. La resistencia y la densidad** son máximas cuando la mezcla llena exactamente los vacíos.

**79 g. Permeabilidad.** Véase la figura 10. « Poco se sabe sobre la acción del concreto para resistir el flujo del agua. » Pues entre « proporciones diferentes y diferentes tamaños de la misma clase de material, las leyes de la impermeabilidad son algo parecidas á las de la resistencia ».

Con un percentaje dado de cemento, las muestras **más densas** serán ordinariamente más impermeables. Cuando las densidades son iguales, las muestras **más ricas** serán las **más impermeables** (véase la figura). Las proporciones, sin embargo, son muy diferentes de las de la densidad ó de la resistencia; una pequeña diferencia en la composición, produce un gran efecto en la impermeabilidad. « **Diferentes clases de agregados** producen resultados muy diferentes en la impermeabilidad. » La figura demuestra el efecto de la **presión** sobre la **permeabilidad**.

**79 h.** El concreto con piedra del Parque Jerome y cerniduras dió proporciones mucho mayores de filtración en todas partes (máxima, 369 gramos por minuto) que el de arena de Cowe Bay y granzón. El concreto de piedra y arena dió, más ó menos, la mitad de las proporciones que se ven en la fig. 10.

**79 i.** La **permeabilidad** es algunas veces mayor con piedras **grandes** y otras veces con las **pequeñas**. Los resultados han sido especialmente erróneos con la piedra quebrada y cerniduras del estanque del Parque Jerome.

**79 j.** La « **permeabilidad** disminuye materialmente con la **edad** », aumentando mucho más rápidamente que lo que disminuye el **espesor del concreto**.

menor con arena y granzón que con piedra y cerniduras;
— — — — y arena;
— — piedra — y cerniduras.

— 80 —

**80. Richd. H. Gaines,** Junta del acueducto de Nueva York, S. A. de I. O., Trans., vol. 59, '07 diciembre, pág. 159.

**80 a. Permeabilidad y resistencia; arcilla y alumbre.**

**Muestras.** Mortero, 1 : 3, Portland, arena de Cowe Bay. Pruebas de tensión con probetas corrientes; pruebas de compresión y tensión con cubos de 5 cm. Edad de las muestras, de 20 á 30 días. Presiones, 2.80 á 5.60 kg/cm cuad.

**Resultados.** (1) Reemplazo del agua de la mezcla por una solución de **alumbre** de 2.5 á 5% (1 á 2% es suficiente). y dió una **impermeabilidad** casi completa.

(2) Reemplazo de 5 á 10% de la arena por **arcilla** molida fina; y (3) combinación (1) y (2) que dió aún mejores resultados.

Las muestras de arcilla (con alumbre y sin él) mostraron de 12 á 18% de ganancia en **resistencia** sobre las que no tenían arcilla.

El procedimiento se basa en **una teoría de la acción físicoquímica** entre los iones del electrólito (el alumbre) y las moléculas coloides (gelatinosas) de la arcilla.

Ninguno de los **procedimientos en uso hasta el presente,** que se han examinado, ha resultado conveniente para uso general.

**La cal apagada disminuye** ligeramente la **permeabilidad**; pero esta ventaja está más que compensada con la pérdida en **resistencia.** No hay razón química para que esto sea de otra manera.

— 81 —

**81. Profesor E. Morsch,** Zurich, para Wayss 'y Freitag A-G, Neustadt. « Der Eisenbetonbau », Stuttgart, Konrad, Wittwer, '08, citaremos las págs de esta obra.

**81 a. Relación de elasticidades,** págs. 27-32. **Muestras**; prismas cuadrados, largo medido, 35 centímetros. 1 parte de cemento Portland Manheim con 3 partes de una mezcla de arena del Rhin y granzón compuesta de 3 partes de arena, 0-5 mm; 2 partes de granzón, 5-20 mm. Agua, 14%. Cada esfuerzo se mantuvo durante 3 minutos. Algunas de las muestras se probaron en tensión; las otras en compresión.

*Compresión en millonésimas de la longitud primitiva.*

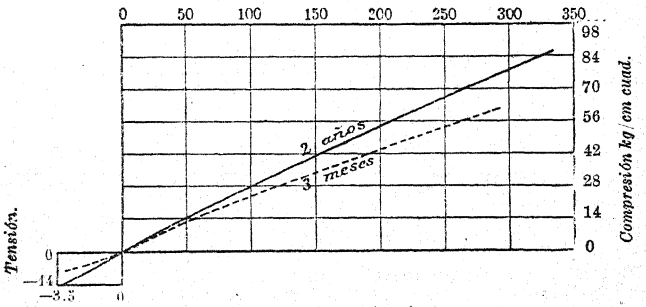



**Fig. 11.** Esfuerzo y estiramiento.

Los 2 años y los 3 meses de la línea continua y de puntos se refieren á la edad de las muestras.)

Resultados. **Las unidades de los esfuerzos y del estiramiento** como en la fig. 11. Las tensiones máximas, kg/cm cuad : 3 meses, 4.10; 2 años, 15.7.

**Coef de elasticidad, E.** Véase fig. 12.

**Con mezclas de 1 : 4,** para un esfuerzo dado en compresión, **E** fué en genera de 15 á 20% menos que con los de 1 : 3. En la tensión, **E** fué más igual en ambas mezclas.

**Con 8% de agua,** para una comp dada, **E** fué en general de 10 á 20% mayor que con 14% de agua.

**Para el índice**, véanse págs. 1346, etc.

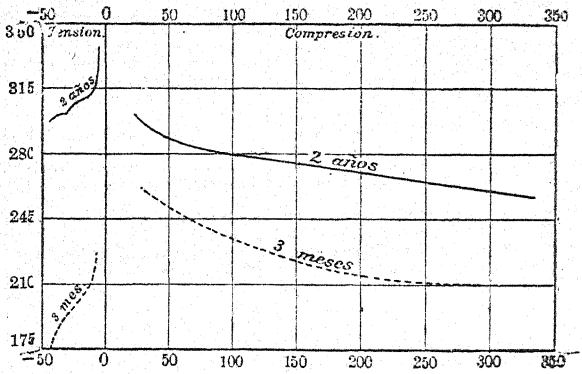



**Fig. 12.** Módulo ó coef de elasticidad.

(Los números verticales á la izquierda, de 175 á 350, dan en **miles** de kg/cm cuad los coeficientes E de elasticidad. Los meses y años de las curvas se refieren á la edad de las muestras.)

**81 b. Esfuerzo cortante.** Fig. 13. Las dimensiones en centímetros. Prismas de 18 cm en cuadro, 40 cm de largo. Mezcla de arena y granzón como en el ensayo 81 *a*.

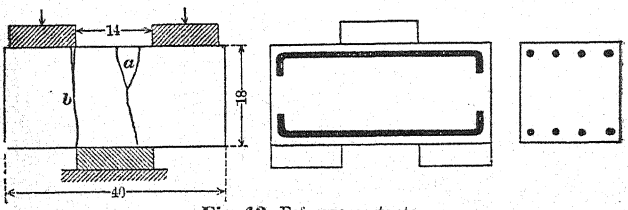



**Fig. 13.** Esfuerzo cortante.

**Sencillo.** La muestra se agrietó al principio, como la viga, en *a*. La presión se aumentó luego hasta aparecer la grieta del esfuerzo cortante, *b*.

| Mezcla. | Agua %. | Edad. | N.° de muestras. | Promedio del esfuerzo máximo cortante, kg/cm cuad *. | |
|---|---|---|---|---|---|
| | | | | Observado. | Calculado**. |
| 1 : 3 | 14 | 2.5 años. | 3 | 65.5 | 68.6 |
| 1 : 4 | 14 | 1.5 meses. | 3 | 37.1 | 38.5 |

* = 1/2 de la fuerza total aplicada ÷ el área de la superficie del esfuerzo cortante.
** Dada la resistencia á la tensión final, $t$, y la resistencia á la compresión final, $c$, de las piezas de prueba de la misma mezcla, edad y fórmula, el esfuerzo cortante = $V\!tc$.

**Armado.** Las barras (de 1 cm de diámetro) sirvieron solamente para mantener unidas las muestras, de manera de poder aumentar las presiones á voluntad. El concreto cedió primero al esfuerzo cortante.

| Mezcla. | Agua %. | Edad. | N.° de muestras. | Promedio del esfuerzo máximo cortante, kg/cm cuad. | |
|---|---|---|---|---|---|
| | | | | Concreto. | Acero. |
| 1 : 4 | 14 | 1.5 meses. | 2 | 36.5 | 3,248 |
| 1 : 4 | 14 | 1.5 — | 3 | 33.9 | 3,556 |

**81 c. Torsión**, pág. 45. Mezcla, 1 : 4. **4 cilindros sólidos**, de 79 á 98 días, de 26 cm de diámetro; largo sometido al experimento, 34 cm con cabezas hexagonales. M=el momento de torsión, R=radio del cilindro.

$t$ = al esfuerzo de torsión en las fibras extremas (véase pág. 528 *h* de este libro) $=2M/\pi R^3$.

$t$, en kg/cm cuad; máx, 19; media, 17; mín, 13.

**3 cilindros huecos**, como anteriormente, de 52 á 55 días; diámetro interior como de 15 cm, $r$=al radio interior.

$t=2MR/\pi\ (R^4-r^4)$,

$t$, en kg/cm cuad; máx, 94; media, 86; mín, 7.8.

**La unidad de resistencias mucho mayor de los cilindros sólidos** según las dan las fórmulas, se atribuye en parte á su mayor edad, pero especialmente al aumento en la unidad de presión de la circunferencia hacia dentro, debido á la cual el material más próximo al centro transmite más de lo que le corresponde del esfuerzo cort de torsión y alivia las partes exteriores.

**81 d. Adhesión**, pág. 49, figs. 14 y 15.

**Muestras.** Cubos de 20 cm. Mezcla, 1 : 4; 10 á15% de agua; edad, 4 semanas Barras redondas de 2 cm de diámetro, fig. 15, espiral de 10 cm de diámetro; alambre, .45 cm de diámetro.

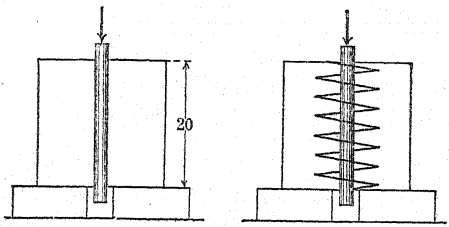



**Fig 14.**

Adhesión.

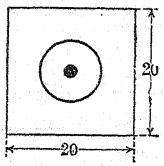



**Fig 15.**

**Tratamiento.** Las barras se **empujaron** hacia afuera. La presión aumentó rápidamente al máximo.

**Resultados. Adhesión**, término medio de 12 pruebas cada una, kg/cm cuad; fig. 14, adhesión=36.3; fig. 15, adhesión =50.

Después de vencer la adhesión, quedó una considerable resistencia á la fricción.

**Para el índice,** véanse págs. 1346, etc.

**81 e. Ductibilidad y esfuerzo cortante en el concreto armado,** pág. 60.
**Muestras. 4 cilindros huecos** armados en torsión, como en el ensayo 81 *c*, armados con espirales en el medio del espesor de las paredes. Los espirales á 45°, colocados de manera que estaban en tensión bajo el momento de torsión. 2 cilindros, cada uno con 3 espirales de hierro redondo de 7 mm; 2 cilindros, cada uno con 10 espirales de hierro redondo de 10 mm. Diámetro de la espiral, 21 cm.
**Esfuerzos sobre el hierro** en el momento del primer agrietamiento en el concreto, kg/cm cuad; máx, 627; medio, 584; mín, 539.
**Estiramiento del hierro** y del concreto en el momento del primer agrietamiento en el concreto, término medio : .00027 × largo original.
Lo anterior se dedujo por la comparación con los resultados obtenidos con cilindros sencillos en torsión. Ensayo 81 *c*.
**Esfuerzo cortante,** kg/cm cuad.

| | Máx. | Media. | Mín. |
|---|---|---|---|
| En el primer agrietamiento..... | 43.0 | 33.6 | 24.3 |
| En la ruptura..... | 53.7 | 43.7 | 30.1 |

**81 f. Muestras. 6 vigas armadas,** de 15 × 30 cm, 2 metros de luz, pág. 62, fig. 16. Dimensiones en centímetros. Espesor de las barras de la armazón como más abajo. 2 cargas concentradas, PP, equidistantes del centro y á 1 metro de separación. Mezcla 1 : 4; edad 3 meses. Medidas sobre la longitud central de 80 cm. El momento de flexión fué constante por todo este largo. El estiramiento del acero se observó por medio de dos botones en A, A, atornillados en las barras. Se colocaron estribos cerca de los extremos de las vigas. Las vigas se mantuvieron húmedas, pero se probaron secas.

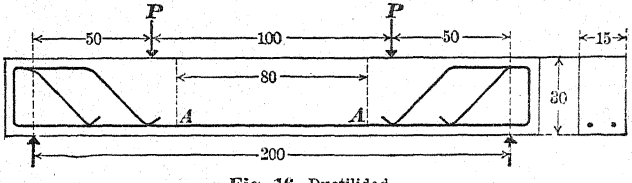
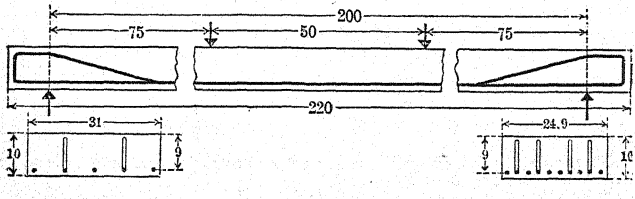



**Fig. 16.** Ductilidad.

**Resultados.** El estiramiento por unidad de longitud en el momento del primer agrietamiento del concreto.

| | Acero. | El concreto baj tensión máx. |
|---|---|---|
| Barras de 10 mm; diámetro = .4%..... | .00042 | .00050 |
| — de 16 — — =1.0%..... | .00033 | .00040 |
| — de 22 — — =1.9%..... | .00030 | .00033 |

**81 g. Esfuerzos del acero y del concreto,** pág. 97.
**Muestras. Vigas armadas chatas,** fig. 17. (En las figs. 17 y 18 las dimensiones son en centímetros.)




A, 3 *vigas.* A *y* B, 6 *vigas.* B, 3 *vigas.*

**Fig. 17.** Esfuerzos.

El momento de flexión fué constante entre las cargas. Mezcla, 1 : 4; largo, 2.2 m; luz, 2 m.

**Resultados. Fracasaron por la trituración del concreto** cerca y entre las 2 cargas. El acero de 10 mm de diámetro.

**Las unidades del esfuerzo.** $s$, en el acero, y $c$ en el concreto, en kg/cm cuad, se dedujeron bajo la suposición de que $n = E./E.; = 15$.

| | Edad. | Acero. | Después que aparece la primera grieta. | | En la ruptura. | |
|---|---|---|---|---|---|---|
| | | | $s$ | $c$ | $s$ | $c$ |
| 3 vigas A, figura 17 | 13 meses. | 1.4% | 1,568 | 92 | 3,796 | 223 |
| 3 — B, — 17 | 13 — | 3.3% | 1,469 | 158 | 2,749 | 296 |
| 3 — A, — 17 | 2 — | 1.4% | 1,307 | 77 | 3,149 | 185 |
| 3 — B, — 17 | 2 — | 3.3% | 1,195 | 128 | 1,968 | 121 |

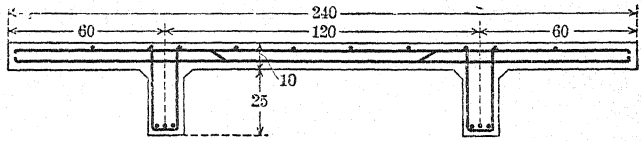



**Fig. 18.** Esfuerzo cortante. Dimensiones en centímetros.

**81 h. Esfuerzo cortante en las vigas. 12 muestras,** cada una compuesta de una plancha chata con dos costillas armadas de la misma manera, fig. 18. Las costillas de 2.7 m de luz normal al papel. « Der Eisenbetonbau », pág. 158.

**Tipos de tejidos de armazones,** despreciando las variaciones ligeras. Véase fig. 19 y la tercera columna de la tabla siguiente.

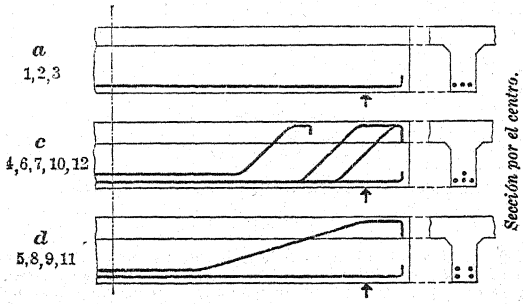



**Fig. 19.** Esfuerzo cortante.

**Estribos** : 4.ª columna de la tabla siguiente : *a*, luz corrida; *b*, en una mitad de la luz; *c*, ningún estribo.

**Barras : diámetro** en mm : *a*, 18; *b*, 16; *c*, 3 barras 15, y 1 barra 18; *d*, 2 barras 15, y 2 barras 16. La viga n.º 3 tenía 3 barras « Thacher » rectas de 18 mm de diámetro.

**Extremos** : 6.ª columna de la tabla siguiente : *a*, gancho; *b*, liso; *c*, 3 barras 45° 1 con gancho; *d*, 2 barras dobladas, 2 con gancho; *e*, 3 barras 45°, 1 lisa.

**Para el índice**, véanse págs. 1346, etc.

En el n.° 2 las **almas** eran de .28 m de ancho; en el n.° 8, .10 m; en los otros, .14 m,

Edad, como 3 meses. Cemento Heidelberg 1 : 4.5 (72% de arena del Rhin de 0-7 mm; 28% de granzón de 7-20 mm).

**Resultados.**

Las grietas se desarrollaron, siguiendo, en general, las curvas convexas hacia arriba, fig. 20.

Esfuerzos en kg/cm cuad.

$s$=tensión en el acero; $c$=compresión en el concreto, $a$=adherencia; $v$=esfuerzo cortante en el soporte.

| Carga. | Viga n.° | Tipo. | Estribo. | Barra, diám. | Extremo. | Al aparecer las grietas diagonales que producen la ruptura. | | | En la ruptura. | | | | Viga. | Carga. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | s | a | v | c | s | a | v | | |
| Uniforme. | 1 | *a* | *b* | *a* | *a* | 1258 | 8.6 | 10.4 | 37.9 | 2059 | 13.2 | 16.8 | 1 | Uniforme. |
| | 2 | *a* | *b* | *a* | *a* | 2411 | 16.4 | 10.0 | 57.5 | 3149 | 21.2 | 12.9 | 2 | |
| | 3 | *a* | *b* | .. | *b* | 1371 | 7.2 | 9.3 | 28.0 | 1954 | 10.3 | 13.1 | 3 | |
| | 4 | *c* | *c* | *c* | *c* | 2573 | 26.8 | 21.7 | 61.9 | 3255 | 33.5 | 27.0 | 4 | |
| | 5 | *d* | *b* | *d* | *d* | 1258 | 14.4 | 10.3 | 48.4 | 2601 | 29.4 | 21.0 | 5 | |
| | 6 | *c* | *a* | *c* | *e* | .... | 16.3 | 13.1 | 55.9 | 2953 | 30.4 | 24.5 | 6 | |
| 2 concentradas.* | 7 | *c* | *a* | *b* | *e* | .... | .... | .... | 65.0 | 3416 | 31.5 | 22.3 | 7 | 2 concentradas. |
| | 8 | *d* | *b* | *b* | *d* | 1110 | 10.6 | 10.6 | 47.5 | 2446 | 22.8 | 22.8 | 8 | |
| | 9 | *d* | *b* | *b* | *d* | 1581 | 15.1 | 9.9 | 52.2 | 2685 | 24.7 | 17.6 | 9 | |
| 1 central. | 10 | *c* | *b* | *b* | *e* | .... | .... | .... | 77.3 | 3866 | 25.4 | 18.1 | 10 | 1 central. |
| | 11 | *d* | *c* | *b* | *d* | .... | .... | .... | 82.9 | 3796 | 25.1 | 17.9 | 11 | |
| | 12 | *c* | *c* | *b* | *e* | .... | .... | .... | 74.5 | 3740 | 24.5 | 17.5 | 12 | |

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**82. Sandford E. Thomson.** A. S. T. M., Procs., '08, vol. 8, pág. 500.

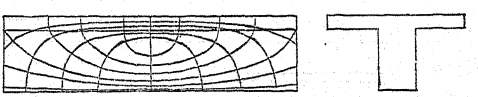


**Fig. 20.** Esfuerzos diagonales.

**82 a. Permeabilidad.** Efecto de la incorporación de la **cal apagada.**

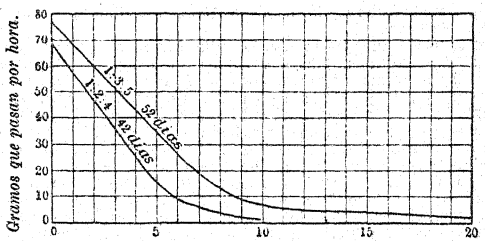
**Muestras.** Bloques cilíndricos, de 50 cm de diámetro, 40 de espesor; cemento Lehigh, arena buena mediana de rivera, roca de conglomeración parecida al basalto en su naturaleza; « una mezcla blanda esponjosa, como para usarla en la construcción ». Cal de Piñón de Rockland, Maine. La cal está expresada en % de peso de cemento seco. Las mezclas fueron como sigue :

| | | | |
|---|---|---|---|
| 1 : 2 : 4 | concreto con 0%, 4%, 7% y 10% cal; | 8% se prefiere; | |
| 1 : 2.5 : 4.5 | — con 0%, 6%, 10% y 14% | — 12% | — |
| 1 : 3 : 5 | — con 0%, 8%, 14% y 20% | — 16% | — |

**Tratamiento.** El agua bajo presión se introdujo al centro del bloque.

* Las dos cargas concentradas dividen la luz en tres partes iguales.

**Resultados**, 1 : 2 : 4 y 1 : 3 : 5, véase fig. 21. 1 : 2.5 : 4 dieron resultados intermedios entre los otros dos.




*Percentaje en peso del cemento, de cal hidratada.*

**Fig. 21.** Permeabilidad; Cal.

**82 b. La arena más ordinaria** requiere más cal, y viceversa.

**82 c. Si la presión va á ser aplicada dentro de un mes**, será mejor usar 10%, 15% y 20% respectivamente, en lugar de 8%, 12% y 16% como se recomienda en la Experiencia 82 *a*.

**82 d.** La pasta de cal ocupa como 2¼ veces el **volumen de la pasta** hecha con peso igual del cemento Portland, « y es por consiguiente muy eficaz para llenar las vacíos. El **costo de los trabajos grandes á prueba de agua** puede reducirse usando, con la cal, un concreto más fino del que sería necesario bajo otras condiciones.

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**83. Richard L. Humphrey**, vigas de concreto sencillas, cubos y cilindros, resistencias de compresión y transversales y relaciones de elasticidad. « La resistencia de las vigas de concreto », U. S. G. S., Boletín n.° 344, '08. Pruebas para determinar el efecto, sobre **la resistencia transversal y de compresión**, de (1) **edad** de la muestra, (2) **consistencia** de la mezcla, (3) naturaleza del **agregado**.

**83 a. Muestras.** Vigas de concreto sin armar, cubos y cilindros. Cemento, una mezcla de 9 cementos Portland. Arena del río Meramec, « compuesta de granos de pedernal con superficies relativamente lisas ». « El análisis granulométrico demuestra que la arena resultó un poco más fina de lo que era de desearse ».

**Propiedades de la arena y del agregado usados.**

| | Densidad. | Vacíos, %. | Mallas por cm del cedazo. | | | | | Tamaño de mallas. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 80 | 40 | 20 | 12 | 4 | 6 mm. | 12 mm. | 19 mm. | 38 mm. |
| | | | Percentaje que pasó el cernidor ó cedazo. | | | | | | | | |
| Cenizas..... | 1.53 | 51 | 2.84 | 4.17 | 6.5 | 10.5 | 21.1 | 37 | 60 | 81 | 100 |
| Granito..... | 2.59 | 41 | 1.59 | 2.29 | 3.2 | 4.4 | 8.5 | 20 | 58 | 99 | 100 |
| Granzón..... | 2.45 | 33 | 0 | 0 | 0 | 0 | 1.0 | 43 | 79 | 95 | 100 |
| Piedra caliza.. | 2.49 | 37 | 2.96 | 3.48 | 4.2 | 5.2 | 10.7 | 29 | 61 | 96 | 100 |
| Arena..... | 2.60 | 38 | 0.20 | 1.30 | 13.9 | 64.0 | 97.0 | 100 | . | .. | .. |

**Proporciones.** 1 : 2 : 4 por volumen, excepto el concreto de ceniza que estuvo más cerca de 1 : 2 : 5. Todo el concreto se mezcló en un mezclador de mortero de impulsión de .75 m cúb. provisto de una tolva de tiro. Se mezcló 2 minutos seco, 3 minutos húmedo; después se apiló en un piso de cemento, se echó con palas en las carretillas y se llevó al piso de moldear. Cada lote suficiente para dos vigas,

**Para el índice**, véanse págs. 1345, etc.

de 20×27 cm y 3.66 m de luz, dos cubos de 15 cm y 2 cilindros de 20 cm de diámetro, 4.80 m de largo.

« **Mojado** »; liso y un poco viscoso inmediatamente antes de verterlo. Corrió hacia atrás del lado ascendente del mezclador sin tendencia á rajarse en su parte superior. Al echarlo no mostró porosidades ni piedras sueltas. Salpicaba al apisonarlo. Al terminarlo, el agua estaba de 6 á 12 mm sobre la superficie del molde.

« **Mediano** »; liso, pero con tendencia á formar terrones. Fluyó con menos suavidad que el « húmedo », una parte retirándose con suavidad, otra partiéndose en terrones. Al vaciarlo parecía terronudo, mostrando piedras, pero no vacíos. Las piedras estaban cubiertas por parejo con concreto. No se reunió el agua en la superficie del molde. La superficie se alisó fácilmente con la cuchara.

« **Húmedo** »; granulado. Poca tendencia á aterronarse. Se llevó á la parte superior del mezclador por el lado ascendente, cayó en piedras sueltas y fragmentos de mortero. Al vaciarlo mostró piedras y vacíos. Resistió al pisón. Se compactó con golpes de mano. No pudo alisarse con la cuchara.

**Concreto colocado** en moldes de acero aceitados, en 3 capas casi iguales y apisonado á mano. « Se tuvo gran cuidado en apisonar todos los concretos de la misma manera. »

**Tratamiento.** Todos los moldes se quitaron á las 24 horas, y las piezas se transportaron á un cuarto húmedo. Se regaron 3 veces diarias.

Las vigas se apoyaron, antes de la prueba, de manera que las sumas de los momentos y esfuerzos existentes entonces en el largo medido, se igualaran, de manera que todas las fibras en esa longitud tuvieran el mismo largo que si no estuvieran comprimidas, y las deformaciones, dentro de la distancia medida, se midieron de esta manera desde cero.

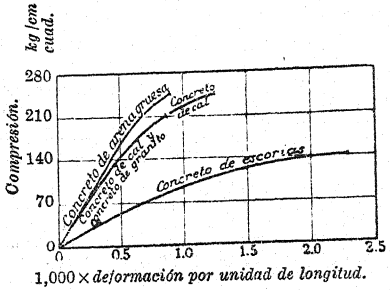


**Fig. 22.** Curvas de esfuerzos y deformaciones para diferentes agregados. Edad de las muestras, 26 semanas.

**Resistencia del concreto.**

**Resultados,** en general, término medio de 3 muestras.

| | Agua %. | Vigas. 20×28 cm y 3.66 m largo. | | | Compresión máx, kg/cm cuad. | | | |
|---|---|---|---|---|---|---|---|---|
| | | Eje neutro *. | Módulo de ruptura †. | | Cubos 15 cm. | | Cilindros. 20 cm diám, 40 cm largo. | |
| | | 100 *m*. | 4 sem. | 26 sem. | 4 sem. | 26 sem. | 4 sem. | 26 sem. |
| **Ceniza.** | | | | | | | | |
| Mojada..... | 21.9 | 43.3 | 12.2 | 17.2 | 87.9 | 162.4 | 75.7 | 141.5 |
| Mediana.... | 20.6 | 39.9 | 13.8 | 19.3 | 83.3 | 193.5 | 84.1 | 154.2 |
| Húmeda.... | 18.9 | 38.2 | 13.8 | 17.5 | 96.5 | 174.2 | 78.3 | 136.1 |
| **Granito.** | | | | | | | | |
| Mojado..... | 9.0 | 49.9 | 26.2 | 37.7 | 220.9 | 332.7 | 187.8 | 277.6≑ |
| Mediano.... | 8.3 | 47.2 | 33.2 | 39.6 | 286.2 | 346.4 | 243.6 | 278.0≑ |
| Húmedo.... | 7.0 | 48.3 | 34.9 | 43.2 | 316.3 | 382.5 | 280.0 | 277.8≑ |
| **Granzón.** | | | | | | | | |
| Mojado..... | 9.7 | 49.9 | 27.3 | 30.4 | 160.9 | 267.0 | 144.2 | 244.0 |
| Mediano.... | 8.9 | 48.4 | 31.5 | 36.4 | 248.3 | 336.6 | 207.3 | 278.0≑ |
| Húmedo.... | 7.9 | 47.5 | 29.8 | 34.7 | 322.8 | 341.9 | 238.5 | 277.8≑ |
| **Piedra caliza.** | | | | | | | | |
| Mojada..... | 10.9 | 48.8 | 29.5 | 35.4 | 357.9 | 242.2 | 215.0 | 225.1 |
| Mediana.... | 10.0 | 50.7 | 32.0 | 39.6 | 202.2 | 272.2 | 203.7 | 258.4 |
| Húmeda.... | 8.5 | 48.1 | 37.5 | 41.2 | 305.7 | 351.7 | 202.6 | 275.9≑ |

(*Obs. del T.* Esta tabla, como todas las demás, la hemos convertido al sistema métrico. Para convertir lbs pulg cuad á kg/cm cuad, hemos multiplicado aquélla por .07, pues nos parece suficiente aproximación.)

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**84. R. G. Clark,** Instituto de Ingeniería civil, Procs., vol. 171, '08, pág 11.5.

**84 a. El tiempo de fragua** se aumentó por la **aeración** y por el **aumento del agregado.** Un cemento puro que fragua en una hora, forma un concreto que requiere 4 ó 5 horas para fraguar.

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**85. Hanisch y Spitzer,** Morsch, « Der Eisenbetonbau », '08, págs. 32-33.

**85 a. Módulo de ruptura,** $6M/bd^2$, **y resistencia de la tensión y compresión directas.**

**Muestras.**

Concreto, 1 : 3.5. Seis planchas de 268 días, de 60 cm de ancho, 7.8 á 11 cm de espesor; luz 150 cm.

**Tratamiento.** La plancha se quebró transversalmente; con los fragmentos se hicieron piezas para ensayos de compresión y tensión.

**Resultados. Esfuerzos** en kg/cm cuad.

| | Módulo de ruptura. | Compresión. | Tensión. |
|---|---|---|---|
| Máx..... | 54.5 | 351.5 | 29.0 |
| Mediana..... | 47.9 | 307.9 | 25.0 |
| Mín..... | 43.2 | 255.9 | 20.0 |

La comparación de los valores de la tensión con el módulo de ruptura demuestra

* $m$ = (espesor del eje neutro debajo de la parte superior de la viga) ÷ (espesor total de la viga).

† « Módulo de ruptura » = $6M/bd^2$, kg/cm cuad; M = momento bajo la carga máxima.

≑ El cilindro no se quebró.

**Para el índice**, véanse págs. 1346, etc.

que la **fórmula**, módulo de ruptura $=6M/bd^2$, no se aplica á los materiales en que, como en el concreto, el módulo de elasticidad varía mucho, y que los módulos de ruptura que se obtengan por esta fórmula deben usarse sólo como medios de comparación.

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**86. Richard L. Humphrey y Wm. Jordán, Jr., U. S. G. S.**, Boletín n.º 331, '08. Resultados de los ensayos hechos en los Laboratorios para Experimentos con Materiales de Construcción, St. Louis, '05-7.

**86 a. Cerniduras de granzón.** Las resistencias á la compresión y tensión de los morteros parecen, en general, aumentar con la densidad de las cerniduras.

**86 b. Cerniduras de piedra.** En general, la resistencia de la mezcla fué mayor con las cerniduras de más uniformidad en la graduación. **La resistencia de la piedra** misma, de donde se obtienen las cerniduras, tiene una influencia importante en la resistencia del mortero resultante.

**86 c. La densidad** de los morteros es mayor con las arenas más densas.

**86 d. Morteros de arenas. Las resistencias** á la tensión, á la compresión y transversal fueron invariablemente mucho **mayores, con arenas densas** que con las que tenían grandes proporciones de vacíos.

**86 e.** La resistencia mayor se obtuvo cuando la arena estaba uniformemente graduada.

**86 f. Una « mezcla típica »** de 7 cementos Portland, con marca separada, llegó á la mayor **resistencia** á la tensión en 90 días. Como el mejor de ellos, mantuvo este máximo 180 días, y la pérdida subsiguiente, un año, y más tarde, no fué mayor que con el mejor de las marcas por separado.

**86 g. Edad de la probeta.** Los ensayos después de 180 días mostraron mayor uniformidad que á los 90 días, y períodos menores.

**86 h.** Después de ensayos á 180 y 360 días, las resistencias de todos los morteros de arena eran muy aproximadas una de otra, demostrando que la considerable variación de la **resistencia primitiva** no afecta seriamente la resistencia posterior.

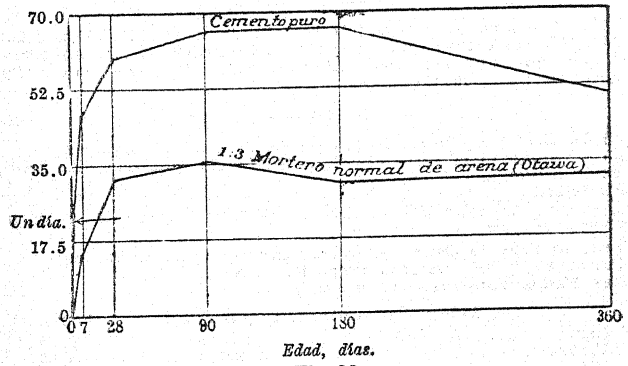



**Fig. 23.**

(*N. del T.* — Las cifras verticales de 0 á 70.0 son kgs/cm cuad, de tensión.)

**86 i. Resistencias á la tensión y compresión de los morteros de cemento Portland** puro y 1 : 3 arena superior de Otawa. Véanse figs. 23 y 24. Cada curva representa un promedio de 10 ensayos.

**Muestras.** El cemento era una mezcla de partes iguales de 7 marcas diferentes. Véanse Ensayos 86 *f*, 86 *g* y 86 *h*.

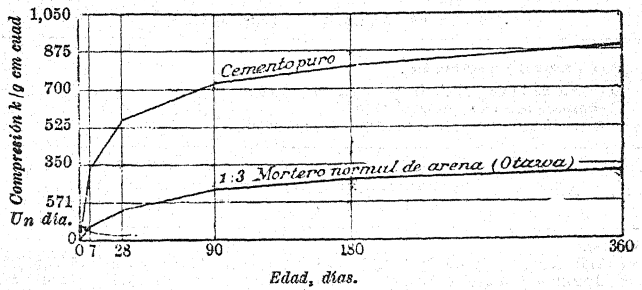



**Fig. 24.**

Las piezas para los ensayos en los moldes, se tuvieron en el gabinete húmedo 24 horas, después se conservaron en agua corriente, como á 21° C, hasta que se probaron. Probetas de tensión de 25 mm en cuadro. Ensayos de compresión, cubos de 5 cm.

**Resultados** como en las figs. 23 y 24.

— 87 —

**87. W. N. Willis.** Ferrocarril del Sur y Oeste. E. R., '08, enero 18, E. N., '08, febrero 6, pág. 145.

**87 a. Mica; agua que se requiere; resistencia.**

**Muestras.**

| | | | | |
|---|---|---|---|---|
| Cedazo n.°..... | 10 | 20 | 50 | 100 |
| % de mica que pasa..... | 100 | 29 | 10 | 4.5 |

Arena superior de Otawa, mezcla 1 : 3 de arena, ó 1 : 3 arena y mica por peso.

**Resultados.**

| | | | | | |
|---|---|---|---|---|---|
| **Mica**; % de peso de arena..... | 0 | 5 | 10 | 15 | 20 |
| **Vacíos**, % en arena de Otawa..... | 37 | ... | ... | ... | 67 |
| **Peso** específico relativo de la arena de Otawa..... | 100 | ... | ... | ... | 80 |
| **Agua de mezcla** que se necesita (relativa). | 100 | ... | ... | ... | 300 |
| **Resistencia á la tensión**, 6 meses (relativa)..... | 100 | 64 | 62 | 59 | 40 |

Lo liso de la superficie de la mica hace que su adhesión sea pequeña.

— 88 —

**88. Profesor J. L. Van Ornum,** Universidad de Wáshington, San Luis, para la Compañía de construcción de concreto armado, San Luis, '08, febrero 6, pág. 142.

**88 a. Adhesión.**

**Muestras.** Barras de acero redondas lisas, diámetros, 12 á 30 mm, incrustadas en bloques prismáticos de 30×30 cm de concreto de 1 : 2 : 4, de 90 días. Barras de acero mediano incrustadas 25 diáms. Barras de acero de alta carbonización, 40 diámetros.

**Resultados.** Véase la tabla más abajo, en que

**para el acero :**

$s$ = resistencia máx, en miles de kg/cm cuad;
$s_e$ = límite de elasticidad, en miles de kg/cm cuad;
$e$ = estiramiento (%);
E = módulo de elasticidad, en millones de kg/cm cuad.

**Para el índice**, véase págs. 1346, etc.

**Para acero y concreto :**

$a$=área de la superficie incrustada, en cm cuad;
B=adhesión, en kg/cm cuad de $a$;
F=fricción, después de resbalarse, kg/cm cuad.

| Acero. | Acero. | | | | Acero y concreto. | | | |
|---|---|---|---|---|---|---|---|---|
| | $s$ | $s_c$ | $e$ | E | $a$ | B | F | F/B |
| Mediano | | | | | | | | |
| Máx..... | 4.28 | 2.85 | 29.0 | 2.10 | 817.86 | 32.34 | 26.71 | .826 |
| Término medio.. | 4.12 | 2.75 | 26.1 | 2.07 | 400.54 | 28.77 | 24.04 | .838 |
| Mín..... | 3.91 | 2.70 | 22.5 | 2.01 | 139.92 | 26.01 | 21.79 | .838 |
| Mucho carbón | | | | | | | | |
| Máx..... | 7.70 | 4.27 | 20.7 | 2.15 | 1279.03 | 33.04 | 19.68 | .596 |
| Término medio.. | 6.51 | 3.94 | 17.6 | 2.09 | 594.04 | 27.56 | 16.87 | .613 |
| Mín..... | 5.90 | 3.73 | 15.7 | 2.03 | 210.91 | 23.20 | 14.06 | .606 |

En todos los casos, la tracción total que vencía la adhesión excedió á la que llevaba el acero á su límite de elasticidad.

— 89 —

**89. W. S. Reed.** Club de Ingenieros de Filadelfia. Procs., vol. 25, n.° 3, pág. 290, '08, julio.

**89 a. Fricción de la arena.** Ensayos por More y Harris Tubor. Presión superior, kg/cm cuad, que se necesitaron para dar .7 kg/cm cuad en el fondo de la caja.

| | Espesor de la arena, en cm. | | | |
|---|---|---|---|---|
| | 6.3 | 12.5 | 19 | 25 |
| Caja. | Presión superior, kg/cm cuad. | | | |
| 10×10 cm.... | .87 | 1.22 | 2.38 | 2.94 |
| 15×15 cm.... | .80 | ... | ... | 1.62 |

**89 b. Punto de fusión de las arenas de cuarzo.** Ensayo por el profesor Heinrich Ries, Universidad de Cornell. 3254° F=1790° C.

— 90 —

**90. Eng. News**, '08, agosto 27, pág. 238.

**90 a. Agua de mar.** Charlestown, Mass. Arsenal.

**Arcos no armados**, construídos '01, por la Oficina de Depósitos y Muelles. **Agua salada de marea,** no muy sucia, pero **helada** á menudo, diferencia de marea 3 m. Especificaciones requeridas para la « construcción continua de muelle á muelle de los arcos ». Cara exterior del mortero de 7.5 cm, 1 : 1. Masa de concreto de 1 : 2 : 4, en 60 cm, detrás de la cara exterior, 1 : 3 : 6, al interior; « cemento superior y granzón local ». Probablemente poroso. No hubo esmero especial para la densidad ó impermeabilidad. Especificaciones que se indicaron : « El contratista debe presentar prueba satisfactoria de la durabilidad en el agua de mar de la marca de cemento que intenta suministrar. » Las enjutas que están á la vista se construyeron después de terminados los arcos. Seco, bien pisado. Desintegración grande. El daño principalmente entre alta y baja marea. El concreto de la parte posterior sufrió considerablemente.

— 91 —

**91. U. James Nichols.** Melbourne, Victoria, E. W., '08, dic. 24, pág. 710

**91 a. Electrolisis** en los morteros de cemento.

**Muestras.** 16 cilindros de 20 cm de diámetro y 20 cm de altura. Cemento Portland superior, arena gruesa con 51% de vacíos. El mortero se apisonó por capas

de 4 cm, hasta que un poco de agua fluyó á la superficie. El electrodo positivo normal á un tubo de acero de 2.5 cm diám y 30 cm largo con el extremo inferior tapado con un corcho, y sumergido, en el eje del cilindro, á una profundidad de 12 cm en el concreto.

**Tratamiento.** Los cilindros se colocaron en agua dulce por más de 28 días. Se probaron 8 cilindros con **corriente constante** de .1 de amperio; 5 con tensión **constante** como de 115 voltios (corrientes altas, uno con corriente inversa), 3 no se sometieron á corriente. Para la corriente, los cilindros se colocaron en una solución de sal de 3% en baldes de metal separados (que normalmente formaban los electros negativos), y montados en serie. Los cilindros tenían de 29 á 57 días de edad al principio de la prueba.

**Resultados.**

**Todos los cilindros** sometidos á la corriente se **agrietaron.** Las grietas se atribuyeron á la acumulación y presión de los gases que se formaron. Al principio las grietas eran de forma capilar y exudaban humedad, humedeciendo la superficie adyacente. Las grietas se ancharon bajo la corriente continua. Bajo la corriente constante las grietas aparecieron al llegar la resistencia al máximo. La resistencia, en general, fué inversamente proporcional **al percentaje de arena.** Los cilindros n.os 1 y 2 se abrieron fácilmente al forzarlos. En los n.os 2 y 9 **el tubo de acero se enmoheció,** y se formaron concavidades en el exterior, adyacentes á las grietas. Con corriente inversa (de potencial constante) (el n.° 12) no se enmoheció ó agujereó.

**Los cilindros que no se sometieron á la corriente** no se agrietaron. Necesitaron como veinte golpes con un martillo pesado y un cincel fino para quebrarlos. No hubo moho.

| | Corriente constante, .1 amperio. Número de muestras. | | | | | | | Potencial constante, 115 voltios. Número de pruebas. | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 9 | 10 | 13 | 14 | 5 | 6 | 3 | 11 | 12 | 15 | 7 |
| Mezclas..... | 1:3 | 1:3 | 1:1 | 1:1 | 1: ⅛ | 1: ⅓ | 1:0 | 1:0 | 1:3 | 1:1 | 1:1 | 1: ¼ | 1:0 |
| Arena, %.... | 75 | 75 | 50 | 50 | 25 | 25 | 0 | 0 | 75 | 50 | 50 | 25 | 0 |
| Días *..... | 7 | 7 | 10 | 16 | 15 | 15 | 28 | 15 | ... | ... | ... | ... | ... |
| Minutos *.... | ... | ... | ... | ... | ... | ... | ... | ... | 5 | 19 | 20 | 9 | 9 |
| Omios**..... | 80 | 90 | 420 | 270 | 230 | 270 | 2900 | 1080 | 120 | 130 | 240 | 163 | 190 |

— 92 —

**92.** « **H** », de Lafayette, Indiana, Carta en E. N., '08, diciembre 31, pág. 751.

**92 a. Arcilla.** En concreto para columnas, el granzón contenía 5% de arcilla que flotó en la superficie al batirlo, y dejó 4 cm de material inútil cerca de la parte superior de la columna

— 93 —

**93. A. Q. Campbell,** Ogden, Uttah, E. N., '08, dic. 31, pág. 751.

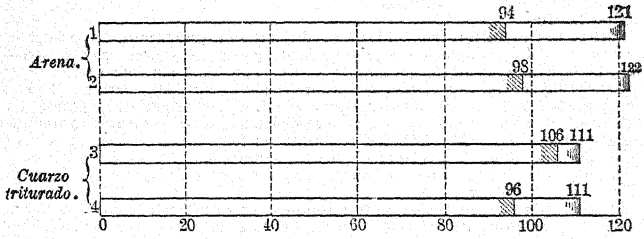
**93 a. Graduación é impermeabilidad. Acabado.** Estanque de concreto armado para agua, rectangular, de 7,400 m cúb (m. ó m.), 6 m de profundidad. El fondo de 15 cm de espesor; las paredes de 20 á 45 cm. 1 cemento, 2 arena ordinaria, 4 piedra (bloques de cuarzo, pórfiro y piedra caliza silícea), trituradas á 2.5 cm con polvo con « percentaje grande de arena y polvo triturados », mezclado en máquina, « con una consistencia que casi fluía ». El piso se colocó en bloques como de 1.5 m cuadrados, « dejando una unión á medio corte de 60 cm ». Las paredes en capas continuas de 50 cm. Acabado (enlucido), con 1 : 1 de cemento y polvo del triturador, que se aplicó con una escoba ordinaria recortada. Agua clara. No hubo cuarteo perceptible en la superficie. Ningún escape aparente.

* Hasta la primera grieta.
** Resistencia máx aproximada.

— 94 —

**94. John C. Trautwine, Jr.** '09.

**94 a. Densidad de la arena, forma del grano.** 100 medidas de granos de arena redondeados, ó de granos de cuarzo triturado que se vertieron muy despacio en 60 medidas de agua. Los ensayos 1 y 2 se hicieron con granos de arena, los n.os 3 y 4 con granos de cuarzo triturado. El lado izquierdo de cada diagrama, fig. 25, representa el fondo del envase; y los números, 94, 121, etc., muestran las elevaciones de la superficie de la arena y del agua respectivamente, después de verter los granos de arena en el agua.




*Elevación de las superficies de la arena y del agua sobre el fondo del depósito.*

**Fig. 25.**

En el n.º 4, el cuarzo triturado se removió en el agua, de vez en cuando, mientras se introducía, para desalojar el aire, que, á pesar de lo despacio de la introducción, los granos de arena hubieran arrastrado en el agua. El hecho de que el agua llega á la misma altura en 4 que en 3, indica que no arrastró más aire en 3 que en 4 y que el movimiento puso los granos en mayor contacto que cuando se dejaron quietos.

# RESUMEN DE LAS ESPECIFICACIONES, ETC.

## PARA LOS TRABAJOS DE CONCRETO EN GENERAL

Páginas 1403 á 1420.

## LISTA DE LAS ESPECIFICACIONES, ETC., QUE SE USAN

### Lista alfabética.

Véase la Lista clasificada, pág 1402.

**AH**, Puerto de Algoma, Wisconsin, Tajamar de cajones, etc., Ing. de los E. U., '08, enero 24.

**BB**, Tajamar, Búffalo, N. Y., Emile Low, Ing. de los E. U., A. S. C. E., Trans., '04 junio, vol. 52, pág. 73.

**BR**, Puerto y Canal de Black Rock, Búffalo, N. Y., Muros de Esclusa, Ing. de los E. U., '07, dic. 19.

**Bu**, Burlington, Vermont, Planta de Filtro Mecánico, Hering y Fuller, '07.

**Ch**, Chicago, '08; enmiendas propuestas al Código de Construcción de '05-6.

**Ci**, Cincinnati, O., Geo. H. Benzenberg;
**a**, Filtros, etc., '05; **b**, Depósito Principal, etc., '06.

**Co**, Columbus, Ohio, John H. Gregory;
**a**, Filtros, etc., '05; Estación de Bomba, '06.

**CR**, Mejoras del río Columbia, Oregón y Wáshington, Canal. Ing. de los E. U., agosto 1.

**CS**, Compañía de Ingeniería de Obras de concreto-acero, Edwin Thacher, especificaciones generales, Melan, Thacher y von Emperger, patentes, '08.

**F**, Wm. B. Fuller, Filtros, especificaciones recibidas, '08.

**FP**, Pensacola, Florida, composición y protección de muros marítimos. Ing. de los E. U., '08, abril 18.

**FW**, Fort Williams, Maine, Muelle en Ship Cove. Ing. de los E. U., '08, abril 14.

**G**, Práctica general.

**Hb**, Harrisonburg, Luisiana, Esclusa y Dique n.° 2. Ing. de los E. U., '08, mayo 13.

**IM**, Canal de Illinois y Misisipí, Esclusas, Sección Oriental. Ing. de los E. U., Jas. C. Long, Sociedad Occidental de Ing., '01, abril, vol. n.° 2, pág. 132.

**JC**, Recomendaciones en el Informe de la Comisión Unida de las A. S. C. E., A. S. T. M., Ingeniería de Ferrocarril americano y Asociación M. W, y Asociación de Manufactureros americanos de Cemento Portland, '09, enero.

**L**, Louisville, Kentucky, Ordenanza de Construcción, '07.

**Lp**, Mejoras en el Puerto de Liverpool, Geo. Cecil, Kenyon, A. S. C. E., Trans., '04, junio, vol. 52, pág. 36.

**Lv**, Louisville, Kentucky, Cloaca de Salida del Sur, '07.

**Mc**, Dique del Pasaje de McCall, río Susquehana, Pensilvania, '08.

**Mh**, Departamento de Manhattan, Regulación de la Oficina de Edificios, '03, sept.

**Ms**, Legislatura de Massáchusets, Actas y Resoluciones de —; '07.

**NO**, Nueva Orleáns. Louisiana, Estaciones para la Purificación del Agua, '06, septiembre 5.

**NY**, Nueva York, Código de Construcción aprobado, '99, octubre 24, con enmiendas á, '06, abril 12.

**OD**, Río Ohio, Pensilvania, Dique n.° 19, Empotramiento. Ing. de los E. U., '08, julio 25.

**Ph**, Filadelfia, Reglamento de la Oficina de Inspección de Edificios, aprobadas '07, octubre 8. Club de Ing. de Filad., octubre 7, vol. 24, n.° 4.

**SE**, Superior Entry, Wisconsin, Desembarcadero del Sur, Clarence Coleman, Ing. asistente. Informe, Jefe de los Ing. de los E. U., '04. Parte 4, págs. 3779, etc.

**TR**, Río Tennessee, más abajo de Chattanooga, Tennesee, Muro del río. Ing. de los E. U., '08, mayo 27.

**T y T**, Taylor y Thomson, « Concreto sencillo y armado », publicado por John Wiley é hijos, Nueva York, '05, págs. 33-37.

**Un**, Aseguradores Marítimos, Junta Nacional de Fuego —; Código de Construcción Recomendado, Nueva York, '07.

**WH**, Waddel y Harrington, especificaciones generales, recibidas, '07 diciembre.

**Wv**, Welsville, Ohio, Pase de Navegación, Dique n.° 8. Ing. de los E. U., '08, febrero 27.

**Yo**, Yonkers, N. Y., Filtros de Mampostería Cubierta, '07.

## Lista clasificada.

Véase la lista alfabética, pág. 1401.

Trabajos del Gobierno de los Estados Unidos, **AH, BB, BR, CR, FP, FW, Hb, IM, SE, Wv.**
Tajamares, **AH, BR, SE.**
Muros marítimos, **FP, SE, TR.**
Esclusas y Canales, **BR, CR, Hb, IM.**
Mejoras de Puerto, **Lp, SE.**
Muelles, **FW, Lp.**
Diques (Presas), **Hb, MC, OD, Wv.**
Estaciones de Bombas, etc., **Ci b, Co b.**
Plantas de Filtro, **Bu, Ci a, Co a, F, NO, Yo.**
Cloacas, **Lv.**
Puentes, **CS.**
Códigos de Construcción, **Ch, L, Mh, Ms, NY, Ph, Un.**
General, **CS, JC, T y T, WH.**

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**Para lista** de especificaciones para concreto, véase págs 1401-02.

Para comparar de una manera comprensible los requisitos de las diferentes Especificaciones, se debe tomar en cuenta la naturaleza del trabajo á que se refieren.

## RESUMEN

### Cemento.

**1. Marca.** Portland ó natural, **NY**; el Portland debajo del quicio, el natural en todas las otras partes del cimiento, Portland en las paredes de trabazón, pero no como respaldo, 60 cm de espesor en la base, Portland y natural ligados, **IM**; para cemento armado Portland, **G**; Portland americano, **CS, BR, Hb, FW**; cemento Portland « Universal », **SE**; cemento hecho por manufactureros de reputación establecida (trabajando con éxito no menos de dos años, **F**), marca de éxito continuo (en América, **F**) durante los últimos 5 años (3 años, **CS**) **G**; en uso satisfactorio en cantidades semejantes por el Departamento de Ing. de los E. U., **TR**; de uniformidad probada, en uso, no menos de 3 años, en un clima parecido, **CR, Hb**, se debe usar solamente una marca, **G**; excepto cuando haya razón para lo contrario, **F**; sólo una marca en los monolitos, **FP**. El Portland en cemento armado y donde esté sometido á choque ó vibraciones ó presiones que no sean la compresión directa, el natural en el trabajo macizo en que el peso es de más importancia que la resistencia, y cuando la economía es el factor que rige; el puzolano solamente en la parte subterránea de los cimientos, no expuesto al aire ó al agua corriente, **JC**.

**2. Requisitos. Para resistencias**, etc., véase Resumen de las Especificaciones para el cemento, por la A. S. T. M., pág. 940, Informe de la Junta de Ingenieros de los E. U., Estudios Profesionales, n.º 28, Cuerpo de Ing. de los E. U., '019 37, y Resumen de las Especificaciones por las Comisiones de Ingeniería Mod to de la Gran Bretaña, pág. 940. **Para las pruebas**, véase el Resumen de la A. S. C. E., pág. 942. Fragua lenta, **FP**; ha debido probarse por más de 6 meses, más de la meses antes de la publicación del permiso, **L**; debe llenar los requisitos de los Estudios Profesionales n.º 28, Cuerpo de Ing. de los E. U., '01, pág. 940, **BR, AH, TR, CR, FW, Wv, FP, Hb.**

**3. Embarque.** Los « bultos deben contener 380 lbs (172 kg) ó algún divisor par de 172 kg », **Lv**; en barriles ó sacos de tela, **NO**; saco, 42 kg, neto, un barril=4 sacos, **NO**; en barriles, forrados con papel, **CR, WH**; en sacos de tela, **Ci**; puede entregarse en sacos de papel, **Wv.**

**4. Almacenaje** en la obra. En edificio ó prueba de intemperie con el piso levantado sobre el suelo ($<$ 15 cm, **T y T**) 2 semanas bajo condiciones análogas á las del trabajo, **Ci**; el cemento en sacos puede usarse después de 3 meses de almacenaje, recháceselo si se pone terronudo ó si se daña de cualquier otra manera dentro de ese tiempo, **BR**; el cemento que se conserva hasta después del invierno se vuelve á probar antes de usarlo, **Wv.**

### Arena.

**5. General.** Sílica, dura, limpia, aguda, **G**. Medianamente limpia, ordinaria, **F**; gastada por el agua, vacíos=35%, **SE**. La « agudeza » omitida intencionalmente, **T y T**. Arena de río, **Ci, a**.

**6. Tamaño.** Bien graduado con granos finos, medianos y gruesos, **F, Lv, NO, CO**. Gruesa, ó fina y gruesa, mezcladas, **CS, T y T**. Predominando la gruesa; prefiriendo la gruesa á doble ó triple costo, **T y T**. Mediana, **Ci, a**. La más grande que pase por un cernidor de 6 mm, **G**. $>$ 10% más gruesa de 3 mm, **NO**; $<$ 50% retenida en el cedazo n.º 30 (agujeros de 14 mm cuad), **WH**. $>$ 40% que pasó cedazo n.º 50 (400 mallas por cm cuad), **Hb**. $>$ 3% muy fina, **NO, Co, Ci, a**. $>$ 5% muy fina, **Bu**.

**Materia extraña** (arcilla, marga, palos). Ninguna, **CS, T y T**; $>$ 2% **NO**, $>$ 3 %, **Co, Lv**; $>$ 5%, **Wv, OD, TR, CR, Bu**, $>$ 10% arcillosa **AH**. $>$ 3% arcilla, etc., $>$ 2% mica, **FW**; $>$ 4% de marga libre, **Hb**; la arena puede estar húmeda, no mojada, **TR**; almacenada en una plataforma de madera, **CR**, ó en almacén, **Wv.**

**7. Cerniduras.** El polvo de piedra triturada, que pase por un cernidor de 6 mm, puede sustituir parte de la arena ó toda ella, **T y T**; « las cerniduras y piedras

trituradas pueden sustituirse por arena y granzón, en condiciones especiales », **F**; se permiten las cerniduras, **BR, CR**; si pasan el cernidor de 6 mm, **TR**; se prefieren las cerniduras á la arena, **AH.**

### Agregado (« Balasto »).

**8. Clase.** Polvo de arena, granzón ó piedra quebrada, **BB**; granzón ó piedra quebrada, **G**; ó ambos, **BR**; granzón, **Lv**; (véase Cerniduras); granzón lavado por el mar, **Lp**; guijarros de piedra volcánica gastados por el agua, **SE**; piedra limpia, granzón, ladrillos duros quebrados, terra cotta, escoria de hornalla ó cenizas duras limpias, **Un**; se prefirió la piedra quebrada, se permitió el granzón en el interior de los muelles, pedestales y empotramientos, **WH**; piedra quebrada, **AH.**

**9. Requisitos.** Limpio, duro, durable; libre de polvo, de marga, arcilla y materia descomponible; lavado ó cernido si fuese necesario, **G**; próximamente cúbico, **CS, AH**; libre de pedazos largos delgados, **BR, NO, CS**; $<$ 2,000 kg /m cúb, **FP**; $<$ 2,080 kg, **Hb**; vacíos=31%, **SE**; empapado antes de usarlo, **G**; pero que no contenga agua, **Wv**; consérvese bien regado, **IM, Hb.**

**10. Tamaños :** mín, 6 mm, **G**; 3 mm, **FW, Mc**; máx, 19 mm, **Un**; 38 mm, **Bu**; 50 mm, **G**; 63 mm, **Hb**; 75 mm, **NO, Co, Ci, a, FP, SE**; granzón, 75 mm, **F**; piedra, como sale del triturador, **F, Mc, AH**; 25 á 63 mm, de acuerdo con la clase de la obra, **AH**; para los cimientos, 50 mm; para la superstructura, 40 mm; para vigas y columnas, 25 mm, **L**; granzón, $<$ 90% sobre 40 mm, $>$ 10% de arena, **Hb.**

| | Granzón ó arena contenidos en un m cúb de piedra. | | | | | |
|---|---|---|---|---|---|---|
| Agregado. | m cúb. | kg. | | m cúb. | kg. | m cúb. kg. |
| Piedra;... gruesa, | .63 | 861; | fina, | .33 | 486; | polvo, .11 176 |
| Granzón;. guijarros, 3 mm. | .80 | 1,304; | arena, | .29 | 469; | ... ... |
| Arena;... granzón, 3 á 6 mm. | .47 | 755; | arena, | .59 | 949; | ... ... |

**11. Almacenaje.** Almacenado en plataformas de madera, **CR, Wv;** ó en arcones de madera, **Wv.**

**12. Concreto de ceniza.** Se permite sólo para pisos, techos y rellenos, **Ms.** Concreto de ceniza armado se usa solamente con el permiso del inspector de edificios, **L.**

**13.** « Se puede usar en todo edificio en que la construcción á prueba de fuego es obligatoria, ó cuando se use en construcciones ordinarias, ó que arda despacio », pero no en columnas, muelles ó muros. Cenizas de caldera de vapor completamente quemada, limpia; mezcladas con cemento Portland á no menos de 1 : 7. Las cenizas deben pasar la malla de 25 mm en cuadro, **Ch.**

**14.** « Todos los otros requisitos especiales y sistemas de cálculo para el concreto armado como se requieren en este capítulo modifican y reglamentan el uso del concreto de ceniza en los edificios », **Ch.**

**15. Piedras grandes.**

Duras, enteras, durables, tan grandes como se puedan manejar con comodidad, bien lavadas, colocadas mojadas; una dimensión $<$ 30 cm, ninguna dimensión menor de 10 cm; ninguna de las piedras á menos de 5 cm de las superficies expuestas en el enlucido. El concreto se comprime en su lugar con pisones livianos, **Co.**

**16.** $>$ 46 kg, $<$ 75 mm distante de las armaduras ó de otras piedras grandes. (De las especificaciones para el Asilo de soldados.)

**17.** Se permite en los muros $>$ 45 cm de espesor, diámetro $>$ un cuarto del espesor de la pared, el volumen de la piedra $>$ que un quinto del volumen de la pared, **Yo.**

**18.** Piedras que pueda un hombre cargarlas y más grandes, más ó menos cúbicas; los pedazos largos y chatos se deben rechazar ó quebrarse; las piedras se distribuyeron con alguna uniformidad en toda la obra; $<$ 20 cm de separación, $<$ 60 cm del tope ó cara hacia la corriente; se dejan caer separadamente en el lecho de concreto húmedo, apisonadas si fuese necesario; se acuña con pala alrededor de las piedras y debajo de ellas el concreto; cada piedra se cubre con concreto antes de colocar las otras. Usense tantas piedras como sea posible sin faltar á estas condiciones, **Mc.**

**Para lista** de especificaciones para concreto, véanse págs. 1401-02.

**19.** « Peñas ». Piedras desde una carga de hombre hasta de varias toneladas (algunas veces de mampostería vieja), que forman como el 30% de toda la obra < 30 cm de la superficie del muro. Se colocaron en la capa superior de concreto de manera de formar trabazón con la capa siguiente prolongándose dentro de ella, **Lp.**

**20. Proporciones.** Véanse págs. 1297 á 1301.

### Medidas de los ingredientes.

**21.** El cemento se mide « como si estuviera compactado de manera que, 172 kg de cemento Portland seco tangan un volumen de .108 m cúb », **Lv**; el cemento se mide flojo, **CS, WH**; 1 saco de cemento < 42 kg=.028 m cúbico, **NO, Ci.** El cemento se mide como viene embalado por el fabricante, **OD, L, T y T.** La arena y el agregado se miden echándolos flojos en la caja de medir, **G.** Todo se mide flojo, **CS, WH**; 46 kg de cemento se suponen que miden .028 m cúb, **F.**

### Consistencia.

**22** En general, « muy húmedo », **NO**; agua como para que suba á la superficie por compresión moderada, **CS**; sin gran trepidación, **OD, TR**; suficientemente fluido que no necesite pisón, **Mc**: poco ó ningún pisón, **Hb.**

**23.** (**a**) Para masas ordinarias de concreto, como cimientos, muros fuertes, arcos grandes, pilas y estribos, mezcla **mediana** de una consistencia tenaz gelatinosa, que trepide al pisarla, **T y T.**

(**b**) **Para el concreto de ripios y para el concreto armado,** como muros delgados de edificios, columnas, puertas, conductos, estanques; **muy húmedo ó pastoso**, tan blando que hay que andar ligero para evitar que se derrame en la pala, **T y T.**

(**c**) **En lugares secos para masas de cimientos**, que deben soportar gran compresión dentro de un mes después de su colocación, úsese « el concreto » **seco** », con consistencia de tierra húmeda, con tal que se extienda en capas de 15 cm comprimiéndolas hasta que el agua brote á la superficie. No se permite en el con creto armado, **T y T.**

**24.** « **Aguado** ». Los trabajadores al extender el concreto en su posición final, usan botas de cuero á prueba de agua y se meten hasta el tobillo en el concreto, **Lp.**

**25. En los cimientos**, « suficiente agua para que haya cohesión al apisonarlos, con pisones de 14 kg forrados de hierro »; en **los muros de esclusas,** bastante para la completa hidratación del cemento; bastante para que haya coherencia después de mezclado completamente; más plástico que arena húmeda; el apisonado completo debe traer el agua á la superficie; la trepidación incipiente marca el límite; un exceso de agua en una carga puede corregirse en la próxima; consistencia variada; de vez en cuando, para llenar las condiciones de temperatura **IM.**

**26.** El concreto para subestructuras mucho más seco que para la superstructura, **SE.** El concreto que se va á colocar debajo del agua debe estar medio seco, **Ph.**

**27.** Agua por carga, aproximadamente :

3.5 m cúbicos por lote de 43.2 m cúbicos, reducidos á 28.5 m cúbicos al apisonarlo;

2.5 m cúbicos por lote de 28.8 m cúbicos, reducidos á 20.0 m cúbicos al apisonarlo, **BB.**

### Mezcla.

**28.** A mano para los cimientos, con mezcladores cúbicos para los muros de esclusa, **IM**; con mezclador cúbico, **SE**; en máquina, **F, BR, AH, NO, Bu, Co, Ci, b**; á máquina cuando la cantidad de la obra pase de 1,000 m cúbicos, **CS**; por máquina en general, **TR, Hb, WH**; « con preferencia por los mezcladores del tipo continuo que miden automáticamente y vierten las proporciones justas en pequeñas corrientes dentro de la cámara de mezclar », **F**; por máquina de lote fijo, **Bu, Ci, b**; « por mezclador mecánico de lote fijo. excepto cuando se necesitan cantidades pequeñas ó cuando la condición del trabajo hace que la mezcla á mano se prefiera; mezcla á mano... solamente cuando la aprueba la Oficina de Inspección de Edificios », **Ph**; mezclador de lote fijo, **Hb, CR, W'v,**

**FW**; $<$ 100 m cúbicos por día de 8 horas, **FW**; se prefieren los mezcladores de lote fijo, los mezcladores continuos solamente con permiso escrito del ingeniero, **WH**.

**Método**. Los materiales se mezclaron secos antes de añadirles el agua, **CS**, **NY**; se remueven $<$ 100 veces, **Ci**, **b**. « En toda mezcla se medirá el material para cada lote »; el agregado, si está caliente y seco, se humedecerá antes de ir al mezclador, **Ph**. Un lote se descarga completamente antes de introducir el otro. No menos de 25 revoluciones por cada lote, removiendo el concreto no menos de una vez en cada revolución, **Un**; orden de la carga : 1.º el granzón, 2.º el cemento, 3.º la piedra, 4.º el agua, cada carga se hace girar $<$ 2 minutos, $>$ 9 revoluciones por minuto, se le darán revoluciones adicionales cuando el tiempo lo permita, **IM**.

**29. Mezcla del lote.** Cemento (60 lit por lote) mezclado en una pasta gruesa en la plataforma. Primero guijarros, después arena y pasta de cemento, después piedra quebrada, vaciada, por el agujero de la plataforma, en la caja que está debajo en el carro. La caja se vacía en el mezclador; 5 á 10 revoluciones, de 7 á 14 lotes por hora. Se necesitan 12 hombres para apisonar 14 lotes, **BB**; primero arena, después cemento, después agregado, después agua, **TR**, **OD**.

**30. Mezcla á mano**. El cemento y la arena se mezclan secos; se añade piedra mojada; se añade agua, **CS**. El cemento y la arena se mezclan secos, se añade agua, el agregado se extiende en un espesor de no más de 5 cm, se riega, se extiende mortero sobre el agregado y se mezcla, **Ph**. El cemento y la arena se mezclan secos, se añade agua, se le mezcla mortero, se le añade agregado (mojado), se mezcla todo, **Hb**; la mezcla de arena y agregado se extiende primero en una capa delgada sobre una plataforma de madera, se le extiende cemento encima, se mezcla seca, se revuelve al ir añadiendo el agua; la piedra quebrada, si se usa con granzón, se añade húmeda á la masa húmeda, **WH**.

**31**. Sobre una plataforma sin rendijas, bastante grande para dos cargas de no más de 1 m cúbico cada una. El cemento y la arena se extienden en capas delgadas y se mezclan secas hasta que tengan un color uniforme.

Después úsese uno de los 3 sistemas, como sigue :

(1) Mezcla de cemento y arena extendida sobre la capa de piedra;

(2) La piedra se echa con pala sobre la mezcla de cemento y arena. En (1) ó (2) revuélvase 3 veces, añadiendo agua en la primera revolución;

(3) El mortero hecho con la mezcla del cemento y la arena se extiende sobre la piedra. La masa de mortero y piedra se revuelve dos veces, **T y T**.

**32**. En todos los casos, el resultado debe ser un concreto suelto de color y apariencia uniformes, con las piedras penetrando completamente en el mortero. La consistencia debe ser uniforme en todo él, **T y T**.

**33**. « A medida que se llenaba la caja del granzón, se le agregaba el cemento gradualmente, de manera que, cuando aquélla estaba llena, la del cemento estaba vacía. Se quitó la caja entonces, y se niveló el montón, dándole un espesor uniforme de $>$ 30 cm, y se mezcló luego echándola hacia atrás y hacia adelante dos veces », se añadió el agua al final de la segunda removida, **Lp**.

## Armaduras (tapialeras *).

**34. Forro**. De tablas bien sazonadas, de 5 cm de espesor, todas acepilladas, con lenguetas y ranuras para trabarlas, **Co**, **b**; 5 × 15 cm de pino, acepilladas en todos los lados, **Hb**; las tablas acepilladas en un lado y dos cantos; un canto ligeramente biselado y colocado contra el canto cuadrado de la plancha siguiente, **Yó**; tablas de pino con preferencia de 5 × 15 cm, acepilladas y machihembradas, **WH**; las formas para las faces descubiertas, con lenguetas y ranuras ó biseladas; las armaduras para muros deben reforzarse y, si es posible, que tengan los lados unidos con alambres, **Ci**; las juntas de los extremos cuadradas y reforzadas con postes, **Hb**; las juntas que queden abiertas se llenarán con arcilla dura inmediatamente antes de colocar el concreto, **Hb**.

Si las **tablas usadas** no están maltratadas, pueden usarse otra vez; pero para trabajo visible deben limpiarse y aceitarse, **Hb**.

**Postes**. Generalmente de 7.5 × 20 cm de pino, acepillados en ambos cantos del mismo alto de la pared, $>$ 1.20 m de distancia, **Hb**.

Las cimbras y las armaduras se deben humedecer, **IM**; si fuese necesario, antes

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* (*N. del T.* — Así las llaman en España cuando se usan en los muros).

**Para lista** de especificaciones para concreto, véanse págs. 1401-02.

de colocarlas, **NO, Ci, b**; ó aceitarlas, **NO**. De acuerdo con las circunstancias, deben mojarse las armaduras (excepto en temperaturas de congelación) ó engrasarlas con aceite crudo antes de colocar el concreto, **T y T**; aceitadas inmediatamente antes de usarlas, **Hb**; pintadas ó aceitadas antes de volverlas á usar, **CR**; humedecidas inmediatamente antes de poner el concreto, conservándolas húmedas hasta que el trabajo se haya endurecido, **TR, Wv**.

Para la quitada de las armaduras, véase p. 1250.

**35.** En el lado del muro que da contra la corriente en un dique, las armaduras deben ser sólo lo suficientemente lisas para que trabajen bien, y no deben tener huecos. En el borde del vertedero y la cara del muro que da hacia la corriente, las armaduras deben tener las superficies acepilladas de manera que el trabajo quede liso al terminarlo, **Mc**.

**36.** Las agujas ó varillas de ligazón que se dejen en el concreto no deben llegar más cerca de 5 cm de la superficie del concreto, **CR**; los extremos salientes de los pernos y varillas de hierro deben cortarse y taparse con concreto, **BR, AH**; no cincelados, sino aserrados ó quitarlos de cualquier otra manera sin sacudir la obra, **AH**; los postes para sujetar las armaduras no se deben insertar á más de 1.20 m de la parte superior de las paredes, **BR**; no deben aparecer pernos, etc., en la obra terminada, **OD**.

## Colocación, removida y apisonado.

**37. El trabajo nocturno** se prohibe en general, **TR. Tiempo de colocación**; el concreto debe colocarse dentro de 30 minutos después de mezclado, **AH, NO, Ci, b**; $>$ 30 minutos « entre la mojada del cemento y la colocación final del concreto en el lugar », **F**; antes de la fragua inicial, **TR, OD, CR, Wv, FW, Hb, Bu**; después de la mezcla, la masa se mantiene en movimiento hasta colocarla en un vehículo para el transporte, **TR**. No se permite **retemplada** ó nueva manipulación, **TR, CR, NO, Bu, Co, Ci, b, JC**. El concreto en que se hayan separado los materiales, debe volverse á mezclar (á mano, **BR, AH**); antes de colocarlo, **T y T**.

**Manipulación.** En el concreto muy mojado, se deberá extraer el aire, removiéndolo, empujando las piedras hacia adentro, y removiendo el concreto **debajo de las barras**, etc., **G**; por medio de barras de acero ó hierro, como de 10×15 cm con mangos largos ajustables, de modo que los trabajadores no tengan que pararse en el concreto, **NO, Ci, b**. El concreto debe removerse ó acomodarlo en su lugar con un apisonado suave, **Bu, Co**; apisónese hasta que el mortero salga á la superficie, **AH, BR**; hasta que todos los vacíos estén llenos y el agua fluya á la superficie, **CS**; un apisonador no debe hacer más de dos m cúbicos por hora, **BR**; el pisón debe tener una superficie de golpeo no menor de 200 cm cuad, que no pese más de 5 kg, **Co**; faz 15 cm en cuadro, peso, con el mango, como 9 kg, **CR**; pisones de 14 kg con aro de hierro, área de la faz no más de 30 ", **IM**; pisones de 18 kg, **SE**; el concreto se coloca sin pisarlo, **FP**.

**38. El concreto seco se humedeció**, regándolo, pero no con chorros, **CR**.

**39.** El concreto se debe remover continuamente **alrededor de la armazón** con hierros aparentes, á medida que se coloca. Se prohibe la llenada completa de las armaduras y el empozamiento consiguiente. El concreto parcialmente en fragua no debe sufrir choques, **Ch**.

**40. Colocación en capas.** Se debe tener cuidado de quitar toda la espuma, que salga en el concreto, antes de colocar la próxima capa, **Lp, JC**.

**41.** Se prohibe que el concreto se descargue directamente de la caja ó carro, ó se eche con palas en su lugar, así como el uso de canales, **OD, Wv, FP, TR, CR**; no se debe dejar caer de más de 1.80 m, **FP**; 1 m, **Wv**.

**42.** No se debe **caminar** en la parte terminada hasta que frague, **OD, Co**.

**43. Espesor de las capas.** No más de 15 cm, **Wv, BR, OD**; como 15, **CR**; como 15 después de pisadas, **TR**; 15 á 20, **CS**; $>$ 15, **F**; $>$ 10 cm, **SE**; con mezcla seca en los declives $>$ 10, **F**; $>$ 10 en los cimientos, como 15 en las paredes de atrás, **JM**; $>$ 22, **Hb**; $>$ 30, **WH**; de manera que cada capa pueda incorporarse á la anterior, **T y T**.

**44.** No se permitieron capas, **Bu, Co**; las capas no deben concluir por una orilla delgada, **FP**; cada capa completa (apisonada, **CR**) antes de colocar la

siguiente, **FP, CR**; cada capa de un día de trabajo se coloca antes que fragüe la capa de abajo, **TR**.

**45. En cimiento de roca viva.** La roca se limpia y lava con escobas de alambre, se hace escabrosa si fuese necesario, se cubre con una capa espesa de una mezcla de cemento puro, **CR**; se hace un lecho de mortero húmedo, **FW**; de 12 mm de espesor, **TR**; el concreto se ancla á la roca con barras de acero, si fuere preciso, **CR**.

## Juntas.

**46. Modos de evitar las juntas horizontales.** Los muros, etc., se construyen en secciones alternadas, tan cortas que puedan quedar monolíticas; estas secciones se traban por juntas verticales que se alternen, **G**. Las juntas continuas desde el cimiento hasta la parte de arriba, **CR**; « las juntas se deben formar entre las secciones contiguas de concreto á 1.20 m hacia abajo, desde la superficie, por una capa de papel alquitranado », **BR**; el machihembrado debe tener una capa delgada de mortero, de 1 : 5, ó más débil, que se pone antes que se coloque el concreto contra ella, **Hb**.

**47. Juntas entre el trabajo viejo y el nuevo.** Las superficies descubiertas se cubren y se conservan húmedas hasta continuar el trabajo, **CR**; los filos astillados ó quebrados, se deben rebajar, **CR**; la superficie vieja se debe dejar escalonada para formar trabazones, limpiándola y mojándola antes de añadir el trabajo nuevo, **FW, G**; se limpia con cepillo de alambres tiesos y vapor de agua, **FP, BR, Hb**; si fuese preciso, **F, Lv**; se le hace una sup áspera con un pico, si fuese preciso, **BR**; listones de madera, de 10 á 15 cm de ancho, con los lados en bisel, que se incrustan $\lessdot$ 7 cm, quitándolos antes que el concreto se endurezca completamente, **NO**; entre el trabajo viejo y el nuevo un lecho de mortero de 1 : 3 de 2.5 cm de espesor, **NO, Co**; capa de mortero de 12 mm, **FP**; la superficie vieja se cubre con un sedimento de cemento puro con consistencia de melado, **BR**; ó cemento seco, **OD**; cemento seco esparcido con un cepillo, **Hb**; con una capa de mortero, **TR, FW**; la superficie vieja, restregada con mortero de 1 : 2. **CS**; con pasta gruesa de cemento puro introducido en la superficie con escobas **CR**; calzado como se ha indicado, **FW**.

**48.** En las juntas horizontales de muros delgados, ó en muros que deben soportar **presión de agua**, ó en otros lugares importantes se puede necesitar junta de mortero. Los estanques, etc., con muros delgados que deben contener agua, se deben fabricar como monolitos, continuando el trabajo, si fuese necesario, de día y de noche, **T y T**.

**49. Cuando se suspende el trabajo** por más de una hora, se deben nivelar los bordes exteriores de la última capa, dejando la parte central de la superficie 15 cm más baja que los bordes, **CR**.

**50.** Unión entre el concreto nuevo y un muro viejo. Cavidades para machihembrado, 60 cm de ancho en la cara, 80 cm atrás, 40 cm de profundidad, se cortan verticalmente en la mampostería, á 1.20 m de separación, **Lp**.

**51.** La última capa que se coloque se dejará tan áspera como sea posible, de manera que sobresalgan las piedras incrustadas. La superficie se limpiará, lavará y se rociará con cemento puro, **Mc**.

## Colocación debajo del agua.

**52. Debajo del agua.** No debe colocarse concreto debajo del agua (sin permiso explícito, **F**; excepto para tapar filtraciones y manantiales, **TR**); no se debe permitir que el agua llegue sobre una obra nueva hasta que esté enteramente fraguado, **IM, Wv, TR, OD**; no menos de 24 horas después de fraguado, **Lv, NO, Co, Ci, b**; si se coloca en el agua antes de fraguar, la mezcla deberá ser 1 : 2 : 3, **WH**; 80% del trabajo debe construirse bajo el nivel del agua (dulce), **SE**; el concreto para colocarlo en el agua debe estar medio seco, **Ph**; los sacos se deben bajar hasta unos pocos cm de la superficie en que se va á depositar el concreto, **FW**.

**53.** Cuando las armaduras llegan debajo de la baja marea se taparán las filtraciones debajo de aquéllas, para impedir las socavaciones antes de la fragua; para esto, se colocan sacos, llenos de arena, ó lona de jute, debajo del concreto á 30 cm, clavada á lo largo de la parte inferior de la armadura, por el lado dentro, **FW**.

**Para lista** de especificaciones para concreto, véanse págs. 1401-02.

### Lluvia.

**54. Lluvia.** Durante la lluvia no se debe hacer trabajo nuevo, **IM, Bu, CR, AH, FP**; el trabajo recientemente hecho se debe proteger con lona **Bu.**

### Heladas.

**55. Congelación.** No se debe hacer concreto ó mortero cuando la temperatura es menor de 2° C, en la sombra; el trabajo de concreto se debe suspender desde el 20 de noviembre hasta el 1.° de abril; durante la temperatura de congelación, no se debe mezclar ó depositar ningún concreto sin permiso del ingeniero, **IM, Bu**; el hielo y la escarcha deben quitarse, el agua y la arena se deben calentar, el granzón se pondrá al vapor, el trabajo debe cubrirse conservándolo caliente por tubos de vapor, **Lv**; el concreto no se debe colocar cuando está helado; si está armado, se debe conservar á más de 0° C, durante $<$ 48 horas después de colocado, el uso de arena y agregado helados está prohibido, **Ch.** No se permite la colocación cuando la temperatura es $>$ 0° C, **Un, AH, BR,** $<$ 0° C, **OD**; $<$ — 1° C, **CR,** $<$ — 2° C, **TR, FP**; cuando haya probabilidad de que hiele antes de la fragua, **Wv**; antes de la fragua final, **OD**; antes de que fragüe lo suficiente para impedir que se dañe, **BR, CR.** El concreto que se hiele después de colocado debe quitarse, **Un.** No se debe colocar el concreto cuando la temperatura sea menor de — 7° C; el agua se debe calentar cuando la temperatura sea menor de — 2° C, **MC.** El empleo de materiales helados está prohibido; la colocación del concreto debe protegerse de la congelación, **Ph.**

**56. El concreto de cemento natural** no debe nunca estar expuesto á la helada hasta después de estar completamente endurecido y seco, **T y T.**

**57.** « Ningún concreto, excepto el que se coloca en grandes masas, ó muros fuertes cuya sup no importe que luzca bien, debe exponerse á la helada hasta que no esté seco y duro. Los materiales que se emplean en la masa de concreto en tiempo de congelación no deben contener escarcha. Las superficies deben protegerse de la escarcha. Las partes de superficie de concreto que se hayan helado, se deberán quitar antes de colocar concreto nuevo. » **T y T.**

**58. Las armaduras** bajo el concreto que se hayan colocado en tiempo de congelación, « deben **permanecer** hasta que todas las trazas de la helada hayan desaparecido del concreto, y que el endurecimiento natural del concreto haya llegado al punto de seguridad », **CH, Ph.**

### Humedecimiento.

**59. Humedecimiento.** El concreto nuevamente colocado debe protegerse del sol (con tablas ó tela alquitranada, **FP, Hb, IM**); conservándole húmedo; **Mc, IM**; $<$ dos semanas ó hasta que se cubra de tierra, **F**; $<$ 10 días, **SE, AH**; 6 días, **CR**; 3 días, **EW**; 48 horas, **BR**; hasta que fragüe, **Wv**; hasta que fragüe fuertemente, **Hb**; las superficies no deben alisarse hasta que se continúe el trabajo, **CR**; con tela alquitranada húmeda $<$ 3 días. Al completarse una sección del muro cúbrase la parte superior con una capa gruesa de arena húmeda, regando con agua la masa del muro hasta que el concreto haya fraguado completamente, **IM**; el concreto debe empaparse dos veces por día, incluyendo los domingos, durante una semana después de la colocación, en época de calor, **Ch, Ph.**

**60.** Humedézcase regando con una regadera fina á cortos intervalos ó cubriéndolo con tela húmeda, etc., **G.**

### Quitada de las armaduras.

**61.** Las armaduras deben dejarse en su lugar $<$ 4 días, **IM**; $<$ 7 días; más tiempo si lo requiere el ingeniero, **Lv**; 72 horas, **OD**; 48 horas, **AH, BR**; hasta que el concreto haya estado en ellas por lo menos 36 horas, **WH**; hasta que el ingeniero autorice la quitada, ó hasta que el concreto se haya endurecido, **Ci, b**; hasta que el concreto pueda soportar su carga con seguridad, **Ms**; las armaduras se quitarán después de 48 horas, **SE.**

**62.** Los puntales debajo de los pisos y techos deben permanecer en su lugar $<$ 2 semanas. Las armaduras de columnas, $<$ 4 días; para planchas, vigas y tra-

viesas, < 1 semana y por lo menos hasta que el piso pueda soportar su propio peso. « Ninguna carga ó peso se colocará en parte alguna de la construcción en que dichos apoyos ó cimbras se hayan quitado. » **Ch, Ph.**

**63.** El tiempo para la quitada de las armaduras y apoyo es de 24 horas á 60 días, dependiendo de la temperatura y otras condiciones atmosféricas y del inspector de edificios, **Un.**

**64.** Hasta que el concreto no esté duro.

| | Tiempo mínimo, días: abril 1.° á dic. 1.°; | dic. 1.° á abril 1.° |
|---|---|---|
| Planchas y dinteles, columnas y paredes monolíticas | 10 | 15 |
| Postes y apoyos inferiores para maderas y vigas | 14 | 21 **L.** |

**65.** Las armaduras bajo concreto colocadas **en tiempo de congelación** « deben permanecer hasta que todas las señales de escarcha desaparezcan del concreto y el endurecimiento natural de éste haya llegado al punto de seguridad », **Ch, Ph.**

## Acabado de las superficies, construcciones á prueba de agua, etc.

**66.** El acabado se conserva terso por manipulación durante la colocación, etc. El concreto, libre de agregado grueso, debe colocarse pegado á las armaduras, despegándolo de ellas con una pala chata, que se introduce entre el concreto y la armadura (la armadura se humedece con agua, **BR**), el concreto se apisona con un pisón de hierro, con faz inferior de 5×15 cm, **AH, BR**; el acabado se hace retirando el granzón hacia atrás por medio de tenedores, **HB**; ó **palas, FP**; las faces se alisan por frotamiento, **TR, Hb**; con un pedazo de madera, ó piedra blanda, **TR**; los vacíos se rellenan con mezcla, **Hb, TR, CR**; el enlucido se permite solamente en una cavidad ocasional y accidental en que el enlucido no esté expuesto al efecto de la escarcha, **CR**. Véase § 79. El mortero de cemento Portland de 1:3, se coloca simultáneamente con el grueso del muro, **CR**. Para muros, el mortero de cemento Portland, muy seco, de 3.5 cm de espesor, **TR**.

**67.** En las caras descubiertas, se deben quitar las armaduras antes que el concreto se endurezca; la superficie (1) se frota con mortero de 1 volumen de cemento Portland, 2 volúmenes de arena, aplicado con un tapón de coleta, y alisado con una brocha de encalador, ó (2) frotado con una brocha de alambre duro, y una mano delgada de pasta fina de cemento Portland, alisada con una brocha de encalador, **NO, Co**; el acabado pulido de los lados se produce apisonándolos muy bien contra las superficies de las armaduras, **SE.**

**68.** Las superficies que no queden contra las armaduras, se cepillarán y alisarán con cuchara, **NO.**

**69.** Los vacíos ú otras imperfecciones que aparezcan al quitar las armaduras, se corregirán á expensas del contratista, que debe quitar y reemplazar el trabajo que no sea satisfactorio, si fuese preciso, **F.**

**70. Para pisos y techos del estanque de mezclar.** Mortero duro, de 1 volumen de Portland, 1 volumen de cernidura finas de piedra que pasan por un anillo de 10 mm, libre de polvo, marga, etc., de 2.5 cm de espesor, colocado antes de la fragua inicial del concreto. Se raspa y se alisa la superficie con cuchara. Se cubre y riega durante 3 días, **CO.**

**71. Los andenes y partes superiores de los parapetos** se alisan con una capa de mortero > 6 mm de espesor, consolidada con el concreto « sobreponiéndole planchas fuertes de 10 cm de espesor y apisonándolas con pisones de hierro fundido de 18 kg, hasta que sus extremos estuvieron en contacto con los extremos de los armaduras », **SE.**

**72. Para los muelles, pedestales y estribos.** En las superficies expuestas al aire ó al agua, mortero de cemento Portland de 4 cm de espesor, 1 de cemento 2 de arena, subido simultáneamente con el concreto en zonas de 25 á 28 cm de ancho por medio de armaduras de láminas de acero de 6 mm, 30 cm de ancho, 1.20 á 1.50 m de largo, colocadas rodeando la obra, á 4 cm de las armaduras, y separadas á cada 30 cm por bloques de madera, cubriéndose los extremos de las planchas, ligeramente, unos sobre otros, **WH.**

**Para lista** de especificaciones para concreto, véanse págs. 1401-02.

**73. Secciones de cloacas**, 1 de cemento, 2 de arena, no más de 13 mm de espesor, colocada al mismo tiempo que el concreto, **Lv**.

**74. En las molduras, cornisas, etc.** Mortero plástico colocado contra las armaduras de construcción fina, á medida que se coloca el concreto; no se permite enlucido exterior, **SE, T y T**; no se hará enlucido á menos que sea especialmente permitido.

**75. Acabado de la parte superior.** El concreto se llevó hasta 9 cm menos de la elevación requerida, y mientras éste estaba todavía sin fraguar y blando, se añadieron 7.5 cm de concreto más fino, que se apisonó y se amasó para formar un monolito con el concreto de abajo; luego se añadió 12 mm de 1 : 3 (1 : 2, **AH**) y se rebajó á la altura requerida con una vara larga de madera, de filo recto, **AH, BR**.

**76. Coronamiento final.** Mientras que la base de concreto está aún blanda, sin fraguar y pegajosa, se extiende mortero (que deberá tener 2.5 cm de espesor al alisarlo), se nivela, y se golpea con madera; se rebaja, y se alisa con una regla y se pule con una cuchara fina de enlucir; se cubre con tablas ó tela alquitranada hasta fraguar firmemente, luego se cubre con arena, se debe conservar húmeda por varios días, **FP**; el mortero de $<$ 2.5 cm de espesor, de 170 kg de cemento Portland para 300 lit de arena, se aprieta encima del concreto apisonado antes que éste empiece á fraguar, se rebaja con un filo derecho, se cepilla y se alisa con cuchara de encalador, **CR**; mortero de cemento Portland de 1 : 2, de 2.5 cm de espesor, **TR**; la superficie se forma empujando las piedras de la superficie hacia atrás, **Hb**.

**77. Acabado de superficies imitando granito**, para la parte superior de las pilas, pedestales y estribos; 1 parte de Portland, 2 partes de arena de granito gruesa limpia ó cerniduras finas de granito, 3 partes de astillas de granito, que pasen por un anillo de 12 mm. Se alisa con una regla de enlucido, **WH**.

**78. Para hacerlo impermeable.** Una mano fuerte de mortero semilíquido de 1 parte de cemento, 1/2 parte de cal apagada, 3 partes de arena. Esta mano se debe dar como un enlucido fino. Cuando haya fraguado, añádase una mano gruesa de pasta líquida de cemento puro, **CS**.

**79. Enlucido con cemento.** No se permite en las caras descubiertas, **AH, CS**. Las caras interiores de las enjutas rellenas, bien humedecidas y enlucidas con mortero de 1 de cemento : 2, 5 arena, **CS**. Véase § 66.

## Piedra artificial.

**80. (a) Para las molduras finas, etc.** Los moldes se enlucen con mortero semilíquido 1 de cemento, 2 de arena fina aguda, respaldada con concreto semihúmedo de 1 : 2 : 4, ó 1 de cemento para 6 de granzón que pase por el anillo de 10 mm. El respaldo de concreto apisonado por capas delgadas. **(b) Para las superficies planas sencillas.** El concreto se apisona en los moldes. Se quitan los moldes. Las superficies descubiertas se aplanan con mortero hasta un acabado liso como en (a). No se debe dejar ningún mortero sobre la cara. Usese solamente lo suficiente para llenar los poros y darle un acabado terso, **CS**.

## Resistencia, etc., que se requieren.

(Las resistencias, etc., en kg/cm cuad, á menos que se especifique de otro modo.)

**81. Compresión máxima** después de endurecerse durante 28 días, $<$ 140, **Un, Mh**.

**82. Esfuerzo cortante máximo** correspondiente á la compresión de 140, **14, Un**.

## Cargas máximas permitidas.

**83. Para las cargas estáticas** sobre un concreto de cemento Portland de 1 : 6.

| | Carga máxima permitida kg cm cuad ** |
|---|---|
| Compresión, superficie de concreto > el área cargada | .325.s* = 45.7 |
| — en columnas, largo > 12 diámetro | .225.s = 31.6 |
| — — armadas sólo longitudinalmente | .225.s = 31.6 |
| — — con sunchos | .270.s = 38.0 |
| — — con sunchos, con barras longitudinales de 1 á 4% | .325.s = 45.7 |
| — — con unidades de columnas de acero que cubren enteramente el corazón de concreto | .325.s = 45.7 |
| Módulo de ruptura (módulo de elasticidad, E, constante) | .325.s = 45.7 |
| — — adyacente á los soportes (E constante) | .375.s = 52.7 |
| Esfuerzo cortante (ninguna compresión normal á la superficie del esfuerzo cortante, recibiendo la armadura la tensión normal) | .060.s = 8.4 |
| — — combinado con compresión igual | .162.s = 22.8 |
| Adhesión, barras lisas | .040.s = 5.6 |
| — alambre desplegado | .020.s = 2.8 |

**JC.**

**84. Compresión.** Véase también § 146.
A, excluyendo los esfuerzos de la temperatura,
B, incluyendo los esfuerzos debidos á los cambios de temperatura de 22° C.

| En los arcos de puentes, kg/cm cuad : | A | B |
|---|---|---|
| para carreteras y ferrocarriles eléctricos | 35 | 42 |
| para ferrocarriles de vapor | 28 | 35 |

**CS.**

**85.** En el concreto de cemento Portland de primera clase, con agregado bien graduado de :

| | |
|---|---|
| 1 : 6 ó menos | 29 kg/cm cuad. |
| 1 : 5 ó menos, en vigas ó planchas | 35 — |

« En caso que se use un concreto más rico, este esfuerzo podrá aumentarse con la aprobación del comisionado á no más de » 42 kg/cm cuad, **Ms.**

| | |
|---|---|
| **86.** Portland, 1 : 2 : 4 | 16 kg/cm cuad. |
| 1 : 2 : 5 | 14.5 — |
| Rosendale ó igual, | |
| 1 : 2 : 4 | 8.7 — |
| 1 : 2 : 5 | 7.7 — |

**N Y.**

| **87.** Portland, kg/cm cuad. Mezcla, | 1 : 2 : 4 | 1 : 2.5 : 5 | 1 : 3 : 6 |
|---|---|---|---|
| mezclado por máquina | 28 | 24.5 | 21 |
| mezclado á mano | 24.5 | 21.0 | 17.5 |
| Natural | | 10.5 | ... |

Portland en el concreto armado, directa, .2 × máxima; en la flexión, .35 × máxima, **Ch.**

**88.** Portland, directa, 24.5 kg/cm cuad; en el cemento armado, 24.5 kg/cm cuad « simultáneamente con 420 kg/cm cuad » de tensión en el acero, **Un.**
**89.** Portland, directa, 24.5; en la flexión, 35, **Mh.**

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* s = Resistencia compresora máxima á 28 días, en kg/cm cuad cuando se probaron, bajo condiciones de laboratorio, en la forma de cilindros de 20 cm de diámetro, 40 cm de largo, de la misma consistencia que los que se usaron en la obra.
** Cuando s = 140 kg/cm cuad.

**Para lista** de especificaciones para concreto, véanse págs. 1401-02.

| | Piedra ó granzón. | Agregado. Escoria. | Ceniza. |
|---|---|---|---|
| **90.** Portland, En la flexión | 42 | 28 | 17.5 kg/cm cuad. |
| Directa, en las columnas largo > 15 diámetros | 35 | 21 | 10.5 — |

En las columnas con sunchos, 70 kg/cm cuad sobre el área dentro de los sunchos, **Ph.**

| | 1:2:4 | 1:2:5 | 1:3:6 | |
|---|---|---|---|---|
| Portland | 49 | 45.5 | 42 | kg/cm cuad. |
| Natural | 28 | ... | ... | — **L.** |

**91. Tensión**, kg/cm cuad.
A, excluyendo los esfuerzos de la temperatura,
B, incluyendo los esfuerzos debidos á los cambios de temperatura de 23° C.

| | A | B | |
|---|---|---|---|
| En arcos armados | 3.5 | 5.25 | |
| En las planchas armadas, vigas, etc | 0 | 0 | **CS.** |

En plano diagonal, .02 × la resistencia máx de compresión, **Ch.**

**92. Esfuerzo cortante**, kg/cm cuad.
5.25, **CS**; 3.5, **Mh**; 4.2 cuando no está combinado con compresión sobre el mismo plano « á menos que el inspector de edificios con el consentimiento de la junta de apelaciones fije algún otro valor », **Ms**; concreto de piedra ó granzón, 5.25; escoria, 3.5; ceniza, 1.75, **Ph.**

### Coeficiente de elasticidad.

**93.** 105,000 kg/cm cuad, **CS.**

### Adhesión.

**94** Véase p. 1322, y pág. 1415, § 113.

### Factores de seguridad.

$$\text{Factor de seguridad} = \frac{\text{carga máxima}}{\text{carga permitida}}.$$

**95.** Al fin de un mes, en las vías subterráneas y puentes de vigas armadas para carreteras y ferrocarriles eléctricos, también edificios, techos, alcantarillas, cloacas, 4; en las vías subterráneas y puentes de vigas armadas para ferrocarriles de vapor, 5, **CS.**
Portland en el concreto armado, compresión directa, 5; en las vigas, 1/.35; **Ch.**
En las vigas armadas, 1 para la carga muerta, más 4 por la carga viva, =5.
En las columnas de enrejado ó de obra abierta de acero ó de hierro, vigas rodeadas por el concreto que llegue hasta < 5 cm más allá del metal (sin concesión para el concreto), 3, **L.**

### Armazón.

**96. Barras**, sin pintar, pero libres de escamas, moho y grasa, **G.**
**97. Forma.** Lisa redonda ó cuadrada, ó corrugada, **Lv**; lisas ó torcidas, **NO**; deformadas, **AH**; torcidas ó deformadas, **Bu**; cuadradas, torcidas por máquina, **Co**; las Ransome cuadradas torcidas se prefieren, **F**; Ransome ó iguales, **Hb**; barra Thacher, **CS**; cuadradas, torcidas en frío ó barra Johnson corrugada; en la barra Johnson, la sección neta=la requerida por los planos, para las barras torcidas; las barras lisas se deben usar en la compresión solamente, **Ci.**
**98. Barras torcidas.**

| Tamaño, mm. | (6) | (9) | (12) | (16) | (19) | (22) | (25.5) | (29) | (32), **NO, Co;** |
|---|---|---|---|---|---|---|---|---|---|
| Torceduras por m. | 39 | 26 | 16 | 11.5 | 8 | 6.5 | 5.75 | 5 | 5 |

Una vuelta en 5 á 7 veces el tamaño nominal, **F.**
Torcidas uniformemente con maquina, variando el área de sección transversal mínima en no más de 2.5%, **NO, Co.**

**99. Barras redondas, corrugadas, etc.**, que tengan la misma área total de sección neta que las barras cuadradas ó torcidas, **NO.**

**Requisitos.**

**100. El hierro y acero** « deben satisfacer las Especificaciones Modelo de los Manufactureros, revisadas feb. 3, '03 », **Ph.** Véase pág. 946 y 947.

**101. Acero, manufactura y dureza.** Mediano, Martín Siemens, **NO, Bu, Co, Ci**; suave, **Lv**; blando ó mediano, **CS.**

**102. Resistencia á la tensión máxima**, en miles de kg/cm cuad: 3.64 á 4.32, **F**; 3.8 á 4.48, **Un, Mh**; mediano, 3.50 á 4.55, **Ci, a**; mediano, 4.20 á 4.76, **CS**; blando, 3.78 á 4.34, **CS**; 3.85 á 4.55, **Lv, T y T**; < 3.85, **NO**; 3.99 á 4.55, **Co, a**; 4.20 á 4.90 antes de torcer, **Co, b**; 4.20 á 4.90, **Bu.**

**103. Compresión máxima.**

| Mezcla. | 1:1:2 | 1:1.5:3 | 1:2:4 | 1:2.5:5 | 1:3:6 |
|---|---|---|---|---|---|
| kg/cm cuad = | 203 | 168 | 140 | 122 | 105 |
| $n = E_s/E_c =$ | 10 | 12 | 15 | 18 | 20 |

**Ch.**

**104. Fractura**, sedosa, uniforme en color y contextura, **Co.**

**105. Límite de elasticidad** < que la mitad de la resistencia máx de tensión, **G.**

**106. Coef de elasticidad**, 2,100,000 kg/cm cuad, **CS.**

**107. Cociente, n, de los coef de elasticidad.**

$$n = \frac{E_s}{E_c} = \frac{\text{coef de elasticidad del acero}}{\text{coef de elasticidad del concreto}}.$$

$n=12$, **Mh.** « Si no se encuentra por pruebas directas », en las vigas y planchas $n=15$; en las columnas, $n=10$, **MS**; con la resistencia de compresión máxima = 140 kg/cm cuad, $n=18$, **Un.**

Concreto de granzón ó piedra, $n=12$; escoria, $n=15$, **Ph**; ceniza, $n=30$, **Ph, Ch.**

**108. Estiramiento**, % mínimo, en 20 cm, 25, **F, Lv, NO, Co, a**; 22, **Co, b, Ci, a**; 20, **Un, Mh**; blando, 25; mediano, 22, **CS** $\frac{98{,}000}{\text{resistencia á la tensión}}$, **T y T.**

**109. Prueba de flexión.** Frío, **F, Lv, Bu, CS**; caliente, frío, ó apagado, **NO, Co, a**; 180° alrededor de un diámetro = al espesor de la barra, **F, NO, Bu, Co, CS**; (antes de deformarlos, **F**); como un diámetro = al doble del grueso de la barra, **Lv**; (después de deformarla, **F**); acero blando, chato, **CS**; frío, 90° sobre un diámetro = dos veces el espesor de la barra en acero > 19 mm de diámetro; más de un diámetro = 3 × el espesor de la barra en acero > 19 mm de diámetro, **Ch.**

**Presiones máximas permitidas en el acero.**

Las presiones en kg/cm cuad á menos que se indique de otra manera.

**110. Tensión**, 1,120, **Mh, Ph, JC**; (hierro 840, **Ph**); un tercio del límite de elasticidad, pero no más de 1,260, **Ch**; blando, 840; mediano, 1,050; muy rico en carbón, 1,260, **L.**

**111. Esfuerzo cortante**, 700, **Mh**; 840, **Ch.**

**112. Compresión** = compresión en el concreto × $\frac{\text{coef de elasticidad del acero}}{\text{coef de elasticid del concreto}}$, **Ch.**

« En los arcos, los anillos de acero que están bajo una presión que no exceda de 1,260 kg/cm cuad, deben estar en capacidad de soportar todo el momento de flexión del arco sin la ayuda del concreto, y tener un área de rebordes ó proyecciones de < la 150.ª parte del área total del arco en la clave. La presión real cuando están incrustadas y obran en combinación con el concreto no debe exceder de 20 veces la presión permitida sobre el concreto. »

« En las planchas, vigas, vigas armadas, pisos y muros, sometidos á esfuerzos transversales, supondremos que el acero soporta toda la presión sin ayuda del concreto, y que tendrá un área suficiente que iguale á la resistencia de compresión del concreto de 6 meses compuesto de 1 parte de cemento Portland, 3 partes de arena y 6 partes de piedra quebrada. »

« En los muros ó postes sometidos á compresión solamente, no se hará concesión en la resistencia del acero incrustado que se usará solamente como una precaución contra las cuarteaduras debidas á la contracción ó cambios de temperatura.

**Para lista** de especificaciones para concreto, véanse págs. 1401-02.

« En los estanques, el acero incrustado bajo una presión que no exceda de 1,050 kg/cm cuad deberá estar en capacidad de soportar toda la presión del agua sin ayuda del concreto », **CS.**

El estiramiento en el servicio de no más de .2%. **Ch.**

**113. Adhesión entre el acero y el concreto.** Se supone $>$ que el esfuerzo cortante permitido en el concreto, **Mh, Ms**; $<$ que el esfuerzo cortante en el concreto, **Un**; en el concreto de granzón ó piedra, 3.5 kg/cm cuad; en el de escoria, 2.8; en el de ceniza, 1.05, **Ph.**

**114.** En el concreto de 1 : 2 : 4, máxima, kg/cm cuad :

| | |
|---|---|
| en las barras cuadradas ó redondas, lisas, acero de construcción. | 4.9 |
| acero muy rico en carbón..... | 3.5 |
| en barras chatas, lisas, razón de los lados $>$ 2 : 1..... | 3.5 |
| en barras torcidas, $<$ 1 torcida en 8 diámetros..... | 5.6 |

en barras de forma especial,

.25 × la adhesión máxima determinada por la prueba; máxima.... = 7 **Ch.**

**115.** « **Cuando se excede la presión permitida,** debe hacerse una concesión para transferir la resistencia del acero al concreto », **Un, Mh, Ph.**

**116. Largo y recubrimiento.**

Las barras longitudinales **de no menos de 9 m**, si es posible, **Lv.**

En las vigas, barras de **largo sencillo**, si es posible, **MO, Co, Ci.**

**Si están recubiertas :**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Lado de la barra en mm. | 6 | 10 | 12.5 | 15 | 19 | 22 | 25 | 28 | 32 | |
| **Recubrimiento** en cm. | 15 | 25 | 33 | 45 | 50 | 55 | 65 | 75 | 80 | **NO.** |
| | 15 | 23 | 30 | 37. | 45 | 50 | 55 | 60 | 67 | **Co.** |

Recubrimiento = $<$ 20 × diámetros de la barra $<$ 30 cm, **Ci.**

**Extremos**, no menos de 5 cm fuera de cualquier superficie, **Lv.**

Las barras se llevan hasta las orillas extremas de las superficies no acabadas.

Las barras se prolongan hasta 2.5 cm de las superficies acabadas, **Co.**

Las barras del piso se prolongan 10 cm más allá de la cara de la pared que soporta el piso.

Las barras de las vigas $<$ 20 cm más allá de la cara de la pared que soporta el piso, **NO, Ci.** Véase Espacio libre más abajo.

**117. Protección.** Si se interrumpe el trabajo, las barras, ya colocadas, deben protegerse con coleta ó papel alquitranado. Los extremos que vayan á estar descubiertos largo tiempo, deberán cubrirse con una mano gruesa de mezcla líquida de cemento puro, **F, Lv.**

## Permisos.

**118.** Planos y especificaciones, completamente detallados, de la composición de concreto, se deben presentar al inspector de edificios, **Ch, Un, Mh, Ph.**

La concesión del permiso no implica la aceptación de la construcción, **Ch.** Para las pruebas que se requieren, véanse págs. 1413-14.

**Espacio libre.** Véanse también §§ 116, 134, 144, 149.

**Distancia, t, entre el acero y la superficie del concreto.**

**119. En las columnas y vigas,** $t$ $<$ 4 cm, **Ch, Ms**; en las planchas $t$ $<$ 12 mm $<$ el diámetro de la barra, **Ch**; $t$ $<$ 2 cm, **Ms**; $t$ $<$ 1.5 × el diámetro de la barra, **JC.**

**Ejes de las barras** separados del exterior del concreto $<$ diámetro de barra, **CS.**

Edificios á prueba de fuego, véanse §§ 120-128.

**Distancia libre entre las barras** $<$ 1.5 × la dimensión máxima de sección de la barra, **Ch, JC. Distancia libre entre dos capas de barras,** $<$ 12 mm, **JC.**

**120. En los edificios á prueba de fuego** (§§ 120-128), la construcion de concreto armado no se debe aprobar « á menos que se hayan hecho pruebas satisfactorias con fuego y agua bajo la inspección de esta Oficina », **Mh.**

Puede aceptarse si se proyecta como lo prescribe el Código, con tal que :

(1) El agregado sea de « ladrillos quebrados muy bien cocidos, ó terracotta, escoria de hornos limpia libre de materia combustible, piedra limpia, quebrada ó escoria de hornos, ó granzón limpio, junto con arena silícica, limpia, si se requiere la arena para producir una mezcla densa y compacta »; **Un.** (Los otros Códigos que se citan especifican menos variedades permitidas de agregados.) El agregado que pase por mallas de 10 mm **Ch**; por anillo de 25 mm, y 25% de agregado $>$ la mitad del tamaño máximo, **Ph.**

(2) El espesor mínimo, $t$, del concreto que rodea las piezas de la armazón, será como sigue, en que $d$=el diámetro paralelo á $t$ :

**121.** Cuando $d > 6$ mm, $t=25$ mm; cuando $d > 6$ mm, $t=4\ d$. En todo caso $t > 10$ cm, $t <$ el espesor requerido para las construcciones más $a=25$ mm en las columnas y vigas laminadas, $a=20$ mm en las planchas ó baldosas para piso; « pero por esto no se entenderá que se aumenta el espesor total del concreto protector como se indica en la presente », **Un.**

**122. En las vigas laminadas y columnas,** $t=5$ cm; en las vigas, $t=4$ cm; en las planchas ó baldosas para pisos, $t=2.5$ cm, **JC.**

**123.** En las columnas monolíticas, los 4 cm exteriores se considerarán como cubierta protectora, y no se incluirá en la sección efectiva, **JC.**

**124. En las vigas armadas y laminadas,** en la parte inferior, $t=5$ cm; en los lados, $t=4$ cm. Barras de baldosas inferiores, $t=2.5$ cm. En las columnas, $=5$ cm, **Ch, Ph.**

**125.** « Si se coloca una estructura de metal suplementaria en el concreto que rodea la armazón, simplemente para sujetar el concreto, el espesor del concreto que está debajo de la armazón podrá reducirse en 12 mm, y no se considerará la estructura como metal de la armazón », **Ch.**

**126.** En las vigas de piso y de techo, $t=2.5$ cm; en las vigas laminadas de piso y de techo, y en las vigas que soportan mampostería, $t=2.5$ cm; en cualquiera otra parte, 5 cm; en las columnas que solamente soportan pisos, $t=7.5$ cm; en las columnas construídas dentro de los muros ó que los soporten, 10 cm, **Ms.**

**127. Concreto de ceniza,** para construcción á prueba de fuego, $t$ lo mismo que para el concreto de piedra; para la construcción de grandes talleres ó de edificios que han de arder con lentitud, en las columnas, $t=5$ cm; « en las vigas armadas, laminadas y otras piezas de construcción de acero ó de hierro », $t=4$ cm. La cubierta debe tener « ligazones de metal ó tela de alambre incrustada en y alrededor » de aquellas piezas; las ligazones, si son de alambre, no deben ser menores que el n.º 8, y á no menos de 40 cm de separación, **Ch.**

**128.** Los ángulos de las columnas, vigas armadas, vigas laminadas, deben biselarse ó redondearse, **JC.**

## Columnas.

**129.** A las columnas se les debe dar $<$ de 2 horas para asentarse y contraerse antes de colocar las vigas laminadas sobre ellas, **JC.**

**130.** « Las **reglas** para calcular las columnas de concreto armado pueden **formularse de vez en cuando** por el inspector de edificios con la aprobación de la Junta de apelación », **Ms.**

**131. Se supone que el concreto y el acero se contraen** « en la misma proporción », **Ms.**

**132. El concreto y el acero se comprimieron** en la relación, $n$, de sus coef de elasticidad, **JC.**

**133.** Las barras se sujetan á intervalos suficientemente cortos para impedir que se doblen, **Ms.** Véase § 136.

**134.** Los 4 cm exteriores deben considerarse como cubierta protectora y no incluirse en la sección efectiva, **JC.**

### Columnas armadas.

L=largo; $d$=diámetro ó lado menor.

**135.** El concreto armado puede usarse en las columnas cuando L $>$ 12 $d$, **Ch, Un, Mh**; $>$ 15 $d$, **JC**; y cuando el área de la sección transversal $<$ 413 cm cuad, **Ch.** Si L $>$ 15 $d$, se deberá disminuir en proporción la presión permitida **Ph.**

**136. Requisitos. Las barras deben ligarse á intervalos** no más grandes que $d$, **Un, Mh, Ph**; no más grandes que 12 $d$, no más de 45 cm, **Ch.** Véase § 133.

**Para lista** de especificaciones para concreto, véanse págs. 1401-02.

**137.** Las barras longitudinales no se deben considerar como recibiendo **compresión directa, Ph.**

**138. El área de la sección transversal** combinada de las barras de compresión $>$ 3% del área de la sección transversal de la columna, **Ch.**

**139.** Cuando no se requieren barras de compresión, el área de la sección transvesal de las barras deberá ser $<$ .5% del área de sección transversal de la columna; no menos de 6 cm cuad, **Ch.**

**140.** La dimensión mínima de la barra menor no debe ser menos de 12 mm, **Ch.**

**141.** Las barras deben prolongarse en la columna por arriba ó por abajo, recubriendo las barras ahí situadas lo suficiente para desarrollar la resistencia en la barra por la unidad de adhesión permitida, **Ch.**

**142. Carga excéntrica ó transversal.** Esfuerzo máximo de las fibras, incluyendo (1) la compresión directa, (2) la flexión debida á la compresión directa, (3) la excentricidad, y (4) la carga transversal que no sea mayor que la resistencia permitida á la compresión. La carga excéntrica se « considerará como afectando excéntricamente tan sólo el largo de la columna que se prolonga hasta el punto inmediato inferior en el cual la columna esté fuertemente sujeta en la dirección de la excentricidad », **Ms.**

**143. Una columna** monolítica con una viga armada ó laminada **rígidamente unida á ella**, debe resistir, además de las cargas directas, un momento=al momento no equilibrado en la viga sobre la columna, **Ch.**

**144. Columnas con sunchos.** El concreto puede someterse á esfuerzos hasta el 25% de la resistencia máxima con tal que :

(1) El área de sección transversal de la armazón vertical $<$ el área de la armazón espiral $>$ 5% del área entre los sunchos;
(2) Percentaje de los sunchos en espiral $<$ .5, $>$ 1.5;
(3) Separación de los sunchos en espiral uniforme y $>$ .1 × diámetro de la columna, $>$ 7.5 cm;
(4) Los espirales sujetos de tal manera á las verticales, en cada intersección, que conserven sus formas y posición;
(5) La distancia entre las verticales $>$ 22 cm, $>$ $\frac{1}{8}$ de la circunferencia de la columna entre los aros.

Se puede « suponer que los sunchos aumentan la resistencia del concreto en un equivalente de 2.5 × la cantidad de sunchos espirales que se consideren como refuerzo vertical ». El concreto, fuera de los sunchos, no se considera como parte de la sección efectiva de la columna, **Ch.**

**145.** « Los esfuerzos que obran serán asunto de consideración especial para el comisionado ó inspector de edificios », **Un.**

**146. Unidad de compresión** permitida=70 kg /cm cuad del área entre los aros, **Ph.**

**147.** El percentaje de barras longitudinales y la distancia entre los aros deben ser tales que el concreto pueda desarrollar este esfuerzo con un factor de seguridad de 4, **Ph.**

**148.** « No debe considerarse que los sunchos ó bandas aumenten directamente la resistencia de la columna », **JC.**

**149.** Espacio libre entre las bandas y aros $>$ .25 × diámetro de la columna incluída, **JC.**

**150. Columnas armadas con acero de construcción.** El concreto puede someterse á $\frac{1}{4}$ de la resistencia máxima, con tal que: (1) el área de sección transversal del acero no sea menos de 6 cm cuad; (2) los espacios del enrejado ó listones no deben ser mayores que el ancho mínimo de la columna, **Ch.**

## Vigas y pisos.

**151.** La teoría común de las vigas es aplicable, **Un, Ch, Mh, Ph.**

**152.** Se supone que el acero soporta todas las tensiones directas, **L, Un, Ch, Ms, Mh, Ph.** Las tensiones en el concreto deben tomarse en cuenta al calcular las deflexiones, **JC.**

**153. La curva del esfuerzo de estiramiento** del concreto en compresión se supone que sea una línea recta, **Ch, Ph,** $n = E_s/E_c = 15$; para las deflexiones, $n = 8$ á 12, **JC.**

**154.** Con 140 kg/cm cuad de esfuerzo máximo en fibras, esta curva puede tomarse como: (*a*) una línea recta; (*b*) una parábola, con eje vertical y el vértice en el eje neutro de la viga; ó bien (*c*) una curva empírica, que incluye un área ¼ mayor que si fuera una línea recta, y con el centro de gravedad á la misma altura que el área en (*b*), **Un.**

**155. Esfuerzos.** Una carga=4×la carga total de trabajo, comprime el acero hasta su límite elástico, y el concreto á 140 kg/cm cuad, **Un.** Proyecto « fundado en la suposición de una carga cuádruple que la carga total », **Ph.** (La carga total= carga muerta ordinaria más la carga viva ordinaria, **Un, Ph.**)

**156. La adhesión**, entre el concreto y el acero, se supone que es suficiente para hacerlos obrar conjuntamente, **Un, Ch, Mh, Ph.**

**157. El metal descubierto** no se considera al calcular la resistencia, **Un, Ch, Ph.**

**158. La luz**=distancia de centro á centro de las planchas de cimiento ú otros soportes, **Ms, JC.** Si la viga está sujeta al lado de una columna, se mide la luz hasta el centro de la columna, **Ms.** La luz > (la luz neta+la altura de la viga ó plancha), **JC.**

**159. Los esfuerzos térmicos y la contracción** se deben prever con la introducción del acero, **Ch, Ph.** « Los esfuerzos iniciales en la armazón, debidos á la contracción ó expansión del concreto, pueden despreciarse », **JC.**

**160.** Cuando el **esfuerzo cortante** que se desarrolle exceda el límite concedido al concreto, se debe introducir el acero para que reciba el exceso, **Un, Mh, Ph, JC.**

**161.** Valores permitidos para las resistencias al esfuerzo cortante: kg/cm cuad.

| | |
|---|---|
| (*a*) Con barras horizontales solamente | 2.8 |
| (*b*) Con parte de la armazón horizontal en forma de barras encorvadas, « dispuestas debidamente con respecto á las resistencias al esfuerzo cortante » | > 4.20 |
| (*c*) Con armazón completa para el esfuerzo cortante | > 8.40 |

**JC.**

En (*c*) se puede suponer que el concreto recibe ⅓ del esfuerzo cortante, los ⅔ restantes los recibirán las barras encorvadas ó los estribos (con preferencia ambos) que soportan su parte en una distancia horizontal=el espesor de la viga, **JC.**

**162.** Los intervalos longitudinales de los estribos ó barras encorvadas > .75×el espesor de la viga, **JC.**

**163. La pega del acabado de cemento,** añadida en la parte superior de las planchas, vigas armadas y laminadas, **no se debe incluir** al calcular la resistencia « á menos que se coloque íntegramente con la masa del concreto », y sin calcularle mayor unidad de resistencia que la del concreto, **Ch.**

**164. Armazón de tejido.** « Cuando el esfuerzo cortante vertical, medido en la sección de una viga sencilla ó laminada, entre los centros de acción de los esfuerzos horizontales, es > .02×la presión máxima directa por cm cuad, se empleará la armazón suficiente de tejido para que reciba el exceso. La armazón de tejido deberá extenderse desde la parte superior hasta la parte inferior de la viga y arrollarse ó unirse á la armazón horizontal. La armazón horizontal, que recibe las presiones directas, no se considerará como armazón de tejido », **Ch.**

**165. El acero en los lados de compresión de las vigas sencillas y laminadas.** « Cuando se use el acero en el lado de compresión de las vigas sencillas y laminadas, se deberán ligar las barras de acuerdo con los requisitos de las columnas verticales armadas, con estribos que las liguen á las barras de tensión de las vigas », **Ch.**

**166.** Cuando el acero, ó el hierro, están del lado de la compresión de la viga, la porporción del esfuerzo recibida por el acero, ó el hierro estará en la relación que existe entre el módulo ó coef de elasticidad del acero, ó del hierro, y el módulo de elasticidad del concreto, con tal que las barras estén bien ligadas á los estribos que las conectan con las barras inferiores de las vigas », **Ph.**

**167.** Cuando se empleen planchas con vigas sencillas y laminadas, se tratarán éstas como si fuesen vigas en T, obrando entonces una parte de la plancha como un reborde, **G.**

**Para lista** de especificaciones para concreto, véanse págs. 1401-02.

**168. Parte, F, del ancho de la plancha ó losa, que obra como reborde,**

$t$=espesor de la losa; L=luz de la viga, armada ó laminada;
$b$=ancho de la viga; S=distancia de centro á centro entre las vigas.

F se « determinará suponiendo que en cualquier sección plana horizontal del reborde, los esfuerzos están distribuídos como las ordenadas de una parábola, con el vértice en la curva del esfuerzo de alargamiento y su eje en un plano vertical longitudinal por el centro de la parte vertical de la T ». Dicha parte debe reforzarse con barras cerca de la parte superior, en ángulo recto con la viga. **Un.**

**169.** F depende de la resistencia al esfuerzo cortante, $F > 20\,t$, **Ph**; $F > 10\,b$, **Mh.**

**170.** F regida por la resistencia al esfuerzo cortante entre la plancha y la parte vertical; $F > S\left(1-\frac{S^2}{L^2}\right) > L/3$, $> \frac{3}{4}$ S. Para suponer que obra de aquella manera, la plancha debe vaciarse al mismo tiempo que la parte vertical, **Ch.**

$$F > L/3, > S, \textbf{Ms}; > L/4 > 8\,t + b, \textbf{JC}.$$

**171.** Las vigas en T deben armarse contra el esfuerzo cortante á lo largo del plano de unión entre la parte vertical y el reborde, **Un, Ph**; usando estribos en todo el largo de la viga, **Ph.**

**172.** Las partes verticales de las vigas deben ser *monolíticas* con las losas del piso, **Un, Ph.**

**173.** « Cuando las vigas laminadas de concreto armado soportan vigas sólidas de cóncreto armado, la parte superior de las losas, ó planchas, del piso, que obran como reborde de la viga, debe ir reforzadas con barras, cerca de su superficie superior, colocadas en ángulo recto á la viga, para que aquella parte de la plancha pueda transmitir las cargas locales directamente á la viga laminada y no á las vigas sostenidas por éstas, para impedir que se sumen los esfuerzos de compresión producidos por la acción simultánea de la plancha del piso y del reborde propio de la viga. » **Un, Ph.**

**Momento, M.** Véase también §§ 178, 179.

**174.** W=carga por m cuadrado; L=luz, en m. En las planchas soportadas libremente, L=abertura libre+espesor; en las planchas continuas, L=distancia entre los centros de los soportes.

**175.** Con cargas concentradas, ó especiales, calcúlese y provéase para los momentos y esfuerzos cortantes en la posición peligrosa de la carga, **Ch.**

Para la carga muerta; M obtenida de la carga muerta efectiva.....
Para la carga viva, sobre los soportes; M obtenida de la viva efectiva..... } cubriendo todas las luces al mismo tiempo.

Para la carga viva, entre los soportes, M = máx obtenido de la carga viva cubriendo 2 luces consecutivas ó 2 alternadas al mismo tiempo.

Cuando todas las luces son iguales, sea $M_1$=mínimo del momento de la carga viva en el centro de la luz. Entonces,

para las luces intermedias, $M_1 = \frac{WL^2}{12}$

para las luces extremas, $M_1 = \frac{WL^2}{10}$

La suma de los momentos de carga viva en un soporte en el centro de la luz, $< \frac{WL^2}{6}$, **Ch.**

**Continuidad.** Véase también § 175.

**176. Las vigas sencillas y laminadas,** se supone simplemente apoyadas en los extremos; no se hace concesión por la continuidad, **Un, Mh.**

**177.** Las vigas, etc., se calculan como si estuviesen soportadas simplemente, ó como continuas, según los casos, **Ch, Ms.**

**178. Las planchas de piso continuas,** armadas en la parte superior sobre los apoyos, pueden considerarse como vigas continuas. Con cargas uniformemente distribuídas, el momento, M, se calculará en no menos de .1 WL; .05 WL para las planchas cuadradas de piso armadas en ambas direcciones y apoyadas en todos sus lados, **Un, Mh, Ph.**

**179. En las losas ó planchas del piso adyacente á los muros**; si la losa está armada en una dirección, $M=\frac{WL}{8}$; si es cuadrada y está armada en ambas direcciones, $M=\frac{WL}{16}$; **Ph.**

**180. Las losas** para pisos calculadas y armadas como continuas sobre los soportes. Si el largo de la losa, $>$ 1.5 × su ancho, la armazón transversal soportará la carga entera. « Las losas ó planchas cuadradas pueden muy bien armarse en ambas direcciones », **J. C.**

**181.** En las vigas y planchas continuas $>$ 2 luces, los momentos de flexión en el centro y en los apoyos, para ambas cargas, viva y muerta, son como sigue :

En las losas á planchas para piso y las luces interiores de vigas continuas..... $M=wL^2/12$
En las luces extremas de las vigas continuas..... $M=wL^2/10$

$w$=carga por unidad de luz; L=luz, **JC.**

**182.** En las luces continuas, provéase, en los soportes, para el momento negativo=.8 del momento positivo en el centro de una luz simplemente apoyada. El momento positivo en el centro de la luz contínua, puede calcularse=al momento negativo en el apoyo, **Ms.**

### Pruebas.

**183.** El comisionado ó inspector de edificios puede exigir las pruebas de los materiales antes ó después de ser empleados en los edificios, **Ms.** El contratista debe estar preparado para hacer pruebas de carga en cualquier parte del edificio dentro de un tiempo racional después de la erección y con tanta frecuencia como lo requiera el ingeniero, **Ch, Ph, Mh, Un.** Las pruebas deben demostrar que la construcción resistirá cargas como sigue :

carga=2 × la suma de las cargas viva y muerta presupuestas, **Ch;**
carga=2 × la carga viva presupuesta, **Ph;**
carga=3 × la carga presupuesta, **Mh.**

**184.** La construcción puede considerarse como una parte de la prueba de carga, **Ch.**

**185.** Cada prueba de carga deberá cubrir 2 ó más tram s, permaneciendo en un lugar no menos de 24 horas, **Ch.**

**186.** La deflección de las planchas de no más de $\frac{\text{luz}}{800}$.

La deflección de las vigas armadas $>\frac{\text{la luz}}{800}\times$ la relación entre el espesor de la losa ó plancha y el espesor de la viga, **Ch.**

**187.** La prueba á 45 días después de terminar.

Carga=1.5 × la carga viva + 1.5 × la carga muerta del área terminada.
Deflección $>$ .001 × el largo de la pieza, **Ci, b.**

## ACERAS DE CONCRETO

### Resumen de especificaciones.

Adoptado por

### La Asociación Nacional de los consumidores de cemento.

Filadelfia, enero, 1908.

**1. Cemento** Portland que llene las condiciones de la A. S. T. M., adoptada en enero, 1906. Véase pág. 1278.

**2. Arena.** Que pase el cernidor n.º 4. Puede contener $>$ 5% de marga y arcilla, éstas no envuelven los granos de arena.

$<$ 60% de la arena que pase por el cedazo n.º 10, ó
35% que pase los cedazos n.ºs 10, 20, 3[illegible], 40,
y el resto se quede en los n.ºs 20, 30, 40, 50.
$>$ 20% de la arena que pase el cedazo n.º 50, ó
70% que passe los n.ºs 10, 20,
y el resto se quede en los n.ºs 40, 50 respectivamente.

**3. Cerniduras** de piedra triturada, como se indica más abajo, y que llenen los requisitos para la arena, se pueden sustituir en lugar de la arena.

**4. Agregado. Piedra triturada**, de roca limpia, buena, dura, cernida seca por mallas de 19 mm, que la retenga la malla de 6 mm.

**5. Granzón**, limpio, duro que varíe entre el retenido por la malla de 6 mm y el que pasa por la de 19 mm.

**6. Granzón sin cernir**, limpio, duro. Que no tenga partículas mayores de 6 mm. La proporción de las partículas finas y gruesas debe obedecer á los requisitos que damos más abajo para el concreto.

**7. Agua**, « bastante limpia, libre de aceite, ácido sulfúrico y álcalis fuertes »

## Sub-base.

**8.** La subbase debe **apisonarse muy bien**, quitando las **partes blandas** y reemplazándolas con material duro.

**9. Rellenos** $>$ 30 cm de espesor, que debe compactarse completamente apisonando las capas de $>$ 15 cm de espesor, y « deben tener una inclinación de $<$ 1 : 1.5. » « La parte superior de todos los rellenos debe prolongarse $<$ 30 cm más allá de la acera. »

**10.** « Mientras se compacta la subbase debe **mojársela completamente** manteniéndola así hasta depositar el concreto. »

## Base.

**11. Vacíos.** El cemento debe rellenar los vacíos de la arena en $<$ 5%.

**12.** El mortero debe rellenar los vacíos del agregado en $<$ 10%. Proporciones 1 : $>$ 3 arena y agregado.

**13.** Cuando los vacíos no están determinados, 1 : 3 arena ó cerniduras : 5 piedra ó granzón. « Un saco de cemento de 42.5 kg se supone que ocupa un volumen de 28 litros. »

## Mezcla.

**14. Á mano.** La arena se extiende pareja sobre una plataforma horizontal impermeable, el cemento se esparce sobre la arena. Mézclase seco hasta que tenga un color uniforme. Se riega el agua y revuelve la masa hasta que tenga una consistencia homogénea y uniforme. Se añade el agregado empapapado y se mezcla todo hasta que el agregado esté completamente cubierto de mortero.

**15. Á mano. Con granzón sin cernir.** El cemento y el granzón se « mezclan secos hasta que no se vean vetas de cemento ». Se le riega el agua y se mezcla. El mortero debe ser igual al que se ha especificado más arriba.

**16.** El agua puede añadirse mientras se está mezclando, pero el concreto debe revolverse $<$ una vez inmediatamente después.

**17. La mezcla por máquina** puede aceptarse cuando se obtiene por ella un concreto equivalente al que se especifica más arriba.

**18. Se prohibe la retemplada.**

## Declive

**19. El declive de la acera** $<$ suficiente para el desague $>$ $^1/_{50}$, « excepto donde esta inclinación sea paralela al largo de la acera ».

## Armaduras.

**20. Madera de construcción**, limpia, libre de torceduras, $<$ 4 cm de espesor.

**21. Los bordes superiores deben conformarse** al declive definitivo de la acera.

**22. Armaduras transversales.** « En cada división de bloque se pondrán armaduras transversales en todo el ancho de la acera y en ángulo recto con el lado de la armadura », excepto como se dice en el § 23.

**23. Juntas de dilatación.** Una tira de metal, de separación de 12 mm, que reemplace la armadura transversal $<$ una vez en cada 15 m. « Cuando la acera esté bastante dura, se quitará esta tira de división llenando la junta con material adecuado antes de abrir la acera al tráfico. Se drpondrán juntas de esta especie cuando la nueva acera se empata con aceras de piedra artificial, á curvas ú otras. »

**24. « Todas las armaduras deben mojarse completamente** antes de depositar ningún material contra ellas. »

**25. Dimensiones de los bloques.**

| Tamaños en centímetros..... | 15×15 | 12·5×12.5 | 11×11 | 10×10 | 7.5×7.5 |
|---|---|---|---|---|---|
| Espesor (centims) : | | | | | |
| En los barrios de tráfico..... | 15 | 14 | 12.5 | 10 | ... |
| En los barrios de habitación.... | 15 | 12.5 | ... | 10 | 7.5 |

En las aceras de los barrios de habitaciones los bordes pueden ser 25% más delgados que el centro, mínimum = 7.5 cm.

**26. El hierro de separación** > 15 cm de ancho, 6 mm de espesor. La ranura llega hasta la subbase; la ranura se llena de arena seca antes de extender la capa superior; la capa superior se corta hasta la arena después de emparejar y alisar con la cuchara.

## Colocación.

**27.** El concreto se llevará á las armaduras en carretillas impermeables. El concreto no debe rebosarlas. Las carretillas no deben pasar sobre el concreto colocado recientemente.

**28.** El concreto debe depositarse dentro de una hora después de la mezcla, extendiéndolo parejo, y apisonándolo hasta que el agua fluya á la superficie.

## Protección.

**29.** Los trabajadores no deben caminar sobre el concreto acabado de colocar.

**30.** La arena ó polvo que se reuna en la base, se debe « quitar con cuidado antes de aplicar la superficie que va á sufrir el tráfico ».

## Superficie de tráfico.

**31.** Espesor mínimo, 2 cm.

**32.** « Mortero, 1 : 2 de arena ó cerniduras, mezcladas como para la base, pero bastante mojadas para que no haya que apisonarlas para que puedan aplanarse con barras de filo recto. « Se echará una capa de mortero delgada y se alisará sobre la base, antes de extender la superficie de tráfico. »

El mortero se debe extender en la base dentro de 30 minutos después de mezclado, alisándolo dentro de 50 minutos.

**33. Marcas.** « Después de haber casi terminado la superficie, se harán las marcas de los bloques directamente encima de las juntas de la base, con un hierro que corte hasta la base y separe completamente las superficies de tráfico de los bloques adyacentes. »

**34. Orillas de la superficie,** se redondean con un radio < 6 mm.

**35.** « Cuando estén parcialmente fraguadas, las superficies se **alisarán** con **la cuchara.** »

**36.** En los declives > 5% se **hace áspera** la superficie con instrumento adecuado « ó incrustando arena gruesa en la superficie ».

**37.** Se emplearán únicamente **colores minerales,** y éstos se **incorporarán** al mortero de toda la superficie de tráfico.

## Capa de una sola pega.

**38. Proporciones,** 1 : 2 de arena : 4 granzón ó piedra triturada. **Los bloques se separan** como en el trabajo de doble capa. El concreto debe **compactarse** firmemente **apisonándolo, encerrándolo y alisándolo** dentro de los bordes de la armadura. « Después, con un hierro adecuado, se comprimirán las partículas más gruesas del concreto á la profundidad necesaria de modo que quede lisa como en el trabajo de doble capa. »

## Protección.

**39** Después de terminada se conservará la acera **húmeda y protegida** contra el tráfico y los elementos durante tres días, por lo menos. Las armaduras se quitarán con gran cuidado, y al quitarlas se amontonará **tierra contra la orilla de la acera.** »

## Declive adyacente á la acera.

**40.** En la orilla de la acera 4 cm, por debajo de la acera se hará un declive < $^1/_{50}$. En el lado de las habitaciones « se le dará al suelo un declive hacia atrás < 60 cm pero no más bajo que la acera.

## BLOQUES DE CONCRETO

**1. Puerto de Búfalo.** Bloques de 1.80 m de largo, como 1.2 m en cuadro, como 2.5 m cúb, hechos en moldes de madera. ½ barril de Portland, 70 lit de arena, 212 lit de guijarros, 212 lit de piedra quebrada, para hacer una capa de concreto, en molde, como de 15 cm de espesor. Las caras de 15 cm de espesor de los bloques del tajamar que da al lago, se hicieron del mejor material. El frente se colocó primero; el respaldo antes de fraguar éste. (Emile Low, A. S. C. E., Trans., junio '04, vol. LII, pág. 96.)

**2. Tajamar de Zeebrugge,** Bélgica. Bloques de 25 metros de largo. 9 metros de ancho, 8.75 metros de alto, como 2,000 metros cúbicos, 4,500 toneladas cada uno. La capa exterior de concreto, con un corte inferior, de tres compartimentos, se hicieron en una armadura de hierro y se llevaron en balsas á su lugar, se colocaron entre las guías hundiendo últimamente el bloque; se undió al entrarle el agua, y se rellenó con concreto, 1 de cemento : 2.5 de arena : 6.1 de pórfiro quebrado, por medio de cestones de 10 metros cúbicos. La medida superior, rica en cemento, se colocó sobre el agua en la marea baja. La punta que de hacia el mar se protegió inmediatamente con ripios de piedra.

La superstructura de bloques de 55 toneladas colocados sobre el nivel del agua, y sobre ellos bloques de concreto formados en el lugar.

**3. Moldes para bloques monolíticos bajo el agua, aislados, de concreto,** de 110 á 166 m cúbicos, que forman la pila de la sección transversal de forma trapezoidal. Los moldes son cajas sin fondo, de sección transversal compuesta de dos lados y dos piezas extremas, sujetas por traviesas de torniquete de 38 mm que obran como vigas del lado afuera del molde. Las traviesas tienen, en cada extremo, anillos en que se introducen tornillos-cuñas al tiempo de la erección. Para quitar los moldes se sacan los tornillos destornillando una tuerca en traviesas que forman parte integral de los tornillos-cuñas. Esto, suelta el tornillo de los anillos de las traviesas y deja libre las paredes del molde, que las recoge el mecánico, juntándolas para colocarlas otra vez. Peso del molde, 40 toneladas. Tiempo que se necesita para quitar el molde del bloque y rearmarlo para volverlo á erigir, de 45 á 60 minutos. La flotación de la madera se vence con lastres de hierro. Se colocaron primero bloques alternados. En los bloques intermedios se usaron solamente las piezas de los lados de un molde. Éstas se sujetan en su lugar con su inclinación requerida, por medio de seis travielias de torniquete, que pasa por una caja hueca de planchas de 2.5 cm, y que obra como puntal. (Muelle del Sur, en Superior Entry, Wisconsin, Informe de Clarenee Coleman, ingeniero asistente, Informe del ingeniero en jefe, U. S. A., 1904, parte IV, p. 3781.)

**4. Las clavijas de grapas** deberán ponerse en los bloques donde sea posible, y de tal manera que « no causen exceso de presión en el concreto, especialmente en la cara del concreto ó cerca de los ángulos del bloque ». Las clavijas y los ganchos se pueden poner en los bloques frescos. Usense bloques de madera y cojines de trapos para voltear los bloques, de otra manera se dañarán las esquinas.

**5. Posición para el moldeo.** Los bloques deben moldearse con la cara más importante hacia abajo, estando las caras visibles tan verticales como sea posible, y la parte de atrás del bloque hacia arriba, de manera que la lechosidad, etc., que salga aparezca en la superficie.

## BLOQUES HUECOS DE CONCRETO PARA CONSTRUCCIONES

### Resumen de las especificaciones.

Adoptado por

**La Asociación Nacional de los consumidores de cemento,**

Filadelfia, enero, 1908.

**1. Cemento** Portland que reuna las condiciones de la A. S. T. M., adoptadas enero, 1906. Véase pág. 1278.

**2. Arena,** silícica, limpia, granosa, que pase el cedazo de 6 mm.

**3. Agregado,** piedra quebrada limpia, libre de polvo, ó granzón limpio cernido. que pase el cedazo de malla de 20 mm, retenido en el de 6 mm.

**4. Unidad de medida** para el cemento. Barril = 172 kg netos, el litro = 1.60 kg. El cemento se mide en el embalaje original ó se pesa, no se mide por volumen suelto.

**5. Proporciones.** Para las paredes exteriores descubiertas ó muros de apoyo :
(*a*) Hechos en máquina. Medio húmedo, 1 : > 3 de arena : > 4 de agregado.
(*b*) Concreto aguado (trepidando ó fluyente) hecho en moldes separados dejándolo endurecerse en ellos, 1 : > 3 de arena : > 5 de agregado.
Si se omite la piedra, se podrá aumentar la proporción de arena si las pruebas no indican aumento en los vacíos ó en la absorción ó ninguna pérdida de la resistencia.

**6. Agua suficiente** para completar la cristalización del cemento.

**7. Mezcla.** « La mezcla estricta y completa es de la mayor importancia. »
(*a*) A mano. El cemento y la arena se mezclan secos. El agua se añade despacio y se revuelve. El agregado húmedo se extiende sobre la mezcla, ó la mezcla sobre el agregado y se mezcla.
(*b*) Se prefiere con máquina. El cemento y la arena, ó cemento, arena y agregado, se mezclan secos. Se añade el agua y se revuelve para unirlos. En el concreto mojado, « se puede variar este procedimiento con el consentimiento de la oficina, etc. ».

**8. Moldes.** La parte superior de los bloques apisonados, y después de nives lados, se « alisará con la cuchara ó de otra manera para obtener densidad y ángulos rectos y agudos ».

**9. Protección.** Después de moldeados, los bloques se « protegerán cuidadosamente de las corrientes de aire, el sol, el calor seco ó la helada durante 5 días por lo menos », dándoles mayor humedad durante ese tiempo « y después de cuando en cuando hasta que estén listos para el uso ».

**10. Edad mínima** antes de usarlos. 1 : 3 de arena, 3 semanas; 1 : 2 de arena, 2 semanas, « con consentimiento especial de la oficina, etc. »; bloques especiales para rematar el trabajo, 7 días « con consentimiento especial de la oficina, etc. ».

**11. Marcas.** Todos los bloques deben marcarse con el nombre ó la marca del fabricante, día, mes y año de la hechura y las proporciones, « 1 : 2 : 3 », etc.

**12. Mortero.** « En todos los muros en que se empleen los bloques, se pegarán éstos con mortero de cemento Portland.

**13. La carga máxima,** incluyendo el peso de la pared, 88 toneladas por metro cuadrado del área de los bloques.

**14. Espesor de los muros.** Los muros de apoyo « pueden tener 10% menos de lo que exige la ley para muros de ladrillo ». En las paredes de división ó tabiques lo mismo que en las baldosas huecas ó de terracotta ó de bloques de yeso.

**15. Disminución de espesor.** « Cuando las paredes disminuyen en espesor, la última hilada del muro más grueso deberá presentar un apoyo suficiente y sólido para las armazones ó muros de la serie de bloques de arriba. »

**16. Debajo de las vigas,** los bloques deben hacese sólidos en < 20 cm de la cara interior. Si la carga concentrada, W, en el bloque, es > 2 toneladas, ésta se aplica á los bloques que soportan la viga, etc.; si W, > 5 toneladas, se aplica á los bloques de < 3 hiladas de abajo, y á una distancia de < 45 cm á cada lado de la viga, etc.

**17. En los muros de división,** se harán los bloques sólidos.

**18. Sujeción.** « Cuando los muros se hacen enteramente de bloques de concreto, y que dichos bloques no tengan el mismo ancho que el muro, en cada 5.ª hilada se prolongará por todo el ancho del muro, formando una sujeción segura, cuando no estén suficientemente sujetos de otra manera. »

**19. Las caras de los bloques,** en el respaldo de ladrillo, « deben estar sujetas fuertemente al ladrillo con dientes que se introduzcan 10 cm en la obra de ladrillo, considerando cada cuarta hilera como muro de cabeza; ningún respaldo de ladrillos deberá ser de menos de 20 cm.

**20. Espesor del alma ó armazón** del bloque (en los muros de apoyo) < .25 × por la altura del bloque.

**21. Espacio hueco.** En los muros de apoyo el percentaje mínimo de espacio hueco :

| Edificios de | 1.º | 2.º | 3.º | 4.º | 5.º | 6.º pisos. |
|---|---|---|---|---|---|---|
| 1 y 2 pisos..... | 33 | 33 | | | | |
| 3 y 4 — ..... | 25 | 33 | 33 | 33 | | |
| 5 y 6 — ..... | 20 | 25 | 25 | 33 | 33 | 33 |

**22. Los quicios y dinteles** deben « reforzarse con barras de hierro ó acero de un modo que satisfaga á la oficina, etc. ». Cuando la luz sea > 1.35 m, el quicio « deberá descansar sobre un bloque sólido en < 20 cm de distancia de la

cara próxima á la abertura y en $\leq$ 3 hileras por debajo de la parte inferior de dintel ».

**23.** Antes de usar los materiales se **hará una solicitud** á la oficina ó jefe del departamento del ramo, dando « una descripción de ellos y un bosquejo breve de su manufactura y las proporciones que se van á usar », con « el nombre de la firma ó corporación, y los empleados responsables de ellos », dando inmediata cuenta de los cambios que se tengan que hacer.

**24. El certificado de aprobación** permanecerá en vigor $\geq$ 4 meses, « á menos que se deposite en la Oficina de inspección de edificios, lo menos una vez cada cuatro meses, un certificado de un laboratorio de pruebas físicas, competente, que indique que el término medio de $\leq$ 3 pruebas de compresión y $\leq$ 3 pruebas transversales llenan los requisitos; « dichas muestras se elegirán (por un inspector de edificios ó por el laboratorio) de los bloques que se vayan á colocar en la construcción ».

**25. Prueba preliminar.** El fabricante someterá el producto á las pruebas requeridas, depositando un certificado de un laboratorio competente de pruebas, que dé los detalles de las pruebas que se hayan hecho. Los resultados de todas las pruebas, satisfactorios ó no, se depositarán en la oficina, expuestos á la inspección, pero no de obligatoria publicación.

**26. Pruebas adicionales.** El fabricante, ó el que los usa, ó ambos, « harán en cualquier tiempo tales pruebas de los cementos que se hayan empleado para hacer estos bloques, ó las mismas pruebas posteriores con los bloques terminados, ó con cada uno de ellos, por su propia cuenta, y bajo la inspección de la oficina de inspección de edificios, como el jefe de dicha oficina lo exija ».

El no resistir estas pruebas, implica la revocación inmediata del certificado concedido al fabricante.

**27. Requisitos para las pruebas.** Los bloques deben someterse á pruebas transversales, de compresión y de absorción, « y podrán someterse á pruebas de congelación y de fuego ». Las pruebas de congelación y de fuego no serán por cuenta del fabricante.

**28. Las pruebas de aprobación** se harán por cuenta del solicitante.

**29.** No se elegirán **menos de 12 muestras** por la oficina, etc.

**30.** « Las muestras deben representar el **producto comercial corriente,** del tamaño regular y la forma que se usen en la construcción. Las muestras se probarán tan pronto como lo desee el solicitante », pero $\geq$ 60 días después de fabricadas.

**31. Los bloques que fallen en las pruebas,** se marcarán « condenados » por el fabricante ó el que los usa, y se destruirán.

**32.** « **Las pruebas se harán en series** de 3 por lo menos, exceptuando las pruebas de fuego en que series de 2 (4 muestras) son suficientes. »

**33.** « **Las medias muestras** se podrán usar para las pruebas de fuego, congelación y trituración. Las muestras restantes se conservarán en reserva para el caso en que se requieran pruebas duplicadas ó confirmatorias. »

**34.** « **Todas las muestras deben marcarse** para su identificación y comparación. »

**35. Pruebas transversales.** La muestra (de tamaño completo) se colocará de plano en soportes de filos redondeados paralelos, á 18 cm de separación. Se aplica la carga, en la mitad de la distancia, entre los soportes, por entre los filos redondeados.

Coeficiente de ruptura $= 3WL / 2d^2$; en que W=la carga en kg; L=la luz en dcm; $b$=ancho del bloque en cm; $d$=espesor del bloque, en cm. « No se deberá hacer concesión... por los espacios huecos. » A los 28 días el módulo de ruptura será por término medio de 10.5 kg/cm cuad, mínimo 7.

**36. Prueba de compresión.** « Las muestras se deben cortar de los bloques de manera que contengan una sección completa de alma. Se debe medir la muestra con cuidado, colocándola después en un lecho de yeso, para obtener un soporte uniforme en la máquina de prueba, y luego triturarla. La carga total de ruptura se divide entonces por el área comprimida, sin deducir los espacios huecos; el área se considerará como el producto del ancho por el largo. »

**37. Esfuerzo máximo de compresión** á los 28 días, promedio 70 kg/cm cuad mínimo 49.

**38. En las paredes ó muros de apoyo,** mínima 70 kg/cm cuad. No se hace deducción por los espacios huecos.

**39. Absorción.** La muestra se seca hasta el peso constante á 100° C. Se pesa, se coloca en agua con la cara hacia abajo, se sumerge á $\leq$ 5 cm. Se pesa á los

30 minutos, las 4 horas, las 48 horas, y se vuelve á colocar en el agua inmediatamente después de cada pesada. Al fin de las 48 horas se determinará la resistencia de compresión de la muestra mojada, según el § 36.

$$\text{Absorción} = \frac{\text{peso del agua absorbida}}{\text{peso del bloque seco}}. \text{ Promedio} > .15 \text{ máxima, } .22.$$

**40. La reducción de la resistencia á la compresión**, por la absorción, $> \frac{1}{3}$ *.

**41. Prueba de congelación.** Se sumerge la muestra, como en el § 39, durante 4 horas, y se pesa. Se somete á $<$ — 10° C, durante $<$ 12 horas. 1 hora en agua de $>$ 67° C. Se repite la operación diez veces. Se toma el peso todavía mojada de la última deshelada. « Y se determina su resistencia á la trituración » como en el § 36.

**42 Pérdida de peso** máxima 10%; **pérdida de resistencia**, máxima $\frac{1}{4}$ *.

**43. Prueba de fuego.** Se colocan dos muestras frías en el horno. Se eleva la temperatura gradualmente á 927° C. Se mantiene por 30 minutos. Una de las muestras se sumerge en agua como de 10° á 16° C. La otra muestra se deja enfriar gradualmente al aire. « El material no debe desintegrarse. »

**44. El ladrillo de cemento**, como sustituto del ladrillo de greda. 1 : $>$ 4 de arena limpia aguda; ó 1 : $>$ 3 de piedra quebrada ó granzón que pase el cedazo de 12 mm y se detenga en el de 6 mm. En otros respectos los ladrillos de cemento deben llenar los requisitos de los bloques huecos de concreto.

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* « Excepto que cuando la cifra inferior esté todavía sobre 70 kg/cm cuad, se podrá despreciar la pérdida de resistencia.

# COSTO *

**1.** Los siguientes datos respecto á los precios y costo se han completado con los informes de construcciones recientes ejecutadas por individuos que se tienen por peritos en el arte, y calculando la obra de mano con los jornales de costumbre. Ellos dan solamente los cálculos aproximados de lo que se puede esperar en circunstancias ordinarias. El costo de los materiales, transporte y especialmente de la obra de mano, varía con el tiempo y la localidad.

**2.** No sólo varía el precio de la obra de mano por hora, sino que la cantidad misma de trabajo ejecutado en un tiempo dado, varía mucho más. Una cuadrilla bien organizada, presidida por un vigilante competente, produce ordinariamente de dos á cuatro veces lo que una cuadrilla cualquiera. Aun los buenos trabajadores dejan decaer su labor, con frecuencia, al 75% de lo que se podría esperar de ellos; los trabajadores sin interés dan solamente del 30 al 20%. Los sistemas del pago, el carácter del superintendente, y el modo de distribuir y manejar el trabajo, son todos de gran importancia, y un trabajador sin práctica ó que no esté familiarizado con las obras de concreto encontrará probablemente dificultad en conservar el costo total dentro del doble de lo que damos.

**3.** Las **principales partidas**, que forman el costo del concreto (simple y armado), pueden clasificarse como sigue:

Materiales: cemento, arena, granzón, piedra, armazón.
Transporte al almacén; acarreos, fletes.
Almacenaje.
Cernidura, lavado.
Mezcla, carga y transporte al mezclador, máquina de mezclar y fuerza, obra de mano y depreciación relacionada con ella, aparatos auxiliares, tales como: tablas para mezclar, carretillas, palas, etc., y el transporte del concreto á los moldes, armaduras, etc.
Armaduras; erección, cambios de lugar, depreciación, material, mano de obra.
Colocación; vaciado, extensión y apisonado.
Acabado; enlucido, acepillado, etc.
Inspección y superintendencia.
Planta (además del mezclador y las armaduras); interés, depreciación, refracciones, aseguro.

## Costo de los materiales.

**4.** El costo de cualquier material, *por metros cúbicos* de concreto, varía mucho en los diferentes casos, debido á la gran variación en los percentajes que se emplean en las diferentes especies de concreto, y por consiguiente sólo se pueden dar aproximadamente entre límites muy amplios.

**5.** Estimando aproximadamente, el **costo total**, para los **materiales** solamente, puede caer entre $3.25 y $9.75 el metro cúbico de concreto. El término medio será probablemente de $5.20 ó un poco más, sin incluir la armazón **.

**6. Cemento.** Por metro cúbico de concreto, entre $1.95 y $5.20; $2.60 y $3.90 son los límites más usados. Varían principalmente por el grado de cemento y la riqueza de la mezcla **.

**7. Arena.** Por metro cúbico de concreto, entre 19 centavos y $1.36 ordinaria-

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* *N. del T.* — Aunque el costo de materiales, mano de obra, etc., varía mucho, y con multitud de circunstancias, trae el autor en estas páginas tanto dato importante en la práctica, que no hemos vacilado en dejar el capítulo. Además, casi siempre podemos dar el precio que él supone (en una lista de precios cuya inserción no creemos necesaria en su totalidad) para la unidad del material ó de la mano de obra, y así se hace muy fácil calcular, aunque sea aproximadamente, los costos correspondientes en otras localidades con diversos jornales, etc. Es claro que todo lo hemos convertido al sistema métrico.

** *N. del T.* — El autor supone los precios siguientes (en dólares): **Cemento**: americanos, 1.40 á 4.60; alemanes, 2.25 á 3.00 (barriles de 180 kg brutos). Natural Rosendale (barriles de 136 k), .85 á 1 dólar. Por cada 1,000 ladrillos se gastan de 1.5 á 3 dólars. **Cal**, .60 á .90 los 100 kg. Se gasta de 1 á 1.50 dólars por 1,000 ladrillos. **Yeso**, 1.5 á 2 dólars por barril. **Piedras**, .28 á .66 por m cúb; piedra común de construcción, 1.30 á 7.00 por m cúb. **Granito**, 20 á 60 por m cúb. **Asfalto**, 2 á 5 por tonelada.

mente. Varía según la clase, la distancia del depósito, el monopolio natural, y la proporción que se use en la mezcla.

**8. Granzón.** En la cantera, excluyendo la cernidura, carga y acarreo, de 25 á 75 centavos por carro; los afectan principalmente la calidad y el monopolio natural.

**9. Piedra.** Precio medio de la piedra, quebrada al tamaño que se requiera, en la cantera, excluyendo el acarreo, cerca de $1.30 ó $1.95 por metro cúbico de piedra. Por metro cúbico de concreto se gasta entre 65 centavos y $1.30. Los afectan principalmente la calidad, distancia de la cantera, el monopolio natural y la proporción de la mezcla.

**10. Armazón.** El costo varía con el diseño y el tipo que se emplea *.

Barras lisas, lotes de 50 toneladas, en la fábrica, centavos por kg aproximadamente: < ¾", 3 cs; < ½", 3½ cs; < ⅜", 4 cs; < ¼", 4½ cs.

Barras torcidas Ramsome, como ²/₇ más de centavo por kg.

Otras barras deformadas, 1½ á ²/₃ más de centavo por kg.

**11.** El percentaje en la armazón varía ordinariamente de ½% á 1½%, más ó menos, de la sección transversal de la viga ó losa.

## Costo del transporte al almacén.

**12. Flete.** El cemento por ferrocarril. Los fletes varían mucho en las varias localidades, debido con frecuencia solamente á causa de la descriminación arbitraria, variando desde ½ centavo por tonelada-milla á más de 2 centavos; en general de 1 á 2 centavos.

**13.** Por canal. Cargas por lotes de 100 toneladas de 2,000 libras cada una; cemento, 1 á 2 centavos /tonelada-milla, de acuerdo con la distancia; piedra y arena, ¾ á 1½.

**14. Flete costanero.** En lotes de cargas de vagones, .4 á .6 centavos /tonelada-milla, aproximadamente.

## Costo de almacenaje, etc.

**15. Almacenaje.** Ordinariamente, los barriles de cemento se pueden almacenar en hileras de 5 barriles que requiere como ⅛ de m cuad de espacio por barril.

**16. Cernidura.** Costo, á mano, entre 13 y 32 centavos ó más /metro cúbico de material trabajado. Cernidura por máquina, entre 5 y 8 centavos /metro cúbico. Para obtener el costo por metro cúbico de material cernido, multipliquese el costo por metro cúbico por la relación de la cantidad total trabajada á la cantidad aceptada.

**17. Lavado.** El costo del lavado de la arena, granzón y piedra triturada puede ser de 6½ centavos ó más por metro cúbico de material trabajado en lavadores mecánicos que lavan grandes cantidades. En cantidades pequeñas, lavadas bajo condiciones favorables, hasta 52 centavos.

## Costo de la mezcla y colocación.

**18. Mezcla y colocación. Costo total,** excluyendo las armaduras, de $1.30 á $3.25 /m cúb.

**19.** La obra de mano que se requiere, para cantidades bastante grandes, es, por término medio, un hombre por cada 1½ ó 2 metros cúbicos mezclados y colocados por día. En los trabajos pequeños un hombre ejecutará mucho menos.

**20.** El concreto seco cuesta como $1.30 más por metro cúbico para mezclarlo y colocarlo que el concreto mojado. Herman Conrow, Jr., A. S. C. E., Trans., vol. 42, 1899, pág. 124.

**21. Carga.** Se cargan en los carros de 9 á 18 m cúbicos de arena por hombre y por día; 9 parece ser lo usual, pero 18 no es exagerado.

**22. Transporte.** Carga media de piedra quebrada, granzón ó arena.

| | |
|---|---|
| Carretillas de madera..... | =68 litros. |
| — de hierro..... | =53 — |

El costo de transporte por metro cúbico de concreto ordinario es entre 14 y

* Hierro refinado y acero en barras de tamaños ordinarios; ángulos y T, vigas ángulos, placas de formas comunes y acero Bessemer para máquinas, de 2 á 2.66 centavos por kg. Carriles de acero, $28 por tonelada; viejos, $15.

32 centavos, dependiendo en gran parte de la distancia y mérito de los trabajadores.

## Costo de la mezcla.

**23. Mezcla (solamente).** Mucho depende del interés de los trabajadores y del tamaño del mezclador. Varios ejemplos indican costos menores de 13 centavos/metro cúbico, contando la obra de mano solamente, mientras que otros indican, con bastante frecuencia, como 32.5 centavos. Sabin dice : « El costo de la mezcla del concreto en grandes cantidades es rara vez menos de 39 centavos/metro cúbico si se hace concesión por la planta ó instalación. »

**24.** Hasta donde sea posible, el material debe rodar hacia abajo, conservando el mezclador por encima de la obra, si se puede. Si se usa un ascensor para el concreto, su entrada deberá estar por debajo del mezclador. En los trabajos subterráneos ó de cloacas, se puede colocar algunas veces el mezclador debajo del nivel de la calle, pero siempre sobre el nivel de la obra, de modo que no sea necesario levantar el material después de vaciarlo de los carros á la calle. Se pierde mucho si el suministro del cemento en la obra y la demanda del concreto no se conservan casi iguales, ó si las condiciones son tales que los hombres no puedan moverse con libertad.

**25.** Generalmente, no se puede dedicar más de media docena de hombres á un mezclador, de manera que puedan manejarlo con ventaja, medir material, limpiar las plataformas, etc. (además de los que estén ocupados en llenar y traer los materiales al mezclador ó á la obra). El costo, por la obra de mano solamente, no debe ser mucho más de 20 centavos por metro cúbico de concreto, aun con máquinas pequeñas.

**26. Los mezcladores** que den 280 á 1,120 litros de concreto por carga (ó, suponiendo una carga cada 2 minutos, 280 á 1,120 litros por hora), costarán de $500 á $1,000, y necesitarán de 5 á 10 HP. para moverla. Las máquinas movidas á brazo, con una capacidad de 140 litros por carga, como $250.

**27. El costo de montar** un mezclador, y de desmontarlo, incluyendo el acarreo á unas pocas millas, y su depreciación, entre $50 y $100.

Hasta 75 ó 150 m cúbicos de concreto, la mezcla á brazo es ordinariamente más económica que por máquina.

**28.** El costo inicial de una **planta de mezclar á brazo** que la manejen 8 ó 10 hombres, se calcula como sigue :

| | |
|---|---|
| 8 palas de punta cuadrada, tamaño n.º 3..... | $ 10 |
| 3 carretillas de hierro..... | 35 |
| 2 pisones..... | 5 |
| 1 plataforma de mezclar, 4.5 × 4.5 m..... | 10 |
| | $ 60 |

**29. Ejecución.** Cuando el material se entrega prontamente, los mezcladores de carga darán, por término medio, un lote cada 2 ó 3 minutos. Un lote en un minuto es un trabajo muy rápido. Algunas veces se requieren de 4 á 5 minutos. Para las capacidades y fuerza que requieren, véase « Mezcladores », § 26.

**30.** El costo de una planta para mezclar concreto se calcula entre 3 y 5% ó más, del costo de la obra.

**31. La duración de un mezclador,** en condiciones ordinarias, es de 30,000 á 40,000 cargas. De este modo, un mezclador que dé 120 cargas por día, necesitará renovarse al año. Se necesitará un tambor nuevo (generalmente) después de mezclar dos tercios de la cantidad total.

**32. Del mezclador á las armaduras.** El tiempo que se necesita para llenar una carretilla en el mezclador, es como de 10 segundos; para descargar todo el mezclador en una operación, de 15 á 20 segundos.

**33.** La carga media de concreto mezclado en una carretilla es de 42 á 50 litros. Un carro de un caballo carga como ½ m cúbico; de dos, ¾ á 1½ m cúbicos. Para computar los gastos de acarreo, etc., véase el art. 4, pág. 864.

**34** Un hombre puede palear de 7.5 á 10 m cúbicos de concreto en 10 horas por día.

## Costo de las armaduras.

**35** El costo, incluyendo el material y la obra de mano, varía principalmente con el carácter de la obra; las armaduras simples para trabajar en masa, son rela-

tivamente baratas, mientras que las de detalles de muros y pisos de edificios son más costosas, especialmente en el cemento armado.

**36. El material** para las armaduras, entre 13 centavos y $1 por metro cúbico de concreto, y en el lugar.

**37.** La **construcción y erección** solamente, cuestan de **$4 á $10 por 1,000 pies B. M.** (más ó menos 100 m cuad de tabla de 2.5 cm de espesor) en las construcciones más simples; en los edificios, de $10 á $20.

**38.** El costo de las armaduras puede ser de 10 á 50 **por ciento** del costo total del concreto en el lugar; 25 á 35% por las armaduras para trabajo ordinario de concreto armado; 50% ó más para los trabajos de detalles en los edificios.

**39.** El costo, **por metro cuadrado de superficie** (como un lado de una pared), puede computarse mejor para el trabajo que se esté haciendo, dados el valor de la madera y la obra de mano de que se disponga, pero ordinariamente es de .4 á 2 dólares.

**40.** El costo de las armaduras, **por metro cúbico de concreto**, en la construcción de edificios, se calcula entre $3.90 y $13; de $5.20 á $7.80 siendo suficiente para la construcción de pisos, y $6.5 á $9.10 es lo ordinario para las armaduras en los trabajos de concreto armado.

**41. Cambios y depreciación.** En los números que se han dado para el costo de las armaduras, se ha supuesto que el material no se va á usar otra vez. En los trabajos especiales que tengan detalles extraordinarios y difíciles, las armaduras son prácticamente inútiles después que se han usado. Generalmente la madera puede usarse 2 ó 3 veces antes de desecharla. En los edificios grandes, cuyas armaduras se diseñan esmeradamente y que en los detalles son las mismas en todo el edificio, se pueden usar las armaduras hasta seis veces.

**42.** El trabajo de mano para cambiar los moldes no será mucho menos que el de construirlos.

**43.** El costo de la **obra de mano, para colocar** las armaduras, está entre 3 á 4% y 20% del costo del concreto en el lugar.

## Costo de la colocación.

**44.** El costo de la **fabricación** (arcos, armada, etc.) **y colocación de la armazón**, es como de 1 á 3.3 centavos /kg de armazón.

**45. Depósito.** La obra de mano efectiva que se necesita, para el depósito solamente, rara vez llega á más de un hombre para ayudar á descargar los carros, cambiar las canales de descarga, etc., no más de unos pocos centavos por metro cúbico de concreto colocado. En los informes se da de 7 centavos para arriba, pero en esto quizás se incluya el transporte del mezclador á las armaduras.

**46. Esparcimiento y apisonado.** El costo varía mucho con la clase de la obra; siendo tan bajo como de 19 centavos por metro cúbico en los trabajos ordinarios (6.5 centavos si la mezcla está muy húmeda), hasta $1.30 ó más cuando se tiene gran esmero en la colocación, apisonado, compresión, etc. Menos, si el concreto se vacía de los carros ó baldes en grandes cantidades.

**47.** Por el apisonado solamente, de 6 á 19 ó 26 centavos /metro cúbico, rara vez más de 52 centavos.

## Gastos varios.

**48. Inspección y superintendencia**, como se hace generalmente, de 1 á 3% del costo de la obra. En vista de las grandes dificultades que resultan si el trabajo no está bien arreglado ó que los hombres no se mantienen en orden, puede dejar cuenta gastar 5 ó 10% más.

**49. Acabado.** Los datos son muy variables, debido probablemente á la diferencia en los sistemas.

**50.** Lavado con brocha, 3 á 10 centavos /m cuadrado de superficie, con ácido hidroclórico diluído para quitar la eflorescencia, como $2 m cuadrado.

**51.** Preparado á martillo de las superficies, .30 á 2.60 dólares por m cuadrado. Con aparato neumático, menos de 1 centavo. Acabado de las juntas y enlucido á brocha, $2.5 /m cuadrado ó más.

## Costos totales.

**52. Sencillo.** Para el costo total, véase « Masa », etc., § 55.

**53. Concreto seco,** como $1.30 más /metro cúbico que el concreto mojado, debido al trabajo adicional de apisonarlo.

**54. Concreto de granzón**, de $1.20 á $2.60 por m cúb más barato que el concreto de piedra dada la misma proporción entre (arena+piedra) y el cemento, la mayor diferencia se obtiene en mezclas pobres en cemento.

**55. Masa.** Los tajamares, fortificaciones, etc., cuestan entre $6.50 y $9.10/metro cúbico de concreto en el lugar, siendo el término medio $7.80. Los extremos, entre $5.20 y $10.40.

**56. Armazón.** Cuando el trabajo está bien organizado, los edificios de cemento armado pueden construirse desde $13/metro cúbico de concreto en el lugar, pero el término medio general es más cerca de $23.40, aunque algunos constructores calculan en redondo en $35.00 el metro cúbico, pero en muy pocos casos han costado tanto.

**57.** El costo depende principalmente de las armaduras (véase § 36). Si éstas están bien diseñadas de manera que puedan cambiarse fácilmente y que se puedan usar varias veces, el costo es bajo, comparado con los trabajos especiales, en que el refinamiento en el diseño no dejaría cuenta.

**58. Muros de sostenimiento**, los muros de cimientos, los empotramientos, las esclusas, los muelles, etc., varían mucho, debido aparentemente á las dificultades que pueden presentarse en la construcción. Los extremos varían entre $5.20 y $20.80 metro cúbico de concreto en el lugar. Muy á menudo, sin embargo, el precio puede ser de $7.80 y $10.70. En las paredes armadas, de $3.90 á $13 más.

**39. Los arcos** de luz moderada, digamos hasta 9 m en los trabajos de cloacas, etc., de $6.50 á $13 metro cúbico.

**60. Edificios.** El costo puede caer entre $7.80 y $15 por metro cúbico de concreto en el lugar, con término medio de cerca de $10.40 para el sencillo, y de $13 á $19.50 ó $26 para la construcción armada.

**61.** Para cualquier tipo dado de construcción, todas las partes de un edificio (exceptuando los cimientos), como los pisos, paredes, y columnas, cuestan prácticamente lo mismo por metro cúbico.

**62.** Mr. L. C. Wason (E. R., '09, febrero 27, pág. 233) da como **costo de los edificios**:

| | $ por metro cúbico de espacio cerrado. | | | $ por metro cuadrado de piso. | | |
|---|---|---|---|---|---|---|
| | Máximo. | Promedio. | Mínimo. | Máximo. | Promedio. | Mínimo. |
| Oficinas y almacenes.. | 6.95 | 4.62 | 2.96 | 26.04 | 19.04 | 12.05 |
| Fábricas..... | 4.55 | 3.60 | 2.12 | 18.29 | 14.42 | 9.68 |
| Garages..... | 4.16 | 3.60 | 3.00 | ... | ... | 13.23 |
| Filtros..... | 11.76 | 8.22 | 4.73 | 41.10 | 26.15 | 11.19 |
| Casas de almacenaje... | 2.93 | 2.68 | 2.43 | 9.04 | 7.64 | 6.24 |
| Molinos, etc., de 2.ª cl. | 4.30 | 2.43 | 1.59 | 16.25 | 9.68 | 5.81 |

# ENLUCIDOS

El enlucido de las paredes interiores de los edificios, ya sea que se haga sobre listones, ladrillos ó piedra, consiste generalmente en tres capas separadas de mortero. La primera se compone de 1 medida de cal viva para 4 de arena (la que no necesita ser de la clase más pura) y ½ de medida de pelo de buey ó caballo, el cual sirve para aumentar la cohesión del mortero y hacerlo menos capaz de agrietarse. Esta capa es como de 9 á 12 mm de espesor. Se coloca toscamente y se comprime con la llana con suficiente fuerza para que se introduzca perfectamente entre y por detrás de los listones, los cuales, para facilitar la operación, no deben tener entre sí una separación menor de 12 mm. En los edificios rústicos ó en los sótanos, etc., ésta es generalmente la única capa que se usa. Cuando esta primera capa se ha dejado secar ligeramente uno ó más días, según el estado del aire, se araña ó raspa con una astilla de madera puntiaguda, penetrando casi todo su espesor y formando líneas diagonales que se crucen y separadas de 5 á 10 cm. Esto facilita la adherencia de la segunda capa, la que se descascaría si no fuese por esto. Si la primera capa se ha secado demasiado, es conveniente humedecerla un poco al poner la segunda.

La segunda capa tiene de 6 á 10 mm de espesor de la misma argamasa de pelo ú otra materia burda. Antes de que se endurezca se la hace áspera por medio de una escobilla ó algún otro utensilio para que la tercera capa se adhiera mejor.

La tercera capa, como de 12 mm de espesor, no contiene pelo, y para darle una apariencia todavía más blanca y más limpia, se le pone más cal (1 de cal para 2 de arena), y se emplea la arena más pura.

En lugar de estuco la tercera capa puede ser, y lo es generalmente, de una mezcla compuesta de una de yeso en polvo para 2 de cal viva, sin arena. Esta mezcla se hace de más fácil manejo, pero no es tan buena como el encalado para paredes que han de pintarse al óleo. El yeso acelera el endurecimiento.

Cualquiera de estas terceras capas se pule ó alisa más ó menos, según que se vaya á empapelar, á pintar ó dejar descubierta. Los instrumentos para pulir se reducen á la *llana*, la regla de alisar y la brocha de agua (una brocha de mango corto con que se humedece la parte de la superficie que se trabaja á fin de pulimentarla más fácilmente). Para un pulido más fino se usa una regla hecha de corcho. La pieza lisa de madera, como de .25 á .30 m en cuadro, que tiene un mango por debajo para agarrarla y en la que pone el obrero la argamasa, para extenderla con la llana en la pared, se llama *gamella* (*hawk*, Ponce de León).

Mientras más se repase con la brocha de agua y la llana cada capa, más firme y más fuerte quedará. A menudo sólo se hacen enlucidos con dos capas en los cuartos inferiores ó donde no se requiere muy buena apariencia. La primera es de argamasa con pelo ú otra materia áspera. Ésta se araña con la escobilla y luego se cubre con la capa final de mortero más fino. Si esta última es casi toda de cal ó con muy poca arena para trabajarla más fácilmente, se llama *encalado* ó *enlucido ligero*. Sin ninguna arena se llama *enlucido fino*. Ninguno de los dos es tan bueno como el encalado si se va á empapelar la pared.

Cuando es este el caso, puede ponérsele á la tercera capa un poco de pelo para darle más consistencia; pero esto no es absolutamente necesario.

(*Obs. del T.* — Entre nosotros se empapela con buen éxito aplicando el papel directamente sobre la capa del encalado, que como es una capa formada de una mezcla de iguales partes de cal y arena fina, si se deja sin alisar mucho la superficie, al fraguar la mezcla forma ésta un solo cuerpo de bastante cohesión, á cuyas asperezas se adhiere el papel. Siempre se evita, ó mejor dicho, nunca se empapela sobre la capa de lechada, pues siendo ésta un agua de cal casi pura, al secar se desprende la cal con el papel no obstante el mucílago que se le agregue para darle consistencia.)

Se puede producir muy buen efecto en las estaciones de ferrocarril, iglesias, etc., con sólo dos capas en las cuales se emplea cascajo fino, cernido, en lugar de arena. Cuando se le trazan estrías ó líneas regulares, presenta el aspecto de una piedra arenisca color de ante muy agradable á la vista.

Cuando se compre pelo para estucar debe tenerse cuidado de que no sea de cueros

salados, pues la sal hará la pared húmeda. Por esta misma razón no debe emplearse la arena del mar. Es casi imposible quitarle la sal por completo.

Si se pretende enlucir ó encalar una pared de ladrillos, las juntas del mortero deben dejarse bien ásperas para que la mezcla se adhiera. Si se aplica á paredes lisas sin raspar antes el mortero hasta una profundidad de cerca de 2½ cm, es muy fácil que se caiga, especialmente en las paredes exteriores, como se puede ver diariamente en cualquiera de nuestras ciudades. Como el escarbar ó descubrir las juntas de los ladrillos es fastidioso y costoso, sería mejor usar pintura en lugar del enlucido. A las paredes debe quitárseles todo polvo y humedecerlas un poco á proporción que se pone el enlucido.

Para imitar el granito en las paredes exteriores se procede así: Después que la segunda capa se seca se le da una mano de lechada con un pequeño tinte de tierra siena ú ocre, etc. Después que ésta se seca, y en caso de que aparezca muy obscura ó demasiado clara, puede aplicársele otra con más ó menos colorido. Finalmente, con una brocha chata, se le da una rociada de agua de cal y negro mineral, para imitar las manchas negras del granito. Por este simple medio, un obrero hábil puede producir excelentes imitaciones. Las juntas horizontales y verticales de la imitación de mampostería, pueden rayarse con una brocha pequeña usando la misma solución negra y una regla larga recta.

Las superficies toscas de todas las paredes son más ó menos onduladas y fuera de línea y al aparejador le es imposible rectificarlas con exactitud al ojo, como se ve en casi todas las casas. Hasta en las que se titulan de primer orden, un ojo experimentado puede generalmente descubrir ondulaciones de mal efecto en las superficies enlucidas ó encaladas.

Para impedir esto se ha apelado al procedimiento de **la plantilla**. Las plantillas son una especie de guías formadas por la aplicación á la primera capa basta del aljorozado, cuando está medio seca, de bandas horizontales de la mezcla de enlucir como de 20 cm de ancho y de 2 á 4 pies (.60 á 1.20 m) de separación alrededor de todo el cuarto. Éstas se hacen de manera que sobresalgan de la primera capa hasta la superficie de la segunda, y mientras están blandas se hacen con cuidado perfectamente derechas y paralelas, por medio de la plomada, etc. Cuando se secan, se coloca la segunda capa, rellenando los espacios anchos horizontales entre ellas, y se hace fácilmente una superficie plana que corresponde con la de las plantillas ó guías por medio de largas reglas rectas que se extienden á la vez sobre dos ó más de éstas.

**Un día de trabajo en enlucido.** Un albañil ayudado por uno ó dos obreros que le mezclen el mortero y se lo tengan á la mano, puede hacer de 80 á 160 m cuad de la primera capa por día; como ⅔ de dicha cantidad de la segunda, y como la mitad de la tercera, que requiere más cuidado. La cantidad que haga dependerá del número de ángulos, tamaño de las piezas ó de que trabaje en techos ó en paredes, etc.

**Presupuesto del general Gillmore sobre el costo del encalado de 80 m cuad** con 2 ó 3 capas. Salario ordinario de un obrero, 1 dólar por día.

| Material. | Tres capas. Obra acabada. | | Dos capas. Enlucido con poca arena | |
|---|---|---|---|---|
| Cal viva | 4 pipas. | $ 4.00 | 3½ pipas. | $ 3.33 |
| — para trabajo fino | ⅔ pipa. | .85 | | |
| Yeso de París | ½ pipa. | .70 | | |
| Listones | 2,000. | 4.00 | 2,000. | 4.00 |
| Pelo | 145 litros. | .80 | 109 litros. | .60 |
| Arena común | 7 cargas. | 2.00 | 6 cargas. | 1.80 |
| Arena blanca | 91 litros. | .25 | | |
| Clavos | 13 lbs. | .90 | 13 lbs. | .90 |
| Obra de mano de albañil | 4 días. | 7.00 | 3½ días. | 6.12 |
| Obreros | 3 días. | 3.00 | 2 días. | 2.00 |
| Acarreo | ..... | 2.00 | ..... | 1.20 |
| Costo de 80 m cuad | ..... | $25.50 | ..... | $19.95 |

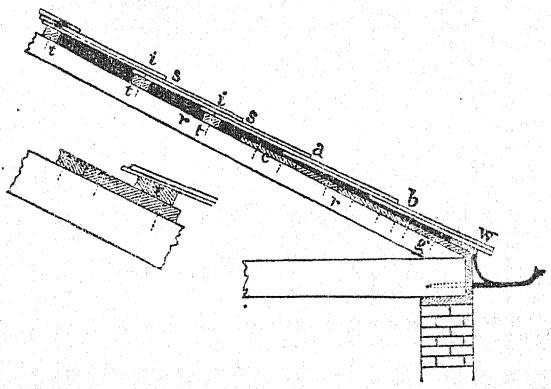
Esto alcanza á 32 cent por m cuad para tres capas y como 25 cent para dos capas.

**Los listones para enlucidos** son generalmente de pino blanco ó amarillo como de 1 á 1.20 metros; tienen como 4 cm de ancho por 3 cm de grueso. Se clavan

horizontalmente á un poco más de 1 cm de distancia. Los postes verticales de los tabiques se colocan á la distancia necesaria (generalmente á unos 40 cm de centro á centro) para que los extremos de los listones se puedan clavar en ellos. Los listones se venden en atados de 1,000, que, colocados como se ha dicho, cubren 62 m cuad. Se pueden obtener aserrados, de cualquier longitud. Un carpintero puede clavar listones para 32 á 50 m cuad de enlucido en un día de 10 horas, según el número de ángulos que tenga el cuarto, etc.

# EMPIZARRADO

**Las pizarras para techos** son generalmente de 3 á 6 milímetros; término medio 4½ milímetros de espesor. Se pueden clavar sobre un forro de tablas ordinarias (*e*, *g*, en la figura) con un espesor de 2 á 3 cm (las que deben estar, y lo están rara vez, machihembreadas), colocadas horizontalmente de par á par ó inclinadas




de parecillo á parecillo, según sea el caso; ó en listones fuertes *ttt* como de 5 á 8 cm de ancho y de 2½ á 3 cm de grueso, clavados sobre los pares á distancia, de modo que se adapten á los bordes de las pizarras. Se emplean dos clavos para cada pizarra, uno cerca de cada esquina superior. Deben ser de cobre (que son más durables, pero más caros), de cinc, ó de hierro galvanizado ó estañado. Los dos últimos se emplean generalmente en trabajos inferiores. Los de hierro ordinario se hacen hervir previamente en aceite de linaza como un preservativo parcial contra la oxidación. El óxido, sin embargo, los debilita á veces tanto, que se rompen y las pizarras son arrastradas por el viento con peligro para los transeuntes. Como las pizarras buenas duran muchos años, es una verdadera economía el uso de clavos que sean igualmente durables. En armaduras de hierro, las pizarras en lugar de ser clavadas sobre tablas se atan á veces directamente á las piezas de hierro por medio de alambres. Un cuadrado (*square*) de empizarrado, de teja manila, etc., tiene 100 pies cuadrados (9.29 m cuad).

En los laboratorios, factorías químicas, etc., que están expuestos á emanaciones ácidas, es difícil encontrar, para amarrar, un metal que no se corroa. En tales casos es mejor colocar entre pizarra y pizarra una capa de mortero, la cual se endurecerá antes de que las ligaduras de metal cedan y mantendrá las pizarras en su lugar mientras se ponen ligaduras nuevas.

El menor declive que se le debe dar á un techo para impedir que la lluvia y la nieve se introduzcan por los intersticios de las pizarras, es como de 26 1/2°, ó sea 1 de altura para 2 de base, lo cual corresponde á una elevación de ½ de la luz en un techo común de dos aguas. Pero aun con mayores declives, la lluvia y la nieve sobre todo penetran á través del techo impulsadas por el viento fuerte, especialmente si se usan sólo listones y aun usando sólo entablado. Para evitar esto se puede poner una capa de mortero como de 6 mm de grueso sobre las superficies de contacto de las pizarras, si están sobre listones. Si sobre tablas, puede aplicarse el mismo procedimiento, ó el más común de cubrir primero las tablas con una capa de lo que se llama fieltro de empizarrar (*slating felt*), pero que en realidad es papel grueso obscuro remojado en alquitrán. Éste se vende en largos rollos de 71 cm de ancho, que pesan de 18 á 23 kg. Un rollo de 23 kg cubrirá como 28 metros cuadrados de techo. Tomando precauciones especiales contra la lluvia y la nieve, puede adoptarse una inclinación de 1 en 2½ ó hasta 1 en 3.

El espesor de pizarras en el techo es doble, excepto en los bordes *is*, *is*, etc., donde es triple. Los bordes que forman la junta se miden desde el hueco del clavo (debajo de *i*) de la pizarra de abajo, hasta el borde inferior ó *cabo s* de la superior, y es generalmente como de 7½ cm. A fin de que los bordes bajos visibles de las pizarras formen al colocarlos líneas rectas regulares á lo largo del techo, se hacen los huecos de los clavos á iguales distancias de la orilla inferior de las pizarras, de modo que cualquier irregularidad de tamaño quede escondida en la cabeza cubierta de la pizarra. El empizarrador calcula el tamaño de una pizarra desde el hueco del clavo en la parte superior hasta el borde inferior, despreciando la faja angosta comprendida entre el hueco del clavo y el extremo superior. Si de esta reducida longitud se deduce el borde de la junta, entonces la mitad de lo que queda será el *escantillón* ó margen de las hiladas de pizarra. El margen en m, multiplicado por el ancho de una pizarra en m da el área en m cuad de techo cubierto por una sola pizarra, y si se divide un m cuad por esta área, el cociente será el número de pizarras que se necesitan para un m cuad de techo. La parte superior de una pizarra se llama *dorso* y la parte inferior la *cama*.

El empizarrado, así como los techos de tejamaní, deben comenzarse del alero hacia arriba. Desde luego, las camas de las pizarras no son exactamente paralelas al entablado, y por consiguiente no descansan de plano sobre él; las situadas en el borde inferior del tejado ó en el alero *w* están espuestas á romperse fácilmente. Para impedir esto se clava primero cerca del alero una tira inclinada (un listón fuerte y ancho con su cara superior en bisel para que ajuste con la inclinación de las pizarras) á fin de que los extremos de la hilera más baja de pizarras descanse sobre ella. Esto se ve en una escala mayor en T.

Las pizarras de mejor calidad tienen una apariencia brillante semimetálica, algo semejante á la de la superficie de un papel frotado con lápiz negro. Las de aspecto opaco terroso son más blandas, más absorbentes y por consiguiente más susceptibles de ceder á las influencias atmosféricas, como las lluvias, heladas, etc. Se encuentran piritas de hierro frecuentemente en las pizarras, y como siempre se descomponen y dejan huecos, no deben admitirse nunca en los techos. Entre dos clases de pizarra, la mejor es generalmente la que, sumergidos dos pedazos de igual tamaño en un poco de agua durante una ó dos horas, absorba menor peso de dicho líquido, pues es menos susceptible de romperse por las heladas y descomponerse por la temperatura. Esta prueba se hace fácilmente.

En Inglaterra los diferentes tamaños se distinguen por nombres absurdos sin significación alguna. En los Estados Unidos se llaman pizarras de 6 por 12, de 16 por 24, etc., según sus dimensiones en pulgadas. Pueden cortarse, según se pidan, de cualquiera dimensión ó forma. Las de uso común varían de 18×36 á 30×46 cm. La primera forma hileras de 12½ á 15 cm y la última de 18 á 20 cm, según la distancia á que se han hecho los agujeros para los clavos de la cabeza. Mientras más distantes, más firme será el empizarrado.

Los techos de pizarra, así como los de hierro, calientan mucho las habitaciones que cubren. Esto disminuye algo cuando las pizarras reposan sobre tablas en lugar de estar sobre listones, y todavía más, cuando hay debajo una capa de yeso ó de encalado. También son susceptibles de quebrarse cuando se camina sobre ellas, pero esto sucede menos cuando están colocadas sobre mortero.

### Peso de los techos de pizarra.

El metro cuadrado de pizarra de .003 m de espesor pesa $1 \times .003 \times 2{,}800 = 8.4$ kg; el de .0047 de espesor pesa $1 \times .0047 \times 2{,}800 = 13.16$ kg; y de un espesor de .0063 pesa $.0063 \times 1 \times 2{,}800 = 17.64$ kg. Su densidad es 2.8. Pero debido á la superpo-

sición de los bordes, un metro cuadrado de techo requiere como 2½ m cuadrados de pizarra de tamaño ordinario, y si las pizarras están colocadas sobre tablas de 2½ cm de grueso, el peso por m cuadrado de techo aumentará como en 11 kg; si son tablas de 3 cm, como en 14 kg. Los listones pesarán como 1.22 kg por m cuadrado.

| Peso aproximado del empizarrado. | | | | | | Peso en kg por m cuad. |
|---|---|---|---|---|---|---|
| Pizarras de | ⅛ | pulg (3 mm) | de espesor | sobre | listones | 23.192 |
| — | — | — | — | — | tablas de 1 pulg (25 mm) | 32.958 |
| — | — | — | — | — | — de 1¼ pulg (32 mm) | 35.643 |
| — | 3/16 | — (5 mm) | — | — | listones | 34.178 |
| — | — | — | — | — | tablas de 1 pulg (25 mm) | 43.944 |
| — | — | — | — | — | — de 1¼ pulg (32 mm) | 46.629 |
| — | ¼ | — (6 mm) | — | — | listones | 45.164 |
| — | — | — | — | — | tablas de 1 pulg (25 mm) | 54.929 |
| — | — | — | — | — | — de 1¼ pulg (32 mm) | 57.61 |

Si se emplea el fieltro de empizarrar (*slating felt*), agréguese 1.22 kg por m cuad, y si se colocan las pizarras sobre 6 mm de mortero, agréguense 14.65 kg por m cuad.

Al peso total soportado por la armadura de un techo debe agregarse también el peso de los parecillos. Éste variará de 7.32 á 14.64 kilogramos por metro cuadrado en techos de luz moderada. Agréguese por el viento y la nieve unos 98 kg por m cuadrado y agréguese finalmente el peso de la misma armadura.

Para coger las juntas de las pizarras (ó tejamaní) y chimeneas, claraboyas, etc., es muy buena una mezcla fuerte de pintura de albayalde tal como se vende por cuñetes, y con arena suficiente para impedir que se esparza, especialmente si se protege por una cubierta de tiras de plomo ó cobre, hojalata, etc., clavadas á las juntas del mortero de la chimenea después de doblarlas, para que penetren en dichas juntas, las cuales deben escarbarse hasta 2½ cm de profundidad y rellenarse luego. A menudo se usa también con este objeto el mortero, protegido de la misma manera y aun sin proteger, pero no es tan bueno como la pintura y la arena. Se usa con igual fin el mortero de algunos días de preparado (para dejar que absorba algunas partículas refractarias de cal) mezclado con cenizas de herrería y melaza (*molasses*) y así se endurece mucho y es muy eficaz.

# TEJAMANÍ

Las tejamanís de cedro blanco son las que se usan con mejores resultados y cuando son de buena calidad duran de 40 á 50 años en nuestros Estados del Norte. Son generalmente de 69 cm de largo por 15 á 18 cm de ancho; como 6 mm de espesor en el extremo superior y como 15 mm en el extremo inferior ó punta. Se colocan en hileras como de 20 á 21 cm de ancho, de modo que sólo ⅓ de la tejamaní queda expuesta á la intemperie. Generalmente se colocan en capas triples, excepto en un espacio de 2½ á 5 cm en los extremos superiores donde entran cuatro. Se clavan sobre listones aserrados de roble ó de pino amarillo como de 4.88 m de largo, 6 cm de ancho y 2½ de grueso, colocados en hileras horizontales á distancias de 22 cm. Éstos se clavan en los pares los que, para listones de dimensiones anteriores, no deben estar separados más de 60 cm de centro á centro. Se emplean dos clavos para cada teja, los cuales se clavan cerca de su extremo superior. No deben ser más pequeños que de á 880 en kg. Los clavos forjados son los mejores por ser los más fuertes; los cortados se quiebran al cimbrarse la tejamaní. Son suficientes, para cien m cuadrados, unos 10 kg de clavos, incluyendo desperdicios. Una tejamaní mediana de 19 cm de ancho en hileras de 21½ cm deja ver cuatro decímetros cuadrados y requiere como 25 tejas por m cuadrado. Pero, tomando en cuenta los desperdicios y las tejas angostas, es conveniente calcular en la práctica unas 10 tejas más por m cuadrado. Tanto techando con tejamaní como empizarrando, es natural empezar

de los aleros hacia arriba. Para coger las juntas entre las tejas y las chimeneas, claraboyas, etc., véase el final del artículo Empizarrado.

El ciprés y el pino blanco son también muy usados para tejamaní, pues son mucho más baratos; pero apenas duran la mitad de aquéllos. Todas las tejamanís se adelgazan con el tiempo por la lluvia y la intemperie. En los climas cálidos y húmedos se pudren de los 6 á los 12 años.

# PINTURA

El principal material usado en la pintura de los edificios es el blanco de plomo (albayade) ó el óxido de cinc mezclado con aceite de linaza crudo sin hervir, con la consistencia de una pasta gruesa. Así lo venden en cuñetes los fabricantes. Para prepararlo cuando se va á usar, sólo se necesita agregarle más aceite de linaza: como .38 á .50 litros por kg de la pintura en cuñete para adelgazarlo lo suficiente á fin de que corra fácilmente la brocha.

Una pintura buena necesita de 4 á 5 manos; pero sólo se usan generalmente 4 para las habitaciones principales y 3 para las inferiores. Cada mano debe dejarse secar perfectamente antes de que se dé la otra.

Un kilogramo de pintura del cuñete, adelgazada como queda dicho, da 3.70 m cuad de primera mano; 5.50 de segunda mano y 7.40 de cada una de las subsiguientes, ó lo que es lo mismo, 1 m cuad con tres manos requiere 580 gramos de pintura: 1 m cuad de 4 manos, 720 gramos, y uno de 5 manos, 860 gramos. La primera mano necesita más que las subsiguientes porque la superficie desnuda de la madera absorbe más pintura.

Cuando, como es costumbre, se usa aceite *crudo* de linaza para adelgazar, se le deben agregar *secantes*, porque de otro modo la pintura necesita varias semanas para endurecerse, mientras que con secantes sólo de 2 á 3 días (según la temperatura) serán suficientes para que cada mano se endurezca y pueda recibir la otra. Los secantes que se usan más generalmente son litargirio pulverizado en la proporción de una cucharadita de te colmada; ó barniz japonés, 1 cucharada de las de sopa para 4½ kg de la pintura. Azúcar de plomo (sal de saturno) ó sulfato de cinc, pueden usarse también en lugar de litargirio, y en la misma proporción. Aun cuando el litargirio y el barniz japonés son obscuros, la cantidad es tan pequeña que no afecta apreciablemente la blancura de la pintura. Si se usa el barniz con exceso, como á menudo se hace para terminar el trabajo de prisa, produce agrietamientos en toda la superficie. No es necesario ningún secante si se usa aceite crudo para adelgazar. No basta solamente hervir el aceite; para que se endurezca más rápidamente, los pintores le echan previamente litargirio antes de hervirlo, en la proporción de 15 gramos por litro. En algunas obras escritas para el uso de los pintores de edificios, se asegura que el aceite se pone demasiado espeso para cualquier trabajo que no sea exterior, cuando se hierve. Pero esto es un error; porque si se hierve bien el aceite, quedará suficientemente delgado para el más fino trabajo interior, y será además más transparente que cuando está crudo, dando á la superficie que se pinta una apariencia más brillante. El calor debe ser tan sólo el necesario para que hierva, es decir, como 355° C. El hervor debe continuar como 1½ horas, removiendo el aceite de cuando en cuando para que el litargirio no se asiente. Se puede dejar entonces apaciguar el fuego y la operación estará terminada. Se habrá formado un sedimento en el fondo, el cual se dejará al trasegar el aceite. Aun cuando con este aceite no se necesita emplear ningún secante, se le puede agregar, sin embargo, un poquito de litargirio cuando se requiera gran prontitud. Los pintores usan rara vez este aceite, debido á que les resulta un poco más costoso.

Otra substancia muy usada para adelgazar el aceite (excepto en la primera mano) es el espíritu de trementina. La cantidad de aceite puede disminuirse en proporción á la trementina que se agrega. Siendo ésta más fluida que el aceite, hace que la pintura corra mejor con la brocha. Disminuye además la tendencia que tiene á ponerse amarilla, especialmente en los cuartos que se conservan cerrados por algún tiempo. Es además más barata que el aceite. No debe emplearse para trabajos exteriores expuestos á la intemperie, ó debe limitársela moderadamente, puesto que tiende á endurecer la pintura, y aun cuando sus efectos se pueden apreciar

apenas en el interior, son verdaderamente visibles cuando el trabajo tiene que estar á la intemperie. Según el gusto, en la pintura de los edificios á veces se desea que la superficie sea de una apariencia brillante y otras veces está en boga un color apagado ó mate. La apariencia brillante es la que tendrá naturalmente la pintura siempre que no se use al adelgazarla más trementina que aceite. El color mate se obtendrá no usando aceite, sino trementina sola, para la última mano, que se llama mano *apagada*. Aun cuando la trementina no es propiamente un secante, como se evapora pronto, ayuda á aclarar la pintura.

Para trabajos exteriores se recomienda generalmente el uso de más secante que para los interiores á fin de que la pintura pueda endurecerse ó secar bastante pronto y para que el polvo y la lluvia no la dañen. En otros casos es mejor emplear menos.

Cuando en vez de una mano final se quiere dar otro color, la substancia colorante se mezcla con la pintura blanca que va á usarse en la última mano solamente; hay veces que son necesarias dos manos de pintura de color, antes de producir un efecto satisfactorio. Las materias colorantes pueden ser añil, negro humo, tierra de siena, tierra de sombra, ocre amarillo, cromo, rojo de venecia, minio, etc., etc., las cuales van mezcladas con aceite y preparadas para la venta por los fabricantes de pinturas de albayalde y cinc. Aquellas están simplemente bien unidas con pintura blanca.

Toda superficie que se va á pintar debe primero secarse perfectamente y quitársele el polvo. Si es sobre madera, debe quitársele á ésta las marcas del cepillo y otras pequeñas irregularidades con papel de lija cuando se desea un pulimento más fino. También se deben hacer penetrar las cabezas de los clavos como 3 mm debajo de la superficie. Para impedir que los nudos se vean á través del pulimento (como sucedería con el pino blanco y amarillo debido á la trementina que contienen), se deben tapar por medio de dos manos de goma laca, que al secarse se pule con lija fina. Otro modo, no tan seguro, consiste en una ó dos capas de albayalde mezclado con agua de cola delgada ó « encolado », como se dice.

Después de estos preparativos, se pone la primera capa, ó capa de aparejo, en la cual no debe haber trementina; porque penetraría de pronto en la madera, dejando sólo el albayalde en estado casi seco y quebradizo. Hecho esto, los huecos de los clavos, rendijas, etc., deben rellenarse con mastique hecho de blanco de España (fino y lavado) y aceite de linaza crudo; el aceite cocido no da buen resultado; la pasta saldría friable. Dicha pasta estaría expuesta á caerse si se pusiera antes de la capa de aparejo; porque la madera absorbería el aceite y se contraería aquélla. Después de que la primera capa esté perfectamente seca, se pondrá la segunda, y para ella, se mezclará uno de trementina con tres del aceite de adelgazar. En la tercera y cualesquiera de las capas subsiguientes se pueden usar iguales partes de trementina y aceite, para adelgazar, si se quiere que la pintura al secar quede con *lustre;* pero si se desea de color mate, la última capa debe ser apagada ó enteramente sin aceite, sustituyendo éste por la trementina.

Los pintores limpian sus brochas, generalmente, quitándoles casi toda la pintura con un cuchillo, y conservándolas en agua hasta que las vuelvan á usar. Si se tienen que guardar por algún tiempo, deben limpiarse muy bien con trementina ó con jabón y agua. Para impedir que se forme una costra dura sobre la pintura; cuando se deja de usar la brocha por algunos días, se le pone un poco de aceite.

**Las mejores pinturas para preservar el hierro expuesto á la intemperie** parecen ser los óxidos de hierro pulverizados, tales como el ocre de hierro amarillo y el rojo, ó hematita obscura de hierro mineral, bien pulverizada y mezclada simplemente con aceite de linaza y un secante. El albayalde aplicado directamente al hierro hace necesario renovarlo constantemente y es probable que ejerza una acción corrosiva. Puede aplicarse, sin embargo, sobre colores más durables cuando las apariencias lo exijan. Se dice que el almagre es muy durable cuando está puro. Se nos presentan como un ejemplo, las varillas de una bomba en un aljibe de 61 m de profundidad que está cerca de Londres, que habiendo sido pintadas así primero, estuvieron en uso por espacio de 45 años, á cuyo término se encontró que su peso era exactamente el mismo que cuando nuevas, demostrando con esto que el moho no las había afectado.

Cuando las dimensiones de la superficie del hierro expuesto lo permitan, puede prevenirse en gran manera, contra el moho, calentándole muy bien primero y luego lavándole bien con aceite de linaza caliente, el cual penetrará entonces en el interior del hierro. Para el hierro estañado expuesto á la intemperie, en techos, canales, etc., el ocre obscuro es un color muy durable. El estaño se encuentra á menudo en per-

fecto estado de conservación y pulimento, después de 40 y 50 años de uso. La pintura blanca se cae con la lluvia en pocos años.

Las paredes encaladas ó enlucidas deben dejarse secar por lo menos un año, si es posible, antes de pintarlas al óleo, porque de lo contrario la pintura está expuesta á ampollarse. Si se quiere, durante este intervalo puede pintarse al fresco.

La pintura sobre maderas verdes ó no bien secas, contribuye á que se piquen. Si la superficie que se quiere pintar es grasosa, la grasa debe quitarse primero con agua en la cual se disuelve un poco de cal.

**Pinturas al temple para trabajos exteriores.** Downing, en su obra sobre casas de campo, recomienda lo siguiente. (*Obs. del T.* — Hemos hecho las conversiones.) Para construcciones de madera: Tómense 20 litros de cal viva y apáguense en un envase de capacidad doble hechándole agua hirviendo hasta cubrirla con una capa de líquido de 10 á 12 cm, se mueve bien hasta apagarla y se le agrega 1 kilogramo de sulfato de cinc, luego se agrega agua como se dijo hasta que tenga la consistencia de una lechada gruesa y se aplica con una brocha de dar lechada. Esta lechada es blanca, pero puede colorarse agregándole ocre en polvo, almagre, tierra de sombra, etc. Si se agrega negro humo á las acuarelas (pinturas líquidas), debe primero disolverse perfectamente en alcohol. El sulfato de cinc hace que la lechada se endurezca en pocas semanas.

**Para ladrillo, mampostería ó fundición en bruto.** Apáguense 20 litros de cal como se dijo anteriormente, llénese entonces ¾ partes del depósito con agua y agréguese otro volumen igual de cemento hidráulico, agréguense 1 ½ kilogramos de sulfato de cinc previamente disuelto en agua. Todo debe tener la consistencia de la pintura y se puede poner con una brocha de dar lechada. Esta pintura se mejora mezclándole 10 lit de arena blanca antes de usarla. Se le puede dar color como se hizo antes.

El mismo autor da también la siguiente pintura al óleo, barata, para madera, ladrillo y piedra, á la intemperie: Una medida de cal viva fresca molida, agréguese la misma cantidad de arena fina, ó ceniza fina de carbón mineral, y el doble de ceniza fresca de leña; todos estos ingredientes se pasan por un cedazo fino. Mézclese todo en seco. Póngasele suficiente aceite crudo de linaza para que la mezcla sea tan delgada como la pintura. Aplíquese con una brocha de pintor. Se le puede dar color como las anteriores, teniendo cuidado de mezclar bien los colores antes de agregárselos. Sería conveniente dar dos manos; la primera delgada y la segunda espesa.

Otra pintura que se dice, dura de 15 á 20 años, es la siguiente: 23 kg de albayalde de superior calidad, 9.45 litros de aceite de linaza crudo, 230 gramos de secante, 23 kg de arena aguda cernida fina, 900 gr de tierra de sombra cruda. Agréguesele un poco de trementina; como ½ litro. Aplíquese con una brocha *ancha*.

**Cemento para coger las juntas** alrededor de las chimeneas, etc., etc. Albayalde en aceite, como se vende en cuñetes, mezclado con arena suficiente para formar una pasta dura que no corra. Se endurece al aire y resiste el calor, el frío y el agua. Se pueden pegar fuertemente pedazos de piedra con este cemento dejándolo secar bien durante algunos meses para que se endurezca.

**Lechada para trabajos al interior.** Según Mr. Downing, se hace más firme y permanente agregándole 2 lit de *aparejo* delgado á una lata de lechada, poco antes de usarla. El mejor aparejo para este objeto es el agua de cola. Debemos añadir que no se debe permitir, como comúnmente se hace, ponerle sal á la lechada. El papel pegado á un muro al que se le ha dado lechada salada, se humedece, se despega y se cae en tiempo de humedad. Se debe raspar la lechada, y luego cubrir las paredes con una ó dos manos de agua de cola para proteger el papel del efecto de la sal que se haya quedado adherida al encalado.

# VIDRIOS Y VIDRIERAS

Los vidrios para ventanas se venden por cajas. Cualquiera que sea el tamaño de las láminas, una caja contiene tan cerca de 50 pies cuadrados=4.64 m cuad de vidrio como lo permitan las dimensiones de aquéllas.

Se fabrican en láminas de cualquier tamaño según se pidan. El vidrio ordinario tiene más ó menos 1.6 mm de grueso, y se sobrentiende que tiene ese grueso mientras no se especifique mayor. El vidrio de *doble espesor* tiene casi 3.2 mm y su precio es 50% más que el de *espesor sencillo*. Por supuesto que es mucho más fuerte que aquél

Las láminas se fijan en los marcos con pasta hecha de blanco de España y aceite de linaza, y con pequeños pedazos de hojalata triangulares como de 6 mm por lado, que mantienen fijo el vidrio mientras se le pone la pasta y se le dejan para que le sirvan de mayor protección mientras la pasta está blanda.

Las mejores clases de vidrios americanos se hacen en las cercanías de Filadelfia, Boston y Pittsburg, etc., y son tan buenos para uso común como los fabricados en el extranjero; pero cuando se desea un alto grado de *belleza*, como en las ventanas de los pisos bajos de las casas lujosas, los escaparates de las tiendas, etc., se deben emplear los vidrios pulidos fabricados en Inglaterra, Francia y Alemania, aunque los precios por láminas de tamaño moderado son de 5 á 8 veces mayores que los mejores de espesor sencillo hechos en los Estados Unidos. Su superficie, perfectamente *lisa*, libre de reflejos deformadores, también los recomiendan como los mejores para cubrir dibujos, pinturas, etc.; sin embargo, si se emplean vidrios americanos cuidadosamente escogidos para este último objeto, pocos conocedores podrían distinguir la diferencia.

**Se fabrica un vidrio grueso especialmente para pisos**, hasta de 2½ cm de grueso y hasta de 1.27×2.75 m de superficie; también se hace para claraboyas de 6 á 12 mm de espesor. Este último se hace á la orden de cualquier tamaño hasta de 1 m×2.4 ó 3 m. Los tamaños pequeños se pueden conseguir *esmerilados*. La opacidad (esmerilado) impide la entrada de toda la luz del sol, y sin embargo difunde la luz en mucho mayor espacio.

**Resistencia del vidrio.** Tensión de 175 á 630 kg por cm cuad. Barras de Boston, probadas por el autor, 245 á 364. Resistencia á la trituración 420 á 700 kg por cm cuad. Transversalmente (según los ensayos del autor), vidrio de piso de 2½ ×2½ cm y 30 cm de luz, entre los apoyos, se rompe con una carga central de unos 77 kg; por consiguiente, es considerablemente más fuerte que el granito, excepto en lo que respecta á la trituración, en la cual los dos son casi iguales.

*Observación.* El vidrio para ventanas y otros, que contienen un exceso de potasa ó de soda, están expuestos á hacerse opacos con el tiempo, debido á la descomposición de aquellos ingredientes por las influencias atmosféricas.

## CUERDAS Y CABLES

**La resistencia** de las cuerdas varía mucho. Pedazos del mismo rollo pueden variar en 25%. La siguiente tabla se basa en un promedio de calidad **Manila**. El buen **cáñamo italiano** es considerablemente más fuerte. Se dice que el **alquitranado** en los cordeles disminuye su resistencia, y que cuando están á la intemperie, también les reduce su durabilidad. Creemos que su uso en aparejos verticales tiene por resultado disminuir su contracción y su expansión por el cambio de humedad y sequedad en la atmósfera. Algunos meses de **trabajo á la intemperie** debilita los cables de 20 á 50 por ciento.

(*N. del T.* — Hemos convertido la tabla del autor al sistema métrico.)

**Tabla de cuerdas de Manila.**

| Diám. | Circunf. | Peso por m. | Carga de ruptura. | Diám. | Circunf. | Peso por m. | Carga de ruptura. |
|---|---|---|---|---|---|---|---|
| mm. | mm. | kg. | kg. | mm. | mm. | kg. | kg. |
| 6.0 | 19.0 | .03 | 254 | 48.5 | 152.4 | 1.77 | 11580 |
| 8.1 | 25.4 | .05 | 355 | 52.6 | 165.1 | 2.07 | 13206 |
| 12.1 | 38.1 | .11 | 711 | 56.6 | 177.8 | 2.41 | 14831 |
| 16.1 | 50.8 | .20 | 1239 | 60.7 | 190.5 | 2.77 | 16457 |
| 20.2 | 63.5 | .31 | 1940 | 64.8 | 203.2 | 3.14 | 18082 |
| 24.3 | 76.2 | .44 | 2773 | 72.6 | 228.6 | 3.98 | 21333 |
| 28.2 | 88.9 | .60 | 3870 | 80.8 | 254.0 | 4.92 | 24583 |
| 32.3 | 101.6 | .79 | 5243 | 88.9 | 279.4 | 5.94 | 27839 |
| 36.3 | 114.3 | 1.00 | 6706 | 97.0 | 304.8 | 7.08 | 31091 |
| 39.4 | 127.0 | 1.23 | 8330 | 105.1 | 330.2 | 8.31 | 34342 |
| 44.4 | 139.7 | 1.49 | 9957 | 113.0 | 355.6 | 9.63 | 37590 |

**Cargas de trabajo.** Para cables de Manila, de 25 á 45 mm de diámetro, corriendo á varias velocidades sobre poleas de diámetros fijos, Mr. C. IV. Hunt (Trans. Am. Soc. Mechl. Engrs., vol. XXIII, 1901) da una tabla conteniendo aproximadamente los siguientes resultados de su experiencia. Carga de trabajo = C × resistencia máxima de un cable nuevo. D = diámetro mínimo de polea, en mm. (Convertidas por el T.)

| Velocidad. | m por min. | en trabajo de | C | Cable de 25 mm. D | Cable de 45 mm. D |
|---|---|---|---|---|---|
| Despacio... | 15 á 30 | grúa, martinete, etc., muelle. | .140 | 200 | 350 |
| Mediano... | 45 á 90 | de cantera, carga..... | .056 | 300 | 450 |
| Rápido.... | 120 á 240 | | .028 | 1000 | 1780 |

Estos cables se destruyen rápidamente. Un cable de 37 mm de diám se destruye cargando de 7,000 á 10,000 toneladas de carbón. Por otra parte, cables de transmisión de 37 mm, corriendo á la velocidad de 1,500 m por minuto y llevando 1,000 caballos de vapor por poleas de 1.50 y 5 m de diámetro, duran años.

Las cifras de Mr. Hunt para la resistencia máxima, basadas en pruebas hechas con cuerdas de Manila, de tres diferentes fabricantes y compradas en el mercado público, son prácticamente idénticas á las que damos en nuestra tabla anterior, así como las del prof. B. Kirsch, del Musco Imperial y Real de Tecnología Industrial, de Viena, citadas por Mr. Hunt.

## PESOS Y RESISTENCIAS DE LOS CABLES DE ALAMBRES

Cables fabricados por **John A. Roebling Sons Co., de Trenton, N. J.,** y otros. Los precios y los pesos dados se refieren á cables con el eje de cáñamo Cuando se fabrican con ejes de alambre los pesos aumentan en un 10%. El cable «**Hércules**» de **A. Leschen and Sons Rope Co., de San Luis, Mo.,** tiene una resistencia y precio 50% mayor que el cable de acero colado.

(*N. del T.* — Las tablas del autor convertidas al sistema métrico.)

| Marca. N.° | Diám. mm. | Peso por metro. kg. | Esfuerzo de ruptura aproximado* en toneladas de 2,000lbs=906kg. | | Diám. mín del tambor en metros. | | Precio en centavos por metro. | |
|---|---|---|---|---|---|---|---|---|
| | | | Hierro. | Acero colado. | Hierro. | Acero colado. | Hierro. | Acero colado. |
| **Cable típico de suspensión con 6 torcidos de 19 alambres cada uno** | | | | | | | | |
| 1 | 57 | 11.92 | 78 | 156 | 3.96 | 2.59 | 383 | 465 |
| 2 | 50 | 9.38 | 62 | 124 | 3.65 | 2.43 | 301 | 364 |
| 3 | 44 | 7.22 | 48 | 96 | 3.04 | 2.21 | 262 | 305 |
| 4 | 41 | 6.18 | 42 | 84 | 2.59 | 1.90 | 206 | 242 |
| 5 | 38 | 5.29 | 36 | 72 | 2.28 | 1.75 | 186 | 216 |
| 5½ | 34 | 4.47 | 31 | 62 | 2.13 | 1.67 | 157 | 183 |
| 6 | 31 | 3.65 | 25 | 50 | 1.98 | 1.52 | 131 | 150 |
| 7 | 28 | 2.98 | 21 | 42 | 1.82 | 1.37 | 108 | 124 |
| 8 | 25 | 2.35 | 17 | 34 | 1.60 | 1.21 | 85 | 98 |
| 9 | 22 | 1.78 | 13 | 26 | 1.47 | 1.06 | 65 | 75 |
| 10 | 19 | 1.32 | 9.7 | 19.4 | 1.21 | .91 | 52 | 59 |
| 10¼ | 16 | .92 | 6.8 | 13.6 | 1.06 | .68 | 39 | 45 |
| 10½ | 14 | .74 | 5.5 | 11.10 | .83 | .53 | 32 | 39 |
| 10¾ | 13 | .58 | 4.4 | 8.8 | .68 | .45 | 26 | 36 |
| 10*a* | 11 | .44 | 3.4 | 6.8 | .61 | .38 | 24 | 32 |
| 10*b* | 9 | .32 | 2.5 | 5.0 | .45 | .30 | 22 | 31 |
| **Cables de transmisión** ó tracción con 6 torcidos de 7 alambres cada uno | | | | | | | | |
| 11 | 38.1 | 5.29 | 34 | 68 | 3.96 | 2.59 | 167 | 196 |
| 12 | 34.92 | 4.47 | 29 | 58 | 3.65 | 2.43 | 141 | 167 |
| 13 | 31.75 | 3.65 | 24 | 48 | 3.27 | 2.21 | 118 | 141 |
| 14 | 28.57 | 2.98 | 20 | 40 | 2.89 | 1.90 | 95 | 118 |
| 15 | 25.4 | 2.35 | 16 | 32 | 2.59 | 1.75 | 75 | 91 |
| 16 | 22.22 | 1.78 | 12 | 24 | 2.28 | 1.52 | 57 | 72 |
| 17 | 19.05 | 1.32 | 9.3 | 18.6 | 2.05 | 1.37 | 45 | 52 |
| 18 | 17.46 | 1.11 | 7.9 | 15.8 | 1.82 | 1.21 | 39 | 44 |
| 19 | 15.87 | .92 | 6.6 | 13.2 | 1.60 | 1.06 | 32 | 36 |
| 20 | 14.28 | .74 | 5.3 | 10.6 | 1.37 | .91 | 26 | 29 |
| 21 | 12.7 | .58 | 4.2 | 8.4 | 1.21 | .76 | 21 | 24 |
| 22 | 11.11 | .44 | 3.3 | 6.6 | .99 | .68 | 18 | 21 |
| 23 | 9.52 | .32 | 2.4 | 4.8 | .83 | .61 | 14 | 18 |
| 24 | 7.93 | .22 | 1.7 | 3.4 | .76 | .53 | 12 | 14 |
| 25 | 7.14 | .18 | 1.4 | 2.8 | .68 | .45 | 10 | 13 |

**Notas sobre el empleo del cable de alambres,** por la Compañía Roebling.

Los cables con 19 alambres por torcido son los más flexibles y por consiguiente los más apropiados para **suspender** y **deslizarse.** Los otros son más rígidos y más apropiados para **vientos** ó **tirantes,** etc. Los cables de hierro y acero,

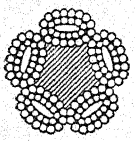
* **Para carga de trabajo y de seguridad,** tómese un quinto ó un séptimo de la de ruptura de acuerdo con la velocidad.

hasta de 76 mm de diámetro, se hacen á la orden. Los cables de ejes de henequén son más flexibles que los de eje de acero. Los cables de alambres no se deben **arrollar** ó **desarrollar** como los de cañamo. Cuando están en un carrete, éstos deben montarse en un eje ó en una mesa giratoria para ir desarrollando el cable. Cuando se envía en un bulto, sin carrete, arróllese el cable en el suelo como una rueda, y asi mismo se debe desarrollar. Evítese destejerlo y hacer dobleces pequeños. **Para conservar los cables de alambre**, pónganseles aceite de linaza crudo (que se puede mezclar con una cantidad igual de negro humo ó pintura obscura) con un pedazo de cuero de carnero, con los pelos hacia el cable. **Si se va á usar bajo el agua ó subterráneo**, agréguesele 36 lit de cal acabada de apagar ó un poco de aserrín para 1 barril de alquitrán. Hiérvase bien la mezcla y satúrese con ella el cable, mientras aquélla está caliente.

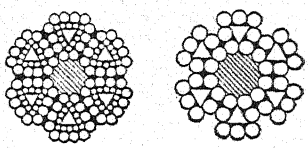
Los cables de **alambre galvanizado** para aparejos son más durables y más baratos que los de cáñamo, y no se estiran constantemente con los grandes esfuerzos. Su volumen es $^1/_6$ y su peso ½ del de cáñamo. Los cables de acero colado en crisol son mucho más durables que los de hierro. Deben mantenerse bien lubricados.

## Cable patentado de acero achatado y estirado.

**Fabricado por A. Leschen and Sons Rope Co., de San Luis, Mo**



Cables de suspensión.



Cables de tracción y de transmisión.

*(Obs. del T.* — Hemos convertido las tablas que trae el autor al sistema métrico.)

| Diám en mm. | Peso medio por metro en kg. | Carga de ruptura * en ton de 2,000 lbs = 906 kg. | | | Diám mín del tambor en metros. | | | Lista de precios por metro, en centavos. | | | Diám en mm. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | « Hércules » **. | Acero colado de crisol. | Hierro sueco. | « Hércules ». ** | Acero colado de crisol. | Hierro sueco. | « Hércules » **. | Acero colado de crisol. | Hierro sueco. | |
| **Cables de suspensión.** | | | | | | | | | | | |
| 57 | 13.78 | 260 | 176 | 75 | 3.65 | 2.59 | 3.50 | 934 | 596 | 498 | 57 |
| 51 | 11.17 | 211 | 140 | 66 | 3.35 | 2.43 | 3.27 | 738 | 472 | 393 | 50 |
| 44 | 8.04 | 168 | 109 | 54 | 2.74 | 2.21 | 2.74 | 682 | 396 | 341 | 44 |
| 41 | 7.07 | 140 | 94 | 45 | 2.59 | 1.90 | 2.28 | 511 | 314 | 268 | 41 |
| 38 | 5.96 | 124 | 81 | 40 | 2.43 | 1.75 | 2.05 | 449 | 282 | 242 | 38 |
| 35 | 5.06 | 106 | 69 | 34 | 2.28 | 1.67 | 1.90 | 367 | 239 | 205 | 34 |
| 32 | 4.17 | 84 | 56 | 28 | 2.13 | 1.52 | 1.75 | 291 | 195 | 170 | 31 |
| 28 | 3.42 | 67 | 47 | 21 | 1.82 | 1.37 | 1.60 | 232 | 164 | 141 | 28 |
| 25 | 2.68 | 56 | 38 | 17 | 1.52 | 1.21 | 1.44 | 196 | 129 | 111 | 25 |
| 22 | 2.01 | 40½ | 29 | 13 | 1.37 | 1.06 | 1.21 | 160 | 98 | 85 | 22 |
| 19 | 1.49 | 32 | 21 | 9 | 1.21 | .91 | 1.06 | 123 | 78 | 68 | 19 |
| 16 | 1.08 | 22½ | 15 | 6 | 1.06 | .68 | .91 | 91 | 60 | 50 | 15 |
| 14 | .80 | 19 | » | » | .91 | » | .76 | 82 | » | 42 | 14 |
| 13 | .65 | 13½ | 9½ | 4 | .83 | .45 | .61 | 68 | 47 | 34 | 12 |
| **Cables de tracción y de transmisión.** | | | | | | | | | | | |
| 32 | 4.17 | 80 | 54 | 27 | 2.81 | 2.21 | 2.89 | 288 | 177 | 147 | 32 |
| 28 | 3.42 | 64 | 45 | 22½ | 2.43 | 1.90 | 2.59 | 229 | 147 | 119 | 28 |
| 25 | 2.68 | 53 | 36 | 18 | 2.05 | 1.75 | 2.36 | 190 | 114 | 95 | 25 |
| 22 | 2.01 | 38 | 27 | 13½ | 1.82 | 1.52 | 2.05 | 144 | 90 | 72 | 22 |
| 19 | 1.49 | 30 | 20 | 10 | 1.60 | 1.37 | 1.82 | 114 | 67 | 57 | 19 |
| 16 | 1.08 | 21 | 14 | 7½ | 1.37 | 1.06 | 1.44 | 82 | 45 | 41 | 16 |
| 13 | .65 | 13 | 9 | 4½ | 1.14 | .76 | 1.06 | 53 | 32 | 27 | 13 |

* Carga de trabajo = 1/5 × carga de ruptura.

** « Hércules » es un cable de marca industrial patentada « fabricado de un acero estirado y patentado especial, que se hace únicamente para esta marca de cable ».

# PAPEL

1 manilla 24 pliegos. 20 manillas una resma.

**Dimensiones del papel de dibujo.**

| | Pulgs. | | Pulgs. |
|---|---|---|---|
| Antiquarian (78.7×132) *.... | 31×52 | Super Royal (48.3×68.6).... | 19×27 |
| Doble elefante (66×101.6)... | 26×40 | Royal (48.3×61)..... | 19×24 |
| Atlas (66×86.4)..... | 26×34 | Medium (43.2×81.3)..... | 17×32 |
| Imperial (53.3×76.2)..... | 21×30 | Demy (38.1×50.8)..... | 15×20 |
| | | Cap (33×43.2)..... | 13×17 |

El papel de dibujo inglés es más fuerte y mejor que el americano. El de Whatman tiene una gran reputación; lo hay, sin embargo, de diferentes calidades. Cuando el papel está pegado en muselina, la diferencia en la calidad no es tan importante. Para el papel en rollos, las fábricas alemanas son las mejores. Se importa poco de otras fábricas.

**Tanto el papel blanco como el de color**, para el uso de los ingenieros, se hace en rollos continuos sin empates. Ancho, 91.4, 106.7, 137.2, 147.3 y 157.5 cm; longitud ordinaria, 36.57 m; pero se pueden obtener, encargándolos, hasta de 365.75 m de largo ó más. Éste puede comprarse ya pegado en muselina, en rollos de 9 á 36 m de largo.

El papel de **cartuchos** ó el papel **de modelos** se venden en rollos largos de la misma longitud que el papel blanco, montado ó no en tela; ancho, hasta 1.37 m. Color de ante claro.

**Papel de calcar.** La mayor parte del que se vende, ya sea del país (E. U.) ó del extranjero, es tan fácil de romper, que relativamente es de poca utilidad.

El papel de pergamino, de 94 y 96½ cm de ancho, en rollos de 18.28 y 21 m, es mejor, pero no coge la tinta perfectamente.

**La tela de calcar** es muy preferible al papel de calcar, por su gran resistencia. Ancho, 45.7, 76.2, 91.4 y 106.7 cm; longitud hasta 22 m.

**El papel para perfiles** se hace en anchos de 22.9 y 50.8 cm y en pliegos sencillos ó en rollos largos continuos.

**Papel para secciones** transversales montado ó sin montar, así como el de calcar y la tela, se venden en pliegos ó en rollos, cuadriculados en quintos, octavos, décimos y duodécimos de pulgada, ó en milímetros.

**Colores.** Desde que se introdujo el uso de las impresiones azules, se hacen pocos dibujos en colores, excepto para efectos arquitectónicos; pero los colores se usan con ventaja en las impresiones de líneas negras calcadas, pág. 1076. Un buen dibujante necesita pocos colores, como rojo de almagre, azul de Prusia, laca ó carmín rojo claro, tierra sombra quemada, tierra siena quemada, tierra siena cruda, gutagamba, ocre romano y verde vegetal. No se debe comprar sino la mejor tinta de China. Las pastillas de colores deben siempre secarse en papel, después de haberlas lavado con agua, y para esta operación se debe emplear poca agua, y luego aumentarla.

**Lápices.** Los *legítimos* de A. W. Faber, n.º 2, 3 y 4, son muy buenos. Su dureza aumenta con el número. Los n.º 3 y 4 sirven para la cartera de mensuras, dependiendo la preferencia de la clase del papel. Sus lápices marcados con letras son de la mejor calidad y mejores para el dibujo. « H » significa « duro » (« *hard* »), « B », « blando » (« *soft* »). Los grados de dureza ó blandura se indica por el número de « H » y de « B ». « F » (intermedio) corresponde al n.º 3. Los lápices americanos de Dixon son buenos. El dibujante en su oficina debe tener una lima plana, ó un pedazo de papel de esmeril pegado en una tira de madera, en la cual afile con facilidad la punta del lápiz, después de usar la cuchilla.

* *N. del T.* — Los números entre paréntesis son las dimensiones en centímetros.

# IMPRESIONES EN AZUL, ETC.

**Art. 1.** (a) Para obtener los mejores resultados, se debe evitar toda **exposición innecesaria** ya sea del papel sensibilizado ó de la solución, á la luz del sol ú otra luz blanca.

(b) La **limpieza** es de la mayor importancia. Los envases en que se hacen y se mezclan las soluciones deben estar escrupulosamente limpios, y, si se lavan con jabón, se deben enjuagar muy cuidadosamente con agua limpia. Deben mantenerse llenos de agua cuando no estén en uso. La presencia de álcali libre de cualquier clase, es fatal para el buen resultado, y destruye inmediatamente el color azul de la impresión. Véase art. 19 (b). No se debe permitir que las soluciones se pongan en contacto con hierro.

**Art. 2.** (a) **La solución** que se emplea para sensibilizar el papel para impresiones en azul es generalmente la de *ferro*cianuro de potasio (ó sea prusiato rojo de potasa) * y amoniocitrato de hierro (citrato de hierro y amoníaco) en agua.

(b) Generalmente se **disuelven las sales por separado** y se mezclan luego las dos soluciones. La sal de potasio se debe quebrantar en partes muy pequeñas. La sal de hierro es generalmente pura y se disuelve muy pronto. Se puede conservar indefinidamente en su estado sólido si está perfectamente seca, pero absorbe muy fácilmente la humedad y se hace pegajosa é inútil; la solución se puede espesar á los pocos días, ya esté sola ó mezclada con la solución de potasio. Por esto se debe preparar (en un cuarto obscuro) en pequeñas cantidades según se necesite.

**Art. 3.** (a) El siguiente es un promedio de varias **recetas** que han dado excelentes resultados:

Solución A. 1 onza de prusiato rojo de potasa para 6 onzas de agua, ó 2½ onzas de sal para una pinta (0.47 lit) de agua.

Disuélvase muy bien y filtrese. La solución se puede filtrar suficientemente en algodón crudo y mucho más pronto que en papel.

Solución B. 1½ onzas de amoniocitrato de hierro para 6 onzas de agua, ó 4 onzas de sal para ½ litro de agua.

Disuélvase muy bien. Filtrese, á menos que la solución esté perfectamente clara.

(b) Manténganse las dos soluciones en frascos distintos provistos de tapas de vidrio, en un lugar obscuro hasta que se vayan á usar. Luego mézclense en partes iguales y filtrese la mezcla. Cuídese de que ninguna partícula sólida del prusiato rojo pase á la solución doble. Debe rechazarse cuando su color marrón se cambie en verde azulado.

(c) La solución combinada **cuesta** á los *amateurs* de 1 á 2 centavos por onza. Unas 4 onzas **bastarán** para cubrir 9 m cuad de papel.

(d) Si se agregan unas pocas gotas de una solución fuerte de amoníaco á la de citrato, B, hasta que el olor se haga perceptible, la adición de una solución saturada de **ácido oxálico** en agua á la doble solución, **precipita la impresión en tiempo nublado.** 10% de la solución de ácido oxálico aumenta la rapidez de la impresión como 2½ veces; 20%, 5 veces; 30 %, 10 veces; pero con más de 20 % es difícil obtener líneas blancas claras. En tiempo de sol la diferencia es mucho menos marcada. (*Engineering News*, dic. 15, 1892.)

**Art. 4.** (a) Cuando no se requiere un trabajo muy fino, cualquier **papel** bien preparado y suficientemente duro para soportar el lavado servirá. Para trabajos de importancia empléese papel de contextura uniforme, fina, lisa y de superficie dura, libre de substancias químicas dañosas. Si la solución penetra debajo de la superficie, una parte de los agentes químicos se queda en el papel, á pesar del lavado, y dañaría el resultado. Se hacen muchas clases de papeles para este objeto. El de Sajonia (alemán) y el de Rives (francés) se consideran como de los mejores. El papel de Johannot y Steinbach da buenas impresiones, pero no es muy fuerte. Los papeles de lino de Weston y de Escocia son más fuertes y dan excelentes impresiones. Antes de sensibilizar una gran cantidad de papel de una nueva clase, pruébese una pequeña hoja. La tela de lino para sensibilizar también se vende en las tiendas de materiales para fotografía y en las de materiales de ingeniería.

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* *No* el *ferrocianuro* ó prusiato *amarillo*.

**Art. 5.** (a) La solución se aplica (en el cuarto obscuro, por supuesto) á un solo lado del papel. Esto se hace á veces haciendo *flotar* el papel sobre la solución, teniendo cuidado de que ésta no toque el *respaldo* del papel. Se sostiene el papel por dos esquinas opuestas diagonalmente y la línea que se forma se deja descansar en la superficie del líquido, luego se dejan caer las esquinas una después de otra. Se levanta una mitad á la vez para ver si se ha formado alguna burbuja de aire debajo de él. Si es así, se saca arrastrando sobre la solución aquella mitad del papel en la cual esté la burbuja, mientras se mantiene la otra mitad suspendida sobre el líquido. La operación de *flotar* se hace en uno ó dos minutos y luego se saca el papel haciéndolo resbalar sobre una de las orillas del depósito, y escurriendo el exceso de líquido. Este procedimiento requiere una cubeta mayor que el papel y el interior de ella debe no sólo ser impermeable, sino también á prueba de la acción química de la solución. Esta manipulación requiere un gran cuidado.

**Art. 6.** (a) La solución se aplica generalmente por medio de una *brocha* suave y ancha (semejante á la que se usa con los libros de copiar cartas) ó una *esponja* grande y suave libre de arena ú otros granitos de suciedad.

**Art. 7.** (a) Al aplicar la solución, puede colocarse el papel sobre una tabla cubierta de un *hule* suave, el cual se debe secar muy bien después de sensibilizar cada hoja, para evitar que se manche la otra.

(b) **La operación** debe hacerse rápidamente, de manera que ninguna parte de la hoja se seque antes que toda la superficie haya sido cubierta con la solución. Por esta razón, si las hojas son muy grandes las deben manejar dos personas. Cúbrase la hoja primero con pases de la esponja ó brocha mojada, muévase á lo largo del papel, é inmediatamente, con pases en ángulos rectos y con la esponja ó la brocha exprimida, de manera que la solución quede uniforme y finamente distribuída sobre toda la superficie. Lávese la esponja inmediatamente en el cuarto obscuro.

**Art. 8.** (a) Se cuelga el papel para que se **seque** en el cuarto obscuro, sujeto por agarradoras de resorte, ú otro medio, que no tengan partes de hierro. Las hojas pequeñas se pueden colgar por una punta; las grandes, por dos ángulos adyacentes ó por dos ó tres puntos más de la misma orilla, según su tamaño, teniendo cuidado de doblar ligeramente aquella orilla, de manera que el papel no se estire al secarse. Si se cuelgan las hojas en una barra, la solución se secará en el doblez con desigualdad. A fin de que los blancos de la impresión salgan claros, el aire debe estar caliente, para que el papel pueda secarse prontamente é impedir así que la solución penetre mucho.

**Art. 9.** (a) Téngase mucho cuidado de que el papel esté perfectamente seco antes de usarlo ó guardarlo y que se conserve *seco* y *en la obscuridad* hasta que se quiera usar. Si se prepara y resguarda cuidadosamente, conservará su sensibilidad por mucho tiempo, pero los mejores resultados se obtienen con papel fresco; no es conveniente conservarlo por más de un mes ó dos.

**Art. 10.** (a) **El papel de calcar**, ó la tela de calcar, debe ser de un tinte azulado y delgado (el papel amarillo retarda la impresión; véase el art. 15), debe ser tan transparente como fuese posible. Debe preservarse, antes y después del dibujo, de una larga exposición á la luz, la cual tiende á hacerlo opaco.

(b) Tanto antes como después de dibujado se debe mantener en plano ó arrollado, y no doblado, pues los dobleces hacen difícil que el dibujo se ponga en perfecto contacto con el papel sensibilizado al hacerse la impresión.

**Art. 11.** (a) **El dibujo** ó trazado debe hacerse con la mejor tinta de China, muy negra. Agregándole un poco de gutagamba ó amarillo cromo aumenta su opacidad. Las líneas trazadas con amarillo cromo y gutagamba se imprimen muy bien; pero el azul de Prusia ó el carmín deben hacerse más opacos agregándoles un poco de blanco de China ú otro blanco opaco. Pónganse los dibujos ante una luz fuerte, para descubrir en ellos las partes débiles de las líneas.

**Art. 12.** (a) **La impresión** consiste en exponer el papel sensibilizado á la acción de la luz, colocando el dibujo entre la luz y la superficie sensibilizada. **La luz eléctrica de arco** imprime más despacio que la luz solar directa, pero tiene la ventaja de su constancia en todo tiempo y á todas horas y de la fijeza de su posición. (Véase art. 16 a).

(b) Colóquese el marco con la cara perpendicular á los rayos de la luz, lo más que se pueda, y téngase cuidado de que no caigan sobre el dibujo sombras de árboles, edificios, etc.

(c) El papel debe manejarse en una luz débil para recortarlo ó ponerlo en el marco.

**Art. 13.** (a) Para obtener un contacto exacto entre el dibujo y el papel sensibilizado, se colocan generalmente en un *chassis* ó marco de impresión. (Véase art. 15.) Las partes esenciales de un marco común son : el marco mismo, una lámina de

vidrio transparente para el paso de la luz y un respaldo acolchonado, que por medio de agarradoras y resortes, comprimen las dos hojas de papel muy unidas, contra el vidrio.

(b) Se coloca el trazado en el marco con el lado del dibujo hacia el vidrio y luego el papel sensibilizado en contacto con el dibujo por su cara sensibilizada. (Pero véase art. 16 b.) Por último, se coloca el respaldo acolchonado en el marco.

(c) Generalmente se hace el respaldo en dos piezas unidas por una visagra y cada una provista de un resorte, de manera que una mitad se pueda levantar para examinar el progreso de la impresión, mientras que la otra mitad permanece fija por la agarradora y mantiene fijos el papel sensibilizado y el trazado.

(d) Empleando un marco abierto en ambos extremos se pueden usar largas tiras de papel sensibilizado, por partes, arrollando el resto en los extremos del marco y envolviéndolo para protegerlo de la luz.

(e) Es muy importante que todos los marcos tengan vidrios suficientemente gruesos para soportar la presión necesaria á fin de obtener el contacto exacto de los dos papeles; deben ser de excelente calidad y sin defectos que puedan obstruir ó refractar imperfectamente la luz. (Véase art. 15., más abajo). Debe limpiarse muy bien el vidrio antes de imprimir.

(f) Los marcos de impresión mejorados tienen cojinetes de caucho y aire comprimido en lugar de acolchonado de lana al respaldo. En otros se hace la presión necesaria por medio del vacío producido entre el trazado y el vidrio por una bomba.

(g) Los marcos de impresión se compran en los almacenes del ramo. Sus precios, incluso el vidrio, varían de $2 por marcos de 25×30 cm á $30 ó $45 por los de 90×150 cm. Se pueden también obtener marcos montados en rodillos, con accesorios para exponerlos fuera de las ventanas, á precios que varían con las dimensiones y requisitos.

(h) Para impresiones grandes en azul, el prof. E. C. Cleaves, de la Universidad de Cornell, usa, en lugar de un marco, un cilindro de madera cubierto con fieltro, y que gira sobre su eje. Sobre este cilindro se extienden el trazado y el papel sensibilizado por medio de un sistema de agarradoras apropiado, y se hace girar el cilindro á la luz del sol. Este método evita el uso del vidrio. Necesita, naturalmente, mayor exposición que el método común. (*Trans. Am. Soc. Mech. Eng.*, vol. VIII, pág. 722.)

(i) Para impresiones aún mayores, el profesor R. H. Thurston extiende los dos papeles sobre una tabla delgada, la cual se cimbra y se mantiene en esa posición, sosteniendo así los papeles distendidos sobre el lado convexo. Por este método también se prescinde del vidrio, y como la curvatura de la tabla y los papeles es ligera, todo el papel queda expuesto á la vez á la luz. (*Trans. Am. Soc. Mech. Eng.*, vol. IX, pág. 696.)

**Art. 14.** (a) **El tiempo que se requiere para la exposición** varía según el color, la dirección é intensidad de la luz, de la opacidad y grueso del papel de calcar, de la negrura del dibujo, de los materiales, del esmero que se haya tenido en la sensibilización del papel y de la frescura de éste; desde dos ó tres minutos hasta horas y aun días. Aproximadamente se puede decir que en plena luz solar, en Filadelfia, unos tres minutos bastan generalmente desde el mediodía hasta las 2 p. m. y 10 minutos á las 10 a. m. ó las 4 p. m.; en la sombra, 13 á 40 minutos, á mediodía; pero no se puede establecer regla fija. La práctica debe decidir en cada caso. Si el respaldo del marco está dividido en dos ó más partes, se puede examinar el proceso de tiempo en tiempo.

(b) Si se ha hecho buen uso de una tinta perfectamente opaca, se puede imprimir el fondo de un azul muy obscuro; pero el exceso de exposición para la impresión hace el fondo negruzco y luego adquiere un tinte sucio. (Véase art. 17 c y d). Los trazados en tinta pálida deben imprimirse muy suavemente, á fin de que las líneas puedan permanecer blancas, y sería conveniente emplear para ellas una luz débil ó protegerlas con papel de seda ó un vidro esmerilado. (Véase art. 18 a.)

**Art. 15.** (a) Para obtener impresiones perfectamente claras, el lado del papel en que está el trazado debe estar en **perfecto contacto** con la superficie sensibilizada del papel, especialmente si, como sucede con la luz del sol, la dirección de la luz es variable; pues si alguna separación apreciable existe entre ellos, como cuando se imprime á través de cartón (véase art. 16 abajo), las sombras producidas por las líneas del trazado se mueven sobre la superficie sensibilizada según cambie la dirección de la luz, produciéndose una impresión borrosa. En la generalidad de los casos, sin embargo, es prácticamente imposible colocar de esta manera las dos superficies, pues esta posición da una impresión invertida con respecto á la derecha

ó izquierda *. Por consiguiente, se recomienda, en el art. 10 (a), el uso de un papel ó tela de calcar que sea delgado. Por la misma razón es imprescindible que los dos papeles estén firme y uniformemente comprimidos contra el vidrio.

**Art. 16.** (a) Empleando una luz de posición permanente con relación á la superficie del trazado, como una lámpara eléctrica de arco, es posible, prolongando la exposición por horas y aun días, obtener impresiones azules de **trazados hechos sobre papel grueso de dibujar**, y aun en una cartulina bristol (*bristol board*).

(b) Se puede lograr el mismo resultado con la luz del sol, ya sea colocando el original de espaldas al vidrio, y el papel sensibilizado (que debe ser muy delgado) de espaldas á la luz solar, ó colocando el marco de impresión en el fondo de una caja honda y angosta, de manera que la luz pueda caer directamente sobre el marco, sólo cuando esté aproximadamente en posición paralela á los costados más largos de la caja. Para imprimir rápidamente, se debe mantener la luz solar de lleno sobre el marco cambiando frecuentemente la posición de la caja.

**Art. 17.** (a) La impresión después de haber tenido suficiente exposición se quita del marco y se **lava muy bien** con agua limpia, por ambos lados, hasta que se desarrolla por completo el color azul característico.

(b) El lavado debe hacerse en una cubeta con el fondo plano, y que sea más grande que la mayor de las impresiones que se van á lavar; debe tenerse cuidado de no dañar la superficie de las impresiones frotándolas muy duro, ó por dobleces muy marcados.

(c) Se puede hacer el lavado más rápidamente, y también aclarar algo las impresiones muy obscuras ó « demasiado expuestas » empleando agua caliente, de 32° á 38° C.

(d) También se pueden aclarar las impresiones obscurecidas por demasiada exposición sumergiéndolas en agua, ligeramente alcalina, con amoníaco. En este baño adquieren un tinte purpúreo que á poco se debilita. En el momento preciso, que sólo se aprende en la práctica, se debe detener la acción alcalina pasando rápidamente la impresión por una solución de 1 parte de ácido hidroclórico (« muriático ») (H. Cl.) en 100 de agua.

(e) Continúese el lavado hasta que el agua quede enteramente clara y cuélguese la impresión bien extendida para que se seque.

**Art. 18.** (a) Después del lavado, la aplicación de una solución de 1 á 5 por ciento de ácido hidroclórico, ó ácido oxálico, en agua, intensifica el color azul, y por consiguiente hace más visible las impresiones pálidas ó que no hayan tenido suficiente exposición; pero éstas deben volverse á lavar después en agua pura. El ácido hidroclórico aplicado *antes* del lavado, ó á impresiones lavadas imperfectamente, hace que las *líneas* aparezcan azules.

**Art. 19.** (a) **Para borrar** una línea (blanca) en una impresión azul, repásese la línea con la solución sensibilizadora empleando para ello un pincel limpio ó una **pluma de ave**. Esto debe hacerse á una luz débil y luego se debe exponer de nuevo toda la impresión y volver á lavar.

(b) **Se agregan líneas blancas** á las impresiones azules generalmente con blanco de China; pero antes se debe quitar el color azul hasta que se vea el blanco del papel, por medio de una solución saturada de lejía concentrada (soda cáustica ó potasa) ó de carbonato de soda **, ó carbonato de potasa, aplicada con una pluma limpia y fina, casi seca. Si se aplica demasiada cantidad, se esparce muy rápidamente. Aun cuando la pluma esté perfectamente limpia, la superficie adquiere un tinte amarillento comparado con el blanco del papel. Las soluciones de carbonato obran más despacio que la lejía, pero no son menos seguras, y no dañan la superficie, mientras que la lejía quema mucho. El agua de cal vendida por los boticarios hace poca ó ninguna impresión sobre el color azul. Si se desean **líneas rojas en vez de blancas**, mézclese con la solución de soda ó de potasa tinta común de escribir color carmín, en la cantidad que dé el color deseado; ésta se fija por tanteos.

**Art. 20.** (a) Las impresiones azules que hayan de someterse á mucho uso deben **montarse** en tela, ó hacerse la impresión desde el principio en tela de calcar sensibilizada.

**Art. 21.** (a) **Los procedimientos que dan un fondo blanco**, con líneas azules ó negras, son generalmente tan complicados que están fuera del alcance de la

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* Una impresión así invertida, puede, por supuesto, leerse fácilmente en un espejo. Esto se hace generalmente con los dibujos de la Oficina de Patentes.

** El mismo resultado da el carbonato (« soda de lavar ») que el bicarbonato.

mayor parte de los ingenieros. Sus resultados son, también, inseguros, aun hechos por expertos, pues á menudo queda el fondo poco blanco.

(b) **El papel Vandyke** (de Eugene Dietzgen Co, de Chicago) y el **Maduro**, dan excelentes **líneas color pardo** en papel bueno, liso y duro. El « Nigrosine » y otros llamados de impresiones negras, dan generalmente líneas purpúreas efímeras sobre un fondo gris, algo brillante, en un papel quebradizo, que no dura.

(c) **El procedimiento Le Clère** da excelentes impresiones en **líneas negras**. Las líneas son perfectamente negras y permanentes, y las impresiones se hacen en un papel de dibujo bueno, cuyo color y duración no se afectan por el procedimiento. También da bellas líneas azules en papel semejante al anterior.

# EXPLOSIVOS MODERNOS

**Art. 1. La mayor parte de los explosivos** que durante los últimos años han reemplazado á la pólvora, se componen de una substancia en estado de polvo, saturada parcialmente de nitroglicerina, que es un fluido producido por la mezcla de la glicerina con ácidos nítrico y sulfúrico.

**Art. 2. La nitroglicerina** pura, á 15°5 C, tiene un peso específico de 1.6. Es inodora, casi ó enteramente incolora, y tiene un gusto dulce y acre. Es venenosa, aun en muy pequeñas cantidades, y su manejo puede producir dolores de cabeza. Es insoluble en el agua. Se inflama á unos 152° C, y no estando encerrada arde sin producir explosión; á menos que sea en tal cantidad que alguna parte de ella, antes de ponerse en contacto con el aire, se caliente hasta el punto de producirla, la cual se efectúa próximamente á los 193° C.

La nitroglicerina y los polvos que la contienen **estallan siempre por medio** de una percusión aguda. Véanse arts. 36, etc. Después de hecha la nitroglicerina se requiere mucho cuidado para despojarla completamente de los ácidos sobrantes que le quedan al prepararla. La presencia de estos ácidos en la nitroglicerina líquida, ó en los polvos que la contienen, la hacen susceptible de descomposición espontánea, la cual por el alza de la temperatura aumenta el peligro de la explosión.

**Art. 3. La N-G se hiela** á 7°2 C, más ó menos. En este estado es **muy difícil la explosión** y debe deshelarse **gradualmente**, dejándola por tiempo suficiente en una habitación que tenga un calor moderado, ó poniendo el envase que la con-

tiene dentro de un segundo envase que contenga agua caliente á no más de 38° C; pero nunca debe exponerse á un calor intenso, tal como colocándola delante del fuego, ó sobre una estufa ó caldera. Se hacen fulminantes extrafuertes para estallar la nitroglicerina y sus polvos cuando está helada.

**Art. 4. La N-G**, debido á su incompresibilidad, está **expuesta á explosiones por percusiones accidentales**. Esta circunstancia, y la facilidad que tiene para filtrarse, la hacen poco á propósito para su transporte y manejo. Por lo que rara vez se usa en su estado líquido en las canteras y otros lugares donde haya que aplicar explosivos. En las regiones petrolíferas de Pensilvania se usa mucho **en los pozos de petróleo** para aumentar la descarga. Con este objeto se mete en tubos cilíndricos de hojalata, de 2½ á 12 cm de diámetro, llamados bombas de torpedo. Éstas se suspenden y se bajan al pozo por medio de una cuerda ó alambre arrollado en un carrete y se destruyen al estallar la carga. Tienen 2½ cm menos de diámetro que la perforación del pozo y contienen de 1 á 20 lit, ó sea de 2.30 á 30 kg de N-G. Su extremo inferior es puntiagudo para facilitar su paso á través del petróleo ó agua que pueda contener el pozo. Cuando se requiere una carga mayor de 30 kg, se colocan dos ó más de estas bombas en el pozo, una sobre la otra, de manera que la punta inferior cónica de cada una de ellas ajuste la parte superior de la siguiente. En este caso se hace estallar la nitroglicerina por medio de uno ó varios fulminantes colocados en la parte superior de la carga antes de bajarla al pozo. Cuando la carga está en su lugar, se hacen estallar los fulminantes por medio de la electricidad comunicada por alambres conductores, como en el art. 37, ó (según el método más usado) dejando caer un peso sobre ellos.

Cuando en un pozo han estallado varios torpedos, y de este modo se ha formado una cavidad tan grande que el espacio que rodea al torpedo disminuye los efectos de la explosión sobre las paredes del pozo, entonces se coloca directamente la N-G en el pozo, bajando un cilindro lleno de ella, provisto de una disposición automática que permite su salida cuando llega el cilindro al fondo del pozo, y se la hace estallar por medio de un torpedo suspendido de una cuerda y provisto de cápsulas fulminantes colocadas en su parte superior. Estas cápsulas se hacen estallar por medio de la electricidad, ó deslizando hacia abajo, por la cuerda, pesos de plomo ó de hierro. Cuando la roca se compone de vetas ó filones, se encierra la N-G en envases cilíndricos cortos de hojalata, que se bajan á la cavidad y se hacen estallar por medio de un torpedo. La nitroglicerina se emplea también para aumentar la corriente de los manantiales de agua. Se comprende naturalmente que aquélla no puede usarse en perforaciones **horizontales ó inclinadas** hacia arriba, como las que se hacen en los túneles, etc.

**Art. 5.** La nitroglicerina hace explosión tan de repente que **no es necesario atacar mucho la carga**. La tierra ó la arena mojada, ó aun el agua, bastan al efecto. Esta circunstancia y el hecho de que no la afecta el agua y de que es más pesada que ella, la hacen particularmente **útil para obras debajo del agua**, ó para perforaciones que contienen agua, con tal de que la roca no tenga grietas que permitan el escape de la N-G. Cuando la roca está **compuesta de vetas ó filones**, debe encerrársela herméticamente en cilindros; estos cilindros ó cajas, sin embargo, dejan necesariamente algún espacio entre la roca y el explosivo, lo cual disminuye considerablemente el efecto de este último.

**Art. 6. La gran fuerza explosiva de la nitroglicerina** se debe en parte al volumen muy grande de gas en que se convierte, por la explosión, una pequeña cantidad de ella, y en parte á la rapidez con que se efectúa aquella conversión, de modo que los gases se ponen en libertad casi instantáneamente, mientras que con la pólvora la producción de los gases se efectúa en mayor tiempo. La acción repentina de la explosión aumenta el efecto, no solamente porque aplica toda su fuerza en un instante, sino también porque se calientan mucho los gases producidos y aumentan así considerablemente su volumen.

**Art. 7.** El estado líquido de la nitroglicerina es conveniente, por que **permite llenar completamente la perforación**, de modo que no queden espacios vacíos con los cuales se disminuye la fuerza de la explosión. Por otra parte, la forma líquida tiene también su desventaja, porque cuando se usa sin envase en rocas estratificadas, parte de ella se filtra y se queda sin hacer explosión y hasta sin notarse, pudiendo más tarde causar una explosión accidental.

**Art. 8. La N-G se guarda en envase de hojalata ó de loza.** Si está bien limpia de ácidos, no daña á la hojalata. Para su transporte, estos envases se embalan en cajas con aserrín, ó en cajas forradas por dentro, y éstas se cargan en carretas. Los ferrocarriles no las reciben como flete.

**Art. 9.** Cuando la N-G y sus componentes han hecho explosión **completamente, los gases resultantes** no son molestos, pero los que provienen de una explosión **incompleta**, como generalmente resulta, ó de la combustión, son muy dañinos.

**Art. 10.** Por conveniencia, damos el nombre de « **dinamita** » á cualquier explosivo que contenga N-G mezclada con un absorbente granular; de « **dinamita verdadera** » á aquella en que el absorbente de la nitroglicerina sea « Kieselguhr » * ú otro polvo **inerte** que no toma parte alguna en la explosión; y de « **dinamita falsa** » á aquellas en las cuales el absorbente mismo contiene otros explosivos que no sea N-G.

**Art. 11. El absorbente**, por su condición compresible, y granular, **obra como cojín**, resorte protector (*cushion*), de la N-G y la defiende de las percusiones y del peligro consiguiente á una explosión accidental.

La N-G no cambia su naturaleza porque sea absorbida : se hiela, arde, estalla, bajo las mismas condiciones de presión, temperatura, etc., que cuando está en forma líquida. El efecto de **cojín** ó defensa del absorbente sólo aumenta la dificultad de producir la presión suficiente para causar la explosión. La absorción de la N-G en la dinamita hace á ésta apta para ser usada en perforaciones **horizontales**, ó inclinadas hacia **arriba**.

**Art. 12.** La N-G y la dinamita hacen explosión mucho más ligero cuando están limitadas por una materia rígida como en un envase **metálico**, ó por las paredes de una perforación hecha en la roca, que cuando están encerradas en una substancia **que cede**, como la madera. Por lo tanto, el hecho de que la dinamita, no siendo líquida, puede embalarse en cajas de madera, la hace menos peligrosa que la N-G, la cual debe guardarse en envases de loza ó metal.

**Art. 13. La dinamita verdadera debe contener por lo menos 50 por ciento** de nitroglicerina. De no ser así, el absorbente la protegería demasiado y sería muy difícil hacerla estallar. Las dinamitas falsas, al contrario, pueden contener un tanto por ciento tan pequeño de N-G como se desee; algunas de ellas no contienen sino 15%. Las substancias explosivas que se agregan en la falsa dinamita contienen grandes cantidades de oxígeno, que se desprende con la explosión, y contribuye á efectuar la combustión completa de los gases nocivos.

**Art. 14. Las dinamitas que contienen un tanto por ciento grande de N-G** estallan (como la N-G líquida, art. 6) muy repentinamente, con tendencia á destrozar ó romper en pedazos pequeños la roca cercana. Son las más útiles en la roca muy dura; la **dinamita n.º 1**, ó la que contiene 75 por ciento de N-G, se considera que, en esta clase de roca, **tiene más ó menos 6 veces la fuerza de un peso igual de pólvora de cañón.**

**Para rocas blandas ó descompuestas**, arena y tierra, **las clases menos fuertes** de dinamita, ó las que contienen un percentaje menor de N-G, son las más á propósito. Hacen explosión con menos rapidez, y su tendencia es más bien á levantar grandes masas de roca que á fracturar pequeñas masas de ella. Por esto, se parecen más bien en su acción á la pólvora de cañón.

Debe calcularse con discreción, en cada caso, la **calidad y cantidad** del explosivo que ha de usarse. Cuando no haya inconveniente en fracturar la roca en pedazos pequeños, ó cuando se desea hacerlo así para mayor facilidad del acarreo, es más útil emplear las más fuertes y destructivas. Cuando se quiere sacar la roca en grandes masas, como en las canteras, son preferibles las menos fuertes.

Para trabajos difíciles en roca dura, y para explosiones submarinas, se usan las calidades más fuertes, que contienen de 70 á 75 por ciento de N-G. Una pequeña carga de ésta produce el mismo efecto que una carga grande de la clase menos fuerte y, por supuesto, no requiere una perforación tan grande.

En los trabajos submarinos la explosión rápida no es disminuída por el agua.

Para trabajos comunes de ferrocarriles ó para perforaciones ordinarias de túneles, excavaciones de minas de minerales, se emplea la clase mediana que contiene 40% de N-G; en las canteras, una que contenga 35%; para volar troncos, árboles, estacas, etc., 30%, y para arena y tierra, 15%.

**Art. 15.** La dinamita, lo mismo que la N-G, **puede hacerse estallar fácilmente debajo del agua** con tal que esté sumergida de manera que no pueda **esparcirse; pero una permanencia prolongada en el agua la daña.**

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* « Kieselguhr » es una piedra calcárea silícica terrosa, compuesta de los restos fósiles de pequeñas conchas. Cada conchita obra como un receptáculo minúsculo para la nitroglicerina. Kieselguhr se encuentra en Hanóver (Alemania) y en New Jersey.

En las clases más fuertes, el agua, por su mayor afinidad con el absorbente, expulsa la N-G. En las clases inferiores el agua puede disolver las sales empleadas como explosivos adicionales.

**Art. 16.** En las dinamitas que contienen un tanto por ciento elevado de N-G, esta última está expuesta á rezumarse ó á filtrarse, sobre todo en época de calores, y á estallar por cualquiera percusión accidental. Existe el mismo peligro, aun cuando el tanto por ciento de la nitroglicerina sea pequeño, si el absorbente no tiene sino un pequeño poder de absorción y por consiguiente se satura fácilmente.

**Art. 17. La dinamita verdadera** se parece al azúcar moscabado. Sus **propiedades** son generalmente las de la N-G que contiene. De manera que arde á 177° C más ó menos y quema libremente. Se hiela á 7°2 C y entonces es difícil hacerla estallar. No hace explosión por el frotamiento ó por percusión ordinaria; requiere en trabajos comunes una cápsula fuerte ó detonador, que contenga pólvora fulminante (véanse arts. 36, 38, etc.). Puede, sin embargo, estallarse con una mecha de pólvora de cañón, atacada fuertemente y encendida por medio de una mecha de seguridad.

**Art. 18. La carga debe llenar la sección transversal de la perforación** tan completamente como sea posible. Si no hay agua dentro del agujero, debe hacerse al cartucho una incisión antes de colocarlo, de modo que el polvo pueda salir y llenar la perforación, ó simplemente vaciarlo.

**Art. 19. Para volar el hielo** en su propio lugar, se abren en él orificios, se ata un número de cartuchos de dinamita (uno de ellos provisto de su cápsula fulminante) y se hacen descender de .30 á 1.50 m dentro del agua. Se hacen estallar tan pronto como sea posible después de la inmersión, para evitar el peligro de que se hielen. Las detonadoras eléctricas (art. 37, etc.) son las mejores para obras debajo del agua.

**Art. 20.** La dinamita es útil para **quebrar piezas** de metal, como cañones viejos, maquinarias desechadas, « salamandras » (masas de escoria endurecida) de hornos de fundición, etc. En los cañones, naturalmente, la dinamita hace explosión en el alma. En otras piezas se hacen generalmente pequeños agujeros para introducirla; pero las planchas de metal, aun de un espesor considerable, pueden romperse, haciendo simplemente estallar la dinamita sobre su superficie.

**Art. 21. Para volar árboles ó troncones**, se hacen estallar uno ó más cartuchos en un orificio practicado en el tronco ó raíz ó debajo de esta última. Así estalla el tronco y las raíces. Puede tumbarse un árbol practicando dentro de él perforaciones de pequeño radio á distancias cortas é iguales en una línea horizontal alrededor de su circunferencia y por medio de una batería eléctrica (arts. 37, etc.) que haga estallar simultáneamente una pequeña carga de dinamita en cada taladro hecho. En un árbol pequeño basta amarrar un solo cartucho largo alrededor del tronco y hacerlo estallar.

**Art. 22. Las estacas pueden volarse del mismo modo que los árboles**; puede también hacerse una perforación en el eje para alojar el cartucho, ó éste puede simplemente amarrarse á cualquiera altura.

**Art. 23.** Las clases superiores de dinamita, lo mismo que la nitroglicerina, **no es preciso que se ataquen mucho.** Usese siempre una **barra de atacar de madera, nunca de metal**, para cualquiera explosivo. Es peligroso sacar una carga de dinamita que « arde lentamente » sin estallar. Sáquesele casi todo el **taco**, dejando sólo algunos cm de grueso encima de la carga; sobre ésta colóquese otro cartucho que contenga el detonador, y hágase la prueba nuevamente. Véase detonadores eléctricos, arts. 37, etc. Si la dinamita está helada, lo mismo que la N-G, entonces debe deshelarse *gradualmente*, dejándola en un cuarto caliente, lejos del fuego, ó colocándola en un envase metálico que luego se coloca dentro de otro envase que contenga agua caliente. El agua no debe estar más caliente de lo que pueda soportar la mano. De lo contrario la N-G puede separarse del absorbente. La N-G contenida en la dinamita puede helarse sin pegarse ó unirse á las moléculas del absorbente, en cuyo caso el polvo es blando al tacto. Una **carga excesiva** de N-G ó de dinamita está expuesta á quemarse y de este modo á desperdiciarse, dando salida á gases nocivos.

**Art. 24. La dinamita se vende en cartuchos cilíndricos, cubiertos de papel**, de 20 á 50 mm de diámetro y de 15 á 20 cm ó más de largo. Se suministran de cualquier tamaño, según pedidos, y están embalados en cajas que contienen 25 ó 50 lbs (11.3 á 22.6 kg) cada una. Las hileras de cartuchos están separadas por aserrín.

**Art. 25.** Algunas **Compañías de ferrocarriles** se niegan á transportar nitroglicerina ó dinamita en cualquier forma. Otras llevan dinamita bajo ciertas

restricciones, basadas sobre leyes de Estado; exigiendo que esté seca (es decir, que no esté filtrando ó sudando nitroglicerina); que las cajas y los carros que la contienen deben marcarse claramente con algunas palabras de precaución, como « explosivos », « peligroso », etc; que los cartuchos estén empaquetados de tal modo en las cajas y éstas colocadas en los carros de tal manera, que ambos descansen sobre sus costados y que no haya peligro de que las cajas se volteen dentro del carro; que las cápsulas fulminantes, etc., no se lleven en el mismo carro junto con la dinamita, etc.

**Art. 26. Se fabrica una gran variedad de clases de dinamita.** Varían (en general muy poco) en la composición del absorbente y en el modo de fabricación. Cada fabricante hace generalmente un número de clases que contienen tantos por ciento diferentes de nitroglicerina, etc., dando á sus polvos un nombre de fantasía.

**Art. 27.** La siguiente **tabla de explosivos** hecha por « The Repauno Chemical Company, Thompson's Point, N. J., Office Wilmington, Delaware », y conocida bajo el nombre de « **Atlas** » **powders** (pólvoras « Atlas »), da el tanto por ciento de N-G que entra en cada una de ellas.

| Marca. | Tanto por ciento de nitroglicerina. | Marca. | Tanto por ciento de nitroglicerina. |
|---|---|---|---|
| A | 75 | D+ | 33 |
| B+ | 60 | | |
| B | 50 | E+ | 27 |
| C+ | 45 | E | 20 |
| C | 40 | | |

**Los absorbentes contienen** en la marca « A » 18 por ciento de pulpa de madera y 7 por ciento de carbonato de magnesia; en la marca « C » (clase media), 46 por ciento de nitrato de soda (soda y sal de nitro), 11 por ciento de pulpa de madera y 3 por ciento de carbonato de magnesia; en la marca « E », 62 por ciento de nitrato de soda, 16 por ciento de pulpa de madera, etc., y 2 por ciento de carbonato de magnesia.

**Art. 28. La pólvora « Miner's Friend »** (Amiga del Minero), contiene nitrato de soda, pulpa de madera, resina y carbonato de magnesia. Se hiela á 5°5 C, y es, entonces, como otras dinamitas, difícil para estallar. Cuando se usa debajo del agua, no deben romperse los cartuchos, porque los ingredientes sufren con el contacto directo del agua. La pólvora « Hecla » es de clase inferior. Es de forma granulada, lo mismo que la pólvora ordinaria de barrenos; pero se dice que es más fuerte. Se emplea como un sustituto de la pólvora ordinaria.

**Art. 29. La pólvora « Giant »** (Gigante) es propiamente una dinamita, y contiene 75 por ciento de nitroglicerina y 25 por ciento de Kieselguhr (tierra de conchas fósiles) obtenida cerca de la fábrica en New Jersey. La clase más baja, marca « M », contiene 20 por ciento de nitroglicerina. Se daba originalmente el nombre de « pólvora gigante » á la dinamita en general.

**Art. 30.** Hay otras marcas, á saber: « Pólvora de Hércules », y « Pólvora R.R.P. de Judson », que reemplaza la pólvora ordinaria de barreno. Está empaquetada en sacos impermeables de papel de 6¼, 12½ y 25 libras (2.83, 5.66 y 11.34 kg) cada uno, y éstos están embalados en cajas de madera que contienen 50 lbs (22.6 kg) cada una. La **dinamita Judson F.F.F.** es de calidad superior, viene en cartuchos de forma usual, en cajas de á 50 lbs (22.6 kg).

**Art. 31.** Los cartuchos de « **Rackarock** » se dice que no contienen nitroglicerina y que son enteramente inexplosivos, hasta que se les sumerge, por algunos segundos, en un líquido inexplosivo suministrado por los fabricantes. Al sacarlos del líquido se dejan parados 15 minutos, después de los cuales se pueden usar en cualquier tiempo. Se encienden lo mismo que la dinamita y pueden usarse debajo del agua. Los fabricantes sostienen que Rackarock « se aproxima en fuerza á la nitroglicerina, y que es más fuerte que la dinamita ».

**Art. 32. Los siguientes explosivos se fabrican y se usan en Europa,** pero no han sido importados todavía con regularidad á los Estados Unidos.

**El algodón pólvora comprimido** es un algodón sumergido en una mezcla de ácidos nítrico y sulfúrico, luego reducido á una pulpa fina; se le da la forma de

discos de 2½ á 5 cm de grueso y de 2 á 5 cm de diámetro, ó más grandes. Generalmente se emplea mojado, para mayor seguridad. En este estado requiere fulminantes bastante fuertes. Hablando en general, es casi tan fuerte como la dinamita n.° 1, pero su efecto es menos *desgarrador*. Como es menos pesado que la dinamita, requiere perforaciones más grandes; y debido á su rigidez, no puede colocarse tan fácilmente, y no se adapta tan completamente á la perforación. Seco es muy inflamable; pero no estando encerrado, arde sin peligro. No contiene líquido y por lo tanto no se hiela ni exuda y su manejo no es peligroso.

**Art. 33. La Tonita** se compone de algodón pólvora dividido finamente, mezclado con nitrato de barita. Está comprimida en cartuchos en forma de velas, que tienen en un extremo un espacio para recibir un detonador con fulminante de mercurio. Los cartuchos pesan más ó menos lo mismo que la dinamita, y generalmente se hacen impermeables.

**Art. 34. Forcite, Lithofractor y Dualin** son productos extranjeros de explosivos de nitroglicerina. En el Dualin el absorbente consiste en aserrín. Presenta un volumen mayor que la dinamita y requiere perforaciones más grandes.

**Art. 35. La gelatina explosiva** es una substancia transparente de color amarillo pálido y elástica, y está compuesta de 90 por ciento de nitroglicerina y 10 por ciento de algodón pólvora. Es menos sensible á la percusión, fricción ó compresión que la dinamita y no la afecta el agua. Su peso específico es 1.6; arde al aire libre. Para su completa explosión se requiere un fulminante especial. La adición de una pequeña cantidad de alcanfor la hace menos sensible todavía y aumenta su fuerza explosiva. El alcanfor se evapora en cierto grado.

En algunos experimentos hechos sobre la fuerza de diferentes explosivos para aumentar la capacidad de una pequeña cavidad practicada en un pedazo de plomo, la gelatina explosiva produjo un aumento 50% mayor que el causado por la dinamita n.° 1. En roca dura la diferencia hubiera sido probablemente mayor. El aumento fué un 10% menor que el causado por la nitroglicerina.

**Art. 36. La cápsula fulminante, ó detonador,** usada con la mecha de seguridad ordinaria para estallar la nitroglicerina y la dinamita, es un cilindro hueco de cobre, de más ó menos 6 mm de diámetro y de 25 á 50 mm de largo. Contiene de 15 á 20%, ó más, de fulminante de mercurio, mezclado con otros ingredientes y hecho un cemento, que llena la parte cerrada de la cápsula. Se dice que las cápsulas son de « fuerza sencilla », « fuerza triple », etc., según la cantidad del explosivo que contienen.

La punta de la mecha, cortada á escuadra, se inserta en la extremidad abierta del fulminante, á suficiente profundidad para que toque la mixtura que contiene. Al hacer esto debe tenerse cuidado de no rozar esta última. El cuello de la cápsula se aprieta cerca de la extremidad abierta, para sujetar bien la mecha. La cápsula, con la mecha fija de este modo, se mete entre la carga de nitroglicerina, ó dinamita, teniendo cuidado de que la mecha no se ponga en contacto con el explosivo, porque éste se quemaría y se inutilizaría. Si se usa un cartucho de dinamita, se coloca primero la mecha con su cápsula. La parte superior ó cuello del cartucho se amarra con una cuerda alrededor de la mecha y de este modo está ya listo para colocarlo en la perforación y estallarlo.

**Art. 37. La caja electromagnética de Siemens,** hoy en uso general, se compone de una caja de madera del tamaño de una caja de teodolito. En el exterior tiene dos postes metálicos con tornillos para la conexión de los dos alambres que conducen al detonador. De la parte superior de la caja sobresale un mango con la punta de una barra vertical. Esta barra, más ó menos tan larga como alta es la caja, está construída de manera que se deslice en ella hacia arriba y hacia abajo, y está provista de una cremallera que engrana en un pequeño piñón en el interior de la caja. Cuando se va á prender el barreno, se tira de la barra hacia arriba por el mango hasta donde puede llegar, luego se aprieta hacia abajo con rapidez, hasta el fondo de la caja. En su descenso, pone en acción, por medio del piñón, una máquina electromagnética que se encuentra en el interior de la caja. Ésta produce una corriente eléctrica que aumenta en intensidad con el movimiento de la barra hacia abajo, pero que está limitada á un corto circuito del alambre dentro de la caja, hasta que el extremo inferior de la barra toca un resorte situado cerca del fondo de la caja, el cual rompe el corto circuito y obliga á la corriente á pasar por los dos alambres conductores que la llevan de los dos postes del exterior de la caja á la cápsula fulminante ó detonador colocado en la carga.

**Art. 38. La cápsula** que se usa en esta máquina es semejante á la usada con la mecha de seguridad (art. 36), con la diferencia de que ésta tiene el fondo cerrado con un cemento de azufre por el cual pasan los alambres conductores de la máquina

eléctrica. Los extremos de estos alambres entran en la mezcla fulminante de la cápsula. Tienen una separación de 3 mm; pero están conectados por un alambre de platino tan delgado, que la corriente de la máquina lo calienta hasta el rojo; su calor enciende el fulminante y hace estallar la cápsula.

**Art. 39. Para hacer estallar simultáneamente un número de barrenos** (aumentando así su efecto), se coloca en cada uno de ellos una carga con su cápsula de platino, y uno de los alambres cortos, fijado á cada cápsula, se pone en conexión con uno de los de la próxima cápsula, de modo que en cada serie de cápsulas hay una punta libre de un alambre corto. Cada una de estas dos puntas se fija al extremo de uno de los alambres conductores, poniendo toda la serie « en circuito ». Cuando las perforaciones están demasiado separadas unas de otras para conectar las cápsulas, como se ha dicho por el alambre corto unido á ellas, las extremidades de este último se unen por un **alambre aislado**.

**Art. 40. La máquina electromagnética** pesa unos 7 kg. Puede hacer estallar unas 12 cápsulas simultáneamente.

Las cápsulas para las cebas ordinarias y para hacer estallar por la electricidad, los alambres, y, además, las máquinas eléctricas, etc., las venden la mayor parte de los que comercian en explosivos, máquinas de barrenar, etc.

**Art. 41.** Solamente por medio de la electricidad pueden inflamarse simultánea y convenientemente un número de perforaciones. El aparato eléctrico de estallar es especialmente útil para explosiones debajo del agua, donde la mecha ordinaria, sobre todo á grandes profundidades, está expuesta á humedecerse é inutilizarse.

Cuando una máquina eléctrica no logra inflamar la carga, se sabe que ésta ya no puede estallar hasta que no se repita la operación. Por lo tanto no hay que perder tiempo ni correr ningún riesgo.

## PÓLVORA DE CAÑÓN

La fuerza explosiva de la pólvora es de unos 2,800 kg por cm cuad. Su **peso** medio es más ó menos el del agua, ó sea 1 kg por litro. En los trabajos de cantera, un metro cúbico de roca maciza en su sitio original (ó m ó m 1.9 m cúb amontonada después de sacada de la cantera), requiere de 113 á 340 gramos. En rocas duras en exceso y mal situadas para sacarlas de la cantera, un metro cúb macizo puede requerir de 450 á 900 gramos. En algunas de las más grandes voladuras de roca y de más satisfactorios resultados, alcanzados en la construcción del Tajamar de Holyhead, en Wales (en que varios millares de libras de pólvora se hacían estallar generalmente por medio de la electricidad en una sola explosión), se **desprendieron** de 1.5 á 3 metros cúbicos macizos por cada libra de pólvora; pero en muchos casos no se volaron más de ¾ á 1½ m cúbs. En la construcción de túneles y pozos se requiere de 1 á 2½ kg por metro macizo; pero más generalmente de 1 ¼ á 2 ¼. La roca blanda, en parte descompuesta, requiere frecuentemente más que las duras. Se vende generalmente en cuñetes de 11.4 kg.

### Peso de la pólvora por 30 cm de profundidad de barreno.

(*N. del T.* — Hemos convertido la tabla del autor al sistema métrico.)

| | | | | | | |
|---|---|---|---|---|---|---|
| **Diám del barreno en mm** | 25 | 30 | 37 | 50 | 63 | 75 |
| **Peso de la pólvora en kg** | .142 | .227 | .312 | .567 | .907 | 1.276 |
| **Diám del barreno en mm** | 90 | 100 | 114 | 150 | 163 | 180 |
| **Peso de la pólvora en kg** | 1.758 | 2.268 | 2.892 | 3.572 | 4.309 | 5.131 |

# APUNTAMIENTOS SOBRE INGENIERÍA LEGAL

Por el doctor J.-L. ARISMENDI, abogado, miembro de la Academia de Ciencias Políticas y Sociales de Venezuela, de las Sociedades de Derecho Comparado, de Paris, de la Derecho Internacional, de Washington, del Instituto Ibero Americano de Derecho Comparado de Madrid y Catedrático de la Universidad de Caracas.

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## Nota del Traductor.

Los ingenieros y jefes de obras en el estudio y construcción de éstas, son quizás entre todos los profesionales los que más necesitan conocer los preceptos del Derecho que se relacionan con el ejercicio de su profesión. Desde el emplazamiento de sus construcciones, durante la ejecución y luego en la explotación, están expuestos a vulnerar el derecho ajeno por la falta de conocimientos, siquiera generales y someros de aquellas prescripciones. Cuantos litigios han resultado de tales deficiencias.

Es por esto que hemos creido conducente y eficaz agregar en esta nueva edición las muy breves apuntaciones, excesivamente sintéticas, que ha tenido la bondad de suministrarnos el notable Jurisconsulto venezolano, Doctor José Loreto Arismendi, cuya ilustrada colaboración jurídica hemos solicitado y a quien reiteramos nuestra gratitud por su trabajo.

Los principios expuestos por el Doctor Arismendi están basados sobre el Derecho Francés, en atención a que éste es el que ha servido generalmente de modelo para calcar sus leyes las naciones de habla española.

Así como, por la índole de este Manual, no se entra en razonamientos y conexiones extensas constitutivos de un estudio completo de la técnica de la ingenieria dando solo las fórmulas y la exposición sintética de las teorías y principios, así también, en el apéndice jurídico de que tratamos, sólo se formulan mui brevemente algunos dictámenes que, íntimamente ligados con las materias de esta obra sirvan como de advertencias útiles para orientar y provocar consultas y más amplios estudios, cuando fuere necesario.

## Contrato de arrendamiento de obra.

Cuando un ingeniero, o cualquiera otra persona, según los casos, se encarga de hacer una obra para otro, puede convenirse en que preste únicamente su trabajo o su industria, o bien que, ademas de ésta, suministre tambien los materiales. — En el primer caso el contrato constituye, sin ninguna duda, un arrendamiento de obra puro y simple; pero, en el segundo, hay que distinguir, si la materia suministrada por el ingeniero es la parte principal y no un simple accesorio del trabajo porque, el contrato tiene entonces el carácter de una venta. Es arrendamiento de industria y no venta, el contrato por el cual un ingeniero se encarga de construir, aun suministrando sus propios materiales una obra sobre el terreno perteneciente al propietario para quien construye.

Las obligaciones del locador (que en este caso es el ingeniero) consisten :

*a*) Ejecutar la obra de que está encargado;

*b*) Terminarla en el plazo convenido, so pena de daños y perjuicios, a menos que el retardo no se deba a un caso de fuerza mayor o a un hecho del locatario;

*c*) A ejecutar las obras segun los principios de la Ciencia.

El locador puede ser condenado a hacer inmediatamente las refacciones necesarias debidamente comprobadas para que la obra quede libre de defectos y tenga las cualidades que requiere una buena ejecución de los trabajos.

El locador es responsable del hecho de las personas que emplea en la ejecución de la obra. El empresario responde especialmente del hecho de los sub-empresarios con los cuales ha contratado las diversas partes de la obra que está encargado de ejecutar.

El dueño (que en este caso es el conductor o locatario), es decir, aquel para quien se construye la obra, está obligado principalmente a pagar el precio convenido. Dicho precio puede determinarse, sea de una manera invariable por el conjunto de la obra, sea por medio de una cuenta detallada de los trabajos ejecutados con fijación del precio de cada artículo.

El ingeniero o empresario, no puede exigir, salvo convenio especial, el pago de la obra, sino cuando esté enteramente terminada; y, si estuviere defectuosa, despues de practicada la refacción necesaria para perfeccionarla. El tiene un privilegio para el pago de su crédito sobre el valor del inmueble, y está garantizado por un derecho de retención sobre el objeto del contrato.

Cuando se ha estipulado un precio fijo por la construcción de la obra, no puede el ingeniero o empresario exigir ningun aumento so pretexto de estar mas caros los materiales y la mano de obra, ni por cambio o aumentos hechos sobre el plano primitivo, no aprobados previamente por el propietario del suelo.

## Trabajos públicos.

Se entiende por trabajos públicos los que son emprendidos con un fin de interés general por el Estado o por alguna persona moral que le esté asimilada, sobre este punto, por la ley o la jurisprudencia. Tales son los trabajos de construcción, modificación, reparación y conservación de los caminos, puertos, canales, rios navegables, ferrocarriles, lineas telegráficas, fortificaciones, establecimientos militares, etc.

Los trabajos ejecutados por los empresarios o concesionarios subrogados en los derechos de la Administración, tienen el mismo carácter que si fueran ejecutados por ella misma. Asi los trabajos ejecutados por las compañias concesionarias de ferrocariles, teniendo por objeto la creación o conservación de las vias férreas y de sus anexos, son trabajos públicos.

**La ejecución de los trabajos públicos** necesita la formación previa de planos y presupuestos, sucesivamente aprobados, desde el punto de vista técnico, por la autoridad competente, segun que se trate de trabajos del Estado, de los departamentos, de las comunas o de los establecimientos públicos. Es menester luego que los trabajos sean regularmente autorizados.

Caso de que, para la ejecución de los trabajos se necesite disponer de la propiedad privada, debe procederse a la expropiación, por causa de utilidad pública, de acuerdo con la ley.

El contrato de trabajos tiene el carácter de arrendamiento de obra, por el cual un ingeniero o empresario se obliga a ejecutar ciertos trabajos mediante un precio determinado. Son necesarios los documentos o piezas siguientes :

*a*) Planos y presupuestos; *b*) Pliego de condiciones con la descripción detallada

de los trabajos; *c*) Resumen de cuentas indicando el precio que debe pagarse por cada unidad de medida de los diferentes trabajos por ejecutar; *d*) Detalle estimativo de los gastos para la ejecución de los trabajos descritos en el presupuesto; *e*) Avalúo de la cantidad de cada naturaleza de obra y el volumen aproximado.

Los trabajos públicos pueden ser ejecutados por la Administración directamente, o bien por vía de concesión o por contrato. Todos los tratos para trabajos del Estado en la forma de adjudicación pública o de convenciones, están sometidos a las condiciones establecidas en el Memorial de los trabajos.

**Obligaciones relativas a los materiales.** — Los materiales para las construcciones deben ser de buena calidad y consistencia conforme a las reglas de la Ciencia. El empleo de éllos debe ser precedido de una verificación y aceptación por el ingeniero en Jefe o por sus encargados, y, la recepción de los materiales, hace fe de la buena calidad de éstos, hasta prueba en contrario.

Los materiales malos o defectuosos pueden ser rechazados y dejarse por cuenta del Empresario, quien no puede reclamar su valor ni los gastos de transporte.

Si ya han sido empleados los materiales, no obstante su mala calidad, pueden hacerse demoler por el Empresario las construcciones en que hayan entrado para reemplazarlos por otros de buena calidad que él mismo debe suministrar.

El empresario no puede hacer ningun cambio en los proyectos, y debe inmediatamente reemplazar, por orden del ingeniero, los materiales o reconstruir las obras cuyas dimensiones o disposiciones no sean conformes al Presupuesto o a las órdenes del servicio. Sin embargo, los ingenieros pueden conservar los trabajos verificados, sin que el empresario tenga derecho, en este caso, a ningún aumento de precio en razón de las dimensiones mas fuertes o de mayor valor de los materiales. Al contrario cuando estas dimensiones sean mas débiles, debe sufrir una disminución de precio, a menos que la Administración haya hecho imposibles las verificaciones a las cuales el empresario tenia derecho, segun el presupuesto, o que los materiales no hayan sido empleados por órden del ingeniero.

**Plazo para la ejecución de los trabajos.** — El empresario debe ejecutar los trabajos dentro del plazo convenido, y, en defecto de cláusula expresa sobre este punto, dentro de un plazo proporcional a la naturaleza e importancia de la obra.

Las cláusulas penales establecidas a este respecto en la contrata o pliego de cargas, no pueden ser aplicadas al empresario sino cuando el retardo proviene de su falta, y no cuando es imputable a la Administración o a las dificultades excepcionales que no habian podido preverse.

**Fijación de precios.** — El empresario puede pedir un precio mas elevado del convenido, siempre que en virtud de las órdenes recibidas en el servicio se haya aumetando el volumen de los trabajos, agravándose para él las condiciones estipuladas; pero si ningun precio ha sido indicado previamente para una obra imprevista o por cambios verificados en las obras previstas para variar la naturaleza de éstas, el ingeniero o empresario tiene el derecho de hacer fijar un precio especial, segun el de las obras análogas.

**Obligaciones relativas a los obreros.** — El empresario debe tomar por empleados y jefes de talleres hombres capaces de ayudarlo y reemplazarlo en caso de necesidad en la conducción de los trabajos. El ingeniero tiene sobre este personal y sobre los obreros un derecho de *control*.

El empresario tiene algunas obligaciones relativas al número de los obreros que debe conservar en los talleres, a la policía de los trabajos, al pago periódico de los salarios, etc., y está sometido a la responsabilidad de los accidentes sobrevenidos a los obreros en la ejecución de los trabajos.

**Modificación en los trabajos.** — La Administración, contrariamente al derecho común, puede, por motivos de interés general, hacer algunas modificaciones a los trabajos en ejecución. Estas modificaciones pueden consistir en un aumento o disminución del volumen de los trabajos, o en un cambio en cuanto a la importancia de ciertas obras o en la ejecución de otras nuevas.

En los trabajos de puentes y calzadas, si el aumento no excede de la sexta parte del montante de la obra, el empresario puede solamente reclamar el montante de los trabajos suplementarios; pero, si excede, tiene el derecho a rescindir el contrato sin indemnización. El aumento del sexto se calcula, segun los precios del presupuesto, sin tener en cuenta las rebajas, así como tampoco el precio de trabajos que hayan sido el objeto de un contrato distinto.

La rescisión debe ser demandada dentro de los dos meses siguientes a la notificación de la orden de servicio al empresario que es el que tiene ese derecho. La Administración no puede prevalerse del aumento de mas de un sexto para confiar una parte de los trabajos a otro; ella debe en este caso indemnizar al empresario. Rescindido el contrato, el Estado, si lo juzga util, puede adquirir el material, pero el empresario no puede obligarlo a ello.

**Indemnización en caso de fuerza mayor.** — Contrariamente a los principios generales de los artículos 1788 y siguientes del Código civil, las pérdidas, averías o daños ocasionados por fuerza mayor pueden, salvo estipulación contraria en el pliego de condiciones o contrato, dar lugar a una partida a favor del empresario. En esta materia, se entiende por fuerza mayor, de una manera general, un acontecimiento calamitoso de la naturaleza, que era imposible prever en el momento de la celebración del contrato y que hace los trabajos mas difíciles y costosos que lo que hubieran sido en condiciones normales, tales como las tempestades, lluvias torrenciales, crecida extraordinaria de los ríos, epidemias que hayan llevado la desorganización a los talleres y el encarecimiento de la mano de obra, las huelgas, etc.

Los hechos de guerra no entran, generalmente, en los casos de fuerza mayor; ellos dan lugar simplemente a la rescisión de las obligaciones respectivas de los contratantes. Esto mismo resultará cuando en ausencia de todo acontecimiento extraordinario e imprevisto, ocurra una elevación excepcional en el precio de la mano de obra, a menos que este aumento no sea causado por una falta de la Administración.

**Medidas coercitivas contra el empresario.** — Cuando el empresario no se conforme con las condiciones de su contrato, ni observe las disposiciones del Memorandum contentivo de las órdenes de servicio, la Administración puede usar de medidas coercitivas que consisten : 1º en tomar ella misma directamente la ejecución de los trabajos; 2º en la readjudicación de éstos en subasta pública; 3º en la rescisión o resolución del contrato. Entre estas medidas puede escoger libremente la que mas le convenga.

**Recepción provisoria de los trabajos.** — Inmediatamente despues de terminados los trabajos se debe proceder a la recepción provisoria de éstos, con asistencia del empresario. La recepción provisoria no puede, por regla general, ser tácita ni presumirse por la toma de posesión de los trabajos independientemente de circunstancias que la hagan equivalente a la recepción.

**Plazo de garantía.** — La recepción provisoria tiene por efecto hacer correr el plazo llamado de garantía, y el empresario tiene el derecho de exigir a la Administración llevar a cabo dicha recepción, una vez terminados los trabajos.

A falta de estipulación expresa entre las partes, el plazo de garantía es de seis meses, a partir de la fecha de la recepción provisoria, para cierta clase de trabajos, y de un año para las obras de arte. — Durante estos plazos, el empresario permanece responsable de sus obras y está obligado a conservarlas.

**Recepción definitiva.** — Esta tiene lugar a la expiración del plazo de garantía; pero ella no resulta necesariamente del solo vencimiento de este plazo, sino que debe emanar de un acto de la Administración en la misma forma que la recepción provisoria.

La expiración del plazo de garantía, no implica la recepción definitiva de los trabajos, sino que permite al empresario poner en mora a la Administración para proceder a dicha recepción, a efecto de que élla no pueda formular reclamaciones ulteriores por los trabajos de que ya ha tomado posesión sin reservas. No obstante este procedimiento y toma de posesión, la Administración, cuando el empresario ha incurrido en falta, modificando los planes, puede rehusar la recepción definitiva hasta que no se restablezcan las obras conforme a éstos, y hasta puede operar la retención del saldo de la cuenta cuando tal circunstancia esté debidamente comprobada.

La recepción definitiva dá, al empresario, el derecho de reclamar el saldo de lo que le es debido; la restitución de la caución cuando la hubiere entregado y el reembolso de lo retenido como garantía, a condición de justificar que ha cumplido con todas sus obligaciones provenientes de la ejecución de las obras.

La recepción definitiva, tiene por efecto libertar al empresario de toda responsabilidad, fuera de la decenal prevista por los artículos 1972 y 1270 del Código Civil.

**Honorarios de los ingenieros y arquitectos.** — Estos reciben a título de remuneración, ora un sueldo fijo, ora algunos emolumentos proporcionales, a las erogaciones ocasionadas por las obras. Cuando el ingeniero o arquitecto ha cumplido todos los actos de su misión o no ha podido cumplirlos por el hecho de la Administración, tiene derecho a la totalidad de sus honorarios.

Si no ha podido cumplir todas sus obligaciones, como, por ejemplo, si ha muerto antes de haber procedido al arreglo de cuentas, los honorarios deben reducirse proporcionalmente. El abono de los honorarios no tiene lugar, en principio, sino por las erogaciones que han sido hechas conforme a las previsiones del presupuesto y no por los trabajos que no hayan sido previstos en él, a menos que el pliego de condiciones no le haya dejado bastante latitud para suplir las omisiones del presupuesto y proveer a las necesidades que se presenten en el curso de los trabajos.

Las faltas de que el arquitecto puede ser responsable, pueden ocasionar la reducción y aun la supresión de sus honorarios.

Cuando los proyectos sobre las obras hayan sido abandonados, el arquitecto o ingeniero, tiene derecho, en ausencia de una convención en contrario, a percibir algunos honorarios por los planos, proyectos y presupuestos que ha elaborado, a menos que la inejecución de éstos no haya provenido de su falta, como en el caso, por ejemplo, de haber sido rechazado el proyecto por demasiado costoso.

Caso de revocación del mandato remunerado por algunos abonos proporcionales, el ingeniero o arquitecto, tiene derecho, por resolución del contrato, a una indemnización equivalente a sus gastos y a los beneficios de que ha sido privado, a condición de que la revocatoria no sea motivada por las faltas que él haya cometido. En este caso no tiene mas derecho que a los honorarios devengados por los trabajos reconocidos útiles.

## Responsabilidad de los ingenieros, arquitectos o empresarios.

**Responsabilidad decenal.** — La responsabilidad de los empresarios, ingenieros o arquitectos, existe : *a*) durante el plazo de garantía; *b*) posteriormente, durante diez años, conforme al artículo 1792 del Código civil, salvo el caso de dolo o de fraude, en que la acción de responsabilidad dura treinta años.

Durante el plazo de garantía, la responsabilidad por las imperfecciones de las obras, pesa sobre el empresario hasta la recepción definitiva de los trabajos. La responsabilidad posterior a este plazo de garantía, reglamentada por los artículos 1792 y 1270 (Cod. civ.), subsiste aun despues de la verificación y recepción definitiva de los trabajos.

**Prescripción Treintenal.** — En caso de dolo o fraude, la responsabilidad no se extingue sino a los treinta años, que es el lapso fijado por la ley para la prescripción de las acciones por daños y perjuicios.

## Daños causados a la propiedad privada.

Se entiende por daños a la propiedad, todo ataque directo y material a los bienes de otro a consecuencia de los trabajos públicos, sin que haya incorporación al dominio público de ninguna fracción del suelo. No es indispensable, sin embargo, que el daño sea directamente al inmueble mismo para que haya lugar a la indemnización, siendo suficiente que resulte de los trabajos la violación de un derecho de propiedad, esto es, que su ejercicio le sea menoscabado.

Los propietarios cuyas fincas dan a las vias públicas, deben soportar, sin indemnización, los trabajos de refacción o de reparación de las vias públicas que no les causen sino una molestia momentánea, y no constituyan un ataque a los derechos de vista, de acceso, y de pasaje de que gocen. — Es menester, sin embargo, que esta molestia no exceda, por su duración y gravedad un límite normal, y que la Administración haya tomado todas las medidas necesarias para atenuar los inconvenientes resultantes de los trabajos. — Deben soportar, tambien sin indemnización, los trabajos cuya ejecución comporten solamente la pérdida de una simple ventaja de que el propietario habria podido ser privado por el hecho de un tercero sin responsabilidad para éste, como por ejemplo, si los trabajos han dado por resultado privar a un inmueble de las vistas de que gozaba, de la facultad de recoger las aguas pluviales que caen sobre los caminos, etc. — El perjuicio proveniente de una servidumbre legal, tal como la que grava las propiedades vecinas a los cementerios, no da lugar a ninguna indemnización a favor de los dueños de éstas.

Fuera de los casos expresados, los vecinos de las vias públicas, que experimenten un daño a consecuencia de la ejecución de los trabajos en las vias de comunicación, pueden, en principio, reclamar una indemnización; tienen este derecho, por ejemplo, cuando les hayan cerrado una via pública a cuyo uso tenian derecho, así como tambien cuando, por los cambios de nivelación, se obstruyan las vistas y claridad de un inmueble; se produzca su descalzamiento o enterramiento, quedando aquél mas alto o mas bajo y por supuesto si ademas queda comprometida su solidez, y, por tales motivos, depreciada la propiedad.

**Indemnización.** — Para reclamar una indemnización por los daños ocasionados por la ejecución de los trabajos públicos, es menester, en principio, haber sufrido personalmente un perjuicio. Esta indemnización es debida generalmente al que era propietario en el momento del daño.

La indemnización debe comprender todo el perjuicio causado al propietario por la depreciación del inmueble y la utilidad de que ha sido privado, tales como la pérdida de arrendamientos, gastos de mudanzas, etc.

## Trabajos privados o particulares.

Todos los trabajos de que esten encargados los ingenieros o arquitectos, bien sean públicos o privados, deben practicarse de acuerdo con los principios de la ciencia. La impericia en su ejecución, afecta su responsabilidad, pues las relaciones de las partes en materia de arrendamiento de servicios o de obras, en general, estan determinadas por las reglas del derecho en lo referente a obligaciones.

En virtud del principio establecido sobre libertad de contratos, los particulares pueden celebrar libremente sus convenciones para la construcción de sus obras, siendo en estos casos el contrato la ley que regula sus relaciones juridicas. — Este derecho de contratación, está limitado únicamente por las leyes en cuya observancia estan interesados el órden público y las buenas costumbres, las cuales no pueden derogarse ni relajarse por convenios particulares.

Para la validez del contrato, es menester, que los contratantes sean capaz de obligarse, conforme al derecho comun. Asi el menor, no emancipado, no puede arrendar sus servicios sin el consentimiento de su padre, si está bajo la patria potestad, o de su tutor si está bajo tutela. La mujer casada tiene necesidad, a este efecto, de la autorización de su marido. — La ausencia de consentimiento o los vicios de éste (error, violencia, dolo) entrañan la nulidad del contrato de arrendamiento de obras o servicios, conforme al derecho comun.

Nadie puede arrendar sus servicios sino por un tiempo limitado o para una obra determinada, pues está prohibida toda convención por la cual una persona arriende sus servicios por toda la duración de su vida.

## Accidentes del trabajo.

Se entiende por accidentes de trabajo, un golpe al cuerpo humano, proveniente de la acción súbita y violenta de una fuerza exterior que le cause daño. Asi, toda lesión del organismo, aparente o no aparente, interna o externa, superficial o profunda, constituye un accidente.

Los accidentes sobrevenidos a los obreros y empleados en los trabajos de que se ocupan, están sometidos, desde el punto de vista de las reparaciones civiles, a un régimen especial y excepcional. El principio en que descanza dicho régimen, es que la victima del accidente o sus representantes, tienen derecho a demandar del jefe de la Empresa, una indemnización sin tener que justificar que éste ha incurrido en falta, apartandose asi del principio general, establecido por el Código civil, que requiere que haya habido falta de parte de la persona de quien se pretende exigir alguna responsabilidad.

Por virtud del régimen excepcional de que tratamos, la responsabilidad existe por el solo hecho del accidente, considerado como una consecuencia directa del ejercicio de la profesión; es lo que se llama el principio del riesgo profesional. Inversamente, y siempre en virtud del mismo principio, la falta de la victima no es motivo para exonerar de esta indemnización al Jefe de la Empresa.

Para que un accidente de trabajo de lugar a una indemnización, es menester que sobrevenga por el hecho del trabajo de que estaba encargado la victima. La falta de ésta, o del empresario, no se toma en consideración, sino para modificar el *quantum* de la indemnización, la cual será reducida, si la falta emana del obrero, o mayor, si emana del patrón.

Las indemnizaciones comprenden dos elementos : *a*) El pago del precio de las medicinas, honorarios medicos y funerarios; *b*) El pago de los salarios perdidos por el hecho del accidente.

Para el cálculo de las indemnizaciones destinadas a suplir el salario perdido, hay que atender a la naturaleza y consecuencias del accidente. Si se trata de una incapacidad de trabajo puramente temporal, el obrero tiene derecho, hasta su curación, a la mitad del salario que ganaba en el momento del accidente; pero si la incapacidad es permanente, la victima tiene derecho a una renta vitalicia. En caso de muerte, el conyuge, los descendientes y ascendientes, tienen tambien derecho a esta renta.

Debe servir de base para la renta citada, en caso de incapacidad permanente parcial, la mitad de la reducción sufrida por el salario a consecuencia del accidente; en caso de incapacidad permanente absoluta, la base es igual a las dos terceras partes del salario perdido.

Los encargados, empresarios, administradores, etc., cubren generalmente a sus obreros y empleados y se descargan a su vez de dichas responsabilidades, por medio de polizas, de seguros (por accidentes, etc.) tomadas en una compañia bien reputada.

## Vias de comunicación.

Las vias de comunicación han sido establecidas en interes público y comprenden las calles, rutas y caminos de diferentes clases.

En principio, una via no es pública, sino cuando ha sido el objeto de un acto administrativo que la coloca en la categoria de tal. Este acto, que lleva el nombre de clasificación, determina el estado civil de la via pública, el régimen administrativo al cual será sometida y la persona moral a quien pertenezca.

Las vias son principales ó secundarias; las primeras las forman las rutas nacionales y departamentales, y, las otras, los caminos vecinales. A excepción de los caminos rurales de los particulares, todas las vias de comunicación forman parte del dominio público y son, por consiguiente, inalienables e imprescriptibles. Las rutas nacionales son del dominio público del Estado, y, las departamentales, del dominio público departamental. Todas las otras vias de comunicación son del dominio público municipal.

La apertura de las vias públicas no puede verificarse sino despues del cumplimiento de las formalidades establecida por la ley sobre expropriación por causa de utilidad pública, es decir : despues de investigación, declaración de utilidad pública, juicio de expropiación y pago previo de la indemnización consiguiente.

### Anchura de las vias públicas. — Delimitación. — Ensanches.

Como las vias son del dominio público, es a la Administración a quien corresponde fijar el ancho y los limites de las diversas vias públicas. La anchura se determina por las autoridades competentes quienes generalmente los fijan por el acto mismo de clasificación, de acuerdo con las necesidades del tráfico.

**Delimitación.** — Los limites de las vias públicas son determinados por los alineamientos trazados por las autoridades administrativas competentes. Estos tienen por objeto principal trazar la linea separativa del dominio público del de las heredades privadas, y pueden ser fijados, sea por actos especiales individuales, cuando un vecino pide a la Administración indicarle el limite del dominio público con su propiedad, sea por la Administración misma cuando ésta ordena, como medida general, trazar los limites de una via pública en toda su extensión, o de todas las vias existentes en una Municipalidad.

Es importante distinguir los alineamientos individuales de los planos generales de alineación, porque los poderes de la Administración en el segundo caso son mas extensos que en el primero; en éste el alineamiento individual no puede hacerse, sino conforme a los limites actuales de la via pública, mientras que, por los planos generales de alineamientos, la Administración tiene un medio para operar el ensanche de las vias públicas, gravando la propiedad privada con muy onerosas servidumbres. — En este último caso, la alineación implica una verdadera expropiación, pues el ensanche se hace a costa de la propiedad del vecino una parte de la cual va unirse a la del dominio público :

**Efectos de los proyectos generales de alineación.** — Los efectos de estos planes sobre las propiedades vecinas difieren, segun que se trate de simples

terrenos o de terrenos donde existan construcciones, y segun la naturaleza de las vias públicas.

**Simples terrenos.** — Si se trata de vias urbanas, la publicación del proyecto tiene por efecto inmediato someter los terrenos comprendidos en los alineamientos a los reglamentos de la grande o pequeña via, gravandolos con una servidumbre. De aquí resulta que todo acto ejecutado por el propietario sobre esos terrenos, sin autorización, constituye una contravención de grande o de pequeña via, segun su categoria. Importa poco que el propietario no haya recibido notificación individual del proyecto de alineación, o que la Administración no haya pagado aun la debida indemnización. La publicación del proyecto tiene los efectos de una declaración de útilidad pública, pero no constituye un juicio de expropiación, por el cual la propiedad del vecino pasa al dominio del expropiante; el derecho de propiedad de aquel, sobre los referidos terrenos, subsiste hasta el dia en que, en ejecución del proyecto, el propietario recibe de la autoridad competente un decreto especial de alineación trazando el limite entre su heredad y la via pública. — Es entonces que nace el derecho a la indemnización; ésta se pagará al propietario de acuerdo con el valor del terreno cedido a la via publica, sin tomarse en cuenta la depreciación que haya podido sufrir la otra parte de la misma propiedad que resta en poder del expropiado.

**Terrenos donde se edifica o construye.** — Los efectos del plan de alineamiento en cuanto a los terrenos donde existan construcciones, son siempre los mismos ya expresados, cualquiera que sea la via pública de que se trate. — En todos los casos en que, por efecto de un plan general de alineación, una construcción se encuentre en pie sobre la via pública, la aprobación general de este plan no entraña la incorporación inmediata de dicha construcción a la via pública. Esta incorporación no se produce y el derecho a la indemnización no nace en provecho del propietario, sino cuando la construcción ha sido demolida voluntariamente o por causa de vetustez.

## Regimen de las propiedades limitrofes a las vias públicas.

La contigüedad a las vias públicas entraña para los inmuebles limitrofes, ventajas, por una parte, y por otra obligaciones. Estas últimas, conocidas con el nombre de servidumbres de via, se aplican tanto a las construcciones como a los terrenos y varian segun la naturaleza de la via pública. Las unas son generales; las otras son especiales a ciertas vias. — Estas cargas, impuestas a los dueños de esas propiedades contiguas, tienen por objeto : las unas, asegurar la conservación del dominio público en toda su integridad; las otras, asegurar a los transeuntes que usen esas vias públicas la seguridad y comodidad de la circulación.

La mas importante de estas servidumbres es la de la alineación de que ya hemos hablado.

**Derechos sobre las vias públicas.** — Los dueños de las propiedades contiguas o limitrofes a las vias públicas, tienen sobre éstas ciertos derechos especiales que consisten : derecho de acceso, derecho de vista y derecho de desagüe.

El derecho de vista y de acceso los facultan para abrir en los muros de la fachada de sus inmuebles algunas puertas y ventanas, sujetandose para tales operaciones a lo que tengan dispuesto sobre el particular las Ordenanzas o leyes Municipales. — El derecho de salida de las aguas, se entiende solamente de las pluviales, que los propietarios pueden derramar sobre las vias públicas (las que caigan sobre sus fincas), bajo las condiciones determinadas por los reglamentos de policia.

## Servidumbre en general. — Definición. — Caractéres.

La servidumbre es una carga impuesta a un predio para uso y utilidad de otro perteneciente a diferente dueño. Ella es un derecho con relación al propietario del fundo dominante, y una obligación o carga con relación al propietario del fundo sirviente. — Este derecho y esta carga tienen un caracter real y de tal suerte estan ligados a la existencia de uno y otro inmueble, que no les afecta las mutaciones que pueda sufrir la propiedad de alguno de éllos. Son tres las condiciones caracteristicas de toda servidumbre predial : *a*) la existencia de dos heredades, de las cuales la una presta el servicio, y la otra lo recibe; *b*) la existencia de dos propietarios, dueño el uno del predio dominante, y dueño el otro del predio sirviente; y *c*) la causa o el fin de la servidumbre, esto es, el uso y utilidad del predio que goza de la misma.

Tras de estas nociones generales, viene la clasificación de las servidumbres derivadas ya de la situación natural de los lugares, ya de las obligaciones impuestas por la ley, ya de los convenios celebrados entre los particulares, distinción esencial, y que constituye el objeto de varias disposiciones legales.

**Servidumbres que se derivan de la situación de los lugares.** — Por el orden natural de las cosas, los campos inferiores tienen que recibir las aguas que huyen de los campos superiores. Por tanto el dueño del campo inferior no podrá librarse de esta servidumbre que es una carga impuesta por la naturaleza, no pudiendo levantar ningun dique que impida la corriente. El dueño del campo mas alto no puede agravar la servidumbre, ni cambiar el curso de las aguas de modo que perjudique al propietario del campo mas bajo. Esto por lo que respecta a las aguas que corren naturalmente y sin trabajo del hombre. El que tiene un manantial en su predio, puede usar de él como mejor le parezca, salvo el derecho que tal vez hubiese adquirido el dueño del predio inferior por convenio o por prescripción. — El dueño del manantial no podrá, empero, cambiar su curso, cuando la fuente suministra a los habitantes de un pueblo, aldea o caserio, el agua que les es necesaria; solo tendrá facultad para pedir una justa indemnización, que será regulada por expertos, con tal que los habitantes no hubiesen adquirido o prescrito el uso que tienen de la fuente.

Aquel por los lindes de cuya heredad pase un rio, puede aprovecharse de sus aguas a su paso para regar sus tierras. Si la corriente atraviesa la heredad, el dueño de ésta podrá tambien aprovecharse de ellas en todo el espacio que atraviesen, pero con obligación de volverla a su curso ordinario a la salida de sus propiedades. Estan exceptuadas aquellas aguas que la ley sobre la distinción de los bienes declara que pertenecen al dominio público. — El uso de estas diversas facultades puede encontrar varios obstáculos, pero siempre que se promuevan contiendas y litigios entre los diversos dueños de las tierras que intenten beneficiarse de las aguas, se procurará conciliar los intereses de la agricultura y de la industria con el respeto debido a la propiedad, no perdiendo jamas de vista los reglamentos locales y particulares acerca el curso y el uso de las aguas.

Todos los dueños pueden cerras sus fundos, a menos que deban la servidumbre de paso. La necesidad de los lindes produce naturalmente la facultad de acotar las heredades; y esto deberá hacerse a expensas comunes.

**Servidumbres establecidas por la ley.** — Las servidumbres que establece la ley tienen por objeto la utilidad pública o comunal o la de los particulares. Las primeras constituyen obligaciones o cargas impuestas, en interes general, a las propiedades y a los propietarios mismos, por las leyes o los reglamentos. El Código civil no menciona sino dos servidumbres de utilidad pública, pero esta enumeración no es mas que enunciativa. Las principales servidumbres de utilidad publica son 1º las que tienen por objeto las calzadas que ha de haber a lo largo de los rios navegables o flotables; 2º las que se refieren a la construcción o reparación de caminos y otras obras públicas o comunales; 3º la prohibición de construir a cierta distancia de los cementerios y plazas de guerra; 4º la de observar las distancias prescritas relativas a ciertos trabajos cerca de las lineas ferroviarias; 5º la de dejar ejecutar los trabajos necesarios para la explotación de las minas; 6º la prohibición de hacer exploraciones de minas cerca de las habitaciones; 7º la de practicar excavaciones en cierto perimetro alrededor de las fuentes minerales o termales; 8º la de alineación, etc., etc.

En principio, a menos de una disposición formal, el establecimiento de las servidumbres de utilidad pública no da derecho a ninguna indemnización, pues la regla segun la cual nadie puede ser expropiado sin justa y previa indemnización no se aplica sino a la cesión de la propiedad por causa de utilidad pública.

Las servidumbres establecidas por la ley en provecho de los particulares son, segun el Codigo civil, las relativas a la medianeria de las paredes, zanjas, etc.; la distancia y obras intermediarias requeridas para ciertas construcciones; a las vistas sobre la propiedad del vecino; a la caida de las aguas de los techos y al derecho de pasaje en caso de un predio enclavado.

**Medianeria.** — La medianeria es una especie particular de copropiedad aplicada a los objetos que sirven de separación entre dos fundos contiguos, especialmente a las paredes, muros, zanjas, setos y vallados. Estos estan gravados, en provecho de dichos fundos, de una servidumbre de indivisión forzada que constituye una excepción al principio general que establece que nadie está obligado a permanecer en comunidad.

Así en las poblaciones como en los campos, se presume medianera, a no ser que haya prueba en contrario, toda pared divisoria de patios y jardines o bien sea cerca de los campos, y en los edificios hasta donde llega el menos alto. — La reparación y construcción de una pared medianera corre a cargo de todos los que tienen derecho en ella y en proporción a este derecho. — Sin embargo todo comunero de una pared medianera puede excusarse de contribuir a su reparación o construcción, renunciando al derecho que le compete, con tal que la pared medianera no sostenga un edificio que le pertenezca.

Todo dueño de una heredad contigua a una pared o muro, tiene la facultad de hacerlo medianero, reembolsando al propietario del muro la mitad de su valor y la mitad del valor del suelo sobre que ha sido construido. — Todo condueño puede edificar sobre la pared medianera y apoyar en ella vigas; puede levantarla a su costa, pero sino está en estado de sostener la elevación que quiere dársele, el que quiera levantarla deberá renovarla por entero a expensas suyas; y si quiere darle mayor anchura, debe tomarla de su terreno. El vecino que no contribuyó a la elevación de la pared, puede hacerla medianera pagando la mitad de su costo y del suelo que debió poner su vecino, si la hizo más ancha.

Las obligaciones de los propietarios de un muro medianero son, de una manera general, las mismas de toda comunidad; ellos deben velar por la conservación del muro y abstenerse de todo lo que pueda perjudicarlo.

Cuando los diferentes pisos de una casa pertenecen a diversos dueños, si los títulos de pertenencia no establecen el modo como deben hacerse las reparaciones y construcciones, se haran del modo siguiente : las paredes maestras y el tejado quedan a cargo de todos los dueños a proporción del valor que tenga el piso que a cada uno de ellos pertenece; el dueño de cada piso reparará y reconstruirá el suelo que pisa; el dueño del primer piso debe hacer la escalera que a él conduce; el dueño del segundo piso debe hacer la que vá desde el primero hasta el suyo, y asi en los demás.

Cada uno de los propietarios tiene un derecho de propiedad exclusiva y absoluta sobre el piso que le pertenece, y puede, por consiguiente, hacer todos los cambios que le convengan, a condición de abstenerse de ejecutar todo aquello que sea perjudicial a los otros copropietarios de la casa.

**Distancias y obras intermedias.** — No se permite plantar arboles de tronco alto, cerca de una pared medianera, que no sea a la distancia prescrita por las leyes. El vecino podrá exigir que sean arrancados los plantados a menor distancia. Aquel sobre cuya heredad se extiendan las ramas de los arboles del vecino, puede obligar a éste a que las corte. — Si las raices se extienden por su heredad, tiene derecho de cortarlas por sí mismo .

Los arboles que se hallan en el seto medianero, son comunes como el mismo seto, y uno y otro vecino pueden exigir que sean cortados.

No se pueden construir letrinas, chimeneas, fraguas, hornos, abrir pozos ni poner materias corrosivas, etc., cerca de una pared sin dejar las distancias prescritas en las leyes y reglamentos.

**Vistas sobre la propiedad del vecino.** — Ningun vecino puede, sin consentimiento del otro vecino, hacer en la pared medianera ventana alguna ni otra clase de abertura. El dueño de una pared que no sea medianera, aun cuando ella se levante al lado mismo de la heredad de otro, puede abrir en dicha pared lumbreras o ventanas con red de alambre y vidriera que no pueda abrirse. Las mallas de la red de alambre deberan tener a lo mas un decimetro y el marco de la vidriera deberá ser clavado; estas aberturas deberan hacerse a veinte y seis decimetros sobre el piso del cuarto que quiera alumbrarse, si es al nivel de la calle; y a diez y nueve decimetros sobre el piso, si es para los superiores.

Las vistas propiamente dichas son las que dan sobre la heredad del vecino; pueden ser rectas u oblicuas. Las primeras son las que existen en un muro paralelo a la linea de separación de las dos heredades; las segundas, son las que se encuentran en un muro que forma angulo con esta linea. Cuando los dos fundos estan situados sobre el mismo alineamiento, no hay vista recta ni oblicua; sin embargo, si uno de los propietarios construye un balcon, resultará una vista oblicua. — Las vistas no pueden abrirse sobre un muro sino cuando este se encuentre a cierta distancia de la linea separativa de las dos heredades; esta distancia varia segun que la vista sea recta u oblicua : diez y nueve decimetros para las primeras y seis decimetros para las segundas. — Las distancias de que hablamos, deben contarse desde el fundamento exterior de la pared en que se ha hecho la abertura, y si hay balcones u otras obras de vuelo, desde su linea exterior hasta la linea divisoria de las dos propiedades.

**De las aguas que caen de los tejados.** — Los propietarios deben hacer sus tejados de manera que las aguas lluvias caigan sobre su terreno o sobre un camino publico; no pueden dirijirlas sobre el predio de su vecino, pero una vez caidas en su propio suelo, el propietario de éste no está obligado a impedir que se extiendan sobre la propiedad del vecino por efecto de la pendiente natural del terreno.

**Derecho de paso.** — La servidumbre legal de paso existe en provecho del propietario cuyo fundo está enclavado. El dueño de una heredad, que por hallarse cercada por otras, no tiene ninguna salida al camino público, puede reclamar que se le conceda un paso por las heredades de sus vecinos para poderla cuidar, con la obligación de indemnizarles a proporción del daño que les pueda ocasionar. — Por lo regular se abrirá el paso por donde sea menor la distancia para llegar al camino público, debiendo atenderse a que cause el menor daño posible en las heredades por donde debe pasar.

La servidumbre de paso no dá al propietario del fundo enclavado la propiedad del terreno que sirve de pasaje; el dueño del fundo gravado conserva la libre disposición de su terreno, con la sola limitación de no interrumpir el ejercicio del derecho de paso. — La extensión de este derecho se regla de acuerdo con las necesidades actuales de la explotación del fundo enclavado; caso de desacuerdo entre las partes debe ocurrirse a una experticia al efecto de verificar estas necesidades y saber si dicha explotación reclama un camino carretero o simplemente uno mas restringido.

**Servidumbres establecidas por el hecho del hombre.** — Pueden los propietarios establecer sobre o en favor de sus heredades las servidumbres que quieran con tal que éstas no sean contrarias al órden público. El uso y extensión de estas servidumbres se regulan a tenor de lo prescrito en las escrituras de su constitución, y en su defecto por las reglas siguientes :

Las servidumbres se establecen para el uso de los edificios o para el de las tierras. Las primeras se llaman servidumbres urbanas, sea que los edificios a cuyo favor fueron constituidas, esten en o fuera de población; las segundas se llaman rústicas. Las servidumbres son continuas o discontinuas; las continuas son aquellas cuyo uso puede ser continuo sin necesidad de un hecho positivo del hombre : tales son los acueductos, las vistas y otras de semejante especie. Las discontinuas son las que, para cuyo ejercicio no se necesita un hecho positivo del hombre : tales son los derechos de pasaje, sacar agua, pastaje y otros semejantes.

Las servidumbres son visibles o no visibles. Las primeras son las que se descubren por alguna obra exterior, como una puerta, una ventana, un acueducto; las segundas aquellas que no se manifiestan por ninguna señal visible, como por ejemplo, la de no edificar en una heredad, o de no poder edificar mas que hasta cierta altura.

Las servidumbres continuas y visibles se adquieren por título o por la posesión de treinta años. Las continuas no visibles y las discontinuas, sean o no visibles, pueden solo establecerse por titulos.

Al establecerse una servidumbre se reputa concederse todo lo necesario para su uso. Asi la servidumbre de sacar agua de una fuente agena comprende necesariamente el derecho de pasaje.

Aquel a quien se debe una servidumbre tiene derecho a hacer todas las obras necesarias para usar de ella y conservarla.

## Ingeniero arquitecto.

Los arquitectos estan obligados a observar las leyes y reglamentos administrativos sobre construcciones. Son responsables para con el propietario de las consecuencias que resulten de la inobservancia de las leyes y reglamentos concernientes a las vias públicas y a la contigüedad de los predios.

Como autor de sus planos y presupuestos y director de trabajos, tiene un derecho privativo sobre su obra en virtud de las leyes relativas a la propiedad intelectual. — Cuando sus honorarios no han sido convenidos de antemano por contrato, quedan éstos sujetos a la apreciación de los Tribunales.

## Deslinde de tierras.

El deslinde es una operación que tiene por objeto fijar la linea divisoria de dos fundos contiguos y determinar con la ayuda de signos materiales llamados mojones, la linea de separación que se busca, de acuerdo con los títulos respectivos exhibidos por las partes.

Todo propietario puede obligar a su vecino al deslinde de sus propiedades contiguas, ocurriendo a la autoridad judicial respectiva a quien incumba conocer del asunto, segun las leyes de procedimiento civil. — La acción de deslinde puede ser intentada por cualquiera que tenga un derecho real sobre el inmueble, especialmente por el propietario, el usufructuario, el nudo propietario, el enfiteuta, el usuario y el copropietario indiviso.

Dos hipotesis pueden presentarse : 1º las partes estan de acuerdo sobre el limite de sus heredades, procediendose entonces solamente a fijar la delimitación por la colocación de los mojones; 2º sin discutir la propiedad ni los titulos presentados, las partes no estan de acuerdo sobre los límites de sus fundos. En este último caso el Juez ordena el deslinde por los expertos que él designe, y, el dia señalado previamente para proceder, que será notificado a todos los colindantes, el Juez concurrirá en persona al deslinde y designará los lugares donde deban situarse los mojones que señalen los linderos despues de haber recibido el informe de los expertos.

Cuando alguno de los colindantes se oponga a la designación de algun lindero, presentará en el acto del deslinde el titulo de sus tierras, o algun instrumento supletorio suficiente, y el Juez, si no pudiere cortar en conciliación la disputa, despues de examinar los titulos y oir a los prácticos, si le pareciere necesario, fijará un lindero provisional, haciendo la debida apreciación del mérito de los autos y del resultado de sus observaciones sobre el terreno; lindero que se respetará mientras se decida la cuestión suscitada en el respectivo juicio motivado por la referida oposición del colindante.

La diligencia del deslinde, haya o no oposición, la firmaran el interesado y todos los colindantes que hubieren concurrido, con el Juez y el Secretario; y si alguno de aquellos no supiere o no quisiere firmar, se expresará asi, advirtiendose al que no quiera firmar que esta falta no le favorecerá de modo alguno.

En la acción de deslinde, todas las partes son respectivamente demandantes, y cada una de ellas debe suministrar la prueba de sus pretensiones. Si existe un titulo emanado del autor comun de las partes, el Juez debe dictaminar de acuerdo con lo expresado en dicho título, a menos que los limites no hayan sido modificados por una convención ulterior. A falta de tal título, se admite que cada parte pueda invocar cualquiera otro documento, en el caso mismo de que su adversario no figure en él, salvo al Juez determinar si halla razon para ello.

Los gastos de compra y de plantación de mojones se dividen de por mitad entre los propietarios interesados, pero los de mensura deben, segun la opinión dominante, repartirse entre los propietarios proporcionalmente a la extensión de sus fundos. Si ha habido oposición durante la operación de deslinde, los gastos que ocasione este juicio debe soportarlo únicamente la parte que sucumbe en el pleito. Los otros gastos del deslinde se repartirán igualmente entre todos los interesados.

La supresión o alteración de los mojones que determinan los linderos, dá lugar a una acción interdictal y a otra correccional; y tambien a la indemnización de los perjuicios ocasionados.

## Metrologia general y su aplicación.

La metrologia general tiene por objeto la medición racional y sistemática de las diversas cantidades que intervienen en los fenomenos naturales, siendo las leyes y principios que la rigen aplicadas a la medición de las materias objeto de la industria y del comercio.

La metrologia aplicada se funda en el sistema métrico decimal, cuyas unidades fundamentales son el metro y el gramo. El primero, llamado metro patrón, es una regla de platino irradiado al 10 % con sección en forma de X, de 20 mm. del lado, que a 0º C define esa longitud tipo; el segundo, o sea el gramo, es la unidad de la masa (equivalente al peso, en un mismo lugar de la Tierra) que se define como la masa de un centimetro cúbico de agua químicamente pura a 4º C, y a la presión normal.

El sistema métrico decimal es obligatorio, que sepamos, en Francia, Alemania, España, Belgica, Italia, Grecia, Holanda, Portugal, Rumania, Servia, Suiza, Austria, Hungria, Suecia y Noruega, Venezuela, Argentina, Perú, Méjico, etc. Es facultativo en los Estados Unidos de America, en el Reino Unido de la Gran Bretaña e Irlanda, en Japón, Turquia, Egipto y Rusia.

Para las diversas unidades del sistema métrico decimal con los símbolos que aconseja y emplea el Comité internacional de Pesas y Medidas, con sus valores en unidades fundamentales y sus relaciones con las de los diversos paises. Véanse pags 229 á 238.

## PENALIDADES

Como el uso de otras pesas y medidas que no sean las establecidas por las leyes constitutivas del sistema métrico decimal, está prohibido; los infractores de las disposiciones legales que lo rigen incurren en las diversas penas que las mismas leyes determinan en sus casos.

### Agua.

El agua, considerada en su sustancia, entra en la categoria de las cosas comunes, no susceptible de apropiación privada; pero las fuentes, los estanques, los lagos, los rios o los mares, que constituyen algunas porciones de la superficie terrestre pueden pertenecer a la colectividad o a los particulares.

De estas diversas categorias del agua, se pueden hacer multiples clasificaciones, segun el punto de vista en que uno se coloque : aguas públicas o privadas, vivas o pluviales, corrientes o estancadas, etc., cuyo régimen está organizado por numerosas leyes.

**Aguas marítimas.** — La mar, no siendo susceptible de ocupación, está clasificada en el número de las cosas comunes. El derecho público reconoce la libertad de la alta mar. Cuanto al mar territorial, se admite al contrario que ella hace parte del territorio del pais que bañan sus aguas; lo mismo está admitido respecto de los mares cerrados, abras, puertos y radas.

Sobre el mar territorial, el Estado ejerce todos sus derechos de soberania, jurisdicción, cabotaje, pesca, alta policia, etc.; él organiza las aduanas y los peajes; reglamenta la navegación, la admisión de navios en los puertos y radas, etc. Se admite que el derecho de jurisdicción se ejerce sobre el mar territorial hasta el punto donde puede llegar una bala de cañon.

Las playas del mar pertenecen al dominio público, lo mismo que el aluvión y el terreno que deja en seco.

**Rios navegables o flotables.** — Son los que tienen caudal suficiente para el tráfico de barcos aunque esta capacidad provenga de la naturaleza misma o de trabajos de arte.

La navegabilidad tiene como consecuencia necesaria la propiedad, y asi estos rios forman parte del dominio público, siendo por tal motivo inalienables e imprescriptibles.

Sólo a la Administración corresponde el derecho de comprobar si un rio es navegable.

Los limites de un curso de agua navegable, hasta su desembocadura en el mar, estan determinados por la autoridad competente. Las bases son complexas : se atiende especialmente al volumen de las aguas, saladura, constitución geológica de sus orillas, etc. El principio de la delimitación es que el lecho del curso de agua comprende todo el terreno que cubren las aguas corrientes hasta su borde antes de desbordarse. Poco importa que esta elevación de las aguas sea debida a las lluvias, o, a la influencia de las mareas ordinarias.

Los terrenos recubiertos por un desbordamiento no son adquiridos por el dominio publico; el ribereño conserva sus derechos para retomar su ejercicio cuando las aguas se retiren. Si la inundación se prolonga largo tiempo, resulta una cuestión de hecho que debe apreciarse para saber si en este caso hay simple desbordamiento o cambio de lecho. Los ribereños tienen contra dicho decreto un recurso contencioso por incompetencia, inobservancia de formas legales, exceso de poder, si el trazado de los limites es incorrecto, por torcimiento de poder, si la delimitación es hecha con otro interes que no sea el libre curso de las aguas y de la navegación.

Un decreto de delimitación no liga a las jurisdicciones administrativas que tienen el derecho de declarar que los limites han sido mal apreciados o que ellos han sido modificados con posterioridad al decreto.

Determinados por una situación de hecho los limites de un rio, son susceptibles de modificarse por el cambio de esta misma situación. El ribereño soporta las mudanzas del curso del agua que le son perjudiciales, como se beneficia de aquellas que le son ventajosas, cuando la causa de estas mudanzas son naturales. Si, al contrario, la modificación de los limites proviene de una elevación de la ribera debida a trabajos de arte, constituye una expropiación que dá lugar a indemnización. El Estado puede tambien ser obligado a indemnizar en razon de las corrosiones de la ribera y otros desgastes resultantes de trabajos públicos; pero, en estos casos, la indemnización no es por expropiación, sino simplemente por daños. Si el inmueble, al propio tiempo que sufre un daño, se beneficia de un mayor valor

por haberse aprovechado, por ejemplo, de un aluvión resultado de los mismos trabajos públicos, hay lugar a efectuar una compensación.

**Trabajos diversos, ocupaciones, construcciones.** — Los rios navegables y sus dependencias, como pertenecen al dominio público, su guarda esta confiada a la Administración, y, sin la autorización de ésta, no puede ejecutarse ningun trabajo y practicar ninguna toma de agua. — El público puede ejercer libremente sus derechos naturales, tales como el de sacar agua, lavar, dar de beber al ganado, etc., a condición de observar los reglamentos de policia y de no hacer ningun establecimiento. Los ribereños no tienen, por su calidad de tales, ningun derecho privilegiado.

Los Prefectos tienen el derecho de autorizar, de acuerdo con la opinión o la proposición de los ingenieros, y bajo la autoridad del Ministro : 1º las tomas de agua por medio de máquinas que no tengan por efecto modificar el regimen de las aguas; 2º los establecimientos temporales aun cuando tengan por efecto modificar el regimen o el nivel de las aguas. — La duración de estas autorizaciones temporales debe ser fijada en el acta de concesión y no puede pasar de dos años. Los otros establecimientos sobre un curso de agua navegable no comprendidos en la categoria de los indicados, no pueden ser autorizados sino por el Presidente de la Republica.

No es permitido estacionar u ocupar un lugar cualquiera sobre el dominio público, sin autorización acordada por el Prefecto en debida forma.

Una instalación no autorizada sobre el dominio fluvial, constituye una infracción y cualquiera que sea su duración, no cambia de carácter ni podria engendrar ningun derecho. La supresión o modificación de un establecimiento, puede ser ordenada por la autoridad competente cuya autorización era necesaria para la obra, debiendo proceder en la misma forma y con las mismas garantias exigidas para dicha autorización. Es menester, sin embargo, hacer reserva de aquellos casos en que la medida se impone por una urgencia absoluta. — La decisión administrativa no impide a los interesados hacer valer los derechos que crean tener a una indemnización.

Si, en principio, la Administración tiene un poder discrecional para suprimir o modificar los establecimientos autorizados, la jurisprudencia mas reciente admite que aquella no debe hacer uso de ese poder, sino en interes de la navegación o del libre curso de las aguas y aun para la conservación del Dominio. Un acto que se inspire en otros motivos, puede ser anulado por desvio de poder.

**Caminos de sarga.** — Los ribereños de una corriente de agua navegable o flotable, deben soportar algunas servidumbres que se ejercen, en principio, sobre ambas riberas. Estas servidumbres consisten principalmente en dejar un espacio libre en las riberas de 7 metros 80 para camino de sarga, y libre de construcciones, plantaciones, etc., un espacio suplementario de 1 metro 95 sobre el borde interno del camino. Estas distancias pueden reducirse, cuando el interes de la circulación fluvial lo permite. Corresponde al Ministro de Trabajos Públicos, tomar las medidas a este efecto. El camino de sarga sirve para la circulación de los animales necesarios para remorcar las naves. Tambien debe dejarse sin construir, cerrar o plantar, un espacio de 3 metros 25 que permita a los marineros llegar a tierra si las necesidades lo requieren.

El camino de sarga, no pertenece al dominio público, sino al ribereño, quien readquiere su libre uso cuando el rio deje de ser navegable; él readquiere tambien el libre uso de la parte que quede exenta de la referida servidumbre, cuando la autoridad respectiva haya reducido los limites de aquella; pero, en todo caso, tiene derecho a los productos del suelo y del subsuelo.

La servidumbre debe limitarse a su objeto, que es el de permitir la sarga, no pudiendo los marineros agravarla por una instalación permanente o temporal, ni por el depósito de objetos sobre el camino.

Las zonas destinadas al camino se mediran, en principio, a partir de la linea separativa del dominio público y de la propiedad privada, es decir; desde el limite indicado por las aguas corrientes por la orilla antes de cualquier desbordamiento. Cuando la orilla no sea bastante plana para permitir la circulación de los hombres y de los caballos, es, segun la jurisprudencia, a partir de la cresta de las orillas que el ancho debe calcularse.

**Prohibición de extraer materiales.** — Está prohibido extraer sin autorización especial, tierras, arenas y otras materias, a una distancia menor de 11 metros 70 del curso de un rio navegable.

**Contribuciones de los ribereños.** — Los ribereños estan obligados a cooperar a los trabajos de construcción de diques necesarios para la protección de las

regiones vecinas al rio. A este efecto, éllos pueden constituirse en asociaciones sindicales.

Cuando el Estado ejecuta por si mismo dichos trabajos, los ribereños estan obligados a contribuir pecuniariamente; y, en las mismas condiciones, deben contribuir a los trabajos para poner a las poblaciones al abrigo de las inundaciones. Estas cargas incumben a los ribereños de todos los rios sean o no navegables.

Está prohibido construir, sin declaración previa, ningun dique sobre las partes sumergibles de los valles de ciertas riberas especialmente designadas. No se trata en este caso de obtener una autorización, sino de una simple declaración de hacer. — Cuanto a los diques existentes, pueden ser destruidos, mediante una indemnización, si el Ministro los juzga perjudiciales.

**Dragados de rios navegables.** — Estas es una operación que consiste en el conjunto de los trabajos necesarios para devolver su anchura y profundidad naturales al lecho de un curso de agua, embarazado por los casquijos, tierras, enlodamientos, etc. El dragado de los rios navegables y sus dependencias, es de cargo del Estado. Una contribución pecuniaria puede, sin embargo, pedirse a las comunas o municipios, a los propietarios de fábricas y a los ribereños, segun las circunstancias.

Los ribereños estan obligados a soportar los inconvenientes naturales del dragado, las interrupciones de la navegación, paradas de las tomas de agua, los daños que sufran las fábricas, pero nó el depósito sobre sus propiedades, de limos y desechos provenientes de la ejecución de los trabajos. — Ellos tienen derecho a una indemnización cuando les ha sido ocasionado un daño por ejecución defectuosa de los referidos trabajos, y tambien cuando éstos entrañan una modificación en la corriente del rio y un avance sobre la propiedad privada.

**Rios no navegables ni flotables.** — Un curso de agua es de propiedad privada en tanto no haya salido del fundo donde nace la fuente a la cual se asimila; el dueño de dicho fundo puede disponer del agua a su voluntad, pero no le está permitido desviarlas, con perjuicio de los usuarios inferiores.

La jurisprudencia y la mayoria de los autores, consideran que los rios no navegables, en su integridad, es decir, tanto por su lecho como por la masa de las aguas, entran en la categoria de las cosas que no pertenecen a nadie y cuyo uso es comun a todos, reglandose su goce por las leyes de policia. La ley de 1898 no ha sancionado este sistema, manteniendo solo el principio en lo que concierne a las aguas, pero no respecto del lecho del rio que considera accesorio de los fundos ribereños, perteneciendo, por consiguiente, este lecho, a los propietarios de ambas riberas.

Si las dos riberas pertenecen a propietarios diferentes, cada uno de ellos tiene la mitad del lecho del rio. El eje de la corriente de agua, es el limite que separa las heredades ribereñas. Es desde este eje que se cuenta la distancia de dos metros exigidos para las vistas y plantaciones. Si en principio, el eje del curso de agua forma entre ribereños el limite natural, éste es susceptible de modificarse, sea por la convención de las partes, sea por la prescripción. El propietario de una sola ribera puede pues ser, al propio tiempo, propietario de la totalidad del lecho.

El agua ha conservado el caracter de cosa que no pertenece a nadie y cuyo uso es comun a todos; los ribereños tienen sobre el uso de ella derechos privilegiados, pero solamente dentro de los limites establecidos por la ley.

La situación legal de un curso de agua no navegable puede no ser conforme a a ley de 1898. Se reconoce la validez de las concesiones y enagenaciones hechas por los dueños antes de la abolición del régimen feudal. En virtud de estos titulos, un ribereño puede ser propietario del rio, sus aguas y lecho.

El derecho mas importante del ribereño, el es de usar el agua para sus necesidades agricolas e industriales dentro de los limites concedidos por la ley para su ejercicio. El derecho de que aqui se trata, está sometido al control de la autoridad administrativa, cuyos reglamentos estan obligados los ribereños a observar.

La autoridad administrativa interviene en virtud de los reglamentos generales y tambien por decisiones individuales llamadas tambien autorizaciones. Su poder se ejerce con el doble objeto de asegurar : 1º la libre corriente del agua desde el punto de vista de la seguridad pública y de la salubridad; 2º la repartición del agua en interes de la agricultura y de la industria. — Toda intervención con otro fin, está considerada como exceso de poder. — En interes de la seguridad y de la salubridad, el poder de la Administración puede extenderse sobre las aguas que, de una manera general, no esten sometidas al régimen de los rios no navegables : un canal privado, por ejemplo, o un curso de agua antes que haya salido del fundo donde nace la fuente, o a un curso de agua que pertenezca a un particular en virtud de una concesión feudal.

Los reglamentos generales, constituyen la ley del curso del agua o de una sección determinada de aquel. Los ribereños estan obligados a pedir autorización para toda obra, sin mas excepción que para los simples cortes de las orillas que pueden practicar libremente para la irrigación. La autorización no es una manifestación del poder discrecional de la autoridad; ella debe ser acordada cada vez que no se corra el riesgo de lastimar un interes general. Si un derecho es violado por la decisión administrativa, los interesados pueden ocurrir a la via contenciosa...

La autorización acordada atestigua que el derecho pretendido por el ribereño, puede ejercerse sin incovenientes para el interes general. Es desde este punto de vista una especie de visto bueno que nada prejuzga en cuanto al valor del derecho mismo, y no tiene influencia en lo que concierne a los conflictos posibles con los derechos de terceros.

Solo a la autoridad competente corresponde determinar el regimen general de las corrientes de agua no navegables desde el punto de vista de los derrámenes y repartición de las aguas. Sus decretos deben conciliar los intereses de la agricultura y de la industria con el respeto debido a la propiedad.

Los reglamentos de repartición de las aguas no pueden intervenir sino en interes general, para operar una partición entre la agricultura y la industria. Es al Poder Judicial que los ribereños deben pedir los reglamentos de agua concernientes a las situaciones privadas resultantes de titulos, convenciones, prescripciones, etc.

**Dragado de rios no navegables.** — Se procede al dragado de éstos, ora por la acción individual de los ribereños, ora por el empresario, ingeniero o alarife, con quien han tratado los sindicatos o la Administración. Esta tiene siempre un derecho de *control* y puede de oficio hacer ejecutar los trabajos que, dentro del plazo fijado, para la ejecución, no hayan sido comenzados o terminados, debiendo previamente poner en mora, por un requerimiento a los propietarios o empresarios negligentes o recalcitrantes. Durante los trabajos, los ribereños, están obligados a dar paso a los obreros, empresarios, funcionarios y demas agentes encargados de la supervigilancia de los trabajos, y a soportar el depósito de los desechos sobre sus tierras.

Los gastos de dragado comprenden : el costo de los trabajos, las indemnizaciones debidas en razon del ensanche, los honorarios de los que forman los planos y proyectos, los de los agentes y funcionarios de vigilancia. Estos gastos se reparten entre los interesados conforme a los usos y reglamentos, y, si no existen precedentes, proporcionalmente al grado de interes de cada uno.

**Canales.** — Son cursos artificiales de aguas construidos para usos variables. Son publicos o privados y están destinados, ora a la navegación o flotación, ora a la irrigación o al desecamiento, ora al movimiento de maquinarias.

Los canales para la navegación se dividen; en canales propiamente dichos y rios canalizados. — Los trabajos de canalización son ejecutados en virtud de una ley que los declare de utilidad pública.

El Estado puede encargarse de la construcción y explotación del canal, y tambien el Departamento o comuna. En el primer caso, el canal pertenece al dominio nacional, y, en el segundo, al dominio público departamental o comunal. — El Estado puede tambien abandonar su construcción o explotación, o ambas cosas a la vez, a compañias concesionarias.

La autoridad administrativa tiene el derecho de reglamentar los canales.

Un rio no navegable puede ser canalizado en todo o en parte, en su curso. La canalización lo mismo que la creación de un canal, entra en la categoria de los grandes trabajos cuya ejecución no puede tener lugar sin previa declaratoria de utilidad pública.

Los canales de irrigación son de interés general o privado. En el primer caso son construidos como los trabajos públicos, en virtud de una ley o de un Decreto, declarándolos de utilidad pública.

**Estanques y lagos.** — Los estanques y lagos son aglomeraciones de agua, dulces o saladas, no corrientes, teniendo o no desagües naturales o artificiales. La denominación de lagos está reservada a los estanques de vasta extensión.

Los estanques naturales estan sujetos a las leyes de interes general y de salubridad pública. Los estanques artificiales pueden crearse libremente por los particulares dentro de sus heredades, sea con las fuentes que nacen en sus fundos, sea con las aguas pluviales que recojan, etc.

El establecimiento de un estanque no está sometido, en general, a ninguna autorización administrativa. Solo se exige dicha autorización en ciertos casos especiales;

el consentimiento de los propietarios de los fundos inferiores se hace indispensable, cuando la creación del estanque pueda agravar la servidumbre impuesta por el curso natural de las aguas.

**Aguas minerales y termales.** — Las aguas minerales naturales y las aguas minerales artificiales, están reglamentadas por una legislación especial. Su conservación y conducción, están reglamentadas por la ley.

## Vias férreas. — Ferrocarriles y Tranvias.

En principio los ferrocarriles se explotan por concesionarios en virtud de contratos celebrados con el Estado. Esta concesión puede hacerse directamente a la persona o compañia que la solicita o por via de adjudicación pública por medio de una licitación a los que presenten mayores garantias y ventajas.

La duración de las concesiones varia; antiguamente fueron a perpetuidad, pero luego se limitaron a 99 años como máximo de tiempo para el goce de la concesión por el concesionario.

Por contrato celebrado entre el Estado y el concesionario, éste se obliga a ejecutar los trabajos de construcción de la via y sus accesorios, y, una vez terminados los trabajos, tiene derecho el concesionario a indemnizarse percibiendo una remuneración de las personas que se sirvan de la linea para el trasporte y, ordinariamente goza tambien de ciertas ventajas pecuniarias que el Estado le asegura, de acuerdo con la convención celebrada. — En esta se establecen por escrito las disposiciones relativas al objeto de la concesión, a las obligaciones contraidas, a las condiciones de construcción, de mantenimiento y explotación de la via ferrea, a la duración de la concesión, a las condiciones para el rescate y la caducidad, a la tarifa máxima que puede aplicarse para el transporte de los viajeros, de la mercancia, etc.

Las vias férreas son de interés general cuando por su importancia y extensión abarcan varios Departamentos o Provincias; y, de interés local, los establecidos por los Departamentos o los Municipios, quedando comprendidos en estos últimos los tranvias; pero todas están sometidas para la ejecución de sus trabajos, conservación y explotación de la linea al *control* y supervigilancia de la Administración pública nacional, departamental o municipal, segun los casos.

Todos los trabajos principales de construcción de las obras, asi como los complementarios de las mismas, deben ser previamente autorizados por la Administración pública respectiva, despues de haber sido aprobados por ésta los correspondientes planos, proyectos, etc.

La ejecución de los trabajos puede hacerse bajo de una de las tres formas siguientes : 1º el Estado ejecuta por su cuenta los trabajos, sea para explotar él mismo la via, sea para hacerla explotar por una compañia; 2º el Estado adquiere los terrenos y hace los trabajos de *infraestructura*, dejando a una compañia el cuidado de establecer la via y los trabajos de *superestructura ;* 3º una compañia concesionaria ejecuta todos los trabajos y explota élla misma el ferrocarril con la ayuda financiera del Estado. En Francia, el sistema de explotación directa se aplica respecto a la red ferrocarrilera del Estado : el sistema de la concesión está en vigor en lo que concierne a las otras lineas férreas.

Los concesionarios no tienen la propiedad de la obra, sino solamente el goce de la via y de sus accesorios hasta la expiración de la concesión que pasa al Estado, Departamento, o Municipio, segun los casos. Los caminos de hierro son del dominio público y, como las otras vias, son inalienables e imprescriptibles, lo mismo que las Estaciones y otras construcciones necesarias para la explotación que están consideradas como accesorias del suelo. El concesionario que quiera, en el curso de la concesión, ceder sus derechos a un tercero, debe obtener la autorización previa de los Poderes públicos respectivos; una cesión hecha sin esa autorización es radicalmente nula y de ningún efecto.

Las concesiones pueden terminar de cuatro maneras : *a*) por la expiración del plazo por que fué concedida; *b*) por la cesión que se haya hecho de élla; *c*) por la pérdida del derecho por incumplimiento del concesionario; *d*) por el rescate que el Estado haga del ferrocarril y de sus accesorios, de acuerdo con las estipulaciones previstas en el respectivo contrato o en la Ley sobre la materia.

El rescate debe ser efectuado en globo, es decir, que debe ser de la totalidad de las obras. Para fijar el precio del rescate, se suman los productos netos obtenidos anualmente por el concesionario durante los últimos siete años que preceden a la fecha en que va a efectuarse el rescate; luego se deducen los productos netos de los dos años menos favorables, y se busca el producto neto medio de los otros cinco

años; este producto medio forma el montante de una annalidad que deberá ser pagado al concesionario durante cada uno de los años que falten para vencer la concesión.

Las relaciones existentes entre el Estado y los concesionarios exigen que éstos lleven su contabilidad de manera clara y precisa; estas cuentas son de dos especies: unas referentes a los gastos de construcción; y las otras sobre la explotación.

**Medidas de conservación y servidumbres.** — Los caminos de hierro construidos o concedidos por el Estado forman parte de la gran vía, y, por consiguiente, le son aplicables las leyes y reglamentos que la rigen y que tienen por objeto asegurar la conservación de los fosos, taludes, muros y obras de arte dependientes de las rutas; el alineamiento, la corriente de las aguas, la ocupación temporal de los terrenos en caso de reparaciones, las distancias que deben observarse para las plantaciones y altura de los árboles, el modo de explotación de las minas, mineros, hornagueras y arenales, en la zona determinada a este efecto. Son igualmente aplicables a la confección y mantenimiento de los caminos de hierro, las leyes y reglamentos sobre la extracción de los materiales necesarios a los trabajos públicos.

Fuera de un muro de cerca, ninguna otra construcción puede hacerse a una distancia de dos metros de los caminos de hierro. En las localidades donde éstos se encuentren en terraplenes de mas de tres metros sobre el nivel natural del terreno, está prohibido practicar, sin autorización previa, excavaciones en una zona de ancho igual a la altura vertical del terraplén medido a partir del pie de los taludes. Asimismo está prohibido acumular a una distancia menor de veinte metros, rastrojos, cobertizos de paja, pila de heno o cualesquiera otros depósitos de materias inflamables. A una distancia no menor de cinco metros tampoco podrá establecerse depósitos de piedras, u objetos no inflamables, sin la autorización previa de autoridad respectiva. Ella no es necesaria: 1º para formar en las localidades donde el camino de hierro esté en terraplén, depósitos de materias no inflamables cuya altura no exceda al terraplén del camino; 2º para formar depósitos temporales de abonos u otras materias necesarias al cultivo de las tierras.

**Tarifas.** — El Estado interviene en la fijación del precio de las tarifas las que, sin su aprobación, no pueden ponerse en vigor. Una tarifa máxima limita los derechos del concesionario, quien no puede hacer sin homologación ministerial ninguna modificación a aquella.

Las tarifas de los caminos de hierro indican los precios de los transportes por tonelada kilométrica, es decir: el transporte de una tonelada de mercancía por cada kilómetro de distancia. Para los animales, el precio se fija por cabeza y por kilómetro. Se distinguen tres especies de tarifas, a saber: *a*) la tarifa legal al maximum, de la cual el concesionario no puede pasar; *b*) la tarifa general aplicada en el hecho por el concesionario cuyos precios son casi siempre inferiores a los de la tarifa legal máxima; *c*) las taritas especiales que contienen algunas reducciones variables sobre el precio de la tarifa general, segun las circunstancias.

El carácter obligatorio de las tarifas y de las taxas tiene por consecuencia el reembolso de lo que se haya recibido de mas y el derecho para el concesionario al reintegro de lo que haya percibido de menos. El público que no está informado del precio exacto de las tarifas en el momento de la expedición de sus mercancías, puede pagar bajo reserva sin que pueda ser esto un pretexto para rehusarse la expedición de la mercancía.

Entre las tarifas deben distinguirse las referentes a los viajeros, de las de mercancías. También existen para los pasajeros tarifas generales y especiales. Los precios de estas se calculan por kilómetros. Las especiales acuerdan a los viajeros, bajo ciertas condiciones, una reducción del precio de las tarifas generales.

**Conservación y explotación de la vía.** — Los poderes de la Administración pública relacionados con el mantenimiento, la explotación y supervigilancia de los caminos de hierro, continuan durante la existencia de la concesión. En virtud de este principio, el Ministro de Trabajos Públicos, como se llama en Francia, ordena ciertas medidas concernientes al servicio de la vía, al empleo de materiales y al uso de las señales. Solo al Ministro incumbe determinar la necesidad y oportunidad de estas medidas dentro de los limites de sus facultades policiales. En virtud de los reglamentos de esta naturaleza dictados por él, los pasos a nivel, las maniobras de las barreras y de alumbrado, desde el punto de vista policial, se dividen en cinco categorias cada una de las cuales está sometida a reglas especiales.

La explotación de los ferrocarriles, está, generalmente, a cargo de compa-